The Clustering Method Applied in a Fair Division Process

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Abstract

In this paper, we intend to present our two processes for the equitable sharing of resources which use several variables, take into account the type of variables and the origin of the resource, involving in particular the notions of data transformation and of clustering used in Statistics. These processes are: PRRS (for *Procédé de la Répartition des Ressources Sans réduction des inégalités*, in English *Process of the Distribution of Resources without reduction of inequalities*) and PRRC (for *Procédé de la Répartition des Ressourcesà partir des résultats de la Classification*, in English *Process of the Distribution of Resources from Clustering results*). They come to solve the problem of injustice in a sharing of resources (in particular, a sum of money), the injustice due to 1) the use of a single variable (criterion) instead of several, 2) the direct use of homogeneous variables where the same unit of measurement is expressed differently for each variable, 3) the direct use of heterogeneous variables and 4) the lack of reduction of inequalities between individuals, in certain cases. Keywords: Ascending Hierarchical Classification, Equitable sharing, Resource, Process, Reduction of inequalities.

Résumé

Danscet article, nous comptonsprésenternosdeuxprocédés de partageéquitable des ressources qui utilisentplusieurs variables, prennentencompte le type de variables et l'origine de la ressourcefaisantintervenirnotamment les notions de transformation des données et de clustering, notions utiliséesenStatistique. Cesprocédéssont: PRRS (Procédé de la Répartition des Ressources Sans réduction des inégalités) etPRRC (Procédé de la Répartition des Ressources à partir des résultats du Clustering). Ilsviennentrésoudre le problème de l'injusticedans un partage des ressources (enparticulier, unesommed'argent) due: 1) à l'utilisationd'uneseule variable (critère) au lieu de plusieurs, 2) à l'utilisationdirecte des variables homogènesoù la mêmeunité de mesureestexpriméedifféremment pour chaque variable, 3) à l'utilisationdirecte des variables hétérogènes et 4) au manque de réduction des inégalités entre les individus, danscertainscas.

Mots clés: Classification AscendanteHiérarchique, Partageéquitable, Ressource, Procédé, Réduction des inégalités.

1. Introduction

The Democratic Republic of Congo's organic law on the composition, organization and functioning of the Decentralized Territorial Entities (ETDs) and their relationship with the State and the Provinces stipulates that the ETDs distribute 40% of the share of national revenue designated to the provinces, following three criteria: Capacity of production, Area and Population without proposing the division mechanism ([16], art. 115, 116).

There are a few problems with this law, which are : (1) no mechanism for allocating these revenues using the three criteria proposed by the legislator is given. Incidentally, instead of three criteria, only one, "Population", is used in practice assigning to each individual a portion proportionally to its population. Which is unfair, from the point of view of the law. (2) no mechanism is proposed, taking into account the relationships between individuals, in the sense of reducing inequalities between them. This is supported by Jean Salem Kapya [17] and Paulin Punga [29] who noticed the injustice in this divisionbecause of the absence of an adequate mechanism for correcting inequalities and the use of a single variable. No solution has been proposed [17], [24], [29].

From the foregoing, also taking into account the problems that arise in Mathematics on the sharing of resources, we generally retain four problems which are grounds for injustice in the sharing, in particular, of a sum of money. These are: (1) The use of a single variable instead of several. Consequence: the imbalance between individuals. (2) The direct use of the starting data from the homogeneous variables for that the same unit of measurement is expressed differently for each variable. Consequence: the influence of the scale of one variable on those of others. (3) The direct use of starting data from heterogeneous variables. Consequence: the influence of one unit of measurement on the others. (4) The absence of the reduction of inequalities between individuals, in certain cases. Consequence: the absence of solidarity between individuals. [24].

We maintain that for a sharing to be fair, it must use an adequate mechanism that employs several variables which takes into account the particularities of the latter and the relationships between individuals allowing, if necessary, the reduction of inequalities between them. A question arises: Which processes should be put in place to solve those problems? We present our two processes using several homogeneous or heterogeneous variables, taking into account the relationships between individuals in the sense of reducing or not the inequalities between them [24].

We will rely on the principle of equity which brings together philosophers, mathematicians and economists and which uses the rule of proportionality to allocate shares as well as on the notions of data transformation and clustering used in Statistics.

2. Materials et method

2.1. Hierarchical Ascending Clustering (HAC)

The Clustering is a method of Statistics which consists in finding classes which are such that the individuals of the same class are the most similar as possible (intra-class homogeneity) while those of different classes are the most dissimilar (inter-class heterogeneity).Among the classification methods, there is the Ascending Hierarchical Classification (HAC) that we will use in the following. The HAC makes it possible to group individuals into a certain number of classes emerging from a hierarchy of partitions.

The HAC is carried out by the following three major steps consisting of: (1) Calculating, (after constituting the data table) the distances between individuals two by two. It is necessary to choose an index among many others, such as the Euclidean distance. ([24], pp. 29-30), [27]:

$$d(X_i, X_k) = \sqrt{\sum_{j=1}^{m} \left(X_{ij} - X_{kj}\right)^2} \quad (2.1)$$

where X_i and X_k are vectors of the values of individuals i and k relating to different variables.

The individuals with the smallest distance between them are grouped together: the 1st grouping. (2) Calculate the distance between the newly formed group and the remaining n-2 isolated individuals using a chosen aggregation criterion, for example the average distance of groups I_1 and I_2([14], p.51):

$$d(I_1, I_2) = \frac{1}{n_1 . n_2} \sum_{i=1}^{n_1} \sum_{k=1}^{n_2} d(X_i, X_k) \quad (2.2)$$

where n_1 and n_2 are the number of individuals in I_1 and I_2 respectively.

This makes it possible to form the 2nd grouping by aggregating the two closest objects (individuals or group of individuals). This procedure will be repeated until all the individuals are grouped in the same group. This step makes the CAH an iterative method. ([4], pp. 38-39), ([33], p. 15) . (3) Graphically represent the hierarchy of partitions obtained through a dendrogram, from the successive groupings determined in the previous steps and cut the dendrogram into the desired number of classes. The latter form a partition of all individuals. The number of classes also depends on the assessment of the researcher ([8], p. 14), [9], [12], ([14], p.90),

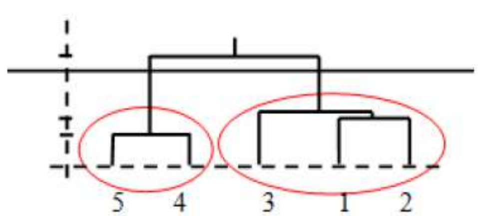

Figure 2.1. Cut of the dendrogram into two classes (Source: Existing theory [24])

After performing a HAC, its results must be interpreted. Various authors, notably Chesneau [9], Husson and Josse [13] limit this interpretation to the calculation of the parameters of the classes resulting from the HAC by determining the most typical individuals: paragons and extremes ([14], p. 91). However, nothing is proposed in the direction of starting from the results of HAC and leading to the determination of the shares of a resource returning to individuals, passing in particular by the reduction of inequalities between them.

The HAC can be preceded by a Factor Analysis, one of the methods of which is Principal Component Analysis (PCA), which uses quantitative data and whose main objective is to reduce the number m of variables to a number q (q < m) of the variables while retaining most of the information contained in the population under study [30]. This allows a better visibility of the graphic representation.

We intend to use the results of the classification to bring out the closest individuals in order to reduce the inequalities between them. Because, as for us, the idea is that the closest individuals, that is to say belonging to the same class, should help each other before seeing others who are further away from them can bring them help. In addition, individuals belonging to the same population should help each other before another population comes to their aid. Thus, individuals unite in their respective class and then in the population as a whole. This justifies the double reduction of inequalities: at the level of classes and then at the level of the whole population. This concerns individuals who did not contribute to create the resource to be shared. Unlike the case where individuals contributed to create the resource. In the latter case, there is no question, as far as we are concerned, of reducing inequalities [24].

2.2. Data transformations

Before carrying out the HAC, it is possible to transform the initial data into centered data or into reduced data or even into centered-reduced data. The data are centered to bring the origin of the axes to the gravity center of the cloud of individuals (case of homogeneous or heterogeneous variables) while they are reduced in order to eliminate the influence of the units of measurement (heterogeneous variables case). Heterogeneous variables are those that are expressed in different units of measurement. Otherwise, they are said to be homogeneous [4], [10], [22], [24].

Let X_j be a variable and X_{ij} the value of individual i relative to this variable. The centered variable, the reduced variable and the centered-reduced variable are determined from X_j by the following formulas:

The centered variable:

$$\hat{X}\text{j} = X_{ij} - \bar{X}\text{j} \qquad (2.3)$$

where

$$\bar{X}\text{j} = \frac{1}{n}\sum_{i=1}^{n} X_{ij} \qquad (2.4)$$

is the arithmetic mean of the variable X_j

The reduced variable:

$tj = \frac{X_{ij}}{\sigma_j}$ (2.5)

where

$\sigma_j = \sqrt{\sum_{i=1}^{n} \frac{(X_{ij}-\bar{X})^2}{n}}$ (2.6)

is the standard deviation (pearson)

The centered-reduced variable:

$\check{X}j = \frac{X_{ij}-\bar{X}}{\sigma_j}$ (2.7)

More particularly, the transformation of the initial data into reduced data solves, according to us, the problem of injustice in the distribution of resources due to the direct use of heterogeneous variables [24].

2.3. Fair division

2.3.1. Definitions of some concepts

A resource C is a finite set of physical objects (or goods), of finite divisible or indivisible quantities. It plays a central role in the sharing problem, hence the expression resource sharing. [5], ([6], pp. 5, 7). A resource can be discrete (or indivisible) or continuous (or divisible). The resource is indivisible if the objects cannot be divided. It is divisible if it can be divided a finite number of times until obtaining indivisible parts [6].

The case of non-homogeneous divisible resources is different from the homogeneous one. In The first one, the individuals do not have the same value of a portion of the resource instead of the second one for which any quantity of the resource has the same value for any agent. Examples of indivisible resources: an inheritance made up of a car and a house, courses to allocate to students, satellite resources, etc. Examples of non-homogeneous divisible resources: a cake, a territory, computing time on a supercomputer, speaking time. Examples of homogeneous divisible resources are: a sum of money, electoral seats between candidates (political and independent parties), a common profit (surplus), an asset between creditors following a bankruptcy, overall cost of common equipment…

Resource division. Let C be a resource, I={1,…,i,…,n} the set of individuals (a person, an object or an entity) beneficiaries of C and P(C) the set of parts of C. A division of C between individuals 1,…,i,…,n is an n-uple $c = (c_1, \ldots, c_i, \ldots, c_n)$ where $c_i \in P(C)$, $\forall i$ and $\bigcup_{i=1}^{n} c_i = C$. The component c_i of c is the share of individual i. ([3], p.16), ([13], p.63), [24].

A division can be fair or unfair. The division of a resource will be said to be fair when each individual is allocated a share and this is accepted by all. As for us, a division will be fair if it uses an adequate mechanism that uses several variables (criteria) while taking into account the particularities of the latter and which takes into account the origin of the resource which determines the relationships between individuals, whether or not they are contributors to the creation of the common resource to be shared, thus making it possible to decide whether or not to reduce inequalities between them. We consider that a share is fair when each person is allocated a share corresponding to the value (proportion) that he verifies.

The distribution of a resource must be done according to criteria which are (intrinsic) characteristics common to individuals and an appropriate method. The sharing criteria influences the sharing according to their number (one or more) and their type (homogeneous or heterogeneous). When several variables are considered, the individuals each verify as many values as there are variables. Therefore, to compare two individuals, a single value should be used for each. This is what we called total value. The total value of an individual is the sum of all its verified values for all variables. The sum of all the total values of all the individuals, that is to say of the whole population, constitutes what we have called global value.

A division method (or rule) Φ assigns to each fair division problem (I, C, w_i) a solution $\Phi(I, C, w_i) = c$ where I is the set of individuals benefiting from the resource C and w_i the different claims of individuals (corresponding to the total values).

2.3.2. Normative principles of distributive justice

Three main normative theories of distributive justice exist, they are: the principle of equity, the welfarism [1], [31], [32] and the absence of envy [6], [38]. Consider the principle of equity.

Aristotle's principle of equity is the first and oldest of the normative theories of distributive justice. It finds its source in Aristotle's maxim in "Nicomachean Ethics, Book V, Chapter 6, Tricot translation": equals must be treated equally and unequals in proportion to their differences. This principle encompasses four principles [6], [18], [23] below: (1) The principle of compensation. The idea underlying this principle is that because of a certain number of involuntary and morally unjustified differences (health, parents' wealth, intellectual capacities, etc.), certain individuals need more large amount of basic resources than other agents in order to achieve the same degree of well-being (ex-post equality is sought). (2) The principle of reward. It is based on the idea that differences in individual characteristics are voluntary, so individuals can be responsible for them. The more an individual has contributed to the creation of the resource, the more he should benefit from it. (3) The principle of exogenous rights. To allocate the resource, this principle takes into account considerations completely external to the consumption of the resource as well as to the questions attached to it relating to the needs and merits of the individuals. A distinction is made between equal exogenous rights (Example: right to vote) (ex ante equality) and unequal exogenous rights (Example: Different sizes of a population): Individuals are not equal with respect to the decision-making procedure for various reasons. (4) The principle of fitness. This principle consists in allocating the resource to the individual who makes the best use of it. It is subdivided into sum fitness and efficiency fitness corresponding respectively to classical utilitarianism and the Pareto principle of efficiency.

Aristotle's principle only works perfectly if the resource to be shared is divisible [39].

2.3.3. Existing methods of fair division

There are different fair division methods distributed according to the types of resources: Indivisible (discrete) and divisible (continuous) [6].

2.3.3.1. Indivisible resource division Methods (Discrete)

2.3.3.1.1. Methods of indivisible resources division without monetary compensation.

In this group of indivisible methods, monetary compensation does not come into play. Fair division methods for the case of discrete resources are based on the principle of fitness and lead to individuals expressing their preferences on the objects of the resource. For example, each individual distributes 100 points on the objects. In this model, the preferences of the individuals are aggregated into a common preference using a collective utility function representing the welfare of the group, thus formally translating the “ethical” criterion chosen by the community. Two models are distinguished: 1) The utilitarian model (classic): we determine the decision that maximizes the collective utility function which is the sum of the individual utilities. The community is interested in the overall utility produced. 2) The egalitarian model: the collective utility function is the minimum of individual utilities. The satisfaction of the group is the least satisfying of group members [5], [6].

2.3.3.1.2. Method of an indivisible resource division with monetary compensation

In this group of methods monetary compensation is allowed. As methods there are: 1) Adjusted winner procedure proposed by Brams and Taylor (1999) and works for the case of two individuals [Brams Taylor 1996]. 2) the Knaster procedure (or Method of sealed offers) proposed by Knaster for a division between several individuals of a small number of objects not necessarily having similar values [Knaster, B (1946)], if they are numerous (For example: an inheritance of jewels) with similar values, it is 3) the method of markers which will be used [43]

2.3.3.2. Methods for divisible resources division

2.3.3.2.1. Inhomogeneous divisible resource division methods

These methods are based on the Cake-cutting model. This includes several methods, these are: 1) Procedure "I cut you choose" or Cut-and-choose or Divide-and-choose proposed by Steinhaus, Hugo (1949). It is used to share a cake, a piece of land, ... between two individuals going through the following steps: (1) One individual (cutter) cuts the cake into 2 parts, (2) the other (chooser) takes one of the 2 parts (which he considers to be the fair share), (3) the cutter takes the remaining part. The generalization of this method for any n of the procedure was made by Banack-Knaster with the procedure Last Diminisher (1948)). The procedure “I cut you choose” has for extension the procedure of Lone-divider. 2) Lone-Divider procedure (n= 3 or more agents) which is used for sharing a resource between n individuals with n-1 choosers. 3) Lone-chooser procedure (single chooser) allows to share a resource between three individuals with two cutters and a chooser. It uses at some level the cut-and-choose method. 3) Last Diminisher method: In this method, each of the individuals is both cutter and chooser [43].

2.3.3.2.2. Homogeneous divisible resources division methods

These division methods concern resources such as amount of money, electoral seats between candidates, common profit, assets of a bankrupt company, overall cost of common equipment… [7], [18], [37], [44].

A fixed divisible resource division methods.

There are different methods of fixed divisible resources division: 1) The equal surplus method which consists in sharing the surplus equally between the beneficiary individuals. The claims of individuals come automatically to them and they share the surplus. 2) The method of uniform losses for which the individuals share the losses in a uniform way. 3) The uniform gains method which favors the individual with the lowest claim because any deficit is initially borne by the individuals with the highest claim. 4) The contested clothing method [18], [26] with its two extensions which are: (1) the random priority method which allocates the resource according to the value of Shapley (of the cooperative game) [2] and (2) the Talmud method which has as its source the Babylonian Talmud. 5) The proportional method which distributes the resource in proportion to the claims of individuals [18]. Our processes are based on the proportional method:

Definition (Proportional method).Let W=(w_1, ..., w_i, …, w_n) be a vector of respective total values (claims) of individuals 1, …, i, …, n belonging to set I; consider C, the resource to be shared between the n individuals of I and (I, C, W) a fair division problem. A division rule Φ is called the proportional rule denoted Φ^p if and only if to any problem (I, C, W) and to any individual $i \in$ I, we associate the share $c_i = \Phi_i(I, C, W) = \frac{w_i}{V} C$ où $V = \sum_{i=1}^{n} w_i > 0$is the global value (the sum of claims).

A divisible variable resource division methods.

This is the case where the resource to be shared is not fixed but is determined by individual demands. That concerns the division of a global cost (resource) created by individual requests. Three categories of cost division methods exist, they are: 1) the average cost method, 2) the sequential distribution method and the methods inspired by the theory of cooperative games: the core, the nucleolus and the Shapley value. On this subject, for more details, see [7], [20], [28], [40].

3. Résultats

The different fair division methods cited above do not pay attention to the cases of plurality of variables, types of variables (homogeneous or heterogeneous) and the origin of the resource (whether it comes from the contributions of individuals or not) which may in some cases, involving problems of proximity or belonging to the same population, thus justifying the reduction of inequalities. The plurality and types of variables as well as the origin of the resource can modify the results of the division if they are not taken into account. As for us, we take into account these elements to treat the two processes that we have proposed: PRRS and PRRC [24]. Let us consider more particularly the case of plurality and types of variables through the example below:

Example 1.3. In 2019, a 37-year-old Internet user with a Master's degree requests a solution to the following question: 1000 Euros to be divided between A, B and C according to the capital (in euros) respectively 300, 200 and 100 invested in the company and the time (in months) respectively 6, 12 and 8 spent within it [42]. This question did not have a convincing solution at the level of Internet users [42].

As for us, the solution to this problem is as follows: We notice that the units of measurement are different "Euro" and "month". The "Capital" and "Time" variables are heterogeneous. They must therefore be transformed into reduced data by dividing each value by the standard deviation of the corresponding variable (Notion used in Statistics). The standard deviation of the "Capital" variable is 100 and that of "Time" is 3.055050463. After dividing each value by the standard deviation of the corresponding variable, the transformed data of individuals A, B and C are presented respectively as follows: Capital: 3, 2 and 1; Time: 1,963,961,012; 3.927922024 and 2.618614683. The total values are: 4.963961012; 5.927922024 and 3.618614683. The standard deviation of each of these variables is transformed into 1. This data can therefore now be used to calculate the parts of the individuals. From the total values, the respective proportions are calculated: $\frac{4.963961012}{14.51049772}$=0.342094469; 0.408526444; 0.249379088. And the respective shares are: 0.342094469x1000=€342.1; €408.5; €249.4 [24] (Cf. 3.1)

3.1. Resource Allocation Process Without Reducing Inequalities (PRRS)

It is, for us, suitable for the case where individuals (such as shareholders of a company) were recovered when the resource to be shared was created. It uses several variables and the principle of equity in the strict sense (each individual is assigned a share proportional to their merit or total value), and does not allow the reduction of inequalities. The sharing rule to be used here is the proportional method. This process follows the following four steps: 1) Determination of the data to be used (renewal, conversion or transformation of the starting variables); 2) calculation of the total values of the individuals; 3) calculation of proportional shares of individuals; 4) Graphical representation of parts of individuals [24].

3.1.1. Determining which data to use

The data can be homogeneous with the same unit of measurement expressed in a unique way, in which case they are used directly. That can be homogeneous but with the same unit of measurement expressed differently for a variable. For this case a conversion is required to have the same notation of the unit of measurement. Finally, the data can be heterogeneous, in which case it is necessary to transform the initial data into reduced data.

3.1.2. Calculation of total values of individuals and globalvalue of the population

It is a question of determining a single value Wi for each individual i (the sum of all its values Yijverified for each variable) Yj instead of several. We have :

$$W_i = \sum_{j=1}^{m} Y_{ij} \qquad (3.1)$$

$$V_k = \sum_{i=1}^{n_k} W_{ik} \qquad (3.2)$$

$$V = \sum_{i=1}^{n} W_i = \sum_{k=1}^{q} V_k \qquad (3.3)$$

With Vk the total value corresponding to the k^{th} class with n_k individuals and V the global value of the entire population with n individuals and q classes.

3.1.3. Calculation of the (proportional) shares of individuals

The share C_i of an individual i of the resource (total share) C, is proportional to its total value W_i:

$$C_i = \frac{W_i}{V}.C \qquad (3.4)$$

where

$$\frac{W_i}{V} = P_i \qquad (3.5)$$

is the proportion of i to the global value.

3.1.4. Graphic representation of the shares of individuals

We can go further in the sense of graphically representing the shares of individuals on a bar or pie chart.

3.2. Resource Allocation Process Based on Clustering Results (PRRC)

This process mainly uses the classification notion which makes it possible to find the closest individuals, who will form the same class and will have to unite through the reduction of inequalities between them, following their close relationships, and then between all of them, forming the whole of the population which will also have to show solidarity following the fact that they belong to the same population.

This process is carried out by the following five main steps: (1) Determination and presentation of the results of the Clustering (the classes). It is assumed that the probable transformation of the data has already taken place. (2) Reduction of inequalities between individuals in their classes and then in the population as a whole. (3) Calculation of corrected proportions of individuals. (4) Calculation of the respective shares of individuals. (5) Graphical representation of parts of individuals. In the case where it is simply a matter of calculating the parts of the individuals, the method can be carried out using the first four steps.

3.2.1. Determination and presentation of clustering results

Assume that the variable centering operation has been performed. It is a question here of finding the individuals distributed in their classes resulting from the cutting of the dendrogram.

3.2.2. Inequality level index and reduction of inequalities

3.2.2.1. Inequality level index

This index that we propose help to measure the level of inequalities between individuals in order to allow their reduction.It is noted J_M and calculated through the following formula:

$$J_M = \frac{\sum_{i=1}^{n}(W_i - W_1)}{\sum_{i=1}^{n} W_i} \qquad (3.6)$$

wherew_i is the total value of individual i and $w_1 \leq ... \leq w_i \leq ... \leq w_n$.

The measurement of inequalities can be done even after sharing, using the shares of individuals.In this case, it suffices to use the formula

$$J_{M'} = \frac{\sum_{i=1}^{n}(C_i - C_1)}{C} \qquad (3.7)$$

where C_1 is the smallest share and C_i ($1 \leq i \leq n$) the share corresponding to an individual i..

3.2.2.2. Function of corrected values or function of reduction of inequalities

1) Formula of corrected values

We have proposed the following expression to calculate the corrected values of individuals.

$$Z_i = W_1 + W_i . J_M \qquad (3.8)$$

where$i \in \{1,2,\ldots,k\}$ and k the number of elements of a given class K.

Theorem 3.1. Let $Y=\{Y_1, \ldots, Y_j, .., Y_m\}$ be the set of m variables;$I=\{1, \ldots, i, \ldots k\}$ the set of k individuals belonging to class K;$X_i = (X_{i1}, \ldots, X_{ij}, .., X_{im})$ the vector of m values verified by individual i.Then the corrected value of an individual i which the total value isW_i:

$$Z_i = \frac{W_1 V + W_i(V_k - kW_1)}{V_k} \qquad (3.9)$$

where$V_k = \sum_{i=1}^{k} W_i$and W_1 the minimum of the distribution of total values of class K.

Proof. Knowing that $J_M = \frac{\sum_{i=1}^{k}(W_i - W_1)}{\sum_{i=1}^{k} W_i} = \frac{\sum_{i=1}^{k} W_i - (kW_1)}{\sum_{i=1}^{k} W_i} = 1 - \frac{kW_1}{\sum_{i=1}^{k} W_i}$,

$Z_i = W_1 + W_i.J_M = W_1 + W_i - \frac{kW_1 W_i}{\sum_{i=1}^{k} W_i} = \frac{W_1 \sum_{i=1}^{k} W_i + W_i \sum_{i=1}^{k} W_i - kW_1 W_i}{\sum_{i=1}^{k} W_i} = \frac{W_1 V_k + W_i (V_k - kW_1)}{V_k}$ $\blacksquare$

Corollary 3.1. The total value of an individual i belonging to class K is determined after reduction of inequalities from its calculated corrected value:

$$W_i = \frac{V_k\,(Z_i - W_1)}{V_k - kW_1} \qquad (3.10)$$

We just have to draw Z_i in the previous equation (3.9) to arrive at this formula.

Notes: 1) The notations Z_i and J_M (respectively $Z_i'$and $J_{M'}$) will be used in the case where the reduction of inequalities is done at the level of classes (respectively of the population). 2) Z_i values fall into two parts. Those of individuals who have transferred values to others (“the rich”) and those of individuals who have received them (“the poor”).The rich will see their values diminished while the poor will see them increased. 3) We calculate the differences between the corrected (total) values and the starting (total) values:

Z_i - W_i= E_i, *(3.11)*

Negative (resp. positive) values of E_i will be seen in rich (resp. poor) individuals. 4) We will check that the sum of all won and lost values equals zero:

$$\sum_{i=1}^{n}(Z_i - W_i) = \sum_{i=1}^{n} E_i = 0 \qquad (3.12)$$

In the case where the level of inequalities is simply calculated, if the index of the level of inequalities is still high, it is necessary to attempt a 2nd, 3rd, … reduction, everything depends on the level of inequalities sought.

The above formulas can also be used in the case of the distribution of seats between different constituencies by allocating them quotas as well as in the case where individuals must make their contributions.

This reduction of inequalities responds to the concern about the lack of a mechanism for reducing inequalities in the sharing of resources, raised by Marcel Kapya [17] and Paulin Punga [29].

Function of corrected values

The function of corrected values is defined as follows: Let A' be the set of total values (at the level of classes or of the population) of individuals and B' the set of corrected values, we have:

$z: A' \subset \mathbb{R} \longrightarrow B' \subset \mathbb{R}$

$$W_i \longrightarrow z(W_i) = Z_i = \frac{W_1 V_k + W_i (V_k - kW_1)}{V_k} \qquad (3.13)$$

where$V_k = \sum_{i=1}^{k} W_i$ and$W_1$the minimum of the total values of the class K (respectively of the population).

Proposition 3.1 (Properties of the inequality level index).If W_1, …, W_i, …, W_n the respective total values of n individuals. Then the level index of inequalities J_M between individuals verifies the following properties:

$J_M = 0$ if $W_i = W_1$, $\forall i \in [1, n]$. It is complete equality: All individuals have the same value

$J_M = 1$ if $\exists i \in [1, n]$, $W_i = 0$. This is total inequality: At least one individual has the value 0

$$J_M = 1 - \frac{W_1 . n}{\sum_{i=1}^{n} W_i}$$

$J_M \in [0, 1]$.

$J_M = \frac{\sum_{i=1}^{n}(W_i - W_1)}{\sum_{i=1}^{n} W_i} = \frac{\overline{(W_\iota - W_1)}_{1\leq i\leq n}}{\overline{\overline{(W_1)}}_{1\leq i\leq n}}$.

Proof.

1) Let us show that $J_M = 0$ if $W_i = W_1$, $\forall i \in [1, n]$. We know that $J_M = \frac{\sum_{i=1}^{n}(W_i - W_1)}{\sum_{i=1}^{n} W_i}$. If $W_i = W_1$, we have : $J_M = \frac{\sum_{i=1}^{n} 0}{\sum_{i=1}^{n} W_i} = 0$.

Let us show that $J_M = 1$ si $\exists i \in [1, n]$, $W_i = 0$. Suppose there is $i' \in [1, n]$ such as $W_{i'} = 0$. Let us consider $W_1 = 0$, we have : $J_M = \frac{\sum_{i=1}^{n}(W_i - W_1)}{\sum_{i=1}^{n} W_i} = \frac{\sum_{i=1}^{n}(W_i - 0)}{\sum_{i=1}^{n} W_i} = 1$. So it suffices to consider $W_{i'} = W_1 = 0$. Hence the existence of i '= 1.

Let us show that $J_M = 1 - \frac{W_1 . n}{\sum_{i=1}^{n} W_i}$. Indeed, $J_M = \frac{\sum_{i=1}^{n}(W_i - W_1)}{\sum_{i=1}^{n} W_i} = \frac{\sum_{i=1}^{n} W_i - W_1 . n}{\sum_{i=1}^{n} W_i} = 1 - \frac{W_1 . n}{\sum_{i=1}^{n} W_i}$.

To show that $J_M \in [0, 1]$, amounts to show that $J_M = 0$, $J_M = 1$ et $0 < J_M < 1$. Indeed, there exists $W_1 = W_i$ pout all i such that $J_M = 0$; there is also $W_1 = 0$ such that $J_M = 1$. In addition, we know that

$\sum_{i=1}^{n} W_i < \sum_{i=1}^{n} W_i +$
W1.n⇔0<i=1nWi−W1.n <i=1nWi⇔0<i=1nWi− W1.ni=1nWi< 1⇔
$0 < \frac{\sum_{i=1}^{n}(W_i - W_1)}{\sum_{i=1}^{n} W_i} < 1 \Leftrightarrow 0 < J_M < 1$

Let us show that $J_M = \frac{\sum_{i=1}^{n}(W_i - W_1)}{\sum_{i=1}^{n} W_i} = \frac{\overline{(W_\iota - W_1)}_{1\leq i\leq n}}{\overline{\overline{(W_1)}}_{1\leq i\leq n}}$. We know, by definition of the arithmeticmean, that $\overline{W_\iota - W_1} = \frac{\sum_{i=1}^{n}(W_i - W_1)}{n}$ (i) and $\overline{W_1} = \frac{\sum_{i=1}^{n} W_i}{n}$ (ii). By dividing (i) by (ii) member to member, we have: $\frac{\overline{W_\iota - W_1}}{\overline{W_1}} = \frac{\sum_{i=1}^{n}(W_i - W_1)}{\sum_{i=1}^{n} W_i} = J_M$

Let us show that

$J_M = \frac{\sum_{i=1}^{n}(w_i - w_1)}{\sum_{i=1}^{n} w_i} = \frac{\overline{(w_\iota - w_1)}_{1\leq i\leq n}}{\overline{\overline{(w_1)}}_{1\leq i\leq n}}$.

We know, through the arithmetic mean definition, that $\overline{w_\iota - w_1} = \frac{\sum_{i=1}^{n}(w_i - w_1)}{n}$ (i)

et $\overline{w_1} = \frac{\sum_{i=1}^{n} w_i}{n}$ (ii)

By dividing (i) by (ii) member by member, we have: $\frac{\overline{w_i - w_1}}{\overline{w_1}} = \frac{\sum_{i=1}^{n}(w_i - w_1)}{\sum_{i=1}^{n} w_i} = J_M$ $\blacksquare$

Proposal 3.2.(Comparison between the starting values and the corrected values). If $W_1, \ldots, W_i, \ldots, W_n$ are the n total values respectively of the n individuals 1,…, i,…, n whose respective corrected values are $Z_1, \ldots, Z_i, \ldots, Z_n$.Then,

$\sum_{i=1}^{n}(Z_i - W_i) = 0$,

$\sum_{i=1}^{n} Z_i = \sum_{i=1}^{n} W_i$,

$\bar{Z}_i = \bar{W}_i$,

$\lim_{J_M \to 0} Z_i = Z' = \bar{W}_i$(Z 'is the constant value of the individuals at the very last reduction of inequality)

Proof.

1) Let us show that $\sum_{i=1}^{n}(Z_i - W_i) = 0$. Indeed, $\sum_{i=1}^{n}(Z_i - W_i) = \sum_{i=1}^{n}(W_1 + W_i J_M - W_i) = \sum_{i=1}^{n}(W_1 + W_i(J_M - 1)) = W_1.\mathrm{n} + (J_M - 1)\sum_{i=1}^{n} W_i = W_1.\mathrm{n} + \sum_{i=1}^{n} W_i - W_1.\mathrm{n} - \sum_{i=1}^{n} W_i = 0$.

2) Let us show that $\sum_{i=1}^{n} Z_i = \sum_{i=1}^{n} W_i$. Indeed, $\sum_{i=1}^{n} Z_i = \sum_{i=1}^{n}(W_1 + W_i J_M) = W_1.\mathrm{n} + J_M \sum_{i=1}^{n} W_i = W_1.\mathrm{n} + \left(\frac{\sum_{i=1}^{n}(W_i - W_1)}{\sum_{i=1}^{n} W_i}\right)\sum_{i=1}^{n} W_i = W_1.\mathrm{n} + \sum_{i=1}^{n}(W_i - W_1) = \sum_{i=1}^{n} W_i$.

3) Let us show that $\bar{Z}_i = \bar{W}_i$. We know that $\sum_{i=1}^{n} Z_i = \sum_{i=1}^{n} W_i$ (i). Let us divide (i) by n (number of individuals), we have: $\frac{\sum_{i=1}^{n} Z_i}{\mathrm{n}} = \frac{\sum_{i=1}^{n} W_i}{n} \Leftrightarrow \bar{Z}_i = \bar{W}_i$.

4) Let us show that $\lim_{J_M \to 0} Z_i = Z' = \bar{W}_i$(Z 'is the constant value of the individuals at the very last reduction of inequality). We know that if $J_M \to 0$, $Z_i \to Z_i'$ (Z_i'being the very last function of the corrected values). We also know that $J_M \in [0, 1]$. Considering $J_M = 0$, $W_1 = W_i$for all i and being in the case of Z_i', we have:$Z_1' = .. = Z_i' = \cdots = Z_n'$. This implies$\lim_{J_M \to 0} Z_i = Z'$ ($Z' = Z_1' = .. = Z_i' = \cdots = Z_n'$). Now $Z' = Z_1' = \frac{\mathrm{n}Z_1'}{\mathrm{n}} = \frac{\sum_{i=1}^{n} Z_i'}{n} = \bar{Z}_i'et\sum_{i=1}^{n} Z_i = \sum_{i=1}^{n} W_i \Rightarrow \bar{Z}_i = \bar{W}_i$. So $\lim_{J_M \to 0} Z_i = Z' = \bar{W}_i$. $\blacksquare$

3.2.3. Calculation of corrected proportions of individuals in the whole population

The corrected proportion or the proportion of the corrected value of i in the population is given by:

$$\tilde{P}(i) = \frac{Z_i'}{\sum_{i=1}^{n} Z_i'} = \frac{T_1 V + T_i(V - nT_1)}{V^2} \qquad (3.14)$$

Indeed, $Z_i' = \frac{T_1 V + T_i(V - nT_1)}{V}$and $\sum_{i=1}^{n} Z_i' = \sum_{i=1}^{k} W_i = V$. Then$\tilde{P}(i) = \frac{T_1 V + T_i(V - nT_1)}{V^2}$

3.2.4. Calculation of the (proportional-corrected) shares of individuals

3.2.4.1. Corrected proportional parts

Corollary 3.2. The corrected proportional share of the resource C of an individual i in the whole population of n individuals according to the reduction index $J_M$is given by:

$$c^*(i) = \frac{T_1V+T_i(V-nT_1)}{V^2}.C \qquad (3.15)$$

where, T_i is the corrected value of an individual i at the population level and $T_1 \leq T_i$, ∀i.

Let us note that this formula can be used directly in the case where it is assumed or proven that all individuals form the same class. It is therefore confused with the whole population. Otherwise, it will be necessary to first reduce the inequalities in the classes before using this formula. In addition, this formula is used to define a sharing rule that we call the corrected proportional method.

3.2.4.2. Function of proportional shares

The function of proportional shares corrected according to the reduction index J_M is defined as follows: Let A' be the set of total corrected values (at the level of the population) of individuals and B' the set of corrected proportional shares, C there source to share, we have:

$c^*: A' \subset \mathbb{R} \longrightarrow B' \subset \mathbb{R}$

$T_i \longrightarrow c^*(T_i) = \frac{T_1V+T_i(V-nT_1)}{V^2}.C$

where $T_1$is the minimum of total values in the population level.

3.2.5.Graphic representations of the shares of individuals

After calculating the different corrected proportions and shares of individuals, it is appropriate to represent them graphically through a bar or pie chart.

3.3. Applications

3.3.1. Applications relating to the PRRS process

Application 1. Either to share between three individuals A, B, D the sum of 5000 Dollars product of their investment. Each had contributed for two months respectively by: 1st Month (in Dollars): 300, 200, 100 and 2nd Month (in CDF): 40000, 300000, 80000. Solution: The two variables can be converted into each other then calculate the shares knowing for example that 20000CDF = 10$.We have: 40000 CDF = 20$, 300000 CDF=150$, 80000 CDF = 140$.The respective total values (in $): 320, 350, 140. The respective proportions: 320/810=0.3950617;0.4320988;0.1728395 and finally the shares are: For A=0.3950617x5000=1975.30864; for B=2160.49383;for D=864,19753**Application 2.**Either to divide between three individuals A, B and D the sum of 5000 Dollars product of their investment. Each having contributed for two months respectively with: 1st Month: Blocks (in Kg): 300, 200, 100;2nd Month (in CDF): 40000, 300000, 80000. Solution: The respective standard deviations of two variables are: 81.6496581 and 114309.521.By dividing each value by the standard deviation of the corresponding variable, the contributions become: 1st Month: 3.67423461;2.44948974;1.22474487 and 2nd Month: 0.3499271;2.6244533;0.6998542.The respective totals are: 4.02416171;5.07394304;1.92459907.The respective proportions are: 4.02416171/11.0227038=0.36507937;0.46031746;0.17460317 and finally the respective shares are: For A= 0.36507937x5000=1825.3968$;B=2301.5873;D=873.0159.The share of A=0.36507937x5000$= 1825.3968$, that of B = 0.46031746x5000$= 2301.5873$ and that of C = 0.17460317x5000= 873.0159$ 3.3.2.Application relating to the PRRC process Either to share between the 24 municipalities of the City-province of Kinshasa considered as ETDs the revenue (C = 144745972822 CDF) of a national nature allocated to them in 2015 according to the three heterogeneous variables: surface area, production capacity and population imposed by the legislator. Determination and presentation of classification results The data table We present below the table of data relating to the three heterogeneous variables which are Area, Production, Population:

Table 1. Table of data for 24 communes of the City-province of Kinshasa

	Superficie	Production	Population
Bandalungwa	6.82	27813458131	273218
Barumbu	4.6	18051309794	111758
Bumbu	5.3	38037720000	365716
Gombe	29.33	18051309800	57308
Kalamu	6.64	42716619500	195385
Kasa-Vubu	5.05	20734456481	72940
Kimbanseke	237.78	115326208025	1036732
Kinshasa	2.87	20803773513	169793
Kintambo	2.7	18051309800	84875
Kisenso	16.6	50774724194	359675
Lemba	23.7	49128444931	356853
Limete	67.6	43409789800	294810
Lingwala	2.88	18051309794	123619
Makala	5.6	29199798725	223502
Maluku	7948.8	21081041631	656672
Masina	69.73	68883798181	631364
Matete	4.88	37431195994	223685
Mont-Ngafula	358.92	22467382225	340378
Ndjili	11.4	45749239550	395890
Ngaba	4	24338942025	208283
Ngaliema	224.3	88812444194	667608
Ngiri-Ngiri	3.4	22120797081	105664
Nsele	898.7	18051309794	555440
Selembao	23.3	45575946975	491794
Total	9964.9	904662330138	8002962
Standard deviation (Pearson)	1582.47384	23892282493	234681.564

Source: [24], [41]

With regard to the Population variable, we should have spoken, for example, of inhabitants/Km2 as the unit expressing the density. This measures the population, the number of inhabitants occupying a given area. Density is calculated by dividing the total population by the area of the region considered. However, in our case the number of inhabitants is given in a rough way and was used to estimate the production capacity of each municipality [24]. 2. Data Transformation: Reduced Data Table The three variables retained are heterogeneous. They cannot therefore be used directly. They must first be transformed into reduced data by dividing each value by the standard deviation of the corresponding variable. After calculations, we present below the table of reduced data relating to these three variables:

Table 2. Table of reduced data of 24 communes of the City-province of Kinshasa

	Superficie	Production	Population	Valeurtotzl
Bandalungwa	0,00430971	1,16411892	1,16420734	2,33263597
Barumbu	0,00290684	0,7555289	0,47621125	1,23464698
Bumbu	0,00334919	1,59205049	1,55834993	3,15374961
Gombe	0,01853427	0,7555289	0,24419473	1,01825789
Kalamu	0,00419596	1,78788358	0,83255368	2,62463322
Kasa-Vubu	0,00319121	0,86783071	0,31080413	1,18182605
Kimbanseke	0,15025841	4,82692301	4,4176116	9,39479302
Kinshasa	0,00181362	0,87073194	0,72350379	1,59604934
Kintambo	0,00170619	0,7555289	0,36166028	1,11889536
Kisenso	0,0104899	2,12515168	1,53260867	3,66825025
Lemba	0,01497655	2,05624745	1,52058387	3,59180787
Limete	0,04271793	1,81689589	1,25621287	3,11582668
Lingwala	0,00181994	0,7555289	0,52675207	1,2841009
Makala	0,00353876	1,22214354	0,95236284	2,17804514
Maluku	5,02302143	0,88233686	2,79814055	8,70349884
Masina	0,04406392	2,88309826	2,6903008	5,61746299
Matete	0,00308378	1,56666472	0,95314262	2,52289111
Mont-Ngafula	0,22680944	0,94036148	1,45038236	2,61755328
Ndjili	0,00720391	1,91481243	1,68692416	3,6089405
Ngaba	0,00252769	1,01869472	0,88751326	1,90873567
Ngaliema	0,1417401	3,71720216	2,84473986	6,70368213
Ngiri-Ngiri	0,00214853	0,92585533	0,45024414	1,37824801
Nsele	0,56790828	0,7555289	2,36678157	3,69021875
Selembao	0,01472378	1,90755935	2,09558003	4,01786317
Total	6,3	37,86	34,1	78,26
Ecart-type	1	1	1	

Source: [24]

It is these reduced data that we will use when calculating the shares of individuals. This will show the possibility and the importance of using several variables, notably heterogeneous, instead of just one. 3. Clustering of individuals The 24 communes of the city/province of Kinshasa are grouped into 6 classes (see the dendrogram built from the R software) by the HAC method passing through a PCA using reduced centered data.

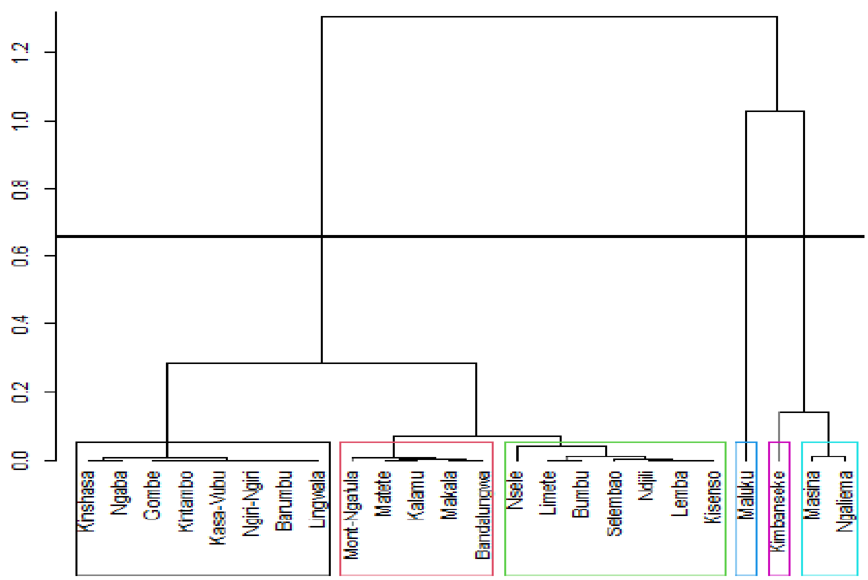

Figure 3.1. Dendrogram of the 24 communes of Kinshasa divided into 6 classes

Source: [24], from R software

Individuals divided into 6 classes: 1st Class: 1) Barumbu, 2) Gombe, 3) Kasa-Vubu, 4) Kinshasa, 5) Kintambo, 6) Lingwala, 7) Ngaba, 8) Ngiri-Ngiri;2nd Class: 1) Bandalungwa, 2) Kalamu, 3) Makala, 4) Matete, 5) Mont-Ngafula;3rd Class: 1) Bumbu, 2) Kisenso, 3) Lemba, 4) Limete, 5) Ndjili, 6) Nsele, 7) Selembao;4th Class: 1) Maluku;5th Class: 1) Masina, 2) Ngaliema;6th Class: 1) Kimbanseke.

3.3.2.2. Reduction of inequalities between individuals in their classes and in the population as a whole

We are going to use the table of reduced data for which the influence of the units of measurement is eliminated.

Reduction of inequalities in classes

We show how to reduce inequalities at the 1st class level with the example of the municipality of Kintambo. For the rest, we will proceed in a similar way ([24], p.115). Knowing that the smallest total value in the 1st class is Gombe: W_1=1.01825789 then:

The level index of inequalities equals: $J_M = \frac{\sum_{i=1}^{n}(W_i - W_1)}{\sum_{i=1}^{n} W_i}$=2.57469708/10.7207602

=0.240159936.So the inequality in the 1st class is 24%.

Kintambo's corrected value is: $Z_i = W_1 + W_i.J_M$ = 1.01825789 + (1.11889536 x 0.240159936) = 1.28697173.

The difference between the corrected value and the total value is: Z_2-W_2 = 1.28697173 - 1.11889536 =0.168076.Which means that the Kintambo individual (Poor) benefited from 0.168076 at the expense of the others (the rich).On the other hand, the individual Ngaba (rich) lost 0.432076 (hence -0.432076) to the benefit of others (the poor).

This gap made it possible to separate the 1st class into two parts: the poor (Gombe, Kintambo, Kasa-Vubu, Barumbu and Lingwala) on one side and the rich (Ngiri-Ngiri, Kinshasa and Ngaba) on the other. After calculations of the corrected values in the rest of the classes.We have: Gombe: 1.26280264;Kintambo: 1.28697173;Kasavubu: 1.30208516;Barumbu: 1.31477063;Lingwala: 1.32664749;Ngiri-Ngiri: 1.34925785;Kinshasa: 1.40156500;Ngaba: 1.47665973;Makala: 2.42387540;Bandalungwa: 2.44132370;Matte: 2.46279730;Mount Ngafula: 2.47348160;Kalamu: 2.47428070;Limete: 3.49653163;Bumbu: 3.50116521;Lemba: 3.55468903;Ndjili: 3.55678237;Kisenso: 3.56402909;Nsele: 3.56671329;Selembao: 3.60674628;Masina: 6.11269257;Ngaliema: 6.20845250;Maluku: 8.70349884;Kimbanseke: 9.39479302 [24].

Reduction of inequalities in the whole population

Let us reduce inequality at the population level ([24], pp. 116-117).Knowing that the smallest corrected value (resulting from the first operation to reduce inequalities in classes) is that of Gombe: Z_1=1.26280264, then: (1) The level index of inequalities for the whole ofequal population:$J'_M = \frac{\sum_{i=1}^{n}(Z_i - Z_1)}{\sum_{i=1}^{n} Z_i}$=47.9553494/78.2626128 =0.61274915.So the inequality in the population is 61.3%.(2) The corrected value, for example, from Kintambo: $Z'_2 = Z_1 + Z_2.J'_M$= 1.26280264 + 1.28697173 x 0.61274915 = 2.05139347 (3) The difference between the corrected valueand the total value: The Kintambo individual (Poor) benefited from: Z'_2-Z_2= 2.05139347 - 1.28697173 = 0.7644217 at the expense of others (the rich).On the other hand, the individual Limete (Rich) lost 0.0912322 (hence -0.0912322) to the benefit of the others (the poor).

The different corrected total values of individuals at the level of the whole population are Gombe: 2.03658388;Kintambo: 2.05139347;Kasavubu: 2.06065422;Barumbu: 2.06842723;Lingwala: 2.07570476;Ngiri-Ngiri: 2.08955924;Kinshasa: 2.12161040;Ngaba: 2.16762463;Makala: 2.74803023;Matete: 2.77187959;Mount Ngafula: 2.77842639;Kalamu: 2.77891604;Limete: 3.40529942;Bumbu: 3.40813865;Lemba: 3.44093532;Ndjili: 3.44221801;Kisenso: 3.44665844;Nsele: 3.44830318;Selembao: 3.47283336;Masina: 5.00834982;Ngaliema: 5.06702663;Maluku: 6.59586416;Kimbanseke: 7.01945408 [24] The gaps calculated above made it possible to divide the population into two parts: the poor who are 13 in number (Gombe, Kintambo, Kasa-Vubu, Barumbu, Lingwala, Ngiri-Ngiri, Kinshasa, Ngaba, Makala, Bandalungwa, Matete, Kalamu and Mont-Ngafula) on one side and on the other the rich, 11 in number (Limete, Bumbu, Lemba, Ndjili, Kisenso, Selembao, Nsele, Masina, NgaliemaKimbanseke and Maluku).

The corrected values of individuals at the level of the whole population come to solve the problem of inequalities between individuals as a whole, knowing that the problem was first solved at the level of classes.These corrected values will be used to calculate the (corrected) proportions of individuals in relation to the whole population.

3.3.2.3. Calculation of the proportions of individuals in relation to the whole population

The proportions of individuals in relation to the whole population are calculated [24] as follows: The (corrected) proportion, for example, of the individual Kintambo is the ratio of its corrected value (at the level of the wholeof the population) and the overall value: p_i = 2.05139347/78.2626128=0.0262, i.e. 2.62%.The different (corrected) proportions of individuals at the level of the whole population are: Gombe: 0.02602244;Kintambo: 0.02621167;Kasavubu: 0.02633;Barumbu: 0.02642931;Lingwala: 0.0265223;Ngiri-Ngiri: 0.02669933;Kinshasa: 0.02710886;Ngaba: 0.02769681;Makala: 0.03511294;Bandalungwa: 0.03524955;Matete: 0.03541767;Mount Ngafula: 0.03550132 Kalamu: 0.03550758;Limete: 0.04351119;Bumbu: 0.04354747;Lemba: 0.04396653;Kisenso: 0.04403965;Nsele: 0.04406067;Selembao: 0.0443741;Masina: 0.06399416;Ngaliema: 0.0647439;Maluku: 0.08427861;Kimbanseke: 0.08969103. 3.3.2.4. Shares of individuals based on their corrected proportions relative to the population

From the corrected proportions p_i found above, we determine the shares of the common resource C = 144745972822 CDF going to the municipalities (C constitutes the national revenue allocated to the ETDs which are in our case the municipalities of the city-province ofKinshasa).The share ci going to individual iis the product of his proportion corrected at the level of the population and the common resource. The corrected shares of individuals (communes) are calculated [24] as follows: For the Kintambo individual, we have: c2 = 0.0262x144745972822 CDF = 3792344488 CDF orutilise$c^*(2) = \frac{T_1 V + T_2(V - nT_1)}{V^2}.C$, with V=78.2626128, n=24, C=144745972822 (See corrected values at class level)

We have thus determined, according to our process, the different (corrected) shares of the 24 municipalities, in CDF, of the national revenue (144745972822 (in CDF)) allocated to them in 2015 as part of the retrocession: Bandalungwa: 5095058243;Barumbu: 3821293683;Bumbu: 6296449818;Gombe: 3763395293;Kalamu: 5138482035;Kasavubu: 3806819085;Kimbanseke: 12983713762;Kinshasa: 3922615863;Kintambo: 3792344488;Kisenso: 6368822804;Lemba: 6368822804;Limete: 6296449818;Lingwala: 3835768280;Makala: 5080583646;Maluku: 12202085509;Masina: 9263742261;Mate: 5124007438;Mount Ngafula: 5138482035;Ndjili: 6368822804;Ngaba: 4009463447;Ngaliema: 9365064442;Ngiri-Ngiri: 3864717474;Nsele: 6383297401;Selembao: 6426721193.

The corrected proportions calculated above can also be used to determine the municipalities' quotas in the distribution of electoral seats. In this case, we will have taken into account three criteria (instead of just one, which is the number of voters or population) and reduce inequalities.They can also be used in the event that the communes have to make their contributions to the city-province of Kinshasa.It follows that the share of Kimbanseke is the largest followed by that of Maluku then Ngaliema then Masina.On the other hand, that of Gombe is the smallest preceded by that of Kintambo, Kasa-vubu, ...

3.3.2.5. Graphic representation of the shares of the municipalities

We will use the table of shares of the 24 communes of the City-province of Kinshasa of the common resource (national revenue allocated to ETDs for the 2015 financial year) to represent them graphically.

Bar chart of shares of municipalities

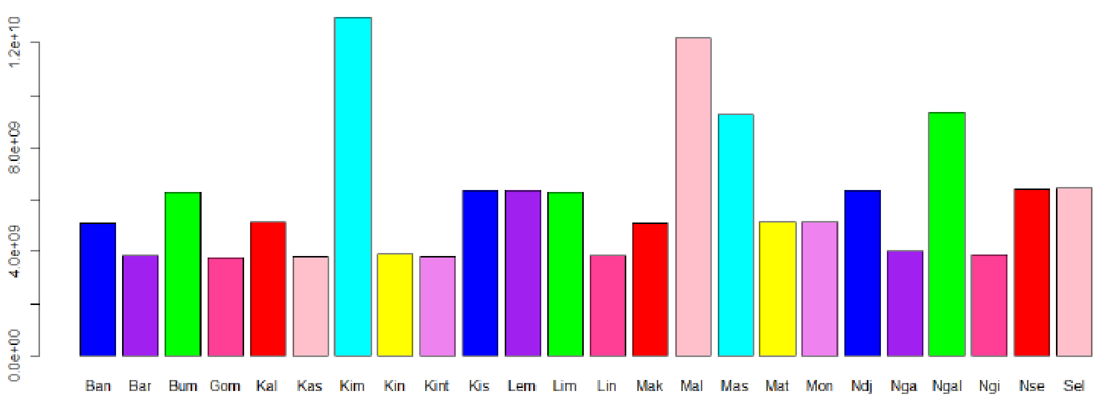

Figure 3.2.Bar graph of the shares of 24 communes of the City-province of Kinshasa.

Source: [24] from R software.

Legend: 1) "Ban": Bandalungwa, 2) "Bar": Barumbu, 3) "Bum": Bumbu, 4) "Gom": Gombe, 5) "Kal": Kalamu, 6) "Kas": Kasa-vubu, 7) "Kim": Kimbanseke, 8) "Kin": Kinshasa, 9) "Kint": Kintambo, 10) "Kis": Kisenso, 11) "Lem": Lemba, 12) "Lim": Limete, 13) "Lin": Lingwala, 14) "Mak": Makala, 15) "Mal": Maluku, 16) "Mas": Masina, 17) "Mat": Matete, 18) "Mon": Mont-Ngafula, 19) "Ndji": Ndjili, 20) "Nga": Ngaba, 21) "Ngal": Ngaliema, 22) "Ngi": Ngiri-Ngiri, 23) "Nse": Nsele, 24) "Sel": Selembao.

2) Pie chart of individual shares

We use the same matrix that was used to construct the bar graph.The Pie chart of the (corrected) shares of the municipalities is as follows:

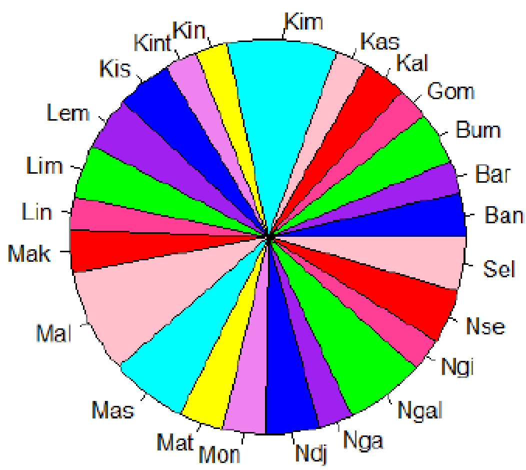

Figure 3.3.: Pie chart of the shares of 24 communes of the City-province of Kinshasa.

Source:[24] from R software

In the light of the graphs above, the share (corrected) of Kimbanseke is the largest followed by that of Maluku then Ngaliema then Masina, … On the other hand, that of Gombe is the smallest preceded by that of Kintambo, Kasa-vubu…

4. Discussion

In the light of the results of our research, it is appropriate to specify the following: Our two methods proposed above (PRRS and PRRC) use of course the proportional method as a sharing rule, a method which already exists and which is even the most widely use daccording to some authors. But in most cases, it is used from a single variable (or a single criterion) to calculate the shares of individuals. We are interested in the case of several variables which lead to check first if they are homogeneous with the same unit of measurement expressed in a unique way or if they are heterogeneous. In the case where the variables are homogeneous but the unit of measurement is expressed differently for each variable and where they are heterogeneous, it will be necessary to think of a preliminary transformation of the data (conversion to the same expression of the unit of measurement or transformation in reduced data) otherwise the results will be erroneous.

In addition, the techniques used for resource sharing emphasize the types of resources (divisible or indivisible) without hinting at the origin of the resource (resource coming from the contributions of individuals or not) which can modify the relations existing between individuals (relationships of proximity or belonging to the same population) and consequently modify the results of a sharing. Taking this aspect into account, we have categorized the individuals who benefit from a given resource into: Contributors to the creation of resources and non-contributors. The first category concerns individuals as shareholders of a company who expect to receive, each, a share proportional to his contribution. No one wants to lose anything to someone else. The second category, for its part, concerns individuals such as state entities which would be prepared to lose part of their proportional share to the benefit of others (the poor).It is the reduction of inequalities. We need criteria based on the relationships between the beneficiaries of the resource and which justify this reduction in inequalities. We opted for the criteria of proximity and belonging to the same population. Which is already justified above. The proximity of individuals is determined by the Clustering method which is a notion of Statistics.

Regarding the Clustering method, most authors limit the interpretation of its results to the determination of the most typical individuals and the most important variables. Our PRRC process extends the interpretation of the results of the Clustering to the determination of the proportions due to individuals and allowing the calculation of their shares of a given resource (amount of money, electoral seats).As for the distribution of electoral seats, if it is done by the PRRC process as we have proposed, the quotas will have been calculated using several variables, even heterogeneous, instead of just one.

5. Conclusion

We have presented and applied our two processes PRRS and PRRC which calculate the shares of individuals of a given divisible resource from several variables, notably heterogeneous, while taking into account the origin of the resource, making it possible in particular to reduce inequalities between individuals. in order to resolve the problems of injustice due to 1) the direct use of data from several homogeneous variables for which the same unit of measurement is expressed differently for each variable, 2) the direct use of data from heterogeneous variables, 3) the absence of a mechanism for distributing resources using several notably heterogeneous variables, 4) the absence of a mechanism for reducing inequalities between individuals.

The first is called the Resource Allocation Process Without Reducing Inequalities (PRRS).It is suitable for the case where individuals (such as shareholders of a company) have contributed to the creation of the common resource to be shared. It does not allow for the reduction of inequalities. Thus, individuals receive shares proportional to their total values. Hence the principle of (gross) multidimensional equity. The second, meanwhile, is called the Resource Allocation Process based on Classification Results (PRRC).It is suitable for the case where individuals have not contributed to the creation of the resource. It admits the reduction of inequalities and uses the notion of clustering to find the closest individuals who will have to show solidarity in their respective classes thanks to their proximity and subsequently with all the others because they belong to a same population. Individuals accordingly receive corrected proportional shares. Hence the principle of reduced (or corrected) multidimensional equity that we have proposed.

Some applications have been proposed more particularly with regard to the PRRC process. It was applied to data from 24 municipalities of the City-Province of Kinshasa based on three heterogeneous variables: Surface area, Production and population, with a view to the distribution of national revenue allocated to them by the province in 2015. City authorities-province of Kinshasa as well as any other researcher will be able to use the results found above to solve the problems mentioned in the introduction.

The corrected proportions calculated in the PRRC process above can also be used to determine the municipalities' quotas in the distribution of electoral seats. In this case, we will have taken into account three criteria (instead of just one, which is the number of voters or population) and reduce inequalities. They can also be used in the event that the communes have to make their contributions to the city-province of Kinshasa. It follows that the share of Kimbanseke is the largest followed by that of Maluku then Ngaliema then Masina.On the other hand, that of Gombe is the smallest preceded by that of Kintambo, Kasa-vubu, ...

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