Unveiling extreme dependencies between oil price shocks and inflation in Tunisia: Insights from a copula dcc garch approach

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Unveiling Extreme Dependencies between Oil Price Shocks and Inflation in Tunisia: Insights from a Copula DCC-GARCH Approach

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Abstract

We follow a non-linear dynamic correlation approach using a combination of a DCC-GARCH model and a copula model to capture the dependence between oil price changes and inflation in Tunisia. The case of Tunisia is particularly instructive since, after having been an exporter and a major producer, it became a net oil importer in the 2000s. The study, based on monthly data spanning decades, selects a Gumbel copula and shows that beyond weak average dependencies, there is a strong correlation between extreme values, suggesting that inflation in Tunisia is more sensitive to extreme (positive) variations in oil prices than to average variations. The implications of these empirical results for economic policy are crucial for the Tunisian economy.

Keywords: : oil price, inflation, copula, dynamic conditional correlation, Tunisia

1 Introduction

The relation between oil prices and inflation caught the attention of economists, especially after the 1970 oil shocks that affected inflation in several countries around the world (Hooker, 2002). In government discussions, oil price increase is considered good news in oil exporting countries and bad news in oil importing countries.

As the impact of oil prices on macroeconomic variables, and in particular inflation, depends on the economic structure and macroeconomic policies of each country, the aim of this study is to examine the Oil-Inflation relationship in Tunisia. This country has gone through a period of enormous structural change in its economy. Tunisia, which was a net-exporter of oil for decades, had to rely on expensive imports to supply its domestic needs. These changes are likely to affect the relationship between inflation and oil price shocks in this country, and makes the study of the Tunisian case very informative.

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From a methodological point of view, our work contributes to the literature by adopting an empirical methodology that implement copulas (Sklar, 1959) to examine the dependency structure and a Dynamic Conditional Correlation GARCH (DCC-GARCH) model (Engle, 2002) to study the dynamic relationship between oil price variations and inflation in Tunisia.

On the one hand, copula functions are suitable tool to examine a multivariate distribution when only marginal distributions are known. Furthermore, such an approach is suitable in situations where multivariate normality does not hold. This is especially the case for oil price variation for which the normality distribution is rejected in many studies (eg. Lee and Cheng, 2007, Choi and Hammoudeh, 2009). Moreover, copulas capabilities of modelling the associated dependency parameter can be conditioned and rendered time varying, even when the marginal dynamics that are being estimated are complex.

On the other hand, in the DCC-GARCH approach the correlation changes over time which allows for time-varying conditional correlation. But this model does not account for a non-linear dependence that may exist between oil price changes and inflation. Moreover, it does not provide information about the tail dependence that characterizes dependency during periods of extreme price behavior.

The Copula-DCC-GARCH approach, adopted in this paper, permits modeling the conditional correlation (via a DCC-GARCH) and the conditional dependence (via a copula) separately and simultaneously for non (necessary) normal multivariate distributions. In particular, the model presented hereafter combine the use of GARCH models and a copula function to allow flexibility on the choice of marginal distributions and dependence structures. Hence, we can examine the tail dependence and dynamic dependence between oil price and inflation in Tunisia with the same approach and without imposing any restrictions such as the normal joint distribution or a linear relationship between oil price changes and inflation. It is clear that the study of oil price shocks on inflation must be coupled with the average effects linking inflation to oil prices. This is partly the purpose of this study.

The link between oil prices and inflation is well documented (e.g. Cuñado and Pérez de Gracia, 2003, Dogrul and Soytas, 2010). Studies tend to focus on oil price shocks and are often limited to developed countries. To the best of our knowledge, there are only a few empirical studies which investigate the relationship between oil price and inflation in Tunisia. Guenichi and Benamou (2010) consider the impact of change in oil price on economic growth and other macroeconomic variables but not specially on inflation. Choi et al. (2018) study the impact of fluctuations in oil prices on inflation using an a panel of developed and developing countries including Tunisia. Brini et al. (2016) deal with the impacts of oil price shocks on inflation and real exchange rate in some MENA countries. All theses studies do not focus specially on Tunisia or not deal with exactly the relationship between oil price fluctuations and inflation, particularly for the analysis of extreme variations.

We examine the dependency structure of oil price variations and domestic inflation in Tunisia by making use of a monthly dataset from 1975 to 2021. Indeed, during these decades, the Tunisian economy has undergone massive structural changes, which in turn, is likely to affect the relationship between oil shocks and inflation. Due to the low refining capacity of the single refinery in Bizerte (North Tunisia), the production capacity of the various oil products has decreased due to the sharp decline in oil reserves. Hence, the output of oil suffered from declining reserves and the absence of new discoveries. In 2014, crude oil production was at about 53,000 barrels per day (bbl/d), with a distinct decline from 97,000 bbl/d in the 1980's, and an overall peak level of 118,00 bbl/d in 1980. Except for three years with high oil production after 2006, Tunisia has been a net importer of oil since 2003 (IEA, 2020).

The remainder of the paper is structured as follows: Section 2 presents the basic features of copula functions, DCC-GARCH models and copula-DCC-GARCH framework used in this study. Section 3 presents the empirical findings, including the descriptive statistics of the data and the results of various copula specifications. Section 4 discusses. The final section concludes the paper and presents some suitable recommendations.

2 Methodology

This section introduces the methodology that we adopt to analyse the dynamic dependence structure between oil prices and inflation in Tunisia. We firstly present the concept of bivariate copulas as well as the copula families considered in this study. We also present measures of non linear dependence related to copulas used. Secondly, we discuss the DCC-GARCH model. Finally, we present the method considered to estimate the copula-DCC-GARCH parameters.

2.1 The Copula approach

The idea behind the concept of "copulas" or "copula functions" as named by Sklar (1959) is the following: for multivariate distributions, the univariate margins and the dependence structure can be separated and the latter may be represented by a copula. In other words, copula describes the function that "joins" one-dimensional distribution functions to form multivariate one, and may serve to characterize several dependence concepts.

The copula theory is an extremely powerful tool because it is able to extract the dependency structure of the joint distribution function and to isolate this dependency structure from univariate marginal distributions. It can also capture asymmetric dependencies. A copula creates a multivariate common distribution that combines marginal distributions and dependencies between variables, which underlines the advantage of this model over conventional methods.

With the copula function, multivariate distributions can be modeled in a simple and extremely flexible way. This function is capable of generating any type of dependency structure, regardless of the marginal distribution by constructing multivariate distribution functions that avoid the assumption of normality (Nelsen, 2006).

The copula of a multivariate distribution can be considered as the part describing its dependence structure as a complement to the behavior of each of its margins. Given that copulas fully describe dependence by providing information on average dependence and dependence for joint distribution tails, we modelled the dependence structure between oil price and inflation in Tunisia for time scales using different copula functions. In this study, we make use of two variables, and hence only bivariate copula theory will be discussed.

Let X and Y two random variables with marginal distributions F and G respectively, and joint distribution H. Then there exist a function C called copula such that:

$$H(x,y) = C(F(x), G(y)) = C(u,v) \tag{1}$$

where $u = F(x)$ is the univariate marginal distribution function of the variable X and $v = G(y)$ is the univariate marginal distribution function of the variable Y.

The copula function in (1) satisfies the following equalities:

[1] $C(u,v) = C(0,v)$ for every $u,v \in [0,1]$

[2] $C(u,1) = u$ and $C(1,v) = v$ for every $u,v \in [0,1]$

[3] $C(u_2,v_2) - C(u_2,v_1) - C(u_1,v_2) + C(u_1,v_1) \geqslant 0$ for every $u_1,u_2,v_1,v_2 \in [0,1]$

Accordind to Sklar theorem, if the marginal distrbutions are continuous, then C is uniquely determined. Conversely, if C is a copula, then the function H in equation (1) is a joint function with margins F and G. The copula C and can also be expressed as $C(x,y) = H\left(F^{-1}(x), G^{-1}(y)\right)$ and so describes the whole dependence structure between X and Y.

The distinction between the variables x and y and the variables u and v transformed respectively by $F(x)$ and $G(y)$ is important. u and v exist in what is called the uniform/rank domain, while x and y exist in the normalized domain. Since this is a copula function, it is simple to generate dependent random variables in the uniform domain, which can then be transformed by the appropriate inverse marginal distribution functions into the normalized domain.

2.2 Copulas and dependencies

Different copula families capture various features of the dependence structure differently, with some copulas being more suitable to model the overall dependence while others are better suited to represent the dependence at the tails of the distribution, which is particularly relevant for modeling extreme events.

2.2.1 Measures of dependence

The common measure of dependence is the linear correlation coefficient. However, this parametric coefficient (Pearsons's correlation) has many shortcomings: not robust to outliers; not invariant to monotone transformations of the variables; can take value 0 whereas variables are strongly dependent; only relevant when variables are jointly normally distributed (Embrechts et al., 1999). This can result in misleading evaluations of the dependence nature between oil price and inflation.

The most widely used non-parametric measures employed in copula modeling for evaluating dependence strength are therefore rank correlations, including Kendall's tau and Spearman's rho (Nelsen, 2006). These two coefficients measure different aspects of dependence.

Spearman's rho is simply Pearson correlation on the rank-transformed marginal variables. In terms of a copula C, it can be expressed as follows (Nelsen, 2006):

$$\rho_S = 12 \iint_{[0,1]^2} C(u,v)\, dC(u,v) - 3 \tag{2}$$

for every $u,v \in [0,1]$ which are the values of marginal distribution functions.

Kendall's tau is a rank correlation measure that can be described as the probability of concordance minus the probability of discordance of two pairs of random variables (Fredricks and Nelsen, 2007). Kendall's tau can be expressed in terms of a copula function C as follows:

$$\tau_C = 4 \iint_{[0,1]^2} C(u,v)\, dC(u,v) - 1 \tag{3}$$

Both Kendall's τ and Spearman's ρ provide measures of strength of the association between random variables, typically referred to as correlation. Thus, they do not fully reflect the complexity of dependence structure between variables, as they are not able to reveal the dependence structure at specific parts of the distribution. For example, they do not specifically capture tail dependence, which refers to the probability that two variables X and Y experience extreme upward or downward movements (Joe, 2014; Nelsen, 2006; Poulin et al., 2007).

In this study, it is very helpful to measure the tendency of inflation to react to a sudden oil price increase or a sudden decrease. Such dependence is usually evaluated through upper- and lower-tail dependence coefficients denoted by λ_U and λ_L, respectively, and are obtained (respectively) from a copula C as:

$$\lambda_U = \lim_{u \to 1} P\left[X \geqslant F^{-1}(u) | Y \geqslant G^{-1}(u)\right] = \lim_{u \to 1} \frac{1 - 2u + C(u,u)}{1 - u} \tag{4}$$

$$\lambda_U = \lim_{u \to 0} P\left[X \leqslant F^{-1}(u) | Y \leqslant G^{-1}(u)\right] = \lim_{u \to 0} \frac{C(u,u)}{u} \tag{5}$$

where $F^{-1}(u)$ and $G^{-1}(v)$ are the marginal quantile functions for variables X and Y respectively, and $\lambda_U, \lambda_L \in [0,1]$.

$\lambda_U = \lambda_L = 0$ corresponds to the absence of dependence between variables in the tails. Two variables exhibit upper (lower) tail dependence if $\lambda_U > 0$ ($\lambda_L > 0$). Larger values of λ_U (λ_L) indicate greater tendency of the data to cluster in the upper (lower) tail of the joint distribution, in which case the variables are said to be upper (lower) tail dependent.

2.2.2 Copula families used

In general, there are two main types of parametric copulas: elliptic (implicit) and archimedean (explicit) copulas. Each of them have distinctive properties. Here, we focus on the presentation of bivariate copulas, which will be used later.

Elliptical copulas are generally used to account for overall dependance structures. They are capable of representing positive and negative dependance. The most popular elliptical copulas are the Gaussian (i.e. normal) copula and the Student's t copula. These copulas do not have a closed-form expressions, however their density functions are available.

The bivariate Gaussian copula is defined by

$$C_\theta(u,v) = \Phi\left(\Phi^{-1}(u) + \Phi^{-1}(v)\right) \tag{6}$$

where Φ is the bivariate normal cumulative distribution function with correlation θ between X and Y and where $\Phi^{-1}(u)$ and $\Phi^{-1}(v)$ are standard normal quantile functions. Gaussian copulas cannot capture tail dependence.

The Student's t copula is characterized by a greater modeling flexibility in terms of tail dependence, which is enabled by an additional degree-of-freedom parameter compared to the Gaussian copula. The Student's t copula is given by

$$C^T_{\theta,\nu}(u,v) = T\left(t_\nu^{-1}(u) + t_\nu^{-1}(v)\right) \tag{7}$$

where T is the bivariate Student-t cumulative distribution function with ν as the degree-of-freedom parameter and correlation θ and where $t_\nu^{-1}(u)$ and $t_\nu^{-1}(v)$ are the quantile functions of the univariate Student distribution with ν degree-of-freedom. This copula converges to the Gaussian one, as ν diverges. The Strudent's t copula allows for symmetric non-zero dependence in the tails, where tail dependence is given by $\lambda_U = \lambda_L = 2t_{\nu+1}\left(-\sqrt{\nu+1}\sqrt{1-\theta}/\sqrt{1+\theta}\right) > 0$.

Archimedean copulas are frequently used in economic and financial applications (Patton, 2012) as they have a simple mathematical form and are specifically easy to sample from, when considering only two variables (Joe, 2014). Furthermore, they allow for different dependence structures, such as concordance and tail dependence (Hofert, 2008). Several Archimedean families exhibit asymptotic dependence in at least one of the two tails (Charpentier and Segers, 2009). This makes them especially suitable for modeling of extreme events such as sudden high variation of oil prices.

In this article, we use the most commonly archimedean copulas in the one-parameter families, namely the Clayton, Frank and the Gumbel Copulas.

The Clayton copula has the following distribution function

$$C_\theta^{\text{Clayton}}(u,v) = \max\left[(u^{-\theta}+v^{-\theta}-1)^{-1/\theta}, 0\right] \tag{8}$$

where $\theta \geqslant 0$. When $\theta = 0$ there is independence. For $\theta > 0$, this copula shows asymmetry, as the degree of dependence is higher in the lower tail ($\lambda_L = 2^{-1/\theta}$) than in the upper tail, where it equals zero ($\lambda_U = 0$).

The Frank copula is a symmetric copula with no tail dependence. It is given by

$$C_\theta^{\text{Frank}}(u,v) = -\frac{1}{\theta}\ln\left(1+(e^{-\theta u}-1)(e^{-\theta v}-1)(e^{-\theta}-1)^{-1}\right) \tag{9}$$

where $\theta \in (-\infty,\infty)\setminus\{0\}$.

The Gumbel copula is given by

$$C_\theta^{\text{Gumbel}}(u,v) = \exp\left(-\left[(-\ln u)^\theta + (-\ln v)^\theta\right]^{1/\theta}\right) \tag{10}$$

where $\theta \in [1,\infty)$. The two variables are independent when $\theta = 1$. It is an asymmetric copula with a higher degree of dependence in the upper tail ($\lambda_U = 2-2^{1/\theta}$) than in the lower tail, where it equals zero ($\lambda_L = 0$).

Table 1 summarize the main characteristics of copula families used in this study: distribution function, Kendall's correlation coefficient τ, Spearman's correlation coefficient ρ_S, upper and lower tail dependence coefficients λ_Y and λ_L, and parameter range (Embrechts et al., 2002) .

2.3 Marginal distribution modelling: the DCC GARCH approach

In order to build the model for bivariate distribution with the copula, the marginal distribution for the series must initially be formed. We will consider a multivariate GARCH (Bollerslev, 1986) model with time varying conditional correlations or DCC GARCH model introduced by Engle (2002). Firstly, let us note $y_t = (y_{1t}, y_{2t})'$ the vector containing the two analyzed series, and consider a first-order autoregressive model for the mean equation for each series:

$$y_t = \mu + Ay_{t-1} + \varepsilon_t \tag{11}$$

where A is a diagonal matrix of dimension 2 comprising the autoregressive coefficients a_1 and a_2 and $\mu = (\mu_1, \mu_2)'$ a vector of the two unconditional means of the two series.

Copula	$C_\theta(u,v)$	τ	ρ_S	λ_U	λ_L	θ range
Gaussian	$\Phi\left(\Phi^{-1}(u)+\Phi^{-1}(v)\right)$	$2\arcsin\theta/\pi$	–	–	–	$(-1,1)$
Student's	$T\left(t_\nu^{-1}(u)+t_\nu^{-1}(v)\right)$	$2\arcsin\theta/\pi$	–	$-2t_{\nu+1}\sqrt{\nu+1}\sqrt{\frac{1-\theta}{1+\theta}}$		$(-1,1)$
Clayton	$(u^{-\theta}+v^{-\theta}-1)^{-1/\theta}$	$2/(2+\theta)$	–	–	$2^{-1/\theta}$	$(0,+\infty)$
Frank	$-\frac{1}{\theta}\ln(1+\frac{(e^{-\theta u}-1)(e^{-\theta v}-1)}{e^{-\theta}-1}$	$1-\frac{4}{\theta}(1-D_1(\theta))$	$1-\frac{12}{\theta}(D_1(\theta)-D_2(\theta))$	–	–	$(-\infty,+\infty)\setminus\{0\}$
Gumbel	$\exp(-[(-\ln u)^\theta+(-\ln v)^\theta]^{1/\theta})$	$1-1/\theta$	–	$2-2^{1/\theta}$	–	$[1,+\infty)$

Note: $D_1(\theta)=\frac{1}{\theta}\int_0^\theta \frac{x}{\exp(x)-1}$ and $D_2(\theta)=\frac{2}{\theta^2}\int_0^\theta \frac{x^2}{\exp(x)-1}$.

Table 1: Main characteristics of copula functions used in this study and links with dependent measures

The error term $\varepsilon_t=(\varepsilon_{1t},\varepsilon_{2t})'$ can be written in the form (following Engle, 2002):

$$\varepsilon_t = H_t^{1/2}\nu_t \tag{12}$$

where $H_t^{1/2}$ is a square matrix of order 2 defined positive and ν_t is a random vector of null mean and variance-covariance matrix equal to the identity matrix of order 2: $E(\nu_t)=0$ and $Var(\nu_t)=I_2$.

The matrix H_t can be decomposed in this way:

$$H_t = D_t R_t D_t \tag{13}$$

where $D_t = diag(h_{1t}^{1/2}, h_{2t}^{1/2})$ and R_t is the matrix of conditional correlations. H_t is defined positive from the moment when the conditional variances h_{1t} and h_{2t} are positive, and that the matrix of conditional correlations R_t is assumed to be defined positive.

A GARCH(1,1) model is then specified for the conditional variances, i.e. for each conditional variance h_{it} $(i=1,2)$:

$$h_{it} = \omega_i + \alpha_i\varepsilon_{i,t-1}^2 + \beta_i h_{i,t-1} \tag{14}$$

These variances are positive under the following conditions: $\omega_i > 0$, $\alpha_i \geqslant 0$ and $\beta_i \geqslant 0$. Moreover, in order to ensure a stationary covariance model, we need to have, for every i, $\alpha_i+\beta_i<1$. The parameter α captures the ARCH effect, β captures the GARCH effect, and $\alpha+\beta$ shows the volatility persistence.

Considering that the conditional correlation between the two series is dynamic (DCC model), the R_t dynamic correlation matrix specified in (13) is written as follows:

$$R_t = P_t Q_t P_t$$

where $P_t = diag(Q_t)^{1/2}$ and $Q_t=(1-\theta_1-\theta_2)\bar{Q}+\theta_1\varepsilon_{t-1}\varepsilon'_{t-1}+\theta_2 Q_{t-1}$ the covariance matrix and $\bar{Q}$ a long term covariance matrix. In the particular case of two series, the elements of the matrix Q_t are then $q_{ij,t}=(1-\theta_1-\theta_2)\bar{\rho}+\theta_1\varepsilon_{i,t-1}\varepsilon_{j,t-1}+\theta_2 q_{ij,t-1}$, $i,j=1,2$ where $\bar{\rho}$ is a constant correlation between ε_1 and ε_2.

When the parameters θ_1 and θ_2 are positive and check the inequality $\theta_1+\theta_2\leqslant 1$, then the correlation matrix R_t is defined positive, in other words $|\rho_t|<1$. On the other hand, if $\theta_1=\theta_2=0$, the correlation is no longer dynamic and we obtain a model with a constant conditional correlation.

Finally, dynamic correlations are obtained by normalizing $q_{12,t}$ according to the following expression:

$$\rho_t = \frac{q_{12,t}}{\sqrt{q_{11,t} q_{22,t}}} \tag{15}$$

In order to estimate all the coefficients in the DCC GARCH model, it is usually used the maximum likelihood method (MLM) or quasi-maximum likelihood method (QMLM) respectively (Engle, 2002). Those methods are based on maximizing the likelihood function.

After specifying the marginal distributions for each variable, the second part consists of modelling the dependence by a copula function. Thus, the task of creating a multivariate distribution is to choose an appropriate copula form that best describes the dependence structure between the variables. In other words, the model removes the linear correlation of the dependent variable and forms uncorrelated dependent errors controlled by a copula, while the correlation is controlled by a DCC GARCH model.

Specifically, the proposed model uses the DCC specification for marginal distributions, and different types of copulas for the joint distribution to allow a wide range of possible dependence structures that will be fitted to the residuals obtained from the DCC model and, for each case, the dependence parameters will be determined. In addition, Spearman's correlation, Kendall's tau and the extreme dependency coefficient will be calculated to measure the effectiveness of the dependency in the use of copula.

3 Data description

The variables used in this study are inflation rate in Tunisia and the growth rate of oil price calculated on a monthly basis. Inflation rate (expressed hereafter in percentage) is deduced from the consumer price index, produced by the "Institut National de la Statistique" (INS) in Tunisia. The data were seasonally adjusted. The data for the oil price (West Texas Intermediate) are from the Energy Information Administration (EIA) and are expressed in dollars. Variations in oil prices and inflation data are observed monthly from January 1975 to December 2021. Monthly series were used because they have sufficient complexity to capture short-term movements over time.

Summary statistics of the two variables used and test statistics are presented in Table 2. These include the mean, standard deviation, skewness, kurtosis, extreme values, and the Jarque-Bera and the Shapiro-Wilk normality tests.

The skewness values of the oil price changes are positive which implies that the distribution of this variable is more skewed towards the right than a normal distribution. The Kurtosis values for the two series exceeds 3, indicating that the distributions are leptokurtic. These results might indicate that the variables are not normally distributed. Indeed, the normality test indicates a clear rejection by the Jarque-Bera normality test under a 1% significant level. Similarly, none of these variables are normally distributed according to the results from the Kolmogorov-Smirnov test of normality, which justifies the choice of copula theory in this study. Moreover, the values of the Ljung-Box statistic for no correlation up to 16th order in the observations suggested the existence of serial correlation, whereas the ARCH (autoregressive conditional heteroscedasticity)-LM (Lagrange multiplier) statistic for serially correlated squared observations indicated that ARCH effects were likely to be found in the inflation variable. The dependence between series may be affected by autoregressive

Table 2: Descriptive statistics

	Oil price monthly variations	Monthly inflation in Tunisia
Mean	0.0093	0.4532
Median	0.0000	0.4135
Maximum	1.3457	4.3255
Minimum	−0.4334	−3.2148
Std. Dev.	0.1004	0.6630
Skewness	4.2033	0.6644
Kurtosis	55.1070	8.0135
Jarque-Bera	80794*	1715.5*
Kolmogorov-Smirnov	0.9679*	0.9871*
Ljung-Box	27.787†	105.51*
ARCH-LM	2.2135	155.10*

Note: Jarque-Bera and Kolmogorov-Smirnov check for normality. Ljung-Box checks for serial correlation (no autocorrelation) computed here with 12 lags. ARCH-LM is Engle's LM test to detect autoregressive conditional heteroscedasticity, conducted using 12 lags (null hypothesis no ARCH effects). * and † imply rejection of the null hypothesis at the 1% and 5% level, respectively.

Table 3: Unit root tests

	Oil price variations	Inflation
ADF[1]	−17.000*	−16.520*
PP[2]	−20.542*	−21.154*
KPSS[3]	0.056*	0.349
ERS[4]	0.282*	1.097*

[1]Augmented Dickey-Fuller test; [2]Phillips-Perron test; [3]Kwiatkowski, Phillips, Schmidt and Shin test; [4]Elliot, Rothenberg and Stock test. Tests are computed with constant and trend. The optimal lag is chosen as per the Akaike Information Criterion (AIC). * imply significance at the 1% level.

and heteroscedastic components, which therefore justifies the use of the standardized residual of the series from the DCC-GARCH.

Furthermore, several unit root tests are used for testing to the series' stationarity. The unit root test statistics are reported in Table 3. We used the Augmented Dickey-Fuller (ADF) test, Phillips-Perron (PP) test, Kwiatkiwski-Phillips-Schmidt-Shin (KPSS) test and Elliot-Rothenberg-Stock test (ERS) (see Patterson, 2012 for a review and details about these tests). ADF, PP and ERS unit root tests have a null hypothesis (H_0) stating that the series in question has a unit root against the alternative that it does not. The null hypotheis (H_0) of KPSS, on the other hand, states that the variable is stationary. All the tests confirm that the two series are stationary at the 99% significance level, except the KPSS test for the inflation variable. Yet, the KPSS test tends to have extreme size distortions when the null hypothesis (H_0) of a stationary series is close to the alternative of a unit root (Caner and Kilian, 2001). Hence, the test may reject H_0 even if the true series is stationary.

Table 4: Results of DCC-GARCH model estimation

	The conditional means and variances equations				
	Parameter	**Estimate**	**Std. Error**	**t value**	**p-value**
Oil price	μ_1	0.0038	0.0050	0.7695	0.4416
	a_1	0.2002	0.0486	4.1234	0.0000
	ω_1	0.0003	0.0009	0.3485	0.7274
	α_1	0.3295	0.1061	3.1040	0.0019
	β_1	0.6695	0.1833	3.6525	0.0003
	υ_1	5.1501	2.8857	1.7847	0.0743
Inflation	μ_1	0.3759	0.0242	15.529	0.0000
	a_2	0.3346	0.0479	6.9866	0.0000
	ω_2	0.0026	0.0021	1.2589	0.2080
	α_2	0.0403	0.0322	1.2486	0.2118
	β_2	0.9413	0.0410	22.930	0.0000
	υ_2	6.6250	1.4405	4.5991	0.0000
	The conditional correlation equation				
DCC	θ_1	0.0049	0.0164	0.2999	0.7642
	θ_2	0.9104	0.0632	14.403	0.0000
	η	7.2361	1.4018	5.1619	0.0000

4 Empirical findings

4.1 Results of the marginal distribution estimation

The estimated model is a DCC-GARCH(1,1) model where the three random errors v_t and ε_{1t} and ε_{2t} given in equation (12) are assumed to follow a Student distribution rather than a normal distribution, given the stylized facts previously observed on the two series studied. The degrees of freedom of these three error terms are denoted by η, υ_1 and υ_2 respectively. The model is estimated by the maximum likelihood method in two steps. In the first step, the conditional volatility of each time series is estimated from a univariate DCC-GARCH model. Then, in a second step, the dynamic correlations are estimated using the standardized residuals from the first step. The results of the estimation are given in Table 4.

All the parameters of Student's distribution are significant, which justifies the adequacy of the selected distribution. The parameters of the mean equation (11), namely the unconditional means and the autocorrelation coefficients are all positive and significant, except μ_1 which is relative to the oil price variable. Furthermore, it can be seen that all the individual GARCH series fulfill the criteria that $\alpha + \beta < 1$ and the sum exceeds 0.9, which allows us to conclude that there is a pronounced GARCH effect. The estimated values of β_1 and β_2 are highly significant which indicates a high degree of volatility persistence specially for the variable inflation in Tunisia. The DCC parameters follow the reversibility condition, since $\theta_1 + \theta_2 < 1$. This confirms the presence of dynamic conditional correlations between the two series.

Figure 1 shows the dynamic conditional correlations between international oil price changes and inflation in Tunisia. These time varying correlations are estimated from the results of the DCC-GARCH model. The correlation between oil price changes and

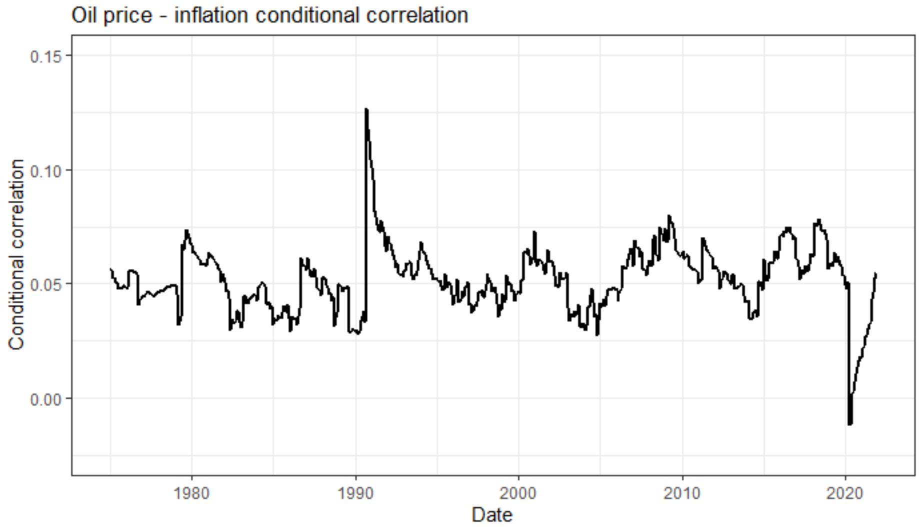

Figure 1: Dynamic correlation between oil price variations and inflation in Tunisia

inflation is generally positive but two particular events are worth mentioning. First, the level of correlation increased sharply in 1990, which is a period of high oil price volatility following Iraq's invasion of Kuwait (Wickham, 1996). Second, this correlation has recently decreased significantly. The Covid-19 pandemic has negatively influenced the price of oil (Bourghelle et al., 2021), while prices in Tunisia have continued to rise due to multiple political and economic factors (Khatat et al., 2020).

We cannot fail to mention that on average, the correlation is quite weak. On the one hand, Tunisia largely subsidizes the price of oil. Therefore, low price volatility will not have an immediate direct effect on domestic prices. On the other hand, the correlation measure is linear and cannot reflect correlations between variables in the presence of high price variability. In other words, Pearson correlation is not robust: outliers can introduce false correlations or mask existing ones. A deeper and different analysis of the link between oil prices and inflation is therefore needed since these variables exhibit higher dependence during turbulent periods than in calm periods. As a consequence, we quantify the dependency through copulas using the results provided by the DCC-GARCH model.

4.2 Dependence estimation using copulas

Selection of the copula The choice of the copula allows a good control of the parts of the distribution to which the variables are most strongly associated. For the selection of the best copula, two criteria will be used. These criteria are based on the maximum likelihood estimation method. The objective is to select the model (the copula) with the highest likelihood while taking into account the number of estimated parameters.

The Akaike Information Criterion (AIC) and the Bayesian Information Criterion (BIC) are used to select the most appropriate copula for our analyzed series. These two criteria are defined as follows:

$$\text{AIC} = -2 \times \ln(\text{likelihood}) + 2k$$

Copula	AIC	BIC
Gaussienne	0.966	5.213
Student	4.273	12.765
Clayton	1.256	5.502
Gumbel	**0.955**	**5.201**
Frank	1.814	6.060
Joe	0.974	5.220

Table 5: AIC and BIC criteria for the selection of the best copula

$$\text{BIC} = -2 \times \ln(\text{likelihood}) + \ln(n)k$$

where k is the number of copula parameters and n is the number of observations. The best model is the one that gives the lowest values for these two criteria.

Table 5 gives the results obtained from the standardized residuals of the DCC-GARCH(1,1) model. The two criteria identify **the Gumbel copula** which is given by:

$$C(u,v) = \exp\left(- \left[(-\log u)^{\theta} + (-\log v)^{\theta} \right]^{\frac{1}{\theta}} \right) \qquad 1 \leq \theta < \infty \tag{16}$$

Remember that Gumbel copula only detects positive dependencies. The degree of dependency is captured by the parameter θ, which is always positive and greater than or equal to 1. The farther the value of θ is from 1, the stronger is the dependence, and when $\theta = 1$ there is independence between the two series.

Therefore, the fact that the two series analyzed here are captured by a Gumbel copula suggests that the dependence between oil price changes and inflation in Tunisia can only be positive. A rise in oil price can only leads to an increase in inflation. The selected copula cannot envisage the case where an increase in the price of oil leads to a fall in the general price level in Tunisia.

Before estimating the degree of this dependency, it should be noted that the Gumbel copula also makes it possible to characterize dependencies at the level of extreme values and in particular at the level of positive extreme values (values at the level of the right tail of the distribution). It is therefore particularly well adapted when it comes to modelling dependencies between extreme values characterized by positive variations. This suggests that if there is a dependency between the two series analyzed, it is largely due to dependencies at the level of positive extreme values.

In practical terms, the findings suggest that a strong positive change in oil price could have an effect on inflation in Tunisia (depending on the degree of dependence captured by θ). On the other hand, inflation in Tunisia would be less affected by moderate variations, i.e. small fluctuations, in oil prices. In particular, the dependence at the extreme values on the left side of the distribution is zero, and if the relationship between these two variables is modeled by the Gumbel copula, it is because any strong but negative variation (significant drop in oil prices) will have no effect on inflation in Tunisia.

Dependence estimation using copulas In order to estimate the dependence between oil price variations and inflation in Tunisia, we will use the Gumbel copula as explained above. This copula has a single parameter θ. It distinguishes a dependence at the level of the right tail of the distribution: $\lambda_U = 2 - 2^{\frac{1}{\theta}}$ ($\lambda_L = 0$). The Kendall correlation coefficient (Kendall τ) is in this case equal to $1 - \dfrac{1}{\theta}$.

However, since the Gumbel copula is symmetrical (in the sense that $C(u,v) = C(v,u)$), it similarly models two random variables X_1 and X_2 independently of the role of each of them. Tawn (1988) has shown that this hypothesis is restrictive in some cases and has proposed an extension of the Gumbel copula by adding two asymmetric parameters α and β allowing the second one to be more flexible. This new extension is named Tawn copula. It is not an archimedean copula, but rather a copula of extreme values.

This type of copula (extreme value copula) consists of analyzing the two tails of the distributions, which constitute only a small part of the entire distribution under examination. Its purpose is not to describe the usual behaviour of stochastic phenomena, but unusual and rarely observed events.

Tawn copula is given by (cf. Tawn, 1988):

$$C(u,v) = u^{1-\alpha} v^{1-\beta} \exp\left(- \left[(-\alpha \log u)^{\theta} + (-\beta \log v)^{\theta} \right]^{\frac{1}{\theta}} \right) \tag{17}$$

where $1 \leq \theta < \infty$ captures the dependence (as in Gumbel copula) and the parameters α and β are such that $0 \leq \alpha$ and $0 \leq \beta$ 1.

For the Tawn copula, when $\theta = 1$ or $\alpha = 0$ or $\beta = 0$, the two variables are independent. The farther θ is from 1, the stronger the dependence is as in the case of Gumbel copula. If $\alpha = \beta = 1$, we find the Gumbel copula. If $\alpha = \beta$, the Tawn copula is symmetrical. When $\alpha = 1$ ($0 \leq \beta \leq 1$), we get the so-called Tawn type 1 copula. When $\beta = 1$ ($0 \leq \alpha \leq 1$), we get the so-called Tawn copula type 2 (Nagler et al., 2019).

In addition, and as for the Gumbel copula, the Tawn copula allows us to distinguish the dependence at the level of the positive extreme values (i.e. at the level of the right tail of the distribution). In this case, we have: $\lambda_U = (\alpha + \beta) - (\alpha^{\theta} + \beta^{\theta})^{\frac{1}{\theta}}$ and we still have $\lambda_L = 0$. Kendall's tau can be obtained from θ, although its expression is not simple as in the case of Gumbel copula (cf. Tawn, 1988).

From the two series analyzed, table 6 gives the results of the estimations of different copula: Gumbel copula, Tawn type 1 and Tawn type 2. The different estimations were also carried out over shorter periods corresponding only to the data for the periods 1990 – 2021, 2000 – 2021 and 2010 – 2021. All the estimations and copula selection were performed using the VineCopula package (Nagler et al., 2019) under R.

First of all, it should be noted that for the whole period, the AIC criterion favors the Tawn type 2 copula, while the BIC criterion favors the Gumbel copula. However, for all the other periods, these two criteria are in line with the choice of the Gumbel copula. We will therefore only interpret the results obtained using the Gumbel copula.

Over the whole period, the estimated coefficient of the Gumbel copula ($\theta = 1.021$ with a standard deviation of 0.023), is not significantly different from 1. Consequently, it seems that there is a little dependence between the two variables if it is estimated over the whole period.

However, this coefficient becomes significantly different from 1, and becomes higher when estimates are made for the most recent periods. Recall that this θ coefficient in Gumbel copula makes it possible to characterize the dependence between the two series: the higher it is compared to 1, the stronger the dependence.

Consequently, if we retain the estimates of this coefficient over the different subperiods, we notice that the Kendall correlation coefficient becomes higher and higher

	Copula parameter(s)	Kendall's τ	λ_U	AIC	BIC
Period 1975 – 2021 (number of observations $n = 565$)					
Gumbel	$\theta = 1.021$ (0.023)	0.020	0.028	0.96	5.20
Tawn type 1	$\theta = 1.025$ (0.051) $\beta = 0.213$ (n.d)	0.010	0.014	3.76	12.25
Tawn type 2	$\theta = 1.511$ (0.398) $\alpha = 0.037$ (0.025)	0.028	0.032	-1.43	7.06
Period 1990 – 2021 (number of observations $n = 384$)					
Gumbel	$\theta = 1.064$ (0.035)	0.060	0.082	-4.12	-0.30
Tawn type 1	$\theta = 1.154$ (0.127) $\beta = 0.249$ (0.236)	0.058	0.077	-1.77	5.87
Tawn type 2	$\theta = 1.129$ (0.115) $\alpha = 0.249$ (0.314)	0.050	0.067	-1.49	6.14
Period 2000 – 2021 (number of observations $n = 264$)					
Gumbel	$\theta = 1.087$ (0.050)	0.080	0.108	-2.35	1.03
Tawn type 1	$\theta = 1.236$ (0.323) $\beta = 0.243$ (0.376)	0.080	0.104	-0.60	6.16
Tawn type 2	$\theta = 1.155$ (0.163) $\alpha = 0.289$ (0.477)	0.064	0.086	0.28	7.03
Period 2010 – 2021 (number of observations $n = 144$)					
Gumbel	$\theta = 1.138$ (0.086)	0.121	0.161	-1.18	1.38
Tawn type 1	$\theta = 1.285$ (0.321) $\beta = 0.329$ (0.431)	0.112	0.147	0.89	6.02
Tawn type 2	$\theta = 1.238$ (0.235) $\alpha = 0.329$ (0.448)	0.098	0.129	1.28	6.41

Note: Values in parentheses indicate the estimated standard deviations. n.d. indicates that the standard deviation is not finite.

Table 6: Results of copula estimations

until it reaches 0.12. This may suggest that the correlation between changes in oil prices and inflation in Tunisia measured by Kendall's τ is not static but dynamic.

This result is consistent with the Pearson correlation coefficient estimates from the dynamic conditional correlation model presented in the previous section (see figure 1). The previous results showed that the Pearson's ρ correlation coefficient is not constant but varies over time. The same result is present for Kendall's ρ when we use copula modelling, although no dynamic coefficient model is estimated here.

Another important correlation coefficient is the one that describes the correlation between extreme values. Note that Gumbel copula is mainly used to model positive extreme dependencies (on the right side of the distribution). The dependence coefficient of extreme values λ_U also gives increasingly larger values depending on the estimation periods. With this copula, the correlation coefficient of positive extreme values reaches the value of 0.16 when it is estimated over the period 2010 - 2021. Thus, it can be deduced that a strong positive variation (a significant increase) in oil prices is correlated with an increase in inflation in Tunisia. For a country such as Tunisia, which became a net oil importer in 2000, any increase in the world price of this resource leads to inflationary pressures on the economy. It is above all sudden and positive variations that have the greatest impact on inflation in Tunisia.

According to table 6, as in the Gumbel copula $\lambda_L = 0$, a strong negative variation (a significant drop) in oil prices is not correlated with inflation in Tunisia, we can therefore deduce that any increase in oil prices can only lead to an increase in inflation, whereas a significant drop in the price of this resource does not affect inflation. This asymmetric effect has recently been highlighted in a number of specific studies (e.g., Raheem et al., 2020, Li and Guo, 2022). However, they have not been accurately quantified and the studies often suffer from the imposition of linearity (or quasi-linearity) assumptions that our methodology does not impose. Our results, although limited to Tunisia, offer complementary perspectives to classic studies with less restrictive modelling.

5 Conclusion

In conclusion, this study highlights the significant relationship between oil price fluctuations and inflation in Tunisia. By employing a non-linear dynamic correlation approach through the combination of a DCC-GARCH model and a copula model, we were able to capture the dependence between extreme oil price changes and inflation. The findings indicate that inflation in Tunisia is more sensitive to extreme positive variations in oil prices than to average variations, underscoring the asymmetrical impact of oil price shocks on the Tunisian economy.

The transition of Tunisia from an oil-exporting to an oil-importing country since the 2000s has further amplified the sensitivity of its inflation rate to global oil price changes. This structural shift has had profound implications for the country's economic stability and policy-making. As demonstrated, the strong correlation between extreme values of oil price changes and inflation suggests that economic policies in Tunisia must account for the volatility and unpredictability of the global oil market.

Moreover, the study's methodological approach offers valuable insights for other developing countries facing similar economic conditions. The use of copula functions to model the dependence structure between oil prices and inflation allows for a more nuanced understanding of their relationship, especially in the presence of extreme events. This approach can be adapted and applied to other contexts, providing a robust framework for analyzing the impact of external shocks on domestic economic variables.

The policy implications of these findings are crucial. For Tunisia, strategies to mitigate the impact of oil price volatility should be prioritized. This could include diversifying energy sources, improving energy efficiency, and implementing fiscal policies that cushion the economy against oil price shocks. Additionally, monetary policies should be designed to respond swiftly to sudden spikes in oil prices to prevent runaway inflation.

In summary, this research contributes to the existing literature by providing a detailed empirical analysis of the oil price-inflation nexus in Tunisia using advanced econometric models. The results emphasize the need for tailored economic policies that address the specific challenges posed by oil price volatility, thereby enhancing the resilience of the Tunisian economy. Future research could extend this analysis to other developing nations, further validating the effectiveness of the copula-DCC-GARCH approach in different economic settings.

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