Ignorance is Strength

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Information asymmetry of source and sink and its effects on superadditivity and subadditivity of the ensemble of both.

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The model transfer space explores how substrate transfers impact the net profit of a source and sink, and the superadditivity or subadditivity of their ensemble. The three coordinates of the transfer space are the substrate concentration in source and sink and the net profit of the ensemble of both. Net profit is the difference between a non-linear benefit function of the substrate concentration and a linear cost function of the substrate concentration in source and sink. Superadditivity and subadditivity emerge in specific areas of the transfer space when comparing the results of a transfer to no transfer.

When an object is transferred from source to sink, it possesses a visible exterior and a hidden value, the latter being a substrate quantity. The amount of substrate and the size of the value are correlated. However, the outward appearance need not correlate with the value of the content. In such a case, if the content´s value is smaller or larger than expected, the result of this ignorance is new activity or inactivity in certain areas of the transfer space or the positive net profit subspace. This will create or miss additional superadditivity and subadditivity for the ensemble. A simple pattern emerges. Source ignorance and collective ignorance dominate when value is overestimated. Sink ignorance and complete information dominate when value is underestimated. Ignorance can indeed be a collective strength. A lack of knowledge - whether intentional or inadvertent - of true value has an effect similar to deception or brute force.

source, sink, ensemble, value, overestimation, underestimation, inflation, deflation, information, symmetry, asymmetry, superadditivity, subadditivity, Cantillon effect, deliberate ignorance

Introduction and initial considerations

The foundational principles of natural sciences and economics share striking similarities. For instance, the principles of conservation of mass and energy, pioneered by Antoine Lavoisier, highlight this parallel. The quote attributed to Baron Amschel Mayer Freiherr von Rothschild (1773 - 1855), "Your money isn't gone, it's just somewhere else," also captures the idea that physical units do not vanish into nothingness; rather, they relocate while still existing. This notion sharply contrasts with the subjective nature of value, which individuals attribute. Value is susceptible to fluctuations based on perception; even in the event of stock market crashes, the physical presence of share certificates and the means of production usually endure. Unlike the elegant pirouette of an ice skater, where expanding or contracting arms alter rotational speed, the Earth's rotation remains impervious to stock market fluctuations, whether they swell or contract by billions. However, knowing the true value or not knowing the true value of something will influence the behaviour and the outcome of an action or not-action. This effect will be investigated.

As my model has been, again, explained in detail in my last publication (1) I only briefly describe the calculations. An ensemble consists of a source and a sink. Everything and every action simultaneously have a benefit and a cost aspect. Both parties want to optimize their benefit (b) and cost (c); that is at b=c. The benefit b in source (so) and sink (si) is a saturating function of the substrate concentration according to Michaelis-Menten:

$$b_{so}=bf_{so}*Vmax_{so}*[S]_{so}/([S]_{so}+Km_{so});\ b_{si}=bf_{si}*Vmax_{si}*[S]_{si}/([S]_{si}+Km_{si})$$

where bf is the benefit factor, here always 1 b*min/µmol; b is a placeholder for other units like KJ or € or $. Vmax is the maximal reaction velocity (µmol/min), [S] is the substrate concentration (mM), and Km is the Michaelis-Menten constant (mM).

The variable cost c in source and sink is a linear function of the substrate concentration (no fixed cost):

$c_{so}=cf_{so}*[S]_{so}$; $c_{si}=cf_{si}*[S]_{si}$

Here, cf is the cost factor (c/mM) and [S] is the substrate concentration (mM). The variable cost c is a placeholder for units like KJ or € or $. The cost function rates and evaluates the benefit function. The cost tells us whether we are observing a true benefit or a “malefit”. That is when a limit is exceeded and there is “too much of a good thing”. The net profit of the ensemble (np_e) or source (np_{so}) or sink (np_{si}) is calculated:

$np_e=np_{so}+np_{si}=(b_{so}-c_{so})+(b_{si}-c_{si})$; np_{max}: $f'(np)=0$ and $f''(np)<0$

The net profit of all concentration pairs within the transfer space manifests as a surface within the transfer space. This surface exhibits a dual nature, with a segment positioned below zero indicating negative net profits, and another segment above zero signifying positive net profits. Superadditivity becomes evident when the surface with transfer surpasses the surface without transfer, while subadditivity occurs in the reverse scenario. Importantly, both superadditivity and subadditivity are independent of whether the net profit for individual parties is positive or negative. The size of the positive net profit is important when single parties compete, the size of the superadditivity - or the ability to minimize or avoid subadditivity - is important when ensembles compete. It is imaginable that an exploited source or sink with a low or even negative net profit is part of a dominating and successful ensemble, as long as the harmed single party is not lost due to some kind of replacement (breeding) or basic support (reciprocity). Replacement and support can be paid for by the superadditivity of the ensemble. Suffering through exploitation is not a part of my considerations, although it may be a cause of low efficiency. In a past examination (2), I utilized the transfer space to explore inflation and deflation driven by changes in money supply. That investigation employed a transfer vehicle - a coin - where the number of coins could increase (inflation) or decrease (deflation) while maintaining a constant total amount of substrate (value) within the transfer space. This process results basically in a symmetric expansion or contraction of the transfer space's coin-based coordinate system.

In the following investigation I want to manipulate the true amount of substrate contained in a coin and the expectation and knowledge of both parties. A one-sided knowledge advantage could be interpreted as an apparent asymmetric expansion or contraction of the coordinate system. I call the true amount of substrate a “value”. Both parties know that there are 10000coins in total and they basically assume that a gain or loss of a coin will increase or decrease their respective concentration by 1µM. However, one party may know that this is no longer the case. In addition, both parties know their respective boundaries (b=c or np_{max}) and behave rationally like a Homo economicus within the boundaries of their own knowledge. Force and deception are no options although they might be considered if expectations are not met. Both parties are equally strong and know all relevant data - with the exception of the true value of the coin. My aim is to observe the outcome of the interaction of source and sink and the superadditivity and subadditivity of the ensemble of both. Besides the knowledge concerning the value of the coin the biochemical symmetry of the ensemble can be varied: symmetric, asymmetric weak and asymmetric strong ensembles. The transfer space and its subspace, the positive net profit subspace, will be investigated (1). Within the transfer space the behaviour of the single party is dominated by the inner motivation to reach an equilibrium between benefit and cost (b=c). In the positive net profit subspace, the behaviour of the single party is guided by the outer motivation to achieve maximal net profit ($f'(np_{so})=0$, $f'(np_{si})=0$). For source this represents a deception according to source´s inner motivation (b=c). This deception, however, is so strong and successful that it appears to source as free will (1). Figure 1 is a top-down view of the three-dimensional transfer space and the positive net profit subspace (area III). For both spaces the positive net profit axis points towards the observer.

Figure 1

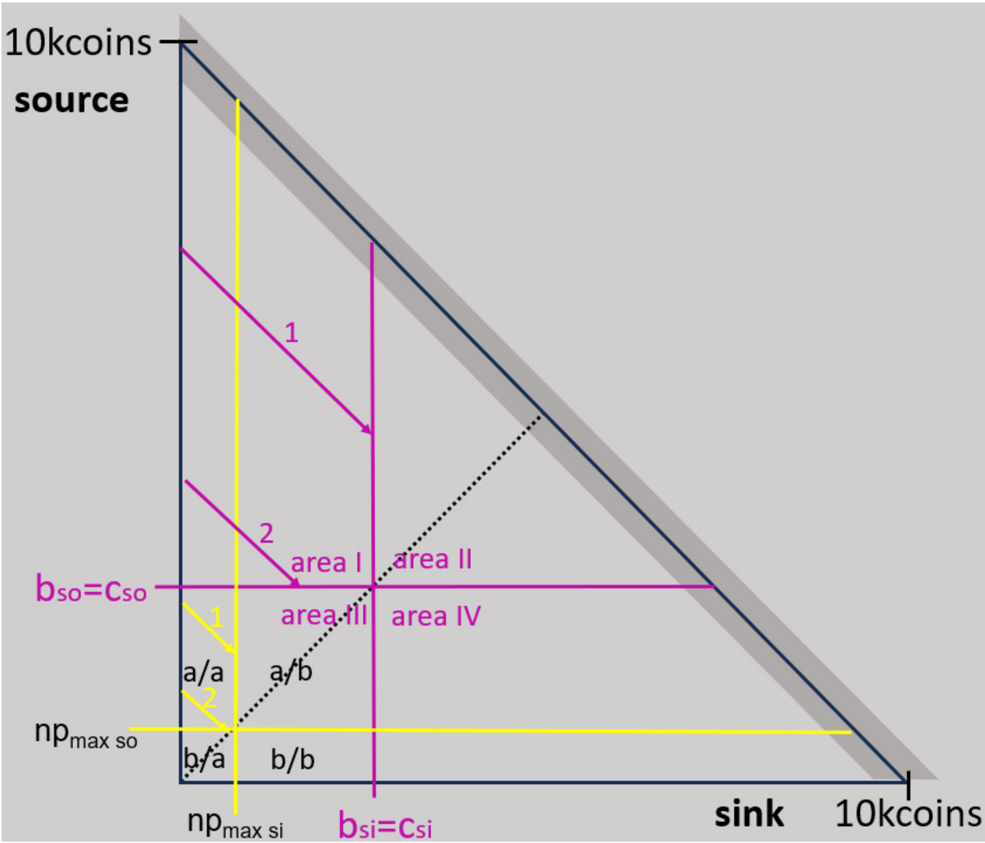

Figure 1

A symmetric ensemble is depicted top down. The ensemble contains 10000coins in total, distributed between source and sink. The purple lines (b=c) separate four areas where either benefit or cost are dominating in either source or sink. The inner motivation activates the ensemble in area I (source b<c, sink b>c). The purple arrow 1 represents a transfer where sink will finally stop to take. The purple arrow 2 represents a transfer where source finally stops to give. The yellow lines mark the maximal net profit of source or sink. The outer motivation activates the ensemble in the subarea a/a of area III, the positive net profit subspace. The yellow arrow 1 represents a transfer where sink will finally stop to take to maximize net profit. The yellow arrow 2 represents a transfer where source finally stops to give to maximize net profit. The black dotted line is the line of equal concentrations. The shadow on both sides of the hypotenuse symbolizes the increase or decrease in coin number at a constant total amount of substrate (10mmol).

According to their inner or outer motivation source and sink will be active either in area I (inner motivation) or in subarea a/a (outer motivation; from the viewpoint of the transfer space this is a successfully implemented deception in area III). The ensemble contains a total of 10000coins and it is assumed by both parties that each coin contains an amount of substrate sufficient to decrease (source) or raise (sink) the concentration by 1µM. Both parties define b=c or np=max by the number of coins. However, this equivalence of coin and concentration is only relevant as long as it is correct. The shortcut to interpret the transferred number of coins as an increase or decrease in concentration is no longer valid when the concentration, *i.e.* value, of the coin has changed. The party with this knowledge is able to optimize its own outcome.

The following list of permutations of ignorance and knowledge and the possibility that a coin contains more or less substrate may seem exaggerated - and this is the case. But it is necessary to put the calculations into context.

The transfer space

In area I of the transfer space, the source aims to achieve $b_{so}=c_{so}$ concerning substrate concentration, which is accomplished by transferring from high concentrations where cost dominates. Conversely, the sink aims to achieve $b_{si}=c_{si}$ by receiving at low concentrations, where benefits dominate.

1. *Both, source and sink do not know* the actual value of the coin. There may be more or less value transported from source to sink than anticipated. If this is the case the true limit b=c in terms of actual concentration of substrate (value), will either not be reached or this limit will be surpassed. The value is a substrate and therefore has

simultaneously a benefit and a cost aspect. The underestimation or overestimation of the value of the coin is similar to deflation or inflation by money supply and has been investigated earlier in detail (2).

More value: When the coin contains more substrate (value), the coins have a larger step size as they transfer a larger amount of substrate. When source or sink orient their behaviour according to the coin number, the concentration limit is overstepped in sink and not reached in source (Figure 2a).

Figure 2a

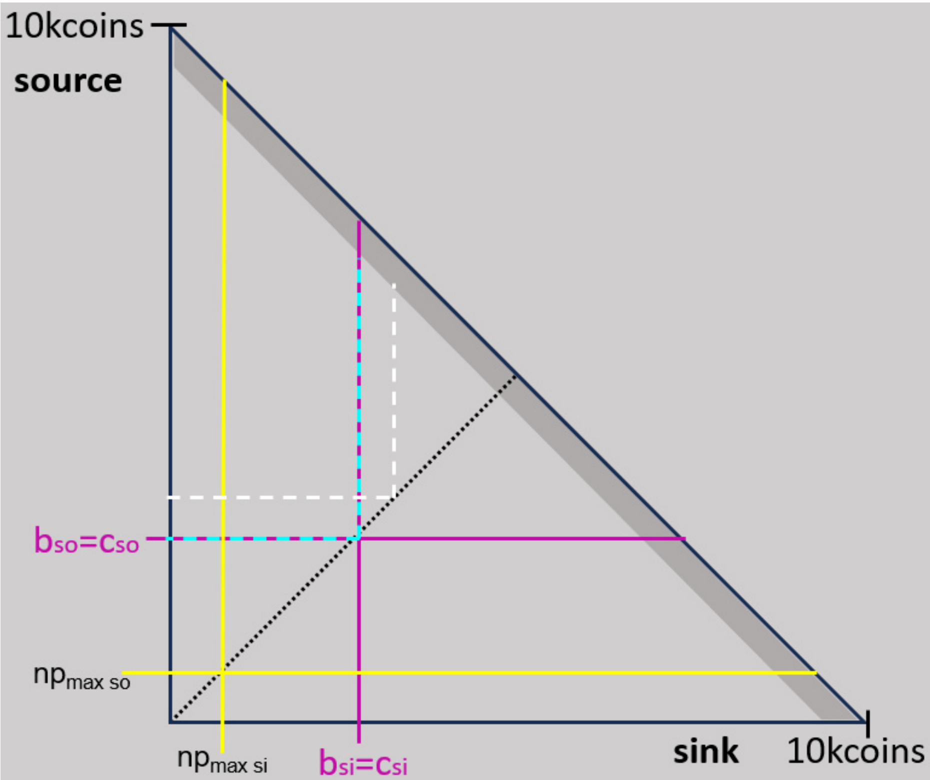

Figure 2a

The coin number has decreased while the total amount of substrate stays constant (10mmol). Both parties do not know that the coin now contains more substrate than expected. The example depicts a symmetric ensemble within the transfer space. The shadow marks the decrease in available coins. The depiction is not to scale. The blue dashed line is the coin target and the equilibrium of benefit and cost (b=c, purple lines), the white dashed line is the resulting value (concentration) that will result from a transfer to the coin target. The black dotted line indicates the line of equal concentration in source and sink.

Less value: When the coin contains less substrate (value), the coins have a smaller step size as they transfer a smaller amount of substrate. When source or sink orient their behaviour according to the coin number, the concentration limit is not reached in sink and overstepped in source (Figure 2b).

Figure 2b

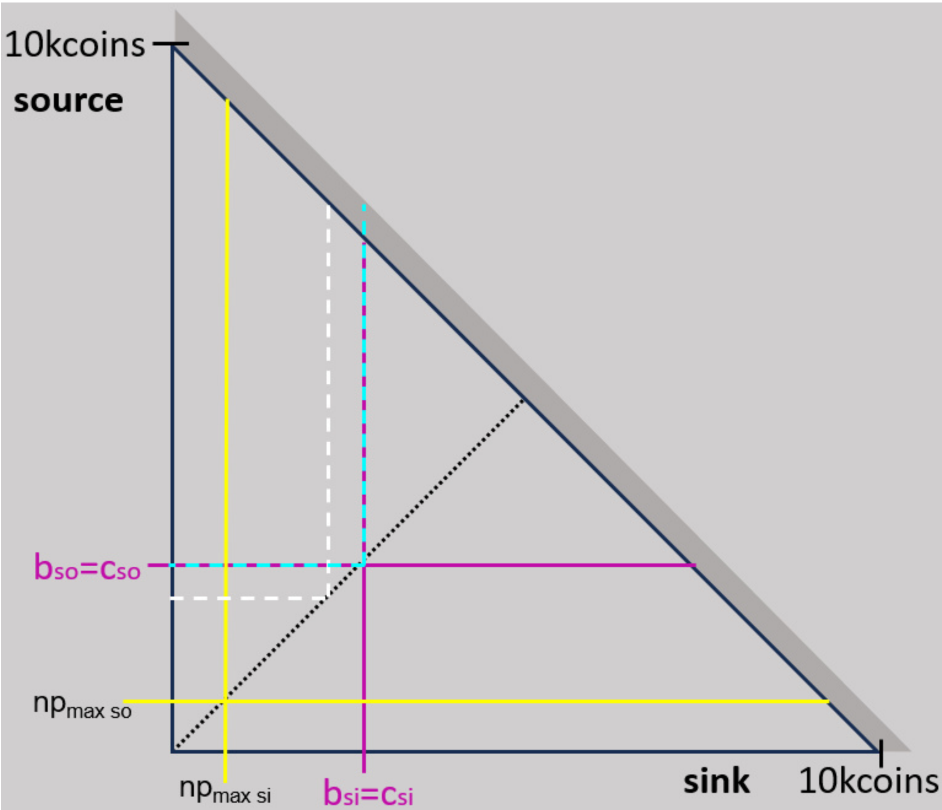

Figure 2b

The coin number has increased while the total amount of substrate stays constant (10mmol). Both parties do not know that the coin now contains less substrate than expected. The example depicts a symmetric ensemble within the transfer space. The shadow marks the increase in available coins. The depiction is not to scale. The blue dashed line is the coin target and the equilibrium of benefit and cost (b=c, purple lines), the white dashed line is the resulting value (concentration) that will result from a transfer to the coin target. The black dotted line indicates the line of equal concentration in source and sink.

2. <u>*Source knows*</u> how much more or less value is in a coin. It is the aim of source to give beyond the concentration limit of sink ($b_{si}=c_{si}$) and to keep its own concentration limit ($b_{so}=c_{so}$). Sink will take at free will in area I according to the number of coins. Transfer is only possible if both agree. Source or sink are able to stop to give or take even within area I.

More value: Source is able to give undetected beyond the concentration limit of sink ($b_{si}=c_{si}$) because the high amount of substrate in the coin is unknown to sink. When source is in equilibrium according to coins, source is still above the equilibrium for the substrate concentration ($b_{so}=c_{so}$). Source has to give more coins to reach the concentration limit of source which is okay for sink. If sink would know the coin equilibrium of source, sink would be surprized but source might hide this as generosity.

Less value: Sink stops taking from source in ignorance of the low substrate amount in the coin at sinks coin limit ($b_{si}=c_{si}$). With respect to value (substrate) this is premature for sink. Source would like to give on but can´t force sink to take. Source is disappointed. If source would stop to give at the coin limit ($b_{so}=c_{so}$) the concentration limit would be overstepped by source, as source starts from a lower concentration level. Therefore, source will stop the transfer ahead of the coin concentration limit. To sink this may appear as frugality of source. The source will agree with this interpretation to hide the real reason - step size and coin no longer match.

3. <u>*Sink knows*</u> how much more or less value is in a coin. It is the aim of sink to take beyond the concentration limit of source ($b_{so}=c_{so}$) and to keep its own concentration limit ($b_{si}=c_{si}$). Source will give at free will in area I according to the number of coins. Transfer is only possible if both agree. Source or sink are able to stop to give or take within area I.

More value: Sink stops source to give beyond the concentration limit of sink ($b_{si}=c_{si}$). However, this is according to coins not sufficient to source. Sink must appear modest to source. Source does not know that the concentration of the coin is larger than anticipated and sink is already in substrate equilibrium ($b_{si}=c_{si}$). Source stops to give at the coin limit but this is ahead of the source concentration limit ($b_{so}=c_{so}$). Sink is disappointed but can do nothing.

Less value: Sink is able to take undetected beyond the concentration limit of source ($b_{so}=c_{so}$). Although the coin, unknowingly to source, transfers less substrate but the starting concentration was closer to the equilibrium of source. As source gives until its own coin limit is reached ($b_{so}=c_{so}$), unknowingly, the concentration limit of source is overstepped. At the equilibrium ($b_{si}=c_{si}$) source wants to overstep sinks limit and give more. Sink seems to act generously letting source overstep the coin limit until sinks concentration limit is met. Sink knows that the step size is too small and more coins have to be transferred to reach sinks concentration limit.

4. *Both know* how much more or less value is in a coin. It is the aim of sink to take beyond the limit of source ($b_{so}=c_{so}$). It is the aim of sink to keep its own limit ($b_{si}=c_{si}$). It is the aim of source to give beyond the limit of sink ($b_{si}=c_{si}$). It is an interest of source to keep its own limit ($b_{so}=c_{so}$). Both are unable to force the other side. Transfer is only possible if both agree. Source or sink are able to stop to give or take within area I. As both parties know the true content of a coin, they no longer orient according to the coin number but according to the real concentration.

If both parties are unaware that the other party also knows the actual value, the following interpretation of the other party's decision may occur.

More value: Sink stops source to give beyond sinks concentration limit ($b_{si}=c_{si}$). The coin limit of source will not be reached. Sink will appear frugal. Source gives beyond the coin limit of source ($b_{so}=c_{so}$) to reach the concentration limit of source. Source appears to be generous.

Less value: Source stops to give ahead of the coin limit of source to avoid to surpass the concentration limit of source ($b_{so}=c_{so}$). Source will appear frugal. Sink will let source give beyond sinks coin limit ($b_{si}=c_{si}$) to reach its own concentration limit with a coin that does not contain sufficient substrate. Sink appears to be generous.

The behaviour within the transfer space appears to be counterintuitive. I have to emphasize that the behaviour of source in area I is guided by the aim to get rid of a cost dominated substrate; difficult to imagine as in our everyday life coins always contain benefit only. The coin could be interpreted as a commodity with an extended or shortened shelf life. The counterintuition will not be very different in area III, the positive net profit subspace, a subspace of the transfer space (figure 1, area III). As this subspace appears to be even more suited to our thinking in terms of maximizing net profit (np_{max}, 1), it may even be more confusing. In the positive net profit subspace source follows an outer motivation. This is essentially a deeply engrained deception of source to give beyond its natural concentration limit $b_{so}=c_{so}$, however, it is perceived by both parties as free will within the limits of the outer motivation.

<u>The positive net profit subspace</u>

5. *<u>Both, source and sink do not know</u>* the actual substrate amount (value) of the coin. There may be more or less value transported from source to sink than anticipated. If this is the case the true limit np_{max} in terms of actual

concentration, will either not be reached or this limit will be surpassed. Because source ends to give at $np_{max\ so}$ according to coins and sink stops to take at $np_{max\ si}$ according to coins, both will be surprized regarding the outcome according to value and the ability to maximize net profit. Both parties start in subarea a/a of area III. There, source will give to reach the maximal net profit and sink will take to maximize net profit.

More value: When the coin contains more substrate (value), the resulting concentration in source and sink is shifted to higher concentrations. The concentration limit is no longer met (figure 3a).

Figure 3a

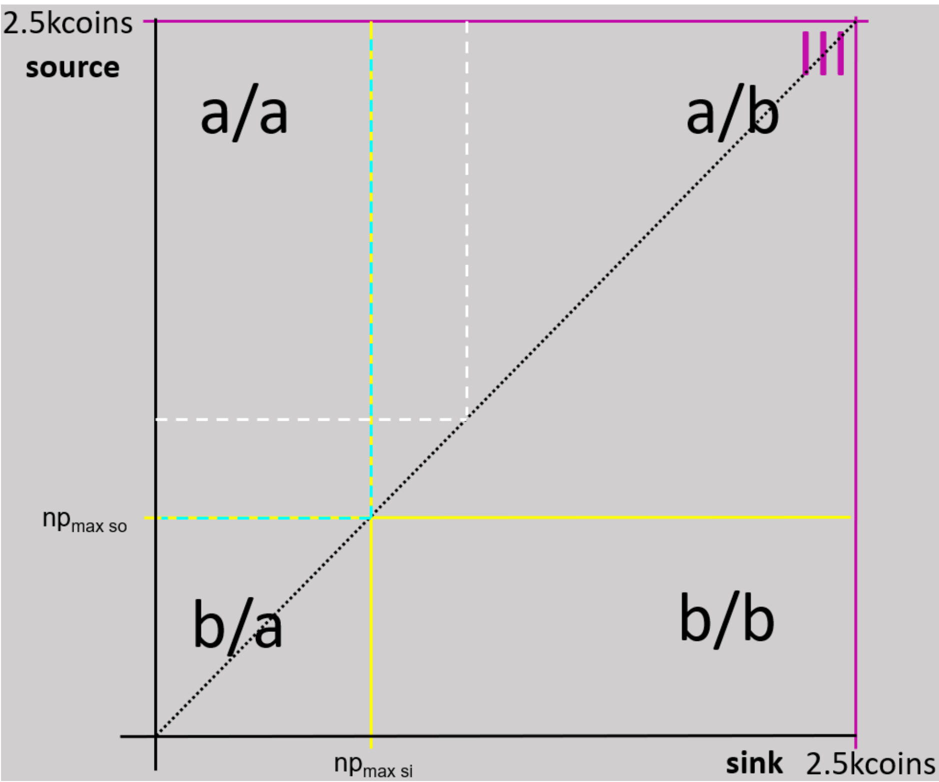

Figure 3a

Both parties do not know that the coin contains more value than expected. The example depicts a symmetric ensemble within the positive net profit subspace (area III of the transfer space). The depiction is not to scale. The blue dashed line is the coin target and the maximal net profit (np_{max}, yellow lines), the white dashed line is the resulting value (concentration) that will result from a transfer to the coin target. The black dotted line indicates the line of equal concentration in source and sink.

Less value: When the coin contains less substrate (value), the resulting concentration in source and sink is shifted to lower concentrations. The concentration limit is no longer met (Figure 3b).

Figure 3b

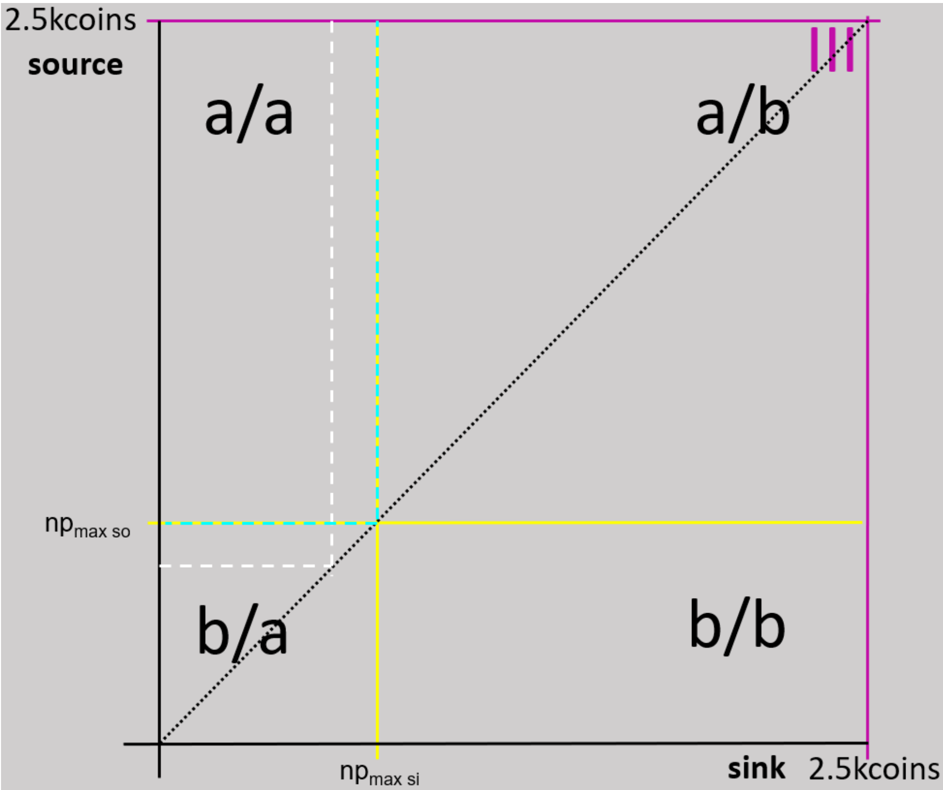

Figure 3b

Both parties do not know that the coin contains less value than expected. The example depicts a symmetric ensemble within the positive net profit subspace (area III of the transfer space). The depiction is not to scale. The blue dashed line is the coin target and the maximal net profit (np_{max}, yellow lines), the white dashed line is the resulting value (concentration) that will result from a transfer to the coin target. The black dotted line indicates the line of equal concentration in source and sink.

6. *Source knows* how much more or less value is in a coin. It is an interest of source to give beyond sink´s limit ($np_{max\ si}$). It is an interest of source to keep its own limit ($np_{max\ so}$). Sink will take at free will in subarea *a/a* with respect to the number of coins. Transfer is possible if both agree. Source or sink are able to stop to give or take within subarea a/a.

More value: Source is able to give substrate undetected beyond the limit of sink ($np_{max\ si}$). Source knows that even the increased step size will not be enough to hit $np_{max\ so}$ due to the high value in the coin in combination with a high start value. Therefore, source will give more coins to hit the maximal net profit and this may appear generous to sink.

Less value: Sink stops to take at sinks coin limit $np_{max\ si}$, but with respect to value it is unknowingly premature for sink. Source would like to give more but is unable without force or deception. If source would stop to give at the coin limit of source the value limit would have been overstepped. Source is now pretending to act frugal. The truth is, source wants to reach the limit $np_{max\ so}$ in value.

7. *Sink knows* how much more or less value is in a coin. It is an interest of sink to take beyond the limit of source ($np_{max\ so}$). It is an interest of sink to keep its own limit ($np_{max\ si}$). Source will give at free will in subarea *a/a* with respect to the number of coins. Transfer is possible if both agree. Source or sink are able to stop to give or take within subarea a/a.

More value: According to the coin number source stops to give at the coin limit of source ($np_{max\ so}$). This is premature regarding the value limit. Sink can´t force source to give more. Sink has to stop source to give at sinks value limit ($np_{max\ si}$). This, however, is ahead of the coin limit of source. Sink appears frugal to source.

Less value: Source stops to give at the coin limit of source ($np_{max\ so}$). This, however, oversteps the value limit of source. Sink is able to take undetected. Source falls short to reach the value limit of sink ($np_{max\ si}$). Sink will now act generously and let source give until sink´s limit is met.

8. Both know how much more or less value is in a coin. It is in the interest of sink to take to maximize the net profit of sink ($np_{max\ si}$). It is the interest of source to give to maximize the net profit of source ($np_{max\ so}$). Both parties do not go beyond their own limit (np=max) according to the true value. Both parties have no interest to respect the limit of the other party. Both parties are unable to force the other side. Essentially, as both parties know the true content of a coin, they no longer orient according to the coin number but according to the real concentration. Both parties do not know that the other side also knows about the changed value per coin.

More value: Sink stops source to give at sinks value limit ($np_{max\ si}$). The coin limit of source is not met. Source will give beyond the coin limit of source to hit $np_{max\ so}$. The deviation from an expected behaviour, coupled with one's own knowledge, leads to mistrust in both parties.

Less value: Sink accepts more coins with less substrate to reach $np_{max\ si}$. Source will not to overstep the value limit of source ($np_{max\ so}$) and therefore give less. Expectation, deviation and knowledge result in mistrust.

In the following example the total amount of 10mmol substrate is either distributed to 10000coins (true value, 1µM per coin) or 9000coins (10% undetected deflation 1,11µM per coin, the value will be underestimated) or 11000coins (10% undetected inflation 0.91µM, the value will be overestimated). The concentration is 1/1±(%/100). The numbers are set to reasonable whole digits in the following examples. The calculations in the figures are accurate. The coin is a featureless vehicle without volume.

Results and discussion

The symmetric ensemble:

The values used for all calculations are: [S] = 0mM to 10mM (1μM steps, 10kcoins); the concentration pairs in source and sink always add up to a maximum of 10mM. Km = 0.5mM, Vmax = 5μmol/min, bf = 1b min/μmol, cf = 5/3 c/mM in the symmetric ensembles. The equilibrium b=c (inner motivation) is at 2.5mM i.e. 2.5kcoins at 1μM per coin, 2252coins at 1.11μM per coin, and 2747coins at 0.91μM per coin symmetrically for source and sink. However, this may not be known to both or one party.

In the symmetric ensemble the maximal net profit of source and sink is at a substrate concentration of 0.7247mM *i.e.* 724.7coins at 1μM, 653coins at 1.11μM, and 796coins at 0.91μM. However, the changed limits will not be known to one or both parties. A transfer will raise the concentration in sink by 1μM and lower the concentration in source by the same value. If the coin contains more or less substrate, a transfer will raise the concentration in sink by 1.11μM or 0.91μM and lower the concentration in source by the same value.

The limit of the positive net profit subspace in a symmetric ensemble is 2.5kcoins in source and 2.5kcoins in sink at 1μM, 2252coins in source or sink at 1.11μM, and 2747coins for source or sink at 0.91μM. The subspace has new inner targets subdividing that subspace again into 4 subareas. The new target (maximal net profit) is a substrate concentration of 0.7247mM (724.7coins at 1μM; 653coins at 1.11μM and 796coins at 0.91μM) However, the change may not be known to both or only one party. This target, again like the transfer space, can only be reached in coordination with the other party. In subarea a/a both parties transfer at free will according to their outer motivation.

Asymmetry of ensembles is adjusted by the variation of the cost factor.

The weak asymmetric ensemble:

The values used for all calculations are: [S] = 0mM to 10mM (1µM steps, 10kcoins); the concentration pairs in source and sink always add up to a maximum of 10mM. Km = 0.5mM, Vmax = 5µmol/min, bf = 1b min/µmol, cf = 10/7 c/mM in source and 2 c/mM in sink. The equilibrium b=c (inner motivation) is 3mM (3000coins at 1µM, 2703coins at 1.11µM and 3297coins at 0.91µM) in source and 2mM (2000coins at 1µM; 1802coins at 1.11µM and 2198coins at 0.91µM) in sink. However, this may not be known to both or one party.

In the weak ensemble the maximal net profit is at a substrate concentration of 0.8229mM (822.9coins at 1µM; 741coins at 1.11µM and 904coins at 0.91µM) in source and at 0.618mM (618coins at 1µM, 556coins at 1.11µM, and 679coins at 0.91µM) for sink. A transfer will raise the concentration in sink by 1µM and lower the concentration in source by the same value. If the coin contains more or less substrate, a transfer will raise the concentration in sink by 1.11µM or 91µM and lower the concentration in source by the same value.

The limit of the positive net profit subspace is 3mM (3000coins at 1µM, 2703coins at 1.11µM and 3297coins at 0.91µM) in source and 2mM (2000coins at 1µM; 1802coins at 1.11µM and 2198coins at 0.91µM) in sink. The subspace has new inner targets subdividing that subspace again into 4 subareas. The target (maximal net profit) is a substrate concentration of 0.8229mM (822.9coins at 1µM; 741coins at 1.11µM and 904coins at 0.91µM) in source and at 0.618mM (618coins at 1µM, 556coins at 1.11µM, and 679coins at 0.91µM) in sink. This, however, is not known to both or only one party. In subarea a/a both parties transfer at free will according to their outer motivation.

The strong asymmetric ensemble:

The values used for all calculations are: [S] = 0mM to 10mM (1µM steps, 10kcoins); the concentration pairs in source and sink always add up to a maximum of 10mM. Km = 0.5mM, Vmax = 5µmol/min, bf = 1b min/µmol, cf = 10/7 c/mM in sink and 2 c/mM in source. The equilibrium b=c (inner motivation) is 3mM (3000coins at 1µM, 2703coins at 1.11µM and 3297coins at 0.91µM) in sink and 2mM (2000coins at 1µM; 1802coins at 1.11µM and 2198coins at 0.91µM) in source. However, this may not be known to both or one party.

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The limit of the positive net profit subspace is 3mM (3000coins at 1µM, 2703coins at 1.11µM and 3297coins at 0.91µM) in sink and 2mM (2000coins at 1µM; 1802coins at 1.11µM and 2198coins at 0.91µM) in source. The subspace has new inner targets subdividing that subspace again into 4 subareas. The target (maximal net profit) is a substrate concentration of 0.8229mM (822.9coins at 1µM; 741coins at 1.11µM and 904coins at 0.91µM) in sink and at 0.618mM (618coins at 1µM, 556coins at 1.11µM, and 679coins at 0.91µM) in source. This, however, is not known to both or only one party. In subarea a/a both parties transfer at free will according to their outer motivation.

From now on the sequence will be weak, symmetric, and strong ensemble.

First, on a qualitative level, I would like to compare an ensemble in which both parties know that the coin is worth more or less than assumed with an ensemble in which both parties underestimate the value (coin contains more value) or overestimate it (coin contains less value). The behaviour of the informed ensemble is such, that this ensemble will stay within its limits; this is either b=c or np=max. The informed ensemble will therefore have always the same result in the case of a coin with less value and with more value. The uninformed ensemble will trust the number of coins as an indicator for the value of the coin and therefore overstep the limits or fall short to the limits. Basically, similar observations have already been made and described in an older paper (2). However, there both parties always did not know the true value and were only lucky when expected value and actual value were in accordance.

The following cases on the next pages demonstrate observable basic phenomena using a typical single example of value underestimation (10% less coins, unchanged amount of substrate, undetected deflation) and value overestimation (10% more coins, unchanged amount of substrate, undetected inflation) as outlined in detail above.

Transfer space, underestimation: weak ensemble figure 4; symmetric ensemble figure 5; strong ensemble figure 6

Transfer space, overestimation weak ensemble figure 7; symmetric ensemble figure 8; strong ensemble figure 9

Positive net profit subspace, underestimation: weak ensemble figure 10; symmetric ensemble figure 11; strong ensemble figure 12

Positive net profit subspace, overestimation: weak ensemble figure 13; symmetric ensemble figure 14; strong ensemble figure 15

Figure 4

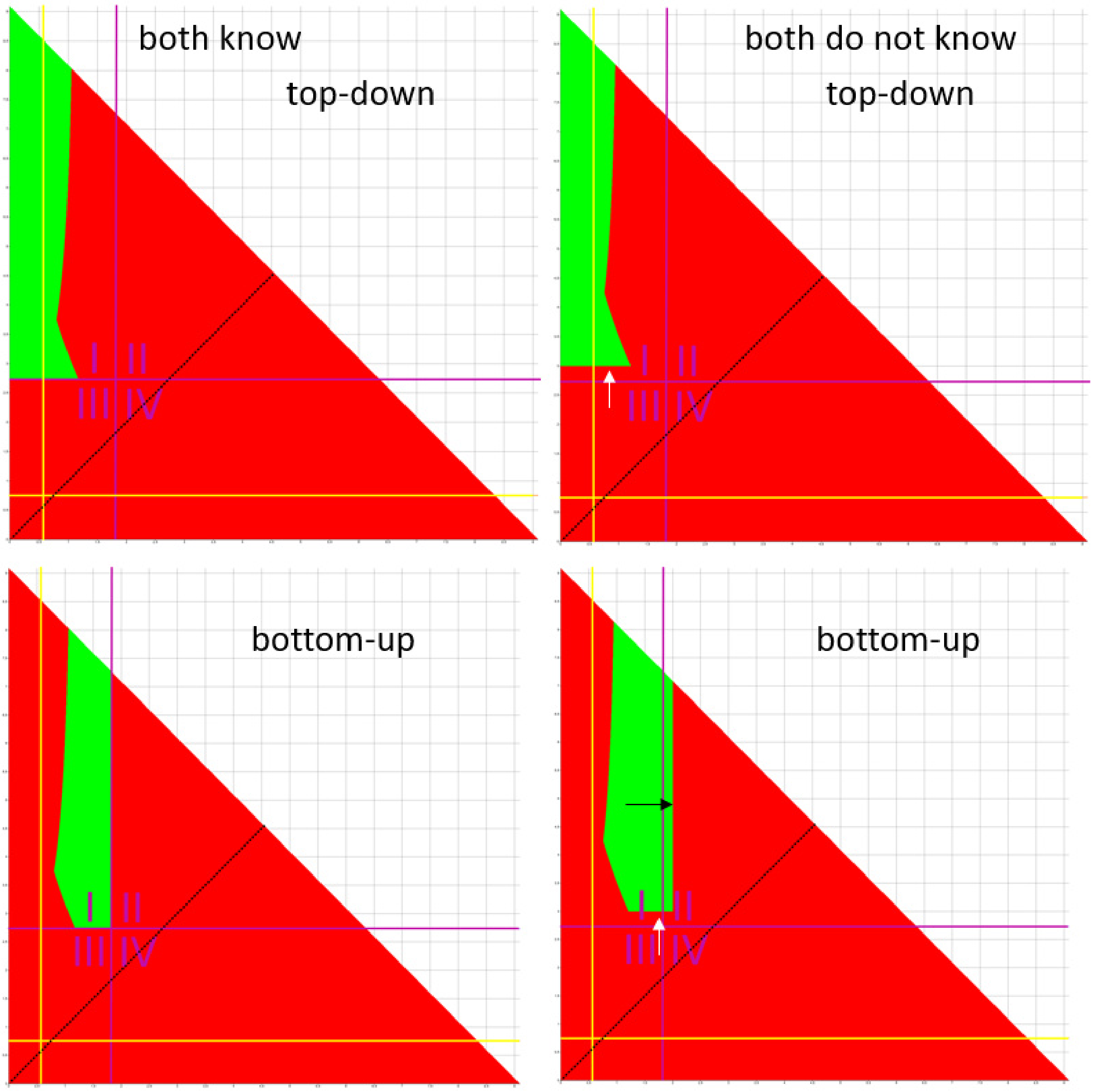


Figure 4

A weak ensemble is observed. On the left side (top-down and bottom-up) both parties of the ensemble know that the coin contains more value. They orient their behaviour according to the true value, the substrate concentration. The ensemble stays in area I. As a weak ensemble is observed a lot of subadditivity appears in area I (bottom-up). On the right side (top-down and bottom-up) both parties do not know that the coin contains more value. They rely on the number of coins as an indicator of concentration. In this case the ensemble oversteps b=c of sink (black arrow) entering area II. The ensemble does not reach b=c of source and stays here in area I (white arrows). Ignorance decreases superadditivity but also subadditivity in area I and increases subadditivity in area II. The purple lines indicate b=c and separate area I, II, III, and IV, the yellow lines mark np=max (investigated later) and the black dotted line is the line of equal concentration in source and sink (compare figure 2a). In this and all following figures the source-axis and sink-axis are given in coins. The result, however, is based on the true concentration. Therefore, the limits (in the transfer space purple and in the positive net profit subspace yellow, later) are overstepped or not reached.

Figure 5

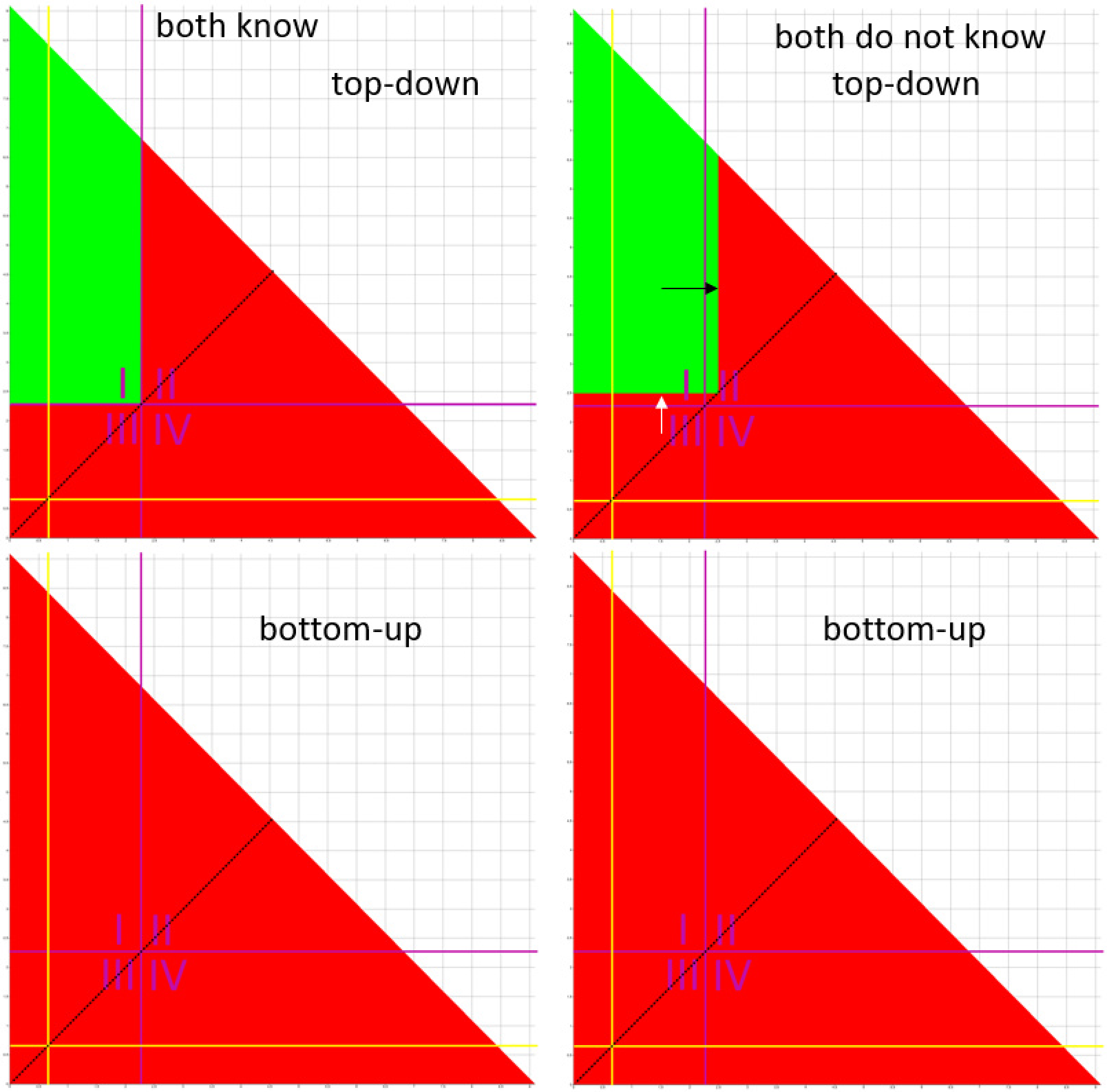


Figure 5

A symmetric ensemble is observed. On the left side (top-down and bottom-up) both parties of the ensemble know that the coin contains more value. They orient their behaviour according to the true value, the substrate concentration. The ensemble stays in area I. On the right side (top-down and bottom-up) both parties do not know that the coin contains more value. They rely on the number of coins as an indicator of the concentration. In this case the ensemble oversteps b=c of sink (black arrow) and enters area II, gaining superadditivity. On the other side b=c of source, is not reached and the ensemble stays in area I (white arrow) missing some superadditivity. The purple lines indicate b=c and separate area I, II, III, and IV, the yellow lines mark np=max and the black dotted line is the line of equal concentration in source and sink (compare figure 2a).

Figure 6

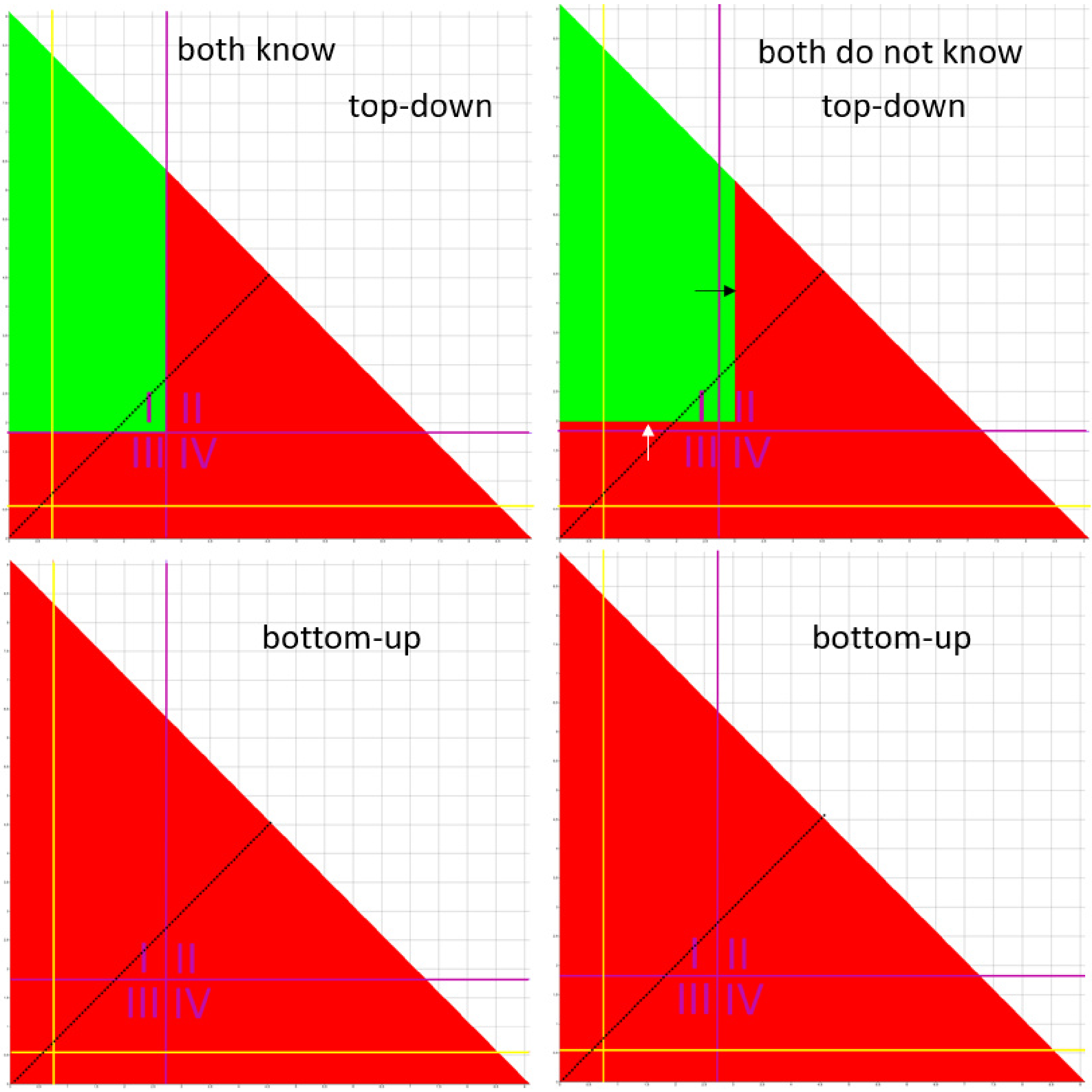


Figure 6

A strong ensemble is observed. On the left side (top-down and bottom-up) both parties of the ensemble know that the coin contains more value. They orient their behaviour according to the true value, the substrate concentration. The ensemble stays in area I. On the right side (top-down and bottom-up) both parties do not know that the coin contains more value. They rely on the number of coins as an indicator of the concentration. In this case the ensemble oversteps b=c of sink (black arrow) and enters area II. This creates more superadditivity. The ensemble does not reach b=c of source and stays in area I (white arrow) missing out on some superadditivity there. The purple lines indicate b=c and separate area I, II, III, and IV, the yellow lines mark np=max and the black dotted line is the line of equal concentration in source and sink (compare figure 2a).

Figure 7

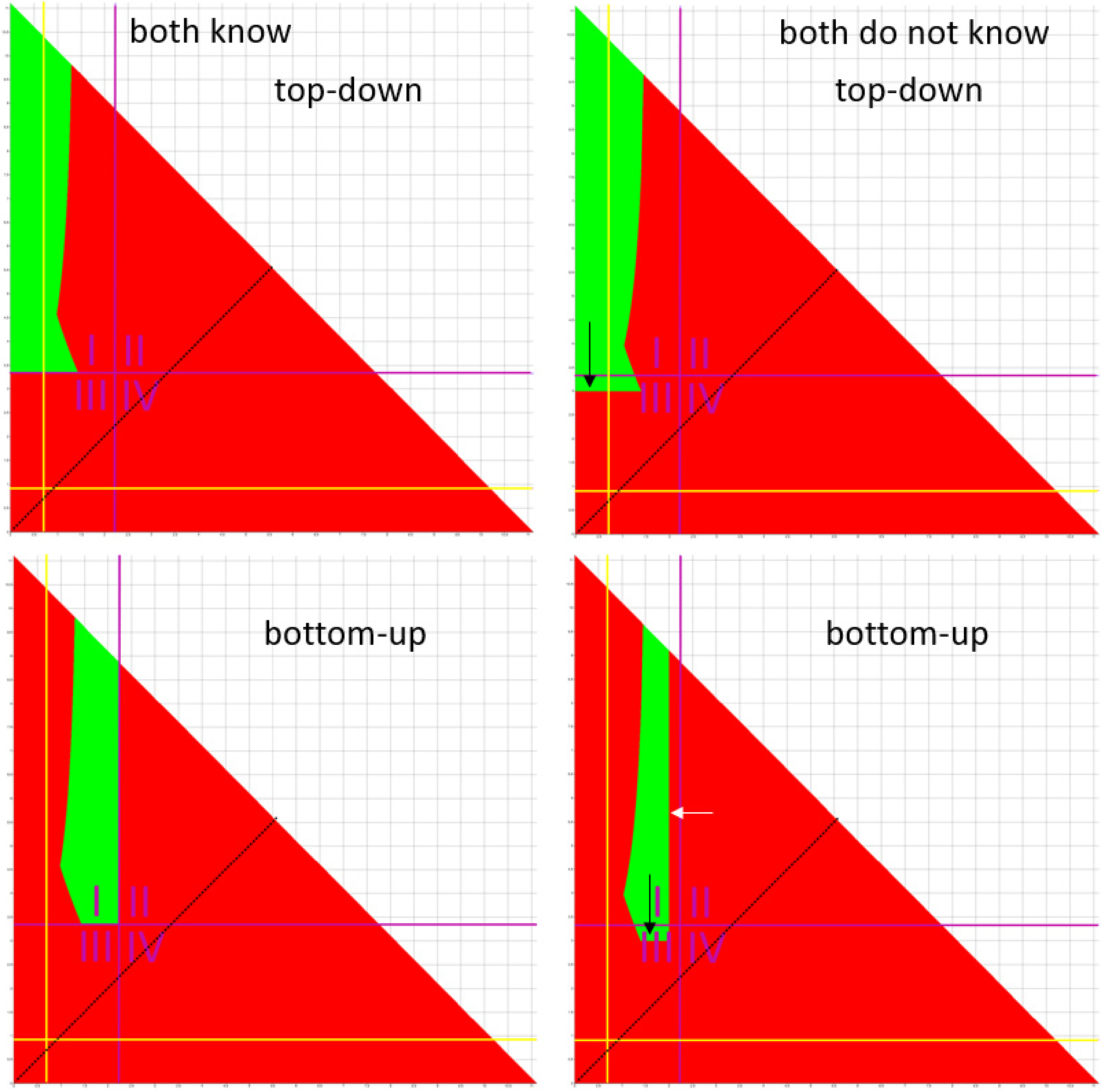


Figure 7

A weak ensemble is observed. On the left side (top-down and bottom-up) both parties of the ensemble know that the coin contains less value. They orient their behaviour according to the true value, the substrate concentration. The ensemble stays in area I. On the right side (top-down and bottom-up) both parties do not know that the coin contains less value. They rely on the number of coins as an indicator of the concentration. In this case the ensemble oversteps b=c (black arrows) of source entering area III creating more superadditivity (top-down) as well as subadditivity (bottom-up). The ensemble does not reach b=c of sink and stays there in area I losing some subadditivity (white arrow). As a weak ensemble is observed still lots of subadditivity is created in area I (bottom-up perspective). The purple lines indicate b=c and separate area I, II, III, and IV, the yellow lines mark np=max and the black dotted line is the line of equal concentration in source and sink (compare figure 2b).

Figure 8

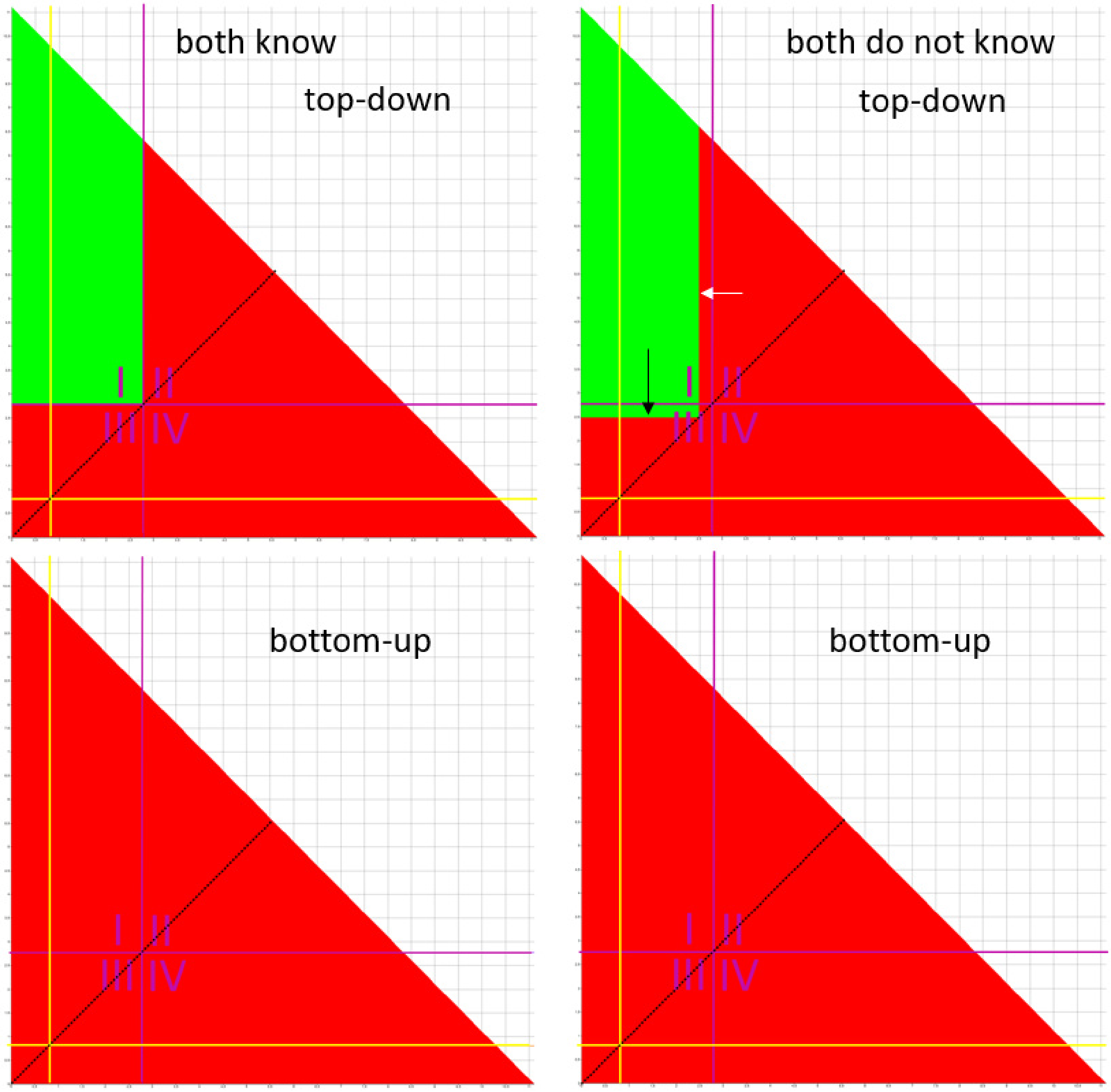


Figure 8

A symmetric ensemble is observed. On the left side (top-down and bottom-up) both parties of the ensemble know that the coin contains less value. They orient their behaviour according to the true value, the substrate concentration. The ensemble stays in area I. On the right side (top-down and bottom-up) both parties do not know that the coin contains less value. They rely on the number of coins as an indicator of the concentration. In this case the ensemble oversteps b=c of source (black arrow) and enters area III. The ensemble does not reach b=c of sink and stays in area I (white arrow). Different amounts of superadditivity are lost and gained. The purple lines indicate b=c and separate area I, II, III, and IV, the yellow lines mark np=max and the black dotted line is the line of equal concentration in source and sink (compare figure 2b).

Figure 9

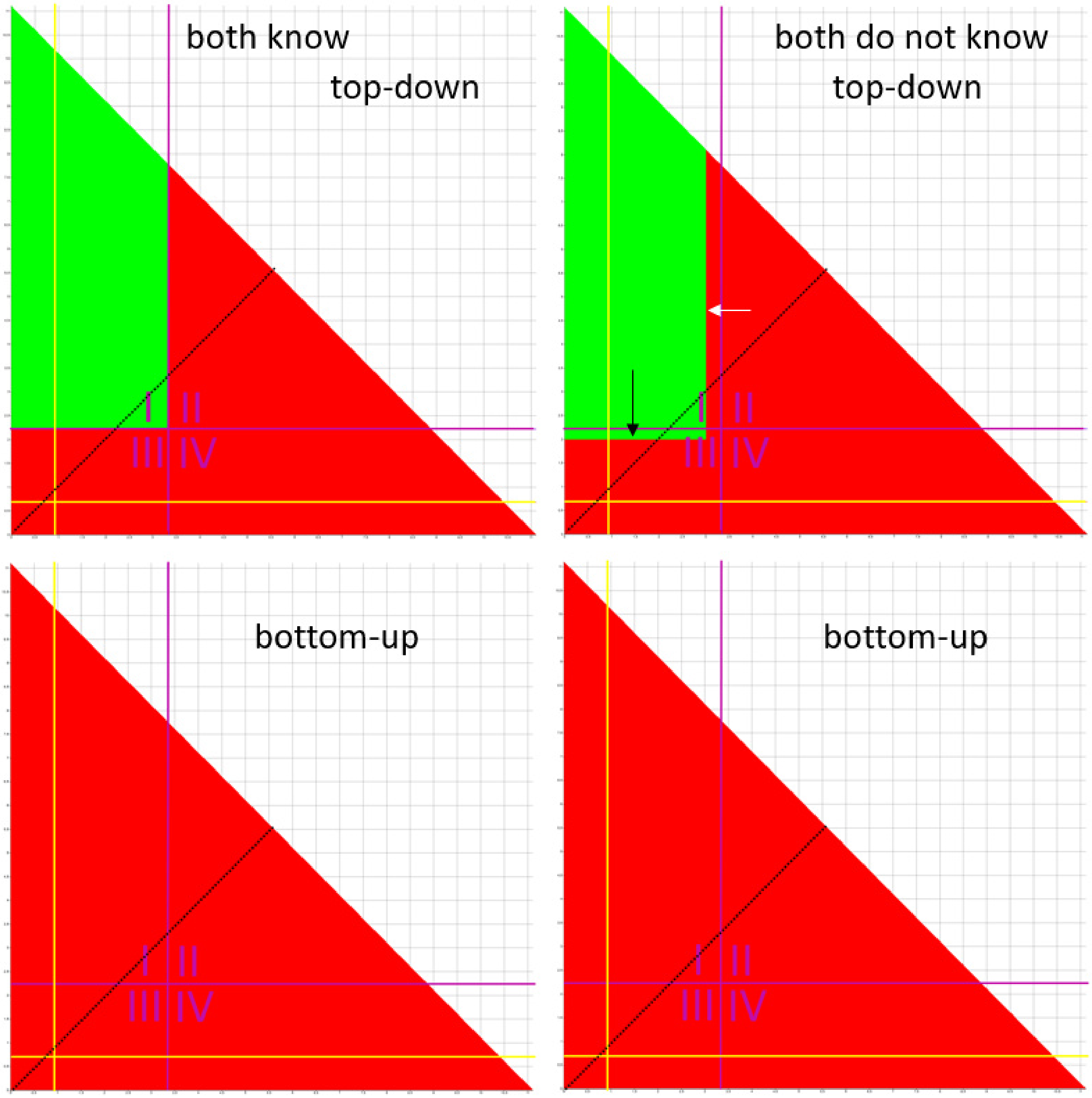


Figure 9

A strong ensemble is observed. On the left side (top-down and bottom-up) both parties of the ensemble know that the coin contains less value. They orient their behaviour according to the true value, the substrate concentration. The ensemble stays in area I. On the right side (top-down and bottom-up) both parties do not know that the coin contains less value. They rely on the number of coins as an indicator of the concentration. In this case the ensemble oversteps b=c of source (black arrow) and enters area III. The ensemble does not reach b=c of sink and stays in area I (white arrow). Different amounts of superadditivity are lost and gained. The purple lines indicate b=c and separate area I, II, III, and IV, the yellow lines mark np=max and the black dotted line is the line of equal concentration in source and sink (compare figure 2b).

Figure 10

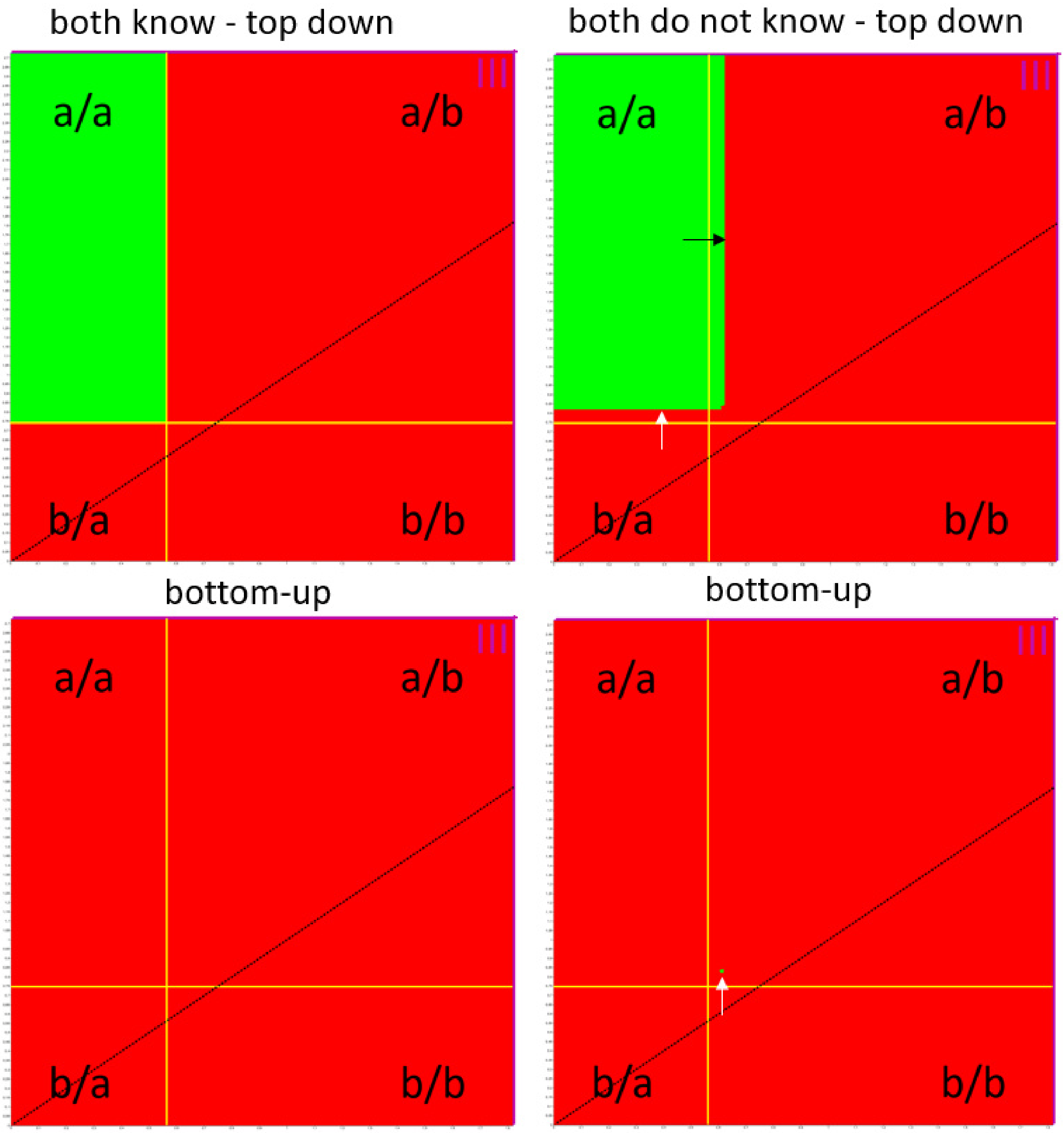


Figure 10

Source and sink separately try to maximize their individual net profit (yellow lines). A weak ensemble is observed. On the left side (top-down and bottom-up) both parties of the ensemble know that the coin contains more value. They orient their behaviour according to the true value, the substrate concentration. The ensemble stays in subarea a/a and in contrast to the weak ensemble of the transfer space no subadditivity appears (figure 4). On the right side (top-down and bottom-up) both parties do not know that the coin contains more value. They rely on the number of coins as an indicator of the concentration. In this case the ensemble oversteps np=max of sink and enters subarea a/b (black arrow). The ensemble does not reach np=max of source and stays within subarea a/a (white arrow, top down). Some subadditivity is created in subarea a/b (white arrow, bottom up). Superadditivity is lost (a/a) and gained (a/b) (white and black arrow, top down). The black dotted line is the line of equal concentration in source and sink (compare figure 3a).

Figure 11

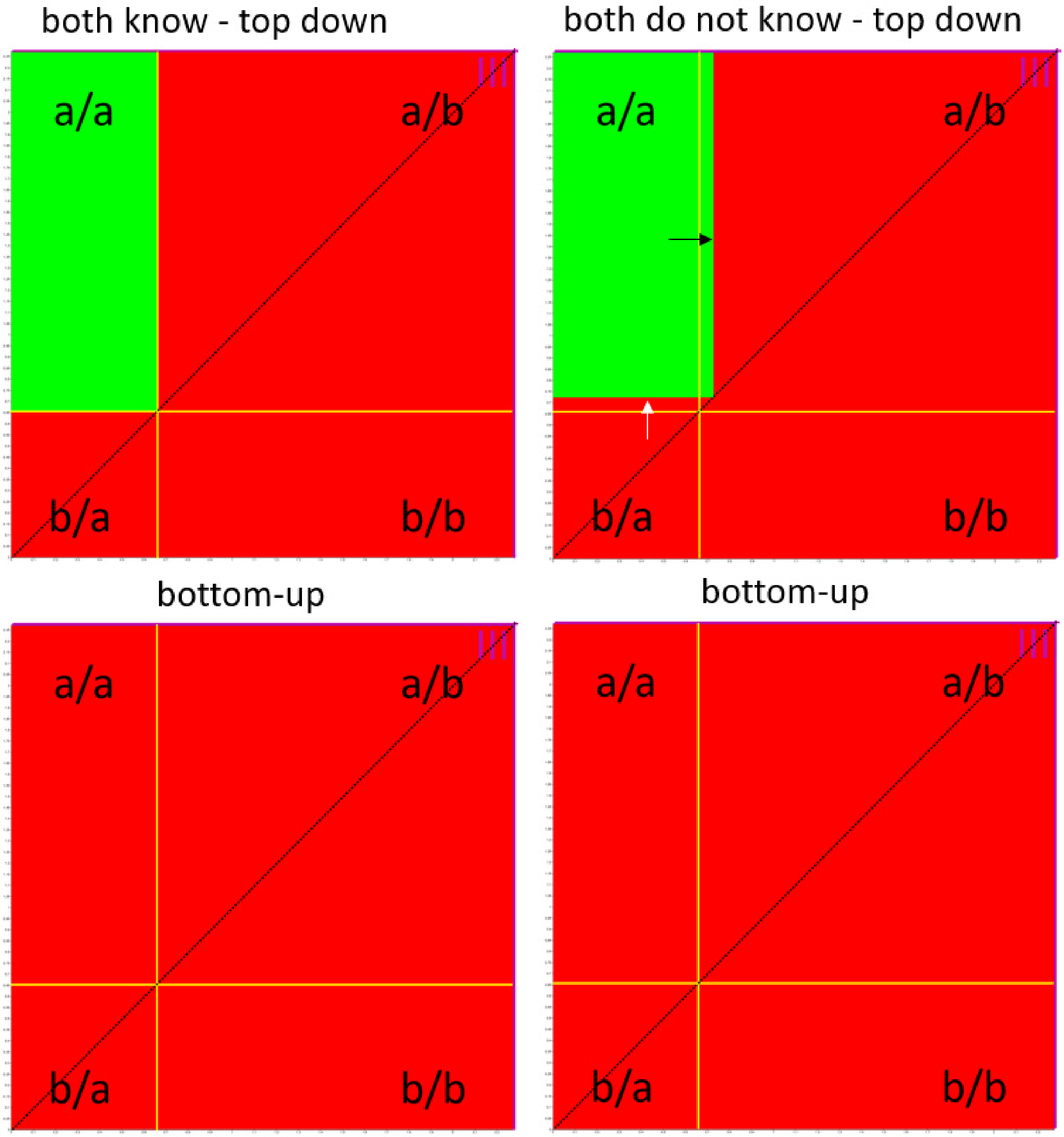


Figure 11

Source and sink separately try to maximize their individual net profit (yellow lines). A symmetric ensemble is observed. On the left side (top-down and bottom-up) both parties of the ensemble know that the coin contains more value. They orient their behaviour according to the true value, the substrate concentration. The ensemble stays in subarea a/a. On the right side (top-down and bottom-up) both parties do not know that the coin contains more value. They rely on the number of coins as an indicator of the concentration. In this case the ensemble oversteps np=max of sink and enters subarea a/b (black arrow). The ensemble does not reach np=max of source and stays within subarea a/a (white arrow). In general, superadditivity is lost and gained. The black dotted line is the line of equal concentration in source and sink (compare figure 3a).

Figure 12

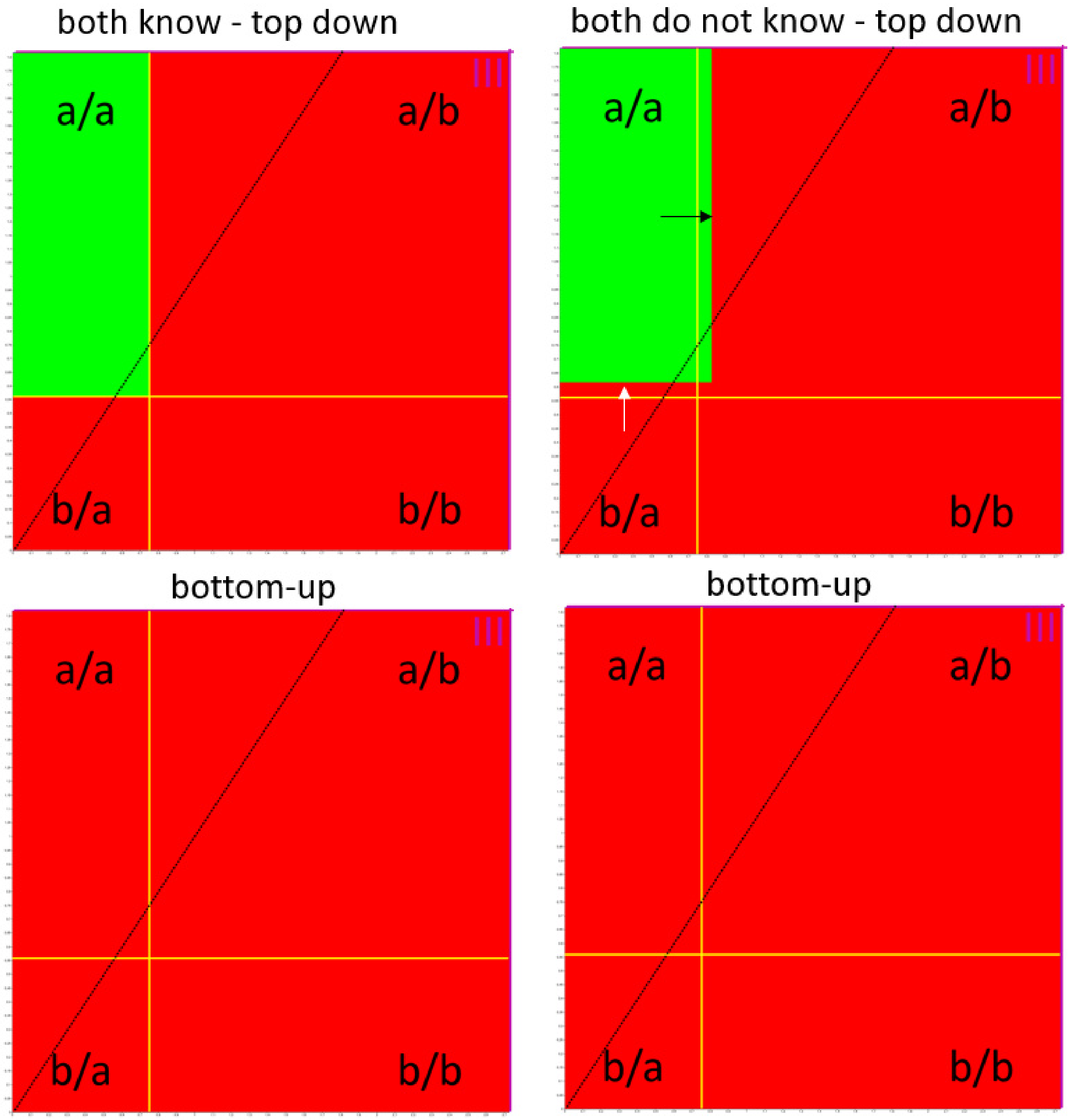


Figure 12

Source and sink separately try to maximize their individual net profit (yellow lines). A strong ensemble is observed. On the left side (top-down and bottom-up) both parties of the ensemble know that the coin contains more value. They orient their behaviour according to the true value, the substrate concentration. The ensemble stays in subarea a/a. On the right side (top-down and bottom-up) both parties do not know that the coin contains more value. They rely on the number of coins as an indicator of the concentration. In this case the ensemble oversteps np=max of sink and enters subarea a/b (black arrow). The ensemble does not reach np=max of source and stays within subarea a/a (white arrow). In general, superadditivity is lost and gained. The black dotted line is the line of equal concentration in source and sink (compare figure 3a).

Figure 13

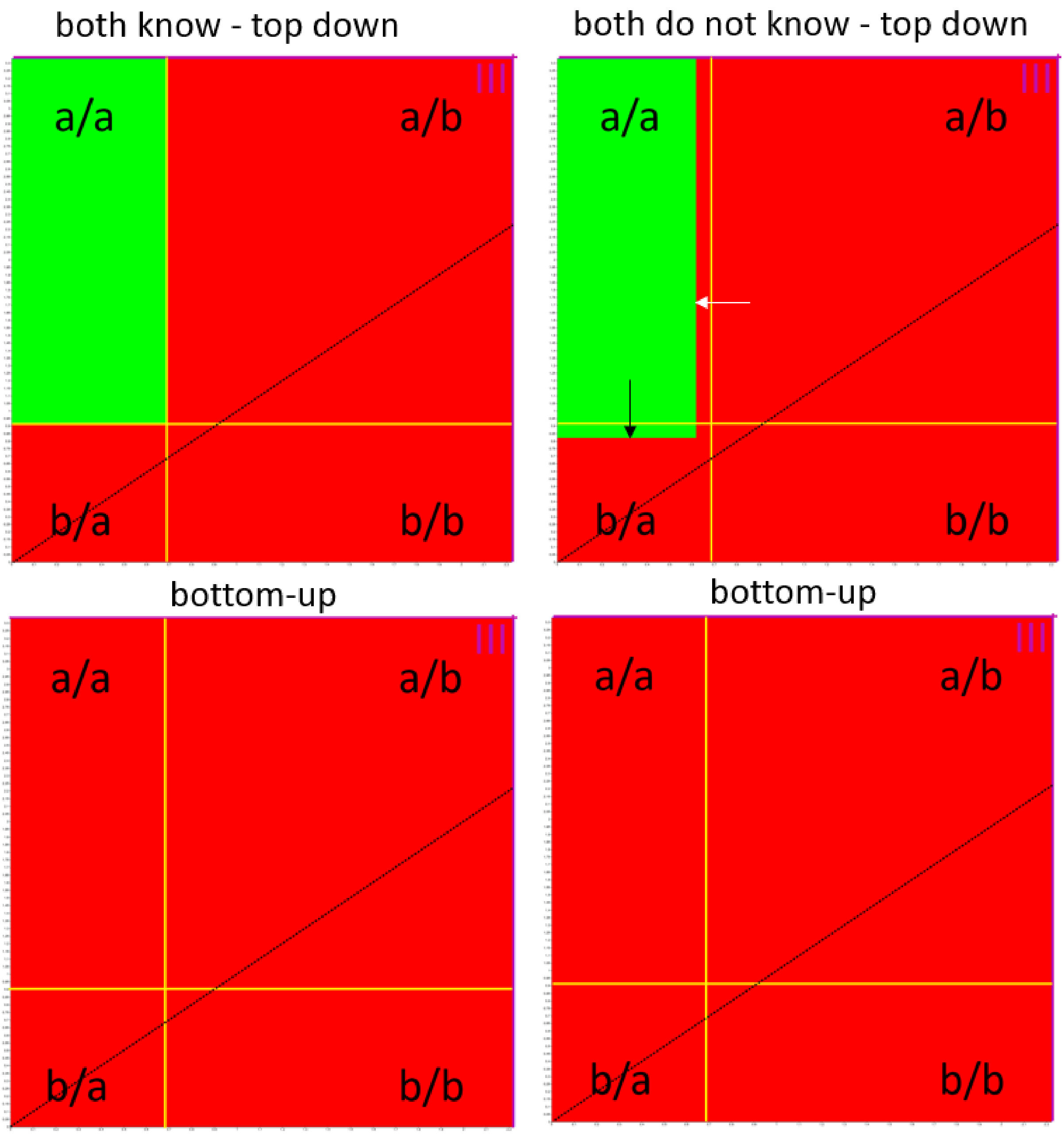


Figure 13

Source and sink separately try to maximize their individual net profit (yellow lines). A weak ensemble is observed. On the left side (top-down and bottom-up) both parties of the ensemble know that the coin contains less value. They orient their behaviour according to the true value, the substrate concentration. The ensemble stays in subarea a/a and in contrast to the weak ensemble of the transfer space no subadditivity appears (figure 7). On the right side (top-down and bottom-up) both parties do not know that the coin contains less value. They rely on the number of coins as an indicator of the concentration. In this case the ensemble oversteps np=max of source and enters subarea b/a (black arrow). The ensemble does not reach np=max of sink and stays within subarea a/a (white arrow). In general, superadditivity is lost and gained. Subadditivity does not appear in contrast to figure 10. The black dotted line is the line of equal concentration in source and sink (compare figure 3b).

Figure 14

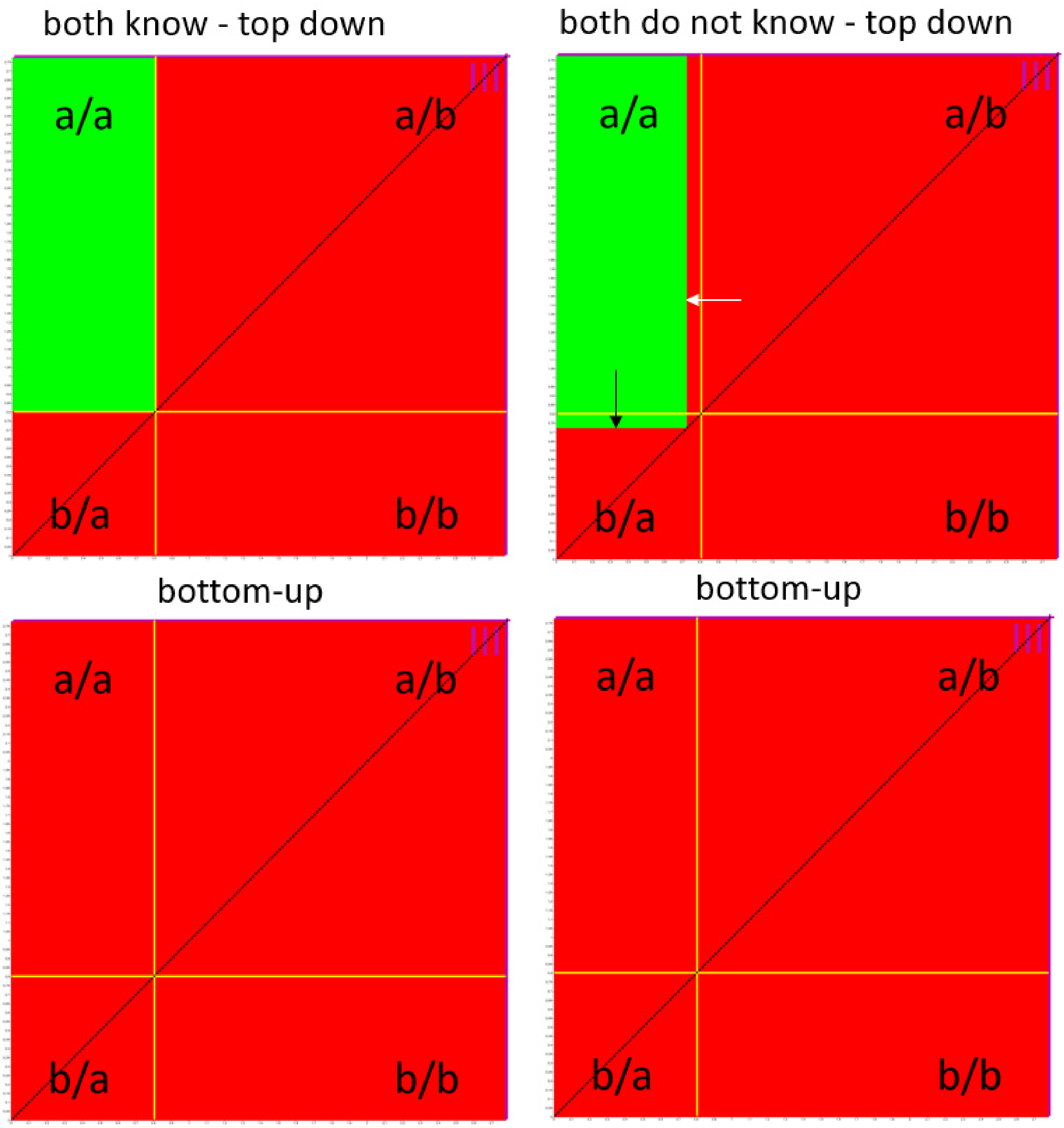


Figure 14

Source and sink separately try to maximize their individual net profit (yellow lines). A symmetric ensemble is observed. On the left side (top-down and bottom-up) both parties of the ensemble know that the coin contains less value. They orient their behaviour according to the true value, the substrate concentration. The ensemble stays in subarea a/a. On the right side (top-down and bottom-up) both parties do not know that the coin contains less value. They rely on the number of coins as an indicator of the concentration. In this case the ensemble oversteps np=max of source and enters subarea b/a (black arrow). The ensemble does not reach np=max of sink and stays within subarea a/a (white arrow). In general, superadditivity is lost and gained. The black dotted line is the line of equal concentration in source and sink (compare figure 3b).

Figure 15

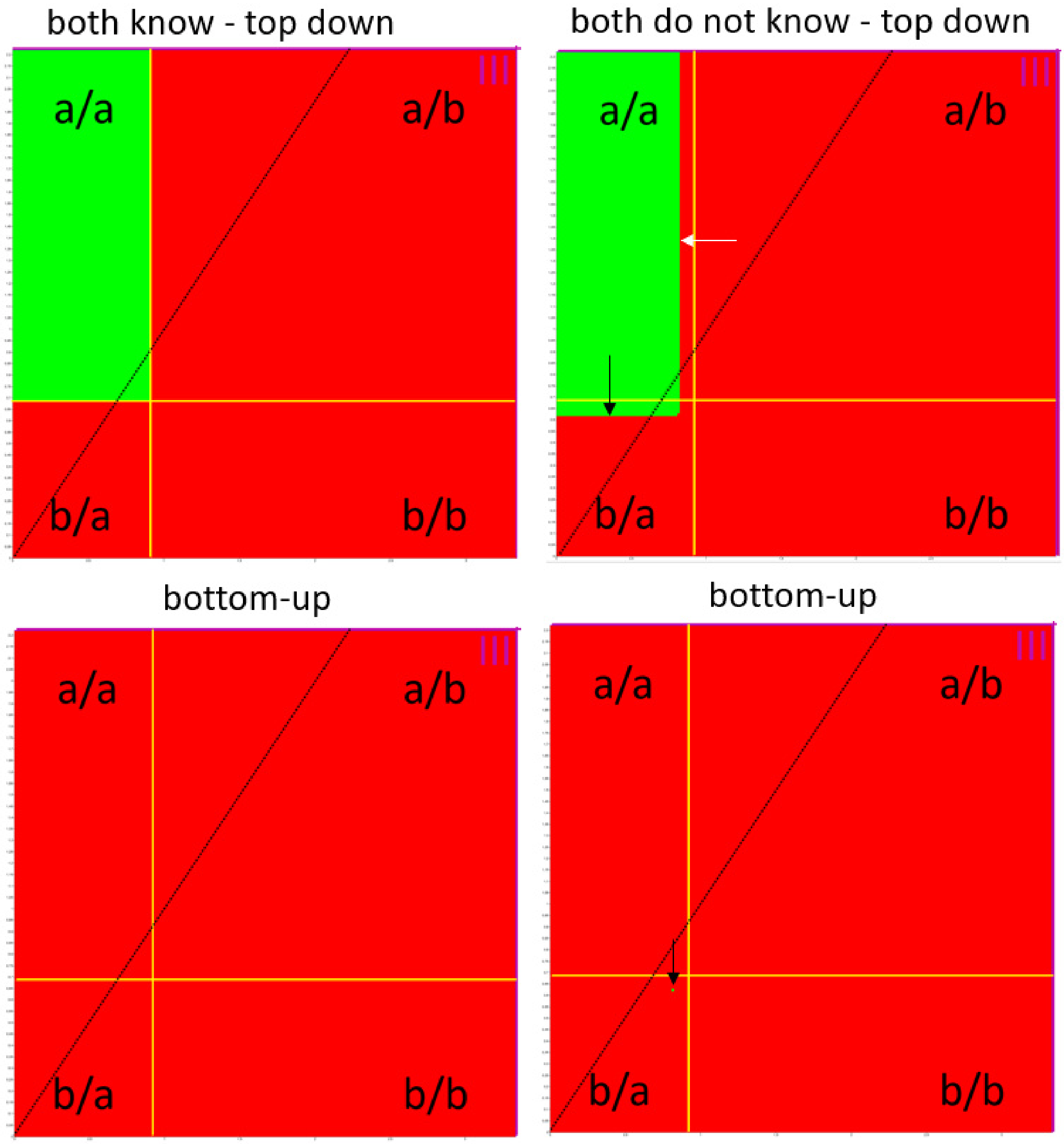


Figure 15

Source and sink separately try to maximize their individual net profit (yellow lines). A strong ensemble is observed. On the left side (top-down and bottom-up) both parties of the ensemble know that the coin contains less value. They orient their behaviour according to the true value, the substrate concentration. The ensemble stays in subarea a/a. On the right side (top-down and bottom-up) both parties do not know that the coin contains less value. They rely on the number of coins as an indicator of the concentration. In this case the ensemble oversteps np=max of source and enters subarea b/a (black arrows). Here, superadditivity and a tiny amount of subadditivity appears. The ensemble does not reach np=max of sink and stays within subarea a/a (white arrow). In general, superadditivity is lost and gained and some subadditivity is gained in contrast to figure 12. The black dotted line is the line of equal concentration in source and sink (compare figure 3b).

In figure 4 to 15 I compare ensembles with knowledge of the true value (*i.e*. substrate concentration) of the coin with ensembles completely unaware of the true value. They believe that the number on the coin is a true information of the value of the coin.

Observations within the transfer space:

When the value of the coin is underestimated the ignorant symmetric and strong ensembles create additional superadditivity in area II of the transfer space. The ignorant weak ensemble is here an exception as subadditivity is created in area I and II. On the other hand, the three types of ignorant ensembles lose superadditivity in area I next to area III. Again, the ignorant weak ensemble sticks out as it also loses in area I subadditivity; this is a gain.

When the value of the coin is overestimated the ignorant symmetric and strong ensemble create superadditivity in area III of the transfer space. The ignorant weak ensemble is here, again, an exception as super and subadditivity are created in area III. On the other hand, the ignorant symmetric and ignorant strong ensembles lose superadditivity in area I next to area II. Again, the ignorant weak ensemble sticks out as it loses a considerable amount of subadditivity in area I; again, a gain.

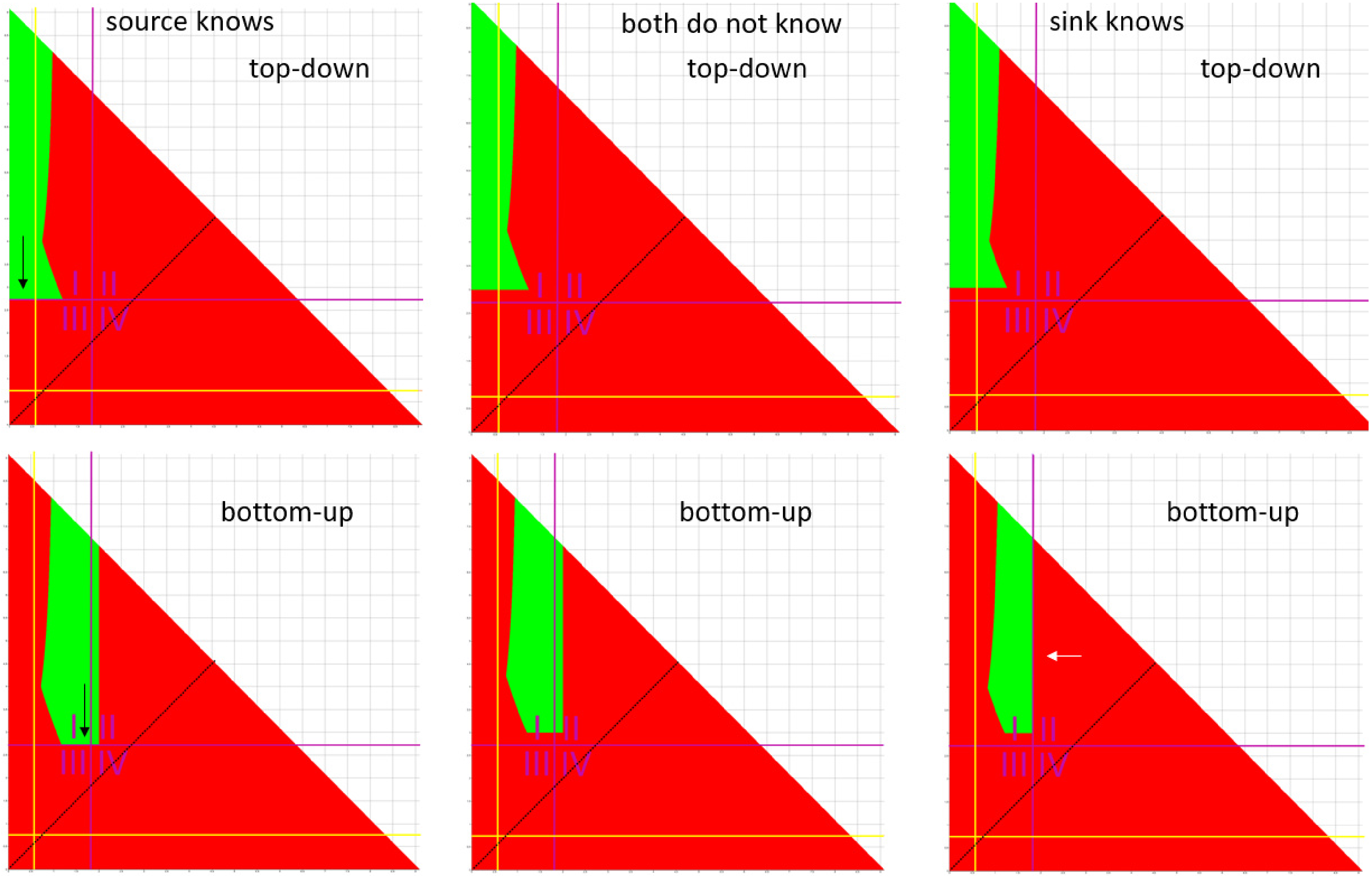
Observations within the positive net profit subspace:

When the value of the coin is underestimated all three types (weak, symmetric, and strong) of ignorant ensembles create superadditivity in subarea *a*/b of the positive net profit subspace. In addition, the ignorant weak ensemble creates a little subadditivity in subarea a/b. All three types of ignorant ensembles lose superadditivity in subarea a/a.

When the value of the coin is overestimated all three types (weak, symmetric, and strong) of ignorant ensembles create superadditivity in subarea b/a of the positive net profit subspace. Here, in contrast to underestimation of value, the ignorant strong ensemble creates a little subadditivity in subarea b/a. All three types of ignorant ensembles lose superadditivity in subarea a/a.

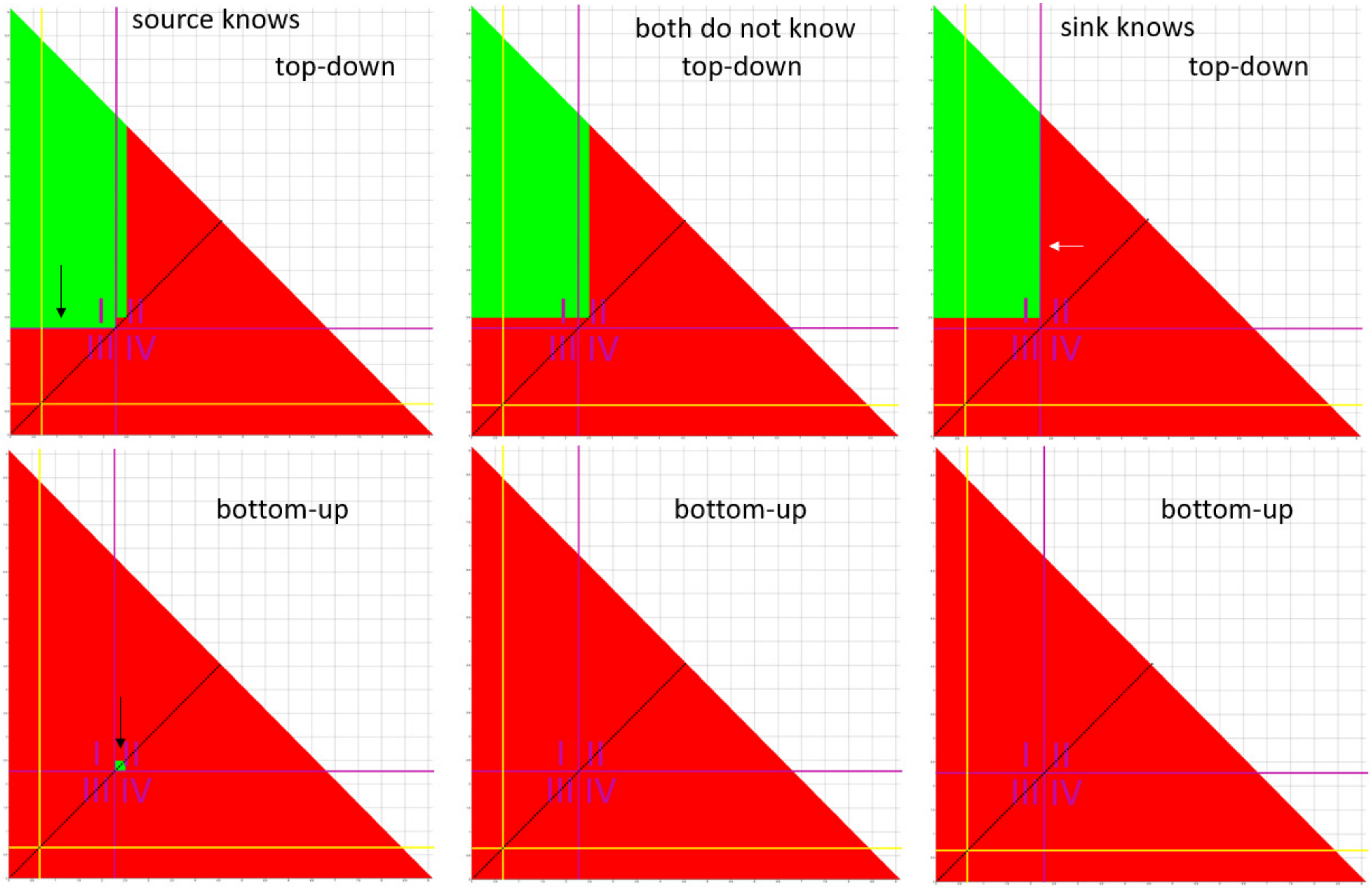
In the following series of figures (figures 16 to 27) I compare qualitatively an ensemble in ignorance of the true value with two ensembles where either source or sink does know the true substrate concentration within the coin. Again, the transfer space and the positive net profit subspace are observed. This is now an asymmetry of information not only between two ensembles but also an asymmetry of information within the ensemble. If the source knows the true value, the sink is ignorant, and if the sink knows the true value, the source is ignorant. The observable gain and loss of the ensemble achieved by knowledge or gain of source and sink is always in comparison to the ignorant ensemble. Basically, a one-sided knowledge-based or ignorance-based gain or loss of superadditivity or subadditivity in the following section is due to the adjustment to the correct limit or overstepping the limit in only one party. Later, when the quantitative aspect will be investigated, the question will be whether one-sided knowledge or ignorance will be more or less harmful than complete knowledge or complete ignorance. One-sided knowledge could avoid a mistake or enhance a mistake. The reason is that beyond the limits of source and sink (b=c, np=max) there is still superadditivity achievable. Especially in weak ensembles subadditivity could be avoided or increased.

Figure 16



source knows
top-down
I II
III IV
both do not know
top-down
I II
III IV
sink knows
top-down
I II
III IV
bottom-up
I II
III IV
bottom-up
I II
III IV
bottom-up
I II
III IV

Figure 17



source knows
top-down
I
II
III
IV
both do not know
top-down
I
II
III
IV
sink knows
top-down
I
II
III
IV
bottom-up
I
II
III
IV
bottom-up
I
II
III
IV
bottom-up
I
II
III
IV

Figure 18

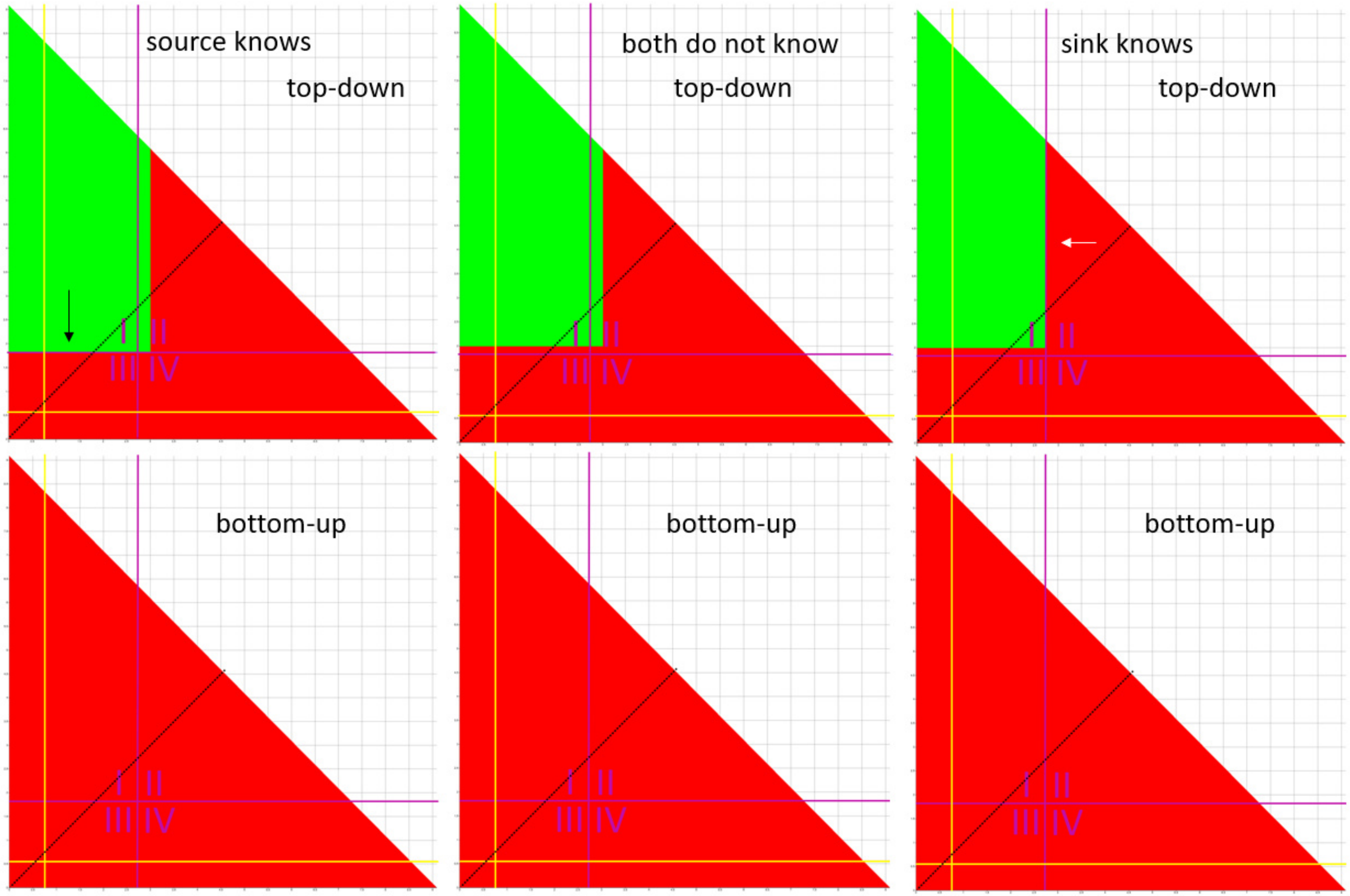

source knows
top-down
I
II
III
IV
both do not know
top-down
I
II
III
IV
sink knows
top-down
I
II
III
IV
bottom-up
I
II
III
IV
bottom-up
I
II
III
IV
bottom-up
I
II
III
IV

Figure 16

A weak ensemble is observed and the value (amount of substrate) within the coin is underestimated. On the left side (top-down and bottom-up) source knows that the coin contains more value. Source acts accordingly within its ensemble. Source will go up to its limit b=c. The ensemble creates additional superadditivity but also additional subadditivity (black arrows) in comparison to the ignorant ensemble. In the middle a completely ignorant ensemble (top-down and bottom-up) is observed, see figure 4. On the right side (top-down and bottom-up) sink knows that the coin contains more value. Sink acts accordingly. Unknowingly that that the ensemble is weak, sink stops at its limit b=c and loses some subadditivity in comparison to the completely ignorant ensemble (white arrow). The purple lines indicate b=c and separate area I, II, III, and IV, the yellow lines mark np=max and the black dotted line is the line of equal concentration in source and sink (compare figure 2a).

Figure 17

A symmetric ensemble is observed and the value (amount of substrate) within the coin is underestimated. On the left side (top-down and bottom-up) source knows that the coin contains more value. Source acts accordingly within its ensemble. Source will go to its limit b=c. The ensemble gets an additional amount of superadditivity (black arrow, top-down) in comparison to the ignorant ensemble but for the price of additional subadditivity in area II (black arrow, bottom up). In the middle a completely ignorant ensemble (top-down and bottom-up) is observed, see figure 5. On the right side (top-down and bottom-up) sink knows that the coin contains more value. Sink acts accordingly. Sink stops at its limit b=c and misses additional superadditivity in area II for the ensemble in comparison to the completely ignorant ensemble (white arrow). The purple lines indicate b=c and separate area I, II, III, and IV, the yellow lines mark np=max and the black dotted line is the line of equal concentration in source and sink (compare figure 2a).

Figure 18

A strong ensemble is observed and the value (amount of substrate) within the coin is underestimated. On the left side (top-down and bottom-up) source knows that the coin contains more value. Source acts accordingly within its ensemble. Source will go up to its limit b=c. The ensemble creates more superadditivity (black arrow) in comparison to the ignorant ensemble. In the middle a completely ignorant ensemble (top-down and bottom-up) is observed, see figure 6. On the right side (top-down and bottom-up) sink knows that the coin contains more value. Sink acts accordingly. Sink stops to take at sinks limit b=c and creates less superadditivity as superadditivity in area II is missed in comparison to the completely ignorant ensemble (white arrow). The purple lines indicate b=c and separate area I, II, III, and IV, the yellow lines mark np=max and the black dotted line is the line of equal concentration in source and sink (compare figure 2a).

Figure 19

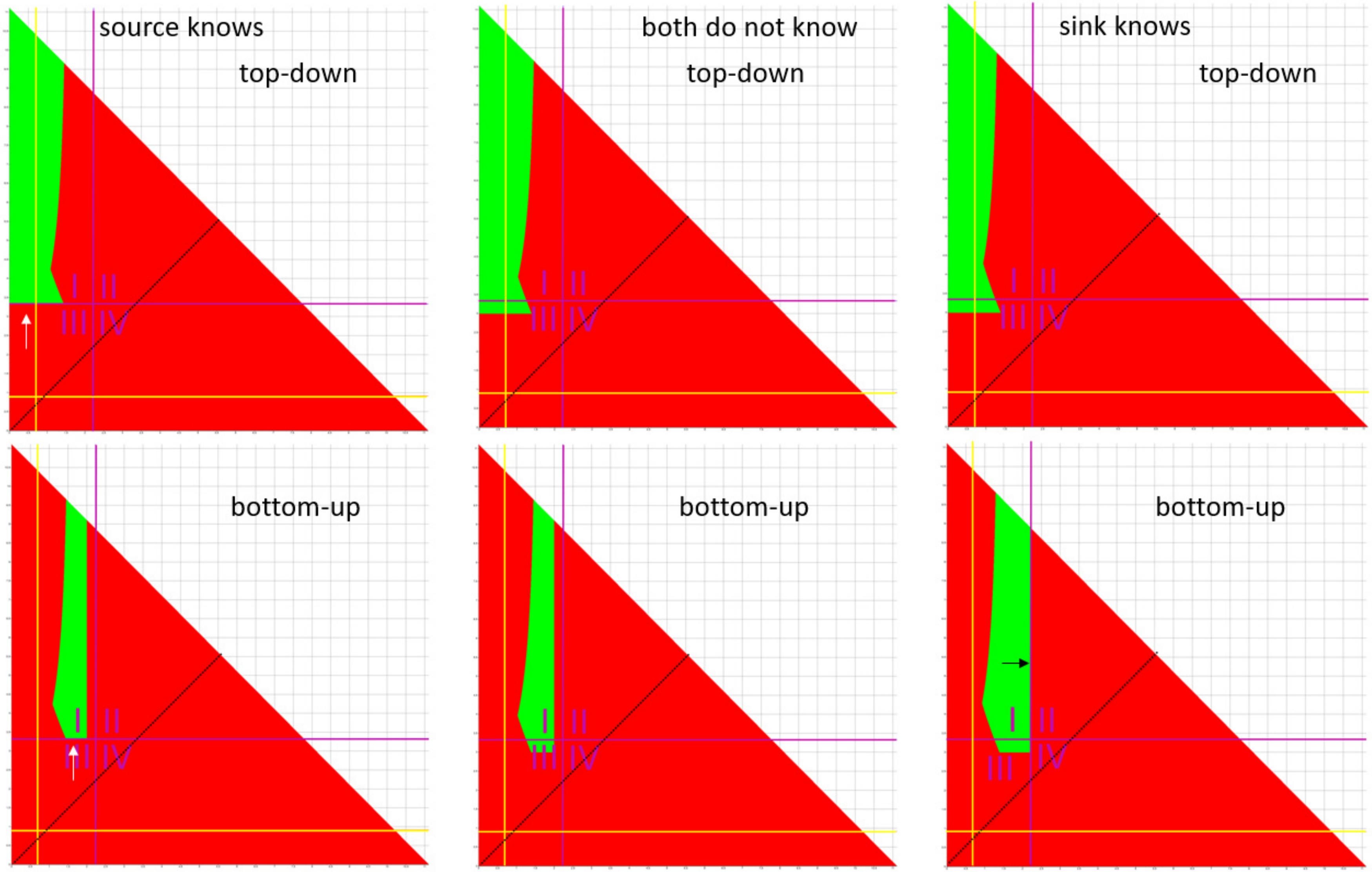
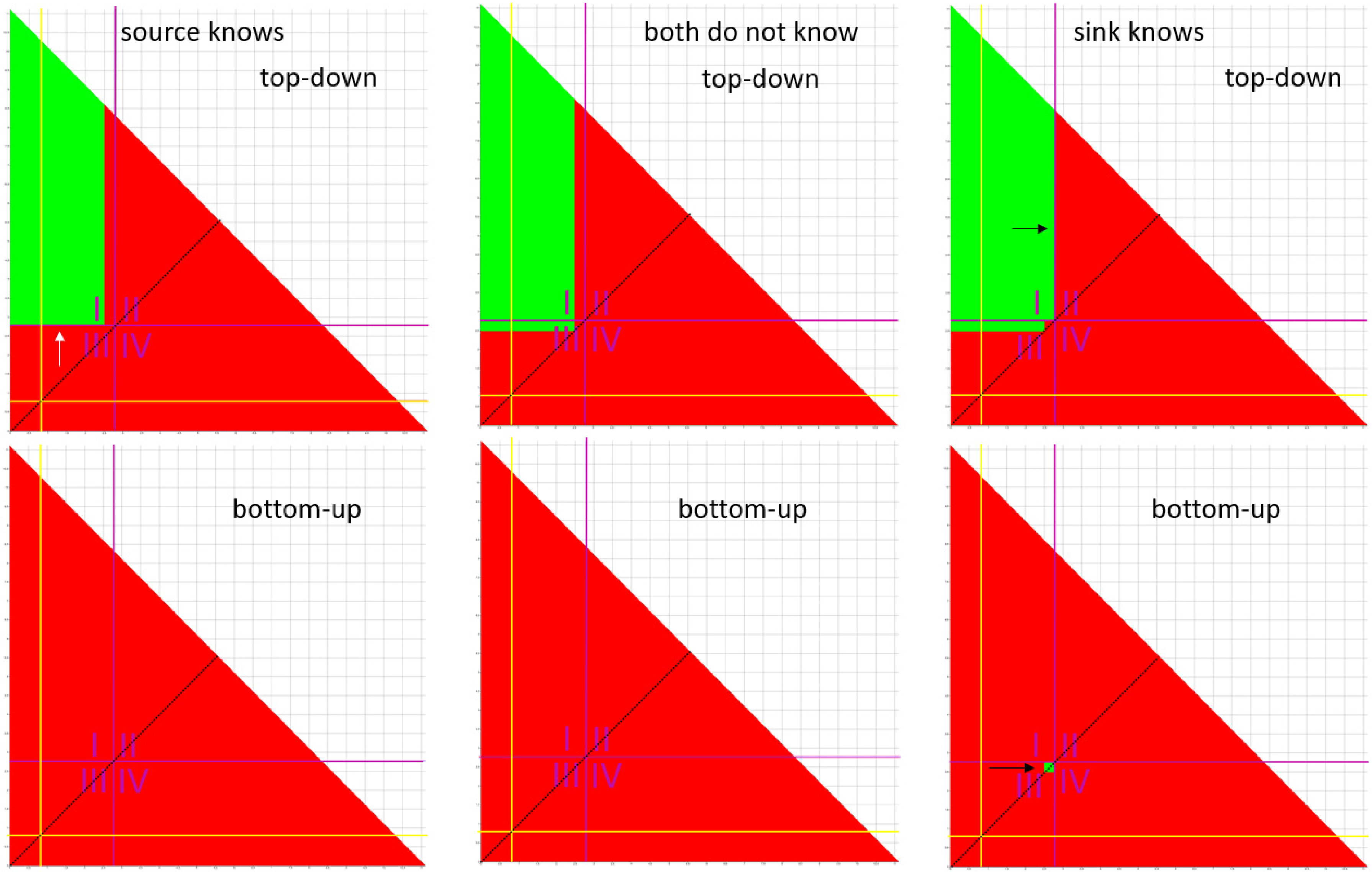

source knows
top-down
I
II
III
IV
both do not know
top-down
I
II
III
IV
sink knows
top-down
I
II
III
IV
bottom-up
I
II
III
IV
bottom-up
I
II
III
IV
bottom-up
I
II
III
IV

Figure 20



source knows
top-down
both do not know
top-down
sink knows
top-down
bottom-up
bottom-up
bottom-up

Figure 21

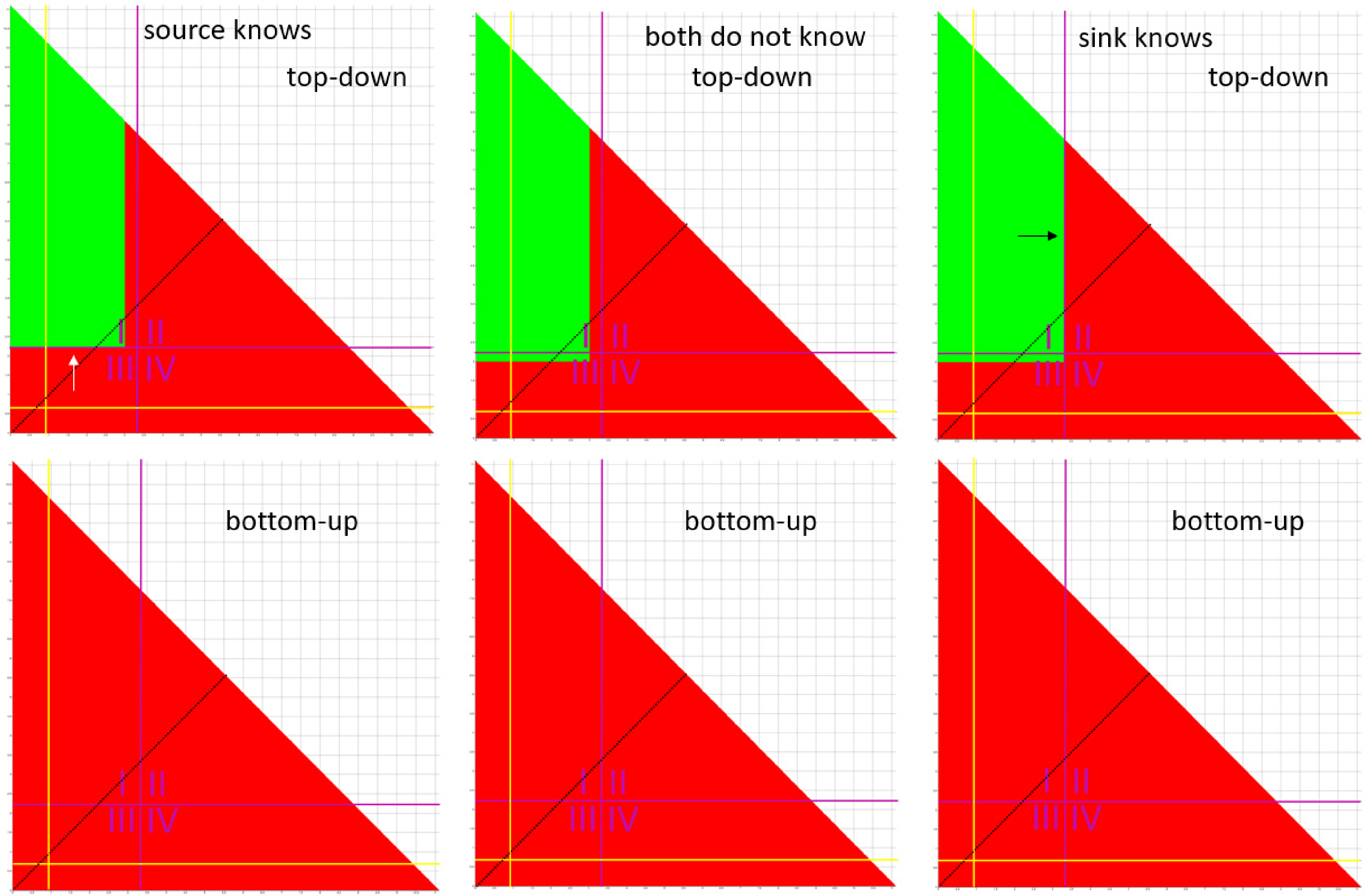

source knows
top-down
I II
III IV
both do not know
top-down
I II
III IV
sink knows
top-down
I II
III IV
bottom-up
I II
III IV
bottom-up
I II
III IV
bottom-up
I II
III IV

Figure 19

A weak ensemble is observed and the value (amount of substrate) within the coin is overestimated. On the left side (top-down and bottom-up) source knows that the coin contains less value. Source acts accordingly within its ensemble. Source will not go beyond its limit b=c. The ensemble misses some superadditivity but also some subadditivity (white arrows) in comparison to the ignorant ensemble. In the middle a completely ignorant ensemble (top-down and bottom-up) is observed, see figure 7. On the right side (top-down and bottom-up) sink knows that the coin contains less value. Sink acts accordingly. Unknowingly that the ensemble is weak, sink reaches its limit b=c and creates more subadditivity in comparison to the completely ignorant ensemble (black arrow). The purple lines indicate b=c and separate area I, II, III, and IV, the yellow lines mark np=max and the black dotted line is the line of equal concentration in source and sink (compare figure 2b).

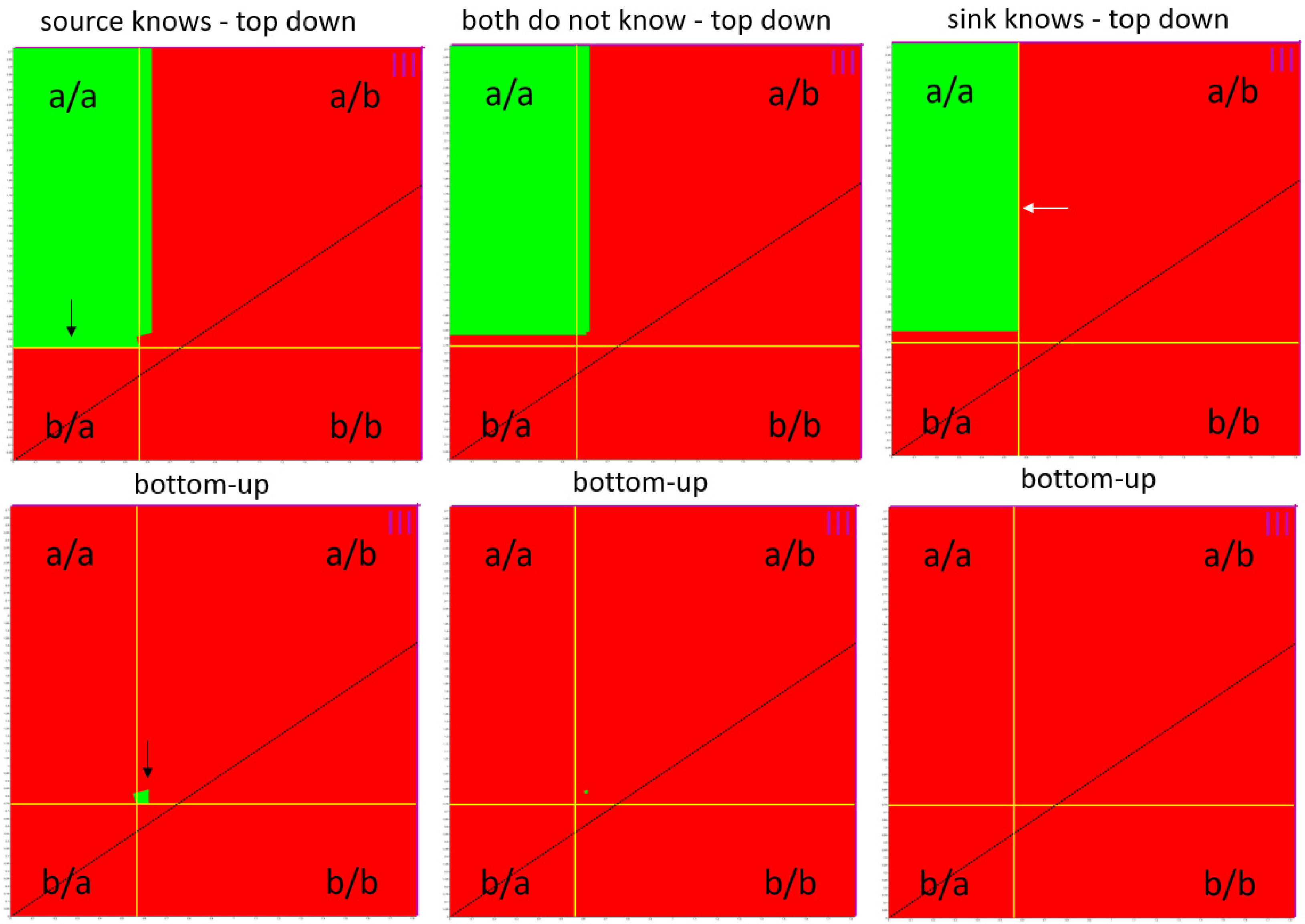
Figure 20

A symmetric ensemble is observed and the value (amount of substrate) within the coin is overestimated. On the left side (top-down and bottom-up) source knows that the coin contains less value. Source acts accordingly within its ensemble. Source will not go beyond its limit b=c. The ensemble misses some superadditivity (white arrow) in comparison to the ignorant ensemble. In the middle a completely ignorant ensemble (top-down and bottom-up) is observed, see figure 8. On the right side (top-down and bottom-up) sink knows that the coin contains less value. Sink acts accordingly. Sink reaches its limit b=c and creates besides a considerable amount of additional superadditivity in area I a little bit of subadditivity in area III in comparison to the completely ignorant ensemble (black arrows). The purple lines indicate b=c and separate area I, II, III, and IV, the yellow lines mark np=max and the black dotted line is the line of equal concentration in source and sink (compare figure 2b).

Figure 21

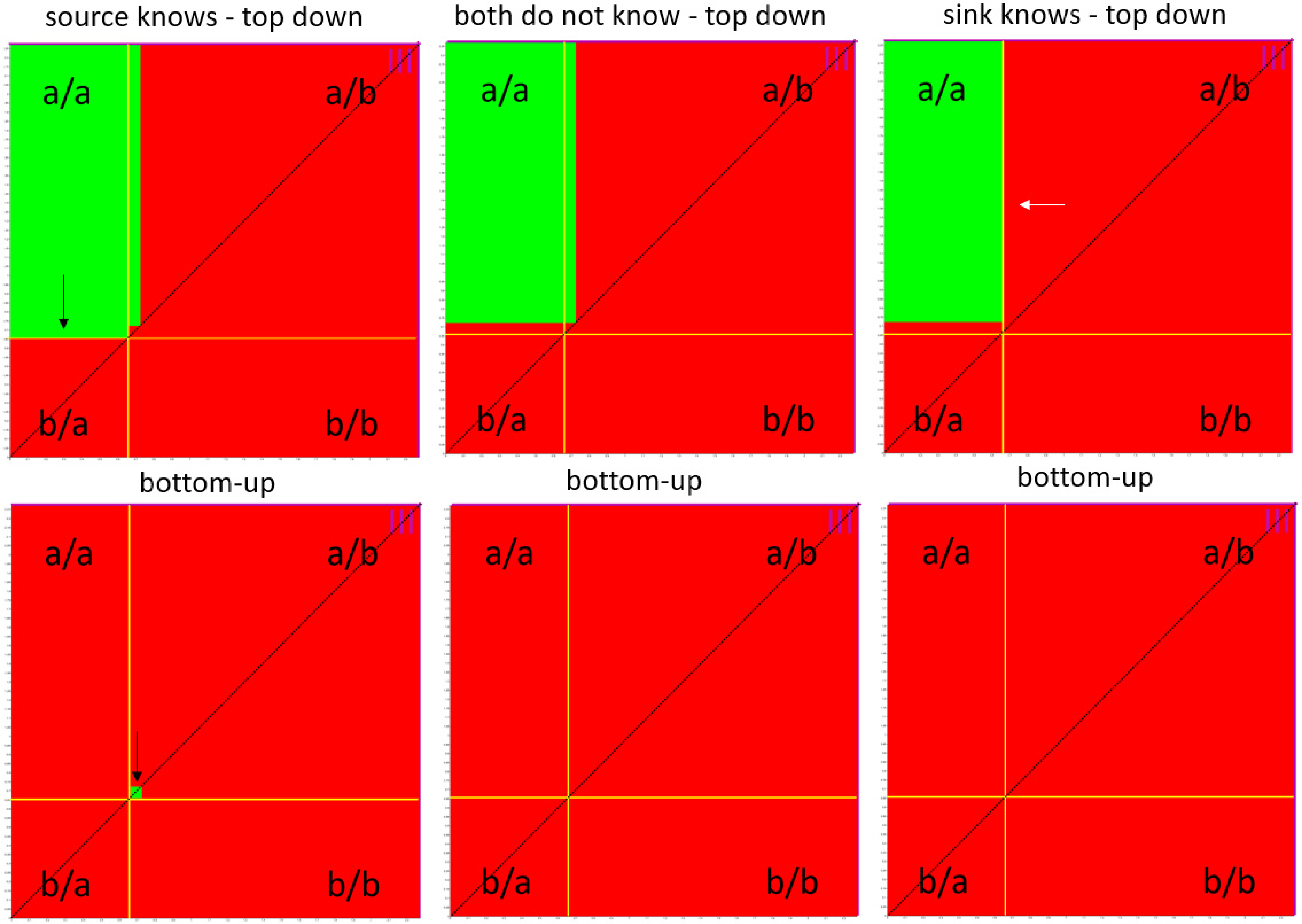
A strong ensemble is observed and the value (amount of substrate) within the coin is overestimated. On the left side (top-down and bottom-up) source knows that the coin contains less value. Source acts accordingly within its ensemble. Source will not go beyond its limit b=c. The ensemble misses some superadditivity (white arrow) in comparison to the ignorant ensemble. In the middle a completely ignorant ensemble (top-down and bottom-up) is observed, see figure 9. On the right side (top-down and bottom-up) sink knows that the coin contains less value. Sink acts accordingly. Sink reaches its limit b=c and creates a considerable amount of additional superadditivity in area I in comparison to the completely ignorant ensemble (black arrow). The purple lines indicate b=c and separate area I, II, III, and IV, the yellow lines mark np=max and the black dotted line is the line of equal concentration in source and sink (compare figure 2b).

Figure 22



source knows - top down
both do not know - top down
sink knows - top down
a/a
a/b
b/a
b/b
bottom-up
bottom-up
bottom-up

Figure 23



source knows - top down
both do not know - top down
sink knows - top down
bottom-up
bottom-up
bottom-up
a/a
a/b
b/a
b/b

Figure 24

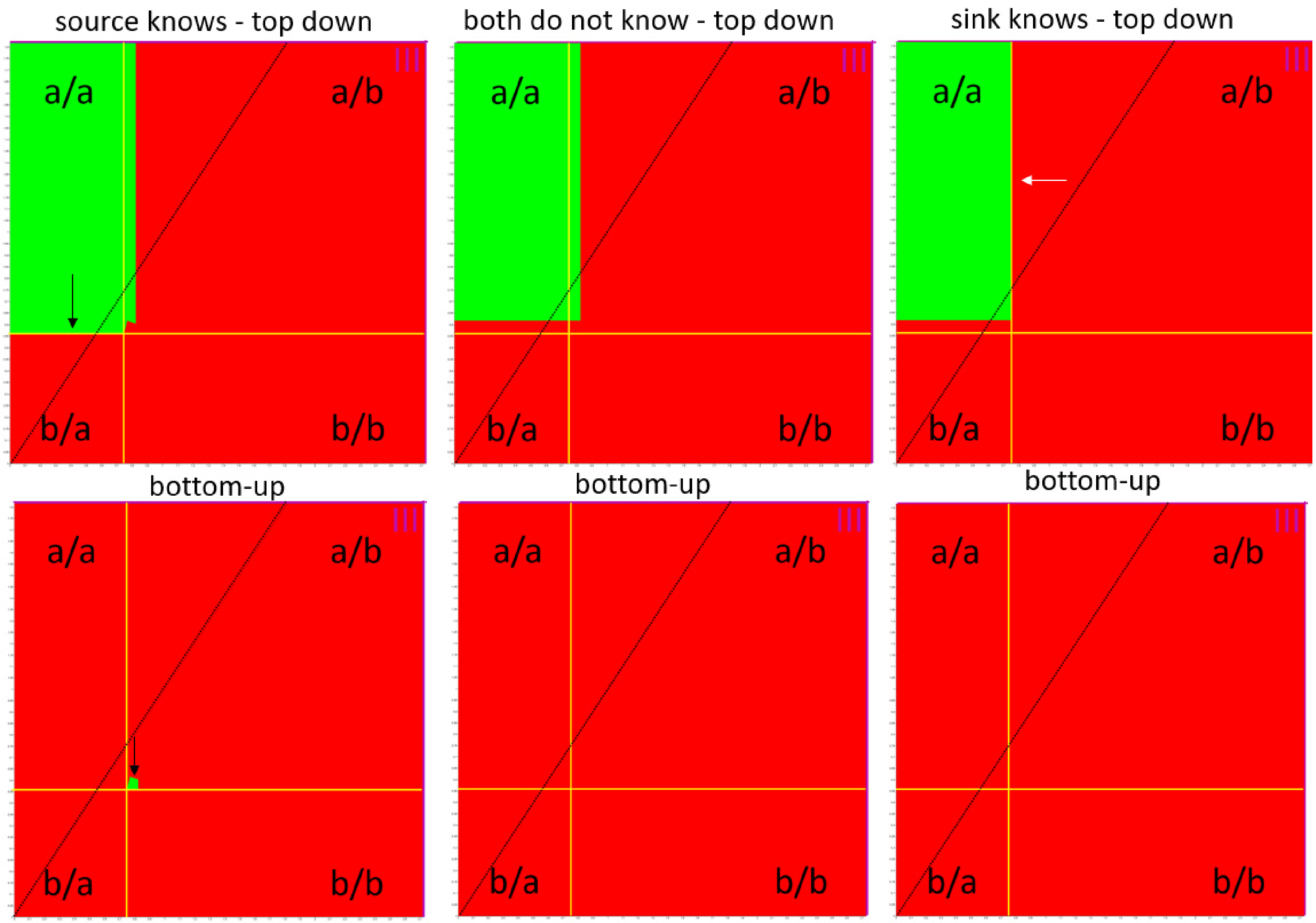

source knows - top down
both do not know - top down
sink knows - top down
bottom-up
bottom-up
bottom-up
a/a
a/b
b/a
b/b

Figure 22

Source and sink separately try to maximize their individual net profit (yellow lines). A weak ensemble is observed and the value (amount of substrate) within the coin is underestimated. On the left side (top-down and bottom-up) source knows that the coin contains more value. Source acts accordingly within its ensemble. Source will go up to its limit np=max. The ensemble creates additional superadditivity but also additional subadditivity in subarea a/a and a/b (black arrows) in comparison to the ignorant ensemble. In the middle a completely ignorant ensemble (top-down and bottom-up) is observed, see figure 10. On the right side (top-down and bottom-up) sink knows that the coin contains more value. Sink acts accordingly. Sink stops at its limit np=max and loses some superadditivity in area a/b in comparison to the completely ignorant ensemble (white arrow). But it loses also a tiny amount of subadditivity in area a/b. The black dotted line is the line of equal concentration in source and sink (compare figure 3a).

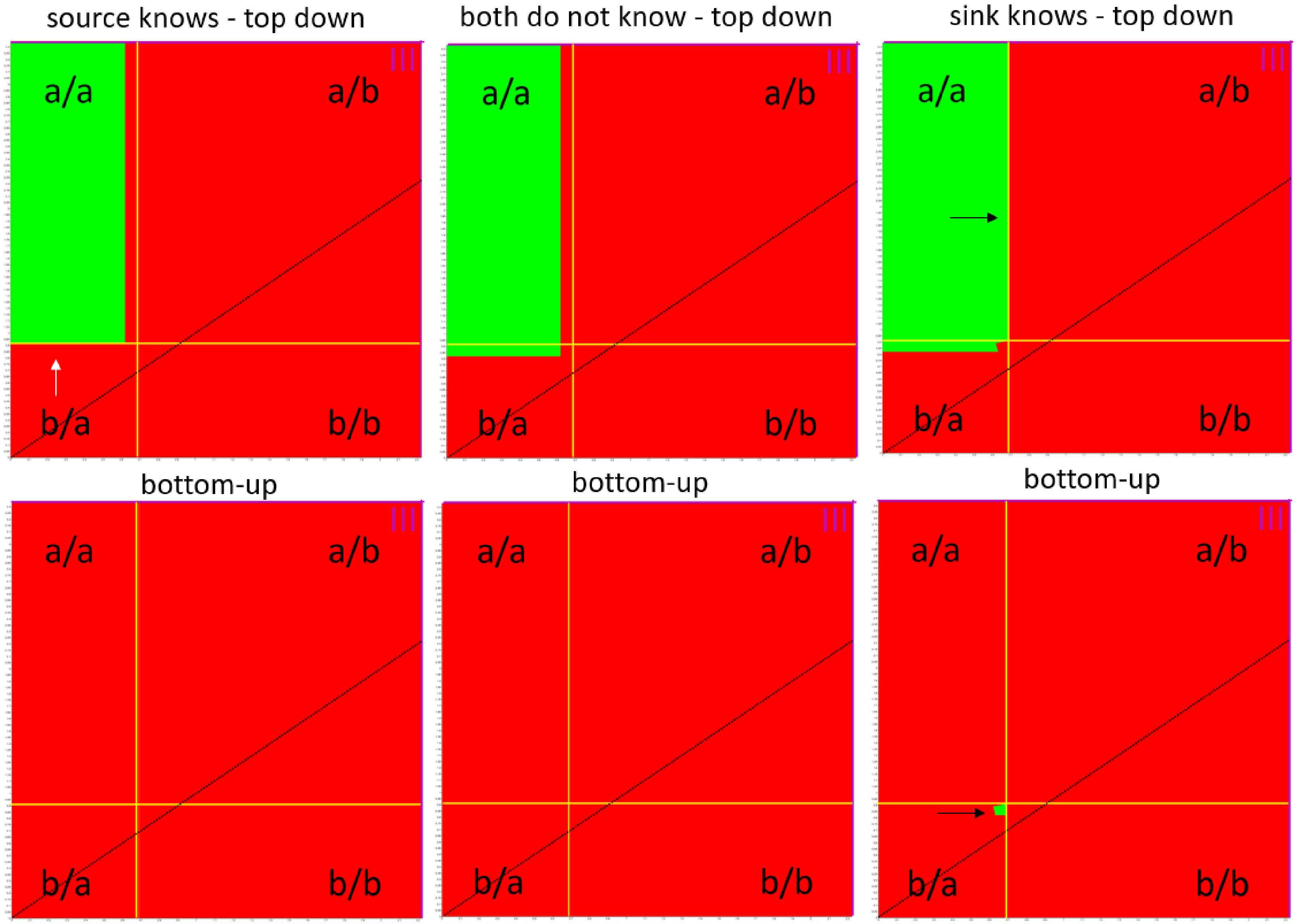
Figure 23

Source and sink separately try to maximize their individual net profit (yellow lines). A symmetric ensemble is observed and the value (amount of substrate) within the coin is underestimated. On the left side (top-down and bottom-up) source knows that the coin contains more value. Source acts accordingly within its ensemble. Source will go up to its limit np=max. The ensemble creates additional superadditivity but also additional subadditivity in subarea a/b (black arrows) in comparison to the ignorant ensemble. In the middle a completely ignorant ensemble (top-down and bottom-up) is observed, see figure 11. On the right side (top-down and bottom-up) sink knows that the coin contains more value. Sink acts accordingly. Sink stops at its limit np=max and loses some superadditivity in area a/b in comparison to the completely ignorant ensemble (white arrow). The black dotted line is the line of equal concentration in source and sink (compare figure 3a).

Figure 24

Source and sink separately try to maximize their individual net profit (yellow lines). A strong ensemble is observed and the value (amount of substrate) within the coin is underestimated. On the left side (top-down and bottom-up) source knows that the coin contains more value. Source acts accordingly within its ensemble. Source will go up to its limit np=max. The ensemble creates additional superadditivity in subarea a/a but also additional subadditivity in subarea a/b (black arrows) in comparison to the ignorant ensemble. In the middle a completely ignorant ensemble (top-down and bottom-up) is observed, see figure 12. On the right side (top-down and bottom-up) sink knows that the coin contains more value. Sink acts accordingly. Sink stops at its limit np=max and loses some superadditivity in area a/b in comparison to the completely ignorant ensemble (white arrow). The black dotted line is the line of equal concentration in source and sink (compare figure 3a).

Figure 25



source knows - top down
both do not know - top down
sink knows - top down
a/a
a/b
b/a
b/b
a/a
a/b
b/a
b/b
a/a
a/b
b/a
b/b
bottom-up
bottom-up
bottom-up
a/a
a/b
b/a
b/b
a/a
a/b
b/a
b/b
a/a
a/b
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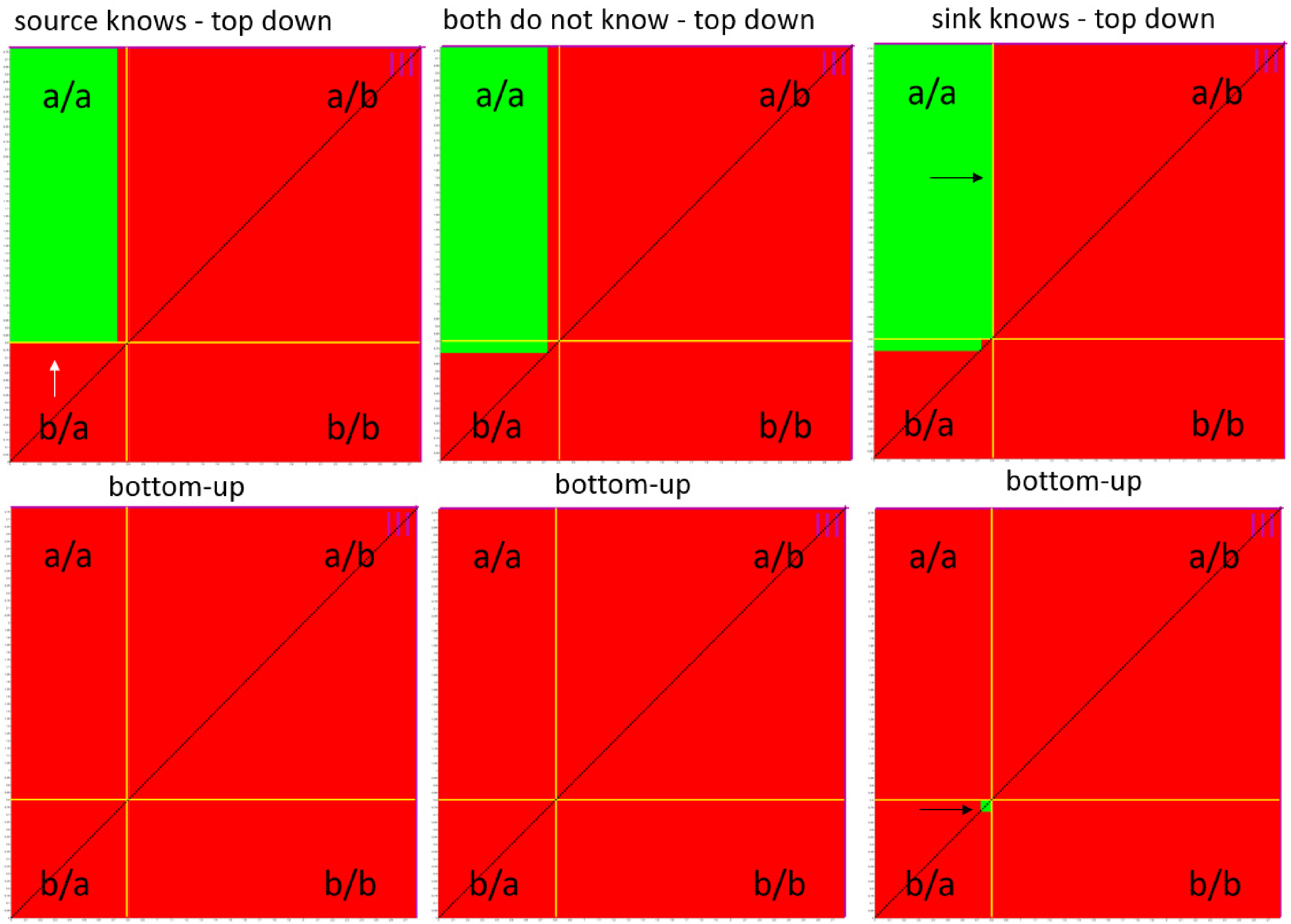
b/b

Figure 26



source knows - top down
a/a
a/b
b/a
b/b
both do not know - top down
a/a
a/b
b/a
b/b
sink knows - top down
a/a
a/b
b/a
b/b
bottom-up
a/a
a/b
b/a
b/b
bottom-up
a/a
a/b
b/a
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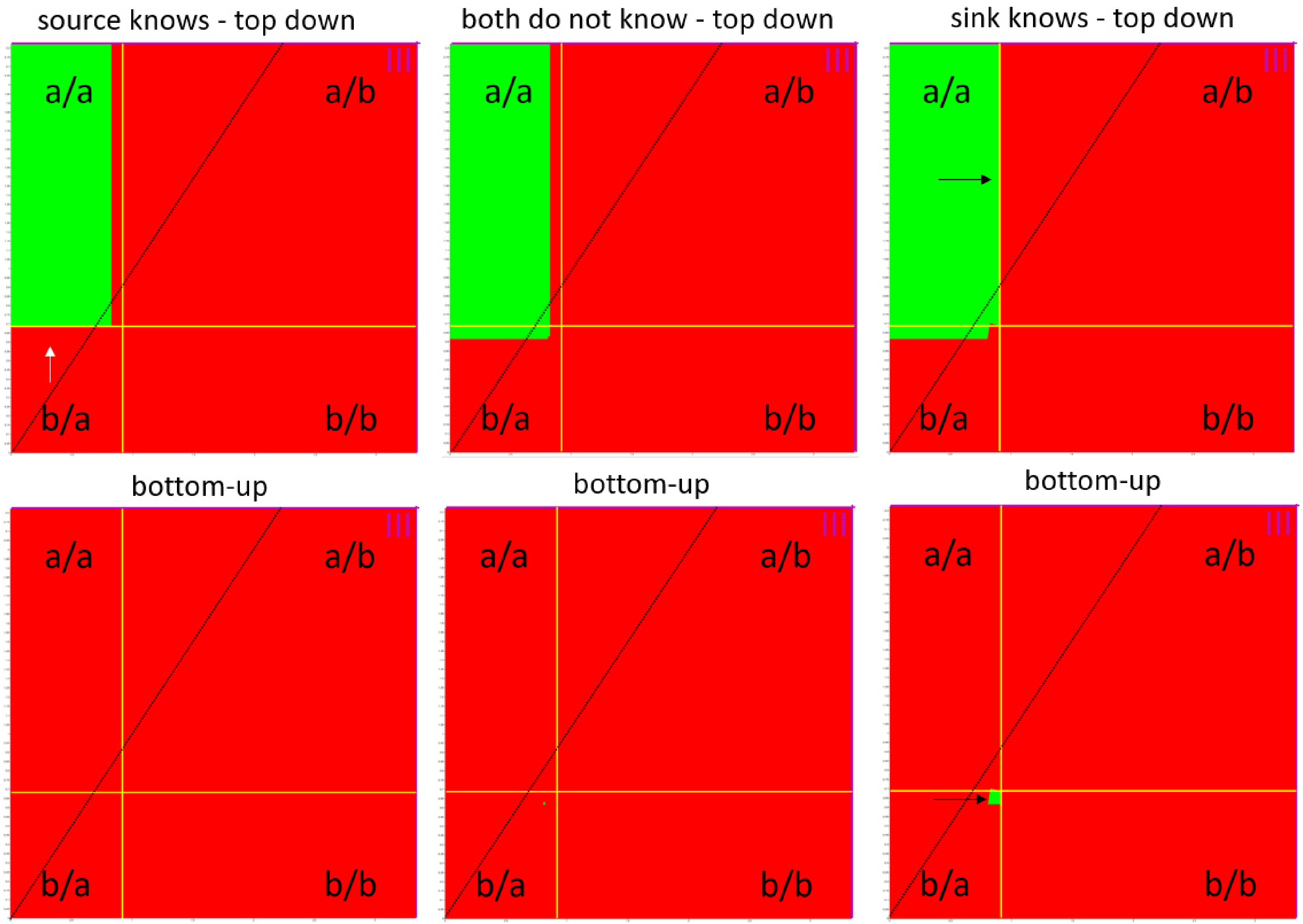
bottom-up
a/a
a/b
b/a
b/b

Figure 27



source knows - top down
both do not know - top down
sink knows - top down
bottom-up
bottom-up
bottom-up
a/a
a/b
b/a
b/b

Figure 25

Source and sink separately try to maximize their individual net profit (yellow lines). A weak ensemble is observed and the value (amount of substrate) within the coin is overestimated. On the left side (top-down and bottom-up) source knows that the coin contains less value. Source acts accordingly within its ensemble. Source will only go up to its limit np=max. The ensemble loses some superadditivity in subarea b/a (white arrow) in comparison to the ignorant ensemble. In the middle a completely ignorant ensemble (top-down and bottom-up) is observed, see figure 13. On the right side (top-down and bottom-up) sink knows that the coin contains less value. Sink acts accordingly. Sink goes to its limit np=max and gains superadditivity in area a/a in comparison to the completely ignorant ensemble but for the price of increased subadditivity in area b/a (black arrows). The black dotted line is the line of equal concentration in source and sink (compare figure 3b).

Figure 26

Source and sink separately try to maximize their individual net profit (yellow lines). A symmetric ensemble is observed and the value (amount of substrate) within the coin is overestimated. On the left side (top-down and bottom-up) source knows that the coin contains less value. Source acts accordingly within its ensemble. Source will only go up to its limit np=max. The ensemble loses some superadditivity in subarea b/a (white arrow) in comparison to the ignorant ensemble. In the middle a completely ignorant ensemble (top-down and bottom-up) is observed, see figure 14. On the right side (top-down and bottom-up) sink knows that the coin contains less value. Sink acts accordingly. Sink goes to its limit np=max and gains superadditivity in area a/a in comparison to the completely ignorant ensemble but for the price of some new subadditivity in area b/a (black arrows). The black dotted line is the line of equal concentration in source and sink (compare figure 3b).

Figure 27

Source and sink separately try to maximize their individual net profit (yellow lines). A strong ensemble is observed and the value (amount of substrate) within the coin is overestimated. On the left side (top-down and bottom-up) source knows that the coin contains less value. Source acts accordingly within its ensemble. Source will only go up to its limit np=max. The ensemble loses some superadditivity in subarea b/a (white arrow) in comparison to the ignorant ensemble. In the middle a completely ignorant ensemble (top-down and bottom-up) is observed, see figure 15. On the right side (top-down and bottom-up) sink knows that the coin contains less value. Sink acts accordingly. Sink goes to its limit np=max and gains superadditivity in area a/a in comparison to the completely ignorant ensemble but for the price of increased subadditivity in area b/a and in subarea a/a in comparison to the completely ignorant ensemble (black arrows). The black dotted line is the line of equal concentration in source and sink (compare figure 3b).

The previous qualitative investigations aimed to visually demonstrate how the interaction between knowledge and ignorance of source and sink leads to various combinations of increased or decreased superadditivity and subadditivity. This applies to weak, symmetric, and strong ensembles with respect to the overestimation and underestimation of value, providing a foundation for the subsequent quantitative results to be understood by the reader. Since area II and area III are not symmetrically balanced in terms of superadditivity and subadditivity (refer to 3, figures 12 to 15, and 4, figure 14), the emergence and reduction of these properties in various regions result in a complex interplay of behaviour-dependent outcomes.

Underestimation of value (deflation): The general observation is that underestimating the value will create additional superadditivity (symmetric and strong ensembles) but also subadditivity (weak ensemble) in area II or subarea a/b if both parties do not know that the value is underestimated. However, if the sink knows this, it will stop at its true boundary and prevent this. This is harmful in strong and symmetric ensembles and good in weak ensembles. On the other hand, underestimating the value has the effect of losing superadditivity (symmetric and strong ensemble) and superadditivity and subadditivity (weak ensemble) in area I next to area III and subarea a/a next to subarea b/a. If source knows that the value is underestimated source will actively approach its true limit and gain additional superadditivity in symmetric and strong ensembles as result. Underestimation of value has additional effects on subadditivity in subarea a/b and b/a. When both, source and sink, are simultaneously near their target (np=max) subadditivity for the ensemble may appear. Even symmetric ensembles are internally asymmetric, since the linear component dominates in region II, while the nonlinear component dominates in region III. Transfers beyond the line of mixing (equal concentrations) are usually not a good idea. The strong ensemble is a bit more forgiving here.

Overestimation of value (inflation): In overestimation of value additional superadditivity (symmetric, weak and strong ensembles) but also subadditivity (weak ensemble) is created in area III or subarea b/a if both parties do not know that there is overestimation of value. However, if the source knows this, it stops at its true limit and this selfish behaviour prevents the gain. On the other side, in area I (border to area II) and subarea a/a (border to subarea a/b), overestimation has the effect that superadditivity (symmetric and strong ensemble) and subadditivity (weak ensemble) are lost. If sink is aware of an overestimation, it will actively approach its true limit and counteract this effect and gain superadditivity but also subadditivity (weak ensemble). Overestimation has additional effects on subadditivity in subarea a/b and b/a. When both, source and sink, are simultaneously near their target (np=max) subadditivity for the ensemble may appear. Again, symmetric ensembles are internally asymmetric as in area II the linear component is dominating while in area III the non-linear component dominates.

The qualitative observations show that when both parties know that there is over- or underestimation of value, knowledge-based activity can prevent a loss of super- and subadditivity as well as simultaneously increase super- and subadditivity in other areas in comparison to complete ignorance. Interestingly, the same is true for partial ignorance. Ignorance of source or sink will not prevent a loss of super- and subadditivity or their ignorance may increase super- and subadditivity in other areas. To assess whether a party's knowledge or ignorance improves or harms the outcome for the ensemble, a quantitative measure is needed.

Quantitative determination of knowledge-based or ignorance-based superadditivity at different degrees of over- or underestimation.

In the following part I distribute a constant amount of substrate (10mmol) to 10000coins (exact estimation) or 8000coins (leading to underestimation of value by 20%, deflation) or 12000coins (leading to overestimation of value by 20%, inflation) by steps of 1%. The knowledge of the parties will vary from “nobody knows (nok)” to “source knows (sok)”, “sink knows (sik)”, and finally “both know (bok)”. Under the condition “sik” source is ignorant, under the condition “sok” sink is ignorant. The condition sik and sok model an asymmetry of information within an ensemble. The amount of superadditivity and subadditivity is determined either as line integral. These are only the largest transfers, the concentration in sink is zero. Or the volume integral. Here transfers of all possible concentration pairs determine the volume between the active surface and the inactive surface. The resulting values are calculated by subtracting subadditivity - if present - from superadditivity. The substrate contained in a coin in underestimation and in overestimation is calculated from 1/(1±%/100).

Similar to my previous report (2) I look at first on the line integral with the unit np*mM (total superadditivity of the ensemble) for the largest transfers from source to sink. Sink, in this case, does not possess any value ahead of the transfer (concentration pairs where sink is at 0mM). A complete value transfer to the limit (b=c and np=max; real or assumed) is reached (figure 26 for the transfer space and figure 27 for the positive net profit subspace). Then I look at the volume integral when all transfer sizes from source to sink up to the limits (b=c, np=max; real or assumed) are calculated. The unit here is np*mM^2 (total superadditivity for the ensemble). In sink different amounts of value besides 0mM are possible (figure 28 for the transfer space and figure 29 for the positive net profit subspace).

Figure 26

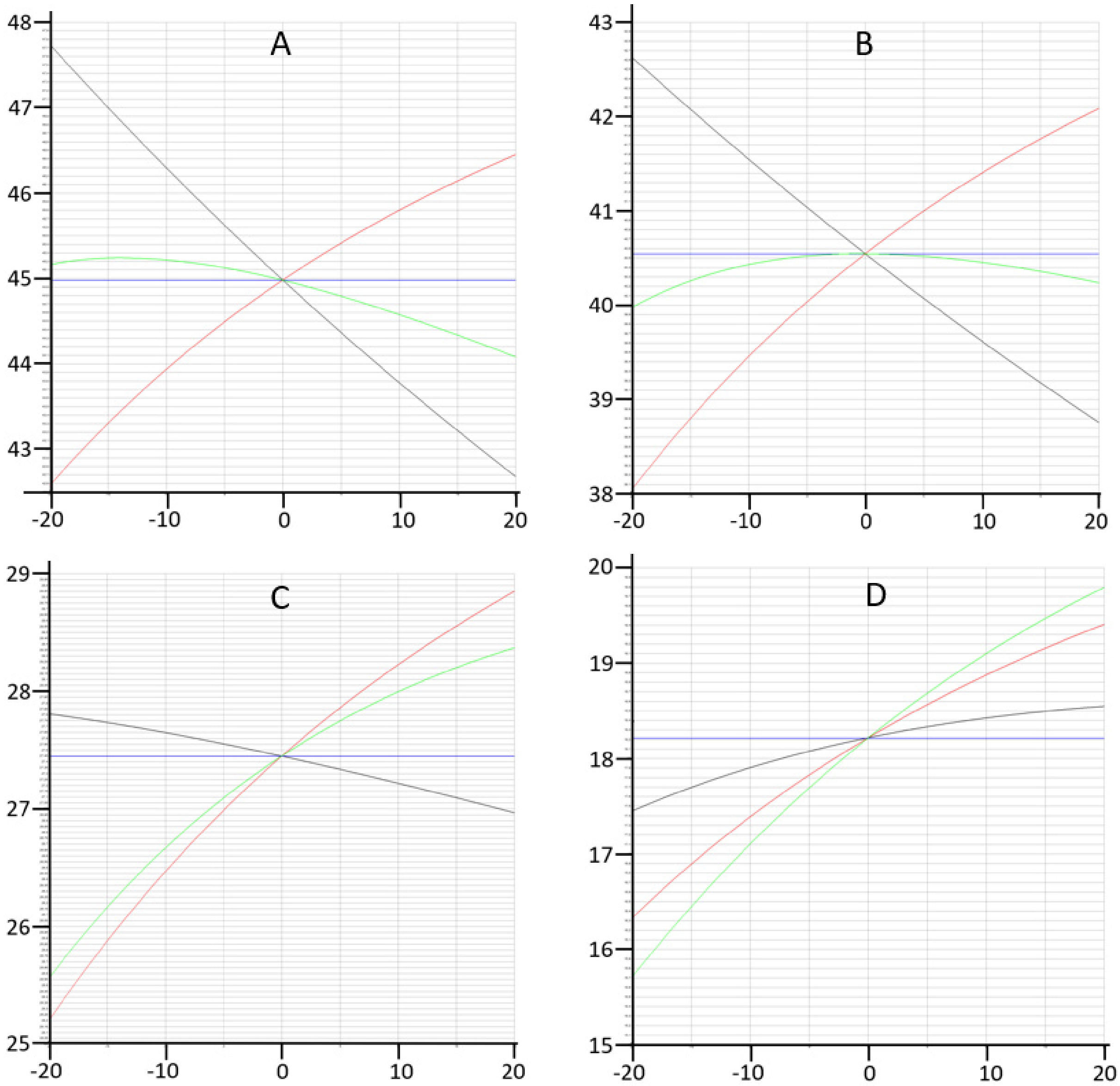


Figure 26

Transfer space: The line integrals (y-axis, np*mM) for 20% deflation (-20, volume decrease, concentration of value or substrate) to 20% inflation (+20, volume increase, dilution of value or substrate) (x-axis). A, strong ensemble (b=c: source 1.8mM, sink 3mM); B, strong ensemble (b=c: source 2mM, sink 3mM); C, symmetric ensemble (b=c: source 2.5mM, sink 2.5mM); D, weak ensemble (b=c: source 3mM, sink 2mM)

bok = blue, sok = black, sik = red, nok = green

Figure 27

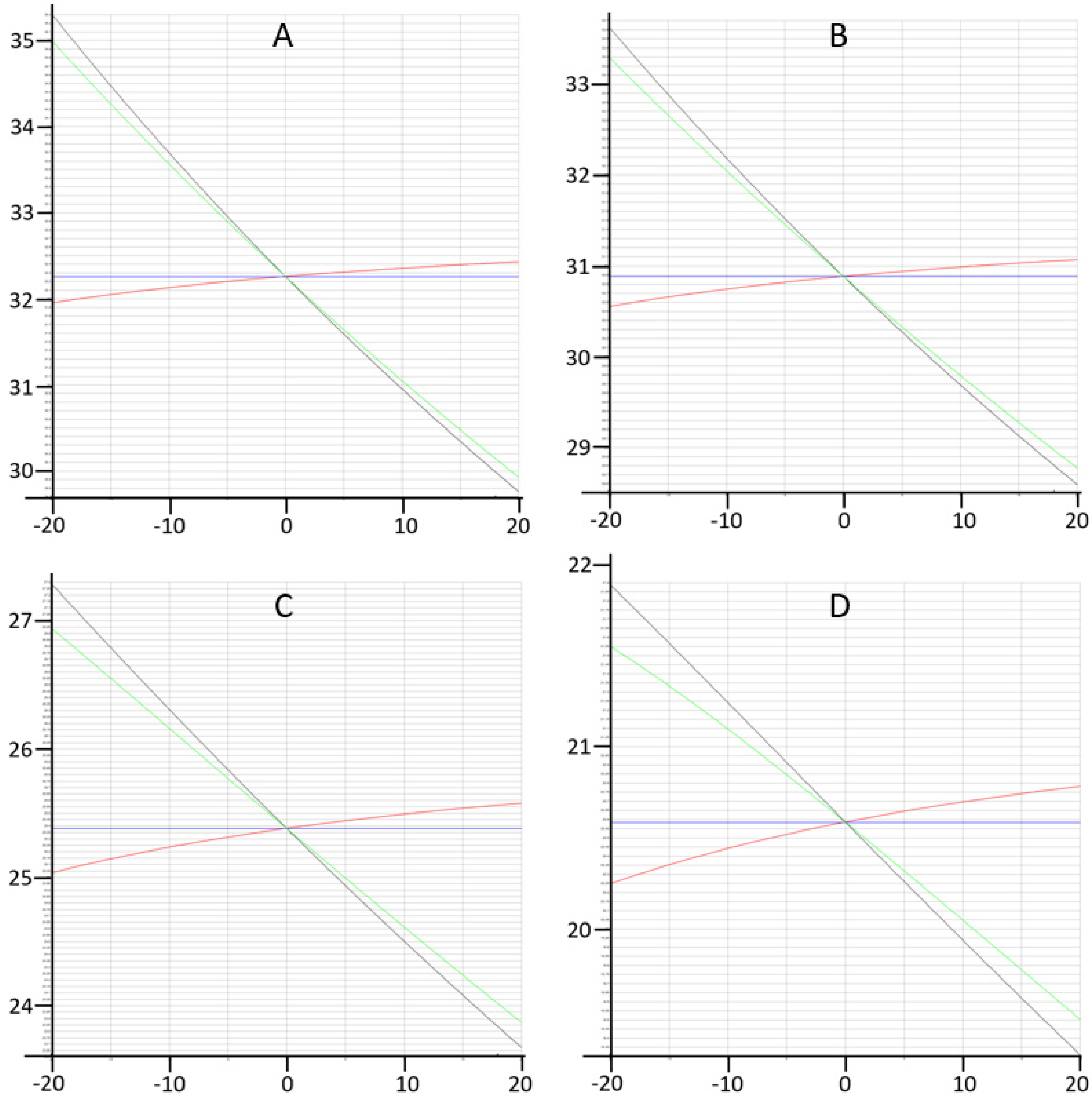


Figure 27

Positive net profit subspace: The line integrals (y-axis, np*mM) for 20% deflation (-20, volume decrease, concentration of value or substrate) to 20% inflation (+20, volume increase, dilution of value or substrate) (x-axis). A, strong ensemble (b=c: source 1.8mM, sink 3mM); B, strong ensemble (b=c: source 2mM, sink 3mM); C, symmetric ensemble (b=c: source 2.5mM, sink 2.5mM); D, weak ensemble (b=c: source 3mM, sink 2mM)

bok = blue, sok = black, sik = red, nok = green

Figure 28

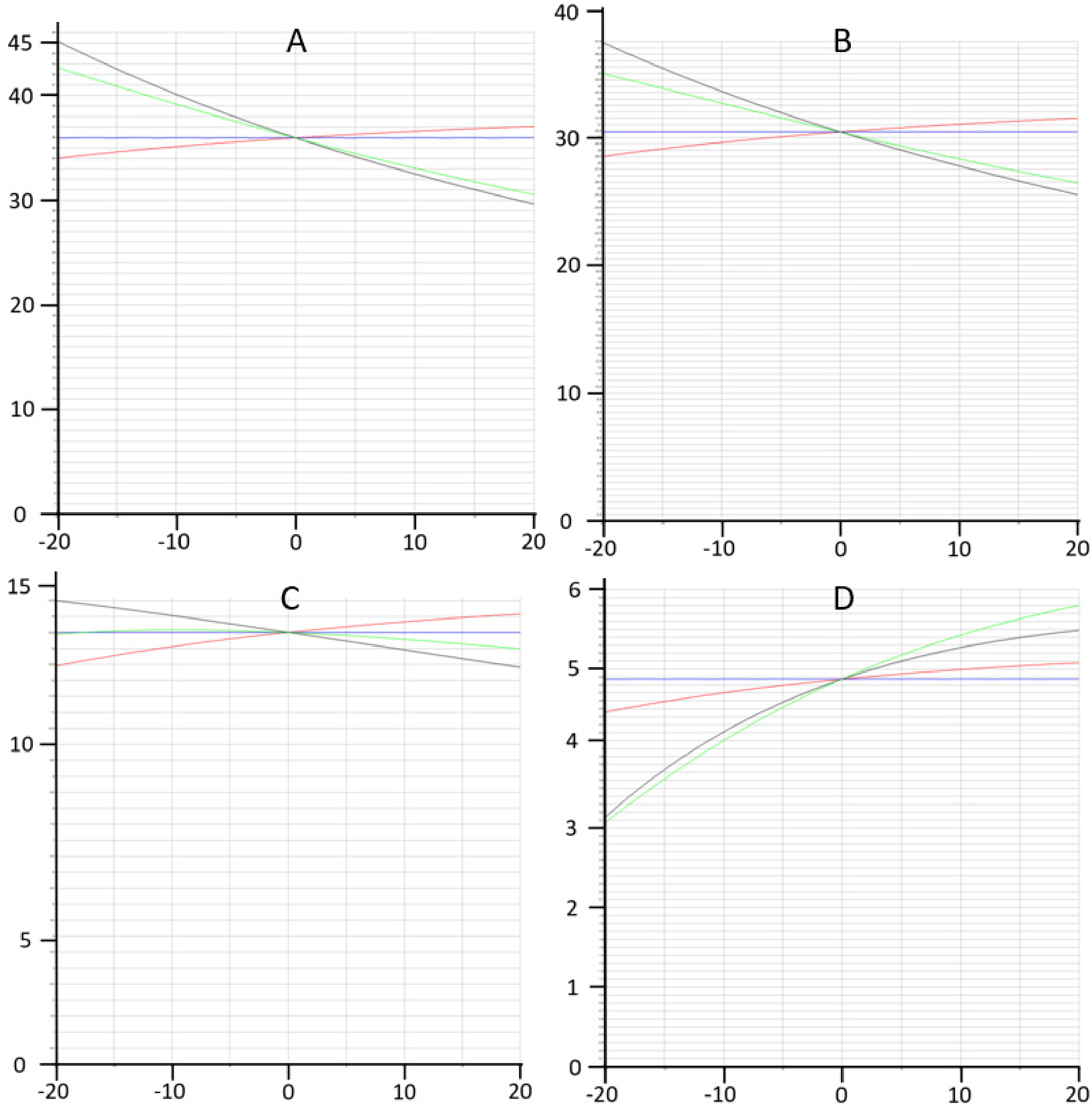


Figure 28

Transfer space: The volume integrals (y-axis, np*mM^2) for 20% deflation (-20) to 20% inflation (+20) (x-axis). A, strong ensemble (b=c: source 1.8mM, sink 3mM); B, strong ensemble (b=c: source 2mM, sink 3mM); C, symmetric ensemble (b=c: source 2.5mM, sink 2.5mM); D, weak ensemble (b=c: source 3mM, sink 2mM)

bok = blue, sok = black, sik = red, nok = green

Figure 29

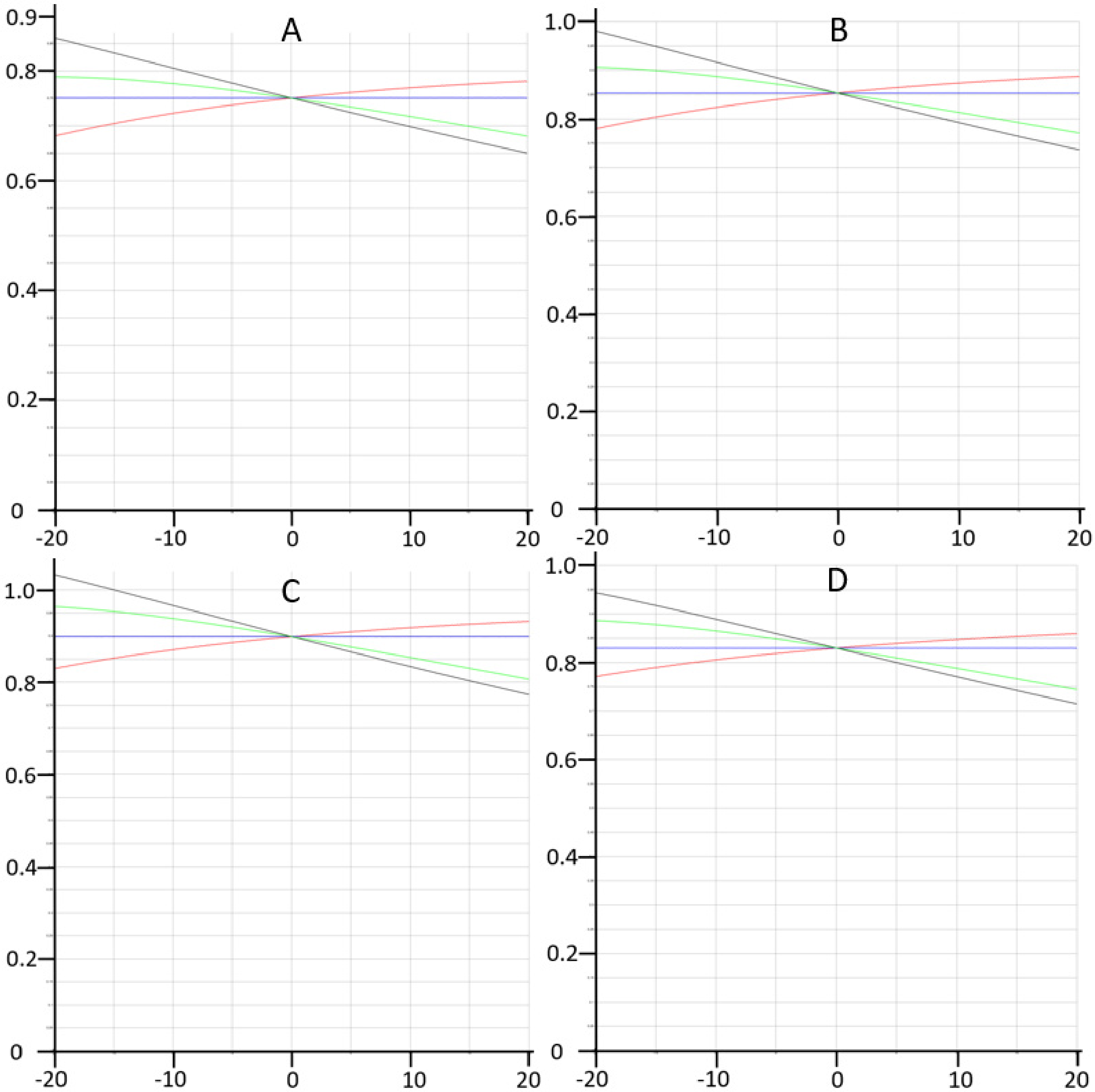


Figure 29

Positive net profit subspace: The volume integrals (y-axis, np*mM^2) for 20% deflation (-20) to 20% inflation (+20) (x-axis). A, strong ensemble (b=c: source 1.8mM, sink 3mM); B, strong ensemble (b=c: source 2mM, sink 3mM); C, symmetric ensemble (b=c: source 2.5mM, sink 2.5mM); D, weak ensemble (b=c: source 3mM, sink 2mM)

bok = blue, sok = black, sik = red, nok = green

The magnitude of superadditivity depends strongly on the symmetry type. To a lesser extent superadditivity is also influenced by the distribution of knowledge within the window of observation. This smaller influence is absent with full information (figures 26 to 29, both know - bok, blue). Here, both parties always rely on the actual value (substrate concentration) and not on the coin count. Therefore, I consider full information as a benchmark. The reason why the amount of superadditivity (np*mM; np*mM^2) varies greatly between symmetry types is already clear: different cost factors. They are responsible for the asymmetry or symmetry. Km, Vmax, and bf are not used here to adjust asymmetry. Now I try to find a general pattern for the factor with the smaller influence on superadditivity: knowledge or the lack of it.

A clear pattern emerges: knowledge that excels when a value is underestimated performs poorly when the value is overestimated, and *vice versa*. This is evident from 20% deflation to 20% inflation. When value and number of coins are equal, knowledge becomes irrelevant. In this scenario, ignorance is advantageous as obtaining information incurs a cost. Here, ignorance truly is strength for the first time.

The transfer space, line integrals (largest transfers, figure 26)

Underestimation: In the strongest ensemble (A) ignorance in sink (source knows, sok) and, to a lesser extent, ignorance in both parties (nok) create more superadditivity than complete information (bok). In the less strong ensemble B only ignorance in sink (sok) creates more superadditivity than complete information. In the symmetric ensemble (C) the amount of superadditivity through ignorance in sink (sok) is only slightly better than complete information (bok). Finally, in the weak ensemble (D) complete information (bok) dominates with respect to superadditivity.

Overestimation: In strong ensembles (A, B) ignorance in source (sink knows, sik) is increasing superadditivity in comparison to the standard (bok). In symmetric ensembles (C) complete ignorance (nok) becomes better than complete information (bok) but is still less effective than ignorance in source (sik). Finally, in the presented weak ensembles (D) the highest superadditivity is a result when both parties are ignorant (nok) followed by source is ignorant (sik) and then sink is ignorant (sok). Nok, sik, and sok all create more superadditivity than complete information (bok). Ignorance is strength.

The positive net profit subspace, line integrals (largest transfers, figure 27)

Underestimation: The ranking here is simple and general. Ignorance in sink (sok) dominates always (A, B, C, and D). The next type of successful ignorance is complete ignorance (nok). In deflation only ignorance in source (sik) is always worse than complete information (bok).

Overestimation: The observation here is very simple, too. In all symmetry types (A, B, C, and D) only ignorance in source (sik) creates more superadditivity than complete information.

The transfer space, volume integrals (all transfers, figure 28)

Underestimation: In the strong ensembles (A, B) ignorance in sink (sok) and, to a lesser extent, ignorance in both parties (nok) create more superadditivity than complete information (bok). In the symmetric ensemble (C) ignorance in both parties (nok) dominates complete information (bok) only at low to medium deflation. Finally, in the weak ensemble (D) complete information (bok) dominates with respect to superadditivity. Surprisingly, here, sok and nok are worse than sik.

Overestimation: In strong ensembles and symmetric ensembles (A, B, and C) ignorance in source (sink knows, sik) is increasing superadditivity a little in comparison to the standard (bok). In the presented weak ensemble (D) the highest superadditivity is a result of ignorance in both parties (nok) followed by ignorance in sink (sok) and finally ignorance in source (sik); this differs compared to figure 26. Nok, sok, and sik all create more superadditivity than complete information (bok). Ignorance is strength.

The positive net profit subspace volume integrals (all transfers, figure 29)

Underestimation: Similar to figure 27 ignorance in sink (sok) dominates always (A, B, C, and D) followed by complete ignorance (nok). Again, ignorance in source (sik) is worse than complete information.

Overestimation: Here too, the result is similar to figure 27. In all symmetry types (A, B, C, and D) only ignorance in source (sik) creates more superadditivity than complete information; nok and sok always stay below bok.

What type of knowledge or ignorance has the upper hand in all possible symmetric and asymmetric ensembles in deflation or inflation?

In figures 26 to 29, I observe four different ensembles: two strong, one symmetric, and one weak, each corresponding to different degrees of value under- or overestimation (0 to 20%). However, within the transfer space and the positive net profit subspace, many more symmetric and significantly more asymmetric ensembles exist. The question is whether there is a coherent pattern that encompasses all possibilities of symmetry and asymmetry caused by varying cost factors. Previously (2), I observed the impact of total ignorance in undetected deflation and inflation compared to no deflation or inflation. This analysis concerned the line integral (representing the largest transfers) of all cost-induced symmetric and asymmetric ensembles (repeated in figure 30, A, top).

Figure 30

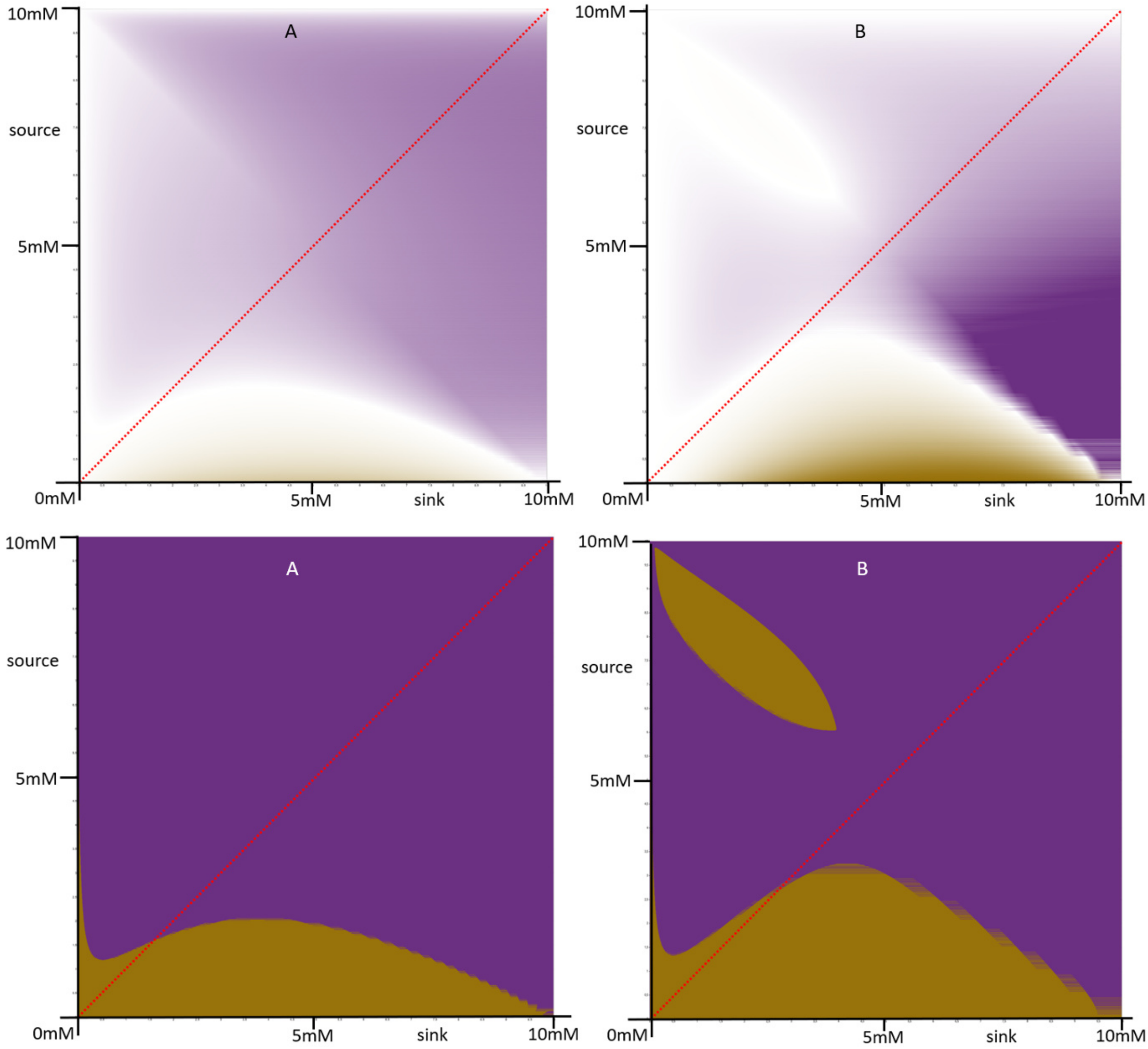


Figure 30

This is an overview of all possible symmetric (red dotted line) and asymmetric ensembles. Line integrals (A, largest transfers; 2) are shown on the left and volume integrals (B, all transfers) on the right. Both axes give the concentration where b=c. The grid is 1µM in A and 5µM in B. The purple region has a positive slope; undetected inflation is better than undetected deflation. In the gold regions, the slope is negative. Undetected deflation is better than undetected inflation (compare figures 26 to 29, green). The colour intensity of the upper images encodes the steepness.

A minor adjustment in inflation or deflation, *i.e.* close to the true value, affects superadditivity. The steepness of this change is shown in figure 30 (top row) by the colour intensity. Purple indicates a positive slope (undetected small inflation is superior to no inflation, as shown in the green curves of figure 26 C and D), while gold indicates a negative slope (undetected small deflation is superior to no deflation, as shown in the green curves of figure 26 A). In the right part of figure 30 (B) the volume integral of all symmetry types is determined. A new pattern emerges, where deflation outperforms inflation in weak ensembles, as all transfer sizes are now considered. The reason is that in weak ensembles, smaller transfers in area I and transfers into area II are subadditive.

In the following figures, I focus exclusively on the volume integral at 10% deflation or inflation of a collection of symmetry types, where b=c in source and sink is spaced at 5µM intervals (2, figure 12 there). In figure 31 the superadditivity at 10% deflation is observed in the positive net profit space (left) and the transfer space (right). Because the superadditivity is different in each of the symmetry types represented by a single point, I only observe which type of knowledge or ignorance is dominating in this point.

Figure 31

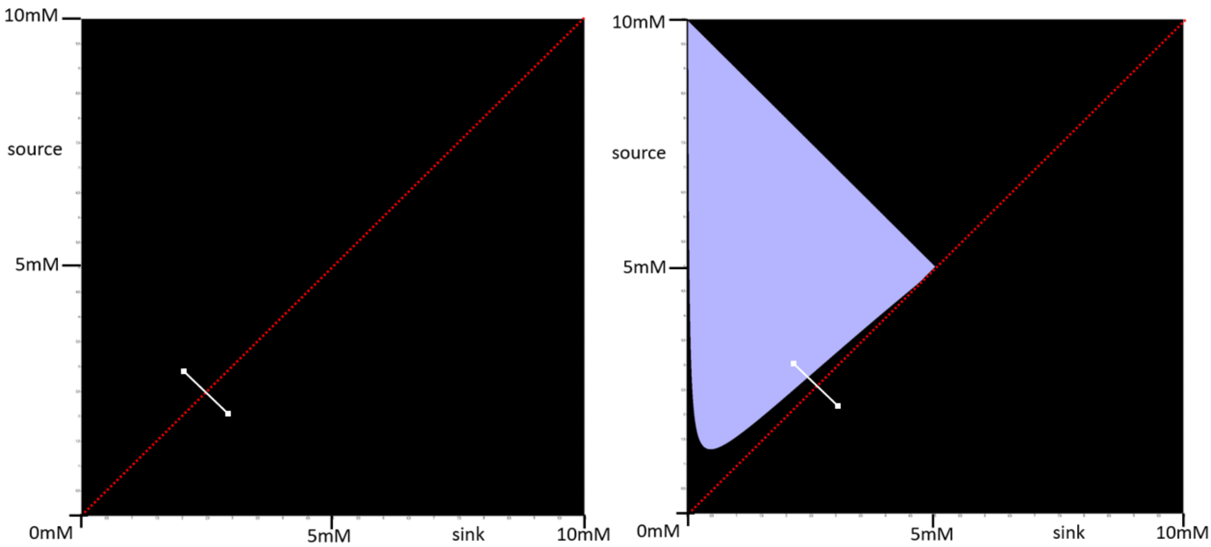


Figure 31

On the left side positive net profit subspaces and on the right side transfer spaces are observed according to their total superadditivity. It is crucial to understand that the axes (mM) do not represent actual concentrations but rather the concentration where b=c in both parties and spaces. At 10% deflation I observe what kind of information asymmetry has a dominating superadditivity. The colour indicates the dominating knowledge (bok = blue, sok = black). The red dotted line is the line of all symmetric ensembles separating weak ensembles (above) from strong ensembles (below).

In the context of a 10% deflation, positive net profit subspaces are dominated by scenarios where “source knows (sok)”. This condition implies that sink remains unaware of the true (high) value. Here, the internal information asymmetry outperforms both complete information and complete ignorance. Among all observed transfer spaces an island appears where in some weak ensembles the best outcome is when “both know (bok)”. In all other symmetry types “source knows (sok)” dominates.

In figure 32, symmetric and asymmetric ensembles are observed as positive net profit subspaces (left) and as transfer spaces (right). When value is overestimated by 10%, I compare which knowledge distribution results in dominating superadditivity. The spacing between b=c in source and sink of all symmetry types is 5µM (2, figure 12).

Figure 32

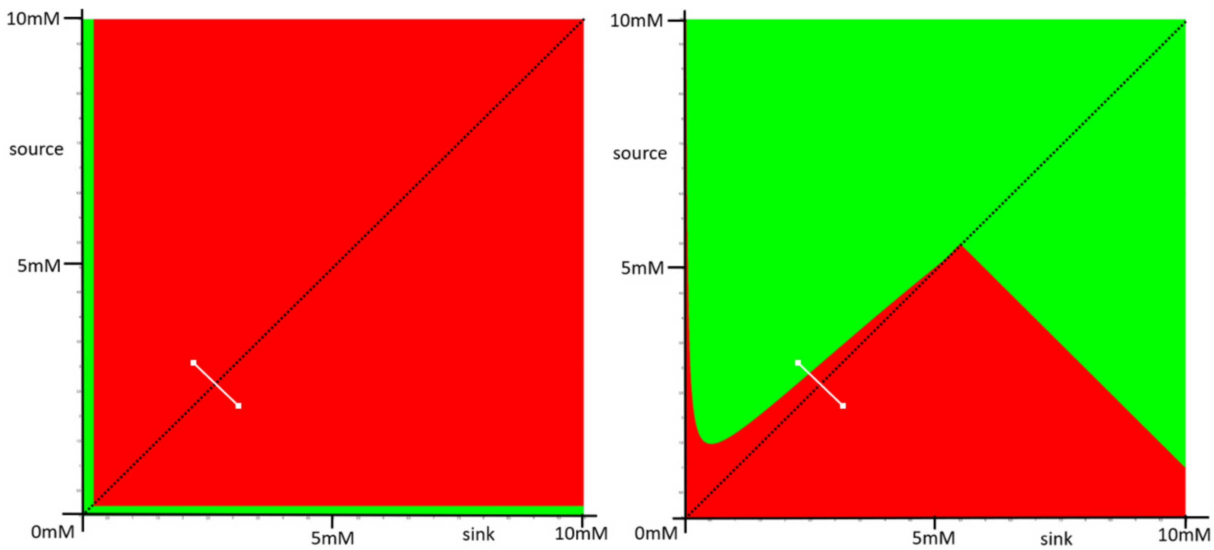


Figure 32

On the left side different symmetry types of positive net profit subspaces and on the right side different symmetry types of transfer spaces are observed. The axes (mM) do not represent actual concentrations but rather the concentration where b=c in both parties and spaces. At 10% inflation I observe what kind of information asymmetry is dominating in the different symmetry types. The visible colour gives the dominating knowledge (sik = red, nok = green). The black dotted line is the line of all symmetric ensembles; weak ensembles above, strong ensembles below that line.

Different symmetry types of the positive net profit subspace in overestimation (10% inflation) are dominated by “sink knows (sik)”. This is equivalent to the condition source does not know about overestimation. In very asymmetric weak and strong ensemble types (very high cost) within the positive net profit subspace the largest superadditivity is obtained when nobody knows. In most of the symmetry types of the transfer space “nobody knows (nok)” performs best. However, in many strong asymmetric ensembles with moderate to high cost in source and a few weak asymmetric ensembles “sink knows (sik)” dominates.

The white lines in figures 31 and 32 mark the path from B to D in figures 26 to 29 and integrate these four figures into the larger picture.

Discussion

The result of the investigation can be summarized as follows: The lack of knowledge about the underestimation or overestimation of the value of a coin activates the ensemble in areas outside of area I or subarea a/a and simultaneously inhibits activity within area I or subarea a/a. Limiting this knowledge to either source or sink can weaken or strengthen this effect. The best result is obtained when additional strength (superadditivity outside of area I or subarea a/a) is combined with inhibition of weakness (loss of superadditivity or gain of subadditivity) within area I or subarea a/a. The effect depends also on the symmetry of the ensemble. Increased superadditivity by activation of the ensemble outside of area I has been observed with the application of brute force and deception by internal and external masters (5). Violence and deception can be compared with knowledge of under- and overestimation. When an internal master (source or sink) applies force and especially successful deception, the outcome is basically identical to under- or overestimation of value when either source or sink know that. One sided ignorance with respect to the true value seems to be a special case of deception (figure 33).

Figure 33

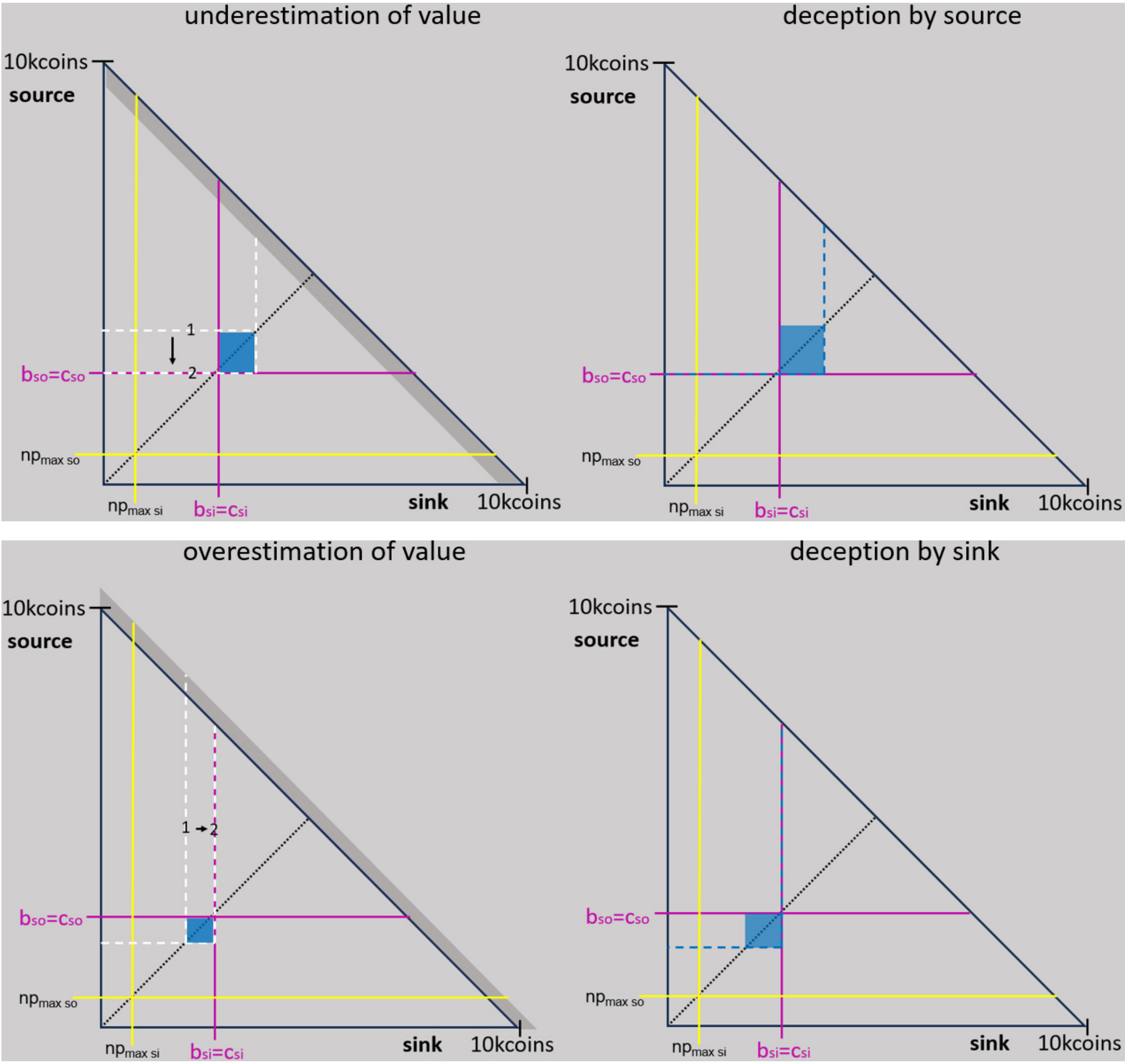


Figure 33

In the top row on the left the value is underestimated by both parties (white dashed line 1) in a symmetric ensemble. As soon as source knows this (white dashed line 2) the situation is indistinguishable from deception or force through source including the location of subadditivity (right, blue dashed line; blue squares indicate subadditive results, see figure 17; compare 5, figure 10b therein).

In the bottom row on the left the value is overestimated by both parties (white dashed line 1) in a symmetric ensemble. As soon as sink knows this (white dashed line 2) the situation is indistinguishable from deception or force through sink including the location of subadditivity (right, blue dashed line; blue squares indicate subadditive results, see figure 20; compare 5, figure 10b therein).

In most of my past considerations I assumed that the external master would use force or deception against source and sink equally. However, it is also possible that an external master has a preference for a single side in an ensemble. This may create a situation similar to one sided information within the ensemble.

Real-life interactions are inherently complex, involving symmetric or asymmetric ensembles of a source, a sink, and a master (internal or external). These entities continuously engage in permutations of varying degrees of force and counterforce, deception, ignorance, and knowledge about real-world changes. This intricate dance within the transfer space and the positive net profit subspace leads to an endless nonlinear array of superadditivity and subadditivity levels, ultimately determining the fate of the ensemble and its components.

Cantillon effect

This idea has been first described by Richard Cantillon in his assay "Essai sur la nature du commerce en general", 1755). Cantillon argues that an increase in money supply by banks (credits) slowly moves through the sectors of an economy. Sectors with the first access to the new money benefit because they purchase goods at still low prices. The rest of the sectors follow later or do not benefit at all in case of consumers. They are not able to benefit while they have to pay the now increased prices.

When money is injected into an economy at a certain location, this location becomes the source of the money in the process of exchanging money for goods. Source knows that there is now more money, *i.e.* less value per coin. This dilution of value (inflation by money supply) displaces source and sink from the equilibrium (b=c or np=max). However, the knowledge of source compensates that and source is able to optimize benefit and cost or maximize net profit. In figure 32 my model suggests that for half of the symmetric and strong ensembles of the transfer space it would be important for sink to know that inflation has occurred to avoid harm to the ensemble (source should not know - an impossible demand). In most of the weak and half of the strong ensembles, inflation should be unknown to both parties to obtain the maximal superadditivity. Within the positive net profit subspace sink always should know about inflation with the exception of very asymmetric ensembles with very high cost in source or very high cost in sink.

Final considerations

In George Orwell's dystopian novel "1984," the slogans of the "Ingsoc" ideology contain the statements: "war is peace", "freedom is slavery", and "ignorance is strength." While "war is peace" and "freedom is slavery" pair opposites to suggest their equivalence, the phrase "ignorance is strength" stands out. If this phrase would be of a similar structure as the two proceeding phrases "ignorance is knowledge" would be a choice. However, the author seems to imply that the two antithetic pairs ignorance/knowledge and weakness/strength are connected. I am convinced that on an individual level ignorance is weakness. However, in the novel the ignorance of the ruled masses according to the true power relation is an important factor of the strength of the elite and their dominating position. This is probable Orwell´s idea here, as external war means internal peace for the elite and slavery of the ruled means freedom to the elite.

My model introduces an economic interpretation. Ignorance is the easiest way to get the members of an ensemble to give a benefit-dominated "thing" or take a cost-dominated "thing" undetected. Other, but costly, alternatives with the same effect are active deception or brute force. All three - ignorance, deception, and force - create additional superadditivity in certain areas of the transfer space and the positive net profit subspace for the master of the ensemble. This is paid for by the emergent cost subadditivity and additional investments. However, ignorance is the cheapest way to achieve this. In my interpretation “ignorance is strength” deals with economic exploitation. Ignorance of the ensemble results in exploitable economic strength and will be harvested by the elite; the master of the ensemble. On the level of an ensemble ignorance of single parties is the strength of the ensemble as a whole. However, this is only true when the supposed value of the coin is not in accordance with the true value of the content! Knowledge here refers to complete information of the true value.

But there is a further idea to consider. The lack of knowledge is a spectrum. This spectrum ranges from unawareness to ignorance. Ignorance implies a wilful or deliberate lack of knowledge. It suggests that a person chooses not to know or acknowledge certain information, even when they have the opportunity to learn it. Ignorance carries a negative connotation of intentional neglect or refusal to learn. Ignorance in this sense can be seen as an active process. Unawareness refers to a more passive state of not knowing. It implies a simple lack of knowledge without the connotation of wilfulness or deliberate avoidance. It is more about being uninformed or unaware. Therefore, George Orwell may also imply that strength through ignorance comes from an active process, a process the protagonist has to painfully learn. Kant speaks of “laziness and cowardice” as the main reasons why many people remain in their immaturity. The process of breaking free from dependence and thinking independently can therefore be painful and unpleasant (6), as it is associated with insecurities and the need to take responsibility for one’s own thoughts and actions. The reversal of Enlightenment through coercive indoctrination with brute force (and drugs, *i.e.* gin) that we have to observe in Orwell's novel is truly more painful than the process of Enlightenment itself. The reversal of Enlightenment in Orwell's novel can only be interpreted as irrational and institutionalized sadism.

My model encompasses both states of not knowing: unawareness and ignorance. However, ignorance as an active process would introduce additional cost but probably less than deception. The cost of deception falls on the deceiving party while ignorance is a cost to the ignorant party (self-deception).

The idea that ignorance as an active process can have a purpose has been investigated and discussed and forms the concept of deliberate ignorance (7). In my model such a purpose is higher superadditivity for the ensemble. However, in light of the semantic discussion above "deliberate ignorance" could be considered a tautology.

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