On the Stationarity of Exhaustible Natural Resource Prices: Misspecification Effects Arising from Incomplete Models

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Abstract

In this paper we examine whether the real prices of eleven natural resource commodities exhibit stochastic or deterministic trends. A common methodological feature in the relevant empirical literature, most of which published in the Journal of Environmental Economics and Management, has been so far the application of univariate tests for unit roots. In these tests the real price for each commodity is tested for unit roots in isolation from all other natural resource commodity prices. We claim that this approach is likely to produce spurious inferences concerning the true number of unit roots, since it ignores any possible dynamic interactions among the available set of nominal prices. We suggest that the hypothesis of stationarity of real commodity prices should be properly defined and tested within a multivariate error correction model, which explicitly accounts for all possible linear interdependencies among the series involved. In such a framework, the stationarity of the real prices that participate in the system depends on whether the system exhibits sufficient cointegration with a specific cointegration matrix. Our empirical results suggest that within this multivariate framework, eight of eleven real prices of exhaustible natural resource commodities satisfy the restrictions for being stationary. On the contrary, all of these eleven real prices appear to be non-stationary, when the unit root hypothesis is tested in the context of incomplete models.

JEL Classification: Q31; C12; C51; C53
Keywords: Natural resource commodity price; Cointegration; Trend stationarity; Vector error correction model

1 Introduction

The implications of increased natural resource scarcity and its effect on economic growth have been discussed since the 18th century. Malthus [15] and Ricardo [20] held that agricultural land scarcity implied strict limits on population growth and the development of living standards. Harold Hotelling [6] offered his well-known counterargument in his seminal article of 1931: competitive firms would manage exhaustible resource stocks to maximize present-value profits, competitive extraction paths would therefore match those chosen by a social planner seeking to maximize intertemporal social surplus, and subject to the caveat of social and private discount rates equality, equivalence between competitive outcome and the work of a rational social planner would be achieved. The Hotelling rule provides the fundamental no-arbitrage condition that every competitive or efficient resource utilization path has to meet. In its basic form it indicates that along such a path the price of an exhaustible resource has to grow with a rate that equals the interest rate.

Hotelling's theory was not empirically tested until the second half of the 20th century. Extant empirical tests show mixed support. As a result its usefulness in describing and predicting the actual behaviour of exhaustible resource markets remains an open question. Slade and Thille [24] categorized the existing empirical tests as (a) price behaviour, (b) shadow price, and (c) Hotelling valuation tests. This paper contributes to the literature on price behaviour tests, which focus on price paths as indicators of scarcity. Included in this group are Barnett and Morse [2], Smith [26] and Slade [22]. Barnett and Morse [2] examined trends in the prices and unit costs of extractive goods (including agricultural, mineral, and forest products) in the United States. Their findings suggested that natural resources were becoming less scarce, not more scarce, in an economic sense. Smith [26] employed an econometric analysis of annual (1900-1973) price data of four aggregate resource groups and concluded that the trend in mineral prices was negative with the rate of decline decreasing over time in absolute magnitude. A similar study of twelve major metals and fuels by Slade [22] concluded that the price paths for nonrenewable natural resources were U-shaped. Slade hypothesized that the declining, flat and increasing price trends implicit in U-shaped price paths, come at different points in the life cycle of the exhaustive resource. Recently, Lin and Wagner [12], suggested that it is unlikely for Hotelling rule to hold since it does not consider stock effects and technological progress. Lin and Wagner extended Hotelling's model in order to capture the effects of these two factors. Then, they tested the modified Hotelling model for fourteen minerals and they concluded that it can not be rejected for the eight of the fourteen commodities. Another recent study by Livernois [13] reviewed the empirical evidence on the behavior of commodity market prices over time, supporting Lin and Wagner's view about the validity of Hotelling rule.

Other recent studies by Slade [23], Berck and Roberts [3] and Ahrens and Sharma [1], among others, find that many non-renewable resource prices have a stochastic trend and suggest that there may be a specification problem in these time series tests. Finally Lee et al. [11], in a very interesting paper, revisit this issue by employing a Lagrangian multiplier unit root test that allows for two endogenously determined structural breaks with and without a quadratic trend. Contrary to previous research, they find evidence against the unit root hypothesis for all price series and support for characterizing natural resource prices as stationary around deterministic trends with structural breaks.[1]

A common methodological point, shared by the majority of the aforementioned studies is that they employ *univariate* tests for unit roots. Specifically, they test for stationarity of the real price, Y_{it}, of commodity i, $i = 1, 2, ..., p$ defined as S_{it}/D_t, where S_{it} and D_t correspond to the nominal price of commodity i and the price deflator, respectively. This approach is equivalent with testing for stationarity the logarithm, y_{it}, of the real price of commodity i, given by $y_{it} = s_{it} - d_t$, where s_{it} and d_t are the logarithms of S_{it} and D_t, respectively. Methodologically, testing for a unit root in y_{it} is equivalent to testing for cointegration between s_{it} and d_t in the context of a bivariate error correction model (ECM) under the maintained assumption that the (unique) cointegration vector is of the form $[1, -1]$. In such a bivariate framework, the real price for each commodity is tested for non-stationarity in isolation from all the other $p - 1$ prices. This means that the adoption of a bivariate than a multivariate $(p+1)$ framework for testing for cointegration ignores any possible dynamic interactions among the p nominal prices, thus producing spurious inferences concerning the true number of cointegrating vectors that exist among the $p + 1$ series under examination. For example, assume that the multivariate stochastic process $\{\mathbf{z}_t\}$, $\mathbf{z}_t = [s_{1t}, s_{2t}, ..., s_{pt}, d_t]^\top$ exhibits cointegration properties consistent with the assumption that the p real prices $s_{it} - d_t$ are stationary. These

[1] Slade and Thille [25] provide an excellent review of the literature that extends and tests the Hotelling [6] model of the optimal depletion of an exhaustible resource.

properties amount to (a) the number of cointegrating vectors is equal to p and (b) the cointegrating vectors satisfy a certain set of parametric restrictions, namely that the ith cointegrating vector is of the form:

$$\mathbf{b}_i = [0, 0, \ldots, 0, 1, 0, \ldots, 0, -1]' \tag{1}$$

where the value 1 appears at the i-th coordinate of $\mathbf{b}_i$. However, instead of testing for the above mentioned restrictions in the context of a $(p+1)-$ dimensional ECM, the applied researcher decides to adopt p bivariate ECMs each one consisting of a nominal price, s_{it}, plus the deflator. Then, as we show in this paper, the total number of cointegrating vectors that will be inferred from these p bivariate systems is likely to be less than p and in some cases even zero. In other words, the evidence of *insufficient cointegrability* which is usually interpreted as evidence against the stationarity of the p real prices under consideration may be solely due to the misspecification effects induced by the adoption of incomplete systems.

In our empirical analysis we use 11 non-renewable natural resource nominal price series plus the producer price index, with the latter being used as the appropriate deflator. Using a twelve-dimensional ECM approach which includes the 11 nominal prices together plus the deflator, we show that the number of the cointegrating relations is indeed eleven and at the same time for most of the cointegrating vectors (eight of them) we cannot reject the theoretically anticipated form **(1)**. Instead if we adopt the bivariate series by series analysis that has been followed in the relevant literature until today, we cannot find any cointegrating relations. Our premise in this paper is that the long literature that tests the price behaviour implications of the Hotelling rule, has not raised the question of stationarity of real prices in the correct framework, namely a multivariate framework that accounts for all possible interactions among the series of interest. In this paper we provide empirical evidence in favour of the stationarity hypothesis of real natural resource prices without resorting to *ad hoc* modifications of the standard unit root tests, such as the ones which appeal to structural breaks.

2 Theoretical Implications

Let us begin by assuming that the hypothesis (H1) that we are interested to test, that is the logarithm of real prices, y_{it}, $i = 1, 2, ..., p$, of p nonrenewable resources is indeed stationary around a linear deterministic trend. We also assume that the corresponding nominal prices, $S_{i,t}$, $i = 1, 2, ..., p$ have been deflated by a common deflator ("a suitable producer index") denoted by D_t. We also make the reasonable assumption (H2) that the logarithm of the employed common deflator, d_t, contains a unit root (there is ample evidence in the literature suggesting that deflators such as PPI or CPI contain at least one unit root). We shall demonstrate that (H1) and (H2) bare strong and testable implications on the cointegrating properties of the vector stochastic process $\{\mathbf{z}_t\}$, $\mathbf{z}_t = [s_{1t}, s_{2t}, ..., s_{pt}, d_t]^\top$.

For every i, set

$$
\begin{aligned}
& s_{it} - d_t = u_{it} \\
\Longrightarrow \; & s_{it} = d_t + u_{it} \ , \qquad (2)
\end{aligned}
$$

where $\{u_{it}\}$ are stationary processes around a linear trend, that is $u_{it} = a_i + \gamma_i t + e_{it}$. Suppose also that d_t is I(1) with a possible linear trend satisfying $d_t = d_{t-1} + v_t$, where $v_t = a_{p+1} + \gamma_{p+1} t + \varepsilon_{p+1,t}$. In vector notation, if we set $\mathbf{s}_t = [s_{1t}, s_{2t}, \ldots, s_{pt}]'$, $\mathbf{u}_t = [u_{1t}, u_{2t}, \ldots, u_{pt}]'$, $\boldsymbol{\varepsilon}_t = [\varepsilon_{1t}, \varepsilon_{2t}, \ldots, \varepsilon_{pt}]'$, $\mathbf{a} = [a_1, a_2, \ldots, a_{p+1}]'$, $\boldsymbol{\gamma} = [\gamma_1, \gamma_2, \ldots, \gamma_{p+1}]$, $\mathbf{i}_p = [1, 1, \ldots, 1]' \in \mathbb{R}^p$ and $\Delta d_t = a_{p+1} + \gamma_{p+1} v_t$, we have

$$\begin{aligned} \begin{bmatrix} \mathbf{s}_t \\ d_t \end{bmatrix} &= \begin{bmatrix} d_t \mathbf{i}_p \\ d_{t-1} \end{bmatrix} + \begin{bmatrix} \mathbf{u}_t \\ v_t \end{bmatrix} \\ &= \mathbf{a} + t\boldsymbol{\gamma} + \begin{bmatrix} d_t \mathbf{i}_p \\ d_{t-1} \end{bmatrix} + \begin{bmatrix} \boldsymbol{\varepsilon}_t \\ \varepsilon_{p+1,t} \end{bmatrix}, \end{aligned} \tag{3}$$

which describes a cointegrated system with p cointegrating vectors given by (1). The vectors $\mathbf{b}_i$ are linearly independent. Therefore, under (H1) and (H2), the rank of system (3), denoted by r, will satisfy $r = p$. If this is not true, then the combined hypotheses (H1) and (H2) must be rejected. Under the maintained assumption that (H2) is likely to be true (given the convincing available evidence) we are led to the inescapable conclusion that (H1) must be false.

2.1 Testing for Cointegration in Incomplete Systems: The Issue of Insufficient Cointegrability

The existing literature of testing whether the real prices are stationary has followed a rather different methodology than the one outlined above (see Ahrens and

Sharma [1], Lee et al. [11], etc.). In particular, they test the stationarity assumption for each of the p real prices under consideration in the context of univariate unit root tests. This testing strategy is equivalent to testing for cointegration in the context of p bivariate sub-systems each one containing only one nominal price plus the deflator, under the imposed restriction that the cointegrating vector in each sub-system is of the form $[1, -1]$. However, this testing strategy is likely to produce misleading inferences on the cointegration properties of the p bivariate systems since it ignores all the dynamics among the nominal prices themselves. Put it differently, the inferences obtained in the context of the full system containing all the p nominal prices plus the deflator are in no way equivalent to the inferences obtained in p bivariate sub-systems each one containing only one nominal price plus the deflator. To explain the sources of misspecifications in the context of testing for cointegration within bivariate systems when the correct model for all the data under consideration is the full system, let us consider the following simplified case:

Assume that we have a trivariate stochastic process, $\mathbf{y}_t = [y_{1t}, y_{2t}, y_{3t}]^\top$, described by

$$\mathbf{y}_t = A_1\mathbf{y}_{t-1} + A_2\mathbf{y}_{t-2} + \mathbf{u}_t \ , \tag{4}$$

where $\mathbf{u}_t = [u_{1t}, u_{2t}, u_{3t}]'$ is a vector white noise process. We also assume that y_{1t}, y_{2t} denote the logarithms of nominal price of two non-renewable resources and y_{3t} denotes the logarithm of the deflator. Moreover, assume that the matrices A_1 and

A_2 are as follows:

$$A_1 = \begin{bmatrix} \frac{3}{2} & 1 & -\frac{3}{2} \\ 1 & -3 & 0 \\ 1 & -1 & \frac{3}{2} \end{bmatrix} \text{ and } A_2 = \begin{bmatrix} -\frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 4 & -1 \\ 0 & 0 & -\frac{1}{2} \end{bmatrix}.$$

First, we express system (4) in vector error-correction form:

$$\begin{aligned} \Delta \mathbf{y}_t &= (A_1 + A_2 - I_3)\, \mathbf{y}_{t-1} + (-A_2)\, \Delta \mathbf{y}_{t-1} + \mathbf{u}_t \\ &= \Pi \mathbf{y}_{t-1} + \Gamma_1 \Delta \mathbf{y}_{t-1} + \mathbf{u}_t \end{aligned} \tag{5}$$

Then,

$$\Pi = A_1 + A_2 - I_3 = \begin{bmatrix} 0 & 1 & -1 \\ 1 & 0 & -1 \\ 1 & -1 & 0 \end{bmatrix}.$$

It can be easily seen that $|\Pi| = 0$ since its last row is obtained by subtracting the first row from the second. This fact implies that (4) has at least one unit root. It can be also verified that

$$\Pi = \mathbf{c}\boldsymbol{\beta}^\top = \begin{bmatrix} 0 & 1 \\ 1 & 0 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \end{bmatrix}, \tag{6}$$

where $\mathbf{c} = (c_{ij})$ and $\boldsymbol{\beta} = \left(\beta_{ij}\right)$, $1 \le i \le 3$, $1 \le j \le 2$. This decomposition of Π implies that two cointegrating relationships are hidden in (4). Namely, the variables $y_{1t} - y_{3t}$ and $y_{2t} - y_{3t}$ are stationary, which in turn implies that both the real prices under consideration are indeed stationary. It is clear also that the decomposition of Π described in (6) is not unique. Nevertheless, it is the only possible decomposition, for which $\beta_{11} = \beta_{22} = 1$ and $\beta_{21} = 0$ hold.

Let us now examine the effects of omitting the second variable y_{2t}, that is omitting one nominal price (which is cointegrated with the deflator) on the inferences that we are likely to draw concerning the cointegration properties of the remaining two variables, y_{1t} and y_{3t}. To this end, define $\mathbf{z}_t = [y_{1t}, y_{3t}]'$ and $\mathbf{v}_t = [u_{1t}, u_{3t}]'$. The vector error correction form implied by a VAR(2) model for $\mathbf{z}_t$ is now

$$\Delta \mathbf{z}_t = Q\mathbf{z}_{t-1} + M\Delta \mathbf{z}_{t-1} + \mathbf{w}_t \ , \tag{7}$$

where it is assumed that $\mathbf{w}_t$ is not endogenous and satisfies the regularity conditions that make equation (7) estimable. Nevertheless, equations (5) and (6) imply that

$$\begin{aligned}
\Delta \mathbf{z}_t &= \begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} y_{1t-1} - y_{3t-1} \\ y_{2t-1} - y_{3t-1} \end{bmatrix} + \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{1}{2} \end{bmatrix} \Delta \mathbf{z}_{t-1} + \mathbf{v}_t \\
&= \begin{bmatrix} y_{2t-1} - y_{3t-1} \\ y_{1t-1} - y_{2t-1} \end{bmatrix} + \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{1}{2} \end{bmatrix} \Delta \mathbf{z}_{t-1} + \mathbf{v}_t \\
&= \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \mathbf{z}_{t-1} + \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{1}{2} \end{bmatrix} \Delta \mathbf{z}_{t-1} + y_{2t-1} \begin{bmatrix} 1 \\ -1 \end{bmatrix} + \mathbf{v}_t
\end{aligned} \tag{8}$$

It can be observed that the omission of y_{2t} from the sub-system results in the inclusion of this variable in the error term of the new model. A first look at equation

(8) may falsely lead us to the conclusion that we have reached an expression where the left hand side is I(0) and the right hand side is I(1) since it includes two I(1) terms, the first one being

$$\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \mathbf{z}_{t-1} = \begin{bmatrix} -y_{3t-1} \\ y_{1t-1} \end{bmatrix}$$

and the second being the term $y_{2t-1}\,[1,-1]'$. It is clear that equation (8) is not estimable in a stable regression framework. Nevertheless, the problems that may arise when estimating equation (7) are not caused by a possible non-stationary nature of the error term (since y_{2t-1} is omitted in the bivariate model), but from the endogeneity of the latter. More specifically, one can transform expression (8) in such a way that it will describe the true contegrating relationship between y_{1t} and y_{3t}. This transformation can be done by making use of the properties of y_{2t}. More specifically, let us define $r_t = y_{2t} - y_{3t}$, which is I(0). Then $y_{2t-1} = y_{3t-1} + r_{t-1}$, and from (8) we obtain

$$\begin{aligned} \Delta \mathbf{z}_t &= \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \mathbf{z}_{t-1} + \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{1}{2} \end{bmatrix} \Delta \mathbf{z}_{t-1} + \begin{bmatrix} 0 & 1 \\ 0 & -1 \end{bmatrix} \mathbf{z}_{t-1} + r_{t-1} \begin{bmatrix} 1 \\ -1 \end{bmatrix} + \mathbf{v}_t \\ & \begin{bmatrix} 0 & 0 \\ 1 & -1 \end{bmatrix} \mathbf{z}_{t-1} + \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{1}{2} \end{bmatrix} \Delta \mathbf{z}_{t-1} + r_{t-1} \begin{bmatrix} 1 \\ -1 \end{bmatrix} + \mathbf{v}_t \\ &= \begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{bmatrix} 1 & -1 \end{bmatrix} \mathbf{z}_{t-1} + \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{1}{2} \end{bmatrix} \Delta \mathbf{z}_{t-1} + r_{t-1} \begin{bmatrix} 1 \\ -1 \end{bmatrix} + \mathbf{v}_t \; . \end{aligned} \tag{9}$$

The right hand side of expression (9) includes

$$\begin{bmatrix} 1 & -1 \end{bmatrix} \mathbf{z}_{t-1} = y_{1t-1} - y_{3t-1} \ ,$$

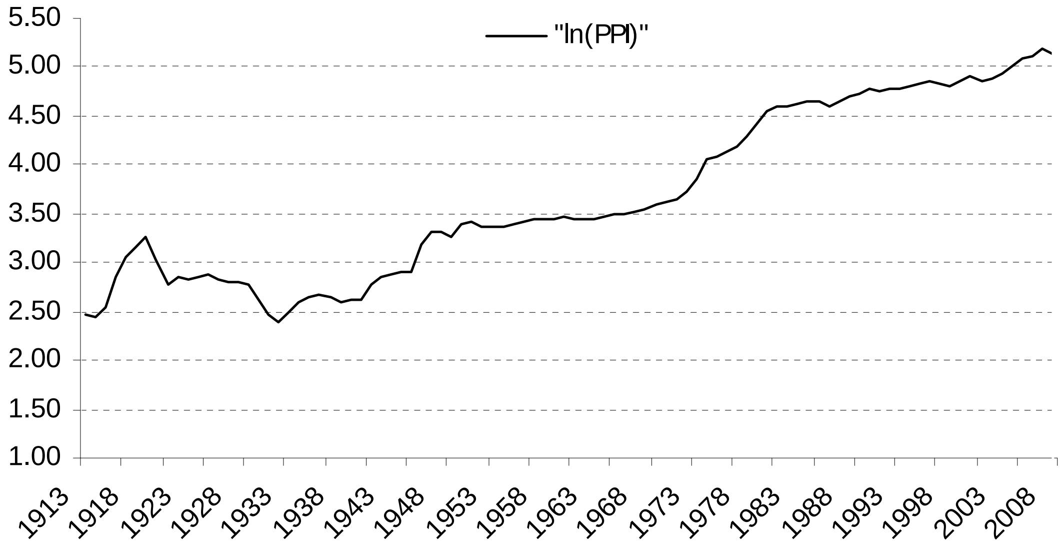
$\Delta \mathbf{z}_{t-1}$, r_{t-1} and $\mathbf{v}_t$, which are all I(0). Nevertheless, the error term, described now by the random vector $w_t := r_{t-1} [1, -1]' + \mathbf{v}_t$, is not only serially correlated but it may also be correlated with any of the regressors. This fact causes estimation problems and makes any inference on the contegrating vector of $\mathbf{z}_t$ unreliable.

The preceding analysis highlights the reasons behind the inability of the univariate approaches to reject the null hypothesis that each individual real price contains a unit root (or equivalently that each of the individual bivariate systems are not cointegrated). In an attempt to rescue the stationarity of real prices, the existing studies attribute the non-rejection of the null to structural breaks not accounted for in the testing procedure (see Lee et al. [11], among others). However, the preceding analysis has demonstrated that non-rejections of the unit root hypothesis are to be expected (even in the absence of structural breaks) if this hypothesis is tested in the context of an incomplete system. In other words, the omission of the interactions of dynamics among the nominal prices themselves in the parametric models, within which the unit root hypothesis is being tested are likely to produce spurious evidence in favour of the null hypothesis. The omission of these interactions is likely to manifest into endogeneity problems in the estimation of each of the bivariate models. When these interactions are properly taken into account in the context of the full model the evidence is highly supportive for the stationarity property of the majority of real prices.

3 Empirical Results

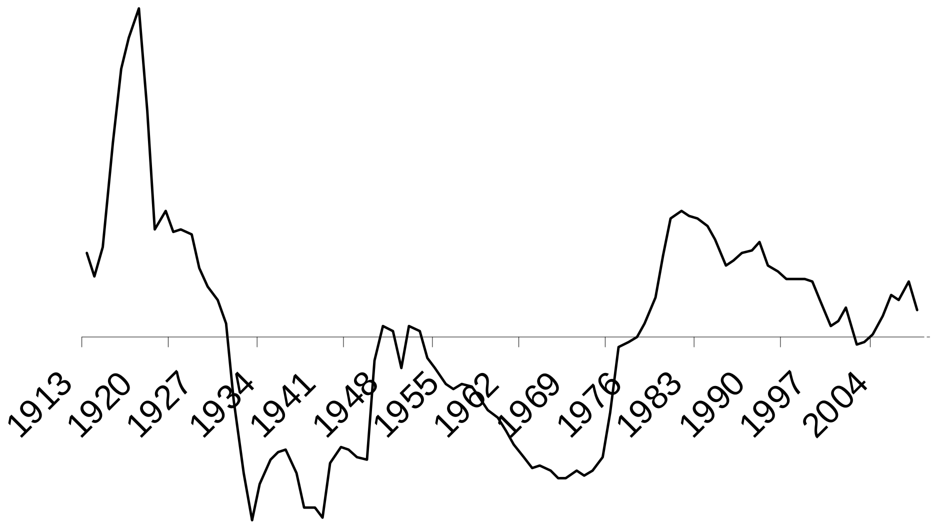
We first focus our attention on the statistical properties of the deflator series D_t. Following the relevant literature (see Lee et al. [11], among others), we take D_t to be the producer price index. Figure 1 depicts the behaviour of the logarithm of this index, d_t, for the period 1913 to 2008.

Figure 1: Logarithm of the Producer Price Index (1913-2008)



Visual inspection of the graph makes clear that this series is far from exhibiting any kind of stationarity. However, it is less clear whether this apparent non-stationarity is of deterministic or stochastic origins. This is the classical dilemma characterising the enormous "unit root literature" that has been developed since the early 1980's. To obtain a first idea concerning this issue, we report the residuals e_t from a regression of d_t on a constant, c, and a time trend, t:

Figure 2: Residuals of the regression of PPI on a constant and a time trend



It can be seen that the residual series, e_t, displays very strong persistence properties which are manifested in the large stochastic cycles present in this figure. Formal tests for the null hypothesis of a unit root in the d_t are in order. We employ the following tests for the null hypothesis of a unit root in d_t : (i) The classical Augmented Dickey-Fuller test with an intercept (ADF-C) and an intercept and a time trend (ADF-CT). The optimal lag-length in the ADF regressions was obtained by means of the Schwarz [21] information criterion. (ii) The semi-parametric Phillips-Perron tests with an intercept (PP-C) and an intercept and a time trend (PP-CT). The test statistics were calculated using a Bartlett kernel and allowing the optimal bandwidth to be endogenously determined by the

Newey and West [16] procedure. (iii) The modified DF tests, proposed by Elliot et al. [5] (ERS). Specifically, we calculate the values of ERS-C and ERS-CT in which an intercept and an intercept together with a time trend respectively have been removed from the original data before the DF regressions are run. (iv) The MZ_a and MZ_t tests of Ng and Perron [17] which are modified versions of the Phillips-Perron tests PP-C and PP-CT, respectively. Finally we employ the KPSS tests proposed by Kwiatkowski et al. [10] (KPSS) for the null hypothesis that d_t is a stationary series. As before, KPSS-C and KPSS-CT correspond to the cases that an intercept and an intercept with a time trend are included in the relevant specifications. The results are reported in Table I:

TABLE I AROUND HERE

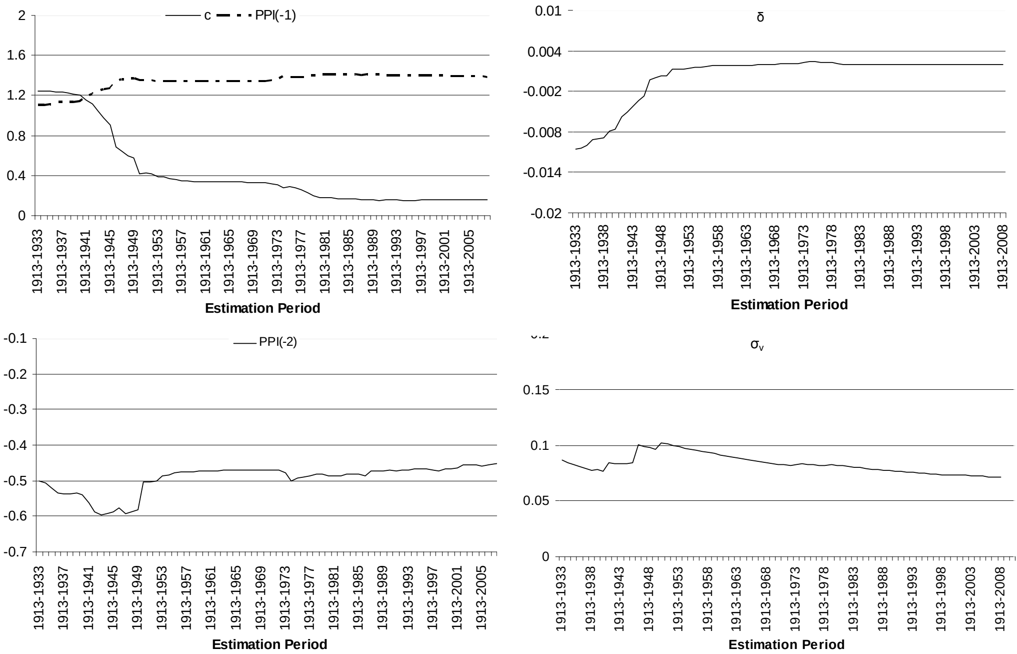
It can be seen that the unit root null is not rejected by any of the employed tests. On the contrary the null hypothesis that d_t is a stationary series is rejected by both KPSS-C and KPSS-CT. Hence, the overall conclusion is that d_t is indeed an I(1) series. Note that quite similar results are obtained for other candidate deflator series such as the Consumer Price Index.

The next question which is usually raised in the context of unit root testing is whether the evidence for the presence of a unit root in the data is due to structural breaks that have not been accounted for in the testing procedure. As a first step towards investigating this possibility we estimate a second-order autoregressive model (AR(2)) for d_t (augmented by a constant and a time trend) recursively, that is we estimate the model

$$d_t = c + \delta t + \rho_1 d_{t-1} + \rho_2 d_{t-2} + \nu_t,\ \nu_t \sim IID(0, \sigma_\nu^2)$$

This procedure is quite powerful in revealing the presence of structural break either in the conditional mean parameters or in the conditional variance of the model. The results are reported in Figure 3:

Figure 3: Recursive OLS Estimates of an AR(2) model for the logarithm of PPI



It can be seen that the parameters of this model display considerable stability after 1950. Before that period, some signs of instability are present especially in the intercept, c, and the coefficient on the deterministic trend, δ. To examine whether the presence of the detected parameter instability is responsible for the evidence of a unit root in d_t we calculate the unit root statistics mentioned above for the period 1950-2010. The results, reported in the second column of Table I, are very similar to those obtained for the whole sample, thus suggesting that the presence of general instability in one the coefficients of the AR(2) model does not affect the inferences on the presence of a unit root in the d_t series.

At this point we should note that various unit root tests under the hypothesis of a structural break have been proposed in the literature (see Perron [18] and [19], Zivot and Andrews [27], and Lumsdaine and Papell [14], among others). These tests concern breaks in the drift and/or trend coefficients. Other tests aim at capturing changes in the error variance of a unit root regression (see for example Cavaliere and Taylor [4]). Nevertheless, in our case we have the particular situation of a possible break in the autoregressive coefficients that probably does not affect the unit root nonstationarity of the series. This type of structural break is not possible in AR(1) models since the coefficient must be constant and equal to 1. In the case, however, of autoregressive models of higher order, coefficients can change in a fashion that will always correspond to a unit root stochastic sequence. As far as d_t is concerned, the unit root or stationarity test statistics are not significantly affected when we split the sample in order to avoid a possible change in the autoregressive coefficients.

Having reached the conclusion that the logarithm of PPI is an I(1) series, it becomes obvious that the only case under which the logarithms of the real nonrenewable resource prices, $s_{it}-d_t$, $i=1,2,...,p$, are stationary around a deterministic trend, is the case in which the logarithms of nominal prices s_{it}, $i = 1, 2, ..., p$ and d_t form a cointegrated system with restrictions described by (3). More specifically, the cointegration rank, r, must be equal to p and the cointegrating vectors $\mathbf{b}_i$ must be of the form given in (1). Following the relevant literature we employ the prices of the same natural resources analyzed by Slade [22], Ahrens and Sharma [1] and Lee et al. [11]. More specifically, our data set contains annual price data on eleven fuel and metal resources collected for the period between 1920 and 2008[2]. Hence our initial system is twelve-dimensional consisting of the logarithms of the eleven nominal prices plus the logarithm of the deflator. In other words, the vector stochastic process, $\{\mathbf{z}_t\}$, whose cointegrating properties we are interested to test can be denoted by $\mathbf{z}_t = [s_{1t}, s_{2t}, ..., s_{11t}, d_t]^\top$.

It must be noted that the presence of a linear trend in the cointegrating relations may be caused by a quadratic trend in the data (see Johansen [9]). On the other hand, the presence of linear trends in the data does not preclude the existence of linear trends in the cointegrating relations even if quadratic trends are absent. As a first step, we assume that the data generated by $\{\mathbf{z}_t\}$ follow a linear trend. More specifically, an application of various model selection criteria, showed that it is reasonable to assume that $\{\mathbf{z}_t\}$ is adequately described by a third-order vector

[2] Our data, downloaded from the U.S. Geological Survey Data Series 140 (http://minerals.usgs.gov/ds/2005/140/), are annual real price series for Aluminium, Coal, Copper, Gas, Iron, Lead, Nickel, Silver, Petroleum, Tin and Zinc. Since our model uses all of the above time series, we focus on the period from 1920 to 2008, during which all series are simultaneously available (note that Gas is available only from 1919 onwards). We also thank Junsoo Lee, John A. List and Mark C. Strazicich for providing access to their data. We decided to use the US Geological Survey data since the series included in their paper stopped in 1990.

autoregressive process (VAR(3)) which contains intercepts. So, let $\mathbf{z}_t$ be given by

$$\mathbf{z}_t = \mathbf{k} + A_1\mathbf{z}_{t-1} + A_2\mathbf{z}_{t-2} + A_3\mathbf{z}_{t-3} + \boldsymbol{\nu}_t \tag{10}$$

Indeed, misspecification tests together with the usual information criteria seem to support this specification. Following the preceding discussion, we can not avoid the possibility of linear trends in the possible cointegrating relations included in (10). Therefore, covering the more general case, we rewrite the model given in (10) in its Error Correction form as follows:

$$\Delta\mathbf{z}_t = \mathbf{k} + \boldsymbol{\gamma}t + \Pi\mathbf{z}_{t-1} + \Gamma_1\Delta\mathbf{z}_{t-1} + \Gamma_2\Delta\mathbf{z}_{t-2} + \boldsymbol{\nu}_t \tag{11}$$

where

$$\Pi = -\left(I - A_1 - A_2 - A_3\right). \tag{12}$$

As it is well known, the cointegration rank of this system is given by the rank, r, of the long-run matrix Π. Assuming that $rank(\Pi) = r < p+1$, the above system can be reduced to

$$\Delta\mathbf{z}_t = \mathbf{k} + \boldsymbol{\gamma}t + \mathbf{c}\boldsymbol{\beta}^\top\mathbf{z}_{t-1} + \Gamma_1\Delta\mathbf{z}_{t-1} + \Gamma_2\Delta\mathbf{z}_{t-2} + \boldsymbol{\nu}_t \tag{13}$$

where $\mathbf{c}$ and $\boldsymbol{\beta}$ are $(n \times r)$ matrices, denoting the adjustment coefficients and the cointegrating vectors respectively. Then, we conduct the usual, trace (tr) and maximum eigenvalue $(\lambda_{\max})$ tests for the determination of the cointegration rank of this system suggested by Johansen [7] and [8]. These tests are reported in Table

II:

TABLE II AROUND HERE

Both the trace and maximum eigenvalue statistics suggest that $r = 3$. In view of the previous discussion, it is clear that the necessary condition for the stationarity of all the eleven real prices (which amounts to $r = 11$) is clearly violated by both the statistics under consideration (these results are robust to alternative orders of the VAR system and alternative specifications of the deterministic components of the system). In other words, the results obtained so far reject the hypothesis that *all* of the real prices under consideration are stationary. What else do these results suggest? They suggest the following:

(i) There might be at most three stationary prices among the set of the eleven prices under examination. Put it differently we can be certain that at least eight real prices are non-stationary.

(ii) The results do not rule out the possibility that all the eleven prices are non-stationary. Indeed, the three cointegrating vectors may correspond solely to nominal non-renewable resource prices, thus leaving outside the cointegration space the deflator.

However, the interpretation of these results is based on the maintained (and so far non-tested) assumption that the cointegration properties of our system are stable over time. As already mentioned, however, one cannot rule out a-priori the possibility that the poor cointegration properties of our system, detected above, may be the result of undetected instability in the parameters of the VAR(3) model.

Indeed, the decision on the cointegration properties of our system is based on the inferences that we draw on the rank of the matrix Π, which, according to (12) is a function of the parameters of the conditional mean of the VAR(3) model. Our central question under investigation is whether there are any structural breaks within our sample that affect our inferences on the cointegration rank of Π. To answer this question, and in view of the fact that the autoregressive representation of the deflator series is relatively stable only for the period 1950-2008, we re-estimate model (11), and hence matrix Π, for this sub-period. The results are reported in Table III.

TABLE III AROUND HERE

The results for the sub-period under consideration tell a completely different story about the cointegration properties of our system. The degree of cointegrability has increased considerably. Indeed, the *tr* statistic suggests the presence of eleven cointegrating vectors which is exactly the number of cointegrating vectors predicted under the assumption that all the eleven real prices under consideration are stationary (around a linear trend). Nevertheless, the $\lambda_{\max}$ statistic is more conservative, suggesting the presence of only seven cointegrating vectors at the 5% significance level. However, even if we accept the results of the *tr* statistic (over those of $\lambda_{\max}$), we cannot reach the conclusion that all the eleven real prices under consideration are stationary without further testing. In order to reach this conclusion we must test whether these eleven cointegrating vectors are of the form described in (1). To this end, we model (11) under the imposed restriction that

$r(\Pi) = 11$. Under this restriction, which means that the rank of the matrix $\boldsymbol{\beta}$ is 11, the algorithmic estimation of the cointegrating relations will provide us with a 11×12 matrix, whose (i, i) elements will be units, while all the elements with coordinates (i, j), $i \neq j$, $j < 12$ will be zeros. These are the normalized estimates of the eleven cointegrating vectors and they are reported in Table IV:

TABLE IV AROUND HERE

The results of Table IV suggest that (under the assumption $r(\Pi) = 11$) the conditions on the values of the cointegrating vectors implied by the stationarity of real prices are rejected for Coal, Silver and Tin. For the last two metals, in particular, the point estimate of b_{12} is positive, thus having the "wrong" sign. Overall, we cannot obtain evidence in favour of the hypothesis that all the eleven real prices are stationary even if we adopt the cointegration results suggest by tr over the outright negative results for our joint hypothesis of the stationarity of all the eleven prices suggested by $\lambda_{\max}$. The results so far suggest that for the period 1950-2008, eight of the eleven real prices under consideration, namely those for aluminium, copper, gas, iron, lead, nickel, petroleum and zinc satisfy the conditions for being considered stationary around a deterministic trend.

3.1 Results from Bivariate Systems

The discussion in section 2 suggests that the employment of bivariate systems instead of the twelve-dimensional setup of the previous section, would probably lead to misleading results. Table V demonstrates the results of $\lambda_{\max}$ and trace tests for cointegration for eleven bivariate systems where the first variable is the logarithm of the price of a nonrenewable resource and the second is the logarithm of the PPI. As in table III, the sample covers the period 1950-2008. Due to the bivariate nature of the systems, the two statistics coincide as well as their critical values.

TABLE V AROUND HERE

The bivariate approach does not identify any cointegrating relation for all nonrenewable resources, except for the case of the bivariate system of coal and PPI, for which, interestingly, it is suggested that the number of cointegrating relations is two. It is clear that this setup, being unable to capture the interactions between the prices of the commodities involved, leads to a totally different conclusion than the one stemming from the twelve-dimensional setup of section 2.

4 Conclusions

In this paper we have investigated the stationarity properties of the real price series for eleven exhaustible natural resource commodities. The main message of our paper is that in order to draw the correct inferences on these properties, we must employ a multivariate model which accounts for all the linear interdependences among the series under consideration. The hypothesis to be tested, carefully defined in the context of a vector error correction model consisting of the logarithms of the eleven nominal price series and that of the deflator, amounts to (a) the number of cointegrating vectors being equal to eleven and (b) the cointegrating vectors satisfying a certain set of parametric restrictions. Our empirical results support the hypothesis that both types of restrictions are satisfied for eight of the eleven prices examined. On the contrary, all the eleven real price series appear to be non-stationary if the unit root hypothesis is tested on a series by series basis in the context of bivariate incomplete models. These results have important implications for academics and policy makers alike, because they contribute to an appropriate understanding of the dynamics of non-renewable natural resource real price series, on which empirical verification of theories, forecasting and proper inference depend.

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Table I

Unit Root Tests for PPI

	1913-2008		1950-2008	
	test-statistic	5% c.v.	test-statistic	5% c.v.
ADF-C	-0.55	-2.89	-0.61	-2.91
ADF-CT	-2.34	-3.45	-1.39	-3.48
PP-C	-0.14	-2.89	-0.38	-2.91
PP-CT	-1.91	-3.45	-1.53	-3.48
ERS-C	71.79	3.10	134.25	3.00
ERS-CT	8.99	5.64	17.71	5.71
$MZ_t - C$	0.93	-1.98	0.73	-1.98
$MZ_{t\_}CT$	-2.28	-2.91	-1.70	-2.91
$KPSS - C$	1.24	0.46	0.91	0.46
KPSS-CT	0.17	0.14	0.11	0.14

Table II

Johansen's Cointegration Tests for the Full System: Period 1920-2008

	Trace	5% c.v.	Max Eigenvalue	5% c.v.
r=0	454.49	374.91	92.55	80.87
r=1	361.93	322.07	76.28	74.84
r=2	285.66	273.19	73.19	68.81
r=3	212.46	228.30	51.43	62.75
r=4	161.04	187.47	40.41	56.71
r=5	120.63	150.56	33.66	50.60
r=6	86.97	117.71	25.39	44.50
r=7	61.57	88.80	22.07	38.33
r=8	39.50	63.88	14.04	32.12
r=9	25.46	42.92	10.12	25.82
r=10	15.34	25.87	9.81	19.39
r=11	5.54	12.52	5.54	12.52

Table III

Johansen's Cointegration Tests for the Full System: Period 1950-2008

	Trace	5% c.v.	Max Eigenvalue	5% c.v.
r=0	714.13	374.91	125.89	80.87
r=1	588.24	322.07	109.07	74.84
r=2	479.17	273.19	101.48	68.81
r=3	377.69	228.30	73.22	62.75
r=4	304.46	187.47	66.22	56.71
r=5	238.25	150.56	64.19	50.60
r=6	174.05	117.71	58.51	44.50
r=7	115.54	88.80	36.71	38.33
r=8	78.84	63.88	27.99	32.12
r=9	50.85	42.92	24.67	25.82
r=10	26.17	25.87	18.59	19.39
r=11	7.58	12.52	7.58	12.52

Table IV

Full System including Eleven Nominal Prices. Sample 1950-2008. Normalized Estimates of the Cointegrating Vectors Under the Restriction $r(\Pi) = 11$

Alum.	Coal	Copp.	Gas	Iron	Lead	Nickel	Silver	Petrol	Tin	Zinc	PPI	t-stat for $H_0 : b_{12} = -1$
1	0	0	0	0	0	0	0	0	0	0	-0.89	0.40
0	1	0	0	0	0	0	0	0	0	0	-1.63	-2.63
0	0	1	0	0	0	0	0	0	0	0	-0.62	1.08
0	0	0	1	0	0	0	0	0	0	0	-1.78	-1.09
0	0	0	0	1	0	0	0	0	0	0	-0.58	0.74
0	0	0	0	0	1	0	0	0	0	0	-0.63	1.35
0	0	0	0	0	0	1	0	0	0	0	-1.01	-0.03
0	0	0	0	0	0	0	1	0	0	0	0.85	2.22
0	0	0	0	0	0	0	0	1	0	0	-1.63	-1.46
0	0	0	0	0	0	0	0	0	1	0	0.95	3.05
0	0	0	0	0	0	0	0	0	0	1	-0.76	0.80

Table V

Johansen's Cointegration Tests for the Eleven Bivariate Systems: Period 1950-2008

	$r(\Pi) =$	Trace or Max Eigenvalue	5% c.v.	Num. of coint. relations
Alum.	0	17.59051	25.87211	0
Coal	1	15.33275	12.51798	2
Copp.	0	14.07237	25.87211	0
Gas	0	8.064657	25.87211	0
Iron	0	17.04925	25.87211	0
Lead	0	10.98429	25.87211	0
Nickel	0	12.62212	25.87211	0
Silver	0	10.49810	25.87211	0
Petrol	0	13.50285	25.87211	0
Tin	0	13.56408	25.87211	0
Zinc	0	22.86470	25.87211	0