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Abstract

In a recent paper, Newell and Pizer (2003) (N&P) build upon Weitzman (1998, 2001) and show how uncertainty about future interest rates leads to 'certainty equivalent' forward rates (CER) that decline with the time horizon. Such Declining Discount Rates (DDR's) have important implications for the economic appraisal of the long-term policy arena (e.g. climate change) and inter-generational equity. This paper discusses the implications of N&P's transition from the theory to practice in the determination of the schedule of discount rates for use in Cost Benefit Analysis (CBA). Using both UK & US data we make the following points concerning this transition: i) to the extent that different econometric models contain different assumptions concerning the distribution of stochastic elements, model selection in terms of *specification* and '*efficiency criteria'* has important implications for operationalising a theory of DDR's that depends upon uncertainty; ii) mispecification testing naturally leads to employing models that account for changes in the interest rate generating mechanism. Lastly, we provide an analysis of the policy implications of DDR's in the context of climate change and nuclear build in the UK and the US.

JEL classification: C13, C53, Q2, Q4

Keywords: Long-run discounting, Interest rate forecasting, State-space models, Regime-switching models, Climate policy

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1 Introduction

The deleterious effects of conventional exponential discounting on present values of costs and benefits that accrue in the distant future, and the issues of intergenerational equity that arise, are well documented (see e.g. Pearce et al 2003). The emergence of a long-term policy arena containing issues as diverse as climate change, nuclear build and decommission, biodiversity conservation, groundwater pollution etc., and the use of social Cost Benefit Analysis (CBA) to guide decision-makers in this arena has brought the discussion of long-run discounting to the fore. Discount rates that decline with the time horizon (Declining Discount Rates or DDRs) have often been touted as an appropriate resolution to what Pigou (1932) described as the 'defective telescopic faculty' of conventional discounting, and there has been much discussion about the moral and theoretical justification for such a strategy (see e.g. Sozou (1998), Dybvig et al (1996), Portney and Weyant (1999), Weitzman (1998, 2001), Gollier (2002a)). Of particular interest are the declining yet socially efficient discount rates resulting from the analysis of Weitzman (1998) and Gollier (2002a, 2002b) both of which appear to offer a theoretical path through the 'dark jungles of the second best' (Baumol 1968) and the intergenerational equity-efficiency trade-off contained therein.

If these theoretical solutions offer even a partial resolution of the problems of conventional discounting then it is clearly important that they can be operationalised and a schedule of DDRs determined. In the case of Gollier (2002a) and Weitzman (1998) it is uncertainty that drives DDRs, with regard to future growth and the discount rate respectively, thus the question of implementation is one of characterizing the uncertainty of these primals in some coherent way. However, of these two approaches it is Weitzman (1998) that has proven to be more amenable to implementation mainly because the informational requirements stop at the characterization of uncertainty, and do not extend to specific attributes of future generations' risk preferences as would be unavoidable in the case of Gollier (2002a, 2002b)[1].

Weitzman's Certainty Equivalent Discount Rate (CER) is derived from the expected discount factor and is therefore a summary statistic of the distribution of the discount rate. The level and behavior over time of this statistic is clearly dependent upon the manner in which uncertainty is characterized and the two applications that exist have taken different approaches stemming from different interpretations of uncertainty. Weitzman (2001) defines uncertainty by the current lack of consensus on the appropriate discount rate for the very long term. His survey of professional economists results in a gamma probability distribution for the discount rate which leads to the so-called 'gamma discounting' approach, a version of which can also be seen in Sozou (1998). More recently, in this journal, Newell and Pizer (2003) (N&P) suggest that while we are relatively certain about the level of discount rates currently, there is considerable uncertainty in future. From this standpoint they assume that the past is informative about the future and characterize interest rate uncertainty econometrically by estimating a reduced form time series process using historical US interest data. This yields a working definition of the CER based upon an econometric model and allows estimation of the CER schedule from a forecasting simulation.

These applications bring to light some interesting issues concerning the characterization of interest rate uncertainty. Firstly it is interesting to note that the decline in discount rates in both of these approaches depends upon the persistence of interest rates over time. The theoretical model of Weitzman (2001) has this persistence in-built, the

[1]Weitzman (1998) assumes risk neutral agents for exposition, but this represents a special case of his general point. For realistic scenarios, determination of DDRs a la Gollier (2002a, 2002b) requires knowledge of the 4th and 5th derivatives of utility functions, something that he admits is very far from being accomplished.

assumption being that each individual discounts the future at their preferred constant rate. I.e. each of the responses that make up the probability distribution remain constant over time. In N&P however, the existence of persistence is an empirical question, and the existence or otherwise of a unit-root in the series determines the rate of decline of the CER. Secondly, beyond choosing a different sample of humanity, it is not immediately clear how one might improve upon the empirical approach taken by Weitzman (2001). However, in the case of N&P there are several additional avenues available for the characterization of interest rate uncertainty and the resulting definition of the CER.

It is these empirical issues that are the main concern of this paper and we build upon the following two points. Firstly it is clear that, if we believe that the past is informative about the future, it is important to characterize the past as accurately as possible. Indeed, the selection of the econometric model is of considerable moment in operationalising a theory of DDRs that depends upon uncertainty and defines the CER in statistical terms, since each specification differs in the assumptions made concerning the time series process. This will affect the attributes of the resulting schedule of CER. Secondly, selection among these models is also an empirical question. Tests for stationarity, model misspecification and comparisons among models based upon efficiency criteria should guide model selection for the practitioner. N&P, for example, specify a simple AR(p) model of interest rate uncertainty, which limits the characterization of uncertainty to a process in which the distribution of the permanent and temporary stochastic components is constant for all time[2]. Such a process guarantees declining CERs whilst ignoring the possibility of structural breaks.

[2]The AR(1) model that they describe provides the following expression for the certainty equivalent discount rate:

$$\widetilde{r} = \overline{\eta} - t\sigma_\eta^2 - \sigma_\varepsilon^2 \Omega\left(\rho, t\right).$$

Since $\Omega_t\left(.\right) > 0$, and the variance of the permanent component of the interest rate, σ_η^2, and the temporary component, σ_ε^2, are constant over time, $\widetilde{r}$ is a declining function of t.

We revisit these issues for US and UK interest rate data and show that in both cases misspecification testing generates a natural progression away from the simple AR(p) specification towards models which explicitly consider changes in the time series process over time. We select among alternative econometric models by comparison of i) their forecasting performance and the associated Mean Square Error (MSE) and ii) efficiency criteria derived from the empirical distribution of the future path of the discount factor: e.g. coefficient of variation, the proximity of upper and lower percentiles, preferring narrower percentiles and lower coefficients of variation.

These points are illustrated using US and UK interest data and we show the policy implications of interest rate uncertainty and model selection in two case studies. The first, the value of carbon damages, allows a direct comparison to the work of N&P. We use identical data and analyze the same policy issue. The second case study is the appraisal of nuclear build in the UK and this brings to light the different econometric specifications that are appropriate in the UK context and highlights the limitations of DDRs in resolving the issues of inter-generational equity.

The paper is organized as follows. In Section 2, we introduce the theory of CER offered by Weitzman (1998), our methodology for model selection and the econometric models used to characterize the uncertainty of interest rates in both the US and UK contexts. The results of the estimation and the simulations are presented in Sections 3 and 4, respectively. Section 5 draws policy implications for model selection in two case studies and Section 6 concludes the paper.

2 From Theory to Practice

2.1 The Certainty Equivalent Discount Factor and Rate

Discounting future consequences in period t back to the present is typically calculated using the discount factor P_t, where $P_t = \exp(-\sum_{i=1}^{t} r_i)$. When r is stochastic, the expected discounted value of a dollar delivered after t years is:

$$E(P_t) = E\left(\exp(-\sum_{i=1}^{t} r_i)\right) \tag{1}$$

Following Weitzman (1998) we define (1) as the *certainty equivalent discount factor*, and the corresponding *certainty-equivalent forward rate* for discounting between adjacent periods at time t as equal to the rate of change of the expected discount factor:

$$-\frac{dE(P_t)/dt}{E(Pt)} = \widetilde{r}_t \leq E\left[r_t\right] \tag{2}$$

where $\widetilde{r}_t$ is the instantaneous period-to-period rate at time t in the future. This definition contains the assumption that individuals are risk neutral, i.e. they are only concerned with the expected value of discounted values, rather than higher order moments. This represents the economic theory of uncertainty causing a DDR, the result coming from noting that (2) is effectively a restatement of Jensen's Inequality. Operationalising this theory requires the determination of the stochastic nature of $\widetilde{r}_t$.

2.2 Parametrization of real interest rates

N&P employed a simulation method to forecast discount rates in the distant future, which was properly designed to account for uncertainty in the future path of interest rates and was mainly based on the estimation results of two econometric models, namely an autoregressive mean-reverting (MR) model and a random walk (RW) model. They estimated the following $AR(p)$ model[3]:

$$r_t = \eta + e_t \tag{3}$$
$$e_t = \sum_{i=1}^{p} a_i e_{t-i} + \xi_t$$

where $\xi_t \sim N(0, \sigma_\xi^2), \eta \sim N\left(\overline{\eta}, \sigma_\eta^2\right)$ and $\sum_{k=1}^{p} \rho_k < 1$ for the mean-reverting model, while $\sum_{k=1}^{p} \rho_k = 1$ for the random walk model[4]. This model gives their definition of the CER as follows[5]:

$$\widetilde{r_t} = \overline{\eta} - t\sigma_\eta^2 - \sigma_\xi^2 f\left(\rho, t\right) \tag{4}$$

where $\overline{\eta}$ is the mean discount rate and (4) is a declining function of t (See N&P (2003) for details).

Before introducing some alternative econometric models which seem to fit our data better, we briefly discuss the importance of model selection in inference and forecasting. The selected model should be able to capture the dynamics of the data generating process in order to achieve an adequate description of the series under scrutiny. The complexity of the model and the restrictions it imposes should correspond to the level of uncertainty of the true data generating process. Otherwise, inference can be misleading and the forecasting performance of the model may be very poor.

Model selection should be based on data observation, statistical and misspecification testing. For example, the results of unit root tests are crucial in determining a class of appropriate models. Furthermore, misspecification testing is always necessary to check the adequacy of econometric models. The existence of autocorrelation, heteroscedasticity

[3]The data used was annual long-term government bonds for the period 1798 to 1999 converted to real rate by subtracting a ten-year moving average of the expected inflation of the CPI.

[4]The estimation results are not reported to save space. More details can be found in N&P (2003).

[5]Where $f\left(\rho, t\right) = \frac{1-\rho^2 - 2\log(\rho)\rho^{t+1}\left(1+\rho-\rho^{t+1}\right)}{2(1-\rho)^3(1+\rho)}$ for MR and $f\left(\rho, t\right) = \frac{1}{12}\left(1 + 6t + 6t^2\right)$ for RW.

or parameter instability is useful information that the researcher should use to select a model that better fits the data. Finally, an out-of-sample forecasting exercise is often very useful to examine the forecasting performance of the model.

We now introduce alternative econometric models that can be used to parametrize the real interest rates. As we will see in the following sections, the results of misspecification tests will indicate how appropriate these models are in the US and UK cases. It turns out that misspecification testing generates a natural progression away from the simple $AR(p)$ specification towards models which exhibit heterogeneity.

First of all, we introduce the $AR(p)$ - $GARCH(l,m)$ model which is often used in empirical studies to describe processes that exhibit heteroscedasticity. Using such a model to describe the real interest rates gives us:

$$\begin{aligned} r_t &= \eta + e_t \\ e_t &= \sum_{i=1}^{p} a_i e_{t-i} + \xi_t \\ \xi_t &= h_t^{1/2} z_t \\ h_t &= c + \sum_{i=1}^{m} \beta_i \xi_{t-i}^2 + \sum_{i=1}^{l} \gamma_i h_{t-i} \end{aligned} \tag{5}$$

where the variables are as before and z_t is an i.i.d. zero-mean normally distributed random variable with unit variance. l and m represent the lags on the terms which make up h_t. This is a more flexible representation of r_t than the $AR\,(p)$ model. Above all the $AR(p)$ - $GARCH(l,m)$ model allows more efficient estimation in the presence of (conditionally) heteroscedastic errors and is often thought to better reflect the processes of financial variables (Harvey 1993).

Both the $AR(p)$ and $AR(p)$ - $GARCH(l,m)$ models assume that the parameters driving the stochastic process are constant over the sample period. This is likely to be an unrealistic assumption for the period for which we have data and certainly for forecasting the CER over the long-term policy horizon in hand which, following N&P, we assume extends for 400 years. For example, the behavior of interest rates is strongly affected by the economic cycles as well as shocks destabilizing them, i.e. periods of economic crisis. In the US, for example, during the period 1979 through 1982, the Federal Reserve (FED) stopped its usual practice of targeting interest rates and decided to use non-borrowed reserves (NBRs) as a target instrument for monetary policy. As a result, the volatility of interest rates increased dramatically during that period. Other factors inducing high volatility to the U.S. interest rates were the OPEC oil crisis (1973-1975), the October 1987 stock market crisis and wars involving the U.S. For this reason a more appropriate econometric model might be one that allows for changes in the behavior of interest rates. Moreover, the strong persistence in the volatility of the estimated GARCH model[6] is an indication of a regime-switching mechanism, as it can be an artifact of changes in the rate generating mechanism (see for example Gray (1996)).

Two possible models are used to account for the possibility of time varying parameters and regime changes. Firstly, we employ a Regime-Switching (RS) model with two regimes. This model provides a more flexible characterization of uncertainty than the simple, single regime, $AR(p)$ model. Each regime incorporates a different speed of mean-reversion, along with a different permanent component, η_k, and error variance. Specifically, the model is as follows:

$$\begin{aligned} r_t &= \eta_k + e_t \\ e_t &= \sum_{i=1}^{p} a_i^k e_{t-i} + \xi_t \end{aligned} \tag{6}$$

where ξ_t is an i.i.d. zero-mean normally distributed random variable with variance σ_k^2, $k = 1, 2$ for the first and second regime, respectively. At any particular point in time

[6] Estimation results are presented in the following sections.

there is uncertainty as to which regime we are in. The probability of being in each regime at time t is specified as a Markov 1 process, i.e. it depends only on the regime at time $t-1$. We define the probability that the process remains at the first regime as P, while the probability that the process remains at the second regime is Q. The matrix with the transition probabilities is assumed to be constant[7].

Secondly, we model time varying parameters using a State Space (SS) (autoregressive random coefficient) model. This is given by the following system of equations:

$$\begin{aligned} r_t &= \eta + \alpha_t r_{t-1} + e_t \\ \alpha_t &= \sum_{i=1}^{p} \eta_i \alpha_{t-i} + u_t \end{aligned} \tag{7}$$

where e_t and u_t are serially independent, zero-mean normal disturbances such that:

$$\begin{pmatrix} e_t \\ u_t \end{pmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_e^2 & 0 \\ 0 & \sigma_u^2 \end{bmatrix} \right). \tag{8}$$

In other words, the interest rate is modelled as an $AR(1)$ model with an $AR(p)$ coefficient. This model represents a more flexible representation of the stochastic process than the "constant parameter" models.

Finally, we allow the possibility of multivariate models in order to exploit covariation between UK and US interest rates. We estimate a VAR model with endogenous variables

[7] The matrix of probabilities can be thought of as follows, where R_t refers to the regime at time t.

$$\begin{aligned} \Pr ob(R_t &= 1 \mid R_{t-1} = 1) = P \\ \Pr ob(R_t &= 2 \mid R_{t-1} = 2) = Q \\ \Pr ob(R_t &= 2 \mid R_{t-1} = 1) = 1 - P \\ \Pr ob(R_t &= 1 \mid R_{t-1} = 2) = 1 - Q \end{aligned}$$

the real UK and US interest rates. The specification of the model is typically the following:

$$\begin{pmatrix} r_t^{uk} \\ \\ r_t^{us} \end{pmatrix} = \begin{pmatrix} n_1 \\ \\ n_2 \end{pmatrix} + \sum_{i=1}^{p} A_i * \begin{pmatrix} r_{t-i}^{uk} \\ \\ r_{t-i}^{us} \end{pmatrix} + \begin{pmatrix} e_{1t} \\ \\ e_{2t} \end{pmatrix} \tag{9}$$

where $E_t = (e_{1t},\, e_{2t})'$ follows a bivariate normal distribution and A_i are $(2x2)$ matrices of coefficients. The VAR models incorporate the interactions between the endogenous variables which is important from the perspective of forecasting.

3 Empirical Results for the US

3.1 Estimation Results

First of all, we test the stationarity of the US real interest rates. The Augmented Dickey-Fuller (ADF) test failed to reject the null hypothesis of a unit-root. In addition, we applied a variety of unit-root tests[8] to examine the stationarity of the series, the details of which can be found in Table A.1 of Appendix 1. The results generally favoured the existence of a unit-root in the series (both levels and logs were examined). However, since it is well-known that unit-root tests often lack the power to reject a false hypothesis of a unit-root for alternatives that lie in the neighborhood of unity, we estimated both a Random Walk (RW) and a mean-reverting (MR) models. Three lags were included in both models ($p = 3$)[9]. Although these models account well for the dependence in the mean of the series (as indicated by the tests for serial correlation in the residuals of the

[8]We used the following unit root tests: the Augmented Dickey-Fuller (Dickey and Fuller (1979)), the Dickey-Fuller test with GLS detrending (Elliott and *al.* (1996)), the Elliot-Rothenberg-Stock Point Optimal test (Elliott and *al.* (1996)), the Phillips-Perron test (Phillips, P.C.B. and P. Perron (1988)), the KPSS test (Kwiatkowski et *al.* (1992)) and the Ng-Perron test (Ng and Perron.(2001)).

[9]Throughout this paper, we use the Scwharz Information Criterion to select the lag-length of the alternative models.

regression), they ignore important properties of the data which determine the properties of the CER..

First of all, the Lagrange Multiplier (LM) test for autoregressive conditional heteroscedasticity (ARCH) in the residuals fails to accept the null hypothesis of homoscedasticity. In order to accommodate this aspect of interest rate uncertainty, we estimated an $AR(3) - IGARCH(1,1)$ model[10]. The estimation results are reported in Table B.1 of Appendix 1.

However, the strong persistence in the volatility of the estimated GARCH model is an indication of a regime-switching mechanism, as we mentioned previously. Therefore, we employed both the RS and SS models to allow for changes in the generating mechanism of the US rates. In the case of RS, each regime was modelled as an $AR(2)$ process. The SS model was characterized as follows:

$$\begin{aligned} r_t &= \eta + a_t * r_{t-1} + e_t \qquad (10) \\ a_t &= \eta_1 * a_{t-1} + u_t \end{aligned}$$

which allows the autoregressive coefficient of the process (a_t) to be an $AR(1)$ process, which turns out to be a random walk. The parameter estimates for each of these models are presented in Tables C.1 and D.1 of Appendix, 1 respectively.

So far, we have estimated five alternative models for the US interest rates. It is important to compare these models on the basis of the level of uncertainty they allow in the generating mechanism of US rates. MR is the simpler model, since it assumes second-order stationarity and constant parameters. RW increases the level of uncertainty by relaxing the assumption of stationarity. However, it still assumes constant variance

[10]Initially, we estimated an $AR(3) - GARCH(1,1)$ model. However, statistical testing indicated that β_1 and γ_1 sum up to unity.

(homoscedasticity) and constant parameters. The AR-GARCH model allows for time-varying conditional variance (heteroscedasticity). On the other hand, the RS model and the SS model entail a higher degree of uncertainty, since they both allow for time-varying coefficients. The RS model describes a non-stationary process with two different regimes. However, the process is "stationary in each regime". In this regard the autoregressive coefficient of SS changes in each period and as a result allows for the higher level of uncertainty.

3.2 Certainty-equivalent discount rates and discount factors (US)

Having specified five alternative models for the US rates, we estimated the schedule of CER associated with each one from simulations of the discount factor[11]. The discount factors and the certainty-equivalent discount rates of the models described so far are presented in Tables 1 and 2 below[12]. We can see that the models produce certainty-equivalent discount rates with substantial differences in their behavior. The RW model and the AR-GARCH model produce lower rates than MR. For example, the certainty-equivalent discount rates of the RW model and the AR-GARCH model fall from 4% to 1.1% and 1.6% after 200 years, respectively. As a result, the discount factors produced by the RW model and the AR-GARCH model are substantially greater than those produced by the mean-reverting model. For example, at the end of the 400-year forecast period, the discount factor of RW is 169 times greater than that of MR. To a great extent this reflects the importance of persistence as a determinant of declining discount rates.

{INSERT TABLE 1 AND 2 HERE: 1: Discount Factors , 2: CERs}

The discount factors and the certainty-equivalent discount rates of the RS model and the SS model are presented in Tables 1 and 2. Initially, the certainty-equivalent discount

[11] The design of the simulation is similar to that of N&P and it is explained in detail in Appendix 4.

[12] An initial value of 4 percent is used in the simulation of the future path of the US interest rates.

rate of RS is substantially greater than that of SS. However, the certainty-equivalent discount rate of RS becomes smaller than that of SS during the last 90 years. Finally, at the end of the forecast period, the discount factor of SS is about 38 times greater than that of RS.

In summary, the forecasts of the alternative models differ substantially. However, the specification tests show that a model with constant coefficients is not able to fully capture the dynamics of the U.S. interest rates over the period examined. Given that we believe that the past is informative about the future, it is important to characterize the past as well as possible. The RS and SS model are properly designed to account for changes over time in the generation mechanism of the interest rates and therefore these two models seem eminently preferable.

3.3 Model Selection

In this subsection, we mainly focus on RS and SS, since we believe that these two models are preferable to the other three models. In addition to calculating the expected discount factor $E(P_t)$ the simulations generated various measures of the empirical distribution of P_t such as the standard deviation and the empirical percentiles of the simulated P_t[13]. These properties of the empirical distributions serve as the basis for the evaluation of and selection among our models, as it is desirable to have not only the "correct" discount factor, but also the one with the minimum deviation. Models with lower coefficients of variation and tighter 5th and 95th quantiles, represent more reliable forecasts. This is especially important for the evaluation of the distant future. We compare all the models on this basis and the results are summarized as follows:

(i) The SS model provides the highest CER for the extreme long-run, i.e. for periods beyond 350 years. On the other hand, the RS model provides the highest CER over the

[13] We calculate the following percentiles: 1%, 2.5%, 5%, 10%, 50%, 90%, 95%, 97.5% and 99%.

first 200 year horizon.

(ii) The RS model exhibits a tighter band between the 5th and 95th percentile than the SS model. Figures B and C in Appendix 3 depict this behavior for the RS and the SS model, respectively.

(iii) On the other hand, judging by the coefficient of variation, the SS model is the one with the lower coefficient of variation suggesting the lower deviation from the mean as a proportion of the mean. Figure A in Appendix 3 shows the relative performance of the models employed.

Evaluating the forecasting performance of these two models for the long run is impossible due to limitation of data, as forward rates exist for a maximum period of 30 years. Next, we attempt to discriminate between these models (along with the remaining three models) on the grounds of their forecasting performance over a 30-year horizon using available real data. We specifically make use of annual nominal forward rates suggested by the term structure of the US government bonds. As a measure of inflation expectations, we extract the implied inflation rate from inflation-indexed US government bonds of similar maturity dates. Then, we calculate the commonly-used Mean Square Error (MSE) and judge the models by this criterion. Alternatively, we calculate four modified MSE criteria by incorporating four kernels[14] which weigh observations by their relevant proximity to the present. The results are presented in Table 3.

{INSERT TABLE 3: Average MSEs: US}

Interestingly, the various specifications of the MSE criterion unanimously rank the SS model first followed by RS model in most of the cases. The AR-GARCH model ranks third followed by MR and then RW. The ranking of the models according to the MSE

[14] The Bartlett, the Parzen, the quadratic-spectral (QS) and the Tukey-Hanning (TK) kernels are the weighting functions used in our evaluation.

criterion is inversely related with the uncertainty notion as incorporated in our models, with the most 'uncertain' model being the best and the most 'certain' being the worst.

In sum, if we select the models on the basis of their ability to characterize the past and their accuracy concerning forecasts of the future we are inclined to accept the SS model for the US case. Our second best choice would be the RS model.

4 Empirical Results for the UK

4.1 Data

To estimate the model of interest rate behavior, we compiled a series of real market interest rates over the two-century period 1800 to 2001. The nominal interest rate used is the United Kingdom 2 1/2% Consol Yield, while inflation is calculated by the annual change in the Consumer Price Index[15]. Our choice of interest rate is limited by the availability of data as well as our desire for the longest time series available. Based on these nominal rates, we calculate real rates by subtracting the 10-year moving average inflation rate, so as to smooth short-term price fluctuations. However, even this technique leads to negative real rates for specific years due to mainly extreme events, such as oil crises or wars. In order to make our model invariant to these economic crises, which affect interest rates temporarily, we estimated the crisis-induced level of inflation by including a dummy in a small model for the inflation rate. The estimated extra-level of inflation is then subtracted from the inflation in the periods of crises and our series of positive real rates is obtained. We then convert these rates to their continuously compounded equivalents. We estimate our models, employed in the simulation of the interest rate, using a 3-year moving average of the real interest rate series to smooth very short-term fluctuations. Moreover, due to the fact that our models employed in the simulation of

[15]Data provided by the Global Financial Data, Inc, available at http://www.globalfindata.com.

the interest rates do not rule out the possibility of persistent negative discount rates, we use the natural logarithms of the series in the estimation procedure.

Regarding the estimation of multivariate model (9), the US real interest rate employed was calculated in a similar mode. The nominal interest rate used is the United States 10-year Bond Constant Maturity Yield, while inflation is calculated by the annual change in the Consumer Price Index[16]. Further calculations were exactly the same as in the UK case, in order to ensure a comparable series used in the estimation of the VAR model.

4.2 Estimation Results

Similarly to the US case, we used a simple AR model as our starting point and undertook specification testing. Once again, this process generated a natural progression away from the simple $AR(p)$ models towards models that incorporate time-varying coefficients. However, in contrast to the US context, the unit root tests revealed the absence of a unit root in the UK interest rate series[17]. As a result, we only estimate a mean-reverting $AR(4)$ model, since the estimation of a Random Walk model is not justified. The parameter estimates are reported in Table B.2 of Appendix 2. The sum of the autoregressive coefficients is 0.85, substantially less than unity. As well as being in contrast to the findings of N&P for the US, this effectively reduces the extent of uncertainty in interest rates and will reduce the extent of the decline in CERs over time.

The Langrange Multiplier (LM) test for autoregressive conditional heteroscedasticity suggests that heteroscedasticity is present in the residuals of the $AR(4)$ model. This suggests that more efficient estimates would be obtained by an $AR(p) - GARCH(l, m)$ model. We estimate an $AR(4) - GARCH(1, 1)$ model. However, the sum of the GARCH coefficients[18] is substantially greater than unity ($\beta_1 + \gamma_1 \cong 1.20$), i.e. the conditional

[16]Data provided by the Global Financial Data, Inc, available at http://www.globalfindata.com.

[17]Table A.2 of Appendix 2 provides details on the unit root tests conducted.

[18]Estimation results are not reported because it seems that the AR-GARCH model is inappropriate to

variance process is explosive. The estimation results lead us to the estimation of a regime-switching model. We estimate the RS model given in (6), where $p = 2$, that is, each regime is an $AR(2)$ mean-reverting process.

The estimation results for the RS model are presented in Table C.2 of Appendix 2. The probability of changing regime while being in the first regime is estimated at 23.3 %. The probability of changing regime falls to 6.8 % when the process is in the second regime. Furthermore, the first regime is more volatile than the second as indicated by the higher variance of the error term, while less persistent as indicated by the sum of the autoregressive coefficients. In addition, the estimated values of the constant and autoregressive terms indicate that the mean of the process in the two regimes varies. Overall, the estimates of this model suggest that periods of low interest rates are quickly mean-reverting, surrounded by greater uncertainty and transit more often to periods of high interest rates which are more persistent and less uncertain.

As an alternative approach to modelling changes in the data generating mechanism, we estimate a SS model identical to that used for the US data. Lastly we estimate a VAR model to account for any interactions between the US and the UK rates. The estimation results for these two models are presented in Table D.2 and E.2, respectively. Table D.2 shows that the state process is highly persistent, almost a random-walk process, as indicated by the estimate of the autoregressive coefficient. Having estimated four alternative models, we simulate and compare the CER for the UK.

4.3 Certainty-equivalent discount rates and discount factors (UK)

We now simulate 100.000 possible future discount rate paths for each model starting in 2002 and extending 400 years into the future. For each model presented and estimated in the previous section the simulations are based on the estimates presented in Tables $B.2$

describe the UK interest rates (as indicated by the estimates).

to $E.2$. Initial values for any lags of the real interest rate necessary for the simulation are set at 3.5 per cent, the rate used for CBA by the UK Treasury (HM Treasury 2003). The simulation design varies considerably with the model used, and the process of picking parameters and shocks is discussed separately for each model in Appendix 4. Moreover, we calculate the certainty-equivalent discount rate employing a discrete approximation to equation (2).

The simulated expected discount factors for the mean reverting AR(4) model are presented in the first column of Table 4 for a time horizon of 400 years, together with a column of discount factors based on a constant rate of 3.5 percent[19]. The discount factor for the $AR(4)$ model halves in first 20 years and falls to less than 10 % of the initial value in the first 80 years. Compared to the constant discounting model, the $AR(4)$ model discount factor is three times higher in the first three quarters of our forecasting horizon, and 22 times higher after 400 years. The certainty-equivalent discount rate is, with the exception of the first 80 years, consistently lower than the constant rate of 3.5 per cent, falling to 0.39% after 400 years. The simulated discount factors of the GARCH model are not reported as the explosive conditional variance yields counter-intuitive results. However, as described above both the mean-reverting model and the GARCH model suffer from estimation problems.

The discount factors for the RS model, reported in Table 4, are comparable to those of the AR(4) model especially during the first 200 years. However, during the second half, the discount factors are lower, leading to a higher terminal value for the discount rate of 2.1 per cent compared with a value of 0.39 % for the AR(4) model.

The SS model is the only one for which the discount factors remain of significant magnitude until the end of the 400-year period. Compared to the constant-discounting model, this model yields increasingly higher valuations, which reach almost 1.500 times

[19] 3.5 percent is the the rate used for CBA by the UK Treasury.

the constant valuation by the end of the period. The SS CER falls relatively slowly from 2.2 per cent in the first 20 years to 1.4 per cent at the period-end.

The expected discount factors of the VAR model are 390 times higher at the end of the 400-year period than those derived by the constant discount rate and the CER declines faster than the other models from 3.5 per cent to 0.35 per cent at the end of the period. The associated discount rates are shown in Table 5 for the UK case.

{INSERT TABLE 4 and 5 HERE: 4: Discount Factors and 5: CERs}

In summary, our main findings are as follows:

(i) Regarding the discount factors, the SS model gives the higher ones followed by the RS, while the lower ones are given by the AR and the VAR model. In any case, these differences are more pronounced during the first half of the forecast horizon. Only the SS and the VAR model sustain some value in the distant future (400 years). Specifically, the SS discount factor 400 years in the future is 0.0016 and 0.00041 for the VAR model.

(ii) Naturally, the certainty-equivalent discount rates implicit in the discount factors simulated reveal the opposite picture. The model that yields the higher rates during the first half of the sample is the AR(4), while during the second half the RS model yields the higher rates. On the other hand, the SS fluctuates in the range of 2.2 to 1.4 per cent. The terminal rates (i.e. after 400 years) range from 0.35 to 2.1 per cent for the VAR and the SS model, respectively.

4.4 Model Selection

The estimation procedure revealed that among the models employed, the RS and SS models are more appropriate characterisations of the data generating process and best fit the data. The question again arises: how do we select among these models? As above, we do this by reference to the empirical distribution generated by each of the models. For comparison purposes, we will comment on the outcomes of all models. Our main findings are summarized as follows:

(i) A measure of the uncertainty of our projections is the standard deviation of the empirical distribution of every simulated path, which is level dependent, though. In this mode, we evaluate our models by the coefficient of variation (i.e. the ratio of the standard deviation over the mean). Figure D of Appendix 3 displays this measure for all our models and reveals that the model with the lowest coefficient is the SS followed by the VAR model , whereas the AR(4) model yields the highest coefficient.

(ii) Alternatively, as a measure of uncertainty, we employ the 5% and 95% empirical percentiles. Figures E and F of Appendix 3 show these percentiles for RS and SS, respectively. This measure seems to favor the RS model, which has the tightest confidence intervals, suggesting that uncertainty over the expected discount factor is considerably reduced. On the other hand, the percentiles of the SS model are relatively wide.

Summing up, our results suggest that long-term forecasting and consequently distant discounting should be carried out by employing a model that can accommodate changes in its structure. Such properties are prevalent in our RS and SS model, which outperform the simple AR(4) model, justifying our preference for them. Of the SS and the RS models, the former has the lowest coefficient of variation and the latter the tightest confidence intervals. Therefore, our preference for either of these models needs to be justified by alternative means.

In this mode, we evaluate the UK models by the alternative MSE criteria described analytically in Section 3.3. The average MSEs for the UK models are presented in Table 6.

{INSERT TABLE 6: Average MSEs: US}

Once again, the various specifications of the MSE criterion unanimously rank the SS model first followed by RS model. The MR model ranks third followed by the bivariate one, justifying our choice for univariate models. The inverse relationship between uncertainty and forecasting performance is valid, once more.

In sum if we select the models on the basis of their ability to characterize the past and their accuracy concerning forecasts of the future we are inclined to accept the SS model for the UK case. Our second best choice would be the RS model.

5 Policy Implications of Model Selection

The foregoing has established the importance of model selection in determining a schedule of declining discount rates for use in CBA. The differences that arise from alternative specifications of the time series process have been revealed and a method for selecting one model above another has been proposed. In this section we highlight the policy implications of declining discount rates and the impact of model misspecification by looking at two case studies relevant to the long-term policy arena. Firstly we follow N&P and consider climate change[20]. We establish the present values of the removal of 1 ton of carbon from the atmosphere, and hence the present value of the benefits of the avoidance of climate change damages for each of the specified models. Secondly, we look at nuclear build in the UK from the perspective of DDRs. This is directly related to the measurement of climate change mitigation above, since nuclear power can benefit from obtaining carbon credits under a system of joint implementation and carbon trading (see Pearce et al. (2003)). The analysis uses the US data in the first case study and the UK data and models in the latter case.

[20] See N&P (2003) for the assumptions concerning the modelling of carbon emissions damages.

5.1 The Value of Carbon Mitigation

Table 7 shows the present value per ton of carbon emissions with respect to the US models described in Section 3.1.

{INSERT TABLE 7 HERE}

The only noticeable difference in values occurs in the case of SS. In this case, the value of carbon emissions reduction is over 150 % larger than that under constant discounting at 4 %. In addition, the RW model values carbon reduction 33.3 % higher than under constant discounting[21]. Similarly, employing the mean reverting model, we find an increase in value of only 12 % compared to the 14% difference noted by N&P under their mean reverting equivalent. The preceding discussion has argued that the RS and SS models are to be preferred over the others since they allow for changes in the interest rate generating process and have desirable efficiency qualities. From the policy perspective we have established that both of these models provide well specified representations of the interest rate series. However, on the one hand the RS model provides roughly equivalent values of carbon to the constant discounting rate values (there is a 9% difference), while on the other the SS produces values up to 150% higher. Comparing the performance of our models to the RW model used by N&P, we find that RW produces larger values of carbon than all models other than the SS model, which exceeds the RW model by about 88.8 %. In our case this represents a 88.8% increase compared to the methodology employed by N&P.

The disparity between the RS and the SS models, and the proximity of the carbon values generated by the former to those generated by conventional constant discounting

[21] The values for the RW model and MR model are nearly bu not exactly the same as those reported by N&P. This is as a result of some of the additional data transformations that we have undertaken and the choice of p for these models.

represents a clear signal of the policy relevance of model selection in determining the CER. It is crucial from a policy perspective to make a clear judgement as to which of the two models is most appropriate to the case in hand. It also highlights the importance of the presence of persistence in this estimation, recalling that the autoregressive process of the SS model parameters was effectively a RW model. In this case we have found that in addition to the lower coefficient of variation, the SS model is also preferable to RS model due to its lower MSE for the 30-year horizon. Hence we suggest it is reasonable to assume that the SS model is preferable in this case. This means that the carbon values are increased by 150% compared to conventional discounting and 88% compared to N&Ps approach.

Given that the value of carbon depends upon model selection for discount rates, it is interesting to examine the implications of this for climate change prevention projects and/or the appraisal of investments in carbon intensive sectors of the economy. For this reason we look at the implications of using the regime switching and state space models in the appraisal of nuclear power investments in the UK.

5.2 The Appraisal of Investments in Nuclear Power

New nuclear build in the UK is still being considered as an option to ensure security of energy supply and adherence to Kyoto targets, and the Performance and Innovation Unit (Performance and Innovation Unit, 2002) recommended that the nuclear option should be kept open. Decommissioning represents a long-term implication of such investments, however the present-value of decommissioning costs is insignificant using conventional discounting. These costs are naturally sensitive to the use of declining discount rates. Following the same cost and price assumptions, and time horizons for construction, operation and decommissioning as Pearce et al. (2003), we compare the NPV of investment in a nuclear power station using the DDRs associated with the state space and regime switching models. Furthermore, following Pearce et al (2003), we investigate the impact of carbon credits given to the nuclear industry based upon the social cost of carbon reflecting the lower intensity of carbon production possible compared to conventional energy. As we have seen above, the use of declining discount rates can improve the relative economics of nuclear generation by raising the social cost of carbon. The implications of these two countervailing effects, and the comparison to conventional constant discounting is presented in Table 8.

{INSERT TABLE 8 HERE}

The aforementioned appraisal shows that although the SS model has significant consequences for the present value of revenues and carbon credits, the present value of decommissioning and operating costs is also increased considerably. Moreover, both the SS and the RS models increase the NPV of the project by more than 8 %. To this extent the present value of nuclear build is affected only marginally by the implementation of these models of declining discount rates.

This case study highlights the limitations of DDRs in accounting for intergenerational equity. There is a tension between benefits and costs that accrue in the far distant future and the use of DDRs raises both of these simultaneously: both carbon credits and decommissioning costs increase since to a large extent they accrue simultaneously. When appraising projects, which have time profiles of costs and benefits of this nature emphasis is perhaps better directed towards a more comprehensive understanding the trade-offs faced intra-temporally, by particular future generations, rather than the inter-temporal trade-off made by the current generation that DDRs address directly[22].

[22] For more on this issue see Horowitz (2002)

6 Conclusions

In response to the need to appraise projects over ever longer time horizons a number of theoretical discussions have arisen concerning the appropriacy of discount rates that fall with the time horizon considered. Such Declining Discount Rates (DDRs) would add greater weight to the costs and benefits that accrue to future generations and thereby at least partially address the issue of inter-generational equity that so often besets the long term policy arena.

Weitzman's (Weitzman 1998) theoretical justification for DDRs depends upon uncertainty of the discount rate and therefore the operationalisation of this theory is highly dependent upon the manner in which one interprets and characterizes uncertainty. Weitzman (2001) suggested that it was the lack of consensus current about the correct discount rate to employ in the far distant future that was the source of uncertainty and his estimated gamma distribution provided the means of operationalising this theory and determining the declining Certainty Equivalent Rate (CER). Newell and Pizer (2003) (N&P) took an alternative view, suggesting that the future is the source of uncertainty and this interpretation lead naturally to an econometric forecasting approach to the measurement of uncertainty and the determination of the CER.

This paper builds on N&Ps approach in determining DDRs and it makes the following points concerning the model selection and the use of DDRs in general. Firstly, N&Ps approach is predicated upon the assumption that the past is informative about the future and therefore characterizing uncertainty in the past can assist us in forecasting the future and determining the path of CERs. We have argued that if one subscribes to this view it is important to characterize the past as well as possible by correctly specifying the model of the time series process. This is particularly so when dealing with lengthy time horizons where the accuracy of forecasts is important. Indeed the selection of the econometric model is of considerable moment in operationalising a theory of DDRs that depends upon uncertainty, because econometric models contain different assumptions concerning the probability distribution of the object of interest. We have shown for US and UK interest rate data that the econometric specification should allow the data generating process to change over time, and that State Space and Regime Switching models are likely to be appropriate. Secondly, selection between well specified models can and should be undertaken by reference to measures of efficiency such as coefficients of variation, confidence bounds and out-of-sample forecast MSEs.

Our estimations, simulations and case studies bear out this assertion. The path of the CER differs considerably from one model to another and therefore each places a different weight upon the future. The policy implications of these estimates is revealed in the estimation of the value of carbon emissions reduction, with values which are up to 150% higher than when using constant discount rates, and up to 88% higher than the Random Walk model employed by N&P.

The assessment of UK nuclear power reveals the limitations of DDRs in accounting for intergenerational equity. The fact that decommissioning costs and the benefits of carbon emissions reductions (for which we assume nuclear power receives credits) both accrue in the distant future means that the use of DDRs does not change the policy prescription: both values are increased by DDRs and the net present value remains negative. This example highlights the importance of the question of valuing static/intra generational as well as intertemporal/intergenerational costs and benefits.

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Table 1. Certainty Equivalent Discount Factors for the US.

Year	4% Constant	N&P (MR)	Random Walk	AR IGARCH	Regime	State space
1	0.96154	0.96154	0.96154	0.96154	0.96154	0.96154
20	0.45639	0.45906	0.45177	0.45876	0.45390	0.56424
40	0.20829	0.21661	0.20917	0.21250	0.19576	0.33136
60	0.09506	0.10471	0.10480	0.10062	0.08458	0.20296
80	0.04338	0.05150	0.05777	0.04894	0.03700	0.12889
100	0.01980	0.02567	0.03482	0.02455	0.01647	0.08408
150	0.00279	0.00476	0.01333	0.00529	0.00238	0.03132
200	0.00039	0.00095	0.00683	0.00178	0.00041	0.01255
250	0.00006	0.00022	0.00419	0.00104	0.00010	0.00526
300	0.00001	0.00006	0.00289	0.00086	0.00003	0.00227
350	0.00000	0.00002	0.00215	0.00080	0.00002	0.00100
400	0.00000	0.00001	0.00169	0.00078	0.00001	0.00044

Table 2. Certainty Equivalent Rates for the US.

Year	N&P(MR)	Random Walk	AR IGARCH	Regime	State space
1	4.00	4.00	4.00	4.00	4.00
20	3.91	4.05	3.96	4.22	2.79
40	3.76	3.76	3.88	4.31	2.59
60	3.65	3.28	3.74	4.26	2.38
80	3.58	2.80	3.60	4.18	2.23
100	3.51	2.37	3.42	4.09	2.10
150	3.36	1.59	2.75	3.79	1.91
200	3.16	1.14	1.62	3.31	1.79
250	2.87	0.85	0.65	2.46	1.72
300	2.43	0.66	0.23	1.83	1.67
350	1.87	0.53	0.09	0.95	1.64
400	1.41	0.44	0.04	0.70	1.61

Table 3. Average MSEs for the US.

Criterion	N&P(MR)	Random Walk	AR IGARCH	Regime	State space
AMSE	2.058	2.171	2.102	2.323	1.832
AMSE (B)	1.692	1.724	1.692	1.687	1.499
AMSE (P)	1.725	1.746	1.720	1.683	1.426
AMSE (QS)	0.842	0.870	0.848	0.879	0.760
AMSE (TH)	1.769	1.797	1.765	1.738	1.550

Table 4. Certainty Equivalent Discount Factors for the UK

Year	3.5% Constant	AR(4)	Regime	State space	VAR
1	0.96618	0.96618	0.96618	0.96618	0.96618
20	0.50257	0.48208	0.51472	0.61857	0.47492
40	0.25257	0.23676	0.26746	0.40678	0.22915
60	0.12693	0.11778	0.13981	0.27722	0.11376
80	0.06379	0.05912	0.07354	0.19368	0.05798
100	0.03206	0.02997	0.03890	0.13775	0.03035
150	0.00574	0.00569	0.00813	0.06172	0.00707
200	0.00103	0.00115	0.00177	0.02882	0.00227
250	0.00018	0.00027	0.00041	0.01379	0.00105
300	0.00003	0.00008	0.00010	0.00669	0.00066
350	0.00001	0.00003	0.00003	0.00328	0.00050
400	0.00000	0.00002	0.00001	0.00161	0.00041

Table 5.Certainty Equivalent Rates for the UK

Year	AR(4)	Regime	State space	VAR
1	3.50	3.50	3.50	3.48
20	3.68	3.35	2.22	3.80
40	3.58	3.31	2.02	3.63
60	3.52	3.28	1.87	3.50
80	3.48	3.25	1.76	3.36
100	3.43	3.22	1.68	3.20
150	3.33	3.14	1.57	2.65
200	3.13	3.05	1.51	1.96
250	2.77	2.93	1.47	1.24
300	2.17	2.75	1.45	0.72
350	1.12	2.45	1.43	0.45
400	0.39	2.14	1.44	0.36

Table 6. Average MSEs for the UK

Criterion	AR(4)	Regime	State space	VAR
AMSE	2.330	1.486	0.195	2.620
AMSE (B)	0.875	0.527	0.135	0.973
AMSE (P)	0.562	0.332	0.132	0.609
AMSE (QS)	0.659	0.407	0.071	0.740
AMSE (TH)	0.818	0.480	0.137	0.905

Table 7. Value of Carbon Damages according to Model Selection (1989$/tonne, Base Year 1995)

Model	Carbon Values ($/tc 400years)	Relative to Constant Rate	Relative to Mean Reverting	Relative to Random Walk
Regime Switch	5.22	-9.0%	-18.7%	-31.7%
Conventional (4.0%)	5.74		-10.7%	-25.0%
AR-IGARCH	6.37	+10.9%	-1.0%	-16.8%
MR	6.43	+12.0%		-16.0%
RW	7.65	+33.3%	+19.0%	
State Space	14.44	+151.7%	+124.7%	+88.8%

Table 8: The Costs and Benefits of Nuclear Build in the UK

(UK pounds/KW)	CAPEX	OPEX	DECOM	Rev/es	C C	NPV	Relative to Flat
3.5% Flat	2173	2336	427	4062	228	-646	—
AR(4)	2167	2245	396	3904	215	-689	-6.6%
Regime Switching	2178	2401	479	4176	249	-633	8.0%
State Space	2196	2973	1126	5170	547	-577	8.9%
VAR	2167	2211	387	3845	215	-705	-22.1%

Appendix 1: US Estimates

Table A.1: Unit Root Tests for the US rates[23]				
US	**Lags [24]/**	**Stat.**	**5%**	**Decision**
TEST	**Bandwidth[25]**		**crit. value**	
ADF	13	-2.314	-2.877	non-stationary
Phillips-Perron	12	-3.251	-2.876	non-stationary
DF-GLS	13	-0.473	-1.942	stationary
ERS Point-Optimal	12	19.733	3.17	non-stationary
Ng-Perron	12	-0.824	-8.100	non-stationary
KPSS	15	1.158	0.463	non-stationary

Table B.1: Estimation results:AR(3)-IGARCH(1,1) model				
Coefficient	Estimate	Std. Error	t-Statistic	p-value
n	1.330	0.104	12.811	0.0000
a_1	1.951	0.085	23.033	0.0000
a_2	-1.322	0.156	-8.472	0.0000
a_3	0.355	0.080	4.441	0.0000
c	8.60E-05	2.66E-05	3.236	0.0012
β	0.442	0.092	4.805	0.0000

[23]The results reported are based on the natural logarithm of the series.

[24]We use SIC to determine the number of lags of the dependent variable in the test specification.

[25]The kernel sum-of covariances estimator with Parzen weights is used. The bandwidth is selected by using the Newey-West bandwidth selection method.

Table C.1 : Estimation results: Regime-switching model				
Coefficient	Estimate	St.Error	t-Statistic	p-value
n_1	1.189	0.128	9.327	0.00
a_1^1	1.589	0.078	20.36	0.00
a_2^1	-0.660	0.086	-7.630	0.00
n_2	1.714	0.238	7.206	0.00
a_1^2	1.787	0.050	35.55	0.00
a_2^2	-0.800	0.049	-16.395	0.00
σ_1^2	0.004	0.0007	5.651	0.00
σ_2^2	0.0003	4.40E-05	6.070	0.00
P	0.867	0.058	14.934	0.00
Q	0.917	0.035	25.976	0.00

Table D.1: Estimation results: State space model				
Coefficient	Estimate	St.Error	t-Statistic	p-value
n	0.510	0.082	6.185	0.00
n_1	0.999	0.002	438.9	0.00
$ln(\sigma_e^2)$	-9.158	1.324	-6.917	0.00
$ln(\sigma_u^2)$	-6.730	0.144	-46.63	0.00

Appendix 2: UK Estimates

Table A.2: Unit Root Tests for the UK interest rates				
UK	**Lags** [26]**/**	**Stat.**	**5%**	**Decision**
TEST	**Bandwidth**[27]		**crit. value**	
ADF	3	-3.189	-2.876	stationary
Phillips-Perron	20	-4.070	-2.876	stationary
DF-GLS	3	-3.186	-1.942	stationary
ERS Point-Optimal	20	0.965	3.164	stationary
Ng-Perron	20	-27.945	-8.100	stationary
KPSS	13	0.0421	0.463	stationary

Table B.2: Estimation results: AR(4) model

Coefficient	Estimate	Std. Error	t-Statistic	Probability
n	1.201	0.177	6.777	0.00
α_1	1.054	0.058	18.165	0.00
α_2	-0.125	0.089	-1.392	0.16
α_3	-0.443	0.070	6.308	0.00
α_4	0.368	0.035	10.452	0.00
σ_ξ^2	0.064	0.005	13.733	0.00

[26] We use SIC to determine the number of lags of the dependent variable in the test specification.

[27] The kernel sum-of covariances estimator with Parzen weights is used. The bandwidth is selected by using the Newey-West bandwidth selection method.

Table C.2: Estimation results: Regime-switching

Coefficient	Estimate	Std. Error	t-Statistic	Prob
n_1	0.760	0.244	3.117	0.002
α_1^1	0.700	0.312	2.249	0.025
α_2^1	-0.212	0.312	-0.679	0.497
n_2	1.306	0.082	15.892	0.000
α_1^2	1.397	0.079	20.573	0.000
a_2^2	-0.530	0.058	-9.094	0.000
σ_1^2	0.219	0.047	4.694	0.000
σ_2^2	0.014	0.002	8.106	0.000
P	0.767	0.101	7.543	0.000
Q	0.933	0.033	28.617	0.000

Table D.2: Estimation results: State space

Coefficient	Estimate	Std. Error	t-Statistic	Prob
n	0.266	0.044	6.091	0.00
n_1	0.999	0.002	438.82	0.00
$ln(\sigma_e^2)$	-2.503	0.104	-24.049	0.00
$ln(\sigma_u^2)$	-6.462	0.594	-10.884	0.00

Table E.2: Estimation results: VAR model

Coefficient	Estimate	Std. Error	t-Statistic
n_1	0.235	0.069	3.387
n_2	0.156	0.045	3.481
α_{11}^1	1.006	0.076	13.204
α_{11}^2	-0.236	0.077	-3.063
α_{12}^1	0.152	0.104	1.462
α_{12}^2	-0.120	0.104	-1.162
α_{21}^1	0.115	0.049	2.335
α_{21}^2	-0.125	0.050	-2.514
α_{22}^1	1.353	0.067	20.096
α_{22}^2	-0.475	0.067	-7.086

Appendix 3: Figures A-F

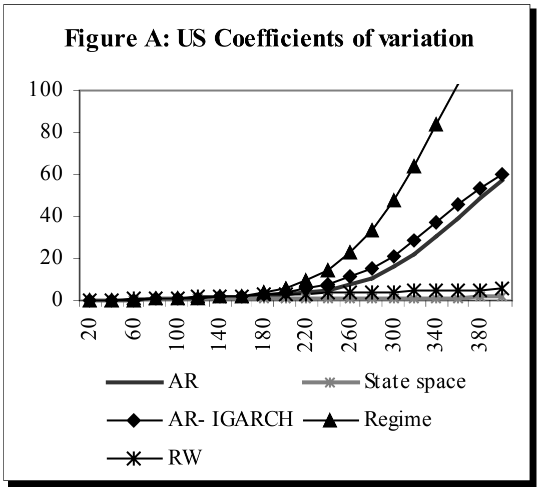

Figure A: US Coefficients of variation

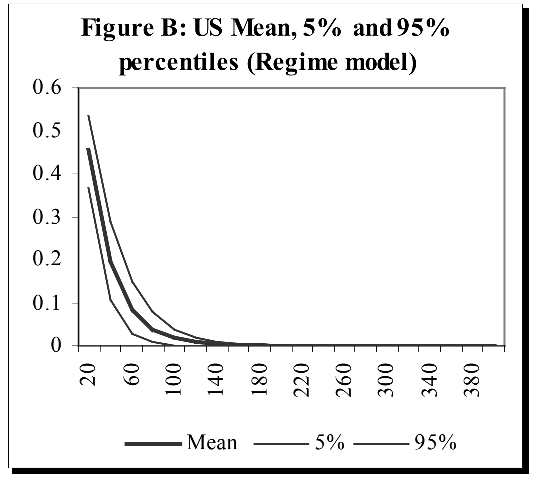

Figure B: US Mean, 5% and 95% percentiles (Regime model)

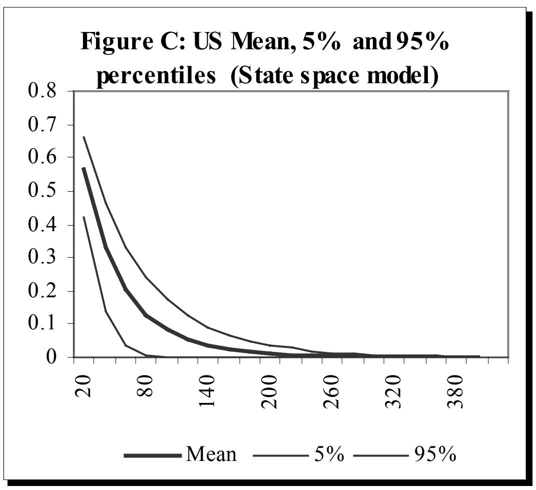

Figure C: US Mean, 5% and 95% percentiles (State space model)

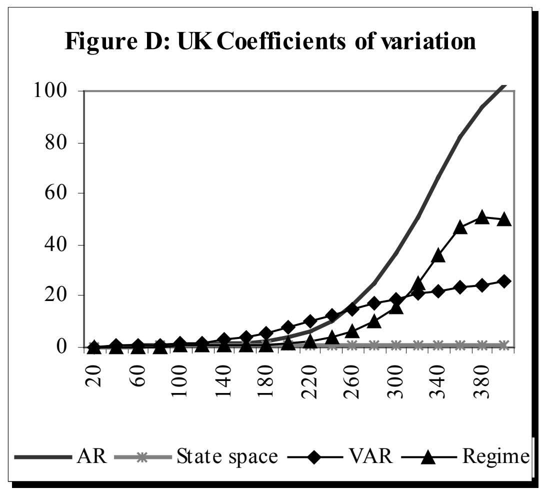

Figure D: UK Coefficients of variation

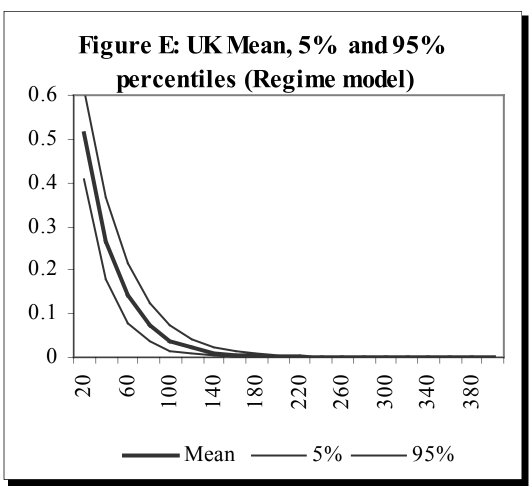

Figure E: UK Mean, 5% and 95% percentiles (Regime model)

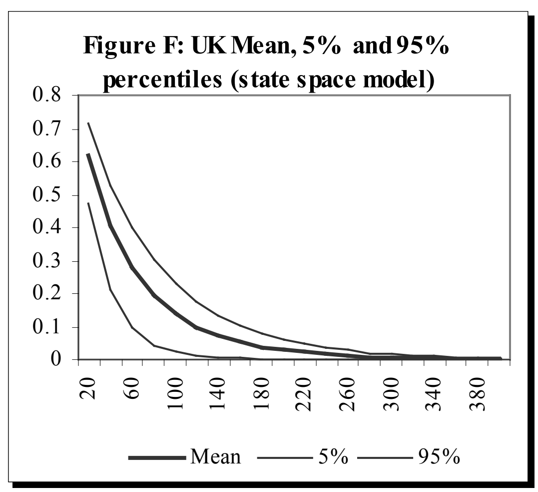

Figure F: UK Mean, 5% and 95% percentiles (state space model)

Appendix 4. Simulation Methodology for each Specification

AR(p) Model: Regarding our first model ($AR(p)$ model), we use the normal distribution to draw random values for the coefficients of (3) taking into account the estimated variance-covariance matrix of the coefficients. Another draw from a normal distribution is employed for the estimated variance.

AR(p)- GARCH (l,m): The simulation methodology is similar to the AR(p) model, except from the fact that the multivariate normal distribution is used to generate random draws for the coefficient values of the GARCH model.

Regime Switching: The RS model offers the most computationally intensive simulation and is conducted as follows. First, we generate random values for the probabilities P and Q from a $Beta(k, j)$ distribution. The values of the parameters k and j of the Beta distribution are properly chosen in order to correspond to a Beta distribution with mean and standard deviation equal to the ones estimated. Specifically, for the US case the parameters k and j are equal to 28.8 and 4.42 for P, respectively. The corresponding values for Q are 55.17 and 5, respectively. Using the values of P and Q, we calculate the probability of being in each regime for each of the future 400 years, namely P_t and Q_t. A univariate normal distribution is used to get random draws for σ_1^2 and σ_2^2 separately according to the estimates presented in Tables C.1 and C.2 for the US and UK case respectively. Similarly to our previous simulations, the random values for the coefficient estimates, n_1, n_2, $a_{1,}^1$ $a_{2,}^1$ a_1^2 and a_2^2 are drawn from a multivariate normal distribution. Then, we simulate the future interest rate path 100.000 times on the grounds of the probabilities P_t and Q_t and the random draws of the coefficients.

State Space: The simulation design for the SS model is straightforward as we randomly draw the coefficient values from univariate normal distributions according to the estimated values.

VAR: The difference between the VAR model and the univariate models is that it demands both UK and US real interest rates to be simulated in the future. The way the simulation is designed follows the line of the previously mentioned experiments.