Patent policy, invention and innovation in the theory of Schumpeterian growth

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Abstract

I develop an endogenous growth model that separates firm decisions to invent, patent, and commercialize new innovations. I use the model to examine how multiple dimensions of patent policy impact economic growth by shaping these relative incentives. I pay particular attention to the role of patenting requirements that dictate how far along the development process an inventor must progress to obtain a patent. The model formalizes how strengthening such requirements generates competing effects on economic growth; stronger requirements reduce ex ante research incentives by increasing the expected cost of patenting, but increase ex post incentives to fully develop patented inventions into commercial innovations by decreasing the additional cost associated with commercialization. Overall, my analysis supports the use of patenting requirements as an effective policy tool to improve economic outcomes by shifting incentives away from invention in the pursuit of patents and towards the development of commercial innovations.

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1 Introduction

The distinction between invention and innovation has a long history in both legal and economic scholarship. Within economics, its modern interpretation is commonly attributed to the influential work of Joseph Schumpeter that "distinguish[es] between the act of invention, which creates a new product or process, and the broader act of innovation, which includes the work necessary to revise, develop, and bring that new product of process to commercial fruition" (Burk and Lemley, 2003). Despite the clear influence of Schumpeterian ideas in the development of contemporary economic growth theory however, endogenous growth models typically ignore this distinction in the process of technical change. Instead, these models routinely treat research and development (R&D) investment as a single input, whose single output is commercially viable innovations.[1]

However, empirical evidence indicates that between 40-60% of patented inventions are never commercially exploited (Mattes *et al.*, 2006; Webster and Jensen, 2011; Torrisi *et al.*, 2016; Walsh *et al.*, 2016). In some cases, firms clearly obtain patents without the intent to develop and commercialize related innovations. Such patents are often used strategically to block competitors from introducing innovations, extract licensing fees from firms in related industries or as collateral in third-party contract negotiations (Cohen *et al.*, 2002; Hall *et al.*, 2014). In many other cases however, patented inventions have potentially valuable commercial applications that are simply not developed. For example, in a large sample of US patents for which inventors indicate a commercial motive was "very important" in their decision to patent, Walsh *et al.* (2016) find that 40% remain uncommercialized as of the time of their study.

Several recent critiques have attributed this "underdevelopment problem" to aspects of the patent system that facilitate patenting early in the development process (Cotropia, 2009; Sichelman, 2009; Lemley, 2016). In particular, although patent applications must include enabling disclosure of the underlying invention, they do not require submission of a working prototype nor evidence that the invention will work for its intended purpose. This implies that patents may be obtained when there remains substantial uncertainty and additional cost associated with commercializing the end innovation. It also encourages inventors to file broad patent claims "simply because they don't actually know what particular implementations of their idea will work" (Lemley, 2016). As a result, inventors may be incentivized to "take the lower-cost avenue of asserting the

[1] See also Fagerberg (2005), who details this "Schumpeterian perspective" and states, "invention is the first occurrence of an idea for a new product or process. Innovation is the first commercialization of the idea." See Schumpeter (1939a,b) for his original work on the topic. I discuss the related endogenous growth literature in Section 1.1.

patent in litigation to extract rents from those who have commercialized in the patented area," instead of proceeding through the costly development process (Cotropia, 2009). Indeed, there is real concern that patent policy has skewed too far towards incentivizing invention in pursuit of patents, rather than the development of commercial innovations.[2]

In this paper, I assess this critique of patent policy by developing an endogenous growth model that incorporates firms' separate incentives to invent, patent, and commercialize new innovations. In the model, economic growth is driven by technological progress in the form of discrete quality improvements to a fixed set of products. Firms invest resources in research races to generate new inventions, which represent technically feasible ideas for quality improvements. Each invention has the potential to become a commercially profitable innovation, if it proceeds through a continuous development process. For example, this process may include drafting blueprints, creating a working prototype, and refining the production process. The associated cost of this development is heterogenous across inventions. As soon as a firm wins a research race, it learns its invention's development cost and separately decides whether to patent the invention and whether to commercialize it. In this way, the model explicitly decouples invention from broader act of fully developing a commercial innovation.

I model three dimensions of patent policy that collectively influence the relative incentives to patent and commercialize new quality improvements. First, patents provide producers of commercialized innovations imperfect protection against competitor imitation. This *backward protection* determines the monopolistic markups charged by technological leaders, and thus, the profits associated with commercializing a new innovation. Second, patents grant an imperfect legal right over future commercial applications that make use of a patented invention. In keeping with the endogenous growth literature, I model this form of patent protection as compulsory licensing agreements between a patent holder and firms that commercialize an innovation that is considered to fall within the claims of the patent. This *forward protection*, also known as blocking patents, implies that patents can generate licensing revenue even when they are not commercialized. Third, I consider *patenting requirements* that dictate how far along the development process an inventor must progress to obtain a patent. Accordingly, patenting requirements determine the development cost with associated with patenting and the corresponding additional cost to fully develop the innovation to the point of commercialization.

[2]See Cotropia (2009), Sichelman (2009), and Lemley (2016) for thoughtful expositions of this critique. For instance, Sichelman (2009) claims that, "patent law is primarily designed to induce invention; any protection it provides to commercialization is mostly an afterthought." Similarly, Lemley (2016) argues "while patent law may encourage racing to a new invention ... [w]e might be better off having inventors race to build something than simply race to come up with new ideas first."

To the best of my knowledge, this paper is the first to analyze the economic impact of patenting requirements of this form in an endogenous growth model. Doing so allows me to evaluate proposed policy changes that are designed to discourage early filing and improve incentives to commercialize innovations. For instance, Cotropia (2009) advocates for returning to the historical "actual reduction to practice" standard that required inventors to build an apparatus, or prototype, that embodied the invention in order to obtain a patent.[3] For Cotropia, this policy promotes commercialization by "moving the inventor further down the development path before [patent] examination," so that "the choice of asserting the patent is not as cheap compared to commercialization as it is under the current system." Lemley (2016) argues for a similar, though less extreme, change of "strengthening the enablement and written description requirements ... [to] nudge inventors toward building and testing their invention first by making it harder to get a patent without doing so." Both authors acknowledge the trade-offs inherent to increasing the standards, and thus cost, associated with obtaining a patent. As Cotropia (2009) notes, "the question is how to change the timing without destroying the incentive to invent."

My analysis formalizes this issue by separating the act of invention, patenting and commercialization within a single endogenous growth framework. For a given set of patent policies, I show that the endogenous patenting and commercialization behavior of inventors is characterized by two interior development cost thresholds. These thresholds partition inventions in three categories: the lowest development cost inventions are patented and commercialized, intermediate cost inventions are patented but not commercialized, and the highest cost inventions are discarded entirely. Patent policy impacts economic growth through its influence on both ex ante research incentives, which determines the economy's rate of invention, and the relative incentive to patent and commercialize new inventions, which determines the proportion of inventions that become innovations. I find that these two channels are indeed often in direct conflict and that their interaction meaningfully influences the economic implications of patent policy. The model thus offers a fresh perspective on the dimensions of patent policy that have received the most attention in the endogenous growth literature, namely backward and forward patent protection.

Moreover, these competing effects are central to the economic impact of policy changes

[3] US patent law held to an actual reduction to practice standard until the late 19th century. Patent law has since moved to a "constructive reduction to practice" standard in which enabling disclosure within a patent application substitutes for actual reduction to practice. This was formalized in the 1952 Patent Act, which gave full parity to actual and constructive reduction to practice, obviating the need to produce a prototype in patent applications. See Cotropia (2009) and Lemley (2016) for details.

to patenting requirements. On the one hand, strengthening patenting requirements decrease the additional cost of commercialization after obtaining a patent. This increases inventors' relative incentive to commercialize, resulting in a greater proportion of inventions that are developed fully into innovations. The model thus captures the main motivation for stronger patenting requirements as emphasized by Cotropia (2009) and Lemley (2016). On the other hand, stronger patenting requirements directly increase the cost associated with obtaining a patent and generating a return on research investment through licensing revenue. This decreases the overall expected return to invention and reduces aggregate research investment. I demonstrate analytically that the policy's overall effect on economic growth depends on the relative size of these two competing mechanisms.

I calibrate the model to US data to explore these competing effects numerically. I find that strengthening patenting requirements is highly effective at shifting inventors' relative incentives from patenting towards commercialization. Although this policy change does decrease ex ante invention incentives and cause the invention rate to fall, this effect is dominated by the more frequent commercialization of inventions. My numerical results show that strengthening patenting requirements increases the rate that new commercial innovations are introduced into the market, which both promotes economic growth and improves welfare. Additionally, I consider an extension of the model to explore the role of spillovers from patent disclosure. I find that strengthening patenting requirements generates a positive effect on research spillovers by improving the quality of disclosed information per patent, which more than offsets the decrease in disclosed information from the associated decline in the flow of new patents. This reinforces the benefits of strengthening patenting requirements. Overall, my analysis provides support for the use of stronger patenting requirements as a policy tool to improve economic outcomes by shifting incentives towards the pursuit of commercial innovations.

1.1 Related literature

This paper relates to several strands of the extensive literature that builds on the canonical Schumpeterian growth framework developed by Segerstrom *et al.* (1990), Grossman and Helpman (1991), and Aghion and Howitt (1992). Numerous studies have adapted this framework to examine multiple dimensions of patent policy. This includes recent analyses that model forward protection (Chu, 2009; Chu *et al.*, 2012; Yang, 2018; Suzuki and Kishimoto, 2023; Klein and Yang, 2024) and backward protection (Chu *et al.*, 2016, 2020, 2021; Klein, 2020) in the context of cumulative innovation as I do.[4] In each case however, these studies treat invention and innovation as a single entity and assume that all inventors patent. I contribute to this literature by analyzing backward and forward patent protection in a framework that decouples invention from innovation. In doing so, my work highlights how patent policy impacts economic growth through two distinct channels: the policy's effect on the ex ante research incentives of potential inventors and its effect on the relative incentive to patent and commercialize new inventions.

To the best of my knowledge, this paper is the first to analyze patenting requirements based on invention development in a model of Schumpeterian growth. Several papers have examined a related policy in the form of minimum innovation size, or the inventive step, requirements to obtain a patent (O'donoghue and Zweimüller, 2004; Koléda, 2008; Kishi, 2018, 2019). The policy is similar to the patenting requirements that I study in the sense that it restricts patent eligibility and can stifle ex ante incentives to invest in research. This literature has also highlighted a potential growth promoting role of innovation size requirements if they lead firms to pursue larger innovations (O'donoghue and Zweimüller, 2004) or alter the distribution of patented innovations towards larger size (Koléda, 2008; Kishi, 2019). In contrast, my analysis of patenting requirements focuses on claims that enabling patenting early in the development process has contributed to weak commercialization incentives.

This paper is also closely related to the relatively small literature that incorporates a post-invention commercialization process into an endogenous growth framework. This includes several studies that consider a two stage innovation process in which basic research firms produce new inventions and distinct applied research firms bring existing inventions to market (Michelacci, 2003; Acs and Sanders, 2012; Cozzi and Galli, 2014, 2017; Gersbach *et al.*, 2018). In particular, Acs and Sanders (2012) and Cozzi and Galli (2014, 2017) focus on the role of patent policy in shaping relative incentives through the division of profit between these distinct basic and applied research firms. I complement these existing studies by modeling the separate invention and commercialization phases of innovation development within a single firm. This structure allows me to endogenize the patenting decision of inventors and jointly analyze how multiple dimensions of patent policy influence firms' relative incentives to invent and commercialize new products.

[4] These forms of patent protection are sometimes referred to as horizontal versus vertical protection and as lagging versus leading patent breadth, but preform the same functions. See Klein and Yang (2024) for a description of how these forms of patent protection correspond to distinct elements of patent law. See also Chu (2022) for a detailed overview of the use of Schumpeterian growth models to analyze patent policy.

In several respects, my theoretical approach is most similar to the work of Chu *et al.* (2017, 2019). These papers develop Schumpeterian growth models featuring endogenous commercial entry of heterogenous innovators to study the economic impact of inflation. In this work, each successful inventor draws the size of their quality improvement from a known probability distribution, then decides whether to pay a fixed entry cost to implement the innovation in the market. The model I develop in this paper follows Chu *et al.* (2017, 2019) in its basic lab-equipment structure and in incorporating a commercialization decision that depends, in part, on the presence of firm heterogeneity. However, I depart from their framework by considering invention heterogeneity in terms of development cost, jointly modeling the patenting and commercialization decision of inventors, and examining how patent policy impacts economic growth through these choices.

The remainder of this paper is structured as follows. I first develop the baseline model in Section 2. I then explore the economic impact of patent policy analytically in Section 3. Section 4 discusses the calibration of the model and presents numerical results. I explore the role of information disclosure in Section 5. Section 6 concludes.

2 The Model

2.1 Households and final good production

The economy is populated by a representative household of fixed size that inelastically supplies one unit of labor at each time t.[5] The household's lifetime utility is given by

$$U \equiv \int_0^\infty e^{-\rho t} \ln c(t) dt, \tag{1}$$

where $\rho > 0$ is the subjective discount rate and $c(t)$ denotes consumption of the final good at time t. The household chooses $c(t)$ to maximize (1) subject to a standard intertemporal budget constraint of $\dot{a}(t) = r(t)a(t) + w(t) - c(t)$, where the final good is treated as the numeraire, $r(t)$ is the instantaneous market interest rate, and $w(t)$ is the wage rate. The resulting Euler equation is

$$\frac{\dot{c}(t)}{c(t)} = r(t) - \rho. \tag{2}$$

The final good $Y(t)$ is produced by perfectly competitive firms according to the pro-

[5]As in Chu *et al.* (2017, 2019), I fix the aggregate supply of labor to avoid the issue of scale-effects that arises in the Schumpeterian growth framework. Allowing for population growth and removing scale-effects complicates the model without contributing additional insight.

duction function $Y(t) = L(t)^{\theta} K(t)^{1-\theta}$, where $\theta \in (0,1)$ captures labor intensity in production and the labor input $L(t) = 1$ for all t. $K(t)$ is a composite of intermediate goods purchased from each structurally identical industry $\omega \in [0,1]$ defined by

$$K(t) \equiv \exp\left(\int_0^1 \ln\left[\sum_j \lambda^j y(\omega, j, t)\right] d\omega\right), \tag{3}$$

where $\lambda > 1$ is the fixed step size of the quality ladder and $y(\omega, j, t)$ denotes the input quantity of intermediate good ω of quality vintage j at time t. Given (3), final goods firms optimally purchase only the intermediate good with the lowest quality adjusted price from each industry. Let j_ω denote the highest quality vintage that has been commercialized in industry ω. As discussed in the following section, the innovator responsible for inventing and commercializing the latest quality vintage will capture its industry's entire market share by setting a limit price, denoted $p(j_\omega, t)$. The conditional demand function for this good is given by

$$y(j_\omega, t) = \frac{(1-\theta)Y(t)}{p(j_\omega, t)} = \frac{(1-\theta)K(t)^{1-\theta}}{p(j_\omega, t)}. \tag{4}$$

Following Chu *et al.* (2017), multiplying both sides of (4) by λ^{j_ω} and aggregating the natural log of the resulting equation with respect to ω yields

$$K(t) = \left[\frac{(1-\theta)Q(t)}{P(t)}\right]^{\frac{1}{\theta}}, \tag{5}$$

where $Q(t) \equiv \exp\left(\int_0^1 \ln \lambda^{j_\omega} d\omega\right)$ and $P(t) \equiv \exp\left(\int_0^1 \ln p(j_\omega, t) d\omega\right)$ denote the aggregate quality and price index of intermediate goods respectively.

2.2 Invention and innovation

Within each industry, competitive firms participate in research races to invent new ideas for quality improvements. Research races follow a standard stochastic Poisson process in which the instantaneous probability of invention depends on the intensity of research investment by firms within each industry. As usual, research firms are risk neutral and research races are characterized by free entry. The winner of a research race for the $j_\omega + 1$ quality iteration discovers an invention that has the potential to improve the state-of-the-art quality of its industry's intermediate good by a fixed $\lambda > 1$ proportion, if the invention is fully developed into a commercial innovation.

I assume that there are $d > 0$ development stages necessary to fully commercialize a new invention. For example, these stages may include drafting blueprints, creating a working prototype, conducting market tests, and refining the production process. I treat d as a continuous parameter. Inventions differ in the cost associated with completing each development stage. As soon as invention occurs, the inventing firm i receives a random draw of δ_i from a known probability distribution $f(\delta)$ that determines this cost. For simplicity, I assume that development costs are uniformly distributed over $[0, 1]$.[6] The total cost of full development to commercialization, $D(\delta_i, t)$, is expressed in units of the final good and is given by

$$D(\delta_i, t) = d(t)\delta_i. \tag{6}$$

Note that $D(\delta_i, t)$ is equivalent to the cost of commercial entry with a new quality improvement. Following Chu *et al.* (2017), I define $d(t) \equiv dQ(t)^{(1-\theta)/\theta}$ so that the cost of development and entry scales with the economy's aggregate quality index. This implies that it is more costly to develop more complex innovations and is necessary to ensure a balanced growth equilibrium.

Immediately after a firm draws its development cost, it separately decides whether to patent the invention and whether to commercialize it. In order to obtain a patent, the firm must partially develop its invention according to the standards set by patent policy. Specifically, firms must complete σd development stages to obtain a patent, where $\sigma \in (0, 1)$ is a policy parameter. Patent policy thus determines how far along the development process an inventor must be to obtain a patent, but does not impose any additional costs beyond those incurred from development. Given σ, an inventor with development cost draw δ_i faces a $\sigma D(\delta_i, t)$ cost to acquire a patent. The remaining development cost associated with commercializing a product after acquiring a patent is $(1 - \sigma)D(\delta_i, t)$.

Technically in this framework, all uncertainty is resolved the moment an invention is discovered and the inventing firm receives their δ development cost draw. That is, firms immediately have full knowledge of the cost of successfully developing their invention both to the point of acquiring a patent and to full commercialization. However, the presence of invention development cost heterogeneity can be interpreted broadly to encompass uncertainty in the development process. Since firms are risk neutral, all that matters for their decision making is the expected cost of invention development. We may interpret a low cost draw ($\delta_i \to 0$) as an invention for which the development path is both obvious and highly certain. Since there is positive value in commercialization,

[6] In Section SM.1 of this paper's supplementary material, I explore an alternate distribution of development costs and show that the paper's main results are not sensitive to this assumption.

firms will always choose to fully develop such inventions. High cost draws on the other hand ($\delta_i \to 1$) may reflect inventions that require significant, costly experimentation to proceed along the development path. As long as $d(t)$ is sufficiently high, firms will optimally choose not to pursue development of such inventions for any $\sigma > 0$.

Free-entry into research implies that the cost of research must exactly offset its expected return in every industry with positive research expenditure. I assume that a firm i that invests $R_i(\omega, t)$ units of the final good in research successfully invents a new idea with instantaneous probability

$$I_i(\omega, t) = \frac{R_i(\omega, t)}{\alpha(t)}, \tag{7}$$

where $\alpha(t)$ measures inverse research productivity and the corresponding expected cost of successful invention. In the baseline model, I follow Chu *et al.* (2017) and define $\alpha(t) = \alpha Q(t)^{(1-\theta)/\theta}$ to ensure balanced growth.[7] Let $\mathbb{E}[V(t)]$ denote the ex ante expected value of a successful invention and random development cost draw. In a symmetric equilibrium with $I(\omega, t) = I(t)$, free-entry implies that firms choose research expenditure such that $R_i(t) = I_i(t)\mathbb{E}[V(t)]$. Using (7) and aggregating to the industry level, the free-entry condition can be written

$$\mathbb{E}[V(t)] = \alpha(t). \tag{8}$$

2.3 Patent protection

Patents grant inventors imperfect legal protection in two dimensions. First, patents protect against direct imitation of commercialized products. Specifically, the innovator responsible for inventing and commercializing the latest quality vintage in each industry, referred to as the quality leader, competes in prices with a competitive fringe of imitative firms. Independent of product quality, each firm can produce one unit of the intermediate good using one unit of the final good. Patent protection determines the degree to which a patent holder can prevent competing firms from commercializing sufficiently similar imitative products. This dimension of patent protection is defined by a single parameter, $\mu > 1$, which specifies the maximum quality lead that a patent holder can effectively maintain over imitative competitors. I refer to μ as backward patent protection. By assumption, unpatented intermediate goods are always imitated fully. Thus,

[7] In Section 5, I consider a more general specification in which the expected cost of successful invention $\alpha(t)$ depends on the flow of information disclosed in patents, and thus, the endogenous patenting decisions of inventors.

each patent holder that commercializes a new quality iteration can drive imitative competitors out of the market by engaging in limit pricing, with $p(j_\omega, t) = \mu$. Imitative firms can do no better than break even and exit the market. Simplifying notation and using (4), each quality leader earns flow profits of

$$\pi(\mu, t) = [\mu - 1]y(j_\omega, t) = \left[\frac{(\mu - 1)}{\mu}\right](1 - \theta)Y(t). \tag{9}$$

Note that backward protection controls the flow profits associated with commercialization.

Second, patents grant an imperfect legal right over future commercial applications that make use of a patented invention. This forward patent protection, also referred to as blocking patents, takes the form of compulsory licensing agreements between a patent holder and firms that subsequently commercialize a product that is considered to fall within the claims of an existing patent. I allow for inter-industry blocking patents following Klein (2022).[8] Specifically, each new commercialized invention incorporates some component or feature that is covered by the claims of the latest patented invention in a $\phi \in (0, 1)$ proportion of industries in the economy. The licensing payment to each infringed patent holder is determined based on the expected value of a flow payment of $s \in (0, 1)$ share of the new innovator's monopoly profits $\pi(\mu, t)$ over its tenure as industry leader. Each firm that commercializes a new invention must pay the present discounted value of this licensing fee to all infringed patent holders as a lump sum as soon as innovation occurs. The two policy parameters, ϕ and s, respectively control the number and the size of licensing payments that are required in order for a firm to commercialize a new product.

Due to their function as blocking patents, patented inventions can generate licensing revenue even when they are not commercialized. Let $v_\ell(t)$ denote the value of the lump sum licensing payment from a new quality leader in a typical industry to a single infringed patent holder. $v_\ell(t)$ is calculated through a standard no-arbitrage condition that equates the risk-free market rate $r(t)$ to the expected return of the licensing deal over the licensee's duration as quality leader. Over a small interval of time dt, the quality leader owes an $s\pi(\mu, t)$ share of profits to the patent holder. The quality leader's licensing obligation continues until it is replaced by the next inventor in its industry that commercializes its invention. With probability $I(t)dt$, a new invention is discovered in the industry. Let $P_c(t)$ denote the probability that an inventor will choose to commercial-

[8]Empirical evidence strongly supports the presence of overlapping patent claims across industries. See Klein (2022) for a detailed discussion.

ize their invention based on their development cost draw. The overall probability that the quality leader is replaced is then $P_c(t)I(t)dt$. If the quality leader is not replaced, the value of the licensing agreement changes by $\dot{v}_\ell(t)dt$. The associated no-arbitrage condition is

$$r(t)v_\ell(t)dt = s\pi(\mu,t)dt - P_c(t)I(t)v_\ell(t)dt + (1 - P_c(t)I(t)dt)\dot{v}_\ell(t)dt. \tag{10}$$

Taking limits as $dt \to 0$ yields

$$v_\ell(t) = \frac{s\pi(\mu,t)}{r(t) + P_c(t)I(t) - \frac{\dot{v}_\ell(t)}{v_\ell(t)}}. \tag{11}$$

The total licensing burden paid by each innovator immediately after commercializing her product is $\phi v_\ell(t)$.

2.4 The patenting and commercialization decision

Let $V_p(\delta_i,t)$ and $V_c(\delta_i,t)$ denote the overall value of obtaining a patent and commercializing an invention with development cost δ_i. Inventors can obtain a patent without commercializing their invention, but cannot profit from commercialization without patenting. This allows us to write the overall value of obtaining a patent in terms of the potential additional value of commercialization,

$$V_p(\delta_i,t) = v_p(t) - \sigma D(\delta_i,t) + \max\{0, V_c(\delta_i,t)\}, \tag{12}$$

where $v_p(t)$ is the value of holding an active patent, which is independent of δ_i. A patent is active in this sense as long as new innovations are commercialized that continue to infringe on the patent. Patents remain active until the next patent arrives in the same industry and replaces it. Over an interval of time dt, new innovations are commercialized in each industry with probability $P_c(t)I(t)dt$ and each such innovation has a ϕ probability of resulting in licensing revenue of $v_\ell(t)$. The patent continues to generate licensing revenue until a new patent arrives in the same industry and replaces it. This replacement occurs with probability $P_p(t)I(t)dt$, and results in a capital loss of $v_p(t)$. If the patent is not replaced, there is a change in valuation of $\dot{v}_p(t)dt$. Equating the overall expected return to the interest rate $r(t)$ and taking limits as $dt \to 0$, we have

$$v_p(t) = \frac{P_c(t)I(t)\phi v_\ell(t)}{r(t) + P_p(t)I(t) - \frac{\dot{v}_p(t)}{v_p(t)}}. \tag{13}$$

The value of commercialization is given by $V_c(\delta_i, t) = v_c(t) - \phi v_\ell(t) - (1-\sigma)D(\delta_i, t)$, where $v_c(t)$ is the value of serving the industry as quality leader, gross of requisite licensing fees and development costs. Over an interval of time dt, a quality leader earns $\pi(\mu, t)$ in profits. It is replaced by the next innovator in the industry with probability $P_c(t)I(t)dt$. The corresponding no-arbitrage condition yields

$$v_c(t) = \frac{\pi(\mu, t)}{r(t) + P_c(t)I(t) - \frac{\dot{v}_c(t)}{v_c(t)}}. \tag{14}$$

Inventors optimally choose to patent if $V_p(\delta_i, t) \geq 0$ and choose to commercialize if $V_c(\delta_i, t) \geq 0$. I assume the following

Assumption 1: Model parameters are such that (i) $V_p(0, t), V_c(0, t) > 0$, (ii) $V_p(1, t), V_c(1, t) < 0$, and (iii) there exists a $\delta^{'} \in (0, 1)$ such that $V_c(\delta^{'}, t) < 0 < V_p(\delta^{'}, t)$.

As discussed previously, the first two parts of Assumption 1 ensure that an inventor that draws the minimum possible development cost will choose to both patent and commercialize its invention, while an inventor that draws the maximum cost will do neither. Note that both $V_c(\delta_i, t)$ and $V_p(\delta_i, t)$ are strictly decreasing in the development cost draw δ_i. This implies that the equilibrium patenting and commercialization behavior of all inventors in terms of two development cost thresholds, $\hat{\delta}_p(t)$, $\hat{\delta}_c(t) \in (0, 1)$, defined implicitly by the cut-off conditions $V_p(\hat{\delta}_p, t) = 0$ and $V_c(\hat{\delta}_c, t) = 0$. All inventors with development cost $\delta_i \leq \hat{\delta}_p(t)$ optimally patent and all inventors with $\delta_i \leq \hat{\delta}_c(t)$ optimally commercialize. Under a uniform distribution of development costs, the probabilities that an invention will result in a patent and commercialized intermediate good are given by $P_p(t) = \hat{\delta}_p(t)$ and $P_c(t) = \hat{\delta}_c(t)$. The final part of Assumption 1 guarantees that there are some development cost draws for which it is profitable to patent the invention, but not to commercialize it. Let $n_c(t)$ denote the proportion of patented inventions that are commercialized, with $n_c(t) \equiv \hat{\delta}_c(t)/\hat{\delta}_p(t)$. Part (iii) implies that $\hat{\delta}_c(t) < \hat{\delta}_p(t)$ and allows us to focus on the empirically relevant case of $n_c(t) < 1$.[9]

Given these threshold values $\hat{\delta}_c(t) < \hat{\delta}_p(t)$ and using the uniform distribution of distribution of development costs, the ex ante expected value of a successful invention

[9]It is straightforward to show that Assumption 1 holds if the development cost parameter, d, is sufficiently large and the patent development requirement, σ, is sufficiently low. See Appendix A.1 for details.

can be written

$$\mathbb{E}[V(t)] = \int_0^{\hat{\delta}_p(t)} [v_p(t) - \sigma d(t)\delta] f(\delta) d\delta + \int_0^{\hat{\delta}_c(t)} [v_c(t) - \phi v_\ell(t) - (1-\sigma) d(t)\delta] f(\delta) d\delta$$
$$= \hat{\delta}_p(t)[v_p(t) - \frac{\sigma d(t)}{2}\hat{\delta}_p(t)] + \hat{\delta}_c(t)[v_c(t) - \phi v_\ell(t) - \frac{(1-\sigma)d(t)}{2}\hat{\delta}_c(t)]. \tag{15}$$

Equation (15) expresses the expected value of a successful invention as the probability weighted average of the value of patenting and commercialization, net of expected development costs.

2.5 Equilibrium

I now solve the model for a balanced growth equilibrium in which the model's endogenous variables $\{Y(t), c(t), I(t), \hat{\delta}_p(t), \hat{\delta}_c(t)\}$ exhibit a constant, possibly zero, growth rate and the following equilibrium conditions are met: the free-entry condition of (25) holds, the final goods market clears, and each inventor optimally chooses to patent and/or commercialize their invention to maximize their value according to the associated cut-off conditions $V_p(\hat{\delta}_p, t) = 0$ and $V_c(\hat{\delta}_c, t) = 0$.

Lemma 1. *Given a constant set of patent policy parameters $\{\mu, \phi, s, \sigma\}$, the economy jumps to a unique and saddle-point stable balanced growth path. On the balanced growth path, output $Y(t)$ and consumption $c(t)$ grow at a common rate $g > 0$, and the invention rate $I(t)$, patenting threshold $\hat{\delta}_p(t)$, and commercialization threshold $\hat{\delta}_c(t)$ are constant.*

Proof. See Appendix A.4. □

Henceforth, I drop the time index for all variables that are constant in equilibrium.

Using (5) and the fact that each quality leader's optimal limit price is determined by patent policy with $p(j_\omega) = \mu$, we have

$$Y(t) = \left[\frac{(1-\theta)Q(t)}{\mu}\right]^{\frac{1-\theta}{\theta}}. \tag{16}$$

The aggregate quality index evolves over time as new inventions arrive at rate I and are commercialized with probability $\hat{\delta}_c$. Each commercialized invention improves the quality of its industry's intermediate good by $\lambda > 1$, but uncommercialized inventions do not directly improve quality. The growth rate of output is given by

$$g \equiv \frac{\dot{Y}(t)}{Y(t)} = \frac{1-\theta}{\theta}\frac{\dot{Q}(t)}{Q(t)} = \frac{1-\theta}{\theta}\ln \lambda \hat{\delta}_c I, \tag{17}$$

where $\hat{\delta}_c I$ is the composite arrival rate of commercialized quality improvements as a function of the commercialization threshold.

Next, define $\overline{\pi}(\mu)$ as the stationary portion of flow profits as a function of backward patent protection. In particular, using (9) and (16), we have

$$\overline{\pi}(\mu) \equiv \frac{\pi(\mu,t)}{Q(t)^{\frac{1-\theta}{\theta}}} = (\mu-1)\left(\frac{1-\theta}{\mu}\right)^{\frac{1}{\theta}}. \tag{18}$$

Note that $\overline{\pi}(\mu)$ captures two competing effects of backward protection on the flow profits of quality leaders. One the one hand, stronger backward protection increases profits because it directly determines each leader's markup over cost. On the other hand, greater markups imply that final goods producers purchase fewer intermediate inputs from each industry, which decreases output at any fixed aggregate quality level $Q(t)$. This markup distortion has a negative effect on each leader's flow profits. It is straightforward to show that flow profits exhibit a non-monotonic, inverted-U shape in backward protection. In particular, $\partial\overline{\pi}(\mu)/\partial\mu > 0$ if and only if $\mu < 1/(1-\theta)$. This implies that flow profits increase in backward protection for sufficiently low levels of protection. Henceforth, I assume $\mu < 1/(1-\theta)$.

As shown in Appendix A.4, it follows from (25), (11), (13), and (14) that $\dot{v}_p/v_p = \dot{v}_c/v_c = \dot{v}_\ell/v_\ell = g$. That is, net of development costs, the value of holding an active patent, commercializing an invention and each licensing agreement grow at the same rate as output. Intuitively, this follows because quality leader flow profits, and associated licensing payments, are proportional to $Y(t)$ according to (9). Using (11), (13), (14), (18), and the Euler equation (2), we can write the separate equilibrium value of patenting and commercializing an invention net of licensing payments as

$$v_p(t) = \left(\frac{s\phi\overline{\pi}(\mu)I\hat{\delta}_c}{(\rho+I\hat{\delta}_c)(\rho+I\hat{\delta}_p)}\right)Q(t)^{\frac{1-\theta}{\theta}}, \qquad v_c(t) - \phi v_\ell(t) = \left(\frac{(1-s\phi)\overline{\pi}(\mu)}{(\rho+I\hat{\delta}_c)}\right)Q(t)^{\frac{1-\theta}{\theta}}. \tag{19}$$

Note that each innovator's licensing obligation enters as a reduction in flow profits over their tenure as quality leader. This specifically reduces the return from commercializing a new invention. The return to patenting is entirely derived from the licensing revenue received from leaders in other industries while the patent remains active. From both perspectives, the overall size of licensing payments is determined by the product of the two policy parameters $s\phi$. Following Klein (2022), I without loss of generality treat $s\phi$ as a single parameter that determines the strength of forward protection.

Using (19), the equilibrium expected value of a successful invention from (15) can be written

$$\mathbb{E}[V(t)] = \left(\frac{\hat{\delta}_c \overline{\pi}(\mu)}{\rho + I\hat{\delta}_c} \left[1 - \frac{\rho s\phi}{\rho + I\hat{\delta}_p} \right] - \frac{d}{2} \left[\hat{\delta}_c^2 + \sigma(\hat{\delta}_p^2 - \hat{\delta}_c^2) \right] \right) Q(t)^{\frac{1-\theta}{\theta}}. \tag{20}$$

As usual, the expected value of inventing a quality improvement depends on quality leader flow profits, discounted at an effective rate that includes the threat of replacement, $\rho + I\hat{\delta}_c$. In the present model, this is further discounted both by the probability that the invention will be commercialized, δ_c, and the effect of licensing payments from forward protection. Even though $\hat{\delta}_c < \hat{\delta}_p$ implies that inventors are more likely to receive fees than pay them, the expected value of invention is strictly decreasing in $s\phi$. This is because licensing fees accrue over time when other firms commercialize their inventions at rate $\delta_c I$ and are discounted accordingly. Thus, the model features the traditional *backloading effect* of forward protection in endogenous growth models such as Chu (2009), Yang (2018), and Klein (2022).

The final term in square brackets reflects the overall expected cost of development for a given patent filing requirement, σ. Since $\hat{\delta}_c < \hat{\delta}_p$, inventors are more likely to partially develop their invention to acquire a patent than fully develop them for commercialization. Thus, by shifting development costs towards inventors that only patent, stronger patenting requirements decrease the expected value of invention. Summarizing, we have the following result,

Lemma 2. *All else equal, the expected value of invention is strictly decreasing in the strength of forward protection, strictly decreasing in patenting requirements, and strictly increasing in backward protection for sufficiently low levels of protection. That is,* $\frac{\partial \mathbb{E}V(t)}{\partial s\phi} < 0$, $\frac{\partial \mathbb{E}V(t)}{\partial \sigma} < 0$, *and* $\frac{\partial \mathbb{E}V(t)}{\partial \mu} > 0$ *for* $\mu < 1/(1-\theta)$.

Finally, imposing balanced growth on (1) results in the following expression for social welfare,

$$U = \frac{1}{\rho} \left(\ln c_0 + \frac{g}{\rho} \right), \tag{21}$$

where c_0 is defined as the level of consumption at the instant the economy jumps to its balanced growth path as a result of any change to patent policy. Consumption is determined by the final goods market clearing condition, $Y(t) = c(t) + X_m(t) + X_r(t) + X_d(t)$, where $X_m(t)$, $X_r(t)$, and $X_d(t)$ denote the amount of the final good used in production of intermediate goods, research, and development respectively. Thus, social welfare reflects both the rate of economic growth and the economy's efficiency in resource use to achieve that level of growth. Normalizing the initial quality index Q_0 to unity, I show in Appendix A.2 that c_0 is determined by parameters and the model's central endogenous variables $\{\hat{\delta}_p, \hat{\delta}_c, I\}$ according to

$$c_0 = \left(1 - \frac{1-\theta}{\mu}\right)\left[\frac{(1-\theta)}{\mu}\right]^{\frac{1-\theta}{\theta}} - I\left[\alpha + \frac{\sigma d}{2}\hat{\delta}_p^2 + \frac{(1-\sigma)d}{2}\hat{\delta}_c^2\right]. \tag{22}$$

2.5.1 Illustrating Equilibrium

The three stationary endogenous variables $\{\hat{\delta}_p, \hat{\delta}_c, I\}$ that characterize the model's equilibrium are determined by the following equilibrium conditions: (1) the patenting cut-off condition $[v_p(t) = \sigma d(t)\hat{\delta}_p]$, (2) the commercialization cut-off condition $[v_c(t) - \phi v_\ell(t) = (1-\sigma)d(t)\hat{\delta}_c]$, and (3) the free-entry condition $[\mathbb{E}V(t) = \alpha(t)]$. To facilitate analytical results, I now combine these conditions into two equilibrium relationships between $\hat{\delta}_c$ and I only.

First, using (19) and the definition of $d(t)$ in (6), the commercialization cut-off condition can be expressed as

$$\frac{(1-s\phi)\overline{\pi}(\mu)}{(\rho + I\hat{\delta}_c)} = (1-\sigma)d\hat{\delta}_c. \qquad \textbf{[CC]} \tag{23}$$

I refer to equation (23) as the "CC" condition, which specifies a downward sloping relationship between $\hat{\delta}_c$ and I. This is because a faster rate of invention I increases the replacement rate of quality leaders and decreases the value of commercialization. The development cost threshold $\hat{\delta}_c$ must decrease to align with this lower value.

Next, substituting the two cut-off conditions into (15) to eliminate $v_p(t)$, $v_c(t)$, and $v_\ell(t)$ allows us to to express the free-entry condition as $[\mathbb{E}V(t) = \alpha(t)]$ in terms of $\hat{\delta}_p$ and $\hat{\delta}_c$ only. Specifically, we have

$$\sigma d\hat{\delta}_p^2 + (1-\sigma)d\hat{\delta}_c^2 = 2\alpha. \tag{24}$$

Free-entry into research requires that the expected cost of invention, $\alpha(t)$, equals the expected value of invention, $\mathbb{E}V(t)$. The $\hat{\delta}_p$ and $\hat{\delta}_c$ threshold values determine the range of the development cost distribution for which there is net profit from patenting and commercialization. Any factor that increases $\hat{\delta}_p$ implies a greater expected profit from patenting, and thus increases $\mathbb{E}V$. To restore the free-entry condition, $\hat{\delta}_c$ must decrease so that the expected profit from commercialization falls. Using (20) and (24), we can write the free-entry condition in its final form of

$$\frac{\hat{\delta}_c \overline{\pi}(\mu)}{(\rho + I\hat{\delta}_c)} \left[1 - \frac{\rho s \phi}{\rho + I\hat{\delta}_p(\hat{\delta}_c)} \right] = 2\alpha, \qquad \textbf{[FE]} \quad (25)$$

Where $\hat{\delta}_p(\hat{\delta}_c)$ is a strictly decreasing function of $\hat{\delta}_c$ as implicitly defined (24). I refer to (25) as the "FE" condition.

For sufficiently small $s\phi$, the FE condition specifies an upward sloping relationship in $(\hat{\delta}_c, I)$ space.[10] To understand why, note that the left-hand side of the FE condition represents the expected value of an invention, net of expected development costs. This is increasing in $\hat{\delta}_c$ since the probability of receiving a development cost draw associated with commercialization increases both the expected profit from commercialization and the expected licensing fees earned by an active patent. However, it decreases in the invention rate I because a more rapid pace of invention increases the rate of replacement for both quality leaders and owners of active patents.

3 Patent policy

In this section, I use equations (23) and (25) to illustrate and analyze the economic implications strengthening patent protection and patenting requirements. I begin by examining the effects of backward and forward protection in Section 3.1. I examine patenting requirements in Section 3.2.

3.1 Backward and forward protection

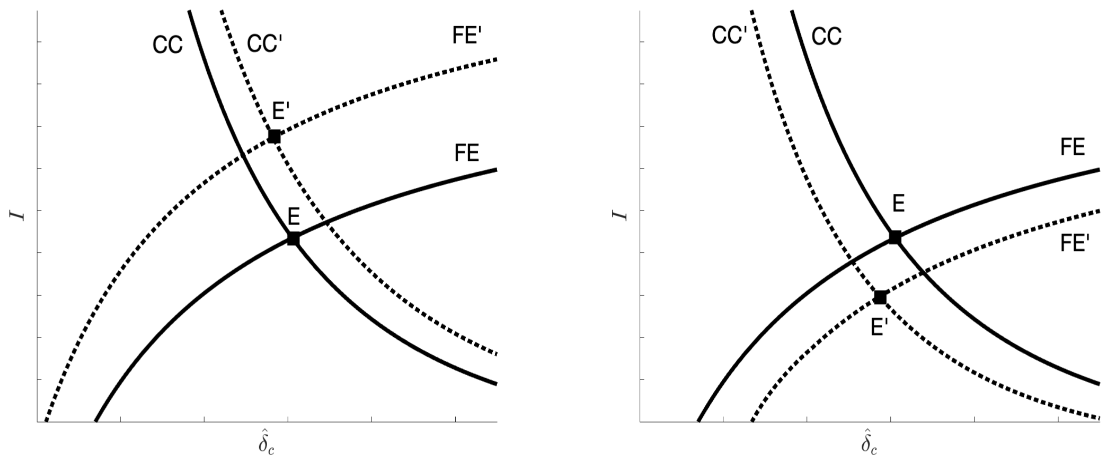
Strengthening backward protection ($\mu \uparrow$) allows each quality leader to raise their mark up over cost and increase their corresponding flow profits. Since the return to both patenting and commercialization are proportional to these flow profits, stronger backward protection directly increases inventor incentives to patent and commercialize quality improvements. As illustrated in Figure 1, panel (a), this causes the CC curve to shift rightward in $(\hat{\delta}_c, I)$ space as inventors find it profitable to commercialize inventions with greater development costs at any invention rate. The increased profitability of patenting and commercialization also increases the expected value of an invention. As

[10]See Appendix A.3 for details.

a result, the FE curve shifts to the left, indicating a greater rate of invention at any commercialization threshold.

Figure 1: Strengthening patent protection

(a) Backward protection ($\mu \uparrow$) (b) Forward protection ($s\phi \uparrow$)



It is apparent from Figure 1 that stronger backward protection increases the equilibrium invention rate and exerts competing effects on the commercialization threshold. However, one can show that $\hat{\delta}_c$ always decreases as a result of this policy change. To see this, consider inventors' *relative* incentive to patent and commercialize their invention by taking the ratio of patenting and commercialization cut-off conditions, $[v_p(t) = \sigma d(t)\hat{\delta}_p]$ and $[v_c(t) - \phi v_\ell(t) = (1-\sigma)d(t)\hat{\delta}_c]$,

$$\frac{I\hat{\delta}_c^2}{\hat{\delta}_p(\rho + I\hat{\delta}_p)} = \frac{(1-s\phi)\sigma}{s\phi(1-\sigma)}, \tag{26}$$

where I have applied (19). Equation (26) does not depend on μ because the return to patenting and commercialization are proportional to $\pi(t)$. However, an increase in the invention rate increases patenting incentives relative to commercialization incentives. This is because a greater I directly decreases the value of commercialization by increasing the replacement rate of quality leaders. Although a greater I similarly increases the replacement rate of active patents, it also increases the frequency of acquiring new licensing deals as new inventions are commercialized. In other words, the return to commercialization is more sensitive to increases in I. Given that $\hat{\delta}_p$ and $\hat{\delta}_c$ are inversely related according to (24), the increase in I resulting from stronger backward protection must be accompanied by an increase in the patenting threshold and a decrease in the commercialization threshold.

Overall, this creates competing effects on the rate of economic growth when backward protection is strengthened such that the impact on growth is ambiguous in the general case. The following proposition summarizes this result

Proposition 1. *Strengthening backward patent protection ($\mu \uparrow$) increases the invention rate ($I \uparrow$), increases the proportion of inventions that are patented ($\hat{\delta}_p \uparrow$), decreases the proportion of inventions that are commercialized ($\hat{\delta}_c \downarrow$), and has an ambiguous impact on economic growth ($g \uparrow\downarrow$).*

Strengthening forward protection ($s\phi \uparrow$) increases the licensing burden of each commercialized invention. This reduces commercialization incentives and shifts the CC curve down and to the left in Figure 1, panel (b), which indicates a lower commercialization threshold $\hat{\delta}_c$ at any invention rate I. As stated in Lemma 1, strengthening forward protection also decreases the expected value of invention through the backloading effect. This shifts the FE curve down and to the right. Thus, strengthening forward protection decreases the invention rate. Note that this generates a competing effect on the commercialization threshold since quality leaders are replaced less frequently. It is clear from inspection of equation (25) that the size of shift to the FE curve is determined by ρ. Intuitively, a smaller ρ implies a smaller backloading effect, and thus, a smaller reduction in invention incentives. For sufficiently small ρ, the corresponding minor shift in the FE curve implies that the commercialization threshold decreases when forward protection is strengthened.[11] Proposition 2 summarizes these findings.

Proposition 2. *Strengthening forward patent protection($s\phi \uparrow$) decreases the invention rate ($I \downarrow$). If ρ is sufficiently small, strengthening forward protection also increases the proportion of inventions that are patented ($\hat{\delta}_p \uparrow$), decreases the proportion of inventions that are commercialized ($\hat{\delta}_c \downarrow$), and decreases economic growth ($g \downarrow$).*

3.2 Patenting requirements

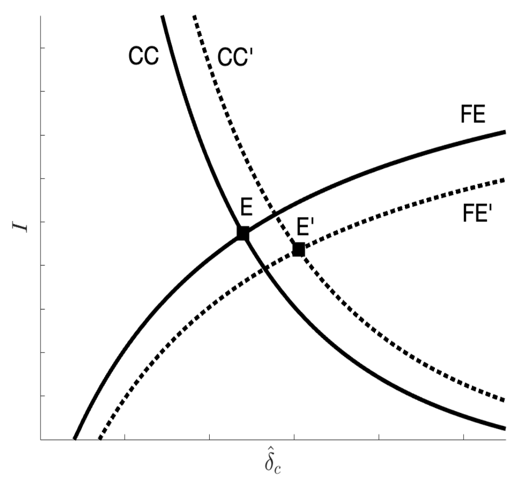
Strengthening patenting requirements ($\sigma \uparrow$) reduces the additional development cost associated with commercialization. This increases commercialization incentives at any invention rate I and shifts the CC curve up and to the right. Indeed, inspection of the CC curve shows that stronger patenting requirements and stronger backward patent protection have an identical impact on commercialization incentives. Stronger requirements

[11] I also note that numerical simulations of the model show that this case obtains for all values of ρ within a reasonable range.

decrease the additional cost of commercialization, while stronger backward protection increase the additional return from commercialization. However, these two policies have a distinct effect on the FE curve. Since more inventors patent than commercialize their invention ($\hat{\delta}_c < \hat{\delta}_p$), stronger patenting requirements increase the expected development costs associated with successful invention. This decreases the overall expected value of invention, and shifts the FE condition up and to the left.[12] This generates competing on the equilibrium invention rate, but the commercialization threshold increases unambiguously. Using (24), the increase in $\hat{\delta}_c$ coupled with the increase in σ implies that the patenting threshold decreases when patenting requirements are strengthened. We have the following.

Proposition 3. *Strengthening patenting requirements ($\sigma \uparrow$) has an ambiguous effect on the invention rate ($I \downarrow\uparrow$), decreases the proportion of inventions that are patented ($\hat{\delta}_p \downarrow$), increases the proportion of inventions that are commercialized ($\hat{\delta}_c \uparrow$), and has an ambiguous effect on economic growth ($g \downarrow\uparrow$).*

Figure 2: Strengthening patenting requirements ($\sigma \uparrow$)



4 Numerical Analysis

In this section, I calibrate the model to aggregate data of the US economy and provide a quantitative illustration of the impact of patent policy. The model features a set of

[12]Note that σ enters the FE condition through the $\hat{\delta}_p(\hat{\delta}_c)$ function that is implicitly defined in (24). Specifically, an increase in σ implies a decrease in $\hat{\delta}_p$ at any level of $\hat{\delta}_c$.

five structural parameters $\{\rho, \theta, \alpha, \delta, \lambda\}$ and three patent policy parameters $\{\mu, s\phi, \sigma\}$. I set the discount factor ρ to a standard value of 0.05 and set $\theta = 0.6$ to reflect recent estimates of the labor share of income in the US (Karabarbounis and Neiman, 2014; Grossman and Oberfield, 2022). Next, each quality leader's markup over marginal cost is directly determined by backward protection, μ. I set the initial value of backward protection to $\mu = 1.1$, which implies a 10% markup, to be consistent with empirical estimates (Gutiérrez and Philippon, 2017; Basu, 2019). Following several studies in the endogenous growth literature, I set $s\phi = 0.15$ so that licensing fees are equal to a 15% share of profits (Chu, 2009; Yang, 2018; Klein and Yang, 2024).

I internally calibrate the remaining four parameters $\{\alpha, \delta, \lambda, \sigma\}$ to match empirical targets for the rate of economic growth, the expected duration of patent licensing revenue, and the proportion of inventions that are patented and commercialized. As mentioned in the introduction, empirical estimates indicate that between 40-60% of patented inventions are eventually commercialized (Mattes *et al.*, 2006; Torrisi *et al.*, 2016; Walsh *et al.*, 2016). I target an intermediate value of 50%, which corresponds to $n_c \equiv \hat{\delta}_c / \hat{\delta}_p = 0.5$ in the model. To separately identify $\hat{\delta}_c$ and $\hat{\delta}_p$, I target a specific value of $\hat{\delta}_p$ based on empirical estimates of firm patent propensity, defined as the proportion of inventions for which a patent application is made. Overall estimates of US firm patent propensity tend to fall in the range of 30 to 55% (Cohen *et al.*, 2002; Hall *et al.*, 2014). However, these estimates purposely include inventions that are protected by alternatives to patents, such as trade secrecy. Since patents are the only means of protection in the model, I use a higher target of $\hat{\delta}_p = 0.667$ to align with estimates of patent propensity within industries where patents are considered a particularly effective method to prevent imitation (Cohen *et al.*, 2002). Together with the $n_c = 0.5$ target, this implies a targeted value of $\hat{\delta}_c = 0.333$.

Next, I use estimates of the profitable lifespan of patents to inform a target for the invention rate I. For instance, Deng (2011), Bilir (2014), and Chen and Shao (2020) find that patent lifespans range from about 6 to 14 years across industries using data on forward patent citations and patent renewals. In the model, each patent actively generates licensing revenue until the next patent arrives in the industry and replaces it. Since patents face a $\hat{\delta}_p I$ rate of replacement, each patent's expected duration of licensing revenue is $1/(\hat{\delta}_p I)$. I target an intermediate value of 11 years, which directly corresponds to a targeted invention rate of $I = 1/(11 \times \hat{\delta}_p) = 0.1377$ given the $\hat{\delta}_p$ target discussed above. Finally, I target a rate of economic growth to match estimates for the share of TFP growth in the US that is attributable to technological progress. I target $g = 0.4\%$, which is consistent with the range of $[0.3, 0.5]$ used in the endogenous growth literature that follows this approach (Chu *et al.*, 2017, 2019; Klein, 2020). I summarize this calibration approach and resulting parameter values in Table 1.

Table 1: Calibration summary

Parameter	Description	Value	Source/Target
External			
ρ	Discount factor	0.05	Standard
θ	Production parameter	0.60	Labor share (60%)
μ	Backward protection	1.1	Price markup (10%)
$s\phi$	Forward protection	0.15	Licensing revenue (15%)
Internal			
α	Innovation difficulty	0.07616	Growth rate (0.4%)
δ	Max develop cost	1.269	% inventions patented ($\hat{\delta}_p = 0.66$)
λ	Innovation size	1.1398	Patent lifespan (11 years)
σ	Patenting requirements	0.02775	% patents commercialized ($n_c = 50.0\%$)

4.1 Numerical results

In Table 2, panel A, I report numerical results from strengthening backward protection from its baseline value of $\mu = 1.1$ in 10% increments to its maximum value of $\mu = 1.14 = \lambda$, which corresponds to perfect backward protection. The greater profits associated with stronger backward protection sharply increase firm incentives to invent and patent new ideas for quality improvements. Increasing backward protection to its maximum value causes the invention rate to increase by 71% over its baseline value. In accordance with Proposition 1, the associated shift in relative incentives towards patenting and away from commercialization causes $\hat{\delta}_c$ to decrease. Although this produces a countervailing negative effect on economic growth since a smaller proportion of inventions are commercialized, the magnitude of this effect is modest. Overall, I find that strengthening backward protection generates a substantial increase in economic growth.

However, the fact that strengthening backward protection decreases the proportion of patents that are commercialized still meaningfully influences the economic impact of this patent policy. This is because each patented invention utilizes the economy's resources but does not contribute to economic growth unless it is further developed into a commercial innovation. The increase in uncommercialized patents thus effectively diverts resources away from consumption. In combination with the standard distortionary effect of higher price markups that further reduces consumption, this generates a negative welfare effect from stronger patent protection that counteracts the positive effect

Table 2: Numerical Results: Patent protection

Panel A: Backward protection	Baseline $\mu = 1.10$	10% ↑ $\mu = 1.11$	20% ↑ $\mu = 1.12$	30% ↑ $\mu = 1.13$	40% ↑ $\mu = 1.14$
Growth rate (g%)	0.400	0.473	0.544	0.612	0.678
Invention rate (I)	0.138	0.164	0.188	0.212	0.236
Patent threshold ($\hat{\delta}_p$)	0.666	0.686	0.701	0.712	0.720
Com. threshold ($\hat{\delta}_c$)	0.333	0.332	0.331	0.330	0.329
Com. proportion (n_c%)	50.00	48.36	47.23	46.39	45.76
Consumption (c_0)	0.303	0.299	0.295	0.291	0.287
Welfare change (ΔU%)	0.000	0.056	0.083	0.083	0.057
Panel B: Forward Protection	Baseline $s\phi = 0.150$	10% ↑ $s\phi = 0.165$	20% ↑ $s\phi = 0.180$	30% ↑ $s\phi = 0.195$	40% ↑ $s\phi = 0.210$
Growth rate (g%)	0.400	0.391	0.382	0.373	0.364
Invention rate (I)	0.138	0.136	0.133	0.131	0.129
Patent threshold ($\hat{\delta}_p$)	0.666	0.705	0.742	0.778	0.812
Com. threshold ($\hat{\delta}_c$)	0.333	0.331	0.328	0.326	0.324
Com. proportion (n_c%)	50.00	46.91	44.24	41.91	39.84
Consumption (c_0)	0.303	0.304	0.304	0.304	0.305
Welfare change (ΔU%)	0.000	- 0.06	- 0.13	- 0.19	- 0.25

of increased economic growth. As a result, I find that welfare exhibits an inverted U-shaped relationship with backward protection and is maximized at an interior value of $\mu = 1.125$, which corresponds to a markup of 12.5%.

I report results from strengthening forward protection in panel B. As with backward protection, strengthening forward protection increases firms' relative incentive to patent versus commercialize new inventions. This leads to a decrease in the proportion of patented inventions that are commercialized. Unlike backward protection however, strengthening forward protection sharply reduces invention incentives through the back-loading effect. As a result of these two forces, economic growth monotonically decreases in forward protection. Although the decrease in invention frees up resources for consumption, this effect is partially offset by the resources needed to support the increase in patenting. The net effect is that consumption increases only slightly and welfare monotonically decreases in forward protection.

I report results from strengthening patenting requirements in Table 3. As described in

Section 1, proposed reforms to patenting requirements range from relatively minor adjustments to existing disclosure and enablement standards to more substantial changes such as requiring proof of a working prototype of the invention. To reflect this range, I consider increases in σ from 10% to 184% of its baseline value. This maximum reform size corresponds to the case where the relative development cost of patenting and commercialization is such that $\hat{\delta}_p \to \hat{\delta}_c$ and all patented inventions are optimally commercialized.

Table 3: Numerical Results: patenting requirements

	Baseline $\sigma = 0.0278$	10% ↑ $\sigma = 0.0305$	50% ↑ $\sigma = 0.0416$	100% ↑ $\sigma = 0.0555$	184% ↑ $\sigma = 0.0789$
Growth rate (g%)	0.4000	0.4003	0.4021	0.4052	0.4119
Invention rate (I)	0.1378	0.1375	0.1368	0.1364	0.1363
Patent threshold ($\hat{\delta}_p$)	0.6660	0.6289	0.5205	0.4347	0.3465
Com. threshold ($\hat{\delta}_c$)	0.3330	0.3337	0.3369	0.3406	0.3465
Com. proportion (n_c%)	50.000	53.064	64.725	78.347	100.00
Consumption (c_0)	0.3032	0.3033	0.3034	0.3034	0.3035
Welfare change (ΔU%)	0.0000	0.0163	0.0791	0.1546	0.2797

I find that strengthening patenting requirements is highly effective at shifting inventors' relative incentives from patenting towards commercialization. Indeed, increasing σ so that the development cost of patenting is only 7.89% of the full cost of commercialization, a 184% in σ from its baseline value, is sufficient to eliminate the presence of uncommercialized patented inventions entirely. This reflects the combined impact of a greater proportion of inventions becoming commercial innovations ($\hat{\delta}_c \uparrow$) and a substantially lower proportion of inventions that are patented ($\hat{\delta}_p \downarrow$). Since this implies that inventions are much less likely to result in positive revenue, the policy does decrease ex ante invention incentives and cause the invention rate to fall. Nevertheless, I find that this effect is dominated by the more frequent commercialization of inventions such that economic growth monotonically increases with σ. Moreover, the decrease in the invention rate coupled with the decrease in patenting implies that more resources are available for consumption resulting in a small increase in consumption. Thus, I find that social welfare is monotonically increasing in patenting requirements since both components of welfare increase in σ.

5 Patent disclosure and research spillovers

In the model, patented inventions provide no direct economic benefit unless they are developed into a commercial innovation. However, a major purported advantage of early patent filing is that it incentivizes rapid disclosure of technical information through patent applications. This disclosed information may generate positive research spillovers and reduce the inefficient duplication of research. Since I find that strengthening patenting requirements leads to fewer patents, the associated reduction in information disclosure represents a potential cost of the policy that is absent from the model.

On the other hand, proponents of strengthening patenting requirements argue that doing so will improve the quality of information disclosed through patents. By requiring patent applications to provide evidence of further development of an invention, stronger patenting requirements increase the amount of useful technical information within each patent and may thus promote research spillovers despite decreasing the overall rate of patenting.[13] Indeed, recent research by Dyer *et al.* (2024) presents compelling evidence that the quality of information disclosed within a patent is causally related to the extent of subsequent research spillovers. However, they also note that,

> Our results do not necessarily imply that stricter enforcement of disclosure requirements will lead to more follow-on innovation in equilibrium. Stricter enforcement may reduce the willingness to invest in innovation and inventors' propensities to patent their innovations, which should be balanced against the benefits of improved disclosure. Subsequent research can provide insight into these other costs and benefits of patent disclosure to paint a more complete picture of the general equilibrium effects of patent disclosure (Dyer *et al.*, 2024, p.19)

In this section, I adapt the model to examine this issue. I adjust the expected cost of successful invention, $\alpha(t)$, to incorporate a congestion externality from duplicative research investment that is mitigated by the quality weighted flow of information disclosed through patents. Specifically, new inventions are patented at a rate of $\hat{\delta}_p I$ and each patent must disclose technical information associated with a σ proportion of the invention development process. Thus, I model the flow of useful disclosed information as $\sigma\hat{\delta}_p I$. I then specify the following general functional form for $\alpha(t)$,

[13] Cotropia (2009) argues that amending patenting requirements such that "the inventor would need to proceed further down the technology development path prior to receiving patent protection ... would generate more technical information about the invention" through disclosure. See Roin (2005), Lemley (2008), and Lemley (2016) for arguments that current, minimal requirements limit the benefits of disclosure; "scientists who are doing research tend to look elsewhere than patents for their learning ... since those patents will lack the level of technical detail and experimental results that an inventor who has actually constructed and tested a prototype can offer" (Lemley, 2016).

$$\alpha(t) \equiv \alpha I^{\beta}[\sigma\hat{\delta}_p I]^{-\eta} Q(t)^{(1-\theta)/\theta}, \tag{27}$$

where $\alpha > 0$ and $0 < \eta \leq \beta < 1$. Note that β imposes decreasing returns to research at the industry level and controls the extent of duplicative research investment among firms competing in research races.[14] The parameter η captures the degree to which this duplication externality is attenuated by the flow of information disclosed through patents. The assumption that $\eta \leq \beta$ ensures that the expected cost of invention increases in the aggregate invention rate.

I now reexamine the impact of strengthening patenting requirements when the expected cost of innovation is given by (27). To do so, I set $\beta = 0.66$, which represents an intermediate value within the endogenous growth literature that analyzes a similar congestion externality (Impullitti, 2010; Chu *et al.*, 2012; Klein, 2022). I consider three separate cases for the importance of information disclosure using $\eta = [0.00, 0.33, 0.66]$. In each case, I recalibrate the parameter α so that the pre-policy reform expected cost of invention is held constant at the baseline level and the baseline equilibrium is identical to that of the prior section. Table 4 reports results from a 100% increase in σ in each of these three cases.

Table 4: patenting requirements and information disclosure

	Baseline $\sigma = 0.0278$	$\eta = 0.00$ $\sigma \uparrow 100\%$	$\eta = 0.33$ $\sigma \uparrow 100\%$	$\eta = 0.66$ $\sigma \uparrow 100\%$
Growth rate ($g\%$)	0.4000	0.4066	0.4309	0.4758
Invention rate (I)	0.1378	0.1371	0.1495	0.1736
Patent threshold ($\hat{\delta}_p$)	0.6660	0.4344	0.4293	0.4192
Com. threshold ($\hat{\delta}_c$)	0.3330	0.3400	0.3304	0.3142
Disclosure ($\sigma\hat{\delta}_p I$)	0.0025	0.0033	0.0036	0.0040
Com. proportion ($n_c\%$)	50.000	78.268	76.966	74.950
Consumption (c_0)	0.3032	0.3034	0.3027	0.3015
Welfare change ($\Delta U\%$)	0.0000	0.1680	0.4012	0.8560

The calibration results in values of $\alpha = [0.2850, 0.0390, 0.0053]$ for the three cases $\eta = [0.00, 0.33, 0.66]$. All other parameters remain at their baseline calibrated value reported in Table 1. The baseline equilibrium is identical across cases.

I find that the presence of duplicative research investment and spillovers from patent

[14] See Jones and Williams (2000) for a similar formulation and a discussion of this "stepping on toes effect" of duplicated research effort in endogenous growth models. See also Impullitti (2010), Chu *et al.* (2012), and Klein (2022) for additional examples.

disclosure reinforces the benefits of strengthening patenting requirements. For example, the $\eta = 0.00$ case isolates the impact of the research congestion externality since information disclosed in patents does not impact the expected cost of invention. Although strengthening patenting requirements generates the same qualitative pattern of results as in the baseline model, the associated decrease in the invention rate reduces the cost of invention and produces an additional channel of efficiency gains from the policy change. Consequently, a 100% increase in σ produces a larger increase in both economic growth and welfare compared to the baseline model as reported in Table 3.

The benefits of strengthening patenting requirements are even more pronounced in the two cases where $\eta > 0$. This is because the quality weighted flow of information disclosed through patents increases under stronger requirements. Necessarily, this implies that the effect of improving the quality of disclosed information ($\sigma \uparrow$) more than offsets the associated decrease in the flow of new patents ($\hat{\delta}_p I \downarrow$).[15] As a result, the expected cost of invention falls sharply. In fact, although strengthening patenting requirements still reduces invention incentives as in the baseline model, this effect is now dominated by the decrease in invention cost.[16] With positive spillovers from information disclosure, I find that the policy change actually increases the invention rate and generates a much large increase in economic growth compared to the baseline model.

6 Conclusion

Recent critiques of the US patent system have emphasized that a large proportion of patented inventions are never commercially exploited. Specifically, there is concern that by facilitating patenting early in the development process, patent policy incentivizes invention in pursuit of patents rather than the development of commercial innovations. In this paper, I assess these claims by developing an endogenous growth model that incorporates firms' separate incentives to invent, patent, and commercialize new innovations. I use the model to evaluate multiple dimensions of patent policy, with a particular emphasis on patenting requirements that control how far along the development process

[15]This result depends on the modeling assumption that the quality weighted flow of information is given by $\sigma\hat{\delta}_p I$. I view this as a natural assumption since each patent must disclose technical information associated with a σ proportion of the invention development process. In Section SM.2 of this paper's supplementary material, I consider a more general specification in which the quality weighted flow of information is given by $\sigma^{\epsilon}[\hat{\delta}_p I]^{1-\epsilon}$, where $0 < \epsilon < 1$. I find that the main results presented in this section continue to hold unless the intensity of information quality ϵ is quite low.

[16]This can be visualized using Figure 2. The decrease in the expected cost of invention in this extended model can now generate a leftward shift in the FE condition, leading to the different qualitative pattern of results from the baseline model.

an inventor must progress to obtain a patent.

The model formalizes how patent policy impacts economic outcomes through two distinct channels: firms' ex ante incentives to invest in research and their ex post incentives to develop the resulting inventions into commercial innovations. I show that the interaction between these two channels generates important policy implications that are necessarily obscured in traditional endogenous growth analyses that treat research and development as a single process. Notably, I find that strengthening backward patent protection from imitation impacts firms' relative incentives such that the economy's rates of invention and patenting increase but the proportion of inventions that are commercialized falls. Although backward patent protection is still generally growth promoting, the rise in uncommercialized patents creates a negative welfare effect through an inefficient use of the economy's resources. This finding suggests that traditional analyses may overstate the case for strong backward patent protection.

In addition, the model allows for an evaluation of patenting requirements within a framework that directly incorporates their equilibrium effect on the research investment incentives. My results show that strengthening patenting requirements is highly effective at shifting inventors' relative incentives from patenting towards commercialization. Although ex ante research investment incentives are reduced leading to a fall in the invention rate, the much higher proportion of inventions that are fully developed causes an increase in the overall rate that new commercial innovations are introduced into the market. This generates both an increase in economic growth and a reduction the inefficient use of resources on uncommercialized inventions, which improves social welfare. My analysis thus provides support for the use of stronger patenting requirements as an underappreciated policy tool to improve economic outcomes by incentivizing the pursuit of commercial innovations.

Appendix A

A.1 Details of Assumption 1

First, using the definition of $V_c(\delta_i, t)$ and $V_p(\delta_i, t)$ from Section 2.4, Assumption 1 parts (i) and (ii) are equivalent to

$$0 < \frac{v_p(t)}{\sigma d(t)} < 1 \qquad \text{and} \qquad 0 < \frac{v_c(t) - \phi v_\ell(t)}{(1-\sigma)d(t)} < 1. \tag{A.1}$$

From (19) and the definition of $d(t)$, it is immediate that these conditions hold for $s\phi \in (0,1)$ and a sufficiently large development cost parameter d. Given (A.1), part (iii) of Assumption 1 is equivalent to the condition that there exists a $\delta' \in (0,1)$ such that

$$\frac{v_c(t) - \phi v_\ell(t)}{(1-\sigma)d(t)} < \delta' < \frac{v_p(t)}{\sigma d(t)}. \tag{A.2}$$

Rearranging, this is identical to

$$\frac{v_c(t) - \phi v_\ell(t)}{v_p(t)} < \frac{1-\sigma}{\sigma}, \tag{A.3}$$

which clearly holds for sufficiently small σ.

A.2 Final goods market clearing

The final good $Y(t)$ is used for consumption $c(t)$, production of intermediate goods $X_m(t)$, research $X_r(t)$, and development $X_d(t)$. Thus, market clearing requires that $Y(t) = c(t) + X_m(t) + X_r(t) + X_d(t)$. Using (4) and the fact that all quality leaders set their price equal to μ, $X_m(t) = Y(t)(1-\theta)/\mu$. Directly from (7), $X_r(t) = \alpha(t)I(t)$. The resources allocated to the development of inventions depend on the threshold values $\hat{\delta}_c(t)$ and $\hat{\delta}_p(t)$ as follows

$$X_d(t) = I(t)\left(\int_0^{\hat{\delta}_p(t)} \sigma d(t)\delta f(\delta)d\delta + \int_0^{\hat{\delta}_c(t)} (1-\sigma)d(t)\delta f(\delta)d\delta\right). \tag{A.4}$$

Evaluating the integrals in (A.4) and combining terms, the market clearing condition can be expressed as

$$Y(t)\left(1-\frac{1-\theta}{\mu}\right)=c(t)+I(t)\left[\alpha(t)+\frac{\sigma d(t)}{2}\hat{\delta}_p^2(t)+\frac{(1-\sigma)d(t)}{2}\hat{\delta}_c^2(t)\right]. \tag{A.5}$$

A.3 Slope of the FE condition

In this section, I derive the parameter condition under which the FE condition specifies an upward sloping relationship in $(\hat{\delta}_c, I)$ space. Rearrange (25) to $g(\hat{\delta}_c, I)=0$ by subtracting 2α from both sides. We have

$$\frac{\partial g(\hat{\delta}_c, I)}{\partial \hat{\delta}_c}=\frac{\overline{\pi}(\mu)}{(\rho+I\hat{\delta}_c)(\rho+I\hat{\delta}_p)}\left(\frac{\rho^2(1-s\phi)+\hat{\delta}_p(\hat{\delta}_c)I\rho}{(\rho+I\hat{\delta}_c)}+\frac{\partial\hat{\delta}_p(\hat{\delta}_c)}{\partial\hat{\delta}_c}\frac{\hat{\delta}_c I\rho s\phi}{(\rho+I\hat{\delta}_p(\hat{\delta}_c))}\right). \tag{A.6}$$

Since $\hat{\delta}_c \leq \hat{\delta}_p(\hat{\delta}_c)$, a sufficient condition for $\partial g(\hat{\delta}_c, I)/\partial\hat{\delta}_c > 0$ is that $\rho(1-s\phi)+\hat{\delta}_p(\hat{\delta}_c)I > -(\partial\hat{\delta}_p(\hat{\delta}_c)/\partial\hat{\delta}_c)\hat{\delta}_c I s\phi$. Using (24),

$$\frac{\partial\hat{\delta}_p(\hat{\delta}_c)}{\partial\hat{\delta}_c}=-\frac{(1-\sigma)}{\sigma}\frac{\hat{\delta}_c}{\hat{\delta}_p(\hat{\delta}_c)}. \tag{A.7}$$

Thus, the sufficient condition is given by

$$\hat{\delta}_p(\hat{\delta}_c)[\rho(1-s\phi)+\hat{\delta}_p(\hat{\delta}_c)I] > \frac{(1-\sigma)}{\sigma}I\hat{\delta}_c^2 s\phi=\hat{\delta}_p(\hat{\delta}_c)[\rho+\hat{\delta}_p(\hat{\delta}_c)I](1-s\phi), \tag{A.8}$$

where the last equality uses (26). Since $s\phi<1$, this condition always obtains. Next, we have

$$\frac{\partial g(\hat{\delta}_c, I)}{\partial I}=\frac{\overline{\pi}(\mu)}{(\rho+I\hat{\delta}_c)}\left(\frac{\rho s\phi\hat{\delta}_p(\hat{\delta}_c)}{[\rho+I\hat{\delta}_p(\hat{\delta}_c)]^2}-\hat{\delta}_c\frac{\rho(1-s\phi)+I\hat{\delta}_p(\hat{\delta}_c)}{\rho+I\hat{\delta}_p(\hat{\delta}_c)}\right). \tag{A.9}$$

This shows that $\frac{\partial g(\hat{\delta}_c, I)}{\partial I}<0$ if and only if

$$\hat{\delta}_c[\rho(1-s\phi)+I\hat{\delta}_p(\hat{\delta}_c)][\rho+I\hat{\delta}_p(\hat{\delta}_c)] > \rho s\phi\hat{\delta}_p(\hat{\delta}_c), \tag{A.10}$$

which holds for sufficiently small $s\phi$. Thus, application of the implicit function theorem shows that the FE condition is upward sloping in $(\hat{\delta}_c, I)$ space under this condition.

A.4 Proof of Lemma 1

First, I will show that $\hat{\delta}_p$ and $\hat{\delta}_c$ are stationary on any balanced growth path. Using the expression for the ex ante value of a successful invention in (15), the free-entry condition can be written

$$\alpha(t) = \hat{\delta}_p(t)[v_p(t) - \frac{\sigma d(t)}{2}\hat{\delta}_p(t)] + \hat{\delta}_c(t)[v_c(t) - \phi v_\ell(t) - \frac{(1-\sigma)d(t)}{2}\hat{\delta}_c(t)]. \tag{A.11}$$

By imposing the two cut-off conditions $V_p(\hat{\delta}_p, t) = 0$ and $V_c(\hat{\delta}_c, t) = 0$, and using the definitions $\alpha(t) = \alpha Q(t)^{\frac{1-\theta}{\theta}}$ and $d(t) = dQ(t)^{\frac{1-\theta}{\theta}}$, we have

$$2\alpha = \sigma d\hat{\delta}_p^2(t) + (1-\sigma)d\hat{\delta}_c^2(t). \tag{A.12}$$

Differentiating with respect to time and rearranging yields

$$g_{\hat{\delta}_p} = -\frac{(1-\sigma)}{\sigma}n_c^2(t)g_{\hat{\delta}_c}, \tag{A.13}$$

where $n_c(t) = \hat{\delta}_c(t)/\hat{\delta}_p(t) > 0$ is the patent commercialization rate and $g_{\hat{\delta}_p}$ and $g_{\hat{\delta}_c}$ denote the equilibrium growth rate of $\hat{\delta}_p$ and $\hat{\delta}_c$ respectively. Differentiating with respect to time once more gives $0 = \dot{n}_c(t)g_{\hat{\delta}_c}$, which implies that $\dot{n}_c(t) = 0$, $g_{\hat{\delta}_c} = 0$, or both. If $g_{\hat{\delta}_c} = 0$, then $g_{\hat{\delta}_p} = 0$ is immediate from (A.13). If $\dot{n}_c(t) = 0$, then $g_{\hat{\delta}_c} = g_{\hat{\delta}_d}$, and it is immediate from (A.13) that both must equal zero. Thus, $\hat{\delta}_p$ and $\hat{\delta}_c$ are stationary.

Next, taking the natural log of the patent cut-off condition and differentiating with respect to time gives $\dot{v}_p(t)/v_p(t) = \dot{d}(t)/d(t) = g$, where g is defined in equation (17). Using (13), this immediately implies that $\dot{v}_\ell(t)/v_\ell(t) = g$. Rearranging the commercialization cut-off condition gives

$$\frac{v_c(t)}{Q(t)^{\frac{1-\theta}{\theta}}} = (1-\sigma)d\hat{\delta}_c + \frac{\phi v_\ell(t)}{Q(t)^{\frac{1-\theta}{\theta}}}. \tag{A.14}$$

Since the right hand side of (A.14) is stationary, we have $\dot{v}_c(t)/v_c(t) = g$.

Finally, I will show that the economy jumps to its stable balanced growth path. Rearranging the market clearing condition (A.5) using (16) and (A.12), we have

$$I(t)2\alpha = \left(\frac{1-\theta}{\mu}\right)^{\frac{1-\theta}{\theta}}\left(1 - \frac{1-\theta}{\mu}\right) - C(t), \tag{A.15}$$

where $C(t) \equiv c(t)/Q(t)^{(1-\theta)/\theta}$ is a stationary transformed variable. Next, use (11), ([14](#)), and the fact that $\dot{v}_\ell(t)/v_\ell(t) = \dot{v}_c(t)/v_c(t) = [(1-\theta)/\theta]\dot{Q}(t)/Q(t)$, to rewrite the commercialization condition $V_c(\hat{\delta}_c, t) = 0$ as

$$r(t) = \frac{(1-s\phi)\overline{\pi}(\mu)}{(1-\sigma)d\hat{\delta}_c} - I(t)\hat{\delta}_c + \frac{(1-\theta)}{\theta}\frac{\dot{Q}(t)}{Q(t)}. \tag{A.16}$$

From the Euler equation (2),

$$\frac{\dot{C}(t)}{C(t)} = r(t) - \rho - \frac{(1-\theta)}{\theta}\frac{\dot{Q}(t)}{Q(t)}. \tag{A.17}$$

Substituting for $r(t)$ from (A.16) gives

$$\frac{\dot{C}(t)}{C(t)} = \frac{(1-s\phi)\overline{\pi}(\mu)}{(1-\sigma)d\hat{\delta}_c} - I(t)\hat{\delta}_c - \rho. \tag{A.18}$$

Note that equation (A.15) defines $I(t)$ as a decreasing function of $C(t)$ and $\hat{\delta}_c$ is both stationary and determined independently of $C(t)$ according to equations (23) and (25). Thus, (A.18) provides a one-dimensional differential equation in $C(t)$ and the right hand side of (A.18) is increasing in $C(t)$. This implies that $C(t)$ is characterized by saddle-point stability and that $C(t)$ must jump to its interior steady state value. Furthermore, since $C(t)$ is stationary, $I(t)$ is also stationary from (A.15).

Supplementary Material - For Online Publication Only

SM.1 Alternate formulation of invention development costs

Throughout the main text, I assume that each new inventor draws a development cost δ_i from a uniform distribution defined on $[0, 1]$. I now consider an alternate formulation to assess the robustness of the paper's main results. In place of equation (6), let the total cost of development be given by

$$D(\gamma_i, t) = \frac{d(t)}{\gamma_i}, \tag{SM.1}$$

where $d(t) = dQ(t)^{(1-\theta)/\theta}$ as before and γ_i now represents an individual inventor's development productivity. Although this is plainly equivalent to the development cost formulation in the main text with $\gamma_i = 1/\delta_i$, it more closely aligns with the common framing of firm heterogeneity in terms of productivity differences. Additionally, I now assume that development productivity is drawn from a Pareto distribution with probability density function

$$f(\gamma) = \frac{1}{\kappa}\gamma^{-(1+\kappa)/\kappa}, \tag{SM.2}$$

where $\kappa \in (0, 1)$ determines the distribution's shape parameter, $1/\kappa$.

Moving to this formulation does not change the basic mechanics of the model or its central analytical results. Reformulating Assumption 1 to reflect $\gamma_i = 1/\delta_i$ still delivers two thresholds ($\hat{\gamma}_p$, $\hat{\gamma}_c$) that characterize the patenting and commercialization decisions of all inventors. The probabilities that an inventor will draw a development productivity associated with patenting and commercialization respectfully are now $P_p = 1 - F(\hat{\gamma}_p) = \hat{\gamma}_p^{-1/\kappa}$ and $P_c = 1 - F(\hat{\gamma}_c) = \hat{\gamma}_c^{-1/\kappa}$, with $P_c < P_p$. Again accounting for $\gamma_i = 1/\delta_i$, it is straightforward to show that the paper's analytical results regarding the impact of patent policy continue to hold as described in Propositions 1-3.

Of course, the use of a Pareto distribution implies that productivity draws are concentrated towards lower values. Relative to the uniform distribution of costs analyzed in the main text, this implies that a greater proportion of inventors face higher than average development costs. I now explore numerically how this change impacts the quantitative effects of patent policy following the approach used Section 4. To do so, I first set the parameter $\kappa = 0.5$, which implies that the mean of the productivity distribution is equal to 2. This aligns with the mean development cost of $\bar{\delta} = 0.5$ in the main text. I then recalibrate the model to match the same empirical targets described in Section 4 and summarized in Table 1. This procedure results in internally calibrated parameter values of $\alpha = 0.0508$, $\delta = 0.7404$, $\lambda = 1.1398$, and $\sigma = 0.0388$.

I present numerical results associated with strengthening backward and forward patent protection in Table SM.1. Overall, these results closely align with those presented in the main text. The most notable difference is that the patenting and commercialization behavior of inventors is marginally more sensitive to changes in patent protection. For instance, strengthening backward protection through a 40% increase in μ causes the proportion of patented inventions that are developed into commercial innovations (n_c) to decrease from 50% to 44.23%. In the baseline model, this same policy generated a smaller decrease from 50% to 45.76%. When forward protection ($s\phi$) is increased by 40%, this proportion falls to 36.37% compared to 39.84% from the corresponding baseline results. This increased sensitivity implies that strengthening patent protection generates greater inefficiency under a Pareto distribution of development productivity and consequently tends to be less desirable. For instance, I find that backward protection still exhibits an inverted-U shaped relationship with welfare. In the baseline results, the welfare maximizing value of protection was $\mu = 1.125$. Under a Pareto distribution, I find a lower optimal value of $\mu = 1.120$.

Table SM.1: Numerical Results with the Pareto Distribution: Patent protection

Panel A: Backward protection	Baseline $\mu = 1.10$	10% ↑ $\mu = 1.11$	20% ↑ $\mu = 1.12$	30% ↑ $\mu = 1.13$	40% ↑ $\mu = 1.14$
Growth rate (g%)	0.400	0.472	0.542	0.609	0.674
Invention rate (I)	0.138	0.163	0.188	0.212	0.235
Patent probability (P_p)	0.666	0.695	0.715	0.731	0.743
Com. probability (P_c)	0.333	0.331	0.330	0.329	0.328
Com. proportion (n_c%)	50.00	47.71	46.18	45.07	44.23
Consumption (c_0)	0.303	0.299	0.295	0.291	0.287
Welfare change (ΔU%)	0.000	0.044	0.059	0.047	0.010
Panel B: Forward Protection	Baseline $s\phi = 0.150$	10% ↑ $s\phi = 0.165$	20% ↑ $s\phi = 0.180$	30% ↑ $s\phi = 0.195$	40% ↑ $s\phi = 0.210$
Growth rate (g%)	0.400	0.389	0.379	0.368	0.357
Invention rate (I)	0.138	0.135	0.133	0.130	0.128
Patent probability (P_p)	0.666	0.722	0.776	0.828	0.879
Com. probability (P_c)	0.333	0.330	0.327	0.323	0.320
Com. proportion (n_c%)	50.00	45.69	42.09	39.02	36.37
Consumption (c_0)	0.303	0.304	0.304	0.304	0.305
Welfare change (ΔU%)	0.000	- 0.09	- 0.17	- 0.25	- 0.33

Finally, I examine the impact of strengthening patenting requirements and report results in Table SM.2. I again find a pattern of results that is very similar to the baseline analysis, with the main difference being an increased sensitivity of n_c to changes in σ. However, since increasing σ strengthens relative incentives to commercialize inventions, this increased sensitivity reinforces the benefit of strengthening patenting requirements. For instance, I find that a 100% increase in σ almost entirely eliminates the presence of uncommercialized patented inventions and generates a larger increase in both economic growth and welfare compared to the baseline model. Indeed, I find that a 105% increase in σ is sufficient to fully eliminate uncommercialized patented inventions, whereas a much larger 184% increase was required in the baseline model.

Interestingly, these numerical results also illustrate that strengthening patenting requirements can lead to an increase in the invention rate, even without considering the impact of information disclosure through patents. This is consistent with Proposition 3, which states that the policy has an ambiguous effect on the invention rate in general. This ambiguity results from the policy's competing effects of increasing commercialization incentives through the CC curve and decreasing research incentives through the FE curve as illustrated in Figure 2. With a Pareto distribution of development productivity, the decrease in the expected value of an invention can be sufficiently small such that the shift in the FE curve is dominated by the shift in the CC curve when σ increases. Although it represents an unusual case and the magnitude is quite small, this results in an increase in the equilibrium invention rate.

Table SM.2: Numerical Results with the Pareto Distribution: Patenting Requirements

	Baseline $\sigma = 0.0388$	10% $\uparrow$ $\sigma = 0.0427$	50% $\uparrow$ $\sigma = 0.0582$	100% $\uparrow$ $\sigma = 0.0776$	105% $\uparrow$ $\sigma = 0.0794$
Growth rate (g%)	0.4000	0.4014	0.4076	0.4163	0.4172
Invention rate (I)	0.1378	0.1375	0.1371	0.1372	0.1372
Patent probability (P_p)	0.6660	0.6141	0.4667	0.3565	0.3486
Com. probability (P_c)	0.3330	0.3346	0.3407	0.3479	0.3486
Com. proportion (n_c%)	50.000	54.488	73.002	97.600	100.00
Consumption (c_0)	0.3032	0.3033	0.3034	0.3034	0.3035
Welfare change (ΔU%)	0.0000	0.0328	0.1606	0.3168	0.3313

SM.2 Generalizing the flow of disclosed information

Section 5 of the main text examines a specification of the model in which the expected cost of invention $\alpha(t)$ decreases in the quality weighted flow of information disclosed through patents. This information flow is specified as $\sigma\hat{\delta}_p I$ to reflect that new inventions are patented at a rate of $\hat{\delta}_p I$ and each patent must disclose technical information associated with a σ proportion of the invention development process. I consider this specification to be neutral in the sense that it gives equal weight to the amount of information disclosed within each patent and the number of new patents that arrive over a small interval of time.

In this section, I consider a more general specification where the quality weighted flow of information is given by $\sigma^\epsilon(\hat{\delta}_p I)^{1-\epsilon}$, with $0 < \epsilon < 1$. The parameter ϵ can be interpreted as the relative intensity of information quality on the research spillovers from patent disclosure. For example, a low ϵ may reflect a case where patent disclosure mitigates duplicative investment simply by signaling competitors' general research direction, whereas a higher ϵ places greater importance of the technical content of information within a patent. A value of $\epsilon = 0.5$ reflects the neutral case examined in the main text.

Given this specification, the expected cost of invention expressed in equation (27) becomes

$$\alpha(t) \equiv \frac{\alpha I^\beta}{[\sigma^\epsilon(\hat{\delta}_p I)^{1-\epsilon}]^\eta}\, Q(t)^{(1-\theta)/\theta}. \tag{SM.3}$$

Using (SM.3), I reevaluate the economic impact of strengthening patenting requirements by increasing σ 100% across different values of ϵ. Specifically, I consider six values $\epsilon = [0.0, 0.1, 0.2, 0.3, 0.4, 0.5]$ and set η such that $(1-\epsilon)\eta = 0.33$ in each case. This ensures that the overall importance of the rate of patenting $\hat{\delta}_p I$ is held constant across cases. I then recalibrate the parameter α such that each case shares common baseline equilibrium values of all endogenous variables, as reported in Table 4. Thus, the $\epsilon = 0.5$ case exactly corresponds to the policy experiment considered in Table 4, column 3.[1]

The results of this exercise are displayed in Figure SM.1. All variables are reported in terms of the change from their baseline values when σ is increased. First, note that strengthening patenting requirements remains highly effective at shifting relative incentives towards commercialization at any value of ϵ. Starting from the baseline where 50% of patented inventions are commercialized ($n_c = 0.50$), the reported increase in n_c of between $[0.27, 0.31]$ implies that greater than 75% of patented inventions are commer-

[1] Although values of ϵ exceeding 0.5 are also plausible, I do not include them in the analysis since the associated results are qualitatively identical to those presented in the main text.

Figure SM.1: Strengthening patent requirements ($\sigma \uparrow 100\%$) across ϵ

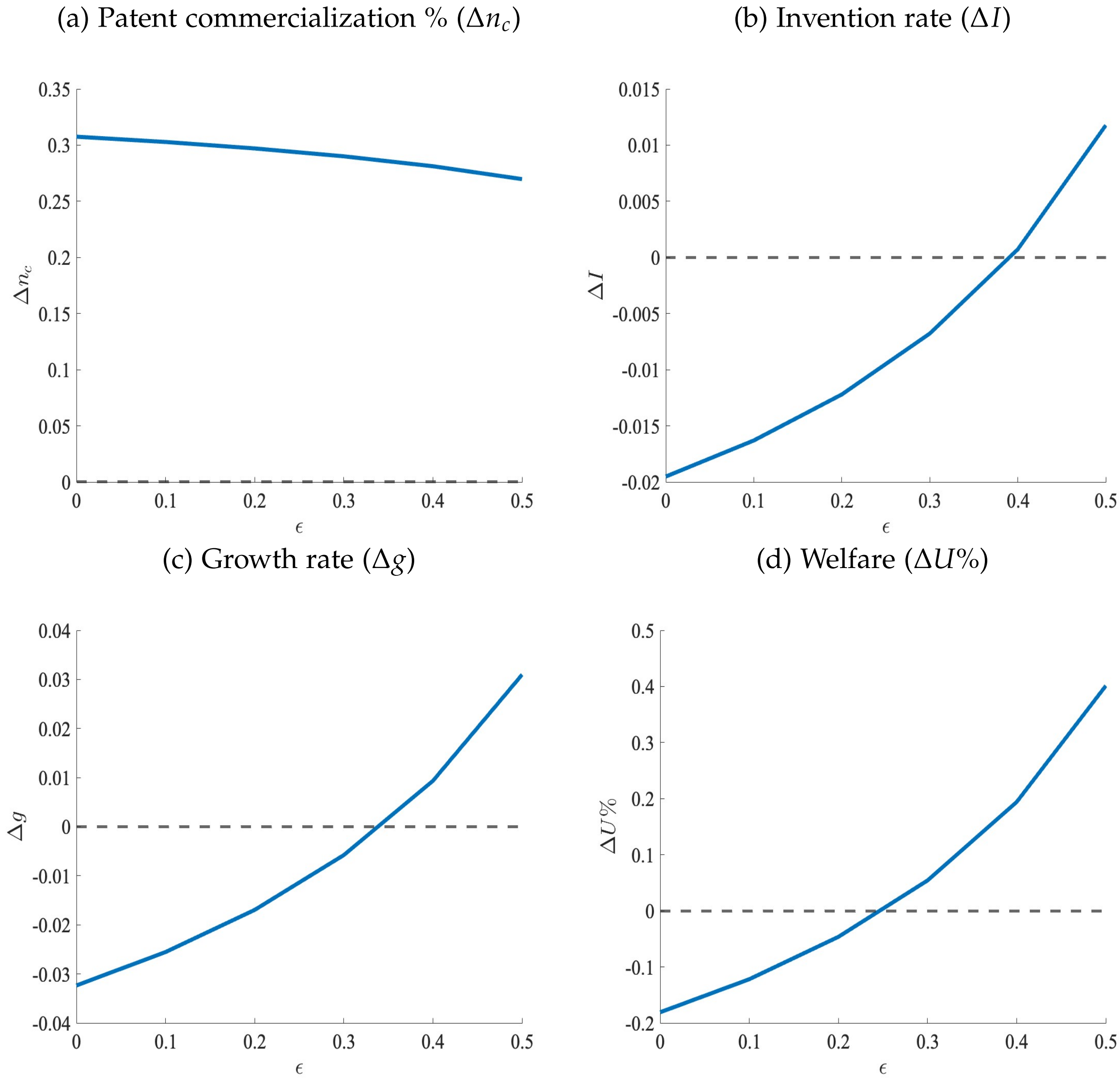


Figure SM.1 displays the change in the indicated variables associated with a 100% increase in σ at six distinct values of $\epsilon = [0.0, 0.1, 0.2, 0.3, 0.4, 0.5]$. For each value of ϵ, I set η such that $(1-\epsilon)\eta = 0.33$ and recalibrate α such that each case shares a common baseline equilibrium. The calibrated values are $\alpha = [0.1287, 0.1127, 0.0955, 0.0772, 0.0581, 0.0390]$. All other parameters remain as in the main text.

cialized in each case after the policy change.[2]

As expected however, the value of ϵ does meaningfully change the policy's impact on the rate of invention, economic growth, and social welfare. This is because ϵ directly

[2]The fact that the magnitude of Δn_c declines slightly with ϵ is a result of the policy's differential impact on the invention rate across values of ϵ. As discussed in Section 3, the return to commercialization is more sensitive to I than the return to patenting. This implies that an increase (decrease) in I exerts a negative (positive) effect on n_c.

controls how the improved quality of disclosed information from stronger patenting requirements influences the expected cost of invention. When ϵ is sufficiently low, the disclosure benefit of increasing σ is dominated by the reduction in the rate that new patents are granted such that the cost of invention rises. This can indeed cause a decrease in economic growth that offsets the efficiency gains from higher rates of commercialization, leading to a decrease in welfare. Nonetheless, my results show that strengthening patenting requirements can still be growth and welfare improving even in cases where ϵ is substantially less than 0.5. I find that the policy begins to increase growth and welfare when ϵ is greater than about 0.35 and 0.25 respectively.

Admittedly, limited empirical evidence makes it difficult to judge exactly what value of ϵ best reflects the real world importance of information quality in research spillovers. However, available evidence does suggest that (1) spillovers through patents are minimal when the quality of disclosed information is low (Roin, 2005; Lemley, 2008), and (2) that higher quality disclosure improves the extent of spillovers through patents (Dyer *et al.*, 2024). Given this, I view the ϵ thresholds identified in this section as highly likely to be exceeded under plausible circumstances.

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