How to successfully apply industrial symbiosis

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2024

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August 7, 2024

Abstract

The premise of industrial symbiosis (IS) is that advancing a circular economy that reuses byproducts as inputs in production is valuable for the environment. We challenge this premise in a simple model. Ceteris paribus, IS is an environmentally friendly approach; however, implementing IS may introduce increased pollution into the market equilibrium. The reason for this is that producers' incentives for recycling can be triggered by the income gained from selling recycled waste in the secondary market, and thereby may not align with environmental protection. That is, producers may boost production (and subsequent pollution) to sell byproducts without internalizing the pollution emitted in the primary industry or the recycling process. We compare the market solution to the social optimum and identify a key technology parameter (the share of reused byproducts) that may have mutual benefits for firms, consumers, and the environment.

Keywords: circular economics; industrial symbiosis; pollution; environmental policy

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[*] We wish to thank Adi Ayal and Arye Hillman for comments and discussion and to Miriam Berliner for research assistance. This research was supported by the Heth Foundation for Regulation Research. The authors report there are no competing interests to declare.

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1 Introduction

The global population has increased significantly in recent decades along with the standard of living, resulting in growing production that incurs economic costs and negative externalities for society and the environment such as waste of natural resources, pollution, the destruction of animal habitats, disease, and climate change. With growing environmental concerns, the concept of the circular economy has gained worldwide support as a promising approach for reducing environmental damage, addressing dwindling global resources, and stabilizing the climate. Advancing the circular economy calls for reducing the need to produce new inputs by extending the life cycle of resources (byproducts, waste, and energy) and products through sustainable consumption, recycling, and reuse (Murray et al., 2017; Wysokińska, 2016). Stahel (2016) estimated that the circular economy could reduce countries' greenhouse gas emissions by up to 70% of current levels. Consequently, substantial funding has been allocated to the adoption of circular economy policies and related regulations have been implemented in developed countries.[1]

Industrial symbiosis (IS) is one of the main tools for achieving a circular economy based on synergy between production lines. Rather than a linear production model, with one-time use of inputs, byproducts can potentially be reused. For example , waste from the food industry can be used to produce biodiesel or compost; agricultural waste can be used as fertilizer to grow other crops or as an additive to animal feed, to produce biogas, and even to treat polluted land; waste from producing animal feed and industrial effluent can both be used to grow algae; copper waste can be used as a raw input material in metal factories; plastic packaging can be reused as raw input material to produce plastic goods; sawdust can

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[1] For example, the European Commission Action Plan allocated more than 10 billion euros to advance the implementation of circular economy approaches in 2016–2020, including 1.4 billion euros allocated to sustainable circular industrial processes and management of resources and waste, 1.8 billion euros to encourage the development of innovative technologies, and 5.3 billion euros for the passage of European Union waste legislation (EC, 2015 and EC, 2020). France adopted an ambitious roadmap for transitioning to a circular economy, including a 30% reduction in natural resource consumption as a percentage of GDP per capita between 2010 and 2030, and a 50% reduction in the amount of nonhazardous waste produced, and full plastic recycling by 2025.

be used as a substrate for raising animals; and diligent oversight of construction sites can minimize waste and promote excess materials' reuse.

Researchers and policymakers describe IS as a mutually beneficial strategy that generates environmental value for the public and profits for producers from selling byproducts, while also saving raw input materials and eliminating the logistic and delivery costs of handling waste (landfill, removal of hazardous materials, incineration, or export), and avoiding fees, fines, or lawsuits concerning waste . This study argues that these claims cannot be taken at face value because the introduction of IS may influence producers' decisions regarding production levels (and associated pollution) in the equilibrium.

Using a simple theoretical model, we demonstrate that producers' interest in maximizing profits may not coincide with environmental protection, and may cause two potential inefficiencies in IS implementation. First, although economic and environmental justifications for adopting IS are compelling, high fixed costs and economies of scale may prevent its implementation in a competitive market. Previous research has predominantly focused on this inefficiency, which has been called the "gap in energy efficiency" (Gerarden et al., 2017). In line with this literature, our results suggest that this gap is more likely to occur when a large number of (identical) firms are in the market. As each firm holds a minor market share, elevated average recycling costs may diminish firms' motivation to introduce IS. In such cases, policies could be implemented to reduce adoption costs, including the costs of acquiring knowledge, making transactions, coordination, searching for potential byproduct buyers, transportation, logistics, regulatory and bureaucratic burdens, and production infrastructure modification (see Ashraf et al., 2016).

We also identify a second form of inefficiency. When producers decide to adopt IS, the extra profit from trade in byproducts and savings in environmental taxes encourages firms to boost production, along with subsequent pollution. The rising production benefits consumers via price reduction, but may also be environmentally harmful. Specifically, if the IS technology is highly polluting or highly ineffective in reusing byproducts, then it cannot fully offset the increased pollution generated by the augmented production. For example, recycling electronics is supposed to protect the environment from the massive amounts of waste created globally each year. Huge volumes of computers, keyboards, cables, and screens are shipped to developing countries for recycling. However, electronic goods are burned during the process of dismantling to retrieve the copper or aluminum that are sold separately, which causes emission of poisonous gases. Some studies have determined that households or firms are sometimes encouraged to adopt new technologies with subsidies, although the technologies lack proven efficacy and economic justification (Gerarden et al., 2017; Fowlie et al., 2018). In these circumstances, it is crucial to engage in cost–benefit analysis before providing subsidies for such technologies (Keiser and Shapiro, 2019).[2]

Notably, we do not suggest refraining from IS altogether, but addressing potential inefficiencies in current approaches and examining how to successfully apply IS. For policymakers who prioritize environmental protection, our suggestion is not to automatically consider all IS initiatives as win–win strategies. Instead, each IS technology must be thoroughly evaluated to determine its environmental impact on the equilibrium. If the available technologies are found to be environmentally harmful or not cost effective, policy efforts should then focus on research and development (R&D) of more effective IS technologies.

Our comparative analysis emphasizes the proportion of reused byproducts as a critical parameter with the potential to be uniquely beneficial to firms, consumers, and the environment. In contrast, policies targeting pollution generated by production processes or environmental taxation potentially cause a tradeoff between firms, consumers, and the environment. Therefore, we contend that a strategically designed environmental policy can foster sustainable economic growth within the equilibrium.

Our study's main contribution is its introduction of a thorough framework to investigate

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[2] Recycling involves the use of energy to modify waste for a new use, and sometimes the recycled material does not replace other production but is added to existing production and encourages increased consumption. It is only possible to obtain positive environmental outcomes based on better resource use and more efficient use of energy when the same production level is maintained (Zink and Geyer, 2017).

the countervailing effects of IS environmental endeavors and analyze their impact concerning thresholds for technology adoption, total surplus, and pollution in the equilibrium. Using comparative statistics, we then map cases where policy intervention is needed and offer policy suggestions. In the final section, we analyze the market solution compared with a social planner that internalizes the environmental cost of production, demonstrating that although pollution is lower in the social optimum, the production and total surplus are larger in the market solution.

The remainder of the paper is organized as follows. Section 2 presents a literature review. Section 3 describes the economic framework, followed by the results of the model in Section 4. Section 6 addresses the social optimum and Section 7 imparts concluding remarks and discussion. Proofs are presented in the Appendix.

2 Literature Review

This study falls within a strand of research in environmental economics that has focused on exploring potential remedies for resource depletion. The literature has largely praised circular production as a sustainable alternative to the standard linear production approach. The main argument of such research is that rather than exhausting available resources, IS significantly reduces waste by establishing closed production loops, where byproducts become valuable inputs for secondary industries (Wysokińska, 2016; Stahel, 2016; Urbinati et al., 2017). Based on a review of more than 500 studies, Merli et al. (2018) argued that additional policy attention must be devoted to designing new approaches for production and consumption that slow existing materials loops.

Nevertheless, although most related articles have strongly advocated IS technologies, they have failed to consider the consequences in the equilibrium. To the best of our knowledge, this study is the first to introduce a theoretical model examining pros and cons of IS adoption to systematically analyze its efficacy in the equilibrium. Our findings reveal potential inefficiencies in which adoption of IS technologies may augment pollution in the equilibrium, indicating the need for a comprehensive evaluation of costs and benefits.

Another important inquiry in the economic literature pertains to the economic feasibility of environmental behavior and the tradeoff between environmental actions and other policy objectives. Can a country simultaneously maintain growth and protect the environment? We aim to answer this question, using our framework to examine the impact of IS environmental endeavors on social welfare. The traditional view in the literature has been that environmental regulations have had a negative impact on competition by placing additional burdens on companies. Companies face the direct costs of preventing pollution and the required investment to comply with regulations negates other profitable opportunities (Xie et al., 2017).

An alternative hypothesis, which is referred to as the Porter hypothesis, asserts that well-designed environmental regulations encourage innovation, providing a competitive advantage that offsets the costs of complying with regulations. Therefore, environmental policy has the potential to contribute to consistent growth while reducing environmental damage (Porter and Van der Linde, 1995, Zhang et al., 2017; Fernández et al., 2018). Several studies have examined the validity of the Porter hypothesis, in a number of European countries (Franco and Marin, 2017) and in the Dutch economy (Van Leeuwen and Mohnen, 2017). Xie et al. (2017), Aghion et al. (2016) and Acemoglu et al. (2016) confirmed that the Porter hypothesis holds in cases of market-based regulations but not in instances of command-and-control regulations. Moreover, ecological innovations that target pollution reduction may have a diminishing effect on total factor productivity (TFP) . Conversely, when regulations incentivize eco-friendly innovations focused on efficient resource use and resource saving, TFP tends to increase .

Our results confirm these findings, indicating that the adoption of IS, particularly when highly effective in reusing byproducts, reduces pollution levels along with increased total surplus in the equilibrium.

3 The Model

We assume that there are n identical firms in the industry. While the firms are competitive, there is no free entry of firms into the market. Each firm i produces a quantity q_i of a product and $Q = \sum_{i=1}^{n} q_i$ is the total product quantity. We make a standard assumption that the marginal cost of production increases with the number of units a firm produces. In our benchmark case, without IS adoption there is no fixed cost. Accordingly, the total cost of firm i is given as cq_i^2, $c > 0$. The demand function is given by

$$p = a - bQ, \tag{1}$$

where p denotes the product price, and $a, b > 0$ are exogenous parameters.

The production process generates pollution, waste, or byproduct emissions that are released into the air, water or ground. The pollution is assumed to be proportional to production,

$$Poll = gQ, \tag{2}$$

where g is a parameter denoting the pollution released per product unit. Accordingly, the production of Q units of a product releases gQ units of pollution. Firms pay a pollution fine (or environmental tax) at a rate of d, and the total fine paid by each producer is dgq_i.[3] In the benchmark case, without IS, the profit of each firm i is as follows:

$$\pi_i = q_i(p - dg) - cq_i^2. \tag{3}$$

Furthermore the consumer surplus, considering demand function (1), is as follows:

$$CS = \frac{bQ^2}{2}. \tag{4}$$

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[3] In many cases, fines are levied for excess pollution. Therefore, g can be considered to represent excess pollution above a certain standard, when aiming below the standard is not feasible for firms. For a discussion on environmental taxes, see Acemoglu et al. (2016) and Barrage (2020).

The total surplus is the sum of firms' profits and consumer surplus, is as follows:

$$TS = n\pi_i + CS. \tag{5}$$

IS is an environmental endeavor with the unique feature of providing firms with an opportunity to earn additional profits if the cost of adoption is not prohibitive. A firm that adopts IS may reduce pollution by selling a portion of its byproducts for recycling or reuse as inputs in other production processes. Denote the *effectiveness of industrial symbiosis*, α, $0 < \alpha < 1$, as the proportion of byproducts that can be reused (or recycled) in other production processes. Accordingly, if a firm adopts IS, then it sells $\alpha g q_i$ units of byproduct in the secondary market. Consequently, the firm's pollution is reduced to $(1-\alpha)gq_i$ units. Note that pollution level is a function of the pollution emitted per unit of production g (how polluting the production technology is) and the effectiveness of the IS technology (α) given the production quantity. Therefore, a production process that emits many pollutants (high g) but uses effective IS technologies (a high proportion of pollutants used in other processes; high α) may be more environmentally friendly than a production process that emits fewer pollutants but IS effectiveness is negligible.

Byproducts' market price is exogenously given by p_g. An IS transaction (selling a byproduct in the market) elicits two gains for a firm. Sales generate a revenue of p_g, while reducing the net pollution, which lowers the associated fine. In addition, IS implementation requires a fixed cost, which is denoted by c_g, including the cost of acquiring knowledge and searching for potential byproduct buyers, transaction, coordination, production infrastructure modification, transportation, logistics, and regulatory and bureaucratic burdens. Accordingly, after adopting IS firm i's profits are determined as follows:

$$\pi_i^{symb} = q_i^{symb}[p^{symb} - (1-\alpha)dg + \alpha p_g g] - c(q_i^{symb})^2 - c_g \tag{6}$$

Rearranging equation (6) reveals that recycling a fraction α of the waste proportional to output decreases marginal cost, generating increased profitability of $\alpha g(d + p_g)$ per unit of production,

$$\pi_i^{symb} = q_i^{symb}[p^{symb} - g(d - \alpha(d + p_g))] - c(q_i^{symb})^2 - c_g. \tag{7}$$

Another important parameter to consider is the share of pollution emitted in the process of IS, $k < 1$. The process of IS may also produce pollution of $k\alpha g q_i$, which is emitted during transportation of byproducts, recycling or modification for reuse, and during reuse in secondary production processes. Accordingly, if IS is adopted, the associated pollution is the sum of pollution emitted in the primary production process and the pollution emitted using the IS technology as follows:

$$Poll_{symb} = [1 - \alpha(1 - k)]gQ_{symb}. \tag{8}$$

Firms are identical; hence, the equilibrium is symmetric, where $q_1 = q_2 = \ldots = q_n = q$, $\pi_1 = \pi_2 = \ldots = \pi_n = \pi$ without IS, and $q_1^{symb} = q_2^{symb} = \ldots = q_n^{symb} = q^{symb}$, $\pi_1^{symb} = \pi_2^{symb} = \ldots = \pi_n^{symb} = \pi^{symb}$ with IS.

4 Results

We begin by solving the equilibrium in a benchmark case without IS. Firms maximize their profits (3) to obtain the equilibrium quantity, as follows:

$$q = \frac{a - dg}{2c + bn}. \tag{9}$$

We assume that $a - dg > 0$, such that a market for the product exists in the equilibrium. Substituting the quantity produced (9) in the demand function (1) yields the market price,

$$p = \frac{2ac + bndg}{2c + bn}. \tag{10}$$

Using the quantity produced (9) and the market price (10), we obtain the firm's profit, consumer surplus, total surplus, and the level of pollution in the equilibrium, as follows:

$$\pi = \frac{c(a-dg)^2}{(2c+bn)^2}. \tag{11}$$

$$CS = \frac{bn^2[a-dg]^2}{2[2c+bn]^2}. \tag{12}$$

$$TS = \frac{n(a-dg)^2}{2[2c+bn]}. \tag{13}$$

$$Poll = gQ = \frac{ng(a-dg)}{2c+bn}. \tag{14}$$

Note that our assumption of an existing market for the product in the equilibrium, $a-dg > 0$, ensures that the firm's profit (11) and consumer surplus (12) are positive in the equilibrium. The equilibrium equations (9)-(14) clearly indicate that a larger tax burden (dg) reduces firms' profits, subsequently compelling firms to reduce their production, shifting the supply curve upward. As a result, price rises and pushes consumer surplus downward, while the pollution level also declines. Therefore, increasing fines reduces the pollution by lowering production, which causes a deadweight loss.

Our next step introduces IS as an alternative approach for reducing pollution. Firms determine whether to adopt IS based on profit considerations. If an individual firm adopts IS, given the quantity produced (9) and the market price (10), its profit (6) can be obtained as follows:

$$\pi^{'} = \pi + q[\alpha g(d+p_g)] - c_g. \tag{15}$$

According to equation (15), when a firm decides whether to adopt IS, it weighs the cost of adoption (c_g) against the potential profitability gain ($\alpha g(d+p_g)$), which represents the reduction in its marginal cost per unit of production. Based on the reusable share of byproducts per unit of production (αg), the company gains $d+p_g$ from the sale of byproducts in the secondary market and fine reduction. Therefore, a firm benefits from adopting IS if the cost of adoption (c_g) is lower than the profit gain, as follows:

$$\pi^{'} > \pi \Leftrightarrow c_g < q[\alpha g(d + p_g)]. \tag{16}$$

Substituting the quantity produced (9) into inequality (16), we find that a firm adopts IS when the adoption cost is slower than a threshold level c_g^*,

Definition 1. *Cutoff level of adoption cost.*

We define the cutoff level of adoption cost as

$$c_g^* = \frac{(a - dg)\alpha g(d + p_g)}{2c + bn},$$

which is positive by our assumption of an existing market for the product in the equilibrium, $a - dg > 0$. Note that the threshold is calculated given the quantity produced by the firm in the benchmark case. However, revealed preference implies that a single firm still prefers to adopt IS when allowed to change production since it will only change quantity to gain profits. Therefore, an adoption cost that is lower than the cutoff threshold ($c_g < c_g^*$) is a sufficient condition for firms to adopt IS, and since all firms are identical, all firms adopt IS.

Proposition 1. *IS adoption*
Assume $c_g < c_g^*$. Then, there exists an equilibrium where all firms adopt IS. The cutoff level of adoption cost c_g^* increases in α and in p_g and declines in n.

The proof for Proposition 1 is presented in the Appendix. The final segment of Proposition 1 outlines three parameters that affect IS adoption by altering the threshold level of adoption cost (c_g^*). First, IS becomes more profitable for firms when byproducts' market price (p_g) rises. Second, when IS effectiveness (α) increases, the firm sells a larger share of its byproducts in the market, providing more profits and reducing the tax burden. Third,

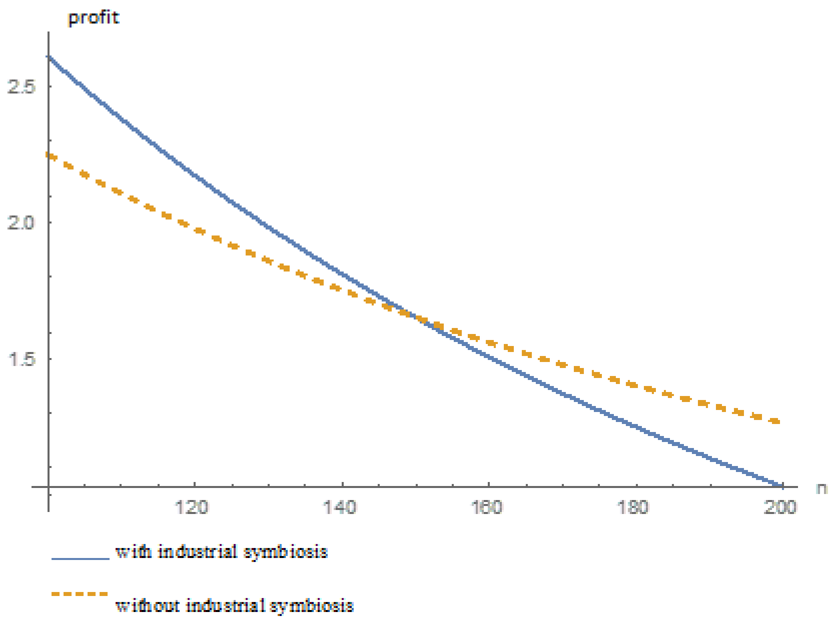

Figure 4.1: Firms' profits with and without industrial symbiosis. Note: The x-axis represents the number of firms in the market (n), and the y-axis represents firms' profits.

the market structure affects the attractiveness of IS adoption attributable to economies of scale. A small number of firms, each holding a large market share, lowers the average cost per firm, which enhances the profitability of IS adoption.

Figure 4.1 illustrates how the number of firms in the market (n) (x-axis) affects each firm's profits (y-axis). The blue line indicates firms' profits in the IS equilibrium and the red line exhibits profits in the benchmark case (without IS). The findings reveal that firms' profits decline in n, regardless of whether IS is implemented. As the number of firms rise, each producer's revenue declines because of reduced market share; however, the slope is steeper in the case of IS because of economies of scale. With a limited number of producers and each firm holding a significant market share, each producer's revenue is ample enough to cover the fixed cost of IS adoption, making IS the preferable option. Nevertheless, as the number of firms rises, smaller market shares amplify the average cost per firm in the IS equilibrium, making IS equilibrium less advantageous for firms compared with the benchmark case.

The effect of the market structure on IS adoption stresses a well-known problem in the adoption of environmental solutions, or more generally, the adoption of technologies with external benefits to society, that has been called the gap in energy efficiency . In some cases, efficient technologies can facilitate energy savings that would benefit society, but they are not adopted because companies exclusively bear the economic cost of adopting the technology. This feature generates a gap between societal and firm benefits wherein the benefit to society is underpriced (Gerarden et al., 2017). An efficiency gap may call for government intervention to encourage firms' IS adoption, particularly when small firms are involved.

However, IS is unique from other environmental endeavors in that firms have internal incentives to implement it and even increase production because of the profitability gain from selling byproducts and reduced fines. Nevertheless, there is no guarantee that the profitability gained from IS causes firms to fully internalize the benefits to society. Notably, in the case of IS, in addition to usual challenges of under-adoption, the profitability gain may cause over-adoption, depending on the market structure and other parameters. Firms may adopt ineffective and highly polluting IS technologies that increase environmental damage in the equilibrium. We expand on this point after solving the equilibrium with IS.

To solve the equilibrium with IS, maximizing the firm's profit (6) yields the following quantity in the equilibrium,

$$q_{symb} = \frac{a - g(d - \alpha(d + p_g))}{2c + bn}. \tag{17}$$

Substituting the quantity produced (17) in the demand function (1) yields the market price, as follows:

$$p_{symb} = \frac{2ac + nbg(d - \alpha(d + p_g))}{2c + bn}. \tag{18}$$

We present the solutions for the remainder of the equilibrium (CS_{symb}, π_{symb}, and TS_{symb}) to the Appendix. In the Appendix, we also verify that a single firm does not choose to deviate from the IS equilibrium (given that the adoption cost is lower than the threshold level c_g^*).

We next analyze the implications of IS. To facilitate notation, denote the gaps between the benchmark case and the IS equilibrium in production, price, consumer surplus, firm's profit, and total surplus by $q_{symb} - q = \Delta q$, $p - p_{symb} = \Delta p$, $CS_{symb} - CS = \Delta CS$, $\pi_{symb} - \pi = \Delta \pi - c_g$, $TS_{symb} - TS = \Delta TS$, respectively. We summarize the comparison between the two equilibria in Proposition 2.

Proposition 2. *Comparison of equilibrium with and without industrial symbiosis*

(1) $q_{symb} > q$, $p_{symb} < p$, and $CS_{symb} > CS$.

(2) Assume $c_g < c_g^*$ and $n < \frac{\alpha g c(d+p_g)}{b(a-dg)}$. Then, $\Delta \pi > 0$ and $\Delta TS > 0$.

(3) Δq, Δp, ΔCS, $\Delta \pi$, ΔTS increase in α, d, and p_g.

(4) Δq, Δp and ΔCS increase in g.

According to Proposition 2, IS adoption increases in production quantity. When firms sell a share (α) of their byproducts for reuse, it is then profitable to increase production because of the revenue received per unit (p_g) and reduced fines (d). In other words, higher profitability gain from IS ($\alpha g(d + p_g)$) increases firms' incentive to raise production. The outward shift in the supply curve causes a quantity increase and a price decrease, which is beneficial for consumers.

As shown in Figure 4.1, Proposition 2 also emphasizes the role of economies of scale in our framework. Let IS adoption cost be below the cutoff level (c_g^*). Subsequently, if the number of firms is sufficiently small (denoted as $n < \frac{\alpha g c(d+p_g)}{b(a-dg)}$), then each firm's market share reduces the average cost in the IS equilibrium, generating higher profits compared with the benchmark scenario.

To complete the analysis, we next measure how IS adoption affects pollution levels, considering that the primary goal of IS is to reduce pollution. To examine whether this goal is achieved in the equilibrium, we compare pollution levels with and without IS. To facilitate the calculation, we denote the pollution gap as $\Delta Poll = Poll_{symb} - Poll$. Substituting pollution as a function of production levels (equations (2) and (8)) in the pollution gap yields the following:

$$\Delta Poll = gn[\Delta q - q_{symb}\alpha(1-k)] \tag{19}$$

Rearranging equation (19) implies the following:

$$\Delta Poll \begin{cases} > 0 & , \frac{\Delta q}{q_{symb}} > \alpha(1-k) \\ \leq 0 & , \frac{\Delta q}{q_{symb}} \leq \alpha(1-k) \end{cases} \tag{20}$$

The resulting inequalities (20) illustrate two opposing effects on the pollution gap. IS adoption achieves its goal of reducing pollution levels when the IS technology is sufficiently green (indicated by a low k) and highly effective (with a high proportion of reusable byproducts (α)). We denote the quality of IS technology as $\alpha(1-k)$ (the right-hand side of inequalities (20)). A subsequent improvement in IS technology reduces the pollution gap.

Nevertheless, as shown in the left-hand side of the inequalities (20), IS adoption affects firms' decisions in the equilibrium. The profitability gain of IS from the sale of byproducts and reduced fines induces firms to increase production ($\Delta q > 0$ according to Proposition 2). Because firms do not internalize their effect on the environment, augmented production can paradoxically increase pollution, surpassing those in the benchmark scenario.

An extreme example of a highly polluting IS technology occurs when $k \to 1$. In this case, the pollution emitted during the IS process completely offsets the environmental gain per unit of production achieved through reusing the byproducts generated by the primary industry. Therefore, the right-hand side of inequalities (20) tends toward 0, and the left-hand side is positive ($\Delta q > 0$) since IS adoption triggers augmented production, elevating pollution above that of the benchmark scenario $\Delta Poll > 0$).

We next substitute the market quantities q_{symb} and q (equations (9), (17)) in equation (19) to obtain the pollution gap as a function of the parameters of the model, as follows:

$$\Delta Poll = \frac{\alpha g n}{2c + bn}[g(d + p_g) - (1 - k)[a - dg + \alpha g(d + p_g)]]. \tag{21}$$

Then, we can easily derive the necessary and sufficient condition for heightened pollution under IS adoption.

Proposition 3. *Comparison of pollution with and without industrial symbiosis*
Let $\phi = \frac{1}{1-k} - \frac{a-dg}{g(d+p_g)}$.

(1)

$$\Delta Poll = \begin{cases} \geq 0 & , \alpha \leq \phi \\ < 0 & , \alpha > \phi \end{cases}$$

(2) $\Delta Poll$ increases in g, d, p_g, and k.

(3)

$$\frac{\partial \Delta Poll}{\partial \alpha} = \begin{cases} \geq 0 & , \alpha \leq \frac{1}{2}\phi \\ < 0 & , \alpha > \frac{1}{2}\phi \end{cases}$$

According to Proposition 3, IS adoption increases pollution if $\alpha < \phi$. In these circumstances, firms' gain from reduced fines and selling byproducts is sufficiently large relative to IS technology quality, $\alpha(1-k)$. Consequently, the augmented production increases pollution levels in the equilibrium relative to the benchmark scenario, $\Delta Poll > 0$. Otherwise, if the IS technology is relatively effective, $\alpha > \phi$, then its implementation successfully reduces pollution in the equilibrium, and increased α further lowers pollution in the IS equilibrium, $\frac{\partial \Delta Poll}{\partial \alpha} < 0$. Note that since $1 > \alpha > 0$, then $\phi \geq 1$ indicates a sufficient condition for $\alpha < \phi$ and $\Delta Poll > 0$. Similarly, $\phi < 0$ implies that $\Delta Poll < 0$ and $\frac{\partial \Delta Poll}{\partial \alpha} < 0$.

The second part of Proposition 3 asserts that the pollution gap expands when primary or secondary production processes are more polluting (g or k rise). Moreover, increased fines or elevated byproduct prices (d or p_g) incentivize firms to raise production, which increases the pollution gap.

The final part of Proposition 3 focuses on the nonmonotonic effect of α on the pollution gap. Apparently, α, representing IS effectiveness (measured by proportion of reusable byproducts), has two contradictory effects on the pollution gap. An increase in α reduces pollution per unit of production while also boosting firms' profitability from IS, which raises production and pollution. Our results indicate that when $\alpha < \frac{1}{2}\phi$, the latter effect occurs; however, when α is sufficiently large, $\alpha > \frac{1}{2}\phi$, an increase in α reduces pollution in the IS equilibrium relative to the benchmark scenario.

To illustrate our results, we present three model simulations with different α values.[4] In Figures 4.2-4.4, the x-axis denotes byproducts' price levels, p_g, and the y-axis describes the corresponding pollution emissions with IS (the blue line) and without IS (the red dashed line). The levels of α in the simulations differ as follows. In Figure 4.2, the IS technology is of low effectiveness, and only 1% of the byproducts can be reused ($\alpha = 0.01$), and in Figure 4.3, IS technology is highly effective, and 90% of the byproducts are reusable ($\alpha = 0.9$), and in Figure 4.4 we assume medium IS effectiveness, with 50% reusable byproducts ($\alpha = 0.5$).

In Figure 4.2, the IS technology is considerably ineffective, $\alpha < \phi$ for all p_g, such that byproducts are only minimally reusable. Therefore, considering firms' incentive to increase production ($\Delta q > 0$), IS implementation raises pollution for all p_g ($\Delta q > 0$). Figure 4.3 illustrates the opposing case of highly effective IS technology, where 90% of the byproducts can be reused ($\alpha > \phi$) for all p_g. In this case, IS implementation compensates for increased production and reduces pollution for all p_g, $\Delta Poll < 0$.

It is clear in both figures that a rise in byproducts' market price in the IS equilibrium (x-axis) induces firms to increase production, elevating pollution relative to the benchmark scenario ($\frac{\partial \Delta Poll}{\partial p_g} > 0$; Proposition 3). However, the slope in Figure 4.3 is less steep. With highly effective IS technology, the remaining pollution from reusing 90% of the byproducts is low, even for high byproduct prices (it is easy to verify that $\frac{\partial \Delta Poll}{\partial p_g \partial \alpha} < 0 \Leftrightarrow \alpha(1-k) > \frac{1}{2}$).

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[4] A preliminary sketch of the model and simulations appear in Hebrew in Heth Foundation series Mehkarei Regulazia.

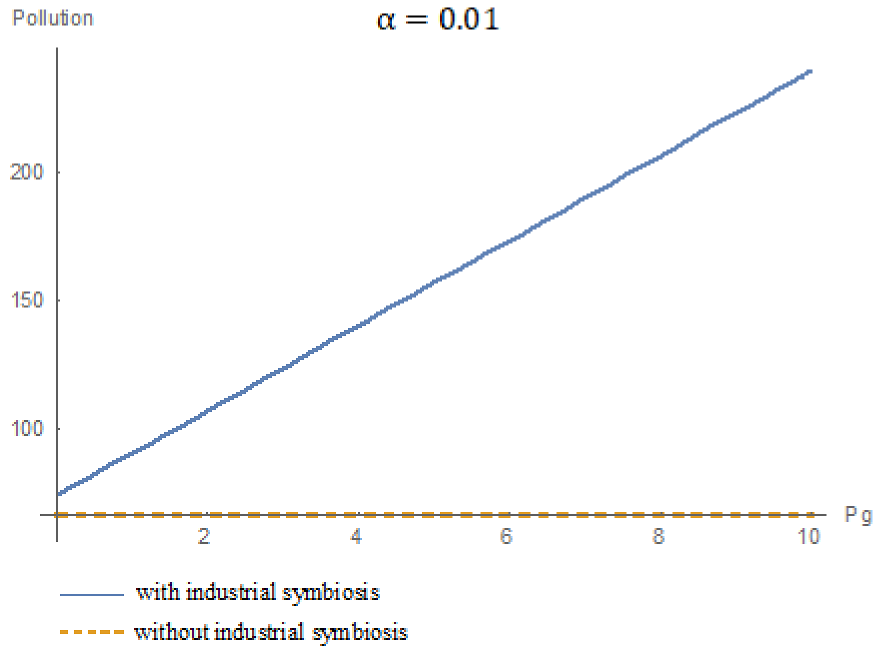

Figure 4.2: Pollution levels with and without IS for $\alpha = 0.01$. Note: Byproducts' market price of p_g is on the x-axis and pollution levels are on the y-axis.

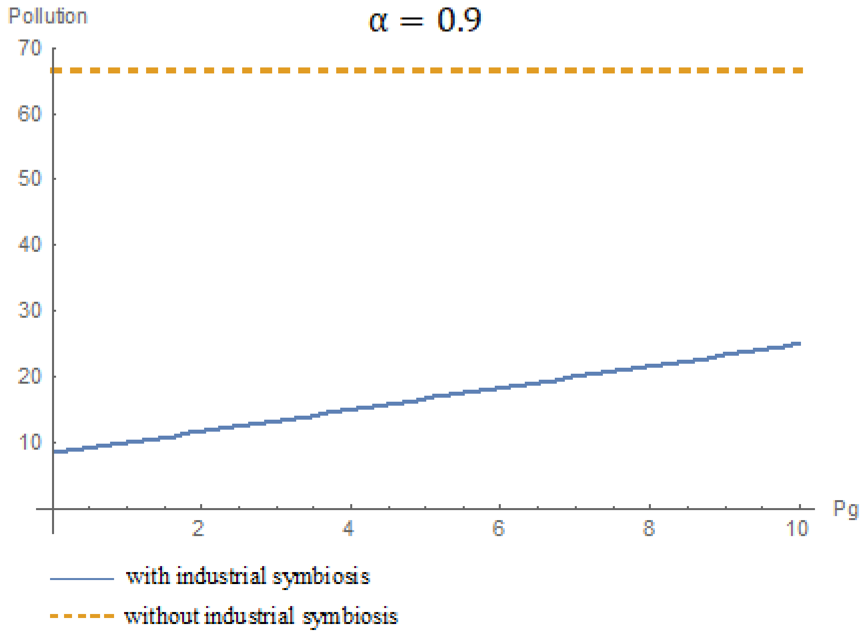

Figure 4.3: Pollution levels with and without IS for $\alpha = 0.9$. Note: Byproducts' market price of p_g is on the x-axis and pollution levels are on the y-axis.

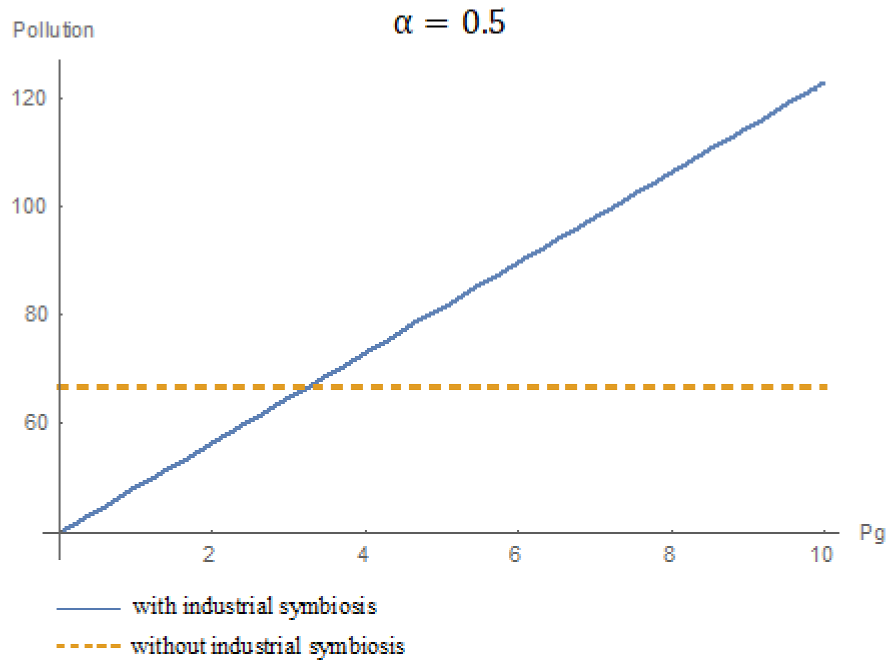

Figure 4.4: Pollution levels with and without IS for $\alpha = 0.5$. Note: Byproducts' market price of p_g is on the x-axis and pollution levels are on the y-axis.

Figure 4.4 illustrates an industry with medium IS effectiveness at 50% byproduct reuse. In this case, the effect of IS adoption on pollution depends on the byproducts' price. If the price is low, then IS implementation reduces pollution, whereas a high price reverses this result. The rationale for this is that a high byproduct price can be highly profitable for the firms, triggering a substantial boost in production and subsequent pollution, which harms the environment relative to the benchmark case.

Notably, IS technology is expected to improve over time with the development of more eco-friendly technologies. In practice, emissions standards for older factories are less strict and allow the use of the Best Practical Technology to maintain profitability. As technological barriers decline and such technologies become less expensive, standards are updated and newer factories are expected to use the Best Available Technology (i.e., greener advanced technologies). Accordingly, we can interpret Figures 4.2-4.4 as describing the effect of technological improvements in IS effectiveness over time. Figure 4.2 describes an initially low effective IS with increased pollution in the equilibrium. In time, R&D investment can promote the innovation of more effective technologies, and IS may increase or decrease pollution depending on byproducts' market price, as shown in Figure 4.4. Eventually, technologies in the industry are sufficiently environmentally friendly, and their adoption reduces pollution for all p_g, as illustrated Figure 4.3.

5 Policy Implications

Significant policy implications arise from our propositions. Adopting IS may establish a tradeoff between total surplus and pollution. For example, implementing IS in industries with relatively high byproduct prices or pollution penalties (p_g or d, respectively) can generate profitability gains for firms. Substantial profits per unit of production are translated into higher production and lower prices, delivering gains for consumers in terms of surplus ΔCS), while also causing environmental losses relative to the benchmark scenario ($\Delta Poll > 0$; Propositions 2 and 3).

Logically, any consideration of social welfare must include total surplus and pollution. Therefore, policy should *not* focus only on implementing IS technologies as much as possible. In contrast to solely focusing on quantity, policymakers must strategically target IS technologies' quality, $\alpha(1-k)$, i.e., adoption and innovation of sufficiently effective and green IS technologies, that satisfy $\alpha > \phi$ to achieve $\Delta Poll < 0$. When IS adoption causes environmental gains ($\alpha > \phi$), our results indicate that further advancement in IS quality ($\alpha(1-k)$) reduces pollution without generating a tradeoff between total surplus and pollution, in contrast to the other parameters. Moreover, IS effectiveness (the proportion of byproducts that can be reused; α) is a key parameter that can benefit all stakeholders. A larger α generates a larger surplus for firms and consumers in addition to lower pollution. Therefore, our results indicate that policy efforts should prioritize enhancing IS quality to reduce pollution without harming total surplus.

A notable finding from our results is that IS should be adopted with caution and on a case-by-case basis, considering all available IS technologies and their potential effect on the equilibrium. When the IS technology is sufficiently green and effective, the regulator's role should be one of reducing the cost of adoption to assure implementation. However, in industries with low-quality IS technologies, policies should provide incentives for technological improvement, considering the response of all stakeholders in the equilibrium so that IS will satisfy its entitled goal of enhancing sustainability without harming economic growth. Next, we complement the market equilibrium analysis with a social welfare analysis, incorporating the external cost of pollution.

6 Social Optimum

Thus far, we analyzed the effect of IS adoption on total surplus and on pollution levels separately. However, a social planner would incorporate the environmental cost of production and total surplus

In our framework, recycling a fraction α of the waste is proportional to output, gq_i, decreases the marginal (private) cost of production by $\alpha g(d + p_g)$, as follows

$$MPC_{symb}(q_i) = 2cq_i + (1 - \alpha)dg - \alpha p_g g = 2cq_i + dg - \alpha g(d + p_g).$$

where

$$MPC_{symb}(Q) = \frac{2cQ}{n} + g(d - \alpha(d + p_g)).$$

For simplicity, we assume that each unit of pollution generates a constant marginal social cost $\gamma > 0$ to the environment. Then, using equation (8), the marginal environmental cost of production is obtained as follows:

$$MEC_{symb}(Q) = \delta Q,$$

where $\delta = \gamma g(1 - \alpha(1 - k))$. Note that $\delta > 0$ since $\gamma > 0$. The marginal social cost is then the sum of $MPC_{symb}(Q)$ and $MEC_{symb}(Q)$, as follows:

$$MSC_{symb}(Q) = (\frac{2c}{n} + \delta)Q + g(d - \alpha(d + p_g).$$

Using the demand function (equation (1)), consumers' marginal benefit (marginal willingness to pay) is obtained as follows:

$$MPB_{symb}(Q) = a - bQ.$$

To optimize social welfare, the social planner equates the marginal social cost to the marginal benefit, $MPB_{symb}(Q) = MSC_{symb}(Q)$. Rearranging this equation obtains the socially optimal output,

$$Q^o_{symb} = \frac{a - g(d - \alpha(d + p_g))}{\frac{2c}{n} + b + \delta}. \tag{22}$$

Since we have n identical firms, the socially optimal quantity per firm i is as follows:

$$q^o_{symb} = \frac{a - g(d - \alpha(d + p_g))}{2c + n(b + \delta)}. \tag{23}$$

We substitute the socially optimal output (22) in equation (8) to obtain the corresponding optimal level of pollution,

$$Poll^o_{symb} = \frac{ng[1 - \alpha(1 - k)][a - g(d - \alpha(d + p_g))]}{2c + n(b + \delta)}. \tag{24}$$

We then calculate the total surplus at the social optimum as the difference between the willingness to pay and the total cost, as follows:

$$TS^o_{symb} = \frac{n[2c + n(b + 2\delta)][a - g(d - \alpha(d + p_g))]^2}{2[2c + n(b + \delta)]^2} - nc_g \tag{25}$$

Social welfare integrates marginal environmental cost MEC_{symb} and total surplus, $SW = TS - \frac{\delta(Q)^2}{2}$. Substituting the socially optimal output and total surplus (equations (23) and (25)), we obtain the social optimum, as follows:

$$SW^o_{symb} = \frac{n[a - g(d - \alpha(d + p_g))]^2}{2(2c + n(b + \delta))} - nc_g \tag{26}$$

We next compare the IS market solution to the IS social optimum.

Proposition 4. *Comparison of IS equilibrium with IS social optimum*

Assume that IS is adopted; then,

1. $Q^o_{symb} < Q_{symb}$, $Poll^o_{symb} < Poll_{symb}$.
2. $TS^o_{symb} < TS_{symb}$, $SW^o_{symb} > SW_{symb}$
3. As γ increases, the market solution further diverts from social optimum.
4. $\frac{\partial TS^o_{symb}}{\partial \alpha} > 0$.
5. $\alpha \leq \frac{1}{2}\phi$ is a sufficient condition for $\frac{\partial Poll^o_{symb}}{\partial \alpha} > 0$. Additionally,

$$\frac{\partial Poll^o_{symb}}{\partial \alpha} = \begin{cases} \geq 0 & , \alpha \leq \alpha^* \\ < 0 & , \alpha > \alpha^* \end{cases}$$

where α^* satisfies $\frac{2c+nb}{2c+n(b+\delta(\alpha^*))}(\alpha^* + \frac{a-dg}{g(d+p_g)}) = \frac{1}{1-k} - \alpha^*$, and $\alpha^* > \frac{1}{2}\phi$.

Firms do not internalize the environmental cost of production; therefore, production exceeds the social optimum and pollution levels in the market solution consequently exceed socially optimal levels, and the social planner achieves a lower total surplus than the market solution. The differences between the social planner's solution and the market solution increase in γ. An augmented γ reflects a larger external cost of pollution. The social planner internalizes the environmental cost and further reduces production relative to the market solution.

We also analyze how the key parameter of the model, the share of reusable byproducts (α), affects the social optimum. Similar to the IS market equilibrium, we find a nonmonotonic effect of α on pollution levels. For a sufficiently large IS technology effectiveness (i.e., $\alpha > \alpha^*$), pollution levels decline in α. Under these circumstances, prioritizing the enhancement of α represents a win–win strategy that will result in environmental benefits and total surplus gains.

7 Discussion

Given the substantial budget allocations for circular economy implementation in developed countries , evaluating IS impact in the equilibrium has significant policy implications, particularly since producers are unlikely to internalize the full impact of their decisions on the environment. Contrary to the prevailing belief that IS can be a panacea for sustainability , our results indicate that the adoption of IS technologies does not guarantee environmental gains. IS technology adoption may generate profitability gains for firms from selling byproducts and environmental tax savings, driving them to boost production, augmenting the amount of byproducts. Consequently, implementation of low-quality IS technology that converts a low share of byproducts for circular use can potentially harm the environment in the equilibrium because it does not offset the negative effect of increased production on the environment.

In these circumstances, policymakers should not encourage IS adoption, instead incentivizing R&D to develop more effective environmentally friendly technologies. This way, considering the reaction of all stakeholders in the equilibrium, IS could satisfy its goal of enhancing sustainability. Policy incentives for R&D could include government subsidies, environmental taxes, or other methods such as compelling companies to allocate a portion of profits to R&D. Another way to push firms to develop improved IS technologies is by raising public awareness and employing informal regulations, which could include publicly disclosing information about polluting companies or initiating consumer boycotts. An apparent public willingness to be part of the effort to protect the environment has emerged. In Switzerland, about 70% of consumers prefer goods that are produced with less carbon emissions and a large proportion would even pay more for such products (Blasch and Farsi, 2014).

Therefore, our policy recommendation for advancing the legal platform for IS is to work on a case-by-case basis. Our results emphasize the necessity of conducting preliminary examinations of the efficiency of each IS technology and its impact on the equilibrium. A thorough cost–benefit analysis of the IS technologies that are currently available in the market is crucial to identify industries where IS can improve sustainability, rather than industries where the available IS technologies may harm the environment. Further technological progress is necessary in industries with low-quality technologies prior to implementation. Therefore, policy efforts should prioritize the promotion of R&D and innovation for more effective eco-friendly IS technologies, rather than aiming to reduce adoption costs.

In some industries, existing IS technologies are sufficiently green and effective to offset the additional pollution generated by increased production in the equilibrium. In these circumstances, environmental policy should focus on reducing adoption cost to ensure the implementation of such technologies, particularly in competitive markets where individual small firms may struggle to bear high average costs. For example, policies could aim to reduce the costs of acquiring knowledge, transactions, coordination, searching for potential byproduct buyers, transportation, logistics, bureaucracy and excessive regulation, and production infrastructure modification (Ashraf et al., 2016). Moreover, firms' increased profitability following the adoption of a certain IS technology may also provide an opportunity for incentivizing the development of even greener and more effective technologies

Considering social welfare, we identify a key technology parameter (the proportion of reused byproducts) that may increase firms' and consumers' surplus, while also providing environmental gains. Accordingly, a win–win government policy for all stakeholders is to promote the advancement of high-quality IS technologies that reuse a higher proportion of byproducts.

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Appendix

The equilibrium when adopting IS. Using the quantity produced (17) and market price (18), we obtain the firm's profit, consumer surplus, total surplus, and pollution in the equilibrium, respectively, as follows:

$$\pi_{symb} = \frac{c[a-g(d-\alpha(d+p_g))]^2}{[2c+bn]^2} - c_g \tag{27}$$

$$CS_{symb} = \frac{bn^2[a-g(d-\alpha(d+p_g))]^2}{2[2c+bn]^2} \tag{28}$$

$$TS_{symb} = \frac{n[a-g(d-\alpha(d+p_g))]^2}{2[2c+bn]} - nc_g \tag{29}$$

$$Poll_{symb} = \frac{ng[1-\alpha(1-k)][a-g(d-\alpha(d+p_g))]}{2c+bn}. \tag{30}$$

Rearranging $Poll_{symb}$, it is easy to see that it is non-monotone in α,

$$Poll_{symb} = \frac{ng^2(1-k)(d+p_g)}{2c+bn}[\frac{1}{1-k}-\alpha][\frac{a-gd}{g(d+p_g)}+\alpha]. \tag{31}$$

We next verify that a single firm would not choose to deviate from the IS equilibrium. Given the quality produced (17) and the market price (18) in the IS equilibrium, the profit of a firm that deviates to not adopting IS is obtained as follows:

$$\pi'_{symb} = \pi_{symb} - q_{symb}[\alpha g(d + p_g)] + c_g. \tag{32}$$

According to equation (32), for $c_g = 0$, $\pi_{symb} > \pi'_{symb}$; therefore a threshold level c'_g exists, such that a firm prefers not to deviate for $c_g < c'_g$. Accordingly, a firm is better off by not deviating if the following holds:

$$\pi_{symb} > \pi'_{symb} \Leftrightarrow c_g < c'_g. \tag{33}$$

where $c'_g = q_{symb}[\alpha g(d+p_g)]$. We next prove that $c_g < c'_g$ is not binding, given our assumption in Proposition 1 that $c_g < c^*_g$, where $c^*_g = q[\alpha g(d + p_g)$ (recall definition (1)). From the equilibrium equations (9) and (17), we deduce that $q_{symb} > q$, because $q_{symb} = q + \Delta q$ and $\Delta q = \frac{\alpha g(d+p_g)}{2c+bn} > 0$. As a result, $c^*_g < c'_g$. This ends the proof with no further assumption needed, because $c_g < c^*_g$ and $c^*_g < c'_g$ imply that $c_g < c'_g$, with no firm deviation.

Proof of proposition 1. A firm deviates and adopts IS when $\pi' > \pi$. Substituting the quantity produced (9) in the inequality (16), we determine that firms adopt IS when $c_g < c^*_g$, where $c^*_g = (\frac{a-dg}{2c+bn})[\alpha g(d + p_g)]$. As we demonstrated above, $c_g < c^*_g$ is sufficient to existence of equilibrium in which all firms adopt IS. Additionally, it is easy to see that c^*_g monotonically increases in α and in p_g and declines in n. □

Proof of proposition 2.

1. Using the equilibrium equations with and without IS, we determine that $q_{symb} > q$ because $\Delta q = \frac{\alpha g(d+p_g)}{2c+bn} > 0$, $p_{symb} < p$, because $\Delta p = \frac{bn[\alpha g(d+p_g)]}{2c+bn} > 0$, and $CS_{symb} > CS$, because $\Delta CS = \frac{b[n\alpha g(d+p_g)]^2}{2[2c+bn]^2} > 0$.

2. $\pi_{symb} - \pi = \Delta\pi - c_g$, where $\Delta\pi = \frac{c[(\alpha g(d+p_g))^2+2\alpha g(d+p_g)(a-dg)]}{[2c+bn]^2}$. Therefore, $\pi_{symb} > \pi \Leftrightarrow c_g < \Delta\pi$.

   A sufficient condition for $c_g < \Delta\pi$, given that $c_g < c^*_g$, is $c^*_g < \Delta\pi$. Substituting c^*_g

(see definition 1) and $\Delta\pi$ and rearranging yields a sufficient condition of $\alpha g(d+p_g)c > bn(a-dg)$. Then, from $\Delta CS > 0$ and $\Delta\pi > 0$ we conclude that $\Delta TS > 0$.

3. $\frac{\partial\Delta\pi}{\partial d} = \frac{2c\alpha g[(a-dg)+(d+p_g)(1-g)]}{[2c+bn]^2}$. This expression is positive, given that the share $g < 1$ and our assumption that $a - dg > 0$. The remainder of the proof is straightforward.

□

Proof of proposition 3. 1. Equation (21), indicates the following:

$\Delta Poll \geq 0 \Leftrightarrow g(d+p_g) \geq (1-k)[a-dg+\alpha g(d+p_g)]$

$\Leftrightarrow g(d+p_g)(1-(1-k)\alpha) \geq (1-k)(a-dg)$. Rearranging this inequality and isolating α yields that $\Delta Poll \geq 0 \Leftrightarrow \alpha \leq \frac{1}{1-k} - \frac{a-dg}{g(d+p_g)}$. Similarly, $\Delta Poll < 0 \Leftrightarrow \alpha > \frac{1}{1-k} - \frac{a-dg}{g(d+p_g)}$.

2. A straightforward derivation of the pollution gap in equation (21) by g, d, p_g, and k obtains positive derivatives.

3. Rearranging the pollution gap (equation (21)) yields

$$\Delta Poll = \frac{gn}{2c+bn}[\alpha g(d+p_g) - (1-k)[\alpha(a-dg) + \alpha^2 g(d+p_g)]].$$

Deriving the pollution gap by α yields

$$\frac{\partial\Delta Poll}{\partial\alpha} = \frac{gn}{2c+bn}[g(d+p_g) - (1-k)[(a-dg) + 2\alpha g(d+p_g)]];$$

therefore,

$$\frac{\partial\Delta Poll}{\partial\alpha} \geq 0 \Leftrightarrow g(d+p_g) \geq (1-k)[(a-dg) + 2\alpha g(d+p_g)]$$

$$\Leftrightarrow g(d+p_g)(1-2\alpha(1-k)) \geq (1-k)(a-dg)$$

$$\Leftrightarrow \frac{1-2\alpha(1-k)}{1-k} \geq \frac{a-dg}{g(d+p_g)}$$

$$\Leftrightarrow \frac{1}{1-k} - 2\alpha \geq \frac{a-dg}{g(d+p_g)}.$$

Isolating α yields $\frac{\partial \Delta Poll}{\partial \alpha} \geq 0 \Leftrightarrow \alpha \leq \frac{1}{2}(\frac{1}{1-k} - \frac{a-dg}{g(d+p_g)})$.

Similarly, it is easy to see that $\frac{\partial \Delta Poll}{\partial \alpha} < 0 \Leftrightarrow \alpha > \frac{1}{2}(\frac{1}{1-k} - \frac{a-dg}{g(d+p_g)})$.

$\square$

Proof of proposition 4. 1. Using equations (17) and (23), $q^o_{symb} < q_{symb}$

$\Leftrightarrow \frac{a-g(d-\alpha(d+p_g))}{2c+n(b+\delta)} < \frac{a-g(d-\alpha(d+p_g))}{2c+bn}$. This is true since $\delta > 0$.

Using equations (24) and (30), $Poll^o_{symb} < Poll_{symb}$

$\Leftrightarrow \frac{ng[1-\alpha(1-k)][a-g(d-\alpha(d+p_g))]}{2c+n(b+\delta)} < \frac{ng[1-\alpha(1-k)][a-g(d-\alpha(d+p_g))]}{2c+bn}$. This is true since $\delta > 0$.

2. Using equations (25) and (29), we obtain the following:

$$TS^o_{symb} < TS_{symb}$$

$$\Leftrightarrow \frac{n[2c+n(2\delta+b)][a-g(d-\alpha(d+p_g))]^2}{2[2c+n(b+\delta)]^2} < \frac{n[a-g(d-\alpha(d+p_g))]^2}{2(2c+bn)}$$

$$\Leftrightarrow \frac{n[a-g(d-\alpha(d+p_g))]^2}{2}\left(\frac{2c+n(2\delta+b)}{[2c+n(b+\delta)]^2} - \frac{1}{2c+bn}\right) < 0$$

$$\Leftrightarrow (2c+bn)((2c+bn)+2\delta n) < ((2c+bn)+\delta n)^2$$

$$\Leftrightarrow (2c+bn)^2 + 2\delta n(2c+bn) < (2c+bn)^2 + 2\delta n(2c+bn) + (\delta n)^2.$$

This is true because $\delta n > 0$.

Substituting the TS_{symb} and Q_{symb} (equations (29) and (17)) yields the social welfare in the market equilibrium, as follows:

$$SW_{symb} = \frac{n[a-g(d-\alpha(d+p_g))]^2[2c+n(b-\delta)]}{2[2c+bn]^2} - nc_g. \tag{34}$$

It is easy to verify that $SW^o_{symb} > SW_{symb}$

$$\frac{n[a-g(d-\alpha(d+p_g))]^2}{2(2c+n(b+\delta))} > \frac{n[a-g(d-\alpha(d+p_g))]^2[2c+n(b-\delta)]}{2[2c+bn]^2}$$

$$\Leftrightarrow \frac{n[a-g(d-\alpha(d+p_g))]^2}{2}\left(\frac{1}{2c+n(b+\delta)} - \frac{2c+n(b-\delta)}{(2c+bn)^2}\right) > 0$$

$$\Leftrightarrow (2c+bn)^2 > (2c+n(b+\delta))(2c+n(b-\delta))$$

$$\Leftrightarrow (2c+bn)^2 > ((2c+nb)+\delta n)((2c+nb)-\delta n))$$

$$\Leftrightarrow (2c+bn)^2 > (2c+bn)^2 - (\delta n)^2,$$

which is true since $\delta n > 0$.

3. The proof is straightforward comparing the IS market equilibrium equations to the IS social optimum equations.

4. It is easy to see that $TS^o_{symb} = \frac{(2c+bn)[2c+n(2\delta+b)]}{[2c+n(b+\delta)]^2} TS_{symb}$. From Proposition 2, $\frac{\partial TS_{symb}}{\partial \alpha} > 0$. A simple derivation obtains that

$$\frac{\partial\left(\frac{(2c+bn)[2c+n(2\delta+b)]}{[2c+n(b+\delta)]^2}\right)}{\partial \alpha} > 0,$$

therefore $\frac{\partial TS^o_{symb}}{\partial \alpha} = \frac{(2c+bn)[2c+n(2\delta+b)]}{[2c+n(b+\delta)]^2}\frac{\partial TS_{symb}}{\partial \alpha} > 0$.

5. Clearly, $Poll^o_{symb} = \frac{2c+bn}{2c+n(b+\delta)} Poll_{symb}$.

Then,

$$\frac{\partial Poll^o_{symb}}{\partial \alpha} > 0$$

$$\Leftrightarrow \frac{2c+bn}{2c+n(b+\delta)}\frac{\partial Poll_{symb}}{\partial \alpha} + \frac{(2c+bn)\gamma g n(1-k)}{[2c+n(b+\delta)]^2} Poll_{symb} > 0.$$

From Proposition 3 we know that

$$\frac{\partial Poll_{symb}}{\partial \alpha} > 0$$

if $\alpha \leq \frac{1}{2}\phi$. All other expressions are positive, which ends the first part of the proof.

For the second part, we derive $Poll^{o}_{symb}$ by α to obtain the following:

$$\frac{\partial Poll^{o}_{symb}}{\partial \alpha} < 0 \Leftrightarrow \frac{2c+nb}{2c+n(b+\delta)} \frac{(a-dg)+\alpha g(d+p_g)}{g(d+p_g)} > \frac{1-\alpha(1-k)}{1-k}$$

$$\Leftrightarrow \frac{2c+nb}{2c+n(b+\delta)}(\alpha + \frac{a-dg}{g(d+p_g)}) > \frac{1}{1-k} - \alpha.$$

Note that for this inequality to hold, the following is insufficient:

$$\alpha + \frac{a-dg}{g(d+p_g)} > \frac{1}{1-k} - \alpha \Leftrightarrow \alpha > \frac{1}{2}\phi,$$

because $\frac{2c+nb}{2c+n(b+\delta)} < 1$. However, since the left-hand side increases in α ($\frac{\partial \delta}{\alpha} < 0$) and the right-hand side declines in α, a threshold $\alpha^* > \frac{1}{2}\phi$ exists, that maximizes $Poll^{o}_{symb}$. Clearly, α^* satisfies the following:

$$\frac{2c+nb}{2c+n(b+\delta(\alpha^*))}(\alpha^* + \frac{a-dg}{g(d+p_g)}) = \frac{1}{1-k} - \alpha^*.$$

$\square$