Inflation and deflation of the transfer space

Friedrich, Thomas

Humboldt-Universität zu Berlin

9 October 2020

Inflation and deflation of the transfer space

T. Friedrich

The transfer space is a model of unidirectional substrate transfers from a source to a sink. Source and sink independently follow linear cost and saturating benefit functions. The transfer of substrate within the ensemble has nonlinear effects on the net profit (benefit minus cost) of the ensemble. Superadditivity or subadditivity are a result in comparison to the condition "no transfer" with simple additivity. In this investigation I observe dilution (inflation) and concentration (deflation) of the transfer space. Inflation and deflation change the substrate concentration and the step size of a featureless transfer vehicle called "coin".

Upon dilution the volume of the transfer space is increased and therefore the concentration of substrate is decreased. This leads to a reduction of the step size of a representative coin in comparison to the starting conditions. After concentration the volume is decreased and therefore the concentration of the substrate as well as the step size of the coin is increased relative to the starting condition. Dilution and concentration cause non-linear effects in source, sink, and the ensemble.

In symmetric and many asymmetric ensembles (strong and weak) inflation is beneficial to the production of superadditivity but deflation is harmful. In asymmetric ensembles with very high cost in source and low to medium cost in sink (very strong ensemble), I observe a limit where the benefit of inflation turns over and becomes harmful. There, deflation is beneficial.

In this model inflation (growth), deflation (scaling down), and division of labour appear to be investment decisions or evolutionary trends coexisting within the transfer space.

Introduction

This paper is a continuation of my past paper (1).

Aim of the investigation

I want to investigate how inflation and deflation of the transfer space affect the superadditive net profit of an ensemble of a source and a sink. The biochemical interpretation of inflation is dilution of substrate (increased volume at constant amount of substrate). Deflation is then a concentration of substrate (decreased volume at constant amount of substrate).

Features of the model "transfer space"

The transfer space (figure 1) is a three-dimensional model of unidirectional substrate transfers between two parties; a source (so) and a sink (si) forming an ensemble (e). The connection of source and sink is set to have no features. Source and sink are two-dimensional entities, the ensemble is a three-dimensional entity and the level where the final balance is calculated. The coordinates of the space are: substrate concentration in source ($[S]_{so}$), substrate concentration in sink ($[S]_{si}$), and net profit of the ensemble (np_e, b_e-c_e,). Source and sink use the same substrate. This substrate has simultaneously a benefit (b) and a cost (c) aspect in source and sink. In symmetric ensembles the benefit functions or cost functions of both parties are identical. In asymmetric ensembles they differ in at least one feature. A substrate is a Janus-headed thing. The cost aspect points to the past and the benefit aspect points to the future.

The production of benefit in source (b_{so}) and sink (b_{si}) follows a saturating function of the Michaelis-Menten type:

source: $b_{so}=V_{so}*bf_{so}$ $\quad$ $V_{so}=([S]_{so}/([S]_{so}+Km_{so}))*Vmax_{so}$

sink: $b_{si}=V_{si}*bf_{si}$ $\quad$ $V_{si}=([S]_{si}/([S]_{si}+Km_{si}))*Vmax_{si}$

V is the reaction velocity (catalytic activity) in µmol/min, Vmax is the maximal reaction velocity (µmol/min); Km (mM) is the Michaelis-Menten constant, an enzyme constant; [S] is the substrate concentration (mM) in source or sink; bf is the benefit factor (b*min/µmol), he is also a complexity factor (2). The benefit factor serves to introduce the unit b as a placeholder for *e.g.* kilojoule or $ or € and may differ in source and sink.

The cost aspect follows a linear function in source (c_{so}) and sink (c_{si}):

source: $c_{so}=[S]_{so}*cf_{so}$ and sink: $c_{si}=[S]_{si}*cf_{si}$

[S] is the substrate concentration (mM) in source or sink and cf is the cost factor (c/mM). The cost factor is used to introduce the unit c as a placeholder for *e.g.* kilojoule or $ or € and may differ in source and sink.
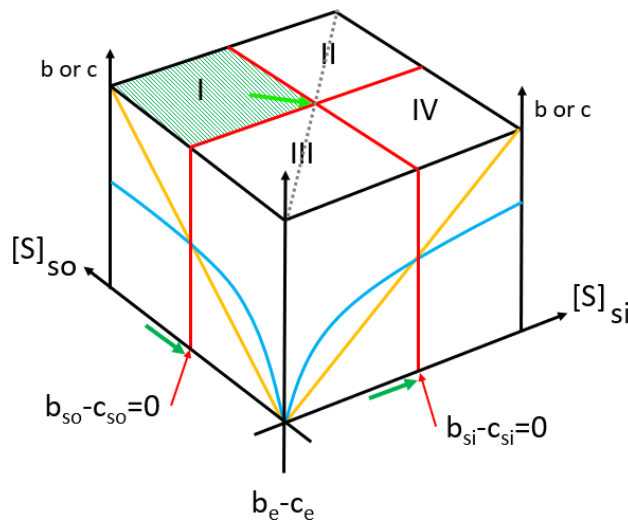
Figure 1

Figure 1
The three axes of the transfer space are substrate concentration in source $[S]_{so}$ and substrate concentration in sink $[S]_{si}$ and net profit of the ensemble (b_e-c_e). The transfer space is cut into 4 areas (I, II, III, IV) by two red limits. The red lines mark b-c=0 in source and b-c=0 in sink. The blue curves are the saturating benefit functions and the orange lines the linear cost functions in source and sink. The dotted grey line is the line of equal substrate concentrations in source and sink. Green arrows mark a single, exemplary substrate decrease in source and substrate increase in sink according to the law of conservation of mass. The light green arrow marks the momentum of the substrate transfer. Here I depict a win-win situation (green area I) of a symmetric ensemble as source gets rid of cost domination and sink gains in benefit domination.

Benefit and cost are always of the same dimensionality and therefore I can subtract them to obtain the net profit for whatever I used them as placeholders (*e.g.* kilojoule or $ or €). The net profit of the ensemble is the sum of the net profit of source and sink: $b_e-c_e=b_{so}-c_{so}+b_{si}-c_{si}$. The net profit of source and sink changes when substrate is transferred from source to sink. Due to non-linear features of this layout the net profit of the ensemble may change, too. The ensemble is observed without transfer of substrate and with transfer of substrate. Simple additivity (b_e-c_e without transfer equals b_e-c_e with transfer), superadditivity (b_e-c_e without transfer is smaller than b_e-c_e with transfer), and subadditivity (b_e-c_e without transfer is larger than b_e-c_e with transfer) are observed.

Benefit domination is observed when b-c>0 in source or sink. Under this condition source does not give substrate and sink takes substrate. Cost domination is observed when b-c<0. There, source gives substrate and sink does not take substrate. However, force or deception can change the behaviour (3). Only when source is cost dominated (b-c<0) and sink is benefit dominated (b-c>0) a transfer occurs at free will and is a win-win situation. This is the case in area I (figure 1). The aim to reach b-c=0 through giving (source) or taking (sink) is the central behavioural aspect of an ensemble and its parts. Therefore, b-c=0 is attractive and stable, an equilibrium point. To reach this point for a single party the use of force and deception in area II and area III by source or sink is a possibility. In addition, there may be a third party involved called "master". He is not active in production of net profit. He is either an honest broker in area I or he will use force and deception to move the ensemble into area II and III and even into area IV. The standard symmetric ensemble observed here has the following features in source and sink: $V_{max}=5\mu mol/min$, $K_m=0.5mM$, bf=1b*min/μmol, cf=5/3c*mM^{-1}, [S]=0-10mM at a total amount of ten millimole. The cost factor is used later to adjust asymmetry.

The transfer space is actually a right prism of an isosceles right triangle

In the beginning of my investigations the transfer space was a cube. In the bird´s eye view he looked like a game theory matrix (figure 1) because the initial idea came from a microbiological paper inspired by game theory (4).

My examples always dealt with equally sized volumes and identical physical and biochemical conditions in source and sink. Sometimes I used different enzymes in source and sink (asymmetric ensemble, same substrate). In most of my calculations I choose to have a concentration of up to 5mM substrate in source and up to 5mM substrate in sink. A cube with a square as surface was the result (figure 1; figure 2, dotted lines at 5mM). However, in that setting the maximal concentration in source **or** sink could be up to 10mM when source or sink possess the whole substrate (10mmol in 1 litre of source or sink) alone. Therefore, the transfer space is in principle of a triangular shape (figure 2). I already made use of this idea when I compared a master of a saturated ensemble (all concentration pairs in area II) with the master of an unsaturated ensemble (all concentration pairs in area III) (5). Now I generally switch to the triangular interpretation of the transfer space in the whole paper. The reason is that I want to change in my standard example (maximal 10mmol substrate in total) the volume of source (1l) and sink (1l) to simulate inflation (dilution of the substrate) and deflation (concentration of the substrate) and assess and compare their effect. In case I increase the volume of source and sink by a factor of 2 (2*1l source and 2*1l sink) the maximal achievable concentration in source or sink will be only 5mM. However, now I have two ensembles of this type. In case I decrease the volume to 50% of the original volume (0.5l source and 0.5l sink) the maximal concentration in source or sink will be 20mM. But in comparison to the size of the original ensemble there will be only half an ensemble left.
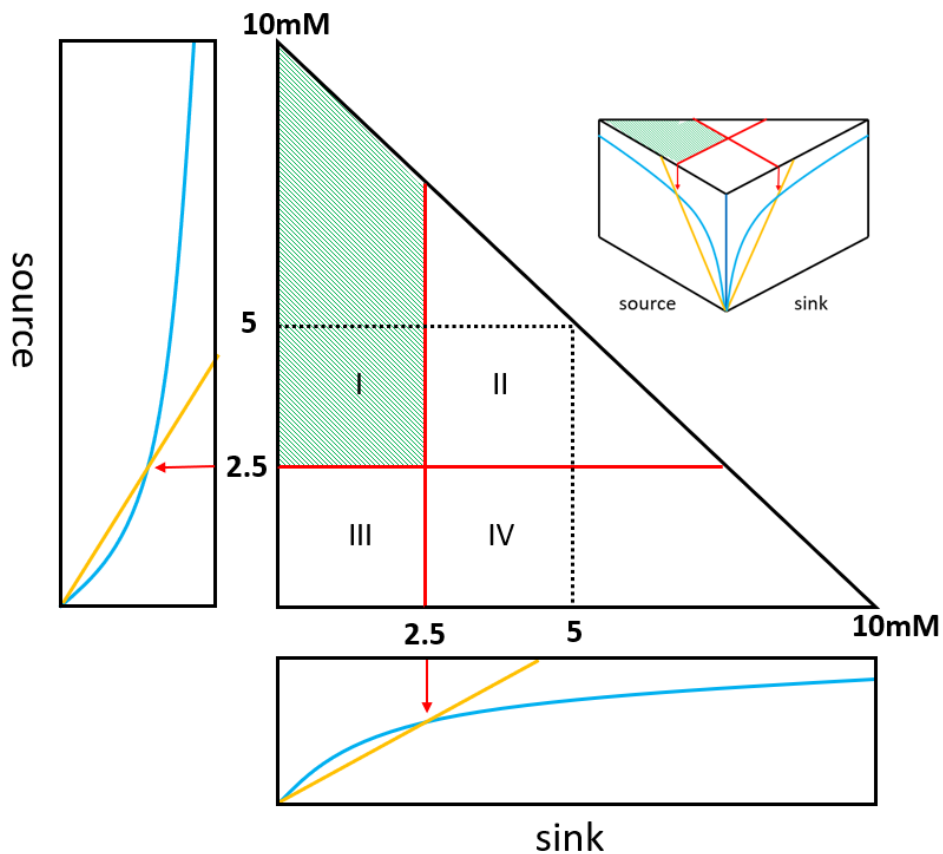
Figure 2

Figure 2
In this figure the three-dimensional transfer space (inset) of a symmetric ensemble (Vmax=5μmol/min, Km=0.5mM, bf=1b*min/μmol, cf=5/3 c*mM^{-1}) is unfolded and we look simultaneously at the source and sink side with the saturating benefit (light blue) and the linear cost (orange) functions. The triangle gives all possible concentrations in source and sink when the maximal amount of substrate is fixed to 10mmol in the ensemble (1litre of source, 1litre of sink). Net profit of the ensemble points towards the observer. 2.5, 5, and 10 are the concentrations of substrate in millimolar (mM). The red lines and arrows indicate that b-c=0 in source and sink is at 2.5mM. I call this an equilibrium concentration. The roman numbers signify the 4 areas of the transfer space with the features: I, source is cost dominated and sink is benefit dominated, transfer is at free will on both sides; II, source and sink are cost dominated, sink does not take; III, source and sink are benefit dominated, source does not give; IV, source is benefit dominated and sink is cost dominated, transfer would be irrational. The green area indicates where transfers at free will are possible. Area I, II and IV are increased in comparison to the old shape of the transfer space (doted black lines).

Three causes of inflation in economy are known: money supply, cost-push, and demand-pull. I interpret money supply as a dilution of the gold content in every coin. The result is inflation by money supply - more coins with less content. I assume that the inverse action leads to deflation.

Increase or decrease in volume at a fixed amount of substrate

In all my past papers "substrate only" was transferred as a pure number from source to sink. This requires the ability to separate substrate and solvent. Beyond the molecular level this is not possible as solvent always will come along. Therefore, the volume in source and sink will change and this has to be considered. In addition, the substrate itself has a measurable volume and the concentration of the substrate will change during catalysis; it becomes consumed. In real experiments a transfer would not be performed but the conditions "no transfer" and "transfer" would be separately mixed, started and measured to determine the reaction velocity (V_{so}, V_{si}) over a range of different substrate concentrations. Super- and subadditivity would be calculated on that basis.

In case substrate and solvent can´t be discriminated and the concentration does change (volume increase or decrease at a constant amount of substrate) a new problem arises. Now the ensemble has to rely on a measure that is no longer the substrate itself. This measure could be the vehicle containing the substrate. I call this vehicle "coin". The true amount of substrate within the coin I call "step size". This is similar to money. Gold coins may consist of pure gold or partially of another metal in a fraudulent intent. In case I rely on counting coins (steps) the true amount (step size) according to the content (gold, value) will be quite differently.

The start condition (figure 3, black triangles, black arrows) is used as bench mark. Dilution and concentration change the concentration (mM) of substrate (figure 3 right, purple and golden numbers). Dilution reduces the step size per coin, concentration increases the step size per coin (figure 3 right, purple and golden arrows). The total amount of substrate is fixed to maximal ten millimole (10mmol). The ensemble always orients according to the number of coins. A coin is a featureless vehicle.
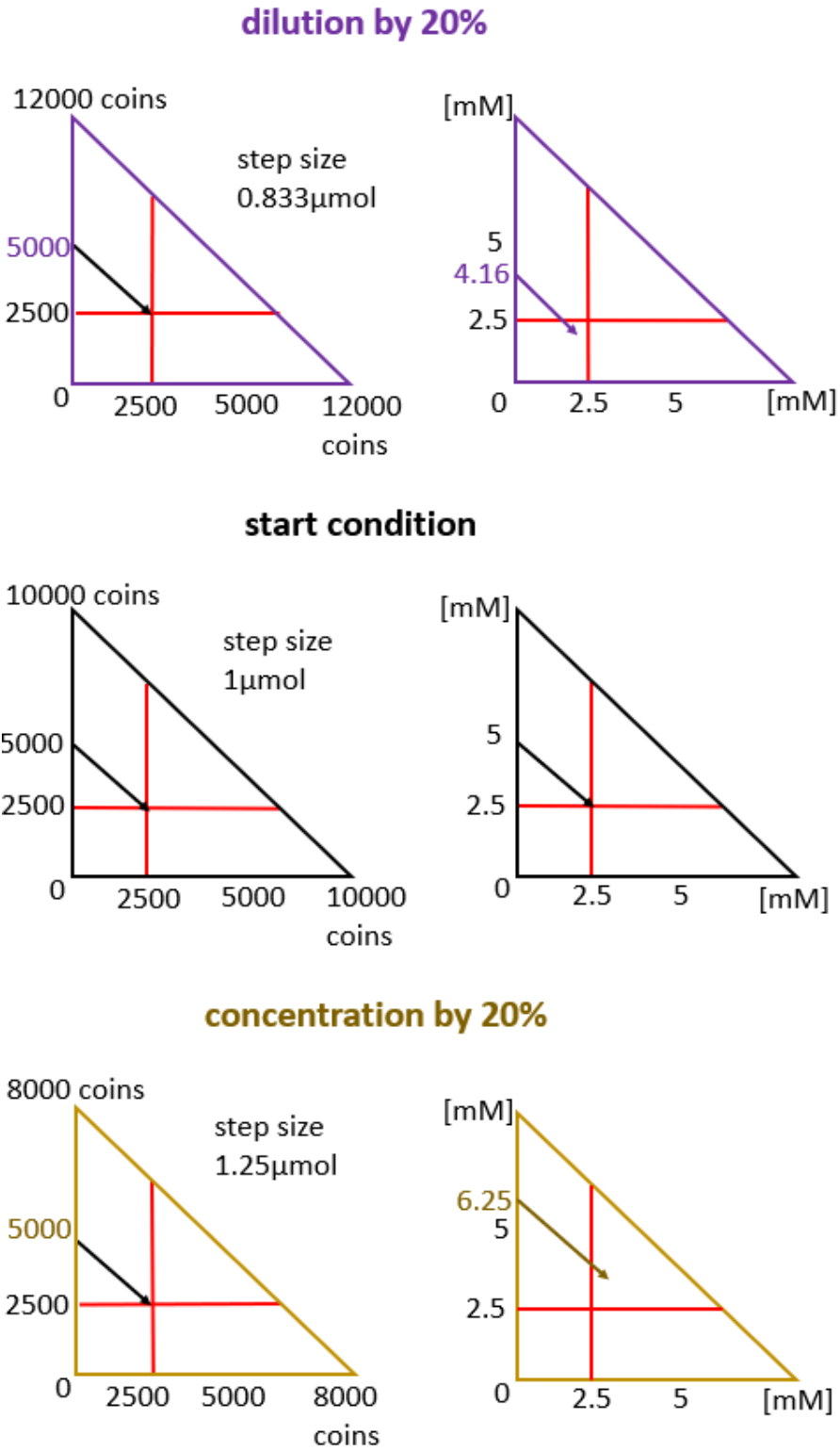
Figure 3

Figure 3
Top down views of transfer spaces. Centre: the start condition of a symmetric ensemble in black; dilution in purple (top) and concentration in gold (bottom). Triangles on the left: the coordinates of the transfer space in coins with different step sizes; triangles on the right: transfer of 2500 coins with different step size from source to sink in ensembles not knowing of a changed step size. The coordinates here are the final concentrations in millimolar [mM]. The red lines are b-c=0 in source and sink.

A dilution by 20% (concentration by 20%) will increase (decrease) the volume in source or sink from 1l to 1.2l (0.8l). The maximal concentration will drop (increase) from 10mM to 8.3¯3mM (12.5mM). In case I coin 0.1ml coins from these volumes I obtain 10000 coins in the start condition and 12000 coins upon dilution and 8000 coins upon concentration (figure 3, left). Now I specify that a coin has no volume and transfers only substrate.

A coin of the start condition transfers 1µmol, after 20% inflation the coin transfers 0.83¯3µmol, and after 20% deflation the coin transfers 1.25µmol substrate. The total amount of substrate does not change; 10mmol (0.83¯3µmol*12000=1µmol*10000=1.25µmol*8000). I could also have used the starting condition (figure 3, black triangle, left) and increase the scale mark from 10000 to 12000 to indicated 20% inflation or reduce the scale mark to 8000 to indicate 20% deflation of the coin number in an identical volume of 1 litre. This is how I proceed now.

Inflation (figure 3 top, 20%): As a change of the substrate amount in a coin or the distance of scale marks can't be measured, the ensemble relies on the original coin out of habit. The ensemble starts *e.g.* at 5000 coins in source (5000coins*0.83¯3µmol/coin = 4.16mmol in 1l = 4.16mM), zero coins in sink and uses 2500 coins of a step size reduced to 0.83¯3µmol to reach b-c=0. This, however, is no longer possible. 2500 coins no longer transfer 2.5mmol of substrate but only 2.08mmol. Because the starting concentration of the example has decreased (from 5mM to 4.16mM) and the step size has dropped, the end point of the transfer is 2.08mM in source and 2.08mM in sink. In inflation the ensemble falls short to the limit of sink but oversteps the limit of source. As the concentration drops by dilution there will be less productivity. However, between 10000 coins and 12000 coins there are now 2000 coin-steps more to transfer coins from source to sink.

Deflation (figure 3 bottom, 20%): The change of substrate concentration or scale mark can´t be measured and the ensembles still relies on the original coin. The ensemble starts *e.g.* at 5000 coins in source (5000 coins*1.25µmol/coin = 6.25mmol in 1l = 6.25mM), zero coins in sink and uses 2500 coins of a step size increased to 1.25µmol to reach b-c=0. This can´t be achieved that way; 2500 coins now transfer 3.125mmol. Because the starting concentration has shifted (from 5mM to 6.25mM) and the step size has changed, the end point of the transfer is 3.125mM in source and 3.125mM in sink. In deflation the ensemble falls short to the limit of source but oversteps the limit of sink. As the substrate concentration increases by deflation there will be more productivity. However, there are now 2000 coin-steps less to transfer coins from source to sink.

The constants Km, Vmax, bf, and cf are not affected by dilution or concentration. The enzyme sees the real concentration and stays unchanged. The cost factor (cf) will be changed later to adjust asymmetry.

Results

The change of superadditivity in area I of a symmetric ensemble after a single step of dilution or concentration

The coin pairs of source and sink in area I come from a grid with a distance of 1 coin. To obtain the net profit (b-c) of the ensemble in the absence of transfer, the coin pairs of area I are recalculated to obtain the concentration in source and sink. The net profits of all coin pairs of area I are added to obtain the net profit (np*mM^2) of area I.

Then the distance in coins of each coin pair to the next limit b-c=0 is determined. In my start condition (figure 3, black triangle, right) source starts to give with free will 1 coin at 2501 coins and up to 2500 coins

between 5000 to 10000 coins because b-c<0. Sink takes with free will starting at zero coins (take what you get up to 2500 coins) to 2499 coins (take 1 coin) because b-c>0. This is the characteristic behaviour in area I; a win-win situation for both parties. Source and sink maximally transfer 2500 coins to reach b-c=0. The coins are transferred from source to sink in every coin pair. The net profit (b-c) after transfer is calculated again and added to obtain the net profit ($np*mM^2$) of the whole area I. As a last step the result without transfer is subtracted from the result with transfer to obtain the final superadditivity in area I. This is the measure (figure 5, y-axes) to judge what happens when conditions change.

After dilution (figure 3, purple triangles, right) source gives with free will between 1 coin and 2500 coins of the maximal amount of coins for the corresponding degree of dilution. Sink takes with free will between 1 coin and 2500 coins. In dilution the maximal amount of coins and the drop in step size are determined by a correction factor: (100+%)/100. The ensemble does not know of dilution and can´t measure the true substrate concentration. Therefore, the ensemble sticks to the old amount of coins and volume before dilution. Further calculations are as above.

After concentration (figure 3, golden triangles) source gives with free will between 1 coin and 2500 coins of the maximal amount of coins for the corresponding degree of concentration. Sink takes with free will between 1 coin and 2500 coins. In concentration the maximal amount of coins and the increase in step size are determined by a correction factor: (100-%)/100. As the ensemble does not know of the concentration the behaviour is unchanged. Further calculations are as above. At first (figure 4) I want to present a view of the resulting local superadditivity in area I for the start condition, 20% inflation, and 20% deflation of a symmetric ensemble. The law of conservation of mass is always obeyed. The same amount of coins given is taken.
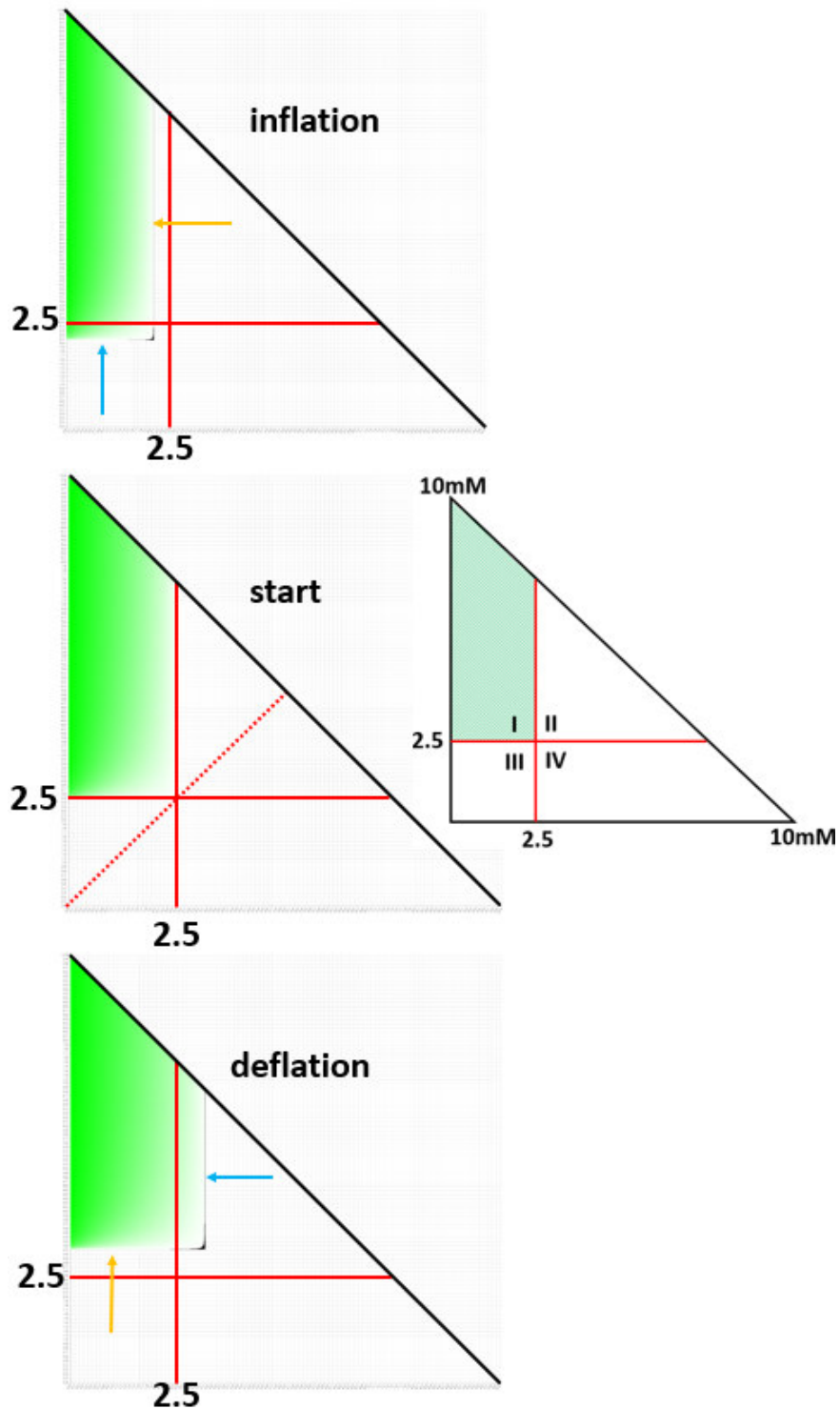
Figure 4

Figure 4
In area I (right schema) transfer is performed at free will, resulting in a superadditive surface above the surface resulting from no transfer. The volume between the two surfaces is the total superadditivity: left, start. The darker the green, the more local superadditivity. All possible symmetric ensembles fall onto the dotted red line (start). In 20% inflation (top) or 20% deflation (bottom) the superadditivity is overstepping (blue arrows) or falls below (orange arrows) the limits b-c=0 (2.5mM in source and 2.5mM in sink, the equilibrium point). The total maximal substrate concentration is 10mM.

The total superadditivity (np*mM^2) is the sum of the local superadditivity (np of all concentration pairs in area I). The local superadditivity in area I is not uniformly distributed. The surface resulting from no transfer and the surface resulting from transfer have different distances in different locations (strong green to nearly white). The total superadditivity (a single point in figure 5) is the total volume (np*mM^2) between two surfaces.

Inflation results in an overstepping of the limit b-c=0 to area III with high local superadditivity. On the other hand, inflation falls short to the limit b-c=0 to area II where local superadditivity is low in area I.

Deflation results in overstepping the limit b-c=0 to area II with low local superadditivity while deflation falls short with respect to the limit b-c=0 to area III where there is a high local superadditivity in area I.

This all together will result in an increasing total superadditivity with increasing dilution in inflation and on the other hand it will result in a decreasing total superadditivity upon increasing concentration in deflation.

This behaviour can be observed in figure 5. We look at the total superadditivity of a symmetric ensemble which experiences single steps of different sizes of dilution (1% to 20% in relation to the start condition) and concentration (negative 1% to negative 20% in relation to the start condition). The starting point (black single dot) separates deflation (left, gold) from inflation (right, purple).

Figure 4 reappears in figure 5. The total superadditivity of 20% inflation in figure 4 is the most right purple point. The total superadditivity of 20% deflation in figure 4 is the most left golden point. And the ensemble named "start" (neither diluted nor concentrated) is the single black dot in the middle.
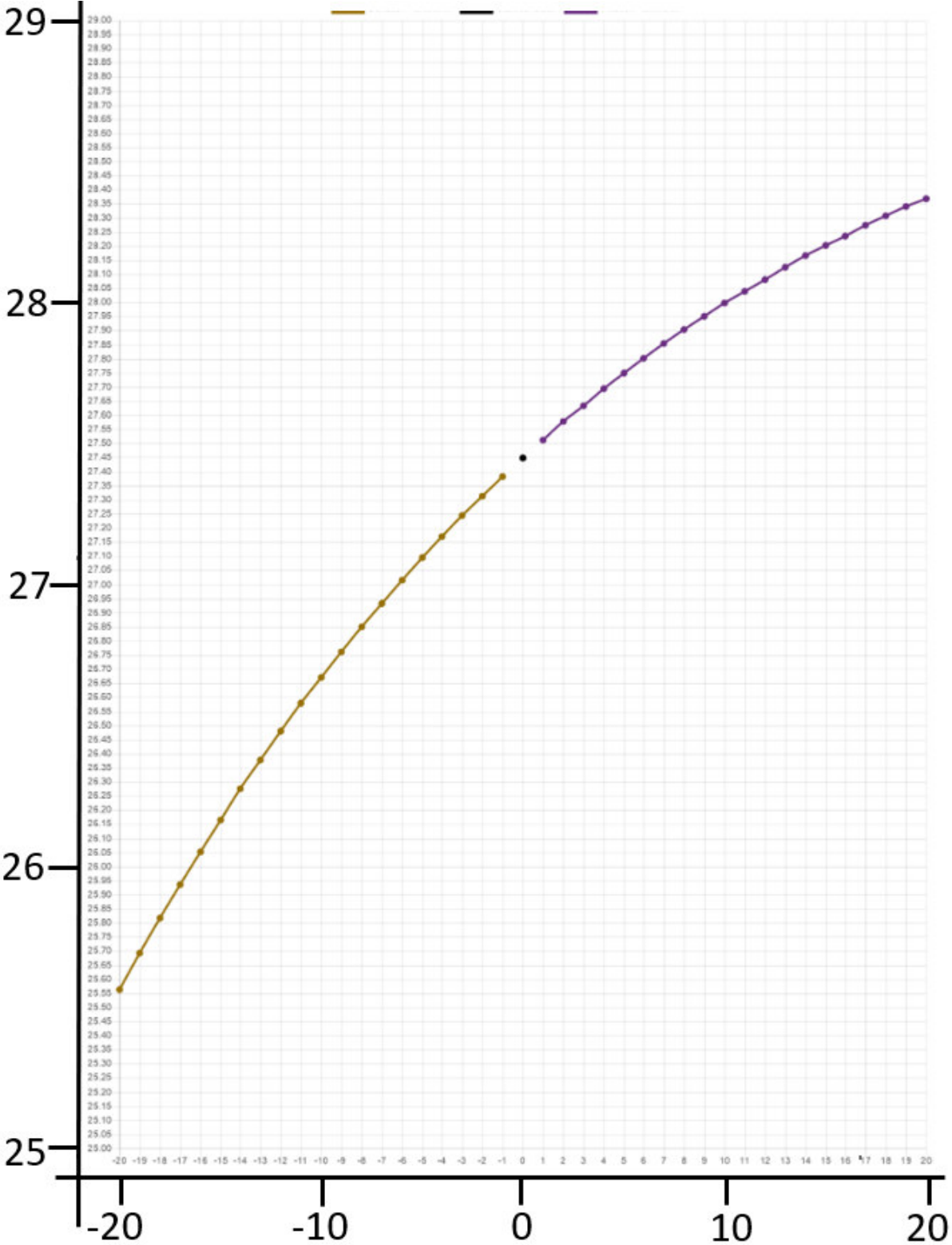
Figure 5

Figure 5
Superadditivity of a symmetric ensemble with the limits b-c=0 at 2.5mM in source and 2.5mM in sink. Single steps of different sizes of dilution (x-axis: 1% to 20%, purple dots) and concentration (x-axis: negative 1% to negative 20%, golden dots) starting at the benchmark (single black dot). The total superadditivity ($np*mM^2$) is given on the y-axis.

The change of superadditivity in area I of two types of asymmetric ensembles after a single step of dilution or concentration

Asymmetric ensembles basically fall into two categories. There are weak ensembles and strong ensembles. Weak ensembles exist left to a line of the concentration coordinates 0mM/0mM – 2.5mM/2.5mM – 5mM/5mM in this transfer space (figure 4, "start", dotted red line). They have *e.g.* a lower cost factor in source than in sink. In addition, there are other possibilities to induce asymmetry but here I concentrate on the cost factor. In weak ensembles source does not like to give (cost is low) and sink does not like to take (cost is high) substrate within certain limits compared to a symmetric ensemble. Strong ensembles exist on the right side of above line. In strong ensembles the cost factor in source is higher than in sink. Therefore, source easily gives substrate and sink easily takes substrate within certain limits compared to a symmetric ensemble (5).

In figure 6 a view of the local superadditivity of a weak asymmetric ensemble (b-c=0 at 2mM in sink and 3mM in source) is presented. The asymmetry is adjusted with the cost factor (source cf=10/7 c/mM, sink cf=2 c/mM); the start condition. The ensemble starts with a concentration of maximal 10mM or 10000 coins in source or sink. In this type of asymmetry there is a considerable amount of local subadditivity visible. In 20% inflation and in 20% deflation the superadditive and subadditive region is shifted and oversteps or falls below the limits b-c=0 in source and sink.

In figure 7 the total superadditivity (superadditivity minus subadditivity) of 20 single steps of inflation or deflation are displayed. The curve is similar but steeper than in the symmetric ensemble (figure 5).

Figure 8 and 9 are calculated as described for a strong ensemble (b-c=0 at 3mM in sink and 2mM in source; source cf=2 mM, sink cf=10/7 c/mM). However, there (figure 9) I observe a new behaviour.
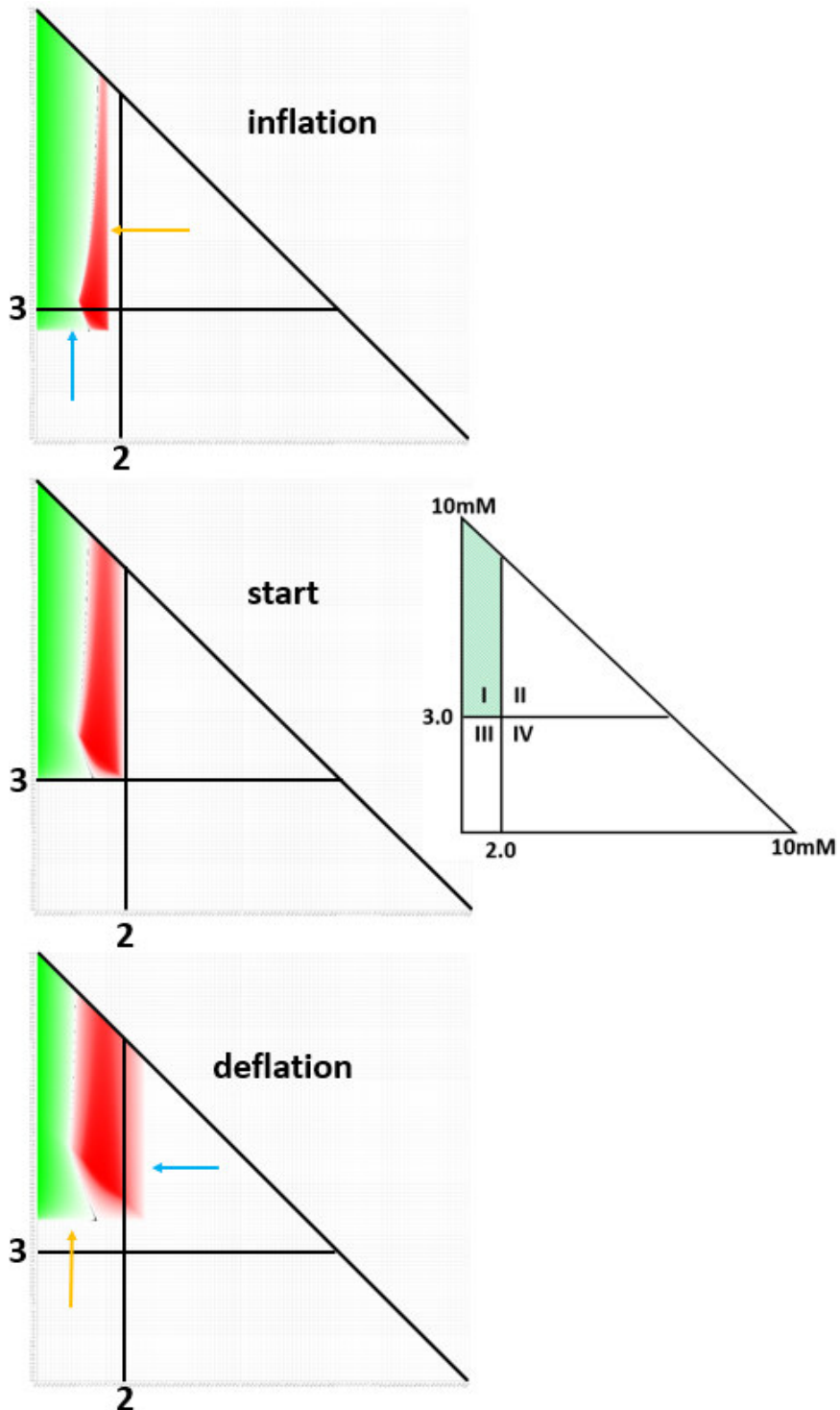
Figure 6

Figure 6
Local superadditivity and subadditivity in area I of a weak asymmetric ensemble. In area I (right schema) transfer is performed at free will. This results here (start) in local superadditivity (green) and subadditivity (red). In 20% inflation (top) or 20% deflation (bottom) the area is overstepping (blue arrows) or falls short of (orange arrows) the limits b-c=0 (3mM in source and 2mM in sink). The darker the green, the larger the local superadditivity. The red area indicates subadditivity; b-c after transfer is smaller than without transfer. The darker the red the larger the subadditivity. The total superadditivity is the sum of the superadditive and (negative) subadditive volume.
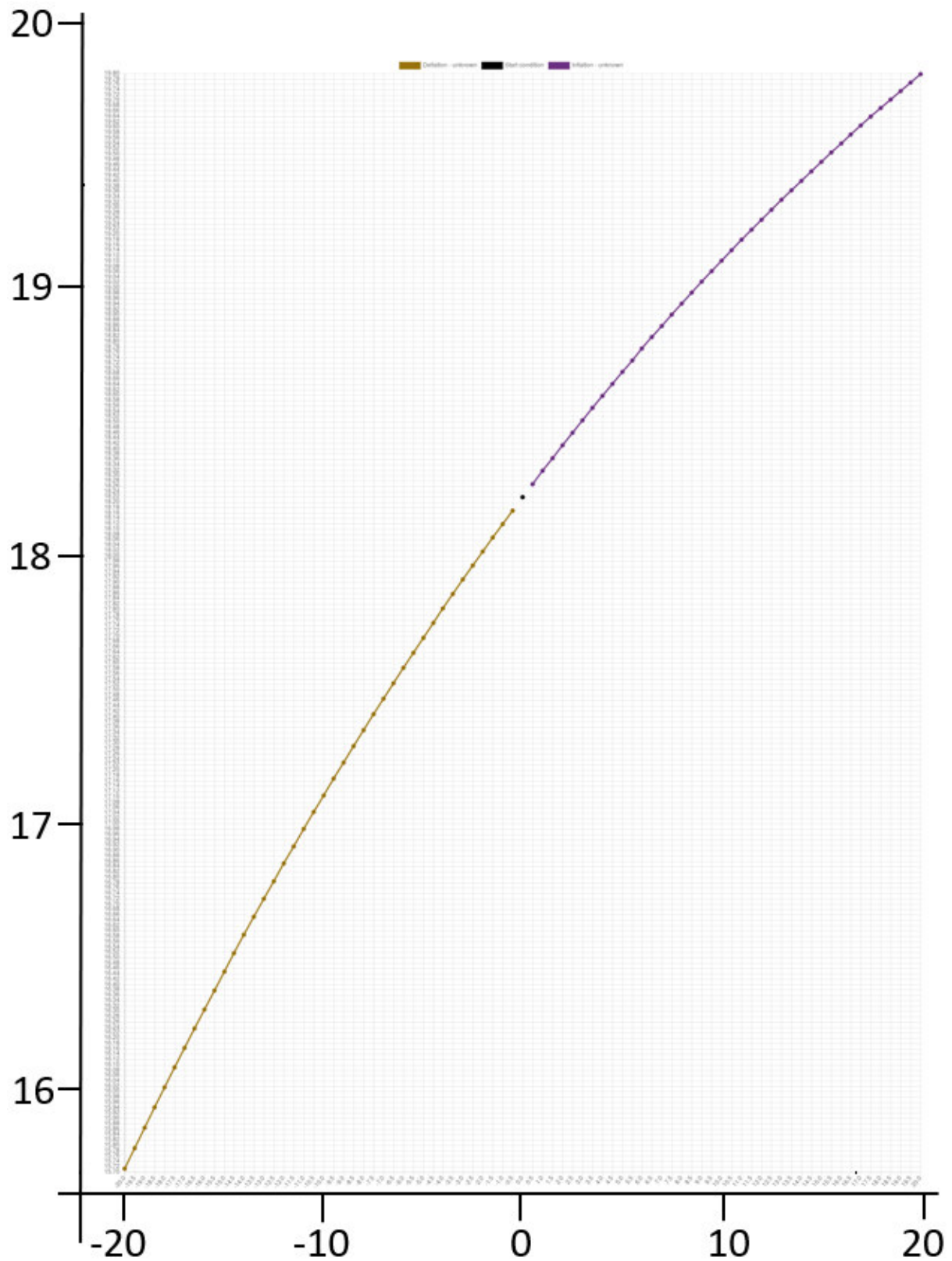
Figure 7

Figure 7
Superadditivity of a weak asymmetric ensemble with the limits 3mM in source and 2mM in sink. Single steps of different sizes of dilution (x-axis: 1% to 20%, purple dots) and concentration (x-axis: negative 1% to negative 20%, golden dots) starting at the benchmark (single black dot). The total superadditivity (np*mM^2) is given on the y-axis.
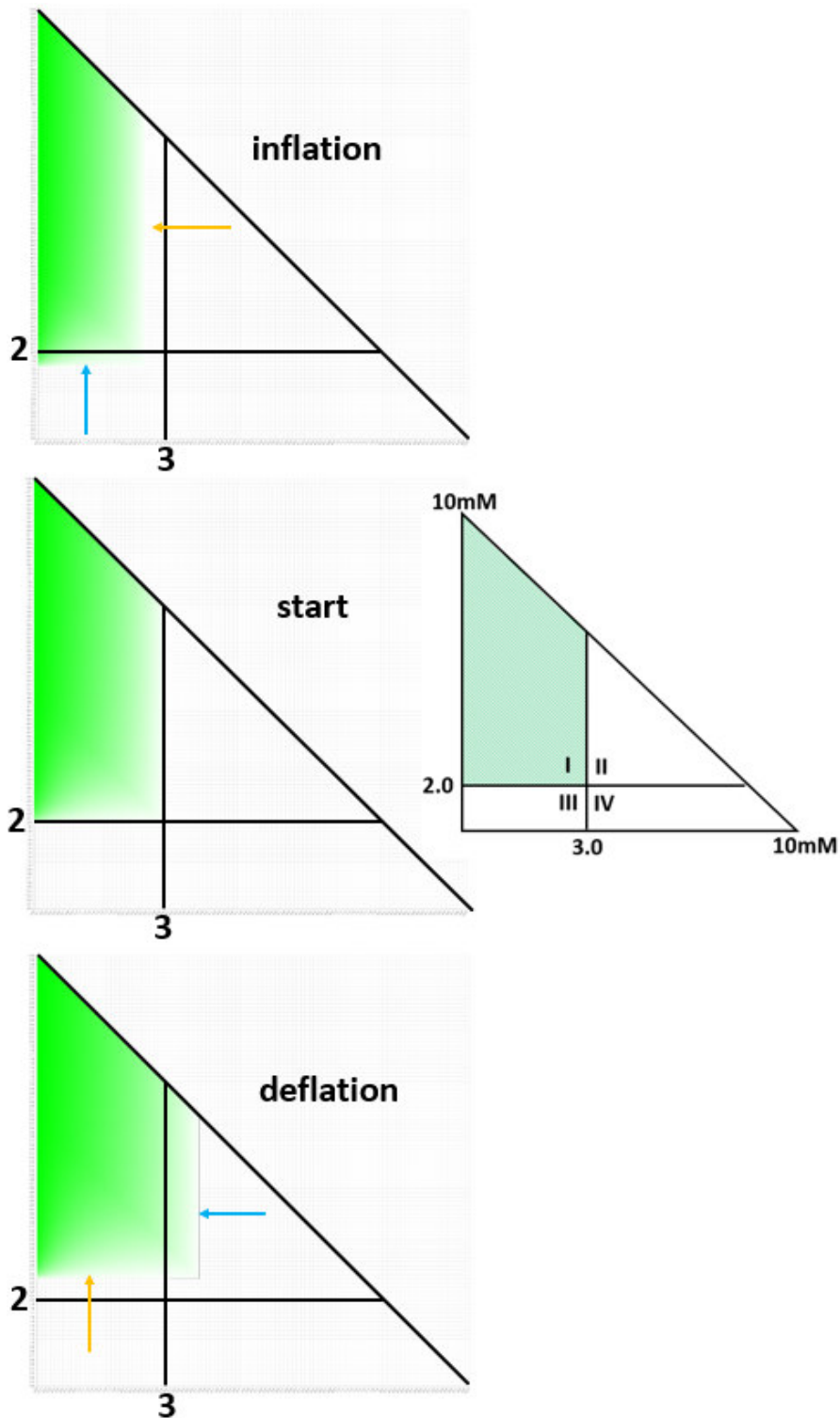
Figure 8

Figure 8
The local superadditivity of a strong, asymmetric ensemble is depicted. In area I (right schema) transfer is performed at free will. This results in local superadditivity (start). The sum of all local superadditivity is the total superadditivity. In 20% inflation (top) or 20% deflation (bottom) the superadditivity is overstepping (blue arrows) or falls short (orange arrows) to the limits b-c=0 (2mM in source and 3mM in sink). The darker the green the larger the local superadditivity.
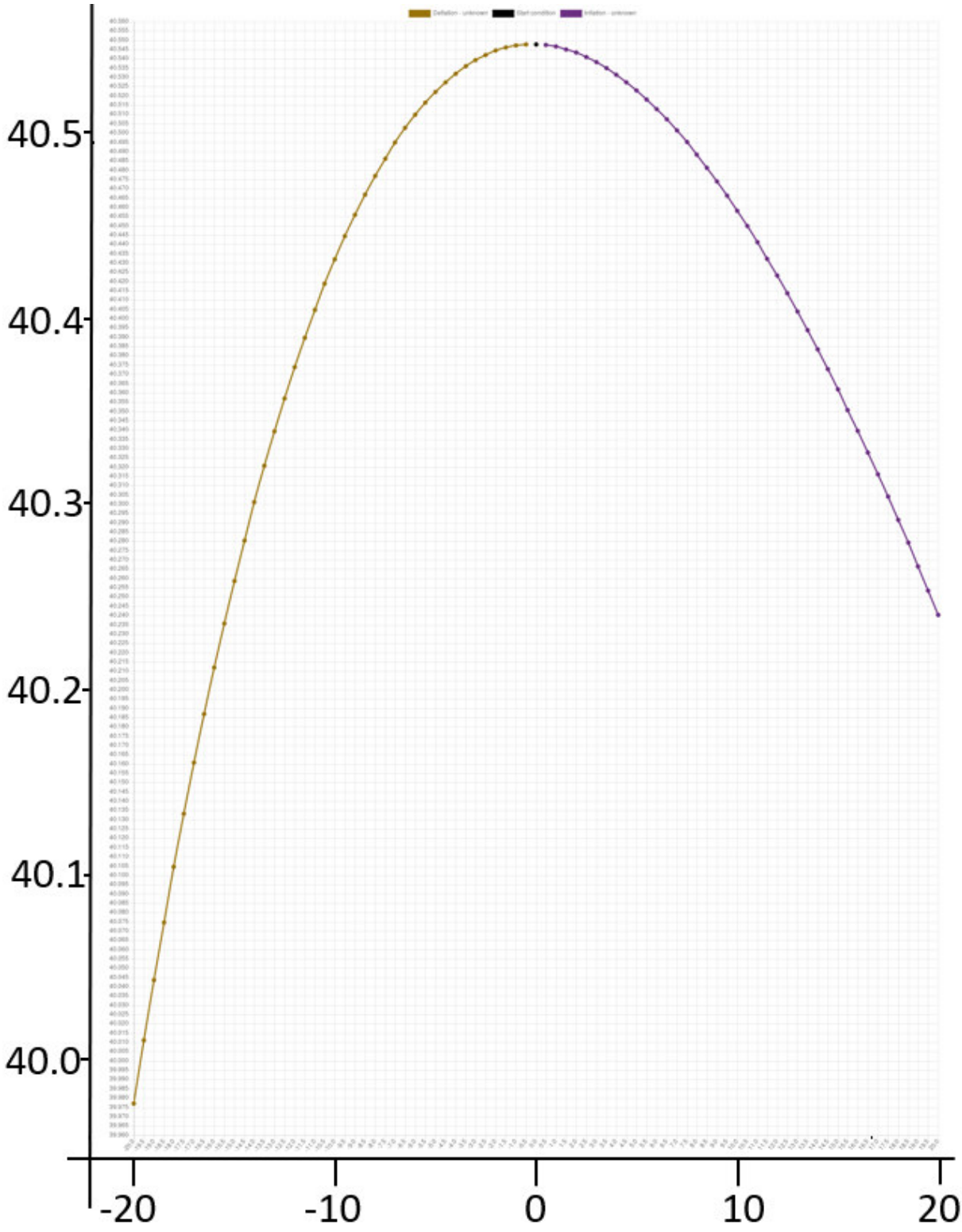
Figure 9

Figure 9
Superadditivity of a strong asymmetric ensemble with the limits 2mM in source and 3mM in sink. Single steps of different sizes of dilution (x-axis: 1% to 20%, purple dots) and concentration (x-axis: negative 1% to negative 20%, golden dots) starting at the benchmark (single black dot). The total superadditivity ($np*mM^2$) is given on the y-axis. This figure reappears as C in figure 10.

Figures 7, 5, and 9 - in this order - in combination with figures 6, 4, and 8 show a pattern. The sequence from weak to strong ensembles is also observable in the evolution of ensembles leading in consequence to division of labour (2). The observable pattern here is the increase in total superadditivity ($np*mM^2$) from weak to strong ensembles accompanied by a decrease in steepness of the slope within the starting condition (black dot). In the beginning the complete curve of diluted, undiluted, and concentrated volume (figure 7) is quite straight and extends over $4np*mM^2$. In the symmetric ensemble the complete curve seems to start to saturate and extends over about $3np*mM^2$. Finally, in figure 9, the curve looks a bit like a negative parabola and extends over only $0.6np*mM^2$.

The weak ensemble shows in area I (transfer is at free will) not only superadditivity but also local subadditivity (figure 6). This is well known and understood (5). Inflation and deflation shift the active area beyond the limits b-c=0 in a typical way. Inflation always oversteps the limit to area III but does not reach the limit b-c=0 to area II. Therefore, the ensemble is missing subadditivity near the border to area II (figure 6, inflation). The loss of a negative contribution is a gain and thus the curve is very steep.

The strong ensemble is superadditive everywhere in area I. Deflation falls short to the limit b-c=0 to area III and oversteps the limit b-c=0 to area II. At high substrate concentrations in sink the additional superadditivity in area II overcompensates the loss of superadditivity in area I in comparison to inflation. The step size of overstepping is much larger in deflation. Therefore, inflation is no longer better than deflation. Inflation is missing a lot of superadditivity near area II which is not by far compensated through the gain in superadditivity in area III (figure 8). This is able to reverse the effect of inflation and deflation on the outcome of total superadditivity as shown in figure 10. The slope in the start condition becomes negative.
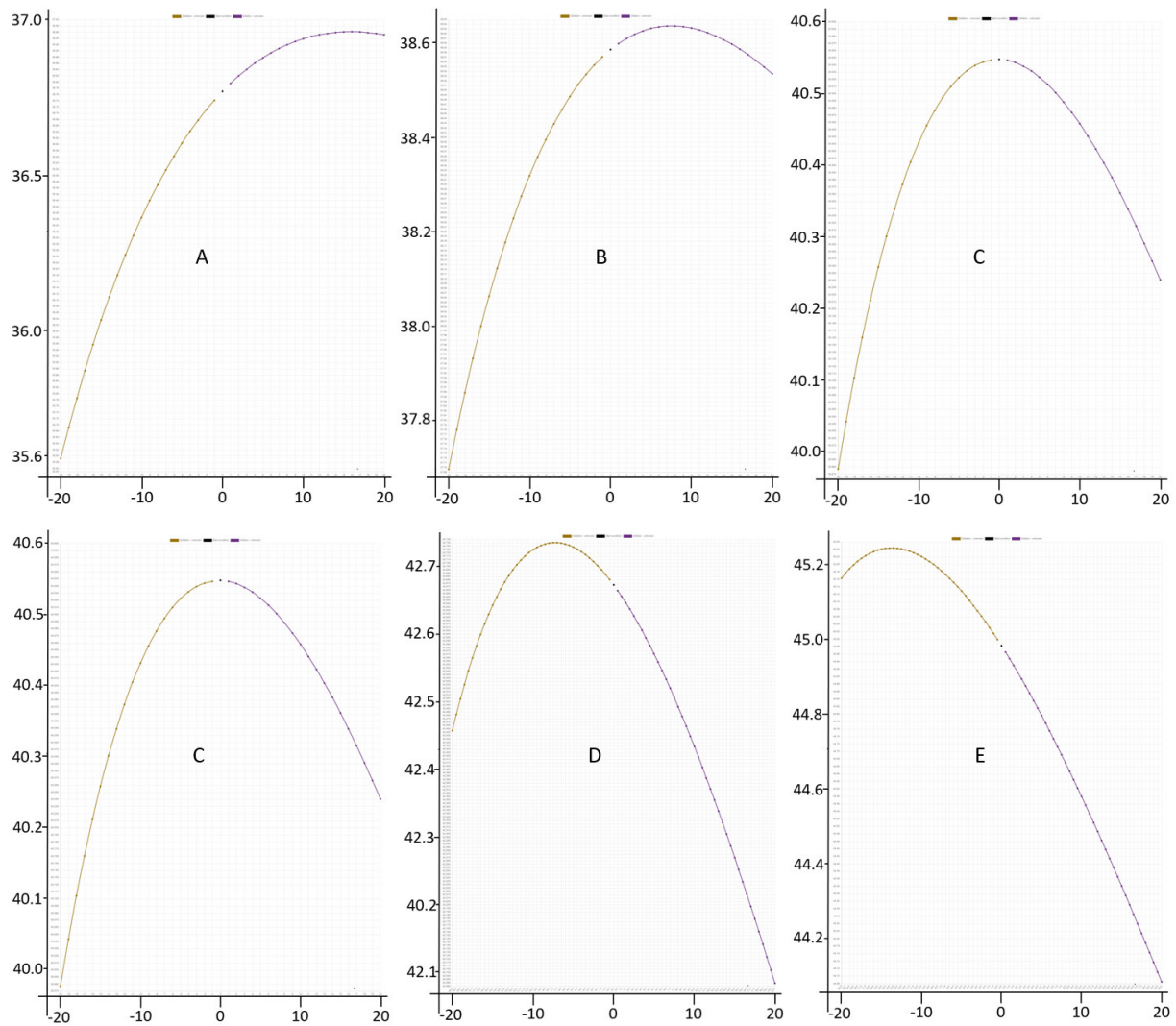
Figure 10

Figure 10
Inflation (purple, 1% to 20%, x-axes) and deflation (gold, minus 1% to minus 20%, x-axes) are observed. The limit b-c=0 in sink is fixed to 3mM and in source b-c=0 is varied from 2.2mM (A) to 2.1mM (B) to 2.0mM (C) and further from 2.0mM (C) to 1.9mM (D) and 1.8mM (E). The domination by inflation (A, B) slowly changes to domination by deflation (D, E). The single black dot is the start condition.

While in figure 10 the limit b-c=0 in sink is fixed at 3mM, the limit b-c=0 in source is systematically lowered in 0.1mM steps from 2.2mM to 1.8mM changing the cost factor. On this way deflation becomes dominating when the total superadditivity is very high (y-axes, $np \cdot mM^2$) and the substrate concentration in source is low and simultaneously higher in sink. It is obvious from this small example that there are conditions when inflation,

deflation or the start situation will dominate. How the different possibilities are distributed over the transfer space is shown in figure 11.
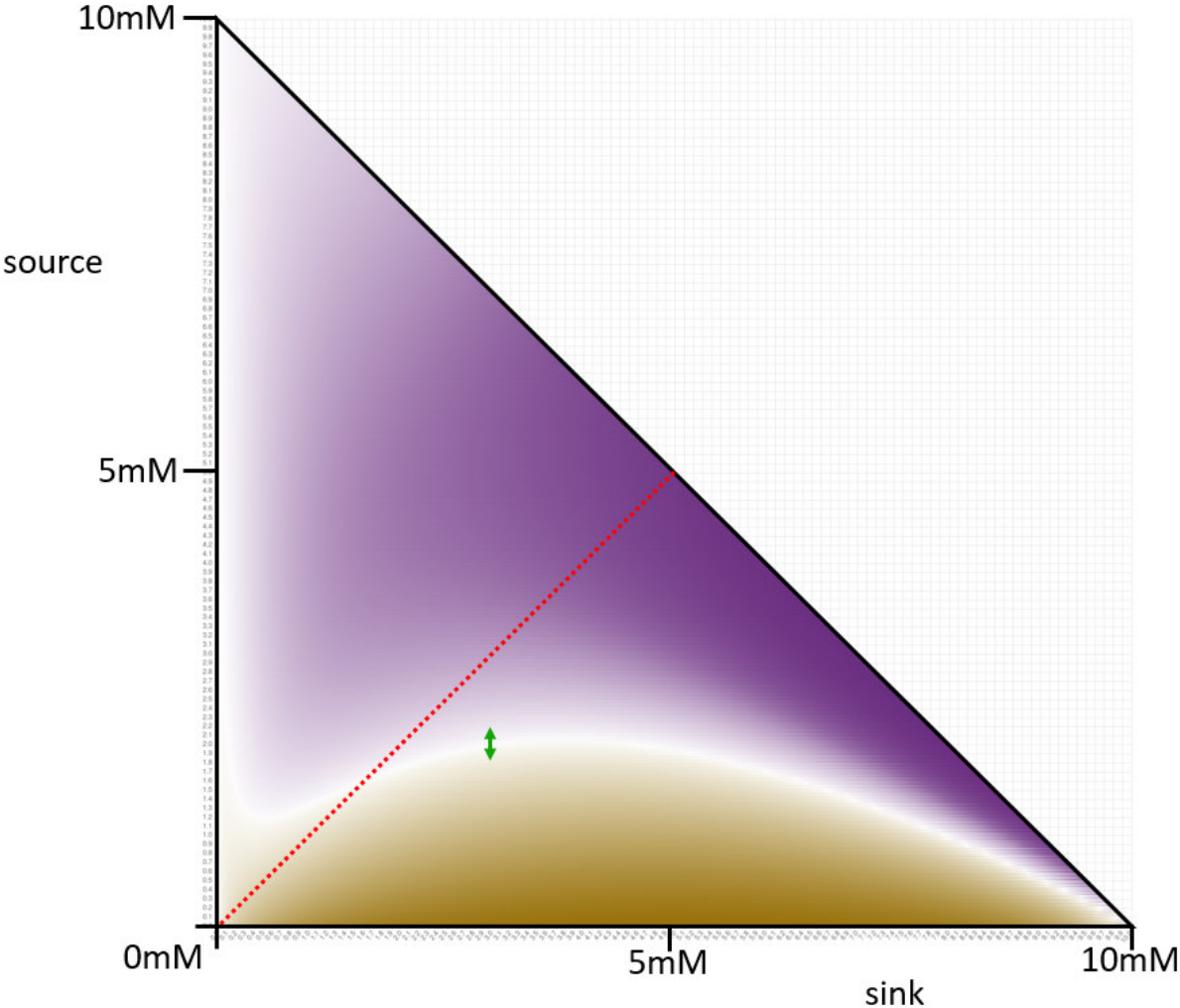
Figure 11

Figure 11
A collection of all kinds of symmetric and asymmetric ensembles is observed. The intersection points where b-c=0 in source and b-c=0 in sink have a distance of 5.55µM. In every intersection point the total superadditivity of the start condition, a small step of inflation, and a small step of deflation are determined. The results are used to calculate the slope of the curve within the start condition for each ensemble. The purple colour marks the area where the slope is positive. Inflation has a higher superadditivity here compared to deflation or the start condition (figure 10A, 10B). The golden colour marks the area where the slope is negative. Deflation results here in a higher superadditivity compared to inflation or the start condition (figure 10D, 10E). The colour intensity translates to a steeper slope. White is any slope near zero. The green double arrow marks the concentration range of figure 10.

Hiding within the white zone of figure 11 are infinitesimal small single black dots forming an invisible line where the start condition is better than inflation or deflation (figure 9 and figure 10C). In figure 11 the general dominance of <u>already</u> inflated or deflated ensembles over the same ensemble within its start conditions is observed. This is an unfair comparison as inflation and deflation are costly investments and these costs are not considered. In the scheme of figure 12 I try to explain how to solve this inequality. The central idea is to go one step back and interpret inflation again as volume increase (a growth in size) and deflation as a volume decrease (decrease in size) and compare the superadditivity there with the superadditivity achievable through increased division of labour.

Every single ensemble in the collection of all ensembles in figure 12 has three options to increase superadditivity. They all start with a certain superadditivity coming from identical Km, Vmax, and bf but different cf and the available substrate in the differently sized areas I. Their possibilities are to increase size (inflation, *in situ*) or to decrease size (deflation, *in situ*). The meaning of *in situ* is that the borders of area I of the observed ensembles stay unchanged. A further possibility is to increase division of labour. Division of labour will increase the size of the present area I to a larger area I. The division of labour will increase when either source starts to give with more ease (increasing cost factor cf; figure 12 A1 and B1) shifting the limit b-c=0 (equilibrium concentration) in source to lower concentrations or sink starts to take with more ease (decreasing cost factor cf; figure 12 A2 and B2) shifting the limit b-c=0 in sink to higher concentrations. Finally, both conditions happen simultaneously (figure 12 A3 and B3).
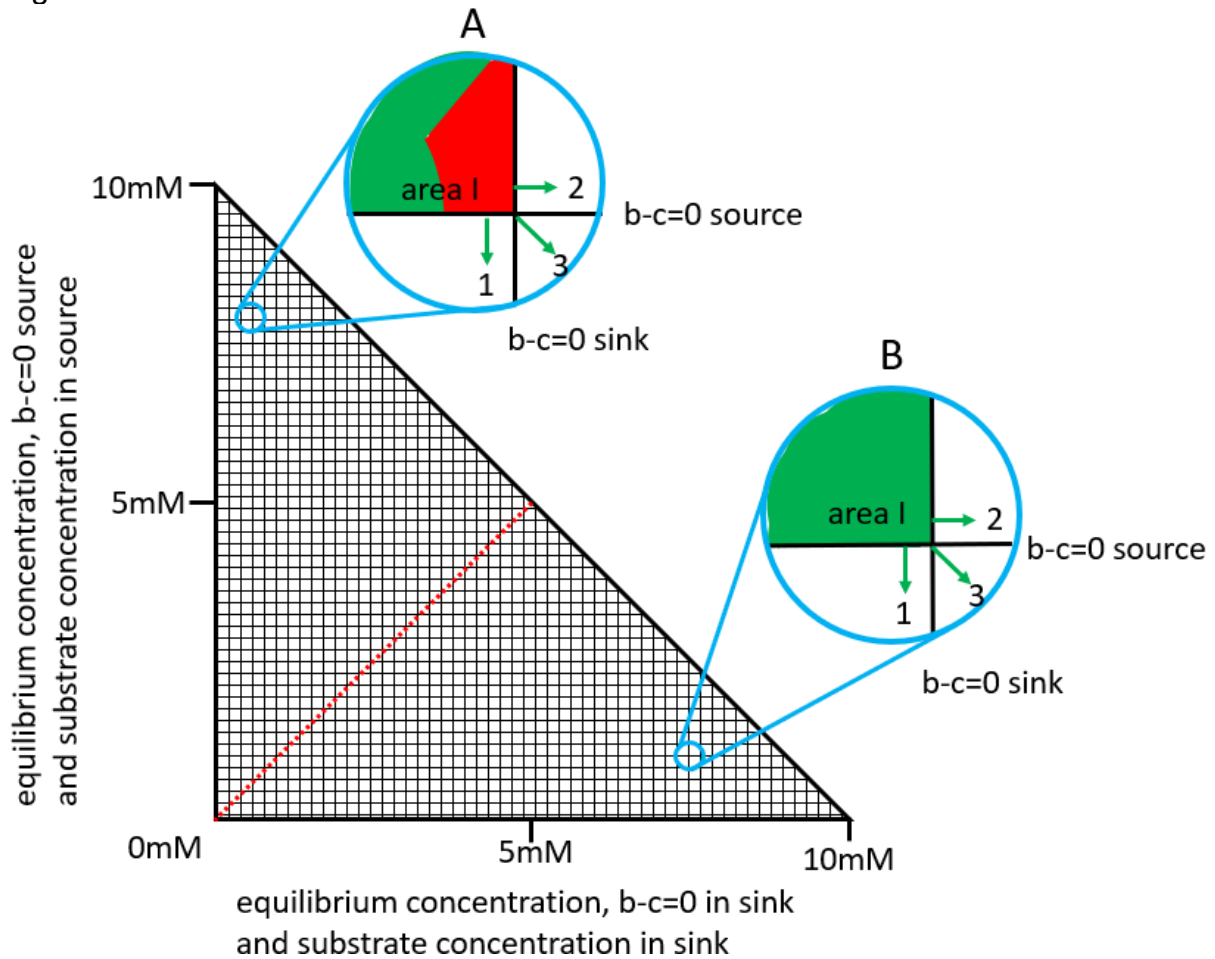
Figure 12

Figure 12
The transfer space is filled with all possible limits b-c=0 in source and sink. In A and B single examples of asymmetric ensembles are shown. The total superadditivity (superadditivity plus negative subadditivity, A) in each area I is determined. For each case the ensemble can make an investment decision: grow (inflation), shrink (deflation) or increase division of labour (A, B). Division of labour can proceed in three ways: 1, decrease the equilibrium concentration b-c=0 in source; 2, increase the equilibrium concentration b-c=0 in sink; 3, do 1 and 2 simultaneously. The collection of ensembles differs by a distance of 5.55µM between two b-c=0 limits. The red dotted line indicates the intersection of b-c=0 in source and b-c=0 in sink in symmetric ensembles.

The three different paths to increase superadditivity are alternative investment decisions or evolutionary development paths. To grow, to shrink and to increase division of labour will cost. Specific benefits are associated with these costs.

Such a specific benefit is in growth *e.g.* the ability to swallow larger food particles and an advantage in aggressive confrontations leading to a self-

reinforcement similar to a preferential attachment effect. In shrinkage thrift will produce other advantages. There will be the advantage of less substrate consumption to build and maintain an ensemble. Less substrate acquisition will result in a smaller predation risk and avoids starvation when food is limited. The best evolutionary path will prevail, but here they all are set to be equal with respect to their specific benefits and costs. In addition, there is an unspecific benefit of these three investments coming from the non-linearity of the transfer space and the production of superadditivity. I want to compare only this unspecific benefit of the three possible investment decisions.

In figure 13 I compare 1% inflation, 1% deflation, and an increased division of labour by a decrease of the equilibrium concentration b-c=0 in source by 0.1mM (1%, source gives more easily). The specific cost and benefit to grow by 1% or the specific cost and benefit to shrink by 1% or the specific cost and benefit to increase division of labour through an increased cost factor in source are set to be equal. Now only the contribution of superadditivity due to the non-linearity of the transfer space may differ and discriminate the three decisions.

Inflation dominates at high and medium substrate concentrations in source and low and medium substrate concentrations in sink. Deflation dominates at low substrate concentrations in source and low to high substrate concentrations in sink. Both areas are separated by an area where division of labour dominates. Division of labour is very strong at very low substrate concentrations in source and very high substrate concentrations in sink.

It appears to be imaginable that in an evolutionary scenario of a long-term continuously decreasing substrate concentration in source a system may successively – in an evolutionary time frame – react with initial growth at high substrate concentrations, then switch to increased internal division of

labour to be followed after further decrease of substrate concentration in source with shrinkage. To predict a possible development when, at low substrate concentrations in source, substrate concentration in sink is slowly rising is much harder as source (in division of labour) moves here perpendicular to the substrate increase.
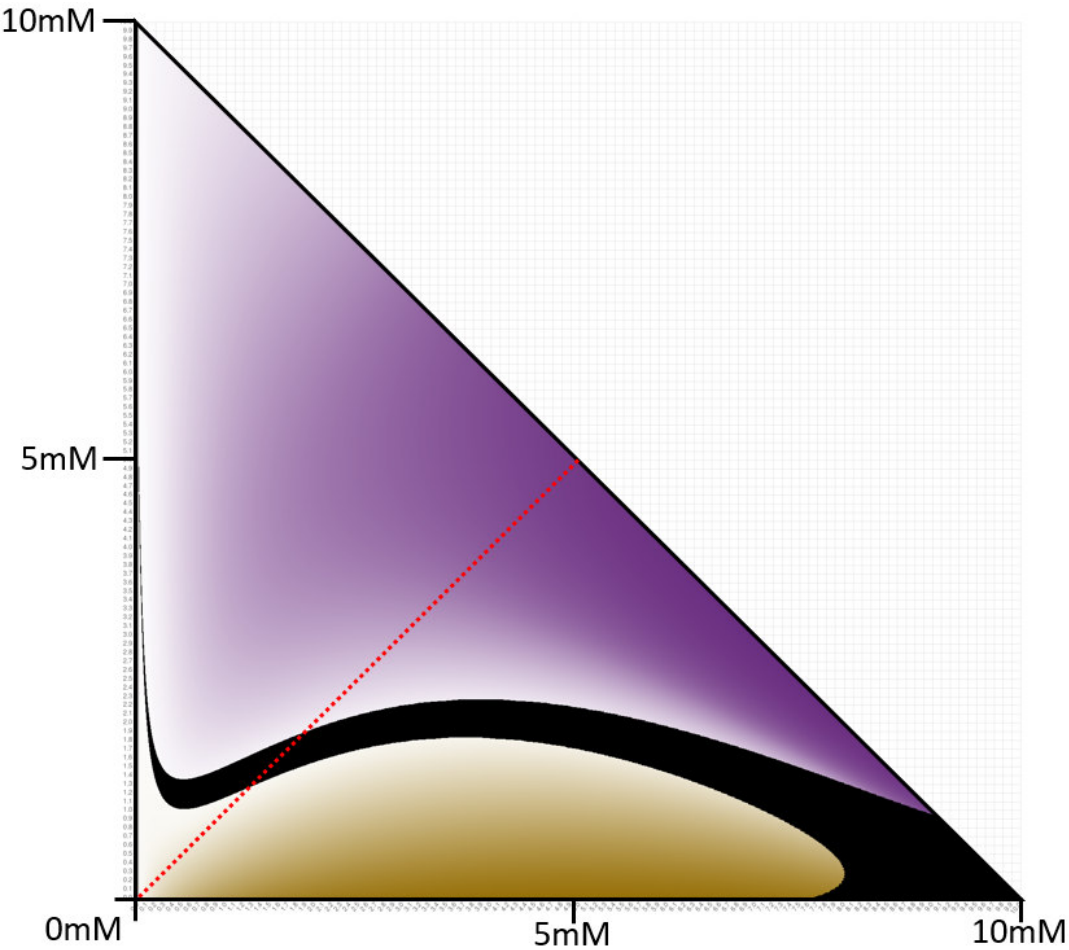
Figure 13

Figure 13
This is a comparison of a 1% inflation step (purple) or 1% deflation step (gold) of a collection of different ensembles with an increase in division of labour (black) by a 0.1mM decrease of the equilibrium concentration (b-c=0) in source (figure 12 A1, B1). The colour intensity correlates with the final amount of superadditivity. This differs from the colour coding in figure 11! The black area is not resolved according to intensity of superadditivity. The red line marks the location of all possible symmetric ensembles. The limits b-c=0 of the collection of ensembles are separated by 5.55µM.

In figure 14 I compare 1% of inflation, 1% deflation, and an increased division of labour by an increase of the equilibrium concentration b-c=0 in sink by 0.1mM. Sink takes with more ease. The reason is that the cost factor in sink becomes smaller. Division of labour is strong at low substrate concentrations in source and sink.
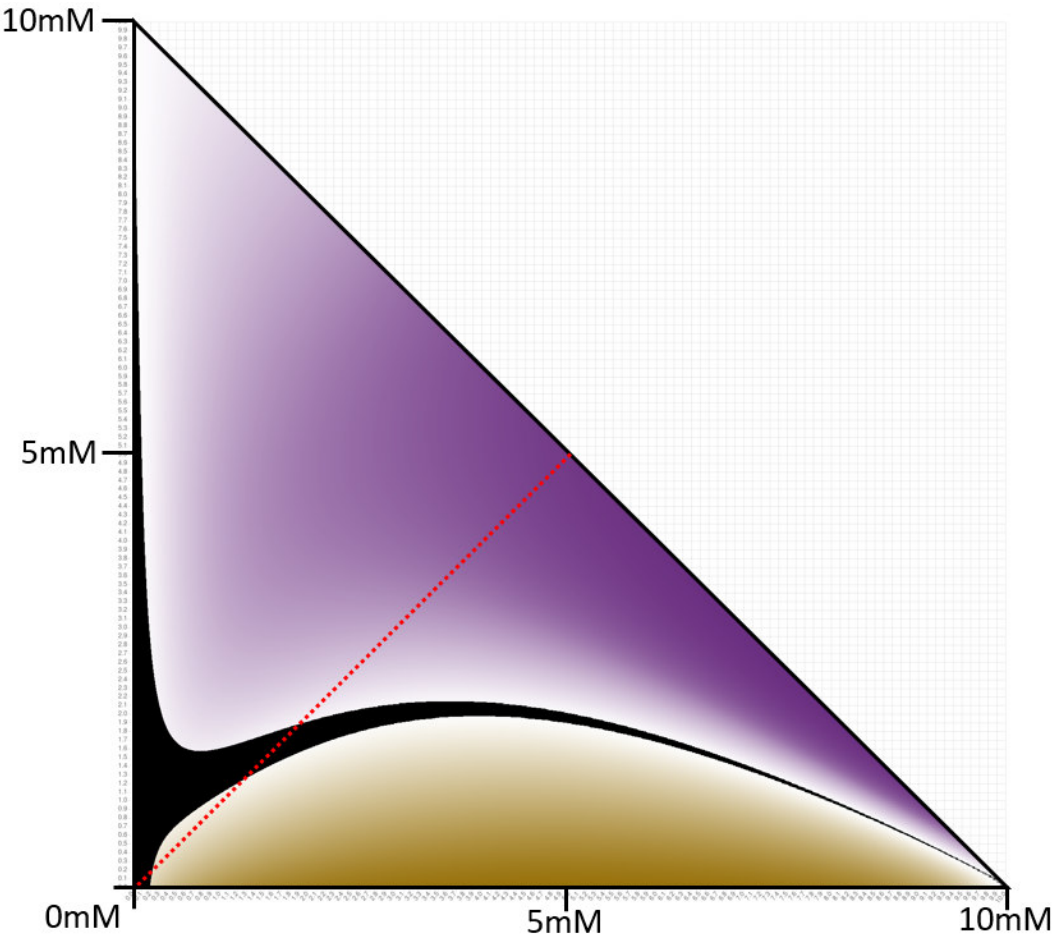
Figure 14

Figure 14
This is a comparison of a 1% inflation step (purple) or 1% deflation step (gold) of a collection of different ensembles with the same collection increasing the division of labour (black) by a 0.1mM increase of the equilibrium concentration (b-c=0) in sink (figure 12 A2, B2). The colour intensity correlates with the final amount of superadditivity. The limits b-c=0 of the collection of ensembles have a distance of 5.55µM.

In figure 15 I compare 1% inflation, 1% deflation, and an increased division of labour by a simultaneous decrease of the equilibrium concentration b-c=0 in source and an increase of the equilibrium concentration b-c=0 in sink. The probability for a simultaneous event may be small, but recombination can easily combine two independent mutations.
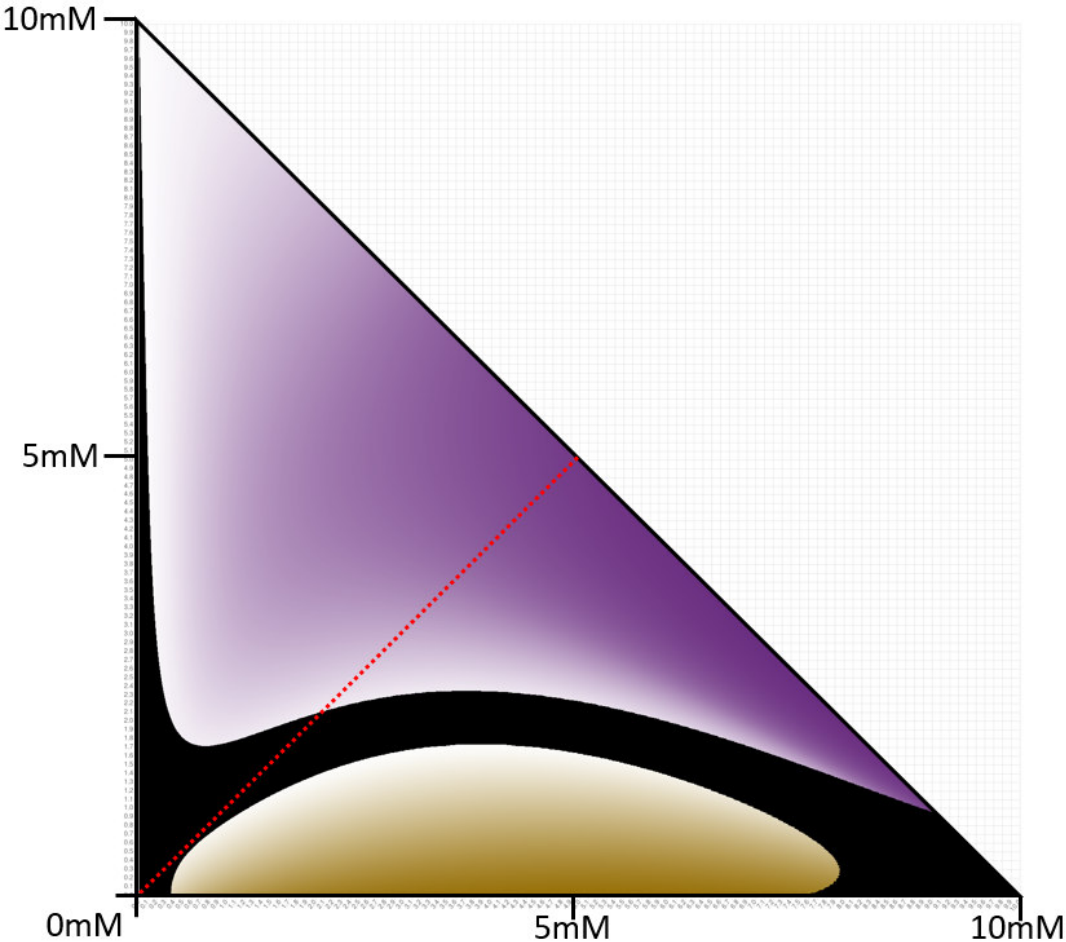
Figure 15

Figure 15
This is a comparison of the superadditivity of a 1% inflation step (purple) or 1% deflation step (gold) of a collection of different ensembles with an increase in division of labour (black) by 0.1mM decrease of the equilibrium concentration (b-c=0) in source and 0.1mM increase of the equilibrium concentration (b-c=0) in sink (figure 12 A3, B3). The colour intensity correlates with the final amount of superadditivity. The limits b-c=0 of the collection of ensembles have a distance of 5.55µM (the pixel size of the figures).

Figure 15 combines the features of figure 13 and 14. In figure 15 a 1% change in inflation or deflation is compared to a 1% change in division of labour by a simultaneous change of the equilibrium point by 0.1mM in source and sink. In general, the golden area of deflation is of lower superadditivity than the purple area of deflation. The black area of division of labour is not resolved according to the size of superadditivity.

Discussion

In my past paper (1) I observed movement through the transfer space with many consecutive steps of 1µM and movement of the space itself, *i.e.* it´s curvature, with a repeated transfer at a single concentration pair in a fast regenerating system with a continuous change of Km or Vmax or cf. Both movements resulted in a decrease in superadditive net profit. This could be interpreted as the action of a force. However, this was an apparent force and the loss in superadditivity originated from the non-linearity of the transfer space. Both settings had in common the ability of source and sink to determine the concentration and adjust the step size accordingly. To keep the superadditivity constant while conditions change it is necessary to be able to determine the substrate concentration. What happens if the parties are, for whatever reason, unable to determine the concentration? They only can count steps. The limits for source to give and sink to take are determined by concentrations. Counting steps is no longer accurate when dilution or concentration has occurred. The steps are now called coins. Changes of the concentration within the transfer space affect the step size of the coin and the number of coins.

Inflation and deflation could be interpreted as an act of force or deception. Force: In the past the volume between the two surfaces (superadditivity, subadditivity: net profit with transfer minus net profit without transfer in

every concentration pair) was calculated not as a continuous integral but discrete. I used within my older papers a constant distance of the grid (1000*1000 for 5mM*5mM concentration). Now I basically compare the effect of different grid sizes on the outcome. Source and sink consider the concentration to be constant. They also consider the substrate to be equally distributed over a constant amount of coins. The change of the concentration changes the grid size, *i.e.* the number of coins and their content. An observer with a fixed measure will have the impression that the coordinates move as if they were compressed or stretched by a force (like a dark energy). I observe a system able to count but unable to measure. To change the number of coins basically changes the number of unit hatch marks on yardstick for that space. However, the internal observer is convinced that the distance between the tick marks is constant. Therefore, it appears as if a force would stretch or compress the space. In my model it is again an apparent force. Only in actual dilution or concentration of a solvent a real force is acting on the space.

Deception: The limit b-c=0 separates self-advantage and self-harm. This limit could be recognized through a concentration determination. However, source and sink either stick to outdated empirical knowledge or are deceived regarding the true value of the coin and their own actual concentration. Therefore, source and sink fall below or overstep the limit b-c=0 (compare figures 4, 6, 8 and literature 3). Outdated information has a similar effect like deceptive information.

A central idea in my model is the consequent application of the law of conservation of mass. Inflation and deflation in the case of money appear as a violation of the law of conservation of mass. As an example: The king has a certain volume of tax revenue but his spending exceeds this revenue by far. How can that be? Is this a violation of the conservation laws? No,

the king dilutes his gold coins with other metals making two out of one; it is deception.

In economics deflation is generally considered to be more harmful than inflation due to the fact that consumers tend to wait for lower prices when they detect deflation. There are also positive effects of deflation. Moderate drops in certain products, such as food or energy, may increase consumer spending in other areas. My model does not encompass psychologic explanations nor does it need deflation or inflation to be detected. The observations within my model are simple and basic. Whether undetected inflation or deflation have positive or negative effects depends on the type of ensemble (weak ensembles and strong ensembles *versus* very strong ensembles; figures 5, 7, 9, and 10). In addition, there are ensembles where there is indeed an optimal inflation (figure 10A, 10B) or deflation (figure 10D, 10E).

Organisms and organisations (living things and companies) are ensembles able to grow and shrink. However, growth and shrinkage are not limitless. Limits are often set by external factors but organisms are also equipped with self-limiting, internal features. The result of an earlier investigation on division of labour was surprising (2). The evolutionary trend for source and sink to increase division of labour (source becoming a collector and sink becoming a point of production) was limitless even in the presence of a fix-cost (literature 2, figures 8 and 9 there). How could this development come to an endpoint?

In the beginning a system of low to intermediate integration may increase superadditivity by growth (inflation) when the superadditivity of a further increase in division of labour is smaller than the superadditivity from an increase in size. The organisation (complexity and asymmetry of source

and sink) of the organism stays the same but now the size increases. This process will reach an external (physical) endpoint when the surface to volume ratio starts to exert negative effects. Now it may be better for the ensemble to switch again to further division of labour. This process may dominate for some time and move the intersection b-c=0 of source and sink through the concentration plane of the transfer space, depending on the size of the black area (figures 13-15). But then a limit for this process is reached in very strong ensembles; shrinking could now contribute better to additional superadditivity. Shrinking, however, will happen on cost of benefits present already in a larger entity. A final equilibrium is reached. However, in the low right corner of the transfer space a different limit has to be found.

The exclusion from this area could be achieved by an appropriately sized fix cost (2). The fix cost could exclude the ensemble from an area of the transfer space characterized by very low substrate concentration in source and very high substrate concentration in sink. The system would be forced onto a higher path leading into an aera where growth dominates. Growth will find an external limit.

Cope´s rule: In the history of life an increase in body size on a geological time scale in many species can be observed. This is called Cope´s rule (Edward Drinker Cope, 1840 - 1897; 6). A result of my model (figure 13, 14, 15) is the dominance of inflation (growth) in large regions of the transfer space. There, growth delivers more superadditivity than further division of labour or shrinkage.

A decrease in size within an otherwise apparently unchanged species (*i.e.* only fossilized remains) is also observable on geological timescale, especially in species suddenly confined to a small island. This is called reverse Cope´s rule. My interpretation would be here that the species is

displaced from a former equilibrium. Superadditivity from a large size is lost. The advantages of a large size are no longer present. A compensation for the lost superadditivity might now come from a decrease in size. However, this is a very theoretical discussion as I do not know the specific contributions of growth, shrinkage or division of labour to the total superadditivity and the proportions of specific and unspecific (*i.e.* superadditive) contributions.

In addition, it appears imaginable that the superadditivity from the decrease in size may compensate decreasing superadditivity from a reduction in division of labour. A reversal of division of labour will decrease the cost factor in source and increase the cost factor in sink. The ensemble will move from down-right to up-left through the concentration plane of the transfer space. In biology this is to my knowledge not observable as an evolutionary trend. However, it may be observable in politics, when larger ensembles disintegrate in the hope for new superadditivity in at least one of the components (source) of the ensemble while existing division of labour is unwound (*e.g.* liquidation of the USSR, Brexit).

Literature

1. Friedrich T (2019) How peaceful is the harmony of source and sink? MPRA_paper_96764
2. Friedrich T (2018) Evolution towards higher net profit in a population of ensembles of ensembles leads to division of labour. MPRA_paper_85517
3. Friedrich T (2015) The limits of wise exploitation in dependent and independent symmetric ensembles. MPRA_paper_68250
4. Turner P E and Chao L (1999) Prisoner´s dilemma in an RNA virus. Nature 398,441-443
5. Friedrich T (2019) Transfers by force and deception lead to stability in an evolutionary process when controlled by net profit but not by turnover. MPRA_paper_92724
6. Bell, M (2014) Trends of body-size evolution in the fossil record – a growing field. Palaeontology [online], 4, 1-9

Addendum: Two additional reasons for inflation are known.

Inflation through increase in cost (cost push):

In my model a change in cost has nothing in common with inflation or deflation of the transfer space. A rise in cost comes from an increased cost factor, a decrease in cost is due to a decrease in the cost factor. The cost factor is a part of the equations to calculate net profit and superadditivity. It is a feature of source and sink and not of the coordinates. When the cost factor changes, the slope of the linear cost function will change. An increase in cost factor will shift the equilibrium point b-c=0 to a lower substrate concentration. Therefore, benefit domination between the old and new limit (b-c=0) will change to cost domination. A source will now start to give and a sink will no longer take. A decrease in cost factor will have the opposite effect (figure A). In case the system wants to keep b-c=0 at the same concentration, the benefit has to be adjusted accordingly.
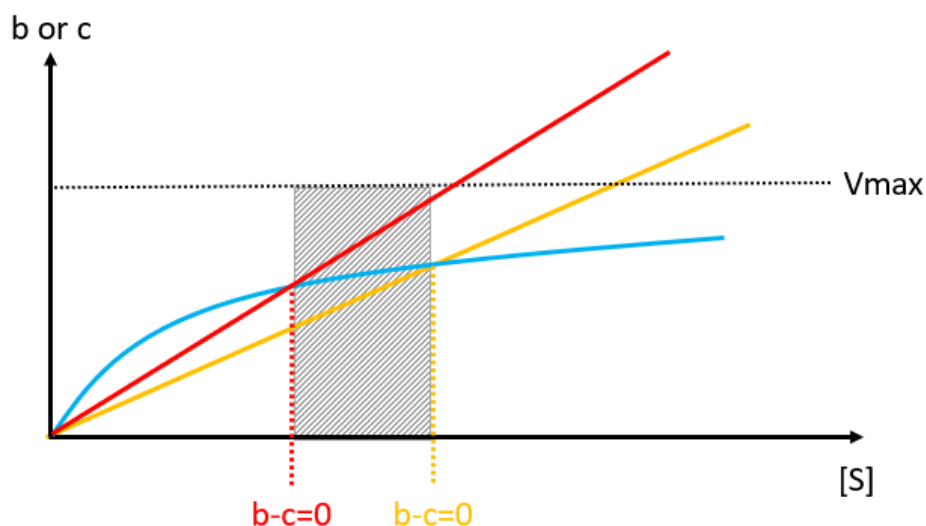
Figure A

Figure A
The grey shaded area is confined by the limits b-c=0 of a small cost factor (orange) and a high cost factor (red) for an identical benefit function (light blue). In case the cost factor changes *e.g.* from low to high the behaviour within the gey area changes from "do not give" to "give" for a source and from "take" to "do not take" for a sink.

Inflation through increase in affinity (demand pull):

In biochemistry Km is the substrate concentration at which the reaction velocity is half maximal (V=([S]/Km+[S])*Vmax; if [S]=Km then V=½Vmax). Km depends on different rates of catalytic steps characterizing the formation and decay of the enzyme-substrate complex. In two step reactions affinity and Km are inverse equivalent when the decay of the enzyme-substrate complex to the product is rate-limiting and much smaller than the association and dissociation of enzyme and substrate. Km is in general a more complex function of several rate constants (Lehninger, Principles of Biochemistry, 5th edition, page 198: Interpreting Vmax and Km). Nevertheless, I am going to use here the simple analogy: affinity ≈ 1/Km. Furthermore, I interpret affinity as demand. The connection between Km and an increase in cost is explained in figure B. If Km drops the point b-c=0 will shift to higher substrate concentrations. To keep the point b-c=0 in place the cost factor has to increase. Km is not a feature of the space.
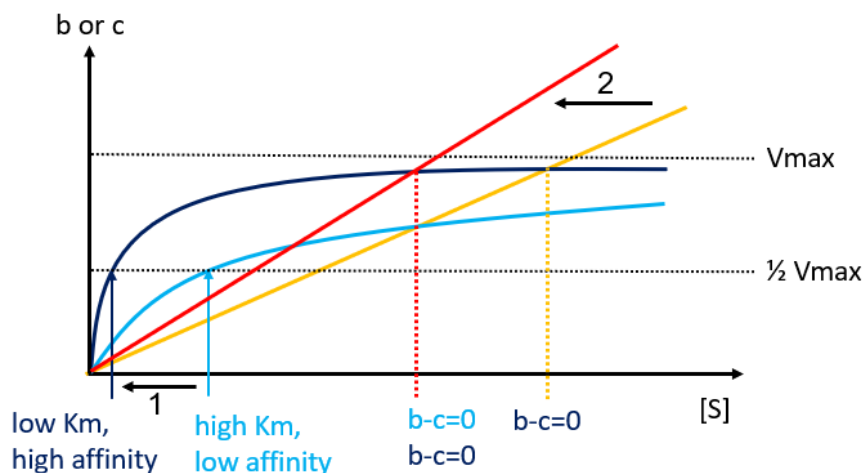
Figure B

Figure B
The light blue saturating benefit function is the starting point. When Km shifts (1) from high Km (low affinity, light blue) to low Km (high affinity, dark blue) b-c=0 shifts to higher substrate concentrations (orange dotted line) while Vmax does not change. Now the cost factor must increase (2) to keep the equilibrium stationary (b-c=0, red dotted line). Increased affinity (demand) will lead to cost inflation. A decrease in affinity would reverse the process and lead to cost deflation.

Finally, an additional factor for inflation or deflation seems imaginable. A decrease or increase in Vmax (figure C).
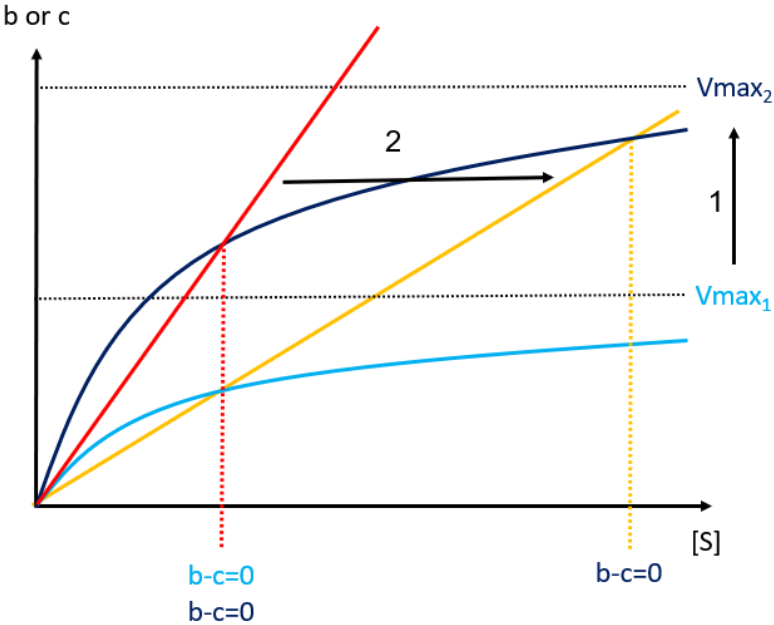
Figure C

Figure C

The light blue saturating benefit function serves as the starting point. The equilibrium b-c=0 has a certain location (red dotted line). When Vmax changes (1) from low Vmax (low productivity, light blue) to high Vmax (high productivity, dark blue) the price per unit may stay unchanged. Demand (1/Km) is unchanged for the same product and the additional products can´t be sold. Now the cost factor must decrease (2) to change the equilibrium (b-c=0, orange dotted line). The area between the red dotted line and the orange dotted line changes in sink from not taking to taking. A lower price might help that the consumers now buy two pieces instead of one. Increased productivity will lead to cost deflation. A decrease in productivity would reverse the process and lead to cost inflation.

Again, Vmax is not a feature of the space. The nature of inflation by cost-push and demand-pull differs very much from inflation by money supply. While cost-push and demand-pull come from intrinsic factors, inflation by money supply comes from an external factor. The transfer space changes in size and the concentration of the substrate - the value - changes. Therefore, paper money and coins seem comparable not to a substrate but to the solvent in which the substrate (value) is dissolved.