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24 October 2024

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October 23, 2024

Abstract

Should public policies address inequality due to heterogeneous life expectancy? Intuitively, taking short life as a disadvantage, such policies should favor those with high mortality. Yet, pension systems implicitly redistribute from low-life-expectancy to high-life-expectancy people. Moreover, this direction of redistribution is optimal from the perspective of the standard utilitarian welfare criterion. We study mortality-related redistribution in a more flexible setting. We start by establishing a formal framework for the analysis by clearly distinguishing between the redistribution along mortality and income dimensions, and thus between mortality and income progressivity. We then show that it is optimal to redistribute towards high-mortality people in two cases. First, when welfare criterion features aversion to lifetime inequality which exceeds aversion to consumption inequality. Second, when income and mortality are negatively correlated, and income redistribution tools are limited.

Keywords: Mortality-related redistribution, Welfare criteria, Pensions, Prioritarianism

JEL Classification Codes: D30, D60, D63, H55

*We thank Varadarajan Chari, Maya Eden, Dirk Krueger, Alexander Ludwig, Shinichi Nishiyama, Yena Park, Paolo Piacquadio, and all participants at the Macroeconomic and Social Insurance workshop at the Barcelona Summer Forum, Macro Public Finance meeting at the NBER SI, Midwest Economic Meeting, Midwest Macro Meeting, Normative Economics and Economic Policy webinar, Society of Economics of the Household Meeting, I-85 conference, GRIPS and University of Kyoto seminars for comments and suggestions. Pashchenko acknowledges financial support from the Terry-Sanford Award.

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1 Introduction

Many public policies aim to reduce economic inequality by redistributing from those who are relatively better-off (rich) to those who are relatively worse-off (poor). There are no explicit measures, however, to address inequality due to differential longevity. Moreover, pension systems implicitly redistribute from low-life-expectancy to high-life-expectancy people. This is because pensions are paid as annuities that are not indexed for individual mortality, and as a result, those who live longer receive more in lifetime payments (everything else equal). Supposing longer life is valuable, the redistribution from low-life-expectancy to high-life-expectancy people is equivalent to redistribution from those who are worse-off to those who are better-off.

This direction of redistribution can, in fact, be optimal. A well-established theoretical result is that when the utilitarian approach is used to assess social welfare, it is optimal to redistribute from low-life-expectancy to high-life-expectancy people (e.g., Fleurbaey et al., 2014). This result has important policy implications. Any welfare analysis with the utilitarian criterion has an implicit bias against high-mortality people, which is particularly relevant when analyzing various pensions reforms.

Interestingly, the optimal redistribution along mortality dimension received relatively little attention in the literature. Yet, the lifespan inequality is large: the difference between the ages of death at the 80th and 20th percentiles of the survivor distribution is more than 20 years (Fuchs and Eggleston, 2018). Moreover, the society places high value on being alive. This is typically measured by the Value of Statistical Life (VSL), which is set in the range between \$1 and \$10 million dollars by the US government agencies in their analysis involving a mortality risk (Robinson, 2007).

In this paper, we aim to investigate the optimal redistribution along the dimension of mortality. It is worth stressing that this question is fundamentally different from the question of optimal redistribution along income dimension when people also differ in mortality (unless income and mortality are perfectly correlated). We thus develop a general framework to analyze the redistribution along mortality dimension in isolation from income redistribution. Our goal is to understand under what conditions the redistribution from low-life-expectancy to high-life-expectancy people is no longer optimal, and hence the implicit bias against high-mortality people is not present in a welfare assessment.

The starting point of our analysis is how to think about mortality-related redistribution. While we are well-equipped to study redistribution along income dimension, there is no formal framework to analyze redistribution along the dimension of mortality (to the best of our knowledge). Mortality inequality is usually discussed within the context of income inequality because mortality-income correlation matters for income progressivity of some public programs.[1]

In contrast, we draw a clear distinction between income- and mortality-related redistribution. To conceptualize ideas, we consider a framework where individuals differ in endowments and mortality. We then examine an average tax imposed by a particular redistributive scheme. If, everything else equal, the average tax is higher (lower) for people with low life expectancy compared to those with high life expectancy, we define this as mortality regressivity (progressivity). We can thus examine the redistribution along mortality dimension for a given degree of income redistribution.

The next two important questions for our analysis are (i) why longevity matters for the welfare assessment, (ii) in what way we need to deviate from the standard utilitarian approach.

In our environment, longevity matters for individual welfare because life is valuable. We assume that being alive brings non-pecuniary benefits in addition to utility from consumption. In other words, being alive creates an additional utility flow not related to one's economic resources. In this, we follow the long tradition in the value of life literature (Hall and Jones, 2007; Murphy and Topel, 2006; Rosen, 1988).

Turning to aggregate welfare, the key point is that when life is valuable, heterogeneity in life expectancy represents an additional source of lifetime inequality. This is because low-mortality people have an utility advantage over those with high mortality even when their consumption is the same.

Importantly, the standard utilitarian welfare criterion is insensitive to this additional source of inequality. The utilitarian social planner aims to reduce consumption inequality among *living* agents (assuming concave utility function). But even if consumption per period is equalized, those with low life expectancy will receive less in terms of *lifetime* consumption. Everything else equal, this creates redistribution from low- to high-life-expectancy people.

Since this result is due to the insensitivity of the utilitarianism to inequality in lifetime utilities, in our analysis, we use a welfare criterion that does not have this limitation. Specifically, we consider social welfare that is a concave (as opposed to linear) function of individual welfare. This approach to aggregating individual welfare is also known as the prioritarian social welfare function (see Adler, 2022). An important advantage of this approach is that by varying the degree of concavity of the function that aggregates individual welfare, we can vary the degree of aversion to lifetime inequality.

Using our theoretical framework, we derive several interesting results. We start by con-

[1] For example, see Coronado et al. (2011) and Goda et al. (2011) for the analysis of progressivity of Social Security when income-mortality correlation is taken into account.

sidering a simple case when people differ in their mortality but not in endowments. We show that in this environment, there is a tension between the aversion to two types of inequality: in consumption and in individual welfare. When aversion to consumption inequality exceeds aversion to inequality in individual welfare, mortality regressivity is always optimal. Moreover, when the aversion to these two types of inequality is the same, mortality regressivity is still optimal. In order for the redistribution from low- to high-mortality people to be optimal, aversion to lifetime inequality must be *stronger* than aversion to consumption inequality. Importantly, the push towards mortality progressivity is stronger when life is more valuable. This is because in this case, the variation in individual welfare due to lifespan variation is higher, calling for larger transfers to those with lower life expectancy.

We next examine a more general framework where there is heterogeneity in both endowments and mortality. In this case, social planner can potentially redistribute along the dimension of endowment and/or the dimension of mortality. We show that as long as social planner can freely redistribute along both dimensions, the optimality of mortality progressivity is determined by the same factors as described above.

We then ask what would be the optimal redistribution along mortality dimension if social planner cannot redistribute along income dimension. We show that in this case, mortality progressivity can be optimal even if there is no aversion to inequality in lifetime utilities. What plays the key role in this environment is the correlation between income and mortality: the stronger is this correlation, the more mortality-progressive is the optimal allocation. This is because it is optimal to redistribute towards low-income people, but since this option is not available, the next best thing is to redistribute towards people with high mortality as they are more likely to be poor. Put differently, mortality progressivity becomes optimal as a substitute for income progressivity which cannot be directly achieved.

This conceptual framework easily lends itself to the study of pension systems' design, and this represents the next step of our analysis. To do this, we modify our approach by introducing working and retirement stages of the life-cycle. People pay contributions during their working life that depend on their labor productivity, and receive pension benefits when retired. We show that the optimal redistribution along mortality dimension embedded into the pension system is largely driven by the the same forces as described above, and the aversion to inequality in consumption versus in lifetime utilities plays the key role. Specifically, in absence of concern for lifetime inequality, optimal pensions are independent of one's life expectancy, while with aversion to lifetime inequality, high-mortality people receive higher pensions.

In the final part of our analysis, we provide a quantitative illustration using a life-cycle model where people differ in their labor income and mortality. We estimate labor income and survival probabilities from the Panel Study of Income Dynamics (PSID) and the Health and Retirement Study (HRS), respectively. We use our quantitative model for the comparative welfare analysis of different spending-neutral pension systems. These systems differ in their mortality progressivity but are restricted in their ability to directly redistribute income. We show that when we shut down the heterogeneity in labor income, the welfare gains from mortality progressivity are relatively low even when aversion to lifetime inequality is high. In contrast, with heterogeneity in labor income, the welfare gains of moving to more mortality-progressive pensions can exceed 2% of annual consumption. This happens because, given the strong estimated correlation between labor income and mortality, it is optimal to use mortality progressivity to increase consumption of low-income people when direct income redistribution is restricted.

Our results thus emphasize that there are two distinct reasons making mortality progressivity optimal. First is the desire to compensate high-mortality people for their potentially short life. This effect is only present when life is valuable and there is strong aversion to inequality in lifetime utilities. Second reason is the desire to redistribute towards low-income people. This effect is present when income and mortality are negatively correlated and when there are limited instruments to directly redistribute income. Importantly, while in both cases, mortality progressivity produces welfare gains, the underlying cause differs. In the first case, optimality of mortality progressivity is driven by the concern for those with low life expectancy, while in the second - by the concern for those who are poor.

Our conclusions have important implications for the comparative pension policy analysis in presence of unequal lifespans. A typical pension system usually involves regressive redistribution along mortality dimension and progressive redistribution along income dimension. We suggest a framework that can be used to analyze the two types of redistribution separately. This allows to do a more detailed comparison of different policies, and to separate the sources of welfare changes between the increase in welfare of people with low income versus that of people with low life expectancy. We show that in this analysis, three modeling assumptions are of crucial importance: the nature of income-mortality correlation, the value of life, and the degree of aversion to lifetime inequality.

The rest of this paper is organized as follows. Section 2 reviews the related literature. Section 3 introduces the model and derives the results for the case when there is only mortality heterogeneity. Section 4 studies the case with both endowment and mortality heterogeneity, and Section 5 extends our framework to analyze different pension designs. Section 6 describes the quantitative illustration, and Section 7 concludes.

2 Related literature

Our paper belongs to the large literature that studies normative questions touching on non-economic aspects of welfare. Among such questions are: How to discount the welfare of future generations (Eden, 2023; Farhi and Werning, 2007)? How to address income inequality when people differ in needs (Atkinson and Bourguignon, 1987; Berg et al., 2023)? How the society should value saving lives of the old versus that of the young (Adler et al. 2021)? Answering this type of questions requires adopting certain value judgments, which in some cases, cannot be accommodated by the standard utilitarian approach.

To incorporate non-utilitarian considerations into normative evaluations, two routes have been taken in the literature. The first is to use an alternative social welfare function (for a review of the common welfare functions, see Adler, 2019; for examples of newer welfare criteria, see Berg and Piacquadio, 2023, Eden and Freitas, 2023, Piacquadio, 2023; Weinzierl, 2014). The second approach is to modify individual welfare weights. In the latter case, one can potentially bypass the social welfare function altogether by assigning each individual the so-called generalized social marginal welfare weight which directly incorporates any relevant value judgment or societal concern for non-economic factors (Saez and Stantcheva, 2016).

In our study, we follow the first route by adopting the prioritarian approach to social welfare. In this approach, aggregate welfare represents a sum of a strictly increasing and strictly concave transformation of individual lifetime utilities.[2] The concave transformation of individual welfare ensures that people who are worse-off in terms of their overall well-being are given higher weight (Adler, 2022). Importantly, the lower well-being may be due to non-pecuniary factors such as lower life expectancy.

We contribute to this literature by showing that while the utilitarianism pre-determines the optimality of mortality-regressivity, with prioritarian approach, both mortality-regressive and mortality-progressive allocations can optimally arise. It is worth noting that an alternative approach would be to directly assign higher welfare weights to people with low life expectancy. However, the advantage of our approach is that it gives the microfoundations for why the welfare weights on high-mortality people are high, and allows these weights to be a function of the degree of aversion to lifetime inequality.

Our paper is also related to the large literature on optimal redistribution. One important focus of this literature is on optimal redistribution along income dimension. Taking into

[2] While the utilitarianism remains the dominant social welfare function, the prioritaninan approach is not uncommon. Tuomala and Weinzierl (2022) review the applications of this approach in the optimal taxation literature. The prioritanian social welfare function was considered in a seminal paper of Mirrless (1971) and in many subsequent contributions (e.g., Atkinson and Stiglitz, 1980; Diamond, 1998; Chone and Laroque, 2005; Kaplow, 2008 and 2010).

account lifespan inequality raises two important questions: (i) How to redistribute along income dimension when people also differ in mortality? (ii) How to redistribute along the dimension of mortality? A number of studies consider an environment when these two questions coincide because high-income people are also the ones with higher life expectancy (Hosseini and Shourideh, 2019; Koehne, 2023; Pestieau and Racionero, 2016). Barigozzi et al. (2023) provides an interesting exemption by studying gender-related redistribution in the environment when high-income people have lower life expectancy (women live longer but earn less than men).

The second question, how to redistribute along mortality dimension, received less attention in the literature, and is often considered in the environment without income heterogeneity. In this area, the standard utilitarian approach is particularly problematic as it implies the optimality of redistribution from low- to high-life expectancy people (Fleurbaey et al., 2014; Leroux et al., 2011; Leroux and Ponthiere, 2013). This theoretical result was labeled by Leroux and Ponthiere (2013) "the paradox of the double penalization of the short-lived". The short-lived are penalized by nature by being endowed with low life expectancy and, in addition, they are penalized by lower lifetime consumption.

To avoid the double penalization paradox, several approaches were suggested. Leroux and Ponthiere (2013) suggest imputing the consumption equivalence of longer life and include it as a part of social planner problem, while Pestieau and Racionero (2016) incorporate higher social weight on the utility of the short-lived. Bommier et al. (2011a, 2011b) use a standard utilitarian welfare function but allow individuals to have temporal risk aversion, i.e., aversion with respect to life duration. Fleurbaey et al. (2014) derive a social objective function of the maximin type that ensures compensating the short-lived for their short life is optimal.

We complement this line of research by drawing a clear distinction between the redistribution along income and mortality dimension. We study the optimal redistribution along the dimension of mortality in two settings: when social planner jointly decides on the optimality of redistribution along both income and mortality dimensions, and when the optimality of mortality-related redistribution is decided upon for a *given* degree of income redistribution.

Our paper can also relate to the quantitative studies of pension systems in presence of heterogeneous life expectancy (see an extensive review in Jones and Li, 2022). These studies differ in their approaches to modeling the relationship between socio-economic variables and life expectancy. Both assumptions of perfect correlation (Bagchi, 2019; Sanchez-Romero, 2019; Sheshinski and Caliendo, 2021) and no correlation (Bagchi and Jung, 2020; Imrohoroglu and Kitao, 2012) are used. Several studies also consider intermediate scenarios with imperfect correlations (Jones and Li, 2022; Laun et al., 2019; Pashchenko and Porapakkarm, 2024). We contribute to this literature by illustrating the role of the assumptions about income-mortality correlation for the welfare assessment with the use of the utilitarian approach.

It is also worth mentioning our relationship with studies related to the value of life. The constraint that continuation utility of being alive exceeds that of being dead is not commonly enforced in studies with individual optimization problems since, in most cases, it does not affect results. However, in certain applications this constraint plays a crucial role. Among them are valuation of changes in life expectancy or health (De Nardi et al., 2022; Hall and Jones, 2007; Murphy and Topel, 2006) or saving and portfolio choice with non-additive preferences (Pashchenko and Porapakkarm, 2022). We contribute to this literature by showing that the value of life can play an important role in welfare evaluations of pension reforms. This happens when these evaluations are done using a more general social welfare criteria that take non-pecuniary factors into account.

3 Only mortality heterogeneity

In this section, we describe the setup of our model when agents only differ in mortality. The framework we use is a small open economy where resources can be transferred intertemporally at the fixed rate r.

We consider two versions of the environment. First is the laissez-faire case when each agent's consumption is determined by his own resources. We assume that in the laissez-faire case, agents have access to acturially-fair annuities. In the second version, consumption is decided upon by a social planner who optimizes ex-ante welfare subject to the aggregate resource constraint.

3.1 Environment

Consider a representative cohort whose initial mass is one. Individuals enter the model at age $t = 0$ with the same endowment A and receive no additional income over their lifetime. Individuals discount the future at the rate β. We assume $\beta(1 + r) = 1$.

Individuals live to the maximum age of T. We denote the probability to survive from age t to $t + 1$ as θ_i. We assume θ_i does not vary with age but differs across individuals with $\theta_i \sim G(\theta_i)$ on the interval $[\theta_{min}, \theta_{max}]$, and the average survival probability is defined as

$$\overline{\theta} = \int_{\theta_{min}}^{\theta_{max}} \theta_i \, dG\left(\theta_i\right)$$

In what follows, as a measure of an agent's i longevity, we use not θ_i but q_i, which is defined as follows:

$$q_i = \sum_{t=1}^{T} \left(\frac{\theta_i}{1+r} \right)^t$$

In the laissez-faire case, q_i represents the actuarially fair price of an annuity. In the social planner economy, q_i is the cost of delivering a unit stream of lifetime consumption to an individual i. While keeping this distinction in mind, we will refer to q_i as "annuity price" in both laissez-faire and social planner versions of our environment for the sake of brevity.

There is a one-to-one relationship between q_i and individual survival probability θ_i: $q_i \in [q_{min}, q_{max}]$, where $q_{min} = q(\theta_{min})$ and $q_{max} = q(\theta_{max})$. We denote the average annuity price as $\overline{q}$, where

$$\overline{q} = \int_{q_{min}}^{q_{max}} q_i \, dG\,(\theta_i).$$

Individuals derive utility from consumption c_{it} based on utility function $u(c_{it})$, where $u(c_{it})$ is strictly increasing, weakly concave and twice continuously differentiable. In addition, individuals derive non-pecuniary benefits from the fact that they are alive, which are captured by a positive constant b. Thus, total utility per period of an individual i, v_{it}, can be represented as follows:

$$v_{it} = u(c_{it}) + b$$

The utility in the state of death is normalized to zero.[3] Thus, life is valued more than death when $v_{it} > 0$.

We assume no consumption takes place at $t = 0$. The ex-ante lifetime utility of an individual i (or lifetime utility from the perspective of age $t = 0$) is

$$V_i = \sum_{t=1}^{T} (\beta\,\theta_i)^t \,(u(c_{it}) + b) = \sum_{t=1}^{T} (\beta\,\theta_i)^t \, v_{it} \tag{1}$$

Given that every agent in the economy gets endowment A, and the total initial mass of people in a representative cohort is one, the aggregate resource constraint for this economy can be represented as follows (to make notations less cluttered, we drop the limits of integration):

$$\int \sum_{t=1}^{T} \left(\frac{\theta_i}{1+r} \right)^t c_{it} \, dG\,(\theta_i) = A \tag{2}$$

In what follows, we will focus on age-invariant consumption allocations such that $c_{it} = c_i \;\; \forall\; t$, i.e., each individual's consumption is constant over time. Consumption is age-

[3] This approach is common in the value of life literature, see, for example, Hall and Jones (2007) and Murphy and Topel (2006). An alternative approach is to re-normalize disutility from being dead instead of assuming extra utility from being alive. Rosen (1988) shows that these two approaches are equivalent.

independent in both laissez-faire and social planner versions of our environment. In the laissez-faire case, consumption is financed by annuity income. In particular, each individual, on entering the model at $t = 0$, converts his entire endowment A into an annuity based on an actuarially fair price.[4] Hence, consumption is the same every period and is equal to $\frac{A}{q_i}$. In the social planner economy, the optimal consumption allocations are also age-independent due to the assumption $\beta(1 + r) = 1$, as we formally show in Section 3.3.

We next introduce two definitions.

Definition 1 We call the consumption allocation $\{c_i\}$ *feasible* if it satisfies the aggregate resource constraint in Eq (2).

A convenient way to describe the relationship between consumption and mortality is by using the concept of elasticity. We introduce the relevant elasticity in the next definition.

Definition 2 Consider a feasible consumption allocation $\{c_i\}$. The elasticity of consumption c_i to actuarially fair annuity price q_i is defined as follows:

$$\varepsilon_{q_i}^{c} = \frac{dc_i}{dq_i} \cdot \frac{q_i}{c_i}$$

3.2 Defining mortality regressivity

Intuitively, mortality regressivity implies that there is regressive redistribution along the dimension of mortality, i.e., from low-life-expectancy to high-life-expectancy people. To understand whether a particular allocation is regressive or progressive, we first need to establish a reference point. We use as a reference point a laissez-faire allocation, when each individual annuitizes his endowment at the actuarially-fair price. In such a situation, an individual does not pool his mortality risk with anyone else, and thus there is no redistribution across mortality dimension. To emphasize the absence of redistribution, we will refer to this as neutral allocation.

Definition 3 A feasible consumption allocation $\left\{c_i^N\right\}$ is *neutral* if

$$c_i^N = \frac{A}{q_i} \quad \forall \ i$$

It is worth noting that this corresponds to the situation when the elasticity of consumption to mortality is equal to -1 , i.e., $\varepsilon_{q_i}^{c} = -1 \quad \forall \ i$.

[4] It is worth noting two points regarding the laissez-faire case. First, full annuitization is optimal since annuities are actuarially fair and thus dominate regular savings. Second, the one-time annuitization in the first period is optimal because there is no uncertainty except that in survival and annuities are actuarially fair. See Pashchenko (2013) for a formal proof.

Consider next a feasible allocation $\{c_i\}$. To understand whether it is mortality-regressive or progressive, we compare it with the neutral (or laissez-faire) allocation $\{c_i^N\}$. We can think of $\{c_i^N\}$ as the allocation before the redistribution takes place, and $\{c_i\}$ as that after the redistribution. We can think of the difference $c_i^N - c_i$ as a tax (possibly negative), and we define the average tax AT_i as follows:

$$AT_i = \frac{c_i^N - c_i}{c_i^N} \; = \; 1 - \frac{c_i}{c_i^N}$$

We define mortality regressivity/progressivity based on how the average tax changes with mortality.

Definition 4 A feasible consumption allocation $\{c_i\}$ is *mortality-regressive* (*mortality-progressive*) if the average tax decreases (increases) with life expectancy or q_i:

$$\frac{\partial AT_i}{\partial q_i} < (>) \; 0.$$

We next are going to formulate the proposition linking mortality regressivity and the concept of elasticity introduced in Definition 2.

Proposition 1 Consider a feasible allocation $\{c_i\}$. If this allocation is mortality-regressive (mortality-progressive) then the following is true:

$$\varepsilon^c_{q_i} \; > (<) - 1 \quad \forall \;\; i$$

Proof We will do the proof for the mortality-regressive case. That for the mortality-progressive case is analogous. Using the definition of the neutral allocation $\{c_i^N\}$, we can rewrite the average tax as follows:

$$AT_i = 1 - \frac{c_i q_i}{A}$$

The derivative of the average tax with respect to q_i is

$$\frac{\partial AT_i}{\partial q_i} = - \frac{1}{A}\frac{\partial (c_i q_i)}{\partial q_i} \; = \; -\frac{c_i}{A}\,(\varepsilon^c_{q_i} + 1),$$

where the last equality follows from the definition of elasticity. Since in the mortality-regressive case $\dfrac{\partial AT_i}{\partial q_i} < 0$, it follows that $\varepsilon_i^{cq} > -1$. This finishes the proof of the proposition.

Example Consider an example of pooled annuitization, that is to say, the feasible allocation that is the same for all agents, $\bar{c} = \dfrac{A}{\bar{q}}$. This allocation is mortality-regressive as the average tax is decreasing in q_i:

$$AT_i = 1 - \frac{q_i}{\bar{q}} \tag{3}$$

Note that the elasticity is equal to zero, $\varepsilon^c_{q_i} = 0$.

3.3 Optimality of mortality progressivity

To understand under what conditions mortality progressivity can optimally arise, in this section, we consider the social planner problem for this economy. In our formulation of this problem, we deviate from the standard utilitarian approach in that we allow for the aversion to inequality in lifetime utilities by using the prioritarian approach. In the standard utilitarian approach, this inequality does not matter because the total welfare is a linear sum of individual lifetime utilities V_i, and thus V_i of different individuals are perfect substitutes. In the prioritanian approach, total welfare is a sum of concave transformations of V_i, which relaxes the assumption of perfect substitution (Adler, 2022).

Social planner maximizes welfare of a representative cohort subject to the aggregate resource constraint:

$$\max_{\{c_{it}\}} \quad \int \Psi(V_i)\, dG\,(\theta_i) \tag{4}$$

$$\text{s.t.} \qquad \int \sum_{t=1}^{T} \left(\frac{\theta_i}{1+r}\right)^t c_{it}\, dG\,(\theta_i) = A \tag{5}$$

Here $\Psi(\cdot)$ represents planner's attitude towards inequality in lifetime utilities V_i , where V_i is defined in Eq (1). Note that in a standard utilitarian welfare case, $\Psi(\cdot)$ is linear. We assume that $\Psi(\cdot)$ is strictly increasing, weakly concave and twice continuously differentiable.

Denoting the Lagrange multiplier on the resource constraint as λ, we can write the first-order conditions as follows:

$$\frac{\partial\, \Psi(V_i)}{\partial\, V_i}\, \beta^t\, \theta_i^t\, \frac{\partial\, u(c_{it})}{\partial\, c_{it}} = \left(\frac{\theta_i}{1+r}\right)^t \lambda$$

Given that $\beta(1+r) = 1$, this can be simplified as follows:

$$\frac{\partial\, \Psi(V_i)}{\partial\, V_i} \cdot \frac{\partial\, u(c_{it})}{\partial\, c_{it}} = \lambda \qquad \forall \;\; i\,, t \tag{6}$$

Two important properties of the optimal allocation follow from Eq (6):

1. $c_{it} = c_i \quad \forall\ i$, i.e., it is optimal to give every individual a constant consumption stream.

2. $\dfrac{dc_i}{dq_i} \leqslant 0$, i.e., optimal consumption is non-increasing in longevity. This follows because for any two individuals i and j, given weak concavity of $\Psi(\cdot)$, Eq (6) implies that if $c_i \geq c_j$ then $V_i \leq V_j$. Since for a given level of consumption, lifetime utility is increasing in longevity or q_i, this can only be true when $\dfrac{dc_i}{dq_i} \leq 0$. This implies that $\varepsilon^c_{q_i} \leq 0 \quad \forall\ i$. In the subsequent analysis, we focus on $\mid \varepsilon^c_{q_i} \mid$.

When consumption is age-invariant, the per-period utility v_i is also the same at each age. We can use this fact together with the definition of q_i and the assumption $\beta(1+r) = 1$, to rewrite the lifetime utility V_i in Eq (1) as follows:

$$V_i = q_i(u(c_i) + b) = q_i v_i$$

Before proceeding, we are going to introduce the following notations:

$$\frac{\partial\, \Psi(V)}{\partial\, V} \equiv \Psi_V\,, \qquad \frac{\partial\, u(c)}{\partial\, c} \equiv u_c$$

$$\frac{\partial^2\, \Psi(V)}{\partial\, V^2} \equiv \Psi_{VV}\,, \qquad \frac{\partial^2\, u(c)}{\partial\, c^2} \equiv u_{cc}$$

Below, we summarize the key assumptions we use for our analysis.

Assumption 1. Both $u(\cdot)$ and $\Psi(\cdot)$ are strictly increasing, weakly concave and twice continuously differentiable

Assumption 2. $\beta(1+r) = 1$

Assumption 3. Life is valuable for all individuals: per-period utility of being alive v_i exceeds zero (utility at death), $v_i = u(c_i) + b > 0$ for all i.

The following proposition formalizes a criteria to determine whether an optimal allocation is mortality-regressive or mortality-progressive.

Proposition 2 Consider the consumption allocation $\{c_i\}$ that represents the solution to the social planner problem described in Eqs (4)-(5). Under Assumptions 1-3, whether this allocation is mortality-regressive/progressive can be determined as follows:

1. If $\Psi(\cdot)$ is linear, $\{c_i\}$ is the same for all individuals, $c_i = \overline{c}$ for all i, and is mortality-regressive.

2. If $\Psi(\cdot)$ is strictly concave, $\{c_i\}$ is mortality-regressive (mortality-progressive) if

$$\frac{u_{c_i} c_i}{v_i} + \frac{R_{ui}}{R_{\Psi i}} > (<)\ 1 \qquad \forall \ \ i \tag{7}$$

where $R_{ui} = -\dfrac{u_{cc_i}}{u_{c_i}} c_i$ and $R_{\Psi i} = -\dfrac{\Psi_{VV_i}}{\Psi_{V_i}} V_i$ are the coefficients of relative risk aversion of functions $u(\cdot)$ and $\Psi(\cdot)$, respectively.

Proof In the subsequent discussion, we will drop an individual's subscript i except in cases when we want to emphasize the difference between individuals.

Using the simplified notation introduced above, we can rewrite the FOC in Eq (6) as follows:

$$\Psi_V \, u_c = \lambda \tag{8}$$

Taking the full differential of this equation around the optimal allocation, we get:

$$\Psi_{VV} \, v \, u_c \, dq + (\Psi_{VV} \, u_c^2 \, q + \Psi_V \, u_{cc}) \, dc = 0$$

Hence

$$\frac{dc}{dq} = -\frac{\Psi_{VV} \, v \, u_c}{\Psi_{VV} \, u_c^2 \, q + \Psi_V \, u_{cc}} = -v \, \frac{\dfrac{\Psi_{VV}}{\Psi_V} V}{\dfrac{\Psi_{VV}}{\Psi_V} V u_c \, q + \dfrac{u_{cc}}{u_c} c \dfrac{V}{c}}$$

Using the definitions of the coefficients of the relative risk aversion and the fact that $V = vq$, we can transform this as follows:

$$\frac{dc}{dq} = -\frac{v}{q} \, \frac{R_\Psi}{R_\Psi \, u_c + R_u \, \dfrac{v}{c}}$$

Rewriting this expression in terms of elasticity, we have:

$$|\, \varepsilon_q^c \,| = \frac{R_\Psi}{R_\Psi \, \dfrac{u_c \, c}{v} + R_u}$$

It directly follows that when $R_\Psi = 0$ ($\Psi(\cdot)$ is linear), $|\, \varepsilon_q^c \,| = 0$, and optimal consumption is the same for all agents. This proves part 1 of the proposition. When $R_\Psi \neq 0$, we can rewrite the elasticity as follows:

$$|\, \varepsilon_q^c \,| = \frac{1}{\dfrac{u_c \, c}{v} + \dfrac{R_u}{R_\Psi}} \tag{9}$$

The consumption allocation is mortality-regressive (-progressive) when $|\varepsilon_q^c| < (>)\, 1$ by Proposition 1. This finishes the proof of the proposition.

Intuition To better understand the intuition, consider the key expression of Proposition 2 in Eq (7), which contains two terms. The first term represents the elasticity of per-period utility v to consumption, $\frac{u_c\, c}{v} = \frac{dv}{dc} \cdot \frac{c}{v}$. It determines how sensitive is per-period utility to the marginal change in consumption. This sensitivity depends on whether consumption is all that matters, or whether there are also non-pecuniary factors that affect utility.

The second term $\frac{R_u}{R_\Psi}$ measures the relative concavity of the functions $u(\cdot)$ and $\Psi(\cdot)$, and thus determines the relative concern for inequality in lifetime utilities versus for that in consumption.

When non-pecuniary benefits of being alive (b) are high, the first term is relatively low, and if it is combined with strong aversion to lifetime inequality (the second term is low), this represents a push for mortality progressivity. This happens because it is optimal to compensate people with low life expectancy for their low lifetime utility with higher consumption. In the opposite case, when consumption is the dominant component in utility, and the planner is less concerned about inequality in lifetime welfare, it is more likely that the optimal allocation is mortality-regressive.

Additional results We next show that when both $u(\cdot)$ and $\Psi(\cdot)$ are the constant relative risk aversion functions, Proposition 2 gives raise to two corollaries.

Assumption 4. The functions $u(\cdot)$ and $\Psi(\cdot)$ have constant relative risk aversion, i.e., $R_{u_i} = R_u$ and $R_{\Psi_i} = R_\Psi$ for all i.

Corollary 1 Suppose Assumptions 1-4 hold. Then the optimal consumption allocation is always mortality-regressive if $R_u \geq R_\Psi$.

Proof: When $R_\Psi = 0$, this follows from part 1 of Proposition 2. When $R_\Psi > 0$, this follows from Eq (7) and the fact that $v > 0$ and $u_c > 0$.

An important implication of Corollary 1 is that even if $u(\cdot)$ and $\Psi(\cdot)$ have the same degree of concavity ($R_u = R_\Psi$), mortality regressivity is still optimal. In other words, in order for mortality progresivity to be optimal, it is not enough to introduce aversion to inequality in lifetime utilities, it is essential that the concern for lifetime inequality is *stronger* than the concern for consumption inequality, i.e., $\Psi(\cdot)$ should be more concave than $u(\cdot)$.

Corollary 2 Suppose Assumptions 1-4 hold. In addition, assume that $u(\cdot)$ and $\Psi(\cdot)$ are strictly concave. Then mortality progressivity of the optimal consumption allocation $\{c_i\}$ is more likely to arise when per-period utility of being alive b is larger.

Proof: Consider the FOC in Eq (8) for two agents, i and j, with $q_i < q_j$. We can combine the FOCs as follows:

$$\frac{\Psi_{V_i}}{\Psi_{V_j}} \frac{u_{c_i}}{u_{c_j}} = \left(\frac{q_i}{q_j}\right)^{-R_\Psi} \left(\frac{u(c_i)+b}{u(c_j)+b}\right)^{-R_\Psi} \frac{u_{c_i}}{u_{c_j}} = 1,$$

Here we used the fact that $V_i = q(u(c_i)+b)$ and the assumption that $\Psi(\cdot)$ is the CRRA function with risk aversion R_ψ. Consider a perturbation of this equation around the optimal allocation when we change b. Since $q_i < q_j$, we have $c_i > c_j$ and the ratio $\left(\frac{u(c_i)+b}{u(c_j)+b}\right)^{-R_\Psi}$ increases in response to the marginal change in b (holding allocation fixed at the optimal level). Hence, we now have $\Psi_{V_i} u_{c_i} > \Psi_{V_j} u_{c_j}$, so it is optimal to rearrange the allocation in a way that c_i increases and c_j decreases. In other words, resources are reallocated from low-mortality to high-mortality agents, which represents a move towards mortality progressivity.

Intuitively, Corollary 2 implies that when the value of being alive increases, unequal lifespans create larger dispersion in lifetime utilities. Social planner who is averse to this type of inequality reduces it by increasing transfers to high-mortality people, thus moving towards mortality progressivity.

3.4 The aversion to inequality in consumption versus in lifetime utilities

The results in the previous section highlight the tension between the two opposite forces when it comes to optimal redistribution along mortality dimension: aversion to consumption inequality and aversion to inequality in lifetime utilities. The first force, aversion to consumption inequality, pushes social planner to equalize consumption. However, the more equal is consumption, the larger is inequality due to heterogeneous life expectancy. The second force, aversion to lifetime inequality, pushes social planner in the opposite direction - to equalize lifetime utilities, which results in unequal consumption. This is because people with high mortality get more consumption to increase their (otherwise low) lifetime utilities.

To better illustrate this tension, we consider two extreme cases when one of these two forces is shut down and hence there is concern for only one type of inequality. In the first extreme case, we remove the concern for lifetime inequality by assuming function $\Psi(\cdot)$ is linear, while $u(\cdot)$ is strictly concave. In the second case, we remove concern for consumption inequality by assuming $u(\cdot)$ is linear, while $\Psi(\cdot)$ is strictly concave.

It is worth noting that aversion to consumption inequality makes the optimal consumption allocation more mortality-regressive, while aversion to lifetime inequality makes it more mortality-progressive. We will thus refer to the optimal consumption allocation in the first extreme case (no concern for lifetime inequality, $\Psi(\cdot)$ is linear) as the *utmost mortality-regressive*; and to the optimal consumption allocation in the second extreme case (no concern for consumption inequality, $u(\cdot)$ is linear) as the *utmost mortality-progressive*.

Only consumption inequality matters: $\Psi(\cdot)$ is linear and $u(\cdot)$ is concave In this case, social planner is concerned only about consumption inequality and is neutral to inequality in lifetime utilities. It is optimal to equalize consumption, $c_i = \overline{c}$. The optimal consumption $\overline{c}$ can be found from the aggregate resource constraint:

$$\int q_i \, \overline{c} \, dG(\theta_i) = \overline{c} \int q_i \, dG(\theta_i) = \overline{c} \, \overline{q} = A,$$

implying $\overline{c} = \dfrac{A}{\overline{q}}$. This is equivalent to the situation when each agent annuitizes his endowment at the same pooled price $\overline{q}$.

Note that the resulting lifetime utility is linearly increasing in life expectancy or q_i:

$$V_i = q_i \left(u(\overline{c}) + b \right)$$

The average tax for this case is given in Eq (3). The elasticity of consumption to longevity or q_i is zero, $|\,\varepsilon^c_{q_i}\,| = 0$ for $\forall \; i$.

Only inequality in lifetime utility matters: $\Psi(\cdot)$ is concave and $u(\cdot)$ is linear In this case, social planner aims to equalize lifetime utilities, while being indifferent to inequality in consumption. Assuming $u(c) = c$, we can solve for the optimal allocation as follows. From the first-order condition in Eq (8) we have:

$$\Psi_{V_i} = \lambda \quad \forall \; i,$$

implying $V_i = \overline{V} \;\; \forall \; i$.

Since $q_i \left(u(c_i) + b\right) = q_i \, c_i + q_i \, b = \overline{V}$, we can transform the aggregate resource constraint:

$$\int q_i \, c_i \, dG(\theta_i) = \int \left(\overline{V} - q_i \, b\right) dG(\theta_i) = \overline{V} - \overline{q} \, b = A$$

Thus $\overline{V} = A + \overline{q}\, b$ and

$$c_i \;=\; \underbrace{\frac{A}{q_i}}_{\text{neutral}} \;+\; \underbrace{b\,\frac{\overline{q} - q_i}{q_i}}_{\text{compensation for short life}} \tag{10}$$

When $b = 0$, the optimal allocation is the same as neutral (or laissez-faire) allocation. When $b > 0$, people with low life expectancy ($q_i < \overline{q}$) receive transfers financed by reduction in consumption of people with high life expectancy ($q_i > \overline{q}$).

We can write the average tax as:

$$AT_i = 1 - \frac{A + b(\overline{q} - q_i)}{A} \;=\; b\,\frac{q_i - \overline{q}}{A},$$

and it is increasing in q_i implying mortality progressivity.

The absolute value of the elasticity of consumption to annuity price is greater than one as long as $b > 0$:

$$|\,\varepsilon^c_{q_i}\,| = \frac{A + b\,\overline{q}}{A + b\,(\overline{q} - q_i)} > 1 \quad \forall \;\; i$$

Numerical illustration The important difference between the utmost mortality-regressive ($\Psi(\cdot)$ is linear) and the utmost mortality-progressive ($u(\cdot)$ is linear) allocations is how consumption changes with longevity type. In the utmost mortality-regressive case, $\dfrac{dc_i}{dq_i} = 0$, while $\dfrac{dc_i}{dq_i} < 0$ for the utmost mortality-progressive case. Overall, the mortality progressivity is determined by the speed at which consumption declines with longevity.

This can be best illustrated with the following numerical example. We consider the example where $A = 5$, $r = 2\%$, $b = 1$, maximum lifespan $T = 20$ and the survival probability being uniformly distributed in the interval $[0.8, 0.95]$. In Figure 1 we plot three allocations: mortality-neutral, the utmost mortality-regressive, and the utmost mortality-progressive. The figure shows that consumption declines much quicker as longevity increases in case of the mortality-progressive case compared to the mortality-neutral case.

4 Mortality and endowment heterogeneity

In this section, we are going to relax the assumption of equal endowments. Instead, we examine the situation when individuals differ in both mortality and endowments, with the two possibly being correlated.

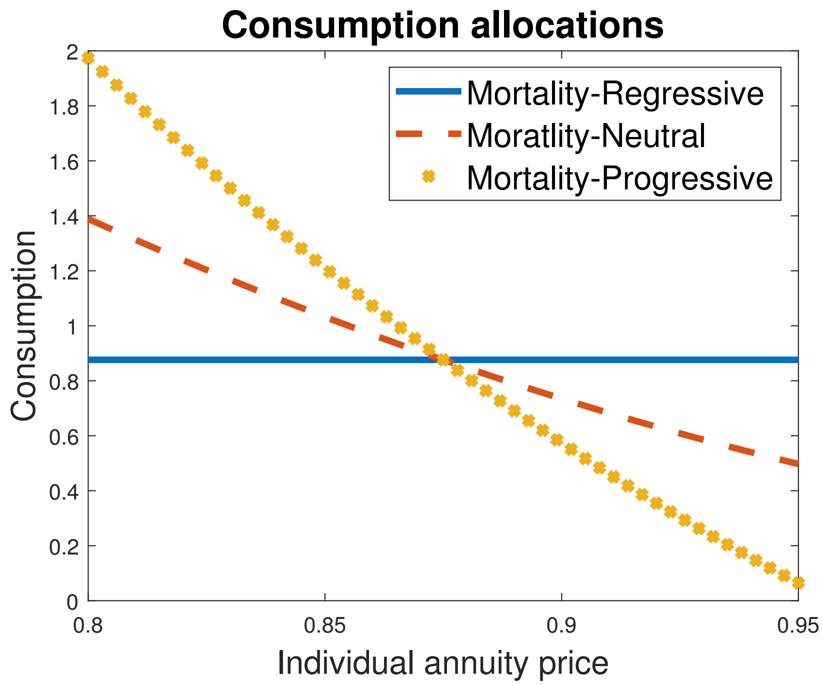

Figure 1: Consumption allocations differing in their mortality progressivity.

4.1 Environment

We denote an endowment of an individual i as a_i with $a_i \sim F(a_i)$ on the interval $[a_{min}, a_{max}]$, and with average endowment $\int a_i \, dF(a_i) = \overline{A}$. We denote the joint distribution of a_i and q_i as $H(a, q)$.

As before, we focus on age-invariant consumption allocations (c_i is the same for all t). Consumption does not vary with age in the laissez-faire case, and it is also the property of the optimal allocation, which can be shown in the same way as in Section 3. We can thus write the aggregate resource constraint as

$$\int_q \int_a \sum_{t=1}^{T} \left(\frac{\theta_i}{1+r} \right)^t c_i \, H(a_i, q_i) \, da_i \, dq_i = \overline{A} \tag{11}$$

We next need to define the reference allocation that can be used to determine mortality regressivity/progressivity. For this, we modify our definition of the neutral or laissez-faire consumption allocation (Definition 3 from Section 3) as follows.

Definition 3.1 A feasible consumption allocation $\{c_i^N\}$ is *neutral* if

$$c_i^N = \frac{a_i}{q_i} \quad \forall \ i$$

This corresponds to the laissez-faire situation when each individual converts his endowment a_i into an annuity based on his actuarially-fair annuity price q_i, and thus does not pool his mortality risk with anyone.

There may be redistribution along both mortality and endowment dimensions, and we wish to disentangle the two, i.e., to analyze redistribution along the mortality dimension for a given endowment distribution. For this, we are going to modify our Definition 4 from Section 3.

As before, we start by comparing a feasible allocation $\{c_i\}$ with the neutral allocation $\{c_i^N\}$, constructing a tax $c_i^N - c_i$, with the average tax AT_i taking the same form as before:

$$AT_i = 1 - \frac{c_i}{c_i^N}$$

Our modified definition of mortality progressivity/regressivity is stated as follows.

Definition 4.1 A feasible consumption allocation $\{c_i\}$ is mortality-regressive/-neutral/-progressive when the average tax decreases/does not change/increases with life expectancy or q_i, *given the endowment* a_i:

$$\left.\frac{\partial AT_i}{\partial q_i}\right|_{a_i} < (=) > \ 0.$$

It is worth noting that this definition also includes a concept of *mortality-neutrality* differing from that of neutrality more generally. When endowments do not differ across agents, these two concepts are the same. With heterogeneous endowments, some feasible allocations can be not neutral (not laissez-faire), while still being mortality-neutral.

One example is the allocation when all endowments are pooled together and each individual gets average endowment $\overline{A}$, which he annuitizes at the actuarially fair annuity price q_i. In this case, $c_i = \dfrac{\overline{A}}{q_i}$. Note that while agents pool together their endowments, there is no pooling of mortality risks. The average tax in this case is $AT_i = 1 - \dfrac{\overline{A}}{a_i}$. This tax does not change with longevity or q_i for a given endowment a_i.

In order to understand whether a particular allocation is mortality-progressive/regressive, we can still apply Proposition 1 but in a slightly modified form.

Proposition 1.1 Consider a feasible allocation $\{c_i\}$. If this allocation is mortality-regressive (mortality-progressive) then the following is true:

$$\varepsilon^c_{q_i\,|\,a_i} > (<) - 1 \quad \forall \ \ i,$$

where $\varepsilon^c_{q_i\,|\,a_i}$ is the partial elasticity of consumption to mortality, i.e., the elasticity for a given level of endowment a_i:

$$\varepsilon^c_{q_i \mid a_i} = \left.\frac{dc_i}{dq_i}\right|_{a_i} \cdot \frac{q_i}{c_i}.$$

Proof See Appendix A.

4.2 Optimal consumption allocation

The social planner's problem in the environment with both mortality and endowment heterogeneity can be formulated as follows:

$$\max_{\{c_i\}} \quad \int \Psi(V_i)\, H(a_i, q_i)\, da_i\, dq_i \tag{12}$$

subject to the aggregate resource constraint in Eq (11). The following proposition compares optimal consumption allocations in this case with the case of homogeneous endowments considered in Section 3.

Proposition 3 Consider the consumption allocation $\{c_i\}$ that represents the solution to the social planner problem described in Eqs (12) and (11). Suppose Assumptions 1-3 hold, and $\int a_i\, dF\,(a_i) = \overline{A} = A$, i.e., the total endowment in the economies with heterogeneous and homogeneous endowments are the same. Then the optimal consumption allocation in the two economies is the same.

Proof Since $\overline{A} = A$, the aggregate resource constraint is the same in the two economies. Denoting the Lagrange multiplier on the aggregate resource constraint as λ, we can write the FOC as follows:

$$\Psi_{V_i}\, u_{c_i} = \lambda \tag{13}$$

This is equivalent to the FOC in Eq (8) for the social planner problem in Eqs (4)-(5). The two social planner problems have the same solution, which finishes the proof of the proposition.

It is worth noting that the FOC in Eq (13) implies that consumption allocation $\{c_i\}$ does not depend on individual endowments a_i but can depend on mortality q_i. Proposition 3 shows that when social planner can freely redistribute endowments, the characterization of the optimal allocation does not change when we allow for heterogeneous endowments. This also means that the conditions for mortality progressivity are the same as described in Proposition 2.

We next examine the two extreme cases considered above, when either function $\Psi(\cdot)$ or $u(\cdot)$ is linear. This results in either the utmost mortality-regressive allocation (when only consumption inequality matters), or the utmost mortality-progressive allocation (when only lifetime inequality matters). Using the same steps as in Section 3.4, we can find the utmost mortality-regressive consumption allocation:

$$c_i = \frac{\overline{A}}{\overline{q}} \tag{14}$$

The utmost mortality-progressive allocation takes the following form:

$$c_i = \frac{\overline{A}}{q_i} + b\,\frac{\overline{q} - q_i}{q_i} \tag{15}$$

In both cases, individual endowments are pooled together and equally distributed across agents (everyone gets $\overline{A}$). In the utmost mortality-regressive case, there is also a pooling of mortality risks across all agents. In the utmost mortality-progressive case, there is no pooling of mortality risks, moreover, people with low(high) life expectancy get positive(negative) transfers whenever $b > 0$.

4.3 Restricted social planner problem

The results above show that in our environment, social planner always chooses to fully redistribute agents' endowments. This is because we abstract from forces that usually prevent full income redistribution, such as asymmetric information or moral hazard (see reviews of these issues in Boadway, 2012; Kaplow, 2008; Mankiw et al., 2009; Salanie, 2011; Tuomala, 2016).

This leaves open two important questions. First, what would be optimal redistribution along mortality dimension in the environment where full income redistribution is not feasible. Second, what is the role of income-mortality correlation. These two questions are closely related. When it is possible to equalize economic resources across agents, mortality-endowment correlation does not play a role in optimal allocations. This is because each agent ends up with an equal share of pooled resources and his starting endowment does not matter. In contrast, when full income redistribution is not feasible, social planner cannot eliminate endowment inequality which can exacerbate inequality in lifetime utilities when mortality and endowments are negatively correlated.

Our goal is thus to examine the optimal redistribution along mortality dimension when income redistribution is restricted. To keep the model tractable, we do not model the underlying forces that prevent social planner from redistribution of economic resources (e.g., moral hazard). Instead, we mechanically restrict the set of instruments that social planner can access. We thus study the optimality of mortality-related redistribution for a *given* degree of income redistribution.

An important question is how to set up an environment when social planner can redistribute along the dimension of mortality but not along the dimension of income. We start by noting that in the unrestricted social planner problem, optimal consumption allocations vary between the utmost mortality-regressive (Eq 14) and the utmost mortality-progressive (Eq 15) cases. We can approximate consumption allocations in between these two extreme cases with the following parametric form:

$$c_i = (1-\alpha_1)\,\frac{\overline{A}}{\overline{q}} + \alpha_1\,\frac{\overline{A}}{q_i} + \alpha_2\, b\,\frac{\overline{q}-q_i}{q_i}, \tag{16}$$

where $\alpha_1 \in [0,1]$ and

$$\begin{cases} \alpha_2 = 0 & ;\ \text{if } \alpha_1 < 1 \\ \alpha_2 \geq 0 & ;\ \text{if } \alpha_1 = 1 \end{cases}$$

When $\alpha_1 = 0$, the allocation is the utmost mortality-regressive. As α_1 increases, the allocation moves away from the utmost-regressive and towards the mortality-neutral case. Once $\alpha_1 = 1$, the allocation is mortality-neutral. Increasing α_2 above zero introduces mortality progressivity. The aggregate resource constraint is satisfied for all described combinations of α_1 and α_2.

It is worth noting that in the consumption allocation rule in Eq (16), varying α_1 and α_2 changes the degree of redistribution along mortality dimension *given* full redistribution of endowments.

We next consider an equivalent consumption allocation rule but *given* no redistribution of endowments. In other words, social planner can change the degree of redistribution along the dimension of mortality while keeping each agent's endowment a_i unchanged. We can represent this consumption allocation rule as follows:

$$c_i = (1-\alpha_1)\,\frac{a_i}{\overline{q}^a} + \alpha_1\,\frac{a_i}{q_i} + \alpha_2\, b\,\frac{\overline{q}-q_i}{q_i}, \tag{17}$$

where $\overline{q}^a$ is the endowment-weighted average annuity price:

$$\overline{q}^a = \frac{\int\limits_q \int\limits_a q_i a_i\ H(a_i, q_i)\, da_i\, dq_i}{\overline{A}},$$

Note that compared to Eq (16), the first term in this equation is divided by $\overline{q}^a$ as opposed to $\overline{q}$ in order to meet the aggregate resource constraint. This way, varying α_1 and α_2 does not change the total spending on consumption allocations.

We can thus formulate the restricted social planner problem as follows:

$$\max_{\alpha_1,\alpha_2} \int\limits_q \int\limits_a \Psi(V_i)\ H(A_i, q_i)\, da_i\, dq_i, \tag{18}$$

where $V_i = q_i(u(c_i) + b)$ and c_i is given in Eq (17).

In our analysis below, we focus on the case when social planner is indifferent to inequality in lifetime utilities by making $\Psi(\cdot)$ linear. Our analysis in the previous sections shows that aversion to lifetime inequality is an important force determining optimality of mortality progressivity. Our goal now is to isolate the role of mortality-endowment correlation in determining mortality-progressivity of optimal allocations. The results are summarized in Proposition 4.

Proposition 4 Consider the constrained social planner problem described in Eq (18), and suppose Assumptions 1-4 hold. In addition, assume $R_\Psi = 0$ and $R_u > 1$. The optimal choice of α_1 and α_2 can be summarized as follows:

(1) If $cov(q, a) = 0$, then at the optimum $\alpha_1 = 0$,

(2) If $cov(q, a) > 0$, then

  (i) At the optimum $\alpha_1 > 0$,

  (ii) If $cov(q, \frac{a}{q}) < 0$, then at the optimum $\alpha_2 = 0$,

  (iii) If $cov(q, \frac{a}{q}) > 0$, then at the optimum $\alpha_2 > 0$.

Proof See Appendix B.

The intuition for the part (i) of Proposition 4 follows from our analysis in the previous section: when social planner's only concern is consumption inequality (as $\Psi(\cdot)$ is linear), it is optimal to equalize consumption. The closest social planner can get to equalizing consumption when he cannot directly redistribute endowments is by making consumption allocation the utmost mortality-regressive, which is equivalent to annuitizing an endowment of each agent at the same pooled price.

Once the covariance between longevity and endowment is positive, optimal allocations change. Specifically, when high-mortality people tend to have lower endowments, it is optimal to move away from the utmost mortality-regressive allocation, and the stronger is the mortality-endowment link, the further it is optimal to move. We formally show that the condition $cov(q, \frac{a}{q}) > 0$ is stronger than $cov(q, a) > 0$ in the Auxiliary proposition in Appendix

B.

It is worth noting that moving away from the utmost mortality-regressive is equivalent to increasing consumption of high-mortality people. In the mortality-neutral case, the sign of the covariance between life expectancy and consumption is determined by $cov(q, \frac{a}{q})$ since $c^N = \frac{a}{q}$. When this covariance is negative, this means that in the mortality-neutral case, people with high life expectancy tend to get lower consumption. In this situation, it is not optimal to further increase consumption of low-mortality people by moving towards mortality progressivity. In contrast, when $cov(q, \frac{a}{q}) > 0$, people with high life expectancy have higher consumption even in the mortality-neutral case, and transferring resources to low-life-expectancy group (moving to mortality progressivity), will further reduce consumption inequality.

It is important to point out that mortality progressivity in this case arises not because social planner wants to equalize lifetime utilities by increasing consumption of high-mortality agents. Instead, mortality progressivity is used as a substitute for income progressivity in absence of direct instruments to redistribute endowments.

5 Mortality regressivity and pension design

In this section, we extend our theoretical framework to analyze pension design. We modify the setup of Sections 3 and 4 in two ways. First, instead of a representative cohort, we consider the overlapping generations model. Second, we introduce two stages of the life-cycle: working and retirement periods. During working period, each individual receives labor income and pays contributions to the pension system, during retirement period, he receives pension benefits. In this environment, we can think of total contributions to the system as one's endowment, and of pension benefits as the annuitized value of the endowment. People may differ both in endowments and mortality, and pension system may thus feature redistribution along both dimensions. We focus on a set of revenue-neutral policies, i.e., policies that are financed by the same tax revenue.

5.1 Environment

We consider an overlapping generations model where each individual lives for T periods: for the first R periods, an agent receives labor income ϵ_i, between periods $R+1$ and T, an agent is retired. The population grows at the rate n.

In our analysis, we maintain Assumptions 1-4 from Section 3. We also assume that the population growth rate is equal to the interest rate, $n = r$. In addition, we assume agents have inelastic labor supply and cannot save. In our quantitative model in the next section, we relax the assumption of no savings.

Agents differ in mortality and labor productivity ϵ_i, with $\epsilon_i \sim F(\epsilon_i)$, and we denote the average productivity $\bar{\epsilon} = \int \epsilon \, dF(\epsilon)$. Agents survive to period R with probability one, and after that, the probability to survive from age t to $t+1$ is θ_i with $\theta_i \sim G(\theta_i)$ and $\overline{\theta} = \int \theta \, dG(\theta)$.

As before, we use actuarially fair annuity price q_i as a marker of an individual's longevity rather than survival probability θ_i. The actuarially fair price q_i of a unit of lifelong annuity income for an individual i acquired before retirement (at age R) is:

$$q_i = \sum_{t=1}^{T-R} \left(\frac{\theta_i}{1+r} \right)^t ,$$

with the average annuity price denoted as $\overline{q}$. We denote their joint distribution of labor productivity and longevity as $H(\epsilon, q)$.

During the working stage of the life-cycle, each agent pays proportional tax τ on his labor income. After retirement, each agent receives benefits ssb_i. Given our assumption that agents cannot save, each period's consumption is equal to income (either labor income or pension income), and the lifetime utility can be represented as follows:

$$\begin{aligned} V_i &= \sum_{t=1}^{R} \; \beta^{t-1}(u(\epsilon_i(1-\tau)) + b) \; + \; \beta^{R-1} \sum_{t=1}^{T-R} \; (\beta\theta)^t (u(ssb_i) + b) \\ &= \sum_{t=1}^{R} \; \beta^{t-1}(u(\epsilon_i(1-\tau)) + b) \; + \; \beta^{R-1} q_i \; (u(ssb_i) + b) \end{aligned} \tag{19}$$

The last equality follows from the assumption $\beta(1+r) = 1$ (Assumption 2) and the definition of the annuity price. Denoting the lifetime utility during working period as $V_i^W = \sum_{t=1}^{R} \beta^{t-1}(u(\epsilon_i(1-\tau))+b)$, and during the retirement period as $V_i^R = q_i \; (u(ssb_i)+b)$, we can also write:

$$V_i = V_i^W + \beta^{R-1} V_i^R$$

5.2 Pension system

There are two ways to set up a pension system in this environment, either on a fully funded or on a pay-as-you-go basis. In the first case, an individual pays contributions to his pension account that is later annuitized. In the second case, working-age people finance pensions of retirees. In this section, we show that under the assumptions $n = r$ and inelastic labor supply, the two systems are equivalent. This result is convenient because it allows us to think of pension contributions in both cases as individuals' notional balances or endowments.

In the fully-funded pension system, an individual i has a notional balance IC_i, which represents his lifelong contributions:

$$IC_i = \tau\epsilon_i \sum_{t=1}^{R} (1+r)^{t-1}$$

Upon retirement, the notional balance IC_i is converted into pensions ssb_i. We can thus write the balance equation for the fully-funded pension system as follows:

$$\int_q \int_\epsilon ssb\ q\ H(\epsilon, q)\, d\epsilon\, dq = \int_\epsilon IC dF(\epsilon), \tag{20}$$

where the left-hand side is the total pension payments and the right-hand side is the total value of contributions.

In the pay-as-you-go system, individuals' contributions are used to finance pensions of the old. Denoting as N the initial size of the cohort who is currently of age T (the oldest cohort), we can write the pension system balance equation as follows:

$$N \int_q \int_\epsilon ssb \sum_{t=1}^{T-R} \theta^t (1+n)^{T-R-t} H(\epsilon, q)\, d\epsilon\, dq = N \int_\epsilon \tau\epsilon \sum_{t=1}^{R} (1+n)^{T-t} dF(\epsilon)$$

Dividing both sides by $N(1+n)^{T-R}$, which is the size of the cohort who is about to retire, we have

$$\int_q \int_\epsilon ssb \sum_{t=1}^{T-R} \left(\frac{\theta}{1+n}\right)^t H(\epsilon, q)\, d\epsilon\, dq = \int_\epsilon \tau\epsilon \sum_{t=1}^{R} (1+n)^{t-1} dF(\epsilon) \tag{21}$$

We can use the assumption $n = r$ and the definition of the actuarially-fair annuity price to show that the balance equations for the fully-funded and pay-as-you-go pension systems in Eqs (20) and (21) are equivalent. Moreover, we can think of the right-hand side of Eq (21) as the average of notional balances IC_i, since when $n = r$ the following is true:

$$IC_i = \tau\epsilon_i \sum_{t=1}^{R} (1+n)^{t-1}$$

In this stylized framework, we will treat IC_i as one's endowment, and pension benefits ssb_i as the annuitized value of this endowment.

It is worth noting that since we focus on a set of revenue-neutral policies, we fix the tax rate τ. Given the assumption of inelastic labor supply, this means the average contributions $\overline{IC} \equiv \int\limits_{\epsilon} ICdF(\epsilon)$ do not vary across policies.

5.3 Redistribution through pensions

The key question in any pension system is how to link pension benefits ssb_i to one's contributions IC_i. Different pension arrangements create different redistribution along income and mortality dimensions. For example, the U.S. Social Security system involves progressive redistribution along income dimension since the replacement rates are higher for people with low lifetime income. At the same time, there is regressive redistribution along mortality dimension since pensions are not indexed for one's life expectancy.

To connect our subsequent analysis to our earlier results, in this section, we formally define the notion of progressive/regressive pensions along the dimension of income or mortality. As before, our starting point is what we refer to as neutral pensions, that is to say, hypothetical pensions that do not involve redistribution along neither mortality nor income dimensions. All pension arrangements we consider are feasible in a sense defined below.

Definition 1.2 We call pension benefits $\{ssb_i\}$ *feasible* if they satisfy the pension system balance equation in Eqs (20) or (21).

Since pension benefits can be thought of as annuitized value of one's endowment, this leads us to the following definition of neutral pension benefits.

Definition 3.2 Feasible pension benefits $\{ssb_i^N\}$ are *neutral* if

$$ssb_i^N = \frac{IC_i}{q_i} \quad \forall \ i$$

This corresponds to the situation when each individual converts his lifelong contributions to the pension system IC_i into pensions based on his actuarially-fair annuity price q_i.

A pension system may involve redistribution along both income and mortality dimensions, and we wish to analyze these two separately. For this, we modify our definition of mortality regressivity (Definition 4.1 from Section 4) and add the definition of endowment regressivity/progressivity.

To do this, we start by comparing feasible pension benefits $\{ssb_i\}$ with neutral benefits $\{ssb_i^N\}$, constructing the average tax AT_i:

$$AT_i = 1 - \frac{ssb_i}{ssb_i^N}$$

Our modified definition of mortality progressivity/regressivity is stated as follows.

Definition 4.2 Feasible pension benefits $\{ssb_i\}$ are mortality-regressive/-neutral/-progressive when the average tax decreases/does not change/increases with life expectancy or q_i, *given the endowment (lifetime pension contributions)* IC_i:

$$\left.\frac{\partial AT_i}{\partial q_i}\right|_{IC_i} < (=) > \; 0.$$

In a similar fashion, we can define the endowment regressivity/progressivity in our environment:

Definition 6 Feasible pension benefits $\{ssb_i\}$ are endowment-regressive/-neutral/-progressive when the average tax decreases/does not change/increases with endowment IC_i, *given the life expectancy or* q_i:

$$\left.\frac{\partial AT_i}{\partial IC_i}\right|_{q_i} < (=) > \; 0.$$

5.4 Optimal pension system

The next question is what is the optimal degree of redistribution along the dimensions of mortality and endowments in the pension system. To answer this, we consider the social planner problem. Since the pension contribution rate τ and the size of the pension program are fixed, social planner only chooses how to set pension benefits subject to the pension system balance equation:

$$\max_{\{ssb_i\}} \int_q \int_\epsilon \Psi(V_i) \; H(\epsilon_i, q_i) \, d\epsilon_i \, dq_i \tag{22}$$

s.t.

$$\int_q \int_\epsilon ssb_i \; q_i \; H(\epsilon_i, q_i) \, d\epsilon_i dq_i = \overline{IC}, \tag{23}$$

where V_i is given in Eq (19).

Denoting the Lagrange multiplier on the constraint as $\beta^{R-1}\lambda$, we can write the FOC as follows:

$$\Psi_{V_i} \, u_{ssb_i} = \lambda, \tag{24}$$

where

$$\frac{\partial \, \Psi(V_i)}{\partial \, V_i} \equiv \Psi_{V_i} \, , \qquad \frac{\partial \, u(ssb_i)}{\partial \, ssb_i} \equiv u_{ssb_i}$$

The FOC in Eq (24) implies the following property of the optimal pension benefits.

Since V_i is increasing in both q_i and ϵ_i, when $\Psi(\cdot)$ is strictly concave, ssb_i is decreasing in q_i (given ϵ_i) and in ϵ_i (given q_i). The later effect is new compared to the environment studied in Sections 3 and 4, where an individual's endowment does not affect optimal consumption. This effect arises because social planner can only change consumption after retirement, while concern for lifetime inequality also makes the utility over working period matter for optimal pensions.

We can now formulate the modified version of Proposition 2.

Proposition 2.1 Consider pension benefits $\{ssb_i\}$ that represent the solution to the social planner problem described in Eqs (22)-(23). Under Assumptions 1-4, whether these benefits are mortality- and endowment-regressive/progressive can be determined as follows.

1. If $\Psi(\cdot)$ is linear, $\{ssb_i\}$ are mortality-regressive.
2. If $\Psi(\cdot)$ is strictly concave, then $\{ssb_i\}$ are mortality-regressive (-progressive) if

$$\frac{u_{ssb_i} ssb_i}{v_i^R} + \frac{R_u}{R_\Psi} \frac{V_i}{\beta^{R-1} V_i^R} > (<)\ 1 \qquad \forall \ i \tag{25}$$

   where $v_i^R = u(ssb_i) + b$ is the flow utility per period after retirement.

3. $\{ssb_i\}$ are always endowment-progressive.

Proof: See Appendix C.

Intuition Consider the condition for mortality regressivity/progressivity in Eq (25). Similarly to Eq (7) in Section 3, it contains two parts. The first term represents the elasticity of per-period utility during retirement period v_i^R to pension benefits ssb_i, i.e., $\frac{u_{ssb_i} ssb_i}{v_i^R} = \frac{d\, v_i^R}{d\, ssb_i} \cdot \frac{ssb_i}{v_i^R}$. It describes how sensitive is per period utility after retirement to the marginal change in pension benefits (or consumption).

The second term measures the relative concavity of the functions $u(\cdot)$ and $\Psi(\cdot)$, and thus captures how important is the concern for consumption inequality compared to the concern for inequality in lifetime utilities. Unlike in Eq (7), here the measure of relative concavity $\frac{R_u}{R_\Psi}$ is multiplied by the term $\frac{V_i}{\beta^{R-1} V_i^R}$, which is the ratio of lifetime utility V_i to the discounted lifetime utility during the retirement period, $\beta^{R-1} V_i^R$. Since the lifetime utility is the sum of utilities during working and retirement periods, this expression is larger than one. This multiplicative term makes the condition for mortality progressivity harder to meet, as it scales up the ratio $\frac{R_u}{R_\Psi}$, thus putting more weight on consumption inequality. Moreover, the smaller is V_i^R relative to V_i, the more pronounced this effect is. This is because inequality in mortality only plays a role after retirement. Thus, the less important is retirement period for individual welfare, the less important is mortality inequality.

It is also worth making a quick note about the relationship between pensions and endowments. In the neutral case (when each individual receives pension benefits based on his own lifetime contribution IC_i), pension benefits are higher for people with high productivity ϵ_i. In contrast, optimal pensions are the same for all productivity types in the standard utilitarian welfare settings ($\Psi(\cdot)$ is linear), implying the redistribution form high- to low-productivity types. Introducing aversion to lifetime inequality ($\Psi(\cdot)$ is concave) makes pension benefits *decreasing* in productivity, thus making redistribution along the endowment dimension more progressive. This is because it is now optimal to compensate low-productivity people for their low consumption during working years.

5.5 Two extreme cases

Our analysis above shows that the tension between aversion to consumption inequality and aversion to inequality in lifetime utilities still plays a key role in determining the optimal mortality progressivity of pensions. As before, the aversion to consumption inequality pushes pensions towards mortality regressivity, while aversion to lifetime inequality makes pension more mortality-progressive. To better illustrate this tension, we once again consider two extreme cases when one of these forces is shut down.

Case 1: Only aversion to consumption inequality matters ($\Psi(\cdot)$ *is linear and* $u(\cdot)$ *is concave).* In this case, optimal social security benefits are the same for all individuals and take the following form:

$$ssb_i = \frac{\overline{IC}}{\overline{q}} \quad \text{for } \forall\ i$$

Thus, social planner equalizes consumption after retirement. This is equivalent to pooling together all individual contributions (or endowments), and then annuitizing equalized endowments at the pooled annuity price $\overline{q}$.

We can write down the average tax in this case as:

$$AT_i = 1 - \frac{\overline{IC}}{IC_i} \frac{q_i}{\overline{q}}$$

The tax increases in endowment IC_i but decreases in life expectancy or q_i, implying endowment progressivity but mortality regressivity.

Case 2: Only aversion to lifetime inequality matters ($\Psi(\cdot)$ *is concave and* $u(\cdot)$ *is linear).* Assuming $u(c) = c$ and using the assumption $\beta(1 + r) = 1$, we can solve for ssb_i with the following result:

$$ssb_i = \underbrace{\frac{\overline{IC}}{q_i}}_{\text{part 1}} + b\underbrace{\frac{\overline{q} - q_i}{q_i}}_{\text{part 2}} + \underbrace{\frac{1-\tau}{\tau}\frac{\overline{IC} - IC_i}{q_i}}_{\text{part 3}}$$

The expression for optimal benefits contains three parts. The first part is the same as the mortality-neutral allocation with fully-redistributed pension contributions, i.e., contributions are pooled together, divided equally among agents, and then converted into an annuity based on the individual actuarially-fair price. The second part is the compensation for short life which is positive (negative) for people with low (high) life expectancy. It is worth noting that these two parts are also present in the expression for optimal consumption when only lifetime inequality matters considered in the previous sections (see Eq (10) in Section 3 and Eq (15) in Section 4).

The third part is new: this is an additional redistributive component that social planner uses to compensate low-income people for low consumption during their working years. This term arises because social planner can only affect consumption after retirement, while aiming to reduce inequality in lifetime utilities.

We can express the average tax in this case as:

$$AT_i = \frac{1}{\tau}\frac{IC_i - \overline{IC}}{IC_i} + b\frac{q_i - \overline{q}}{IC_i}$$

The average tax increases in life expectancy or q_i, and in endowment or IC_i, implying both mortality and endowment progressivity.[5]

6 Quantitative illustration

In this section, we aim to understand the relative quantitative importance of the theoretical mechanisms described above. To do this, we quantitatively solve a life-cycle model with two stages, working and retirement periods, as described in Section 5. We then use the model to assess the welfare effects of different pension designs that vary in the degree of mortality regressivity. We consider a small open economy with the fixed interest rate r.

[5] It can be shown that AT_i increases in IC_i when $\frac{1}{\tau}\overline{IC} + b(\overline{q} - q_i) > 0$. The latter inequality always holds, otherwise ssb_i is negative.

6.1 Model description

Individuals A model period is one year. Individuals enter the model at age $t = 25$. Up to age R, individuals receive labor income, after age R, individuals retire and receive pensions.

Individuals are ex-ante different in their type j, $j \in [1, ..., J]$ which is fixed throughout their life. Type affects individual's survival probability and labor productivity. As in Section 5, we assume agents survive with probability one till age R. For age $t > R$, we denote the type-dependent probability to survive from age t to $t+1$ as θ_t^j. The earnings of an individual are equal to λ_t^j, the idiosyncratic productivity that depends on age (t) and type (j).

During the working period ($t = 1...R$), an individual pays tax τ on his labor earnings, after retirement ($t = R + 1...T$), he receives pension benefits ssb.

The state variables of an individual are assets (k_t), age (t), and type (j). The optimization problem of an individual can be represented as follows:

$$V_t(k_t, j) = \max_{c_t, k_{t+1}} \left\{ u(c_t) + b + \theta_t^j V_{t+1}(k_{t+1}, j) \right\} \tag{26}$$

subject to

$$k_t \, (1 + r) + inc_t = k_{t+1} + c_t, \tag{27}$$

where

$$inc_t = \begin{cases} \lambda_t^j (1 - \tau) & ; \text{ if } \; t \leq R \\ ssb & ; \text{ if } \; t > R \end{cases} \tag{28}$$

Social Security There is a pension system that collects contributions from the young and pays out benefits to the old. As in Section 5, we assume the economy is dynamically efficient, $n = r$. Denoting the distribution of agents over states as $\mathcal{M}(\cdot)$, we can write down the pension system balance equation as follows:

$$\int_{t \leq R} \tau \lambda_t^j \; \mathcal{M}(k, j, t) = \int_{t > R} ssb \; \mathcal{M}(k, j, t)$$

Note that due to the dynamic efficiency assumption, the balance equation is the same for the pay-as-you-go and fully funded systems (as discussed in Section 5.2). Since all agents survive with probability one till age R, the left-hand side of the pension balance equation represents the average of individuals' pension contributions IC_j, where

$$IC_j = \tau \sum_{t=1}^{R} (1 + n)^{t-1} \lambda_t^j = \tau \sum_{t=1}^{R} (1 + r)^{t-1} \lambda_t^j$$

The last equality follows from the dynamic efficiency assumption, and it allows us to think of IC_j as a notional pension balance or individual endowment.

Welfare We aggregate the lifetime utilities of different individuals into the ex-ante measure of welfare as follows:

$$EW = \int \Psi\ (V_1\ (k_1, j))\ \mathcal{M}\,(k_1, j) \tag{29}$$

We assume that both the utility function $u(\cdot)$ and the function used to aggregate individual welfare $\Psi(\cdot)$ are of the CRRA type with risk aversion parameters $R_u = \sigma$ and $R_\Psi = \gamma$:

$$u(c) = \frac{c^{1-\sigma}}{1-\sigma}$$

and

$$\Psi(V_1) = \frac{V_1^{1-\gamma}}{1-\gamma}.$$

The welfare effects of each experiment are computed as follows. We treat the utmost mortality-regressive case (pooled annuitization) as a benchmark, and we denote the corresponding ex-ante welfare as EW^{BS}. Consider a situation when every agent receives cash transfer Δ every period, and denote the corresponding ex-ante welfare as $EW(\Delta)$. Note that if $\Delta = 0$, we have the benchmark welfare: $EW(0) = EW^{BS}$.

Denote the ex-ante welfare in the experimental economy as EW^{Exp}. We compute the cash transfers needed to make average welfare in the baseline and experimental economies the same (Δ^*) by solving the following equation:

$$EW(\Delta^*) = EW^{Exp}$$

Our welfare measure CEV is expressed as a percentage of average consumption:

$$CEV = \frac{\Delta^*}{\overline{c}}$$

6.2 Parameterization

In our model, we focus on male individuals. Within the same gender group, education and race are important determinants of longevity inequality. To capture this, the ex-ante fixed type in our model is a pair of fixed characteristics, education and race, $j = (ed, ra)$. There are three education types and two race types, thus the total number of types J is equal to 6. Three education types correspond to high-school dropouts ($ed = 1$), people with only high-school degree or high-school degree and some college education but no college degree ($ed = 2$), and people with college or higher degree ($ed = 3$). Two race groups correspond to non-white ($ra = 1$) and white ($ra = 2$).

We set the retirement age $R + 1$ to 65. For age $t > R$, the conditional probability to survive from age t to $t + 1$ (θ_t^j) is estimated using the Health and Retirement Study (HRS) dataset. In our estimation, we use a sample of male individuals and estimate a logit model which depends on a set of age, race, and education dummy variables. Our estimated survival probabilities are plotted in the left panel of Figure 2.

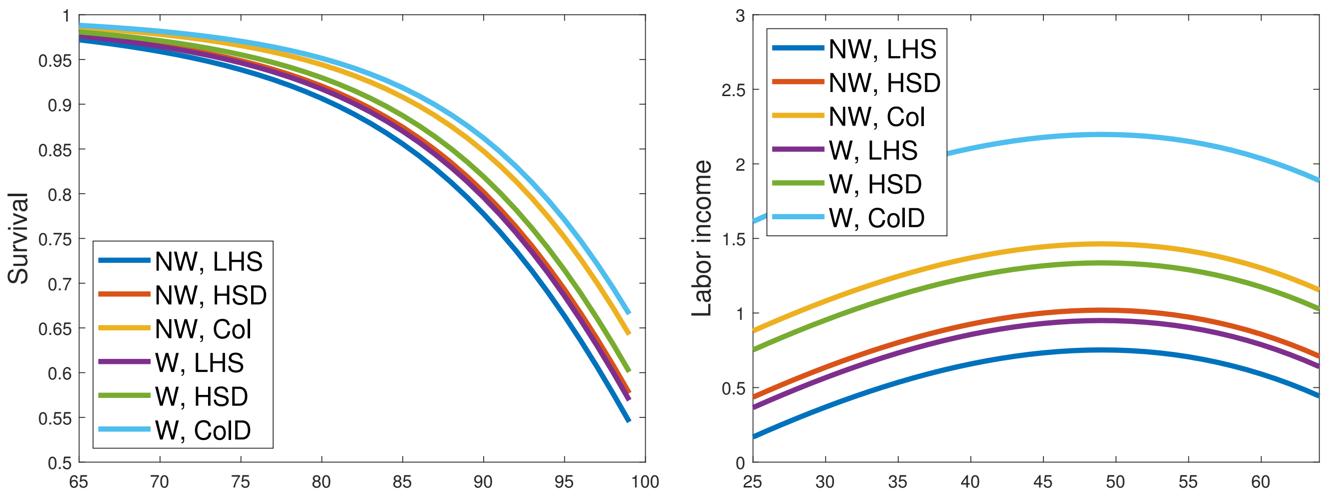

Figure 2: Left panel: survival probabilities by race and education. Right panel: labor income (normalized by average earnings) by race and education. The abbreviations are as follows: LHS - less than high-school degree ($ed = 1$), HSD - high-school degree ($ed = 2$), ColD - college degree and above ($ed = 3$). NW stands for non-whites ($ra = 1$), and W stands for whites ($ra = 2$).

To estimate labor productivity for each type, λ_t^j, we use the Panel Study of Income Dynamics (PSID). We use a sample of male workers, where we define a person as employed if he works at least 520 hours per year, and earns at least the federal minimum wage. We normalize labor income to 2002 base year using the Consumer Price Index (CPI). Our estimated labor income profiles are plotted in the right panel of Figure 2.

We set population growth n and interest rate r to 1%. As in our theoretical section, we maintain the assumption $\beta(1+r) = 1$, hence we set the discount rate β to the inverse of $1+r$. We set risk aversion over consumption, σ, to 2, which is a common value used in structural life-cycle models. We set the initial distribution of each type based on the fraction of people in each race/education group in the HRS at age 65. We consider several alternative values for the aversion to inequality in lifetime utilities (γ), and non-pecuniary utility of being alive (b).

6.3 Results

In this section, we use our quantitative life-cycle model for the comparative welfare analysis of different pension arrangements. We start by describing how we set up our quantitative experiments. We then evaluate the welfare effects of restoring mortality neutrality using the utmost mortality-regressive case as a benchmark. Finally, we compare a wider range of pension policies varying in the degree of mortality regressivity/progressivity.

In each case, we change the degree of redistribution among the dimension of mortality *given* no redistribution along income dimension. In this, we follow the setup in Section 4.3. While typical pension systems involve some degree of income redistribution, no pension system goes all the way towards full redistribution of endowment, which is optimal in our stylized settings. To keep the model tractable while capturing the key features of reality, we mechanically restrict the ability of pension system to redistribute pension contributions. This also allows us to understand the role of income-mortality correlation.

6.3.1 Setup

To better understand the underlying mechanisms when changing the degree of mortality progressivity in a pension system, we consider two versions of our model. The first model, which we refer to as "only mortality heterogeneity", is when people have the same endowments throughout their lifetime. To construct this model, we assume that all types have the same labor productivity equal to the average in their age group: $\lambda_t^j = \overline{\lambda}_t$ for all j. In this case, all individuals have exactly the same pension balances or contributions to the pension system: $IC_j = IC$ for all j. The second model, which we refer to as "mortality and income heterogeneity", corresponds to the full model, i.e., each type has different labor income and mortality.

6.3.2 Moving to mortality-neutrality

The starting point of our analysis is the utmost mortality-regressive case or pooled annuitization, which is a common feature of many pension systems. We first consider the welfare effects of restoring mortality-neutrality. Below we explain how we compute pension benefits in the utmost mortality-regressive and mortality-neutral cases.

The utmost mortality-regressive case In the utmost mortality-regressive case, pension benefits are the same for all people in the model "only mortality heterogeneity". This is because in this model, labor income and thus pension balances do not differ across types, and converting them into an annuity based on the pooled price results in the same pension benefits:

$$ssb_j = \frac{IC}{\overline{q}} \quad \text{for } \forall\ j$$

These pension benefits are plotted as a solid line in the left panel of Figure 3. It is worth noting that in this environment, even though people have the same income and the same pensions, their consumption still differs as they have different optimal savings because of the difference in longevity.

In the "mortality and income heterogeneity" model, pension benefits differ across types because of the difference in pension balances:

$$ssb_j = \frac{IC_j}{\overline{q}}$$

These benefits are plotted as a solid line in the right panel of Figure 3.

Mortality-neutral case In the mortality-neutral case, pension benefits are based on individual mortality. For "only mortality heterogeneity" model, pension benefits take the following form:

$$ssb_j = \frac{IC}{q_j}$$

The benefits are plotted as a dashed line in the left panel of Figure 3. In the "mortality and income heterogeneity" model, we have

$$ssb_j = \frac{IC_j}{q_j}$$

These pension benefits are plotted as a dashed line in the right panel of Figure 3.

An important observation from Figure 3 is that when there is no heterogeneity in labor income, mortality-neutral pension benefits are higher for high-mortality groups. This is because pension balances are the same for all groups, while annuity prices are lower for those with high mortality. With labor income heterogeneity, pension benefits are lower for those with high mortality due to the strong income-mortality correlation. In light of the discussion in Section 4.3 and Proposition 4, this corresponds to the case $cov(q, \frac{a}{q}) > 0$.

The welfare effects of moving to mortality-neutrality for the two versions of the model are presented in Figure 4. Each graph in the figure shows CEV for different degrees of aversion to inequality in lifetime utility γ, varying from 0 to 5, and for different levels of non-pecuniary utility of being alive b, varying from 10 to 50.

Consistent with our results in the previous sections, welfare effects are sensitive to non-

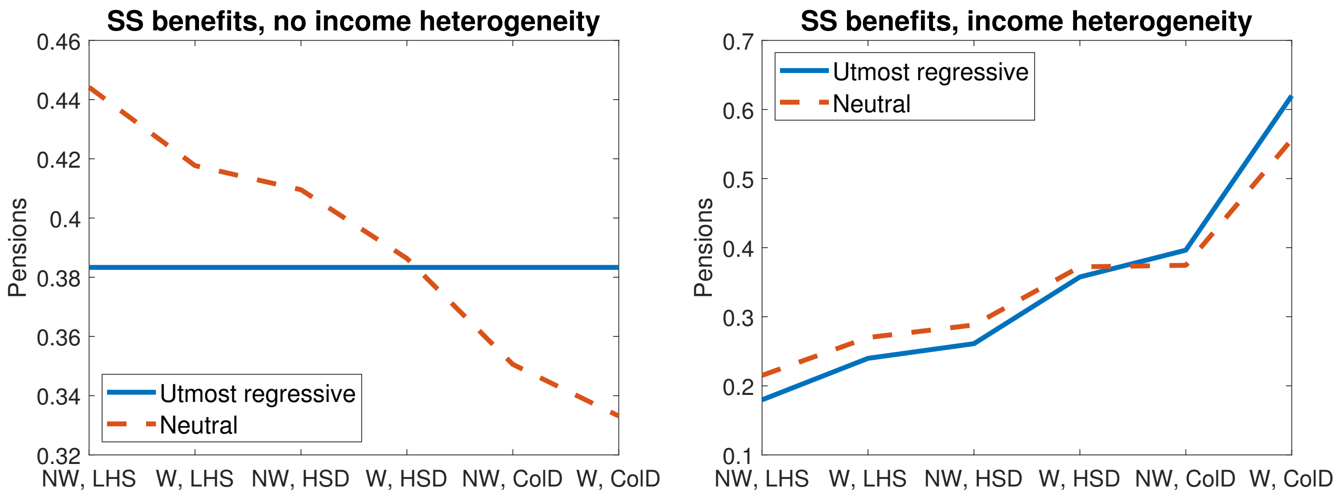

Figure 3: Pensions benefits (normalized by average income), the utmost mortality-regressive (pooled annuitization) versus the mortality-neutral cases. Left panel: only mortality heterogeneity. Right panel: mortality and income heterogeneity. The abbreviations are as follows: LHS - less than high-school degree ($ed = 1$), HSD - high-school degree ($ed = 2$), ColD - college degree and above ($ed = 3$). NW stands for non-whites ($ra = 1$), and W stands for whites ($ra = 2$).

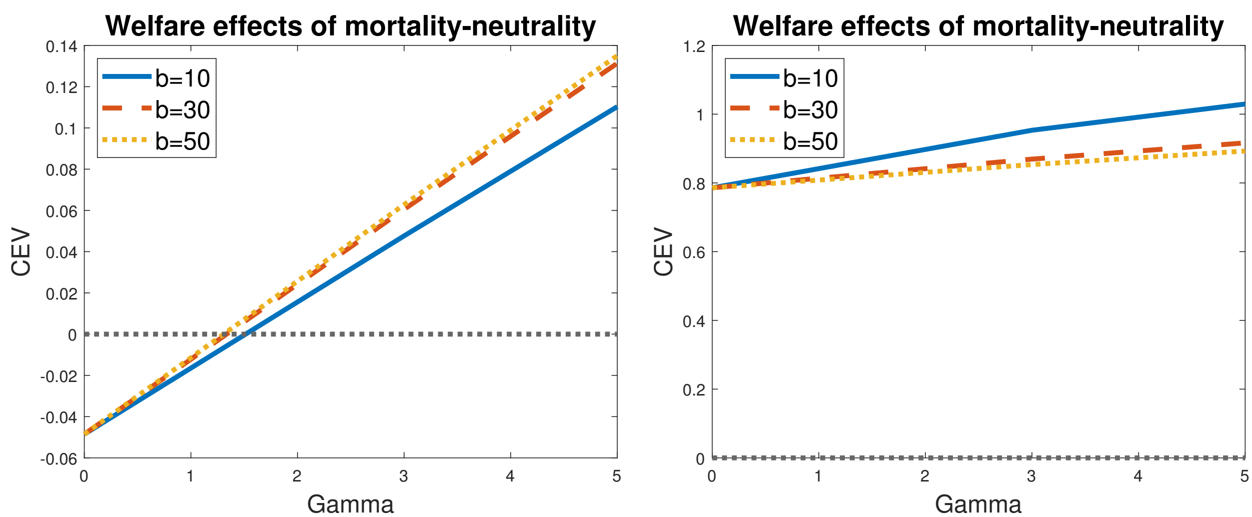

Figure 4: Welfare effects when moving from the utmost mortality-regressive (pooled annuitization) to mortality-neutral pensions. Left panel: only mortality heterogeneity. Right panel: mortality and income heterogeneity.

pecuniary benefits of being alive b, and to the aversion to inequality in lifetime utilities γ. For the model "only mortality heterogeneity" (left panel) welfare effects of mortality-neutrality become positive even before γ exceeds risk aversion over consumption σ, which is equal to 2. This happens because in our quantitative model, we allow for savings. This generates additional consumption inequality across types, and hence increases the push for mortality progressivity.

We next turn to the model "mortality and income heterogeneity" (right panel). Here mortality-neutrality is always welfare-improving, even when there is no aversion to lifetime inequality ($\gamma = 0$). This is consistent with the results in Proposition 4: the endowment inequality between types is large, and moving to mortality-neutrality helps to reduce this inequality. However, it is important to point out that the reduction in inequality when moving to mortality-neutral pensions is small (see the right panel of Figure 3). This is because inequality in pensions due to inequality in labor income is much more important than that generated by difference in mortality, suggesting it can be optimal to move beyond mortality-neutrality and towards a more progressive system.

6.3.3 Moving to mortality progressivity

We next turn to evaluating welfare effects of different pension designs ranging from the utmost mortality-regressive to mortality-progressive. Specifically, following the analysis in Section 4, we consider pension systems where benefits are determined as follows:

$$ssb_j = (1 - \alpha_1)\,\frac{IC_j}{\overline{q}^{IC}} + \alpha_1\,\frac{IC_j}{q_j} \;+\; \alpha_2\,\frac{\overline{q} - q_j}{q_j}$$

Here $\overline{q}^{IC}$ is the endowment-weighted average annuity price.[6] In this expression, α_2 represents the compensation coefficient that increases consumption of people with low life expectancy ($q_i < \overline{q}$) at the cost of decreasing consumption of people with high life expectancy. By varying α_1 and α_2, we can move from the utmost mortality-regressive to mortality-progressive case as summarized in Table 1. Importantly, the pension system balance equation is met for all considered combinations of α_1 and α_2.

	α_1	α_2
Utmost regressive (pooled price)	0	0
Weakly regressive	0.5	0
Neutral	1	0
Progressive	1	0.75

Table 1: Values of α_1 and α_2 corresponding to different degrees of mortality progressivity

The results of this exercise are presented in Figure 5. To construct this figure, we fix the level of non-pecuniary utility of life b at 30. Each graph in the figure reports CEV when moving from mortality-regressive to mortality-progressive cases described in Table 1

[6] As in Section 4.3, we use the endowment-weighted average annuity price so that the total pension payments are the same, and the pension balance equation is met in each case.

for different levels of aversion to lifetime inequality γ. As in the previous analysis, all welfare results are reported using the utmost mortality-regressive case as a benchmark.

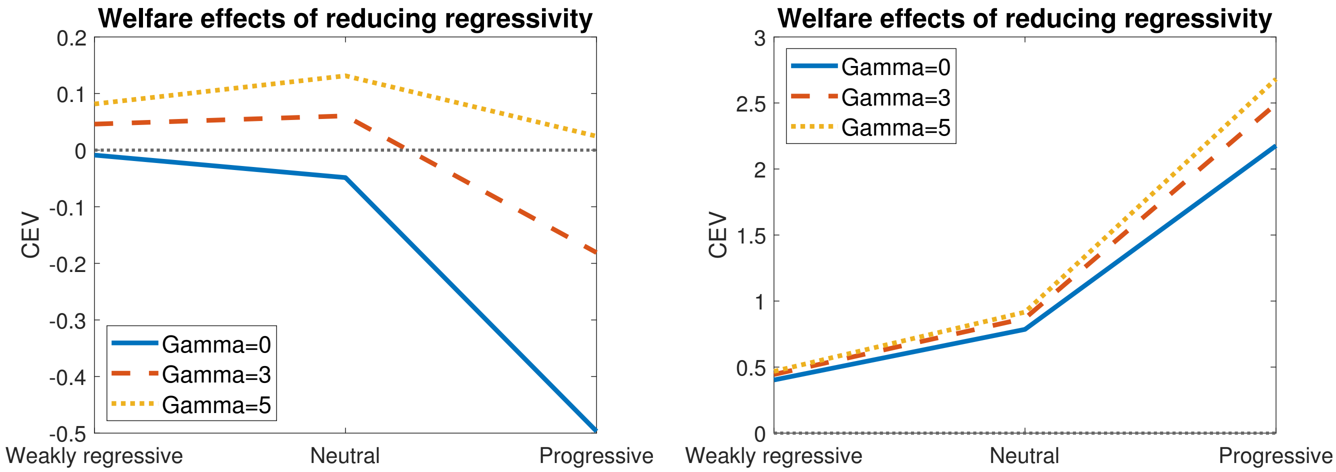

Figure 5: Welfare effects when changing mortality progressivity. Left panel: only mortality heterogeneity. Right panel: mortality and income heterogeneity.

Several important observations from Figure 5 are as follows. For the model "only mortality heterogeneity" (left panel), mortality progressivity is optimal when aversion to lifetime inequality is high. For the model "mortality and income heterogeneity", mortality progressivity is welfare improving for all the combinations of parameters considered, including the case with no aversion to lifetime inequality ($\gamma = 0$). Moreover, the welfare gains of moving to mortality progressivity in the full model can be quite substantial. For example, the CEV is equal to 2.5% when the aversion to lifetime inequality γ is equal to 5. It is important to point out, however, that these gains arise not because high-mortality types get compensated for their potentially short life. Instead, the gains are due to the ability of mortality progressivity to reduce inequality coming from dispersion in pension contributions. This is because mortality and labor income are highly correlated, and there is a constraint that pension contributions cannot be directly redistributed.

7 Conclusion

We develop a framework for understanding the optimality of redistribution along the mortality dimension in isolation from the issue of income redistribution. While income and mortality are correlated, the correlation is not perfect, and it is important to have a tool for a separate analysis of redistribution along these two dimensions. We are particularly interested in the optimality of mortality progressivity, that is to say, the redistribution from low-mortality to high-mortality people. The standard utilitarianism implies the reverse type of redistribution is optimal, meaning that any welfare assessment with this welfare criterion has an implicit bias against high-mortality people.

Three important features of our analysis are as follows. First, we deviate from the standard utilitarian welfare criterion by introducing aversion to inequality in lifetime utilities in addition to aversion to consumption inequality. Second, we define a neutral case which involves no redistribution along the mortality dimension and use it as a benchmark to assess mortality regressivity/progressivity. Third, we treat life as valuable, which implies that low-mortality people have an utility advantage over high-mortality people even when there is no difference in their consumption.

Using this framework, we derive several interesting results. First, when people differ in life expectancy but not in income, mortality progressivity is optimal only when life is valuable and aversion to lifetime inequality is stronger than aversion to consumption inequality. Put differently, adding concern about lifetime inequality is not enough to make mortality progressivity optimal unless this concern dominates aversion to consumption inequality.

Second, when people differ in their income, and income and mortality are negatively correlated, mortality progressivity can also optimally arise when income-redistributive tools are limited. This happens because in this case, mortality progressivity can partially substitute for income progressivity.

We then apply our framework to the analysis of pension design. We show that optimal pension benefits are mortality-regressive unless social planner has a strong aversion to lifetime inequality. At the same time, when concern for lifetime inequality enters pension design, it creates an additional effect of increasing the degree of optimal income progressivity. This happens because it becomes optimal not only to compensate high-mortality people for their potentially short life, but also to compensate the poor for their low consumption during working life.

In the final part of our analysis, we construct a quantitative life-cycle model to numerically illustrate the welfare effects of increasing mortality progressivity of pension benefits, while restricting income redistribution and preserving revenue-neutrality. We show that welfare effects of mortality-related redistribution crucially depend on the degree of aversion to lifetime inequality, and (even more so) on the income-mortality correlation.

Our results thus emphasize the distinction between the two reasons for why mortality progressivity may be optimal: (i) to compensate high-mortality people for their short life; (ii) as a substitute for income progressivity. The first case arises when life is valuable and social welfare incorporates strong aversion to lifetime inequality, and the second - when income and mortality are correlated and there are limited instruments to redistribute income. While similar in its final effect (welfare gains from mortality-progressive pensions), these two cases differ fundamentally. In the first case, the results are driven by the concern for those with high mortality, and in the second - by the concern for the poor.

Our results thus create an important framework for better understanding welfare effects of pension policies when people differ in life expectancy. Pension systems in many countries, including in the U.S., are under strain from the changing demographics, and various ways to maintain their sustainability are widely discussed. Many considered policies are judged based on the standard utilitarian welfare criteria, and thus are favoring low-mortality people over those with high mortality. We propose a way to refine the measurement of welfare consequences of pension reforms, and to separate the gains accruing to people with low income from the gains to those with high mortality.

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Online Appendix

A Proof of Proposition 1.1

In this section, we provide the poof of Proposition 1.1. We first restate the proposition.

Proposition 1.1 Consider a feasible allocation $\{c_i\}$. If this allocation is mortality-regressive (mortality-progressive) then the following is true:

$$\varepsilon^c_{q_i\,|\,a_i} > (<) -1 \quad \forall \;\; i,$$

where $\varepsilon^c_{q_i\,|\,a_i}$ is the partial elasticity of consumption to mortality, i.e., the elasticity for a given level of endowment a_i:

$$\varepsilon^c_{q_i\,|\,a_i} = \left.\frac{dc_i}{dq_i}\right|_{a_i} \cdot \frac{q_i}{c_i}.$$

Proof We will do the proof for the mortality-regressive case. That for the mortality-progressive case is analogous. Using the definition of the neutral allocation $\{c_i^N\}$, we can rewrite the average tax as follows:

$$AT_i = 1 - \frac{c_i q_i}{a_i}$$

The partial derivative of the average tax with respect to q_i for a given level of endowment a_i takes the form:

$$\left.\frac{\partial AT_i}{\partial q_i}\right|_{a_i} = -\frac{1}{a_i}\left.\frac{\partial (c_i q_i)}{\partial q_i}\right|_{a_i} \;=\; -\frac{c_i}{a_i}\,(\varepsilon^c_{q_i\,|\,a_i} + 1),$$

where the last equality follows from the definition of the partial elasticity. Since in the mortality-regressive case $\left.\frac{\partial AT_i}{\partial q_i}\right|_{a_i} < 0$, it follows that $\varepsilon^c_{q_i\,|\,a_i} > -1$. This finished the proof of the proposition.

B Proof of Proposition 4

In this section, we provide the poof of Proposition 4. We first restate the proposition.

Proposition 4 Consider the constrained social planner problem described in Eq (18), and suppose Assumptions 1-4 hold. In addition, assume $R_\Psi = 0$ and $R_u > 1$. The optimal choice of α_1 and α_2 can be summarized as follows:

(1) If $cov(q,a) = 0$, then at the optimum $\alpha_1 = 0$,

(2) If $cov(q,a) > 0$, then

(i) At the optimum $\alpha_1 > 0$,

(ii) If $cov(q, \frac{a}{q}) < 0$, then at the optimum $\alpha_2 = 0$,

(iii) If $cov(q, \frac{a}{q}) > 0$, then at the optimum $\alpha_2 > 0$.

Proof The first-order condition for the choice of α_1 can be written as follows:

$$(\alpha_1): \qquad \int\limits_q \int\limits_a u_c\, q \left(\frac{a}{q} - \frac{a}{\overline{q}^a}\right) H(a,q)\, da\, dq \; = \; 0 \tag{30}$$

Consider this equation evaluated at $\alpha_1 = 0$. Based on the consumption allocation rule in Eq(17), we have $c_i = \frac{a_i}{\overline{q}^a}$ (since when $\alpha_1 = 0$, we have $\alpha_2 = 0$ as well).

Consider first the case when $cov(q,a) = 0$. In this case, c_i varies with a_i but not with q_i. Thus, $cov(q,c) = 0$ and hence $cov(q,u_c) = 0$. Based on this, we can transform Eq(30) as follows:

$$\int\limits_q \int\limits_A u_c a\, H(a,q)\, da\, dq - \frac{1}{\overline{q}^a} \int\limits_q \int\limits_a u_c a\, q\, H(a,q)\, da\, dq =$$

$$= \int\limits_q \int\limits_a u_c a\, H(a,q)\, da\, dq - \frac{\overline{q}}{\overline{q}^a} \int\limits_q \int\limits_a u_c a\, H(a,q)\, da\, dq = 0$$

where in the last equality we used the fact that $\overline{q} = \overline{q}^a$ when $cov(q,a) = 0$. The first-order condition holds with equality when $\alpha_1 = 0$ implying that this represents the optimum. This finishes the proof of part (1) of the proposition.

Consider next the case when $cov(q,a) > 0$. Using the fact that $u(\cdot)$ is the CRRA function with risk aversion R_u (Assumption 4), we have $u_c = c^{-R_u}$, and hence we can rewrite the left-hand side of the FOC as follows:

$$(\overline{q}^a)^{R_u} \int\limits_q \int\limits_a a^{1-R_u} \left(1 - \frac{q}{\overline{q}^a}\right) H(a,q)\, da\, dq \; = (\overline{q}^a)^{R_u} \int\limits_q \int\limits_a \phi(a)\mu(q) H(a,q)\, da\, dq, \tag{31}$$

where $\phi(a) \equiv a^{1-R_u}$ and $\mu(q) \equiv 1 - \frac{q}{\overline{q}^a}$. Both functions are monotone decreasing in its respective arguments (for the first function it follows from $R_u > 1$), and have positive means (for the second function it follows from the positive correlation between q and

a, which implies $\overline{q} < \overline{q}^a$). Since $cov(q,a) > 0$, we thus have $cov(\mu(q),\phi(a)) > 0$, and $\int\limits_q \int\limits_a \phi(a)\mu(q)H(a,q)\,da\,dq > 0$. Thus, the FOC in Eq (31) is positive when evaluated at $\alpha_1 = 0$, meaning it is optimal to increase the value of this variable. This finishes the proof of part 2(i) of Proposition 4.

To prove parts (ii) and (iii) of the proposition, we turn to the FOC for the choice of α_2, which can be written as follows:

$$\int\limits_q \int\limits_a u_c\,(\overline{q} - q)\,H(a,q)\,da\,dq \;=\; 0 \tag{32}$$

Consider this equation evaluated at $\alpha_1 = 1$ and $\alpha_2 = 0$. In this case, the consumption allocation rule in Eq (17) becomes $c_i = \dfrac{a_i}{q_i}$. The FOC in Eq (32) can be transformed as follows:

$$\int\limits_q \int\limits_a u_c\,(\overline{q} - q)\,H(a,q)\,da\,dq \;=\; -cov(u_c, q)$$

Since $u_c = \left(\dfrac{a}{q}\right)^{-R_u}$, the correlation between u_{ci} and q_i depends on the sign of $cov(q, \dfrac{a}{q})$. Let us consider two cases.

First, when $cov(q, \dfrac{a}{q}) < 0$, we have $cov(u_c, q) > 0$. This means that the left-hand side of the FOC in Eq (32) is negative, implying that it is not optimal to increase α_2 above zero.[7] This finishes the proof of part 2(ii) of Proposition 4.

Second, when $cov(q, \dfrac{a}{q}) > 0$, we have $cov(u_c, q) < 0$, implying that the left-hand side of the FOC in Eq(32) is greater than zero, thus it is optimal to set $\alpha_2 > 0$. This finishes the proof of part 2(iii) of Proposition 4.

Comparing conditions $cov(q,a) > 0$ **and** $cov(q, \dfrac{a}{q}) > 0$ We argue that the condition $cov(q, \dfrac{a}{q}) > 0$ is stronger than the condition $cov(q,a) > 0$, and we formally prove it in the auxiliary proposition below.

Auxiliary proposition If $cov(q, \dfrac{a}{q}) > 0$ then $cov(q,a) > 0$, while the reverse is not true.

[7] In fact, based on the FOC in Eq (32) it is optimal to make α_2 negative. This happens because in this case, setting $\alpha_1 = 1$ is not optimal.

Proof We can use the definition of covariance to express $cov(q, \frac{a}{q})$ as follows:

$$cov(q, \frac{a}{q}) = \int\limits_q \int\limits_a q \cdot \frac{a}{q}\ H(a,q)\, da\, dq \ - \ \overline{q} \int\limits_q \int\limits_a \frac{a}{q}\ H(a,q)\, da\, dq \ =$$

$$= \overline{q} \left(\frac{\overline{A}}{\overline{q}} - \overline{\left[\frac{a}{q}\right]} \right),$$

where

$$\overline{\left[\frac{a}{q}\right]} \equiv \int\limits_q \int\limits_a \frac{a}{q}\ H(a,q)\, da\, dq$$

To approximate $\overline{\left[\frac{a}{q}\right]}$, we can use Taylor expansion around the means of a and q, $\overline{A}$ and $\overline{q}$, respectively. This results in the following expression:

$$\overline{\left[\frac{a}{q}\right]} \approx \frac{\overline{A}}{\overline{q}} + \frac{\overline{A}}{\overline{q}} \cdot \frac{Var(q)}{\overline{q}^2} - \frac{cov(q,a)}{\overline{q}^2}$$

If $cov(q, \frac{a}{q}) > 0$, we have $\frac{\overline{A}}{\overline{q}} > \overline{\left[\frac{a}{q}\right]}$. This implies

$$\frac{cov(q,a)}{\overline{q}^2} > \frac{\overline{A}}{\overline{q}} \cdot \frac{Var(q)}{\overline{q}^2} > 0$$

Thus, $cov(q,a) > 0$.

On the other hand, when $cov(q,a) > 0$, it is still possible to have $\frac{\overline{A}}{\overline{q}} < \overline{\left[\frac{a}{q}\right]}$, and thus $cov(q, \frac{a}{q}) < 0$. This finishes the proof of the auxiliary proposition.

C Proof of Proposition 2.1

In this section, we provide the poof of Proposition 2.1. We are going to start by giving two additional definitions and by formulating and proving an additional proposition.

We define two partial elasticities of pension benefits, with respect to mortality and endowment (or lifetime pension contributions). These elasticities show how pension benefits change with mortality (endowment), while keeping endowment (mortality) fixed:

$$\varepsilon^{ssb}_{q_i\,|\,IC_i} = \left.\frac{dssb_i}{dq_i}\right|_{IC_i} \cdot \frac{q_i}{ssb_i}$$

$$\varepsilon^{ssb}_{IC_i\,|\,q_i} = \left.\frac{dssb_i}{dIC_i}\right|_{q_i} \cdot \frac{IC_i}{ssb_i}$$

To understand whether pensions benefits are progressive/regressive along mortality and endowment dimensions, we can use the modified version of Proposition 1 from Section 3, which is stated below.

Proposition 1.2 Consider feasible pension benefits $\{ssb_i\}$. If these benefits are mortality-regressive/progressive then the following is true:

$$\varepsilon^{ssb}_{q_i\,|\,IC_i} > (<) -1 \quad \forall \;\; i$$

If these benefits are endowment-regressive/progressive then the following is true:

$$\varepsilon^{ssb}_{IC_i\,|\,q_i} > (<)\; 1 \quad \forall \;\; i$$

Proof We start by doing the proof for the mortality-regressive case. That for the mortality-progressivity case is analogous. In the mortality-regressive case, we have

$$\left.\frac{\partial AT_i}{\partial q_i}\right|_{IC_i} = -\frac{1}{IC_i}\left.\frac{\partial(ssb_i q_i)}{\partial q_i}\right|_{IC_i} = -\frac{ssb_i}{IC_i}(\varepsilon^{ssb}_{q_i\,|\,IC_i} + 1) < 0.$$

From here it follows that $\varepsilon^{ssb}_{q_i\,|\,IC_i} > -1$, which finishes the proof of the proposition.

We next consider the case of endowment regressivity. That for the endowment progressivity case is analogous. In the endowment-regressive case, we have

$$\left.\frac{\partial AT_i}{\partial IC_i}\right|_{q_i} = -q_i \left.\frac{\partial\left(\dfrac{ssb_i}{IC_i}\right)}{\partial IC_i}\right|_{q_i} = -\frac{q_i\;ssb_i}{IC_i^2}(\varepsilon^{ssb}_{IC_i\,|\,q_i} - 1) < 0.$$

From here it follows that $\varepsilon^{ssb}_{IC_i\,|\,q_i} > 1$, which finishes the proof of the proposition.

We next will restate and then prove Proposition 2.1.

Proposition 2.1 Consider pension benefits $\{ssb_i\}$ that represent the solution to the social planner problem described in Eqs (22)-(23). Under Assumptions 1-4, whether these benefits are mortality- and endowment-regressive/progressive can be determined as follows:

1. If $\Psi(\cdot)$ is linear, $\{ssb_i\}$ are mortality-regressive.

2. If $\Psi(\cdot)$ is strictly concave, then $\{ssb_i\}$ are mortality-regressive (-progressive) if

$$\frac{u_{ssb_i} ssb_i}{v_i^R} + \frac{R_u}{R_\Psi} \frac{V_i}{\beta^{R-1} V_i^R} > (<) \; 1 \qquad \forall \;\; i \tag{33}$$

3. $\{ssb_i\}$ are always endowment-progressive.

Proof:

Following the same steps as when proving Proposition 2, we can take a full differential of Eq (24) around the optimal allocation while keeping either IC_i or q_i fixed. This allows us to obtain the following expressions for partial elasticities:

$$\varepsilon^{ssb}_{q_i \,|\, IC_i} = -\frac{R_\Psi}{R_\Psi \dfrac{u_{ssb_i} ssb_i}{v_i^R} + R_u \dfrac{V_i}{\beta^{R-1} V_i^R}}$$

$$\varepsilon^{ssb}_{IC_i \,|\, q_i} = -\frac{R_\Psi \dfrac{u^W_{c_i} c^W_i}{v^W_i} \dfrac{V^W_i}{V_i}}{R_\Psi \dfrac{u_{ssb_i} ssb_i}{v_i^R} \dfrac{\beta^{R-1} V_i^R}{V_i} + R_u}$$

Consider first the partial elasticity of pension benefits with respect to q, $\varepsilon^{ssb}_{q_i \,|\, IC_i}$. Based on Proposition 1.2, $\{ssb_i\}$ are mortality-regressive(-progressive) if this expression is greater (less) than negative one. From here, parts 1 and 2 follow directly.

Consider next the partial elasticity of pension benefits with respect to IC, $\varepsilon^{ssb}_{IC_i \,|\, q_i}$. Based on Proposition 1.2, $\{ssb_i\}$ are endowment-regressive(-progressive) if this expression is greater (less) than one. Given that all agents have positive flow utility of being alive every period ($v_i^W > 0$ and $v_i^R > 0$ for all i), $\varepsilon^{ssb}_{IC_i \,|\, q_i}$ is always negative and hence less than one, implying endowment progressivity. This proves part 3 of the proposition.