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Immigration, Demand, Supply and Sectoral Heterogeneity in the UK Labor Market: A Time Series Approach*

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Abstract

Should we think of immigration as an exogenous shock to labor supply in the receiving economy? The time series of the share of migrant labor is 'Granger caused' by that of total hours worked and the average real wage in the UK economy. This suggests that immigration is, in part, determined by demand and supply effects in the labor market. In this paper we model, for the first time, immigration, wages and hours worked, as responding to demand, supply and immigration shocks at the aggregate and sectoral levels. The labor market is therefore modelled as being subject to multiple shocks at any one time, with individual shocks reinforcing and offsetting each other. We use a vector autoregression (VAR) on a time series of UK labor market variables from 2001-2019 for 35 different sectors. We find, across different identification techniques taken from the macroeconomics literature, that a fundamental component of the estimated time series process has a negative association between immigration and native wages. This shock, which can be interpreted as an 'immigration shock', plays a significant role in the determination of wage growth, although its influence varies across sectors and identification methods. Indeed, immigration itself is also determined, in part, by aggregate demand and supply shocks. (206 Words)

Keywords: Immigration, Demand, Supply, VAR, Sectoral Heterogeneity

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1 Introduction

The effect of immigration on the labor market is of intrinsic economic interest. It is also the focus of longstanding political attention, with immigration linked empirically to the rise of counter-globalization voting patterns across the world, see Rodrik (2021) and Docquier et al (2023). In this context, establishing whether there may exist adverse labor market effects associated with immigration becomes an even more important concern. The empirical literature to date has consistently found only small effects of immigration on wages.[1] Manacorda, Manning, and Wadsworth (2012), and Ottoviano and Peri (2012) offer imperfect substitutability of immigrant and native labor as one possible explanation for these small wage effects. In this paper, we argue instead that wages, hours and indeed immigration, are determined simultaneously in a labor market subject to multiple shocks at the aggregate and sectoral level. Thus, for example, the observed weak association between wages and immigration may be due to a negative association between wage growth and immigration being offset by a positive association between immigration, wages and aggregate demand. By the same token, effects which have been attributed to migration may in fact be the result of other shocks to the system.

We investigate empirically the relative importance of different types of shocks in explaining the variation in key UK labor market variables. The UK is an important exemplar in this regard, exhibiting as it does, large variations in immigration rates, wages and employment across sectors and over time. Our analysis employs a vector autoregression (VAR) approach which models immigration as part of a multivariate stochastic process evolving throughout the sample period. This contrasts with much of the literature which models immigration as an assumed exogenous shock to the labor supply.

Moreover, even if immigration at the aggregate level is driven by an exogenous shock, immigration at the sectoral level may not be exogenous. Migrants will tend to flow to those sectors with high demand for their labor all else being equal. Sectors will also likely differ in the nature of the production process, and thereby in their use of different types of labor, and for other supply side reasons. Therefore to analyze the labor market effects of immigration one also needs to take account of likely sectoral heterogeneity as well as the multiplicity of shocks.

It has long been acknowledged that modeling immigration solely as a labor supply shock has limitations. Borjas (1994) noted that "The size and composition of the immigrant flow are jointly determined by supply side considerations . . . .as well as by factors that determine the host country's demand for immigrants". Borjas' comments relate to the demand constraints at country-level imposed by visa quotas, which are common across industrialized economies.[2] The existence of shortage occupation lists as in Australia, Canada and the UK, can also be viewed as evidence of the importance of demand in influencing the level and type of immigration. There may also be dynamic demand responses to immigration. Ottoviano and

[1] See for example Borjas (2004), or Ottoviano and Peri (2012) for the United States or Dustmann, Frattini and Preston (2013), Manacorda, Manning, and Wadsworth, (2012) for the UK).

[2] The EU of course allows unrestricted mobility of individuals between member states.

Peri (2012) acknowledge this possibility, stating “We treat immigration as a labor supply shock, omitting any productivity impact that it may produce due, for example, to improved efficiency, choice of better technologies, or scale externalities”. Peri, Rury and Wiltshire (2023) state that their results on the effect of immigration following Hurricane Marie are also consistent with a negative labor supply shock, offset by positive consumer demand shocks.

The assumption of exogenous labor supply shocks nevertheless still underlies the identification of immigration's effects on native workers' labor market outcomes in much of the empirical literature. Dustmann, Schönberg, and Stuhler (2016) state, “Any of the approaches we discuss slices the labor market into different sub-labor-markets and uses variation in the inflow of immigrants into these sub-labor-markets as an identification device. We assume here that the allocation of immigrants to these sub-labor-markets is (conditionally) independent of shocks to wages or employment of native workers (which could be achieved either through random allocation of immigrants, or by use of an appropriate instrument)... Studies that slice the labor market into skill groups instead typically assume that immigrant inflows are exogenous, an assumption that may be violated (Llull 2014).” Campo et al (2018) similarly argue “ While there is significant consensus that immigrants select into labor markets with more favorable conditions (lower unemployment, higher wages) thus... immigration flows might be higher... to high productivity sectors which are more attractive and likely to be growing.”

Alongside this issue of identification there also exists the possibility that immigration effects differ across skill levels. The existing literature has acknowledged the possibility of heterogeneous effects of immigration. Largely this has focussed on different effects by migrant skill level and geographical origin, see e.g. Dustmann et al (2016), Ottoviano and Peri (2012) and Manacorda, Manning, and Wadsworth, (2012). In this paper, we also distinguish between skill levels across sectors. Mountford and Wadsworth (2023) find that the effects of skilled immigration on training of the native workforce differs significantly across sectors, in particular between traded and non-traded sectors.[3]

In this paper we show that these concerns about abstracting away from the effects of demand and sectoral heterogeneity may be well placed. We use a VAR approach where demand, supply and immigration effects can occur simultaneously in every time period so that, potentially, a negative association between wage growth and immigration may be offset by a positive association between immigration, wages and aggregate demand. The VAR framework has been previously used to study the effects of migration most notably by Blanchard and Katz (1992) for internal migration, and more recently Furlanetto and Robstad (2019) on Norwegian data. However to our knowledge our paper is the first to employ a VAR framework that explicitly incorporates shocks at the sectoral as well as aggregate level.

VARs have long been regarded as a good way of describing the dynamic correlations in the data, see e.g. Sims (2003), Baumeister and Hamilton (2024). This has typically been

[3]Negative effects of immigration on native training in the skilled non-traded sectors and positive effects in the traded sector, which they attribute to the limited ability of the non-traded sector to increase output compared to the traded sector.

done using an arbitrary decomposition, a Cholesky factorization, of the variance-covariance matrix of the residuals of the VAR. However, determining causality based on the Cholesky factorization is problematic due to the strong restrictions it imposes on the responses of the identified shocks, as discussed in, for example Uhlig (2005) and Baumeister and Hamilton (2015, 2019, 2024). We therefore also employ the less restrictive sign restriction identification methodology where, for example, a positive sectoral labor demand shock is identified as having positive effects on sectoral wages and hours worked. This approach follows the macroeconomic literature, of *inter alia*, Canova and De Nicoló (2002), Uhlig (2005), Mumtaz and Surico (2009) and Baumeister and Hamilton (2015, 2019, 2024) and which has been applied to immigration by Furlanetto and Robstad (2019) and Kiguchi and Mountford (2017).

We employ a six variable VAR using UK data from 2001-2019 for each of 35 different labor market sectors to identfy demand, supply and immigration effects at both the aggregate and sectoral level. The six variables are the economy-wide migration share, hours worked and real wage of natives, along with the same variables for each sector. These six variables are chosen to permit the identification of the six aggregate and sectoral shocks. This is described below in section 4. The combination of aggregate and sectoral variables in the same VAR echoes the approach of Canova (2005) in modeling the effects of US shocks on smaller Latin American economies and Mumtaz and Surico (2009) on the effects of international shocks on the domestic UK, economy.

Sign restrictions can be imposed on either structural or reduced form VARs. In this paper we focus on the structural approach of Baumeister and Hamilton (2015, 2019) who show how one can incorporate beliefs and incomplete information about the effects of different shocks into the priors for these parameters in a Bayesian estimation procedure. However the key results do not depend on this choice. We also estimate the model using the Cholesky and reduced form sign restriction approaches. In all three approaches one of the identified fundamental shocks has a negative correlation between immigration and native wage growth. Furthermore, this shock plays a significant adverse role in the determination of wage growth in the economy.

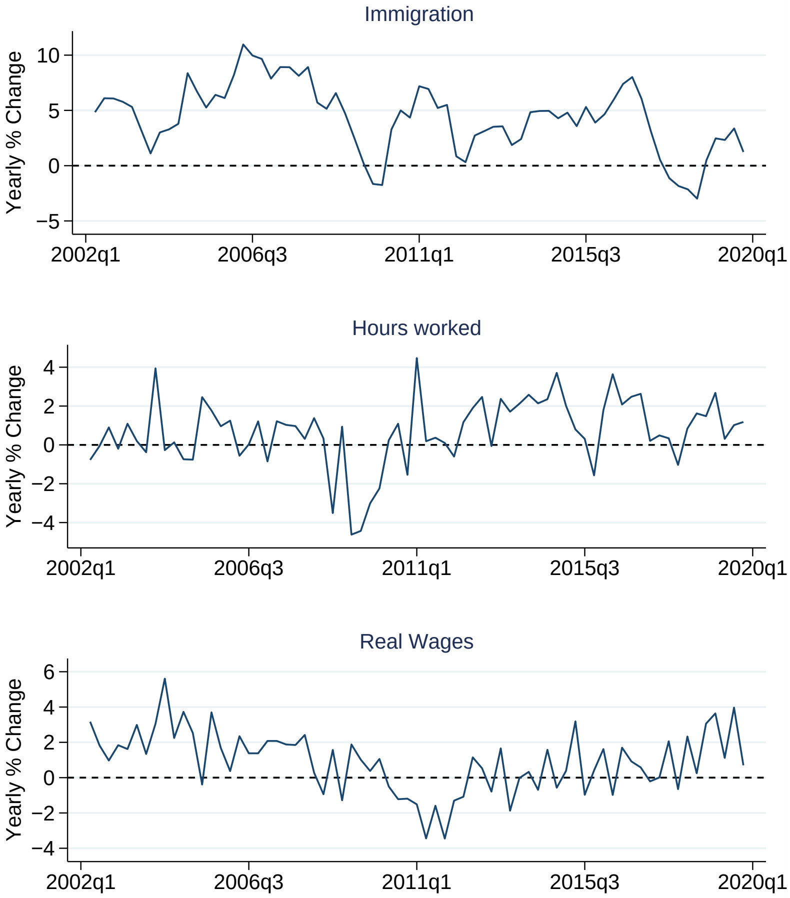


Figure 1: Growth rates of immigration share in the working population, hours worked and average wages in the UK 2001-2020.

2 Data and Motivation

In order to estimate our VAR models we need sectoral level data on both the employment and wages of UK-born workers and the concentration of immigrants working in each sector. The requisite information is contained in the UK Labour Force Survey (LFS). The LFS is a quarterly random sample of around 40,000 households and the individuals therein. We use data starting in the first quarter of 2001 and end our sample in the last quarter of 2019 so as not to include data subject to the effects of the COVID pandemic.[4]

The LFS contains details of the country of birth of every individual in the sample. An immigrant is defined as anyone who is born outside the UK. The LFS also gives the 3-digit industry and occupation codes of employed workers. Since specific industries contain many occupations and a given occupation can be found across different industries, the definition of a sector in our analysis combines individual occupation and industry affiliation. Sample size constraints determine that a sector is built as a combination of four possible occupations, (Professional/Managerial, Other Non-Manual, Skilled Manual and Manual), and 13 indus-

[4]The LFS sample response rates also decline significantly during the pandemic which adversely affects data analysis using disaggregated units.

tries.[5] For example sector 112 is a professional(1-digit SOC code = 1) working in the health industry, (2-digit SIC code = 12). One complication with pooling LFS data over time is that the occupational codes change approximately every 10 years.[6] The industry classifications also change in 2009 but we are able to correct for this using the mapping of Smith.[7] We collate the data by sector for each quarter in each year. This ensures that there is a minimum of 100 observations in each of 35 sectors in each quarter with a median sample cell size of 1122 for hours and 267 for wages. The hours variable we use in our analysis is 'Total Hours Worked', in the survey reference week includes paid and unpaid overtime. We observe hourly wages for 40% of the survey respondents and use the median of this at the sectoral level deflated by the CPI price index. The aggregate versions of these variables are the aggregates of the sectoral variables weighted by their LFS population weights.

The idea that the amount of immigrant labor employed in an economy will depend on the demand for and supply of labor is extremely intuitive. Figure 1 plots the year on year growth rates of the share of immigrants in the working age population, the total number of hours worked and the average real wage of natives between 2001-2020 using UK LFS data. The time series appear related most noticeably after the financial crisis of 2007-2008 and the subsequent recovery period. This is borne out by Granger causality tests, reported in Table 1, which show that immigration is Granger caused by the total number of hours worked and that wages are Granger caused by immigration. These results are generated using a VAR of these three aggregate variables with 4 lags both with and without a time trend, estimated on the entire sample and for the shorter sub-sample 2004q1-2016q2, to demonstrate that the results are not due to Brexit or the sample's initial conditions. Interestingly total hours worked are not Granger caused by either real wages or immigration in any specification or sample.

These results suggest that the level of immigration and real wages are related to the total hours worked in the economy, which itself is surely affected by the macroeconomic environment, including demand and supply effects. Empirical macroeconomics, as explained below in section 4, has developed methods to untangle the individual effects from the multiple influences on a variable's time series. We apply these methods to identify the contribution of labor demand and supply at both the aggregate and sectoral level, on immigration share, hours worked and native wages in the UK economy.

[5] Production, Construction, Retail, Transport, Food & Hospitality, Media&IT, Finance, Scientific, Transport&Support Services, Public Admin, Education, Health , Other Services.

[6] The latest industry recoding was 2008 and there were 2010 and 2020 recodings for occupations. The occupational classifications also change much more significantly in 2001, which makes matching before this period difficult. Using 4 broad occupation codes facilitates comparability over time.

[7] The change in the industry codes is less substantial, see https://warwick.ac.uk/fac/soc/economics/staff/jcsmith/sicmapping/

Table 1

Granger Causality Tests For Aggregate Labor Market Variables

	Time Period 2003q1-2019q4		**Time Period 2004q1-2016q2**	
Model:	**VAR(4)**	**VAR(4) with trend**	**VAR(4)**	**VAR(4) with trend**
	Chi-Sq	Chi-Sq	Chi-Sq	Chi-Sq
~~Agg~~ Immigration				
Exclude Hours	21.89***	18.97***	23.91***	23.87***
Exclude Real wage	6.234	4.654	11.48**	8.031*
Exclude both	25.86***	20.90***	33.23***	28.11***
Total Hours				
Exclude Immig.	3.230	1.342	3.161	1.773
Exclude Real wage	3.961	3.703	5.145	5.484
Exclude both	5.974	4.268	6.594	6.053
Real Wages				
Exclude Immig	12.95**	14.44***	11.15**	11.15**
Exclude Hours	5.592	7.017	4.141	6.918
Exclude both	14.53*	16.26**	13.34	15.27*

Notes: The table reports, the Chi-squared values for the Granger causality tests from VARs of the year on year growth rates of the share of immigrants in the working population, the total number of hours worked and the average hourly wage in the UK. The VARs use 4 lags and are run for the time periods, 2003q1-2019q4 and 2004q1-2016q2. ***,**, and * indicate significance at the 1%,5% and 10% level respectively.

3 Sectoral Variation in Immigrant Labor

To illustrate the degree of heterogeneity in the use of immigrant labor across sectors, Figure 2 graphs the 18-year change in employment of both UK-born workers and immigrants in each of the 35 sectors in the data set. The backward sloping 45 degree line separates occupations that experience net growth in employment in this period from those that are declining. Any occupation that lies above and to the right of this line is growing. The forward sloping 45 degree line separates occupations that are growing primarily because of immigration - those sectors above the line - from those that are growing mainly due to growth in UK-born employment - those sectors below the line. The figure shows that most sectors grow over this period, but a minority decline (e.g. 401 Unskilled Workers in Production or 403 Unskilled Workers

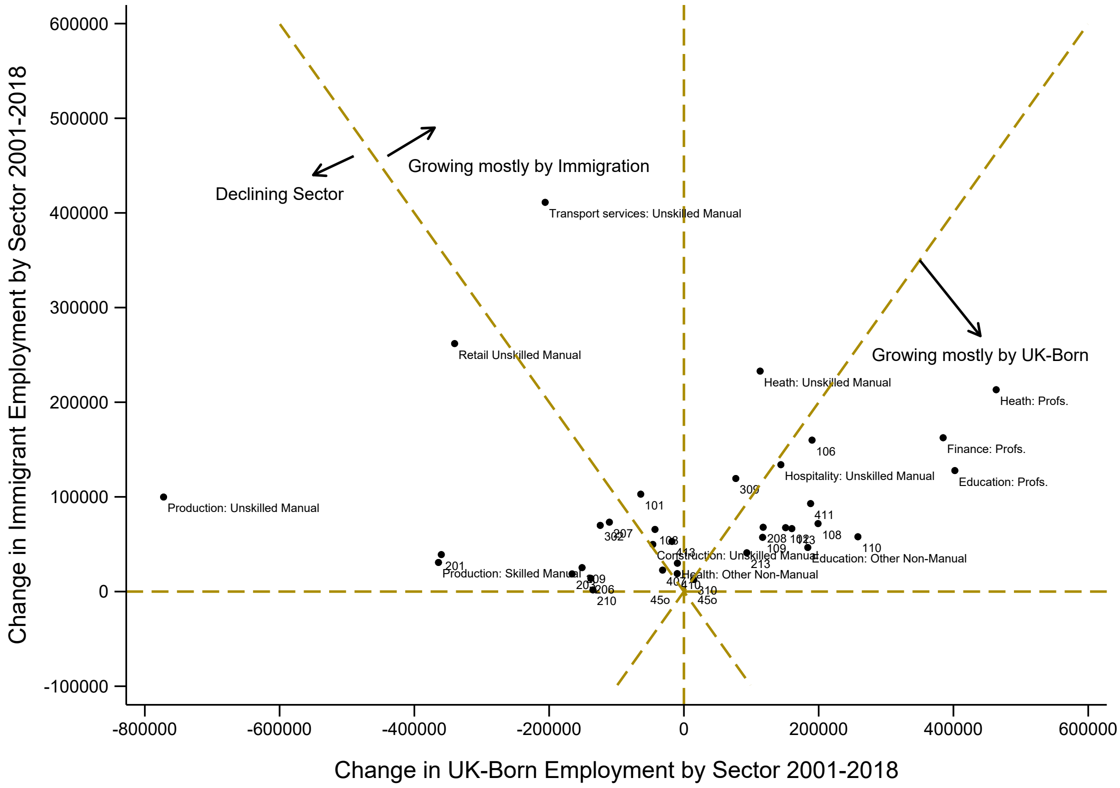


Figure 2: Change in UK-Born & Immigrant Employment by Sector 2001-2018

in Retail). In all these declining sectors, the number of immigrants rises while the number of UK-born workers falls. This means that the share of immigrants has risen in all sectors with a net decline in employment.[8] Of the sectors with net employment growth over the period, some grow exclusively because of rising immigrant numbers, (e.g. 404 Unskilled Workers in Transport) while numbers of UK-born employed fall. Others grow through approximately equal numbers of immigrants and UK-born, (e.g. 223 Unskilled workers in Hospitality) and some grow primarily, though not exclusively, through rising numbers of UK-born workers, (e.g. 111 Teaching Professionals). There is no sector in which the level of immigration falls over this period. Overall the Figure shows that there is substantial heterogeneity in changes in both employment and the immigrant share across sectors over the sample period.

Figure 3 indicates another facet of heterogeneity of experience across sectors by plotting the change in (log) wages of UK-born workers in each sector over the sample period against the change in the sectoral log immigrant share. For a given change in immigrant share, the graph shows a large variation in wage growth across sectors. In some sectors wages fall, while in other sectors, for the same immigrant change, wages rise. Again this suggests that the association between immigration and the labor market experience of native-born workers is unlikely to be the same in all sectors. However, despite this heterogeneity, there does appear to be a positive relationship overall between changes in immigration and wages. The extent to which this is caused by supply and demand factors is the focus of the rest of the paper.

[8]This finding also indicates that the immigrant share, a common measure of immigrant concentration in the literature, can also change because of changes in the size of the native workforce.

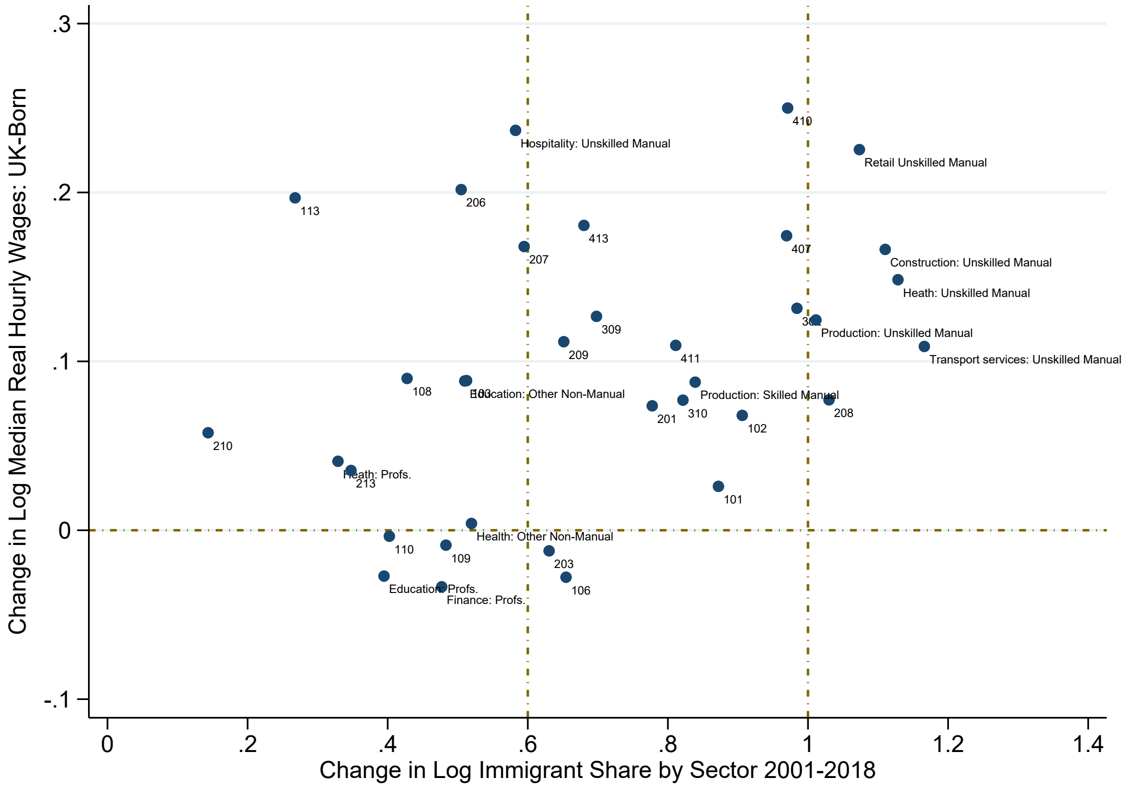


Figure 3: Log Change in UK-Born Log Real Hourly Wage & Immigrant Employment Share by Sector 2001-2018

4 Sectoral Labor Market Dynamics with Multiple Causal Factors

In this paper we argue that it is useful to think of sectoral labor markets as being subject to multiple forces at any one time. For example, sectoral wages of native workers in period t, w_t^{sec}, may be subject to shocks from a combination of aggregate migration, $\epsilon_{aggM,t}$, aggregate supply, $\epsilon_{aggS,t}$, aggregate demand, $\epsilon_{aggD,t}$, sectoral migration, $\epsilon_{secM,t}$, sectoral supply, $\epsilon_{secS,t}$, and sectoral demand, $\epsilon_{secD,t}$, as in the following equation,

$$w_t^{sec} = \beta x_{t-1} + \alpha_1 \epsilon_{aggM,t} + \alpha_2 \epsilon_{aggS,t} + \alpha_3 \epsilon_{aggD,t} + \alpha_4 \epsilon_{secM,t} + \alpha_5 \epsilon_{secS,t} + \alpha_6 \epsilon_{secD,t} \tag{1}$$

where α_i are parameters indicating the strength of each shock in determining wages in this sector and where x_{t-1} is a vector of predetermined variables.

This type of wage equation corresponds to one of the equations in a structural VAR, where the predetermined variables are the constant term and the lags of the variables included in the VAR, denoted y, so that in equation (1), $x_{t-1} = [1, y'_{t-1}, \ldots, y'_{t-p}]'$, and x_{t-1} is an $((np+1) \times 1)$ vector where n is the number of variables in the VAR and p is the lag length.

A structural VAR is described by equation (2),

$$Sy_t = C + B_1 y_{t-1} + B_2 y_{t-2} + \cdots + B_p y_{t-p} + \epsilon_t \tag{2}$$

where S, is the $(n \times n)$ contemporaneous effects matrix and ϵ_t, is the vector of the fundamental shocks on the VAR. The $(n \times n)$ variance covariance matrix for these shocks, $E[\epsilon_t \epsilon_t'] = D$, is assumed to be diagonal. The diagonal structure implies the shocks are 'fundamental' in the sense of not being associated with each other.

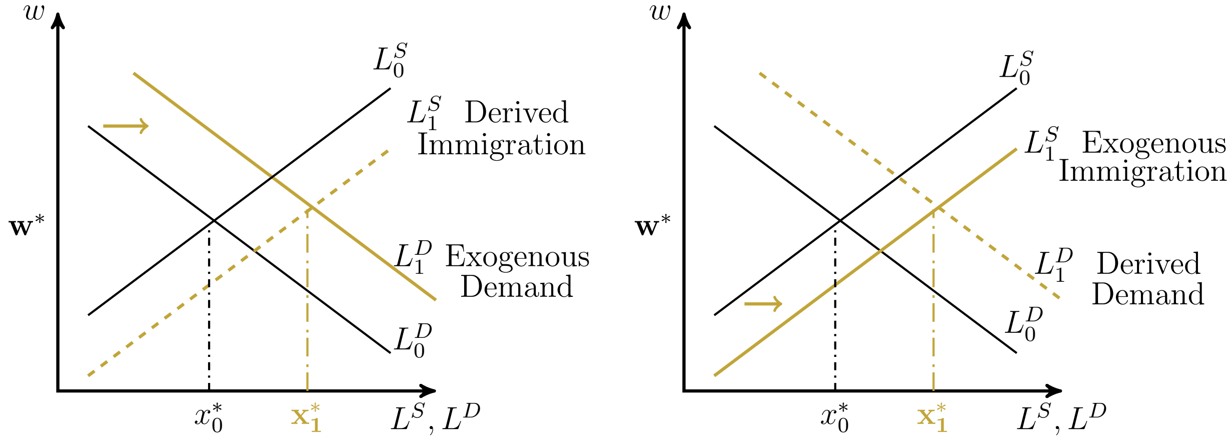
In our VAR model, for each sector we choose $p = 4$, for quarterly data, and $n = 6$, as we are interested in the interaction between 3 aggregate and 3 sectoral shocks on aggregate and sectoral variables. The six variables are the year on year difference of logs of economy-wide migration share, M_t^{agg}, economy-wide total hours worked, H_t^{agg}, economy-wide real wages of native workers, W_t^{agg}, the sectoral migration share, m_t^{sec}, total hours worked in the sector, h_t^{sec}, and sectoral real wages of natives, denoted w_t^{sec}. Thus $y_t = (M_t^{\text{agg}}, H_t^{\text{agg}}, W_t^{\text{agg}}, m_t^{\text{sec}}, h_,^{\text{sec}} w_t^{\text{sec}})'$. The aggregate variables allow for the identification of economy-wide aggregate demand, supply and migration shocks while the presence of the sectoral variables allows for the possibility of sectoral demand, supply and migration shocks to also affect the outcome variables.

Baumeister and Hamilton (2015, 2019) show how one can incorporate prior beliefs about the signs and size of the coefficients in the contemporaneous effects matrix, S, into a Bayesian estimation procedure for the VAR. They derive a Metropolis Hasting algorithm for drawing from the posterior distribution for the parameters resulting from these priors. Baumeister and Hamilton (2015) describes this method in detail and and so our exposition here is brief. We use the programs supplied by Baumeister and Hamilton for the replication of Baumeister and Hamilton (2019) to estimate the model.[9] We also impose an additional assumption in the prior for the B parameters. We set the prior to be very tight around zero for the parameters associated with the lagged sectoral variables in the aggregate variable equations, following e.g. Blake and Mumtaz (2017), and also impose stationarity. This implies, as we will see below, that the dynamics of the aggregate variables are almost entirely determined by the 3×3 VAR of aggregate variables.

Identifying Different Structural Shocks

Given the estimated model parameters and variance covariance matrix, the aim of identification is to define the different fundamental shocks which underlie the movement of each variable. This amounts to choosing the matrix, S, in the structural VAR described in equation (2). Each identification method is then be able to see which shocks are most important for the variation of each variable, sector by sector, and also whether they offset each other. For example, as illustrated in Figure 4, one may be unsure whether demand shocks or migration shocks are most important in determining sectoral wage rates. An observed significant rise in hours worked occurring alongside little or no change in sectoral wages may be the result of a positive shock to the demand for labor which is responded to by an increase in immigrant labor supply. This case is depicted in Panel a) of Figure 4. Equally, an exogenous shock to immigration which increases domestic labor demand in response could generate the same effect. This is depicted in Panel b) of Figure 4.

[9] We make 2,000,000 draws from the posterior for each sector discarding 1,000,000 draws as 'burn in' and retain every 25th draw as the 'thinning' process. We set $\lambda_0 = 100$, $\lambda_1 = 1$, $\lambda_2 = 1$ and $\lambda_3 = 100$ in the Minnesota prior, use the identity matrix for the covariance of the proposal distribution and adjust the jump size during the burn-in phase when it deviates too far from an acceptance rate of 0.35.



(a) Exogenous demand shock causing increased immigration leaving wages unchanged

(b) Exogenous Immigration leading to increased demand for labor leaving wages unchanged

Figure 4: Identification: which shocks matter most for changes in sectoral wages?

Clearly this is not an exhaustive list of potential explanations for this relationship. In Section 5 we compare the contribution of the six identified shocks to the variation in wage growth in each sector using impulse response functions and historical decompositions which are described below.

4.1 Structural Identification: Using the Cholesky Factor

The Cholesky factorization of the reduced form's variance covariance matrix, Σ, is widely seen as a useful and transparent, if arbitrary, way of summarizing the data's dynamics. The reduced form VAR is given by multiplying equation (2) by the inverse of the S matrix,

$$y_t = S^{-1}C + S^{-1}B_1 y_{t-1} + S^{-1}B_2 y_{t-2} + \cdots + S^{-1}B_p y_{t-p} + u_t \tag{3}$$

where, therefore, $u_t = S^{-1}\epsilon_t$. Any symmetric positive-definite matrix, such as a variance covariance matrix, Σ, can be written as the product of a lower triangular matrix, L, known as the Cholesky factor, and its transpose, such that $LL' = \Sigma$, which implies that $S^{-1} = L$. This allows the dynamics of the data to be summarized by six independent shocks, $\epsilon_1 \dots \epsilon_6$ which each have assumed zero mean and unit variance. Defining $\epsilon_t = [\epsilon_{1,t} \dots \epsilon_{6,t}]$, the independence of the shocks and their normalization implies that $E[\epsilon_t \epsilon_t'] = I_6$ and thus that the actual prediction errors of the VAR, u_t can be mapped into these independent shocks via the equation $u_t = L\epsilon_t$, since $E[u_t u'] = LE[\epsilon_t \epsilon_t']L' = \Sigma$. The Cholesky factorization thus decomposes each variable's time series into the sum of responses to multiple independent shocks, as discussed in e.g. Baumeister and Hamilton (2024) or Sims (2003).

Assuming that S^{-1} is lower triangular implies that only one shock, $\epsilon_{1,t}$, affects all variables contemporaneously.[10] This is often seen as a very strong restriction. The sign restriction approach, described below, imposes looser restrictions so that, in our case, all sectoral variables can be affected contemporaneously by all shocks.

[10] See Appendix A.2

4.2 Structural Identification: Using Sign Restrictions

Instead of assuming that S^{-1} is lower triangular, we can impose the looser restriction that the VAR is lower block diagonal. Given that y_t is ordered, $y_t = (M_t^{\text{agg}}, H_t^{\text{agg}}, W_t^{\text{agg}}, m_t^{\text{sec}}, h_{,}^{\text{sec}} w_t^{\text{sec}})'$, this implies that aggregate variables are not contemporaneously affected by sectoral variables, following e.g. the intuition of Liu, Mumtaz and Theophilopoulou (2014). The S matrix therefore has the form

$$S = \begin{bmatrix} 1 & s_{MH} & s_{MW} & 0 & 0 & 0 \\ s_{HM} & 1 & s_{HW} & 0 & 0 & 0 \\ s_{WM} & s_{WH} & 1 & 0 & 0 & 0 \\ s_{mM} & s_{mH} & s_{mW} & 1 & s_{mh} & s_{mw} \\ s_{hM} & s_{hH} & s_{hW} & s_{hm} & 1 & s_{hw} \\ s_{wM} & s_{wH} & s_{wW} & s_{wm} & s_{wh} & 1 \end{bmatrix} = \begin{bmatrix} S_1 & 0 \\ S_2 & S_3 \end{bmatrix} \tag{4}$$

where S_1, S_2 and S_3 are the upper left, bottom left, and bottom right 3×3 submatrices of S. The first column of S gives the contemporaneous effect of aggregate migration on the other variables. Thus s_{HM} is the contemporaneous effect of aggregate migration on aggregate hours. Similarly the first row is the contemporaneous effects of other variables on aggregate migration, so that s_{MH} is the contemporaneous effect of aggregate hours on aggregate migration. Note that in the top left 3×3 block all three aggregate variables can contemporaneously effect each other, and the sectoral variables can be contemporaneously effected by all, aggregate and sectoral, variables.

The impact matrix of the fundamental shocks in equation (3), the reduced form VAR, is the inverse of the S matrix, S^{-1}. The lower block triangular nature of S implies that the determinant of S, $\det(S)$, is given by $\det(S) = \det(S_1)\det(S_3)$ and that S^{-1} can be decomposed into the product of 3×3 matrices.

$$S^{-1} = \begin{bmatrix} S_1^{-1} & 0 \\ -S_3^{-1} S_2 S_1^{-1} & S_3^{-1} \end{bmatrix}$$

Thus S^{-1} can be written

$$S^{-1} = \left[\begin{array}{cccccc} \frac{1 - s_{HW}s_{WH}}{\det(S_1)} & \frac{s_{MW}s_{WH} - s_{MH}}{\det(S_1)} & \frac{s_{MH}s_{HW} - s_{MW}}{\det(S_1)} & 0 & 0 & 0 \\ \frac{s_{WM}s_{HW} - s_{HM}}{\det(S_1)} & \frac{1 - s_{MW}s_{WM}}{\det(S_1)} & \frac{s_{MW}s_{HM} - s_{HW}}{\det(S_1)} & 0 & 0 & 0 \\ \frac{s_{HM}s_{WH} - s_{WM}}{\det(S_1)} & \frac{s_{MH}s_{WM} - s_{WH}}{\det(S_1)} & \frac{1 - s_{MH}s_{HM}}{\det(S_1)} & 0 & 0 & 0 \\ & & & \frac{1 - s_{hw}s_{wh}}{\det(S_3)} & \frac{s_{mw}s_{wh} - s_{mh}}{\det(S_3)} & \frac{s_{mh}s_{hw} - s_{mw}}{\det(S_3)} \\ & -S_3^{-1} S_2 S_1^{-1} & & \frac{s_{wm}s_{hw} - s_{hm}}{\det(S_3)} & \frac{1 - s_{mw}s_{wm}}{\det(S_3)} & \frac{s_{mw}s_{hm} - s_{hw}}{\det(S_3)} \\ & & & \frac{s_{hm}s_{wh} - s_{wm}}{\det(S_3)} & \frac{s_{mh}s_{wm} - s_{wh}}{\det(S_3)} & \frac{1 - s_{mh}s_{hm}}{\det(S_3)} \end{array} \right] \tag{5}$$

Equation (5) demonstrates that a sign restriction on any one element of the S matrix does not necessarily imply a sign for the impact of any shock on any variable. Nevertheless sign restrictions can be imposed using the formulas for impact given in equation (5) during the sampling procedure for the elements of S, so that e.g. the restriction that the 4th ordered shock has a positive impact on sectoral hours and a positive impact on sectoral wages are the restrictions on the elements of S that $\frac{s_{wm}s_{hw}-s_{hm}}{\det(S_3)} > 0$ and $\frac{s_{hm}s_{wh}-s_{wm}}{\det(S_3)} > 0$. We order the shocks so that the aggregate migration shock is placed first in the ϵ_t vector, so that $\epsilon_t = (\epsilon_{aggM,t}, \epsilon_{aggS,t}, \epsilon_{aggD,t}, \epsilon_{secM,t}, \epsilon_{secS,t}, \epsilon_{secD,t})'$. Thus the sign restrictions for the aggregate migration shock are in the first column of S^{-1}.

The sign restrictions we impose in the structural model are described in equation (6), which omits the constant and lagged terms in equation (3) for ease of exposition. Thus the sign restrictions for a positive aggregate demand shock are that it has a positive impact effect on aggregate immigration, aggregate hours worked and aggregate real wages of natives. The aggregate supply shock in column 2 is identified as a shock which increases aggregate hours worked and reduces the aggregate real wage of natives on impact. There are no restrictions on the responses of other variables which is often referred to as being 'agnostic' about the responses of unrestricted variables to a shock.

$$\underbrace{\begin{bmatrix} \text{Aggregate Migration Share} \\ \text{Aggregate Hours Worked} \\ \text{Aggregate Native Wage} \\ \text{Sectoral Migration Share} \\ \text{Sectoral Hours Worked} \\ \text{Sectoral Native Wage} \end{bmatrix}}_{y_t} = \underbrace{\begin{bmatrix} + & \text{none} & + & \text{none} & \text{none} & \text{none} \\ + & + & + & \text{none} & \text{none} & \text{none} \\ \text{none} & - & + & \text{none} & \text{none} & \text{none} \\ \text{none} & \text{none} & \text{none} & + & \text{none} & \text{none} \\ \text{none} & \text{none} & \text{none} & \text{none} & + & + \\ \text{none} & \text{none} & \text{none} & \text{none} & - & + \end{bmatrix}}_{S^{-1}} \underbrace{\begin{bmatrix} \epsilon_{aggM,t} \text{ (Agg. Migration Shock)} \\ \epsilon_{aggS,t} \text{ (Agg. Supply Shock)} \\ \epsilon_{aggD,t} \text{ (Agg. Demand Shock)} \\ \epsilon_{secM,t} \text{ (Sec. Migration Shock)} \\ \epsilon_{secS,t} \text{ (Sec. Supply Shock)} \\ \epsilon_{secD,t} \text{ (Sec. Demand Shock)} \end{bmatrix}}_{\epsilon_t} \quad (6)$$

We do not impose the same sign restrictions at the sectoral level as at the aggregate level. Thus for immigration, at the aggregate level it is intuitive that an increase in aggregate imigration should increase aggregate hours. However this need not be the case for every sector i.e. as described in section 3 in some sectors increased immigration may be associated with reduced total hours as migrants are replacing an outflow of domestic labor. Thus the sectoral migration shock is simply restricted to increase the sectoral migrant share in equation (6). Similarly sectoral demand shocks are not required to increase sectoral migration shares.

Note that the sign restrictions described by equation (6) do not completely pin down the interpretation of these shocks. In particular, as we discuss below in section 5, the sign restrictions do not restrict the response of migration to a supply shock. Thus it may be that the second shock could be interpreted as a migration shock. This isn't a problem for our analysis. What we are investigating is whether a decomposition of the data makes sense in which there is a possibility of a shock with a negative association between migration and native wage growth, and if so, how important this shock is in determining wage growth. Note this is not explicitly imposed on the data in any of the identification approaches we use. We regard the restrictions in equation (6) as certainly plausible, but nevertheless it is just one set of plausible assumptions. Different identifying assumptions, such as the Cholesky factor and the reduced form sign restriction approach described below, will imply different roles for the identified shocks in contributing to the observed time series. Baumeister and Hamilton (2015, 2019) show how one can go further and tighten the identification by incorporating knowledge about the likely sizes of these S matrix parameters (elasticities) into the priors for these parameters. We do not do this in this paper as the incorporation of sector specific information for 35 sectors is beyond the scope of this paper. We therefore employ loose priors for all these coefficients, and leave the analysis employing more informative tighter priors for future work.[11]

Interpretation of the structural model

The structural VAR of equation (2) has a natural interpretation in terms of the elasticities of the variables with respect to the other variables in the system. For example, using the parameterization of the S matrix in equation (5), the first row of equation (2) can be written as the following equation

$$M_t^{agg} = \beta x_{t-1} - S_{MH} H_t^{agg} - S_{MW} W_t^{agg} + \epsilon_t[1] \tag{7}$$

where $\epsilon_t[1]$ is the first element of ϵ_t.

Thus S_{MH} is the short run elasticity of migration labor share growth with respect to hours growth. The shock $\epsilon_t[1]$ affects M_t^{agg} directly. It affects H_t^{agg} and W_t^{agg}, indirectly via its effect on M_t^{agg} multiplied by S_{HM} and S_{WM} in the second and third rows of equation (2) respectively. The sign restrictions in equation (6) restrict the size of these elasticities as, e.g. the restriction that the first shock has a positive impact on immigration share in the labor force implies that $\frac{1-s_{HW}s_{WH}}{\det(S_1)} > 0$.

4.3 Reduced form Approach Using Penalty Functions

Sign restrictions can also be imposed on reduced form VARs. In this section we show how this approach can implement an alternative set of identifying assumptions. The reduced form VAR is given by equation (3) which we rewrite as

$$y_t = \tilde{C} + \tilde{B}_1 y_{t-1} + \tilde{B}_2 y_{t-2} + \ldots \tilde{B}_p y_{t-p} + u_t$$

where $\tilde{C}$ corresponds to $S^{-1}C$ and $\tilde{B}_i$ corresponds to $S^{-1}B_i$ in equation (3) and $E[u_t u'] = \Sigma$. For the reduced form model we use standard assumption for Bayesian VARs that the prior for (β, Σ) belongs to the Normal-Wishart family, where $\beta = [\tilde{C}, \tilde{B}_1, \ldots, \tilde{B}_p]$, following e.g. Uhlig (2005). We also use the prior to shrink the β coefficients for the sectoral variables effect on aggregate variables as in Section 4.2, and use the same variables and lag lengths, $p = 4$.

Given the estimated parameters, identification is achieved by finding a matrix A, such that $u_t = A\epsilon_t$ and where $AA' = \Sigma$. This is equivalent to S^{-1} in Section 4.2. There are infinitely many such matrices, A, including the Cholesky factor for Σ. The sign restriction approach chooses a matrix A that generates impulse responses which agree qualitatively, i.e. in terms of their signs, with economic theory.[12] There are different methods of applying these sign restrictions. One approach is the 'pure sign restriction' approach of Uhlig (2005), generalized to the identification of multiple shocks by Rubio-Ramirez et al (2010), which selects A using random draws.

[11] The priors are a students t distribution, $t_\nu(\mu, \tau^2, \nu)$ with $\mu = 0, \tau = 100$ and $\nu = 3$.

[12] See the Appendix, or Hamilton (1994) for the calculation of impulse responses.

The penalty function approach, that we use here, chooses the matrix A matrix which best satisfies the imposed sign restrictions according to a numerical score given by this penalty function. The form of the penalty function follows Uhlig (2005) and is given in the Appendix. As the method chooses A matrices with high scores, as measured by the penalty function, then this method will choose A matrices with larger impulse responses for the restricted variables over those with smaller responses.

The purpose of demonstrating this alternative approach is to show the implications of different identifying assumptions. We therefore choose a radically different identification scheme which gives a lot of weight to the role of aggregate demand. Nevertheless, as described in section 5.1.1, the impulse responses display a definite similarity and in all three approaches, (structural Cholesky, structural sign restrictions and penalty function) there exists a significant role for an identified shock with a negative correlation between immigration and wages.

In the penalty function approach the order that shocks are identified in matters. The approach first imposes sign restrictions for the first shock and then imposes the sign restrictions for the second shock together with the additional assumption that the second shock is orthogonal to the first and so on, so that the last shock is restricted to be orthogonal to all the previously identified shocks. Note that the sign restrictions imposed are not mutually exclusive in themselves, but are uniquely identified by this extra restriction that they be orthogonal to the previously identified shocks, as discussed in Mountford and Uhlig (2009).

In our analysis we place the demand shock first, identified as increasing aggregate hours worked and aggregate real wages of natives on impact. There are no restrictions on the responses of other variables, i.e. no 'zero' restrictions. The aggregate migration shock is identified as a shock which increases aggregate migration share and which is orthogonal to the demand shock. The aggregate supply shock is identified as a shock which increases aggregate hours worked and reduces the aggregate real wage of natives and which is orthogonal to the two previously identified shocks. Identifying the demand shock first gives it the greatest degrees of freedom to explain the variation in the data and so thereby answers the question of 'What is the most variation in wages and hours worked that can be attributed to demand shocks?', where the metric is provided by the penalty function. This is a very different identification meachanism to that in section 4.2 but is also plausible, which is why we have chosen to highlight it.

5 Results

In this section we discuss the results from the VARs described above. We first discuss the results for aggregate variables in section 5.1 and then the results at the sectoral level in section 5.2. In each case we first describe the impulse response functions. These are the building blocks for the discussion of the historical decompositions of the data which will be focus our attention. A key motivation for this paper is the idea that different contemporaneous shocks may offset each other, as illustrated in Figure 4. Historical decompositions are the empirical counterpart of Figure 4. They show in each time period the contribution of each of the six identified shocks, as well as the initial conditions amd constant term, to each data series including whether they are offsetting or reinforcing each other.

5.1 Results for Aggregate Variables

The impulse responses for each of the three identification methods - structural Cholesky, structural sign restrictions and reduced form sign restriction - are shown in Figures 5, 6 and 7 and the historical decomposition of aggregate wages for all three identification methods is shown in Figure 9. We first describe the salient aspects of the impulse responses in section 5.1.1 which are then used in our interpretation of the historical decompositions in section 5.1.2.

5.1.1 Impulse Response Functions of Aggregate Variables

Impulse response functions map out the dynamic paths of the six variables in the VAR in response to a standard deviation innovation for each of the identified shocks.[13] This is done for each draw from the posterior distribution of the parameters and so results in a distribution of impulse responses. In our discussion below we focus on the median response from this distribution.

Figures 5, 6 and 7 plot the median impulse responses from the 10 VARs estimated for the 10 unskilled manual sectors in the dataset.[14] In Figure 5 the shocks are identified using the structural Cholesky ordering which imposes zero impact effects on the lower triangular elements of the 6×6 grid. In Figure 6 the shocks are identified using the structural sign restrictions described in section 4.2 and in Figure 7 the shocks are identified using the reduced form sign restrictions approach described in section 4.3. The sectoral variables play very little role in aggregate dynamics and so the top left 3×3 submatrices of Figures 5, 6 and 7 are close to those of an independent 3 dimensional VAR in the aggregate variables. The aggregate responses here are all very similar to each other. In Figures 5 and 7 they appear for the most part to be one thick line line. For the structural sign restrictions approach the median aggregate responses do differ across sectors for the first two aggregate shocks. In particular some of the impulses are near mirror images of the others. Thus in Figure 6 we have multiplied such impulses by -1 to demonstrate how close they are to each other. Nevertheless there are minor, though noticeable, differences between some sectors. This is because the aggregate sign restrictions influence the sectoral sign restrictions, though the determinant of S^{-1} in equation (6), and this in turn influences the acceptance probability of the draw. However qualitatively the aggregate impulses are very similar.

In each of Figures 5, 6 and 7 there is a shock in which aggregate migration has a negative association with aggregate wages. In Figure 5, this is the first identified shock, in Figure 6 this is the shock labelled the supply shock and Figure 7 this is the identified migration

[13]See the Appendix A.1, or Hamilton (1994), for description of impulse responses.

[14]The impulse responses for the other sectors are available on request.

shock. This is of key importance. It shows that there exists a decomposition of the data where one of the key components has negative association between immigration and wages. One, although not the only, interpretation of this shocks is an 'immigration shock'. The historical decompositions described in the section 5.1.2 show how important these shocks are in describing the underlying data. As we will show in some sectors this influence is considerable.

The top left 3×3 submatrices of Figures 5, 6 and 7 are similar to each other in many ways. As discussed, the first shock in the Cholesky ordering in Figures 5, looks similar to a supply shock in Figure 6, although with differences in aggregate hours responses, and to the immigration shock in Figures 7. In addition, the third shock in the Cholesky ordering in Figures 5 has positive impact on wages and immigrations, as in the case for the demand shocks in both Figures 6 and Figures 7.

We asked at the outset whether immigration could be thought of as an exogenous shock. The impulses responses here under all three identification scheme suggest that aggregate immigration at least in part is influenced by shocks that could be considered aggregate supply and aggregate demand. However in each case there is also a shock which could be thought of as an exogenous migration shock, the first shock in the Cholesky case in Figure, the second shock in the reduced from sign restrictions case in Figure 7 and either the first or second shocks in the structural sign restrictions Figures 5, 6. The difference in the labelling of the underlying structural shocks demonstrates, as in Uhlig (2005), that macroeconomic theory by itself is not sufficient to tightly pin down the effects of any particular shock. Thus if one is wanting a definitive result on the percentage of wages 'caused' by migration then you will need to find more information and/or impose tighter restrictions. As described in section 4.2, the structural approach of Baumeister and Hamilton (2015, 2019) is designed to be able to incorporate such additional information, although doing so for 35 different sectors would be a large task.

Figure 5: Impulse Responses For the Unskilled Manual Sectors using the Cholesky Factor

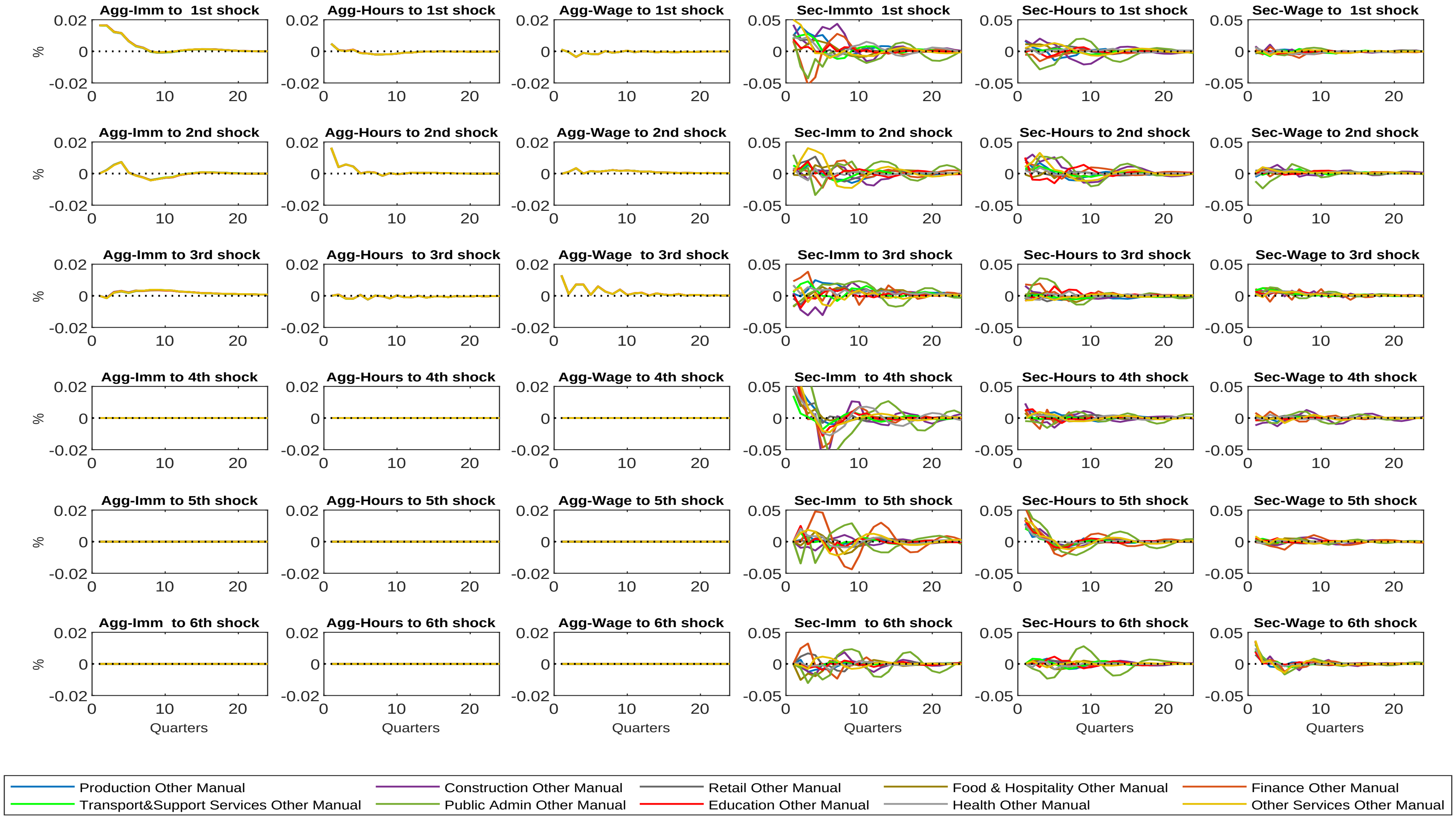


Figure 5 plots the median impulse responses for the unskilled manual sectors using the Cholesky factorization. Each row are the responses to a particular shocks (row 1 is the first shock, row 2 is the second shock, ...) and each column is the responses of a particlar variable (column1 is aggregate migration, column 2 is aggregate hours, ...). The responses of all 10 sectors are plotted together. The sectoral variables play very little role in aggregate dynamics and so the aggregate responses are all very similar to each and appear for the most part to be a thick line line.

Figure 6: Impulse Responses For Unskilled Manual Sectors using the Structural Sign Restriction Approach

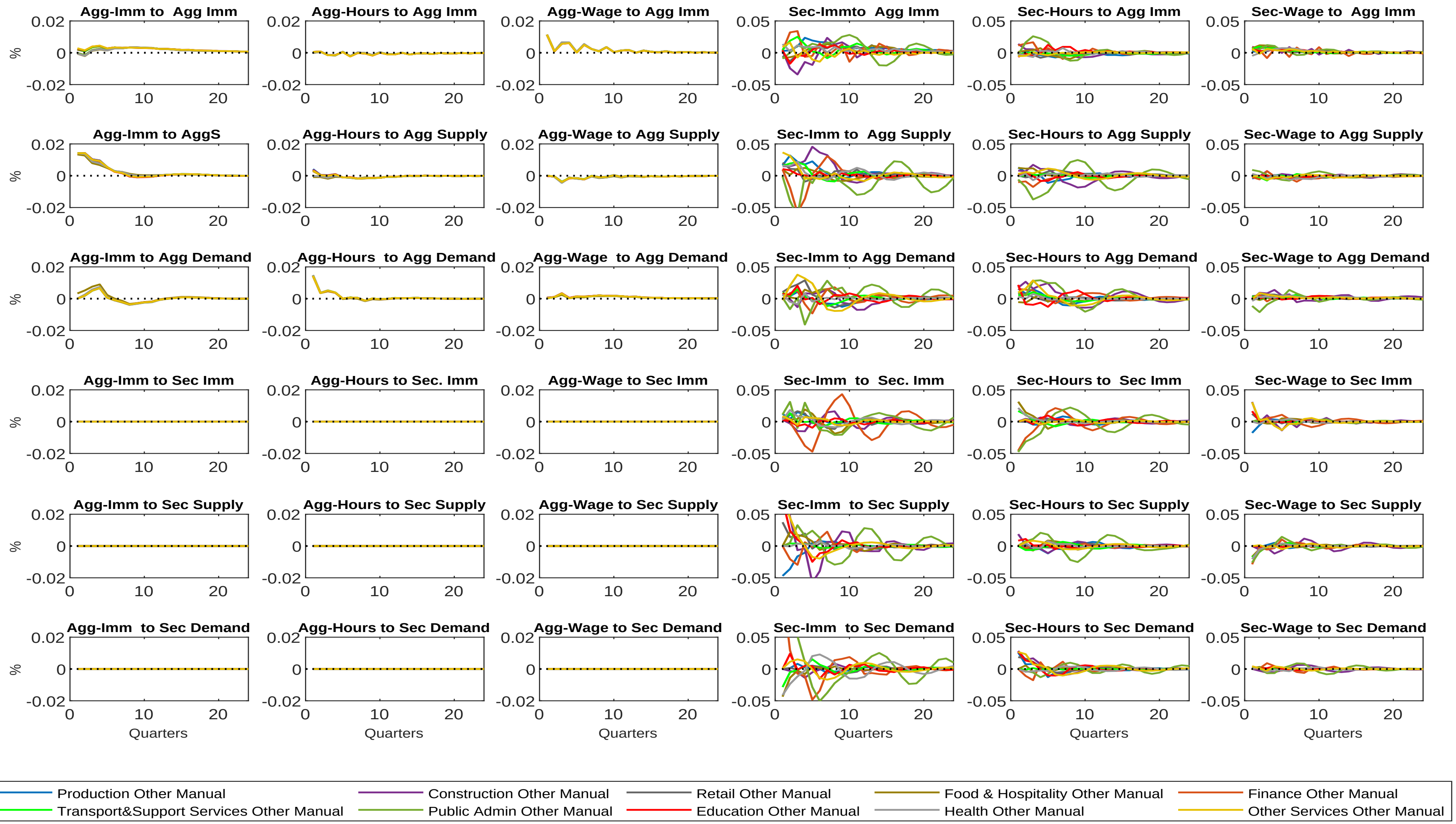


Figure 6 plots the median impulse responses for the unskilled manual sectors using the Baumeister and Hamilton sign restriction approach of section 4.2. Each row are the responses to a particular shocks (row 1 is the first identified shocks, aggregate migration, row 2 is the second identified shock, aggregate supply,...) and each column is the responses of a particlar variable (column1 is aggregate migration, column 2 is aggregate hours, ...).

Figure 7: Impulse Responses For Unskilled Manual Sectors - using the Reduced Form Sign Restrictions Approach

Figure 7 plots the median impulse responses for the unskilled manual sectors using the penalty function approach of section 4.3. Each row are the responses to a particular shocks (row 1 is the first identified shock, aggregate demand, row 2 is the second identified shock, aggregate supply ...) and each column is the responses of a particlar variable (column1 is aggregate migration, column 2 is aggregate hours, ...). The responses of all 10 sectors are plotted together. The sectoral variables play very little role in aggregate dynamics and so the aggregate responses are all very similar to each and appear for the most part to be a thick line line.

5.1.2 Historical Decomposition of Aggregate Wages

Given the estimated parameters, the time series of the data used in the VAR can be decomposed into the contributions from each fundamental shock by iterating on the estimated VAR.[15] These contributions depend on raised powers of the matrix of β parameters and will therefore differ across sectors. The contribution of each shock is highly related to its impulse response function, as described in e.g. Hamilton (1994). Therefore the similarity in the shape of the implulse response functions across the different identification methods and the differences in the labelling of the fundamental shocks that we saw in section 5.1.1, will also be present in the historical decompositions. We first discuss the aggregate series and then proceed to the sectoral series.

Figure 8 displays the confidence bands for the historical decomposition of the aggregate wage series using the structural sign restriction approach described in section 4.2.[16] As explained above sectoral shocks play very little role in the variation of aggregate sectors and so the series are explained by the three aggregate shocks as well as the constant and initial conditions. Figure 8 plots the region between the 16% and 84% quantiles from the posterior distribution for the contribution of each shock to the aggregate wage growth series. This shows that the growth rate of wages share is mostly determined by the first shock in equation (6) which is labelled an aggregate migration shock and the constant term.[17] The initial conditions play some role at the beginning of the sample and the third shock, labeled aggregate demand, plays a role in the period following the financial crisis of 2008. It is worth re-emphasising here that the supply shock in this identification scheme could be reinterpreted as a migration shock and vice versa.

These findings are illustrated in Figure 9 which shows the historical decomposition for the aggregate wage series for all three identification methods in a stacked var chart, using the median responses.[18] In each panel of Figure 9 we have coloured in red the contribution of the shock which most resembles an aggregate demand shock. We have used blue to color the contribution of the shock where there is a negative association between migration and wages. Figure 9b tell much the same story as Figure 8. The growth rate of wages share is mostly determined by the 'aggregate migration' shock and the constant term, although 'aggregate demand' plays an important role in the period following the financial crisis of 2008. The Cholesky identification scheme in Figure 9a tells a similar story although with a greater role for the 'demand' where, as suggested in section 5.1.1, the first ordered shock is labeled a migration shock and the third ordered shock is the demand shock. Figure 9c for the penalty function approach, gives an even larger role for demand, as would be expected from its identification method and assumptions.

[15] This is briefly described in the Appendix A.1, where the historical decomposition formula is given by equation (A3).

[16] Decompositions based on the other methods are available on request.

[17] See Bergholt *et al.* (2024) for the importance of modeling the uncertainty in the deterministic terms, although we do not place additional assumptions on the constant's prior in our analysis.

[18] The sum of the median contributions do not necessarily sum to the original but as Figure 9 shows the match is quite close.

Thus the different panels of Figure 9 give different plausible accounts of the history of aggregate wage growth in the UK economy from 2003-2019. Views will differ as to which restrictions are most reasonable to place on the data. However what is important is that the blue colored shock has a significant role in all three panels. Notably negative around in the time of the Brexit referendum (June 2016) and then a positive contribution afterwards. This is intuitive. Note that a positive contribution of the blue shocks means that this shock, is lower than expected and so its effects on wages are less negative than expected.

Figure 8: Historical Decomposition of Aggregate Wage Growth

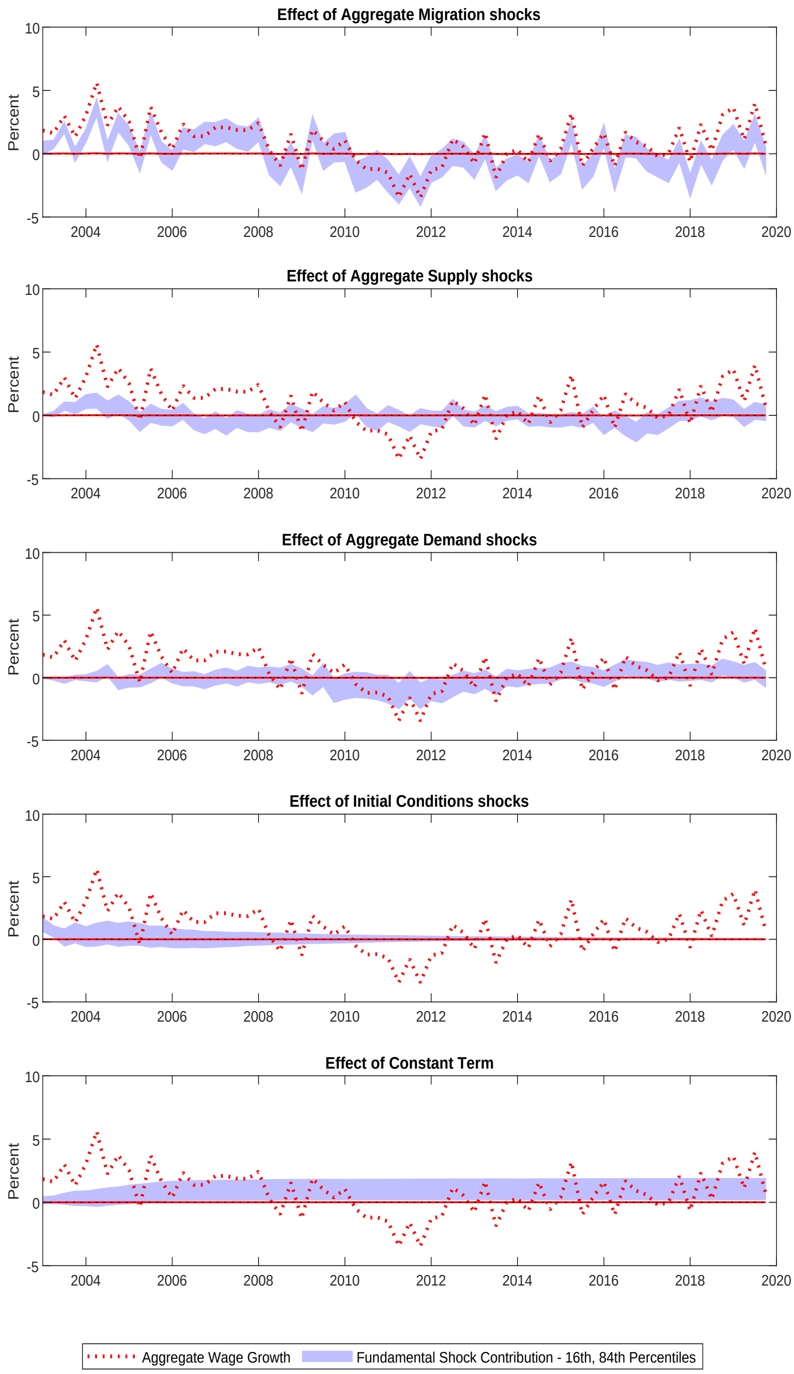
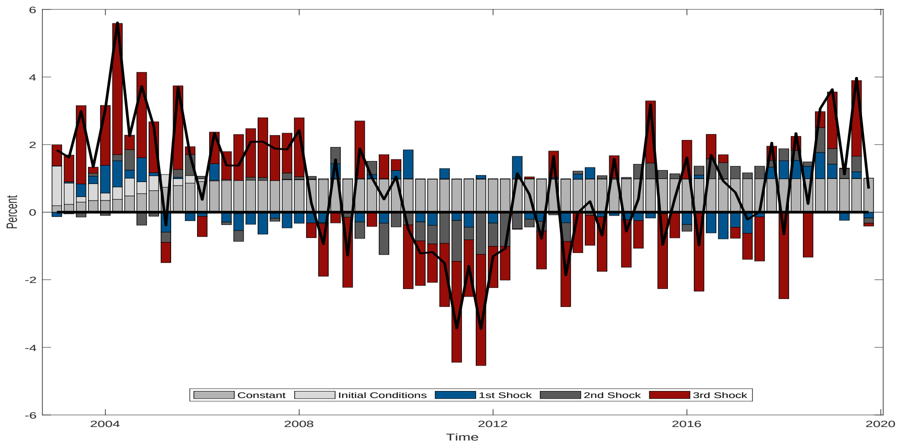
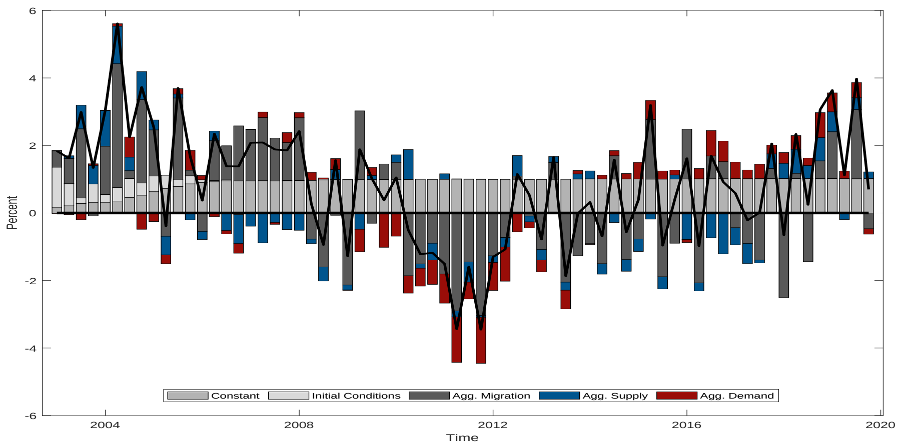


Figure 8 plots the area between the 16% and 84% quantiles from the posterior distribution for the contribution of each shock and the initial conditions and constant term, to the dynamics of Aggregate wage growth in each period, using the structural sign restriction approach of section 4.2. This is decomposition is taken from the unskilled manual transport workers sector

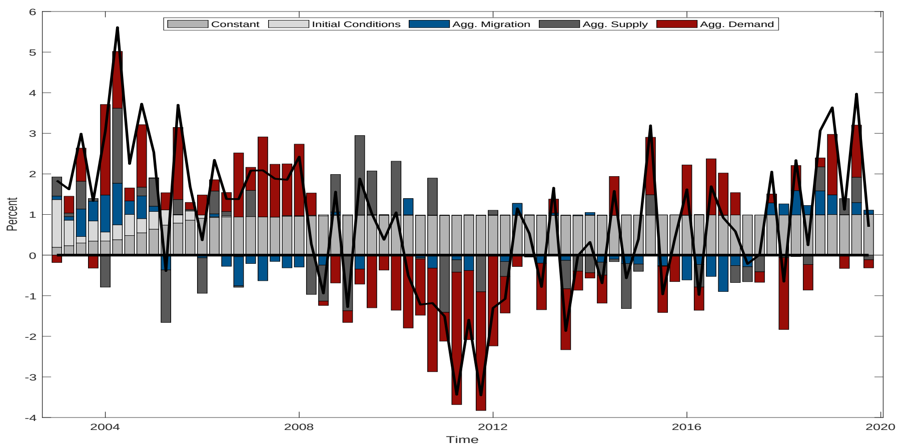
Figure 9: Historical decomposition of aggregate wage growth using the three identification methods



(a) Aggregate Wages - Structural Cholesky approach



(b) Aggregate Wages - Structural sign restrictions approach



(c) Aggregate wages -Reduced form sign restrictions approach

Figure 9 plots the median estimate of the historical contribution to aggregate wage growth of the aggregate identified shocks, the initial conditions and the constant term, using the three identification techniques described in section 4. Panel a) plots the Cholesky case, Panel b) plots the structural sign rtestrictions case which is taken from the unskilled manual transport workers sector's results, and Panel c) plots the penalty function each. In each panel the black line plots data for aggregate wage growth.

5.2 Results for Sectoral Variables

The last three columns of Figures 5, 6 and 7 plot the median impulses of the sectoral variables. The first three rows show the responses to aggregate shocks and the last three rows show the responses to sectoral shocks. The responses of sectoral variables to sectoral shocks are displayed in the lower right 3×3 submatrix of Figures 5, 6 and 7. In the Cholesky case Figure 5 shows that the strongest responses are on the diagonals, which naturally leads one to think of the fourth ordered shock as a sectoral migration shock and the fifth and sixth ordered shocks as sectoral hours and sectoral wage shocks respectively. In the structural sign restrictions case Figure 6 shows that the responses to the sectoral shocks are much more varied than the responses to the aggregate shocks, in particular, the responses of hours and wages are negative in some sectors and positive in others. This is also true in the penalty function approach in Figure 7 where sectoral demand shocks are in some sectors associated with negative responses of sectoral migration, although here the responses for many sectors are better described as being cyclical around zero.

There are two broad features in these figures which immediately stand out. Firstly that, as one would expect, the responses of sectoral variables tend to have a higher variance than the aggregate variables. The second is that the aggregate shocks explain a lot of the variation in the sectoral labor market variables. This is intuitive. It is what one would expect macroeconomic shocks should do.

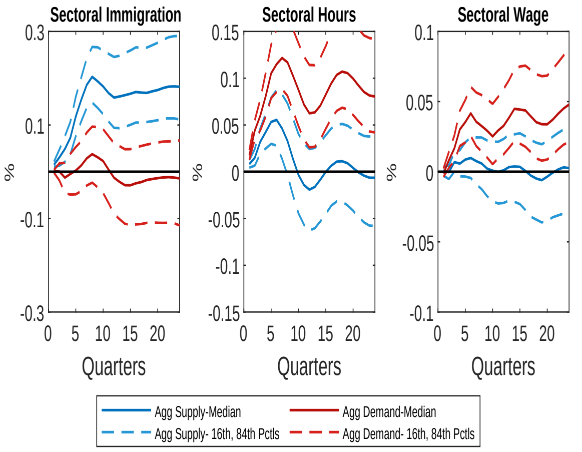
Sectoral shocks nevertheless still account for a lot of variation in sectoral variables. The heterogeneity across sectors is also apparent and is also in line with expectations. Some sectors appear pro-cyclical, others counter-cyclical. Similarly some sectors experience an increase in sectoral migration share in responses to an aggregate migration shock and other a decrease. However, while intuitive, this makes the interpretation of the sectoral decompositions less straightforward because one cannot assume, for example, that an aggregate supply shock should be colored blue in the historical decompositions figures as in section 5.1, because in some sectors aggregate supply shocks may not have a negative association between sectoral migration and sectoral wages.

Our approach therefore is to look at the impulse responses of two example sectors, which have experienced large inflows of migrants over the sample period; the unskilled Food and Hospitality and Construction sectors, these are sectors where one might expect to find a large response of wage growth to sectoral immigration. We highlight the impulses in these sectors with a negative association between sectoral migration and sectoral wages. This will then allow us to see the contribution of these shocks in the historical decomposition of wage growth in section 5.2.1.

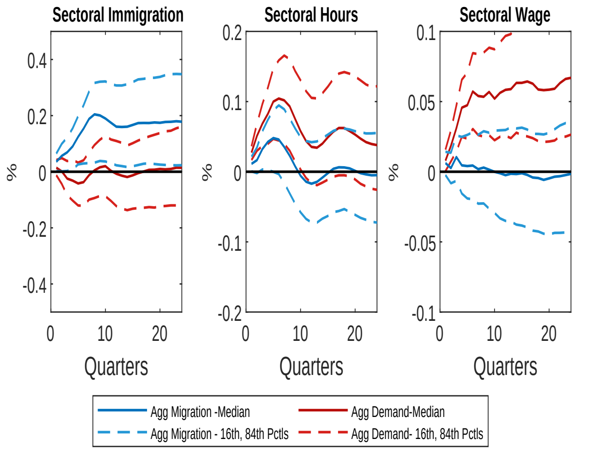
The Unskilled Construction Sector

Figure 10 plots median cumulative impulse responses, i.e. the levels, - solid line - as well as the 16th and 84th percentiles - dashed lines- of the sectoral variables to two aggregate and sectoral shocks identified by two different approaches. We choose to plot the cumulative

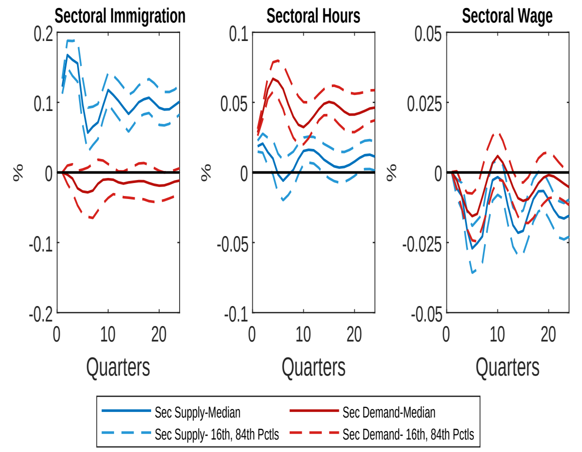
Figure 10: Sectoral Impulse Responses Responses in the Unskilled Construction Sector



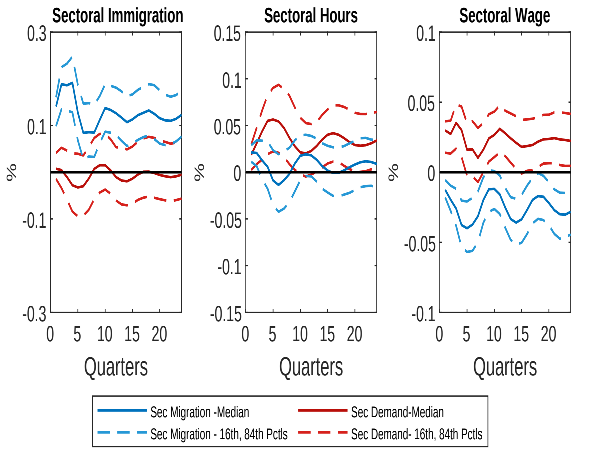
(a) Aggregate Shocks: Structural Sign Restrictions



(b) Aggregate Shocks: Reduced Form Sign Restrictions



(c) Sectoral Shocks: Structural Sign Restrictions

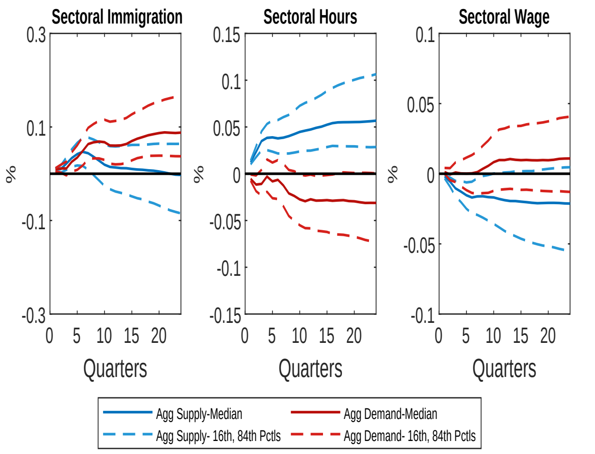


(d) Sectoral Shocks: Reduced Form Sign Restrictions

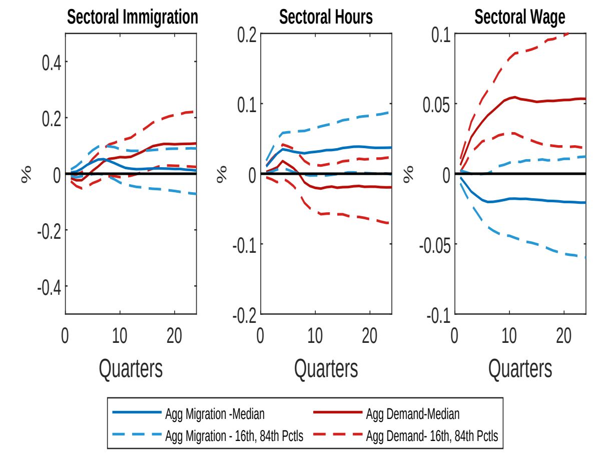
responses as the non-cumulative responses sometimes oscillate which makes the longer run effects difficult to make out. In contrast the cumulative responses are clear. The responses to aggregate shocks are displayed in panels a) and b) and those to sectoral shocks in panels c) and d). In panel a) the shocks are identified using the structural sign restrictions, with the identified shocks being aggregate supply -colored blue - and aggregate demand - colored red. In this case the impulses do not display a large correlation between sectoral migration and wages. Sectoral migration responds positively to the aggregate supply shocks but the sectoral wage response looks centered around zero. For the aggregate demand shock, sectoral wages respond positively but sectoral migration seems centred around zero. The second panel - panel b) - the shocks are identified using the reduced form sign restrictions approach and the identified shocks are aggregate migration -colored blue - and aggregate demand - colored red. These responses are very similar to those in panel a) and so in this case too aggregate shocks are not shocks with a large correlation between sectoral migration and wages.

However for the sectoral impulse responses there are shocks with strong negative correlations between the response of sectoral migration and sectoral wage. For the structural sign

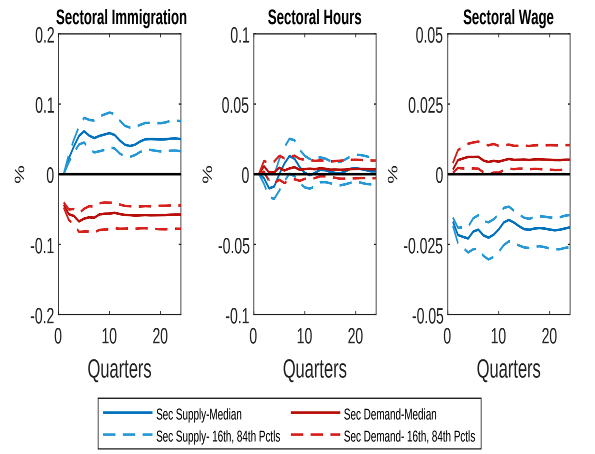
Figure 11: Sectoral Impulse Responses Responses in the Unskilled Food and Hospitality Sector



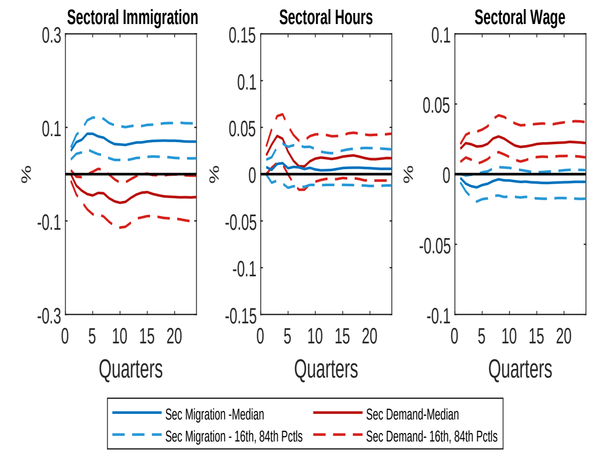
(a) Aggregate Shocks: Structural Sign Restrictions



(b) Aggregate Shocks: Reduced Form Sign Restrictions



(c) Sectoral Shocks: Structural Sign Restrictions



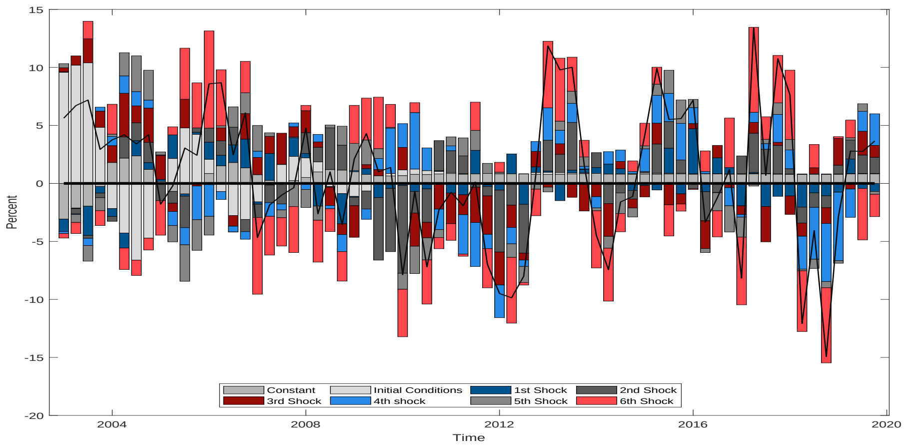
(d) Sectoral Shocks: Reduced Form Sign Restrictions

restrictions identification method in panel c) this is the sectoral supply shock, and in panel d) for the reduced form sign restrictions this is the sectoral migration shock. Therefore when looking at the historical decomposition of wage growth in the unskilled construction sector in section 5.2.1, these are the shocks to look out for. As we will see they play a significant part in both identification approaches.

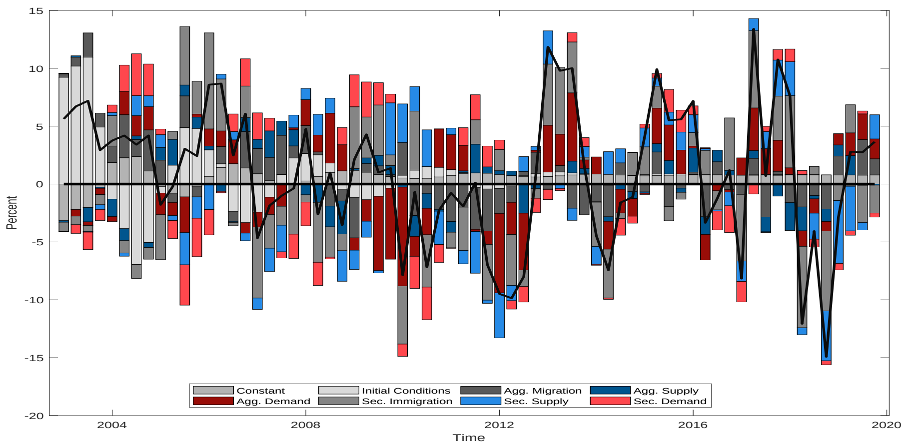
The Unskilled Food and Hospitality Sector

Figure 11 has the same structure as Figure 10 but plots the responses for the unskilled Food and Hospitality sector. In contrast to the unskilled Construction sector, there are both aggregate and sectoral shocks with a negative association between sectoral migration and sectoral wages. For the structural sign restrictions approach, this is the aggregate supply shock - in panel a) - and the sectoral supply shock -panel c). For the reduced form sign restrictions approach, this is the aggregate migration shock - panel b) and the sectoral migration shock - panel d). These shocks play a significant role in the historical decomposition of wage growth in the unskilled Food and hospitality sectors as we discuss in section 5.2.1 below.

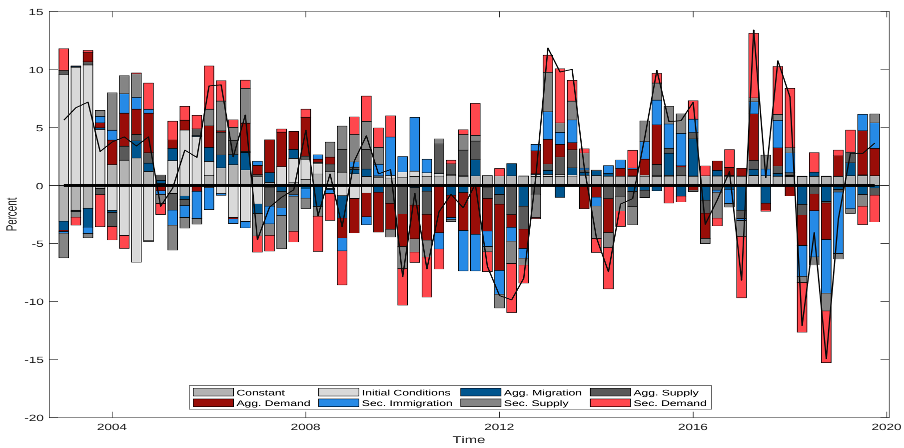
Figure 12: Historical Decomposition of Wages in Unskilled Construction Sector



(a) Structural Cholesky



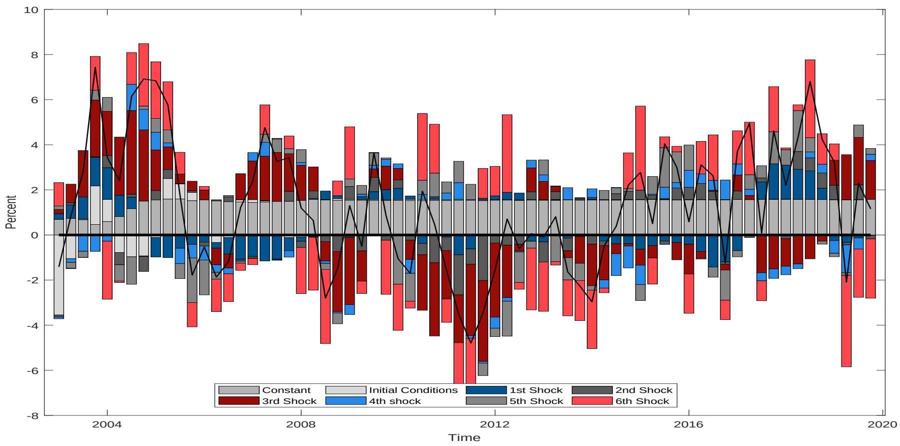
(b) Structural Sign Restrictions



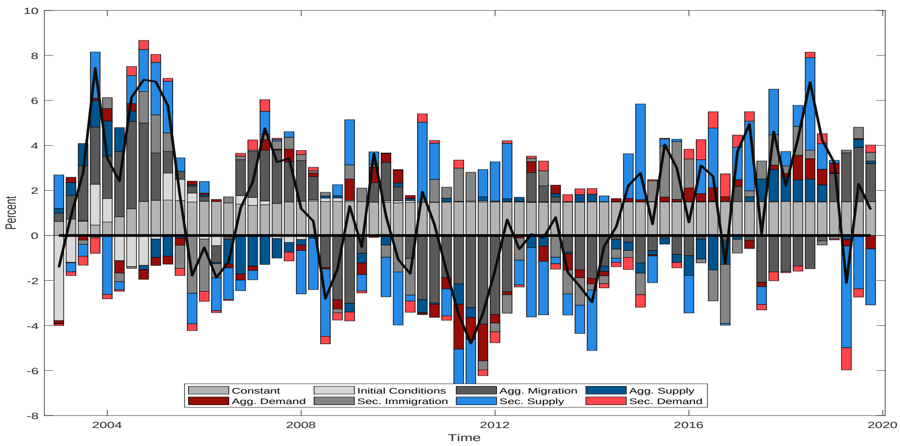
(c) Reduced Form Sign Restrictions

Figure 12 plots the median estimate of the historical contribution to wage growth in the unskilled manual construction sector of the identified aggregate and sectoral shocks, the initial conditions and the constant term, using the three identification techniques described in section 4. Panel a) plots the Cholesky case, Panel b) plots the Baumeister and Hamilton case, and Panel c) plots the penalty function each. In each panel the black line plots data for sectoral wage growth.

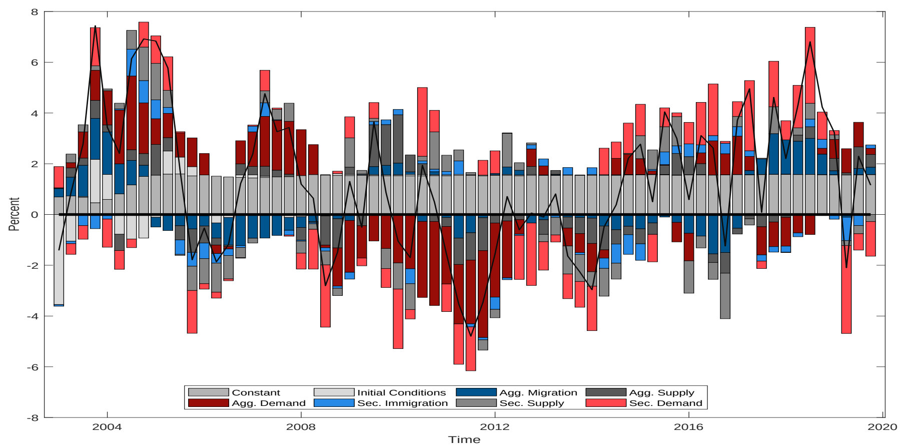
Figure 13: Historical Decomposition of Wages in Unskilled Food and Hospitality Sector



(a) Structural Cholesky



(b) Structural Sign Restrictions



(c) Reduced Form Sign Restrictions

Figure 13 plots the median estimate of the historical contribution to wage growth in the unskilled manual Food and Hospitality sector of the identified aggregate and sectoral shocks, the initial conditions and the constant term, using the three identification techniques described in section 4. Panel a) plots the Cholesky case, Panel b) plots the Baumeisterand Hamilton case, and Panel c) plots the penalty function each. In each panel the black line plots data for sectoral wage growth.

5.2.1 Sectoral Historical Decompositions

The historical decompositions at the sectoral level have eight different contributing factors, as now the three sectoral shocks play a significant role in the dynamics.

Figure 12 for the wage growth in the unskilled construction sector shows a similar narrative across all three identification approaches. All three panels show aggregate demand shocks, colored red, having a very significant contribution to wage growth in this sector, with a large negative effect following the 2008 financial crisis and wages growth remaining low until 2013. This is intuitive. Section **??** showed that the shock with a negative correlation between migration and wages in the structural sign restrictions case is the sectoral supply shock and is the sectoral migration shock in the reduced form sign restrictions case. These are colored light blue in panels b) and c) of Figure 12 respectively. The light blue color is very evident through all three panels being particularly prominent around 2012 and 2018.

Figure 13 for the decomposition of the wage growth in the unskilled Food and Hospitality sector, the impulse responses above showed that both aggregate and sectoral supply shocks in the structural sign restrictions case, and both the aggregate and sectoral migration shocks in the reduced form sign restrictions case had a negative association between sectoral migration and sectoral wages. These shocks are colored dark blue and light blue in Figure 13 and evidently play a very significant role in wage variation. This is especially so in panel b) for the structural sign restriction case where blue shocks appear to be a dominant force. In contrast identified demand shocks appear dominant in panels a) and c) but the light and dark blue shocks are nevertheless very significant components.

We have shown how the contribution of aggregate and sectoral demand, supply and migration shocks to wage growth differs across sectors and across identification methods. However Figures 12 and 13 also illustrate a lot of similarities. In each approach there are shocks where migration and wages are positively associated and shocks where migration and wages are negatively associated. Macroeconomic theory would suggest that demand is the driver of the positive association. One interpretation of the negative association is that migration is causing, directly or indirectly, a reduction in wages. In all three identification approaches this shock plays a significant role in determining wage growth, although its effect in the data is often obscured by the other contemporaneous shocks.

We have focused here on the historical decompositions of wage growth in the unskilled Construction, and Food and Hospitality sectors. In other sectors the effects are less strong but as the aggregate wage decompositions in Figure 9 show they are important in the economy as a whole.[19]

[19]Decompositions based on the other methods are available on request.

6 Conclusion

We asked at the outset whether immigration could be thought of as an exogenous shock, whether immigration can be plausibly associated with adverse labor market effects, and if so, whether these effects are similar across different sectors of the economy. We have applied established methods of multiple time series analysis to decompose a time series of UK labor market variables into 'fundamental' constituent parts. As argued by Uhlig (2005) and Baumeister and Hamilton (2015), economic theory does not provide sufficient information to definitively identify these fundamental parts. We have therefore applied three different, though plausible, approaches for characterizing them, across 35 different sectors of the UK labor market. We have found, under all three identification schemes, that aggregate immigration is, in part, determined by shocks that could be considered aggregate supply and aggregate demand shocks. Thus what have previously been considered the effects of exogenous shocks to immigration may in fact be the result of multiple underlying causes.

In answer to the question of whether there are adverse labor market effects of immigration, in each identification approach we have found that there are shocks where immigration and wages are positively associated, and shocks where immigration and wages are negatively associated. A natural interpretation for the positive association at the aggregate level is a macroeconomic demand shock. One interpretation of the negative association is that migration is causing, directly or indirectly, a reduction in wages. We have shown that this shock plays a significant role in the determination of wage growth in all identification methods, and that the size of its effect can vary considerably across sectors.

The literature on the labor market effects of immigration has frequently noted that its results are subject to the proviso that they are abstracting away from the effects of demand shocks and sectoral heterogeneities. Our approach has shown that this proviso is indeed justified and that the literature may be missing significant adverse effects of immigration. Our conclusion therefore echoes that of Uhlig (2005) regarding monetary policy. It is that there are good reasons for being uncertain about the labor market effects of immigration.

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A Appendix

A.1 Formulas used in Calculation

The formulas we use to calculate the impulse response functions and historical decompositions follow those of the literature see e.g. Hamilton (1994), Uhlig (2005) and Baumeister and Hamilton (2018).

The reduced form VAR of n dimensions and p lags can be written

$$\underset{(n\times1)}{y_t} = \underset{(n\times1)}{C} + \underset{(n\times n)}{B_1}\underset{(n\times1)}{y_{t-1}} + \underset{(n\times n)}{B_2}\underset{(n\times1)}{y_{t-2}} + \dots \underset{(n\times n)}{B_p}\underset{(n\times1)}{y_{t-p}} + \underset{(n\times1)}{u_t} \qquad u_t \sim \mathcal{N}(0, \underset{(n\times n)}{\Sigma}) \tag{A1}$$

This can be stacked and written as 1 lag VAR,

$$\underset{(np\times1)}{\widehat{Y_t}} = \underset{(np\times1)}{\widehat{C}} + \underset{(np\times np)}{F}\underset{(np\times1)}{\widehat{Y_{t-1}}} + \underset{(np\times1)}{u_t} \tag{A2}$$

where

$$\widehat{Y_t} = \begin{bmatrix} y_t \\ \vdots \\ y_{t-p+1} \end{bmatrix} \widehat{c} = \begin{bmatrix} C_{n\times1} \\ 0_{(n(p-1)\times1)} \end{bmatrix} \quad F = \begin{bmatrix} B_1 & \dots & B_p \\ I_{n(p-1)} & & 0_{n(p-1),n} \end{bmatrix} \widehat{u_t} = \begin{bmatrix} u_t \\ 0_{n(p-1),1} \end{bmatrix}$$

Iteration of equation (A2) forward implies that the observation $\widehat{Y_{t+s}}$ in period $t+s$ can be decomposed into three contributions, the initial conditions,the constant terms and the innovations in the previous s periods , i.e.

$$\widehat{Y_{t+s}} = \underbrace{F^s\widehat{Y_t}}_{\text{Initial Conditions}} + \underbrace{F^{s-1}\widehat{c} + F^{s-2}\widehat{c} + \dots + \widehat{c}}_{\text{constant terms}} + \underbrace{F^{s-1}\widehat{u_{t+1}} + F^{s-2}\widehat{u_{t+2}} + \dots + \widehat{u_{t+s}}}_{\text{Innovations}} \tag{A3}$$

The historical decomposition and counterfactual exercizes are produced using equation (A3) where the contribution of the fundamental innovations use the formula $u_t = A\epsilon_t$ in equation (4.3), where $A = S^{-1}$ following equation (3) in the text.

Iterating backwards into infinite history, $\widehat{Y_t}$ can be expressed as an $MA(\infty)$ process

$$\begin{aligned} \widehat{Y_t} =& \widehat{c} + F\widehat{c} + F^2\widehat{c} + \dots + \widehat{u_t} + F\widehat{u_{t-1}} + F^2\widehat{u_{t-2}} + \dots + \\ \widehat{Y_t} =& \widehat{\mu} + \Psi(L)\widehat{u_t} \end{aligned} \tag{A4}$$

Where $\mu = (I_n - A_1 - \dots A_p)^{-1}c$ and $\widehat{\mu} = [\mu' \; 0_{1\times n(p-1)}]'$ and where $\Psi(L)$ is an $MA(\infty)$ process. These can be written

$$\begin{aligned} y_t =& \mu + u_t + \Psi_1 u_{t-1} + \Psi_2 u_{t-2} + \Psi_3 u_{t-2} + \\ \widehat{Y_t} =& \mu + \Psi(L)u_t \end{aligned} \tag{A5}$$

where Ψ_j is the upper left $n \times n$ clock of matrix F^j, following Hamilton (1994).

A.2 Formulas used in the Penalty Function Approach

For the penalty function we follow Uhlig (2005) and use the function, $f(x) = 100x$ if $x \geq 0$ and $f(x) = x$ if $x \leq 0$, where x is the impulse response of a variable to a specific shock. If s_j is the the standard error of variable j and $J_{S,+}$ is the index set of variables whose responses are restricted to be positive and $J_{S,-}$ the index set of variableswhose responses are restricted to be negative, then the sign restrictions are imposed by solving for a column of the A matrix associated with the relevant shock, a, so that $a = \text{argmin}_{a=\tilde{A}q, Rq=0}\Psi(a)$, where the criterion function $\Psi(a)$ is given by

$$\Psi(a) = \sum_{j\epsilon J_{S,+}} \sum_{k=0}^{p} f(-\frac{x_{ja}(k)}{s_j}) + \sum_{j\epsilon J_{S,-}} \sum_{k=0}^{p} f(\frac{x_{ja}(k)}{s_j})$$

where p is the number of periods after impact that the sign restriction are imposed and $x_{ja}(k)$ is the imp[ulse response of variable j to impulse vector a, at time horison, k. Computationally, we implement this minimization, using MATLAB fmincon function.

The impulse response functions are also derived from equation (A3). The fundamental innovations, ϵ_t are related to the errors from the VAR, u_t by the matrix A so that $u_t = A\epsilon_t$. Equation (A3) shows that the impacts of the VAR innovations in period t are given by first m elements of $F^{s-1}\widehat{u_{t+1}}$. The impulse responses of the fundamental shocks are thus linear combinations of these impulses via the formula $\epsilon_t = A^{-1}u_t$.

The variance covariance matrix in any period is

$$E[u_t u_t'] = A\epsilon_t\epsilon_t' A = a_1 a_1' + \cdots + a_m a_m' \tag{A6}$$

where a_i is the i'th column of matrix A, which is equation (11.5.5) of Hamilton (1994) where we have normalized the variance of the fundamental shocks to 1. Thus the variance decomposition at any period is the share of each shock's squared impulses responses in the sum of squared impulses responses in that period

A.2 Cholesky Factorization - Structural Interpretation

Cholesky factorization of the reduced form variance-covariance matrix, Σ, also has a structural interpretation. Since $u_t = L\epsilon_t$ premultiplying the reduced form VAR, equation (4.3), by the Cholesky factor L^{-1} gives

$$L^{-1}y_t = L^{-1}\tilde{C} + L^{-1}\tilde{B}_1 y_{t-1} + L^{-1}\tilde{B}_2 y_{t-2} + \ldots L^{-1}\tilde{B}_p y_{t-p} + \epsilon_t \tag{A7}$$

Since L is lower triangular then L^{-1} is also lower triangular and equation (A7) has the form

$$\begin{bmatrix} c_{11} & 0 & 0 & 0 & 0 & 0 \\ c_{21} & c_{22} & 0 & 0 & 0 & 0 \\ c_{31} & c_{32} & c_{33} & 0 & 0 & 0 \\ c_{41} & c_{42} & c_{43} & c_{44} & 0 & 0 \\ c_{51} & c_{52} & c_{53} & c_{54} & c_{55} & 0 \\ c_{61} & c_{62} & c_{63} & c_{64} & c_{65} & c_{66} \end{bmatrix} \begin{bmatrix} \text{Aggregate Migration Share} \\ \text{Aggregate Hours Worked} \\ \text{Aggregate Native Wage} \\ \text{Sectoral Migration Share} \\ \text{Sectoral Hours Worked} \\ \text{Sectoral Native Wage} \end{bmatrix} = L^{-1}C + \text{Lagged terms} + \begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_3 \\ \epsilon_4 \\ \epsilon_5 \\ \epsilon_6 \end{bmatrix} \tag{A8}$$

Equation (A8) implies that the variable ordered first in the VAR - aggregate migration share in this case - is only a function of lagged values of the other variables and $\epsilon_{1,t}$. Thus the order that the variables are placed in the VAR has a very significant impact on the interpretation and properties of each shock. In equation (A8), a natural interpretation of the first shock is as an exogenous aggregate migration shock. Similarly, the variable ordered second - Aggregate Hours Worked - is only a function of lagged values of the other variables and the contemporaneous value of aggregate migration (which is a function of $\epsilon_{1,t}$) and also of $\epsilon_{2,t}$. One interpretation of the second shock is therefore as a shock to aggregate hours net of the effects of the shock to aggregate migration. The same logic can be applied to the other shocks so that for the variable ordered last - the sectoral real wage in this case-, depends on all the current values of all variables and is therefore dependent on all contemporanous shocks $\epsilon_{1,t}$, $\epsilon_{2,t}$, $\epsilon_{3,t}$, $\epsilon_{4,t}$, $\epsilon_{5,t}$ and $\epsilon_{6,t}$. This has the same form as equation (1) above. However the property of the Cholesky factorization that only one shock's impulse responses, $\epsilon_{1,t}$, affects all the variables contemporaneously is often seen as a very strong restriction.