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Search and Competition in Expert Markets

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**Abstract**. We analyze a model where consumers sequentially search experts for treatment

recommendations and prices, facing either zero or a positive search cost, while experts simulta-

neously compete in these two dimensions. In equilibrium, experts may "cheat" by overstating

the severity of a consumer's problem and recommending an unnecessary treatment, prices fol-

low distributions depending on the problem type and the treatment, and consumers employ

Bayesian belief updating about their problem types during search. Paradoxically, as search

cost decreases, expert cheating and prices can both increase stochastically. However, if search

cost is sufficiently small, competition will force all experts to behave honestly.

**Keywords**: search, experts, competition, credence good

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#### 1 Introduction

In markets for expert services, such as auto repair, home improvements, healthcare, and financial services, sellers often possess superior information about the service or product that a consumer needs, even after the consumer's purchase. A prevalent issue in these markets, often referred to as credence-good or expert markets, is that expert sellers may "cheat" by recommending unnecessary treatments to consumers with inflated prices. Consumers, aware of this risk, may search multiple experts, hoping that competitive pressures would yield honest recommendations and lower prices. But it can be costly to conduct search, and the impact of search cost on market dynamics is not well understood, particularly when competition among experts is based not only on price but also on the quality of their recommendations.

When sellers offer homogeneous products with known values to consumers, Stahl (1989) provides a seminal analysis of oligopoly price competition under consumer sequential search. As search frictions decrease, competition intensifies monotonically, and the classical Bertrand outcome (marginal cost pricing) and Diamond outcome (monopoly pricing) are obtained as the limiting cases of his model, respectively when search cost is zero and the fraction of consumers with zero search cost is zero.<sup>1</sup> In expert markets, if the nature of a consumer's problem were public information, competition could be analogous to that for homogeneous products. However, because only the experts may learn a consumer's problem and the appropriate treatment, an expert's recommendation and price can reveal information about whether he is being honest, and a consumer may perform Bayesian belief updating about her problem during search. This can substantially complicate the strategic choices of consumers and experts, but—as we will show in this paper—they can be fruitfully analyzed in a model building on Stahl (1989).

We consider an expert market in which each consumer has a problem that can be either major or minor. A major treatment can fix both types of the problem, but a minor treatment can only fix a minor problem. When visiting an expert, a consumer's problem is learned by

<sup>&</sup>lt;sup>1</sup>For differentiated products, a seminal contribution is Wolinsky (1986), in which market power also increases with search friction but equilibrium price is above marginal cost even as search cost goes to zero, because product differentiation also softens price competition.

the expert, who can then offer the consumer a recommended treatment at a certain price. At no time can a consumer observe her problem type, so the treatment by an expert is a credence good.<sup>2</sup> The consumer can either accept the offer or search other experts sequentially for additional offers. The expert is obligated to solve the consumer's problem if his offer is accepted, and the treatment performed is verifiable by the consumer. However, there can be higher profits from the major treatment than from the minor treatment, which provides an incentive for experts to overstate the severity of a consumer's problem and recommend the major treatment—possibly with some probability—even for the minor problem. We extend Stahl (1989) to study consumer search and expert competition in this environment, focusing especially on how search frictions shape equilibrium expert behavior. As in Stahl (1989), we assume that a fraction of the consumers are *shoppers* who have zero search cost to visit any expert, whereas the rest of the consumers are *searchers* who must incur a positive search cost to visit an expert.

In a symmetric perfect Bayesian equilibrium of the model, a shopper will purchase from the expert who can solve her problem at the lowest price, whereas searchers will adopt an optimal reservation price for each recommended treatment. The tension between attracting the shoppers and exploiting the searchers implies that, as in Stahl (1989), experts will choose treatment prices with mixed strategies, and they will always recommend the major treatment for a major problem but may cheat by recommending the major treatment also for the minor problem. Specifically, when search cost is above some threshold, initially experts will cheat when facing the minor problem with a probability that is strictly between 0 and 1, which we term as the hybrid equilibrium; whereas when search cost is high enough, the pooling equilibrium prevails where experts will always recommend the major treatment. However, when search cost is sufficiently small, the model has a unique separating equilibrium, where experts always make honest recommendations for both problem types.

Therefore, the magnitude of search cost plays a critical role in shaping equilibrium expert

<sup>&</sup>lt;sup>2</sup>For example, the air conditioner in a consumer's car is not cooling. The problem could be either a faulty compressor or inadequate refrigerant. Replacing the compressor will fix both types of the problem, but adding refrigerant can only fix the latter. An auto mechanic will know what the problem is but the consumer does not.

behavior. The hybrid equilibrium exhibits especially complex interactions between experts' cheating probability, prices, and consumer search, revealing a nuanced relationship between experts' cheating probability and search cost. In particular, an increase in experts' cheating probability negatively impacts consumers' search benefit because it reduces the likelihood to encounter an honest expert from another search, but it also positively impacts search benefit because equilibrium prices and their dispersions are higher. Either effect may dominate, and thus a consumer's search benefit can be a decreasing, increasing, or non-monotonic function of the cheating probability. Consequently, there exists a threshold of this probability such that as search cost decreases, expert cheating rises (falls) if the equilibrium cheating probability is below (above) this threshold.<sup>3</sup> While this result may seem surprising at first glance, it has the following simple intuition: When cheating is sufficiently common in the market (i.e., when the cheating probability is above the threshold), a marginal reduction in search cost leads to relatively more competition for dishonest experts, motivating experts to behave more honestly; but when cheating is not as common in the market, a marginal reduction in search cost leads to relatively more competition for honest experts, motivating experts to behave less honestly.<sup>4</sup>

It is also interesting that when search cost is small enough, despite experts' information advantage regarding the consumers' problem type, the equilibrium outcome in our model reduces to that in Stahl (1989). In this case, where the separating equilibrium prevails, the expected profits for the two treatments are the same and experts always report consumers' problems truthfully.<sup>5</sup> Then, the equilibrium price distribution for each treatment has the

<sup>&</sup>lt;sup>3</sup>The threshold is 1, 0, or strictly between 0 and 1 if search benefit is monotonically decreasing, monotonically increasing, or first decreasing and then increasing in cheating probability, respectively. The result holds also for the pooling and the separating equilibrium, if the "decreases" and "increases" are interpreted as "weakly decreases" and "weakly increases".

<sup>&</sup>lt;sup>4</sup>The threshold of the cheating probability depends on the extra cost for the major treatment and the number of sellers in the market, both of which can affect the relationship between a consumer's search benefit and the experts' cheating probability.

<sup>&</sup>lt;sup>5</sup>In the credence-goods literature, an important insight is that experts will not cheat if the price markups for the two treatments are equalized (Emons, 1997; Dulleck and Kerschbamer, 2006). Our result generalizes this insight to situations where prices follow mixed strategies and expected profits are equalized.

same form as in Stahl (1989). Therefore, although competition in expert markets with search cost generally works very differently from competition in other search markets, when search friction is sufficiently small, competition can effectively discipline experts, and the market operates as if consumers could observe their problem types. Furthermore, same as in Stahl (1989), the prices for the two treatments both approach their respective marginal costs when search cost approaches zero.

To the best of our knowledge, this is the first paper to study consumer search for both recommendations and prices in expert markets. It contributes to the literature on credence goods and expert markets by providing a framework to understand expert behavior when consumers can conduct costly searches.<sup>6</sup> The literature has studied how various mechanisms may stop experts from cheating and improve efficiency, such as separating diagnosis from treatment (e.g., Wolinsky, 1993), liability (e.g., Fong, 2005; Dulleck et al., 2011; Bester and Dahm, 2018; Chen et al., 2022), and reputation (e.g., Schneider, 2012; Fong et al., forthcoming). Several papers (Wolinsky, 1993, 1995; Pesendorfer and Wolinsky, 2003) have also examined the role of second opinions and expert competition, but in these studies prices are predetermined and observable to all consumers without costly search. Our model allows experts to compete in—and consumers to search for—both recommendations and prices, and we demonstrate that the interplays between consumer search and competition in these two dimensions can substantially change how expert markets function. As we shall discuss later, the equilibrium patterns of expert recommendations and prices that are revealed from our analysis, especially those in the hybrid equilibrium, are broadly consistent with the (anecdotal) evidence that experts sometimes recommend unnecessary treatments and prices for a treatment can vary widely in expert markets.

<sup>&</sup>lt;sup>6</sup>The literature often considers products or services in expert markets as credence goods (e.g., Darby and Karni, 1973; Taylor, 1995; Emons, 1997, 2001; Fong, 2005; Alger and Salanie, 2006; Liu, 2011). See Dulleck and Kerschbamer (2006) for a review of the earlier literature, and Balafoutas and Kerschbamer (2020) for more recent contributions.

<sup>&</sup>lt;sup>7</sup>Obradovits and Plaickner (forthcoming) also study expert markets with costly consumer search. In their model, consumers can seek treatment from an informed expert or purchase minor treatments from fringe firms, with costly search.

Our paper also contributes to the consumer search literature by studying search and competition when only sellers can observe product features that match buyers' needs. In the extant literature, consumers either know the product value before price search for a homogeneous product (e.g., Stigler, 1961; Stahl, 1989; Janssen et al., 2011), or they also search for a product's value either under horizontal differentiation (e.g., Wolinsky, 1986; Anderson and Renault, 1999; Haan and Moraga-González, 2011; Rhodes, 2011) or (additionally) under vertical differentiation (e.g., Bar-Issac et al., 2012; Chen and Zhang, 2018, Moraga-González and Sun, forthcoming). An exception is Chen et al. (2022), in which consumers search for product matches without observing product quality before purchase, but in their model of experience goods each seller offers only one product with a predetermined quality, there is no role for product recommendation, and all sellers set the same deterministic price in equilibrium. By contrast, in our model each seller may produce two products (either a major or a minor treatment), and his choice of recommendation may interact with prices to influence consumers' search and purchase decisions. Our finding that changes in search cost can have non-monotonic effects on prices is consistent with the results in the literature, but the channel through which this happens in our model is novel: lower search cost can increase false recommendations, which in turn leads to higher prices.

Our paper is closely related to Janssen et al. (2011), in which sellers have identical but stochastic production costs and the cost realization is unknown to consumers. In both papers, consumers update their beliefs in a Bayesian fashion when sequentially searching sellers, and in equilibrium all searchers purchase from their first-visited seller. One notable difference between the two papers is that sellers' production cost is unknown to consumers in Janssen et al. (2011), while experts' treatment cost is verifiable in our setting. Also, in our model

<sup>&</sup>lt;sup>8</sup>While prices unambiguously increase in search cost in seminal papers such as Stahl (1989) and Wolinsky (1986), later contributions have shown that reductions in search frictions can sometimes increase price for homogeneous products (e.g., Chen and Zhang, 2011) or for differentiated products (e.g., Bar-Isaac, Caruana, and Cuñat, 2012; Zhou, 2014; Moraga-González, Sandor, and Wildenbeest, 2017; Choi, Dai, and Kim, 2018; Chen et al., 2022).

<sup>&</sup>lt;sup>9</sup>The credence-goods literature has considered two alternative assumptions: the treatment is verifiable (e.g., Emons, 1997; Alger and Salanie, 2006; Chen et al., 2022), or non-verifiable (e.g., Wolinsky, 1993; Taylor, 1995; Fong, 2005; Liu, 2011). We adopt the former to focus on situations where an expert may be unethical

experts choose treatment (i.e., product) recommendations, whereas no such choice is made in Janssen et al. (2011); consequently, costs are exogenously determined in their model but depend on the experts' recommendation choice in ours. Moreover, Janssen et al. (2011) focus on how production cost uncertainty matters for market outcomes and welfare, whereas we emphasize the effects of search frictions on experts' cheating behavior. These differences make Janssen et al. (2011) especially suitable for retail markets such as gasoline, on which their analysis offers important insights; whereas our setting is more relevant for expert markets such as those for auto or home repairs, medical/dental treatment, and financial services.

In the rest of the paper, we present our model in Section 2, which also contains results in the benchmark where consumers know their problem types so that for each treatment our model reduces to a version of Stahl (1989)'s model. Section 3 analyzes the hybrid equilibrium. Section 4 studies how changes in search cost and the number of competing experts may affect the cheating probability and prices at the hybrid equilibrium. Section 5 characterizes the separating and pooling equilibria, and provides the conditions for their existence. Section 6 concludes.

#### 2 The Model

The market contains a unit mass of consumers and  $N \geq 2$  experts. Each consumer has a problem that needs to be treated by an expert. A consumer's problem can be one of two types: major (i = M) or minor (i = m), each occurring with probability  $\theta$  or  $1 - \theta$ . The realization of the problem type (i) is independent across consumers. Any expert can solve the consumer's problem by a major treatment  $(T = T_M)$  for  $i \in \{M, m\}$  or by a minor treatment  $(T = T_m)$  for i = m. We assume that each consumer is willing to pay at most  $V_i$  to have problem  $i \in \{M, m\}$  solved, with  $V_M \equiv V$  and  $V_m \equiv v$ . One natural interpretation of this in recommending an unnecessary treatment, but he does not commit a crime (theft) by billing customers a service that is not performed.

<sup>10</sup>This assumption of two problem types with two possible treatments is commonly made in the credence-goods literature. We also maintain this assumption for analytical tractability. Liu and Ma (2021), however, study a more general model in which a consumer's problem types are a continuum, though their analysis focuses on a monopoly expert.

assumption is that if problem i is not treated, the consumer will suffer a loss of  $-V_i$ . The costs of the treatments are  $c_i$  for i = M, m, where  $c_M = C > 0$  and  $c_m$  is normalized to 0. We assume  $V - C \ge v$ , so that an expert may potentially obtain a higher markup from the major treatment, which provides an incentive for the expert to cheat: recommend  $T_M$  when only  $T_m$  is needed to treat a consumer's problem.

Consumers, who do not know their problem types and initially also do not observe experts' prices, may sequentially search experts in random order for recommendations and prices. Following Stahl (1989), we assume that portion  $\lambda$  of consumers are shoppers who have zero cost to visit any expert, whereas proportion  $1 - \lambda$  of consumers are searchers who incur a search cost s > 0 to visit any expert except for a first visit. Whether a consumer is a shopper or a searcher is her private information.

The assumptions above aim to capture in a stylized way the key incentive issues in some familiar expert markets, in which multiple experts all can treat a range of consumer problems, without specialization for the treatment of a minor or a major problem alone. The experts are able to diagnose the problem and recommend the required treatment, such as why an air conditioner is not working and how to fix it, with price variations not only for different treatments but also for the same treatment from different providers; and the experts may sometimes recommend an unnecessary (more expensive) treatment. Some consumers—the shoppers—will search all available sellers to find the lowest price to treat a problem, possibly because they are "savvy" individuals who can access a price-comparison platform<sup>11</sup> or because, as argued in Stahl (1989), shoppers may enjoy searching for the lowest price; while other consumers—the searchers—will visit an additional seller only if the expected search benefit exceeds the search cost.<sup>12</sup>

<sup>&</sup>lt;sup>11</sup>For example, consumers may submit home repair or other service requests on a platform (e.g., hellotoby.com in Hong Kong and HomeAdvisor.com in the U.S.) to obtain price quotes simultaneously from multiple service providers, but only some consumers may know how to access such a platform.

<sup>&</sup>lt;sup>12</sup>Experimental and empirical evidence for expert service indicates that while some consumers conduct only a single search, others engage in multiple searches (Mimra et al., 2016; Wagner and Wagner, 1999); and some consumers may continue to search even after receiving a minor-treatment recommendation (Schneider et al., 2021).

An expert is obligated to solve a consumer's problem if his offer is accepted, and the type of treatment is verifiable, implying that an expert needs to incur cost C if he recommends a major treatment (even to fix the minor problem). Since problem M can be solved only with  $T_M$ , any expert will always recommend  $T_M$  for M. However, an expert may recommend either  $T_M$  or  $T_m$  for m. A strategy of expert j, j = 1, ..., N, can thus be denoted by  $\gamma^j = (\mathcal{F}^{ij}(p), \mathcal{F}^j(q), \alpha^j)$ , where  $\mathcal{F}^{ij}(p)$  is j's price distribution for  $T_M$  when  $i \in \{M, m\}$ ,  $\mathcal{F}^j(q)$  is j's price distribution for  $T_M$  when i = m, and  $\alpha^j$  is j's probability to recommend  $T_M$  when i = m. Notice that we allow experts to choose pure strategies in prices, in which case mixed-strategy price distributions  $\mathcal{F}^{ij}(p)$  and  $\mathcal{F}^j(q)$  would degenerate to singletons.

Upon visiting the  $t^{th}$  expert in her random search, if the expert recommends  $T_M$ , a consumer holds the belief that her problem is M or m respectively with probabilities  $\mu_t$  and  $1 - \mu_t$ , for t = 1, ..., N, where  $\mu_t$  may also depend on the expert's price for  $T_M$  and on offers from previously-visited experts (if t > 1). Because an expert cannot solve a major problem with  $T_m$ , a consumer will hold belief  $\mu_t = 0$  once she has received recommendation  $T_m$ . A shopper's strategy is to search all experts and then decide which expert's offer to accept (if she accepts an offer at all). As in Stahl (1989) and Janssen et al. (2011), each searcher follows a reservation price strategy, which specifies a pair of reservation prices  $(r(\mu_t), r_m)$  for  $(T_M, T_m)$  in her  $t^{th}$  visit under belief  $\mu_t$ : she will accept recommendation  $T_M$  at price  $p \le r(\mu_t)$ , and she will accept recommendation  $T_m$  at price  $q \le r_m$ . Clearly,  $r(\mu_t) \le V$  and  $r_m \le v$ . Notice that  $r(\mu_t)$  is generally a function of  $\mu_t$ . As we shall argue later, a reservation price strategy will indeed be optimal for the searchers, given the optimal strategy of the experts.

The timing of the game is as follows: First, experts simultaneously choose their strategies. Next, shoppers search all experts, while searchers may sequentially search experts. When seeing a consumer and learning her problem, expert j offers his recommendation and price to the consumer according to  $\gamma^j$ . The consumer may (a) accept the offer, (b) search another expert, (c) possibly return to accept the offer from a previously-visited expert with no additional search cost, or (d) exit the market without receiving a treatment. The game ends if (a), (c), or (d) occurs for every consumer.<sup>13</sup>

<sup>&</sup>lt;sup>13</sup>Notice that although a consumer can observe two prices respectively for major and minor treatments when

By assumption, an expert cannot publicly commit to treating only one problem before being visited by consumers. This, together with experts and consumers both being ex ante identical, allows us to focus on symmetric perfect Bayesian equilibrium (PBE) where all experts choose the same strategy and so do all consumers.<sup>14</sup> We can thus simplify notations by writing  $\gamma^j$  as  $\gamma$ . A PBE of our game is a profile of strategies by the experts and consumers, together with consumer beliefs, that satisfies:

- (i) For j = 1, ..., N, given each consumer's strategy and all other experts' strategies,  $\gamma$  maximizes expert j's expected profit and j has no incentive to change his strategy upon seeing any consumer.
- (ii) Given  $\gamma$  for j=1,...,N, each consumer chooses her strategy to maximize her expected surplus under her belief. Clearly, the optimal strategy for any shopper is to accept the offer from the expert with the lowest price, provided that this is better than no treatment (which—as it will become clear—must be true in equilibrium). Our equilibrium analysis will thus focus on the optimal strategy of the searchers, for whom the PBE imposes two requirements: given her belief and the experts' strategies, each searcher chooses her reservation price optimally, and it is indeed optimal for each searcher to follow a reservation price search strategy.
- (iii) Consumers' beliefs are derived from the Bayes' rule along the equilibrium path. As argued in Janssen et al. (2011), a "reasonable" assumption for off-equilibrium beliefs, which we shall also make, is the following: if  $p^*$  is an equilibrium price, then when a consumer observes an off-equilibrium price p' in a small neighborhood of  $p^*$ , i.e.,  $p' \in (p \epsilon, p + \epsilon)$ , her belief about the type of her problem associated with p' would be the same as that with  $p^*$ , i.e.,  $\mu_t(p') = \mu_t(p^*)$ . This assumption will play an important role in the construction of our visiting an expert, the expert will recommend only one treatment and is obligated to solve the consumer's problem if the recommended offer is accepted.

<sup>14</sup>If experts could make public commitments before consumers search, then experts might specialize in only treating the minor or the major problem, which could remove their cheating incentive (Wolinsky, 1993). In practice, an expert often treats a range of problems with different levels of severity, possibly due to reasons such as lacking commitment ability and economies of scope in product offerings.

<sup>15</sup>As pointed out in Janssen et al. (2017), the reservation price equilibria in Janssen et al. (2011) may not exist and whenever they exist, they require out-of-equilibrium beliefs that may be inconsistent with D1 criterion. In our paper, the possibility of having a separating equilibrium helps avoid the non-existence problem,

PBE, a key feature of which is that prices for both treatments are non-deterministic; however, as we will explain later, any PBE of our model must involve some randomization in prices for the minor treatment  $(T_m)$  under any off-equilibrium beliefs.

Our model may have three types of equilibria, in which experts always recommend  $T_M$  if i=M but differ in their recommendation for i=m: (i) a hybrid equilibrium where experts recommend  $T_M$  for m with probability  $\alpha \in (0,1)$ ; (ii) a separating equilibrium where experts always recommend  $T_M$  for m (i.e.,  $\alpha=0$ ); and (iii) a pooling equilibrium where experts always recommend  $T_M$  for m (i.e.,  $\alpha=1$ ). These equilibria may prevail in different regions of parameter values, and in each of them experts always adopt mixed strategies in prices, choosing prices from non-degenerate probability distributions. We will focus on the hybrid equilibrium but will also provide results for the separating and pooling equilibria. It seems more plausible that experts may sometimes—but not always—behave dishonestly in their recommendations, and we will discuss some (anecdotal) evidence consistent with the hybrid equilibrium after characterizing its main features in the next section.  $^{16}$ 

We conclude this section by considering a benchmark where each consumer can observe her  $i = \{M, m\}$ .

#### Benchmark: Problem Types are Observable to Consumers

In this case, there is no possibility of expert cheating. For each  $i \in \{M, m\}$ , our model is then the same as that in Stahl (1989). Following Stahl (1989), there is a unique symmetric equilibrium where experts price according to price distribution  $F_i(p)$  for  $i = \{M, m\}$  and consumers search with reservation price  $r_i^o \leq V_i$  for  $T_i$ . The equilibrium can be derived as follows.

First, notice that, as in Stahl (1989), there can be no symmetric equilibrium where experts adopt a pure strategy. Suppose that, to the contrary, in equilibrium  $p = p^*$  for  $T_M$ . Then, if  $p^* > C$ , an expert can profitably deviate by lowering his price slightly to attract all the and consistency is not an issue because each seller has identical costs to provide the major treatment to both consumer types so that D1 has no bite here.

<sup>&</sup>lt;sup>16</sup>Another motivation for our focus on the hybrid equilibrium is that it is more robust, in the sense that it is the only equilibrium if treatment is non-verifiable (see our discussion at the end of section 5 about how our results might change if treatment were not verifiable).

shoppers; while if  $p^* = C$ , an expert can profitably deviate by slightly raising the price which will be accepted by any searcher (who has a search cost s > 0). Similarly a deterministic price  $q = q^*$  for  $T_m$  cannot be sustained in equilibrium. Next,  $F_i(p)$  must be atomless, as any price associated with a probability mass will also induce profitable deviations. Moreover, the upper bound of  $F_i(p)$  is  $r_i^o$  for i = M, m, the reservation price of searchers in their sequential search.

For  $i \in \{M, m\}$  and for any price p generated from  $F_i(p)$ , in equilibrium

$$(p - c_i) \left[ \frac{1 - \lambda}{N} + \lambda \left( 1 - F_i \left( p \right) \right)^{N-1} \right] = (r_i^o - c_i) \frac{1 - \lambda}{N},$$

where  $(1 - F_i(p))^{N-1}$  is the probability that an expert can sell to a shopper and  $r_i^o$  is the highest price in the support of  $F_i(p)$ . The equilibrium price distribution is

$$F_{i}(p) = 1 - \left[ \frac{\left(r_{i}^{o} - p\right)\left(1 - \lambda\right)}{\left(p - c_{i}\right)\lambda N} \right]^{\frac{1}{N-1}} \quad \text{with} \quad p \in \left[b_{i}^{o}, r_{i}^{o}\right],$$
 (1)

where  $b_M^o = \frac{r_M^o(1-\lambda) + C\lambda N}{\lambda N + 1 - \lambda}$  and  $b_m^o = \frac{r_m^o(1-\lambda)}{\lambda N + 1 - \lambda}$ .

Define  $r_M^o$  as the solution to

$$\int_{b_M^o}^{r_M^o} (r_M^o - p) \, dF_M(p) = s. \tag{2}$$

We can rearrange the term on the left-hand side in the above equation, which is the search benefit—a consumer's benefit from another search—as

$$\int_{b_{M}^{o}}^{r_{M}^{o}} (r_{M}^{o} - p) dF_{M}(p) = r_{M}^{o} + \int_{b_{M}^{o}}^{r_{M}^{o}} pd[1 - F_{M}(p)].$$

Define  $x = 1 - F_M(p)$ , and rewriting p as a function of x by (1), the search benefit under  $F_M(p)$  becomes

$$r_M^o + \int_{b_M^o}^{r_M^o} pd[1 - F_M(p)] = (r_M^o - C)(1 - \phi), \tag{3}$$

where

$$\phi \equiv \int_0^1 \frac{1 - \lambda}{\lambda N x^{N-1} + 1 - \lambda} dx < 1,\tag{4}$$

and  $\phi$  is a constant for given  $\lambda$  and N. Notice that  $\phi$  is lower when  $\lambda$  is higher or N is lower; and  $\phi \to 0$  when  $\lambda \to 1$  while  $\phi \to 1$  when  $\lambda \to 0$ .

Throughout the paper, we assume:

$$s \le \bar{s} \equiv (1 - \phi) \left[\theta V + (1 - \theta)v - C\right]. \tag{5}$$

For given V and v, from (3),  $r_M^o = C + \frac{s}{1-\phi} \le \theta V + (1-\theta)v < V$ . As it will become clear later, when  $s \le \bar{s}$ , the consumer's reservation price is no higher than her expected willingness-to-pay for  $T_M$ , even in all equilibria of our model where problem types are not observable to consumers.

Furthermore, let  $r_m^o = \min\{v, \omega\}$ , where  $\omega$  solves

$$\int_{\frac{\omega(1-\lambda)}{\lambda N+1-\lambda}}^{\omega} (\omega - p) dF_m(p) = s$$
 (6)

or  $\omega = \frac{s}{1-\phi}$ . Then  $r_m^o$  uniquely exists. The unique equilibrium  $F_i(p)$  is characterized by (1), (2) and (6). Notice that search benefit is strictly increasing in reservation prices  $r_M^o$  for  $T_M$  and  $r_m^o$  for  $T_m$ , which implies that it is optimal for searchers to adopt a reservation price strategy under both  $T_M$  and  $T_m$ . The above discussion can be summarized in the following result that is due to Stahl (1989):

**Proposition 0 (Stahl, 1989)** Suppose that the problem types are observable to consumers. For  $s \leq \bar{s}$ , there exists a unique symmetric equilibrium such that each expert's equilibrium strategy is to recommend  $T_i$  and price according to  $F_i(p)$  for i = M, m. Searchers sequentially sample experts with reservation prices  $r_i^o \leq V_i$  for  $T_i$ .

We next return to the equilibrium analysis of our main model in which consumers do not observe their problem types.

# 3 Hybrid Equilibrium: Probabilistic Cheating

When only experts can privately learn a consumer's problem, they may cheat by recommending  $T_M$  even when i = m. This section analyzes the hybrid equilibrium where each expert cheats with probability  $\alpha \in (0,1)$ .

As in the benchmark case, here there is also no equilibrium in which experts choose deterministic prices. To see this, consider a candidate equilibrium where the price is  $q = q^*$ 

for  $T_m$  and  $p = p^*$  for  $T_M$ . Then, given that  $T_M$  can fix both types of the problem and C > 0, clearly  $q^* \le p^*$ . Next, any  $q^* > 0$  cannot be supported in equilibrium because an expert can profit from a deviation to a slightly lower price, while  $q^* = 0$  also cannot be supported in equilibrium because an expert can profitably deviate to a slightly higher price. Furthermore, this argument holds for any off-equilibrium belief, not just under our assumption that consumers maintain the equilibrium belief about the problem type (which in this case is i = m) for small price deviations, because even for a small upward price deviation at  $q^* = 0$ , the deviating seller could sell to all the searchers who visit it—under any belief they may have—given that they have search cost s to visit another expert.

Next, consider the candidate equilibrium price  $p = p^*$  for  $T_M$ . With all experts recommending  $T_M$  with probability 1 if i = M and with probability  $\alpha$  if i = m, each shopper, after seeing the recommendations from all experts, will form certain belief along the equilibrium path about the probability that her i = M. Clearly, some consumers must be willing to pay  $p^*$  for  $T_M$  in order for  $p^*$  to be an equilibrium price. Then, if  $p^* > C$ , an expert can deviate to a slightly lower price, for which consumers will still have the same belief as before under our assumption. It follows that the deviation is profitable to the expert by attracting all shoppers who would have purchased from other experts under  $p^*$ , and the deviation would not reduce the expert's demand from searchers. On the other hand, if  $p^* = C$ , an expert can profitably deviate to a slightly higher price to sell to searchers for whom he happens to be the first expert they visit.<sup>17</sup> Notice that our off-equilibrium belief assumption is important for the non-existence of deterministic prices for  $T_M$ : if, for example, the belief is i = m for a deviating price (slightly) below  $p^* > C$ , the searchers who would have purchased from the deviating seller may no longer do so, because the deviating price might exceed their reservation price for  $T_m$ , which could make the deviation unprofitable.

In a potential symmetric mixed-strategy equilibrium, suppose that experts choose p according to distribution F(p) when recommending  $T_M$  for i = M, choose p according to distribution G(p) when recommending  $T_M$  for i = m, and choose q according to distribution

<sup>&</sup>lt;sup>17</sup>For  $T_M$ , the experts may also choose a deterministic price  $p_1$  when i = M and  $p_2$  when i = m. From arguments similar to the above, there can be no deterministic equilibrium prices  $p_1^*$  or  $p_2^*$ .

H(q) when recommending  $T_m$  for i=m. From familiar arguments, the equilibrium price distributions are atomless.

Suppose that all searchers' reservation price for  $T_m$  is  $r_m = \min\{v, \omega\}$ , where  $\omega$  is defined in (6). We construct the equilibrium under the assumption that  $v < \omega$  and will later show that  $v \geq \omega$  is not consistent with any hybrid equilibrium. Because a searcher can return to a previously-searched expert without cost, in equilibrium her reservation price for  $T_M$  or  $T_{m}$  under a certain belief must not increase in t. Let  $\mu \equiv \mu_{1} \in [0,1]$ , and let  $\{r(\mu), r_{m}\}$ be the searchers' reservation prices for  $\{T_M, T_m\}$  in their first round of search. As we shall confirm later, in equilibrium consumers will indeed hold stationary belief  $\mu_t = \mu$  for all t and searchers will adopt a reservation-price search strategy. Then, the upper bound B of distributions F(p) and G(p) must be  $r \equiv r(\mu)$ , and the upper bound of H(q) must be  $r_m$ . To see this, suppose to the contrary that  $B \neq r$ . If B > r, then price p = B will not yield any sale to searchers during their first and possible future rounds of searches, and it will also not yield any sales to shoppers. By deviating to B=r, an expert will have a positive profit and hence the deviation is profitable. If B < r, since there is zero probability that B is the lowest price, an expert can profitably raise p = B to p = r, for which he will not lose any sales to shoppers but will have a higher profit from searchers who visit him and who will pay r instead of B < r. A similar argument establishes that the upper bound of H(q) must be  $r_m$  when the recommended treatment is  $T_m$  (noticing  $r_m = v$ ). Therefore, in equilibrium all searchers will purchase at their first visit.

In subsection 3.1 below, we derive the equilibrium price distributions and cheating probability  $\alpha$ , given the consumers' strategies. In subsection 3.2, we then derive the optimal consumer strategy under the equilibrium expert strategy and fully characterize the hybrid equilibrium.

 $<sup>^{18}</sup>$ In equilibrium, consumers will correctly infer that they have a minor problem when receiving the  $T_m$  recommendation, but because in another search they may encounter a dishonest expert who recommends  $T_M$ , which lowers their search benefit, their reservation price is weakly higher under asymmetric information than that under full information.

#### 3.1 Price Distributions and Cheating Probability

We start by deriving the equilibrium price distributions, given that shoppers will purchase from the lowest-priced expert and searchers will search with reservation prices  $(r, r_m)$  for  $(T_M, T_m)$ . We consider in turn the cases where a consumer has a major problem (i = M) and where she has a minor problem (i = m).

First, suppose that i = M. Then, upon seeing the consumer, any expert will recommend  $T_M$  with a price p randomly drawn from F(p). To determine F(p), notice that an expert earns the same expected profit for any  $p \in [b_f, r]$  in the symmetric mixed strategy equilibrium:

$$(p-C)\left[\frac{1-\lambda}{N} + \lambda \left(1 - F(p)\right)^{N-1}\right] = (r-C)\frac{1-\lambda}{N},$$

where the expert can sell only to searchers if he sets p = r. Thus, the equilibrium price distribution is

$$F(p) = 1 - \left[ \frac{(r-p)(1-\lambda)}{(p-C)\lambda N} \right]^{\frac{1}{N-1}} \quad \text{with} \quad p \in [b_f, r],$$
 (7)

where  $b_f = \frac{r(1-\lambda)+C\lambda N}{\lambda N+1-\lambda}$ . We have F(r) = 1,  $F(b_f) = 0$ , and the probability density is

$$f(p) = \frac{1}{N-1} \left[ \frac{(r-p)(1-\lambda)}{(p-C)\lambda N} \right]^{\frac{1}{N-1}-1} \frac{r-C}{(p-C)^2} \left( \frac{1-\lambda}{\lambda N} \right).$$
 (8)

Notice that the price distribution has the same form as that for i = M when consumers can observe their problem types. However, the equilibrium r (to be derived) will differ from  $r_M^o$  in the benchmark case, because in optimally choosing r a consumer will now take into account the possibility that an expert may cheat by recommending  $T_M$  even when i = m.

Next, suppose that i = m. For such a consumer, an expert will recommend  $T_M$  with probability  $\alpha$  under a price p that is randomly drawn from G(p). The expert earns equal profits from offering  $T_M$  to such consumers with any price p drawn from G(p) if

$$(p-C)\left[\frac{1-\lambda}{N}+\lambda\alpha^{N-1}\left(1-G\left(p\right)\right)^{N-1}\right]=\left(r-C\right)\frac{1-\lambda}{N},$$

where  $\frac{1-\lambda}{N}$  of the *m*-type searchers will first visit the expert and will pay for  $T_M$  at  $p \leq r$ , while  $\alpha^{N-1} (1 - G(p))^{N-1}$  is the probability that the expert can sell to a shopper with i = m

when other experts also cheat and price higher. Hence, G(p) is given by

$$G(p) = 1 - \frac{1}{\alpha} \left[ \frac{(r-p)(1-\lambda)}{(p-C)\lambda N} \right]^{\frac{1}{N-1}} \quad \text{for } p \in [b_g, r],$$
 (9)

where  $b_g = \frac{r(1-\lambda)+C\alpha^{N-1}\lambda N}{1-\lambda+\alpha^{N-1}\lambda N}$ . Moreover, G(r) = 1,  $G(b_g) = 0$ , and the probability density is

$$g(p) = \frac{1}{\alpha} \frac{1}{N-1} \left[ \frac{(r-p)(1-\lambda)}{(p-C)\lambda N} \right]^{\frac{1}{N-1}-1} \frac{r-C}{(p-C)^2} \left( \frac{1-\lambda}{\lambda N} \right). \tag{10}$$

Next, with i = m, any expert will recommend  $T_m$  with probability  $1 - \alpha$  under a price q randomly drawn from H(q). An expert earns equal profits from recommending  $T_m$  to consumers with i = m under any price q drawn from H(q) if

$$q\left\{\frac{1-\lambda}{N} + \lambda \left[\alpha + (1-\alpha)\left(1 - H\left(q\right)\right)\right]^{N-1}\right\} = v\left(\frac{1-\lambda}{N} + \lambda \alpha^{N-1}\right),$$

where we have further assumed that  $v < b_g$  so that prices under G(p) by experts who cheat are all higher than v.<sup>19</sup> Thus,

$$H\left(q\right) = \frac{1}{1-\alpha} \left\{ 1 - \left[ \frac{v(1-\lambda+\lambda N\alpha^{N-1}) - (1-\lambda)q}{\lambda Nq} \right]^{\frac{1}{N-1}} \right\} \text{ with } q \in [b_h, v], \qquad (11)$$

where  $b_h = \frac{1-\lambda+\lambda N\alpha^{N-1}}{1-\lambda+\lambda N}v$ . Moreover, H(v) = 1,  $H(b_h) = 0$ , and the probability density is

$$h\left(q\right) = \frac{1}{1-\alpha} \left[ \frac{v(1-\lambda+\lambda N\alpha^{N-1})}{\lambda Nq} - \frac{1-\lambda}{\lambda N} \right]^{\frac{1}{N-1}-1} \frac{v(1-\lambda+\lambda N\alpha^{N-1})}{(N-1)\lambda Nq^2}.$$

Finally, the experts must earn the same expected profit from recommending either  $T_M$  or  $T_m$  for i=m, 20 and hence  $v\left(\frac{1-\lambda}{N}+\lambda\alpha^{N-1}\right)=(r-C)\frac{1-\lambda}{N}$ , which implies that r and  $\alpha$  are positively related in the following way:

$$r = \frac{1 - \lambda + \lambda N \alpha^{N-1}}{1 - \lambda} v + C \equiv \tilde{r}(\alpha).$$
 (12)

Intuitively, from an expert's point of view, a higher r provides a higher incentive to recommend  $T_M$  for i=m, corresponding to a higher  $\alpha$ .

We summarize the price distributions, their properties, and the cheating probability for given r at a hybrid equilibrium as follows:

<sup>&</sup>lt;sup>19</sup>As we will see shortly in Lemma 1, this assumption is always satisfied in equilibrium.

<sup>&</sup>lt;sup>20</sup>Recall that in equilibrium all searchers will accept the first offer received. Hence, searchers with the minor problem will not have different search experiences that may affect their acceptance probability for major treatment.

**Lemma 1** In a hybrid equilibrium, the price distributions F(p), G(p), and H(q) are given by (7), (9), and (11), with the following properties: (i)  $b_f < b_g$  with  $F(b_g) = 1 - \alpha$ ; (ii)  $g(p) = \frac{1}{\alpha} f(p)$  for  $p \in [b_g, r]$ ; (iii)  $h(q) = \frac{1}{1-\alpha} f(q+C)$  for  $q \in [b_h, v]$ ; and (iv)  $b_g = v + C$  and  $b_f = b_h + C$ . Furthermore,  $\alpha$  is determined by (12) for given r.

#### **Proof.** See the appendix.

Part (i) in Lemma 1 implies that when a consumer receives a recommendation for  $T_M$  at a price  $p \in [b_f, b_g)$ , she can infer that the expert has made an honest recommendation: i = M, whereas when she receives a recommendation for  $T_M$  at a price  $p \in [b_g, r]$ , the true state can be either i = M or i = m.

Part (ii) in Lemma 1 implies that when  $T_M$  is being recommended at price  $p \in [b_g, r]$ , density g(p) is larger than f(p) with  $g(p) = \frac{1}{\alpha}f(p)$ . However, since an expert will recommend  $T_M$  when i = m only with probability  $\alpha$ , from the Bayes' rule a consumer's posterior belief when receiving recommendation  $T_M$  under a price  $p \in [b_g, r]$  is the same as her prior belief:

$$\mu(p) = \frac{\theta f(p)}{\theta f(p) + (1 - \theta)\alpha g(p)} = \theta \quad \text{for } p \in [b_g, r].$$
(13)

Interestingly, under recommendation  $T_M$ , a lower price,  $p \in [b_f, b_g)$ , signals that the problem is indeed M, whereas a higher price,  $p \in [b_g, r]$ , does not provide useful information. This is because if an expert chooses to cheat—recommending  $T_M$  when i = m—he is unlikely to sell to shoppers and would thus rather charge a higher price to earn a higher profit when selling to searchers.

Part (iii) suggests that the price density function h(q) for  $T_m$  under i=m is a shift to the left by C from the density function f(p) for  $T_M$  under i=M on  $[b_f,b_g]$ . Part (iv) is based on the idea that when an expert recommends  $T_M$  with a price  $p \in [b_g,r]$ , a searcher's belief is  $\mu = \theta$  (as indicated in (ii) above), and hence her reservation price r for  $T_M$  is the same under both F(p) and G(p). Therefore, both  $b_g$  and  $b_f$  are determined by the same r satisfying (12) that makes the expert indifferent between recommending  $T_M$  and  $T_m$  for i=m.

Figure 1 below illustrates the relations between F(p), G(p), and H(q).

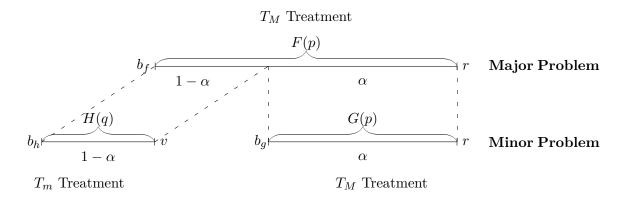


Figure 1: Price distributions.

#### 3.2 Optimal Consumer Search

We now characterize optimal consumer search given the experts' strategy described in the previous subsection. From the analysis of price distributions for our proposed equilibrium, a consumer's beliefs after receiving experts' offers of treatment and prices can be summarized in the following.

**Lemma 2** Upon receiving the offer from the  $t^{th}$  expert that she visits, a consumer's belief  $\mu_t$  is consistent with the experts' equilibrium strategies when, for all  $t \geq 1$ : (i)  $\mu_t = 0$  if at least one of her visited experts recommends  $T_m$  with price  $q \leq v$ ; (ii)  $\mu_t = 1$  if at least one of her visited experts recommends  $T_M$  with price  $p \in [b_f, b_g)$ ; and (iii)  $\mu_t = \frac{\theta_f(p_1)...f(p_t)}{\theta_f(p_1)...f(p_t) + (1-\theta)\alpha^t g(p_1)...g(p_t)} = \theta$  if all her visited experts recommend  $T_M$  with prices  $p_1, ..., p_t \in [b_g, r]$ .

Since  $\mu_t$  is either 0, 1, or  $\theta$ , independent of t, we can simply denote a consumer's belief by  $\mu$ . Thus, despite the dynamic nature of consumers' Bayesian belief updating, their beliefs are stationary given the experts' equilibrium strategy, which substantially facilitates the analysis. The analysis of our model is made tractable also by the observation that because sellers will optimally choose not to price above the searchers' reservation prices, in equilibrium all searchers will purchase during their first visit when undertaking sequential searches. Given  $\alpha$ , the price distributions, and belief  $\mu$  from Lemma 2, we can describe the optimal sequential search rule of a searcher as follows. (1) She will accept an offer that recommends  $T_m$  with price  $q \leq v$ . (2) She will accept an offer that recommends  $T_m$  with price  $p \leq v = r(\mu)$ , and

r satisfies

$$\mu \int_{b_{f}}^{r} (r-p) dF(p) + (1-\mu) \alpha \int_{b_{g}}^{r} (r-p) dG(p) + (1-\mu) (1-\alpha) \int_{b_{h}}^{v} (r-q) dH(q) = s.$$
(14)

In the left-hand side of (14), which is the search benefit from visiting another expert, the first term is the expected benefit from finding a lower price when i = M, the second term is the expected benefit from finding a lower price when i = m but the expert recommends  $T_M$ , and the third term is the expected benefit from finding a lower price when i = m and the expert recommends  $T_m$ . Equation (14) says that at the optimal r the search benefit is equal to the search cost (s).

So far, given experts' strategy, we have derived the searchers' (stationary) reservation prices, under the presumption that they follow a reservation price search strategy. We now argue that given experts' strategy, it is indeed optimal for searchers to adopt a reservation price strategy, which would be true if the search benefit is increasing in a sampled price under both  $T_m$  and  $T_M$ . Suppose first that a searcher is recommended  $T_m$ . Then, her belief is  $\mu = 0$ , and at the current offer  $\{T_m, q'\}$ , her benefit from another search is

$$(1-\alpha)\int_{q\leq q'}(q'-q)dH(q),$$

which clearly increases with the sampled price q'. Next, suppose that a searcher is recommended  $T_M$ . At the current offer  $\{T_M, p'\}$ , the potential complication is that as the sampled price p' increases, a searcher's belief may change. In particular, if a lower p' were associated with a lower  $\mu$ , then the search benefit could be higher at a lower p', because the lower  $\mu$  associated with p' would imply that with another search, it could be more likely for the searcher to encounter an honest expert that recommends  $T_m$  with a lower price. Fortunately, given the experts' strategy,  $\mu$  is weakly higher for lower p', which ensures that search benefit increases in p'. Therefore, it is indeed optimal for searchers to adopt a reservation price search strategy under both  $T_m$  and  $T_M$ .<sup>21</sup>

<sup>&</sup>lt;sup>21</sup>We may consider each shopper's expected value from having her problem solved, which depends on her belief about her problem type, as her reservation price: after searching all experts, she will purchase at the lowest price if it does not exceed her reservation price. For convenience, we sometimes also say that each shopper adopts a reservation price in search.

Utilizing Lemma 1, noticing  $\mu = \theta$  when a searcher receives recommendation  $T_M$  with a price  $p \in [b_g, r]$ , we can simplify the left-hand side of equation (14) and rewrite the equation when  $\mu = \theta$  as

$$\tau(\alpha) \equiv \int_{b_f}^{r} (r - p) dF(p) + (1 - \theta) (1 - \alpha) C = s, \tag{15}$$

where  $r = \tilde{r}(\alpha)$  satisfies (12). The search benefit  $\tau(\alpha)$  in the above equation has an intuitive interpretation. The first term, which we call the *price benefit*, is the benefit to a consumer from finding a lower price if i = M. The second term, which we call the *honesty benefit*, is the additional benefit from encountering an honest expert if i = m: the consumer then expects to pay a price that is lower by C than under  $T_M$ .

From (15),  $\int_{b_f}^r (r-p) dF(p) \leq s$ . It follows that  $r_M^o \geq r$ , where  $r_M^o$  satisfies (2).<sup>22</sup> Therefore, if a consumer receives a recommendation for  $T_M$  with a price  $p \in [b_f, b_g) < r$ , her updated belief is  $\mu = 1$  and she will pay for  $T_M$  without searching further. Also, since  $v < \omega$  (as we have assumed), the consumer will also pay for the treatment without further searching when she is recommended  $T_m$  with a price  $q \in [b_h, v] < r$ , under which her updated belief is  $\mu = 0$ .

To solve the equilibrium, it remains to examine how  $\alpha^*$  is determined in (15), where r and the price distributions are all functions of  $\alpha$ . It is useful to note that an increase in  $\alpha$  has two opposing effects on the search benefit  $\tau(\alpha)$ :

$$\frac{d\tau(\alpha)}{d\alpha} = \underbrace{\frac{\partial \tau(\alpha)}{\partial \alpha}}_{\text{lower honesty benefit(-)}} + \underbrace{\frac{\partial \tau(\alpha)}{\partial r} \frac{\partial r}{\partial \alpha}}_{\text{higher price benefit(+)}}.$$
(16)

An increase in  $\alpha$  has a negative direct effect on the search benefit: As  $\alpha$  rises, experts are more likely to cheat, which reduces the honesty benefit of search, as can be seen from the first term of (16). On the other hand, an increase in  $\alpha$  has a positive indirect effect on search benefit: As  $\alpha$  rises, so does  $r = \tilde{r}(\alpha)$  given by (12), which in turn stochastically increases the equilibrium prices under  $T_M$  and hence also the price benefit of search, as can be seen from the second term of (16) where  $\frac{\partial \tau(\alpha)}{\partial r} > 0$  from (15). We next show that, depending on parameter values,

<sup>&</sup>lt;sup>22</sup>Hence, if a consumer can observe her type, she will search with a higher reservation price for  $T_M$  than when she is recommended  $T_M$  but cannot observe whether her type is indeed i = M. In the latter case, there is a chance that her true type is i = m and she will receive a lower price if encountering an honest expert, which motivates her to lower the reservation price.

 $\tau(\alpha)$  can be a monotonically decreasing, monotonically increasing, or U-shaped function of  $\alpha$ . The result below refers to  $\hat{C}$  and  $\hat{\alpha}$  defined by

$$\hat{C} = \frac{(1-\phi)\lambda N(N-1)}{(1-\theta)(1-\lambda)}v, \qquad \hat{\alpha} = \left(\frac{C(1-\theta)(1-\lambda)}{v\lambda N(N-1)(1-\phi)}\right)^{\frac{1}{N-2}}, \tag{17}$$

where  $\phi < 1$  is given by (4) and  $\hat{\alpha}$  is defined only if N > 2.

Lemma 3 The search benefit function in (15) can be written as

$$\tau(\alpha) = (1 - \phi) \frac{1 - \lambda + \lambda N \alpha^{N-1}}{1 - \lambda} v + (1 - \theta)(1 - \alpha)C. \tag{18}$$

For all  $\alpha \in (0,1)$ : when  $C \geq \hat{C}$ ,  $\tau(\alpha)$  monotonically decreases; when  $C < \hat{C}$ ,  $\tau(\alpha)$  monotonically increases if N=2, but it first decreases and then increases—minimizing at  $\hat{\alpha} \in (0,1)$ —if N>2.

#### **Proof.** See the appendix.

To see the intuition about how  $\tau(\alpha)$  varies, we notice first that the honesty benefit is higher if C is larger (and it is independent of N). The price benefit is independent of C but depends on the number of competing experts. Therefore, if C is sufficiently large  $(C \ge \hat{C})$  for given N, then when  $\alpha$  increases, the reduction of the honesty benefit dominates, and hence  $\tau(\alpha)$  is decreasing for any  $N \ge 2$ , as illustrated in panel 2(a) of Figure 2.

Second, similar to Stahl (1989), the price benefit of search is high when prices are (stochastically) high. When  $\alpha$  is higher, so are  $r = \tilde{r}(\alpha)$  and prices. Hence, when  $C < \hat{C}$  and as  $\alpha$  increases, the price benefit of search dominates the reduction of honesty benefit either if N=2 or if N>2 and  $\alpha>\hat{\alpha}$ , so that  $\tau(\alpha)$  is increasing; but the price benefit is dominated if N>2 and  $\alpha<\hat{\alpha}$ , so that  $\tau(\alpha)$  is decreasing. In Figure 2, panels 2(b) illustrates the case where  $C<\hat{C}$  and N=2, whereas the bottom two panels illustrate the other cases, with 2(c) corresponding to the case where  $\tau(0)<\tau(1)$  and 2(d) to the case where  $\tau(0)>\tau(1)$ . Notice that when  $C<\hat{C}$ , the shape of  $\tau(\alpha)$  depends on N, because N affects the equilibrium price distribution and hence also the price benefit of search.

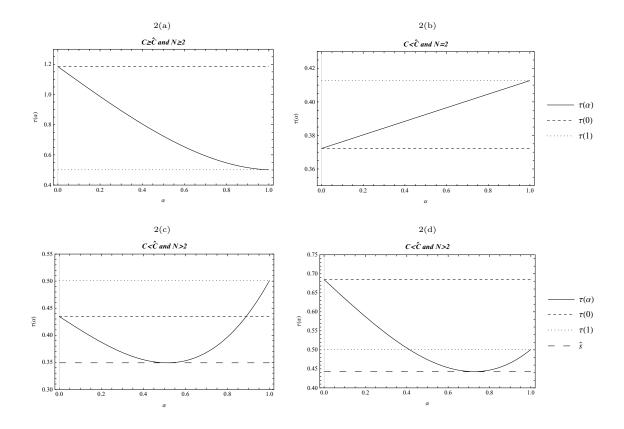


Figure 2: Search benefit  $\tau(\alpha)$  varies with  $\alpha$ . (Parameter values:  $\lambda = 0.3, \theta = 0.5, v = 0.8$ .)

We are now in a position to fully characterize the hybrid equilibrium. Define

$$\hat{s} \equiv \min_{\alpha \in [0,1]} \tau \left( \alpha \right). \tag{19}$$

By Lemma 3, when  $C \geq \hat{C}$ ,  $\tau(\alpha)$  decreases in  $\alpha$  and thus  $\hat{s} = \tau(1)$ ; when  $C < \hat{C}$  and N = 2,  $\tau(\alpha)$  increases in  $\alpha$  and thus  $\hat{s} = \tau(0)$ ; but when  $C < \hat{C}$  and N > 2,  $\tau(\alpha)$  is minimized at  $\hat{\alpha} \in (0,1)$  and thus  $\hat{s} = \tau(\hat{\alpha})$ .

**Proposition 1** Suppose that  $\hat{s} < s < \max\{\tau(0), \tau(1)\}$ . There exists a hybrid equilibrium, where  $\alpha^* \in (0,1)$  and  $r^*$  satisfy (12) and (15). Moreover,  $\alpha^*$  is unique if  $C \ge \hat{C}$  or if  $C < \hat{C}$  and N = 2; while  $\alpha^*$  may have either one or two values if  $C < \hat{C}$  and N > 2. Each expert's equilibrium strategy is: for i = M, recommend  $T_M$  and price from F(p); for i = m, recommend  $T_M$  and price from G(p) with probability  $\alpha^*$  but recommend  $T_m$  and price from G(p) with

probability  $1 - \alpha^*$ . Searchers sequentially search experts with reservation price  $r^*$  for  $T_M$  with  $p \in [b_g, r^*]$ ,  $r_M^o$  for  $T_M$  with  $p \in [b_f, b_g)$ , and v for  $T_m$ .

#### **Proof.** See the appendix.

The hybrid equilibrium has several notable features. First, each expert randomizes between recommending  $T_M$  and  $T_m$  for m, with the corresponding prices drawn from different distributions. While probabilistic cheating is a familiar equilibrium feature in the credencegoods literature, it is usually accompanied by random rejection of an expert's offer by consumers (as in the early contribution of Pitchik and Schotter, 1987). In our model, consumers adopt pure strategies, and an expert's indifference between honesty and dishonesty in equilibrium is due to the competition with other experts to balance the incentives to attract shoppers and to exploit searchers. Second, a price in the interval  $[b_f, b_g]$  indicates that the problem is M while a price in the interval  $[b_g, r^*]$  indicates the problem is M only with prior belief  $\theta$ . Hence, a lower price for  $T_M$  can be a signal that the expert is being truthful.<sup>23</sup> This result may seem surprising, but it actually has a simple intuition: if an expert chooses to recommend  $T_M$  when i=m, he is unlikely to sell to shoppers and thus prefers to charge a higher price that would allow him to earn a higher profit when selling to searchers. Third, there is a gap between H(q) and G(p):  $b_g = v + C$ , so that the prices for  $T_M$  when i = m are higher than the prices for  $T_m$  by at least C. This is because under our assumption of verifiable treatment, when recommending  $T_M$  for m, an expert needs to incur C and will thus generally charge a price that is higher by more than C than the price for  $T_m$  to compensate also for the reduced probability of sale.

The hybrid equilibrium is consistent with anecdotal evidence that experts sometimes recommend unnecessary treatments and charge widely different prices for the same service. For example, Emons (1997) cites reports in the U.S. and Germany that unnecessary repairs are frequently—but not always—recommended to car owners by service providers, and the price for similar auto bodywork at one shop can be as high as twice of that at another. Empirical

<sup>&</sup>lt;sup>23</sup>When consumers lack information about product quality, a well-known result in the literature on pricing under asymmetric quality information is that a high price can serve as a signal for high quality (e.g., Bagwell and Riordan, 1991).

studies have also found substantial price dispersions in expert markets, such as those for financial services including S&P 500 index funds (Hortaçsu and Syverson, 2004), mortgages (Allen et al., 2014), and insurance products (Brown and Goolsbee, 2002). While these studies do not focus on practices by experts to recommend unnecessary products to consumers in order to profit from higher commissions and fees, casual observations suggest that such practices are not uncommon, and future empirical work could investigate and potentially quantify the prevalence of such practices for different products. The equilibrium suggests two additional empirical predictions that can be potentially tested: (i) when a more expensive treatment is recommended, a dishonest expert—whose recommended treatment is unnecessary—is more likely to charge a higher price than an honest expert, and (ii) the price increase for the more expensive treatment is more than its cost increase.

# 4 Comparative Statics of Search Frictions and the Number of Experts

In this section, we provide some comparative statics with respect to search frictions and the number of experts at the hybrid equilibrium, under the assumption that  $\hat{s} < s < \max\{\tau(0), \tau(1)\}$ . Subsections 4.1 and 4.2 respectively study how search frictions affect equilibrium cheating probability and prices, while subsection 4.3 analyzes how equilibrium cheating probability is impacted by the number of experts in the market.

#### 4.1 Search Frictions and Equilibrium Cheating Probability

We are interested in whether under lower search frictions, in the sense that s is lower or  $\lambda$  is higher, competition by experts would reduce cheating in the market, with a lower  $\alpha^*$ .

From (15) and (18), the equilibrium expert cheating probability ( $\alpha^*$ ) and search cost (s) satisfy consumers' optimal search rule:  $\tau(\alpha^*) - s = 0$ , or

$$\Psi\left(\alpha^*,s\right) \equiv (1-\phi)\frac{v(1-\lambda+\lambda N\alpha^{*N-1})}{1-\lambda} + (1-\theta)(1-\alpha^*)C - s = 0.$$

We have  $\frac{\partial \Psi}{\partial s} = -1 < 0$ ,

$$\frac{\partial \Psi}{\partial \lambda} = -\frac{\partial \phi}{\partial \lambda} \frac{v(1 - \lambda + \lambda N \alpha^{*N-1})}{1 - \lambda} + (1 - \phi) v \frac{N \alpha^{*N-1}}{(1 - \lambda)^2} > 0$$

because  $\frac{\partial \phi}{\partial \lambda} < 0$ , and

$$\frac{\partial \Psi}{\partial \alpha^*} = \frac{\partial \tau}{\partial \alpha^*} = (1 - \phi) \frac{v \lambda N(N - 1) \alpha^{*N - 2}}{1 - \lambda} - (1 - \theta) C$$

may be either negative or positive, depending on the values of C and N. In particular,

(1) When  $C \ge \hat{C}$  (corresponding to the situation illustrated in 2(a) of Figure 2),  $\frac{\partial \Psi}{\partial \alpha^*} < 0$  and thus

$$\frac{\partial \alpha^*}{\partial s} = -\frac{\frac{\partial \Psi}{\partial s}}{\frac{\partial \Psi}{\partial \alpha^*}} < 0 , \quad \frac{\partial \alpha^*}{\partial \lambda} = -\frac{\frac{\partial \Psi}{\partial \lambda}}{\frac{\partial \Psi}{\partial \alpha^*}} > 0.$$

(2) When  $C < \hat{C} \equiv \frac{(1-\phi)\lambda N(N-1)}{(1-\theta)(1-\lambda)}v$  and N=2 (corresponding to the situation illustrated in 2(b) of Figure 2),  $\frac{\partial \Psi}{\partial \alpha^*} > 0$  and thus

$$\frac{\partial \alpha^*}{\partial s} > 0, \quad \frac{\partial \alpha^*}{\partial \lambda} < 0.$$

(3) When  $C < \hat{C}$  and N > 2 (corresponding to the situation illustrated in 2(c) and 2(d) of Figure 2) with  $\hat{\alpha}$  as defined in (17), we have

$$\frac{\partial \Psi}{\partial \alpha^*} < 0 \text{ and } \frac{\partial \alpha^*}{\partial s} < 0 \text{ if } \alpha^* < \hat{\alpha}, \qquad \frac{\partial \Psi}{\partial \alpha^*} > 0 \text{ and } \frac{\partial \alpha^*}{\partial s} > 0 \text{ if } \hat{\alpha} < \alpha^* < 1.$$

Cases (1) and (2) above provide sufficient conditions on exogenous parameter values under which equilibrium cheating probability decreases or increases as search frictions become more severe, summarized in the result below:

**Lemma 4** At the hybrid equilibrium, if  $C \ge \hat{C}$ , then  $\alpha^*$  decreases as s rises or  $\lambda$  falls; while if  $C < \hat{C}$  and N = 2, then  $\alpha^*$  increases as s rises or  $\lambda$  falls.

The shape of the search benefit function,  $\tau(\alpha)$ , is crucial for understanding Lemma 4. When  $C \geq \hat{C}$ , the loss in the honesty benefit of search from a higher  $\alpha$  dominates so that search benefit  $\tau(\alpha)$  monotonically decreases in  $\alpha$ . Hence, when s increases,  $\alpha^*$  falls to restore the condition that  $\tau(\alpha^*) = s$ . On the other hand, when  $C < \hat{C}$  and N = 2, as  $\alpha$  increases,

the gain in the price benefit dominates the loss in the honesty benefit of search, so that  $\tau(\alpha)$  increases. Then, when s increases,  $\alpha^*$  must also rise to restore optimal search.

Case (3) above provides additional sufficient conditions on the relation between equilibrium cheating probability and search frictions, but it involves the value of the endogenous variable  $\alpha^*$ . It turns out that we can state the relation, based on the value of  $\alpha^*$ , for all three cases above in a way that contains clear economic intuition. To do this, we define

$$\alpha^{c} = \begin{cases} \min \left\{ \hat{\alpha}, N-2 \right\} & if \quad C < \hat{C} \\ 1 & if \quad C \ge \hat{C} \end{cases}, \tag{20}$$

where  $0 < \alpha^c < 1$  if  $C < \hat{C}$  and N > 2. Then, when  $C < \hat{C}$  and either N = 2 or  $\alpha^* > \hat{\alpha}$ , which is equivalent to  $\alpha^* > \alpha^c$ ,

$$\frac{\partial \alpha^*}{\partial s} = -\frac{\frac{\partial \Psi}{\partial s}}{\frac{\partial \Psi}{\partial \alpha^*}} > 0, \quad \frac{\partial \alpha^*}{\partial \lambda} = -\frac{\frac{\partial \Psi}{\partial \lambda}}{\frac{\partial \Psi}{\partial \alpha^*}} < 0;$$

whereas if  $C \geq \hat{C}$  or if  $C < \hat{C}$  but N > 2 and  $\alpha^* < \hat{\alpha}$ , which is equivalent to  $\alpha^* < \alpha^c$ ,

$$\frac{\partial \alpha^*}{\partial s} < 0 , \quad \frac{\partial \alpha^*}{\partial \lambda} > 0.$$

From Lemma 4 and Case (3) above, we then immediately have the following:

**Proposition 2** Suppose that  $\alpha^* \in (0,1)$ . Then,  $\alpha^*$  increases in s and decreases in  $\lambda$  if  $\alpha^* > \alpha^c$ , but  $\alpha^*$  decreases in s and increases in  $\lambda$  if  $\alpha^* < \alpha^c$ .

Thus, at the hybrid equilibrium, if expert cheating in the market is pervasive enough  $(\alpha^* > \alpha^c)$ , increased competition due to lower search frictions can discipline experts, as one might expect. However, if cheating is rare enough in the market  $(\alpha^* < \alpha^c)$ , lower search frictions actually increase expert cheating (i.e.,  $\alpha^*$  rises).

To see the intuition for these results, first notice that as  $\lambda$  increases, there are more shoppers in the market who will purchase from the lowest-priced seller, and offering  $T_m$  for i=m is more likely to have the lowest price if more experts are currently cheating by offering  $T_M$  for m. Hence, if the cheating probability in the market is currently above a critical level  $(\alpha^* > \alpha^c)$ , a higher  $\lambda$  makes it relatively more attractive for an expert to be honest, decreasing

equilibrium cheating probability  $\alpha^*$ ; whereas if the cheating activity is currently below the critical level, competition for honest experts who offer  $T_m$  for m will be relatively more intense, and a higher  $\lambda$  increases the attractiveness of cheating, leading to a higher  $\alpha^*$ .<sup>24</sup>

Next, suppose that at a hybrid equilibrium associated with some  $(\alpha^*, r^*)$  there is a marginal decrease in search cost s. This will have similar effects on experts' cheating as an increase in  $\lambda$ , but through somewhat different mechanisms. When  $\alpha^* > \alpha^c$ , cheating is sufficiently common in the market and search benefit increases in  $\alpha$ . A decrease in s then leads to relatively more competition for dishonesty experts, as reflected by a reduction in the searchers' reservation price for  $T_M$  but not for  $T_m$  (i.e.,  $r^*$  falls but  $r_m^* = v$  is unchanged). This motivates experts to be more honest, resulting in a decrease in  $\alpha^*$ . On the other hand, when  $\alpha^* < \alpha^c$ , cheating is sufficiently uncommon in the market and search benefit decreases in  $\alpha$ . A reduction in s then leads to relatively more competition for honest experts who recommend  $T_m$  for m, as reflected by a rise in  $r^*$  while  $r_m^* = v$  is unchanged, causing  $r_m^*/r^*$  to fall. This motivates experts to cheat—recommending  $T_M$  instead of  $T_m$  for m—more, resulting in an increase in  $\alpha^*$ .

As Proposition 2 suggests, search friction impacts expert behavior through complex interactions between experts' cheating probability, consumers' reservation price in sequential search, and experts' prices. Importantly, price competition can change the relation between search cost and expert cheating through the effects on consumer search benefit. Sulzle and Wambach (2005) and Wolinsky (1993) have also explored this relationship and noted its possible non-monotonicity, but in these studies prices are (essentially) exogenously given and there is no price dispersion for each treatment. As such, search benefit tends to be low either when most experts are cheating or when they are honest, so that search benefit may be a concave function of cheating probability, implying that as search cost rises, equilibrium cheating probability can increase (decrease) when it is below (above) some critical level. In our model,

<sup>&</sup>lt;sup>24</sup>The change in  $\alpha^*$  depends on both the optimal search condition given by (15) and the experts' optimal choice between honesty and dishonesty given by (12). Because the effect of  $\lambda$  is more pronounced if  $\alpha$  is higher in (12), when  $\alpha$  is sufficiently high, an increase in  $\lambda$  would cause the right-hand side of (12) to exceed the endogenous r on the left-hand side, leading to a decrease in  $\alpha$  that restores (12); otherwise, the opposite change in  $\alpha$  can occur.

by contrast, the endogenous prices play a critical role in determining the benefit of search. In particular, equilibrium price and price dispersion tend to be high when expert cheating is high, so that search benefit can be high when cheating probability is high. Furthermore, when the equilibrium cheating probability is low, a searcher who is recommended  $T_M$  and maintains a posterior belief  $\theta$  has a high expected search benefit. Therefore, roughly speaking, search benefit is convex in cheating probability (as illustrated by Figure 2). Consequently, as search cost rises, equilibrium cheating probability decreases (increases) when it is below (above) some critical level.

#### 4.2 Search Frictions and Equilibrium Prices

To examine the effects of search frictions on equilibrium prices, it is convenient to denote the equilibrium price distribution when i = m as

$$\Phi(p) = \begin{cases} (1 - \alpha) H(p) & if \quad p \in [b_h, v] \\ 1 - \alpha & if \quad p \in (v, b_g) \\ 1 - \alpha + \alpha G(p) & if \quad p \in [b_g, r] \end{cases}$$

Since H(v) = 1,  $G(b_g) = 0$ , and both H(p) and G(p) increase in p,  $\Phi(p)$  is continuous and weakly increases in p. Moreover,  $\Phi(r) = 1 - \alpha + \alpha G(r) = 1$ , and  $\Phi(b_h) = (1 - \alpha)H(b_h) = 0$ . Therefore,  $\Phi(p;\alpha)$  is a continuous c.d.f.

The following lemma is helpful for understanding the comparative statics on prices.

**Lemma 5** Both F(p) and  $\Phi(p)$  decrease in  $\alpha$ .

#### **Proof.** See the appendix.

Lemma 5 indicates that equilibrium prices are increasing in  $\alpha$  in the sense of first-order stochastic dominance (FSD). Notice that F(p) and  $\Phi(p)$  depend on s only through  $\alpha$  from (12) and (15). Thus, from Proposition 2, as s increases,  $\alpha^*$  and hence equilibrium prices are higher if  $\alpha^* > \alpha^c$  but lower if  $\alpha^* < \alpha^c$ . We thus have:

**Proposition 3** Suppose that prices are compared in the sense of FSD. Then, at a hybrid equilibrium, as search cost increases, both equilibrium prices and cheating probabilities are higher if  $\alpha^* > \alpha^c$  and both are lower if  $\alpha^* < \alpha^c$ .

The effects of search cost on equilibrium prices and cheating probabilities are connected in interesting ways: they either both fall or both rise as search friction increases. Intriguingly, in an expert market, a reduction in search cost can hurt all consumers when the current level of expert cheating is relatively low, because in this case lower search cost will increase competition relatively more for honest experts and thus motivate experts to increase the frequency of recommending  $T_M$  for m, resulting in a higher  $\alpha^*$ . This in turn reduces consumers' search incentive, leading to higher equilibrium prices.

#### 4.3 Effects of Changes in the Number of Experts

Consider first the effects of a change in the number of experts on the equilibrium cheating probability. The analysis is complicated, partly because from Lemma 3 search benefit  $\tau(\alpha)$  can be a monotonically increasing, monotonically decreasing, or U-shaped function of  $\alpha$ , depending on whether N=2 or N>2. To obtain clear statements about the relation between  $\alpha^*$  and N, we treat N as a continuous variable and impose condition

$$1 + N \ln \alpha^* < 0, \tag{A1}$$

which ensures that  $\tau\left(\alpha\right)$  shifts down as N increases. The condition is equivalent to  $\alpha^* < e^{-\frac{1}{N}}$  and is satisfied if, for example, N=2 and  $\alpha^*<0.61$ , N=3 and  $\alpha^*<0.72$ , N=4 and  $\alpha^*<0.77$ , or N=5 and  $\alpha^*<0.82$ .

**Proposition 4** Suppose (A1) holds. Then: (i)  $\alpha^*$  decreases in N if  $C \geq \hat{C}(N)$  or if  $C < \hat{C}(N)$ ,  $\alpha^* < \hat{\alpha}$ , and  $N \geq 3$ .(ii)  $\alpha^*$  increases in N if  $C < \hat{C}(N)$  and N = 2 or if  $\alpha^* > \hat{\alpha}$  and  $N \geq 3$ .

#### **Proof.** See the appendix.

Condition (A1) ensures that  $\tau(\alpha)$  shifts down as N increases. On the other hand, from Lemma 4 and the discussion leading to Proposition 2, (i)  $\tau'(\alpha) > 0$  if  $C < \hat{C}(N)$  and N = 2 or if  $\alpha^* > \hat{\alpha}$  (when  $N \ge 3$ ), while (ii)  $\tau'(\alpha) < 0$  if  $C \ge \hat{C}(N)$  or if  $C < \hat{C}(N)$  and  $\alpha^* < \hat{\alpha}$  (when  $N \ge 3$ ). Since  $\alpha^*$  satisfies  $\tau(\alpha^*) = s$ , an increase in N, which shifts down  $\tau(\alpha)$ , leads to a higher  $\alpha^*$  in case (i) but to a lower  $\alpha^*$  in case (ii).

The effects of changes in N on equilibrium prices are even more complex, because N not only affects prices indirectly through  $\alpha$ , but it also enters directly in the equilibrium price distributions given by (7), (9) and (11). Nevertheless we are able to shed light on how N impacts the equilibrium expected prices through numerical analysis. At the hybrid equilibrium, let  $P_M$  and  $P_m$  be the expected prices paid by a searcher when her problem type is M or m, and let  $p_M$  and  $p_m$  be the expected prices paid by a shopper when her problem type is M or m. We first have:

**Lemma 6**  $P_M = (r - C)\phi + C$ ,  $P_m = (r - C)\phi + \alpha C$ ,  $p_M = (r - C)\rho + C$ , and  $p_m = (r - C)\rho + \alpha^N C$ , where  $\phi$  is given by (4) and

$$\rho = \int_0^1 \frac{(1-\lambda)Nx^{N-1}}{1-\lambda + \lambda Nx^{N-1}} dx.$$
 (21)

#### **Proof.** See the appendix.

Considering that equilibrium prices and expert cheating probability are simultaneously determined, we illustrate their comparative statics with respect to N jointly in Table 1 below, where  $\lambda = 0.3$ ,  $\theta = 0.5$ , and v = 1.

Table 1: Cheating Probability and Prices Change with N

 $C = 0.1 < \widehat{C}, \ s = 0.35$ 

N	$\alpha^*$	$P_M$	$P_m$	$p_M$	$p_m$
2	0.458	4.006	2.379	3.903	1.532
3	0.403	3.904	2.114	3.711	0.908
4	0.367	3.834	1.935	3.584	0.639
5	0.347	3.811	1.851	3.513	0.528

 $C = 3 > \hat{C}, \ s = 1.2$ 

As indicated in Table 1, when  $C < \widehat{C}$ ,  $\alpha^*$ ,  $P_M$ ,  $P_m$ ,  $p_M$  and  $p_m$  all increase in N, while when  $C \ge \widehat{C}$ , they all decrease in N. Intriguingly, an increase in the number of sellers can increase both expert cheating and prices. The intuition for this is somewhat similar to that in Stahl (1989) where sellers compete only in prices: as N rises, an honest expert that offers  $T_m$  for m is less likely to be the lowest-priced seller, which provides incentives for experts to increase

cheating and raise prices. However, in our model experts also choose product offerings (i.e., recommendations), and the cheating incentive is influenced also by the values of both C (the cost of  $T_M$ ) and search cost (s).<sup>25</sup> Therefore, unlike in Stahl (1989), where an increase in the number of sellers always increases expected prices, here the cheating probability and expected prices can both be lower as N increases. It is worth noting that our result differs sharply from Wolinsky (1993), where the number of experts does not affect equilibrium cheating probability. By considering consumer search and expert competition in both prices and recommendations, our analysis yields new insight on how competition may affect expert behavior.

## 5 Separating and Pooling Equilibria

When search cost is sufficiently low or high, the model has equilibria that differ from the hybrid equilibrium.

### 5.1 Separating Equilibrium

When search cost is sufficiently low, there is a separating equilibrium where all experts recommend  $T_i$  for  $i \in \{M, m\}$ . At the equilibrium, experts will recommend  $T_M$  for i = M with prices drawn from  $F_M(p)$  given by (1), and searchers who are recommended  $T_M$  will search with reservation price  $r_M^o$  given by (2); whereas experts will recommend  $T_m$  for i = m with prices drawn from  $F_m(p)$  given by (1), and searchers who are recommended  $T_m$  will search with reservation price  $r_m^o = \min\{v, \omega\}$ .

To establish the equilibrium, it remains to show that no expert can benefit from choosing  $T = T_M$  for i = m. Suppose that a searcher with i = m, who is willing to pay  $p = r_M^o$  for  $T_M$ , visits such a deviating expert and mistakenly believes that her problem is i = M. Since other experts will still recommend  $T_m$  for i = m and price according to  $F_m(p)$ , resulting in stochastically lower prices than under  $F_M(p)$ , the deviating expert is less likely to sell to shoppers than the other experts. This implies that the most profitable deviation is for the

<sup>&</sup>lt;sup>25</sup>When C and s are high, prices for minor treatment  $(T_m)$  are relatively high and cheating has relatively high opportunity costs, so that the increased competition due to an increase in N tends to reduce average prices and boost expert honesty.

deviating expert to offer  $T = T_M$  for i = m with price  $p = r_M^o$ . The expert's profit under this deviation is

$$(r_M^o - C) \frac{1-\lambda}{N},$$

whereas his profit when following the equilibrium strategy is

$$r_m^o \frac{1-\lambda}{N}$$
.

Hence, the separating equilibrium can be sustained if and only if

$$r_m^o \ge r_M^o - C.$$

**Proposition 5** If  $s \leq v(1-\phi)$ , there is a unique symmetric equilibrium in which experts are honest  $(\alpha^* = 0)$ . Equilibrium price distribution and optimal consumer search rules are the same as in the case where consumers can observe  $i \in \{M, m\}$ .

#### **Proof.** See the appendix.

Although the market in our model has search frictions, Proposition 5 indicates that if search cost is low enough, competition can effectively discipline experts so that they will all behave honestly.<sup>26</sup> Intuitively, as s becomes small enough, the price distribution H(q) shrinks so that its upper bound becomes  $r_m = \omega$ , under which the expected profit under  $T_m$  is the same as that under  $T_M$ . Experts will then have no incentive to offer  $T_M$  for m. Recall that  $\phi$  is lower when  $\lambda$  is higher or N is lower, and  $\phi \to 1$  as  $\lambda \to 0$ . Hence, the region of parameter values under which the separating equilibrium prevails is larger when  $\lambda$  is higher or N is lower, but the region vanishes as  $\lambda \to 0$ . This confirms that the presence of shoppers who can search without cost is essential for the existence of the separating equilibrium.

A key insight in the literature on credence goods is that experts will provide honest recommendations if there are equal markups for  $T_M$  and  $T_m$ . Our result extends this insight to situations under mixed-strategy pricing with consumer search: the experts will behave

<sup>&</sup>lt;sup>26</sup>Notice that in our model a fraction of consumers have no search cost. If all consumers have a positive search cost, then the Diamond (1971) result holds: no matter how small the search cost is, experts will charge the monopoly price  $\theta V + (1 - \theta) v$ , and the separating equilibrium does not exist.

honestly when they expect to receive the same expected profit from  $T_M$  and  $T_m$  for problem m. Notice that from (18) and (19),  $\min \{\tau (0), \tau (1), \hat{s}\} > v (1 - \phi)$ .

Proposition 5 also extends Stahl (1989) to expert markets: When s is sufficiently small, experts will price and consumers will search in the same ways as in Stahl (1989), even though—unlike in Stahl (1989)—here consumers do not observe the value of the service they receive. Moreover, as  $s \to 0$ ,  $r_M^o = C + \frac{s}{1-\phi}$  and  $\omega = \frac{s}{1-\phi}$  approach C and 0, the respective marginal costs for  $T_M$  and  $T_m$ . Hence, same as in Stahl (1989), the Bertrand outcome is the limiting case of our model of expert markets when search cost tends to zero.

#### 5.2 Pooling Equilibrium

When search cost is high enough, experts will always cheat, which yields a pooling equilibrium where experts always recommend  $T_M$  and follow the same pricing strategy for  $i \in \{M, m\}$ . Similar to the result in Lemma 1, experts would then price according to F(p) and G(p) respectively for i = M and i = m, which have the same upper bound. Setting  $\alpha = 1$  in G(p), we have  $b_g = b_f$  and  $G(p) = F(p) = F_M(p)$ . The equilibrium upper bound for the common price distribution is then  $r_M^o$ , as given by (2).

At the proposed pooling equilibrium, if an expert deviates to  $T_m$  when i = m, it can save cost C and potentially capture all shoppers. The expert's optimal deviating price in this case is v, while he still prices according to  $F_M(p)$  if i = M. If s is high enough, such a deviation would not be profitable because the price reduction to v would be too large.

**Proposition 6** If  $\tau(1) \leq s \leq \bar{s} = (1-\phi)[\theta V + (1-\theta)v - C]$ , there is a symmetric equilibrium in which experts always recommend  $T_M$  (i.e.,  $\alpha^* = 1$ ) and price according to  $F_M(p)$  for  $i \in \{M, m\}$ . All searchers will search with reservation price  $r_M^o \leq \theta V + (1-\theta)v$ .

#### **Proof.** See the Appendix.

Intuitively, if s is high enough, search benefit is likely below s, which means that searchers have low incentives to search. Then, experts will charge high prices for treatment  $T_M$  (but a price only up to v for treatment  $T_m$ ). This motivates experts to always recommend  $T_M$  for m in equilibrium ( $\alpha^* = 1$ ), resulting in the pooling equilibrium. As s decreases, consumers

search more intensively, which imposes downward pressure to the prices for both treatments; and when s is low enough, experts will cheat only with probabilities  $\alpha^* < 1$ , resulting in a hybrid equilibrium. Notice that the hybrid equilibrium and the pooling equilibrium coexist if  $\tau(1) < s < \tau(0)$ . When s is sufficiently low, v will not be a binding constraint for the price of  $T_m$ , and recommending  $T_i$  for  $i \in \{M, m\}$  is then optimal for experts because they need to incur C without getting a much higher price from  $T_M$  than from  $T_m$ , making cheating not profitable. However, there may be a (small) region of s for which a symmetric equilibrium fails to exist. Notice that for given s,  $\tau(1) \leq s$  holds if s is small enough. Hence, a pooling equilibrium exists as s approaches s, and in this case the outcome in Stahl (1989) is also a limiting case of our model.

Notice that in the interior regions of s and  $\lambda$  in which the separating or pooling equilibrium exists, a marginal change in search frictions (either s or  $\lambda$ ) has no effect on  $\alpha^*$ , which is either 0 or 1. Hence, our results on the effects of search frictions on  $\alpha^*$ , based on the hybrid equilibrium, holds weakly at all equilibria of the model.

Finally, returning to the verifiability issue of treatment, we have assumed that an expert's treatment is verifiable to focus on situations where an expert may give biased or untruthful recommendations but he does not engage in outright theft by billing a service that is not performed. While both activities may occur in expert markets, we feel that our framework is more suitable to analyze the former. Although the treatment needed for the consumer's problem is a credence good in nature, the actual treatment—such as a new compressor for the air conditioner or a new dental crown placed on a tooth—is likely verifiable ex post, and legal liability can be imposed on false treatment claims.

Nevertheless, if we modify our model to assume instead that treatment is non-verifiable, then it can be shown that neither a separating nor a pooling equilibrium would exist,<sup>27</sup> but a hybrid equilibrium would. However, in this case the equilibrium price distributions for  $T_M$  and  $T_m$  under m will have no gap, because the experts would use treatment  $T_m$ , without incurring

<sup>&</sup>lt;sup>27</sup>In this case, at a candidate separating equilibrium, an expert would always deviate to recommending  $T_M$  for m. At a candidate pooling equilibrium,  $T_M$  and  $T_m$  would have same prices. This would lead to different expected profits under M and m because only  $T_M$  costs C, which would in turn invalidate the equilibrium.

C, even when recommending  $T_M$  for m.<sup>28</sup> The waste of C to treat a minor problem is then avoided. Therefore, within our framework, if s is sufficiently small, in equilibrium experts will behave honestly when treatment is verifiable but not when it is unverifiable, whereas welfare is the same under the two alternative assumptions because in neither case experts would incur C for m. If s is higher, there is cheating in equilibrium under both assumptions. Not surprisingly, experts are more likely to cheat and earn higher expected profits—but welfare is weakly higher due to the avoidance of C for m—if treatment is not verifiable than when it is.

## 6 Conclusion

This paper has developed and analyzed a model of search and competition in expert markets. We extend Stahl (1989) to introduce sellers' private information about the appropriate treatment/service for consumers. The model shows that, due to search cost, for the same problem consumers may receive divergent recommendations and prices from different experts. Some experts may cheat by recommending an unnecessary treatment, and the dishonest experts also charge higher prices on average. Consumers search experts sequentially under Bayesian belief updating and with an optimal reservation price for each recommended treatment. The model further shows that search frictions can affect expert behavior non-monotonically: as they decrease, expert cheating can fall if it already occurs frequently enough in the market, but it can rise otherwise.

Despite the central importance of competition for economic efficiency, it is not surprising that competition may not work well when sellers possess superior product information relative to consumers. A novel insight of this paper, however, is that search cost can be an especially greater barrier to effective competition in expert markets. In fact, in our model if search cost is below some critical level, competition will drive all experts to make honest recommendations, and the equilibrium outcome coincides with that of Stahl (1989). Thus, a clear way to achieve efficiency gains from competition in expert markets is to make consumer

<sup>&</sup>lt;sup>28</sup>The analysis is lengthy but largely parallels the analysis when treatment is verifiable and also the analysis in Janssen et al. (2011). We thus spare the readers from the detailed analysis under this alternative assumption. An appendix that contains this detailed analysis is available upon request.

search sufficiently convenient, even if it does not entirely eliminate search cost.<sup>29</sup> However, in practice, because search cost may often be relatively high and its (marginal) reductions can—as we have shown—have non-monotonic effects, the role that competition plays in disciplining expert behavior is likely to be limited. This sentiment is echoed further by our finding that increases in the number of competing experts can result in more cheating.

We have built on Stahl's classical model of homogenous product as a first attempt to study expert markets with consumer search. For future research, it would be desirable to consider a setting with differentiated products such as in Wolinsky (1986). It is also desirable to relax some of the assumptions in our model. For example, it would be interesting to study models in which some consumers know which type of treatments they need (Jost et al., 2021), consumers have more dispersed search costs (Stahl, 1996), or experts' incentives to overprescribe services depend on their queues (Chiu and Karni, 2021; Karni, 2022).

Although not considered in our model, many products in expert markets may not be pure credence goods, in the sense that there is a (small) probability that a dishonest expert will be found to have been untruthful. Extending our model to include such a possibility will not change the analysis and results if consumers have no recourse ex post after detecting an expert's cheating, but it suggests that regulations can improve the performance of expert markets. For instance, regulators may be able to promote or set higher standards for professional services, inspect or gather information about the works performed by experts, and warn consumers about dishonest experts, especially when experts may interact with different consumers over time but each individual consumer lacks the knowledge about them. Regulations may also impose liabilities for experts who fail to fulfil obligations and for unethical practices (as for instance in medical malpractices). By showing the limits to effective competition in expert markets due to search frictions, our paper suggests the need for regulation in such markets, even when they may appear to be highly competitive with numerous providers.

<sup>&</sup>lt;sup>29</sup>In particular, digital platforms that enable consumers to easily make price and quality comparisons for expert services can alleviate the adverse effects of search cost.

## **APPENDIX**

The appendix contains proofs for Lemmas 1, 3, 5, and 6, and Propositions 1, 4, 5, and 6. **Proof of Lemma 1.** It suffices to prove properties (i)-(iv).

(i) Since  $b_g$  decreases in  $\alpha$  we have

$$b_g = \frac{r(1-\lambda) + C\alpha^{N-1}\lambda N}{1-\lambda + \alpha^{N-1}\lambda N} > \frac{r(1-\lambda) + C\lambda N}{1-\lambda + \lambda N} = b_f$$

for r > C. It is straightforward to also verify that  $F(b_g) = 1 - \alpha$  from (7).

- (ii) From comparing (8) and (10), we immediately have  $g(p) = \frac{1}{\alpha} f(p)$  for  $p \in [b_g, r]$ .
- (iii) From (11) and (12),

$$h(q) = \frac{1}{1-\alpha} \left[ \frac{v(1-\lambda+\lambda N\alpha^{N-1})}{\lambda Nq} - \frac{1-\lambda}{\lambda N} \right]^{\frac{1}{N-1}-1} \frac{v(1-\lambda+\lambda N\alpha^{N-1})}{(N-1)\lambda Nq^2}$$

$$= \frac{1}{1-\alpha} \left[ \frac{(r-q-C)(1-\lambda)}{\lambda Nq} \right]^{\frac{1}{N-1}-1} \frac{(r-C)(1-\lambda)}{(N-1)\lambda Nq^2}$$

$$= \frac{1}{1-\alpha} f(q+C).$$

(iv) Substituting the r from (12) into

$$b_g = \frac{r(1-\lambda) + C\alpha^{N-1}\lambda N}{1-\lambda + \alpha^{N-1}\lambda N}$$
 and  $b_f = \frac{r(1-\lambda) + C\lambda N}{\lambda N + 1 - \lambda}$ ,

we obtain  $b_g = v + C$  and  $b_f = b_h + C$ .

**Proof of Lemma 3.** By the argument leading to (3) and from (12),

$$\int_{b_f}^{r} (r - p) dF(p) = (r - C)(1 - \phi)$$
$$= (1 - \phi) \frac{v(1 - \lambda + \lambda N \alpha^{N-1})}{1 - \lambda}.$$

Hence, the search benefit in (15) can be rewritten as (18), which is clearly positive. We then have

$$\tau'(\alpha) = -(1 - \theta)C + (1 - \phi)\frac{\lambda N(N - 1)\alpha^{N - 2}}{1 - \lambda}v$$
(22)

and

$$\tau''(\alpha) = (1 - \phi) \frac{\lambda N(N - 1)(N - 2)\alpha^{N - 3}}{1 - \lambda} v \ge 0,$$

where the weak inequality holds strictly if N > 2. Hence,  $\tau(\alpha)$  is a (weakly) convex function. When  $C \ge \hat{C}$ ,  $\tau'(\alpha) < 0$  for all  $\alpha \in (0,1)$ . When  $C < \hat{C}$ ,  $\tau'(\alpha) > 0$  for all  $\alpha \in [0,1]$  if N = 2; but if N > 2,  $\tau(\alpha)$  is minimized at

$$\hat{\alpha} = \left(\frac{C(1-\theta)(1-\lambda)}{(1-\phi)\lambda N(N-1)v}\right)^{\frac{1}{N-2}} < \left(\frac{\hat{C}(1-\theta)(1-\lambda)}{(1-\phi)\lambda N(N-1)v}\right)^{\frac{1}{N-2}} = 1.$$

Obviously  $\hat{\alpha} > 0$ .

**Proof of Proposition 1.** The equilibrium strategies of the experts and consumers follow directly from the construction of the equilibrium. Notice that, from (18),  $\tau(\alpha) > v(1 - \phi) \frac{1-\lambda+\lambda N\alpha^{N-1}}{1-\lambda} > v(1-\phi)$ . Hence, for  $s > \hat{s} = \min_{\alpha \in [0,1]} \tau(\alpha) > v(1-\phi)$ ,  $\omega = \frac{s}{1-\phi} > \frac{v(1-\phi)}{1-\phi} = v$  and  $r_m^* = \min\{v, \omega\} = v$ . Thus, it suffices to show the existence and possible uniqueness of  $(\alpha^*, r^*)$ . We consider in turn three possible cases.

- (i) When  $C \geq \hat{C}$ ,  $\tau(\alpha)$  decreases in  $\alpha$  for all  $\alpha \in (0,1)$  by Lemma 3. For  $\tau(1) = \hat{s} < s < \tau(0)$ , there is a unique  $\alpha^* \in (0,1)$  such that  $\tau(\alpha^*) = s$ , and the unique equilibrium  $r^*$  is then given by (12) with  $\alpha = \alpha^*$ .
- (ii) When  $C < \hat{C}$ , by Lemma 3, if N = 2,  $\tau(\alpha)$  monotonically increases, and hence for  $\tau(0) = \hat{s} < s < \tau(1)$ , there is a unique  $\alpha^*$  such that  $\tau(\alpha^*) = s$ , and the unique  $r^*$  is then given by (12) with  $\alpha = \alpha^*$ .

If N > 2,  $\tau(\alpha)$  first decreases and then increases, reaching its minimum at  $\hat{\alpha} \in (0,1)$ . Then

$$\hat{s} = \tau(\hat{\alpha}) = (1 - \theta)(1 - \hat{\alpha})C + (1 - \phi)\frac{(1 - \lambda + \lambda N\hat{\alpha}^{N-1})}{1 - \lambda}v > 0.$$

If  $\min \{\tau(0), \tau(1)\} < s < \max \{\tau(0), \tau(1)\}$ , there is a unique  $\alpha^*$  such that  $\tau(\alpha^*) = s$ ; whereas if  $\hat{s} < s < \min \{\tau(0), \tau(1)\}$ , there are two values of  $\alpha^*$ ,  $\alpha_1^* \in (0, \hat{\alpha})$  and  $\alpha_2^* \in (\hat{\alpha}, 1)$ , such that  $\tau(\alpha_1^*) = s$  and  $\tau(\alpha_2^*) = s$ .

**Proof of Lemma 5.** First, from (12), r increases in  $\alpha$ . From (7),  $\frac{\partial F(p)}{\partial r} < 0$  and thus

$$\frac{\partial F(p)}{\partial \alpha} = \frac{\partial F(p)}{\partial r} \frac{\partial r}{\partial \alpha} < 0.$$

Second,

$$1 - \alpha + \alpha G(p) = 1 - \left[ \frac{(r-p)(1-\lambda)}{(p-C)\lambda N} \right]^{\frac{1}{N-1}} \quad \text{for } p \in [b_g, r],$$

which decreases in r and thus decreases in  $\alpha$ . From (11),

$$(1-\alpha)H(q) = 1 - \left[\frac{v(1-\lambda+\lambda N\alpha^{N-1})}{q\lambda N} - \frac{1-\lambda}{\lambda N}\right]^{\frac{1}{N-1}} \quad \text{for } q \in [b_h, v],$$

which decreases in  $\alpha$ .

**Proof of Proposition 4.** From (18), treating N as a continuous variable, we have

$$\frac{\partial \tau(\alpha)}{\partial N} = -\frac{\partial \phi}{\partial N} \frac{1 - \lambda + \lambda N \alpha^{N-1}}{1 - \lambda} v + \frac{\lambda (1 + N \ln \alpha) \alpha^{N-1}}{1 - \lambda} (1 - \phi) v. \tag{23}$$

From (4),

$$\frac{\partial \phi}{\partial N} = -\lambda (1 - \lambda) \int_0^1 \frac{x^{N-1} (1 + N \ln x)}{(1 - \lambda + N \lambda x^{N-1})^2} dx.$$

Since  $\int x^{N-1}(1+N\ln x)dx = x^N\ln x$ , integrating by parts we have

$$\frac{\partial \phi}{\partial N} = -\lambda (1 - \lambda) \left\{ \frac{x^N \ln x}{(1 - \lambda + N\lambda x^{N-1})^2} \Big|_0^1 + 2 \int_0^1 \frac{N(N-1)\lambda x^{N-2} x^N \ln x}{(1 - \lambda + N\lambda x^{N-1})^3} dx \right\}.$$

By L'Hospital rule,  $\lim_{x\to 0} x^N \ln x = 0$  given that  $N \ge 2$ . Thus,

$$\frac{\partial \phi}{\partial N} = -\lambda (1 - \lambda) 2 \int_0^1 \frac{N(N - 1)\lambda x^{N - 2} x^N \ln x}{(1 - \lambda + N\lambda x^{N - 1})^3} dx > 0,$$

where the last inequality follows from the fact that  $\ln x < 0$  for x < 1. Thus, under A1,  $\frac{\partial \tau}{\partial N} < 0$ , which implies  $\tau(\alpha)$  shifts down as N increases.

From Lemma 3, when  $C < \hat{C}(N)$  and N = 2,  $\frac{\partial \tau}{\partial \alpha} > 0$ . It follows that  $\alpha^*$ , where  $\tau(\alpha^*) = s$ , increases as N increases. Moreover, when  $C < \hat{C}(N)$  and  $N \ge 3$ ,

$$\frac{\partial \tau}{\partial \alpha} < 0 \text{ if } \alpha^* < \hat{\alpha}, \text{ and } \frac{\partial \tau}{\partial \alpha} > 0 \text{ if } \hat{\alpha} < \alpha^* < 1.$$

Hence  $\alpha^*$  decreases in N if  $\alpha^* < \hat{\alpha}$  but increases in N if  $\alpha^* \ge \hat{\alpha}$ . Finally, when  $C \ge \hat{C}(N)$ ,  $\frac{\partial \tau}{\partial \alpha} < 0$ , and thus  $\alpha^*$  decreases in N.

**Proof of Lemma 6.** For a searcher with i = M,

$$P_M = E[F(p)] = \int_{b_f}^{r} p dF(p) = (r - C)\phi + C.$$

A searcher with i = m will buy at the first search with a price randomly drawn from  $\Phi(p)$ , and the expected price he pays is:

$$P_{m} = E[\Phi(p)] = \alpha \int_{bg}^{r} p dG(p) + (1 - \alpha) \int_{b_{h}}^{v} p dH(p)$$

$$= \int_{b_{g}}^{r} p d[\alpha - \alpha G(p)] + \int_{b_{h}}^{v} p d[1 - (1 - \alpha)H(p)]$$

$$= \int_{0}^{\alpha} C + \frac{(r - C)(1 - \lambda)}{1 - \lambda + \lambda Nx^{N-1}} dx + \int_{\alpha}^{1} \frac{(r - C)(1 - \lambda)}{1 - \lambda + \lambda Nx^{N-1}} dx$$

$$= (r - C)\phi + \alpha C.$$

For a shopper with i = M, he will purchase from the lowest price among all experts charging a price from F(p) which follows a distribution:

$$F^{l}(p) = 1 - [1 - F(p)]^{N}$$

Thus, a shopper with i = M pays an expected price

$$p_{M} = E[F^{l}(p)] = \int_{bf}^{r} p dF^{l}(p) = -\int_{0}^{1} p d\left[1 - x^{N}\right]$$

$$= \int_{0}^{1} Nx^{N-1} \left(\frac{(r - C)(1 - \lambda)}{1 - \lambda + \lambda Nx^{N-1}} + C\right) dx$$

$$= \int_{0}^{1} \frac{(r - C)(1 - \lambda)Nx^{N-1}}{1 - \lambda + \lambda Nx^{N-1}} dx + C \int_{0}^{1} Nx^{N-1} dx$$

$$= (r - C) \int_{0}^{1} \frac{(1 - \lambda)Nx^{N-1}}{1 - \lambda + \lambda Nx^{N-1}} dx + C$$

$$= (r - C)\rho + C.$$

Finally, for a shopper with i = m, he will purchase from the lowest price among all experts charging a price from  $\Phi^l(p)$  which follows distribution

$$\Phi^{l}(p) = \begin{cases} 1 - [1 - (1 - \alpha)H(p)]^{N} & \text{if } p \in [b_{h}, v] \\ 1 - \alpha^{N} & \text{if } p \in [v, b_{g}] \\ 1 - [\alpha(1 - G(p))]^{N} & \text{if } p \in [b_{g}, r] \end{cases}.$$

Thus, a shopper with i = m pays an expected price

$$\begin{split} p_m &= E[\Phi^l(p)] = \int_{bg}^r pd \left\{ 1 - \left[ \alpha (1 - G(p)) \right]^N \right\} + \int_{b_h}^v pd \left\{ 1 - \left[ 1 - (1 - \alpha) H(p) \right]^N \right\} \\ &= -\int_{bg}^r pN [\alpha (1 - G(p))]^{N-1} d[\alpha (1 - G(p))] - \int_{b_h}^v pN [1 - (1 - \alpha) H(p)]^{N-1} d[1 - (1 - \alpha) H(p)] \\ &= \int_0^\alpha \left[ \frac{(r - C)(1 - \lambda)}{1 - \lambda + \lambda N x^{N-1}} + C \right] N x^{N-1} dx + \int_\alpha^1 \left[ \frac{(r - C)(1 - \lambda)}{1 - \lambda + \lambda N x^{N-1}} \right] N x^{N-1} dx \\ &= (r - C) \int_0^1 \frac{(1 - \lambda) N x^{N-1}}{1 - \lambda + \lambda N x^{N-1}} dx + C \int_0^\alpha N x^{N-1} dx \\ &= (r - C) \rho + \alpha^N C. \end{split}$$

**Proof of Proposition 5.** When  $v \ge \omega$ ,  $r_m^o = \omega$ , which is determined by (6). Since

$$r_M^o = C + \frac{s}{1 - \phi}$$
 and  $\omega = \frac{s}{1 - \phi}$ ,

experts receive the same profit from recommending  $T_M$  or  $T_m$  when i=m. Hence recommending  $T_i$  for  $i \in \{M, m\}$  is optimal for experts.

Notice that  $\omega$  increases in s, and  $\omega = v$  when  $s = v (1 - \phi)$ . Thus, if  $s > v (1 - \phi)$ ,  $r_m^o = \min\{v, \omega\} = v < r_M^o - C$ . In this case, experts would deviate to recommending  $T_M$  for i = m with price  $r_M^o$ .

Therefore, if  $s \leq v (1 - \phi)$  and  $\omega$  solves (6), then there is a symmetric separating equilibrium in which  $\alpha^* = 0$ , and the equilibrium is unique because  $F_i(p)$  is unique.

**Proof of Proposition 6.** At the proposed equilibrium, each expert's profit is

$$(r_M^o - C) \frac{1-\lambda}{N}.$$

If an expert deviates to offering  $T_m$  for i = m with p = v, his profit is

$$v(\lambda + \frac{1-\lambda}{N}).$$

Therefore, the equilibrium can be sustained if and only if

$$r_M^o = C + \frac{s}{1 - \phi} \ge C + \frac{1 - \lambda + \lambda N}{1 - \lambda} v,$$

or

$$s \ge \frac{(1-\phi)(1-\lambda+\lambda N)}{1-\lambda}v = \tau(1).$$

In addition, to ensure the existence of the reservation price, we need  $r_M^o \leq \theta V + (1-\theta)v$ , which is equivalent to

$$s \le (1 - \phi)[\theta V + (1 - \theta)v - C] = \bar{s},$$

which holds under assumption (5). Notice that  $\tau(1) < \bar{s}$  when V is sufficiently large.

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