Disutility caused by remote work in urban system

Aizawa, Hiroki and Saka, Takuhiro

Kanazawa University, Chuo University

8 December 2024

Disutility caused by remote work in urban system

Hiroki Aizawa*[1] and Takuhiro Saka*[2]

*[1] *College of Transdisciplinary Sciences for Innovation, Kanazawa University, Sciences Administration Department, Kakuma-machi, Kanazawa, Ishikawa 920-1192, Japan*

*[2] *Department of Civil and Environmental Engineering, Chuo University, Kasuga 1-13-27, Bunkyo-ku, Tokyo 112-8551, Japan*

Abstract

Remote work can affect population distribution in an urban system (i.e., a chain of cities). The current paper explores the effects of remote work on population distribution in an urban system, welfare, and utilities of workers. To examine these effects, we explore the equilibria of a New Economic Geography model that expresses remote workers. We elucidate the bifurcation mechanism of the full agglomeration in a narrow corridor with NEG models with two industries and remote work in order to investigate the effect of remote work on equilibrium. We demonstrate that the introduction of remote work can decrease the utilities of workers. We show that remote workers with myopic behavior themselves decrease their own utilities. With myopic behavior, it is not necessarily ensured that remote workers can obtain higher utilities compared to those with population distribution before the introduction of remote work.

Keywords: Agglomeration; Bifurcation; Economic geography; Remote work; Welfare.

*[1] Corresponding author; E-mail: hirokiaizawam1em2@gmail.com
*[2] E-mail: a20.85hj@g.chuo-u.ac.jp
We are grateful to Masayoshi Tanishita for very detailed and useful comments. We thank participants at several conferences for insightful comments. The authors gratefully acknowledge financial support from JSPS KAKENHI (23K18790).

1. Introduction

The effect of remote work in a city has been explored (e.g., Delventhal et al., 2022; Monte et al., 2023; Behrens et al., 2024). The introduction of remote work can potentially change population distribution in urban systems. Actually, some remote workers changed their residential locations across cities.[1] Remote work can affect population distribution in an urban system (i.e., a chain of cities).

Who obtains benefits from the introduction of remote work in the urban system? All of the workers cannot necessarily conduct remote work. Migration of workers between cities generated by remote work (i.e., change in population distribution) affects prices and masses of varieties and wages in cities. Remote workers may obtain higher utility due to the introduction of remote work, whereas non-remote workers are affected by their location changes through market mechanisms.

We aim to elucidate the effects of remote work on population distribution in an urban system, welfare, and utilities of workers. To examine these effects, we explore the equilibria of a New Economic Geography (NEG) model that expresses remote workers. In the model, there are two industries and one industry can introduce remote work and the other cannot. Workers of an industry can remotely work across cities and workers of the other industry cannot remotely work. We explore how the introduction of remote work affects the population distribution and utilities of the workers.

We elucidate a mathematical mechanism of a change in population distribution caused by the introduction of remote work. We explore this change with a spatial platform where multiple cities are along a narrow corridor (i.e., odd number of places evenly distributed over a line segment). Full agglomeration, which is population distribution in which all the mobile workers gather in one place, expresses one large city surrounded by peripheral cities in a narrow corridor. The bifurcation mechanism of the full agglomeration of NEG models with remote work is investigated in the current paper. The bifurcation explored in the current paper expresses population distribution in which some workers conduct remote work. This bifurcation mechanism is the mathematical mechanism of the change in equilibrium with the full agglomeration due to the introduction of remote work. We demonstrate the introduction of remote work can change the full agglomeration to the following population distributions:

(i) the population distribution with remote workers reside in cities around the large central city and supply labor to firms in the central city in the narrow corridor (see Fig. 1(a)),

(ii) the population distribution with remote workers reside in the large central city and supply labor to firms in cities around the central city in the narrow corridor (see Fig. 1(b)).

It is a theoretical contribution of the current paper that this paper elucidates the bifurcation mechanism in terms of remote work. Economic mechanisms (e.g., a change in a transportation cost) that change the stability of full agglomeration in a two-places economy with replicator dynamics have been investigated with many NEG models (Krugman, 1991; Fujita et al., 2001; Baldwin et al., 2011). The bifurcation mechanism with NEG models that express one industry has been theoretically investigated based on an equidistant economy (Aizawa et al., 2020), a hexagonal lattice economy (Aizawa et al., 2023), and a long narrow economy (Ikeda et al., 2024). In contrast to these previous analyses, the current paper elucidates the bifurcation mechanism of

[1] Such remote workers are reported on https://www.livemint.com/companies/news/tech-workers-take-to-the-mountains-bringing-silicon-valley-with-them-11604301993812.html (accessed on December 8, 2024).

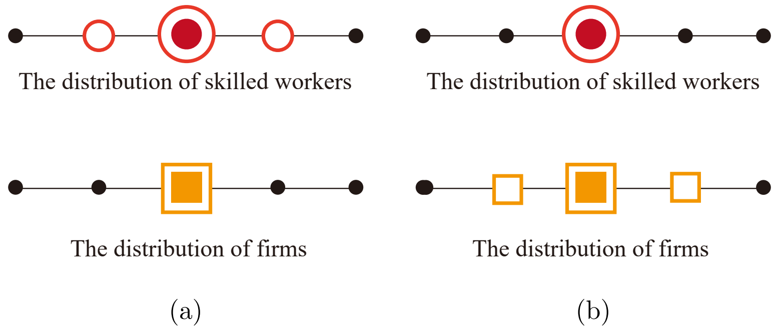


Figure 1: Population distribution in the narrow-corridor with five places. ◯ colored with red: the locations where remote workers are distributed: • colored with red: the location where non-remote workers are distributed; □ colored with yellow: the locations where firms to which remote workers supply labor are distributed; ■ colored with yellow: the location where firm to which non-remote workers supply labor are distributed.

the full agglomeration in the narrow corridor with NEG models with two industries and remote work in order to investigate the effect of remote work on equilibrium.

We demonstrate the introduction of remote work can decrease the utilities of workers. This introduction changes the stability of full agglomeration when workers can obtain higher wages or avoid incurring a high cost of housing. With unstable full agglomeration, workers myopically change their location, and stable population distribution shown in Fig. 1 emerge. We demonstrate that although the utilities of remote workers with the distribution shown in Fig. 1(a) increase, those with the distribution shown in Fig. 1(b) decrease compared to utilities with the full agglomeration. Interestingly, remote workers with myopic behavior themselves decrease their own utilities. The changes in prices and wages from those with the full agglomeration due to the myopic behavior of remote workers; it is not necessarily ensured that remote workers can obtain higher utilities compared to those with population distribution before the introduction of remote work.

The current paper is organized as follows. An NEG model with remote work is explained in Section 2. The bifurcation mechanism of full agglomeration in the narrow corridor with the remote work is explored in Section 3. Bifurcation from the full agglomeration is investigated in Section 4. Social welfare and utilities of workers are investigated in Section 5. Section 6 concludes.

Related literature. Several studies investigate how remote work affects the economy. Intracity effects of remote work have been investigated in urban economics literature (Safirova, 2002; Rhee, 2008; Larson and Zhao, 2017; Dingel and Neiman, 2020; Gokan et al., 2021; Delventhal et al., 2022; Monte et al., 2023; Parkhomenko and Delventhal, 2023). As a theoretical study of remote work, Behrens et al. (2024) investigate the effects of remote work on housing prices and the productivity of workers. Remote work potentially enables workers to move to other cities without changing the locations of offices to which they commute. On the other hand, Brueckner and Sayantani (2023) and Brueckner et al. (2023) investigate how remote work affects the location choices of workers and housing prices in cities with different amenity levels. These studies focus on the intercity effects of remote work. Brueckner and Sayantani (2023) focus on the effects of remote work on population distribution in cities and utilities of workers in terms of exogenous amenity levels among cities rather than a change in agglomeration economy. In contrast to these previous analyses, the current paper focuses on the intercity effect of remote work on the agglomeration economy generated by the scale economy.

The agglomeration mechanism of the population has been investigated in terms of the agglomeration economy and scale economy in NEG literature pioneered by Krugman (1991). While the framework of the model of Krugman (1991) is based on one industry and two-places economy, it is extended by several studies. Tabuchi and Thisse (2011) and Takatsuka and Zeng (2013) develop NEG models with multiple industries. Several theoretical studies (e.g., Gaspar et al., 2018; Takayama et al., 2020) explore population distribution of equilibrium with NEG models with spatial platforms that express multiple places. Ikeda et al. (2018) and Aizawa et al. (2023) explore changes in the population distribution of equilibrium by conducting bifurcation analysis with a change in transportation cost. In contrast to these previous analyses, the current paper explores the bifurcation from the full agglomeration with an NEG model with multiple places and two industries to elucidate the effect of remote work on population distribution and utilities of workers.

2. Model

We explore how the introduction of remote work changes spatial population distribution. Following Brueckner and Sayantani (2023), we explore the equilibria of two theoretical models. One model is a general equilibrium model that expresses the migration of workers and scale economy, but does not include remote work. The other model is a general equilibrium model that expresses the remote work as well as the migration and scale economy. To examine how remote work affects workers, we explore population distribution with equilibria with these models. Moreover, to assess whether remote work is desirable for workers, we compare the social welfare with the population distribution of one model and that of the other model.

The theoretical models in our paper are based on a New Economic Geography (NEG) model. The common framework for these models is the framework of the footloose entrepreneur model which has been employed in theoretical analyses (Baldwin et al., 2011). We explore equilibria of footloose entrepreneur models with two industries, multiple places, and the land consumption of workers.

2.1. Footloose entrepreneur model with two industries

2.1.1. Basic framework

In this subsection, we explain the assumptions of a footloose entrepreneur model with two industries, multiple places, and the land consumption of workers. In our paper, this model is called the FE model. The FE model is the multiple-places and two-industries version of the footloose entrepreneur model proposed by Pflüger and Südekum (2008).

The economy of the FE model consists of K (≥ 2) places, skilled and unskilled workers, and three types of sectors: a manufacturing sector (M-sector), a housing sector (H-sector), and an agriculture sector (A-sector). Skilled and unskilled workers are the factors of production of varieties produced in M-sector. All the workers inelastically supply one unit of labor. Skilled workers choose where to reside among the places in the economy, whereas unskilled workers cannot choose their place of residence. The population distribution of the unskilled workers is exogenous.

The manufacturing sector is composed of two types of manufacturing, denoted as M^1 and M^2 sectors. Each skilled worker is a factor of production in one sector. We assume that they are immobile between the sectors. The number of the skilled workers belonging to M^m sector $(m = 1, 2)$ in place i is denoted λ_i^m and the total number of the skilled workers belonging to each sector is normalized to be one: $\sum_{i\in\mathcal{K}} \lambda_i^m = 1$, where $\mathcal{K}$ is the set of the places in the economy. Unskilled workers are immobile and equally distributed across all places. The total number of the unskilled workers in each place is represented by L. The labor of each skilled worker is the fixed input of manufacturing production, whereas that of each unskilled worker is the variable input of both manufacturing and agricultural production.

We explain the other sectors. Agricultural production is constant returns to scale. Housing is a non-produced consumption good supplied in each place. The total amount of the housing stock in each place is H, which is a constant value.

2.1.2. Preferences and demands

All workers have the same preference. The utility of each worker residing in place $i \in \mathcal{K}$ is quasi-linear:

$$U(C_i^{\mathrm{M}_1}, C_i^{\mathrm{M}_2}, C_i^{\mathrm{H}}, C_i^{\mathrm{A}}) = \alpha_1 \ln C_i^{\mathrm{M}_1} + \alpha_2 \ln C_i^{\mathrm{M}_2} + \beta \ln C_i^{\mathrm{H}} + C_i^{\mathrm{A}}, \tag{1}$$

with $\alpha_1, \alpha_2, \beta > 0$. C_i^{H} denotes the consumption of housing measured in floor space in place i, C_i^{A} denotes the consumption of the agricultural good, and $C_i^{\mathrm{M}_m}$ denotes the CES-consumption index of varieties produced in M^m-sector:

$$C_i^{\mathrm{M}_m} = \left(\sum_{j \in \mathcal{K}} \int_0^{n_j^m} q_{ji}^m(\ell)^{(\sigma^m - 1)/\sigma^m} d\ell \right)^{\sigma^m/(\sigma^m - 1)}.$$

The consumption in place i of variety $\ell \in [0, n_j^m]$ produced in place $j \in \mathcal{K}$ is denoted $q_{ji}^m(\ell)$. n_k^m and $\sigma^m > 1$ are the mass of varieties produced in place k and the constant elasticity of substitution between any two varieties, respectively. The budget constraint of each worker residing in place i is given by

$$\sum_{k \in \mathcal{K}} \left(\int_0^{n_k^1} p_{ki}^1(\ell) q_{ki}^1(\ell) \mathrm{d}\ell + \int_0^{n_k^2} p_{ki}^2(\ell) q_{ki}^2(\ell) \mathrm{d}\ell \right) + p_i^{\mathrm{H}} C_i^{\mathrm{H}} + p_i^{\mathrm{A}} C_i^{\mathrm{A}} = Y_i, \tag{2}$$

where p_i^{A} denotes the price of the agricultural good in place i, $p_{ki}^m(\ell)$ denotes the price of variety ℓ in place i produced in place k, and Y_i denotes the income of an worker residing in place i. The income of each worker consists of wage and land revenue. We assume public ownership of land for simplicity. This income is expressed as

$$Y_i = \begin{cases} Y_i^{\mathrm{u}} \equiv w_i^{\mathrm{u}} + R & \text{(unskilled worker)}, \\ Y_i^1 \equiv w_i^1 + R & \text{(skilled worker of } \mathrm{M}^1 \text{ sector)}, \\ Y_i^2 \equiv w_i^2 + R & \text{(skilled worker of } \mathrm{M}^2 \text{ sector)}, \end{cases} \tag{3}$$

where R is the equally divided land revenue, w_i^{u} is the wage of each unskilled worker in place i, and w_i^m is the wage of each skilled worker supplying labor to a firm of M^m sector.

Each worker residing in place i maximizes the utility (1) subject to the budget constraint (2). Since the utility is quasi-linear, the demand functions other than the demand of the agricultural good are the same across all workers. Using the first-order condition for the utility maximization problem, we obtain the demand functions:

$$C_i^{\mathrm{M}_m} = \frac{\alpha_m}{\rho_i^m}, \quad C_i^{\mathrm{H}} = \frac{\beta}{p_i^{\mathrm{H}}}, \quad C_i^{\mathrm{A}} = Y_i - \alpha_1 - \alpha_2 - \beta, \quad q_{ji}^m(\ell) = \frac{\alpha_m (\rho_i^m)^{\sigma_m - 1}}{p_{ji}^m(\ell)^{\sigma_m}},$$

where ρ_i^m denotes the price index of the varieties consumed in place i:

$$\rho_i^m = \left(\sum_{j \in \mathcal{K}} \int_0^{n_j^m} p_{ji}^m(\ell)^{1 - \sigma_m} d\ell \right)^{1/(1 - \sigma_m)}. \tag{4}$$

Substituting the demand functions into the utility (1), we obtain the indirect utility of a worker who reside in place i:

$$U_i = -\alpha_1 \ln \rho_i^1 - \alpha_2 \ln \rho_i^2 - \beta \ln p_i^H + Y_i + \zeta, \tag{5}$$

where $\zeta = \alpha_1 \ln \alpha_1 + \alpha_2 \ln \alpha_2 + \beta \ln \beta - (\alpha_1 + \alpha_2 + \beta)$ is a constant value that consists of exogenous parameters.

2.1.3. Production

The A-sector is perfectly competitive and each firm of this sector produces agricultural good with constant returns to scale. Each firm needs one unit of unskilled worker to produce one unit of the agricultural good. The agricultural good is assumed to be freely traded across places and to be the numéraire. In market equilibrium, $p_i^{\mathrm{A}} = w_i^{\mathrm{u}} = 1$ $(\forall i \in \mathcal{K})$ holds.

Varieties of M^m sector $(m = 1, 2)$ are produced with increasing returns to scale under monopolistic competition. Each firm of M^m sector needs one unit of skilled worker as the fixed input and c^m units of unskilled workers as the marginal labor requirement to produce one unit of variety. The operating profit of the firm in place i is given by

$$\Pi_i^m(\ell) = \sum_{k \in \mathcal{K}} p_{ik}^m(\ell) Q_{ik}^m(\ell) - \left(c^m x_i^m(\ell) + w_i^m\right), \tag{6}$$

where $Q_{ik}^m(\ell)$ denotes the total demand at place k for variety ℓ produced in place i, and $x_i^m(\ell)$ denotes the total production of this variety produced in place i. $c^m x_i^m(\ell) + w_i^m$ expresses the cost function with the firm. The firm in place i determine price $p_{ik}^m(\ell)$ $(k \in \mathcal{K})$ so that it maximizes the profit (6). Since the total number of workers in place k is $\lambda_k^1 + \lambda_k^2 + L$, $Q_{ik}^m(\ell)$ is given by

$$Q_{ik}^m(\ell) = q_{ik}^m(\ell)(\lambda_k^1 + \lambda_k^2 + L). \tag{7}$$

Varieties are transported between places with transport costs assumed to take the iceberg form. For one unit of each variety transported from place i to place j, only a fraction $1/\tau_{ij} < 1$ arrives $(\tau_{ii} = 1)$. As in Ikeda et al. (2018), we assume $\tau_{ij} = \exp(\tau\, l(i, j))$ which is a function of transport cost parameter $\tau > 0$, integer $l(i, j)$ that expresses the distance between places i and j. The iceberg form of the transport cost determines the total production of each variety: $x_i^m(\ell) = \sum_{k \in \mathcal{K}} \tau_{ik} Q_{ik}^m(\ell)$.

The firm in place i determines price $p_{ik}^m(\ell)$ $(k \in \mathcal{K})$ so that it maximizes the profit (6). The first-order condition for the maximization problem with the profit (6) is given by

$$\frac{Q_{ik}^m(\ell)}{p_{ik}^m(\ell)} \left(p_{ik}^m(\ell) + (p_{ik}^m(\ell) - c^m \tau_{ik}) \eta_{ik}^m(\ell)\right) = 0,$$

where $\eta_{ik}^m(\ell)$ is the price elasticity of the total demand: $\eta_{ik}^m(\ell) = \partial \log Q_{ik}^m(\ell) / \partial \log p_{ik}^m(\ell) = -\sigma_m$ $(\forall \ell \in [0, n_i^m], \forall i, k \in \mathcal{K})$. Using the above first-order condition yields the optimal price of the variety:

$$p_{ik}^m = \frac{\sigma_m}{\sigma_m - 1} \times c\tau_{ik}, \tag{8}$$

where we omit ℓ since the prices of the varieties do not depend on ℓ. This equation implies that $Q_{ik}^m(\ell)$ and $x_i^m(\ell)$ are independent of ℓ. Argument ℓ, thus, is omitted subsequently.

2.1.4. Short-run equilibrium

Following standard procedure for theoretical analyses with NEG models (e.g., Baldwin et al., 2011), we explore market equilibrium with short- and long-run equilibrium conditions. In the short-run equilibrium, skilled workers are immobile between places. Spatial distribution of skilled workers $\boldsymbol{\lambda}^m = (\lambda_i^m)_{i\in\mathcal{K}}$ $(m = 1, 2)$ is assumed to be given in the short-run. The other endogenous variables are determined in the short-run equilibrium. The market equilibrium condition for the short-run consists of four conditions: the housing market clearing condition, the variety market clearing condition, the zero-profit condition under the free entry of firms of M-sector, and the labor market clearing condition.

The housing market clearing condition is given by

$$H = C_i^H(\lambda_i^1 + \lambda_i^2 + L). \tag{9}$$

The variety market clearing condition is given by Eq. (7). The zero-profit condition requires that the operating profit of each firm, given by Eq. (6), is entirely absorbed by the wage payment for its skilled worker:

$$w_i^m = \sum_{k\in\mathcal{K}} p_{ik}^m Q_{ik}^m - c^m x_i^m. \tag{10}$$

The labor market clearing condition is expressed as $n_i^m = \lambda_i^m$ which implies that the mass of firms in each place is equals to that of skilled workers who reside in the place.

The housing market clearing condition (9) and demand for housing C_i^{H} yield the equilibrium price of the housing:

$$p_i^{\mathrm{H}} = \frac{\beta(\lambda_i^1 + \lambda_i^2 + L)}{H}. \tag{11}$$

Using Eq. (8), we obtain price index ρ_i^m, shown in (4), as a function of population distribution $\boldsymbol{\lambda}^m$ and transportation costs:

$$\rho_i^m = \frac{\sigma_m c}{\sigma_m - 1}\left(\sum_{k\in\mathcal{K}} d_{ki}^m \lambda_k^m\right)^{1/(1-\sigma_m)}, \tag{12}$$

where d_{ji}^m is the spatial discounting factor that represents the friction due to transportation between places j and i:

$$d_{ji}^m = \tau_{ji}^{1-\sigma_m} = \exp\left[-\tau\, l(i,j)(\sigma_m - 1)\right] \in (0, 1). \tag{13}$$

As the above equation shows, this factor decreases in an increase in transportation distance $l(i, j)$. Substituting Eqs. (7), (8), (12), and (13) into Eq. (10), we obtain the wage of each skilled worker in the short-run equilibrium:

$$w_i^m = \frac{\alpha_m}{\sigma_m}\sum_{j\in\mathcal{K}} \frac{d_{ij}^m}{\Delta_j^m}(\lambda_j^1 + \lambda_j^2 + L), \tag{14}$$

where Δ_j^m is a function of the population distribution and the spatial discounting factor:

$$\Delta_j^m = \sum_{k\in\mathcal{K}} d_{kj}^m \lambda_k^m.$$

We can rewrite the price index with this factor: $\rho_i^m = (\Delta_i^m)^{1/(1-\sigma_m)}\sigma_m c/(\sigma_m - 1)$.

Indirect utility with the short-run equilibrium is a function of population distribution $\boldsymbol{\lambda} = (\boldsymbol{\lambda}^1, \boldsymbol{\lambda}^2)$. Let $U_i^{\mathrm{u}}(\boldsymbol{\lambda}^1, \boldsymbol{\lambda}^2, \tau)$ and $U_i^m(\boldsymbol{\lambda}^1, \boldsymbol{\lambda}^2, \tau)$ $(m = 1, 2)$ denote the indirect utility of the unskilled worker and the skilled worker of M^m sector who reside in place i, respectively. We explicitly show transportation cost τ in the arguments as a representative exogenous parameter that affects the market equilibrium. Substituting the price of the housing (11), the price index (12) into the indirect utility (5) and using Eq. (3), we obtain the indirect utility with the FE model:

$$U_i^{\mathrm{u}}(\boldsymbol{\lambda}^1, \boldsymbol{\lambda}^2, \tau) = \frac{\alpha_1}{\sigma_1 - 1} \ln \Delta_i^1 + \frac{\alpha_2}{\sigma_2 - 1} \ln \Delta_i^2 - \beta \ln \Lambda_i + 1 + \xi, \tag{15a}$$

$$U_i^1(\boldsymbol{\lambda}^1, \boldsymbol{\lambda}^2, \tau) = \frac{\alpha_1}{\sigma_1 - 1} \ln \Delta_i^1 + \frac{\alpha_2}{\sigma_2 - 1} \ln \Delta_i^2 - \beta \ln \Lambda_i + w_i^1 + \xi, \tag{15b}$$

$$U_i^2(\boldsymbol{\lambda}^1, \boldsymbol{\lambda}^2, \tau) = \frac{\alpha_1}{\sigma_1 - 1} \ln \Delta_i^1 + \frac{\alpha_2}{\sigma_2 - 1} \ln \Delta_i^2 - \beta \ln \Lambda_i + w_i^2 + \xi, \tag{15c}$$

where $\Lambda_i = \lambda_i^1 + \lambda_i^2 + L$ and ξ is a constant value that consists of exogenous parameters. The first and second terms in each equation in (15) are the effects of the price indices on utility, the third term is the effect of the housing price, and the fourth term is the effect of the wage. Note that the wage of the unskilled workers is one.

2.1.5. Long-run equilibrium

In the long-run, each skilled worker can decide where to reside. Long-run equilibrium is defined as a spatial distribution of skilled workers $(\boldsymbol{\lambda}^1, \boldsymbol{\lambda}^2) = (\boldsymbol{\lambda}^{1*}, \boldsymbol{\lambda}^{2*})$ that satisfies the following conditions:

$$\begin{cases} U^{m*} - U_i^m(\boldsymbol{\lambda}^{1*}, \boldsymbol{\lambda}^{2*}, \tau) = 0 & \text{if } \lambda_i^{m*} > 0, \\ U^{m*} - U_i^m(\boldsymbol{\lambda}^{1*}, \boldsymbol{\lambda}^{2*}, \tau) \geq 0 & \text{if } \lambda_i^{m*} = 0, \end{cases} \tag{16}$$

and $\sum_{i \in \mathcal{K}} \lambda_i^m = 1$ $(m = 1, 2)$. U^{m*} denotes the equilibrium utility level for skilled workers of M^m sector.

We can explore the long-run equilibrium by solving a dynamics that expresses a change in the spatial distribution (Sandholm, 2010). We explore long-run equilibria with the replicator dynamics, which is employed in theoretical studies with NEG models in which there are multiple places (e.g., Gaspar et al., 2018):

$$\frac{\mathrm{d}\boldsymbol{\lambda}}{\mathrm{d}t} = \boldsymbol{F}(\boldsymbol{\lambda}^1, \boldsymbol{\lambda}^2, \tau), \tag{17}$$

where $\boldsymbol{F}(\boldsymbol{\lambda}^1, \boldsymbol{\lambda}^2, \tau) = (F_i^m(\boldsymbol{\lambda}^1, \boldsymbol{\lambda}^2, \tau))_{i \in \mathcal{K}, m \in \{1,2\}}$, and

$$F_i^m(\boldsymbol{\lambda}^1, \boldsymbol{\lambda}^2, \tau) = \lambda_i^m \left[U_i^m(\boldsymbol{\lambda}^1, \boldsymbol{\lambda}^2, \tau) - \overline{U^m}(\boldsymbol{\lambda}^1, \boldsymbol{\lambda}^2, \tau) \right] \quad i \in \mathcal{K}, m \in \{1, 2\}. \tag{18}$$

Here, $\overline{U^m} = \sum_{i \in \mathcal{K}} \lambda_i^m U_i^m$ is the weighted average utility with skilled workers of M^m sector. Stationary points are defined as solutions to the following equation:

$$\boldsymbol{F}(\boldsymbol{\lambda}^1, \boldsymbol{\lambda}^2, \tau) = \boldsymbol{0}. \tag{19}$$

The stable stationary points are stable long-run equilibria (Sandholm, 2010). The stability of a stationary point is determined with the eigenvalues of the Jacobian matrix of $\boldsymbol{F}$: $J = \partial \boldsymbol{F} / \partial \boldsymbol{\lambda}$. If all the real parts of the eigenvalues of J are negative, then the associated stationary point is stable, implying that this point is a stable long-run equilibrium. This Jacobian matrix with the FE model is shown in Appendix A.1.

2.2. Footloose entrepreneur model with remote work

To elucidate how equilibrium changes after skilled workers can remotely supply labor (i.e., remote work) across places, we develop a footloose entrepreneur model with remote work. We call this model the FER model. Since basic framework of the FER model is the same as the FE model, the FER model is a theoretically extended FE model in terms of location choices of skilled workers due to remote work. We compare an equilibrium of the FER model and that of the FE model to explore the impacts of remote work on workers (Sections 4 and 5).

2.2.1. Framework of the FER model

The differences between the FER model and the FE model are only the location choices of skilled workers in terms of remote work. The other assumptions of the FER model are the same as those of the FE model. Following Brueckner and Sayantani (2023), we express remote work in the FER model as follows. In the FER model, we assume that only the skilled workers of M^1 sector can work remotely. Skilled workers of M^1 sector can freely supply their labor to any place, whereas skilled workers of M^2 sector can supply their labor to only the place where they reside. Such an assumption can be interpreted as a heterogeneity among industries regarding the possibility of introducing remote work in the real world.

Preference and budget constraint of each worker in the FER model are the same as those of the FE model (i.e., Eqs. (1) and (2)). On the other hand, we consider that skilled workers of M^1 sector can supply their labor to places where they do not reside. Let w^1_{ij} denote the wage of skilled workers of M^1 sector who reside in place i and supply their labor force to firms which operate in place j. The difference in the expression of the wage between the FE and FER models can alter the population distribution in the long-run equilibrium with the FE model (Section 2.2.3). The number of skilled workers who reside in place i and supply labor to firms which operate in place j is denoted by λ^1_{ij} under the normalizing constant $\sum_{i,j\in\mathcal{K}} \lambda^1_{ij} = 1$. The income of each worker is given by

$$Y_i = \begin{cases} Y^{\mathrm{u}}_i \equiv w^{\mathrm{u}}_i + R & \text{(unskilled worker)}, \\ Y^1_{ij} \equiv w^1_{ij} + R & \text{(skilled worker of } \mathrm{M}^1 \text{ sector)}, \\ Y^2_i \equiv w^2_i + R & \text{(skilled worker of } \mathrm{M}^2 \text{ sector)}. \end{cases} \tag{20}$$

As the above definition indicates, the total of the location choices of skilled workers of M^1 sector with the FER model is $K \times K$, whereas the total with the FE model is K (see Eq. (3)).

The production of firms of M^1 sector in the FER model differs from that in the FE model. We assume that firms of M^1 sector can produce varieties with labor not dependent on the location of the labor. Let $\Pi^1_{ij}(\ell)$ denote the profit of firm which operates in place i, employs a skilled worker who reside j, and supplies variety ℓ. Note that the definition of the profit implies that $\Pi^1_{ii}(\ell)$ is the operating profit of the firm which employs a skilled labor who does not work remotely, and $\Pi^1_{ij}(\ell)$ $(i \neq j)$ is that of the firm which employs a remote worker. The profit of a firm of M^m sector is given by

$$\Pi^1_{ij}(\ell) = \sum_{k\in\mathcal{K}} p^1_{ik}(\ell) Q^1_{ik}(\ell) - \left(c^1 x^1_i(\ell) + w^1_{ji}\right), \tag{21a}$$

$$\Pi^2_{i}(\ell) = \sum_{k\in\mathcal{K}} p^2_{ik}(\ell) Q^2_{ik}(\ell) - \left(c^2 x^2_i(\ell) + w^2_{i}\right). \tag{21b}$$

where $c^1 x^1_i(\ell) + w^1_{ji}$ and $c^2 x^2_i(\ell) + w^2_i$ are cost functions of firms of M^1 and M^2 sectors, respectively.

2.2.2. Short-run equilibrium

As in the FE model, in the short-run equilibrium, the population distribution of skilled worker is given. The population distribution of skilled workers of M^1 and that of M^2 are given by $\boldsymbol{\lambda}^1 = (\lambda^1_{ij})_{i,j\in\mathcal{K}}$ and $\boldsymbol{\lambda}^2 = (\lambda^2_i)_{i\in\mathcal{K}}$, respectively. The market equilibrium condition for the short-run equilibrium consists of four conditions: the housing market clearing condition, the variety market clearing condition, the zero-profit condition under the free entry of firms, and the labor market clearing condition.

The first and the second conditions are the same as the conditions introduced in Section 2.1.4 (i.e., Eqs. (7) and (9)). The third condition requires that the operating profits of the firms, given by Eqs. (21a) and (21b), are entirely absorbed by the wage payment for the skilled workers:

$$w_{ij}^1 = \sum_{k\in\mathcal{K}} p_{jk}^1 Q_{jk}^1 - c^1 x_j^1, \tag{22a}$$

$$w_i^2 = \sum_{k\in\mathcal{K}} p_{ik}^2 Q_{ik}^2 - c^2 x_i^2. \tag{22b}$$

Eq. (22a) indicates that the wage paid to skilled workers who supply labor force to firms in place j is not dependent on places where they reside. The fourth condition is expressed as $n_i^1 = \sum_{k\in\mathcal{K}} \lambda_{ki}^1$ and $n_i^2 = \lambda_i^2$.

Indirect utility can be expressed as a function of the spatial distribution of workers as the FE model. Let $V_i^{\mathrm{u}}(\boldsymbol{\lambda}^1, \boldsymbol{\lambda}^2, \tau)$, $V_{ij}^1(\boldsymbol{\lambda}^1, \boldsymbol{\lambda}^2, \tau)$, and $V_i^2(\boldsymbol{\lambda}^1, \boldsymbol{\lambda}^2, \tau)$ denote the indirect utility of the unskilled worker, that of the skilled worker of M^1 sector who resides in place i and supplies his labor to a firm in place j, and that of the skilled worker of M^2 sector who resides in place i, respectively. Using the market equilibrium conditions, we can analytically obtain these functions (see Appendix A.2 for detail of the derivation)

$$V_i^{\mathrm{u}}(\boldsymbol{\lambda}^1, \boldsymbol{\lambda}^2, \tau) = \frac{\alpha_1}{\sigma_1 - 1}\ln N_i^1 + \frac{\alpha_2}{\sigma_2 - 1}\ln \Delta_i^2 - \beta \ln \Lambda_i + 1 + \xi, \tag{23a}$$

$$V_{ij}^1(\boldsymbol{\lambda}^1, \boldsymbol{\lambda}^2, \tau) = \frac{\alpha_1}{\sigma_1 - 1}\ln N_i^1 + \frac{\alpha_2}{\sigma_2 - 1}\ln \Delta_i^2 - \beta \ln \Lambda_i + w_{ij}^1 + \xi, \tag{23b}$$

$$V_i^2(\boldsymbol{\lambda}^1, \boldsymbol{\lambda}^2, \tau) = \frac{\alpha_1}{\sigma_1 - 1}\ln N_i^1 + \frac{\alpha_2}{\sigma_2 - 1}\ln \Delta_i^2 - \beta \ln \Lambda_i + w_i^2 + \xi, \tag{23c}$$

where $N_i^1 = \sum_{o,k\in\mathcal{K}} d_{oi}^1 \lambda_{ko}^1$,

$$w_{ij}^1 = \frac{\alpha_1}{\sigma_1}\sum_{k\in\mathcal{K}}\left[\frac{d_{jk}^1}{N_k^1}\left(\lambda_k^1 + \lambda_k^2 + L\right)\right], \tag{24a}$$

$$w_i^2 = \frac{\alpha_2}{\sigma_2}\sum_{k\in\mathcal{K}}\frac{d_{ik}^2}{\Delta_k^2}(\lambda_k^1 + \lambda_k^2 + L). \tag{24b}$$

Here, $\lambda_j^1 = \sum_{m\in\mathcal{K}} \lambda_{jm}^1$ is the total of skilled workers of M^1 sector who reside in place i. Note that $\lambda_i^1 = \sum_{o\in\mathcal{K}} \lambda_{io}^1$ denotes the total number of skilled workers belonging to M^1-sector and residing in place i. As in the FE model, the first and second terms in each equation in (24) are the effects of the price indices on utility, the third term is the effect of the housing price, and the fourth term is the effect of the wage.

2.2.3. Long-run equilibrium

The long-run equilibrium is defined as a spatial distribution of skilled workers $(\boldsymbol{\lambda}^1, \boldsymbol{\lambda}^2) = (\boldsymbol{\lambda}^{1*}, \boldsymbol{\lambda}^{2*})$ that satisfies the conditions of

$$\begin{cases} V^{1*} - V_{ij}^1(\boldsymbol{\lambda}^{1*}, \boldsymbol{\lambda}^{2*}, \tau) = 0 & \text{if } \lambda_{ij}^{1*} > 0, \\ V^{1*} - V_{ij}^1(\boldsymbol{\lambda}^{1*}, \boldsymbol{\lambda}^{2*}, \tau) \geq 0 & \text{if } \lambda_{ij}^{1*} = 0, \end{cases} \tag{25a}$$

$$\begin{cases} V^{2*} - V_i^2(\boldsymbol{\lambda}^{1*}, \boldsymbol{\lambda}^{2*}, \tau) = 0 & \text{if } \lambda_i^{2*} > 0, \\ V^{2*} - V_i^2(\boldsymbol{\lambda}^{1*}, \boldsymbol{\lambda}^{2*}, \tau) \geq 0 & \text{if } \lambda_i^{2*} = 0, \end{cases} \tag{25b}$$

$\sum_{i,j\in\mathcal{K}} \lambda_{ij}^1 = 1$, and $\sum_{i\in\mathcal{K}} \lambda_i^2 = 1$. V^{m*} denotes the equilibrium utility level for skilled workers of M^m-sector. We explore stable spatial equilibria with the replicator dynamics:

$$\frac{\mathrm{d}\boldsymbol{\lambda}}{\mathrm{d}t} = \boldsymbol{F}(\boldsymbol{\lambda}^1, \boldsymbol{\lambda}^2, \tau), \tag{26}$$

where $\boldsymbol{F}(\lambda, \tau) = (F_{ij}^1, F_i^2)_{i,j\in\mathcal{K}, m\in\{1,2\}}$ and

$$F_{ij}^1(\boldsymbol{\lambda}, \tau) = \lambda_{ij}^1 \left[V_{ij}^1(\boldsymbol{\lambda}^1, \boldsymbol{\lambda}^2, \tau) - \overline{V^1}(\boldsymbol{\lambda}^1, \boldsymbol{\lambda}^2, \tau)\right] \quad i, j \in \mathcal{K}, \tag{27a}$$

$$F_i^2(\boldsymbol{\lambda}, \tau) = \lambda_i^2 \left[V_i^2(\boldsymbol{\lambda}^1, \boldsymbol{\lambda}^2, \tau) - \overline{V^2}(\boldsymbol{\lambda}^1, \boldsymbol{\lambda}^2, \tau)\right] \quad i \in \mathcal{K}. \tag{27b}$$

Here, $\overline{V^1} = \sum_{i,j\in\mathcal{K}} \lambda_{ij}^1 V_{ij}^1$ and $\overline{V^2} = \sum_{i\in\mathcal{K}} \lambda_i^2 V_i^2$ are the weighted average utilities with the FER model. Stationary points are defined as solutions to $\boldsymbol{F}(\boldsymbol{\lambda}^1, \boldsymbol{\lambda}^2, \tau) = \mathbf{0}$.

A market equilibrium with the FE model is not necessarily that with the FER model, since the Jacobian matrix differs from the matrix with the FE model. At an unstable equilibrium, some skilled workers have incentive to change their location choices in terms of places where they reside and to which they supplies labor. We can observe this incentive as follows. Let $\left(\widetilde{\boldsymbol{\lambda}}_{\text{FE}}^1, \widetilde{\boldsymbol{\lambda}}_{\text{FE}}^2\right)$ denote a market equilibrium with the FE model. We define the components of $\widetilde{\boldsymbol{\lambda}}_{\text{FE}}^1$ and $\widetilde{\boldsymbol{\lambda}}_{\text{FE}}^2$ as follows.

$$\widetilde{\boldsymbol{\lambda}}_{\text{FE}}^1 = \left(\widetilde{\lambda}_1^1, \widetilde{\lambda}_2^1, \ldots, \widetilde{\lambda}_K^1\right), \quad \widetilde{\boldsymbol{\lambda}}_{\text{FE}}^2 = \left(\widetilde{\lambda}_1^2, \widetilde{\lambda}_2^2, \ldots, \widetilde{\lambda}_K^2\right).$$

Using these components, we define the associated population distribution with the FER model:

$$\widetilde{\boldsymbol{\lambda}}_{\text{FER}}^1 = \left(\widetilde{\boldsymbol{\lambda}}_1^1, \widetilde{\boldsymbol{\lambda}}_2^1, \ldots, \widetilde{\boldsymbol{\lambda}}_K^1\right), \quad \widetilde{\boldsymbol{\lambda}}_i^1 = \left(\underbrace{0, \ldots, 0}_{i-1}, \widetilde{\lambda_i^1}, 0, \ldots, 0\right), \quad \widetilde{\boldsymbol{\lambda}}_{\text{FER}}^2 = \widetilde{\boldsymbol{\lambda}}_{\text{FE}}^2.$$

Using these vectors and the definitions of the indirect utilities in the FE and FER models, we obtain indirect utility that skilled workers of M^1 sector who reside in place i, but does not remotely work: $V_{ii}^1\left(\widetilde{\boldsymbol{\lambda}}_{\text{FER}}^1, \widetilde{\boldsymbol{\lambda}}_{\text{FER}}^2, \tau\right) = U_i^1\left(\widetilde{\boldsymbol{\lambda}}_{\text{FE}}^1, \widetilde{\boldsymbol{\lambda}}_{\text{FE}}^2, \tau\right)$. If a remote worker can obtain higher indirect utility than the indirect utility of a non-remote worker with spatial distribution $\left(\widetilde{\boldsymbol{\lambda}}_{\text{FER}}^1, \widetilde{\boldsymbol{\lambda}}_{\text{FER}}^2\right)$ (i.e., $V_{ii}^1\left(\widetilde{\boldsymbol{\lambda}}_{\text{FER}}^1, \widetilde{\boldsymbol{\lambda}}_{\text{FER}}^2, \tau\right) < V_{ij}^1\left(\widetilde{\boldsymbol{\lambda}}_{\text{FER}}^1, \widetilde{\boldsymbol{\lambda}}_{\text{FER}}^2, \tau\right)$ $(\exists j \neq i)$), then this spatial distribution does not satisfy the equilibrium condition (25a). For example, if a remote worker can obtain a high wage or avoid to incur a high cost of housing, then equilibrium condition (25a) does not necessarily hold with spatial distribution $\left(\widetilde{\boldsymbol{\lambda}}_{\text{FER}}^1, \widetilde{\boldsymbol{\lambda}}_{\text{FER}}^2\right)$. We numerically demonstrate that a stable equilibrium with the FE model is unstable with the FER model (Section 4.2).

2.2.4. Social welfare

We evaluate the welfare impact caused by the introduction of remote work. Social welfare is defined as the Bentham welfare function:

$$W(\boldsymbol{\lambda}^1, \boldsymbol{\lambda}^2, \tau) = \sum_{i\in\mathcal{K}} \left(L V_i^{\mathrm{u}}(\boldsymbol{\lambda}^1, \boldsymbol{\lambda}^2, \tau) + \sum_{k\in\mathcal{K}} \left(\lambda_{ik}^1 V_{ik}^1(\boldsymbol{\lambda}^1, \boldsymbol{\lambda}^2, \tau)\right) + \lambda_i^2 V_i^2(\boldsymbol{\lambda}^1, \boldsymbol{\lambda}^2, \tau)\right). \tag{28}$$

The Bentham social welfare has been employed in welfare analyses with NEG models (e.g., Ottaviano et al., 2002; Pflüger and Südekum, 2008).

3. Emergence of remote workers from an unstable equilibrium of the FER model

We explore the emergence of remote workers after the introduction of remote work. Our paper regards this emergence as the change from an unstable equilibrium such that there are no remote workers to a stable equilibrium such that there are remote workers. In this section, we explore a condition of the emergence of the FER model. We show that this condition relates to the stability condition for the equilibrium of the FER model.

We focus on the emergence of remote workers who reside in large city in an economy before the introduction of remote work. To examine this emergence, we explore equilibrium such that some skilled workers remotely

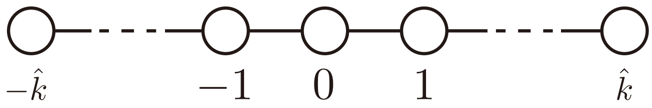


Figure 2: Long narrow economy

work and the other skilled workers reside in the large city. The remote work in our paper implies that a skilled worker of M^1 sector chooses location choice such that the locations of his residence and the firm to which he supplies labor differ.

As a spatial setting, we employ a long narrow economy which is employed in theoretical analyses in NEG models with multiple places (Ikeda et al., 2017, 2024). In this economy, places are equally spread on a line segment. The long narrow economy has an odd number of places as shown in Figure 2. The set of places $\mathcal{K}$ is expressed as

$$\mathcal{K} = \left\{-\hat{k}, -\hat{k}+1, \ldots, -1, 0, 1, \ldots, \hat{k}-1, \hat{k}\right\} \quad \left(\hat{k} = 1, 2, \ldots\right).$$

These assumptions imply the economy has the center. Distance between places and the spatial discount factor are given by, respectively,

$$l(i,j) = |i-j| \quad (i, j \in \mathcal{K}), \quad d^m_{ji} = \exp\left(-\tau\,|i-j|\,(\sigma_m - 1)\right).$$

We examine the emergence of remote workers from the full agglomeration where all the skilled workers reside in one place. This population distribution has been focused on in theoretical analyses with NEG models. The mathematical definition of the full agglomeration is $\boldsymbol{\lambda}_{\mathrm{FA}} = (\boldsymbol{\lambda}^1_{\mathrm{FA}}, \boldsymbol{\lambda}^2_{\mathrm{FA}})$, where

$$\boldsymbol{\lambda}^m_{\mathrm{FA}} = (\underbrace{0, \ldots, 0}_{\hat{k}}, 1, \underbrace{0, \ldots, 0}_{\hat{k}}).$$

With the full agglomeration, all the skilled workers reside in the central place. This central place is interpreted as the large city in the economy with the FER model.

We explore the emergence of remote workers in the economy not having any advantage other than geographical advantage. This economy can be expressed as the following symmetric conditions in terms of the indirect utilities of the skilled workers:

$$V^1_{-i,-i}(\boldsymbol{\lambda}_{\mathrm{FA}}, \tau) = V^1_{i,i}(\boldsymbol{\lambda}_{\mathrm{FA}}, \tau) \quad \forall i \in \mathcal{K}, \tag{29a}$$

$$V^2_{-i}(\boldsymbol{\lambda}_{\mathrm{FA}}, \tau) = V^2_{i}(\boldsymbol{\lambda}_{\mathrm{FA}}, \tau) \quad \forall i \in \mathcal{K}, \tag{29b}$$

$$V^1_{-i,0}(\boldsymbol{\lambda}_{\mathrm{FA}}, \tau) = V^1_{i,0}(\boldsymbol{\lambda}_{\mathrm{FA}}, \tau) \quad \forall i \in \mathcal{K}, \tag{29c}$$

$$V^1_{0,-i}(\boldsymbol{\lambda}_{\mathrm{FA}}, \tau) = V^1_{0,i}(\boldsymbol{\lambda}_{\mathrm{FA}}, \tau) \quad \forall i \in \mathcal{K}, \tag{29d}$$

where we insert a comma in indirect utilities in the above equations in order to discriminate whether the notation i denotes the place where a worker lives or the location of the firm to which he belongs. The conditions (29a) and (29b) imply that the levels of indirect utilities that skilled workers can obtain are determined by the distance from the central place with the full agglomeration. The condition (29c) implies that the levels of indirect utilities that remote workers who reside in peripheral places can obtain are also determined by the distance between the central place and the places where they reside, whereas the condition (29d) implies that those that remote workers who resided in the central place can obtain are also determined by the distance of the central place and the places where the firms to which remote workers operated.

Full agglomeration $\boldsymbol{\lambda}_{\mathrm{FA}}$ not only expresses the agglomeration of the skilled workers in the economy but also possesses desirable attributes that allow us to investigate market equilibria. A population distribution is called an invariant pattern if this distribution retains a stationary point for any values of exogenous parameter (Ikeda et al., 2018). Full agglomeration $\boldsymbol{\lambda}_{\mathrm{FA}}$ is an invariant pattern as shown in the following.

$$\begin{aligned}
&F^1_{00}(\boldsymbol{\lambda}_{\mathrm{FA}}, \tau) = V^1_{00}(\boldsymbol{\lambda}_{\mathrm{FA}}, \tau) - V^1_{00}(\boldsymbol{\lambda}_{\mathrm{FA}}, \tau) = 0,\\
&F^1_{ij}(\boldsymbol{\lambda}_{\mathrm{FA}}, \tau) = 0 \times [V^1_{ij}(\boldsymbol{\lambda}_{\mathrm{FA}}, \tau) - V^1_{00}(\boldsymbol{\lambda}_{\mathrm{FA}}, \tau)] = 0 \quad (i,j) \neq (0,0),\\
&F^2_{0}(\boldsymbol{\lambda}_{\mathrm{FA}}, \tau) = V^2_{0}(\boldsymbol{\lambda}_{\mathrm{FA}}, \tau) - V^2_{0}(\boldsymbol{\lambda}_{\mathrm{FA}}, \tau) = 0,\\
&F^2_{i}(\boldsymbol{\lambda}_{\mathrm{FA}}, \tau) = 0 \times [V^2_{i}(\boldsymbol{\lambda}_{\mathrm{FA}}, \tau) - V^2_{0}(\boldsymbol{\lambda}_{\mathrm{FA}}, \tau)] = 0 \quad (i \neq 0).
\end{aligned}$$

A stable full agglomeration is a stable market equilibrium (Sandholm, 2010). The stability of full agglomeration $\boldsymbol{\lambda}_{\mathrm{FA}}$ is determined by the signs of the real parts of the eigenvalues of Jacobian matrix J. We can appropriately permute the components of $\boldsymbol{\lambda}_{\mathrm{FA}}$ and $\boldsymbol{F}$ to arrive at $\hat{\boldsymbol{\lambda}} = (1,1,0,\dots,0)$ and $\hat{\boldsymbol{F}} = (F^1_{00}, F^2_0, \dots)$. For these vectors, the associated Jacobian matrix can be rearranged as (Ikeda et al., 2018)

$$\hat{J} = \frac{\partial \hat{\boldsymbol{F}}}{\partial \hat{\boldsymbol{\lambda}}} = \begin{pmatrix} J_+ & J_{+0} \\ O & J_0 \end{pmatrix},$$

where J_+ and J_{+0} are matrices whose components are non zero, and

$$\begin{aligned}
J_0 = \mathrm{diag}\Big(&V^1_{-\hat{k},0} - V_{00}, V^1_{\hat{k},0} - V_{00}, \dots, V^1_{-1,0} - V_{00}, V^1_{1,0} - V_{00},\\
&V^2_{-\hat{k}} - V^2_0, V^2_{\hat{k}} - V^2_0, \dots, V^2_{-1} - V^2_0, V^2_1 - V^2_0\Big),
\end{aligned}$$

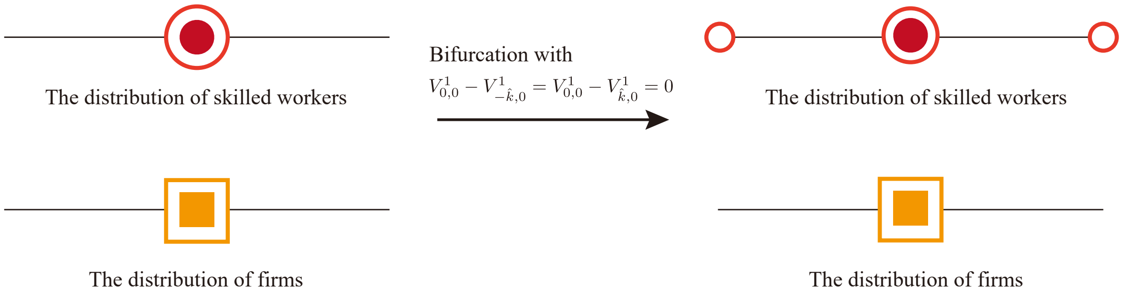
where $\mathrm{diag}(\cdot)$ denotes a diagonal matrix with the elements in the parentheses. Full agglomeration $\boldsymbol{\lambda}_{\mathrm{FA}}$ is stable if all the eigenvalues of both J_+ and J_0 have negative real part.

As J_0 shows, the stability of full agglomeration $\boldsymbol{\lambda}_{\mathrm{FA}}$ is determined by the differences in the utilities. With the FER model, differences between indirect utilities with non-remote work and remote work affect the stability of full agglomeration $\boldsymbol{\lambda}_{\mathrm{FA}}$ (i.e., $V^1_{i,0} - V_{00}$ and $V^1_{0,i} - V_{00}$). If the indirect utility with remote work exceeds the indirect utility with the central large city, then full agglomeration $\boldsymbol{\lambda}_{\mathrm{FA}}$ is unstable. Hence, the stability condition of full agglomeration with the FER model differs from that with the FE model. An unstable full agglomeration is changed to other population distribution by a slight shock and the myopic behavior of skilled workers over time.

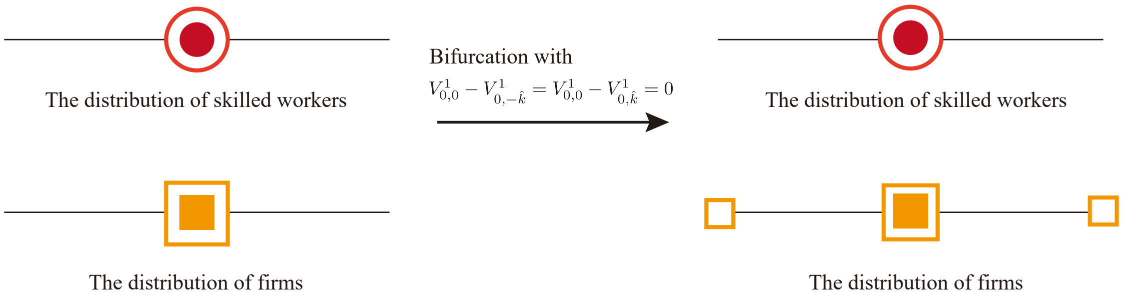
To explore the emergence of remote workers, we conduct a bifurcation analysis for full agglomeration $\boldsymbol{\lambda}_{\mathrm{FA}}$. If the difference in indirect utilities $V^1_{i,j} - V^1_{00}$ $(\exists i, j)$ is zero, then the Jacobian matrix is singular. Actually, with the replicator dynamics, the following proposition shows that this singularity is also the sufficient condition for the emergence of a bifurcating solution; the condition of stability with equilibrium is related to the condition of emergence of bifurcation solution from the full agglomeration.

Proposition 1. *We have the following theoretical results:*

(i) *If $V_{j,0} - V_{0,0} = 0$ holds at $(\boldsymbol{\lambda}_{\mathrm{FA}}, \tau)$, then a bifurcation solution with $\lambda_{0,0}, \lambda_{j,0} = \lambda_{-j,0} > 0$ emerges from $(\boldsymbol{\lambda}_{\mathrm{FA}}, \tau)$.*



(a) Emergence remote workers supplying labor to firms in the central city



(b) Emergence remote workers supplying labor to firms in peripheral cities

Figure 3: Examples of theoretically predicted bifurcation behavior

(ii) *If* $V_{0,j} - V_{0,0} = 0$ *holds at* $(\boldsymbol{\lambda}_{\mathrm{FA}}, \tau)$*, then a bifurcation solution with* $\lambda_{0,0}, \lambda_{0,j} = \lambda_{0,-j} > 0$ *emerges from* $(\boldsymbol{\lambda}_{\mathrm{FA}}, \tau)$*.*

Proof. See Appendix B. □

An intuitive explanation of Proposition 1 is as follows. Each skilled worker changes their location choices to obtain larger indirect utility under myopic behavior. Such a myopic behavior changes the population distribution of the economy. If workers can obtain higher utility by conducting remote work with the full agglomeration, remote workers increase over time.

Predicted bifurcation solutions which have an important role in our paper are shown in Figure 3. If $V_{0,0} - V_{j,0} = 0$, then the bifurcation solution expresses the population distribution with remote workers who reside in place $-j$ and j emerges (Fig. 3(a)). With this solution, remote workers supply labor to firms operating in the central large city (i.e., place 0). On the other hand, if $V_{0,0} - V_{0,j} = 0$, then the bifurcation solution expresses the population distribution with remote workers who supply labor to firms operating place $-j$ and j (Fig. 3(b)). With this solution, remote workers belong to firms operating in places around the center.

We refer to theoretical aspects of Proposition 1. First, this proposition, which shows the existence of the bifurcation solution, is an extension of theoretical bifurcation analysis of NEG models with one industry (Ikeda et al., 2024). Second, this proposition is proved by using only the differentiability and the symmetry of the indirect utility function in terms of places (see Appendix B for detail). The theoretical result, thus, is applicable to other economic models which express the location choices of economic agents.

4. Bifurcation solution from the full agglomeration

4.1. Qualitative analysis with the full agglomeration

Using Proposition 1, we can conduct the bifurcation analysis with the difference in the indirect utility, which is a desirable attribute of the full agglomeration. This difference depends on exogenous parameters. Remote workers can emerge with a change in transportation cost parameter τ as theoretical studies (e.g., Baldwin et al., 2011) have shown that the transportation cost has an important role for the change in stable population distribution in the urban system.

We examine whether remote workers who reside in peripheral cities and supply labor to the central city appear. Using the indirect utility (23b), we obtain the difference in the utility of remote work and no remote work with the full agglomeration:

$$V_{j0}^1(\boldsymbol{\lambda}_{\mathrm{FA}},\tau) - V_{00}^1(\boldsymbol{\lambda}_{\mathrm{FA}},\tau) = \frac{\alpha_1}{\sigma_1 - 1}\ln d_{0j}^1 + \frac{\alpha_2}{\sigma_2 - 1}\ln d_{0j}^2 + \ \beta \ln\left(\frac{L+2}{L}\right). \tag{30}$$

The first and second terms are the effects of relocating to a different residence on the price index, and the third term is the effect on housing prices. The transportation cost does not affect the third term, but the first and second terms. As the third term shows, the relocation has a positive impact in terms of housing price (i.e., $\ln(L+2/L) > 0$). Transportation cost affects the stability of the full agglomeration since the sign of the difference in the effects of the price indices changes with a change in the level of this cost. We can decide on these signs with extreme cases for the transportation cost parameter:

$$\begin{aligned}\lim_{\tau\to 0}\left[V_{j0}^1(\boldsymbol{\lambda}_{\mathrm{FA}},\tau) - V_{00}^1(\boldsymbol{\lambda}_{\mathrm{FA}},\tau)\right] &= \beta\ln\left(\frac{L+2}{L}\right) > 0,\\ \lim_{\tau\to\infty}\left[V_{j0}^1(\boldsymbol{\lambda}_{\mathrm{FA}},\tau) - V_{00}^1(\boldsymbol{\lambda}_{\mathrm{FA}},\tau)\right] &= -\infty < 0.\end{aligned}$$

As the above inequalities show, the full agglomeration is unstable with low transportation costs. This intuitively implies that, with low transportation costs, the negative effect of remote work on the price index is relatively small compared to the positive effect on housing prices. Moreover, the intermediate value theorem ensures that there exists transportation cost such that $V_{j0}^1 - V_{00}^1 = 0$ holds. At this cost, a bifurcation solution such that remote workers appear emerges (Proposition 1 (i)). Skilled workers have an incentive to relocate their locations.

Next, we explore whether remote workers who reside in the central city and supply labor to firms in peripheral places appear. Using the indirect utility (23b), we obtain the following difference in utility with the full agglomeration:

$$V_{0j}^1(\boldsymbol{\lambda}_{\mathrm{FA}},\tau) - V_{00}^1(\boldsymbol{\lambda}_{\mathrm{FA}},\tau) = w_{0j}^1 - w_{00}^1. \tag{31}$$

The above equation implies that the difference in the wage affects the stability of full agglomeration $\boldsymbol{\lambda}_{\mathrm{FA}}$, whereas the price index and housing price do not affect that of this agglomeration. Using Eq. (24a), we obtain the RHS of Eq. (31) with full agglomeration $\boldsymbol{\lambda}_{\mathrm{FA}}$:

$$w_{0j}^1 - w_{00}^1 = \frac{\alpha_1}{\sigma_1}\left[\underbrace{2(d_{j0}^1 - 1)}_{<0} + \underbrace{L\sum_{k\in\mathcal{K}}\left(\frac{d_{jk}^1}{d_{0k}^1} - 1\right)}_{\gtreqless 0}\right]. \tag{32}$$

As the above equation shows, transportation cost τ and the number of unskilled workers L determine whether the wage of remote work in a peripheral city takes a higher value than the central city. Full agglomeration $\boldsymbol{\lambda}_{\mathrm{FA}}$ is unstable with high transportation costs since the wage with remote work exceeds that with no remote work:

$$\lim_{\tau\to\infty}(w_{0j}^1 - w_{00}^1) = \frac{\alpha_1}{\sigma_1}\left[\lim_{\tau\to\infty} \quad 2(d_{j0}^1 - 1) + \lim_{\tau\to\infty} L\sum_{k\in\mathcal{K}}\left(\frac{d_{jk}^1}{d_{0k}^1} - 1\right)\right] = \frac{\alpha_1}{\sigma_1}(-2+\infty) > 0 \quad (j \neq 0). \tag{33}$$

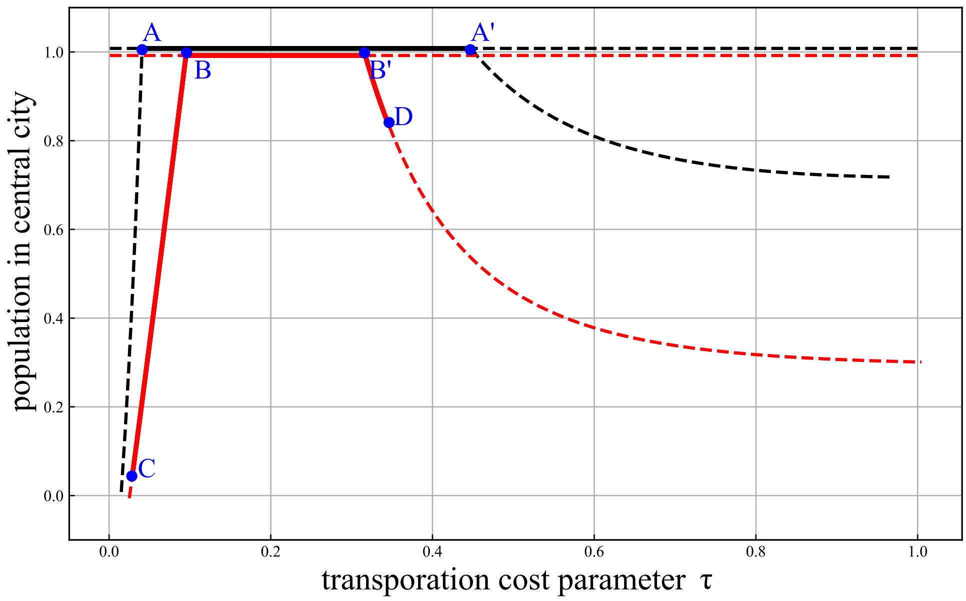
This equation intuitively implies that skilled workers who conduct remote work can get higher revenue from the unskilled workers residing in peripheral cities with high transportation costs than in the central city. We find stable equilibria such that remote workers who reside in peripheral places by conducting numerical analysis of equilibria with the FER model (Section 4.2).

4.2. Numerical bifurcation analysis

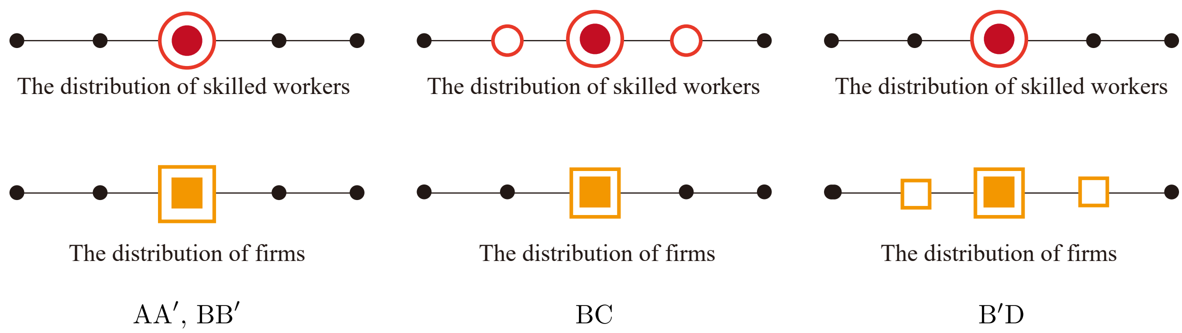
Conducting numerical analysis for stable equilibria with the FE and FER models, we examine the impact of remote work on population distribution $\boldsymbol{\lambda}$ of equilibrium. As explained in Sections 2.2.3 and 3, the stability conditions of equilibria with the FE and FER models differ. That is, the stable full agglomeration of the FE model is not necessarily stable of the FER model. In order to explore how remote work changes population distribution with the full agglomeration which is the distribution such that the skilled workers and the firms agglomerate in the central city, we conduct bifurcation analysis for full agglomeration $\boldsymbol{\lambda}_{\mathrm{FA}}$.

In an economy where skilled workers can conduct remote work, the full agglomeration with the FE model is not necessarily stable. Fig. 4 shows the solution curves of the dynamics with the FE and FER models with $\alpha_i = 0.3$, $\beta = 0.1$, $\sigma_i = 3.0$, $L = 2.6$. With this parameter setting, the wage of skilled workers in the central city w_{00}^1 with the full agglomeration is solved as 1.5 (see Appendix C.1 for detail). Full agglomeration $\boldsymbol{\lambda}_{\mathrm{FA}}$ is stable with $\tau \in (0.0409, 0.447)$ and the FE model, as shown with path AA$'$. On the other hand, full agglomeration $\boldsymbol{\lambda}_{\mathrm{FA}}$ is stable with $\tau \in (0.0959, 0.316)$ for the FER model, as shown with path BB$'$. With $\tau \in (0.0281, 0.0959) \cup (0.316, 0.346)$ and the FER model, full agglomeration $\boldsymbol{\lambda}_{\mathrm{FA}}$ is unstable.

Population distribution with skilled workers who conduct remote work can emerge from unstable full agglomeration $\boldsymbol{\lambda}_{\mathrm{FA}}$. As shown in Fig. 4(a), bifurcation solutions emerge from points B and B$'$. These are the points at which the stability of full agglomeration $\boldsymbol{\lambda}_{\mathrm{FA}}$ changes. From point B with the FER model, population distribution in which skilled workers reside in place 1 (or -1) and belong to firms operating in place 0 emerges (see Fig. 4(b)). Some skilled workers relocate from the central city to peripheral cities with $\tau \in (0.0281, 0.0959)$. On the other hand, from point B$'$ with the FER model, population distribution in which skilled workers reside in place 0 and belong to firms operating in place 1 (or -1) emerges. All the skilled workers do not relocate, whereas some firms relocate from the central city to peripheral cities with $\tau \in (0.316, 0.346)$.

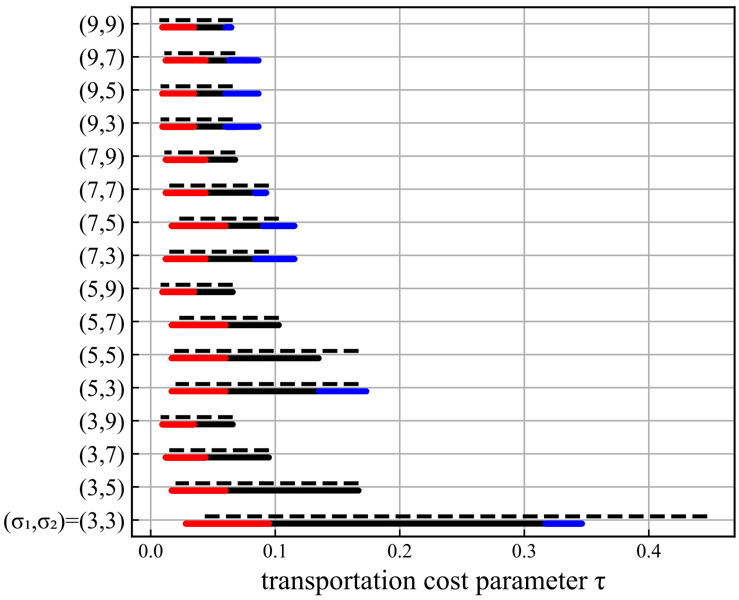


(a) Solutions curves of the dynamics

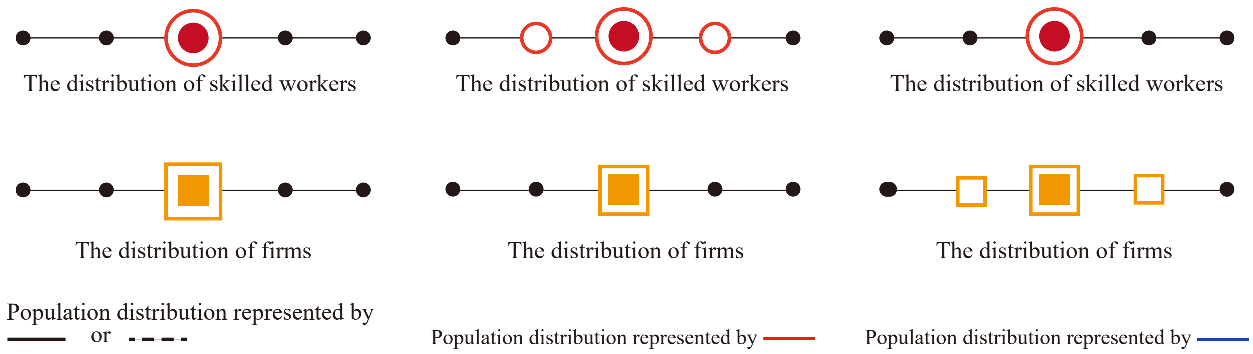


(b) Population disribution with stable equilibria

Figure 4: Bifurcating solutions from full agglomeration $\boldsymbol{\lambda}_{\mathrm{FA}}$ with 5 places economy. —: a stable equilibrium with the FE model; – – –: an unstable equilibrium with the FE model; —: a stable equilibrium with the FER model; – – –: an unstable equilibrium with the FER model.



(a) Stable equilibria



(b) Population disribution with stable equilibrium

Figure 5: Sensitivity analysis for stable equilibria (solid line in (a): stable equilibrium with the FER model; dashed line in (a): stable equilibrium with the FE model)

Conducting sensitivity analysis for stable equilibria of the FE and FER models, we examine how exogenous parameters affect stable equilibria of the FE and FER models. We analyze for several values with elasticity of substitution σ_1 and σ_2 and mass of unskilled worker L. We choose σ_1 and σ_2 from $\{3, 5, 7, 9\}$. L and other parameters affect the wage of skilled workers w^1_{ij}. α_1, α_2, and β are held constant: $\alpha_1 = 0.3, \alpha_2 = 0.3, \beta = 0.1$. Given chosen values σ_1, σ_2, α_1, α_2, and β, we choose L so that w^1_{i0} belongs to $\{1.05, 1.5, 2.0\}$.

Stable equilibria in which some skilled workers conduct remote work can exist with various parameter patterns. Fig. 5 shows the stable equilibria of the FE and FER models with parameter patterns and $w^1_{i0} = 1.5$. Population distribution where remote workers reside in peripheral cities stably exists with low transportation cost τ with which the full agglomeration is unstable for all the parameter patterns shown in Fig. 5(a). Population distribution where remote workers reside in the central city stably exists with high transportation cost τ with which the full agglomeration is unstable for several parameter patterns. The results with $w^1_{i0} = 1.05$ and

$w_{i0}^1 = 2.0$ are similar to the result shown in Fig. 5(a) (see Appendix C.2 for detail).

5. The change in efficiency and disutility generated by the introduction of remote work

5.1. Qualitative analysis with the full agglomeration

The introduction of remote work can marginally affect changes in the utilities of workers. Welfare change generated by remote work is the total of the changes in utilities of unskilled and skilled workers in the FER model. As shown in the indirect utilities (23a)–(23c), a change in population distribution caused by remote work affects price indices, housing prices, and wages of skilled workers.

We examine a marginal change in the utility of an unskilled worker caused by the emergence of remote workers with full agglomeration $\boldsymbol{\lambda}_{\mathrm{FA}}$. The emergence of remote workers from the full agglomeration can be expressed by $-\mathrm{d}\lambda_{00}^1 e_{00} + \mathrm{d}\lambda_j e_{j0} + \mathrm{d}\lambda_j e_{-j,0}$ $(\mathrm{d}\lambda_{00}^1, \mathrm{d}\lambda_j > 0)$, where e_{ij} denotes the standard basis whose the component associated with a remote worker who resides in place i and supplies labor to a firm in place j is one. $-\mathrm{d}\lambda_{00}^1 e_{00} + \mathrm{d}\lambda_j e_{j0} + \mathrm{d}\lambda_j e_{-j,0}$ implies remote workers reside in city j and $-j$ and belong to firms in the central city. Using Eqs. (26) and (27a), we have the population constraint in terms of the marginal change in population distribution: $2\mathrm{d}\lambda_j e_{j0} - \mathrm{d}\lambda_{00}^1 = 0$. The indirect utility of each unskilled worker V_i^{u} is composed of the price index and the housing price and exogenous variables: $V_i^{\mathrm{u}} = V_{ij}^1 - w_{ij}^1 + 1$. Using this equation and Eq. (23b), we obtain the marginal change in utility of each unskilled worker in place i generated by marginal change $-\mathrm{d}\lambda_{00}^1 e_{00} + \mathrm{d}\lambda_j e_{j0} + \mathrm{d}\lambda_j e_{-j,0}$:

$$\mathrm{d}V_0^{\mathrm{u}} = \frac{\beta}{\Lambda_i} \frac{\partial \Lambda_i}{\partial \lambda_{00}^1} \mathrm{d}\lambda_{00}^1 > 0, \tag{34a}$$

$$\mathrm{d}V_i^{\mathrm{u}} = -\frac{\beta}{\Lambda_i} \frac{\partial \Lambda_i}{\partial \lambda_{j,0}^1} \mathrm{d}\lambda_j^1 \quad (i \neq 0). \tag{34b}$$

These equations show that marginal change $-\mathrm{d}\lambda_{00}^1 e_{00} + \mathrm{d}\lambda_j e_{j0} + \mathrm{d}\lambda_j e_{-j,0}$ does not affect the price indices, but the housing prices. As the above equations show, the marginal change in unskilled workers in the central city is positive. Moreover, the marginal change in the utility of unskilled workers who reside in the city where remote workers exist is negative, whereas otherwise, this change is zero.

Next, we focus on the marginal change in population distribution in terms of remote workers who relocate the location of firm: $-\mathrm{d}\lambda_{00}^1 e_{00} + \mathrm{d}\lambda_j e_{0j} + \mathrm{d}\lambda_j e_{0,-j}$ $(\mathrm{d}\lambda_{00}^1, \mathrm{d}\lambda_j > 0, 2\mathrm{d}\lambda_j - \mathrm{d}\lambda_{00}^1 = 0)$. This implies remote workers reside in city 0 and belong to firms in city j or $-j$. The marginal change in utilities of unskilled workers in the central city associated with $-\mathrm{d}\lambda_{00}^1 e_{00} + \mathrm{d}\lambda_j e_{0j} + \mathrm{d}\lambda_j e_{0,-j}$ is given by

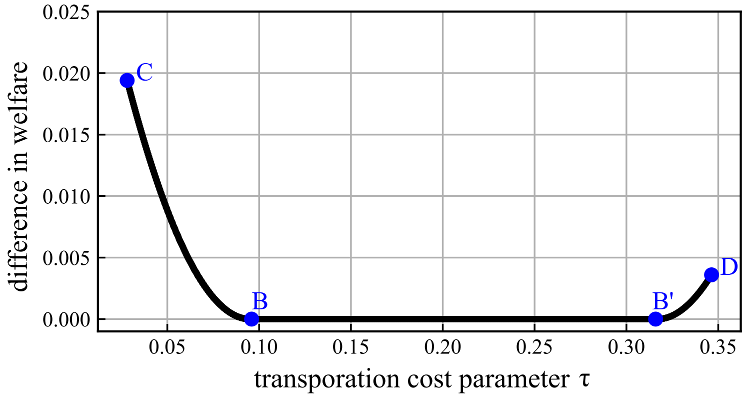
$$\mathrm{d}V_0^{\mathrm{u}} = \frac{\alpha_1}{\sigma_1 - 1} \frac{1}{N_0^1} (-d_{00}^1 \mathrm{d}\lambda_0 + d_{j0}^1 \mathrm{d}\lambda_j + d_{-j,0}^1 \mathrm{d}\lambda_j) - \frac{\beta}{\Lambda_0}(-\mathrm{d}\lambda_0 + 2\mathrm{d}\lambda_j). \tag{35}$$

The first and second terms are the changes in the price index and the change in the housing price, respectively. The second term is zero because of the population constraint, whereas the first term is negative as shown in the following inequality:

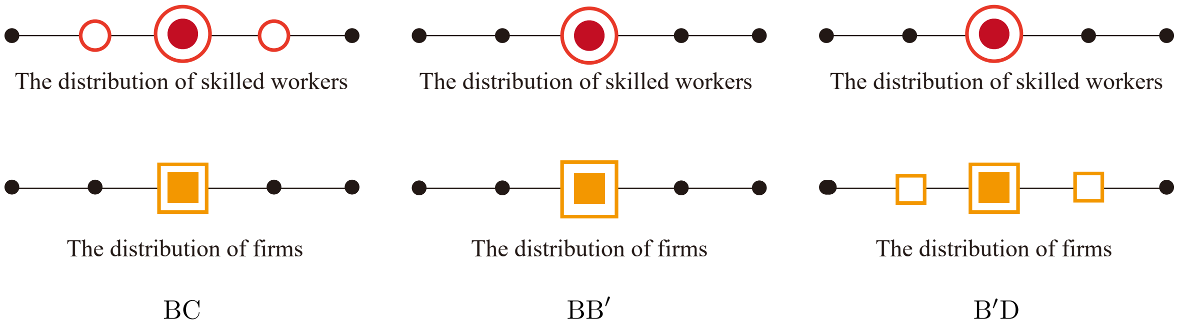
$$-d_{00}^1 \mathrm{d}\lambda_0 + d_{j0}^1 \mathrm{d}\lambda_j + d_{-j,0}^1 \mathrm{d}\lambda_j < -d_{00}^1 \mathrm{d}\lambda_0 + d_{00}^1 \mathrm{d}\lambda_j + d_{00}^1 \mathrm{d}\lambda_j = 0.$$

Using this inequality, we obtain $\mathrm{d}V_0^{\mathrm{u}} < 0$. On the other hand, the marginal change in utilities of unskilled workers in a peripheral city is given by

$$\mathrm{d}V_i^{\mathrm{u}} = \frac{\alpha_1}{\sigma_1 - 1} \frac{1}{N_i^1} (-d_{0i}^1 \mathrm{d}\lambda_0 + d_{ji}^1 \mathrm{d}\lambda_j + d_{-j,i}^1 \mathrm{d}\lambda_j) - \frac{\beta}{\Lambda_i} \underbrace{(-\mathrm{d}\lambda_0 + 2\mathrm{d}\lambda_j)}_{=0} \quad (i \neq 0). \tag{36}$$



(a) Change in social welfare: $W(\boldsymbol{\lambda}, \tau) - W(\boldsymbol{\lambda}^{\mathrm{FA}}, \tau)$



(b) Population disribution

Figure 6: Change in the social welfare with a change in population distribution

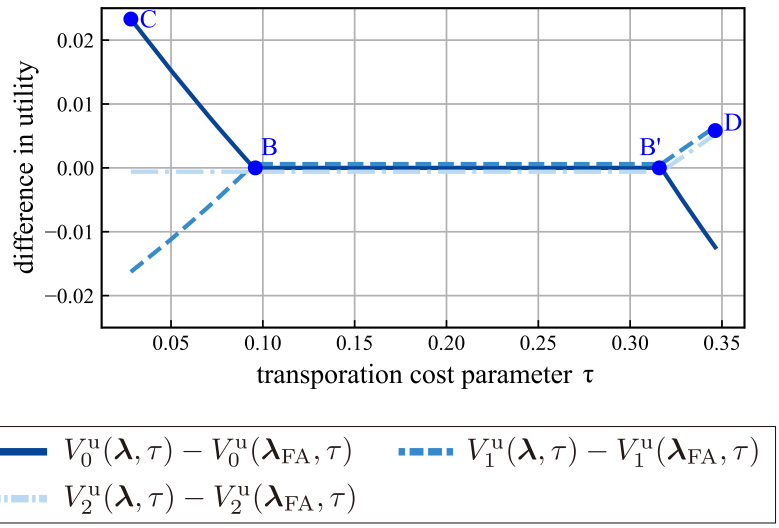
The sign of the above marginal change depends on transportation cost τ and the level of population migration (i.e., $\mathrm{d}\lambda_0$ and $\mathrm{d}\lambda_j$).

The marginal change in utility of each skilled worker consists of the marginal changes in price indices, housing price, and wage. As the indirect utility (23b) indicates, the effects of the price index and housing price on this utility are the same as those on the utility of unskilled workers, whereas the effect of the wage is added to the change in the utility. As the wage (24a) shows, this change depends on transportation costs τ_{ij} and population distribution $\boldsymbol{\lambda}$. We numerically explore this change (Section 5.2).

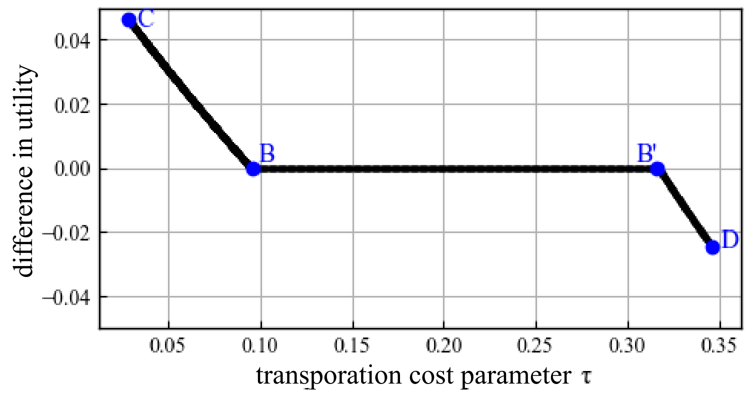
5.2. Numerical welfare analysis

We focus on how social welfare and utilities of workers change when remote work becomes widespread and the city size distribution changes. We explore the effect of the introduction of remote work on the social welfare and the utilities of workers. We compare the welfare of full agglomeration and that of stable equilibrium with the FER model in order to elucidate whether the introduction of remote work is desirable. The stable equilibria with the FER model are shown in Fig. 4 in Section 4.2.

Fig. 6 shows the change in welfare after the introduction of remote work compared to before its introduction, within the range of stable equilibrium. Exogenous parameters are the same as those for the numerical analysis shown in Fig. 4. Paths BC and B′D show the changes in the social welfare generated by the emergence of remote workers. In path BC (B′D), the social welfare with the population distribution with the remote work



(a) Change in the utility of an unskilled worker



(b) Change in the utility of a skilled worker of sector 1: $V^1_{0,0}(\boldsymbol{\lambda}, \tau) - V^1_{0,0}(\boldsymbol{\lambda}_{\mathrm{FA}}, \tau)$

Figure 7: Change in utility of a worker with a change in population distribution (i.e., $V_i^{\mathrm{u}}(\boldsymbol{\lambda}, \tau) - V_i^{\mathrm{u}}(\boldsymbol{\lambda}_{\mathrm{FA}}, \tau)$)

exceeds that with the full agglomeration from zero to 0.0194 (0.00360). The spread of remote work, thus, can be desirable from the perspective of the social welfare.

Some unskilled workers can suffer from decreases in their utilities as shown in Section 5.1. Fig. 7(a) provides the change in the utility of unskilled workers in each place generated by remote work. When only workers move (path BC), the utility of workers in the central city increases, while the utility of workers in the two neighboring cities decreases. There is no change in the utility of the unskilled workers in the two cities on the outskirts (i.e., place ± 2). These changes are due to the changes in the housing prices (see Eqs. (34a) and (34b)). That is, the price of floor space in the central city decreases, whereas the prices in the neighboring cities increase. On the other hand, when only firms move (path B′D), the utility of unskilled workers in a city other than the central city increases, but that of unskilled workers in the central city decreases (see Eq. (35)).

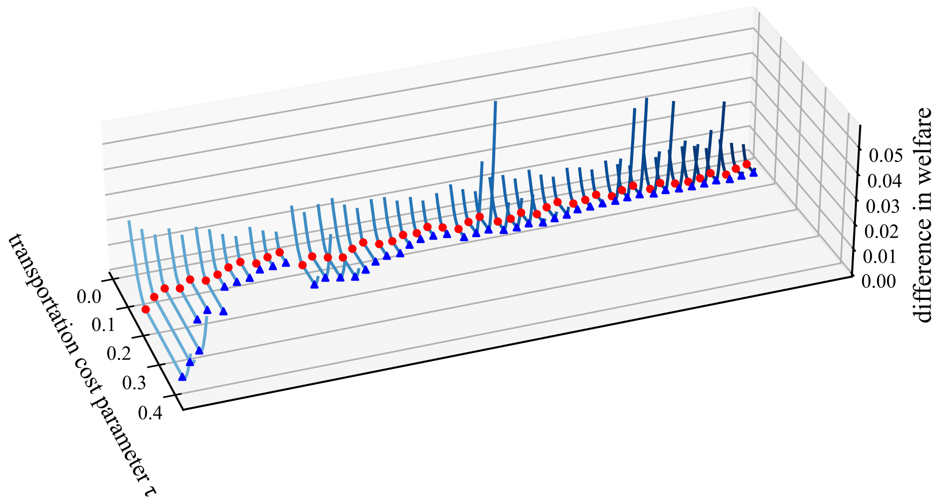
Skilled workers can experience lower utility depending on how the city size distribution changes. Fig. 7(b) shows the change in the utility of skilled workers. Path AB in the figure shows that the utility increases when only workers move to the surrounding cities with $\tau \in (0.0281, 0.0959)$, whereas path B′D shows that the utility decreases when only firms move to the surrounding cities with $\tau \in (0.316, 0.346)$. Those changes are similar to the changes in the utility of unskilled workers in the central city.

5.3. Sensitivity analysis

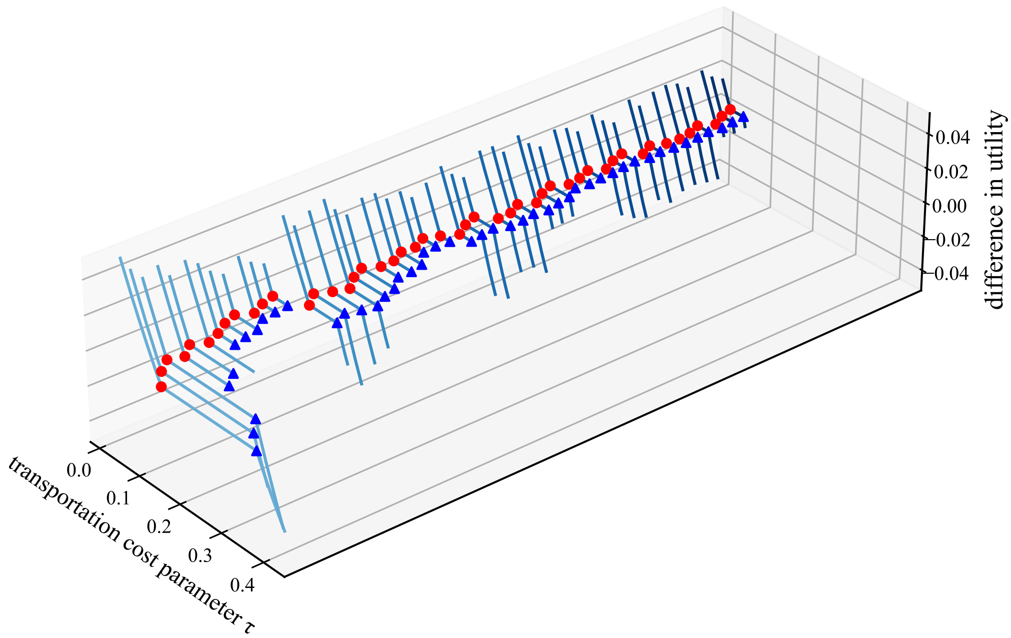
Conducting sensitivity analyses for the social welfare and the utility that each worker obtains, we explore whether exogenous parameters affect the result such that remote work can not be necessarily desirable for some workers. As the analysis shown in Fig. 5, we choose σ_1 and σ_2 from $\{3, 5, 7, 9\}$. α_1, α_2, and β are held constant: $\alpha_1 = 0.3, \alpha_2 = 0.3, \beta = 0.1$. Given chosen values σ_1, σ_2, α_1, α_2, and β, we choose L so that w_{i0}^1 belongs to $\{1.05, 1.5, 2.0\}$ (see Eq. (C3) in Appendix C.1) with full agglomeration $\boldsymbol{\lambda}_{FA}$. The number of the parameter values patterns is 48.

The introduction of remote work can be beneficial from the perspective of social welfare. Fig. 8(a) shows the change in welfare after the introduction of remote work compared to before its introduction, within the range of stable equilibrium. Each solid line shown in this figure expresses the difference with a chosen pattern of parameter values. The straight lines with endpoints labeled with the circle ($\circ$) and triangle ($\triangle$) markers refer to the full agglomeration. As shown in Fig. 5, bifurcation solutions can emerge for low transport costs. The curves with the end point labeled with the circle marker ($\circ$) refer to the population distribution in which skilled workers reside in peripheral cities and belong to firms operating in place 0. For all the parameter values patterns, these patterns emerge as bifurcation solutions with low transport costs (i.e., the low cost indicated by the circle marker). The curves with the end point labeled with the triangle marker ($\triangle$) refer to the population distribution in which skilled workers reside in place 0 and belong to firms operating in peripheral cities. For 9 patterns of parameter values, this population distribution pattern emerges as a bifurcation solution with high transport cost (i.e., the high cost indicated by the triangle marker). As shown in Fig. 8(a), the social welfare with the population distribution in which some skilled workers conduct remote work exceeds that with the full agglomeration.

As shown in Fig. 7(a), the introduction of remote work can not be necessarily beneficial for skilled workers. Fig. 8(b) shows that the change in indirect utility of a skilled worker $V_{0,0}^1$ after the introduction of remote work compared to before its introduction, within the range of stable equilibrium. The utility of skilled workers increases for the population distribution in which skilled workers reside in peripheral places and belong to firms operating in place 0, whereas it decreases for the population distribution in which skilled workers reside in place 0 and belong to firms operating in peripheral places.



(a) The sensitivity of the change in the social welfare: $W(\boldsymbol{\lambda}, \tau) - W(\boldsymbol{\lambda}^{\mathrm{FA}}, \tau)$



(b) The sensitivity of the change in the change in utility of skilled worker: $V^1_{0,0}(\boldsymbol{\lambda}, \tau) - V^1_{0,0}(\boldsymbol{\lambda}_{\mathrm{FA}}, \tau)$

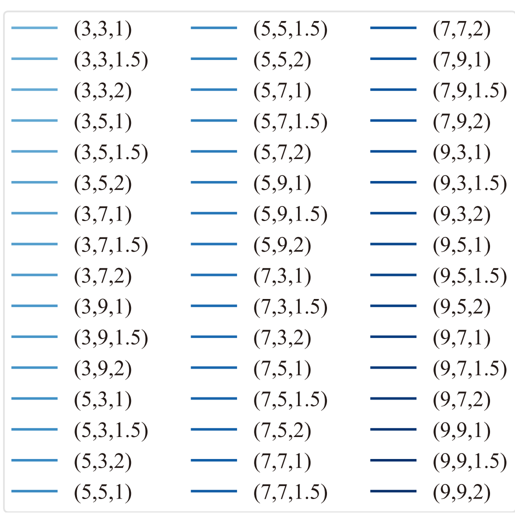


Figure 8: The sensitivities of changes in the welfare and the utility from the full agglomeration with $(\sigma_1, \sigma_2, w^1_{00})$

6. Conclusion

We have explored how remote work affects population distribution in an urban system, social welfare, and utilities of workers. Conducting bifurcation analysis of equilibrium for Footloose Entrepreneur models with and without remote work, we examine the stability of the full agglomeration and the bifurcation solutions that express population distributions in which some workers conduct remote work. We obtain two main findings: (1) remote work can generate two types of population distributions: population distribution in which only workers migrate and only firms migrate to small cities, and (2) with myopic behavior of skilled workers, remote work can cause to decrease in utilities of some workers, whereas the social welfare increases.

From the perspective of equity, policymakers need to implement policies that control population distribution. Location-based policies are possible choices to relax the decline in the utility. These policies are theoretically investigated in the literature of land use regulation (Kono and Joshi, 2019) and location-based subsidy policies (Aizawa and Kono, 2023). The welfare and utility analyses for such policies are future works.

Appendix

A. Details with the models

A.1. Jacobian matrix of indirect utility of the FE model

Let $\boldsymbol{w}^s = (w_1^s, \ldots, w_K^s)$ denotes the wage vector. Using the above equation yields the wage vector as the following matrix form:

$$\boldsymbol{w}^s = \frac{\alpha_s}{\sigma_s}(D^s)(\Delta^s)^{-1}(\boldsymbol{\lambda}^1 + \boldsymbol{\lambda}^2 + L\mathbf{1}), \tag{A1}$$

with identity matrix I, $\mathbf{1} = (1, \ldots, 1)^\top$, $D^s = (d_{ij}^s)$, and $\Delta^s = \operatorname{diag}(\Delta_1^s, \ldots, \Delta_K^s)$. Let $\boldsymbol{V}^s \equiv (V_1^s, V_2^s, \ldots, V_K^s)$ denote the vector of the indirect utility. The Jacobian matrix of the vector is given by

$$\frac{\partial \boldsymbol{V}}{\partial \boldsymbol{\lambda}} = \begin{pmatrix} \partial \boldsymbol{V}^1/\partial \boldsymbol{\lambda}^1 & \partial \boldsymbol{V}^1/\partial \boldsymbol{\lambda}^2 \\ \partial \boldsymbol{V}^2/\partial \boldsymbol{\lambda}^1 & \partial \boldsymbol{V}^2/\partial \boldsymbol{\lambda}^2 \end{pmatrix}, \tag{A2}$$

where

$$\begin{aligned}
\frac{\partial \boldsymbol{V}^1}{\partial \boldsymbol{\lambda}^1} &= \frac{\alpha_1}{\sigma_1 - 1}\Phi^1(D^1)^\top - \beta\Psi + \frac{\partial \boldsymbol{w}^1}{\partial \boldsymbol{\lambda}^1}, \\
\frac{\partial \boldsymbol{V}^1}{\partial \boldsymbol{\lambda}^2} &= \frac{\alpha_2}{\sigma_2 - 1}\Phi^2(D^2)^\top - \beta\Psi + \frac{\partial \boldsymbol{w}^1}{\partial \boldsymbol{\lambda}^2}, \\
\frac{\partial \boldsymbol{V}^2}{\partial \boldsymbol{\lambda}^1} &= \frac{\alpha_1}{\sigma_1 - 1}\Phi^1(D^1)^\top - \beta\Psi + \frac{\partial \boldsymbol{w}^2}{\partial \boldsymbol{\lambda}^1}, \\
\frac{\partial \boldsymbol{V}^2}{\partial \boldsymbol{\lambda}^2} &= \frac{\alpha_2}{\sigma_2 - 1}\Phi^2(D^2)^\top - \beta\Psi + \frac{\partial \boldsymbol{w}^2}{\partial \boldsymbol{\lambda}^2}.
\end{aligned}$$

Here, $\Phi^s = \operatorname{diag}(1/\Delta_1^s, 1/\Delta_2^s, \ldots, 1/\Delta_K^s)$, $\Psi = \operatorname{diag}(1/\Lambda_1, 1/\Lambda_2, \ldots, 1/\Lambda_K)$, and $\partial \boldsymbol{w}^s/\partial \boldsymbol{\lambda}^j$ is the Jacobian matrix of the wage with respect to the population distribution. $\partial \boldsymbol{w}^s/\partial \boldsymbol{\lambda}^j$ is given by

$$\begin{aligned}
\frac{\partial \boldsymbol{w}^1}{\partial \boldsymbol{\lambda}^1} &= \frac{\alpha_1}{\sigma_1} D^1\Phi^1 \left[I - \operatorname{diag}(\Lambda)(D^1\Phi^1)^\top\right], \\
\frac{\partial \boldsymbol{w}^1}{\partial \boldsymbol{\lambda}^2} &= \frac{\alpha_1}{\sigma_1} D^1\Phi^1, \\
\frac{\partial \boldsymbol{w}^2}{\partial \boldsymbol{\lambda}^1} &= \frac{\alpha_2}{\sigma_2} D^2\Phi^2, \\
\frac{\partial \boldsymbol{w}^2}{\partial \boldsymbol{\lambda}^2} &= \frac{\alpha_2}{\sigma_2} D^2\Phi^2 \left[I - \operatorname{diag}(\Lambda)(D^2\Phi^2)^\top\right],
\end{aligned}$$

where $\Lambda = (\Lambda_1, \Lambda_2, \ldots, \Lambda_K)$.

A.2. FER model with specified utility function

A.2.1. Derivation of the indirect utility with the FER model

Because the total number of workers in place i is $\sum_{k\in\mathcal{K}} \lambda_{ik}^1 + \lambda_i^2 + L$, $Q_{ji}(\ell)$ is given by

$$Q_{ji}^s(\ell) = \frac{\alpha_s(\rho_i^s)^{\sigma_s - 1}}{p_{ji}^s(\ell)^{\sigma_s}}\left(\sum_{k\in\mathcal{K}} \lambda_{ik}^1 + \lambda_i^2 + L\right). \tag{A3}$$

The first-order condition for this profit maximization yields the following optimal price

$$p_{ij}^s(\ell) = \frac{\sigma_s}{\sigma_s - 1} \times c\tau_{ij}. \tag{A4}$$

The first condition and the demand for housing yields the equilibrium price of the housing

$$p_i^{\mathrm{H}} = \frac{\beta\left(\sum_{j\in\mathcal{K}} \lambda_{ij}^1 + \lambda_i^2 + L\right)}{H}. \tag{A5}$$

The price index ρ_i^s is, thus, given by

$$\rho_i^1 = \frac{\sigma_1 c}{\sigma_1 - 1}\left(\sum_{k\in\mathcal{K}} d_{ki}^1\left(\sum_{m\in\mathcal{K}} \lambda_{mk}^1\right)\right)^{1/(1-\sigma_1)}, \tag{A6}$$

$$\rho_i^2 = \frac{\sigma_2 c}{\sigma_2 - 1}\left(\sum_{k\in\mathcal{K}} \lambda_k^2 d_{ki}^2\right)^{1/(1-\sigma_2)}, \tag{A7}$$

where $d_{ji}^s = \tau_{ji}^{1-\sigma_s}$. Wages in the short-run equilibrium w_{ij}^1, w_i^2 are

$$w_{ij}^1 = \frac{\alpha_1}{\sigma_1}\sum_{k\in\mathcal{K}}\left[\frac{d_{jk}^1}{\sum_{o\in\mathcal{K}} d_{ok}^1(\sum_{m\in P}\lambda_{mo}^1)}\left(\lambda_k^1 + \lambda_k^2 + L\right)\right], \tag{A8}$$

$$w_i^2 = \frac{\alpha_2}{\sigma_2}\sum_{k\in\mathcal{K}}\frac{d_{ik}^2}{\Delta_k^2}(\lambda_k^1 + \lambda_k^2 + L), \tag{A9}$$

where $\Delta_j^s = \sum_{m\in\mathcal{K}} d_{mj}^s \lambda_m^s$ and $\lambda_j^1 = \sum_{m\in\mathcal{K}} \lambda_{jm}^1$. Substituting the price indices and the wages in the short-run equilibrium into the indirect utility yields the indirect utility as a function of population distribution (i.e., Eqs. (23a)–(23c)).

A.2.2. The Jacobian matrix of the indirect utility with the FER model

The wage can be expressed as the following matrix form:

$$\boldsymbol{w}_i^1 = \frac{\alpha_1}{\sigma_1} D^1 \Psi^1(\boldsymbol{\lambda}^1 + \boldsymbol{\lambda}^2 + L\mathbf{1}), \tag{A10}$$

$$\boldsymbol{w}^2 = \frac{\alpha_2}{\sigma_2} D^2 \Phi^2(\boldsymbol{\lambda}^1 + \boldsymbol{\lambda}^2 + L\mathbf{1}), \tag{A11}$$

where $\Psi^1 = \mathrm{diag}(1/N_1^1, 1/N_2^1, \ldots, 1/N_K^1)$ and $\Phi^2 = \mathrm{diag}(1/\Delta_1^2, 1/\Delta_2^2, \ldots, 1/\Delta_K^2)$. Let $\boldsymbol{\lambda}_i^1 = (\lambda_{i1}^1, \lambda_{i2}^1, \ldots, \lambda_{iK}^1)$ denote the population distribution of skilled workers of M^1 sector. We have $\boldsymbol{\lambda}^1 = (\boldsymbol{\lambda}_1^1, \boldsymbol{\lambda}_2^1, \ldots, \boldsymbol{\lambda}_K^1)$. Similarly, we have $\boldsymbol{w}^1 = (\boldsymbol{w}_1^1, \boldsymbol{w}_2^1, \ldots, \boldsymbol{w}_K^1)$, $\boldsymbol{w}^2 = (w_1^2, w_2^2, \ldots, w_K^2)$, $\boldsymbol{V}^1 = (\boldsymbol{V}_1^1, \boldsymbol{V}_2^1, \ldots, \boldsymbol{V}_K^1)$, and $\boldsymbol{V}^2 = (V_1^2, V_2^2, \ldots, V_K^2)$. The Jacobian matrix of the indirect utility is given by

$$\frac{\partial \boldsymbol{V}}{\partial \boldsymbol{\lambda}} = \left(\begin{array}{c|c} \partial \boldsymbol{V}^1/\partial \boldsymbol{\lambda}^1 & \partial \boldsymbol{V}^1/\partial \boldsymbol{\lambda}^2 \\ \hline \partial \boldsymbol{V}^2/\partial \boldsymbol{\lambda}^1 & \partial \boldsymbol{V}^2/\partial \boldsymbol{\lambda}^2 \end{array}\right)$$

$$= \left(\begin{array}{cccc|c} \partial \boldsymbol{V}_1^1/\partial \boldsymbol{\lambda}_1^1 & \partial \boldsymbol{V}_1^1/\partial \boldsymbol{\lambda}_2^1 & \cdots & \partial \boldsymbol{V}_1^1/\partial \boldsymbol{\lambda}_K^1 & \partial \boldsymbol{V}_1^1/\partial \boldsymbol{\lambda}^2 \\ \partial \boldsymbol{V}_2^1/\partial \boldsymbol{\lambda}_1^1 & \partial \boldsymbol{V}_2^1/\partial \boldsymbol{\lambda}_2^1 & & \partial \boldsymbol{V}_2^1/\partial \boldsymbol{\lambda}_K^1 & \partial \boldsymbol{V}_2^1/\partial \boldsymbol{\lambda}^2 \\ \vdots & & \ddots & & \vdots \\ \partial \boldsymbol{V}_K^1/\partial \boldsymbol{\lambda}_1^1 & \partial \boldsymbol{V}_K^1/\partial \boldsymbol{\lambda}_2^1 & \cdots & \partial \boldsymbol{V}_K^1/\partial \boldsymbol{\lambda}_K^1 & \partial \boldsymbol{V}_K^1/\partial \boldsymbol{\lambda}^2 \\ \hline \partial \boldsymbol{V}^2/\partial \boldsymbol{\lambda}_1^1 & \partial \boldsymbol{V}^2/\partial \boldsymbol{\lambda}_2^1 & \cdots & \partial \boldsymbol{V}^2/\partial \boldsymbol{\lambda}_K^1 & \partial \boldsymbol{V}^2/\partial \boldsymbol{\lambda}^2 \end{array}\right). \tag{A12}$$

The Jacobian matrices of the indirect utilities with respect to the population distribution are given by

$$\frac{\partial \boldsymbol{V}_i^1}{\partial \boldsymbol{\lambda}_j^1} = \frac{\alpha_1}{\sigma_1 - 1}\frac{1}{N_i^1}(D^1 E_i)^\top - \frac{\beta}{\Lambda_i} I + \frac{\partial \boldsymbol{w}_i^1}{\partial \boldsymbol{\lambda}_j^1},$$

$$\frac{\partial \boldsymbol{V}_i^1}{\partial \boldsymbol{\lambda}^2} = \frac{\alpha_2}{\sigma_2 - 1}\frac{1}{\Delta_i^2}(D^2 E_i)^\top - \frac{\beta}{\Lambda_i} E_i^\top + \frac{\partial \boldsymbol{w}_i^1}{\partial \boldsymbol{\lambda}^2},$$

$$\frac{\partial \boldsymbol{V}^2}{\partial \boldsymbol{\lambda}_j^1} = \frac{\alpha_1}{\sigma_1 - 1}\Phi^1(D^1)^\top - \frac{\beta}{\Lambda_j}E_j + \frac{\partial \boldsymbol{w}^2}{\partial \boldsymbol{\lambda}_j^1}.$$
$$\frac{\partial \boldsymbol{V}^2}{\partial \boldsymbol{\lambda}^2} = \frac{\alpha_2}{\sigma_2 - 1}\Phi^2(D^2)^\top - \beta\Psi + \frac{\partial \boldsymbol{w}^2}{\partial \boldsymbol{\lambda}^2}.$$

where $\Psi = \mathrm{diag}(1/\Lambda_1, 1/\Lambda_2, \ldots, 1/\Lambda_K)$, and $E_i = e_i\mathbf{1}^\top$. The Jacobian matrices of the wage vectors with respect to the population distribution are given by

$$\frac{\partial \boldsymbol{w}_i^1}{\partial \boldsymbol{\lambda}_j^1} = \frac{\alpha_1}{\sigma_1}\left[\frac{1}{N_j^1}D^1E_j - D^1\Psi^1\mathrm{diag}(\Lambda)(D^1\Psi^1)^\top\right],$$
$$\frac{\partial \boldsymbol{w}_i^1}{\partial \boldsymbol{\lambda}^2} = \frac{\alpha_1}{\sigma_1}D^1\Psi^1,$$
$$\frac{\partial \boldsymbol{w}^2}{\partial \boldsymbol{\lambda}_j^1} = \frac{\alpha_2}{\sigma_2}D^2\Phi^2E_j,$$
$$\frac{\partial \boldsymbol{w}^2}{\partial \boldsymbol{\lambda}^2} = \frac{\alpha_2}{\sigma_2}\left[D^2\Phi^2 - D^2\Phi^2\mathrm{diag}(\Lambda)(D^2\Phi^2)^\top\right],$$

where $\Lambda = (\Lambda_1, \Lambda_2, \ldots, \Lambda_K)$.

B. Proof of Proposition 1

We explain a proof of only Proposition 1 (i) since that of Proposition 1 (ii) is almost the same as this proof. The proof in our paper is similar to the proof of the existence of a bifurcation solution conducted by Ikeda et al. (2024).

Around bifurcation point $(\boldsymbol{\lambda}_{\mathrm{FA}}, \tau)$, there exists a bifurcation equation which is a reduced function for the replicator dynamics (Ikeda and Murota, 2019). This bifurcation equation is given by

$$\lambda_{-i,0}^1\left[V_{-i,0}^1(\lambda_{-i,0}^1, \lambda_{i,0}^1, \tau) - \overline{V^1}(\lambda_{-i,0}^1, \lambda_{i,0}^1, \tau)\right] = 0, \tag{B1}$$
$$\lambda_{i,0}^1\left[V_{i,0}^1(\lambda_{-i,0}^1, \lambda_{i,0}^1, \tau) - \overline{V^1}(\lambda_{-i,0}^1, \lambda_{i,0}^1, \tau)\right] = 0, \tag{B2}$$

for two nonzero components $\lambda_{-i,0}^1$ and $\lambda_{i,0}^1$. By the bilateral symmetry of places $-i$ and i, we have the symmetry conditions:

$$\overline{V^1}(\lambda_{i,0}^1, \lambda_{-i,0}^1, \tau) = \overline{V^1}(\lambda_{-i,0}^1, \lambda_{i,0}^1, \tau), \tag{B3}$$
$$V_{-i,0}^1(\lambda_{i,0}^1, \lambda_{-i,0}^1, \tau) = V_{i,0}^1(\lambda_{-i,0}^1, \lambda_{i,0}^1, \tau). \tag{B4}$$

Since the above conditions yield $V_{-i,0}^1(\lambda, \lambda, \tau) = V_{i,0}^1(\lambda, \lambda, \tau)$ $(\lambda > 0)$, bifurcation equations (B1) and (B2) become identical. Solution curve $(\lambda_{-i,0}^1, \lambda_{i,0}^1, \tau) = (\lambda, \lambda, \tau)$ satisfies these bifurcation equations since $V_{-i,0}^1(\lambda, \lambda, \tau) - \overline{V^1}(\lambda, \lambda, \tau) = 0$ holds.

C. Supplements to numerical analyses in Sections 4 and 5

C.1. Wage with the full agglomeration

Substituting population distribution with full agglomeration $\boldsymbol{\lambda}_{\mathrm{FA}}$ into wage (24a), we obtain the wage of skilled workers:

$$w_{ij}^1 = \frac{\alpha_1}{\sigma_1}\sum_{k\in\mathcal{K}}\frac{d_{jk}^1}{d_{0k}^1}(\lambda_k^1 + \lambda_k^2 + L). \tag{C1}$$

As the above equation shows, the wages of skilled workers belonging to firms operating in the central city are the same:

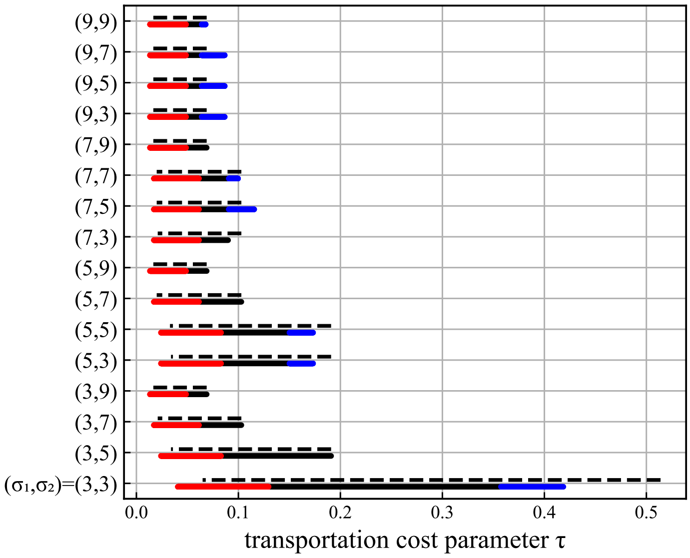
$$w_{i0}^1 = \frac{\alpha_1}{\sigma_1}((2K+1)L+2). \tag{C2}$$

For the full agglomeration, the wage is determined by α_1, σ_1, K, and L. Moreover, given chosen values, the number of unskilled workers is uniquely determined:

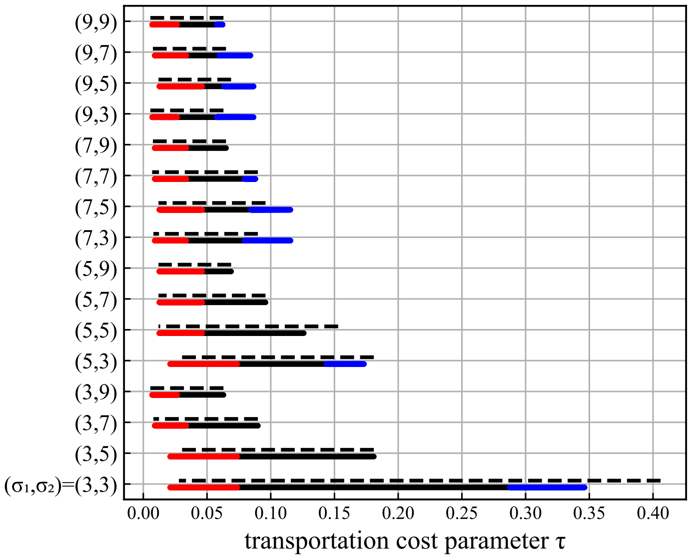
$$L = \frac{1}{2K+1}\left(\frac{\sigma_1 w_{i0}^1}{\alpha_1} - 2\right). \tag{C3}$$

C.2. Sensitivity of existence of equilibrium

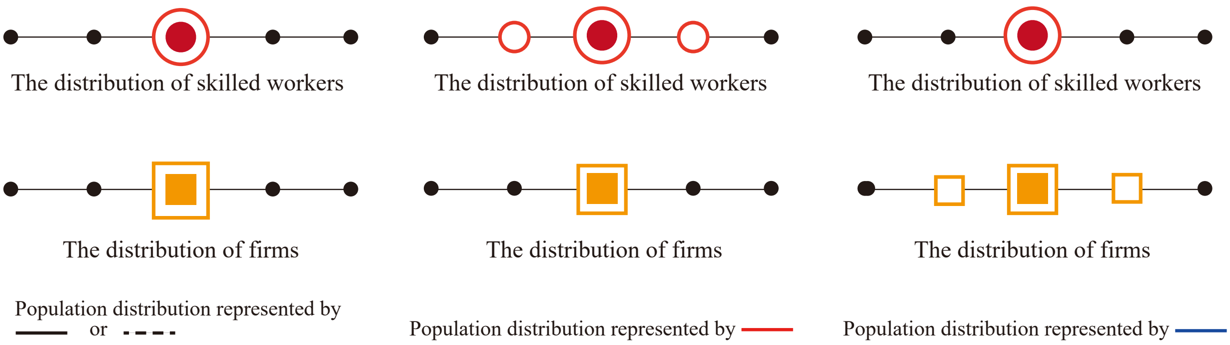
Fig. C1(a) and (b) show the stable equilibria of the FE and FER models with parameter patterns and $w_{i0}^1 = 1.05$ and $w_{i0}^1 = 2.0$ with the full agglomeration, respectively. The results shown in this figure are similar to the result shown in Fig. 5(a).



(a) Stable equilibria with $w_{i0}^1 = 1.05$ at the full agglomeration



(b) Stable equilibria with $w_{i0}^1 = 2.0$ at the full agglomeration



(b) Population disribution with stable equilibria

Figure C1: Sensitivity analysis for stable equilibria (solid line in (a) and (b): stable equilibrium with the FER model; dashed line in (a) and (b): stable equilibrium with the FE model)

References

Aizawa, H., Ikeda, K., Kogure, Y., 2023. Satellite city formation for a spatial economic model. Networks and Spatial Economics, 1–30.

Aizawa, H., Ikeda, K., Osawa, M., Gaspar, J.M., 2020. Breaking and sustaining bifurcations in sn-invariant equidistant economy. International Journal of Bifurcation and Chaos 30, 2050240.

Aizawa, H., Kono, T., 2023. How should place-based policies be designed to efficiently promote retail agglomeration? Available at SSRN 4737293 .

Baldwin, R., Forslid, R., Martin, P., Ottaviano, G., Robert-Nicoud, F., 2011. Economic geography and public policy. Princeton University Press.

Behrens, K., Kichko, S., Thisse, J.F., 2024. Working from home: Too much of a good thing? Regional Science and Urban Economics 105, 103990.

Brueckner, J.K., Kahn, M.E., Lin, G.C., 2023. A new spatial hedonic equilibrium in the emerging work-from-home economy? American Economic Journal: Applied Economics 15, 285–319.

Brueckner, J.K., Sayantani, S., 2023. Intercity impacts of work-from-home with both remote and non-remote workers. Journal of Housing Economics 59, 101910.

Delventhal, M.J., Kwon, E., Parkhomenko, A., 2022. Jue insight: How do cities change when we work from home? Journal of Urban Economics 127, 103331.

Dingel, J.I., Neiman, B., 2020. How many jobs can be done at home? Journal of Public Economics 189, 104235.

Fujita, M., Krugman, P.R., Venables, A., 2001. The spatial economy: Cities, regions, and international trade. MIT press.

Gaspar, J.M., Castro, S.B., Correia-da Silva, J., 2018. Agglomeration patterns in a multi-regional economy without income effects. Economic Theory 66, 863–899.

Gokan, T., Kichko, S., Thisse, J.F., 2021. On the impact of telecommuting on cities. Unpublished paper .

Ikeda, K., Aizawa, H., Gaspar, J., 2024. Bifurcation mechanism at a sustain point of a long narrow economy. International Journal of Bifurcation and Chaos 34, 2430021.

Ikeda, K., Murota, K., 2019. Imperfect bifurcation in structures and materials. Springer.

Ikeda, K., Murota, K., Akamatsu, T., Takayama, Y., 2017. Agglomeration patterns in a long narrow economy of a new economic geography model: Analogy to a racetrack economy. International Journal of Economic Theory 13, 113–145.

Ikeda, K., Onda, M., Takayama, Y., 2018. Spatial period doubling, invariant pattern, and break point in economic agglomeration in two dimensions. Journal of Economic Dynamics and Control 92, 129–152.

Kono, T., Joshi, K.K., 2019. Traffic congestion and land use regulations: Theory and policy analysis. Elsevier.

Krugman, P., 1991. Increasing returns and economic geography. Journal of political economy 99, 483–499.

Larson, W., Zhao, W., 2017. Telework: Urban form, energy consumption, and greenhouse gas implications. Economic Inquiry 55, 714–735.

Monte, F., Porcher, C., Rossi-Hansberg, E., 2023. Remote work and city Structure. Technical Report. Working Paper.

Ottaviano, G., Tabuchi, T., Thisse, J.F., 2002. Agglomeration and trade revisited. International Economic Review, 409–435.

Parkhomenko, A., Delventhal, M.J., 2023. Spatial implications of telecommuting in the united states. Working Paper .

Pflüger, M., Südekum, J., 2008. Integration, agglomeration and welfare. Journal of Urban Economics 63, 544–566.

Rhee, H.J., 2008. Home-based telecommuting and commuting behavior. Journal of Urban Economics 63, 198–216.

Safirova, E., 2002. Telecommuting, traffic congestion, and agglomeration: a general equilibrium model. Journal of Urban Economics 52, 26–52.

Sandholm, W.H., 2010. Population games and evolutionary dynamics. MIT press.

Tabuchi, T., Thisse, J.F., 2011. A new economic geography model of central places. Journal of Urban Economics 69, 240–252.

Takatsuka, H., Zeng, D.Z., 2013. Industrial configuration in an economy with low transportation costs. The Annals of Regional Science 51, 593–620.

Takayama, Y., Ikeda, K., Thisse, J.F., 2020. Stability and sustainability of urban systems under commuting and transportation costs. Regional Science and Urban Economics 84, 103553.