Trade Variations Due to Distance and Delaying Costs across Time Zones

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Abstract

This paper constructs a competitive trade model involving two nations, with distinct time zone locations. Our results suggest that geographical distance could positively impact service trade, in contrast to its harmful nature for goods trade. These results partly disagree with the gravity arguments of international trade. Our model also reveals an intriguing relationship: an increase in distance between trading nations results in higher skilled labour wages and diminished capital rent for service products, while goods trade experienced the opposite effect. We then connect distance with delaying cost and find that an escalation in delaying cost led to a decline in skilled labour wages and an increase in rent. We further extend our basic model to introduce two additional sectors—informal and Government manufacturing—along with unorganized labour and land as extra production factors. Despite these additions, the consistency in the effects on factor prices and output persists.

JEL Classification: D24, E26, F1, J3, J31, O14

Key words: Trade; Time Zone; Factor prices; Output Changes; Informality: Manufacturing Sector

1. Introduction

We start the essay by acknowledging the fact that various costs are involved in international trade when countries are located at different geographical distances. These costs may include transaction costs, transportation costs, communication costs, delaying costs etc., all linked to the geographical separation of countries. Therefore, our objective is to investigate the relationship between geographical distance and associated delaying costs with trade. To strengthen our study, we introduce another essential factor—time zones (TZ). Consequently, we examine the impact of Time Zone related distance and delaying cost on factor prices and output. To achieve this, we systematically introduce four sectors. The first is the service sector, the second is the goods sector, the third is the informal sector, and the fourth is the government manufacturing sector which is included in the extended model.

In traditional trade theory, the gravity model of international trade, introduced by Walter Isard (1954), predicts bilateral trade flows based on the economic sizes and distance between two countries. This model says that when the distance between two trading countries increases then the trading cost should increase. Some notable references in this line are Samuelson (1954), Falvey (1976), Deardorff (2014), Laussel and Riezman (2008), and Marjit and Mandal (2012). Conventional literature, such as Benedictis & Taglioni (2011), Gómez-Herrera (2013), Melitz (2007), Taglioni & Baldwin (2014), and Rudolph (2009) often posit that geographical distance is detrimental to trade, supported by both theoretical and empirical evidence. However, recent studies like Mandal (2015), Marjit, Mandal, Nakanishi (2020), and Mandal and Prasad (2021) challenge this notion, particularly in the context of service trade. Unlike goods trade, service trade may benefit from non-overlapping time zones, as highlighted by Marjit (2007), Kikuchi (2009), Kikuchi and Marjit (2011) and Kikuchi et al. (2013) etc.

This paper investigates whether distance and associated delaying costs affect goods and service trade differently or symmetrically, exploring the impact on factor prices, output changes, and the structural composition of the economy. Building on Marjit's (2007) argument, the study considers time zone differences as a factor influencing distance-related costs. The interesting implication of such trade is that unlike goods trade, service trade, reliant on the internet and not physical shipments, may not incur increased trading costs with greater distance. In this backdrop our paper aims to provide insights into the effects of distance on goods and service trade, questioning the applicability of standard gravity model results in this context.

Here we formulate a model, which deals with the distance and effects of distance on factor prices and output in the presence of time zone differences. In this context we consider two types of trade; one is trade in the goods sector and another is trade in the service sector. Further, we will see the effects simultaneously. Then we move to the effects of delaying costs on the trading nations. We further examine the effect of distance on trade in a stylized economy consisting of both government and unorganised or informal sector. Therefore, we essentially try to establish a relationship between distance, and delaying cost in the presence of time zones based on their effects on trade.

Before delving into model development, it is prudent if we examine empirical data provided by the OECD as real-life examples from the years 1995 to 2020, which reveals intriguing trends. When analysing India's trade dynamics with nearby Asian nations, encompassing both the goods trade ratio and service trade ratio, it becomes evident that both the goods trade ratio and service trade ratio have remained relatively static, goods trade ratio is fluctuating between 0.60 to 0.71 while the service trade ratio is fluctuating between 0.28 to 0.39 over time (shown in the figure 1).

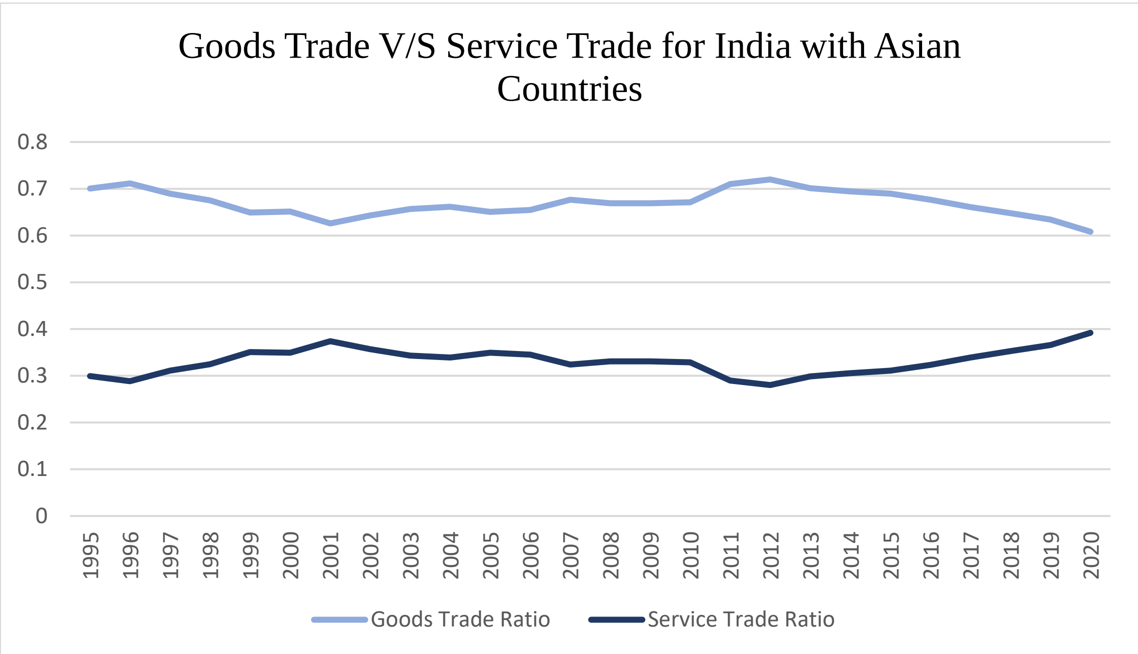

Figure 1: The vertical axis represents the Trade in goods (or services)/Total trade. All the data are calculated for India over the time period 1995 to 2020. Data Source: https://www.oecd.org/sti/ind/measuring-trade-in-value-added.htm#access.

Conversely, when scrutinizing trade relations between India and geographically distant countries like the USA and Canada, a distinct pattern emerges: the goods trade ratio demonstrates a downward trajectory, while the service trade ratio exhibits an upward trend. Notably, post-2016, the service trade ratio surpasses the goods trade ratio, reaching a ratio to 0.51 from 0.30, while the goods trade ratio ranges from 0.70 to 0.49 (shown in the figure 2).

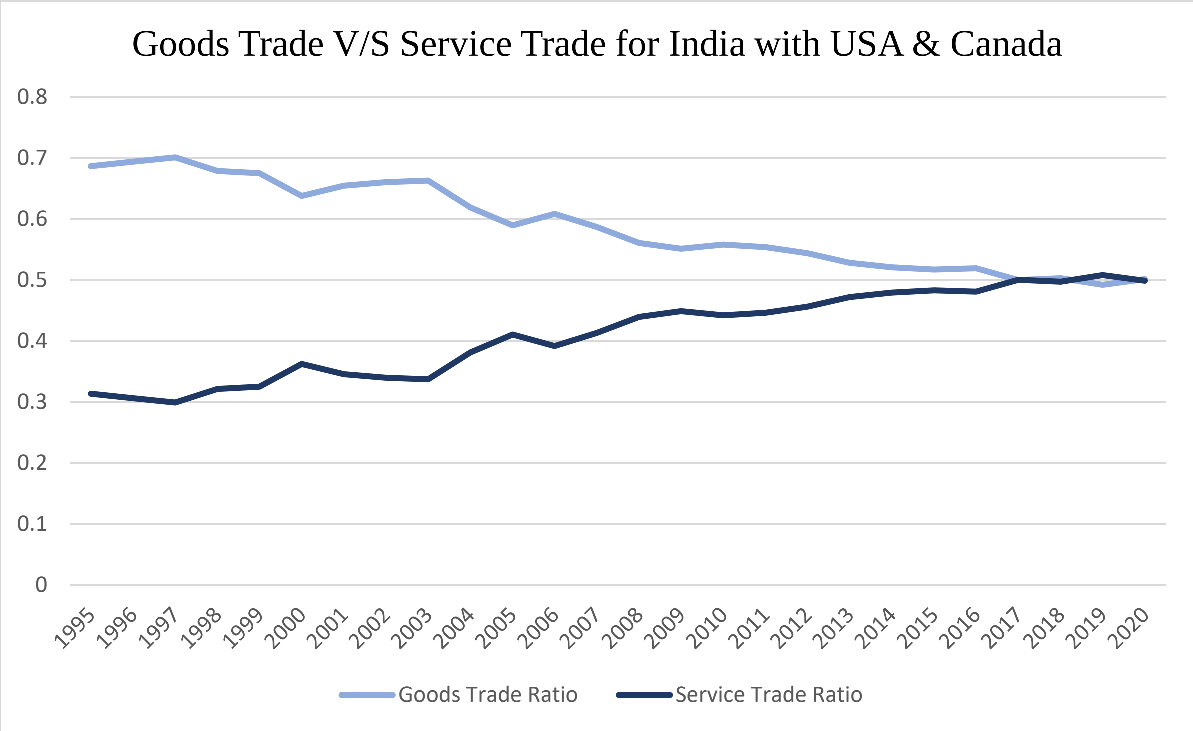

Figure 2: The vertical axis represents the Trade in goods (or services)/Total trade. All the data are calculated for India over the time period 1995 to 2020. Data Source: https://www.oecd.org/sti/ind/measuring-trade-in-value-added.htm#access.

This analysis suggests a correlation between trade dynamics and geographical distance, with the nature of trade—whether in goods or services—playing a pivotal role in the presence of time zones.

The remaining paper is arranged as follows. Section 2 outlines the basic model with the story and formulates the model using the Heckscher–Ohlin setup. Then it examines the effects of distance on trade across different Time Zones and shows its impact on factor prices both in the case of goods trade and in the case of service trade. After that, we replace the distance with delaying cost and see the effects on factor prices and output across different Time Zones. In Section 3 we extend the model with another two sectors; one is the informal sector and another is the government manufacturing sector. Finally, Section 4 concludes the paper. Mathematical derivations are shown in the Appendix for reference.

2. The Model

We are considering two distinct countries located at a reasonable distance in the world. Our primary interest lies in examining the behaviours of one country as it tries to engage in trade with the other country. It's important to note that these countries are situated in completely non-overlapping Time Zones. As a result, there exists a considerable geographical distance between these two nations.

In this context, we consider a competitive economy that produces two commodities: X and Y. Commodity X pertains to services, while commodity Y is a physical, tangible good. Commodity X, being a service, can be effortlessly transported between the two geographically distant countries at minimal expenses as the cost of information communication technology is negligible. On the other hand, the trading of commodity Y involves significant costs and time due to its tangible nature.

In this scenario, both X and Y, are manufactured utilizing skilled labour (S) and capital (K). The production process adheres to constant returns to scale (CRS) while experiencing diminishing marginal productivity (DMP) concerning these factors. The market functions under conditions of perfect competition, with trade occurring in a 24-hour cycle. For the production of X, a time frame of 12 hours is required, and a_{sx} a unit of skilled labour (S) and a_{kx} unit of capital (K) is sufficient to yield one unit of X. For simplicity we assume, $a_{sx} = a_{kx} = 1$, i.e., one unit of skilled labour and one unit of capital is required to produce one unit of X. As a result, X becomes available for sale after 12 hours of production. However, whether X is now saleable or not depends on if the market is open. We will come back to this issue again in a few minutes.

Due to its small size, the country's commodity prices are determined by the global market. The country accepts the prevailing prices without influencing them. If no other factors are at play, the entire price of a commodity should be allocated among the factors of production. The assumption of a competitive market further states that, here the commodity price should equate to the average cost of production. However, if there are additional considerations such as trading costs, delays, depreciation, and related factors, a concept referred to as the "disposable price" comes into action. This price reflects adjustments for these factors and is the sum that can be divided among the various productive factors. Therefore, the commodity price should be multiplied by a factor that accounts for aspects like transportation, delays, depreciation, and similar influences.

Let us denote it by δ. As a result, the price that the producer receives is $P_x\delta$, where, P_x is the price of X and $(1-\delta)$ denotes the transportation cost where δ is the fraction of the price which is retained. Also, note that δ is a function of geographical distance (D). The discount factor δ $(0 < \delta \leq 1)$ captures both the transportation cost and time preference or time cost of the consumers. For further details on this, readers may consult Marjit, Mandal and Nakanishi (2020), and Mandal and Das (2023).

Now, δ has different interpretations for goods trade and service trade cases. We consider the service trade scenario first. When X is fully produced and ready for sale after 12 hours of production, it coincides with nighttime in the producing country, causing the local market to be closed. If the intention is to sell the product within the same country of production, it remains inactive for an additional 12 hours until the local market reopens. This necessitates the relocation of the product to other countries where the market remains open during that time period. For trade to yield advantageous outcomes, the product must be transferred to a country where the market opens within a timeframe shorter than 12 hours. Optimal profit maximization dictates exporting the product to a country positioned in exactly opposite directions on the globe. Hence, a substantial geographical gap separates the trading countries, as this strategy exploits the time zone differences to ensure continuous market access and capitalize on maximum selling opportunities.

In this situation, both nations are located in completely non-overlapping time zones, and the distance (D) becomes maximum, signifying minimal transportation costs $(1-\delta)=0$ and δ also reaches its maximum value of $\delta = 1$. In such a scenario, the effective price of the product (δP_x) is at its peak. Such costs may define transportation costs, delaying costs, costs associated with time preference etc. In our case, the sole transportation cost incurred is typically related to Internet communication, which is negligible in today's interconnected world. Remember that, this is possible only for service which can be shipped from one place to another in a split of a second through the Internet. So, we can say that there is an increase in δ with the increase in Distance (D) in the case of service trade.

When we consider exporting goods instead of services as we explained in the previous paragraph, it shows the opposite effects. When goods are produced for export to another country situated in a non-overlapping time zone, a significant geographical distance separates the trading nations. This distance gives rise to two distinct types of costs. Firstly, there is the transportation cost: As the distance (D) increases, there will be huge transportation costs $(1-\delta)$ associated with huge aerial distance. This leads to a decrease in δ. With rising transportation costs, the effective factor prices must decrease, as the contracted price remains fixed, encompassing transportation expenses. The transportation cost, thus includes both the actual travel expenses and the additional costs incurred due to delays. Secondly, there is the delaying cost. Upon reaching the target country where the goods are to be marketed, there inevitably exists a cost associated with any delays in transit. For instance, if the transit time is 12 hours, there is a travel cost incurred, essentially representing the cost of time. This opportunity cost of time is essentially included in δ, which is already inclusive of transportation costs in our model. Hence, it can be inferred that transportation costs comprise two distinct components: the direct expenses of travel to another country and the additional costs attributable to delays in transit. So, we can say that there is a decrease in δ with the increase in Distance (D) in the case of goods trade.

Commodity Y is a pure tangible good. a_{sy} is the amount of skilled labour and a_{ky} is the amount of capital required to produce one unit of Y. P_y is the price of Y.

Here is a crucial point to remember that, we measure X as service/good and Y is always a tangible good. So, when X is service then an increase in distance leads to an increase in δ. And, when we measure X as good then δ is decreasing with the increase in Distance (D).

Now in this section, we try to check how the trade across time zone differences affects the changes in the factor prices. Furthermore, we also analyse the impact on production output. In a perfect competition setting, the cost per unit aligns with its price. Therefore, in a competitive framework the cost-price equations will be:

$$a_{sx}w_s + a_{kx}r = P_x\delta \tag{1}$$

$$a_{sy}w_s + a_{ky}r = P_y \tag{2}$$

Where, w_s is skilled wage and r is rent. P_x and P_y denote prices of X and Y, respectively. The technological coefficients are fixed[1] for the production of X while a_{sy} and a_{ky} are variable. Moreover, we assume that X is a skilled labour (S) intensive service, whereas Y is a capital (K) intensive good.

Both skilled labour (S) and capital (K) are fully employed. Hence, the endowment constraints are given as (we also assume that S and K endowment are fixed)

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[1] The co-efficients are fixed as per the assumptions of this model. One unit of capital (K) and one unit of skilled labour wage (S) are required to produce one unit of X.

$$a_{sx}X + a_{sy}Y = \bar{S} \tag{3}$$

$$a_{kx}X + a_{ky}Y = \overline{K} \tag{4}$$

Equations (1) to (4) illustrate the basic structure of the model. Here we get four equations and four variables, X, Y, w_s and r . Wage (w_s) and rent (r) are determined using equations (1) and (2). Since a competitive framework is assumed, input coefficients of Y can be calculated using the factor prices. With the values of input coefficients obtained, one can get the values of X and Y by solving equations (3) and (4).

2.1. Replacing distance with delaying cost

When examining the impact of time zones on trade, a crucial discount factor is to consider that is associated with it, namely the delaying cost. This cost varies depending on the nature of the trade, whether it involves goods or services. Goods trade, requiring physical shipment, incurs higher delaying costs for exports. The more the distance of the destination from the exporting country, the more expensive the trading process becomes. These delaying costs can be categorized into two types. Firstly, there is the cost incurred during shipment, which denotes the time necessary to transport the product to the distant country. Secondly, there is the waiting or storage cost, which accrues after the product has reached its destination country. Upon arrival, the product might encounter nighttime in the destination country, leading to idle time and incurring additional delaying costs. So, it is obvious that an escalation in distance correlates with an increase in delaying costs concerning goods trade.

On the other hand, service trade entails minimal costs in the face of the very low cost of ICT. When production is completed domestically and the product is exported to somewhere in a just-opened market, optimal time zone utilization is achieved. Therefore, there are no additional costs. However, if trading occurs in a time zone where the day has not yet started or is about to close within 12 hours, suboptimal time zone utilization leads to the need for an additional trade to maximize efficiency. In both scenarios, it is apparent that there is an increased delaying cost as products wait for a sale or remain idle, compared to the situation where full time zone utilization is possible. In summary, non-overlapping time zones result in optimal utilization, maximizing profits and making trade most beneficial. Consequently, delaying costs are higher for nearer countries and lower for distant ones. Hence, delaying costs are inversely related to distance in the case of service trade

Let us examine the outcomes with respect to delaying costs. The delaying cost is represented by ρ, which always takes a positive value with a minimum value of zero (0). When both the countries are situated in entirely non-overlapping time zones, the distance (D) becomes as extensive as possible, and δ takes the maximum value 1[2]. This indicates a minimal delaying cost with $\rho = 0$.

As we have argued before, the effective price of the commodity depends on transportation costs, discount factors and delays associated with shipment, marketing, nighttime etc. It is also clear that, across time zones the effective price of the commodity must reflect the delay associated with distance and shipment. Whereas, if the commodity is sold in the same time zone, the delaying cost comes through wastage due to nighttime waiting. In both cases, the effective price of the product has a negative shock, which must trickle down to the factors of production. Hence, the factor prices will have to absorb the price shock. Let us assume that, the price shock is distributed proportionately among the factors of production.

Therefore, in equation (1), we have a new set of w_s and r . Assume that these are $w_s{'}$ and r' . Notice that, this $w_s{'}$ and r' are the skilled wage and rental rate respectively due to discount factors or delaying costs. Hence, equation (1) can be rewritten as,

$$a_{sx} w_s{'} + a_{kx} r' = P_x \delta$$

$$a_{sx} \left(\frac{w_s{'}}{\delta} \right) + a_{kx} \left(\frac{r'}{\delta} \right) = P_x$$

Comparing the above equation with equation (1), one must understand that,

$$\frac{w_s{'}}{\delta} = w_s \quad and, \qquad \frac{r'}{\delta} = r$$

This intuitively indicates that as a result of any shock the commodity price must induce a negative effect on the factor prices. Therefore, we can replace the discount factor δ by delaying cost ρ, which allows us to:[3]

$$a_{sx} w_s{'} + a_{kx} r' + 2\rho = P_x \qquad (5)$$

[2] Note that δ is the discount parameter where discount cost is $(1 - \delta)$, and after paying for the discount or transportation δ proportion of the value of the product is retained.

[3] For detailed calculations check the Appendix 1

Since our prime motive is to check the effect of either δ or ρ on the factor prices and sectoral composition, we should use the original notations for skilled wage and rental. A 'prime' over a variable is used just to drive home the basic intuition behind possible price effects.

For a similar reason. if we distribute the cost proportionally, then another equation can be formed as:[4]

$$a_{sx}w_s'(1+\rho) + a_{kx}r'(1+\rho) = P_x \qquad (5')$$

But it should not change our basic results. So, we are considering equation (5) for further calculations.

2.2.A. Effect on Factor Prices due to distance

Now let us examine the changes in factor prices due to the change(increase/decrease) in distance between the trading countries, i.e., the change in skilled wage rate ($\widehat{w_s}$) and the change in rent ($\hat{r}$). To solve the above equations for the change in skilled wage rate ($\widehat{w_s}$) and the change in rent ($\hat{r}$), at first, we have to differentiate the equations and after that, we have to apply the crammer's rule to solve the equations for $\widehat{w_s}$ and $\hat{r}$. Therefore, we get the following results:[5]

$$\widehat{w_s} = \frac{\theta_{ky}\hat{\delta}}{|\theta|} \qquad (6)$$

$$\text{And, } \hat{r} = (-)\frac{\theta_{sy}\hat{\delta}}{|\theta|} \qquad (7)$$

Where, $\theta_{ky} > 0, \theta_{sy} > 0$ and $|\theta| > 0$[6]

Since, X is a service, if geographical distance (D) increases then δ will also increase. Therefore, when there is an increase in the distance between the trading countries, then the wage for skilled labour will increase in this type of trade. On the other hand, the rent for capital will decrease due to an increase in the distance between the trading countries.

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[4] Effective price of the commodity is going down by the proportion $(1-\rho)$. So, the factor prices (w_s *and* r) fall down by $(1+\rho)$ proportion.

[5] Since, $a_{sx} = a_{kx} = 1$. Detailed calculations for all these values are given in the Appendix 1

[6] Here, $\theta_{sx} = \frac{w_s}{P_x\delta}$, $\theta_{kx} = \frac{r}{P_x\delta}$, $\theta_{sy} = \frac{w_s a_{sy}}{P_y}$, $\theta_{ky} = \frac{r a_{ky}}{P_y}$. Since, X is S (Skilled labour) intensive and Y is K (capital) intensive, then, $\theta_{sx} > \theta_{sy}$ and $\theta_{ky} > \theta_{kx}$ $\therefore \theta_{sx}\theta_{ky} > \theta_{kx}\theta_{sy}$ Or, $(\theta_{sx}\theta_{ky} - \theta_{kx}\theta_{sy}) > 0$, and, $|\theta| = \theta_{sx}\theta_{ky} - \theta_{kx}\theta_{sy}$ or, $|\theta| > 0$, $\hat{\imath}$ denotes the change in the parameter i.

Now, if distance (D) increases, transportation costs will also increase for goods trade and, therefore δ will decrease. If the cost increases, then the effective factor prices should be decreased since the commodity price is fixed. Therefore, when there is an increase in the distance between the trading countries, then the wage for the skilled labour will decrease and the rent for capital will increase in case of goods trade.

The implication of time zone difference associated distance on δ is not symmetric for services and goods trade. In the case of services δ rises with D whereas in the case of goods, an increase in D leads to an increase in $(1-\delta)$. The reason is the rise in cost which is a loss due to transportation. Therefore, for goods trade δ falls with an increase in D.

So, we propose that,

Proposition I: An increase in distance between trading countries leads to an increase in the wage of skilled labour (w_s) and a fall in rent (r) if the product is a service, whereas the same reason leads to a decrease in skilled wage (w_s) and a rise in rent (r) if the product is a good.

Proof: See discussion above.

Explanation: When there is a positive outcome for service trade, it results in an upward shift in the prices of factors extensively utilized in service production. Given the advantageous nature of service trade with distant countries, this leads to a rise in the wage rates for skilled labour, heavily employed in service production. Conversely, there is a decline in capital, predominantly utilized in goods production and vice versa.

2.2.B. Effect on Factor Prices due to delaying cost

We now aim to investigate the alterations in factor prices caused by variations (increases or decreases) in the distance as well as the delaying cost between the trading nations. These variations lead to changes in the skilled wage rate ($\widehat{w_s}$) and rent ($\hat{r}$). To address this, we need to differentiate the equations (2) and (5) and subsequently employ Cramer's rule to solve for the changes in skilled wage rate ($\widehat{w_s}$) and rent ($\hat{r}$). As a result of these calculations, we obtain the following outcomes:[7]

$$\widehat{w_s} = (-)\frac{\theta_{ky}\theta_{\rho x}\hat{\rho}}{|\theta|} \tag{8}$$

[7] Since, $a_{sx} = a_{kx} = 1$. Detailed calculations for all these values are given in the Appendix 1

And, $\hat{r} = \frac{\theta_{sy}\theta_{\rho x}\hat{\rho}}{|\theta|}$ (9)

$\theta_{ky} > 0, \theta_{\rho x} > 0, \theta_{sy} > 0, |\theta| > 0$[8]

Since everything is positive here, the change in the skilled wage rate and the change in rent solely depends upon the change in delaying cost, i.e., $\hat{\rho}$. Now we observe that if ρ goes down; i.e., $\hat{\rho} < 0$. This leads to an increase in the skilled wage rate, $\widehat{w_s} > 0$. In the other words, w_s goes up along with the decrease in the delaying cost.

On the other hand, when ρ goes down, $\hat{r} < 0$. This means that rent decreases with the decrease in delaying cost and vice versa.

We state the second proposition as:

Proposition II: An increase in delaying cost due to an increase in distance between trading countries leads to a decrease in the wage of skilled labour (w_s) and a rise in rent (r) if the product is a good, and a decrease in delaying cost due to an increase in distance between trading countries leads to an increase in skilled wage (w_s) and a fall in rent (r) if the product is a service.

Proof: See discussion above.

Explanation: When the trade is advantageous for service then there will be an increase in the factor prices which is used intensively for service production. So, there will be a decrease in the factor prices which is used intensively for goods production.

2.3.A. Effect on Output due to distance

Let's examine the situation where there are changes in service trade along with the changes in delaying costs while keeping skilled labour (S) and capital (K) fully employed and their endowments fixed. When we consider service trade, it involves the exchange of services across international borders. In this context, if both skilled labour (S) and capital (K) are already fully employed, it indicates that the available workforce and capital are being utilized to their maximum potential. This state ensures that the production capabilities are optimized.

Now, if there have to be any changes in service trade, such as increased production of certain services or shifts in service specialization, these changes need to occur within the

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[8] Here, $\theta_{\rho x} = \frac{2\rho}{P_x}$ and $\theta_{sx} + \theta_{kx} + \theta_{\rho x} = 1$

existing framework of skilled labour and capital endowments. Any adjustments must work within the constraints of the available human resources and capital investment. For example, if a country experiences a surge in demand for its software development services, it would need to allocate its skilled labour and capital efficiently to meet this demand. This could involve training more individuals in software development or adopting more efficient software development tools. However, we are not concerned with these dimensions. In our framework, changes in outputs, if any, must be triggered through factor substitution which we will explain later.

Now, we examine the effects on output changes in the case of both service trade and good trade.

To get the changes in the output for X ($\hat{X}$) and the change in the output of Y ($\hat{Y}$), we have to introduce the variation in the coefficients, the elasticity of substitution (σ_Y) between two factors (S and K) which is given as

$$\sigma_Y = (-)\frac{\widehat{a_{sy}} - \widehat{a_{ky}}}{\widehat{w_s} - \hat{r}} \qquad (10)^{9}$$

We solve the values for $\widehat{a_{sy}}$ and $\widehat{a_{ky}}$ with the help of envelop theorem[10] and the equation (10). After that, we differentiate the conditions for full employment equations (3 and 4), and apply the Crammer's rule to get the values for the changes in the output for X ($\hat{X}$) and the change in the output of Y ($\hat{Y}$) as,[11]

$$\hat{X} = \frac{1}{|\lambda||\theta|}\left(\lambda_{ky}\lambda_{sy}\theta_{ky} + \lambda_{sy}\lambda_{ky}\theta_{sy}\right)\sigma_Y\widehat{\delta(D)} \qquad (11)$$

And, $\hat{Y} = (-)\frac{1}{|\lambda||\theta|}\left(\lambda_{kx}\lambda_{sy}\theta_{ky} + \lambda_{sx}\lambda_{ky}\theta_{sy}\right)\sigma_Y\widehat{\delta(D)}$ (12)[12]

Therefore, when there is an increase in the distance between the trading countries, the output for service X will increase while the output for good Y will decrease. Thus, when, X is service, if D increases, $\delta(D)$ will increase. Therefore, $\widehat{\delta(D)} > 0$. Hence, $\hat{X} > 0$ and $\hat{Y} < 0$

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[9] We know that, $(\widehat{w_s} - \hat{r}) = -\frac{\theta_{\rho x}\hat{\rho}}{|\theta|}$

[10] Envelop theorem implies that, $\widehat{a_{sy}}\theta_{sy} + \widehat{a_{ky}}\theta_{ky} = 0$

[11] Since, $a_{sx} = a_{kx} = 1$. Detailed calculations for all these values are given in the Appendix 2

[12] Here, $\lambda_{sx} = \frac{x}{S}$, $\lambda_{kx} = \frac{x}{K}$, $\lambda_{sy} = \frac{a_{sy}y}{S}$, $\lambda_{ky} = \frac{a_{ky}y}{K}$. Since, X is S (Skilled labour) intensive and Y is K (capital) intensive, then, $\lambda_{sx} > \lambda_{sy}$ and $\lambda_{ky} > \lambda_{kx}$ $\therefore \lambda_{sx}\lambda_{ky} > \lambda_{kx}\lambda_{sy}$ Or, $(\lambda_{sx}\lambda_{ky} - \lambda_{kx}\lambda_{sy}) > 0$, and, $|\lambda| = \lambda_{sx}\lambda_{ky} - \lambda_{kx}\lambda_{sy}$ or, $|\lambda| > 0$

As the geographical distance (D) between these countries rises, the transportation costs associated with trading the goods also increase, leading to a reduction in the parameter δ. Consequently, as the distance between the trading nations increases, the production output of good X experiences a decline and the production output of good Y expands. Thus, when, X is a tangible good, if D increases, $\delta(D)$ will decrease. Therefore, $\widehat{\delta(D)} < 0$. Hence, $\hat{X} < 0$ and $\hat{Y} > 0$

Hence, we have the following Proposition.

Proposition III: An increase in distance between trading countries leads to an increase in the output of X and a decrease in the output of Y if X is a service and leads to a decrease in the output of X and an increase in the output of Y if X is a good.

Proof: See discussion above.

Explanation: Since in the case of service trade, there is an increase in the skilled labour wage and a fall in the rent of capital, the service production will be increasing. In this situation, the skilled workers are engaged in service production resulting in a decrease in the output of the other product which is capital-intensive in nature and vice versa.

2.3.B. Effect on Output due to delaying costs

When analysing the impact of time zones on trade, one crucial factor that must be taken into account is the delaying cost associated with it. This cost varies depending on the type of trade being conducted, whether it involves goods or services. Goods trade, involving physical shipment, faces higher delaying costs for exports. As the distance between the exporting and destination countries increases, the trading process becomes more expensive due to these delaying costs. These costs can be divided into two categories. Firstly, there are the expenses incurred during shipment, representing the time needed to transport the product to the distant country. Secondly, there are waiting or storage costs, which accumulate after the product reaches its destination country. When the product reaches to its destination country, it may experience nighttime in that country, resulting in idle time and additional delaying costs. Therefore, it's evident that an increase in distance correlates with a rise in delaying costs in goods trade.

But if we look at the service trade scenario, then, delaying costs in service trade pertain to the expenses and obstacles related to transmitting information over geographic distances.

These costs play a crucial role in shaping how services are traded. Changes in delaying costs, influenced by technological advancements and shifts in communication infrastructure, can significantly impact service trade dynamics. A decrease in delaying costs facilitates easier communication and collaboration between service providers and clients across borders. This, in turn, enhances the coordination of services, expands access to global markets, and fosters specialization in remotely deliverable services. Reduced delaying costs promote more extensive and efficient service trade, allowing countries to leverage their specialized service strengths. However, it's essential to manage these changes within the constraints of fixed skilled labour and capital resources, necessitating optimal resource allocation and technological strategies to maximize benefits.

Here also we get the similar endowment constraints as previous. We solve the values for $\widehat{a_{sy}}$ and $\widehat{a_{ky}}$ with the help of envelop theorem[13] and the equation (10). After that, we differentiate the conditions for full employment equations (8 and 9), and apply the Crammer's rule to get the values for the changes in the output for X ($\hat{X}$) and the change in the output of Y ($\hat{Y}$) as,[14]

$$\hat{X} = (-)\frac{1}{|\lambda||\theta|}\left(\lambda_{ky}\lambda_{sy}\theta_{ky} + \lambda_{sy}\lambda_{ky}\theta_{sy}\right)\sigma_Y\theta_{\rho x}\hat{\rho} \quad (13)$$

And, $$\hat{Y} = \frac{1}{|\lambda||\theta|}\left(\lambda_{kx}\lambda_{sy}\theta_{ky} + \lambda_{sx}\lambda_{ky}\theta_{sy}\right)\sigma_Y\theta_{\rho x}\hat{\rho} \quad (14)$$

$Where, \lambda_{ky} > 0, \lambda_{sy} > 0, \theta_{ky} > 0, \theta_{sy} > 0, \sigma_Y > 0, \theta_{\rho x} > 0, \theta_{sy} > 0, |\theta| > 0, |\lambda| > 0$[15]

If, ρ goes down; i.e., $\hat{\rho} \downarrow$, i.e., $\hat{\rho} < 0$, this leads to $\hat{x} > 0$, x goes up. On the other hand, if, $\hat{\rho} < 0, \hat{y} < 0$, then $y \downarrow$, i.e., y falls and vice versa.

Therefore, when there is a decrease in the delaying cost between the trading countries, then the output for service X will increase while the output for good Y will decrease.

Hence, we have the following Proposition.

Proposition IV: A decrease in the delaying cost due to an increase in distance between trading countries leads to a rise in the production of X and a fall in the production of Y if the product is a service whereas, an increase in the delaying cost due to an increase in distance between

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[13] Envelop theorem states that, $\widehat{a_{sy}}\theta_{sy} + \widehat{a_{ky}}\theta_{ky} = 0$

[14] Since, $a_{sx} = a_{kx} = 1$.Detailed calculations for all these values are given in the Appendix 3

[15] Here, $\lambda_{sx} = \frac{x}{S}$, $\lambda_{kx} = \frac{x}{K}$, $\lambda_{sy} = \frac{a_{sy}y}{S}$, $\lambda_{ky} = \frac{a_{ky}y}{K}$. Since, X is S (Skilled labour) intensive and Y is K (capital) intensive, then, $\lambda_{sx} > \lambda_{sy}$ and $\lambda_{ky} > \lambda_{kx}$ $\therefore \lambda_{sx}\lambda_{ky} > \lambda_{kx}\lambda_{sy}$ Or, $(\lambda_{sx}\lambda_{ky} - \lambda_{kx}\lambda_{sy}) > 0$, and, $|\lambda| = \lambda_{sx}\lambda_{ky} - \lambda_{kx}\lambda_{sy}$ or, $|\lambda| > 0$

trading countries leads to a fall in the production of X and a rise in the production of Y if the product is a good.

Proof: See discussion above.

Explanation: Similar to proposition III.

3. Extended Model

In this section, we extend the basic model by introducing additional two sectors. One of these sectors pertains to the informal sector, denoted by I, while the other is the government manufacturing sector, labelled as Z. Therefore, besides X and Y, we have two new sectors I and Z. X, Y and Z are formal sectors, where I is an informal sector. Further, we assume that the government manufacturing sector uses unorganised labour (wage w) and capital (r) for production and the Informal sector uses unorganised labour (wage w) and land (R).

When we are considering a developing country like India, it is evident that there must exist a huge informal sector in the Economy. For this reason, we have to incorporate informality into our model. The informality comes through the capital market which privately provides capital, T to I. This could be local money lenders or land or any specific type of capital. There is also an important factor which has to be taken into account which is the nature of the labour. Normally, the informal sector uses unorganised or unskilled labour. Therefore, there must be some wage inequality in this economy in the presence of an informal sector between skilled labour wage and unskilled labour wage.

X and Y are produced using resources S and K, as previously mentioned. Additionally, sector Z is characterized as being unorganized labour intensive, while sector I is known for its high reliance on resource R. In this section, we aim to investigate the impact of trade across different time zones, considering delaying costs. This analysis specifically focuses on the scenario where one of the formal products (X) is a service. Furthermore, we will examine the implications for factor prices. Therefore, along with the previous price equations here, we have two new price equations for sector Z and I, which are given as:

$$a_{uz}w + a_{kz}r = P_z \tag{15}$$

$$a_{ui}w + a_{ti}R = P_i \tag{16}$$

Where, w is unorganised wage and r is rent. P_z denotes the price of Z. a_{uz} is the amount of unorganised labour required to produce one unit of Z and a_{kz} is the amount of capital required to produce one unit of Z. On the other hand, a_{ui} is the amount of unorganised labour required to produce one unit of I and a_{ti} is the amount of land required to produce one unit of I. Here, P_i denotes the price of I.

The skilled labour (S), unorganised labour (U), land (T) and capital (K) all are fully employed. So, the full employment conditions are:

$$a_{sx}X + a_{sy}Y = \bar{S} \tag{17}$$

$$a_{kx}X + a_{ky}Y + a_{kz}Z = \bar{K} \tag{18}$$

$$a_{uz}Z + a_{ui}I = \bar{U} \tag{19}$$

$$a_{ti}I = \bar{T} \tag{20}$$

There are six unknown variables (w, R, X, Y, Z and I) and six equations. Given the values of w_s and r, solutions for w and R are obtained using equations (15) and (16). Equation (20) can be solved by using the value of input coefficients a_{ti} . The value of I and the values of the co-efficient obtained are used to substitute the value in equation (19) and get the value for Z. Finally, substitute the value for Z in equation (18) and then using the modified (18) and (17) the values of X and Y are obtained. Thus, the system is solved.

3.1. Delaying Costs and Factor Prices

We are now interested in examining how the change in delaying costs between trading nations—whether they increase or decrease—affects factor prices. Specifically, we intend to analyse the variations in the unorganized wage rate ($\hat{w}$) and rent of land ($\hat{R}$) due to these shifts. To address this, we begin by solving equations (15) and (16) for the changes in the unorganized wage rate ($\hat{w}$) and the change in land ($\hat{R}$). To achieve this, we start by differentiating the equations (15) and (16). After that, we substitute the resultant value of the parameter $\hat{r}$ into the relevant equations. This process enables us to determine the changes in the unorganized wage rate ($\hat{w}$) and the rent of land ($\hat{R}$). Therefore, we get the following results:[16]

$$\hat{w} = -\frac{\theta_{kz}}{\theta_{uz}}\frac{\theta_{sy}\theta_{\rho x}\hat{\rho}}{|\theta|} \tag{21}$$

[16] Detailed calculations for all these values are given in the Appendix 4

$$\hat{R} = (+)\frac{\theta_{ui}}{\theta_{ti}}\frac{\theta_{kz}}{\theta_{uz}}\frac{\theta_{sy}\theta_{\rho x}\hat{\rho}}{|\theta|} \quad (22)^{17}$$

Since, $\theta_{kz} > 0, \theta_{uz} > 0, \theta_{sy} > 0, \theta_{\rho x} > 0, |\theta| > 0$

When delaying cost decreases between two trading countries, this means $\hat{\rho} < 0, then,\ \hat{w} > 0$. On the other hand, if $\hat{\rho} < 0$, then $\hat{R} < 0$.

Since, $\theta_{ui} > 0, \theta_{ti} > 0, \theta_{kz} > 0, \theta_{uz} > 0, \theta_{sy} > 0, \theta_{\rho x} > 0, |\theta| > 0$

Therefore, we can say that, an increase in delaying cost due to an increase in distance between trading countries leads to a decrease in wage of unorganised labour **(**w) and wage of skilled labour (w_s) and a rise in rent of land (R) and rent of capital (r) if the product is a good. On the other hand, a decrease in delaying cost due to an increase in distance between trading countries leads to an increase in the wage of unorganised labour (w) and skilled wage rate (w_s) and a fall in rent of land (R) and rent of capital (r) if the product is a service.

3.1.A. Wage inequality

Since, the informal wage is usually less than the formal wage where the minimum wage rule is followed, then we can see that;[18]

$$(\widehat{w_s} - \hat{w}) = -\frac{\theta_{\rho x}\hat{\rho}}{|\theta|}\left(\frac{\theta_{ky}\theta_{uz}-\theta_{kz}\theta_{sy}}{\theta_{uz}}\right)$$

When X is a service, then if there is an increase in geographical distance (D), then there will be a decrease in ρ and hence the wage difference between skilled labour and unskilled labour increases. On the other hand, if X is a good, then the wage difference between skilled labour and unskilled labour decreases because there is an increase in ρ with the increase in distance between the two trading countries.

Hence, we have the following Proposition.

Proposition V: If delaying cost decreases due to the increase in distance between trading countries then wage inequality will increase if the product (X) is a service and when delaying

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[17] $\widehat{w_s} = -\frac{\theta_{ky}\theta_{\rho x}\hat{\rho}}{|\theta|}, \hat{r} = \frac{\theta_{sy}\theta_{\rho x}\hat{\rho}}{|\theta|}$ and, $\theta_{uz} = \frac{wa_{uz}}{P_z}, \theta_{kz} = \frac{ra_{kz}}{P_z}, \theta_{ui} = \frac{wa_{ui}}{P_i}, \theta_{ti} = \frac{Ra_{ti}}{P_i}$

[18] Detailed calculations for all these values are given in the Appendix 4

cost increases with the increase in distance between trading countries then wage inequality will decrease if the product (X) is a good.

Proof: See Appendix 4

3.2. Output Effect of Delaying Cost

In this section, our focus will be directed towards examining the repercussions of alterations in output of the sectors; service sector (X), goods sector (Y), informal sector (I) and government manufacturing sector (Z). Also remember that, the skilled labour (S), unorganised labour (U), land (T) and capital (K) are all operating at full employment levels. We will explore a detailed study on the changes in output and the consequences while considering the changes in the distance-related delaying cost between the trading countries. This will enable us to gain a comprehensive understanding of how these shifts impact the various factors of production, including skilled labour, unorganized labour, land and capital, all of which are fully engaged in their respective roles.

To assess the impact of variations in the output levels of X, Y, I and Z, it becomes necessary to apply the notion of elasticity of substitution between two factors (U and T) in I (σ_i). The elasticity of substitution in Z is mathematically expressed as,

$$\sigma_i = (-)\frac{\widehat{a_{ui}}-\widehat{a_{ti}}}{\widehat{w}-\widehat{R}} \qquad (23)^{19}$$

We solve the values for $\widehat{a_{ui}}$ and $\widehat{a_{ti}}$ with the help of envelop theorem[20] and the equation (23). After that, we differentiate the condition for full employment equation (20) to get the value for the changes in the output of I ($\hat{I}$) as,[21]

$$\hat{I} = (+)\sigma_i\theta_{ui}\frac{\theta_{kz}}{\theta_{uz}\theta_{ti}}\frac{\theta_{sy}\theta_{\rho x}\hat{\rho}}{|\theta|} \qquad (24)$$

Since, $\sigma_i > 0, \theta_{ui} > 0, \theta_{kz} > 0, \theta_{uz} > 0, \theta_{ti} > 0, \theta_{sy} > 0, \theta_{\rho x} > 0, |\theta| > 0$

When distance related delaying costs rises between the trading countries then, $\hat{\rho} > 0$. This leads to a rise in the output for I, i.e., $\hat{I} > 0$.

[19] We know that, $\widehat{w} - \widehat{R} = -\frac{\theta_{kz}}{\theta_{uz}\theta_{ti}}\frac{\theta_{sy}\theta_{\rho x}\hat{\rho}}{|\theta|}$

[20] Envelop theorem says that, $\theta_{ui}\widehat{a_{ui}} + \theta_{ti}\widehat{a_{ti}} = 0$

[21] Detailed calculations for all these values are given in the Appendix 5

To determine the effect on Z, we need to calculate the elasticity of substitution (σ_z) between U and K which is given as,

$$\sigma_z = (-)\frac{\widehat{a_{uz}} - \widehat{a_{kz}}}{\hat{w} - \hat{r}} \quad (25)^{22}$$

In what follows, we solve for the values of $\widehat{a_{uz}}$ and $\widehat{a_{kz}}$ [23]. To get the value for the changes in the output of Z ($\hat{Z}$), we differentiate the full employment condition, equation (19) and substitute the value of $\hat{I}$as,[24]

$$\hat{Z} = (-)\frac{\theta_{kz}\theta_{sy}\theta_{\rho x}\hat{\rho}}{\lambda_{uz}\theta_{uz}|\theta|}\left\{\sigma_z\lambda_{uz} + \sigma_i\lambda_{ui} + \sigma_i\frac{\theta_{ui}}{\theta_{ti}}\lambda_{ui}\right\} \quad (26)^{25}$$

Since, $\theta_{kz} > 0, \theta_{sy} > 0, \theta_{\rho x} > 0, \lambda_{uz} > 0, \theta_{uz} > 0, |\theta| > 0, \sigma_z > 0, \sigma_i > 0, \lambda_{ui} > 0,$

$\theta_{ui} > 0, \theta_{ti} > 0$

Thus, when, delaying cost increases between the trading nations then, sector Z contracts.

Then we move to the output effects on X and Y[26]. These are shown as,

$$\hat{X} = -\frac{\lambda_{sy}\theta_{\rho x}\hat{\rho}}{|\lambda||\theta|}\left[\lambda_{ky}\sigma_Y + \lambda_{kz}\frac{\theta_{kz}\theta_{sy}}{\lambda_{uz}\theta_{uz}}\left(\sigma_z\lambda_{uz} + \sigma_i\lambda_{ui} + \sigma_i\frac{\theta_{ui}}{\theta_{ti}}\lambda_{ui}\right) + \lambda_{kz}\sigma_z\theta_{sy}\right] \quad (27)$$

And, $$\hat{Y} = \frac{1}{|\lambda|}\frac{\theta_{\rho x}\hat{\rho}}{|\theta|}\left[\lambda_{kx}\lambda_{sy}\sigma_Y\theta_{ky} + \lambda_{sx}\left\{\lambda_{ky}\sigma_Y\theta_{sy} + \lambda_{kz}\frac{\theta_{kz}\theta_{sy}}{\lambda_{uz}\theta_{uz}}\left(\sigma_z\lambda_{uz} + \sigma_i\lambda_{ui} + \sigma_i\frac{\theta_{ui}}{\theta_{ti}}\lambda_{ui}\right) + \lambda_{kz}\sigma_z\theta_{sy}\right\}\right] \quad (28)^{27}$$

Everything is positive in the bracket and $\theta_{\rho x} > 0, \lambda_{sy} > 0, |\lambda| > 0, |\theta| > 0$ for both the equations.

Therefore, when, delaying cost increases between the trading countries then, X decreases while Y increases.

Hence, we have the following Proposition.

Proposition VI: An increase in the delaying cost due to the increase in distance between trading countries leads to a fall in the production of X and Z and an expansion in the production of Y and I if the product is a good. On the other hand, a decrease in the delaying cost due to the

[22] We know that, $\hat{w} - \hat{r} = -\frac{\theta_{sy}\theta_{\rho x}\hat{\rho}}{|\theta|\theta_{uz}}$

[23] Envelop theorem says that, $\theta_{uz}\widehat{a_{uz}} + \theta_{kz}\widehat{a_{kz}} = 0$

[24] Detailed calculations for all these values are given in the Appendix 5

[25] Here, $\lambda_{uz} = \frac{za_{uz}}{U}$, $\lambda_{ui} = \frac{ia_{ui}}{U}$

[26] Since, $a_{sx} = a_{kx} = 1$. Detailed calculations for all these values are given in the Appendix 6

[27] Here, $\lambda_{kz} = \frac{a_{kz}z}{K}$

increase in distance between trading countries leads to a rise in the production of X and Z and a contraction in the production of Y and I.

Proof: See discussion above.

4. Concluding Remarks

This paper is initiated by formulating a fundamental model involving two countries, under the crucial assumption that these countries are situated in distinct time zones. Notably, we observed that geographical distance could yield a positive impact on service trade, challenging conventional gravity model arguments that typically view distance as harmful to trade. Interestingly, while goods trade tends to suffer under increased distance, service trade appears to benefit. We noted a relationship in which an increase in the distance between trading nations results in a rise in skilled labour wages and a decline in capital rent for service products. However, in contrast for goods, geographical distance between trading countries leads to a reduction in skilled labour wages and an increase in capital rent. Thus, our model effectively demonstrates that the impact of aerial distance on trade outcomes is dependent upon the nature of the product—whether it is a good or a service.

Expanding upon this basic model, substituting distance with delaying cost reveals the following outcomes: an escalation in delaying cost between trading nations results in a decline in skilled labour wages and an increase in rent. On the other hand, for the changes in output, we can see that a rise in delaying cost between trading nations triggers an upswing in the production of service trade and a downturn in the production of goods trade.

We further extend the model by incorporating two supplementary sectors: the informal sector and the government manufacturing sector. Additionally, we introduce unorganized labour and a specific capital as two additional factors of production. Our model affirms that, despite the introduction of both the informal sector and the government manufacturing sector, the consistency in the evolution of factor prices and output persists as delaying costs change. Thus, we can assert the asymmetric impact of distance and delaying costs between trading countries depending on whether we talk about goods or services.

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Appendix 1

$a_{sx}w_s + a_{kx}r = P_x\delta$ (1.1)

$a_{sy}w_s + a_{ky}r = P_y$ (1.2)

Differentiating totally equation (1.1) we get,

$\theta_{sx}\widehat{w_s} + \theta_{kx}\hat{r} = \hat{\delta}$ (1.3)

Differentiating totally equation (1.2) we get,

$\theta_{sy}\widehat{w_s} + \theta_{ky}\hat{r} = 0$ (1.4)

Since, $\theta_{sx} + \theta_{kx} = 1$, $\widehat{P_x} = 0, \widehat{P_y} = 0$ and, envelop theorem says that, $\theta_{sy}\widehat{a_{sy}} + \theta_{ky}\widehat{a_{ky}} = 0$

Solving equations (1.3) and (1.4) by using Crammer's rule, we get,

$\widehat{w_s} = \frac{\theta_{ky}\widehat{\delta(D)}}{|\theta|}$ (1.5)

And, $\hat{r} = -\frac{\theta_{sy}\widehat{\delta(D)}}{|\theta|}$ (1.6)

Since, $a_{sx} = a_{kx} = 1$

If, X is S (Skilled labour) intensive and Y is K (capital) intensive, then,

$\theta_{sx} > \theta_{sy}$ and $\theta_{ky} > \theta_{kx}$

$\therefore \theta_{sx}\theta_{ky} > \theta_{kx}\theta_{sy}$

Or, $(\theta_{sx}\theta_{ky} - \theta_{kx}\theta_{sy}) > 0$

Or, $|\theta| > 0$

Extended Model: Replacing distance with delaying cost

$a_{sx}w_s' + a_{kx}r' = P_x\delta$

$or, a_{sx}\left(w_s'\frac{1}{\delta}\right) + a_{kx}\left(r'\frac{1}{\delta}\right) = P_x$

$w_s'\frac{1}{\delta} = w_s,\ Similarly\ for\ r, r'\frac{1}{\delta} = r$

$\therefore a_{sx}(w_s' + \rho) + a_{kx}(r' + \rho) = P_x$

$or, a_{sx}w_s' + a_{kx}r' + 2\rho = P_x$ (1.7)

Since, $a_{sx} = a_{kx} = 1$

If we distribute the cost proportionally then, another equation can be formed as:

$a_{sx}w_s' + a_{kx}r' = P_x(1 - \rho)$

$or, a_{sx}w_s' + a_{kx}r' = P_x - \rho(a_{sx}w_s' + a_{kx}r')$

$or, a_{sx}w_s' + a_{kx}r' + \rho a_{sx}w_s' + \rho a_{kx}r' = P_x$

$or, a_{sx}w_s'(1 + \rho) + a_{kx}r'(1 + \rho) = P_x$ (1.8)

Differentiating totally equation (1.7) we get,

$$\theta_{sx}\widehat{w_s} + \theta_{kx}\hat{r} = -\theta_{\rho x}\hat{\rho} \tag{1.9}$$

Here, $\widehat{P_x} = 0$ and $\theta_{sx} + \theta_{kx} + \theta_{\rho x} = 1$

Solving equations (1.4) and (1.9) by using Crammer's rule, we get,

$$\therefore \widehat{w_s} = -\frac{\theta_{ky}\theta_{\rho x}\hat{\rho}}{|\theta|} \tag{1.10}$$

And, $\hat{r} = \frac{\theta_{sy}\theta_{\rho x}\hat{\rho}}{|\theta|}$ (1.11)

Since, $a_{sx} = a_{kx} = 1$

Appendix 2

$$\sigma_Y = -\frac{\widehat{a_{sy}} - \widehat{a_{ky}}}{\widehat{w_s} - \hat{r}}$$

$$\therefore \widehat{a_{sy}} = \widehat{a_{ky}} - \sigma_Y(\widehat{w_s} - \hat{r}) \tag{2.1}$$

And, $\widehat{a_{ky}} = \widehat{a_{sy}} + \sigma_Y(\widehat{w_s} - \hat{r})$ (2.2)

Envelop theorem states that, $\widehat{a_{sy}}\theta_{sy} + \widehat{a_{ky}}\theta_{ky} = 0$

$$\therefore \widehat{a_{sy}} = -\widehat{a_{ky}}\frac{\theta_{ky}}{\theta_{sy}} \tag{2.3}$$

And, $\widehat{a_{ky}} = -\widehat{a_{sy}}\frac{\theta_{sy}}{\theta_{ky}}$ (2.4)

Comparing Equation (2.1) and (2.3) we get, and using equations (1.5) and (1.6) we get,

$$\widehat{a_{ky}} = \sigma_Y\theta_{sy}\frac{\widehat{\delta(D)}}{|\theta|} \tag{2.5}$$

Similarly, comparing Equations (2.2) and (2.4) we get,

$$\widehat{a_{sy}} = -\sigma_Y\theta_{ky}\frac{\widehat{\delta(D)}}{|\theta|} \tag{2.6}$$

The endowment constraints are given as (since we assume that S and K endowment are fixed)

$$a_{sx}\,X + a_{sy}Y = \bar{S} \tag{2.7}$$

$$a_{kx}\,X + a_{ky}Y = \bar{K} \tag{2.8}$$

Differentiating totally equation (2.7) we get,

$$\lambda_{sx}\hat{X} + \lambda_{sy}\hat{Y} = \lambda_{sy}\sigma_Y\theta_{ky}\frac{\widehat{\delta(D)}}{|\theta|} \tag{2.9}$$

Where, $\lambda_{sx} = \frac{1\,X}{S}$ *and* $\lambda_{sy} = \frac{a_{sy}Y}{S}$ *and* $\lambda_{sx} + \lambda_{sy} = 1$

$\left[\text{Since}, \hat{S} = 0 \text{ and } \widehat{a_{sy}} = -\sigma_Y\theta_{ky}\frac{\widehat{\delta(D)}}{|\theta|}\right]$

Similarly, from equation (2.8),

$$\lambda_{kx}\hat{X} + \lambda_{ky}\hat{Y} = -\lambda_{ky}\sigma_Y\theta_{sy}\frac{\widehat{\delta(D)}}{|\theta|} \tag{2.10}$$

Where, $\lambda_{kx} = \frac{1\,X}{K}$ $and\ \lambda_{ky} = \frac{a_{ky}Y}{K}$ $and\ \lambda_{kx} + \lambda_{ky} = 1$

$\left[\text{Since}, \hat{K} = 0 \text{ and } \widehat{a_{ky}} = \sigma_Y \theta_{sy} \frac{\widehat{\delta(D)}}{|\theta|}\right]$

Solving equation (2.9) and (2.10) by using Cramer's rule, we get,

$$\hat{X} = \frac{1}{|\lambda||\theta|}\left(\lambda_{ky}\lambda_{sy}\theta_{ky} + \lambda_{sy}\lambda_{ky}\theta_{sy}\right)\sigma_Y\widehat{\delta(D)} \quad (2.11)$$

And, $\hat{Y} = -\frac{1}{|\lambda||\theta|}\left(\lambda_{kx}\lambda_{sy}\theta_{ky} + \lambda_{sx}\lambda_{ky}\theta_{sy}\right)\sigma_Y\widehat{\delta(D)}$ (2.12)

If, X is S (Skilled labour) intensive and Y is K (capital) intensive, then,

$(\lambda_{sx}\lambda_{ky} - \lambda_{kx}\lambda_{sy}) > 0$, $|\lambda| > 0$

Since, $a_{sx} = a_{kx} = 1$

Appendix 3

Subtracting equation (1.11) from equation (1.10), we get,

$$(\widehat{w_s} - \hat{r}) = -\frac{\theta_{\rho x}\hat{\rho}}{|\theta|} \quad (3.1)$$

Full employment conditions:

$$a_{sx}X + a_{sy}Y = \bar{S} \quad (3.2)$$

$$a_{kx}X + a_{ky}Y = \bar{K} \quad (3.3)$$

To determine the variation in the coefficients, the elasticity of substitution (σ_Y) between two factors is given as,

$$\sigma_Y = -\frac{\widehat{a_{sy}} - \widehat{a_{ky}}}{\widehat{w_s} - \hat{r}}$$

$$\therefore \widehat{a_{sy}} = \widehat{a_{ky}} - \sigma_Y(\widehat{w_s} - \hat{r}) \quad (3.4)$$

And, $\widehat{a_{ky}} = \widehat{a_{sy}} + \sigma_Y(\widehat{w_s} - \hat{r})$ (3.5)

Envelop theorem states that,

$$\widehat{a_{sy}}\theta_{sy} + \widehat{a_{ky}}\theta_{ky} = 0$$

$$\therefore \widehat{a_{sy}} = -\widehat{a_{ky}}\frac{\theta_{ky}}{\theta_{sy}} \quad (3.6)$$

And, $\widehat{a_{ky}} = -\widehat{a_{sy}}\frac{\theta_{sy}}{\theta_{ky}}$ (3.7)

Comparing Equations (3.4) and (3.6) we get,

$$\widehat{a_{ky}} = -\sigma_Y\theta_{sy}\frac{\theta_{\rho x}\hat{\rho}}{|\theta|} \quad (3.8)$$

[From equation (3.1), we get, $(\widehat{w_s} - \hat{r}) = -\frac{\theta_{\rho x}\hat{\rho}}{|\theta|}$ and $\theta_{ky} + \theta_{sy} = 1$]

Similarly, Comparing Equations (3.5) and (3.7) we get,

$$\widehat{a_{sy}} = (+)\sigma_Y \theta_{ky} \frac{\theta_{\rho x}\hat{\rho}}{|\theta|} \tag{3.9}$$

[From equation (3.1), we get, $(\widehat{w_s} - \hat{r}) = -\frac{\theta_{\rho x}\hat{\rho}}{|\theta|}$ and $\theta_{ky} + \theta_{sy} = 1$]

Differentiating totally equation (3.2) we get,

$$\lambda_{sx}\hat{X} + \lambda_{sy}\hat{Y} = -\lambda_{sy}\sigma_Y\theta_{ky}\frac{\theta_{\rho x}\hat{\rho}}{|\theta|} \tag{3.10}$$

$\left[\text{Since}, \hat{S} = 0, \lambda_{sx} + \lambda_{sy} = 1 \text{ and from equation (3.9), we get}, \widehat{a_{sy}} = \sigma_Y\theta_{ky}\frac{\theta_{\rho x}\hat{\rho}}{|\theta|}\right]$

Differentiating totally equation (3.3) we get,

$$\lambda_{kx}\hat{X} + \lambda_{ky}\hat{Y} = \lambda_{ky}\sigma_Y\theta_{sy}\frac{\theta_{\rho x}\hat{\rho}}{|\theta|} \tag{3.11}$$

$\left[\text{Since}, \hat{K} = 0, \lambda_{kx} + \lambda_{ky} = 1 \text{ and from equation (3.8), we get}, \widehat{a_{ky}} = -\sigma_Y\theta_{sy}\frac{\theta_{\rho x}\hat{\rho}}{|\theta|}\right]$

Solving equations (3.10) and (3.11) by using Cramer's rule, we get,

$$\therefore \hat{X} = -\frac{1}{|\lambda||\theta|}\left(\lambda_{ky}\lambda_{sy}\theta_{ky} + \lambda_{sy}\lambda_{ky}\theta_{sy}\right)\sigma_Y\theta_{\rho x}\hat{\rho} \tag{3.12}$$

And, $\hat{Y} = \frac{1}{|\lambda||\theta|}\left(\lambda_{kx}\lambda_{sy}\theta_{ky} + \lambda_{sx}\lambda_{ky}\theta_{sy}\right)\sigma_Y\theta_{\rho x}\hat{\rho}$ (3.13)

Since, $a_{sx} = a_{kx} = 1$

If, X is S (Skilled labour) intensive and Y is K (capital) intensive, then,

$\frac{a_{sx}}{a_{kx}} > \frac{a_{sy}}{a_{ky}}$ or, $\frac{\lambda_{sx}}{\lambda_{kx}} > \frac{\lambda_{sy}}{\lambda_{ky}}$ or, $(\lambda_{sx}\lambda_{ky} - \lambda_{kx}\lambda_{sy}) > 0$ or, $|\lambda| > 0$

Appendix 4

Introducing another two sectors: Price equations,

$$a_{sx}w_s + a_{kx}r = P_x - 2\rho \tag{4.1}$$

$$a_{sy}w_s + a_{ky}r = P_y \tag{4.2}$$

$$a_{uz}w + a_{kz}r = P_z \tag{4.3}$$

$$a_{ui}w + a_{ti}R = P_i \tag{4.4}$$

$Since, \widehat{w_s} = -\frac{\theta_{ky}\theta_{\rho x}\hat{\rho}}{|\theta|}$ $and,$ $\hat{r} = \frac{\theta_{sy}\theta_{\rho x}\hat{\rho}}{|\theta|}$

Differentiating totally equation (4.3) we get,

$$\theta_{uz}\hat{w} + \theta_{uz}\widehat{a_{uz}} + \theta_{kz}\hat{r} + \theta_{kz}\widehat{a_{kz}} = \widehat{P_z}$$

Since, envelop theorem, $\theta_{uz}\widehat{a_{uz}} + \theta_{kz}\widehat{a_{kz}} = 0, \hat{r} = \frac{\theta_{sy}\theta_{\rho x}\hat{\rho}}{|\theta|}$ $and,$ $\widehat{P_z} = 0$

$$\therefore \hat{w} = -\frac{\theta_{kz}}{\theta_{uz}}\frac{\theta_{sy}\theta_{\rho x}\hat{\rho}}{|\theta|} \tag{4.5}$$

Differentiating totally equation (4.4) we get,

$\theta_{ui}\widehat{w} + \theta_{ui}\widehat{a_{ui}} + \theta_{ti}\widehat{R} + \theta_{ti}\widehat{a_{ti}} = \widehat{P_i}$

Since, envelop theorem, $\theta_{ui}\widehat{a_{ui}} + \theta_{ti}\widehat{a_{ti}} = 0, \widehat{w} = -\frac{\theta_{kz}}{\theta_{uz}}\frac{\theta_{sy}\theta_{\rho x}\widehat{\rho}}{|\theta|}$ $and,$ $\widehat{P_i} = 0$

$$\widehat{R} = (+)\frac{\theta_{ui}}{\theta_{ti}}\frac{\theta_{kz}}{\theta_{uz}}\frac{\theta_{sy}\theta_{\rho x}\widehat{\rho}}{|\theta|} \tag{4.6}$$

Wage inequality:

$$\widehat{w_s} = -\frac{\theta_{ky}\theta_{\rho x}\widehat{\rho}}{|\theta|}$$

$$\widehat{w} = -\frac{\theta_{kz}}{\theta_{uz}}\frac{\theta_{sy}\theta_{\rho x}\widehat{\rho}}{|\theta|}$$

$$\therefore (\widehat{w_s} - \widehat{w}) = -\frac{\theta_{ky}\theta_{\rho x}\widehat{\rho}}{|\theta|} - \left(-\frac{\theta_{kz}}{\theta_{uz}}\frac{\theta_{sy}\theta_{\rho x}\widehat{\rho}}{|\theta|}\right)$$

Or, $(\widehat{w_s} - \widehat{w}) = -\frac{\theta_{\rho x}\widehat{\rho}}{|\theta|}\left(\theta_{ky} - \frac{\theta_{kz}\theta_{sy}}{\theta_{uz}}\right)$

Or, $(\widehat{w_s} - \widehat{w}) = -\frac{\theta_{\rho x}\widehat{\rho}}{|\theta|}\left(\frac{\theta_{ky}\theta_{uz} - \theta_{kz}\theta_{sy}}{\theta_{uz}}\right)$ (4.7)

Since, Z is unorganised labour intensive and Y is capital intensive,

$\therefore \left(\theta_{ky}\theta_{uz} - \theta_{kz}\theta_{sy}\right) > 0$ and, $\theta_{\rho x} > 0$, $\theta_{uz} > 0, |\theta| > 0$

In case of service, $(\widehat{w_s} - \widehat{w}) > 0$; if $\hat{\rho} < 0$ and in case of good, $(\widehat{w_s} - \widehat{w}) < 0$; if $\hat{\rho} > 0$.

Appendix 5

From equation (4.5) and (4.6) we get,

$$\widehat{w} - \widehat{R} = -\frac{\theta_{kz}}{\theta_{uz}\theta_{ti}}\frac{\theta_{sy}\theta_{\rho x}\widehat{\rho}}{|\theta|} \tag{5.1}$$

Since, $\theta_{ti} + \theta_{ui} = 1$

From equation (1.11) and (4.5) we get,

$$\widehat{w} - \hat{r} = -\frac{\theta_{sy}\theta_{\rho x}\widehat{\rho}}{|\theta|\theta_{uz}} \tag{5.2}$$

Since, $\theta_{kz} + \theta_{uz} = 1$

From equation (3.1), we get, $(\widehat{w_s} - \hat{r}) = -\frac{\theta_{\rho x}\widehat{\rho}}{|\theta|}$

Full employment conditions:

$$a_{sx}X + a_{sy}Y = \bar{S} \tag{5.3}$$

$$a_{kx}X + a_{ky}Y + a_{kz}Z = \overline{K} \tag{5.4}$$

$$a_{uz}Z + a_{ui}I = \overline{U} \tag{5.5}$$

$$a_{ti}I = \bar{T} \tag{5.6}$$

Differentiating totally equation (5.6) we get,

$\hat{I} = -\widehat{a_{ti}}$ Since, $\hat{T} = 0$,

To determine the variation in the coefficients, the elasticity of substitution (σ_i) between two factors is given as,

$$\sigma_i = -\frac{\widehat{a_{ui}} - \widehat{a_{ti}}}{\hat{w} - \hat{R}}$$

$$or, \widehat{a_{ti}} = \widehat{a_{ui}} + \sigma_i(\hat{w} - \hat{R}) \quad (5.7)$$

$$and, \widehat{a_{ui}} = \widehat{a_{ti}} - \sigma_i(\hat{w} - \hat{R}) \quad (5.8)$$

Envelop theorem says that,

$$\theta_{ui}\widehat{a_{ui}} + \theta_{ti}\widehat{a_{ti}} = 0$$

$$or, \widehat{a_{ti}} = -\frac{\theta_{ui}}{\theta_{ti}}\widehat{a_{ui}} \quad (5.9)$$

$$and, \widehat{a_{ui}} = -\frac{\theta_{ti}}{\theta_{ui}}\widehat{a_{ti}} \quad (5.10)$$

From equation (5.7) and (5.9) we get,

$$\widehat{a_{ui}} = (+)\sigma_i\theta_{ti}\frac{\theta_{kz}}{\theta_{uz}\theta_{ti}}\frac{\theta_{sy}\theta_{\rho x}\hat{\rho}}{|\theta|}$$

Since, $\theta_{ti} + \theta_{ui} = 1$ and $\left(\hat{w} - \hat{R}\right) = -\frac{\theta_{kz}}{\theta_{uz}\theta_{ti}}\frac{\theta_{sy}\theta_{\rho x}\hat{\rho}}{|\theta|}$

From equation (5.8) and (5.10) we get,

$$\widehat{a_{ti}} = (-)\sigma_i\theta_{ui}\frac{\theta_{kz}}{\theta_{uz}\theta_{ti}}\frac{\theta_{sy}\theta_{\rho x}\hat{\rho}}{|\theta|}$$

Since, $\theta_{ui} + \theta_{ti} = 1$ and $\left(\hat{w} - \hat{R}\right) = -\frac{\theta_{kz}}{\theta_{uz}\theta_{ti}}\frac{\theta_{sy}\theta_{\rho x}\hat{\rho}}{|\theta|}$

$$\therefore \hat{I} = (+)\sigma_i\theta_{ui}\frac{\theta_{kz}}{\theta_{uz}\theta_{ti}}\frac{\theta_{sy}\theta_{\rho x}\hat{\rho}}{|\theta|} \quad (5.11)$$

To determine the variation in the coefficients, the elasticity of substitution (σ_z) between two factors is given as,

$$\sigma_z = -\frac{\widehat{a_{uz}} - \widehat{a_{kz}}}{\hat{w} - \hat{r}}$$

$$or, \widehat{a_{uz}} = \widehat{a_{kz}} - \sigma_z(\hat{w} - \hat{r}) \quad (5.12)$$

$$and, \widehat{a_{kz}} = \widehat{a_{uz}} + \sigma_z(\hat{w} - \hat{r}) \quad (5.13)$$

Envelop theorem says that,

$$\theta_{uz}\widehat{a_{uz}} + \theta_{kz}\widehat{a_{kz}} = 0$$

$$or, \widehat{a_{uz}} = -\frac{\theta_{kz}}{\theta_{uz}}\widehat{a_{kz}} \quad (5.14)$$

$$and, \widehat{a_{kz}} = -\frac{\theta_{uz}}{\theta_{kz}}\widehat{a_{uz}} \quad (5.15)$$

From equation (5.13) and (5.15) we get,

$$\widehat{a_{uz}} = (+)\sigma_z\theta_{kz}\frac{\theta_{sy}\theta_{\rho x}\hat{\rho}}{|\theta|\theta_{uz}}$$

Since, $\theta_{kz} + \theta_{uz} = 1$ and $(\widehat{w} - \hat{r}) = -\frac{\theta_{sy}\theta_{\rho x}\hat{\rho}}{|\theta|\theta_{uz}}$

From equation (5.12) and (5.14) we get,

$\widehat{a_{kz}} = (-)\sigma_z\theta_{uz}\frac{\theta_{sy}\theta_{\rho x}\hat{\rho}}{|\theta|\theta_{uz}}$

Since, $\theta_{uz} + \theta_{kz} = 1$ and $(\widehat{w} - \hat{r}) = -\frac{\theta_{sy}\theta_{\rho x}\hat{\rho}}{|\theta|\theta_{uz}}$

Differentiating totally equation (5.5) and substituting the values for $\widehat{a_{uz}}, \widehat{a_{ui}}$ and $\hat{\imath}$ we get,

$$\hat{Z} = (-)\frac{\theta_{kz}\theta_{sy}\theta_{\rho x}\hat{\rho}}{\lambda_{uz}\theta_{uz}|\theta|}\left\{\sigma_z\lambda_{uz} + \sigma_i\lambda_{ui} + \sigma_i\frac{\theta_{ui}}{\theta_{ti}}\lambda_{ui}\right\} \quad (5.16)$$

Appendix 6

Differentiating totally equation (5.3) we get,

$$\lambda_{sx}\hat{X} + \lambda_{sy}\hat{Y} = -\lambda_{sy}\sigma_Y\theta_{ky}\frac{\theta_{\rho x}\hat{\rho}}{|\theta|} \quad (6.1)$$

Since, $\hat{S} = 0, \lambda_{sx} + \lambda_{sy} = 1$ and from the previous calculation we know that, $\widehat{a_{sy}} = \sigma_Y\theta_{ky}\frac{\theta_{\rho x}\hat{\rho}}{|\theta|}$

Differentiating totally equation (5.4) we get,

$$\lambda_{kx}\hat{X} + \lambda_{ky}\hat{Y} = \lambda_{ky}\sigma_Y\theta_{sy}\frac{\theta_{\rho x}\hat{\rho}}{|\theta|} + \lambda_{kz}\frac{\theta_{kz}\theta_{sy}\theta_{\rho x}\hat{\rho}}{\lambda_{uz}\theta_{uz}|\theta|}\left(\sigma_z\lambda_{uz} + \sigma_i\lambda_{ui} + \sigma_i\frac{\theta_{ui}}{\theta_{ti}}\lambda_{ui}\right) + \lambda_{kz}\sigma_z\theta_{uz}\frac{\theta_{sy}\theta_{\rho x}\hat{\rho}}{|\theta|\theta_{uz}} \quad (6.2)$$

$$\left[\begin{array}{c}\text{Since, } \hat{K} = 0, \lambda_{kx} + \lambda_{ky} + \lambda_{kz} = 1 \text{ and from previous calculation we know that, } \widehat{a_{ky}} = -\sigma_Y\theta_{sy}\frac{\theta_{\rho x}\hat{\rho}}{|\theta|} \\ and, \hat{Z} = (-)\frac{\theta_{kz}\theta_{sy}\theta_{\rho x}\hat{\rho}}{\lambda_{uz}\theta_{uz}|\theta|}\left\{\sigma_z\lambda_{uz} + \sigma_i\lambda_{ui} + \sigma_i\frac{\theta_{ui}}{\theta_{ti}}\lambda_{ui}\right\} \text{ } and \text{ } \widehat{a_{kz}} = (-)\sigma_z\theta_{uz}\frac{\theta_{sy}\theta_{\rho x}\hat{\rho}}{|\theta|\theta_{uz}}\end{array}\right]$$

Solving equation (6.1) and (6.2) by using Cramer's rule, we get,

$$\hat{X} = -\frac{\lambda_{sy}\theta_{\rho x}\hat{\rho}}{|\lambda||\theta|}\left[\lambda_{ky}\sigma_Y + \lambda_{kz}\frac{\theta_{kz}\theta_{sy}}{\lambda_{uz}\theta_{uz}}\left(\sigma_z\lambda_{uz} + \sigma_i\lambda_{ui} + \sigma_i\frac{\theta_{ui}}{\theta_{ti}}\lambda_{ui}\right) + \lambda_{kz}\sigma_z\theta_{sy}\right] \quad (6.3)$$

$$\hat{Y} = \frac{1}{|\lambda|}\frac{\theta_{\rho x}\hat{\rho}}{|\theta|}\left[\lambda_{kx}\lambda_{sy}\sigma_Y\theta_{ky} + \lambda_{sx}\left\{\lambda_{ky}\sigma_Y\theta_{sy} + \lambda_{kz}\frac{\theta_{kz}\theta_{sy}}{\lambda_{uz}\theta_{uz}}\left(\sigma_z\lambda_{uz} + \sigma_i\lambda_{ui} + \sigma_i\frac{\theta_{ui}}{\theta_{ti}}\lambda_{ui}\right) + \lambda_{kz}\sigma_z\theta_{sy}\right\}\right] \quad (6.4)$$

Since, $a_{sx} = a_{kx} = 1$