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Berliant, Marcus and Tabuchi, Takatoshi

Washington University in St. Louis, Chuo University

2 December 2024

Online at https://mpra.ub.uni-muenchen.de/122854/
MPRA Paper No. 122854, posted 03 Dec 2024 09:22 UTC

Market Size, Trade, and Productivity Reconsidered: Poverty Traps and the Home Market Effect*

Marcus Berliant† and Takatoshi Tabuchi‡

December 1, 2024

Abstract

To investigate questions related to migration and trade, a model of regional or international development is created by altering Melitz and Ottaviano (2008) to include a labor market. The model is then applied to analyze poverty traps and the home market effect. We find that in the spatial economics context of migration but no trade, poverty can persist unless population in one region of many is pushed past a threshold. Then growth commences. In the context of trade but no migration, the home market effect holds for a range of parameters, similar to previous literature. However, unlike previous literature, we find that if populations in the countries are large, the home market effect can be reversed.

JEL Codes: F12, R11

Keywords: Monopolistic competition; Poverty trap; Home market effect; Labor market clearing

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*The authors thank two anonymous referees, Costas Arkolakis, Carsten Eckel, Theodore Papageorgiou, Kohei Takeda, and Pierre-Olivier Weill for helpful comments, but retain responsibility for any errors. The first author thanks the Whiteley Center at the University of Washington for their kind hospitality during the research for this paper.

†Corresponding Author. Department of Economics, Washington University, MSC 1208-228-308, 1 Brookings Drive, St. Louis, MO 63130-4899 USA. Fax: (314) 935-4156. E-mail: berliant@wustl.edu

‡Faculty of Global Management, Chuo University, 742-1, Higashi-Nakano, Hachioji 192-0393, Japan. Fax: +81-42-674-3716. Tel: +81-42-674-3362. E-mail: ttabuchi@tamacc.chuo-u.ac.jp

1 Introduction

Can an insufficient labor supply cause a poverty trap?

We build on Ottaviano et al. (2002) and Melitz and Ottaviano (2008). Our simplest economy comprises one country and involves two sectors, a manufacturing sector that produces a differentiated product under monopolistic competition, and a traditional sector that produces under constant returns to scale and perfect competition. Our twist on Melitz and Ottaviano (2008) is that we introduce a simple labor market clearing condition, absent there, to close the model. Surprisingly, this twist yields equilibrium behavior, including firm selection, that is different from and more complex than the earlier models even when there is only one location.

The model is then extended to multiple locations in order to examine the following applications.

We address the issue of poverty traps in a version of this model where workers can migrate into a city from the hinterlands or other cities if it increases their utility. In contrast, most models of poverty traps, as surveyed in Azariadis (1996) for example, are based on aspatial models of growth.

We find that if the population is small, there is an equilibrium with an active traditional sector but no manufacturing sector. Utility of consumers is relatively small. But if labor supply is pushed upward past a threshold, for example by subsidizing in-migration, utility in the region increases and more workers migrate into it. The manufacturing sector initiates production and growth. With an even larger population, wage increases and the traditional sector ceases production, so there is a big manufacturing sector in the city but no traditional sector. Eventually, wage is reduced to its original level, and traditional production appears in conjunction with manufacturing. Above the lowest population threshold, indirect utility is monotonically increasing in population, which is the same as labor supply. So workers will continually migrate into the region once population is pushed past the lowest threshold. (In contrast, Melitz and Ottaviano (2008) only consider the case where the traditional good and the manufactured good are always produced in the equilibrium of their basic model.)

This represents a poverty trap in the following sense. Intervention by an entity such as a government is necessary to improve welfare if the agents are myopic in that they take the utility level in a region as given. If they are forward looking, they will not know which region or regions will have expanding population, so they will wait to migrate and the economy will experience a hold up problem. Either way, a region can become caught in a low utility poverty trap that can be avoided only by encouraging immigration past a threshold.

Next we turn to the home market effect (HME) for two countries in our context. The HME states in our model that the larger country should have a larger ratio of goods or firms to population. The model has two countries where trade but no migration is possible. We find that the HME holds for some parameter values where the populations in the two countries are relatively balanced, whereas it does not hold for parameter values where the populations are unbalanced. This is different from standard models of trade with increasing returns and trade cost, such as Krugman (1980), where the HME always holds. And as detailed in Medin (2017, p. 304), "The empirical evidence of the HME is also ambiguous."

There is nothing wrong with Melitz and Ottaviano (2008) given their assumptions. But it's natural to ask what the results look like when an explicit labor market is added and the assumptions are weakened. Equilibrium behavior is more complex.

Finally, in this entire class of models (including ours), notice that it is possible to reinterpret the traditional sector as a service sector with a homogeneous good and a constant returns to scale production technology, provided that it is not traded.

The outline of the remainder of the paper is as follows. In Section 2 we provide our basic model, altering the Melitz and Ottaviano (2008) model to account for a labor market. In Section 3 we provide our applications to poverty traps and the HME. Each requires that we modify our basic model slightly. Section 4 gives our conclusions and suggestions for future research. Appendix A contains a discussion of our basic model with no endowment of homogeneous traditional good. Appendices B and C contain comparative statics of interest beyond those derived in the main text. Appendix D contains the proof of a proposition. Appendix E contains a proof that HME always holds for an alternate definition of the HME.

2 The Closed Economy Model

2.1 Melitz and Ottaviano (2008)

We build on Melitz and Ottaviano (2008). The economy comprises one country and involves two sectors. The mass of consumers (or workers) is L. Each worker supplies exactly one unit of labor.

The preferences of a typical consumer are represented by the following utility function:

$$U = q_0 + \alpha \int_0^N q_i \, \mathrm{d}i - \frac{\gamma}{2} \int_0^N (q_i)^2 \, \mathrm{d}i - \frac{\eta}{2} \left(\int_0^N q_i \, \mathrm{d}i \right)^2, \tag{1}$$

where q_0 is the consumption of the homogeneous traditional good, q_i is the consumption of a differentiated manufacturing good of variety i, N is the mass of varieties, whereas $\alpha > 0$, $\gamma > 0$, and $\eta > 0$ are fixed utility parameters.[1] Each individual maximizes her utility subject to the budget constraint:

$$q_0 + \int_0^N p_i q_i \, \mathrm{d}i = \widehat{q}_0 + w, \tag{2}$$

where p_i represents the price of the differentiated manufactured good i, w is the wage of a consumer, and $\widehat{q}_0$ is an endowment of the homogeneous traditional good, which is chosen as the numéraire. The endowment is supposed to be sufficiently large so that the equilibrium consumption of the numéraire is positive for each worker. The purpose of this assumption is to avoid corner solutions to the consumer optimization problem, and will be relaxed in Appendix A.

The first-order condition to maximize individual utility subject to the budget yields market demand for each variety i of a manufactured good:

$$q_i L = \frac{\alpha L}{\gamma + \eta N} - \frac{L}{\gamma} p_i + \frac{\eta N L \overline{p}}{\gamma (\gamma + \eta N)}, \tag{3}$$

where

$$\overline{p} \equiv \frac{1}{N} \int_{i \in \Omega^*} p_i \, \mathrm{d}i$$

is the average price and Ω^* is the set of varieties with nonnegative demand $q_i \geq 0$.

[1] We cannot obtain analytical results if we alter the utility function, for example to a Stone-Geary subutility.

Variety i has nonnegative demand if and only if $q_i \geq 0$ in (3), or

$$p_i \leq \frac{\alpha\gamma + \eta N\overline{p}}{\gamma + \eta N} \equiv p_{\max} \tag{4}$$

holds. Because product differentiation ensures a one-to-one relation between firms and varieties, the mass of firms and varieties is the same and is endogenously determined in equilibrium by free entry and exit of firms. Due to *ex-post* symmetry between varieties, we drop subscript i hereafter.

Turning next to the production side of the model, firms in the numéraire or traditional sector produce a homogenous traditional good using labor under perfect competition and constant returns to scale. Units are chosen such that one unit of output requires one unit of labor. Assuming costless trade of the homogeneous traditional good, the equilibrium wage of workers is equal to 1. However, for later use we will retain notation w as the wage paid in the manufacturing sector, for example when traditional good is not produced.

A monopolistically competitive firm produces one variety of the differentiated good under a technology that requires a fixed cost followed by constant returns to scale. The production technology of any variety requires labor input c per unit and fixed labor input f_E following Krugman (1980). Each firm, after payment of their entry cost, draws their unit labor requirement c from a Pareto distribution

$$G(c;k) = \left(\frac{c}{c_M}\right)^k$$

with support $[0, c_M]$, where $k > 1$ is an exogenous parameter and $1/c_M$ is the lower productivity bound. Firms that cannot cover their *marginal* cost exit, whereas all other firms survive.

Let c_D represent the (endogenous) unit labor requirement of the type of firm that is indifferent between exiting and staying in the industry, namely the type of firm that earns exactly zero profit. All firms who draw a higher unit labor requirement exit. Calculation of equilibrium proceeds exactly as in Melitz and Ottaviano (2008).

The total operating profit of a firm is given by

$$\pi(N, c) = (p(c) - wc)\, q(c), \tag{5}$$

where $q(c) = q_i L$ is determined by (3).

The equilibrium operating profit of a firm with unit labor requirement c is

$$\pi(c) = \frac{w^2 L}{4\gamma} (c_D - c)^2 . \tag{6}$$

The variables whose equilibrium values are most important to us are c_D and N:

$$c_D^* = \left(\frac{\gamma\phi}{wL}\right)^{\frac{1}{k+2}}, \tag{7}$$

$$N^* = \frac{2(k+1)\gamma}{\eta} \frac{\alpha - wc_D^*}{wc_D^*}, \tag{8}$$

where $\phi \equiv 2(k+1)(k+2) c_M^k f_E$. Then, the indirect utility of a consumer is computed as

$$U^* = \widehat{q}_0 + w + \frac{1}{2\eta}(\alpha - wc_D^*)\left(\alpha - \frac{k+1}{k+2} wc_D^*\right). \tag{9}$$

Since free entry is assumed, in equilibrium the firms must have zero expected profit. This condition is

$$\int_0^{c_D} \pi(c) \; \mathrm{d}G(c) = wf_E. \tag{10}$$

Melitz and Ottaviano (2008) assume $c_M > c_D^*|_{w=1} = \sqrt{2(k+1)(k+2)\gamma f_E / wL}$, or

$$L > L_{\min} \equiv \frac{\gamma\phi}{c_M^{k+2}}. \tag{11}$$

So far, the derivations parallel those in Melitz and Ottaviano (2008).

2.2 Introduction of the Labor Market Clearing Condition

Melitz and Ottaviano (2008) use a fixed wage and thus, at least implicitly, a supply of labor that is infinitely elastic.

Plugging (7) into (8), the equilibrium number of firms is

$$N^* = \frac{2\gamma(k+1)}{\eta}\left[\frac{\alpha}{w}\left(\frac{wL}{\gamma\phi}\right)^{\frac{1}{k+2}} - 1\right] \tag{12}$$

if $L > L_0 \equiv \gamma\phi/\alpha^{k+2}$. Otherwise, only the traditional good is produced in the economy. Whereas the equilibrium number of active firms is N^*, the equilibrium number of entrants is given by $N_E^* = N^*/G(c_D^*)$. This differs from the number of active firms because some firms enter and then find that their draw of unit labor requirement is too high to produce. We assume that $L_0 > L_{\min}$, which is equivalent to $\alpha < c_M$. Then, only the traditional good is produced when $L_{\min} < L < L_0$.

Using (12) and (7), the aggregate demand for the labor in the manufacturing sector is computed as

$$L_m \equiv N_E^* \int_0^{c_D^*} cq(c)\,\mathrm{d}G(c) + N_E^* f_E = \frac{(w+k)\,L\left(\frac{\gamma\phi}{wL}\right)^{\frac{1}{k+2}}\left[\alpha - w\left(\frac{\gamma\phi}{wL}\right)^{\frac{1}{k+2}}\right]}{(k+2)\,\eta}, \quad (13)$$

which is positive if $L > L_0$, i.e., both L_m and N^* are positive if $L > L_0$.

When both traditional and manufactured goods are produced, the equilibrium wage is equal to 1. Since L is the total supply of labor in the economy, the equilibrium number of traditional workers is given by

$$L_a(L)|_{w=1} \equiv L - L_m$$

if it is positive. This condition does not appear in Melitz and Ottaviano (2008). Thus, we will be quite explicit below when we use it.

Solving $L_a(L)|_{w=1} = 0$ yields two solutions

$$\begin{aligned} L_1 &= \left[\frac{(k+1)\,\alpha - \sqrt{(k+1)\left[(k+1)\,\alpha^2 - 4(k+2)\eta\right]}}{2\,(k+2)\,\eta}\right]^{k+2}\gamma\phi, \\ L_2 &= \left[\frac{(k+1)\,\alpha + \sqrt{(k+1)\left[(k+1)\,\alpha^2 - 4(k+2)\eta\right]}}{2\,(k+2)\,\eta}\right]^{k+2}\gamma\phi, \end{aligned}$$

which are real if $\alpha \geq \overline{\alpha} \equiv 2\sqrt{(k+2)\,\eta/\,(k+1)}$. The traditional good is not produced in equilibrium if and only if $L_a(L)|_{w=1} \leq 0$, which holds if and only if $L \in [L_1, L_2]$ and $\alpha \geq \overline{\alpha}$. Otherwise, the traditional good is produced.[2] From (12), manufactured goods are produced in equilibrium if and only if $L > L_0$. Hence, we have the following proposition concerning a comparative static in L. At this point, it is exogenous, but it will be endogenous later. For the purpose of comparing the following Proposition with Melitz and Ottaviano (2008), both traditional and manufactured goods are always produced in that model.

Proposition 1 *If $\alpha > \overline{\alpha}$, then there are $0 < L_0 < L_1 < L_2$ such that:*

[2] The traditional good is always produced in Melitz and Ottaviano (2008) because they assume $\alpha < \overline{\alpha}$.

(i) only the traditional good is produced for $L \leq L_0$*;*

(ii) both the traditional and manufactured goods are produced for $L_0 < L < L_1$*;*

(iii) only the manufactured goods are produced for $L_1 \leq L \leq L_2$*;*

(iv) both the traditional and manufactured goods are produced for $L > L_2$*.*

If $\alpha \leq \overline{\alpha}$*, then there is* L_0 *such that:*

(i) only the traditional good is produced for $L \leq L_0$*;*

(ii) both the traditional and manufactured goods are produced for $L > L_0$*.*

Remark 2 *We interpret this result as follows. If the marginal utility of manufactured goods is sufficiently high, then there are 4 phases of production as labor supply increases upward from zero. First, only the traditional good is produced, then both types of goods are produced, followed by only manufactured goods, and finally, both goods. The transition between the last two phases is driven by keener competition in the manufacturing sector, which leads to reappearance of the traditional sector.*[3] *We may say that the increasing share of manufacturing labor in phase (ii) is industrialization, whereas the decreasing share in phase (iv) is deindustrialization. If marginal utility of manufactured goods is low, then there are only two phases as labor supply increases. First, only traditional good is produced, then both goods are produced.*

Remark 3 *In contrast with Melitz and Ottaviano (2008), the introduction of a labor market makes a difference in results (iii) and (iv) of Proposition 1. For result (iv), in contrast with Melitz and Ottaviano (2008), labor supply is not infinitely elastic here. Consequently, as seen in Figure 1, in phase (iv) the growth rate of labor use in the traditional sector is faster than that in manufacturing. Such deindustrialization is not found in Melitz and Ottaviano (2008).*

Therefore, the equilibrium use of manufacturing labor is given by

$$L_m^* = \begin{cases} 0 & \text{for } L \leq L_0 \\ L_m & \text{for } L_0 < L < L_1 \text{ or } L > L_2 \\ L & \text{for } L_1 \leq L \leq L_2. \end{cases}$$

[3] That is, as L gets large, the cut-off c_D^* goes down, which decreases the profit of each manufacturing firm, and thus decreases the manufacturing wage, so that the traditional sector reappears for sufficiently large L.

When we analyze equilibrium in applications, we will have to consider the case $L \in [L_1, L_2]$, which occurs when the homogeneous or traditional good, typically agriculture, is not produced due to various endogenous factors, such as a high wage in the manufacturing sector. In this case, the demand system is slightly different from the one in previous literature.

When the traditional good is not produced in equilibrium, wage w is no longer equal to 1 and there is an additional equilibrium condition, $L = L_m$, that determines equilibrium $w^*(> 1)$. The equilibrium c_D^* and N^* for $L \in [L_1, L_2]$ are obtained by substituting w^* into (7) and (12), respectively.

No production of the traditional good can happen in the single country case when utility is quasi-linear with a sufficiently large initial endowment of the traditional good. This does not happen in a single country model if the upper-tier utility is Cobb-Douglas, as in the Dixit-Stiglitz-Krugman model. In this case, the traditional good is always demanded and produced.[4]

When the price of each variety decreases as the mass of firms increases, that is called a *pro-competitive effect*. Our quasi-linear utility function has a procompetitive effect, but no income effect, whereas Cobb-Douglas utility has an income effect, but no pro-competitive effect. Therefore, the presence or absence of these effects are not a necessary condition for no traditional good production in a multi-country economy.

Comparative statics

Following the tradition of growth theory, we examine comparative statics with respect to an exogenous change in population L. We also consider comparative statics with respect to an exogenous change in manufacturing productivity; see Appendix B.

We can show that

$$\frac{\partial c_D^*}{\partial L} < 0, \qquad \frac{\partial N^*}{\partial L} > 0, \qquad \frac{\partial L_m^*}{\partial L} > 0, \qquad \frac{\partial U^*}{\partial L} = 0 \text{ for } L < L_0, \qquad \frac{\partial U^*}{\partial L} > 0 \text{ for } L \geq L_0 \tag{14}$$

The average operating profit $\overline{\pi} \equiv \int_0^{c_D^*} \pi(c)\ \mathrm{d}G(c) / \int_0^{c_D^*} \mathrm{d}G(c)$ increases with L.

[4] Nevertheless, no traditional good production can happen even if the upper-tier utility is Cobb-Douglas if there are multiple countries with trade as in Section 3.2 below. This happens because some countries can import the traditional good rather than produce it. See Appendix 1 of Behrens et al. (2009) for the no-traditional–good-production condition using the Dixit-Stiglitz-Krugman model with homogeneous firms.

On the interval $[L_1, L_2]$, equilibrium w^* as a function of L has an inverted U-shape. This is shown as follows.

Let $w = \omega(L)$ be the implicit function defined by the equation $L_a(L) = 0$.[5] We know that $w = \omega(L_1) = \omega(L_2) = 1$ and that there are at most two solutions L of $L_a(L) = 0$, given w. This implies that $w = \omega(L)$ is either U-shaped or inverted U-shaped on the interval $[L_1, L_2]$. Since manufacturing labor demand L_m is higher than labor supply L when $w = 1$, $w > 1$ holds on the interval $[L_1, L_2]$. Thus, $w = \omega(L)$ has an inverted U-shape.

Solving equation (12) for w yields

$$w = \left[\frac{2(k+1)\alpha\gamma}{\eta N^* + 2(k+1)\gamma}\right]^{\frac{k+2}{k}} \left(\frac{L}{\gamma\phi}\right)^{\frac{1}{k}},$$

and thus,

$$\frac{dw}{dL} = \underset{+}{\frac{\partial w}{\partial L}} + \underset{-}{\frac{\partial w}{\partial N^*}}\underset{+}{\frac{\partial N^*}{\partial L}}.$$

Therefore, the sign of dw/dL depends on the two terms on the right hand side. The first term is the *size effect*. Keeping N^* constant, a larger market size L leads to higher profits and a higher wage. The second term is a *pro-competitive effect*. A larger market size intensifies competition and reduces profits and the wage. As L increases, the former effect dominates the latter in the initial phase of $[L_1, L_2]$ so that $\partial w/\partial L$ is positive, whereas the opposite is true in the later phase of $[L_1, L_2]$.

The excess demand for manufacturing labor is given by $-L_a(L)$. The inverted U-shaped relationship implies that as L gets larger, the excess demand $-L_a(L)$ at $w = 1$ initially increases and raises the wage w. In contrast, as L gets even larger, excess demand decreases and lowers the wage.

The intuition for the comparative static of wage on labor or population is as follows. The exogenous labor supply can be represented by a vertical supply curve. As L increases, the supply curve shifts to the right. Demand for labor by the manufacturing sector is given by a downward sloping derived demand curve. As L increases, it shifts to the right. Whether wage increases or decreases with L depends on how fast demand shifts to the right relative to supply. At levels of population just

[5] From equation (13), $\frac{\partial L_m}{\partial w} = -\frac{L}{w\eta}\left(\alpha\left(\frac{1}{Lw}\phi\right)^{\frac{1}{k+2}} + kw\left(\frac{1}{Lw}\phi\right)^{\frac{2}{k+2}}\right)\frac{k+1}{(k+2)^2} < 0$, so the implicit function theorem yields that ω exists.

over L_1, demand shifts to the right faster than the supply curve, so wage increases. At levels of population just below L_2, supply shifts faster than demand, so wage decreases with L.

Appendix C examines the comparative static of scale elasticities with respect to L, as suggested by a referee.

2.3 Numerical Simulations

Here we present numerical simulations of the model detailed in this section.

Set $\alpha = 2.38$, $f_E = \gamma = \eta = 1$, $k = 2$, and $c_M = 3$. In this case, the calculated values of the thresholds are: $L_{\min} = 2.67$, $L_0 = 6.73$, $L_1 = 45$, $L_2 = 326$, and $\overline{\alpha} = 2.31$. Since $\overline{\alpha} < \alpha < c_M$ is satisfied, the first case in Proposition 1 applies, so both phase (i) $L \in (L_{\min}, L_0)$ and (iii) $L \in (L_1, L_2)$ appear.

We put labor usage L on the horizontal axis in Figure 1, and suppose that we increase L monotonically from 0. As L increases beyond L_0, both N^* and U^* continuously increase whereas c_D^* decreases regardless of whether the traditional good is produced or not. The manufacturing wage w^* is equal to 1 for $L \in (0, L_1] \cup [L_2, \infty)$, whereas it is larger than 1 and inverted U-shaped for $L \in (L_1, L_2)$. For $L < L_0$, only the traditional good is produced, whereas wage and utility are constant. Beginning at L_0, the manufacturing sector initiates production. Then, at L_1, traditional good production ceases and wage rises. Eventually, wages reach a maximum and begin to decline. At L_2, wage returns to its original level, and traditional good production is re-initiated, joining manufacturing. Throughout, utility is (weakly) increasing in L.

3 Applications

3.1 Poverty Traps

Until this point, we have taken L to be an exogenous parameter. Next we consider the case of many regions where L is endogenously determined by the utility level available to consumers in the region, who are free to migrate to the region that offers the highest utility level to them.[6] It is typical in this variety of model to limit migration at a given time to be proportional to the utility differences between regions; see for example Krugman (1991b), Fukao and Bénabou (1993) and Forslid and Ottaviano (2003). There is no trade.

Consider a countable infinity $r = 1, ..., \infty$ of distinct regions.[7] Time is continuous and indexed by $t \in [0, \infty)$. The population in region r at time t is denoted $L(r,t)$, so $L : \mathbb{N} \times [0, \infty) \to \mathbb{R}_+$, where $\mathbb{N}$ is the set of natural numbers. From (9) and (7), indirect utility for location r at time t is given by

$$U(L(r,t)) = \widehat{q}_0 + w + \frac{1}{2\eta}\left[\alpha - w\left(\frac{\gamma\phi}{wL(r,t)}\right)^{\frac{1}{k+2}}\right]\left[\alpha - \frac{k+1}{k+2}w\left(\frac{\gamma\phi}{wL(r,t)}\right)^{\frac{1}{k+2}}\right]$$

$$= \begin{cases} \widehat{q}_0 + 1 + \frac{1}{2\eta}\left[\alpha - \left(\frac{\gamma\phi}{L(r,t)}\right)^{\frac{1}{k+2}}\right]\left[\alpha - \frac{k+1}{k+2}\left(\frac{\gamma\phi}{L(r,t)}\right)^{\frac{1}{k+2}}\right] & \text{for } L(r,t) \notin [L_1, L_2] \\ \widehat{q}_0 + \omega(L(r,t)) + \frac{1}{2\eta}\left[\alpha - \omega(L(r,t))\left(\frac{\gamma\phi}{\omega(L(r,t))L(r,t)}\right)^{\frac{1}{k+2}}\right] & \\ \qquad \cdot \left[\alpha - \frac{k+1}{k+2}\omega(L(r,t))\left(\frac{\gamma\phi}{\omega(L(r,t))L(r,t)}\right)^{\frac{1}{k+2}}\right] & \text{for } L(r,t) \in [L_1, L_2] \end{cases}$$

The parameter $\delta \geq 0$ is the speed of population adjustment across regions. When $\delta = 0$, the population is immobile. When $\delta = \infty$, there is free mobility and population adjustment is immediate.

$$\dot{L}(r,t) \equiv \frac{dL(r,t)}{dt} \tag{15}$$

$$= \sum_{r' \neq r} \frac{1}{2^{|r'-r|}}\delta \cdot [U(L(r,t)) - U(L(r',t))] \cdot \min\left\{1, \frac{L(r,t) - L_{\min}}{L_0 - L_{\min}}, \frac{L(r',t) - L_{\min}}{L_0 - L_{\min}}\right\}$$

The expression is normalized by $\frac{1}{2^{|r'-r|}}$ to keep migration finite.[8] The last expression ensures that population in a region is never below $L_{\min}$.

Fixing $\{L(r,0)\}_{r=1}^{\infty}$ where $L(r,0) > L_{\min}$ for each r and $\sup_{r=1,2,...} L(r,0) < \infty$,[9] a *migration equilibrium* is a population vector profile $\{L(r,t)\}_{r=1}^{\infty}$ such that $L(r,t) = L(r,0) + \int_0^t \dot{L}(r,t')dt'$ for $r = 1, 2, ...$ and $t \geq 0$.

[6] As is common in the urban economics literature, a small, open city or open region model can be used here as well; see Fujita (1989), chapter 3.3.

[7] Of course, we could limit the number of regions to be finite, but that would impose an arbitrary limit on the number of workers.

[8] One way to think about this is as follows. Regions are located geographically on the real line according to r. People are more likely to move to or from closer regions.

[9] This last condition is stronger than necessary.

From this definition, for a given initial state $\{L(r,0)\}_{r=1}^{\infty}$, from Deimling (1977, Corollary 6.2), migration equilibrium exists and is unique for every closed, finite time interval provided that $\delta < \infty$.

Proposition 4 [10] *Consider the case $\alpha > \overline{\alpha}$.*

(i) Consider the case where all regions have the same low population, $L_{\min} < L(r,0) = L \leq L_0$. Then it is a traditional good economy. The migration equilibrium is the (initial) steady state $L_{\min} < L(r,t) = L \leq L_0$.

(ii) Consider the case $\infty > \delta > 0$ where all regions but one have the same low population, $L_{\min} < L(r,0) = L \leq L_0$ for $r > 1$, but $L(1,0) > L_0$. Then population and utility in region 1 rise monotonically with time and the economy in region 1 goes through the phases described in Proposition 1 (ii) - (iv), depending on $L(1,0)$.[11] *Utility is constant over time for consumers living in all regions $r > 1$. If $\delta = \infty$, then all population moves to region 1 immediately and phase (iv) persists forever with $L(1,t) = \infty$ and $L(r,t) = L_{\min}$ for $r > 1$. If $\delta = 0$, the initial population distribution over all time is the migration equilibrium. Indirect utility in each region is constant.*

Remark 5 *We interpret this result as follows. Consider first the situation where all regions have the same population $L_{\min} < L \leq L_0$. Then it is a traditional good economy. Indirect utility as a function of population is the same for all population levels below L_0. To obtain higher utility, any given region (in this case region 1) must be pushed past the L_0 population threshold by encouraging immigration. This could be achieved by subsidizing immigration (at time $t = 0$) into the region from the others. Alternatively, a country can escape a poverty trap if $L > L_0 \equiv \phi/\alpha^{k+2} = 2(k+1)(k+2)c_M^k f_E/\alpha^{k+2}$, or $f_E < \alpha^{k+2}L/\left[2(k+1)(k+2)c_M^k\right]$. This is possible if a country lowers the fixed cost f_E required for entry below $\alpha^{k+2}L/\left[2(k+1)(k+2)c_M^k\right]$, thus reducing L_0. Once population size L_0 is passed, utility is higher in the target region, and utility is strictly increasing in L. Then population and utility growth are self-sustaining for this region, and manufacturing is initiated. Thus, the poverty trap is a result of paucity of population in regions or cities.*

[10] Given the structure of migration equilibrium and Proposition 1, the proof of this proposition is rather obvious.

[11] For example, if $L(1,0) > L_1$, then phase (ii) is skipped.

To this point, we have treated locations as symmetric. A reviewer has pointed out that some locations might be more desirable than others from the standpoint of a consumer/worker, for example due to the weather, and thus it would be useful to treat locations asymmetrically. It is easy to modify the model to account for this extension. We take $s(r)$ $(r = 1, 2, ...)$ to be the desirability of location r in utils, and additive in utility. We assume that $s(r)$ is bounded. Then we can modify equation (15) as follows

$$\begin{aligned}\dot{L}(r,t) &\equiv \frac{dL(r,t)}{dt} \\ &= \sum_{r' \neq r} \frac{1}{2^{|r'-r|}} \delta \cdot [U(L(r,t)) + s(r) - U(L(r',t)) - s(r')] \cdot \min\left\{1, \frac{L(r,t) - L_{\min}}{L_0 - L_{\min}}, \frac{L(r',t) - L_{\min}}{L_0 - L_{\min}}\right\}\end{aligned}$$

If a location r achieves $\sup_{r'} s(r')$, then it can play the role of region 1 in the proposition just above. People will migrate to it in all phases.

The preceding discussion presumes that consumers are myopic, in that they migrate to where utility is highest, without foresight. This is common in the urban economics literature, for example Krugman (1991a). We can account for foresight as follows. Informally, if consumers cannot predict which region will grow, they will wait until they observe growth empirically before moving or, alternatively, they wait for the government to choose the region that will be subsidized. So without an explicit government policy, they will wait until other consumers select a region and migrate. Thus, there can be a bad equilibrium where every consumer is waiting for others to migrate.

Without myopia, there are many self-fulfilling dynamic equilibria in this model. For example, a set of consumers could expect that they will all move to one target region at time 10. They are in a bad allocation for times below 10 where population in each region is too small to initiate manufacturing. Then that expectation is fulfilled, the consumers move to the target region at time 10, and growth begins in the target region. This kind of self-fulfilling equilibrium is examined in detail in Berliant and Wang (2008) in the context of city subcenter formation rather than regional growth. Myopia or uncertainty can exclude these rather arbitrary equilibria.

3.2 The Home Market Effect

3.2.1 The General Case

Next we consider a model of trade, where consumers are completely immobile in the context of two regions or countries. In this subsection, the thresholds will differ from those discussed previously.

The countries will be denoted by l and h. Their respective populations are L^l ($l = 1, 2$), assumed to be fixed in this subsection. The barriers to imports will be denoted by $\tau > 1$, where 1 unit of a commodity arrives at its destination when τ units are shipped.

Repeating the previous calculations for this slightly modified model, the upper cut-off of unit labor requirement for firms that will produce differentiated good for the domestic market in country l at equilibrium is

$$c_D^l = \left\{ \frac{2\gamma\phi\tau^k w^h \left[2w^h \left(w^l\tau^k - w^h\right) - k^2 \left(w^l - w^h\right)^2 - k\left(w^l - w^h\right)\left(w^l - 3w^h\right)\right]}{L^l \left[4\left(\tau^{2k} - 1\right)\left(w^l w^h\right)^2 - k\left(k+1\right)^2\left(k+2\right)\left(w^l - w^h\right)^4\right]} \right\}^{\frac{1}{k+2}}$$

and that for the export market is $c_X^h = c_D^l/\tau$ for $h, l = 1, 2$ and $l \neq h$. The number of firms or varieties active in country l is

$$N^l = \frac{2\left(k+1\right)\gamma\left[\left(c_D^h\right)^{k+1}\left(\alpha - w^l c_D^l\right)\left[\left(\tau^{2k} - 1\right)w^h + k\left(w^l - w^h\right)\right] - \left(kc_D^l\right)^{k+1}\left(\alpha - w^h c_D^h\right)\left(w^l - w^h\right)\right]}{\eta c_D^l \left(c_D^h\right)^{k+1}\left[\left(\tau^{2k} - 1\right)w^l w^h + k\left(k+1\right)\left(w^l - w^h\right)^2\right]}$$

The first threshold, defined in Section 2 by $c_M > c_D^l\big|_{w^l = w^h = 1}$, is the minimum population considered given by $L > L_{\min} = \frac{\gamma\phi}{c_M^{k+2}\left(1+\tau^{-k}\right)}$. The second threshold, defined in Section 2 by $N^l\big|_{w^l = w^h = 1} > 0$, is the lowest population where manufacturing occurs with trade given by $L_0 = \frac{\gamma\phi}{\alpha^{k+2}\left(1+\tau^{-k}\right)}$. Since we are assuming $\alpha < c_M$, $L_0 > L_{\min}$ holds.

Turn next to the other thresholds defining where only manufacturing is active. Unlike $L_{\min}$ and L_0, they will be functions of the (fixed) population in the other country and given by $F^l\left(L^l, L^h\right) \equiv L^l - L_m^l\big|_{w^l = w^h = 1}$ in the current context with trade. Define the equilibrium use of manufacturing labor in country l as:

$$\begin{aligned} L_m^l &\equiv N_E^* \left(\int_0^{c_D^l} cq^l L^l \, \mathrm{d}G(c) + \int_0^{c_X^l} cq^h L^h \, \mathrm{d}G(c) + f_E \right) \\ &= \frac{\left[N^l \left(\tau c_D^h\right)^k - N^h \left(c_D^l\right)^k \right] \left[kL^l \tau^{k+1} \left(c_D^l\right)^{k+2} + kL^h \left(\tau c_D^h\right)^{k+2} + \tau^{k+1} \gamma \phi \right]}{2(k+1)(k+2)\gamma\tau\left(\tau^{2k}-1\right)\left(c_D^l c_D^h\right)^k}. \end{aligned}$$

Then, $w^l = 1$ if $F^l\left(L^l, L^h\right) \geq 0$ and $w^l > 1$ if $F^l\left(L^l, L^h\right) < 0$. The same can be said by exchanging l and h.

Solving $F^l\left(L^l, L^l\right) = 0$ for L^l yields the analogs of L_1 and L_2. The solutions are real if $\alpha < \overline{\alpha}_2$, where

$$\overline{\alpha}_2 \equiv 2\sqrt{\frac{(k+2)\eta\tau\left(\tau^k+1\right)}{k+\tau+(k+1)\tau^{k+1}}}, \tag{16}$$

which is the analog of $\overline{\alpha}$ in Proposition 1.

Furthermore, there is another threshold. Since $L^l = L_0$ is equivalent to $\alpha = c_D^l\big|_{w^l=w^h=1}$, plugging it into $F^h\left(L_0, L^h\right)\big|_{w^l=w^h=1}$, differentiating it by L^h, and solving it for L^h, we have

$$L^h = \left(\frac{2}{\alpha}\right)^{k+2} \frac{\gamma\phi\tau^k}{\tau^k+1}.$$

Then, we can show that

$$F^h\left(L_0, \left(\frac{2}{\alpha}\right)^{k+2} \frac{\gamma\phi\tau^k}{\tau^k+1}\right)\Bigg|_{w^l=w^h=1} \gtreqless 0 \Leftrightarrow \alpha \gtreqless \overline{\alpha}_1,$$

$$\overline{\alpha}_1 \equiv 2\sqrt{\frac{(k+2)\eta\tau\left(\tau^k-\tau^{-k}\right)}{k+\tau+(k+1)\tau^{k+1}}}. \tag{17}$$

This implies that if $\alpha < \overline{\alpha}_1$, then $F^h\left(L^l, L^h\right) = 0$ and $L^l = L_0$ do not intersect.

Since $\overline{\alpha}_1 < \overline{\alpha}_2$, we have established the following.

(a) If $\alpha < \overline{\alpha}_1$, then both F^l and F^h are positive in any domain $\left(L^l, L^h\right)$, so that there is no phase (iii) in both countries;

(b) If $\overline{\alpha}_1 < \alpha < \overline{\alpha}_2$, then either F^l or F^h is negative in some domains, where phase (iii) appears in a country;

(c) If $\alpha > \overline{\alpha}_2$, then both F^l and F^h are negative in some domain, where phase (iii) appears in both countries.

Thus, $\alpha < \overline{\alpha}_1$ *is the necessary and sufficient condition for traditional production for any population* $\left(L^l, L^h\right)$, i.e., there is no phase (iii) in both countries.

Cases (a), (b) and (c) are illustrated in Figures 2, 3, and 4, respectively, where the Roman numerals indicate phases. For example, domain **(i, ii)** means $\{L_{\min} < L^l \leq L_0 \cap L_0 < L^h < L_1\}$, so that only the traditional good is produced in country l, while both traditional and manufactured goods are produced in country h. In the following, we focus on the HME analytically in case (a), whereas we analyze cases (b) and (c) numerically.

According to Crozet and Trionfetti (2008) and Behrens et al. (2009), the HME in this model holds if and only if:

$$L^l < L^h \text{ implies } \frac{N^l}{L^l} < \frac{N^h}{L^h}. \tag{18}$$

In words, the larger country has a larger ratio of firms to population.

There is another definition of the HME:

$$L^l < L^h \text{ implies } X^l < X^h \tag{19}$$

used by Krugman (1980) and Costinot et al. (2019), where $X^l = \int_0^{c_X^l} p^h(c) q^h(c) L^h \, \mathrm{d}G(c)$ is the total export from countries l to h. In words, the larger country has positive net exports.

3.2.2 When $\alpha \leq \overline{\alpha}_1$

In case (a), the traditional good is always produced so that $w^l = w^h = 1$. There are four domains **(i, i)**, **(i, ii)**, **(ii, i)**, and **(ii, ii)** on the $\left(L^l, L^h\right)$-coordinates.

Setting $w^l = w^h = 1$ and solving $N^l = 0$ yields

$$L_0 \equiv \frac{\gamma\phi}{\alpha^{k+2}\left(1+\tau^{-k}\right)},$$

when manufacturing initiates in the case of two countries with trade.

Define

$$\begin{aligned} H\left(L^l, L^h\right) &\equiv \left[\frac{1}{L^l L^h} - \alpha\left(\frac{1-\tau^{-k}}{\gamma\phi}\right)^{\frac{1}{k+1}} \frac{\left(L^l\right)^{-\frac{k+1}{k+2}} - \left(L^h\right)^{-\frac{k+1}{k+2}}}{L^l - L^h}\right] \qquad (20) \\ L_{\mathrm{HME}} &\equiv \left(\frac{k+2}{k+1}\right)^{k+2} \frac{\gamma\phi}{\alpha^{k+2}\left(1+\tau^{-k}\right)} > L_0. \end{aligned}$$

As shown by Lemma 7 in Appendix D, the curve $H\left(L^l, L^h\right) = 0$ is negatively sloped, passes through $(L_{\mathrm{HME}}, L_{\mathrm{HME}})$, and does not intersect the line $N^l = 0$. Then, the following can be proved. The proof is contained in Appendix D.

Proposition 6 *When $L^l > L_0$, $L^h > L_0$, and $\alpha \leq \overline{\alpha}_1$, the HME (18) holds for small population $H\left(L^l, L^h\right) < 0$, but not for large population $H\left(L^l, L^h\right) > 0$.*

In Figure 2, $N^l/L^l = N^h/L^h$ is depicted by the purple curve $H\left(L^l, L^h\right) = 0$ and purple line $L^h = L^l$. Proposition 6 states that the HME (18) holds in the southwest domain of the purple curve (D_{HME}), whereas *the reverse HME (18) holds in the northeast domain of the purple curve* (D_{RHME}). Such a reverse HME occurs for large population size because the aggregate marginal productivity in the manufacturing sector is positive, but decreases as population rises. This implies that as the population increases, the scale economy is positive, but is steadily decreasing. It should be noted that such a reverse HME never occurs in the Krugman (1980) model, where the upper-tier utility is Cobb-Douglas and the lower-tier is CES.

Concerning the second definition of the HME given by (19), we show in Appendix E that it always holds, unlike the first definition of the HME given by (18). That is, net exports are always positive for the larger country. Nevertheless, the two definitions of the HME coincide in the Krugman (1980) model.

Concerning the utility differential, we can show that the utility is always higher in the larger country. This implies that full agglomeration is a stable spatial equilibrium in the presence of migration, which is in accord with Proposition 4 in Section 3.1.

Finally, the HME holds for $L < L_{\mathrm{HME}}$ in the vicinity of $L^h = L^l$. This is true for large τ (high transport costs) and large γ (more differentiated products), which agree with the empirical findings by industry of Hanson and Xiang (2004).

3.2.3 When $\alpha > \overline{\alpha}_1$

In cases (b) and (c), $F^l < 0$ (resp., $F^h < 0$) holds in some domains, where the traditional good is not produced in country l (resp., h) so that $w^l > 1$ (resp., $w^h > 1$).

Figures 3 and 4 illustrate case (b) $\overline{\alpha}_1 < \alpha < \overline{\alpha}_2$ and case (c) $\alpha > \overline{\alpha}_2$. We conducted numerical simulations and found that Proposition 6 is true for all domains, thus confirming validity of the proposition.

In the domain of **(i, iii)** in Figure 3, country l in phase (i) produces the traditional good only, whereas country h in phase (iii) produces the manufactured good only. This is the so-called interindustry trade, which is in contrast to the intraindustry trade in most of the other domains.

In the domain of **(ii, iii)** in Figures 3 and 4, country l is in phase (ii) and country h is in phase (iii), so that $w^l = 1$ and w^h is a solution of $L^l - L_m^l\big|_{w^1=1} = 0$. Because country h does not produce the traditional good, country h, in net, exports the manufactured good to country l. Nevertheless, the reverse HME $N^l/L^l > N^h/L^h$ holds, whereas $N_E^l/L^l < N_E^h/L^h$ holds. This means that although the larger country h attracts a more than proportionate number of entrants, N_E^l, a less than proportionate number of firms, N^h, remains in the market. In fact, the share of remaining firms in country h is smaller $G\left(c_D^l\right) > G\left(c_D^h\right)$ because competition is keener in the larger market.

4 Conclusions

We have reexamined a standard model of monopolistic competition in the frameworks of regional economics and international trade, introducing a simple labor market. In Proposition 1 (iii), we show that when the manufacturing demand for labor exceeds the inelastic supply of labor, the traditional good is not produced . In the context of regional economics, namely of free migration but no trade, complex behavior in the form of a poverty trap is a result (Proposition 4), and policies that encourage immigration can overcome the trap. In the context of international trade, namely of costly trade but no migration, the HME can disappear if populations are large (Proposition 6).

Future work should consider both costly trade and migration in the same model, as well as normative questions such as optimal trade and migration policy. Moreover, dynamic versions of the model with capital accumulation and endogenous technological progress should be examined.

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5 Appendix A: The Model with No Endowment of Traditional Good

In this appendix, we examine our model with no endowment of the traditional commodity, as in Arkolakis (2008) and Demidova (2017). The purpose is twofold. First, it seems like a reasonable assumption relative to the real world. Second, we wonder how robust the model is to such a small alteration.

Assume that, unlike Melitz and Ottaviano (2008), $\widehat{q}_0 = 0$. That is, the preferences of a typical consumer are represented by (1), but the budget constraint (2) is replaced with

$$q_0 + \int_0^N p_i q_i \,\mathrm{d}i = w + \frac{N_E \cdot \Pi}{L}, \tag{21}$$

where Π is the average profit of a firm and N_E is the number of firms that enter, i.e. pay the fixed cost. As is apparent, we assume that consumers in the one country are endowed with equal profit shares in the firms.[12] The firms with $c \in [c_D, c_M]$ earn negative profit after sinking the fixed cost and exiting. Hence the free entry condition (10) and the law of large numbers implies that $\Pi = 0$.

The free entry condition is rewritten as

$$\int_0^{c_D} p(c)q(c)\,\mathrm{d}G(c) - w\int_0^{c_D} cq(c)\,\mathrm{d}G(c) = wf_E. \tag{22}$$

Using the budget constraint $q_0 + \int_0^N p_i q_i \,\mathrm{d}i = w$, the total demand for the traditional good is given by

$$Q_0^{\text{demand}} = q_0 L = \left(w - \int_0^N p_i q_i \,\mathrm{d}i\right) L. \tag{23}$$

The total supply of traditional good is given by

$$Q_0^{\text{supply}} = L - L^{\text{demand}} = L - N\left[\frac{\int_0^{c_D} cq(c)\,\mathrm{d}G(c)}{G(c_D)}\right] - N_E f_E.$$

Using (22), this is rewritten as

$$Q_0^{\text{supply}} = L - \frac{N}{w}\frac{\int_0^{c_D} p(c)q(c)\,\mathrm{d}G(c)}{G(c_D)}. \tag{24}$$

[12] This is actually irrelevant, since we prove in the next few lines that $\Pi = 0$.

Since $\int_0^{c_D} p(c)q(c)\,\mathrm{d}G(c)/G(c_D)$ is the average revenue per firm and $\int_0^N p_i q_i\,\mathrm{d}i$ is the expenditure for the manufactured goods per consumer,

$$N\frac{\int_0^{c_D} p(c)q(c)\,\mathrm{d}G(c)}{G(c_D)} = L\int_0^N p_i q_i\,\mathrm{d}i.$$

Plugging the left hand side of this equation into (24) and noting that $N_E = N/G(c_D)$ yields

$$Q_0^{\text{supply}} = L - \frac{L}{w}\int_0^N p_i q_i\,\mathrm{d}i. \tag{25}$$

Plugging $w = 1$ into (23) and (25), we confirm that $Q_0^{\text{supply}} = Q_0^{\text{demand}}$.

(a) When the traditional good is produced and consumed, $Q_0^{\text{supply}} = Q_0^{\text{demand}}$ should be positive. This is true in phase (ii) of Proposition 1 when $L_0 < L < L_1$ and phase (iv) when $L > L_2$. Insofar as the traditional good is produced and consumed, all the derivations and equilibrium values in the model with sufficiently large endowment are the same as those in the model with no endowment. This means that as long as the traditional good is produced, we don't need the assumption of a sufficiently large endowment.

(b) However, this does not apply when the traditional good is not produced for $L_1 \le L \le L_2$. In the model with a sufficiently large endowment, the endowment of the traditional good is always consumed even when the traditional good is not produced. In contrast, with no endowment of traditional good, when the traditional good is not produced, it cannot be consumed. The latter case is analyzed in Arkolakis (2008).

6 Appendix B: Comparative statics on the upper productivity bound c_M

As technology improves with innovation, the upper bound c_M on the distribution of unit labor requirement in manufacturing decreases. Then, we can show that

$$\frac{\partial c_D^*}{\partial c_M} > 0, \qquad \frac{\partial N^*}{\partial c_M} < 0$$

for all c_M. That is, the impact of reducing the upper productivity bound c_M is opposite that of increasing population L. Fixing the same population size $L_1 =$

$L_2 = L$, we can also show that the marginal change in labor usage by manufacturing firms is given by

$$\frac{\partial L_m^*}{\partial c_M} \begin{cases} = 0 & \text{for } c_M \geq c_{M_0} & \text{phase (i)} \\ < 0 & \text{for } c_{M_0} > c_M > c_{M_1} & \text{phase (ii)} \\ = 0 & \text{for } c_{M_1} \geq c_M \geq c_{M_2} & \text{phase (iii)} \\ > 0 & \text{for } c_M < c_{M_2} & \text{phase (iv),} \end{cases} \tag{26}$$

where c_{M_0}, c_{M_1}, and c_{M_2} ($c_{M_0} > c_{M_1} > c_{M_2}$) correspond to L_0, L_1, and L_2 ($L_0 < L_1 < L_2$). When the upper productivity bound c_M falls to c_{M_0} (phase (i)), manufacturing labor demand L_m^* starts increasing until it hits the labor supply constraint L at $c_M = c_{M_1}$ (phase (ii)). Manufacturing labor demand L_m^* is equal to L until c_M falls to c_{M_2} (phase (iii)). Then, a further fall in c_M decreases manufacturing labor demand L_m^* (phase (iv)). In sum, L_m^* is inverted U-shaped, which is in accord with the empirical findings; see Herrendorf et al. (2014).

Phase (ii) of increasing L_m^* (the third line in (26)) occurs because faster productivity growth in the manufacturing sector induces more workers to abandon the traditional sector. This conforms with the size effect in the comparative statics on population L. In contrast, phase (iv) of decreasing L_m^* (the first line in (26)) occurs because productivity gains in the manufacturing sector push workers out of the manufacturing sector (Matsuyama, 2008). This is consistent with the procompetitive effect in the comparative statics on population.

To make this concrete, consider a developed country 1 and developing country 2, with different production technologies $c_M^1 < c_M^2$. When both are large and $c_M^2 \geq c_{M_0} > c_M^1$, the developing country produces the traditional good only (phase (i)), whereas the developed country produces both the traditional and manufactured goods (phase (ii)). This implies *interindustry trade.* When $c_{M_0} > c_M^1 > c_{M_1} > c_M^2$, the developing country produces the traditional and manufacturing goods (phase (ii)), whereas the developed country produces the manufactured good only (phase (iii)). This is both interindustry and *intraindustry trade* (Grubel and Lloyd, 1975). When $c_{M_1} > c_M^1 > c_{M_2} > c_M^2$, the developing country produces the manufactured good only (phase (iii)), whereas the developed country produces both the traditional and manufactured goods (phase (iv)). This is both interindustry and intraindustry trade. It is worth noting that the transition from phases (iii) to (iv) is *deindustrialization* in the developed country, which is not only due to the shift of manufacturing production from developed to developing countries, but also due to technological progress in artificial intelligence and robotization of manufacturing production in the developing country (Mayer, 2018). Thus, the model in Melitz and Ottaviano (2008) can describe complex and detailed stylized facts in economic development once a labor market clearing condition is introduced, as we have done in Section 2.2.

7 Appendix C: Comparative statics on scale elasticities

How do effective scale elasticities vary with labor usage? We address that here. With this model and notation, as in Melitz and Ottaviano (2008), we compute elasticities of scale using the elasticities of aggregate productivity in quantity and in value with respect to labor, and examine how they vary with the economy's size. Naturally, we are using the basic model from Section 2 and the parameter values for the simulations in Section 2.3 here.

To accomplish this, we define two elasticities:

$$\text{Definition 1:} \qquad \varepsilon_1 = \frac{d\ln(Q_m)}{d\ln(L_{\text{demand}})}$$

$$\text{Definition 2:} \qquad \varepsilon_2 = \frac{d\ln(\text{GDP})}{d\ln(L)}.$$

where $Q_m \equiv N_E^* \int_0^{c_D^*} q(c)\, dG(c)$ is the manufacturing output and $\text{GDP} \equiv N_E^* \int_0^{c_D^*} p(c)q(c)\, dG(c) + (L - L_{\text{demand}})$ is the total value of production in the economy. The first definition is the elasticity of the aggregate output in the manufacturing sector with respect to labor demand in the sector. The second definition is the elasticity of aggregate GDP in the entire economy with respect to labor demand in the entire economy.

Regarding Definition 1, we have

$$\varepsilon_1 = \frac{d\ln(Q_m)}{d\ln(L_{\text{demand}})} = \frac{dQ_m}{dL_{\text{demand}}} \frac{L_{\text{demand}}}{Q_m}.$$

Since

$$Q_m = \frac{L}{\eta}\left[\alpha - w\left(\frac{\gamma\phi}{w^2 L}\right)^{\frac{1}{k+2}}\right] \tag{27}$$

$$L_{\text{demand}} = \frac{(k+w)\,L}{(k+2)\,\eta}\left(\frac{\gamma\phi}{w^2 L}\right)^{\frac{1}{k+2}}\left[\alpha - w\left(\frac{\gamma\phi}{w^2 L}\right)^{\frac{1}{k+2}}\right],$$

solving the former for w and plugging it into the latter, we obtain

$$L_{\text{demand}} = \frac{Q_m}{(k+2)}\left[\alpha - \frac{\eta Q_m}{L} + k\frac{(\gamma\phi L)^{1/k}}{(\alpha L - \eta Q_m)^{2/k}}\right].$$

Differentiating it by Q_m and substituting Q_m in (27) yields

$$\frac{dL_{\text{demand}}}{dQ_m} = \frac{1}{(k+2)\,w}\left[(2-w)\,\alpha + (k+2w-2)\,w\left(\frac{\gamma\phi}{w^2 L}\right)^{\frac{1}{k+2}}\right],$$

and thus

$$\varepsilon_1 = \frac{(k+w)\,w}{(k+2w-2)\,w + (2-w)\,\alpha\left(\frac{\gamma\phi}{w^2 L}\right)^{\frac{1}{k+2}}} > 0,$$

where $w = 1$ in phases (ii) and (iv) and w is a solution of $L = L_{\text{demand}}$ in phase (iii). Then, we can show that ε_1 is monotonically decreasing in L for $\alpha \le \overline{\alpha}$, where $w = 1$ holds. For $\alpha > \overline{\alpha}$, we can numerically show in Figure A1 that ε_1 is monotonically decreasing in L using the same parameter values as in Section 2.3. Therefore, as manufacturing labor demand L_{demand} increases, the aggregate marginal productivity in the manufacturing sector is positive, but decreases. This implies that as the population increases, the scale economy is positive, but steadily decreasing. That is why the reverse HME holds for large population size.

Regarding Definition 2, we have

$$\text{GDP} = \begin{cases} N_E^* \int_0^{c_D^*} p(c)q(c)dG(c) + (L - L_{\text{demand}})\Big|_{w=1} = 1 & \text{in phases (ii) and (iv)} \\ N_E^* \int_0^{c_D^*} c dG(c)\Big|_{w=w^*} = \frac{(k+1)wL}{(k+2)\eta}\left(\frac{\gamma\phi}{w^2L}\right)^{\frac{1}{k+2}}\left[\alpha - w\left(\frac{\gamma\phi}{w^2L}\right)^{\frac{1}{k+2}}\right]\Big|_{w=w^*} & \text{in phase (iii),} \end{cases}$$

where w^* is a solution of $L = L_{\text{demand}}$ and $c_D^* = \left(\frac{\phi}{wL}\right)^{\frac{1}{k+2}}$. The elasticity is given by

$$\varepsilon_2 = \begin{cases} 1 & \text{in phases (ii) and (iv)} \\ \frac{\partial \ln(\text{GDP})}{\partial \ln(L)}\Big|_{w=w^*} & \text{in phase (iii).} \end{cases}$$

Then, we can numerically show in Figure A2 that as the population increases, ε_2 is constant in phases (ii) and (iv) while decreasing in phase (iii) using the same parameter values as in Section 2.3.

Notice that $\varepsilon_2 = 1$ in phases (ii) and (iv), so it is not very interesting.

8 Appendix D: Proof of Proposition 6

First we compute and compare the thresholds corresponding to L_0, L_1, L_2 in Section 2 along the 45 degree line in (L^l, L^h)-coordinates. Setting $L^l = L^h$ and $w^l = w^h = 1$ and solving $N^l = 0$ yields L_0, whereas solving $F^l = 0$ yields L_1, L_2 as follows.

$$\begin{aligned} L_0 &\equiv \frac{\gamma\phi}{\alpha^{k+2}\left(1+\tau^{-k}\right)}, \\ L_1, L_2 &\equiv \frac{\gamma\phi}{\tau^2\left(1+\tau^k\right)^{k+3}}\left[\frac{\alpha z \pm \sqrt{\alpha^2 z^2 - 4(k+2)\eta\tau\left(1+\tau^k\right)z}}{2(k+2)\eta}\right]^{k+2}, \end{aligned}$$

where $z \equiv k + \tau + (k+1)\tau^{k+1}$. Since $F^l(L_1, L_1) = F^l(L_2, L_2) = 0$, $F^l(L_0, L_0) = 1 > 0$, and $\partial F^l(L, L)/\partial L\big|_{L=L_0} < 0$, $L_0 < L_1 < L_2$ always holds.

Since
$$\frac{N^l}{L^l} - \frac{N^h}{L^h} = \frac{2(k+1)\gamma}{\eta}\left(L^l - L^h\right) H\left(L^l, L^h\right),$$
$\frac{N^l}{L^l} - \frac{N^h}{L^h} = 0$ is equivalent to $L^l = L^h$ and $H\left(L^l, L^h\right) = 0$, where $H\left(L^l, L^h\right)$ is given by (20). The former is the 45 degree line, whereas the latter is characterized as follows.

Lemma 7 *The curve $H\left(L^l, L^h\right) = 0$ is negatively sloped, passes through (L_1, L_1), and does not intersect the line $N^l = 0$.*

Proof. Solving $H\left(L^l, L^h\right) = 0$ for α and plugging it into the slope of $H\left(L^l, L^h\right) = 0$, we have

$$\left.\frac{dL^h}{dL^l}\right|_{H\left(L^l, L^h\right)=0} = -\frac{\partial H/\partial L^l}{\partial H/\partial L^h} = \frac{(k+2)r - (kr + r + 1)r^{\frac{1}{k+2}}}{\left[r + k + 1 - (k+2)r^{\frac{1}{k+2}}\right]r^2} = -\frac{\sum_{j=1}^{k+1} j\ z^{\frac{j-1}{k+2}}}{r^{\frac{2k+3}{k+2}}\sum_{i=1}^{k+1} i\ z^{\frac{k+1-i}{k+2}}} < 0,$$

where $r \equiv L^l/L^h$. Thus, $H\left(L^l, L^h\right) = 0$ is negatively sloped. Furthermore, $H\left(L^l, L^h\right) = 0$ passes through $(L_{\mathrm{HME}}, L_{\mathrm{HME}})$, where L_{HME} is the unique solution of $H(L_{\mathrm{HME}}, L_{\mathrm{HME}}) = 0$.

Since $N^l = 0$ means $L^l = L_0$, plugging it into $H\left(L^l, L^h\right)\big|_{L^l=L_0} = 0$ and solving for L^h, we get $L^h = L_0$. This implies that $H\left(L^l, L^h\right) = 0$ does not intersect the line $N^l = 0$ for all $L^l > L_0$. By symmetry, $H\left(L^l, L^h\right) = 0$ does not intersect the line $N^h = 0$ for all $L^h > L_0$. ■

Since the HME (18) holds for $H\left(L^l, L^h\right) < 0$, and the reverse HME holds for $H\left(L^l, L^h\right) > 0$, Lemma 7 implies Proposition 6.

9 Appendix E: Proof that HME always holds for the second definition of HME (19)

The net export from country h is defined by $\Delta X^h \equiv X^h - X^l$, where $X^h = \int_0^{c_X^h} p^l(c) q^l(c) L^l \, \mathrm{d}G(c)$ is the total or gross export from country h to country l. After some cumbersome computations, we can show the net export is zero when

$$\Delta X^h\left(L^h, L^l\right) = A_1\left[\left(L^h\right)^{\frac{k+1}{k+2}} - \left(L^l\right)^{\frac{k+1}{k+2}}\right] - A_2\left[\left(L^h\right)^{\frac{k}{k+2}} - \left(L^l\right)^{\frac{k}{k+2}}\right] = 0,$$

where A_1 and A_2 are positive constants involving α, γ, ϕ, τ, k, and L^h.

The zero net export curve can be rewritten as

$$\begin{aligned}
\Delta X^h\left(L^h, L^l\right) &= \left[\left(L^h\right)^{\frac{k+1}{k+2}} - \left(L^l\right)^{\frac{k+1}{k+2}}\right] Y\left(L^h, L^l\right) \\
Y\left(x^h, x^l\right) &\equiv A_1 - A_2 \frac{\left(L^h\right)^{\frac{k}{k+2}} - \left(L^l\right)^{\frac{k}{k+2}}}{\left(L^h\right)^{\frac{k+1}{k+2}} - \left(L^l\right)^{\frac{k+1}{k+2}}}.
\end{aligned}$$

That is, the zero net export curve can be decomposed into $\left(L^h\right)^{\frac{k+1}{k+2}} = \left(L^l\right)^{\frac{k+1}{k+2}}$ and $Y\left(L^h, L^l\right) = 0$. The former is the 45 degree line in the $\left(L^h, L^l\right)$ plane. The latter curve passes through $(L_0, 0)$ and $(0, L_0)$ because $L^h = (A_2/A_1)^{k+2} = L_0$ is the unique solution to $Y\left(L^h, 0\right) = 0$ and because the two countries are symmetric. It also passes through (L_{NE}, L_{NE}), where $L_{NE} = \left[\frac{k}{\alpha(k+1)}\right]^{k+2} \frac{\gamma\phi}{1+\tau^{-k}}$ is the solution to $\lim_{L^l \to L^h} Y\left(L^h, L^l\right) = 0$. We can readily show that $L_{NE} < L_0$.

We examine phase (ii) in both countries defined by the domain $\{\left(L^h, L^l\right) \mid L^h > L_0$ and $L^l > L_0\}$, and we show next that the zero net export curve $Y\left(L^h, L^l\right) = 0$ does not cross this domain. Thus, a reverse HME cannot occur.

Setting $x^h = \left(L^h\right)^{\frac{k+2}{k+1}}$ and $x^l = \left(L^l\right)^{\frac{k+2}{k+1}}$, $\Delta X^h\left(L^h, L^l\right) = 0$ can be expressed as

$$\Delta X^h\left(x^h, x^l\right) = A_1\left(x^h - x^l\right) - A_2\left[\left(x^h\right)^{\frac{k}{k+1}} - \left(x^l\right)^{\frac{k}{k+1}}\right] = 0.$$

Since

$$\frac{\partial^2 \Delta X^h}{\partial\left(x^h\right)^2} = \frac{A_2 k}{(k+1)^2}\left(x^h\right)^{\frac{-k-2}{k+1}} > 0,$$

ΔX^h is convex, implying that $\Delta X^h\left(x^h, x^l\right) = 0$ has at most two solutions in x^h for a given x^l. That is, $\Delta X^h\left(L^h, L^l\right) = 0$ has at most two solutions in L^h for a given L^l, and thus, $Y\left(L^h, L^l\right) = 0$ has at most one solution in L^h for a given L^l. Furthermore, since $Y\left(L^h, L^l\right) = 0$ is symmetric about the 45 degree line in the $\left(L^h, L^l\right)$ plane, $Y\left(L^h, L^l\right) = 0$ has at most one solution in L^l for a given L^h. Hence, it must be that $Y\left(L^h, L^l\right) = 0$ is strictly downward sloping starting from $(0, L_0)$, passing through (L_{NE}, L_{NE}), and reaching at $(L_0, 0)$. Put differently, the zero net export curve $Y\left(L^h, L^l\right) = 0$ does not enter the domain $\left\{\left(L^h, L^l\right) \mid L^h > L_0 \text{ and } L^l > L_0\right\}$.

In contrast, the zero net export curve $L^h = L^l$ crosses the domain. This implies that the net export X^h is positive whenever $L^h > L^l$ and negative whenever $L^h < L^l$. Therefore, HME according to the second definition given by (19) always holds.

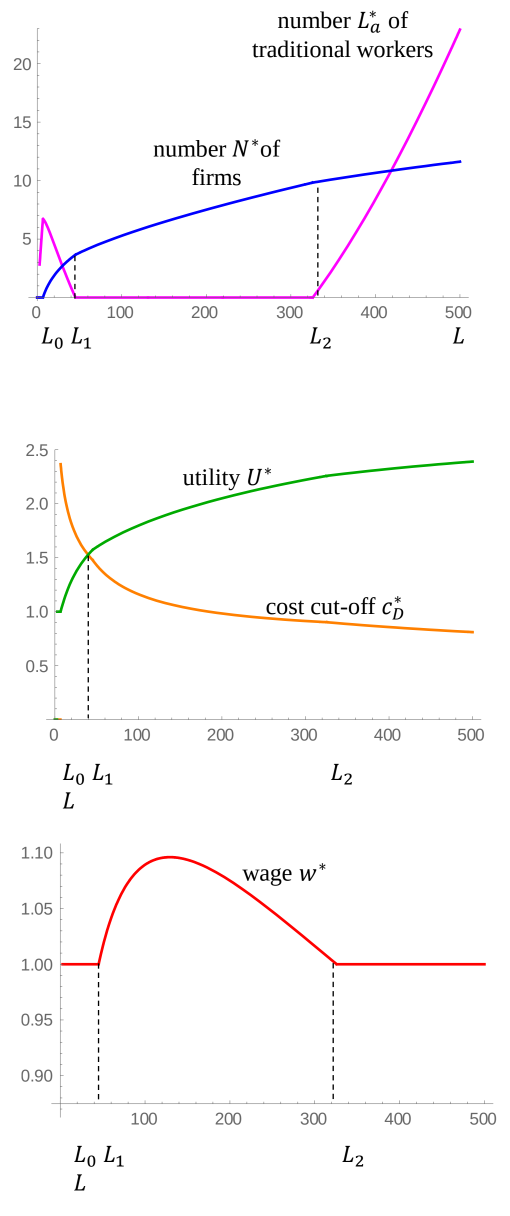

Figure 1: The equilibrium number of firms, number of traditional workers, utility, cost cut-off, and wage (Section 2.3 contains choices of parameter values.)

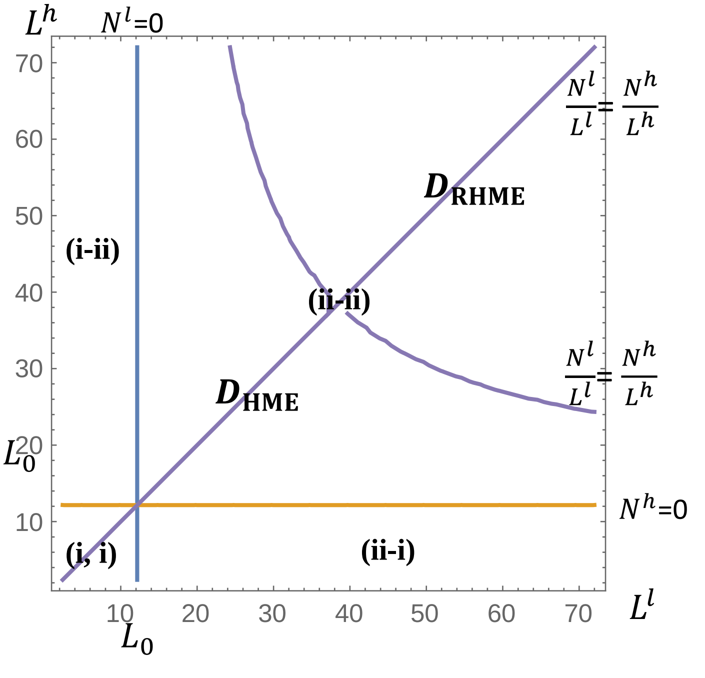

Figure 2: The home market effect when $\alpha < \bar{\alpha}_1$
(blue: $N^l = 0$, orange: $N^h = 0$, purple: $\frac{N^l}{L^l} = \frac{N^h}{L^h}$.
Section 2.3 contains choices of parameter values.)

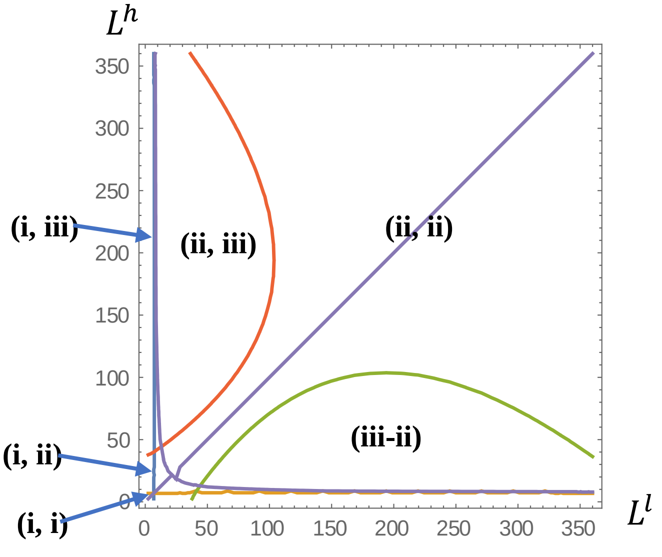

Figure 3: The home market effect when $\bar{\alpha}_1 < \alpha < \bar{\alpha}_2$

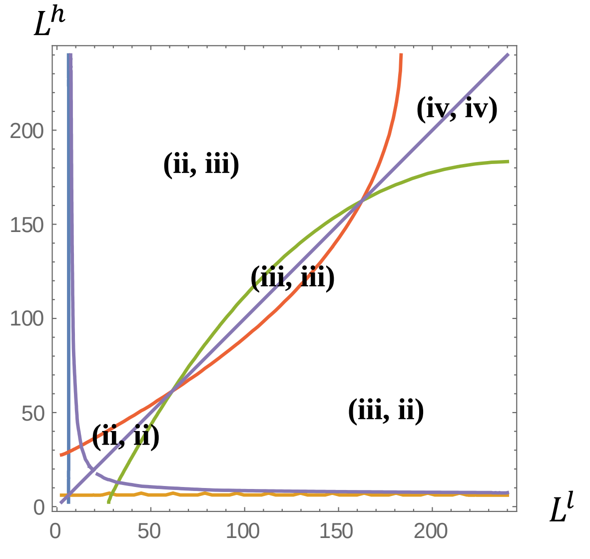

Figure 4: The home market effect when $\alpha > \bar{\alpha}_2$

(Roman numbers are phases in countries l and h. blue: $N^l = 0$, orange: $N^h = 0$, green: $F^l(L^l, L^h) = 0$, red: $F^h(L^l, L^h) = 0$, purple: $\frac{N^l}{L^l} = \frac{N^h}{L^h}$.
Section 2.3 contains choices of parameter values.)

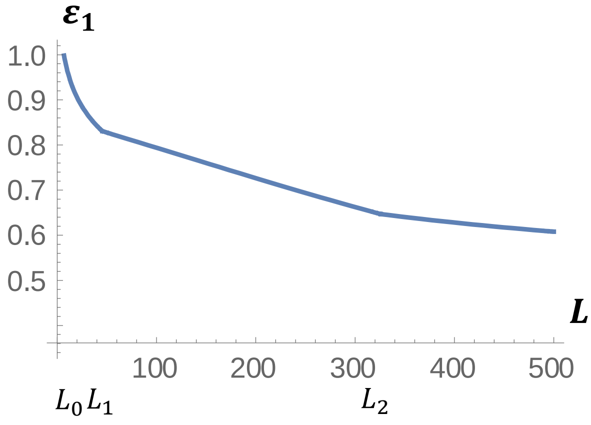

Figure A1: Elasticity of the aggregate manufacturing output sector relative to manufacturing employment
(Section 2.3 contains choices of parameter values.)

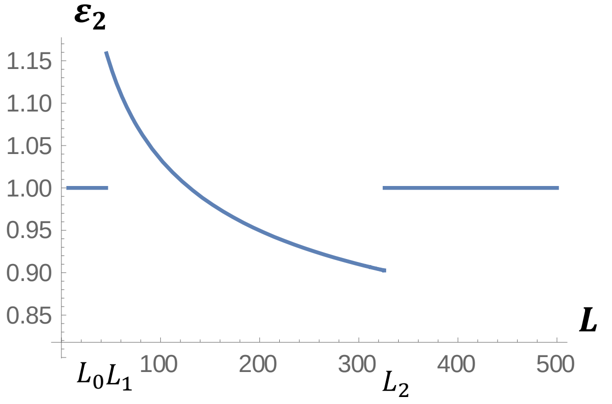

Figure A2: Elasticity of the GDP in the entire economy relative to the population
(Section 2.3 contains choices of parameter values.)