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The spirit of capitalism, innovation and long-run growth*

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Abstract

We develop an endogenous growth model incorporating the spirit of capitalism and examine how it influences innovation and economic growth. In the benchmark homogeneous-ability model, we find that the spirit of capitalism increases both the capital accumulation rate by enhancing consumer patience and the knowledge accumulation rate by reallocating human capital from the final goods sector to the R&D sector, both of which drive the endogenous growth of the macroeconomy. In the extended heterogeneous-ability model, we identify an additional channel through which the spirit of capitalism boosts innovation and economic growth: by lowering the shreshold ability required to become an entrepreneur and increasing the amount of human capital working in the R&D sector.

Keywords: The spirit of capitalism; innovations; endogenous growth; heterogeneous ability.

JEL Classification Numbers: E1, O3, O4.

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1 Introduction

In this paper, we introduce the "spirit of capitalism" into the Romer (1990) model and examine its effects on capital accumulation, innovation, and long-run economic growth. In the benchmark homogeneous-ability model, we demonstrate that the spirit of capitalism stimulates both the capital accumulation rate and the knowledge accumulation rate, both of which drive the endogenous growth of the macroeconomy. Intuitively, the spirit of capitalism decreases the effective time preference rate, thereby increasing consumer patience. As a result, consumers reduce consumption and increase savings, leading to a higher equilibrium savings rate and faster capital accumulation. However, this accelerated capital accumulation exerts downward pressure on equilibrium interest rates. The lower interest rates, in turn, reduce the discount rate applied to profit flows from intermediate goods production, increasing the equilibrium price of patents. Consequently, the higher real wages for human capital in the R&D sector, given the level of the knowledge stock, attract more human capital to work in this sector, boosting the rate of knowledge accumulation. In summary, the spirit of capitalism stimulates both the capital accumulation rate and the knowledge accumulation rate, reinforcing the endogenous growth of the macroeconomy.

In the extended heterogeneous-ability model, we show that stronger sentiments of the spirit of capitalism lower the shreshold ability required to become an entrepreneur, increase the amount of human capital working in the R&D sector, and thus raise the growth rates of innovation and the overall macroeconomy. Compared to the homogeneous-ability model, the heterogeneous-ability model introduces a new channel through which the spirit of capitalism influences the reallocation of human capital between the final goods sector and the R&D sector.

Our model differs from the existing theoretical literature on the spirit of capitalism and economic growth. In a seminal paper, Kurz (1968) introduces the capital stock into the consumer's utility function (referred to as the "wealth effect") and examines the possibility of multiple steady states in the standard optimal growth model. Since the long-run growth rate is zero in this framework, the spirit of capitalism has no growth effects. Motivated by Weber's views on the spirit of capitalism, Zou (1994) describes capital (or wealth) in the utility function as the spirit of capitalism and shows that it induces a growth effect in an AK growth model, where the endogenous growth rate increases with the degree of the spirit of capitalsm. However, because there is no innovation sector in this model, the specific channel through which the spirit of capitalism affects long-run growth is not explored. In an AK model with the spirit of capitalism, Futagami and Shibata (1998) show that the growth effect of the spirit of capitalism depends on the degree of heterogeneity among consumers. Specifically, if all consumers are idential, the growth effect is positive; whereas if agents are heterogeneous, the growth effect may be negative. In a stochastic growth model with generalized isoelastic preferences, Smith (1999) shows that the growth effect of the spirit of capitalism depends both on consumers' willingness to substitute over time and on the range of investments in the economy. In an AK model driven by technological spillovers, Corneo and Jeanne (2001) show that the long-run growth rate increases with the status-seeking motive and the initial equality of the wealth distribution. He et al. (2022) develop a labor reallocation channel within a quality-enhancing innovation model based on Aghion and Howitt (1992), showing how the spirit of capitalism affects growth. Specifically, the spirit of capitalism lowers the real interest rate and the borrowing costs for entrepreneurs, which increases the return to entrepreneurial investment and shifts labor from manufacuring to R&D, leading to higher long-run growth. Complementing the existing literature, we explore the implications of the spirit of capitalism on innovation and growth in Romer's (1990) variety-expanding technological progress model. In the homogeneous-ability model (our benchmark), we recover the human capital reallocation channel through which the spirit of capitalism fosters variety-expanding innovations and, consequently, endogenous growth. Specifically, we find that the spirit of capital increases both the capital accumulation rate by enhancing consumer patience and the knowledge accumulation rate by reallocating human capital from the final goods sector to the R&D sector. Furthermore, in the heterogeneous-ability model, we identify an additional channel through which the spirit of capitalism boosts innovation and economic growth: by lowering the shreshold ability required to become an entrepreneur and increasing the amount of human capital working in the R&D sector.

The remainder of the paper is organized as follows. Section 2 presents the Romer model with the spirit of capitalism. In Section 3, we derive the balanced growth path. Section 4 examines the implications of the spirit of capitalism on innovation and endogenous growth. In Section 5, we extend the model to account for heterogeneous abilities. Finally, Section 6 concludes the paper.

2 The Romer model with the spirit of capitalism

In this section, we introduce the spirit of capitalism (SOC)—a direct preference for wealth—into the Romer (1990) model and examine the competitive equilibrium of the model economy.

Consumer. The representative consumer's optimization problem is maximizing the following lifetime utility function:

$$U = \int_{t=0}^{\infty} e^{-\rho t} \left[\ln (c_t) + \gamma \ln (k_t) \right] dt, \tag{1}$$

subject to the capital accumulation equation:

$$\dot{k}_t = r_t k_t + w_{ht} h_{yt} + w_{lt} l_t + \int_{i=0}^{a_t} \pi_{it} di - c_t, \tag{2}$$

where c_t is consumption, k_t is capital holdings, r_t is the rental rate of physical capital, h_{yt} is human capital employed in the final good sector, and $l_t = l$ is the constant labor force fully employed in the final good sector. Additionally, w_{ht} and w_{lt} are the wage rates of human capital and labor, respectivly, π_{it} represents the profits earned by intermediate good $i \in [0, a_t]$, and $\int_{i=0}^{a_t} \pi_{it} di$ represents the total profits earned by the intermediate goods sector. The variable a_t denotes the number of intermediate goods types (or the stock of knowledge), and $\rho\,(>0)$ is the time discount rate. Similar to Zou (1994), we utilize capital (or wealth) in utility to represent the SOC or the desire for social status, where $\gamma \geq 0$ captures the strength of the SOC. A larger γ reflects a stronger preference for wealth or a stronger SOC.[1] This model strategy is motivated by Weber (1958)'s viewpoints on the spirit of capitalism: The essence of the spirit of capitalism is the continual accumulation of wealth for its own sake, rather than only for the material rewards that it can serve to bring.

Application of the Pontryagin maximum principle leads to the Euler equation:

$$\frac{\dot{c}_t}{c_t} = r_t - \rho + \gamma \frac{c_t}{k_t} = r_t - \left(\rho - \gamma \frac{c_t}{k_t}\right), \tag{3}$$

which shows that a new positive term, $\gamma c_t/k_t$, appears in the Euler equation when the consumer has the spirit of capitalism (i.e., $\gamma > 0$). Define $\widetilde{\rho} \equiv \rho - \gamma c_t/k_t$ $(< \rho)$ as the effective time preference rate.[2] The lower effective time preference rate indicates that a consumer with the spirit of capitalism is more patient and places greater value on future consumption than on current consumption. If the consumer has no sentiment for the spirit of capitalism (i.e., $\gamma = 0$), the model reverts to the original Romer (1990) model. To summarize, equations (2) and (3) describe the consumer's optimal behaviors.

Production. The production side of the economy consists of three sectors: a final goods sector, an intermediate goods sector, and a research and development (R&D) sector. The final good sector uses the total labor (l), human capital (h_{yt}), and all the intermediate goods ($\{x_{it}\}_{i=0}^{a_t}$) to produce the final good. Taking the wage rates of labor and human capital $\{w_{lt}, w_{ht}\}$ and the prices of all intermediate goods $\{p_{it}\}_{i=0}^{a_t}$ as given, the representative firm in the final goods sector solves the problem:

$$\max_{\{h_{yt}, l, \{x_{it}\}\}} \left(y_t - \int_{i=0}^{a_t} p_{it} x_{it} di - h_{yt} w_{ht} - w_{lt} l \right),$$

where $y_t = h_{yt}^{\alpha} l^{\beta} \int_{i=0}^{a_t} x_{it}^{1-\alpha-\beta} di$ is the generalized Cobb-Douglas production technology for the final good, $\alpha, \beta, (1-\alpha-\beta) \in (0,1)$ are income shares parameters. The first-order necessary conditions with respect to h_{yt}, l, and x_{it} are:

$$h_{yt} : \alpha \frac{y_t}{h_{yt}} = w_{ht}, \tag{4}$$

[1] We use absolute wealth (or capital) in the utility function to measure the spirit of capitalism. Using relative wealth (such as $k_{it}/\overline{k}_t$, where k_{it} refers to individual wealth and $\overline{k}_t$ is the average wealth in the economy) does not alter the main results, as the log utility treats the average wealth level as constant, which does not affect the optimal decisions.

[2] Alternatively, define $\widetilde{r}_t \equiv r_t + \gamma c_t/k_t$ as the effective real interest rate. Equation (3) indicates that the SOC increases the real return on capital accumuation, thereby promoting greater capital accumulation.

$$l : \beta \frac{y_t}{l} = w_{lt}, \tag{5}$$

$$x_{it} : h_{yt}^{\alpha} l^{\beta} (1 - \alpha - \beta) x_{it}^{-\alpha-\beta} = p_{it}. \tag{6}$$

Equations (4)-(6) show that the prices of these three production factors equal their respective marginal productivities. Due to the constant returns-to-scale property of the Cobb-Douglas production function, the optimal profits of the representative firm in the final goods sector are zero.

In the intermediate goods sector, firm i purchases the patent for intermediate good i (as a fixed cost) in the competitive patents market and becomes a monopolistic firm. It then rents capital (as a variable cost) to produce intermediate good i. Because the patents market is competitive, the price of each patent (p_{at}^i) will be bid up until it equals the present value of the net revenue that monopolistic firm i can extract. Therefore, at each date t, it must hold that:

$$p_{at}^i = \int_{\tau=t}^{\infty} e^{-\int_{s=t}^{\tau} r_s ds} \pi_{i\tau} d\tau. \tag{7}$$

Taking the demand function in equation (6) as given, firm i, having already incurred the fixed-cost investment in the patent, will choose a level of output x_{it} to maximize its revenue minus variable cost at each date. Specifically, the firm solves:

$$\pi_{it} = \max_{\{p_{it}, x_{it}\}} p_{it} x_{it} - r_t \eta x_{it} = \max_{\{x_{it}\}} h_{yt}^{\alpha} l^{\beta} (1 - \alpha - \beta) x_{it}^{1-\alpha-\beta} - r_t \eta x_{it}.$$

In order to produce one unit of any intermediate good, firm i rents η units of capital (foregone consumption) from consumers and pays interest cost of $r_t \eta$ units of output (as the marginal cost). Solving the first-order condition with respect to x_{it} and combining it with the demand curve in (6) lead to the (symmetric) monopolistic pricing rule:

$$p_{it} = \frac{1}{1 - \alpha - \beta} r_t \eta \equiv p_t, \tag{8}$$

where $r_t \eta$ is the marginal cost of producing an additional unit of any intermediate good, and $1/(1 - \alpha - \beta) (> 1)$ is the markup over marginal cost. To earn monopoly profits, all monopolistic firms price their products above marginal costs. Due to the symmetric nature of intermediate goods entering the final good production function and the symmetric pricing in the intermediate goods sector, each intermediate firm i produces the same quantity of intermediate goods and thus earns the same level of monopoly profits at each date t, namely:

$$x_{it} = \left(\frac{h_{yt}^{\alpha} l^{\beta} (1 - \alpha - \beta)}{p_t} \right)^{\frac{1}{\alpha+\beta}} = x_t, \pi_{it} = (\alpha + \beta) p_t x_t = \pi_t. \tag{9}$$

The research and development (R&D) sector uses human capital h_{at} and the existing stock of knowledge a_t to produce new knowledge, with the following knowledge production function:

$$\dot{a}_t = \delta h_{at} a_t = \delta (h - h_{yt}) a_t, \tag{10}$$

where $\delta\,(>0)$ is the productivity parameter. This function shows that devoting more human capital to R&D leads to a higher production rate of new designs, and the larger the total stock of designs, the higher the productivity of researchers in the R&D sector will be.

The condition determining the allocation of human capital between the final goods and R&D sectors is that the wages paid to human capital in each sector must be equal. Specifically,

$$\alpha h_{yt}^{\alpha-1} l^{\beta} x_t^{1-\alpha-\beta} a_t = w_{ht} = p_{at}\delta a_t. \tag{11}$$

Equilibrium. At any date t, there are six types of goods and/or factors markets: intermediate goods, patents (or knowledge), labor force, human capital, physical capital, and the final good. At any date t, existing patents are purchased by intermediate good producers, and all intermediate goods are inputs to the final good sector. Hence, the markets for patents and intermediate goods clear. The total labor force is employed in the final good sector (by assumption). The market-clearing condition for human capital is:

$$h_{yt} + h_{at} = h. \tag{12}$$

Since it takes η units of foregone consumption to create one unit of any intermediate good, the equilibrium condition for the physical capital market is:

$$k_t = \int_{i=0}^{a_t} \eta x_{it} di = \eta x_t a_t. \tag{13}$$

Finally, because there is no depreciation, the market-clearing for the final goods sector requires:

$$c_t + \dot{k}_t = y_t. \tag{14}$$

Equations (2)-(14) describe the monopolistic competitive equilibrium of the model economy. We summarize the equilirium in the following

Definition 1 *A monopolistic competitive equibrium for the economy consists of allocation sequences* $(c_t, k_t, y_t, l_t, h_{yt}, h_{at}, a_t, \{x_{it}\})$ *and price sequences* $\left(w_{ht}, w_{lt}, r_t, \left\{p_{it}, p_{at}^i\right\}\right)$*, satisfying the following conditions: (1) Consumer optimization: Given prices, consumers choose consumption and accumulate physical capital to maximize their utility (1), subject to the budget constraint (2); (2) Final goods and intermediate goods production: Given prices, final goods producers maximize profits. Each intermediate goods producer, given the demand for its product, purchases a patent from the competitive patents market and rents capital from consumers to maximize profits; (3) R&D sector: The R&D sector uses human capital and the existing knowledge stock to create new knowledge; (4) Market clearing: all markets (for labor, human capital, physical capital, intermediate goods, patents, and the final good) clear.*

3 Balanced growth path

In this section, we derive the balanced growth path of the model economy. By substituting the monopolistic pricing rule (8), the demand function for any intermediate good (6), and the market-clearing condition for physical capital (13) into the Euler equation (3), we obtain:

$$\frac{\dot{c}_t}{c_t} = (1-\alpha-\beta)^2 \eta^{\alpha+\beta-1} h_{yt}^{\alpha} l^{\beta} k_t^{-(\alpha+\beta)} a_t^{-(\alpha+\beta)} - \rho + \gamma \frac{c_t}{k_t}. \tag{15}$$

Substituting equation (13) and the production function of the final good into the market-clearing condition for the final good (14) leads to:

$$\frac{\dot{k}_t}{k_t} = \eta^{\alpha+\beta-1} h_{yt}^{\alpha} l^{\beta} k_t^{-(\alpha+\beta)} a_t^{-(\alpha+\beta)} - \frac{c_t}{k_t}. \tag{16}$$

Putting equation (13) in equation (11) and taking the logarithmic derivative of both sides with respect to t give us:

$$\frac{\dot{p}_{at}}{p_{at}} = (\alpha+\beta-1)\frac{\dot{a}_t}{a_t} + (\alpha-1)\frac{\dot{h}_{yt}}{h_{yt}} + (1-\alpha-\beta)\frac{\dot{k}_t}{k_t}. \tag{17}$$

Taking the time derivative of both sides of equation (7) yields:

$$\dot{p}_{at} = r_t p_{at} - \pi_t. \tag{18}$$

Substituting equations (8), (9), (11), and (13) into equation (18) leads to:

$$\frac{\dot{p}_{at}}{p_{at}} = (1-\alpha-\beta)^2 \eta^{\alpha+\beta-1} h_{yt}^{\alpha} l^{\beta} k_t^{-(\alpha+\beta)} a_t^{-(\alpha+\beta)} - \frac{\delta}{\Lambda} h_{yt}, \tag{19}$$

where $\Lambda \equiv \alpha/(\alpha+\beta)(1-\alpha-\beta)$.

Combining equations (17) and (19) and using equations (10) and (16), we obtain:

$$\frac{\dot{h}_{yt}}{h_{yt}} = \left\{ \begin{array}{c} \frac{\delta\Lambda(1-\alpha-\beta)+\delta}{\Lambda(1-\alpha)} h_{yt} - \frac{(1-\alpha-\beta)}{(1-\alpha)}\frac{c_t}{k_t} - \frac{(1-\alpha-\beta)}{(1-\alpha)}\delta h \\ + \frac{(\alpha+\beta)(1-\alpha-\beta)}{(1-\alpha)} \eta^{\alpha+\beta-1} h_{yt}^{\alpha} l^{\beta} k_t^{-(\alpha+\beta)} a_t^{-(\alpha+\beta)} \end{array} \right\}. \tag{20}$$

Thus, the competitive equilibrium of the model economy is characterized by four differential equations (10), (15), (16), and (20) with respect to two controls (c_t, h_{yt}) and two states (a_t, k_t). By defining $z_t \equiv \eta^{(1-\alpha-\beta)/(\alpha+\beta)} k_t/a_t$ and $q_t \equiv c_t/k_t$, we transform the four-dimensional system about (c_t, h_{yt}, a_t, k_t) into the following three-dimensional system about (z_t, h_{yt}, q_t):

$$\frac{\dot{z}_t}{z_t} = z_t^{-(\alpha+\beta)} h_{yt}^{\alpha} l^{\beta} - q_t - \delta(h - h_{yt}), \tag{21}$$

$$\frac{\dot{h}_{yt}}{h_{yt}} = \left\{ \begin{array}{c} \frac{\delta\Lambda(1-\alpha-\beta)+\delta}{\Lambda(1-\alpha)} h_{yt} - \frac{(1-\alpha-\beta)}{(1-\alpha)} q_t - \frac{(1-\alpha-\beta)}{(1-\alpha)}\delta h \\ + \frac{(\alpha+\beta)(1-\alpha-\beta)}{(1-\alpha)} h_{yt}^{\alpha} l^{\beta} z_t^{-(\alpha+\beta)} \end{array} \right\}, \tag{22}$$

$$\frac{\dot{q}_t}{q_t} = \left[(1-\alpha-\beta)^2 - 1\right] h_{yt}^{\alpha} l^{\beta} z_t^{-(\alpha+\beta)} - \rho + (\gamma+1) q_t, \tag{23}$$

in which z_t is a state variable and h_{yt} and q_t are the two control variables. The two systems have the same balanced growth path and transitional dynamics. Thus, we have the following[3]:

Proposition 1 *The steady state $\left(z^*, h_y^*, q^*\right)$ of the three-dimensional system (21)-(23) exists uniquely and is saddle-point stable, and it is solved as follows:*

$$\left(\left(h_y^{*\alpha} l^{\beta} \left(q^* + \delta\left(h - h_y^*\right)\right)^{-1}\right)^{\frac{1}{\alpha+\beta}}, \frac{\Lambda}{\delta} r^*, \frac{(1-\alpha-\beta)^{-2}\left(\rho + \delta h + (\rho\Lambda - \delta h)(1-\alpha-\beta)^2\right)}{\left[(1+\Lambda) + \gamma\left(\Lambda + (1-\alpha-\beta)^{-2}\right)\right]}\right).$$

In the steady state, some endogenous variables ($r_t, h_{yt}, h_{at}, p_t, x_t, \pi_t, p_{at}$) are constant, namely,

$$\left(r^*, h_y^*, h_a^*, p^*, x^*, \pi^*, p_a^*\right) = \left(\begin{array}{c} \frac{\rho+(1+\gamma)\delta h}{\left(1+\gamma(1-\alpha-\beta)^{-2}\right)+(1+\gamma)\Lambda}, \frac{\Lambda}{\delta} r^*, h - h_y^*, \frac{r^*\eta}{(1-\alpha-\beta)}, \\ \left(h_y^{*\alpha} l^{\beta} (1-\alpha-\beta)/p^*\right)^{\frac{1}{\alpha+\beta}}, (\alpha+\beta) p^* x^*, \frac{\pi^*}{r^*} \end{array}\right), \tag{24}$$

other endogenous variables (c_t, k_t, a_t) have the same growth rate:

$$g_i^* = g^* = \frac{\delta h - \rho\Lambda + \delta h\gamma(1-\alpha-\beta)^{-2}}{1 + \Lambda + \gamma\left(\Lambda + (1-\alpha-\beta)^{-2}\right)}, i = c, k, a, \tag{25}$$

which implies that the economy is on the balanced growth path.

4 Implications for innovation and growth

Now we discuss the equilibrium implications of the spirit of capitalism on savings and capital accumulation, human capital reallocation, innovations, and economic growth. It is shown that the spirit of capitalism creates a new driving force for savings and ultimately generates a new channel (i.e., human capital reallocations) to stimulate innovation and economic growth.

Proposition 2 *In the Romer model with the spirit of capitalism, the spirit of capitalism creates a new driving force of long-run economic growth, i.e., $g^*_{\gamma>0} - g^*_{\gamma=0} > 0$. Furthermore, the stronger the spirit of capitalism, the higher the rate of economic growth, i.e., $\partial g^*/\partial\gamma > 0$.*

As shown in Section 2, the spirit of capitalism decreases the effective time preference rate (i.e., $\widetilde{\rho} - \rho = -\gamma c_t/k_t < 0$), which creates a new savings motive, "saving for its own sake" (i.e., $s^*_{\gamma>0} - s^*_{\gamma=0} > 0$). This new savings motive induces consumers to consume less and save more. In the long run, the equilibrium rate of capital accumulation is higher (i.e., $g^*_{\gamma>0} - g^*_{\gamma=0} > 0$).

[3] The proofs of all propositions in the paper are provided in the online appendix. The proof of Corollary 1 is straightforward.

Furthermore, the endogenous growth rate increases with the degree of the spirit of capitalism. The intuitions are as follows: on one hand, with higher degrees of the spirit of capitalism, consumers become more patient (i.e., $\partial\widetilde{\rho}/\partial\gamma < 0$), leading them to consume less and save more. Thus, the equilibrium savings rate increases (i.e., $\partial s^*/\partial\gamma > 0$), and capital accumulates more quickly (i.e., $\partial g_k^*/\partial\gamma > 0$). On the other hand, the higher rates of capital accumulation depress the equilibrium interest rates (i.e., $\partial r^*/\partial\gamma < 0$). The lower interest rates reduce the discount rate of the profit flow from the production of intermediate goods and, hence, increase the equilibrium prices of the patents (i.e., $\partial p_a^*/\partial\gamma > 0$).[4] Given the level of the knowledge stock (a), real wage rates of human capital employed in the R&D sector will be higher (i.e., $\delta a p_a^*$ increases), which induces more human capital to work in the R&D sector and thus increases the knowledge accumulation rate (i.e, $\partial h_a^*/\partial\gamma > 0$, $\partial g_a^*/\partial\gamma > 0$). Altogether, the spirit of capitalism stimulates the capital accumulation rate and the knowedge innovation rate, both of which enforce endogenous growth of the macroeconomy.

5 Heterogeneous ability

In this section, we introduce heterogeneity in human capital within our model by following Jaimovich and Rebelo (2016). We show that the spirit of capitalism lowers the upper bound of the required abilities of human capital employed in the R&D sector, increases the number of human capital workers in that sector, and enforces technological progress.

In this heterogeneous-ability setting, the representative consumer formulation from the homogeneous-ability model is no longer applicable. We now denote l as the number of consumers and h as the amount of human capital in the economy. Each consumer possesses h/l (units of) human capital with a different entrepreneurial ability n, which follows a Pareto distribution: $\Gamma(n) = 1 - \left(\frac{\underline{n}}{n}\right)^{\kappa}$, $\kappa > 1$. This assumption is consistent with the right tail of the U.S. income distribution (Diamond and Saez, 2011), the right skewness in the cross-sectional distribution of profits from innovation (Scherer, 1998; Grabowski, 2002), and the returns to entrepreneurship (Moskowitz and Vissing-Jorgensen, 2002). To simplify, we assume that consumers with identical abilities have the same initial stock of knowledge and that all agents have the same level of initial capital holdings. Under these assumptions, the only source of heterogeneity in the economy is the agent's abilities.

The maximization problem of a consumer with human capital ability n is summarized as

[4] The higher degrees of the spirit of capitalism increase/decrease the equilibrium quantity/price of any intermediate good (i.e., $\partial x^*/\partial\gamma > 0$, $\partial p^*/\partial\gamma > 0$), while its effects on the equilibrium profit of any intermediate good is indeterminate (i.e., $\partial\pi^*/\partial\gamma$ is indeterminate). However, the equilibrium effects of the increased spirit of capitalism on the discount rate are so large that its net effects on the equilibrium price of any patent are positive (i.e., $\partial p_a^*/\partial\gamma > 0$).

follows:

$$\max_{\{c_t(n),k_t(n)\}} U(n) = \int_{t=0}^{\infty} e^{-\rho t} \left[\log c_t(n) + \gamma \log k_t(n)\right] dt,$$

subject to the flow budget constraint

$$\dot{k_t}(n) = r_t k_t(n) + w_{lt} + w_{ht} l_t(n) \frac{h}{l} + a_t(n)(1 - l_t(n)) \pi_t - c_t(n), \tag{26}$$

where $k_t(n)$ and $c_t(n)$ represent his capital holdings and consumption at time t; his endowments of labor and human capital are 1 and h/l, respectively; $l_t(n)$ is equal to one if the consumer chooses to work in the final goods sector (as a worker) and zero when working in the R&D sector (as an entrepreneur); and $a_t(n)$ denotes the amount of patents owned by a consumer with entrepreneurial ability n at time t. The first-order necessary conditions for $c_t(n)$ and $k_t(n)$ yield the Euler equation:

$$\frac{\dot{c_t}(n)}{c_t(n)} = r_t - \rho + \gamma \frac{c_t(n)}{k_t(n)}, \tag{27}$$

which is similar to equation (3).

The optimal behaviors of the final goods and intermediate goods sectors are the same as those in the homogenous-ability model, as shown in equations (4)-(6) and (7)-(9), respectively. The law of motion for $a_t(n)$ is given by

$$\dot{a_t}(n) = \delta n a_t \left[1 - l_t(n)\right], \tag{28}$$

which implies that workers receives no new patents (when $l_t(n) = 1$), while innovators increase their stock of patents (when $l_t(n) = 0$). The total stock of current patents a_t is beneficial for any innovator to develop new patents; that is, the larger the value of a_t, the easier it is for any innovator to invent new goods. The number of patents in the economy, a_t, evolves according to

$$\dot{a_t} = \int_{n=\underline{n}}^{\infty} \dot{a_t}(n) l d\Gamma(n) = \delta l a_t \int_{n=n^*}^{\infty} n d\Gamma(n). \tag{29}$$

Similar to the homogeneous-ability model, $l_t(n)$ is determined endogenously by the (modified) no-arbitrage condition of human capital allocation:

$$w_{ht} \frac{h}{l} = \delta a_t n p_{at}, \tag{30}$$

which states that a consumer with ability n earns the same level of income whether he works in the final goods sector or in the R&D sector.

We show in Online Appendix 7.3 that the equilibrium of the economy is characterized by a threshold rule. Agents with ability $n \geq n^*$ always work as entrepreneurs in the R&D sector, while the rest always work in the final goods sector. Thus, the amounts of human capital working in the final goods sector and the R&D sector are

$$h_{yt} = h\Gamma(n^*), h_{at} = h\left[1 - \Gamma(n^*)\right], \tag{31}$$

respectively. The growth rate of the economy, g^*, coincides with the growth rate of a_t, namely,

$$g^* = \frac{1 + (1 - \alpha - \beta)^{-2} \gamma}{1 + \gamma} \frac{\delta l}{\Lambda} n^* \Gamma (n^*) - \frac{\rho}{1 + \gamma}, \tag{32}$$

where the threshold value of n^* is pinned down by the following equation:

$$\delta l \int_{n=n^*}^{\infty} n d\Gamma (n) = \frac{1 + (1 - \alpha - \beta)^{-2} \gamma}{1 + \gamma} \frac{\delta l}{\Lambda} n^* \Gamma (n^*) - \frac{\rho}{1 + \gamma}. \tag{33}$$

Therefore, we have the following

Proposition 3 *In the heterogeneous-ability Romer model with the spirit of capitalism, stronger sentiments of the spirit of capitalism decrease the shreshold value of ability required to work as an entrepreneur (i.e., $\partial n^* / \partial \gamma < 0$), increase the amount of human capital working in the R&D sector (i.e., $\partial h_a^* / \partial \gamma > 0$), and thus raise the growth rates of innovations and the macroeconomy (i.e., $\partial g^* / \partial \gamma > 0$).*

Proposition 3 illustrates that, relative to the homogeneous-ability Romer model, the heterogeneous-ability model creates a new channel through which the spirit of capitalism affects the reallocation of human capital between the final goods sector and the R&D sector. Specifically, stronger sentiments of the spirit of capitalism decrease the shreshold value of ability required to work in the R&D sector, increase the amount of human capital employed in that sector, and improve the equilibrium growth of the knowledge stock and the macroeconomy.

6 Conclusion

In this paper, we introduce the "spirit of capitalism" into the Romer (1990) model and examine its effects on capital accumulation, innovation, and long-run economic growth. In the benchmark model, we demonstrate that the spirit of capitalism stimulates both the capital accumulation rate and the knowledge accumulation rate, both of which drive the endogenous growth of the macroeconomy. In the extended heterogeneous-ability model, we show that stronger sentiments of the spirit of capitalism lower the shreshold ability required to become an entrepreneur, increase the amount of human capital working in the R&D sector, and thus raise the growth rates of innovation and the overall macroeconomy.

References

[1] Aghion, P., and Howitt, P., 1992. A model of growth through creative destruction. *Econometrica* 60, 323-351.

[2] Corneo, G. Jeanne, O., 2001. Status, the distribution of wealth, and growth. *Scandinavian Journal of Economics* 103, 283-293.

[3] Diamond, P., Saez, E., 2011. The case for a progressive tax: from basic research to policy recommendations. *Journal of Economic Perspectives* 25(4), 165-190.

[4] Futagami, K., Shibata, A., 1998. Keeping one stem ahead of the Joneses: status, the distribution of wealth, and long run growth. *Journal of Economic Behavior & Organization* 36, 109-126.

[5] Grabowski, H., 2002. Patents and new product development in the pharmaceutical and biotechnology Industries. In *Proceedings of the 2002 Conference on Exploring the Economics of Biotechnology*, edited by John V. Duca and Mine K. Ycel. Dallas: Fed. Reserve Bank.

[6] He, Q., Luo, Y., Nie, J., and Zou, H., 2022. Money, growth, and welfare in a Schumpterian model with the spirit of capitalism. *Review of Economic Dynamics* 47, 346-372.

[7] Jaimovich, N., Rebelo, S., 2016. Nonlinear effects of taxation on growth. *Journal of Political Economy* 125, 265-291.

[8] Kurz, M., 1968. Optimal economic growth and wealth Effects. *International Economic Review* 9, 348-357.

[9] Moskowitz, T., Vissing-Jorgensen, A., 2002. The returns to entrepreneurial investment: A Private Equity Premium Puzzle? *American Economic Review* 92, 745-778.

[10] Romer, P., 1990. Endogenous technological change. *Journal of Political Economy* 98(5), 71-102.

[11] Scherer, F., 1998. The size distribution of profits from innovation. In "The Economics and Econometrics of Innovation," specical issue, *Annales d'Economie et de Statistique* (49/50), 495-516.

[12] Smith, W., 1999. Risk, the spirit of capitalism and growth: the implications of a preference for capital. *Journal of Macroeconomics* 21, 241-262.

[13] Weber, M., 1958. The Protestant ethic and the spirit of capitalism. Charles Scribner's Sons, New York.

[14] Zou, H., 1994. The spirit of capitalism and long-run growth. *European Journal of Political Economy* 10(2), 279-293.

7 Online Appendix (not for publication)

7.1 Proof of Proposition 1

Proof of Proposition 1. The dynamic system describing the competitive equilibrium is summarized as follows:

$$\frac{\dot{z}_t}{z_t} = z_t^{-(\alpha+\beta)} h_{yt}^{\alpha} l^{\beta} - q_t - \delta\left(h - h_{yt}\right), \tag{34}$$

$$\frac{\dot{h}_{yt}}{h_{yt}} = \left\{ \begin{array}{c} \frac{\delta\Lambda(1-\alpha-\beta)+\delta}{\Lambda(1-\alpha)} h_{yt} - \frac{(1-\alpha-\beta)}{(1-\alpha)} q_t - \frac{(1-\alpha-\beta)}{(1-\alpha)} \delta h \\ + \frac{(\alpha+\beta)(1-\alpha-\beta)}{(1-\alpha)} h_{yt}^{\alpha} l^{\beta} z_t^{-(\alpha+\beta)} \end{array} \right\}, \tag{35}$$

$$\frac{\dot{q}_t}{q_t} = \left[(1-\alpha-\beta)^2 - 1\right] h_{yt}^{\alpha} l^{\beta} y_t^{-(\alpha+\beta)} - \rho + (\gamma+1)\, q_t, \tag{36}$$

where $z_t \equiv \eta^{(1-\alpha-\beta)/(\alpha+\beta)} k_t/a_t$ is a state variable, h_{yt} and $q_t \equiv c_t/k_t$ are two control variables. Setting $\dot{z}_t/z_t = \dot{h}_{yt}/h_{yt} = \dot{q}_t/q_t = 0$ in the equations (34), (35), and (36), we obtain three algebraic equations about the steady state $\left(z^*, h_y^*, q^*\right)$:

$$z^{*-(\alpha+\beta)} h_y^{*\alpha} l^{\beta} = q + \delta\left(h - h_y^*\right), \tag{37}$$

$$\frac{\delta\Lambda\left(1-\alpha-\beta\right)+\delta}{\Lambda\left(1-\alpha\right)} h_y^* + \frac{\left(\alpha+\beta\right)\left(1-\alpha-\beta\right)}{\left(1-\alpha\right)} h_y^{*\alpha} l^{\beta} {z^*}^{-(\alpha+\beta)} = \frac{\left(1-\alpha-\beta\right)}{\left(1-\alpha\right)}\left(q+\delta h\right), \tag{38}$$

$$\left[(1-\alpha-\beta)^2 - 1\right] h_y^{*\alpha} l^{\beta} z^{*-(\alpha+\beta)} = \rho - (\gamma+1)\, q^*. \tag{39}$$

Substituting equation (37) into equations (38) and (39), and rearranging the results, we have

$$q^* = -\delta h + \frac{\delta}{(1-\alpha-\beta)}\left[1 + \frac{(1-\alpha)(\alpha+\beta)}{\alpha}\right] h_y^*, \tag{40}$$

$$q^* = \frac{1-(1-\alpha-\beta)^2}{\gamma + (1-\alpha-\beta)^2}\left[\delta\left(h - h_y^*\right) + \frac{\rho}{1-(1-\alpha-\beta)^2}\right]. \tag{41}$$

Combining equations (40) and (41), we solve for

$$h_y^* = \frac{\Lambda}{\delta} \frac{\rho + \delta h\,(1+\gamma)}{1 + \Lambda + \gamma\left[\Lambda + (1-\alpha-\beta)^{-2}\right]}, \tag{42}$$

$$q^* = \frac{\rho + \delta h + (\rho\Lambda - \delta h)\,(1-\alpha-\beta)^2}{(1-\alpha-\beta)^2\left[1 + \Lambda + \gamma\left(\Lambda + (1-\alpha-\beta)^{-2}\right)\right]}. \tag{43}$$

Putting the steady-state values of h_y^* and q^* in (37) leads to the steady-state value of z^* in Proposition 1. Thus, we have established the existence of the unique steady state.

Putting (42) in $\dot{a}_t = \delta\left(h - h_{yt}\right) a_t$, we derive the equilibrium growth rate:

$$g^* = \frac{\delta h - \rho\Lambda + \delta h \gamma\,(1-\alpha-\beta)^{-2}}{1 + \Lambda + \gamma\left(\Lambda + (1-\alpha-\beta)^{-2}\right)}. \tag{44}$$

Substituting (43) and (44) into the Euler eqution, we derive the equilibrium interest rate:

$$r^* = \frac{\rho + (1+\gamma)\,\delta h}{\left(1+\gamma\,(1-\alpha-\beta)^{-2}\right) + (1+\gamma)\,\Lambda}. \tag{45}$$

Putting (42) and (45) in the monopolistic pricing rule and the demand function for intermediate goods, we know that

$$p^* = \frac{r^*\eta}{(1-\alpha-\beta)}, x^* = \left[h_y^{*\alpha} l^\beta\,(1-\alpha-\beta)\,p^{*-1}\right]^{\frac{1}{\alpha+\beta}}. \tag{46}$$

Substituting (46) and $\pi^* = (\alpha+\beta)\,p^* x^*$ into $r^* p_a^* = \pi^*$ gives rise to

$$p_a^* = \frac{\pi^*}{r^*} = (\alpha+\beta)\left(\eta\,(1-\alpha-\beta)^{-1}\right)^{1-1/(\alpha+\beta)}\left(\left(\frac{\Lambda}{\delta}\right)^\alpha l^\beta\,(1-\alpha-\beta)\right)^{1/(\alpha+\beta)} r^{*-\beta/(\alpha+\beta)}. \tag{47}$$

To examine the saddle-point stability of the steady state, we linearize the three-dimensional dynamic system around the steady state and rewrite the linearized system in its matrix form:

$$\begin{bmatrix} \dot{z}_t \\ \dot{h}_{yt} \\ \dot{q}_t \end{bmatrix} = \begin{bmatrix} -(\alpha+\beta)\,w_0 & \left(\frac{\alpha w_0}{h_y^*}+\delta\right) z^* & -z^* \\ -(\alpha+\beta)\,w_1 \frac{w_0 h_y^*}{z^*} & \alpha w_1 w_0 + \frac{\delta\Lambda(1-\alpha-\beta)+\delta}{\Lambda(1-\alpha)} h_y^* & -\frac{1-\alpha-\beta}{1-\alpha} h_y^* \\ -(\alpha+\beta)\,w_2 \frac{w_0 q^*}{z^*} & \alpha w_2 w_0 \frac{q^*}{h_y^*} & (1+\gamma)\,q^* \end{bmatrix} \begin{bmatrix} z_t \\ h_{yt} \\ q_t \end{bmatrix},$$

where $w_0 = z^{*-(\alpha+\beta)} h_y^{*\alpha} l^\beta$, $w_1 = (\alpha+\beta)\,(1-\alpha-\beta)\,/\,(1-\alpha)$, and $w_2 = (1-\alpha-\beta)^2 - 1$. Denote J the Jacobian matrix of the linear system. It is easy to know the determinant of J is negative, namely,

$$\det(J) = -\frac{\delta}{\alpha}(\alpha+\beta)\,w_0 q^* h_y^* \begin{bmatrix} \alpha\,(1-\alpha-\beta)\left(1+\Lambda^{-1}\right) + \\ \gamma\,(\alpha+\beta)\left(1+\Lambda\,(1-\alpha-\beta)^2\right) \end{bmatrix} = \Pi_{i=1}^3 \lambda_i < 0,$$

where $\lambda_i, i = 1,2,3$ are eigenvalues of J. The negativity of the determinant of the Jacobian matrix J establishes that there are two possibilities: (i) J has one negative eigenvalue and the other two eigenvalues have negative real parts; (ii) J has one negative eigenvalue and the other two eigenvalues have positive real parts. If we can prove that the trace of the Jacobian matrix J is positive, then possibility (ii) must be true. Thus, the Jacobian matrix has one stable eigenvalue and two unstable ones. By the definition of the transformed variables, we know that h_{yt} and q_t are two control (or unpredetermined) variables, while y_t is a state (or predetermined) variable. The number of state variables is equal to the number of stable eigenvalues, which establishes that the steady state is saddle-point stable. That is, given the initial values of the state variable, $y_0\,(= k_0/a_0)$, the economy converges to the unique steady state along the unique stable manifold.

Finally, we demonstrate that the trace of the Jacobian matrix is positive. By definition, we know:

$$
\begin{aligned}
&\operatorname{trace}(J) \\
&\equiv \sum_{i=1}^{3} \lambda_i \\
&= -(\alpha+\beta) w_0 + \alpha w_1 w_0 + \frac{\delta\Lambda(1-\alpha-\beta)+\delta}{\Lambda(1-\alpha)} h_y^* + (1+\gamma) q^* \\
&= \frac{\left(\begin{array}{c} \left[-\frac{(1-\alpha)(\alpha+\beta)}{(1-\alpha-\beta)} + \alpha(\alpha+\beta) + \left(\Lambda(1-\alpha-\beta)^2 + (1-\alpha-\beta)\right)\right](\rho+\delta h(1+\gamma)) \\ +\frac{(1-\alpha)(1+\gamma)}{(1-\alpha-\beta)}\left[(\rho+\delta h) + (\rho\Lambda-\delta h)(1-\alpha-\beta)^2\right] \end{array}\right)}{(1-\alpha)(1-\alpha-\beta)\left[(1+\Lambda)+\gamma\left(\Lambda+(1-\alpha-\beta)^{-2}\right)\right]} \\
&> \frac{\left(\begin{array}{c} \left[-\frac{(1-\alpha)(\alpha+\beta)}{(1-\alpha-\beta)} + \alpha(\alpha+\beta) + \left(\Lambda(1-\alpha-\beta)^2 + (1-\alpha-\beta)\right)\right](\rho+\delta h(1+\gamma)) \\ +\frac{(1-\alpha)(1+\gamma)}{(1-\alpha-\beta)}\left[\left(\frac{\rho}{1+\gamma}+\delta h\right) + (\rho\Lambda-\delta h)(1-\alpha-\beta)^2\right] \end{array}\right)}{(1-\alpha)(1-\alpha-\beta)\left[(1+\Lambda)+\gamma\left(\Lambda+(1-\alpha-\beta)^{-2}\right)\right]} \\
&= \frac{\left(\Lambda(1-\alpha-\beta)^2+1\right)(\rho+\delta h(1+\gamma)) + \rho(1-\alpha)(1-\alpha-\beta)(1+\Lambda(1+\gamma))}{(1-\alpha)(1-\alpha-\beta)\left[(1+\Lambda)+\gamma\left(\Lambda+(1-\alpha-\beta)^{-2}\right)\right]} > 0.
\end{aligned}
$$

Thus, we have completed the proof of Proposition 1. □

Additionally, if consumers have no desire for the spirit of capitalism, the model reduces to the original Romer (1990) model. This fact is stated in the following corollary and its proof is straightforward.

Corollary 1 *(Romer 1990): If consumers have no desire for the spirit of capitalism (i.e., $\gamma = 0$), then the model degenerates to the standard Romer model. In this case, the unique saddle-point stable steady state $\left(z^*, h_y^*, q^*\right)$ is solved as follows:*

$$
\left(\left(h_y^{*\alpha} l^\beta \left(q^* + \delta\left(h - h_y^*\right)\right)^{-1}\right)^{\frac{1}{\alpha+\beta}}, \frac{\Lambda}{\delta} r^*, \frac{(1-\alpha-\beta)^{-2}\left(\rho + \delta h + (\rho\Lambda - \delta h)(1-\alpha-\beta)^2\right)}{1+\Lambda}\right).
$$

The state state values of $(r_t, h_{yt}, h_{at}, p_t, x_t, \pi_t, p_{at})$ are

$$
\left(r^*, h_y^*, h_a^*, p^*, x^*, \pi^*, p_a^*\right) = \left(\begin{array}{c} \frac{\rho+\delta h}{1+(1+\gamma)\Lambda}, \frac{\Lambda}{\delta} r^*, h - h_y^*, \frac{r^*\eta}{(1-\alpha-\beta)}, \\ \left(h_y^{*\alpha} l^\beta (1-\alpha-\beta)/p^*\right)^{\frac{1}{\alpha+\beta}}, (\alpha+\beta) p^* x^*, \frac{\pi^*}{r^*} \end{array}\right).
$$

The BGP growth rate is then

$$
g_i^* = g^* = \frac{\delta h - \rho\Lambda}{1+\Lambda}, i = c, k, a.
$$

7.2 Proof of Proposition 2

Proof of Proposition 2. To prove Proposition 2 and its explanations, we need to make some substitutions and derive several partial derivatives. The endogenous growth rate is expressed as follows:

$$g_i^* = g^* = \frac{\delta h - \rho\Lambda + \delta h\gamma\left(1-\alpha-\beta\right)^{-2}}{1+\Lambda+\gamma\left(\Lambda+\left(1-\alpha-\beta\right)^{-2}\right)}, i = c, k, a. \tag{48}$$

Substituting $\gamma > 0$ and $\gamma = 0$ in equation (48) and calculating their differences, we obtain:

$$g_{\gamma>0}^* - g_{\gamma=0}^* = \frac{\gamma\Lambda\left[\delta h\left(1-\left(1-\alpha-\beta\right)^2\right)+\rho\left(\Lambda\left(1-\alpha-\beta\right)^2+1\right)\right]}{\left(1+\Lambda\right)\left[\left(1+\gamma\right)\Lambda\left(1-\alpha-\beta\right)^2+\left(\left(1-\alpha-\beta\right)^2+\gamma\right)\right]} > 0, \tag{49}$$

which shows that the growth rate with the SOC is strictly larger than the growth rate without it. This implies that the spirit of capitalism represents a new driving force for endogenous growth. Taking the partial derivatives with respect to γ on both sides of (48) yields:

$$\frac{\partial g^*}{\partial\gamma} = \frac{\delta h\Lambda\left(1-\left(1-\alpha-\beta\right)^2\right)+\rho\Lambda\left(1+\Lambda\left(1-\alpha-\beta\right)^2\right)}{\left(1-\alpha-\beta\right)^2\left[\left(1+\gamma\left(1-\alpha-\beta\right)^{-2}\right)+\left(1+\gamma\right)\Lambda\right]^2} > 0. \tag{50}$$

By definition, the savings rate is as $s_t \equiv \dot{k}_t/\left(c_t+\dot{k}_t\right) = \left(\dot{k}_t/k_t\right)/\left(c_t/k_t+\dot{k}_t/k_t\right)$. On the balanced growth path, the equilibrium savings rate is

$$s^* = \frac{g^*}{q^*+g^*} = \frac{\left(\delta h-\rho\Lambda\right)\left(1-\alpha-\beta\right)^2+\delta h\gamma}{\rho+\delta h\left(1+\gamma\right)}. \tag{51}$$

Substituting $\gamma > 0$ and $\gamma = 0$ in equation (51) and calculating their differences yield:

$$s_{\gamma>0} - s_{\gamma=0} = \frac{\delta h\gamma\left[\delta h\left(1-\left(1-\alpha-\beta\right)^2\right)+\rho\left(\Lambda\left(1-\alpha-\beta\right)^2+1\right)\right]}{\left(\rho+\delta h\right)\left[\rho+\delta h\left(1+\gamma\right)\right]} > 0.$$

The positive difference $(s_{\gamma>0} - s_{\gamma=0})$ represents a new savings motive for consumers due to the SOC, namely, "saving for its own sake". Taking the partial derivatives with respect to γ on both sides of (51) leads to:

$$\frac{\partial s^*}{\partial\gamma} = \frac{\delta h\left[\rho+\delta h\left(1-\left(1-\alpha-\beta\right)^2\right)+\rho\Lambda\left(1-\alpha-\beta\right)^2\right]}{\left[\rho+\delta h\left(1+\gamma\right)\right]^2} > 0. \tag{52}$$

Taking the partial derivatives with respect to γ on both sides of (45) leads to

$$\frac{\partial r^*}{\partial\gamma} = \frac{\delta h\left(1-\left(1-\alpha-\beta\right)^2\right)-\rho\left(\Lambda+\left(1-\alpha-\beta\right)^{-2}\right)}{\left[\left(1+\gamma\left(1-\alpha-\beta\right)^{-2}\right)+\left(1+\gamma\right)\Lambda\right]^2} < 0. \tag{53a}$$

Plugging (43) in $\widetilde{\rho} \equiv \rho - \gamma q^*$ leads to

$$\widetilde{\rho} = \frac{\rho (1+\Lambda) + \delta h \gamma \left(1 - (1-\alpha-\beta)^{-2}\right)}{(1+\Lambda) + \gamma \left(\Lambda + (1-\alpha-\beta)^{-2}\right)}. \tag{54}$$

Taking the partial derivatives with respect to γ on both sides of (54), we obtain:

$$\frac{\partial \widetilde{\rho}}{\partial \gamma} = \frac{\delta h (1+\Lambda) \left(1 - (1-\alpha-\beta)^{-2}\right) - \rho (1+\Lambda) \left(\Lambda + (1-\alpha-\beta)^{-2}\right)}{\left[(1+\Lambda) + \gamma \left(\Lambda + (1-\alpha-\beta)^{-2}\right)\right]^2} < 0. \tag{55}$$

Taking the partial derivatives with respect to γ on both sides of (46) and (47), we obtain:

$$\frac{\partial p^*}{\partial \gamma} = \frac{\eta}{1-\alpha-\beta} \frac{\partial r^*}{\partial \gamma} < 0, \tag{56}$$

$$\frac{\partial x^*}{\partial \gamma} = \left(\frac{\Lambda}{\delta}\right)^{\alpha} l^{\beta} (1-\alpha-\beta)^2 \eta^{-1} \frac{\alpha - 1}{\alpha+\beta} \frac{\partial r^*}{\partial \gamma} > 0, \tag{57}$$

$$\frac{\partial p_a^*}{\partial \gamma} = \beta \left(\frac{\eta}{1-\alpha-\beta}\right)^{1-1/(\alpha+\beta)} \left(\left(\frac{\Lambda}{\delta}\right)^{\alpha} l^{\beta} (1-\alpha-\beta)\right)^{1/(\alpha+\beta)} r^{*-\beta/(\alpha+\beta)-1} \left(-\frac{\partial r^*}{\partial \gamma}\right) > 0. \tag{58}$$

7.3 Proof of Proposition 3

Proof of Proposition 3. The Hamiltonian of the consumer with ability n is:

$$\mathcal{H} = e^{-\rho t} \left[\log c_t(n) + \gamma \log k_t(n)\right] + \lambda_t(n) \left[r_t k_t(n) + w_{lt} + w_{ht} l_t(n) \frac{h}{l} + a_t(n)(1 - l_t(n)) - c_t(n)\right],$$

where $\lambda_t(n)$ is the Hamiltonian multiplier. The first-order necessary conditions are:

$$c_t(n) : e^{-\rho t} \log c_t(n) = \lambda_t(n), \tag{59}$$

$$k_t(n) : e^{-\rho t} \frac{\gamma}{k_t(n)} + \lambda_t(n) r_t = -\dot{\lambda}_t(n). \tag{60}$$

Combining the two equations above leads to the consumption Euler equation presented in the main text:

$$\frac{\dot{c}_t(n)}{c_t(n)} = r_t - \rho + \gamma \frac{c_t(n)}{k_t(n)}. \tag{61}$$

The optimal behaviors of the final goods sector and the intermediate goods sector are the same as those in the homogenous ability model. They are described by the following equations:

$$\alpha \frac{y_t}{h_{yt}} = w_{ht}, \beta \frac{y_t}{l} = w_{lt}, h_{yt}^{\alpha} l^{\beta} (1-\alpha-\beta) x_{it}^{-\alpha-\beta} = p_{it}, \tag{62}$$

$$p_{at}^i = \int_{\tau=t}^{\infty} e^{-\int_{s=t}^{\tau} r_s ds} \pi_{i\tau} d\tau, p_{it} = \frac{1}{1-\alpha-\beta} r_t \eta \equiv p_t, \tag{63}$$

$$x_{it} = \left(\frac{h_{yt}^{\alpha} l^{\beta} (1-\alpha-\beta)}{p_t} \right)^{\frac{1}{\alpha+\beta}} = x_t, \pi_{it} = (\alpha+\beta) p_t x_t = \pi_t. \tag{64}$$

The no-arbitrage condition of human capital allocations is given by

$$w_{ht} \frac{h}{l} = \delta a_t n p_{at}. \tag{65}$$

The market-clearing condition for the physical capital and the resource constraint are given by

$$k_t = \int_{i=0}^{a_t} \eta x_{it} di = \eta x_t a_t, c_t + \dot{k}_t = y_t, \tag{66}$$

respectively. On the balanced growth path, the growth rates for consumption, physical capital, and knowledge are equal to a constant, g^*, and the equilibrium interest rate r^* and the consumption-capital ratio $\phi^* \, (= c/k)$ are also constant. From equation (61), on the BGP, we have:

$$g^* = r^* - \rho + \gamma \phi^*. \tag{67}$$

Combining equations (62), (63), and (66), on the BGP, we obtain

$$g^* = \frac{r^*}{(1-\alpha-\beta)^2} - \phi^*. \tag{68}$$

Using equations (62)-(66), we solve for

$$h_y^* = \frac{\Lambda h}{\delta l} \frac{r^*}{n^*}. \tag{69}$$

Combining equations (31) and (69) leads to

$$h \Gamma (n^*) = \frac{\Lambda h}{\delta l} \frac{r^*}{n^*}. \tag{70}$$

On the BGP, equation $\dot{a}_t = \delta l a_t \int_{n=\underline{n}}^{\infty} n d\Gamma (n)$ gives us the equilibrium growth rate of knowledge:

$$g^* = \delta l \int_{n=n^*}^{\infty} n d\Gamma (n). \tag{71}$$

Solving equation (68) for ϕ^* and plugging it in equation (67) lead to

$$g^* = \frac{1+\gamma (1-\alpha-\beta)^{-2}}{1+\gamma} r^* - \frac{\rho}{1+\gamma}. \tag{72}$$

Substituting (69) into (72) gives rise to the equilibrium growth rate of the economy:

$$g^* = \frac{1+\gamma (1-\alpha-\beta)^{-2}}{1+\gamma} \frac{\delta l}{\Lambda} n^* \Gamma (n^*) - \frac{\rho}{1+\gamma}. \tag{73}$$

Combining equations (71) and (73) gives rise to the key equation to pin down the threshold value of n^*:

$$\delta l \int_{n=n^*}^{\infty} n d\Gamma (n) = \frac{1+(1-\alpha-\beta)^{-2} \gamma}{1+\gamma} \frac{\delta l}{\Lambda} n^* \Gamma (n^*) - \frac{\rho}{1+\gamma}. \tag{74}$$

In particular, we assume that the ability n follows the Pareto distribution:

$$\Gamma(n) = 1 - \left(\frac{\underline{n}}{n}\right)^{\kappa}, \kappa > 1. \tag{75}$$

Putting (75) in equation (74) and taking the derivatives on both sides w.r.t γ, we obtain

$$\frac{\partial n^*}{\partial \gamma} = -\frac{\delta l \left((1-\alpha-\beta)^{-2} - 1\right) n^* \Lambda^{-1} \left(1 - (\underline{n}/n^*)^{\kappa}\right) + \rho}{\delta l \kappa \left(\underline{n}/n^*\right)^{\kappa} (1+\gamma)^2 + (1+\gamma)\left(1 + (1-\alpha-\beta)^{-2}\right)\left(1 + (\kappa - 1)\left(\underline{n}/n^*\right)^{\kappa}\right)} < 0.$$

Finally, we take the partial derivatives with respect to γ on both sides of equations $h_a^* = h\left[1 - \Gamma(n^*)\right]$ and (71):

$$\frac{\partial h_a^*}{\partial \gamma} = h\kappa \left(\frac{\underline{n}}{n^*}\right)^{\kappa-1} \frac{\underline{n}}{n^{*2}} \left(-\frac{\partial n^*}{\partial \gamma}\right) > 0,$$
$$\frac{\partial g^*}{\partial \gamma} = -\delta l \kappa n^* \left(\frac{\underline{n}}{n^*}\right)^{\kappa} \frac{\partial n^*}{\partial \gamma} > 0.$$

Thus, we have completed the proof of Proposition 3. □