Periodicity in Bitcoin returns: A time-varying volatility approach

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Abstract

We examine if the day-of-the-week effect is present in Bitcoin return series. The model specification in use accounts for conditional heteroscedasticity, which is captured in the form of a stochastic volatility process that allows for periodic time-varying parameters. We find periodicity in Bitcoin returns, which is evidence against the market efficiency of Bitcoin.

Keywords: Bitcoin series, periodicity, stochastic volatility model

JEL CODE: C5, C22, G12

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1 Introduction

Over the last decade, there has been an increasing interest to understand the behaviour of a new type of financial asset that has been labelled cryprocurrency. Given its large capitalisation (Coinmarket, 2016), the market of cryptocurrencies has attracted much attention (governments, policy makers, banks, hedge funds, consumers). The most popular cryprocurrency is the Bitcoin (Nakamoto, 2009).

It has been well documented that cryptocurrency return series exhibit time-varying volatility, also known as conditional heteroscedasticity. Conditional heteroscedasticity in the relevant empirical literature has been modelled as a GARCH-type specification (Glaser et al., 2014; Gronwald, 2014; Dyhrberg, 2016a, 2016; Bouoiyour and Selmi, 2015, 2016; Bouri et al., 2017a, Katsampa, 2017; Naimy and Hayek, 2018) or as a stochastic volatility process (Phillip et al., 2018).

In this paper we turn our attention to the day-of-the-week effect on Bitcoin returns. Although there is a voluminous financial literature that has examined whether the expected return and volatility of a stock are uniformly distributed across the days of the week (thus, rejecting or accepting the efficient market hypothesis, proposed by Fama (1965); see e.g. Aknouche, 2017; Aknouche et al., 2020; Aknouche et al., 2018), no study so far has examined something similar in the case of returns on crypotocurrencies. The present paper aspires to fill this gap.

To our knowledge, the paper by Mbanga (2018) is the only paper that has investigated the day-of-the-week pattern of price clustering in Bitcoin. However, Mbanga (2018) does not analyse potential abnormalities over the week in Bitcoin returns but in price levels. This is the reason for which this paper has ignored conditional heteroscedasticity, which is a main characteristic of cryptocurrency returns. Aknouche et al. (2022) used a periodic autoregressive conditional duration to detect a day-of-the week periodicity in Bitcoin volumes. However, conditional heteroskedasticity has not been explored.

Accounting for conditional heteroscedasticity, we focus on whether the seasonality pattern in the form of day-of-the-week effect is present in the Bitcoin returns (and with what periodicity). To this end, we exploit the periodic autoregressive stochastic volatility model (Aknouche, 2017). It is an extension of the standard stochastic volatility model (Taylor, 1986) that allows the parameters in the stochastic volatility equation to vary periodically over time. In this way, we can identify any periodically changing structure (e.g. Aknouche, 2015) in the time series volatility of Bitcoin returns.

Our paper also contributes to the debate about whether the market of Bitcoin is efficient. The empirical findings about the efficient market hypothesis for Bitcoin are inconclusive (Urquhart, 2016; Nadarajah and Chu, 2017; Bouri et al., 2017a, 2017b; Balcilar et al., 2017). If there is periodicity in Bitcoin returns, this is evidence against that hypothesis.

The paper is organized as follows. In Section 2 we describe the model and in section 3 we present the empirical results.

2 The periodic autoregressive stochastic volatility model

The Gaussian stochastic volatility (SV) model is given by:

$$\begin{cases} y_t = \sqrt{h_t}\eta_t \\ \log(h_t) = \alpha + \beta \log(h_{t-1}) + \sigma e_t \end{cases}, \quad t \in \mathbb{Z}, \tag{1}$$

where y_t is the log-return and h_t is the conditional variance that follows a first-order autoregressive process, where $\alpha \in \mathbb{R}$ is the intercept, $|\beta| < 1$ is the slope parameter, and $\sigma^2 > 0$ is the variance. Also, $\left\{(e_t, \eta_t) \overset{i.i.d}{\sim} N\left((0,0)', I_2\right)\right\}$, where I_2 is the identity matrix of dimension 2.

The Gaussian periodic SV model (Aknouche, 2017) imposes S-periodicity on the parameters over time by setting $t = nS + v$, for $n \in \mathbb{Z}, 1 \leq v \leq S$. The resulting specification reads:

$$\begin{cases} y_{nS+v} = \sqrt{h_{nS+v}}\eta_{nS+v} \\ \log(h_{nS+v}) = \alpha_v + \beta_v \log(h_{nS+v-1}) + \sigma_v e_{nS+v} \end{cases}, n \in \mathbb{Z},\ 1 \leq v \leq S, \tag{2}$$

where the parameters $\alpha_v \in \mathbb{R}$, $|\beta_1 ... \beta_S| < 1$, and $\sigma_v^2 > 0$ $(1 \leq v \leq S)$ are S-periodic over t, and $\{(e_{nS+v}, \eta_{nS+v})\}$ is defined as before. The model in (2) is named periodic autoregressive stochastic volatility ($PAR\text{-}SV_S$). It defines a periodic-time varying dependence structure, where the dependence between successive times is distanced by a multiple of the period S. If there is S-periodicity in the volatility of a daily return series (in our case Bitcoin), this suggests the presence of the day-of-the-week effect in that series. Notice also, that the $PAR\text{-}SV_S$ reduces to the SV model for $S = 1$.

For the estimation of the SV and $PAR\text{-}SV_S$ we adopt Bayesian methods as described in Aknouche (2017). The (reasonably flat) priors used in this paper are displayed in the Appendix of this paper.

3 Empirical analysis: The Bitcoin

3.1 Data and some descriptive statistics

We use 3014 daily closing prices for the Bitcoin Coindesk Index, from July 18, 2010 to October 17, 2018. We transform the prices to returns, by taking the natural logarithm of the ratio of two successive closing prices. The time series plot of Bitcoin returns is given in Figure 1(a).

Based on the simple and partial autocorrelation functions of Bitcoin returns (Figures 1(b) and 1(c), respectively), these returns seem to follow a white noise process. It is difficult to detect a possible periodicity in their conditional distributions, as almost all autocorrelations are insignificant.

However, if, for example, these returns follow a SV (or a GARCH) model, the squared returns will have an autoregressive conditional duration (ACD) representation, which is no longer uncorrelated and a possible periodicity could appear in their autocorrelations. Therefore, we plotted the squared returns of Bitcoin, along with their simple and partial autocorrelation functions (Figures 1(d) and 1(f), respectively).

From the autocorrelations of the squared returns, there is an indication of periodicity, as it can be observed that important picks appear in lags, which are multiples of 7 (7, 21, 28 in Figure 1(f)). There exist of course other more important picks on other lags but they are not so persistent at their multiples.

In Table 1, we provided some descriptive statistics for the Bitcoin data, using the full sample and by each day of the week separately. The average return and the volatility (approximated by the absolute value) are somewhat different from one day to another. Furthermore, the returns exhibit negative skewness and high positive kurtosis that vary notably from Monday to Sunday. Since the distributions of returns over the days are not constant, we suspect that the day-of-the week effect may characterize the Bitcoin return data.

3.2 Estimation results

We design our empirical analysis, by assuming that the conditional distribution of Bitcoin returns is characterized by a periodicity of up to magnitude 7. This approach is different from the one usually used for non-cryptocurrency returns (such as stocks, exchange rates, etc.,), which are characterized by a periodicity of up to magnitude 5, due to the non-trading days at each week (week-end).

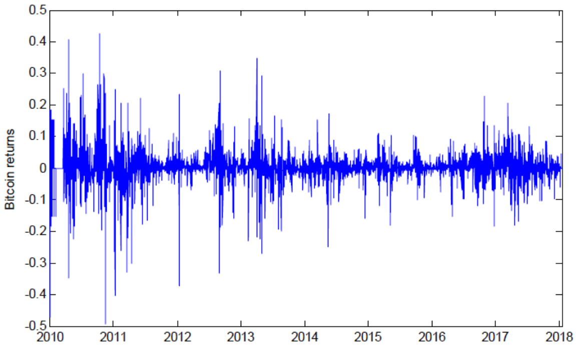
Therefore, we first estimate seven $PAR\text{-}SV_S$ models, corresponding to each $S \in \{1, ..., 7\}$. Then, we conduct model comparison, using the Deviance information Criterion (DIC; Spiegelhalter et al., 2002) in order to identify the period of the best fitting $PAR\text{-}SV_S$ model.

We run the MCMC samplers for 2500 iterations after a burn-in of 2500 draws. To monitor convergence and mixing of the samplers, we use (and report) the relative numerical inefficiency (RNI) and the numerical standard error (NSE); see, for example, Geweke (1989).

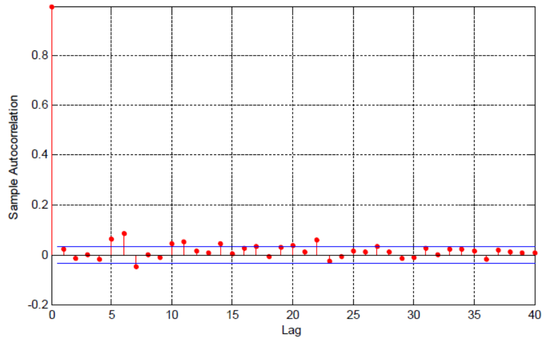
Based on the DIC values (Table 2), $PAR\text{-}SV_7$ is the best model, being followed by the SV model. In Table 3, the results (posterior means and standard deviations) for the SV model show that there is high persistence (0.8626) in the estimated volatilities, which are plotted in Figure 2. In Table 4, the parameters for the $PAR\text{-}SV_7$ model are all significant and different from one day to another especially for the α_v's and β_v's, a fact that supports the use of periodic SV modeling tools. The existence of periodicity in Bitcoin returns is also an evidence that they are market inefficient. Finally, Figure 3 plots the estimated volatilities from the $PAR\text{-}SV_7$ model and Figure 4 portraits the difference in volatilities between the $PAR\text{-}SV_7$ and the SV models.

4 Conclusions

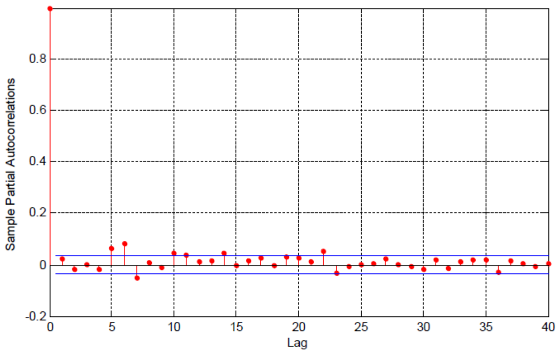
We examined the day-of-the-week effect in Bitcoin and found that the hypothesis that Bitcoin returns exhibit periodicity in their conditional distributions is tenable. Also, this empirical finding does not support the efficient market hypothesis for the data in question.



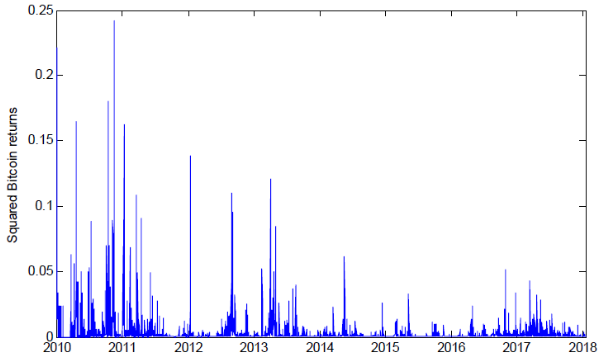
(a) Time series plot of Bitcoin returns.



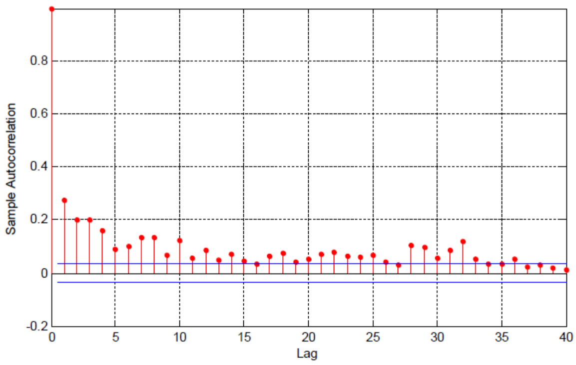
(b) Sample autocorrelation function of the Bitcoin returns.



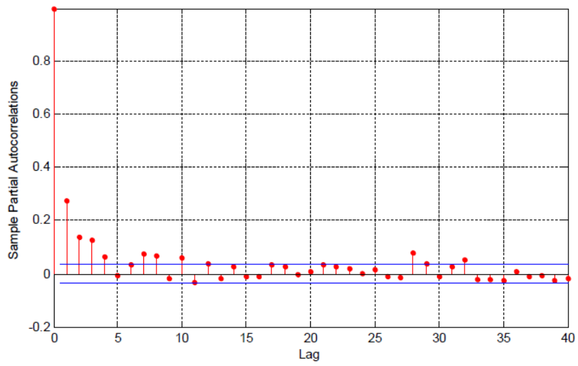
(c) Sample partial autocorrelation function of the Bitcoin returns.



(d) Time series plot of the squared Bitcoin returns.



(e) Sample autocorrelation function of the squared Bitcoin returns.



(f) Sample partial autocorrelation function of the squared Bitcoin returns.

Figure 1: Empirical results. Descriptive plots for Bitcoin.

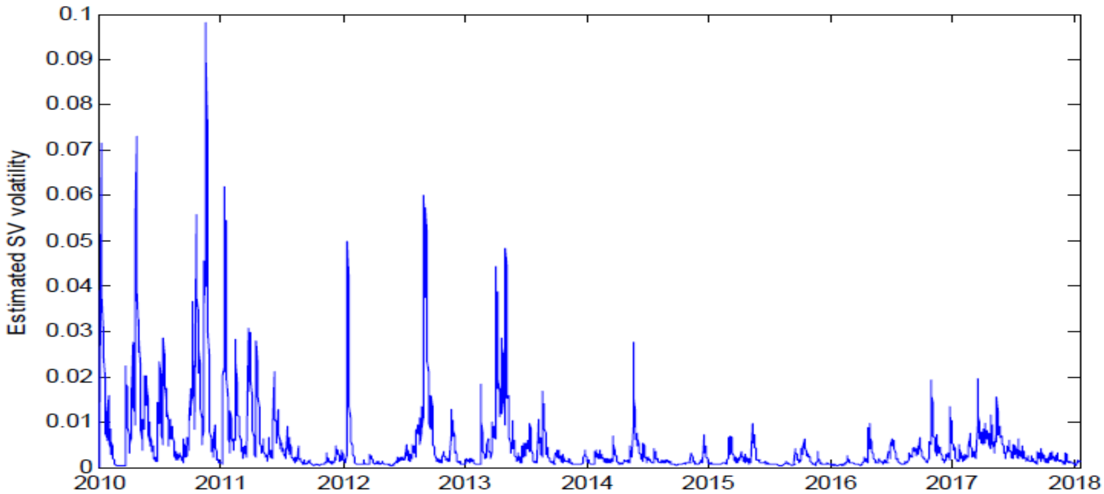


Figure 2: Empirical results. Bitcoin estimated volatilities induced by the SV model.

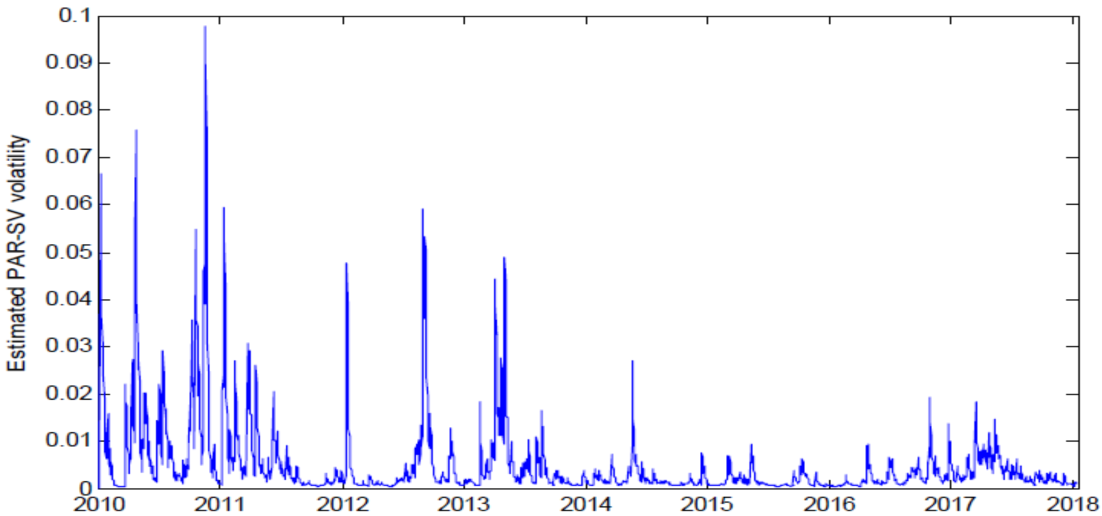


Figure 3: Empirical results. Bitcoin estimated volatilities induced by the $PAR\text{-}SV_7$ model.

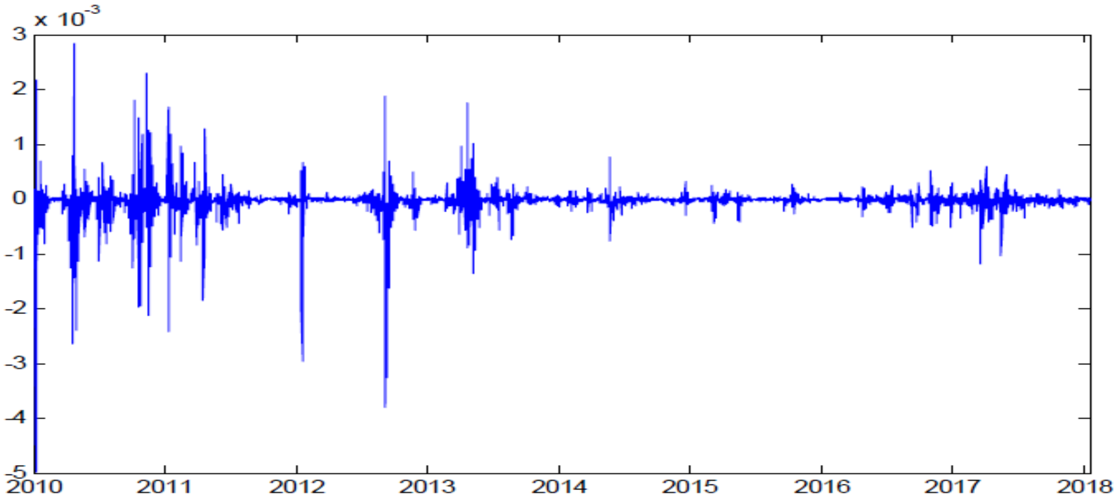


Figure 4: Empirical results. The difference between estimated volatilities $(h_t^{PAR-SV_7} - h_t^{SV})$.

Table 1: Empirical results. Day-of-the-week effect in the daily Bitcoin returns(y_t).

		Mean of y_t	Mean of $\|y_t\|$	Mean of y_t^2	Skewness	Kurtosis	Min	Max
	Full series	0.0037	0.0333	0.0033	-0.3452	15.0201	-0.4915	0.4246
1	Monday	0.0048	0.0346	0.0035	-0.8533	13.6092	-0.3483	0.3478
2	Tuesday	0.0068	0.0353	0.0035	0.9435	12.7229	-0.2188	0.4246
3	Wednesday	0.0043	0.0329	0.0032	-0.5759	11.9987	-0.3321	0.3086
4	Thursday	0.0044	0.0375	0.0040	-0.5852	13.2344	-0.4700	0.2908
5	Friday	0.0025	0.0349	0.0034	0.4227	9.3506	-0.2610	0.2987
6	Saturday	0.0032	0.0312	0.0036	-1.1543	24.9235	-0.4915	0.4055
7	Sunday	7.4187e-05	0.0264	0.0020	-0.8474	17.4367	-0.3724	0.2360

Table 2: DIC values and monodromy parameters (Bitcoin returns).

	$PAR\text{-}SV_1 \equiv$ SV	$PAR\text{-}SV_2$	$PAR\text{-}SV_3$	$PAR\text{-}SV_4$	$PAR\text{-}SV_5$	$PAR\text{-}SV_6$	$PAR\text{-}SV_7$
DIC	−9946.7109	−9883.0069	−9918.1148	−9922.8003	−9905.4721	9902.2034	−9950.8321
(*Std*)	(2.2431)	(2.5607)	(2.8753)	(3.0197)	(2.9816)	(2.8903)	(2.6844)
Rank of DIC values	2	7	4	3	5	6	1
Monod.	0.8626	0.7841	0.6480	0.6229	0.6024	0.5775	0.4621

Notes: The monodromy (Monod.) parameters $\prod_{v=1}^{S} \beta_v$ are quite large, indicating strong volatility persistence. *Std* stands for standard deviation. In computing the standard errors of DIC, we have replicated the algorithm 100 times.

Table 3: Empirical results for the SV model (Bitcoin returns).

	Mean	St. dev	NSE	RNI
α	−0.8324	0.0514	0.0025	0.4080
β	0.8626	0.0097	0.0001	0.5261
σ^2	0.3162	0.0111	0.0005	0.2592

Table 4: Empirical results for the $PAR\text{-}SV_7$ model (Bitcoin returns).

Periodic SV parameters		Mean	St. dev	NSE	RNI
1 Monday	α_1	−1.0069	0.1319	0.0025	0.0458
	β_1	0.8421	0.0214	0.0004	0.0509
	σ_1^2	0.2661	0.0215	0.0060	0.1934
2 Tuesday	α_2	−0.9573	0.1511	0.0012	0.3174
	β_2	0.8398	0.0249	0.0033	0.0644
	σ_2^2	0.3404	0.0287	0.0004	0.0359
3 Wednesday	α_3	−0.7087	0.1469	0.0214	2.5999
	β_3	0.8788	0.0239	0.0032	2.2271
	σ_3^2	0.3328	0.0312	0.0069	0.2151
4 Thursday	α_4	−0.6756	0.1464	0.0010	0.1903
	β_4	0.8877	0.0241	0.0044	0.1281
	σ_4^2	0.3230	0.0330	0.0008	0.1695
5 Friday	α_5	−0.2986	0.1654	0.0117	0.6854
	β_5	0.9434	0.0258	0.0016	0.4957
	σ_5^2	0.3501	0.0293	0.0006	0.0954
6 Saturday	α_6	−0.3996	0.1357	0.0012	0.2397
	β_6	0.9309	0.0234	0.0023	0.6778
	σ_6^2	0.3269	0.0320	0.0037	1.5641
7 Sunday	α_7	−0.2888	0.1564	0.0012	0.2375
	β_7	0.9536	0.0257	0.0013	0.2116
	σ_7^2	0.3043	0.0277	0.0020	0.6830

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Appendix

Table A.1: Prior distributions for the parameters of the SV model (which is equivalent to $PAR\text{-}SV_S$ with S=1).

Priors	Prior for $\omega = (\alpha, \beta)'$, $\omega \sim N(\omega_0, \Sigma_0)$: ω_0	Σ_0	Prior for σ^2 : $\frac{a\lambda}{\sigma^2} \sim \chi^2_a$: a	λ
Hyperparameters	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 0.05 & 0 \\ 0 & 0.5 \end{pmatrix}$	5	0.2

Notes: χ^2_a denotes the chi-square distribution with a degrees of freedom.

The diagonal matrix D_k $(k = 4, 10)$ is defined to be

$$D_k(i,j) = \begin{cases} 0 & \text{if} \quad i \neq j \\ 0.05 & \text{if} \quad i = j \text{ is odd} \\ 0.5 & \text{if} \quad i = j \text{ is even} \end{cases}, \quad 1 \leq i, j \leq k. \tag{A.1}$$

Table A.2: Prior distributions of ω and σ^2 for the candidates $PAR\text{-}SV_S$, $2 \leq v \leq 7$.

Priors	Prior for $\omega = (\alpha, \beta)$: $\omega \sim N(\omega_0, \Sigma_0)$: $\omega_0 =$	$\Sigma_0 =$	Prior for σ^2 : $\frac{a_v \lambda_v}{\sigma_v^2} \sim \chi^2_{a_v}$: $a = (a_2, ..., a_S)$	$\lambda = (\lambda_2, ..., \lambda_S)$
$S = 2$	$0_{4\times1}$	D_4	$10 \times \mathbf{1}_2$	$0.1 \times \mathbf{1}_2$
$S = 3$	$0_{6\times1}$	D_6	$10 \times \mathbf{1}_3$	$0.1 \times \mathbf{1}_3$
$S = 4$	$0_{8\times1}$	D_8	$10 \times \mathbf{1}_4$	$0.1 \times \mathbf{1}_4$
$S = 5$	$0_{10\times1}$	D_{10}	$10 \times \mathbf{1}_5$	$0.1 \times \mathbf{1}_5$
$S = 6$	$0_{12\times1}$	D_{12}	$10 \times \mathbf{1}_6$	$0.1 \times \mathbf{1}_6$
$S = 7$	$0_{14\times1}$	D_{14}	$10 \times \mathbf{1}_7$	$0.1 \times \mathbf{1}_7$

Notes:
D_k, $0_{k\times1}$ and $\mathbf{1}_k$ denote respectively the diagonal matrix given by (1), the null vector with k components and the k-vector with all components equal to 1.