Fair Share of GDP to Mitigate Climate Change Costs (according to DICE)

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The Social Cost of Carbon
does not cover the social cost of carbon (Part 2)

Fair Share of GDP to Mitigate Climate Change Costs (according to DICE)

Version 0.8

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Abstract

This paper investigates the fair share of GDP required to mitigate climate change costs, using an extended DICE model. A dedicated fund, supplied annually by a fixed fraction of GDP, is introduced to cover abatement and damage costs.

Numerical analysis reveals that a funding rate of 2.4% of GDP is sufficient to meet all costs, enabling faster early abatement and reducing total emissions significantly. The proposed approach promotes intergenerational equity by distributing climate-related costs evenly across generations, overcoming the classical DICE model's limitations.

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1. Introduction

Integrated Assessment Models aim to identify optimal emission pathways to mitigate climate change. A well-known example is the DICE model [8], where the optimal emission path is derived by maximizing total (discounted) social welfare. However, this objective function can result in intergenerational inequality [3], as it balances changes in future damage costs against changes in current abatement costs. Notably, the optimization focuses on balancing *cost changes* rather than ensuring equitable *costs* distribution across generations.

While the DICE model provides a qualitative framework for assessing mitigation strategies, its assumptions about damages and social welfare functions may limit its applicability. Assuming that the DICE model gives at least a qualitative assessment of mitigation strategy, we use the model to answer a simple question:

What is the fair share of the GDP to mitigate climate change costs?

We do this by adding a minor element to the DICE model: a fund that is fed by an annual fraction of the GDP, where the fund can be used to cover abatement and damage cost in the model. This alters the optimal emission path, depending on the funding rate.

The principle of measuring debt relative to GDP is common, see e.g. [1]. Also, determining budget as a constant share of the GDP is common, e.g., NATO's 2% target [7].

The principle of using a fund to mitigate climate change costs is common, e.g. the German climate transformation fund (KTF).

Acknowledgement

I am most grateful to Aurel Wünsch for feedback and stimulating discussions.

2. Model

We extend the standard DICE model by incorporating a dedicated fund. Let $t \mapsto F(t)$ denote the amount of cash in the fund at time t measured in trillion USD. The fund operates according to the following rules:

- Initially, the fund is empty, i.e., $F(0) = 0$
- The fund accrues interest at the market rate, equivalent to the model's time-preference rate r (sometimes also denoted as ρ).
- Each year, a fixed fraction q of the GDP is allocated to the fund, increasing its balance while proportionally reducing the GDP.
- The fund is used to cover damage and abatement costs. If total costs exceed the fund's balance, the fund is depleted ($F(t) = 0$), and the excess is covered by the GDP.

See also Section A.

Given that setup, we investigate:

What is the minimal rate $q = q^*$ *such that all climate costs (damage and abatement) will be covered from the fund.*

Here, climate costs refers to climate change mitigation and adaptation costs, in terms of the model

At first, one might estimate q^* a-posterior from the cost and GDP of a calibrated DICE model.

However, the value q has an effect on the optimal abatement strategy. As the rate q applies to all times equally, this alters the optimality of abatement: while with $q = 0$, it may be optimal to spare a reduction of the GDP at early times, hence, to defer early abatement, with $q > 0$ there is an amount available for early abatement, such that early abatement is not impacting the growth of the GDP. Hence, the optimal abatement strategy depends on q.

Determining q^* requires a numerical optimization, where we propose a candidate q, then optimize the abatement strategy that maximizes the total welfare (giving the optimal abatement policy), and then check if the fund was sufficient to cover the total cost. Here, total cost refers to the sum of abatement cost and damage cost.

3. Numerical Results

We take the DICE 2016 model parameters with an interest rate (time-preference) level of 1.5% per annum.

3.1. Total Cost per GDP

Our analysis indicates that a funding rate of $q^* = 2.4\%$ of the GDP is sufficient to cover all associated costs. This funding enables significantly faster abatement during the initial years, stabilizing at a higher level over time. As a result, total costs and emissions are substantially reduced. Figures 1 to 4 provide an overview, with the red curves representing the classical DICE model values.

For low to moderate discount rate, the optimal funding rate is 2.4% of the GDP (Figure 1, blue). Supplied with the fund, the optimal emission path suggests a much higher abatement rate (jumping immediately to around 50%, Figure 2, green).

With that optimal emission path, the total costs are initially smaller than the amount supplied to the fund. After 50 years, however, they become higher (Figure 1, green). This allows the fund to grow before it is eventually depleted.

The limitations of the fund lead to a slowdown in abatement at a high level. Nevertheless, this abatement strategy leads to a significant reduction in emissions (Figure 3).

Remarkably, these findings are almost independent of the discount rate as long as the discount rate stays below the fund rate, see Section 3.3. The results presented in this section are for a discount rate of 1.5%.

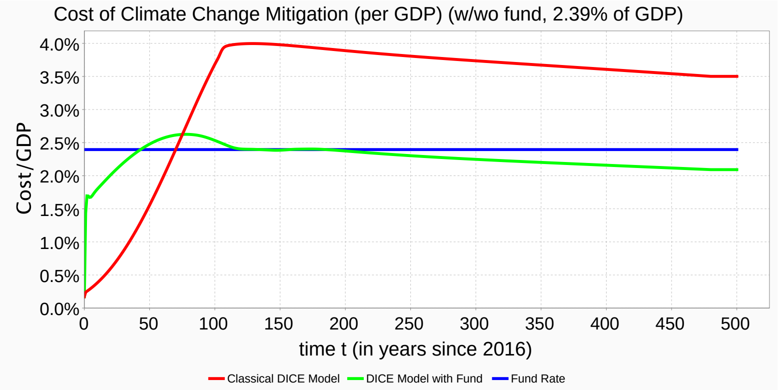

Figure 1: Cost per GDP in the calibrated DICE model, comparing the classical model (red) with the model with emission fund 2.4% of GDP (green).

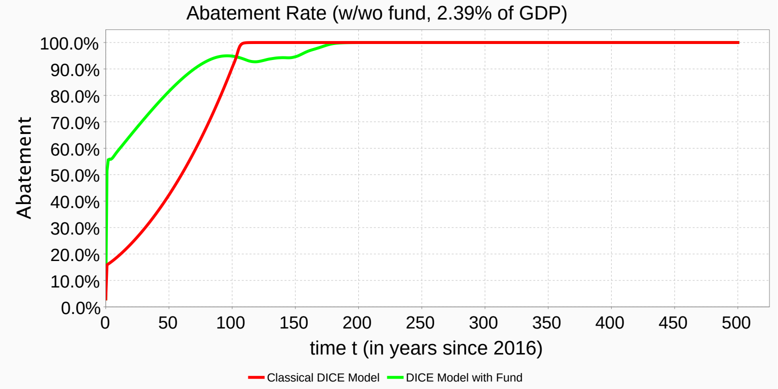

Figure 2: Abatement in the calibrated DICE model, comparing the classical model (red) with the model with emission fund 2.4% of GDP (green).

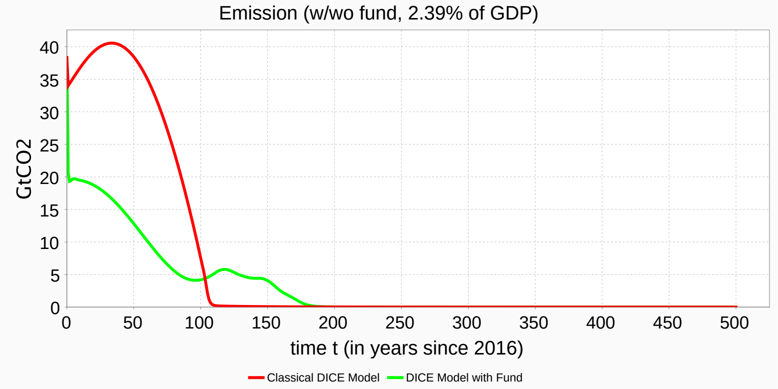

Figure 3: Emission in the calibrated DICE model, comparing the classical model (red) with the model with emission fund 2.4% of GDP (green).

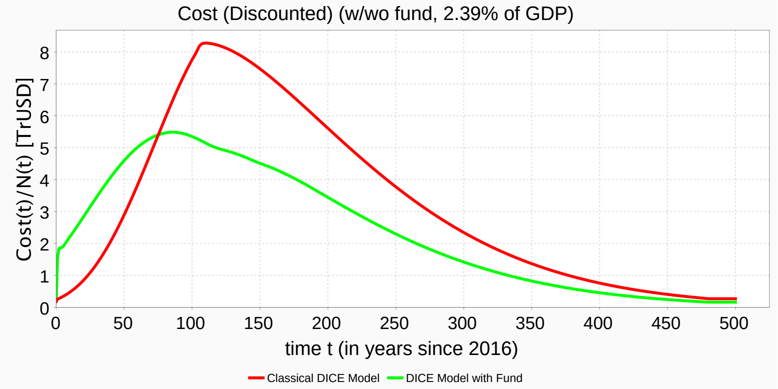

Figure 4: Cost in the calibrated DICE model, comparing the classical model (red) with the model with emission fund 2.4% of GDP (green).

3.2. Burden

The presence of the fund F results in different optimal emission paths, depending on the funding rate q. For the limit case of $q = 0$ we recover the optimal emission path of the classical DICE model.

The emission fund will levelize the burden of costs across generations, levelize in a "per GDP sense".

In other words, if we compare the model equipped with the fund F with the classical model, the question arises how the model with the fund redistributes certain quantities. We compare the output (GDP) and the consumption of the optimal emission path in the respective models.

Figure 5 shows the relative difference of the GDP. The funding puts a strain on the GDP and the GDP is lowered by 0.4% at first, but soon profits from the reduces damage costs.

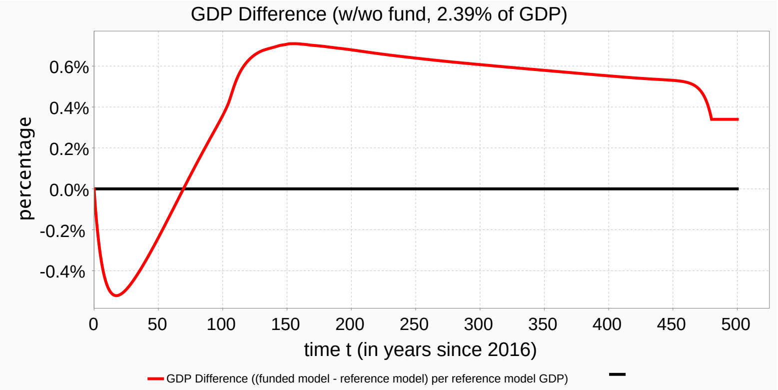

Figure 5: The difference of the GDP: funded model relative to the classical model. The plot shows the function $t \mapsto \frac{GDP^q(t)-GDP^0(t)}{GDP^0(t)}$, where $GDP^q(t)$ denotes the GDP in the model with funding rate q.

The comparable low impact on the GDP can be explained by an adjustment of the savings rate, which is (as in the classical DICE model) optimized. Figure 6 shows the relative difference of the consumption, where we see that the initial supply to the fund corresponds with a reduced consumption.

A breakdown of the total cost into abatement cost and damage cost, Figure 7, shows that an initial increase in the abatement cost results in a large reduction of the damage cost.

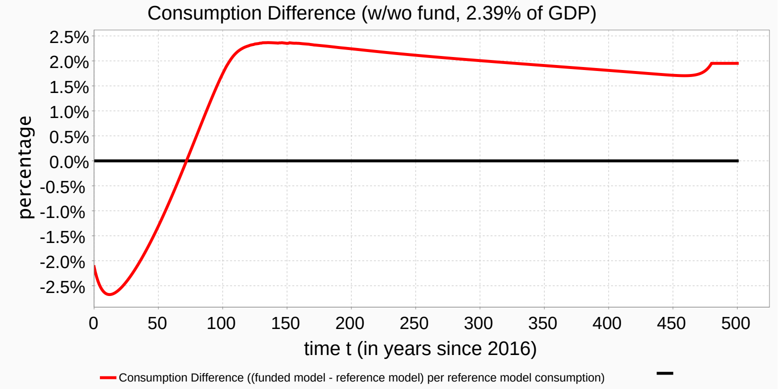

Figure 6: The difference of the Consumption: funded model relative to the classical model.

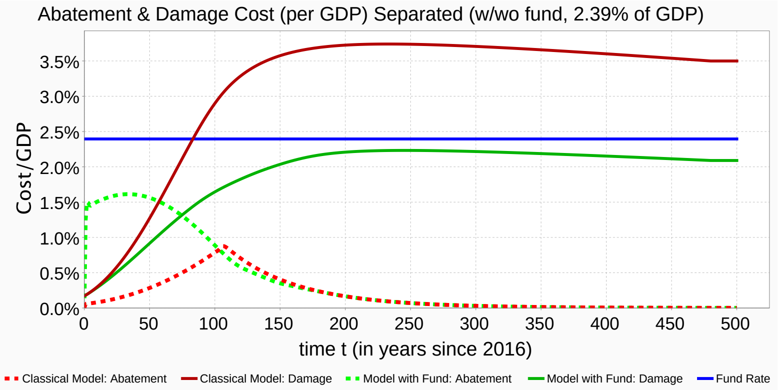

Figure 7: The cost per GDP as in Figure 1 split up between abatement cost and damage cost for the two models.

3.3. Role of the Discount Rate

The discount rate is a much debated exogenous parameter of the DICE model, [5, 6]. In the classical DICE model, it has a strong impact on the optimal abatement strategy.

For low to moderate discount rates we observe that adding a fund as a model extension to the DICE model significantly reduces the sensitivity of the optimal abatement strategy to the discount rate. While it is still qualitatively similar: higher interest rates lead to slower abatement, the presence of the fund will favor early abatement and the differences in the optimal abatement strategies are reduced, Figure 11. Note that for interest rates below 2.5%, the time at which the 85% abatement level is reached is almost independent of the discount rate in the model with fund. For low interest rates, the optimal funding rate q^* to cover all future costs is almost independent of the discount rate.

Figure 12 shows the corresponding emission path for different levels of the discount rate. Figure 13 shows the effective cost per GDP. Table 1 gives an overview of numerical experiments with different discount rate levels.

Discount Rate	Funding Rate that Covers all Costs
0.5%	2.43%
1.0%	2.42%
1.5%	2.39%
2.0%	2.33%
2.5%	2.23%
3.0%	2.98%
3.5%	3.72%
4.0%	4.64%

Table 1: Funding rate required to cover abatement and damage costs, depending on the chosen discount rate.

For discount rates below 2.5%, the abatement strategy and the optimal funding rate depend only weakly on the discount rate. For discount rates higher than 2.5% the discount rate has again a strong impact on the optimal abatement policy, similar to the classical model. This behavior is reasonable, as for high discount rates, deferring abatement is favourable, as future damages are devalued by increasing the discount rate. Such an emission path creates more nominal damages in the future, hence more nominal costs. Thus, the required funding rate rises.

The rise in the funding rate is remarkable, too. A high funding rate is clearly not desirable and illustrates that the optimal emission paths that are associated with higher discount rates could present economic and political challenges.

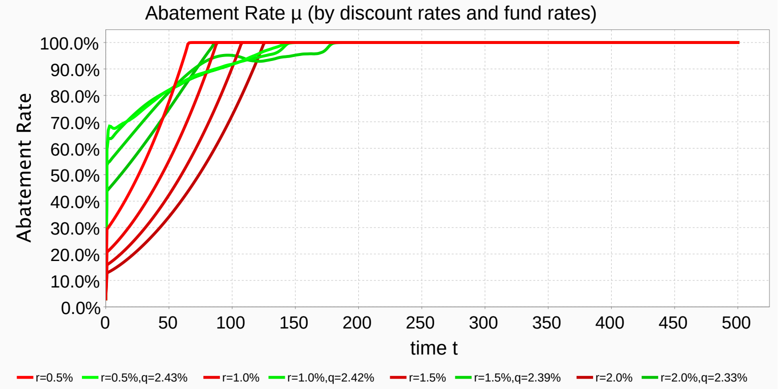

Figure 8: Optimal abatement policy for different discount rate levels, low to moderate discount rates.

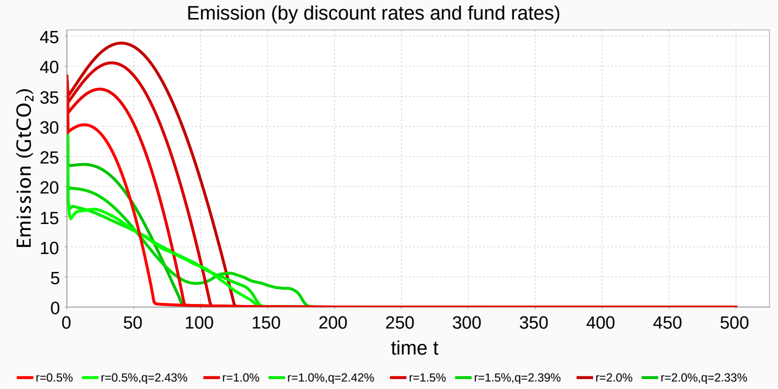

Figure 9: Emissions on the optimal abatement policy for different discount rate levels, low to moderate discount rates.

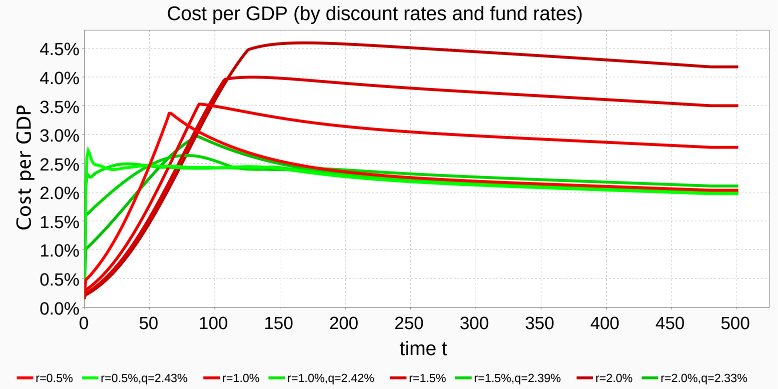

Figure 10: Cost per GDP for different discount rate levels, low to moderate discount rates.

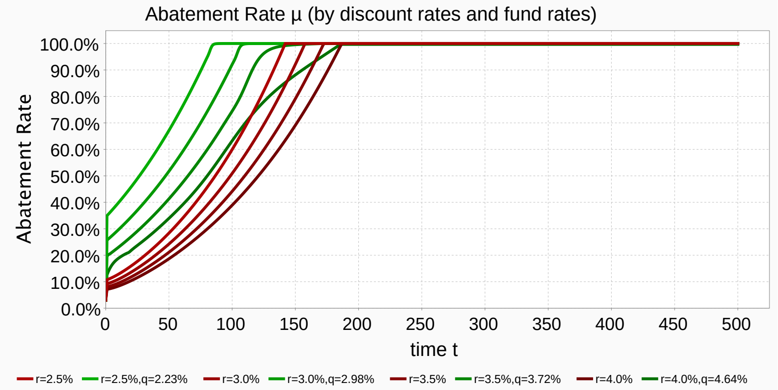

Figure 11: Optimal abatement policy for different discount rate levels, higher discount rates.

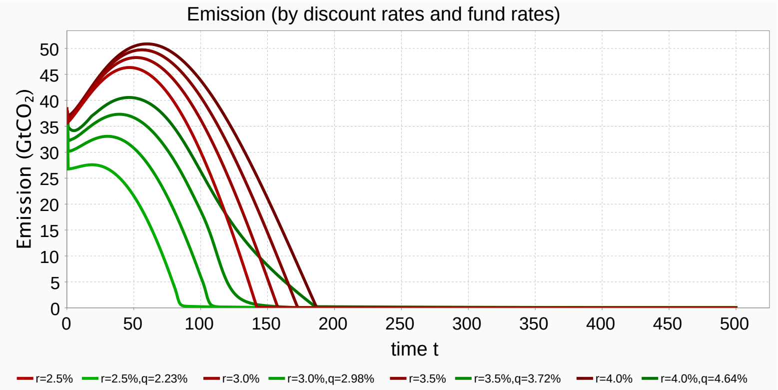

Figure 12: Emissions on the optimal abatement policy for different discount rate levels, higher discount rates.

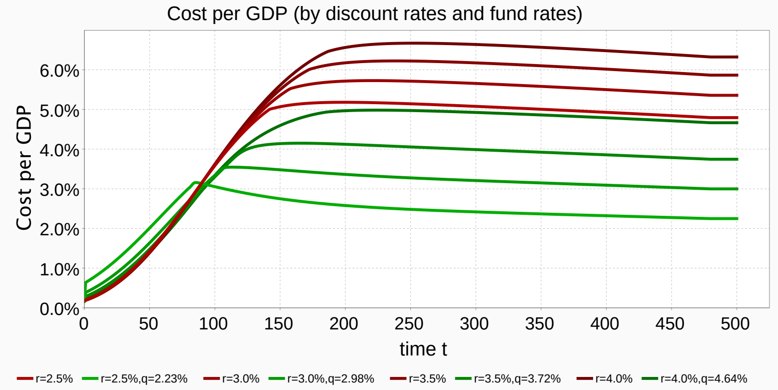

Figure 13: Cost per GDP for different discount rate levels, higher discount rates.

4. Conclusion

With low to moderate discount rates, the DICE 2016 model suggests that a fund supplied with 2.4% of the GDP would be sufficient to cover all costs from climate change, that is, abatement and damage costs. To put this into comparison, the U.S. national defence spending is between 4% and 5% of the U.S. GDP (1990-2012), [9].

In the presence of the fund, the optimal emission path (with the otherwise unmodified objective function to maximize social welfare) results in much stronger emission reductions, which effectively reduces the (undiscounted) costs associated with climate change.

The fund rate is almost independent of the specific discount rate as long as the discount rate stays below 2.5%. This shows that the optimal funding rate is rather an endogenous quantity generated by the mode.

The fund has the advantage of distributing climate change costs equally (in terms of percentage of the GDP) across all generations, while the classical model may exhibit intergenerational inequality [3]. Our extended model gives the minimum funding rate required, and for that, the optimal emission pathways in the presence of the fund.

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5. References

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A. Implementation Details

In [3, 4] and [2] we proposed different extensions to the classical DICE model. In [2], a fund was introduced that could be used to cover abatement and damage costs. There, the fund was supplied by an amount derived from a CO_2 price. For the current numerical experiment, we added the option to supply the fund by a fraction of the GDP.

The equations of the classical DICE model are modified as follows:[1]

Add fund from GDP:

$$\begin{aligned} F(t) &\leftarrow F(t) + q \cdot GDP(t) && \text{to fund} \\ GDP(t) &\leftarrow GDP(t) + q \cdot GDP(t) && \text{from GDP} \end{aligned}$$

Cover damages from fund:

$$F(t) \leftarrow F(t) - (C_{\mathrm{A}}(t) + C_{\mathrm{D}}(t)) \qquad \text{cover damages from fund}$$

Excess damages are covered from GDP (fund cannot become negative):

$$\begin{aligned} GPD(t) &\leftarrow GDP(t) + \min(F(t), 0) \\ F(t) &\leftarrow \max(F(t), 0) \end{aligned}$$

Accrual of the fund (at market rate):

$$F(t + \Delta t) \leftarrow F(t) \cdot (1 + r\Delta t)$$

We then implement an optimization that finds the optimal funding rate q. The objective function is the amount of damage not covered by the fund. So we seek the minimum rate q so that the fund covers all abatement and damages costs.

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[1] Here GDP denotes the GDP (Y in [8]), C_{A} denotes the abatement cost ($\Lambda \cdot Y$ in [8])), C_{D} denotes the damage cost ($\Omega \cdot Y$ in [8]), r denote the time-preference rate per annum and Δt the time step in years.

A.1. Source Code

The implementation can be found at
`https://gitlab.com/finmath/finmath-climate-nonlinear`.

The Figures 1 to 7 can be reproduce with the class

`ClimateModelExperimentEmissionFundAsPercentageOfGDP`

in the package `net.finmath.climate.models.experiments`. The Figures 11 to 13 can be reproduce with the class

`ClimateModelExperimentEmissionFundAsPercentageOfGDPByRate`.

The model implementation can be found in

`SIAModelWithNonlinearFunding`.

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Notes

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