Agglomeration in purely neoclassical and symmetric economies

Berliant, Marcus and Watanabe, Axel

Washington University in St. Louis, Concordia University

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Agglomeration in Purely Neoclassical and Symmetric Economies*

Marcus Berliant† and Axel Watanabe‡

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Abstract

This article demonstrates the emergence of agglomeration unaccompanied by conventional explanatory factors such as scale economies, externalities or comparative advantages. We construct a general equilibrium model with four commodities, four types of households and linear production over two regions. A pair of types behave disassortatively when their endowments complement each other. The resultant distribution involves an intense agglomeration consisting of varied types. In contrast, they behave assortatively when they are in direct competition for endowments that cannot be transported or produced. This results in a moderate agglomeration with a disproportionate presence of selected types. Complementarity and endowment portability are the primary causative factors behind consumer behavior and subsequent equilibrium agglomeration.

Keywords: Agglomeration, general equilibrium, spatial sorting
JEL classification: R13

"Therefore, it follows that if assumptions a1-a4 are upheld, there exists either a trivial solution, or no (price taking) competitive equilibrium. In short, the spatial impossibility theorem says that the smooth market mechanism alone cannot generate spatial agglomeration of activities." (Fujita [Fuj86], pp. 113-114).

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†Washington University in St. Louis. Address: MSC 1208-228-308, 1 Brookings Drive, St. Louis, MO 63130-4899, USA. E-mail: berliant@wustl.edu.

‡Department of Economics, Concordia University, and CIREQ. Address: 1455, boulevard De Maisonneuve Ouest, Montréal (Québec) H3G 1M8, Canada. E-mail: axel.watanabe@concordia.ca.

1 Introduction

1.1 Overview

Here we examine the circumstances underlying equilibrium population agglomeration in the context of a completely standard economy, namely without externalities or imperfect competition, but with ordinary utility functions and constant returns to scale production. Whatever equilibria there are will clearly be Pareto efficient. And symmetric equilibria will be present. In such a situation, what force can possibly cause population to agglomerate, and importantly, can this force complement or substitute for the agglomerative forces more commonly used in the literature, such as the New Economic Geography or externalities?

As we shall explain, it is a bit puzzling and surprising that agglomeration can be generated in such a simple neoclassical model, starting with a completely symmetric situation. In fact, transportation cost can be zero or positive; the results are identical. In equilibrium, the regions or locations are autarkic, but the population distributions can be asymmetric. In the end, it is complementarity of types of consumers through their endowments that causes agglomeration. Next, we detail the strategy for our analysis.

Our focus is on a very specific example for tractability and expository reasons. We adopt and then adapt the example of Kehoe [Keh85]. This classical example is aspatial, so it is best to imagine it to have only one region. There are four commodities and four consumers with different Cobb-Douglas utilities, but two different producers with constant returns to scale technologies. Constant returns to scale simplifies matters, since equilibrium profits must be zero. Thus, there is no need to worry about profit distribution and the zero profit conditions yield restrictions on equilibrium prices, useful for computational purposes. The key properties of this example are that it is quite simple **but features 3 equilibria**. Heterogeneous income effects play a big role both in Kehoe's example and in our work.

Next, we adapt Kehoe's model to the spatial context. There will be 2 identical regions or locations. There will be measure 1 of each of the four types of consumer. The same production technologies are available in each region. There are now 8 commodities, 4 in each region. Consumers can move between regions at no cost, as is standard in the literature. Commuting between regions to work is not allowed.

We consider three versions of the model with differing portability of endowments. In the first version, endowments move with the consumers. An example of a portable endowment is labor. In the second, endowments are not portable but income derived from endowments moves with the consumers. Notice that land is an example of an endowment that is not portable. In the third, both types of endowments are present.

All three versions support a uniform distribution of population in equilibrium. They support deviations from it as well, in their own way. Their differences arise from the way different types of consumers interact with each other. If one type's endowment helps increase other types' utility level and vice versa, they sort into a region at a certain ratio and create agglomeration. On the contrary, if they compete for endowments, they sort into different regions. Though not as prominent as the first case, this too creates agglomeration if they sort assortatively off a one-to-one ratio. Suppose, for example, that there are 50 households of type 1 and another 50 households of type 2 in an economy consisting of two regions. The initial distribution is uniform with 25 of each type in each region. If incoming type 1 drives out not one but six type 2's, the region that one type 1 left becomes urban by a margin of 10.[1] Whereas scale economies or externalities may reinforce agglomeration, consumer heterogeneity alone suffices to disturb spatial parity.

Sorting patterns are determined by several factors. We identify the conditions under which different types attract or repel each other. This in turn determines agglomerative forces in each scenario.

Our model and results are perfectly consistent with the spatial impossibility theorem as stated by Fujita and Thisse ([FT13], p. 39), even though we have a continuum of agents:

> **The Spatial Impossibility Theorem**. Assume a two-region economy with a finite number of consumers and firms. If space is homogeneous, transport is costly, and preferences are locally nonsatiated, there is no competitive equilibrium involving transportation.

1.2 Related Literature

Whereas its impact on the economy is significant, modelling agglomeration poses some challenges. As we quoted at the beginning, agglomeration cannot be formed out of a standard setting. It usually requires exacting groundwork. Scale economies count among the most extensively studied sources of agglomeration. The New Economic Geography forgoes perfectly competitive markets. Increasing returns to scale, paired with low transport costs, give rise to agglomeration. Heterogeneity in preferences or endowments does not play a part in it because consumers are identical in this class of models. See Fujita and Thisse [FT13] for a comprehensive review.

In studying agglomeration, we typically assume that economic outcomes are non linear in population. The 10th and the 100th in-migrants contribute differently to the receiving city's economy. Their entry may manifest its impact within the industry they work in or spread across other industries that the city hosts. Jacobs [Jac69] advocates for the latter, namely, urbanization over localization externalities to promote urban growth. While her point of view pertains to the composition of producers, the current article examines the composition of consumers in a city. Externalities in our model are of pecuniary nature.

[1] Appendix A.1 details the conditions under which the distribution becomes uneven in our model.

Indeed it is useful to disaggregate agglomeration and examine what it is composed of. Agglomeration is exemplified by a non-uniform distribution of economic activities or population; spatial sorting is evidenced by varying compositions of heterogeneous agents by location. An economy may feature agglomeration without spatial sorting, and spatial sorting may take place without agglomeration.[2] Nevertheless, the analysis of sorting patterns enables us to identify the root cause of agglomeration in our model.

The spatial impossibility theorem is predicated on homogeneous agents. Upon introduction of heterogeneity, spatial sorting becomes virtually inevitable. In Eeckhout, Pinheiro, and Schmidheiny [EPS14], households of different skill levels make reference to wages, amenities and housing prices in determining which city to live in. They sort disassortatively in equilibrium for skill complementarities. Behrens, Duranton, and Robert-Nicoud [BDRN14] further weigh the evidence of sorting, selection and agglomeration in explaining productivity differences among cities.

By comparison, productivity does not differ by individual worker nor scale of production in the current article. Sorting is driven by endowment availability. Consumers sort themselves to avoid direct competition with other types vying for the same endowment for consumption or production. They would have nowhere else to go if the economy was aspatial. Having two regions enables them to sort spatially (however, there will be no welfare gains in the end for the reasons we will explain in the subsequent sections). So long as endowments and preferences are heterogeneous, this type of sorting will take place with or without heterogeneity in skill levels, a requisite for the works cited just above.

Agglomeration is not an exclusive product of firm-oriented factors. There are spatial models that examine mechanisms that give rise to agglomeration exclusive of scale economies. Berliant and Wang [BW93] identify the conditions under which a city or cities emerge as a place to trade location-specific commodity endowments. Berliant and Konishi [BK00] further introduce production to the framework. Their agglomeration also arises from gains from trade made possible through endogenously formed marketplaces, connected by a mass transport system if there are more than two of them. Both their and our models capitalize on preferences for a variety of goods in generating agglomeration rather than increasing returns to scale. The current article can be thought of as a distilled version of the antecedent works. The same technology is universally available and endowments are either evenly distributed or move with their owner. Regions are ex ante identical and cannot be a cause of agglomeration. Furthermore, as we will show in section 3.5, there are no gains from inter-regional trade. Agglomeration is purely an outcome of endowment complementarities among heterogeneous households.[3]

Mossay and Picard [MP11] also consider the emergence of agglomerations without imposing specific technology. Consumers derive utility from land and social interactions,

[2] Figure 2 visualizes these situations in an economy consisting of two types.

[3] Also of note, our model has only two discrete locations whereas the cited works build on a continuum.

whose net benefit fades with distance. Social interactions are not traded in markets and cause spatial externalities as they depend on the distribution of population. In our setup, the only way consumers interact with each other is through market transactions. Certain types benefit from the co-presence of other types as they increase the availability of a desired endowment, but they pay the price for it. Our equilibrium is thus Pareto optimal.

The remainder of the paper proceeds as follows. In section 2 we lay out our model. Section 3 outlines baseline findings with a downscaled version of the model with only two types and two commodities. In particular we discuss inter-regional trades in section 3.5. Three versions of the full-fledged model will be presented afterwards: Endowments are portable in section 4.1, not portable in section 4.2, and partially portable in section 4.3. Section 5 concludes.

2 The Model

We build our model on the production economy analyzed by Kehoe [Keh85]. His model features a single region with four commodities $i = 1, \cdots, 4$, four consumers $j = 1, \cdots, 4$, and linear technology. We add one more region to it. As a result, there are eight commodities, four in each region.

There is a unit mass of each of four types of consumers, who take up residence in either region a or b. There is no preference for location. Consumers can freely choose their location of residence, in other words there is no relocation cost. We denote the population distribution by $\lambda = \begin{bmatrix} \lambda_1 & \lambda_2 & \lambda_3 & \lambda_4 \end{bmatrix}$, where $\lambda_j \in [0, 1]$ is a fraction of type-j consumers in region a. In what follows we use a superscript to denote a row and commodity i, and a subscript to denote a column and consumer type j and/or region a or b as is standard in general equilibrium models.

A consumer of type j maximizes Cobb-Douglas utility function $u_j(x_j) = \prod_{i=1}^{4} \left(x_j^i\right)^{\alpha_j^i}$ subject to $\pi \cdot x_j \leq \pi \cdot w_j$, where $x_j = \begin{bmatrix} x_j^1 & x_j^2 & x_j^3 & x_j^4 \end{bmatrix}^\top$ is his consumption bundle, $w_j = \begin{bmatrix} w_j^1 & w_j^2 & w_j^3 & w_j^4 \end{bmatrix}^\top$ is his endowment, and $\pi = \begin{bmatrix} \pi^1 & \pi^2 & \pi^3 & \pi^4 \end{bmatrix}^\top$ is a price vector. Let $z_j := x_j - w_j$ be his net demand. **As we will show below, the equilibrium price vector will be the same in both regions**. Expenditure share α and endowment w are specified as

$$\alpha = \begin{bmatrix} .52 & .86 & .5 & .06 \\ .4 & .1 & .2 & .25 \\ .04 & .02 & .2975 & .0025 \\ .04 & .02 & .0025 & .6875 \end{bmatrix} \quad \text{and} \quad w = \begin{bmatrix} 50 & 0 & 0 & 0 \\ 0 & 50 & 0 & 0 \\ 0 & 0 & 400 & 0 \\ 0 & 0 & 0 & 400 \end{bmatrix}. \tag{1}$$

For instance, type-4 consumer's expenditure share of commodity 1 is .06, and he is endowed with zero units of it.

Technology is linear and specified by technological process

$$A = \begin{bmatrix} -1 & 0 & 0 & 0 & 6 & -1 \\ 0 & -1 & 0 & 0 & -1 & 3 \\ 0 & 0 & -1 & 0 & -4 & -1 \\ 0 & 0 & 0 & -1 & -1 & -1 \end{bmatrix}. \tag{2}$$

The supply is Ay, where y is a 6×1 non-negative vector indicating how much of each column of A is deployed in production. The first four columns are free disposal technologies. Since all commodities are goods, the first four entries of y are zero in equilibrium.

Inter-regional trade does not occur in equilibrium, no matter transport cost (see section 3.5 and appendix A.4 to follow). Markets are cleared by moving people between the regions, not commodities. That is because equilibrium prices equate across regions. This is similar to the Factor Price Equalization Theorem. Here, utility levels play the role of product prices, and goods prices play the role of factor prices.

The distribution of endowments is $\mu \in [0,\ 1]^4$, where μ^i denotes a fraction of w^i_j located in region a for any j. This is exogenous if cross-regional movement of endowment is not allowed. We define equilibrium next:

Definition 2.1 Equilibrium

Intra-regional equilibrium *Region a is in intra-regional equilibrium when each resident maximizes his utility level subject to his budget constraint, and excess demand*

- $z_a \lambda^\top - Ay_a = \mathbb{0}$ *if the endowment travels with the consumer, or*
- $x_a \lambda^\top - \mu \circ (w\mathbb{1}) - Ay_a = \mathbb{0}$ *if fraction μ^i of endowment w^i is allocated to region a.*[4]

Inter-regional equilibrium *Populated regions a and b are in inter-regional equilibrium if*

1. *each region is in intra-regional equilibrium, and*
2. $u(x_{j,\ a}) = u(x_{j,\ b})$ *for any j.*

Remark. Whereas the first requirement for inter-regional equilibrium guarantees that the gains from trade are exhausted in each region, the second requirement guarantees that the utility gains from relocation are exhausted across regions. It will always be the case that in equilibrium, all types are present in both regions.

Proposition 2.1 Orthogonal Prices

Given (1) *and* (2), *the set of potential intra-regional equilibrium price vector is*

$$\Pi^\perp = \left\{ \pi \in \mathbb{R}^4_{++} :\ \pi = \begin{bmatrix} \pi^1 & \frac{1}{4} & \frac{7\pi^1 - 1}{3} & \frac{-10\pi^1}{3} + \frac{13}{12} \end{bmatrix}^\top \text{ and } \pi^1 \in \left(\frac{1}{7},\ \frac{13}{14} \right) \right\}. \tag{3}$$

[4] A number in script font denotes a column or row vector (whichever is appropriate) consisting of repeated entries of a same number, e.g., $\mathbb{0} = \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix}^\top$ and $\mathbb{1} = \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix}^\top$ in the preceding equations. We denote an entry-wise product by a circle: $x \circ y = \begin{bmatrix} x_1 y_1 & x_2 y_2 & \cdots & x_n y_n \end{bmatrix}$.

Proof. Firms earn zero profits in equilibrium because of constant returns to scale. Thus, the intra-regional equilibrium price vector must be orthogonal to the column space of A. In addition, Walras' law enables the normalization of prices, $\sum \pi^i = 1$. Combined, π must be of the form (3) in intra-regional equilibrium. □

Remark. Commodity 2 functions as a numéraire in our economy. Note that π^2, π^3 and π^4 are a linear and thus monotone functions of π^1 over $\Pi^\perp$. Moreover, non-numéraire commodity prices π^3 and π^4 are strictly monotone in π^1, rendering them interchangeable when evaluating the monotonicity of the price function. In what follows we say a function is monotone over $\Pi^\perp$ to mean that within the restricted domain $\Pi^\perp\left(\subset \mathbb{R}^4_{++}\right)$ a function is monotone in terms of a non-numéraire price π^1, π^3 or π^4.

Despite its simplicity, Kehoe [Keh85]'s model entails **three** equilibria with distinct prices and allocations due to income effects. We list them below:

Equilibrium #1

$$
\begin{aligned}
\pi &= \begin{bmatrix} 0.159 & 0.250 & 0.0387 & 0.552 \end{bmatrix}^\top \\
x &= \begin{bmatrix} 26 & 67.43 & 48.49 & 83.09 \\ 12.75 & 5 & 12.37 & 220.77 \\ 8.25 & 6.47 & 119 & 14.28 \\ 0.58 & 0.45 & 0.07 & 27 \end{bmatrix} \\
u &= \begin{bmatrix} 16.0 & 44.9 & 47.4 & 240.5 \end{bmatrix} \\
y &= \begin{bmatrix} 0 & 0 & 0 & 0 & 42.7 & 81.2 \end{bmatrix}^\top
\end{aligned}
$$

Equilibrium #2

$$
\begin{aligned}
\pi &= \begin{bmatrix} 0.250 & 0.250 & 0.250 & 0.250 \end{bmatrix}^\top \\
x &= \begin{bmatrix} 26 & 43 & 200 & 24 \\ 20 & 5 & 80 & 100 \\ 2 & 1 & 119 & 1 \\ 2 & 1 & 1 & 275 \end{bmatrix} \\
u &= \begin{bmatrix} 19.1 & 29.8 & 140.8 & 181.9 \end{bmatrix} \\
y &= \begin{bmatrix} 0 & 0 & 0 & 0 & 52.0 & 69.0 \end{bmatrix}^\top
\end{aligned}
$$

Equilibrium #3

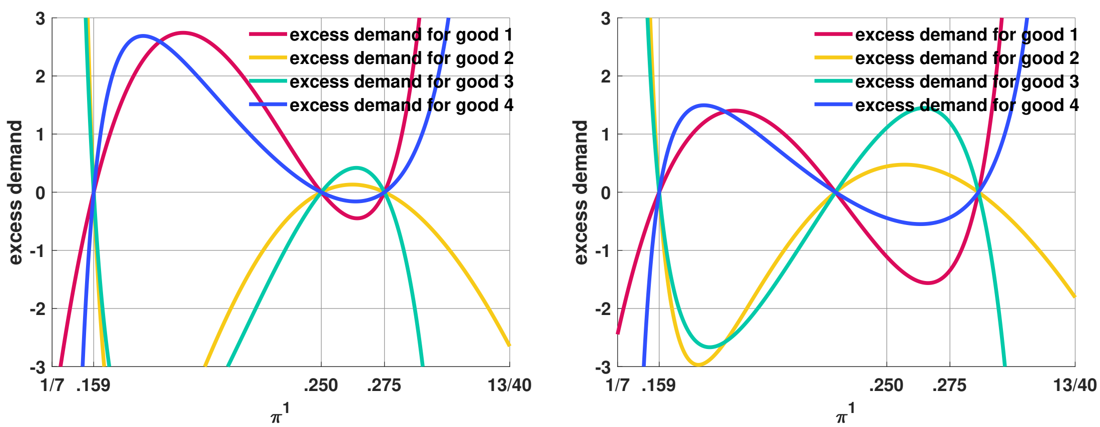
$$\pi = \begin{bmatrix} 0.275 & 0.250 & 0.309 & 0.166 \end{bmatrix}^\top$$

$$x = \begin{bmatrix} 26 & 39.07 & 224.36 & 14.50 \\ 22.01 & 5 & 98.77 & 66.49 \\ 1.78 & 0.81 & 119 & 0.54 \\ 3.31 & 1.50 & 1.86 & 27 \end{bmatrix}$$

$$u = \begin{bmatrix} 20.1 & 27.6 & 155.8 & 159.1 \end{bmatrix}$$

$$y = \begin{bmatrix} 0 & 0 & 0 & 0 & 53.2 & 65.1 \end{bmatrix}^\top.$$

Figure 1(a) plots excess demand. His model is a special case of our model where endowments are portable and everyone congregates in one region, i.e., $\lambda = \mathbb{0}$ or $\mathbb{1}$. By adding another region, aggregate excess demand in region a becomes the sum of individual demand with weight $\lambda \in [0,\ 1]^4$ rather than $\lambda = \mathbb{0}$ or $\mathbb{1}$. The intra-regional markets may clear at a price outside Kehoe's as plotted in figure 1(b).



(a) $\lambda = \mathbb{1}$, namely Kehoe's economy.

(b) $\lambda = \begin{bmatrix} .972 & .126 & .569 & .641 \end{bmatrix}$.

Figure 1. Excess demand in region a. Price is on the x-axis and quantity is on the y-axis.

In rendering Kehoe's aspatial economy spatial, in essence, we are dividing it into two parts with a desired level of interconnection. In the extreme where two regions have no connections, there are simply two aspatial economies. The polar opposite is where there is no friction of any kind between the two. This too is in effect aspatial. We examine the cases that fall between these two extremes, with differing degrees of inter-regional linkage described in table 1.

In practice, there are four ways in which two regions may be spatially connected. To prevent any confusion, we specify our terminologies below and do not use them interchangeably in this article. Term "portable" indicates whether a consumer must or cannot

term	portable	tradable	mobile	transferable
applicable to	endowment	commodity	people	income
sections 3.2 and 4.1	•		•	•
sections 3.3 and 4.2			•	•
sections 3.4 and 4.3	partially		•	•
section 3.5	either	•	•	•
appendix A.4		either	•	•

Table 1. Types of cross-border movements allowed in each section.

take his endowments with him when he relocates,[5] and "tradable" indicates whether commodities (be it endowments or outputs) can be shipped out to another region. Income from endowments is "transferable" if it can be cashed in at a different region than where it was generated. We reserve the term "mobile" to refer to worker's geographic mobility. With the possible exception of tradability, any cross-border movements incur no cost. Table 1 summarizes these four terms.

We assume throughout the paper that workers are perfectly mobile and income is transferable at no cost. Perfect mobility enables us to characterize equilibrium as a state where utility levels equate type by type across regions. It also rationalizes portability: We will not be able to discuss portability unless workers are mobile in the first place. Transferability makes no difference when endowments are portable. However, a lack of it would add inessential constraints to the analysis when endowments are not portable.

Labor is portable and land is not portable for example. Neither one of them is tradable. Their portability or tradability notwithstanding, their owners are perfectly mobile and income generated from them is freely transferable.

Since $u_j\left(x_j\right) > u_j\left(w_j\right) = 0$ for any $x_j \in \mathbb{R}^4_{++}$ for any j, type j's location decision is strongly motivated by where endowment or output $i(\neq j)$ is located. In general, interaction among types is less intense and responsive when endowments are not portable because the migration decision has no impact on μ.

Despite the simple setup, the model above is not analytically solvable. We will preface full-fledged versions in section 4 with a downscaled but tractable variant below. Agglomeration is said to occur when the equilibrium populations of the two regions are not the same; type plays no role here.

[5] In our context, portability does not mean that a consumer decides whether he takes endowment with him at his discretion. Rather, he **must** move with it and **put it to use where he resides**. This eliminates an inter-regional commute as labor cannot be employed outside where a worker lives.

3 Economy with Two Types and Two Commodities

3.1 Overview

Define an $I \times J$ economy to be an economy consisting of I commodities and J types. The current section considers 2×2 economies. We isolate types 1 and 2, and commodities 1 and 2 from section 2, and overwrite A with a 2×1 vector $\hat{A} := \begin{bmatrix} A_5^1 & A_5^2 \end{bmatrix}^\top = \begin{bmatrix} 6 & -1 \end{bmatrix}^\top$. As in proposition 2.1, the firm generates zero profits so that $\pi \cdot \hat{A} = 0$, resulting in a **unique** (up to normalization) equilibrium price vector. If there is only one region (call it E^{1R}), the equilibrium is $x = \begin{bmatrix} 28.3 & 269 \\ 3.62 & 5.21 \end{bmatrix}$, $y = 41.2$, and $\pi = \begin{bmatrix} \frac{1}{7} & \frac{6}{7} \end{bmatrix}^\top$.

Consider two-region economies. Although all markets are perfectly competitive, interaction among types can retrospectively be framed as cooperative and complementary, or competitive and assortative in nature, depending on the assumed portability of endowments. We sort economies by it: E^P with portable endowments, E^{NP} with non-portable endowments, and E^{MP} with mixed portability where w^1 is portable and w^2 is not.

Proposition 3.1 Agglomeration in 2×2 Economies

Suppose that endowments, if not portable, are allocated evenly over two regions. In a 2×2 economy with parameters inherited from (1) and technology $\hat{A}$, the equilibrium distribution is

$$\left\{ \lambda \in [0,\ 1]^2 :\ \lambda_2 \in \left[\frac{||z_1||}{||z_2||}\lambda_1,\ 1 - \frac{||z_1||}{||z_2||}(1-\lambda_1) \right] \right\} \qquad \text{in } E^P, \tag{4}$$

$$\left\{ \lambda \in [0,\ 1]^2 :\ \lambda_2 = .5 \right\} \qquad \text{in } E^{MP}, \text{ and} \tag{5}$$

$$\left\{ \lambda \in [0,\ 1]^2 :\ \frac{w_1^1}{\hat{A}^1}(\lambda_1 - .5) = \frac{w_2^2}{\hat{A}^2}(\lambda_2 - .5) \right\} \qquad \text{in } E^{NP}. \tag{6}$$

Proof. Let $\pi = \begin{bmatrix} \frac{1}{7} & \frac{6}{7} \end{bmatrix}^\top$. Solve $z(\pi)\lambda^\top = Ay_a$ in E^P or $x(\pi)\lambda^\top = Ay_a + \mu \circ (w\mathbb{1})$ in E^{NP} and the corresponding equality in region b for λ, with the constraint $y_a,\ y_b \geq 0$. For E^{MP}, replace μ^1 with λ_1. □

Remark. Figure 2 accompanies the distributions described above. We discuss it next. The measures of types 1 and 2 in region a are on the axes. The downward sloping diagonal represents allocations with no agglomeration.

All three economies inherit the same value of **individual** demand x_j from E^{1R} because there is only one equilibrium price. Their differences arise from the construct of **region-wide** net demand. They all realize agglomeration in equilibrium but for different reasons.

3.2 Portable Endowments (E^P)

As documented in figure 2, there is a two-dimensional continuum of equilibria. Type 1 needs to be accompanied by a sufficient number of type 2 to secure consumption of commodity 2, which can only be supplied through endowment but not through production. If λ_2 falls below $\frac{||z_1||}{||z_2||}\lambda_1$, there will not be enough commodity 2 to go around and type 1's utility level declines. They will move from region a to region b.

3.3 Non-Portable Endowments (E^{NP})

There is a one-dimensional continuum of equilibria. In order to neutralize the effect of spatial inhomogeneity, we assume $\mu = .\mathbb{5}$ in E^{NP}. Types behave assortatively. In contrast to E^P, type 2 appears to type 1 as an unwelcome party that eats into non-portable endowments. They move into a region without bringing commodity 2 with them. The distribution remains uneven so long as the volumes of outgoing type 1 and incoming type 2 do not match.[6]

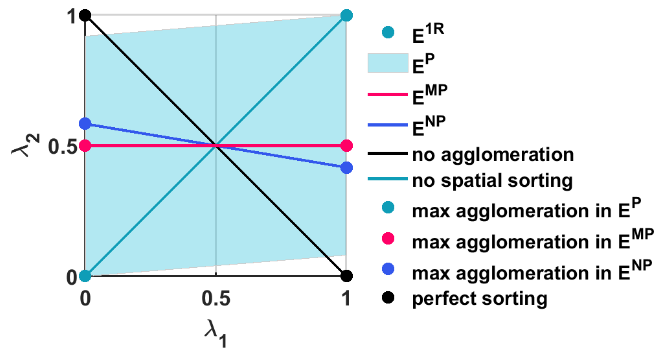


Figure 2. Inter-regional equilibria in 2×2 economies. The population of type 1 in region a is on the horizontal axis and that of type 2 is on the vertical axis. These are the only types in section 3. E^{1R} is a special case of E^P where $\lambda = \mathbb{0}$ or $\mathbb{1}$.

3.4 Mixed Portability (E^{MP})

There is a one-dimensional continuum of equilibria. Standing midway between the two preceding economies, interaction between two types becomes neutral. Type 1 does not bring in nor drive out type 2.

3.5 Inter-Regional Trade

Agglomeration takes place whether inter-regional trade is allowed or not. Let $t \geq 1$ be units of commodity required to be shipped from one region to receive one unit of it in the other region. Preceding sections have assumed $t \to \infty$. Let us consider two other cases: $t > 1$, and $t = 1$.

When $t > 1$, the inter-regional equilibria remain the same. Technology is not heterogeneous by region to warrant comparative advantages, nor does it exhibit increasing returns to scale to warrant exclusive production in a particular region. Imported goods are always priced higher than locally produced goods and thus no one buys them.

When $t = 1$, the economy turns aspatial with a token presence of λ because there is no friction between two regions.

This intuition extends to 4×4 economies to follow as the equilibrium price will not differ by region either.[7]

[6] Appendix A.1 looks into the case where they do, in which case E^{NP} does not support any agglomeration.
[7] With unlikely exceptions described in appendix A.4.

3.6 Limitations of 2×2 Economies

The nature of spatial sorting is disassortative in E^P, assortative in E^{NP} and neutral in E^{MP}. We cannot examine a situation where different kinds of sorting behavior coexist within an economy, because there are **only two types**.

In addition, type 2 attracts or repels type 1 because type 1 wants to consume w_2^2 as a consumption good, but not as an input. We cannot address input-driven assortativeness or disassortativeness because there are **only two commodities**.

With that, we now turn to economies with **four types and four commodities**.

4 Economy with Four Types and Four Commodities

4.1 Portable Endowments (E^P)

This section parallels section 3.2 in a 4×4 setting. The equilibrium price does not differ by region nor from Kehoe's values:

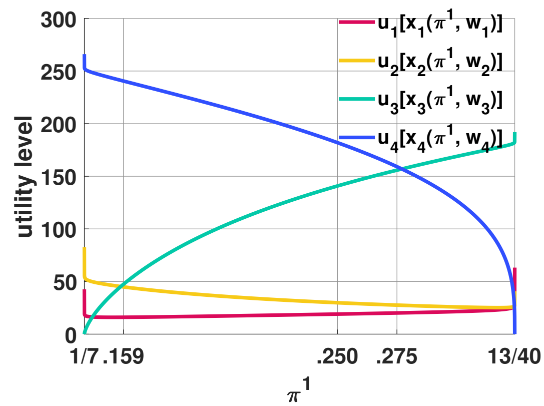


Figure 3. Value functions

Proposition 4.1 Inter-Regional Equilibrium

Suppose that at least one type of consumer has a strictly monotone value function over $\Pi^\perp$. If an inter-regional equilibrium exists, $\pi_a = \pi_b$ and $x_a = x_b$.

Proof. Suppose that a type-j consumer has a strictly monotone value function over $\Pi^\perp$, and that $\pi_a \neq \pi_b$. Then his utility level changes depending on where he is: $u_j\left[x_j\left(\pi_a,\ w_j\right)\right] \neq u_j\left[x_j\left(\pi_b,\ w_j\right)\right]$, and thus π_a and π_b do not make an inter-regional equilibrium price vector (see item 2 in definition 2.1). Therefore, if an inter-regional equilibrium exists, $\pi_a = \pi_b$. Consequently, $x\left(\pi_a,\ w\right) = x\left(\pi_b,\ w\right)$. □

Remark. The value functions or utility levels of types 3 and 4 are strictly monotone over $\Pi^\perp$ (see figure 3. We picked π^1 as the independent variable for illustrative purposes). Thus, proposition 4.1 applies in our case.

Proposition 4.2 Equilibrium Prices in Single- and Two-Region Economies

Let Π^{1R} and Π^P be the set of (inter-regional) equilibrium price vectors in E^{1R} and E^P respectively. Then $(\pi,\ \pi) \in \Pi^P \Leftrightarrow \pi \in \Pi^{1R}$.

Proof. For any $\pi \in \mathbb{R}^4_+$, net demand $z(\pi)$ is identical between E^{1R} and E^P. The budget constraint is the same in both.

(⇒) Consider a pair of price vectors $(\pi,\ \pi) \in \Pi^P$. The material balance condition in each region is $z(\pi)\lambda^\top = Ay_a$ and $z(\pi)(\mathbb{1} - \lambda)^\top = Ay_b$. Aggregate them to obtain

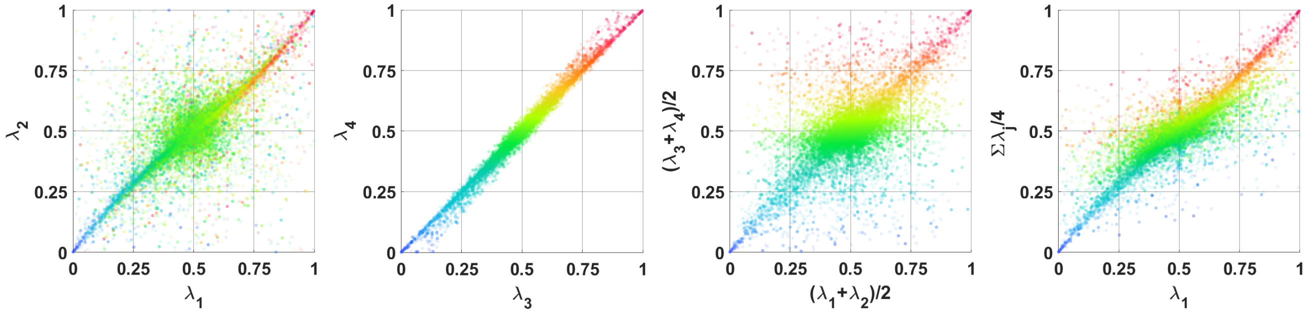
$z(\pi)\mathbb{1} = A(y_a + y_b)$. Then material balance in E^{1R} can be met by setting $y^{1R} = y_a + y_b$. Therefore, $\pi \in \Pi^{1R}$.

($\Leftarrow$) Consider some $\pi \in \Pi^{1R}$. The material balance condition is $z(\pi)\mathbb{1} = Ay^{1R}$. Let $\lambda = c\mathbb{1}$ with $c \in (0,\ 1)$ and set $y_a = cy^{1R}$ and $y_b = (1-c)y^{1R}$. Multiply both sides of the material balance condition by c and $(1-c)$ respectively to obtain $z(\pi)\lambda^\top = Ay_a$ and $z(\pi)(\mathbb{1}-\lambda)^\top = Ay_b$. Thus, material balance is met in each region under π by setting $\lambda = c\mathbb{1}$. Moreover, $u_j\left(x_j(\pi_a)\right) = u_j\left(x_j(\pi_b)\right)$ for any j because $\pi_a = \pi_b = \pi$. Therefore, $(\pi,\ \pi) \in \Pi^P$. □

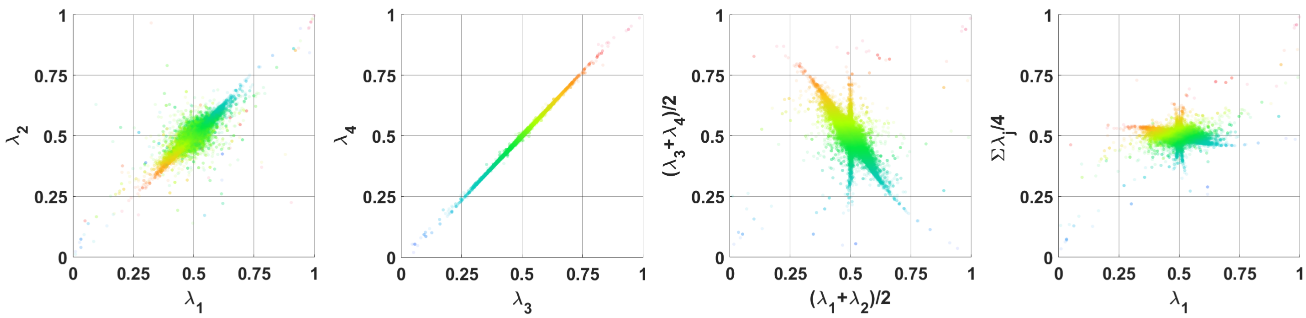
Remark. The second part of the proof indicates that any λ of the form $c\mathbb{1}$ constitutes an inter-regional equilibrium in E^P. Namely, the equilibria in E^{1R} are scalable.

Outside $c\mathbb{1}$, the range of λ is restricted on two grounds. First, there is no equilibrium involving production in an economy that consists of only one type. Thus, only select combinations of λ_j can clear the markets (see appendix A.2). Moreover, even if we find a linear combination $z(\pi)\lambda^\top$ that clears markets in region a, it does not necessarily do so in region b (see appendix A.3).

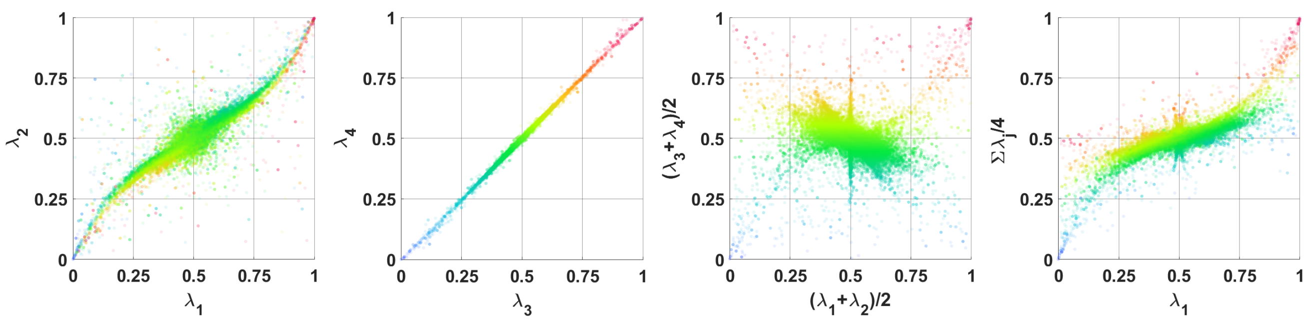
Figure 4 lists inter-regional equilibria sorted by price (see proposition 4.2). Types 1 and 2 behave disassortatively because they would like to consume each other's endowment. Types 3 and 4 behave disassortatively because they would like to use each other's endowment for production of commodities 1 and 2. The former two types and the latter two types behave assortatively or disassortatively depending on the price. Accordingly, the intensity of agglomeration depends on it as well.



(a) $\pi^1 = .159$.



(b) $\pi^1 = .250$.

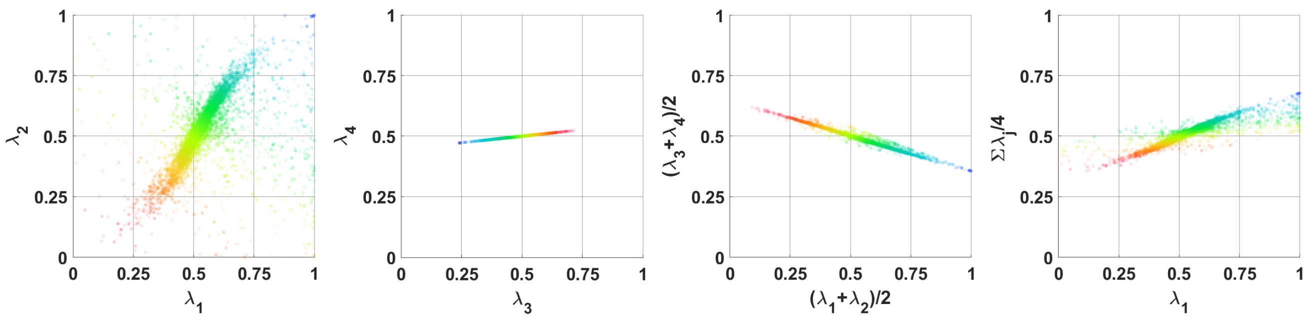


(c) $\pi^1 = .275$.

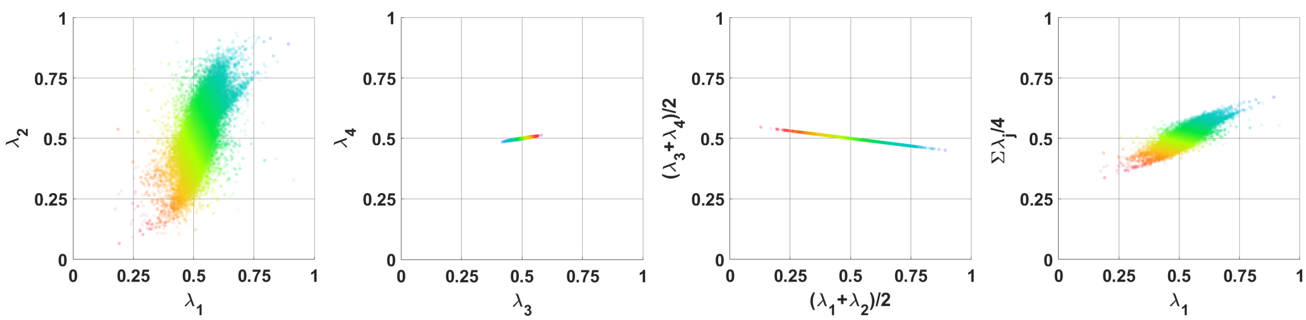
Figure 4. Inter-regional equilibria in E^P. Each equilibrium is colored according to the value of λ_4 to show correspondence among plots at each price.

4.2 Non-Portable Endowments (E^{NP})

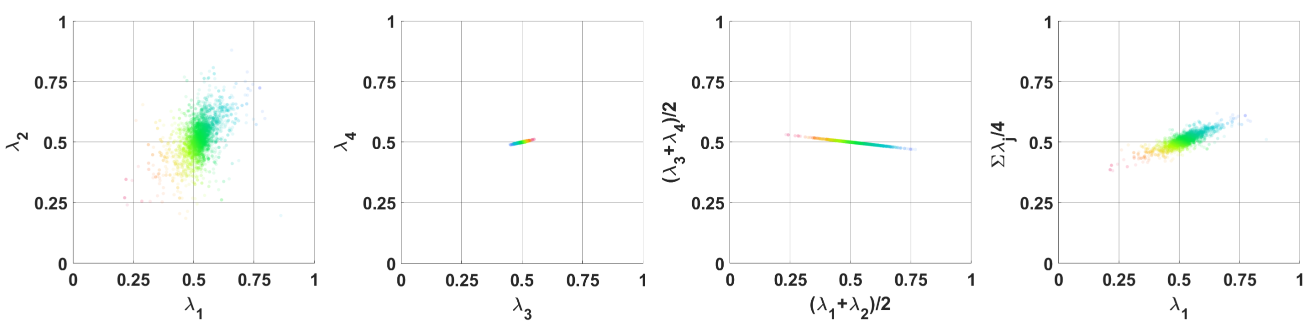
In parallel with section 3.3, this section renders endowments non portable. As in section 4.1, there is no regional variation in price (see appendices A.4 and A.5). Agglomeration takes place but only to a limited extent (cf. appendix A.1). Figure 5 lists inter-regional equilibria. Separation between the locations of an owner and his endowment reduces the need for consumers to co-locate with appropriate **types**. They only need to co-locate with appropriate **endowments** to maximize their utility level.



(a) $\pi^1 = .159$.



(b) $\pi^1 = .250$.



(c) $\pi^1 = .275$.

Figure 5. Inter-regional equilibria in E^{NP}. Each equilibrium is colored according to the value of λ_4 to show correspondence among plots at each price.

4.3 Mixed Portability of Endowments (E^{MP})

Finally we move on to an economy with mixed portability that corresponds to section 3.4. We render endowments of commodities 2 and 4 non-portable, like land. The endowments of commodities 1 and 3 are portable, like labor.

In inter-regional equilibrium, $\pi_a = \pi_b \in \Pi^{1R}$ because proposition 4.1 applies.[8] We present one of the inter-regional equilibria below. It is computed with $\mu = \begin{bmatrix} \lambda_1 & .5 & \lambda_3 & .5 \end{bmatrix}^\top$.

[8] Type 3's income does not depend on the cross-regional price, and their indirect utility function is strictly monotone increasing in the own regional price (cf. appendix A.5).

$$
\begin{aligned}
\pi &= \begin{bmatrix} 0.159 & 0.250 & 0.0387 & 0.552 \end{bmatrix}^\top \\
\lambda &= \begin{bmatrix} 0.821 & 0.577 & 0.512 & 0.496 \end{bmatrix} \\
\begin{bmatrix} \lambda\mathbb{1} & 4-\lambda\mathbb{1} \end{bmatrix}/4 &= \begin{bmatrix} 0.602 & 0.399 \end{bmatrix} \\
\mu &= \begin{bmatrix} 0.821 & 0.500 & 0.512 & 0.500 \end{bmatrix}^\top \\
x &= \begin{bmatrix} 26 & 67.43 & 48.49 & 83.09 \\ 12.75 & 5 & 12.37 & 220.77 \\ 8.25 & 6.47 & 119 & 14.28 \\ 0.58 & 0.45 & 0.07 & 275 \end{bmatrix} \\
u &= \begin{bmatrix} 16.0 & 44.9 & 47.4 & 240.5 \end{bmatrix} \\
y_a &= \begin{bmatrix} 0 & 0 & 0 & 0 & 21.2 & 41.8 \end{bmatrix}^\top \\
y_b &= \begin{bmatrix} 0 & 0 & 0 & 0 & 21.5 & 39.4 \end{bmatrix}^\top \\
y_a + y_b &= \begin{bmatrix} 0 & 0 & 0 & 0 & 42.7 & 81.2 \end{bmatrix}^\top (= y^{1R}).
\end{aligned}
$$

Figure 6 lists a collection of equilibria.

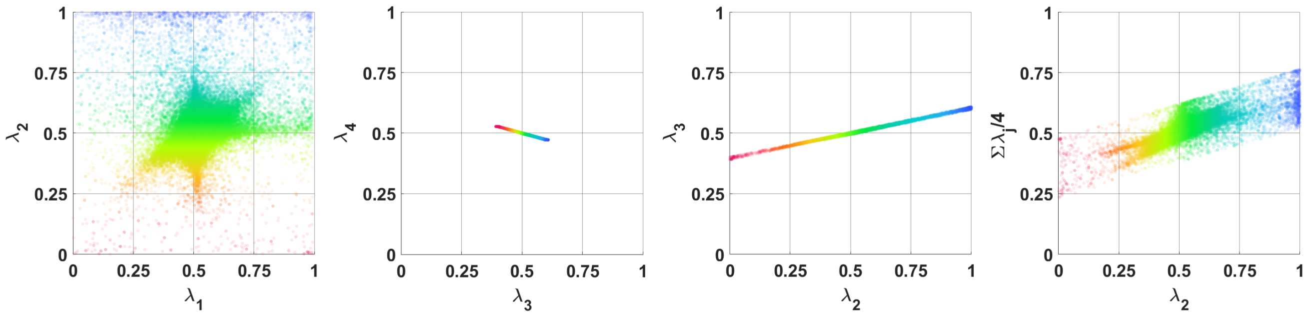
Types 2 and 3 behave disassortatively; types 3 and 4 behave assortatively. As a result, the intensity of agglomeration falls between that of E^P and E^{NP}.

During the production of commodity 1 that types 2 and 3 prefer to consume, the firm employs a high volume of endowment 3. Type 2 is then better off co-locating with type 3 for ease of procurement.

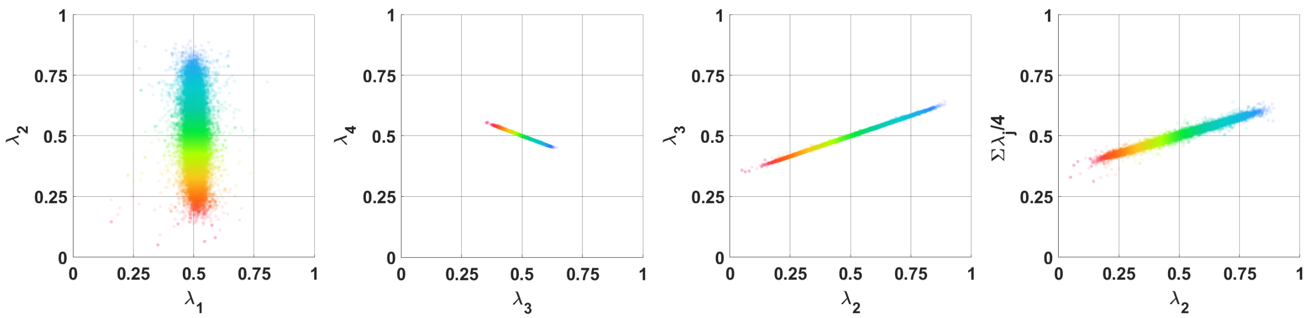
Type 4's presence has nothing to do with production because w_4^4 is 200 in each region whether type 4 is around or not. Thus, type 4 simply eats into w_2^2 with no benefits in return to type 3. Their repulsion waters down the agglomerative force created from interaction between types 2 and 3, but still breaks symmetry (see appendix A.1).

5 Conclusions

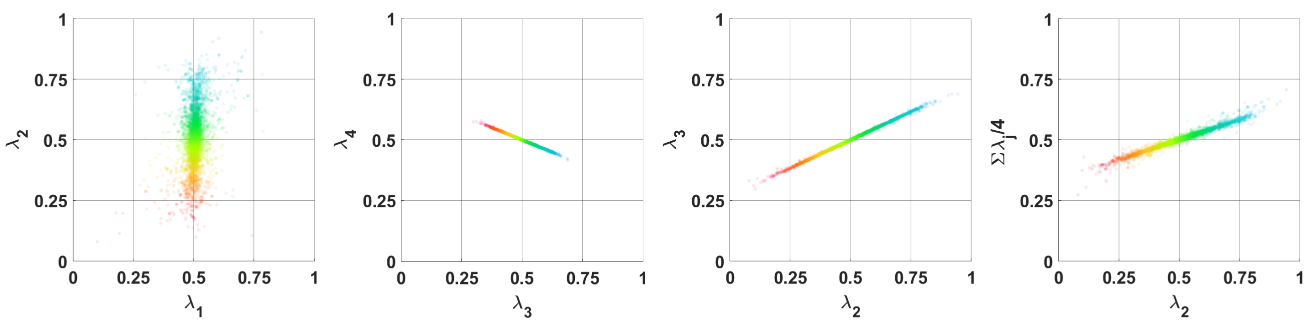
We studied when and how heterogenous households agglomerate in a standard neoclassical framework. Agglomeration is conventionally thought of as a production-driven phenomenon. Scale economies and positive externalities favor a concentration of inputs within close proximity; see, for example, the New Economic Geography literature. Barring scale economies, can there still be agglomeration? To examine whether agglomeration can be driven by consumption rather than production, we adapted a general equilibrium model with constant returns to scale proposed by Kehoe [Keh85] to a spatial context. Our model demonstrates that heterogeneity among consumers creates asymmetry in population distribution and thus agglomeration does not necessitate the presence



(a) $\pi^1 = .159$.



(b) $\pi^1 = .250$.



(c) $\pi^1 = .275$.

Figure 6. Inter-regional equilibria in E^{MP}. Each equilibrium is colored according to the value of λ_4 to show correspondence among plots at each price.

of scale economies.

Agglomeration forms out of spatial sorting in our model. We proposed three different types of economies based on the portability of endowments. Different types co-locate when one type's endowment is in demand by another type either as a consumption good or an input. They may as well settle into separate regions if their co-location would put them in direct competition for commodities that cannot be produced or relocated. Either sorting pattern breaks symmetry, more so under disassortative migration, and results in agglomeration to varying degrees.

Our equilibria are not unique. Agglomeration can be augmented by the introduction of scale economies, externalities, amenities or imperfect mobility.

Our model does not entail any externalities. The equilibria are Pareto efficient. The complementarities are externalities of a pecuniary nature. Mossay and Picard [MP11] also derive agglomeration from the consumers' end. They model return from social interactions that attenuates with distance. Both endowments and preferences are homogenous so that agglomeration is due to externalities from social contacts. If we incorporate their setup in our model, social interactions may enhance spatial sorting, rendering agglomeration even more clear-cut.

We are not intent on overriding the existing body of knowledge about production-oriented agglomeration. Rather, we cast light on the role consumption plays in generating agglomeration, which combined with other forces should illustrate a more realistic mechanism behind agglomeration. An important open empirical question is: What percentage of agglomeration results from each force, including this one? There are welfare and policy implications due to the market failures embedded in other forces but not in ours.

As Kehoe did, we worked on a specific class of preferences in the interest of tractability. We defer to future research for reproduction of our results in a general setting.

A Appendix

A.1 Conditions for Agglomeration in E^{NP}

If types behave disassortatively, the set of equilibria includes non-uniform distributions. When one type relocates, another type goes along with them so that $\lambda\mathbb{1}$ will either increase or decrease. However, if they behave assortatively, the set may only include a uniform distribution. When one type relocates to region a, the other may relocate to region b by the same number. In this case, $\lambda\mathbb{1}$ will not change and agglomeration will not take place.

PROPOSITION A.1 CONDITIONS FOR AGGLOMERATIONS

Suppose $\mu = .\mathbb{5}$ *in* E^{NP}. *If* $\dfrac{-\hat{A}^2}{\hat{A}^1} = \dfrac{w_2^2}{w_1^1}$, *the economy does not support agglomeration in equilibrium.*

Proof. Given the equality above, (6) implies $\lambda\mathbb{1} = 1$. □

Remark. Suppose that the economy is currently in inter-regional equilibrium. When a fraction Δ of type 1 moves in, the region-wide demand increases by Δx_1. The aggregate demand returns to $\mu \circ (w\mathbb{1}) + Ay'_a$ for some other y'_a if a specific amount of x_2 is removed. The required amount to remove is exactly Δx_2 if $\dfrac{-\hat{A}^2}{\hat{A}^1} = \dfrac{w_2^2}{w_1^1}$. Consequently, $\begin{bmatrix}\lambda_1 + \Delta & \lambda_2 - \Delta\end{bmatrix}\mathbb{1} = 1$ so that the distribution remains uniform. In figure 2, the blue line will coincide with the black line.

$\hat{A}$ and w are exogenous. Unless we start with very specific values, even assortative behavior, though not as prominent as disassortative behavor, results in agglomeration.

A.2 Limiting Factors of Distribution in E^P

If $z_j(\pi) - Ay$ were $\mathbb{0}$ for any j, then combining $z_j(\pi)$ with any arbitrary weight λ would make an equilibrium as it did in section 3.2.[9] Figure 7 shows the minimum norm possible of excess demand when there is only type j in E^{1R}. None of them reaches zero on their own over $\Pi^{\perp}$. They do so only when aggregated with an equal weight $\mathbb{1}$ (in broken line) or with an equilibrium weight λ. If this was a 2×2 economy in section 3.2, type 2 would reach zero at $\pi^1 = \frac{1}{7}$ on their own, resulting in a wide array of equilibrium λ.

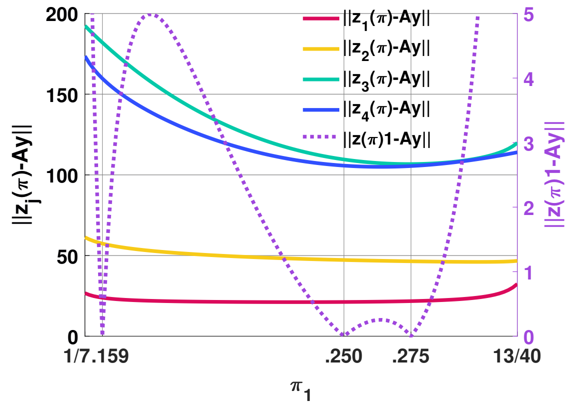


Figure 7. Minimum norm possible.

A.3 Equilibrium Prices in E^P

Expansion of a set of λ from $\{\mathbb{0}, \mathbb{1}\}$ of E^{1R} to $[0, 1]^4$ of E^P unleashes many market-clearing price vectors outside Π^{1R}, but most of them only clear the markets in one region. Figure 8 represents excess demand in region b that corresponds to figure 1(b) in region a. Region a features one intra-regional equilibrium price vector in Π^{1R} and two outside Π^{1R}. The latter two will not make an inter-regional equilibrium because they only clear the markets in region a, but not in b.

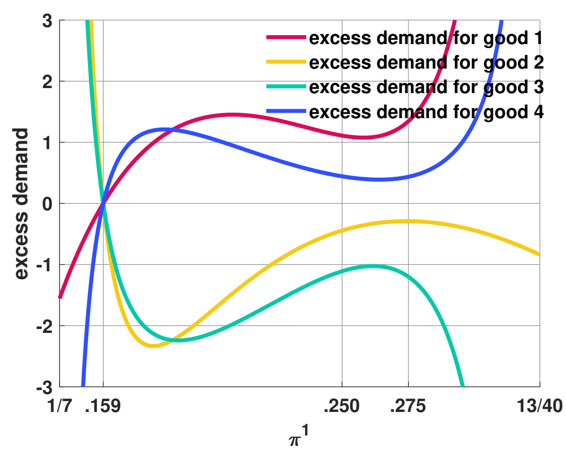


Figure 8. Excess demand in region b.

A.4 Equilibrim Prices in E^{NP}

The following is an E^{NP} counterpart to proposition 4.2:

PROPOSITION A.2 EQUILIBRIUM PRICES IN E^{1R} AND E^{NP}

Suppose $\mu = .\mathbb{5}$. *Let* Π^{NP} *be a set of equilibrium prices in* E^{NP}. *Then* $(\pi,\ \pi) \in \Pi^{NP} \Leftrightarrow \pi \in \Pi^{1R}$.

Proof. The proof of proposition A.2 is similar to that of proposition 4.2 except that c needs to be equal to .5. At any $\pi \in \mathbb{R}^4_{++}$, optimal bundles $x(\pi)$ in E^{1R} and $x(\pi,\ \pi)$ in E^{NP} share the same value because the budget constraint in E^{NP} does not involve λ nor μ. Income $\pi \cdot (\mu \circ w_j) + \pi \cdot ([\mathbb{1} - \mu] \circ w_j)$ in E^{NP} reduces to $\pi \cdot w_j$ in E^{1R} for any j.

[9]Barring non-negativity constraints on y.

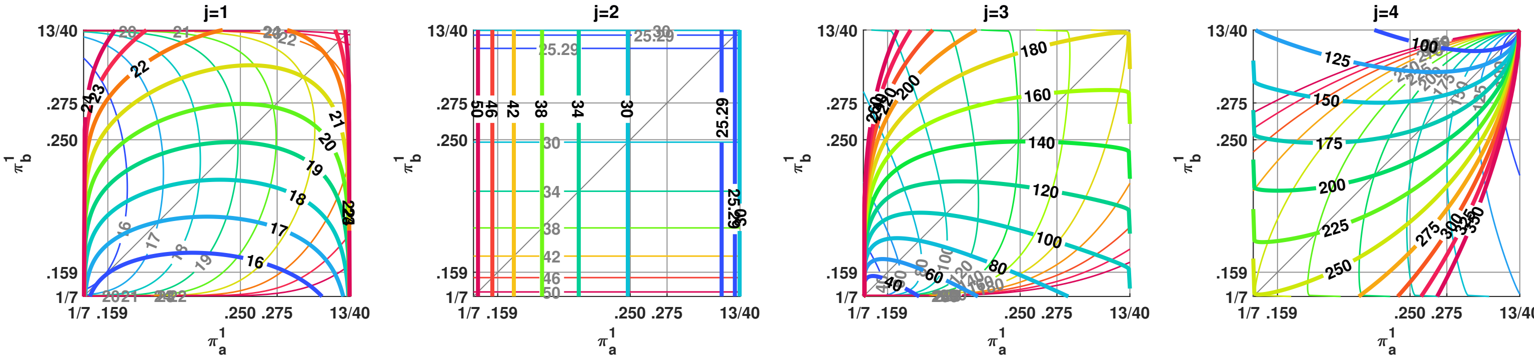


Figure 9. An E^{NP} counterpart to figure 3 in E^P. Level sets of value functions $v_{j,\,a}(\pi_a,\ \pi_b,\ w)$ are represented by bold lines with dark numbers indicating utility levels, and $v_{j,\,b}(\pi_a,\ \pi_b,\ w)$ by thin lines with light numbers. Warmer colors correspond to higher utility levels in either region.

(⇒) Consider a pair of price vectors $(\pi,\ \pi) \in \Pi^{NP}$. The material balance in each region is $x(\pi,\ \pi)\lambda^\top - \mu \circ (w\mathbb{1}) = Ay_a$ and $x(\pi,\ \pi)(\mathbb{1}-\lambda)^\top - (\mathbb{1}-\mu)\circ(w\mathbb{1}) = Ay_b$. Aggregate them and replace $x(\pi,\ \pi)$ with $x(\pi)$ to obtain $x(\pi)\mathbb{1} - w\mathbb{1} = A(y_a + y_b)$. Then the material balance in E^{1R} can be met by setting $y^{1R} = y_a + y_b$. Therefore, $\pi \in \Pi^{1R}$.

(⇐) Consider some $\pi \in \Pi^{1R}$. The material balance is $x(\pi)\mathbb{1} - w\mathbb{1} = Ay^{1R}$. Let $\lambda = .\mathbb{5}$ and set $y_a = .5y^{1R}$ and $y_b = .5y^{1R}$. Multiply both sides of the material balance by .5 and replace $x(\pi)$ with $x(\pi,\ \pi)$ to obtain $x(\pi,\ \pi)\lambda^\top - \mu \circ (w\mathbb{1}) = Ay_a$ and $x(\pi,\ \pi)(\mathbb{1}-\lambda)^\top - (\mathbb{1}-\mu)\circ(w\mathbb{1}) = Ay_b$. Thus, the material balance is met in each region under $(\pi,\ \pi)$ by setting $\lambda = .\mathbb{5}$. Furthermore, $u_j\left[x_j(\pi_a,\ \pi_b)\right] = u_j\left[x_j(\pi_b,\ \pi_a)\right]$ for any j because $\pi_a = \pi_b$. Therefore, $(\pi,\ \pi) \in \Pi^{NP}$. □

Remark. The equilibrium is **not** scalable.

A.5 Indirect Utility Functions in E^{NP} and Inter-Regional Trades

Unlike proposition 4.1 in E^P, a strictly monotone value function does not necessarily result in a shared equilibrium price between regions in E^{NP}. Nevertheless, regional variation in price is not likely in E^{NP}.

Demand in region a depends not only on π_a but also on π_b by way of income collected from region b. Define indirect utility function by $v_{j,\,a}(\pi_a,\ \pi_b,\ w) := u_{j,\,a}\left(x_{j,\,a}(\pi_a,\ \pi_b,\ w)\right)$.[10] Figure 9 represents the level sets of $v_{j,\,a}(\pi_a,\ \pi_b,\ w)$ and $v_{j,\,b}(\pi_b,\ \pi_a,\ w)$.[11]

[10]We list the home price first, followed by the cross-regional price regardless of the region in question to maintain the symmetry. Written in this way, in conjunction with an even allocation of non-portable endowments, value functions appear axially symmetric to each other in figure 9.

[11]Note that if we slice the value function along the 45° line, the cut surface is identical to figure 3: Along the 45° line, $\pi_a^1 = \pi_b^1$ so that type j's income $\pi_a \cdot (\mu \circ w_j) + \pi_b \cdot \left[(\mathbb{1}-\mu)\circ w_j\right]$ reduces to $\pi_a \cdot w_j$ irrespective of the value of μ. Therefore, $v_{j,\,a}(\pi_a,\ \pi_b,\ w) = v_{j,\,a}(\pi_a,\ w)$ in effect, which coincides with figure 3, except that the horizontal axis is stretched by $\sqrt{2}$.

If (π_a, π_b) constitutes an inter-regional equilibrium, $v_{j,\,a}(\pi_a, \pi_b, w) = v_{j,\,b}(\pi_b, \pi_a, w)$ for any j.[12] Whereas this equality holds at some $\pi_a \neq \pi_b$ for **some** type, there is no regionally variant price where the utility value equalizes for **all** types. Utility equalization is realized only where $\pi_a = \pi_b$ in our example, along the 45° line in figure 10.

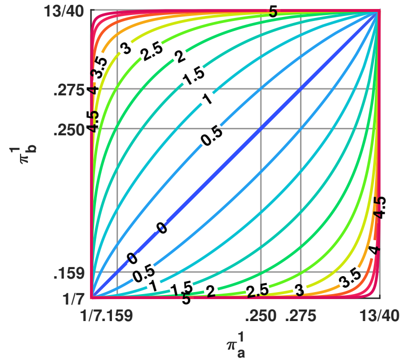


Figure 10. Sum of the utility differential $\sum_j |\log v_{j,\,a}(\cdot) - \log v_{j,\,b}(\cdot)|$. All types are indifferent between two regions when this value is zero.

Whereas $\pi_a \neq \pi_b$ may be made possible in inter-regional equilibrium by modifying parameter (1) in E^{NP}, such an equilibrium is in fact distantly likely. Utility levels will differ by region when the prices do not equate. However, E^{NP} cannot regulate the utility differentials by way of income change.

Whereas E^P does not support regional variations in price, E^{NP}, while highly unlikely, may. If so, there may be inter-regional trades. If in addition $\pi_b^i = t\pi_a^i$, then there could be commodity flow from region a to b (see section 3.5). The firm producing commodity i becomes indifferent between selling it in region a or export it to region b. However, it is already quite difficult to equate utility levels in E^{NP} for the extra constraint to level income across regions. It is increasingly unlikely that E^{NP} satisfies $\pi_b^i = t\pi_a^i$ on top of that.

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[12] On a graph, this translates to contour lines of the same utility level crossing at (π_a, π_b).

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↻ Last updated on September 30, 2024 at 17:08 (ET).