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Abstract

Over the past approximately 30 years, many researchers have examined oligopoly models where firms endogenously select the timing of their action decisions. Therefore, this paper studies a mixed triopoly model featuring competition between a labour-managed firm, a capitalist firm and a state-owned firm. The sequence of events is as follows. In stage 0, each firm independently and simultaneously selects either 'stage 1' or 'stage 2'. In this context, stage 1 denotes that a firm produces in stage 1, whereas stage 2 signifies that it produces in stage 2. In stage 1, if a firm opts for stage 1, it determines its output for this stage. In stage 2, if a firm chooses stage 2, it decides on its output for this stage. Upon the conclusion of the game, the market opens, and all firms sell their outputs. The purpose of this paper is to present the equilibrium outcome of triopoly competition where a state-owned firm, a labour-managed firm and a capitalist firm compete in quantities. As a result of the analysis, this paper reveals that there exists an equilibrium wherein both the labour-managed firm and the capitalist firm assume leadership roles. The paper finds that the state-owned firm is precluded from functioning as the Stackelberg leader.

Keywords: Capitalist firm; Cournot game; Endogenous timing; Labour-managed firm; State-owned firm
JEL classification: C72; D21; L30

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1. Introduction

The theoretical analysis by Hamilton and Slutsky (1990) examines endogenous timing in two two-player games. In a preplay stage, players decide whether to take actions in the basic game at the first opportunity or wait to observe their rivals' first-stage actions. In one extended game, players first decide when to take actions without committing to actions. In another extended game, deciding to act at the first turn necessitates committing to an action. It is shown that in both extended games, sequential play outcomes are pure strategy equilibria only in undominated strategies. Robson (1990) examines a straightforward game model of price-setting duopoly where strategic timing is endogenous. Each firm selects not only a price but also a time at which this price is set. It is shown that the only subgame perfect Nash equilibria with sequential timing arise. Lambertini (1996) investigates the choice of roles by firms in a vertically differentiated duopoly by introducing a preplay stage where firms determine the timing of moves. It is found that the unique subgame perfect Nash equilibrium involves simultaneous play in the quantity stage, followed by sequential play in the price stage, where both firms would prefer to be price leaders. van Damme and Hurkens (2004) examine a linear price-setting duopoly game with differentiated products where firms can endogenously decide whether to lead or follow. While the follower role is most attractive for each firm, it is shown that waiting is more risky for the low-cost firm. Hence, the low-cost firm will emerge as the endogenous price leader. Chen et al. (2024) investigate an endogenous timing game of R&D decisions with research spillovers and compare the effects of output and research subsidies. It is shown that the simultaneous-move (sequential-move) game is an equilibrium if the spillover rate is low (high) under an output subsidy, while this equilibrium is socially beneficial if the spillover rate is either high or sufficiently low. It is also shown that under a research subsidy, the simultaneous-move game is a unique equilibrium regardless of the spillover rate and is always socially beneficial. There are numerous additional studies, such as those by Amir (1995), van Damme and Hurkens (1999), Deneckere and Kovenock (1992), Furth and Kovenock (1993), Hoffmann and Rota-Graziosi (2020), Lambertini (2000), van Leeuwen et al. (2022), Matsumura (1999), Park et al. (2021), Sadanand and Sadanand (1996), von Stengel (2010) and Tasnádi (2003). However, these studies encompass oligopoly models with profit-maximizing capitalist firms and do not include state-owned firms.

Several studies delve into endogenous timing within mixed oligopoly models that include state-owned public firms. For instance, Pal (1998) tackles the matter of the endogenous sequence of actions in a mixed market by employing the observable delay game proposed by Hamilton and Slutsky (1990) in the context of a quantity-setting mixed oligopoly where a state-owned firm and capitalist firms initially select the timing of their quantity decisions. It is demonstrated that in the case of more than two time periods, there exists a unique subgame perfect Nash equilibrium wherein all capitalist firms produce simultaneously in the first period and the state-owned firm produces in the second period. Lu (2007) investigates endogenous timing in a mixed oligopoly comprising a state-owned firm and foreign capitalist firms, and demonstrates that in the game of two time periods for quantity choice, there exists no subgame perfect Nash equilibrium outcome where all firms produce simultaneously in the same time period. Bárcena-Ruiz (2007) considers a mixed duopoly model where a state-owned firm and a capitalist firm decide whether to set prices sequentially or simultaneously, and shows that they opt for simultaneous pricing. Lu and Poddar (2009) explore a two-stage mixed duopoly game of endogenous timing with observable delay in the context of sequential capacity and quantity choice, and reveal that the state-owned and capitalist firms choose capacity and quantity sequentially in all possible equilibria. Ohnishi (2016) examines mixed duopoly games where a state-owned firm and a foreign capitalist firm compete in terms of pricing. Ohnishi (2016) explores the desirable role of the state-owned firm, whether as a leader or a follower, the impact of eliminating the foreign capitalist firm, and the endogenous role in price-setting mixed duopoly by adopting the observable delay game. As a result, it is shown that the state-owned firm cannot become the leader. Bárcena-Ruiz and Sedano (2011) explore the endogenous order of moves in a mixed duopoly for differentiated goods. Firms decide whether to set prices sequentially or simultaneously. It is revealed that the equilibrium outcome is significantly influenced by the weight assigned to consumer surplus in the weighted welfare and the extent to which goods are substitutes or complements. Lee and Xu (2018) investigate an endogenous timing game in product-differentiated duopolies under price competition when an emission tax is applied to environmental externalities, and show that there exists an equilibrium outcome where the state-owned firm is the leader. Haraguchi and Matsumura (2020) explore endogenous timing in a mixed duopoly model that features price competition, along with distinct social and private marginal costs. It is shown that various equilibrium timing patterns—Bertrand, Stackelberg with public leadership, Stackelberg with private leadership, and multiple Stackelberg equilibria—may emerge. However, these studies involve models with state-owned and capitalist firms, excluding labour-managed firms.

A few studies explore endogenous timing in mixed oligopoly models that incorporate labour-managed firms. For instance, Lambertini (1997) investigates endogenous timing in a mixed duopoly involving a capitalist firm and a labour-managed firm, competing either in terms of prices or quantities. He reveals that the Bertrand game leads to multiple equilibria, whereas the Cournot game yields a unique subgame perfect Nash equilibrium, wherein the capitalist firm assumes the leader's role and the labour-managed firm takes on the follower's role. Furthermore, Ohnishi (2012) explores a mixed duopoly model involving competition between state-owned and labour-managed firms. There are two production stages, and the firms initially declare the stage in which they will decide their output. It is shown that the unique equilibrium corresponds to the Stackelberg solution, with the labour-managed firm as the leader.

We explore a mixed triopoly model wherein a labour-managed firm, a capitalist firm and a state-owned firm coexist. The sequence of events is as follows. In stage 0, each firm independently and concurrently selects either 'stage 1' or 'stage 2'. In this context, stage 1 denotes that a firm produces in stage 1, whereas stage 2 signifies that it produces in stage 2. In stage 1, if a firm opts for stage 1, it determines its output for this stage. In stage 2, if a firm chooses stage 2, it decides on its output for this stage. Upon conclusion of the game, the market opens, and all firms sell their outputs. We examine the equilibrium of the endogenous timing triopoly model. To the best of the author's knowledge, no previous work has dealt with such an economic situation. This study aims to present the equilibrium outcome of triopoly competition where a state-owned firm, a labour-managed firm and a capitalist firm compete in quantities.

The remainder of this paper is organized as follows. In Section 2, the model is described in detail. Section 3 provides supplementary explanations of the model. Section 4 discusses fixed timing games. Section 5 presents the equilibrium of the model. Section 6 concludes the paper.

2. Model

Let us consider a market composed of one capitalist firm (firm C), one labour-managed firm (firm L) and one state-owned firm (firm S). In the rest of this study, subscripts C, L and

S denote firm C, firm L and firm S, respectively. In addition, when i, j and k are used to refer to firms in an expression, they should be understood to represent C, L and S with $i \neq j \neq k$. We do not consider the possibility of entry or exit. The triopolists produce perfectly substitutable goods. The given price-demand function is $P(Q)$, where $Q = q_{\mathrm{C}} + q_{\mathrm{L}} + q_{\mathrm{S}}$. We assume that $d^2P/dQ^2 (= \partial^2 P/\partial q_{\mathrm{C}}^2 = \partial^2 P/\partial q_{\mathrm{L}}^2 = \partial^2 P/\partial q_{\mathrm{S}}^2) \geq 0$ and $dP/dQ + q_i\left(d^2P/dQ^2\right) < 0$. Note that this assumption allows a linear price-demand function which was used in many papers of mixed oligopoly (e.g., see Delbono and Rossini, 1992; Lambertini, 1997; Lambertini and Rossini, 1998; Ohnishi, 2009).

The game's sequence is as follows. In stage 0, each firm i independently and simultaneously selects $t_i \in (1,2)$, where t_i indicates the timing of deciding the non-negative output q_i. Specially, $t_i = 1$ means that firm i produces in stage 1, and $t_i = 2$ means that it produces in stage 2. Firm i observes t_j and t_k. In stage 1, firm i that chooses $t_i = 1$ determines its output q_i in this stage. In stage 2, firm i that chooses $t_i = 2$ determines its output q_i in this stage. At the end of the game, the market opens, and all firms sell their outputs.

Hence, each firm's profit is represented by

$$\pi_i = P(Q)q_i - w(q_i) - r(q_i) - f, \tag{1}$$

where w is the labour cost function, r is the capital cost function, and $f > 0$ is the fixed cost. Firm C chooses q_{C} to maximize its own profit. We assume $dw/dq_i > 0$, $d^2w/dq_i^2 > 0$, $dr/dq_i > 0$ and $d^2r/dq_i^2 > 0$. We assume that the three firms face the same cost function and that the marginal cost of production is increasing. This assumption is utilized in many papers studying mixed oligopoly markets (see, e.g. Bárcena-Ruiz and Garzón, 2003; Delbono and Rossini, 1992; Delbono and Scarpa, 1995; Fjell and Heywood, 2002; Fjell and Pal, 1996; Harris and Wiens, 1980; Lee and Xu, 2018; Matsumura, 2003; Ohnishi, 2018, 2020, 2021; Pal and White, 1998; Poyago-Theotoky, 1998; Wang and Wang, 2009; Ware, 1986; White, 1996;). If the marginal cost of production is constant or decreasing, then firm S chooses to produce where the price equals the marginal cost of production. Consequently, neither firm C nor firm L has an incentive to operate in the market, allowing firm S to act as a monopolist.

Economic welfare, which is the sum of consumers' surplus and total profits by the firms, is represented by

$$W = \int_0^Q P(x)dx - w(q_S) - r(q_S) - w(q_C) - r(q_C) - w(q_L) - r(q_L) - 3f. \quad (2)$$

Firm S chooses q_S to maximize economic welfare.

Firm L's income per worker is represented by

$$\varphi_L = \frac{P(Q)q_L - r(q_L) - f}{l(q_L)}, \quad (3)$$

where l is the labour input function. We assume $dl/dq_L > 0$ and $d^2l/dq_L^2 > 0$. Firm L chooses q_L to maximize income per worker. We adopt subgame perfection as our solution concept to solve this game.

3. Supplementary explanations

First, we derive firm C's reaction function in quantities from (1). Firm C's reaction function is defined by

$$R_C(q_L, q_S) = \arg\max_{\{q_C \geq 0\}} \left[P(Q)q_C - w(q_C) - r(q_C) - f\right]. \quad (4)$$

We present the following lemma.

Lemma 1: Under Cournot competition, firm C's reaction functions are downward sloping.

Proof: Firm C aims to maximize its profit with respect to q_C, given q_L and q_S. The equilibrium must satisfy the following conditions: The first-order condition for (1) is

$$q_C \frac{dP}{dQ} + P - \frac{dw}{dq_C} - \frac{dr}{dq_C} = 0, \quad (5)$$

and the second-order condition for (1) is

$$q_C \frac{d^2P}{dQ^2} + 2\frac{dP}{dQ} - \frac{d^2w}{dq_C^2} - \frac{d^2r}{dq_C^2} < 0. \quad (6)$$

Moreover, we obtain

$$\frac{\partial R_C(q_L, q_S)}{\partial q_L} = \frac{\partial R_C(q_L, q_S)}{\partial q_S} = -\frac{\dfrac{dP}{dQ} + q_C \dfrac{d^2P}{dQ^2}}{2\dfrac{dP}{dQ} + q_C \dfrac{d^2P}{dQ^2} - \dfrac{d^2w}{dq_C^2} - \dfrac{d^2r}{dq_C^2}}. \quad (7)$$

Since $dP/dQ + q_{\mathrm{C}}\left(d^2P/dQ^2\right) < 0$, the lemma follows. QED.

This lemma implies that firm C regards quantities as strategic substitutes. The concepts of strategic substitutes and complements are attributed to Bulow, Geanakoplos, and Klemperer (1985).

Second, we derive firm L's reaction function in quantities from (3). Firm L's reaction function is defined by

$$R_{\mathrm{L}}(q_{\mathrm{C}}, q_{\mathrm{S}}) = \arg\max_{\{q_{\mathrm{L}} \geq 0\}} \left[\frac{P(Q)q_{\mathrm{L}} - r(q_{\mathrm{L}}) - f}{l(q_{\mathrm{L}})} \right]. \tag{8}$$

We now state the following lemma.

Lemma 2: Under Cournot competition, firm L's reaction functions are upward sloping.

Proof: Firm L aims to maximize its income per worker with respect to q_{L}, given q_{C} and q_{S}. The equilibrium must satisfy the following conditions: The first-order condition for (3) is

$$\left(q_{\mathrm{L}} \frac{dP}{dQ} + P - \frac{dr}{dq_{\mathrm{L}}} \right) l - \left(Pq_{\mathrm{L}} - r - f \right) \frac{dl}{dq_{\mathrm{L}}} = 0, \tag{9}$$

and the second-order condition is

$$\left(q_{\mathrm{L}} \frac{d^2P}{dQ^2} + 2\frac{dP}{dQ} - \frac{d^2r}{dq_{\mathrm{L}}^2} \right) l - \left(Pq_{\mathrm{L}} - r - f \right) \frac{d^2l}{dq_{\mathrm{L}}^2} < 0. \tag{10}$$

Moreover, we obtain

$$\frac{\partial R_{\mathrm{L}}(q_{\mathrm{C}}, q_{\mathrm{S}})}{\partial q_{\mathrm{C}}} = \frac{\partial R_{\mathrm{L}}(q_{\mathrm{C}}, q_{\mathrm{S}})}{\partial q_{\mathrm{S}}} = -\frac{q_{\mathrm{L}} l \dfrac{d^2P}{dQ^2} + \left(l - q_{\mathrm{L}} \dfrac{dl}{dq_{\mathrm{L}}} \right) \dfrac{dP}{dQ}}{\left(q_{\mathrm{L}} \dfrac{d^2P}{dQ^2} + 2\dfrac{dP}{dQ} - \dfrac{d^2r}{dq_{\mathrm{L}}^2} \right) l - \left(Pq_{\mathrm{L}} - r - f \right) \dfrac{d^2l}{dq_{\mathrm{L}}^2}}. \tag{11}$$

Since $d^2l/dq_{\mathrm{L}}^2 > 0$, $l - q_{\mathrm{L}}\left(dl/dq_{\mathrm{L}}\right) < 0$, and thus $q_{\mathrm{L}} l\left(d^2P/dQ^2\right)$ $+\left[l - q_{\mathrm{L}}\left(dl/dq_{\mathrm{L}}\right)\right] dP/dQ$ is positive. QED.

This lemma asserts that firm L regards quantities as strategic complements.

Third, we derive firm S's reaction function in quantities from (2). Firm S's reaction function is defined by

$$R_{\mathrm{S}}(q_{\mathrm{C}},q_{\mathrm{L}}) \\ = \arg\max_{\{q_{\mathrm{S}}\geq 0\}}\left[\int_0^Q P(x)dx - w(q_{\mathrm{S}}) - r(q_{\mathrm{S}}) - w(q_{\mathrm{L}}) - r(q_{\mathrm{L}}) - w(q_{\mathrm{C}}) - r(q_{\mathrm{C}}) - 3f\right]. \tag{12}$$

We present the following lemma.

Lemma 3: Under Cournot competition, firm S's reaction functions are downward sloping.

Proof: Firm S aims to maximize economic welfare with respect to q_{S}, given q_{C} and q_{L}. The equilibrium needs to satisfy the following conditions: The first-order condition for (2) is

$$P - \frac{dw}{dq_{\mathrm{S}}} - \frac{dr}{dq_{\mathrm{S}}} = 0, \tag{13}$$

and the second-order condition is

$$\frac{dP}{dQ} - \frac{d^2w}{dq_{\mathrm{S}}^2} - \frac{d^2r}{dq_{\mathrm{S}}^2} < 0. \tag{14}$$

Moreover, we have

$$\frac{\partial R_{\mathrm{S}}(q_{\mathrm{C}},q_{\mathrm{L}})}{\partial q_{\mathrm{C}}} = \frac{\partial R_{\mathrm{S}}(q_{\mathrm{C}},q_{\mathrm{L}})}{\partial q_{\mathrm{L}}} = -\frac{\dfrac{dP}{dQ}}{\dfrac{dP}{dQ} - \dfrac{d^2w}{dq_{\mathrm{S}}^2} - \dfrac{d^2r}{dq_{\mathrm{S}}^2}}. \tag{15}$$

Thus, the lemma follows. QED.

We assume that R_{C}, R_{L} and R_{S} intersect at a single point. This assumption is made to eliminate the scenario where one firm is extremely large or small compared to the other firms.

4. Stackelberg games of fixed timing

We begin by considering three Stackelberg duopoly games. If firm i is the Stackelberg leader, then firm i selects q_i, and firm j selects q_j after observing q_i. Firm i maximizes $(q_i, R_j(q_i))$ with respect to q_i. We present the following three lemmas.

Lemma 4: In capitalist and labour-managed duopoly competition, (i) $q_{\mathrm{C}}^{CLDL} < q_{\mathrm{C}}^{CLDN}$ and (ii) $q_{\mathrm{L}}^{CLDL} > q_{\mathrm{L}}^{CLDN}$. Here, '*CLDL*' denotes the Stackelberg leader outcome of the capitalist and labour-managed duopoly game, and '*CLDN*' denotes the Cournot-Nash outcome of the capitalist and labour-managed duopoly game.

Proof: (i) If firm C is the Stackelberg leader, then it maximizes $(q_{\mathrm{C}}, R_{\mathrm{L}}(q_{\mathrm{C}}))$ with respect to q_{C}. Therefore, firm C's Stackelberg leader output satisfies the first-order condition:

$$\frac{\partial \pi_{\mathrm{C}}}{\partial q_{\mathrm{C}}} + \frac{\partial \pi_{\mathrm{C}}}{\partial q_{\mathrm{L}}}\frac{\partial R_{\mathrm{L}}}{\partial q_{\mathrm{C}}} = 0. \tag{16}$$

Here, $\partial \pi_{\mathrm{C}} / \partial q_{\mathrm{L}} = q_{\mathrm{C}}\left(dP/dQ\right)$ is negative, and $\partial R_{\mathrm{L}} / \partial q_{\mathrm{C}}$ is positive (Lemma 2). To satisfy (16), $\partial \pi_{\mathrm{C}} / \partial q_{\mathrm{C}}$ must be positive.

(ii) If firm L is the Stackelberg leader, then it maximizes $(R_{\mathrm{C}}(q_{\mathrm{L}}), q_{\mathrm{L}})$ with respect to q_{L}. Therefore, firm L's Stackelberg leader output satisfies the first-order condition:

$$\frac{\partial \varphi_{\mathrm{L}}}{\partial q_{\mathrm{L}}} + \frac{\partial \varphi_{\mathrm{L}}}{\partial q_{\mathrm{C}}}\frac{\partial R_{\mathrm{C}}}{\partial q_{\mathrm{L}}} = 0. \tag{17}$$

Here, $\partial \varphi_{\mathrm{L}} / \partial q_{\mathrm{C}} = q_{\mathrm{L}}\left(dP/dQ\right)$ is negative, and $\partial R_{\mathrm{C}} / \partial q_{\mathrm{L}}$ is also negative (Lemma 1). To satisfy (17), $\partial \varphi_{\mathrm{L}} / \partial q_{\mathrm{L}}$ must be negative. QED.

Lemma 5: In capitalist and state-owned duopoly competition, (i) $q_{\mathrm{C}}^{CSDL} > q_{\mathrm{C}}^{CSDN}$ and (ii) $q_{\mathrm{S}}^{CSDL} < q_{\mathrm{S}}^{CSDN}$. Here, '*CSDL*' represents the Stackelberg leader outcome of the capitalist and state-owned duopoly game, and '*CSDN*' represents the Cournot-Nash outcome of the capitalist and state-owned duopoly game.

Proof: The proof is similar to that of Lemma 4, and thus we omit it. QED.

Lemma 6: In labour-managed and state-owned duopoly competition, (i) $q_{\mathrm{L}}^{LSDL} > q_{\mathrm{L}}^{LSDN}$ and (ii) $q_{\mathrm{S}}^{LSDL} > q_{\mathrm{S}}^{LSDN}$. Here, '*LSDL*' denotes the Stackelberg leader outcome of the labour-managed and state-owned duopoly game, and '*LSDN*' denotes the Cournot-Nash outcome of the labour-managed and state-owned duopoly game.

Proof: The proof is similar to that of Lemma 4 and thus is omitted. QED.

We now examine three Stackelberg triopoly games with fixed timing. If firm C acts as the Stackelberg leader to the other firms, then it selects q_{C} to maximize its profit $\pi_{\mathrm{C}}(q_{\mathrm{C}}, R_{\mathrm{L}}(q_{\mathrm{C}}), R_{\mathrm{S}}(q_{\mathrm{C}}))$. Conversely, if firm C is the follower to the other firms, then it maximizes its profit $\pi_{\mathrm{C}}(R_{\mathrm{C}}(q_{\mathrm{L}}, q_{\mathrm{S}}), q_{\mathrm{L}}, q_{\mathrm{S}})$ accordingly. We present the following proposition.

Proposition 1: (i) $\pi_C(q_C, R_L(q_C), R_S(q_C)) \geq \pi_C(q_C, q_L, q_S)$;

(ii) $\pi_C(R_C(q_L, q_S), q_L, q_S) > \pi_C(q_C, q_L, q_S)$

if $\pi_C(R_C(q_S), q_L, q_S) - \pi_C(q_C, q_L, q_S) > \pi_C(q_C, q_L, q_S) - \pi_C(R_C(q_L), q_L, q_S)$ and

$\pi_C(R_C(q_L, q_S), q_L, q_S) < \pi_C(q_C, q_L, q_S)$

if $\pi_C(R_C(q_S), q_L, q_S) - \pi_C(q_C, q_L, q_S) < \pi_C(q_C, q_L, q_S) - \pi_C(R_C(q_L), q_L, q_S)$.

Proof: (i) The leader (firm C) chooses q_C to maximize its profit $\pi_C(q_C, R_L(q_C), R_S(q_C))$ and it can choose $q_C = q_C^N$. Here, the superscript 'N' denotes the Cournot-Nash outcome of the triopoly game. Therefore, we obtain $\pi_C(q_C, R_L(q_C), R_S(q_C)) = \pi_C(q_C, q_L, q_S)$.

When firm C is the leader, it maximizes $\pi_C(q_C, R_L(q_C), R_S(q_C))$ with respect to q_C. The first-order condition for profit maximization is

$$q_C \frac{dP}{dQ} + P - \frac{dw}{dq_C} - \frac{dr}{dq_C} + q_C \frac{dP}{dQ}\frac{\partial R_L}{\partial q_C} + q_C \frac{dP}{dQ}\frac{\partial R_S}{\partial q_C} = 0, \tag{18}$$

where $dP/dQ < 0$, $\partial R_L/\partial q_C > 0$ (Lemma 2), and $\partial R_S/\partial q_C < 0$ (Lemma 3). If $q_C\left(dP/dQ\right)\left(\partial R_L/\partial q_C\right) + q_C\left(dP/dQ\right)\left(\partial R_S/\partial q_C\right) < 0$, then $q_C\left(dP/dQ\right) + P - dw/dq_C - dr/dq_C > 0$, and therefore firm C maximizes its profit by choosing $q_C < q_C^N$. Conversely, if $q_C\left(dP/dQ\right)\left(\partial R_L/\partial q_C\right) + q_C\left(dP/dQ\right)\left(\partial R_S/\partial q_C\right) > 0$, then $q_C\left(dP/dQ\right) + P - dw/dq_C - dr/dq_C < 0$, and therefore firm C chooses $q_C > q_C^N$.

(ii) When firm L is the Stackelberg leader, it increases q_L (Lemma 4 (ii)). Since $\partial\pi_C(q_C, q_L, q_S)/\partial q_L = q_C\left(dP/dQ\right) < 0$, increasing q_L decreases π_C given q_C and q_S, and thus $\pi_C(R_C(q_L), q_L, q_S) < \pi_C(q_C, q_L, q_S)$.

When firm S is the Stackelberg leader, it decreases q_S (Lemma 5 (ii)). Since $\partial\pi_C(q_C, q_L, q_S)/\partial q_S = q_C\left(dP/dQ\right) < 0$, decreasing q_S increases π_C given q_C and q_L, and thus $\pi_C(R_C(q_L), q_L, q_S) > \pi_C(q_C, q_L, q_S)$. QED.

Proposition 2: (i) $\varphi_L(R_C(q_L), q_L, R_S(q_L)) > \varphi_L(q_C, q_L, q_S)$;

(ii) $\varphi_L(q_C, R_L(q_C, q_S), q_S) > \varphi_L(q_C, q_L, q_S)$

if $\varphi_L(q_C, R_L(q_C), q_S) - \varphi_L(q_C, q_L, q_S) > \varphi_L(q_C, q_L, q_S) - \varphi_L(q_C, R_L(q_S), q_S)$ and

$\varphi_L(q_C, R_L(q_C, q_S), q_S) < \varphi_L(q_C, q_L, q_S)$

if $\varphi_L(q_C, R_L(q_C), q_S) - \varphi_L(q_C, q_L, q_S) < \varphi_L(q_C, q_L, q_S) - \varphi_L(q_C, R_L(q_S), q_S)$.

Proof: (i) Since the leader (firm L) maximizes income per worker and it can choose $q_L = q_L^N$, we obtain $\varphi_L(R_C(q_L), q_L, R_S(q_L)) \geq \varphi_L(q_C, q_L, q_S)$. We show that

$\varphi_{\mathrm{L}}(R_{\mathrm{C}}(q_{\mathrm{L}}),q_{\mathrm{L}},R_{\mathrm{S}}(q_{\mathrm{L}}))\neq\varphi_{\mathrm{L}}(q_{\mathrm{C}},q_{\mathrm{L}},q_{\mathrm{S}})$ by showing that $q_{\mathrm{C}}^{L}\neq q_{\mathrm{C}}^{N}$. Here, the superscript '*L*' denotes the Stackelberg leader outcome of the triopoly game. If firm L is the leader, then it maximizes income per worker $\varphi_{\mathrm{L}}(R_{\mathrm{C}}(q_{\mathrm{L}}),q_{\mathrm{L}},R_{\mathrm{S}}(q_{\mathrm{L}}))$ with respect to q_{L}. The first-order condition is

$$\left(q_{\mathrm{L}}\frac{dP}{dQ}+P-\frac{dr}{dq_{\mathrm{L}}}\right)l-\left(Pq_{\mathrm{L}}-r-f\right)\frac{dl}{dq_{\mathrm{L}}}+q_{\mathrm{L}}\frac{dP}{dQ}\frac{\partial R_{\mathrm{C}}}{\partial q_{\mathrm{L}}}l+q_{\mathrm{L}}\frac{dP}{dQ}\frac{\partial R_{\mathrm{S}}}{\partial q_{\mathrm{L}}}l=0. \tag{19}$$

Here $dP/dQ<0$, $\partial R_{\mathrm{C}}/\partial q_{\mathrm{L}}<0$ (Lemma 1) and $\partial R_{\mathrm{S}}/\partial q_{\mathrm{L}}<0$ (Lemma 3). To satisfy (19), $\left[q_{\mathrm{L}}\left(dP/dQ\right)+P-dr/dq_{\mathrm{L}}\right]l-\left(Pq_{\mathrm{L}}-r-f\right)dl/dq_{\mathrm{L}}$ needs to be negative, and therefore $q_{\mathrm{L}}^{L}>q_{\mathrm{L}}^{N}$.

(ii) The proof is similar to that of Proposition 1 (ii) and thus is omitted. QED.

Proposition 3: (i) $W(R_{\mathrm{C}}(q_{\mathrm{S}}),R_{\mathrm{L}}(q_{\mathrm{S}}),q_{\mathrm{S}})\geq W(q_{\mathrm{C}},q_{\mathrm{L}},q_{\mathrm{S}})$;
(ii) $W(q_{\mathrm{C}},q_{\mathrm{L}},R_{\mathrm{S}}(q_{\mathrm{C}},q_{\mathrm{L}}))>W(q_{\mathrm{C}},q_{\mathrm{L}},q_{\mathrm{S}})$.

Proof: (i) The leader (firm S) chooses q_{S} to maximize economic welfare $W(R_{\mathrm{C}}(q_{\mathrm{S}}),R_{\mathrm{L}}(q_{\mathrm{S}}),q_{\mathrm{S}})$ and it can choose $q_{\mathrm{S}}=q_{\mathrm{S}}^{N}$. Therefore, we obtain $W(R_{\mathrm{C}}(q_{\mathrm{S}}),R_{\mathrm{L}}(q_{\mathrm{S}}),q_{\mathrm{S}})=W(q_{\mathrm{C}},q_{\mathrm{L}},q_{\mathrm{S}})$.

When firm S is the leader, it maximizes $W(R_{\mathrm{C}}(q_{\mathrm{S}}),R_{\mathrm{L}}(q_{\mathrm{S}}),q_{\mathrm{S}})$ with respect to q_{S}. The first-order condition for welfare maximization is

$$P-\frac{dw}{dq_{\mathrm{S}}}-\frac{dr}{dq_{\mathrm{S}}}+q_{\mathrm{S}}\frac{dP}{dQ}\frac{\partial R_{\mathrm{C}}}{\partial q_{\mathrm{S}}}+q_{\mathrm{S}}\frac{dP}{dQ}\frac{\partial R_{\mathrm{L}}}{\partial q_{\mathrm{S}}}=0, \tag{20}$$

where $dP/dQ<0$, $\partial R_{\mathrm{C}}/\partial q_{\mathrm{S}}<0$ (Lemma 1), and $\partial R_{\mathrm{L}}/\partial q_{\mathrm{S}}>0$ (Lemma 2). If $q_{\mathrm{S}}\left(dP/dQ\right)\left(\partial R_{\mathrm{C}}/\partial q_{\mathrm{S}}\right)+q_{\mathrm{S}}\left(dP/dQ\right)\left(\partial R_{\mathrm{L}}/\partial q_{\mathrm{S}}\right)<0$, then $P-dw/dq_{\mathrm{S}}-dr/dq_{\mathrm{S}}>0$, and therefore firm S maximizes economic welfare by choosing $q_{\mathrm{S}}<q_{\mathrm{S}}^{N}$. Conversely, if $q_{\mathrm{S}}\left(dP/dQ\right)\left(\partial R_{\mathrm{C}}/\partial q_{\mathrm{S}}\right)+q_{\mathrm{S}}\left(dP/dQ\right)\left(\partial R_{\mathrm{L}}/\partial q_{\mathrm{S}}\right)>0$, then $P-dw/dq_{\mathrm{S}}-dr/dq_{\mathrm{S}}<0$, and therefore firm S chooses $q_{\mathrm{S}}>q_{\mathrm{S}}^{N}$.

(ii) When firm C is the leader, it increases q_{C} (Lemma 5 (i)). Since $\partial W(q_{\mathrm{C}},q_{\mathrm{L}},q_{\mathrm{S}})/\partial q_{\mathrm{C}}=P-dw/dq_{\mathrm{C}}-dr/dq_{\mathrm{C}}>0$, increasing q_{C} improves economic welfare given q_{L} and q_{S}.

When firm L is the leader, it increases q_{L} (Lemma 6 (i)). increasing q_{L} improves economic welfare given q_{C} and q_{S}, and thus $W(R_{\mathrm{C}}(q_{\mathrm{L}}),q_{\mathrm{L}},R_{\mathrm{S}}(q_{\mathrm{L}}))>W(q_{\mathrm{C}},q_{\mathrm{L}},q_{\mathrm{S}})$. QED.

5. Equilibrium

In this section, we present the equilibrium of the mixed triopoly model described in Section 2. Firm *i* aims to maximize its objective function value. In stage 0, each firm *i* simultaneously and independently chooses t_i. If firm *i* chooses $t_i = 1$, it selects q_i in stage 1. On the other hand, if firm *i* chooses $t_i = 2$, it selects q_i in stage 2. At the end of the game, the market opens, and each firm's objective function value is decided. The primary finding of this study is stated in the following proposition.

Proposition 4: There exists a unique equilibrium in which $t_{\mathrm{C}} = t_{\mathrm{L}} = 1$ and $t_{\mathrm{S}} = 2$.

Proof: From Proposition 3 (ii), $W(q_{\mathrm{C}}, q_{\mathrm{L}}, R_{\mathrm{S}}(q_{\mathrm{C}}, q_{\mathrm{L}})) > W(q_{\mathrm{C}}, q_{\mathrm{L}}, q_{\mathrm{S}})$. Therefore, firm S prefers to chooses stage 2. In addition, from Proposition 2 (i), $\varphi_{\mathrm{L}}(R_{\mathrm{C}}(q_{\mathrm{L}}), q_{\mathrm{L}}, R_{\mathrm{S}}(q_{\mathrm{L}})) > \varphi_{\mathrm{L}}(q_{\mathrm{C}}, q_{\mathrm{L}}, q_{\mathrm{S}})$. Hence, firm L chooses stage 1.

If firm L is the Stackelberg leader, it increases q_{L} (Lemma 6 (i)). Since $\partial \pi_{\mathrm{C}}(q_{\mathrm{C}}, q_{\mathrm{L}}, q_{\mathrm{S}}) / \partial q_{\mathrm{L}} = q_{\mathrm{C}} \left(dP/dQ \right) < 0$, increasing q_{L} decreases π_{C} given q_{C} and q_{S}, and thus $\pi_{\mathrm{C}}(R_{\mathrm{C}}(q_{\mathrm{L}}), q_{\mathrm{L}}, R_{\mathrm{S}}(q_{\mathrm{L}})) < \pi_{\mathrm{C}}(q_{\mathrm{C}}, q_{\mathrm{L}}, q_{\mathrm{S}})$. Hence, firm C does not prefer to be a follower if firm L is a leader. Furthermore, if firm S is a follower and firm C is a leader, then $q_{\mathrm{C}}^{CSDL} > q_{\mathrm{C}}^{CSDN}$ (Lemma 5 (i)) and firm C can increase its profit. Hence, firm C chooses stage 1. QED.

6. Conclusion

We have examined endogenous timing in a mixed Cournot triopoly model featuring competition between a labour-managed firm, a capitalist firm and a state-owned firm. We have demonstrated that there exists an equilibrium wherein both the labour-managed firm and the capitalist firm assume leadership roles. Our results indicate that the state-owned firm should not function as the Stackelberg leader.

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