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Luke Garrod, Ruochen Li, Antonio Russo and Chris M. Wilson*

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Abstract

There is limited theoretical understanding of cost pass-through within markets where prices are dispersed. Under a general demand function, we analyse the effects of cost changes in a seminal model of price dispersion, where some consumers are captive to particular sellers while others are not (Varian, 1980). To study pass-through in this mixed-strategy context, we employ a novel approach that links well to the pass-through literature in pure-strategy settings. Following an industry-wide cost increase, we show how the magnitudes of price rises faced by different consumer types, as well as the wider effects on price dispersion, depend upon whether demand is log-concave or log-convex. Furthermore, we examine whether the burden of the cost increase is expected to fall more heavily on captive or non-captive consumers. Finally, we show how our results vary with the level of competition and analyse the relationship between pass-through and demand shocks under price dispersion.

Keywords: Cost pass-through, price dispersion, demand curvature, competition, demand shocks

JEL Codes: D43; L13; D83

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1 Introduction

Understanding how cost changes are passed on to consumers through prices is fundamental for many areas of economics (Weyl and Fabinger, 2013). For instance, beyond the immediate application to tax incidence (e.g. Adachi and Fabinger, 2022), cost pass-through has been shown to be important for international trade (Nakamura and Zerom, 2010; Mrázová and Neary, 2017), development economics (Atkin and Donaldson, 2015), environmental regulation (Fabra and Reguant, 2014), monetary policy (Gregor et al., 2021), and labour economics (Harasztosi and Lindner, 2019). Moreover, within industrial organization, pass-through is useful in analysing price discrimination (Cowan, 2012; Miklós-Thal and Shaffer, 2021), the effects of mergers (Jaffe and Weyl, 2013), damages in antitrust cases (Verboven and van Dijk, 2009), and the pattern of rising mark-ups in modern product markets (Döpper et al., 2023).

The theoretical literature has made great strides in uncovering the determinants of cost pass-through in many market settings, often highlighting the importance of the shape of demand (e.g. Bulow and Pfleiderer, 1983; Seade, 1985; Anderson et al., 2001; Weyl and Fabinger, 2013; and Ritz, 2024).[1] However, the previous literature has largely overlooked the fact that, contrary to the 'law' of one price, real-world markets often exhibit price dispersion – where each firm's price differs to its rivals' even though they sell seemingly homogeneous products.[2] Hence, there is limited theoretical understanding of cost pass-through within markets where prices are dispersed and so a number of important questions remain unanswered: How will a cost increase affect price dispersion? How will the burden of a cost rise vary across consumers who pay different prices? What are the effects of competition? Will the answers depend upon the shape of demand?

To help address this gap, this paper analyses cost pass-through under a general demand function within a seminal model of price dispersion (Varian, 1980). This model introduces two consumer types that differ in their willingness or ability to buy from different sellers: 'captives' only buy from their captor firm whereas 'shoppers' buy from the firm with the lowest price. The equilibrium exhibits price dispersion because firms use a mixed-strategy price distribution to balance their incentives of i) offering low prices to attract shoppers, and ii) charging high prices to exploit captives. Thus, the determination of equilibrium prices differs markedly from models with pure-strategy pricing equilibria and so does the mecha-

[1] For instance, in the textbook supply and demand framework, costs are passed through to consumers to a greater extent when demand is less price elastic. More generally, under imperfect competition, pass-through depends upon the curvature of demand; that is, pass-through rates rise as demand becomes more convex, other things equal.

[2] For evidence of price dispersion see Sorensen (2000), Lach (2002), Baye et al. (2006), Kaplan and Menzio (2015), and Gorodnichenko et al. (2018).

nism driving cost pass-through. Nevertheless, we develop a novel approach to understand pass-through in our mixed-strategy context that links well to the existing pass-through literature. Following an industry-wide cost increase, we show how the magnitudes of price rises faced by different consumer types, as well as the wider effects on price dispersion, depend upon whether demand is log-concave or log-convex. Specifically, we demonstrate that when demand is log-concave: i) price dispersion decreases following a cost rise, and ii) the burden of a cost increase is expected to fall less heavily on captives. In contrast, when demand is log-convex: i) prices become more dispersed, and ii) captives expect to face larger price rises than shoppers. Furthermore, we show how our results vary with the level of competition and analyse the relationship between pass-through and demand shocks under price dispersion.

Our findings offer several policy implications regarding which types of consumers will be most affected by cost changes in markets with price dispersion. This is important given the growing policy interest in protecting 'vulnerable' consumers (e.g. OECD, 2023; European Parliament, 2021; CMA, 2019). As consistent with the captives in our model, this term applies to any consumer who is "unable to engage effectively in a market and as a result, is at a particularly high risk of getting a poor deal" (p.5, CMA, 2019).[3] Applying this interpretation to our results suggests that the relative impact of cost changes on vulnerable consumers will differ depending on whether costs rise or fall. For instance, if demand is log-concave demand and costs rise, then the expected price increase and associated fall in consumer surplus is smaller for a vulnerable consumer (i.e. captive) than a non-vulnerable consumer (i.e. shopper). However, if costs fall, then the expected price decrease and associated rise in consumer surplus is *smaller* for a vulnerable consumer. This implies that policymakers should be most concerned about the distributional impact of cost changes for vulnerable consumers when costs decrease rather than increase.

Our paper can also be applied across a range of other areas. As a first wider example, consider the transmission mechanism of monetary policy (e.g. Gregor et al., 2021) in the context of financial retail markets, where price dispersion is well documented (e.g. Allen et al., 2014; and Westphal, 2024). Our results can be used to understand how changes in the base rate will be passed through to consumers. For instance, when demand is log-concave (log-convex), our results suggest that a rise in the base rate will increase the effective retail interest rate to a relatively greater (lesser) extent for consumers who are willing and able to obtain the best deals. As a second example, consider the empirical phenomenon of rising mark-ups, which has been explained by prices not reflecting recent cost decreases (Döpper et

[3] The findings from the empirical literature suggest that such less engaged consumers are most likely to be those with lower education, lower income, and/or a more senior age profile (see Lusardi and Mitchell, 2014; Hortaçsu et al., 2017; Byrne and Martin, 2021; and Stango and Zinman, 2023).

al., 2023). Our findings explain how this effect will vary across heterogeneous consumers. For instance, when demand is log-concave (log-convex), a decrease in costs will raise expected mark-ups to a relatively greater (lesser) extent for captive consumers. Further example applications can be constructed with regards to tax changes, environmental regulations, minimum wage policies, international trade tariffs, and antitrust damages.

To demonstrate the results, our paper takes a novel approach to analyse the effects of a cost rise on the equilibrium price distribution, $F(p)$. Rather than examining the impact on $F(p)$ directly for a given price, p, we instead analyse the effects on the inverse of the equilibrium price distribution (i.e. the quantile function, $F^{-1}(p)$). This approach enhances tractability and allows us to link our results to the existing (pure-strategy) pass-through literature, while also uncovering the underlying economic intuition. Specifically, our approach analyses the extent to which the price, p, has to rise after an increase in costs in order to keep $F(p)$ constant. As an example, consider the median price: our approach characterizes the extent to which this median price must increase to ensure that the probability of pricing below the new level is held constant at 50%. We refer to this as the 'inverse price distribution rate of cost pass-through'. By analysing this pass-through rate across the entire price range, we are able to characterise exactly how the price distribution will shift and understand the economic implications for the expected prices.

Following this approach, we examine the impact of an industry-wide cost change on the price distribution. We begin by explaining how the price elasticity of demand affects pass-through when all else is held constant. While this provides some insight for certain points of the price distribution, we then develop a more general understanding of pass-through for all points by distinguishing between log-concave and log-convex demand. In particular, as consistent with the simple monopoly setting (with constant marginal cost), we find that the inverse price distribution rate of cost pass-through is less (greater) than one across the entire price range when demand is log-concave (log-convex). This implies that the shift in the price distribution in terms of price will always be less (greater) than the cost rise. Moreover, for a wide class of demand curves (that includes any with constant curvature), we show that when demand is log-concave (log-convex) the cost rise will shift the equilibrium price distribution in terms of price to a greater (smaller) extent towards the bottom of the distribution.[4]

Broadly speaking the intuition for these key results is as follows. The shift in the price distribution towards the top is closer to the monopoly pass-through rate, because the upper bound corresponds to the monopoly price. In contrast, the shift in the price distribution towards the bottom is closer to a pass-through rate of one (i.e. full pass-through), because

[4] For this last result, we initially ease exposition by focussing on the large class of demand functions with constant curvature. We later generalize this beyond constant curvature in our extensions section.

the lower bound price is nearer to marginal cost. Given the monopoly pass-through rate is less (greater) than one when demand is log-concave (log-convex), a cost rise will shift the equilibrium price distribution to a larger (smaller) extent towards the bottom.

This result provides the foundation for three important findings regarding cost pass-through under price dispersion. First, despite both the upper and lower bound prices increasing with costs, the difference between the two (i.e. the range of prices) will shrink (expand) when demand is log-concave (log-convex). Second, following a cost rise, the difference in the expected prices paid by the two consumer types (that we call the 'captivity premium') will decrease (increase) when demand is log-concave (log-convex). Intuitively, captives face a relatively smaller (bigger) price increase, because they expect to pay a higher price that is weighed less heavily by the bottom of the distribution. Third, when demand is log-concave, the impact on consumer surplus from a cost rise is always smaller for captives than for shoppers, as captives face a smaller price rise. However, when demand is log-convex, the impact on consumer surplus may not be larger for captives than for shoppers, despite captives facing the bigger price increase.

Next, we then examine the extent to which there is a relationship between the effects of a change in marginal cost under price dispersion and a unit parallel vertical shift in demand. There is a well-known relationship in settings with pure-strategy pricing equilibria which implies an equivalence between the effects of a unit tax on firms and a unit tax on consumers (Weyl and Fabinger, 2013). We establish that the same relationship also applies within our mixed-strategy pricing equilibrium. This offers novel insights into the effects of demand shocks on equilibrium price dispersion. In particular, we find that a unit parallel vertical decrease in demand generates the same price and welfare effects as a unit increase in marginal cost.

Towards the end of the paper, we also examine how pass-through varies with the level of competition. This has been a significant theme in the previous literature (e.g. Weyl and Fabinger, 2013; Miller et al., 2017; Genakos and Pagliero, 2022; and Ritz, 2024). The conventional wisdom is that greater competition will force pass-through rates to become closer to one and so price changes will be more cost reflective. In our alternative setting of price dispersion, we show that an increase in competition in terms of a greater proportion of shoppers is consistent with this conventional wisdom. In contrast, an increase in competition in terms of a greater number of firms can prompt changes in the expected price paid by captives to become *less*, rather than more, cost reflective.

Finally, we extend our analysis in a number of directions. First, we demonstrate that our results can apply to settings beyond constant marginal costs. This is challenging because, to the authors' knowledge, there is no model that has extended Varian (1980) to both downward-sloping demand and non-constant marginal costs. We make some progress by considering a specific cost structure, where each firm also incurs an ad valorem cost. This cost structure is particularly policy-relevant and empirically important in regards to ad valorem taxes (e.g. Häckner and Herzing, 2016; and Adachi and Fabinger, 2022) and revenue-sharing contracts (e.g. Johnson, 2017). We show that our results relating to the (unit) cost pass-through rate continue to apply in this setting with non-constant marginal costs, and are consistent with the effects of an increase in the ad valorem cost. In addition, we also consider extensions that demonstrate i) how any of our results that were presented under constant curvature will also apply for a class of demand curves with non-constant curvature, ii) how our results apply to the special case of unit demand, and iii) how our methodology can be applied to a related search cost framework (Stahl, 1989). In the Supplementary Appendix, we provide detail on some specific demand examples.

Related literature: Our paper is able to explain some mixed empirical evidence on the impact of cost changes on different consumer groups. For instance, recent findings from the German and French retail fuel markets suggest that more informed consumers who buy at the lowest prices can experience either relatively higher or lower rates of pass-through than uninformed consumers (Montag et al., 2024). Our results point to the shape of demand as one possible explanation for these varied findings.[5] Similar mixed results are also found in a previously unconnected empirical literature on inflation (Argente and Lee, 2021; and Broda and Romalis, 2009). Specifically, compared to richer consumers, this literature suggests that poorer consumers can experience either relatively higher or lower levels of price inflation, even when controlling for the same basket of goods. This mixed result could be explained by our model under the proviso that inflation is driven by cost increases and low income consumers are more captive.

On the theoretical side, our paper is broadly connected to a few existing studies. These papers consider some related issues regarding cost changes within settings linked to Varian (1980), but they restrict attention to unit demand and are focused on how cost changes affect the expected price. In particular, the theory section of Westphal (2024) examines the pass-through rate of the expected price in a consumer search framework under the additional complication that consumers are uncertain about firms' production costs. Tappata (2009) presents a dynamic framework where consumer search behaviour leads to the 'rockets and feathers' pattern (where cost increases cause prices to rise more quickly than prices fall when costs decrease). Hence, unlike our paper, they do not explain how cost changes affect the

[5] Fischer et al. (2024) analyse data from the same market over a different time period and find, consistent with our results under log-concave demand, that more informed consumers experience relatively higher rates of pass-through.

equilibrium price dispersion, how the burden falls on different types of consumers, or the important role of the shape of demand.[6]

Methodologically, our approach of analysing cost pass-through under price dispersion through the inverse price distribution employs a change of variables technique. A similar technique has been used in the broader price dispersion literature to analyse a variety of different issues under the assumption of unit demand (e.g. Janssen et al., 2005; Tappata, 2009; Janssen et al., 2011; Pennerstorfer et al., 2020; and Garrod et al., 2023). We expand this method to allow for downward-sloping demand to study how the shape of demand affects cost pass-through under price dispersion.

Finally, Varian's (1980) model of sales gave rise a large body of literature that forms the leading theoretical explanation for price dispersion (e.g. Burdett and Judd, 1983; Stahl, 1989; Baye and Morgan, 2001; Janssen and Moraga-González, 2004; Armstrong and Vickers, 2022). This literature has ample empirical support (e.g. Lach, 2002; Baye et al., 2004; Wildenbeest, 2011; Chandra and Tappata, 2011; Lach and Moraga-González, 2017; and Pennerstorfer et al., 2020) and has also been used to study wider applications including price comparison platforms, choice complexity, and even several issues in finance and macroeconomics (e.g. Moraga-González and Wildenbeest, 2012; Ronayne and Taylor, 2022; Spiegler, 2016; Gavazza and Lizzeri, 2021; and Burdett and Menzio, 2018).

The rest of the paper is organized as follows. Sections 2 and 3 presents the model and equilibrium. Section 4 then characterizes the inverse price distribution rate of cost pass-through. In Section 5, we analyse various properties of the equilibrium pass-through. Section 6 analyses the connection to demand shocks, while Section 7 examines how our results vary with the level of competition in terms of both the number of firms and the proportion of shoppers. Finally, Section 8 presents some extensions and Section 9 concludes. All proofs are relegated to the appendix.

2 Model

Consider the following version of Varian (1980). Suppose there are $n \in [2, \infty)$ identical firms, $i = \{1, ..., n\}$, that compete in prices to sell a single homogeneous product. Each firm's marginal cost is constant and equal to $c > 0$. Fixed costs are normalized to zero.

There is a unit mass of consumers comprising of two types. A proportion $\sigma \in (0, 1)$ of consumers are 'shoppers'. They compare the prices of all firms and will buy from a firm that offers the lowest price (randomizing between any tied firms with equal probability). The

[6] To generate their empirical hypotheses, Montag et al. (2024) also present a numerical analysis of cost pass-through under price dispersion that is limited to unit demand.

remaining $1-\sigma$ consumers are 'captive'. Each captive consumer will only ever buy from their designated firm, where each firm has a symmetric share of captives, $\frac{1-\sigma}{n}$.

When buying from any firm with some price, $p \geq 0$, the demand function of each consumer is $q(p)$, where demand is downward-sloping, $q'(p) < 0 \ \forall p \in [0, \hat{p}]$ with $\hat{p} > c$ and $q(\hat{p}) = 0$. While there are various possible interpretations, we interpret this as each consumer demanding $q(p)$ units.[7] Let $\varepsilon(p) = -\frac{pq'(p)}{q(p)}$ represent the price elasticity of demand, where $\varepsilon(p) > 0 \ \forall p \in [0, \hat{p}]$, and let $\xi(p) = \frac{q(p)q''(p)}{q'(p)^2}$ denote a measure of the curvature of demand. When demand is strictly concave (convex), $\xi(p) < (>) 0$ as $q''(p) < (>) 0$; when demand is linear, $\xi(p) = 0$. Furthermore, when demand is strictly log-concave (log-convex), $\xi(p) < (>) 1$ as $(\ln q(p))'' < (>) 0$; when demand is log-linear, $\xi(p) = 1$.

Throughout the paper, we impose the following standard property on $\varepsilon(p)$ that is sometimes referred to as Marshall's second law of demand.

Assumption 1. $\varepsilon'(p) = -\frac{q'(p)}{q(p)} \left[1 + \varepsilon(p)\left(1 - \xi(p)\right)\right] \geq 0, \ \forall p \in [0, \hat{p}]$

Assumption 1 states that the price elasticity of demand is (weakly) increasing in price. This property allows demand to be log-concave ($\xi(p) \leq 1 \ \forall p$) or strictly log-convex ($\xi(p) > 1 \ \forall p$) provided it is not too log-convex ($\xi(p) \leq 1 + \frac{1}{\varepsilon(p)} \ \forall p$).[8]

Firms can earn per-consumer profits of $\pi(p; c) \equiv (p - c)q(p)$. Let $p^m(c)$ denote the monopoly price that satisfies $\pi'(p^m(c); c) = 0$, where

$$\pi'(p; c) = q(p)\left[1 - \left(\frac{p - c}{p}\right)\varepsilon(p)\right] \tag{1}$$

such that $p^m(c) \in (c, \hat{p})$ as $\pi'(c; c) > 0$ and $\pi'(\hat{p}; c) < 0$.[9] Note that Assumption 1 guarantees that the term in square brackets is strictly decreasing in p, because $\xi(p) \leq 1 + \frac{1}{\varepsilon(p)}$ ensures

$$\left(\frac{p - c}{p}\varepsilon(p)\right)' = \frac{-q'(p)}{q(p)}\left[1 + \left(\frac{p - c}{p}\varepsilon(p)\right)(1 - \xi(p))\right] > 0 \ \forall c > 0 \tag{2}$$

One important implication of this is that, for any p below $p^m(c)$, the per-consumer profits are strictly increasing in price, $\pi'(p; c) > 0$, such that $\frac{p-c}{p} < \frac{1}{\varepsilon(p)}$ for all $p < p^m(c)$ from (1). Furthermore, the existence and uniqueness of $p^m(c)$ are guaranteed, as the per-consumer profits are strictly decreasing in price for any p above $p^m(c)$.[10]

[7] A standard alternative interpretation of $q(p)$ is where each consumer demands one unit with a stochastic valuation, v, that is unknown to firms, such that $q(p)$ represents the probability that the consumer will buy.

[8] For brevity, here and henceforth we use $\forall p$ to refer to all prices where demand is positive, i.e. $p \in [0, \hat{p}]$.

[9] For consistency and to simplify notation, we use Lagrange's notation to denote derivatives with respect to p throughout the paper, even for functions with more than one argument, like in (1).

[10] In Section (8.3), we show that our results also apply if consumers have unit demand and a common (and

The timing of the game is as follows. Firms set their prices simultaneously, and then consumers make their purchase decisions in line with their respective strategies outline above. We study symmetric Nash equilibria. To allow for firms' use of mixed strategies, let $F(p)$ represent each firm's equilibrium price distribution.

3 Equilibrium

Lemma 1 presents the equilibrium. As standard, given $\sigma \in (0,1)$, there is no pure-strategy pricing equilibrium. Instead, following Varian (1980), there is a mixed-strategy pricing equilibrium. For this, and throughout the paper, the following profit ratio will be important

$$\mathcal{L}(p,c) \equiv \frac{\pi\left(p^m(c);c\right) - \pi(p;c)}{\pi(p;c)} \geq 0 \ \forall p \in [c, p^m(c)]. \tag{3}$$

Lemma 1. *For any proportion of shoppers, $\sigma \in (0,1)$, there is a unique symmetric equilibrium where each firm earns $\Pi^N(c,\sigma,n) \equiv \left(\frac{1-\sigma}{n}\right)\pi\left(p^m(c);c\right) > 0$ by using a price distribution $F(p)$ on support $[\underline{p}(c,\sigma,n), p^m(c)]$, where*

$$F(p) = 1 - \left[\frac{1}{n}\left(\frac{1-\sigma}{\sigma}\right)\mathcal{L}(p,c)\right]^{\frac{1}{n-1}}, \tag{4}$$

and where the lower bound price $\underline{p}(c,\sigma,n) \in (c, p^m(c))$ is the unique level of p that satisfies $\pi(p;c)\left(\sigma + \frac{1-\sigma}{n}\right) = \left(\frac{1-\sigma}{n}\right)\pi\left(p^m(c);c\right)$.

Intuitively, when setting any $p \in [\underline{p}(c,\sigma,n), p^m(c)]$, each firm expects to earn profits of

$$\pi\left(p;c\right)\left[\frac{1-\sigma}{n} + \sigma\left(1 - F(p)\right)^{n-1}\right] = \left(\frac{1-\sigma}{n}\right)\pi\left(p^m(c);c\right) \tag{5}$$

The left-hand side is the profits a firm earns from supplying its $\frac{1-\sigma}{n}$ captives with certainty and σ shoppers if it has the lowest price, which occurs with probability $\left(1-F(p)\right)^{n-1}$. The right-hand side is the maximum profit a firm can earn with certainty from its captives, which is equivalent to $\Pi^N(c,\sigma,n)$. The equilibrium price distribution, $F(p)$, equates the two to ensure that each firm is indifferent over any $p \in [\underline{p}(c,\sigma,n), p^m(c)]$. This balances each firm's incentives to supply captives at high prices and compete for shoppers with low prices.

By rearranging (5) in terms of (6) below, we can understand the economic importance of

known) willingness to pay.

the profit ratio in (3):

$$\frac{\sigma\left(1-F(p)\right)^{n-1}}{\frac{1-\sigma}{n}}=\frac{\pi\left(p^{m}(c);c\right)-\pi(p;c)}{\pi(p;c)} \tag{6}$$

Here, the left-hand side represents how a sale that reduces price to some $p < p^m(c)$ increases a firm's expected market share of consumers relative to no sale, $p = p^m(c)$. The right-hand side, equivalent to $\mathcal{L}(p, c)$, represents the extent to which such a sale decreases the per-consumer profits below $\pi\left(p^m(c); c\right)$ relative to the per-consumer profits under the sale, $\pi(p; c)$. Thus, the profit ratio in (3) determines the "relative loss" from a sale (i.e. the right-hand side of (6)). In equilibrium, this must equal the "relative gain" from a sale (i.e. the left-hand side of (6)).

Given the lower bound price is strictly less than the monopoly price, there is price dispersion in equilibrium. As such, captives and shoppers will expect to pay different prices. Respectively, these equal the expected price and the expected minimum price:

$$E(p) = \int_{\underline{p}(c,\sigma,n)}^{p^m(c)} pF'(p)dp \tag{7}$$

$$E(p_{min}) = \int_{\underline{p}(c,\sigma,n)}^{p^m(c)} pn\left(1-F(p)\right)^{n-1}F'(p)dp \tag{8}$$

The difference between the two represents the 'captivity premium', $\kappa(c, \sigma, n) \equiv E(p) - E(p_{min})$; that is, how much more captives expect to pay over shoppers. The captivity premium is strictly positive given $\sigma \in (0, 1)$. However, it approaches zero for extreme values of σ, $\lim_{\sigma\to 0}\kappa(c, \sigma, n) = \lim_{\sigma\to 1}\kappa(c, \sigma, n) = 0$. Intuitively, when (almost) all consumers are captives, $\sigma \to 0$, prices are concentrated close to $p^m(c)$ because firms act like local monopolies, and when (almost) all consumers are shoppers, $\sigma \to 1$, prices are concentrated close to c near the Bertrand equilibrium. It is also useful to define the range of prices, $\rho(c, \sigma, n) \equiv p^m(c) - \underline{p}(c, \sigma, n)$. This approaches zero when almost all consumers are captives, as $\lim_{\sigma\to 0}\underline{p}(c, \sigma, n) = p^m(c)$, but it is largest when almost all consumers are shoppers, as $\lim_{\sigma\to 1}\underline{p}(c, \sigma, n) = c$.

4 Equilibrium Cost Pass-Through

We wish to investigate the equilibrium effects of an industry-wide increase in marginal costs, c. However, examining the direct effect of a cost rise on the equilibrium price distribution, $F(p)$, for a given p, has limited tractability. For instance, any attempt to analyse an increase in c on the expressions in (7) and (8) would have to resolve the difficulties associated with multiple counteracting effects that are not always comparable. Instead, we analyse the effects on the inverse of the equilibrium price distribution (i.e. the quantile function, $F^{-1}(p)$). In particular, we characterise the extent to which p has to change after a cost increase to ensure that $F(p)$ is held constant at some $1 - z \in [0, 1]$. This alternative approach enhances tractability and allows us to link our results to the existing (pure-strategy) pass-through literature, while also uncovering the underlying economic intuition.

This distinction between approaches is illustrated in Figure 1. It depicts how $F(p)$ changes following a cost increase from c to $\widetilde{c} > c$. Intuitively, as we shall formally establish below, the increase in c gives firms an incentive to set higher prices, and so $F(p)$ shifts to the right. Rather than analysing how the change in c affects $F(p)$ for a given p, our approach derives the magnitude of the horizontal shift in the price distribution in terms of p for any point $F(p) = 1 - z$. Specifically, we denote $p^*(z, c, \sigma, n)$ as the price that sets $F(p) = 1 - z$, which is equivalent to the $(1 - z)$-quantile of the price distribution. We then derive the associated cost pass-through rate, $\frac{\partial p^*(z,c,\sigma,n)}{\partial c}$, and refer to it as the "inverse price distribution rate of cost pass-through".

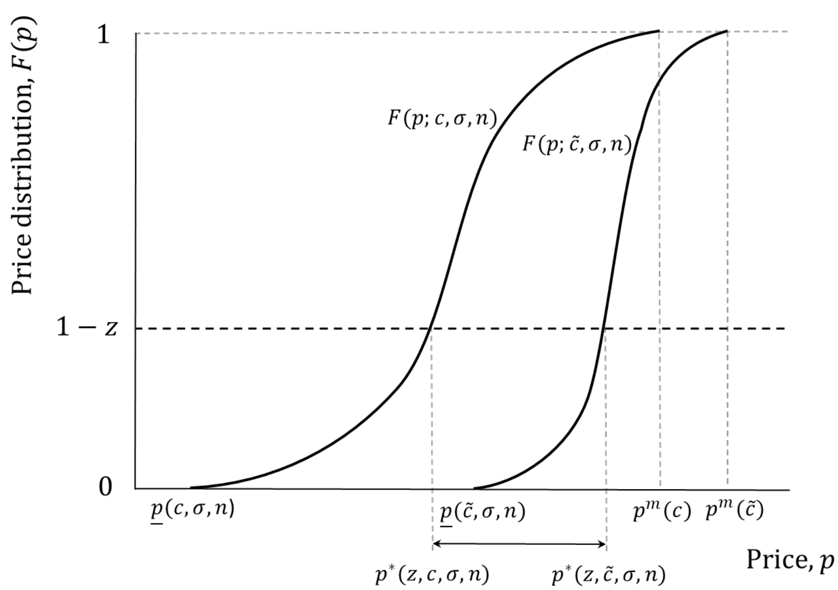


Figure 1: Change to the equilibrium price distribution after a cost rise from c to $\widetilde{c} > c$

There are two main benefits of this approach. First, it has the advantage that $\frac{\partial p^*(z,c,\sigma,n)}{\partial c}$ for all z will be directly comparable to the monopoly pass-through rate, $\frac{\partial p^m(c)}{\partial c}$, because the upper bound of the price distribution is $p^m(c) = p^*(0, c, \sigma, n)$. This allows us to link our results back to the simple monopoly setting. Second, as we explain further in Section 4.2, the prices that captives and shoppers expect to pay in (7) and (8), respectively, can be expressed in terms of $p^*(z,c,\sigma,n)$. Consequently, studying $\frac{\partial p^*(z,c,\sigma,n)}{\partial c}$ also allows us to tractably analyse the changes to the expected pass-through rates, $\frac{\partial E(p)}{\partial c}$ and $\frac{\partial E(p_{min})}{\partial c}$, as well as the lower-bound pass-through rate, $\frac{\partial \underline{p}(c,\sigma,n)}{\partial c} = \frac{\partial p^*(1,c,\sigma,n)}{\partial c}$.

Before moving on, Lemma 2 characterises $p^*(z,c,\sigma,n)$.

Lemma 2. *For any $z \in [0,1]$, there exists a unique $p^*(z,c,\sigma,n) \in \left[\underline{p}(c,\sigma,n), p^m(c)\right]$ that sets $z = 1 - F(p)$. It is decreasing in z, $\frac{\partial p^*(z,c,\sigma,n)}{\partial z} \leq 0$, where the inequality is strict $\forall z > 0$.*

Intuitively, from (6), $p^*(z,c,\sigma,n)$ is the level of p that sets the relative loss from a sale, $\mathcal{L}(p,c)$, equal to the relative gain from a sale, $\frac{\sigma n z^{n-1}}{1-\sigma}$, for a given $z = 1 - F(p)$. It is strictly decreasing in $z > 0$, because a higher z implies a higher relative gain from a sale and so, given Assumption 1, $p^*(z,c,\sigma,n)$ must strictly decrease to re-equate the relative loss from a sale with the relative gain.

4.1 Inverse Price Distribution Pass-Through Rate

We now derive the inverse price distribution rate of cost pass-through.[11]

Proposition 1. *For any $z \in (0,1]$, the inverse price distribution rate of cost pass-through is*

$$\frac{\partial p^*(z,c,\sigma,n)}{\partial c} = \left.\frac{\frac{\partial \mathcal{L}(p,c)}{\partial c}}{-\mathcal{L}'(p,c)}\right|_{p=p^*(z,c,\sigma,n)} \equiv \left.\frac{1 - \frac{p-c}{p^m(c)-c}}{1 - \frac{p-c}{p}\varepsilon(p)}\right|_{p=p^*(z,c,\sigma,n)} > 0, \tag{9}$$

where at the upper bound of the distribution $lim_{z\to 0}\frac{\partial p^*(z,c,\sigma,n)}{\partial c} = \frac{1}{2-\xi(p^m(c))} \equiv \frac{\partial p^m(c)}{\partial c} > 0$.

To understand the determinants of the pass-through rate in Proposition 1, we first explain the two alternative expressions in (9). This is important for the intuition of later results. To begin, consider the first expression in (9). This is derived using $F(p) = 1 - z$ and the implicit function theorem. It shows that, following a change in c, the price change required to keep $F(p)$ constant at $1 - z$ is determined by the relative responsiveness of $\mathcal{L}(p,c)$ to c and p. Intuitively, from (6), a change in c affects the relative loss from a sale, $\mathcal{L}(p,c)$, but it does not affect the relative gain, as $\frac{\sigma z^{n-1}}{(1-\sigma)/n}$ is constant for a given $z = 1 - F(p)$. Thus, p must adjust to re-equate the two by returning the relative loss from a sale, $\mathcal{L}(p,c)$, back to its original level. Importantly, this implies that the determinants of the pass-through rate are i) how responsive $\pi\left(p^m(c);c\right)$ and $\pi(p;c)$ are to c (from $\frac{\partial \mathcal{L}(p,c)}{\partial c}$), and ii) how responsive $\pi(p;c)$ is to p (from $-\mathcal{L}'(p,c)$).

[11] For brevity in the text, we refer to it as the pass-through rate in (9).

To go further, we now explain the second expression of (9). This rewrites the determinants in terms of the underlying parameters relating to demand and cost, which will allow us to later understand how the shape of demand affects the pass-through rate. The numerator captures the relative responsiveness of $\pi\left(p^m(c);c\right)$ and $\pi(p;c)$ to c, where at a given price p,

$$1-\left[\tfrac{1}{\pi(p^m(c);c)}\tfrac{\partial\pi(p^m(c);c)}{\partial c}\right]/\left[\tfrac{1}{\pi(p;c)}\tfrac{\partial\pi(p,c)}{\partial c}\right]=1-\frac{p-c}{p^m(c)-c}>0\ \forall p<p^m(c), \tag{10}$$

and the denominator captures the relative responsiveness of $\pi(p;c)$ to p and c, where at a given price p,

$$-\tfrac{\pi'(p;c)}{\pi(p;c)}/\left(\tfrac{1}{\pi(p;c)}\tfrac{\partial\pi(p,c)}{\partial c}\right)=1-\frac{p-c}{p}\varepsilon(p)>0\ \forall p<p^m(c). \tag{11}$$

Having understood the two expressions and determinants, now note that the pass-through rate in (9) is always strictly positive, as consistent with $F(p)$ shifting to the right in Figure 1. This follows since i) at the upper bound of the price distribution, (9) yields the familiar monopoly pass-through rate, $\lim_{z\to0}\frac{\partial p^*(z,c,\sigma,n)}{\partial c}=\frac{1}{2-\xi(p^m(c))}$, and ii) away from the upper bound, (10) and (11) are strictly positive. Intuitively, using the first expression in (9), note that an increase in c will raise the relative loss from a sale, $\mathcal{L}(p,c)$, so p must rise in order to decrease $\mathcal{L}(p,c)$ back to its original level.[12]

4.2 Pass-Through Rates of Expected Prices

For later, it is useful to present a technical Lemma that shows that the pass-through rates of the expected prices in (7) and (8) can be expressed as a function of (9). These further highlight the advantages of our approach because they only involve the one term of (9), which ensures that the analysis is tractable and able to overcome the previously discussed challenges with using (7) and (8). Given $\frac{\partial p^*(z,c,\sigma,n)}{\partial c}>0\ \forall z$, it is unsurprising that the pass-through rates of the expected price and the expected minimum price are positive.

Lemma 3. *The expected price and expected minimum price rates of cost pass-through are, respectively*

$$\frac{\partial E(p)}{\partial c}=\int_0^1\frac{\partial p^*(z,c,\sigma,n)}{\partial c}dz>0, \tag{12}$$

$$\frac{\partial E(p_{min})}{\partial c}=\int_0^1 nz^{n-1}\frac{\partial p^*(z,c,\sigma,n)}{\partial c}dz>0. \tag{13}$$

[12]This follows since the relative loss from a sale is strictly increasing in c, $\frac{\partial\mathcal{L}(p,c)}{\partial c}>0$, because per-consumer profits are more responsive to c at $p<p^m(c)$ than at $p^m(c)$. Furthermore, the relative loss from a sale is strictly decreasing in p, $\mathcal{L}'(p,c)<0$, because per-consumer profits will rise as p gets closer to $p^m(c)$ given Assumption 1.

5 Properties of Pass-Through under Price Dispersion

In this section, we explore the equilibrium properties of pass-through under price dispersion. We develop important implications for understanding how a change in costs will affect the magnitudes of price changes faced by different consumer types and price dispersion more generally. As an initial step towards uncovering these properties, Section 5.1 explains how the price elasticity of demand affects the pass-through rate in (9). While this gives us some insight into the effects of pass-through for certain points of the price distribution, Section 5.2 builds on Section 5.1 to develop a more general understanding of pass-through for all points by distinguishing between log-concave and log-convex demand. It does this in two steps. First, Section 5.2.1 shows that pass-through rates *across* the entire price range will be higher for log-convex demand than for log-concave demand. This follows since the pass-through rate in (9) is always less than one in the former but always greater than one in the latter. Second, Section 5.2.2 shows how the pass-through rates vary systematically at different points *within* the price range depending upon whether demand is log-concave or log-convex. This provides insights into how pass-through affects the range of prices, the expected prices faced by different consumer types, and the subsequent captivity premium. Finally, Section 5.3 studies how pass-through affects welfare, including the consumer surplus of the different consumer types.

5.1 The Pass-Through Rate and Price Elasticity of Demand

In this subsection, as an initial step towards uncovering the properties of pass-through, we first explain how the price elasticity of demand affects the pass-through rate in (9). To proceed, Figure 2 illustrates two demand curves that are deliberately constructed to share the same $p^m(c)$, $p^*(z,c,\sigma,n)$ and associated quantities for some $z > 0$ and for a given c. Comparing these demand curves at $p^*(z,c,\sigma,n)$ then allows us to isolate the effect of the price elasticity of demand – or more precisely, the slope of demand – on the second expression of (9), because all else is being held constant. Equivalently, in terms of the first expression in (9), such a comparison varies how responsive $\pi(p;c)$ is to p (which determines $-\mathcal{L}'(p,c)$) while holding constant how responsive $\pi(p^m(c);c)$ and $\pi(p;c)$ are to c (which determine $\frac{\partial \mathcal{L}(p,c)}{\partial c}$).

Given the differing slopes of the two demand curves, consider the extents to which a small increase in c will raise $p^*(z,c,\sigma,n)$. The answer, from (9), is that the price rise required to keep $F(p)$ constant at $1-z$ will be smaller for the demand curve that is more elastic (i.e. flatter) at $p^*(z,c,\sigma,n)$, which in Figure 2 happens to be the linear demand curve. Intuitively, in (11), $\pi(p;c)$ will be less responsive to an increase in p at $p^*(z,c,\sigma,n)$ for the linear demand

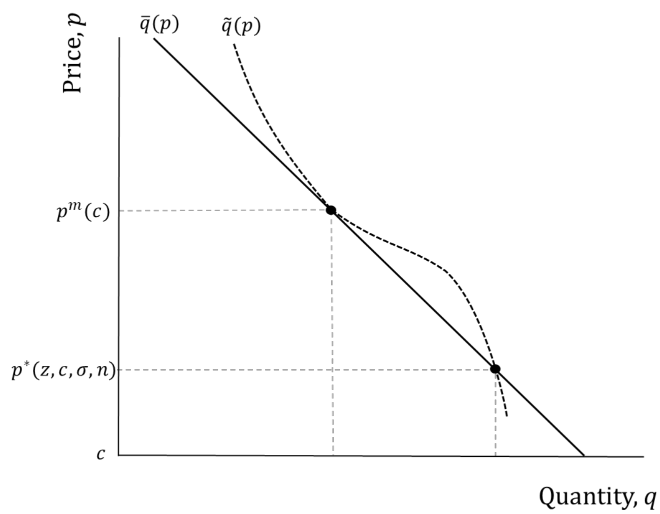


Figure 2: Isolating the role of the price elasticity of demand

curve, because the profit increase from a higher price-cost margin will be offset more by a larger fall in the quantity demanded. Hence, for the linear demand that is more elastic at $p^*(z,c,\sigma,n)$, a cost rise that raises the relative loss from a sale, $\mathcal{L}(p,c)$, requires a larger increase in p to decrease $\mathcal{L}(p,c)$ back to its original level.

In our subsequent analysis, we will go beyond these conditions to understand pass-through more generally across the entire price range, including points at which two demand curves do not intersect. At any non-intersecting point, the two demand curves will have a different $q(p)$ for a given price. Consequently, if the demand curves share the same $p^m(c)$, the two demand curves will differ in their level of $p^*(z,c,\sigma,n)$ for the same $z>0$. In particular, for a given $z=1-F(p)$, it follows from (6) that the higher is $q(p)$, the lower is $p^*(z,c,\sigma,n)$. Thus, the impact of the shape of demand on the pass-through rate in (9) for a given z is not isolated to the price elasticity of demand like it was in our previous example in Figure 2. Instead, as well as affecting how responsive $\pi\left(p;c\right)$ is to p at $p^*(z,c,\sigma,n)$ (which determines $-\mathcal{L}'(p,c)$), the shape of demand will also affect how responsive $\pi\left(p;c\right)$ is to c (which affects $\frac{\partial\mathcal{L}(p,c)}{\partial c}$). While this makes the analysis more difficult at non-intersecting points, the next subsections show how progress can be made by distinguishing between log-concave and log-convex demand.

5.2 The Effects of Pass-Through on the Price Distribution

In this subsection, we develop a more general understanding of pass-through. In particular, Section 5.2.1 shows that pass-through rates *across* the entire price range will be higher for log-convex demand than for log-concave demand. Section 5.2.2 then shows how the pass-through rates vary systematically at different points *within* the price range depending upon whether demand is log-concave or log-convex.

5.2.1 Pass-Through across the Price Range

In this subsection, we show that the pass-through rate in (9) is higher for log-convex demand than for log-concave demand across the entire price range, for all z. To do so, we consider the conditions under which $\frac{\partial p^*(z,c,\sigma,n)}{\partial c}$ is greater or less than one (or equivalently whether $p^*(z,c,\sigma,n)$ rises by more or less than the increase in marginal cost). It is well-known that the monopoly pass-through rate (with constant marginal costs) is greater than one when demand is strictly log-convex, but not when demand is log-concave. Consequently, the same applies to the upper bound of the equilibrium price distribution. Proposition 2 now shows that the same condition also applies to $\frac{\partial p^*(z,c,\sigma,n)}{\partial c}$ at *all* points across the price range.

Proposition 2. *When demand is strictly log-concave (log-convex), $\xi(p) < (>) 1\ \forall p$, the inverse price distribution rate of cost pass-through is strictly less (greater) than one across the entire price range, $\frac{\partial p^*(z,c,\sigma,n)}{\partial c} < (>) 1$ for any $z \in [0,1]$. When demand is log-linear, $\xi(p) = 1\ \forall p$, it equals one across the entire price range.*

To understand the intuition of Proposition 2, it is helpful to compare the magnitudes of the numerator and denominator for the two expressions of the pass-through rate in (9). This leads to two equivalent conditions that determine whether the magnitude of this pass-through rate will be less than or greater than one. From the first expression in (9), the numerator will be less (greater) than the denominator when $\mathcal{L}(p,c)$ is more (less) responsive to changes in p than to changes in c (i.e. $-\mathcal{L}'(p,c) > (<) \frac{\partial \mathcal{L}(p,c)}{\partial c}$). From the second expression, we can derive an equivalent condition in terms of the shape of demand. In particular, by subtracting the numerator, (10), away from the denominator, (11), and manipulating we can obtain

$$\frac{p-c}{p^m(c)-c} - \frac{p-c}{p}\varepsilon(p) = (p-c)\left(\frac{-q'(p^m(c))}{q(p^m(c))} - \frac{-q'(p)}{q(p)}\right), \tag{14}$$

where $\frac{1}{p^m(c)-c} = \frac{-q'(p^m(c))}{q(p^m(c))}$ from the Lerner index. The right-hand side of (14) implies that whether (9) will be less than or greater than one for any $z > 0$ will depend upon the difference between the changes in relative demand at $p^m(c)$ and at $p^*(z,c,\sigma,n) < p^m(c)$.

To complete the intuition, we now link these two equivalent conditions to the curvature of demand in terms of whether demand is log-concave or log-convex. Initially, let us consider the special case where demand is log-linear, $\xi(p) = 1\ \forall p$ (and therefore strictly convex, $q''(p) > 0$).[13] Here, the pass-through rate in (9) is exactly equal to one across the entire

[13] Exponential demand is an example of this special case.

price range. Intuitively, we can see from (14) that a marginal price rise at any price always leads to the same fall in relative demand, $-\left(\frac{q'(p)}{q(p)}\right)' = -(\ln q(p))'' = 0$.[14] An implication of this is that the responsiveness of $\pi(p;c)$ to p always guarantees that the profit ratio $\mathcal{L}(p,c)$ is equally sensitive to p as it is to c. Consequently, in regard to the first expression of (9), an increase in c that raises the relative loss from a sale, $\mathcal{L}(p,c)$, requires an increase in p of the same magnitude to reduce $\mathcal{L}(p,c)$ back to its original level.

Now suppose demand is strictly log-concave (log-convex). Here, the pass-through rate in (9) is strictly below (above) one across the entire price range. Intuitively, as price falls below $p^m(c)$, a marginal price rise leads a smaller (larger) fall in relative demand, $-\left(\frac{q'(p)}{q(p)}\right)' = -(\ln q(p))'' > (<)0$. Consequently, $\pi(p;c)$ becomes more (less) responsive to p as p falls further below $p^m(c)$. This ensures that $\mathcal{L}(p,c)$ is more (less) responsive to changes in p than to changes in c. Therefore, an increase in c that raises the relative loss from a sale, $\mathcal{L}(p,c)$, requires a relatively smaller (larger) increase in p to reduce it back to its prior level.

Finally, given Proposition 2 applies across the entire price range, Corollary 1 follows immediately.

Corollary 1. *If demand is strictly log-concave (log-convex), $\xi(p) < (>)1\ \forall p$, then the cost-pass-through rates of the monopoly price, $\frac{\partial p^m(c)}{\partial c}$, expected price, $\frac{\partial E(p)}{\partial c}$, expected minimum price, $\frac{\partial E(p_{min})}{\partial c}$, and lower bound price, $\frac{\partial \underline{p}(c,\sigma,n)}{\partial c}$ are all strictly less (greater) than one. When demand is log-linear, $\xi(p) = 1\ \forall p$, all such cost-pass through rates always equal one.*

5.2.2 Pass-Through within the Price Range

The previous subsection showed that pass-through is higher across the entire price range when demand is log-convex rather than log-concave. In contrast, this subsection considers how the pass-through rate in (9) varies at different points *within* the price range depending upon whether demand is log-concave or log-convex. This has important implications for understanding how pass-through affects the range of prices, the expected prices faced by different consumer types, and the subsequent captivity premium. Henceforth, we consider demand that is either *strictly* log-concave or *strictly* log-convex. When demand is log-linear, $\xi(p) = 1\ \forall p$, we know from Corollary 1 that the captivity premium and range of prices are independent of c, $\frac{\partial \kappa(c,\sigma,n)}{\partial c} = \frac{\partial \rho(c,\sigma,n)}{\partial c} = 0$. Furthermore, for ease of exposition, we focus on the large class of demand functions that has constant curvature, $\xi'(p) = 0$, and postpone

[14] This occurs due to two counteracting effects that perfectly offset each other: for lower p, there is more quantity demanded (because $q'(p) < 0$)) but a marginal price increase also leads to a larger decrease in the quantity demanded (because the slope will be flatter at lower prices, $q''(p) > 0$)).

discussion of non-constant curvature until Section 8.2.[15]

Now consider how the impact of a cost increase varies across different points of the equilibrium price distribution. This is an important foundation for later results.

Proposition 3. *If demand is strictly log-concave (log-convex) with constant curvature, $\xi(p) = \xi < (>) 1\ \forall p$, then the inverse price distribution rate of cost pass-through increases (decreases) towards one for higher levels of z closer to the bottom of the price distribution, $\frac{\partial^2 p^*(z,c,\sigma,n)}{\partial c \partial z} \geq (\leq) 0$ for any $z \in [0,1]$, where the inequality is strict $\forall z > 0$.*

Roughly speaking, the intuition is as follows. The horizontal shift in $F(p)$ towards the top of the distribution is closer to the monopoly pass-through rate, because the upper bound is the monopoly price, $p^*(0, c, \sigma, n) = p^m(c)$. In contrast, the horizontal shift in $F(p)$ towards the bottom of the distribution is closer to 1, because $p^*(z, c, \sigma, n)$ is closer to c for higher levels of z. To see this, note that, when evaluated at $p = c$, both (10) and (11) equal one, such that $\mathcal{L}(p, c)$ is equally sensitive to p and c. Therefore, given the monopoly pass-through rate is less (greater) than one when demand is log-concave (log-convex), the horizontal shift in $F(p)$ is relatively smaller (larger) at the top of the distribution than towards the bottom. More precisely, as z moves away from zero, the pass-through rate in (9) increases (decreases) away from the monopoly pass-through rate towards one, because the numerator of (9) increases towards one at a relatively faster (slower) rate than the denominator given that i) demand curvature is constant and ii) (2) applies via Assumption 1.[16]

Let us now discuss some implications of Proposition 3. First, we can see that Figure 1 is consistent with a log-concave demand curve with constant curvature, as $F(p)$ shifts to the right more towards the bottom of the distribution. Given this consistency with log-concave demand, it then follows from Proposition 2 that the horizontal shift in $F(p)$ in Figure 1 will also be less than the increase in c for any z, as the pass-through rate will always be strictly less than one. Second, consider the implications for the range of prices and captivity premium in the following result.

Proposition 4. *If demand is strictly log-concave (log-convex) with constant curvature, $\xi(p) = \xi < (>) 1\ \forall p$, then the following ranking of pass-through rates applies:*

$$1 > (<) \frac{\partial \underline{p}(c, \sigma, n)}{\partial c} > (<) \frac{\partial E\left(p_{min}\right)}{\partial c} > (<) \frac{\partial E\left(p\right)}{\partial c} > (<) \frac{\partial p^m\left(c\right)}{\partial c} = \frac{1}{2-\xi}. \tag{15}$$

[15] Examples of demand functions with constant curvature include linear demand (where $\xi(p) = 0$), isoelastic demand (where $\varepsilon(p) = \varepsilon > 0$ and $\xi(p) = 1 + \frac{1}{\varepsilon} > 0$) and exponential demand (where $\xi(p) = 1$).

[16] This follows since, as demonstrated in the proof, the numerator of (9) is more (less) responsive to p than the denominator (e.g. $-\frac{\partial \mathcal{L}'(p,c)}{\partial c} / \frac{\partial \mathcal{L}(p,c)}{\partial c} > (<) \frac{\mathcal{L}''(p,c)}{-\mathcal{L}'(p,c)}$).

Hence, the range of prices and captivity premium are both strictly smaller (larger) after an industry-wide increase in marginal cost, $\frac{\partial \rho(c,\sigma,n)}{\partial c} < (>)0$ *and* $\frac{\partial \kappa(c,\sigma,n)}{\partial c} < (>)0$.

Intuitively, the ranking of pass-through rates in (15) is determined by whether $F(p)$ shifts to the right more towards the bottom of the price distribution or the top. Clearly, this follows immediately for the pass-through rates of the upper and lower bound prices. However, it also follows for the pass-through rates of the expected price and the expected minimum price, because the expected minimum price is weighed relatively more heavily by the bottom of the distribution. Thus, we are able to understand how the range of prices and captivity premium change in response to an increase in costs.

First, consider the range, $\rho(c,\sigma,n)$. An increase in c will make the range of prices smaller (larger) when demand is log-concave (log-convex) with constant curvature, because the lower-bound pass-through rate will be greater (less) than the upper bound (monopoly) pass-through rate, $\frac{\partial \rho(c,\sigma,n)}{\partial c} = \frac{\partial p^m(c)}{\partial c} - \frac{\partial \underline{p}(c,\sigma,n)}{\partial c} < (>)0$. This extends our understanding beyond the simple case of unit demand where such an increase in costs trivially reduces the range of prices because the monopoly price remains fixed while the lower bound price rises. In particular, Proposition 4 provides general conditions under which the range of prices will shrink, despite both the upper and lower bound prices rising, while also establishing conditions under which the range of prices will expand.

Second, consider the captivity premium, $\kappa(c,\sigma,n)$. Under constant curvature, an increase in c will also make the captivity premium smaller (larger) when demand is log-concave (log-convex), $\frac{\partial \kappa(c,\sigma,n)}{\partial c} = \frac{\partial E(p)}{\partial c} - \frac{\partial E(p_{min})}{\partial c} < (>)0$. Hence, depending on the curvature of demand, the cost increase can lead to either captives or shoppers experiencing a relatively larger price hike. As noted in the introduction, this can help explain the mixed empirical results on the impact of cost changes on different consumer groups (e.g. Broda and Romalis, 2009; Argente and Lee, 2021; and Montag et al., 2024).

5.3 The Effects of Pass-Through on Welfare

In this subsection, we consider the effects of cost pass-through on welfare. We begin with consumer surplus before considering profits and total welfare. Previously, we showed that shoppers experience a relatively larger (smaller) price effect than captives when demand is log-concave (log-convex) with constant curvature. We now explore how an increase in costs affects the expected consumer surplus of each consumer type.

Proposition 5. *Suppose demand has constant curvature,* $\xi(p) = \xi \ \forall p$. *When demand is log-concave, an industry-wide increase in marginal cost will reduce the consumer surplus of a* *shopper to a greater extent than a captive. This can also be true when demand is log-convex despite shoppers expecting a smaller price increase than captives.*

To understand this result, first consider when demand is log-linear, $\xi(p) = 1\ \forall p$. Here, we know from Corollary 1 that the cost pass-through rates of the expected price, $\frac{\partial E(p)}{\partial c}$, and the expected minimum price, $\frac{\partial E(p_{min})}{\partial c}$ both equal one. This implies both consumer types face the same expected price rise. However, a shopper demands more than a captive because they expect to buy at a lower price. Therefore, each shopper receives a relatively larger reduction in consumer surplus, because they experience the price rise over a larger number of units. Now suppose demand is strictly log-concave. Here, each shopper still experiences a relatively larger decrease in consumer surplus because, in addition to demanding more, they also face a larger cost pass-through rate than a captive. In contrast, when demand is strictly log-convex, although each shopper is expected to demand more than a captive, they face a relatively smaller cost pass-through rate. Consequently, whether or not a shopper receives a larger reduction in consumer surplus than a captive depends upon the size of these counteracting effects. As discussed in the introduction, Proposition 5 has important implications for the growing interest in protecting 'vulnerable' consumers, who are less able to engage in markets (e.g. OECD 2023, European Parliament 2021, CMA 2019).

Let us now discuss the effects of an industry-wide increase in marginal costs on profits and total welfare. While Proposition 4 implies that the expected price-cost margins that firms expect to earn from shoppers and captives will fall (rise) when demand is strictly log-concave (log-convex), firms' expected profits always decrease. Intuitively, recall that the equilibrium profits are determined by the monopoly profits that firms can earn from their captive consumers, $\Pi^N(\sigma, c, n) = \left(\frac{1-\sigma}{n}\right)\pi(p^m(c); c)$. Thus, a cost rise will have a negative first-order effect on equilibrium profits, $\left(\frac{1-\sigma}{n}\right)\frac{\partial \pi(p^m(c);c)}{\partial c}$, but the associated price rise will not have a first-order effect as $\left(\frac{1-\sigma}{n}\right)\frac{\partial p^m(c)}{\partial c}\pi'\left(p^m(c); c\right) = 0$ from the envelope theorem. While this is consistent with results of monopoly, it contrasts with other oligopoly models where over-shifting can increase equilibrium profits (e.g. Anderson *et al.*, 2001). Finally, given equilibrium profits and expected consumer surplus decrease for both consumer types, it follows that expected total welfare will also strictly decrease.

6 Relation to Demand Shocks

In this section, we show that there is a relationship between the effects of a change in marginal cost under price dispersion and a unit parallel vertical shift in demand. This relationship is well-known in settings with pure-strategy pricing equilibria (Weyl and Fabinger,

2013) implying an equivalence between the effects of a unit tax on firms and a unit tax on consumers. We now establish that the same results apply within our mixed-strategy setting. This offers novel insights into the effects of how demand shocks can affect the equilibrium price dispersion.

To proceed, let the parameter a represent a demand shifter that produces a unit parallel vertical increase in demand such that $\frac{\partial q(p)}{\partial a} = -q'(p) > 0$ for all p.[17]

Proposition 6. *A unit parallel vertical increase in demand shifts the equilibrium price distribution by* $\frac{\partial p^*(z,c,\sigma,n)}{\partial a} = 1 - \frac{\partial p^*(z,c,\sigma,n)}{\partial c}$ *for any* $z \in [0, 1]$.

When combined with our previous analysis, Proposition 6 has several implications. First, it follows from Proposition 2 that if demand is strictly log-concave (log-convex), then the unit parallel vertical increase in demand will cause $F(p)$ to shift horizontally to the right (left). Consequently, as this applies across the entire distribution, it follows that the monopoly price, the expected price, the expected minimum price, and the lower bound price will all strictly increase (decrease). Furthermore, Proposition 3 implies that if demand is strictly log-concave (log-convex) with constant curvature, then the increase in demand causes $F(p)$ to shift horizontally to the right (left) to a *smaller* extent towards the bottom of the distribution, $\frac{\partial^2 p^*(z,c,\sigma,n)}{\partial a \partial z} = -\frac{\partial^2 p^*(z,c,\sigma,n)}{\partial c \partial z} < (>) 0$. Intuitively, $p^*(z, c, \sigma, n)$ is closer to c for higher levels of z, so it is is more cost reflective and therefore less responsive to shifts in demand. This leads immediately to the following.

Corollary 2. *If demand is strictly log-concave (log-convex) with constant curvature,* $\xi(p) = \xi < (>) 1 \ \forall p$, *a unit parallel vertical increase in demand will lead to i) a larger (smaller) range of prices,* $\frac{\partial \rho(c,\sigma,n)}{\partial a} = -\frac{\partial \rho(c,\sigma,n)}{\partial c} > (<) 0$, *and ii) a larger (smaller) captivity premium,* $\frac{\partial \kappa(c,\sigma,n)}{\partial a} = -\frac{\partial \kappa(c,\sigma,n)}{\partial c} > (<) 0$.

This result is interesting because, to our knowledge, the previous literature has not offered any insights about the effects of shifts in downward-sloping demand on price dispersion. Furthermore, we offer the following welfare result.

Proposition 7. *A unit parallel vertical increase in demand has the same welfare impact on firms, captives and shoppers as a unit decrease in marginal cost.*

[17] An increase in the parameter a will lead to a vertical increase in demand if, after a horizontal increase in demand, $\frac{\partial q(p)}{\partial a}$, and an increase in price, $\frac{\partial p}{\partial a}$, the quantity demanded remains constant, $\frac{dq(p)}{da} = \frac{\partial q(p)}{\partial a} + \frac{\partial p}{\partial a} q'(p) = 0$. Thus, there is a *unit* vertical increase in demand if $\frac{\partial p}{\partial a} = 1$, such that $\frac{\partial q(p)}{\partial a} = -q'(p) > 0$. Finally, there will be a unit *parallel* vertical increase if $\frac{\partial q(p)}{\partial a} = -q'(p)$ applies for all p.

One important implication of Proposition 7 is that it verifies that the welfare effects of taxation are independent of which side of the market physically pays the tax, even in settings with equilibrium price dispersion. This follows from interpreting the parallel vertical decrease in demand as a unit tax on consumers and the increase in marginal cost as a unit tax on firms.

7 The Effects of Competition

Understanding the role of competition on cost pass-through forms a key part of the existing literature (e.g. Weyl and Fabinger 2013, Miller et al. 2017, Genakos and Pagliero 2022, Ritz 2024). However, little is known about this issue in settings with price dispersion. Hence, this section explores how our previous results on cost pass-through vary with the level of competition as measured by either the number of firms, n, or the proportion of shoppers, σ. In particular, Section 7.1 examines how these two measures of competition affect the expected pass-through rates, while Section 7.2 analyses how the two measures influence the effects of cost increases on the captivity premium and the range of prices. These results are useful in generating new empirical predictions and guiding policy to assess which sorts of markets are most likely to exhibit large price effects for vulnerable consumers. Throughout the section, we continue to focus on i) constant demand curvature, delaying discussion of non-constant curvature until Section 8.2, and ii) strict log-concavity/convexity because from Corollary 1 the pass-through rate in (9) always equals one for any level of competition when demand is log-linear, $\xi(p) = 1 \; \forall p$.

7.1 The Effects of Competition on the Pass-Through Rate

To begin, we examine how cost pass-through varies with the two measures of competition. The conventional wisdom is that greater competition will lead to pass-through rates that are closer to one and so the price changes will be more reflective of costs.[18] Within our setting of price dispersion, while the upper bound (monopoly) pass-through rate is independent of both measures, $\frac{\partial p^m(c)}{\partial c} = \frac{1}{2-\xi(p^m(c))}$, we show that whether competition makes the pass-through rate in (9) more cost reflective across the rest of the distribution depends upon which measure of competition changes. We begin by considering how $\frac{\partial p^*(z,c,\sigma,n)}{\partial c}$ changes with the proportion of shoppers, σ. Here, the results follow the conventional wisdom.

[18]Specifically, this can involve pass-through rates increasing towards one from below under log-concave demand or decreasing towards from above one under log-convex demand, where both are observed within the empirical literature (e.g. Genakos and Pagliero, 2022; and Miller et al. 2017, respectively).

Proposition 8. *If demand is strictly log-concave (log-convex) with constant curvature,* $\xi(p) = \xi < (>) 1 \ \forall p$*, then as the proportion of shoppers rises, the inverse price distribution rate of cost pass-through strictly increases (decreases) towards one,* $\frac{\partial^2 p^*(z,c,\sigma,n)}{\partial c \partial \sigma} > (<) 0$ *for any* $z \in (0,1]$*. Hence, the cost pass-through rates of the expected price,* $\frac{\partial E(p)}{\partial c}$*, expected minimum price,* $\frac{\partial E(p_{min})}{\partial c}$*, and lower bound price,* $\frac{\partial \underline{p}(c,\sigma,n)}{\partial c}$*, also move closer towards one.*

A change in the proportion of shoppers, σ, will not affect the upper bound (monopoly) pass-through rate, but it will have an impact on the rest of the price distribution. Specifically, Proposition 8 implies that when demand is strictly log-concave (log-convex) with constant curvature, an increase in c will shift $F(p)$ horizontally to the right to a greater (smaller) extent when there is a larger proportion of shoppers. Intuitively, when there are more shoppers, prices become less concentrated around the monopoly price and more concentrated closer to marginal cost. Consequently, the inverse price distribution rate of cost pass-through moves further away from the monopoly pass-through rate and closer to one. This in turn ensures that the prices that captives and shoppers expect to pay will also be more reflective of cost.

Next, consider how $\frac{\partial p^*(z,c,\sigma,n)}{\partial c}$ changes with the number of firms, n. Here, the results are different as they do not always follow the conventional wisdom.

Proposition 9. *There exists a unique* $z^* \in (0,1)$ *such that if demand is strictly log-concave (log-convex) with constant curvature,* $\xi(p) = \xi < (>) 1 \ \forall p$*, then as the number of firms rises, the inverse price distribution rate of cost pass-through:*
i) strictly increases (decreases) towards one if $z > z^*$*,* $\frac{\partial^2 p^*(z,c,\sigma,n)}{\partial c \partial n} > (<) 0$*, yet*
ii) strictly decreases (increases) away from one if $z \in (0, z^*)$*,* $\frac{\partial^2 p^*(z,c,\sigma,n)}{\partial c \partial n} < (>) 0$*.*

As before, a change in the number of firms, n, will not affect the upper bound (monopoly) pass-through rate, but it will have an impact on the rest of the price distribution. In particular, Proposition 9 states that as the number of firms increases $\frac{\partial p^*(z,c,\sigma,n)}{\partial c}$ will move towards one at the bottom of the distribution, where $z > z^*$, yet it will move *away* from one close to the top of the distribution, where $z \in (0, z^*)$. Consequently, when demand is strictly log-concave (log-convex) with constant curvature, as the number of firms rises, an increase in c will shift $F(p)$ horizontally to the right to a greater (smaller) extent towards the bottom of the distribution but to a smaller (greater) extent towards the top. This counteracting effect towards the top of the distribution introduces the possibility that, in contrast to the conventional wisdom, the expected pass-through rates will move away from one. While these counteracting effects limit the scope for general results on the expected pass-through rates, we can still obtain the following result.

Proposition 10. *As the number of firms becomes large, $n \to \infty$, the pass-through rate of the expected price tends towards the monopoly pass-through rate, $lim_{n\to\infty}\frac{\partial E(p)}{\partial c} = \frac{\partial p^m(c)}{\partial c} = \frac{1}{2-\xi(p^m(c))}$, whilst the pass-through rates of the expected minimum price and lower bound price tend towards one, $lim_{n\to\infty}\frac{\partial E(p_{min})}{\partial c} = \frac{\partial p(c,\sigma,n)}{\partial c} = 1$.*

When the number of firms is large, Proposition 10 indicates that the pass-through rates of the expected minimum price and lower bound price are consistent with the conventional wisdom, but the opposite is true for the expected price. These differing results occur due a standard feature of Varian (1980). In particular, when $n \to \infty$, there is a small chance that any one firm will win the shoppers. Consequently, in equilibrium, firms concentrate their prices close to the monopoly level to exploit their captive consumers. This implies that the pass-through rate of the expected price tends to the monopoly pass-through rate. However, with a small probability, each firm will compete for shoppers and when they do so their price will be very close to c. Thus, given there is an infinite number of firms, the expected minimum price tends towards c and its pass-through rate equals one.

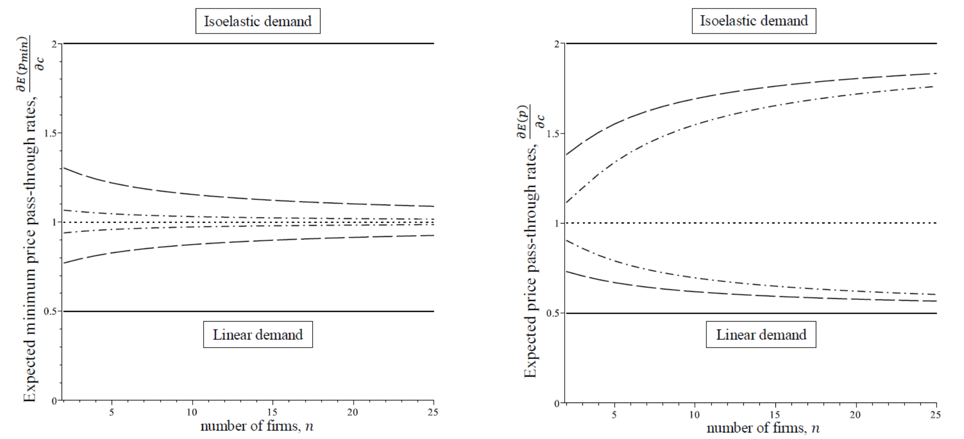
To understand how the pass-through rates of the two expected prices change with the number of firms away from this limit, Figure 3 plots $\frac{\partial E(p_{min})}{\partial c}$ in panel (a) and $\frac{\partial E(p)}{\partial c}$ in panel (b) under linear demand ($q(p) = \frac{a-p}{b}$) and isoelastic demand ($q(p) = \upsilon p^{-\varepsilon}$ with $\varepsilon = 2$).[19] Each panel shows how the pass-through rates change with n for various example proportions of shoppers, σ. The pass-through rates in both panels are below one for the (log-concave) linear demand but above one for the (log-convex) isoelastic demand. For both demands, Figure 3(a) demonstrates how $\frac{\partial E(p_{min})}{\partial c}$ moves towards one as the number of firms increases, while in contrast to the conventional wisdom, Figure 3(b) demonstrates how $\frac{\partial E(p)}{\partial c}$ moves away from one, towards the monopoly pass-through rate. Furthermore, for both demands, $\frac{\partial E(p_{min})}{\partial c}$ and $\frac{\partial E(p)}{\partial c}$ move closer to one as the proportion of shoppers rises, although the effect is most pronounced when the number of firms is small.

7.2 Implications for Price Dispersion

To generate new empirical predictions and to understand the market conditions that are most likely to exhibit large price effects for different types of consumers, this subsection briefly discusses how the two measures of competition affect the impact of a cost change on price dispersion as measured by i) the captivity premium and ii) the range of prices.

First, consider the change in the captivity premium, $\frac{\partial \kappa(c,\sigma,n)}{\partial c} = \frac{\partial E(p)}{\partial c} - \frac{\partial E(p_{min})}{\partial c}$. Given the captivity premium always tends to zero at the extremes when (almost) all consumers are

[19]For more technical details on these example demand curves, see the Supplementary Appendix.



(a) The pass-through rate of the expected minimum price for linear demand (below 1) and isoelastic demand (above 1)

(b) The pass-through rate of the expected price for linear demand (below 1) and isoelastic demand (above 1)

Figure 3: Pass-through when demand is linear ($q(p) = \frac{a-p}{b}$) and isoelastic ($q(p) = \upsilon p^{-2}$)

shoppers or captives, it follows from Proposition 4 that the absolute value of $\frac{\partial \kappa(c,\sigma,n)}{\partial c}$ will be greatest for a moderate proportion of shoppers. While the scope for general results about the number of firms is limited due to the counteracting effects on the price distribution, Figure 3 indicates that under both of the example demand curves $\frac{\partial \kappa(c,\sigma,n)}{\partial c}$ will be greater as the number of firms increases. Together, these indicate that the impact of a cost increase on the captivity premium will be greatest in markets where there is a large number of firms and an intermediate proportion of shoppers and captives.

Now, consider the change in the range of prices, $\frac{\partial \rho(c,\sigma,n)}{\partial c} \equiv \frac{\partial p^m(c)}{\partial c} - \frac{\partial \underline{p}(c,\sigma,n)}{\partial c}$. Here, as the monopoly pass-through rate is independent of σ and n, the effect of competition on $\frac{\partial \rho(c,\sigma,n)}{\partial c}$ is driven entirely by the effect on the lower bound price. Given the lower bound price tends towards marginal cost as either measure of competition increases, we known from Propositions 8 and 9 that the pass-through rate of the lower bound price will always tend to one. Hence, the following result can be stated immediately.

Corollary 3. *When demand is strictly log-concave (log-convex) with constant curvature, $\xi(p) = \xi < (>)1$ $\forall p$, an industry-wide increase in marginal costs will reduce (raise) the range of prices to a greater extent when either the proportion of shoppers rises, $\frac{\partial^2 \rho(c,\sigma,n)}{\partial c \partial \sigma} = -\frac{\partial^2 p^*(1,c,\sigma,n)}{\partial c \partial \sigma} < (>)0$, or the number of firms rises, $\frac{\partial^2 \rho(c,\sigma,n)}{\partial c \partial n} = -\frac{\partial^2 p^*(1,c,\sigma,n)}{\partial c \partial n} < (>)0$.*

This provides an empirically testable prediction about how the impact of a cost increase on the range of prices will vary with competition depending upon the shape of demand.

8 Extensions

In this last section, we offer some extensions of our analysis. Section 8.1 shows how our results can still hold in settings with non-constant marginal costs or ad valorem taxes/fees. Section 8.2 provides conditions under which our results apply to other demand functions with non-constant curvature. Section 8.3 demonstrates how our analysis applies to unit demand as commonly used within the price dispersion literature. Finally, Section 8.4 extends our results Section 8.2 to consider the impact of search costs.

8.1 Non-Constant Marginal Costs: Unit and Ad Valorem Taxes

Up to this point, we have assumed that firms face constant marginal costs. However, it is well-known that the shape of firms' costs can affect pass-through as well as the shape of demand (e.g. Ritz, 2024). In this section, we want to demonstrate that our results can apply to settings beyond constant marginal costs. This is challenging because, to the authors' knowledge, there is no model that has extended Varian (1980) to both downward-sloping demand and non-constant marginal costs.[20]

Despite this, we make some progress by considering a specific cost structure with non-constant marginal costs, where each firm incurs an additional ad valorem cost. This cost structure is policy-relevant and empirically important in many markets. For instance, it is consistent with i) ad valorem taxes (e.g. Häckner and Herzing, 2016; and Adachi and Fabinger, 2022) and iii) revenue-sharing contracts (e.g. Johnson, 2017). In what follows, we focus on the tax interpretation. In particular, consider a per-consumer total cost function, $C(p) \equiv \alpha p q(p) + \tau q(p)$, where $\alpha \in (0, 1 - \frac{\tau}{\hat{p}})$ is the ad valorem tax rate and where $\tau \in (0, \hat{p})$ now denotes the constant unit cost (which can be interpreted as a unit tax rate). Given this total cost function, marginal cost is non-constant and equal to $\tau + \alpha[p(1 - \frac{1}{\varepsilon(p)})]$, where the term in square brackets represents marginal revenue.[21]

Even with non-constant marginal cost and downward-sloping demand, it is easy to show that the equilibrium is the same as in Lemma 1 with the exception that $\widehat{c} \equiv \frac{\tau}{1-\alpha}$ replaces c. This follows since the per-consumer profits can be written as $(p(1-\alpha) - \tau)q(p) \equiv$

[20] Indeed, most of the literature assumes unit demand with constant marginal costs. The few papers that do have non-constant costs assume unit demand (Varian, 1980; and Baye et al., 1992) and the ones with downward-sloping demand assume constant marginal costs (e.g. Stahl, 1989; Baye and Morgan, 2001; Armstrong and Vickers, 2022).

[21] This marginal cost is non-constant because Assumption 1 guarantees $[p(1 - \frac{1}{\varepsilon(p)})]' = 2 - \xi(p) > 0$ (i.e. marginal revenue is strictly increasing in p).

$(1-\alpha)\pi(p;\widehat{c})$. Thus, the new relative loss from a sale is

$$\mathcal{L}(p,\widehat{c}) = \frac{(1-\alpha)[\pi(p^m(\widehat{c});\widehat{c}) - \pi(p;\widehat{c})]}{(1-\alpha)\pi(p;\widehat{c})} = \frac{\pi(p^m(\widehat{c});\widehat{c}) - \pi(p;\widehat{c})}{\pi(p;\widehat{c})},$$

which is the same as (3) with $c = \widehat{c}$.[22] From this, we can then obtain the following Proposition.

Proposition 11. *Suppose there is an ad valorem tax, $\alpha > 0$. The inverse price distribution rate of (unit) cost pass-through is $\frac{dp^*(z,\widehat{c},\sigma,n)}{d\tau} = \frac{1}{1-\alpha}\frac{\partial p^*(z,\widehat{c},\sigma,n)}{\partial c} > 0$. The inverse price distribution rate of ad valorem tax pass-through is $\frac{dp^*(z,\widehat{c},\sigma,n)}{d\alpha} = \frac{\widehat{c}}{1-\alpha}\frac{\partial p^*(z,\widehat{c},\sigma,n)}{\partial c} > 0$.*

Following the approach in Section 4, the inverse price distribution rate of (unit) cost pass-through is determined by the relative responsiveness of $\mathcal{L}(p,\widehat{c})$ to p and the unit cost, τ, and likewise for ad valorem pass-through rate. Now note that τ and α only affect $\mathcal{L}(p,\widehat{c})$ indirectly through $\widehat{c}$. Therefore, their respective pass-through rates are the products of $\frac{\partial p^*(z,c,\sigma,n)}{\partial c}$ and either $\frac{\partial \widehat{c}}{\partial \tau} = \frac{1}{1-\alpha}$ or $\frac{\partial \widehat{c}}{\partial \alpha} = \frac{\widehat{c}}{1-\alpha}$.

It follows from Proposition 11 that the pass-through rates, $\frac{dp^*(z,\widehat{c},\sigma,n)}{d\tau}$ and $\frac{dp^*(z,\widehat{c},\sigma,n)}{d\alpha}$, will have the same properties as $\frac{\partial p^*(z,\widehat{c},\sigma,n)}{\partial c}$ which have been discussed throughout the paper. This has the two following implications. First, our results relating to an increase in unit cost continue to apply in a setting with non-constant marginal costs, $\alpha > 0$. The only minor difference is that the (unit) cost pass-through rate, $\frac{dp^*(z,\widehat{c},\sigma,n)}{d\tau}$, can now be greater than one when demand is log-concave. To see this, note that when demand is log-linear, $\xi(p) = 1\ \forall p$, then $\frac{dp^*(z,\widehat{c},\sigma,n)}{d\tau} = \frac{1}{1-\alpha} > 1$ because $\frac{\partial p^*(z,\widehat{c},\sigma,n)}{\partial c} = 1\ \forall z$. Nevertheless, it is still true that the (unit) cost pass-through rate is higher across the entire price range for log-convex demand than for log-concave demand.

The second implication is that our results relating to an increase in unit cost also apply to an increase in the ad valorem tax (with the same caveat discussed above about how the pass-through rate compares to one). For brevity, consider the most empirically important results from Section 5 concerning the range of prices and captivity premium, and note that $\frac{\partial}{\partial z}\left(\frac{dp^*(z,\widehat{c},\sigma,n)}{d\alpha}\right) = \frac{\partial \widehat{c}}{\partial \alpha}\frac{\partial^2 p^*(z,\widehat{c},\sigma,n)}{\partial c \partial z}$. Then, it follows from Propositions 3 and 4 that, when demand is strictly log-concave (log-convex) with constant curvature, an increase in α will strictly decrease (increase) both the range of prices, $\frac{d\rho(\widehat{c},\sigma,n)}{d\alpha} = \frac{\partial \widehat{c}}{\partial \alpha}\frac{\partial \rho(\widehat{c},\sigma,n)}{\partial c} < (>)0$, and the captivity premium, $\frac{d\kappa(\widehat{c},\sigma,n)}{d\alpha} = \frac{\partial \widehat{c}}{\partial \alpha}\frac{\partial \kappa(\widehat{c},\sigma,n)}{\partial c} < (>)0$.

Together these implications provide empirically testable predictions, highlighting that the impact of an ad valorem (or unit) tax change depend upon the shape of demand.

[22] Equivalently, the equilibrium condition in (5) with $c = \widehat{c}$ still applies because $1-\alpha$ will cancel from both sides.

8.2 Non-Constant Curvature

Many of our results from Section 5.2.2 onwards restricted attention to demand functions with constant curvature, $\xi'(p) = 0 \ \forall p$. This section now provides conditions under which such results will apply for other demand functions with non-constant curvature.

Proposition 12. *All previous results for demand functions with constant curvature will also apply for any log-concave (log-convex) demand function with non-constant curvature provided $\xi'(p)$ is not too positive (negative) at any $p \in [c, p^m(c)]$.*

As an example of this, Proposition 13 shows that this condition is satisfied for a class of demand functions that have non-constant curvature. The defining feature of this example class is that there is a constant superelasticity of demand, given by $\psi(p) \equiv \frac{p\varepsilon'(p)}{\varepsilon(p)} \geq 0$ (that is, the elasticity of the price elasticity of demand is constant). To see how the superelasticity relates to the curvature of demand, note that by manipulating the expression of $\varepsilon'(p)$ in Assumption 1, we can obtain $\xi(p) = 1 + \left(\frac{1-\psi(p)}{\varepsilon(p)}\right)$. Thus, when demand is strictly super-elastic, $\psi(p) > 1$, it is strictly log-concave, yet when demand is super-inelastic, $\psi(p) < 1$, it is strictly log-convex. Furthermore, note that $\xi'(p) > (<)0$ when demand is strictly log-concave (log-convex), because it is easy to check that $\xi'(p) = -\frac{\varepsilon'(p)}{\varepsilon(p)}\left(\frac{1-\psi(p)}{\varepsilon(p)}\right)$ when $\psi'(p) = 0 \ \forall p$.

Proposition 13. *All previous results for demand functions with constant curvature, $\xi'(p) = 0 \ \forall p$, apply to any demand function with constant superelasticity, $\psi'(p) = 0 \ \forall p$.*

8.3 Unit Demand

In this subsection, we show how our approach can also be employed under unit demand. This form of demand is commonly assumed in much of the wider literature on price dispersion and is used as a theoretical basis in all previous empirical papers on cost pass-through under price dispersion (e.g. Montag et al., 2024; Fisher et al., 2024; and Westphal, 2024). Specifically, we now assume that consumers have a common (and known) willingness to pay of $v > c$ and a fixed inelastic demand, $q\left(p\right) = q > 0$ for any $p \leq v$ such that $\varepsilon\left(p\right) = 0$. Unlike in the main text, the monopoly price is not derived from a first-order condition, instead it follows that $p^m\left(c\right) = v$.

Proposition 14. *The inverse price distribution rate of cost pass-through under unit demand is*

$$\frac{\partial p^*(z,c,\sigma,n)}{\partial c} = \left.\frac{\frac{\partial \mathcal{L}(p,c)}{\partial c}}{-\mathcal{L}'\left(p,c\right)}\right|_{p=p^*(z,c,\sigma,n)} = \left.\frac{v-p}{v-c}\right|_{p=p^*(z,c,\sigma,n)} = 1 - \frac{1}{1+\frac{\sigma}{1-\sigma}nz^{n-1}}. \qquad (16)$$

From this, we can show that the results under unit demand are more consistent with those under log-concave demand but remain in stark contrast to those under log-convex demand. In particular, (16) implies that an increase in c will shift $F(p)$ to a greater extent towards the bottom of the distribution, $\frac{\partial^2 p^*(z,c,\sigma,n)}{\partial c \partial z} = -\frac{\partial p^*(z,c,\sigma,n)}{\partial z}\frac{1}{v-c} > 0$. Hence, the following ranking applies

$$1 > \frac{\partial \underline{p}(c,\sigma,n)}{\partial c} > \frac{\partial E\left(p_{min}\right)}{\partial c} > \frac{\partial E\left(p\right)}{\partial c} > \frac{\partial p^m\left(c\right)}{\partial c} = 0.$$

This ranking is consistent with our results in (15) under log-concave demand (except that the upper bound price is now independent of cost, $\frac{\partial p^m(c)}{\partial c} = 0$). Further, the effects of increased competition are consistent with log-concave demand as well, $\frac{\partial^2 p^*(z,c,\sigma,n)}{\partial c \partial \sigma} = -\frac{\partial p^*(z,c,\sigma,n)}{\partial \sigma}\frac{1}{v-c} > 0$ and $\frac{\partial^2 p^*(z,c,\sigma,n)}{\partial c \partial n} = -\frac{\partial p^*(z,c,\sigma,n)}{\partial n}\frac{1}{v-c}$. Hence, theoretical results that are based on the simplifying assumption of unit demand will only be appropriately applied to markets in which demand is log-concave. In contrast, any such results will be misleading if applied to markets with log-convex demand.

8.4 Search Costs

This subsection aims to show how our approach can be used to generate results in other models that exhibit price dispersion beyond Varian (1980). Here, we consider the leading framework from the consumer search literature by Stahl (1989) where consumers are able to undertake costly search to gather price information. In doing so, we show that the common simplification of unit demand applied to Stahl (1989) is highly restrictive in relation to cost pass-through.

The main difference between Stahl (1989) and Varian (1980) is that each captive is now willing to consider buying from an alternative firm. Specifically, while captives know the price of their 'captor' firm and are initially unaware of all other prices, they can learn the prices of other firms. They can do this by searching the other firms sequentially at a cost of $s > 0$ for each firm searched. Once a captive decides to stop searching, they then buy from the firm with the cheapest price they know.

The equilibrium is the same as Varian (1980) apart from the definitions of the upper and lower bound prices. In particular, the upper bound price is now given by $\overline{p} = min\left\{p^r(c,n,s), p^m(c)\right\}$, where $p^r(c,n,s)$ is the "reservation price" that equates a captive's expected marginal benefit and marginal cost of searching. Intuitively, although search is permitted, each firm optimally always prices below $\overline{p}$ to incentivise its captives to buy without searching. Therefore, each firm can guarantee profits of $\left(\frac{1-\sigma}{n}\right)\pi\left(\overline{p};c\right)$, such that $F(p)$ is determined by

$$\pi(p;c)\left[\frac{1-\sigma}{n}+\sigma(1-F(p))\right]=\left(\frac{1-\sigma}{n}\right)\pi(\overline{p};c).$$

This is the same as (5) except that now $\overline{p}$ replaces $p^m(c)$ and so the lower bound price subsequently satisfies $\pi(p;c)\left(\sigma+\frac{1-\sigma}{n}\right)=\left(\frac{1-\sigma}{n}\right)\pi(\overline{p};c)$.

When $p^r(c,n,s)\geq p^m(c)$, the equilibrium is identical to Lemma 1 and so all our results remain unaffected. However, when $p^r(c,n,s)<p^m(c)$, the equilibrium now depends on $p^r(c,n,s)=\overline{p}$. Thus, the price that sets $z=1-F(p)$ is now a function of the search cost, s, and so we denote it as $p^*(z,c,\sigma,n,s)$. While replicating our results for general demand lies out of the scope of this paper, we obtain the following.

Proposition 15. *Let $\mathcal{L}(p,c,s)=\frac{\pi(\overline{p};c)-\pi(p;c)}{\pi(p;c)}$. If $p^r(c,n,s)\geq p^m(c)$, all of our results are unchanged. If $p^r(c,n,s)<p^m(c)$, then i) the inverse price distribution rate of cost pass-through is*

$$\frac{\partial p^*(z,c,\sigma,n,s)}{\partial c}=\left[\frac{\frac{\partial\mathcal{L}(p,c,s)}{\partial c}}{\mathcal{L}'(p,c,s)}\right]_{p=p^*(z,c,\sigma,n,s)}=\left[\frac{1-\frac{p-c}{p^r(c,n,s)-c}\left(1-\frac{\partial p^r(c,n,s)}{\partial c}\left[1-\frac{p^r(c,n,s)-c}{p^r(c,s)}\varepsilon(p^r(c,n,s))\right]\right)}{1-\frac{p-c}{p}\varepsilon(p)}\right]_{p=p^*(z,c,\sigma,n,s)} \tag{17}$$

and ii) under unit demand, (17) collapses to $\frac{\partial p^*(z,c,\sigma,n,s)}{\partial c}=1$ for any $z\in[0,1]$.

Following the approach in Section 4, (17) is still determined by the relative responsiveness of $\mathcal{L}(p,c,s)$ to p and c. However, compared to (9), the numerator of the second expression in (17) now has an extra term that depends upon $\frac{\partial p^r(c,n,s)}{\partial c}$, because when deriving $\frac{\partial\mathcal{L}(p,c,s)}{\partial c}$ the envelope theorem no longer applies given $p^r(c,n,s)<p^m(c)$. Under unit demand, there is complete pass-through across the entire price range when $p^r(c,n,s)<p^m(c)$, such that $\frac{\partial p^r(c,n,s)}{\partial c}=\frac{\partial E(p)}{\partial c}=\frac{\partial E(p_{min})}{\partial c}=\frac{\partial \underline{p}(c,\sigma,n)}{\partial c}=1$. This is noted by Janssen et al. (2011) within a different application of Stahl (1989), but (17) demonstrates that it is not a general result and we can now understand the underlying reason. In particular, (17) collapses to one because $\mathcal{L}(p,c,s)$ becomes equally responsive to p and c given $\varepsilon(p)=0$ and $\frac{\partial p^r(c,n,s)}{\partial c}=1$. This demonstrates that the common assumption of unit demand within the consumer search literature is highly restrictive in relation to cost pass-through.

9 Conclusion

Understanding how cost changes are passed on to consumers through prices is fundamental for many areas of economics. However, there is limited theoretical understanding of cost pass-through within markets where prices are dispersed. Under a general demand function, we have analysed the effects of cost changes in a seminal model of price dispersion where some consumers are captive to particular sellers while others are not (Varian, 1980). To study pass-through in this mixed-strategy context, we have developed a novel approach that links well to the existing pass-through literature in pure-strategy settings. Following a cost change, we showed how the magnitudes of price changes faced by different consumer types, as well as the wider effects on price dispersion, vary depending upon whether demand is log-concave or log-convex. Specifically, we demonstrated that when demand is log-concave: i) price dispersion decreases following a cost rise, and ii) the burden of a cost increase is expected to fall less heavily on captives than non-captive consumers. In contrast, when demand is log-convex: i) prices become more dispersed, and ii) captives expect to face larger price rises than non-captives. Furthermore, we showed how our results vary with the level of competition and analysed the relationship between pass-through and demand shocks under price dispersion.

Our findings highlight the important factors in identifying which types of consumers are most affected by cost changes. This has implications for the growing interest in protecting 'vulnerable' consumers, who engage less in markets and so are more captive (e.g. OECD 2023, European Parliament 2021, CMA 2019). For instance, suppose demand is log-concave with constant curvature. If there is an increase in unit cost, then a captive consumer will experience a relatively smaller price increase and smaller reduction in consumer surplus than a non-captive consumer. However, this is reversed with respect to cost decreases. In particular, if there is a decrease in unit cost, then a captive will experience a relatively smaller price decrease and smaller increase in consumer surplus. Therefore, this suggests that policymakers should be most concerned about the distributional impact of cost changes for vulnerable consumers when costs decrease rather than increase.

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Appendix

Proof of Lemma 1. The equilibrium follows easily from Varian (1980) together with elements of Baye and Morgan (2001) to allow for downward-sloping demand. As consistent with the implications of our Assumption 1, Baye and Morgan assume that per-consumer profits are strictly increasing up to the monopoly price. Using standard arguments, one can then show that a pure-strategy pricing equilibrium cannot exist, and that the price distribution in a symmetric (mixed-strategy) equilibrium does not have any gaps or atoms. Given this, in line with the commentary in the main text, one can then verify that the proposed equilibrium is uniquely defined, with no profitable deviations. Finally, given that Assumption 1 ensures $\pi'(p;c) > 0$ for all $p \in [\underline{p}(c,\sigma,n), p^m(c))$, it follows that $\underline{p}(c,\sigma,n) \in (c, p^m(c))$ is uniquely defined, and that $F(p)$ is well-behaved with $F(\underline{p}(c,\sigma,n)) = 0$, $F(p^m(c)) = 1$, and that

$$F'(p) \quad = \quad -\frac{1}{n-1}\left[\left(\frac{1-\sigma}{\sigma n}\right)\mathcal{L}(p,c)\right]^{\frac{1}{n-1}}\frac{\mathcal{L}'(p,c)}{\mathcal{L}(p,c)} > 0 \;\forall p \in [c, p^m(c)), \tag{18}$$

since $\mathcal{L}(p,c) > 0$ and

$$\mathcal{L}'(p,c) = -\frac{\pi(p^m(c);c)}{\pi(p;c)}\frac{\pi'(p;c)}{\pi(p;c)} = -\frac{\pi(p^m(c);c)}{\pi(p;c)}\left[\frac{1}{p-c} - \frac{-q'(p)}{q(p)}\right] < 0 \;\forall p \in [c, p^m(c)). \tag{19}$$

□

Proof of Lemma 2. Denote

$$\Omega(p,z) \equiv 1 - F(p) - z = \left[\frac{1}{n}\left(\frac{1-\sigma}{\sigma}\right)\mathcal{L}(p,c)\right]^{\frac{1}{n-1}} - z, \tag{20}$$

such that $p^*(z,c,\sigma,n)$ is the level of p that sets $\Omega(p,z) = 0 \; \forall z \in [0,1]$, where $p^*(0,c,\sigma,n) = p^m(c)$ and $p^*(1,c,\sigma,n) = \underline{p}(c,\sigma,n)$. Then note that $p^*(z,c,\sigma,n)$ exists and is unique, satisfying $p^*(z,c,\sigma,n) \in \left(\underline{p}(c,\sigma,n), p^m(c)\right) \; \forall z \in (0,1)$, because i) $\Omega(p^m(c), z) = -z < 0$, ii) $\Omega\left(\underline{p}(c,\sigma,n), z\right) = 1 - z > 0$, and iii) $\Omega'(p,z) = -F'(p) < 0 \; \forall p < p^m(c)$ from (18). Applying the implicit function theorem to $\Omega(p,z) = 0$ with $\frac{\partial \Omega(p,z)}{\partial z} = -1$ shows $\frac{\partial p^*(z,c,\sigma,n)}{\partial z} = \frac{1}{\Omega'(p^*(z,c,\sigma,n),z)} = \frac{1}{-F'(p)} < 0 \; \forall z \in [0,1)$. For $z = 0$, given that $p^*(0,c,\sigma,n) = p^m(c)$ and that $p^*(z,c,\sigma,n) < p^m(c) \; \forall z > 0$, it must be that $\lim_{z \to 0} \frac{\partial p^*(z,c,\sigma,n)}{\partial z} \leq 0$. □

Proof of Proposition 1. Using the implicit function theorem on $\Omega(p,z) = 0$ from (20) yields

$$\frac{\partial p^*(z,c,\sigma,n)}{\partial c} = -\left.\frac{1}{\Omega'(p)} \frac{\partial \Omega(p,z)}{\partial c}\right|_{p=p^*(z,c,\sigma,n)} = \left.\frac{-\frac{\partial F(p)}{\partial c}}{F'(p)}\right|_{p=p^*(z,c,\sigma,n)} \tag{21}$$

where $F'(p)$ is given by (18) and differentiating (4) with respect to c yields

$$\frac{\partial F(p)}{\partial c} = -\frac{1}{n-1}\left[\left(\frac{1-\sigma}{\sigma n}\right)\mathcal{L}(p,c)\right]^{\frac{1}{n-1}} \frac{1}{\mathcal{L}(p,c)} \frac{\partial \mathcal{L}(p,c)}{\partial c} \tag{22}$$

Substituting (18) and (22) into (21) confirms that the first term in (9) is as claimed, where $\frac{\partial p^*(z,c,\sigma,n)}{\partial c} > 0 \; \forall z > 0$, as $\mathcal{L}'(p) < 0 \;\; \forall p \in [c, p^m(c))$ from (19) and differentiating (3) with respect to c yields

$$\begin{aligned} \frac{\partial \mathcal{L}(p,c)}{\partial c} &= \frac{\pi(p^m(c);c)}{\pi(p;c)}\left[\frac{1}{\pi(p^m(c);c)}\frac{\partial \pi(p^m(c),c)}{\partial c} - \frac{1}{\pi(p;c)}\frac{\partial \pi(p,c)}{\partial c}\right] \\ &= \frac{\pi(p^m(c);c)}{\pi(p;c)}\left[\frac{1}{p-c} - \frac{1}{p^m(c)-c}\right] > 0, \;\; \forall p \in [c, p^m(c)). \end{aligned} \tag{23}$$

Finally, substituting (19) and (23) into the first term in (9) and manipulating obtains the second term in (9). For $z = 0$, note that the numerator and denominator of the second term in (9) equal zero when p is evaluated at $p^m(c)$. Thus, applying L'Hôpital's rule yields $\lim_{z \to 0} \frac{\partial p^*(z,c,\sigma,n)}{\partial c} = \frac{1}{2-\xi(p^m(c))} > 0$, because $\left(\frac{p-c}{p^m(c)-c}\right)' = \frac{1}{p^m(c)-c} = \frac{-q'(p^m(c))}{q(p^m(c))} > 0$ from the Lerner index and $\left.\left(\frac{p-c}{p}\varepsilon(p)\right)'\right|_{p=p^m(c)} = \frac{-q'(p^m(c))}{q(p^m(c))}\left[2 - \xi(p^m(c))\right] > 0$ given $\xi(p^m(c)) < 1 + \frac{1}{\varepsilon(p^m(c))} < 2$. □

Proof of Lemma 3. First, we change the variables of (7) and (8) from p to z. Specifically, differentiating $z = 1 - F(p)$ with respect to p and manipulating yields $dz = -F'(p)dp$. Furthermore, given $1 - F(p^m(c)) = 0 \equiv z(p^m(c))$ and $1 - F(\underline{p}(c,\sigma,n)) = 1 \equiv z(\underline{p}(c,\sigma,n))$, we can rewrite (7) and (8) as

$$E(p)=\int_{\underline{p}(c,\sigma,n)}^{p^m} pF'(p)dp=-\int_{z(\underline{p}(c,\sigma,n))=1}^{z(p^m(c))=0} p^*(z,c,\sigma,n)dz \tag{24}$$

and

$$E\left(p_{min}\right)=\int_{\underline{p}(c,\sigma,n)}^{p^m(c)} pn\left(1-F\left(p\right)\right)^{n-1}F'(p)dp=-\int_{z(\underline{p}(c,\sigma,n))=1}^{z(p^m(c))=0} p^*(z,c,\sigma,n)nz^{n-1}dz \tag{25}$$

Finally, differentiating both (24) and (25) with respect to c obtains (12) and (13). □

Proof of Proposition 2. First, given $\lim_{z\to 0}\frac{\partial p^*(z,c,\sigma,n)}{\partial c}=\frac{1}{2-\xi(p^m(c))}$, note that $\lim_{z\to 0}\frac{\partial p^*(z,c,\sigma,n)}{\partial c}\lesseqqgtr 1\iff \xi(p^m(c))\gtreqqless 1$. Next, for any $z>0$, we can manipulate the second expression in (9) to obtain

$$\frac{\partial p^*(z,c,\sigma,n)}{\partial c} = 1-(p-c)\left(\frac{\frac{-q'(p^m(c))}{q(p^m(c))}-\frac{-q'(p)}{q(p)}}{1-\frac{p-c}{p}\varepsilon(p)}\right)\Bigg|_{p=p^*(z,c,\sigma,n)} \tag{26}$$

where $\frac{1}{p^m(c)-c}=\frac{-q'(p^m(c))}{q(p^m(c))}$ from the Lerner index. Given $\frac{p-c}{p}\varepsilon(p)<1\ \forall p<p^m(c)$, it follows from the above that whether (9) is less or greater than one is determined by the sign of $\frac{-q'(p^m(c))}{q(p^m(c))}-\frac{-q'(p)}{q(p)}$. Thus, the proof is completed by noting that $\lim_{p\to p^m(c)}\left[\frac{-q'(p^m(c))}{q(p^m(c))}-\frac{-q'(p)}{q(p)}\right]=0$ and $-\left(\frac{q'(p)}{q(p)}\right)'=-(\ln q(p))''=(1-\xi(p))\left(\frac{q'(p)}{q(p)}\right)^2$. So, if demand is strictly log-concave (log-convex), $\xi(p)<(>)1\ \forall p$, then (9) is strictly less (greater) than one, because $\frac{-q'(p^m(c))}{q(p^m(c))}>(<)\frac{-q'(p)}{q(p)}\ \forall p<p^m(c)$ as $\frac{-q'(p)}{q(p)}$ strictly decreases (increases) as p falls away from $p^m\left(c\right)$. In terms of the first expression of (9), this ensures that (19) is greater (smaller) than (23). When demand is log-linear, $\xi(p)=1\ \forall p$, (9) equals one because $\frac{-q'(p^m(c))}{q(p^m(c))}=\frac{-q'(p)}{q(p)}\ \forall p$, such that (19) equals (23). □

Proof of Proposition 3. Differentiating the second expression of $\frac{\partial p^*(z,c,\sigma,n)}{\partial c}$ in (9) with respect to z yields

$$\begin{aligned}\frac{\partial^2 p^*(z,c,\sigma,n)}{\partial c\partial z} &= -\frac{\partial p^*(z,c,\sigma,n)}{\partial z}\left[\frac{\frac{1}{p^m(c)-c}\left(1-\frac{p-c}{p}\varepsilon(p)\right)-\left(1-\frac{p-c}{p^m(c)-c}\right)\left(\frac{p-c}{p}\varepsilon(p)\right)'}{\left(1-\frac{p-c}{p}\varepsilon(p)\right)^2}\right]_{p=p^*(z,c,\sigma,n)}\\ &= -\frac{\partial p^*(z,c,\sigma,n)}{\partial z}\frac{\partial p^*(z,c,\sigma,n)}{\partial c}\left[\frac{1-\frac{p-c}{p}\varepsilon(p)-(p^m(c)-p)\left(\frac{p-c}{p}\varepsilon(p)\right)'}{(p^m(c)-p)\left(1-\frac{p-c}{p}\varepsilon(p)\right)}\right]_{p=p^*(z,c,\sigma,n)}\end{aligned} \tag{27}$$

We want to sign (27) $\forall z$. Given the signs of $\frac{\partial p^*(z,c,\sigma,n)}{\partial z}$ and $\frac{\partial p^*(z,c,\sigma,n)}{\partial c}$ are known, we need to sign the term in square brackets. (This term is equivalent to the difference between $-\frac{\partial \mathcal{L}'(p,c)}{\partial c} / \frac{\partial \mathcal{L}(p,c)}{\partial c}$ and $\frac{\mathcal{L}''(p,c)}{-\mathcal{L}'(p,c)}$, as can be seen by differentiating the first expression in (9) with respect to z). However, from inspection, the sign of this term is not always clear. For instance, while its denominator is strictly positive $\forall p < p^m(c)$, substituting (2) into the numerator yields, for any given p,

$$1 - (p^m(c) - c)\frac{-q'(p)}{q(p)} - (p^m(c) - p)\frac{-q'(p)}{q(p)}\left(\frac{p-c}{p}\varepsilon(p)\right)(1 - \xi(p)) \ . \tag{28}$$

Thus, when demand is strictly log-concave (log-convex), $\xi(p) < (>) 1 \ \forall p$, the sign of (28) is unclear $\forall p \in (c, p^m(c))$, because $\xi(p) < (>) 1 \ \forall p$ guarantees that the third term will be subtracted from (added to) the first two terms, which combined are strictly positive (negative) given $p^m(c) - c = 1 / \frac{-q'(p^m(c))}{q(p^m(c))}$ and $-\left(\frac{q'(p)}{q(p)}\right)' = -(\ln q(p))'' > (<) 0$.

Towards signing the term in square brackets in (27), first note that its numerator is always zero at $p = p^m(c)$. Thus, it follows that the numerator will be strictly positive (negative) $\forall p < p^m(c)$ if it is strictly decreasing (increasing) in p. The derivative of this numerator with respect to p is $-(p^m(c) - p)\left(\frac{p-c}{p}\varepsilon(p)\right)''$, where

$$\left(\frac{p-c}{p}\varepsilon(p)\right)'' = \frac{-q'(p)}{q(p)}\left[2\left(\frac{p-c}{p}\varepsilon(p)\right)'(1 - \xi(p)) - \frac{p-c}{p}\varepsilon(p)\xi'(p)\right]. \tag{29}$$

Hence, given Assumption 1 guarantees $\left(\frac{p-c}{p}\varepsilon(p)\right)' > 0 \ \forall c > 0$, the sign of (29) is determined by the sign of $1 - \xi(p)$ when demand curvature is constant, $\xi'(p) = 0$. This implies that if $\xi < (>) 1 \ \forall p$, then the numerator of (27) is strictly positive (negative) $\forall p < p^m(c)$. Thus, $\frac{\partial^2 p^*(z,c,\sigma,n)}{\partial c \partial z} > (<) 0 \ \forall z \in (0, 1]$ when $\xi < (>) 1 \ \forall p$ given $\frac{\partial p^*(z,c,\sigma,n)}{\partial c} > 0$ and $\frac{\partial p^*(z,c,\sigma,n)}{\partial z} < 0$.

To sign (27) at $z = 0$, note that the term in the square brackets in (27) is an indeterminate form when evaluated at $p = p^m(c)$, because both the numerator and denominator equal zero. So, it follows from L'Hôpital's rule that

$$\lim_{p \to p^m(c)}\left[\frac{1 - \frac{p-c}{p}\varepsilon(p) - (p^m(c)-p)\left(\frac{p-c}{p}\varepsilon(p)\right)'}{(p^m(c)-p)\left(1 - \frac{p-c}{p}\varepsilon(p)\right)}\right] = \lim_{p \to p^m(c)}\left[\frac{-(p^m(c)-p)\left(\frac{p-c}{p}\varepsilon(p)\right)''}{-\left(1 - \left(\frac{p-c}{p}\varepsilon(p)\right)\right) - (p^m(c)-p)\left(\frac{p-c}{p}\varepsilon(p)\right)'}\right]$$

As this is still an indeterminate form, applying L'Hôpital's rule again yields

$$\begin{aligned}\lim_{p \to p^m(c)}\left[\frac{1 - \frac{p-c}{p}\varepsilon(p) - (p^m(c)-p)\left(\frac{p-c}{p}\varepsilon(p)\right)'}{(p^m(c)-p)\left(1 - \frac{p-c}{p}\varepsilon(p)\right)}\right] &= \lim_{p \to p^m(c)}\left[\frac{\left(\frac{p-c}{p}\varepsilon(p)\right)'' - (p^m(c)-p)\left(\frac{p-c}{p}\varepsilon(p)\right)'''}{2\left(\frac{p-c}{p}\varepsilon(p)\right)' - (p^m(c)-p)\left(\frac{p-c}{p}\varepsilon(p)\right)''}\right] \\ &= \lim_{p \to p^m(c)}\left[\frac{\left(\frac{p-c}{p}\varepsilon(p)\right)''}{2\left(\frac{p-c}{p}\varepsilon(p)\right)'}\right]\end{aligned} \tag{30}$$

Thus, given $\left(\frac{p-c}{p}\varepsilon(p)\right)' > 0 \; \forall c > 0$ from Assumption 1, (30) confirms that as $p \to p^m(c)$ the sign of the square brackets in (27) is also determined by the sign of (29). Then given $\lim_{z\to 0}\frac{\partial p^*(z,c,\sigma,n)}{\partial c} > 0$ and $\lim_{z\to 0}\frac{\partial p^*(z,c,\sigma,n)}{\partial z} \leq 0$, it follows that $\lim_{z\to 0}\frac{\partial^2 p^*(z,c,\sigma,n)}{\partial c \partial z} \geq (\leq) 0$ when $\xi < (>) 1 \; \forall p$. □

Proof of Proposition 4. First, note that the effects on the captivity premium and range of prices follows trivially from the ranking in (15). Thus, the rest of the proof establishes that the ranking in (15) follows when $\frac{\partial^2 p^*(z,c,\sigma,n)}{\partial c \partial z}$ has the same sign $\forall z > 0$. We already know from Proposition 3 that sign$\left\{\frac{\partial p^*(z,c,\sigma,n)}{\partial c \partial z}\right\}$ = sign$\{1-\xi\}$ $\forall z > 0$ such that $\frac{\partial^2 p(z,c,\sigma,n)}{\partial c \partial z} > (<) 0$ $\forall z > 0$ when demand is strictly log-concave (log-convex) with constant curvature.

Let us first compare the pass-through rates of the expected price with the upper bound price. Given $\int_0^1 1dz = 1$, it follows from Lemma 3 that we can write

$$\frac{\partial p^m(c)}{\partial c} - \frac{\partial E(p)}{\partial c} = \int_0^1 \left(\lim_{z\to 0}\frac{\partial p^*(z,c,\sigma,n)}{\partial c} - \frac{\partial p^*(z,c,\sigma,n)}{\partial c}\right) dz \tag{31}$$

Next, we compare the pass-through rate of the expected minimum price and the lower bound. Given $\int_0^1 nz^{n-1}dz = 1$, we can write

$$\frac{\partial E(p_{min})}{\partial c} - \frac{\partial \underline{p}(c,\sigma,n)}{\partial c} = \int_0^1 nz^{n-1}\left(\frac{\partial p^*(z,c,\sigma,n)}{\partial c} - \frac{\partial p^*(1,c,\sigma,n)}{\partial c}\right) dz \tag{32}$$

Thus, it is trivial to see that (31) and (32) have the opposite sign of sign$\left\{\frac{\partial p^*(z,c,\sigma,n)}{\partial c \partial z}\right\}$ = sign$\{1-\xi\}$ $\forall z > 0$.

Next, compare the pass-through rates of the expected price and the expected minimum price. We first show that if $\frac{\partial^2 p^*(z,c,\sigma,n)}{\partial c \partial z} > 0 \; \forall z > 0$, such that $\xi < 1$, then $\frac{\partial E(p_{min})}{\partial c} > \frac{\partial E(p)}{\partial c}$. To prove this, let $\overline{z} \equiv \left(\frac{1}{n}\right)^{\frac{1}{n-1}} > 0$ and note from (12) and (13) that

$$\begin{aligned}
\frac{\partial\left[E(p) - E(p_{min})\right]}{\partial c} &= \int_0^1 \frac{\partial p^*(z,c,\sigma,n)}{\partial c}\left(1 - nz^{n-1}\right) dz \\
&= \int_0^{\overline{z}} \frac{\partial p^*(z,c,\sigma,n)}{\partial c}\left(1 - nz^{n-1}\right) dz - \int_{\overline{z}}^1 \frac{\partial p^*(z,c,\sigma,n)}{\partial c}\left(nz^{n-1} - 1\right) dz \\
&< \int_0^{\overline{z}} \frac{\partial p^*(\overline{z},c,\sigma,n)}{\partial c}\left(1 - nz^{n-1}\right) dz - \int_{\overline{z}}^1 \frac{\partial p^*(\overline{z},c,\sigma,n)}{\partial c}\left(nz^{n-1} - 1\right) dz \\
&= \frac{\partial p^*(\overline{z},c,\sigma,n)}{\partial c}\int_0^1 \left(1 - nz^{n-1}\right) dz = 0
\end{aligned}$$

Thus, if $\frac{\partial^2 p^*(z,c,\sigma,n)}{\partial c \partial z} > 0 \; \forall z > 0$, then $\frac{\partial E(p_{min})}{\partial c} > \frac{\partial E(p)}{\partial c}$. Instead, if $\frac{\partial^2 p^*(z,c,\sigma,n)}{\partial c \partial z} < 0 \; \forall z > 0$, such that $\xi > 1$, repeating the above analysis yields $\frac{\partial E(p_{min})}{\partial c} < \frac{\partial E(p)}{\partial c}$. $\square$

Proof of Proposition 5. Denote the per-consumer surplus at a given p be $s(p) = \int_p^{\hat{p}} q(x)dx$, where $s'(p) = -q(p)$. By changing the variables, we can write the expected per-consumer surplus of each captive and shopper as, respectively,

$$CS^K \equiv \int_{\underline{p}(c,\sigma,n)}^{p^m(c)} s(p)F'(p)dp = \int_0^1 s(p^*(z,c,\sigma,n))dz \tag{33}$$

$$CS^S \equiv \int_{\underline{p}(c,\sigma,n)}^{p^m(c)} s(p)n(1-F(p))^{n-1}F'(p)dp = \int_0^1 s(p^*(z,c,\sigma,n))nz^{n-1}dz \tag{34}$$

Differentiating with respect to c yields

$$\frac{\partial CS^K}{\partial c} = -\int_0^1 \frac{\partial p^*(z,c,\sigma,n)}{\partial c} q(p^*(z,c,\sigma,n))dz < 0$$

$$\frac{\partial CS^S}{\partial c} = -\int_0^1 \frac{\partial p^*(z,c,\sigma,n)}{\partial c} q(p^*(z,c,\sigma,n))nz^{n-1}dz < 0$$

We want to sign $\frac{\partial CS^K}{\partial c} - \frac{\partial CS^S}{\partial c}$. We first show that if $\frac{\partial}{\partial z}\left(\frac{\partial p^*(z,c,\sigma,n)}{\partial c} q(p^*(z,c,\sigma,n))\right) > 0$ $\forall z > 0$, then $0 > \frac{\partial CS^K}{\partial c} > \frac{\partial CS^S}{\partial c}$, such that $\frac{\partial CS^K}{\partial c} - \frac{\partial CS^S}{\partial c} > 0$. To prove this, recall that $\overline{z} \equiv \left(\frac{1}{n}\right)^{\frac{1}{n-1}} > 0$ and suppose $\frac{\partial}{\partial z}\left(\frac{\partial p^*(z,c,\sigma,n)}{\partial c} q(p^*(z,c,\sigma,n))\right) > 0$ $\forall z$. It then follows that $\frac{\partial [CS^K - \partial CS^S]}{\partial c}$ equals

$$\begin{aligned}
& -\int_0^1 \frac{\partial p^*(z,c,\sigma,n)}{\partial c} q(p^*(z,c,\sigma,n))\left(1-nz^{n-1}\right)dz \\
= & -\left[\int_0^{\overline{z}} \frac{\partial p^*(z,c,\sigma,n)}{\partial c} q(p^*(z,c,\sigma,n))\left(1-nz^{n-1}\right)dz - \int_{\overline{z}}^1 \frac{\partial p^*(z,c,\sigma,n)}{\partial c} q(p^*(z,c,\sigma,n))\left(nz^{n-1}-1\right)dz\right] \\
> & -\left[\int_0^{\overline{z}} \frac{\partial p^*(\overline{z},c,\sigma,n)}{\partial c} q(p^*(\overline{z},c,\sigma,n))\left(1-nz^{n-1}\right)dz - \int_{\overline{z}}^1 \frac{\partial p^*(\overline{z},c,\sigma,n)}{\partial c} q(p^*(\overline{z},c,\sigma,n))\left(nz^{n-1}-1\right)dz\right] \\
= & -\frac{\partial p^*(\overline{z},c,\sigma,n)}{\partial c} q(p^*(\overline{z},c,\sigma,n)) \int_0^1 \left(1-nz^{n-1}\right)dz = 0
\end{aligned}$$

Thus, if $\frac{\partial}{\partial z}\left(\frac{\partial p^*(z,c,\sigma,n)}{\partial c} q(p^*(z,c,\sigma,n))\right) > 0$ $\forall z$, then $\frac{\partial CS^K}{\partial c} > \frac{\partial CS^S}{\partial c}$. Repeating the above analysis for $\frac{\partial}{\partial z}\left(\frac{\partial p^*(z,c,\sigma,n)}{\partial c} q(p^*(z,c,\sigma,n))\right) < 0$ $\forall z$ yields $\frac{\partial CS^K}{\partial c} < \frac{\partial CS^S}{\partial c}$. Likewise, it follows that if $\frac{\partial}{\partial z}\left(\frac{\partial p^*(z,c,\sigma,n)}{\partial c} q(p^*(z,c,\sigma,n))\right) = 0$ $\forall z$, then $\frac{\partial CS^K}{\partial c} = \frac{\partial CS^S}{\partial c}$.

Next, we need to find the conditions under which $\frac{\partial p^*(z,c,\sigma,n)}{\partial c} q(p^*(z,c,\sigma,n))$ is monotonic in $z > 0$. Differentiating $\frac{\partial p^*(z,c,\sigma,n)}{\partial c} q(p^*(z,c,\sigma,n))$ with respect to z yields

$$\frac{\partial^2 p^*(z,c,\sigma,n)}{\partial c \partial z} q(p^*(z,c,\sigma,n)) + \frac{\partial p^*(z,c,\sigma,n)}{\partial c}\frac{\partial p^*(z,c,\sigma,n)}{\partial z} q'(p^*(z,c,\sigma,n)) \tag{35}$$

The second term in (35) is strictly positive given $\frac{\partial p^*(z,c,\sigma,n)}{\partial c} > 0$, $\frac{\partial p^*(z,c,\sigma,n)}{\partial z} < 0$ and $q'(p^*(z,c,\sigma,n)) < 0\ \forall z > 0$. Thus, it follows from the first term in (35) that $\frac{\partial^2 p^*(z,c,\sigma,n)}{\partial c \partial z} > 0$ is a sufficient condition to guarantee $\frac{\partial}{\partial z}\left(\frac{\partial p^*(z,c,\sigma,n)}{\partial c} q(p^*(z,c,\sigma,n))\right) > 0\ \forall z > 0$. This, together with Proposition 3, implies that $\frac{\partial CS^K}{\partial c} > \frac{\partial CS^S}{\partial c}$ when demand is (weakly) log-concave $\left(\frac{\partial^2 p^*(z,c,\sigma,n)}{\partial c \partial z} \geq 0\ \forall z > 0\right)$ with constant curvature. In contrast, when demand is strictly log-convex with constant curvature $\left(\frac{\partial^2 p^*(z,c,\sigma,n)}{\partial c \partial z} < 0\ \forall z\right)$, $\frac{\partial CS^K}{\partial c} > \frac{\partial CS^S}{\partial c}$ if $\frac{\partial^2 p^*(z,c,\sigma,n)}{\partial c \partial z}$ is sufficiently close to $0\ \forall z > 0$. $\square$

Proof of Proposition 6. Note that $\mathcal{L}(p,c) \equiv \frac{\pi(p^m(c);c)}{\pi(p;c)}$ is the only term in (4) that is a function of p and a. Thus, it follows that

$$\frac{\partial p^*(z,c,\sigma,n)}{\partial a} = \left.\frac{\frac{\partial \mathcal{L}(p,c)}{\partial a}}{-\mathcal{L}'(p,c)}\right|_{p=p^*(z,c,\sigma,n)}$$

where $\mathcal{L}'(p,c)$ is given in (19) and

$$\begin{aligned}\frac{\partial \mathcal{L}(p,c)}{\partial a} &= \frac{\pi(p^m(c);c)}{\pi(p;c)}\left[\frac{1}{\pi(p^m(c);c)}\frac{\partial \pi(p^m(c);c)}{\partial a} - \frac{1}{\pi(p;c)}\frac{\partial \pi(p,c)}{\partial a}\right] \\ &= \frac{\pi(p^m(c);c)}{\pi(p;c)}\left[\frac{1}{q(p^m(c))}\frac{\partial q(p^m(c))}{\partial a} - \frac{1}{q(p)}\frac{\partial q(p)}{\partial a}\right]\end{aligned} \quad (36)$$

Substituting in (19) and (36) and manipulating yields

$$\frac{\partial p^*(z,c,\sigma,n)}{\partial a} = \left.\frac{(p-c)\left(\frac{1}{q(p^m(c))}\frac{\partial q(p^m(c))}{\partial a} - \frac{1}{q(p)}\frac{\partial q(p)}{\partial a}\right)}{1 - \frac{p-c}{p}\varepsilon(p)}\right|_{p=p^*(z,c,\sigma,n)}.$$

Given $\frac{\partial q(p)}{\partial a} = -q'(p)\ \forall p$, it follows from (26) that $\frac{\partial p^*(z,c,\sigma,n)}{\partial a} = 1 - \frac{\partial p^*(z,c,\sigma,n)}{\partial c}\ \forall z$. $\square$

Proof of Proposition 7. First, consider $s(p^*(z,c,\sigma,n)+\tau)$ that represents the per-consumer surplus for an effective price of $p^*(z,c,\sigma,n)+\tau$ for a given z. Differentiating $s(p^*(z,c,\sigma,n)+\tau)$ with respect to c and τ yields, respectively,

$$\frac{ds(p^*(z,c,\sigma,n)+\tau)}{dc} = \frac{\partial p^*(z,c,\sigma,n)}{\partial c}s'(p^*(z,c,\sigma,n)+\tau) = -\frac{\partial p^*(z,c,\sigma,n)}{\partial c}q(p^*(z,c,\sigma,n)+\tau)$$

and

$$\frac{ds(p^*(z,c,\sigma,n)+\tau)}{d\tau} = \left(\frac{\partial p^*(z,c,\sigma,n)}{\partial \tau}+1\right)s'(p^*(z,c,\sigma,n)+\tau) = -\left(\frac{\partial p^*(z,c,\sigma,n)}{\partial \tau}+1\right)q(p^*(z,c,\sigma,n)+\tau)$$

Thus, given an increase in τ is equivalent to a unit parallel downward shift in demand, it follows that $\frac{\partial p^*(z,c,\sigma,n)}{\partial \tau} = -\frac{\partial p^*(z,c,\sigma,n)}{\partial a} = -\left(1 - \frac{\partial p^*(z,c,\sigma,n)}{\partial c}\right)$, so substituting in yields $\frac{ds(p^*(z,c,\sigma,n)+\tau)}{d\tau} = -\frac{ds(p^*(z,c,\sigma,n)+\tau)}{da} = \frac{ds(p^*(z,c,\sigma,n)+\tau)}{dc}$. Then it follows from (33) that $\frac{\partial CS^K}{\partial \tau} = -\frac{\partial CS^K}{\partial a} = \frac{\partial CS^K}{\partial c}$ and from (34) that $\frac{\partial CS^S}{\partial \tau} = -\frac{\partial CS^S}{\partial a} = \frac{\partial CS^S}{\partial c}$.

Finally, the effect on the sum of firms' equilibrium profits, $\Pi^N(c,\sigma,n) = (1-\sigma)\pi(p^m(c);c)$, operates only through the increase in demand (from the envelop theorem). Thus,

$$\frac{d\Pi^N(c,\sigma,n)}{da} = (1-\sigma)(p^m(c)-c)\frac{\partial q(p^m(c))}{\partial a} = (1-\sigma)q(p^m(c)) \equiv -\frac{d\Pi^N(c,\sigma,n)}{dc}$$

as $\frac{\partial q(p^m(c))}{\partial a} = -q'(p^m(c))$ and $p^m(c) - c = \frac{q(p^m(c))}{-q'(p^m(c))}$ from the Lerner index. □

Proof of Proposition 8. Differentiating the second expression of $\frac{\partial p^*(z,c,\sigma,n)}{\partial c}$ in (9) with respect to σ yields

$$\begin{aligned}\frac{\partial^2 p^*(z,c,\sigma,n)}{\partial c \partial \sigma} &= -\frac{\partial p^*(z,c,\sigma,n)}{\partial \sigma}\left[\frac{\frac{1}{p^m(c)-c}\left(1-\frac{p-c}{p}\varepsilon(p)\right)-\left(1-\frac{p-c}{p^m(c)-c}\right)\left(\frac{p-c}{p}\varepsilon(p)\right)'}{\left(1-\frac{p-c}{p}\varepsilon(p)\right)^2}\right]_{p=p^*(z,c,\sigma,n)} \\ &= -\frac{\partial p^*(z,c,\sigma,n)}{\partial \sigma}\frac{\partial p^*(z,c,\sigma,n)}{\partial c}\left[\frac{1-\frac{p-c}{p}\varepsilon(p)-(p^m(c)-p)\left(\frac{p-c}{p}\varepsilon(p)\right)'}{(p^m(c)-p)\left(1-\frac{p-c}{p}\varepsilon(p)\right)}\right]_{p=p^*(z,c,\sigma,n)} \quad (37)\end{aligned}$$

In the proof of Proposition 3, we showed that the term in square brackets is strictly positive (negative) when $\xi(p) = \xi < (>) 1\ \forall p$, because the sign of (29) is determined by the sign of $1 - \xi(p)$. Thus, given $\frac{\partial p^*(z,c,\sigma,n)}{\partial c} > 0\ \forall z > 0$, it remains to prove that $\frac{\partial p^*(z,c,\sigma,n)}{\partial \sigma} < 0\ \forall z > 0$ to show that the sign of (37) is as claimed. Applying the implicit function theorem to (20) yields

$$\frac{\partial p^*(z,c,\sigma,n)}{\partial \sigma} = -\left.\frac{\partial F(p)}{\partial \sigma}\frac{1}{F'(p)}\right|_{p=p^*(z,c,\sigma,n)} = -\left.\frac{\mathcal{L}(p,c)}{\sigma(1-\sigma)(-\mathcal{L}'(p,c))}\right|_{p=p^*(z,c,\sigma,n)}$$

Then it follows from $\mathcal{L}(p,c) > 0$ and $\mathcal{L}'(p,c) < 0$ from (19) $\forall p \in [c, p^m(c))$ that $\frac{\partial p^*(z,c,\sigma,n)}{\partial \sigma} < 0$ $\forall z \in (0,1]$. □

Proof of Proposition 9. Differentiating the second expression of $\frac{\partial p^*(z,c,\sigma,n)}{\partial c}$ in (9) with respect to n yields

$$\begin{aligned}\frac{\partial^2 p^*(z,c,\sigma,n)}{\partial c \partial n} &= -\frac{\partial p^*(z,c,\sigma,n)}{\partial n}\left[\frac{\frac{1}{p^m(c)-c}\left(1-\frac{p-c}{p}\varepsilon(p)\right)-\left(1-\frac{p-c}{p^m(c)-c}\right)\left(\frac{p-c}{p}\varepsilon(p)\right)'}{\left(1-\frac{p-c}{p}\varepsilon(p)\right)^2}\right]_{p=p^*(z,c,\sigma,n)} \\ &= -\frac{\partial p^*(z,c,\sigma,n)}{\partial n}\frac{\partial p^*(z,c,\sigma,n)}{\partial c}\left[\frac{1-\frac{p-c}{p}\varepsilon(p)-(p^m(c)-p)\left(\frac{p-c}{p}\varepsilon(p)\right)'}{(p^m(c)-p)\left(1-\frac{p-c}{p}\varepsilon(p)\right)}\right]_{p=p^*(z,c,\sigma,n)} \quad (38)\end{aligned}$$

In the proof of Proposition 3, we showed that the term in square brackets is strictly positive (negative) when $\xi(p) = \xi < (>)1$ $\forall p$, because the sign of (29) is determined by the sign of $1 - \xi(p)$. Thus, given $\frac{\partial p^*(z,c,\sigma,n)}{\partial c} > 0$ $\forall z$, it remains to sign $\frac{\partial p^*(z,c,\sigma,n)}{\partial n}$ to show that the sign of (38) is as claimed. Applying the implicit function theorem to (20) yields

$$\frac{\partial p^*(z,c,\sigma,n)}{\partial n} = - \left. \frac{\partial F(p)}{\partial n} \frac{1}{F'(p)} \right|_{p=p^*(z,c,\sigma,n)} = \left. \frac{\mathcal{L}(p,c)}{\mathcal{L}'(p,c)} \left[\frac{1}{n} + \frac{1}{n-1} \ln \left((1-F(p))^{n-1} \right) \right] \right|_{p=p^*(z,c,\sigma,n)}$$

where $(1-F(p))^{n-1} \in (0,1]$ $\forall p \in [\underline{p}(c,\sigma,n), p^m(c))$. Given that the $\mathcal{L}(p,c) > 0$ and $\mathcal{L}'(p,c) < 0$ from (19) $\forall p \in [c, p^m(c))$, the sign of $\frac{\partial p^*(z,c,\sigma,n)}{\partial n}$ has the opposite sign of the term in square brackets. To sign this, let z^* denote the level of z that sets the term in square bracket equals zero such that $(1 - F(p^*(z,c,\sigma,n)))^{n-1} = e^{-\frac{n-1}{n}}$, where $z^* \in (0,1)$ as $e^{-\frac{n-1}{n}} \in (0,1)$. Then if $z > (<) z^*$ such that $(1 - F(p^*(z,c,\sigma,n)))^{n-1} > (<) e^{-\frac{n-1}{n}}$, it follows that the term in square brackets is strictly positive (negative) so $\frac{\partial p^*(z,c,\sigma,n)}{\partial n} < (>)0$. Hence, if $\xi(p) = \xi < (>)1$ $\forall p$, then $\frac{\partial^2 p^*(z,c,\sigma,n)}{\partial c \partial n} > (<)0$ for any $z > z^*$ yet $\frac{\partial^2 p^*(z,c,\sigma,n)}{\partial c \partial n} < (>)0$ for any $z < z^*$. □

Proof of Proposition 10. Integrating (7) by parts, with $u = p$ and $dv = F'(p)dp$, yields $E(p) = p^m(c) - \int_{\underline{p}(c,\sigma,n)}^{p^m(c)} F(p)dp$. Then, since $\lim_{n\to\infty} F(p) = 0$ from (4) and $\lim_{n\to\infty} \underline{p}(c,\sigma,n) = c$ from the definition of $\underline{p}(c,\sigma,n)$ in Lemma 1, it follows that $\lim_{n\to\infty} E(p) = p^m(c)$ such that $\lim_{n\to\infty} \frac{\partial E(p)}{\partial c} = \frac{\partial p^m(c)}{\partial c}$. Similarly, integrating (8) by parts, where in this case $u = p$ and $dv = n(1 - F(p))^{n-1} F'(p)\, dp$, yields $E(p_{min}) = \underline{p}(c,\sigma,n) + \int_{\underline{p}(c,\sigma,n)}^{p^m(c)} (1 - F(p))^n \, dp$. Then, since $\lim_{n\to\infty} (1 - F(p))^n = \lim_{n\to\infty} \left(\frac{1}{n} \left(\frac{1-\sigma}{\sigma} \right) \mathcal{L}(p,c) \right)^{\frac{n}{n-1}} = 0$ from (4), it follows that $\lim_{n\to\infty} E(p_{min}) = \lim_{n\to\infty} \underline{p}(c,\sigma,n) = c$ such that $\lim_{n\to\infty} \frac{\partial E(p_{min})}{\partial c} = 1$. □

Proof of Proposition 11. Using the implicit function theorem on (20) with $c = \widehat{c} = \frac{\tau}{1-\alpha}$ for some parameter $\gamma = \{\alpha, \tau\}$ yields

$$\frac{\partial p^*(z,c,\sigma,n)}{\partial \gamma} = \left. \frac{-\frac{\partial \widehat{c}}{\partial \gamma} \frac{\partial F(p)}{\partial c}}{F'(p)} \right|_{p=p^*(z,c,\sigma,n)} = \left. \frac{\frac{\partial \widehat{c}}{\partial \gamma} \frac{\partial \mathcal{L}(p)}{\partial c}}{-\mathcal{L}'(p)} \right|_{p=p^*(z,c,\sigma,n)} = \frac{\partial \widehat{c}}{\partial \gamma} \frac{\partial p^*(z,c,\sigma,n)}{\partial c}$$

where $F'(p)$ is given by (18), $\frac{\partial F(p)}{\partial c}$ by (23), and the final equality follows from (9). Thus, given $\frac{\partial \widehat{c}}{\partial \tau} = \frac{1}{1-\alpha}$ and $\frac{\partial \widehat{c}}{\partial \alpha} = \frac{\tau}{(1-\alpha)^2}$, it follows that $\frac{\partial p^*(z,c,\sigma,n)}{\partial \tau}$ and $\frac{\partial p^*(z,c,\sigma,n)}{\partial \alpha}$ are as claimed. □

Proof of Proposition 12. In the proof of Proposition 3 we showed that the sign of $\left(\frac{p-c}{p}\varepsilon(p)\right)''$ in (29) was determined by the sign of $1-\xi(p)$ when demand curvature is constant, $\xi'(p)=0$. This was critical in our method to sign $\frac{\partial^2 p^*(z,c,\sigma,n)}{\partial c \partial z}$, $\frac{\partial^2 p^*(z,c,\sigma,n)}{\partial c \partial \sigma}$ and $\frac{\partial^2 p^*(z,c,\sigma,n)}{\partial c \partial n}$ in Propositions 3, 8 and 9, respectively. Furthermore, the sign of $\frac{\partial^2 p^*(z,c,\sigma,n)}{\partial c \partial z}$ was used in the proofs of Propositions 4 and 5 and for Corollary 2; and the signs of $\frac{\partial^2 p^*(z,c,\sigma,n)}{\partial c \partial \sigma}$ and $\frac{\partial^2 p^*(z,c,\sigma,n)}{\partial c \partial n}$ were used for Corollary 3. Thus, it follows that all previous results for demand functions with constant curvature will apply for any log-concave (log-convex) demand function with non-constant curvature, if $\xi'(p)$ is not too positive (negative) at any $p \in [c, p^m(c)]$ such that $\left(\frac{p-c}{p}\varepsilon(p)\right)''$ in (29) remains strictly positive (negative) for all $p \in [c, p^m(c)]$. □

Proof of Proposition 13. Differentiating $\xi(p) = 1 + \left(\frac{1-\psi(p)}{\varepsilon(p)}\right)$ with respect to p yields $\xi'(p) = -\left[\frac{\varepsilon'(p)}{\varepsilon(p)}\left(\frac{1-\psi(p)}{\varepsilon(p)}\right) + \frac{\psi'(p)}{\varepsilon(p)}\right]$. Substituting both into (29) and manipulating yields

$$\left(\frac{p-c}{p}\varepsilon(p)\right)'' = \frac{-q'(p)}{q(p)}\left[-\left(\frac{1-\psi(p)}{\varepsilon(p)}\right)\left(\frac{-q'(p)}{q(p)}\right)\left(2\frac{c}{p} + \left(\frac{p-c}{p}\right)\psi(p)\right) + \left(\frac{p-c}{p}\right)\psi'(p)\right]$$

Thus, if $\psi'(p) = 0$, then sign $\left(\frac{p-c}{p}\varepsilon(p)\right)'' = -\text{sign}\{1-\psi(p)\}\ \forall p \geq c$, because $\varepsilon(p) > 0$, $\frac{-q'(p)}{q(p)} > 0$ and $\psi(p) \geq 0$. Then given $\psi(p) > (<)1$ implies demand is strictly log-concave (log-convex) $\xi(p) < (>)1$, it follows from Proposition 12 that all previous results for demand functions with constant curvature will apply. □

Proof of Proposition 14. Following Proposition 1, the pass-through rate in (9) still applies under unit demand. By substituting $p^m(c) = v$ and $\varepsilon(p) = 0$ into the second expression of (9), we obtain the second expression of (16). Furthermore, substituting into (6) and rearranging provides the explicit expression $p^*(z,c,\sigma,n) = c + \frac{v-c}{1+\frac{\sigma}{1-\sigma}nz^{n-1}}$, from which we obtain the explicit pass-through rate in the third expression of (16). □

Proof of Proposition 15. The equilibrium follows from Stahl (1989, Proposition 1). By following the same approach described in Section 4, the pass-through rate in (17) can be derived from the relative responsiveness of $\mathcal{L}(p,c,s)$ to c and p. Intuitively, a change in c affects the relative loss from a sale, $\mathcal{L}(p,c,s)$, but it does not affect the relative gain, as $\frac{\sigma n z^{n-1}}{1-\sigma}$ is constant for a given $z = 1-F(p)$. Thus, p must adjust to re-equate the two by returning $\mathcal{L}(p,c,s) = \frac{\pi(\overline{p};c)-\pi(p;c)}{\pi(p;c)}$ back to its original level. To obtain the second expression of (17), one can substitute the following into the first expression of

$$\mathcal{L}'(p,c,s) = -\frac{\pi(\overline{p};c)}{\pi(p;c)}\frac{\pi'(p;c)}{\pi(p;c)} = -\frac{\pi(\overline{p};c)}{\pi(p;c)}\left[\frac{1}{p-c} - \frac{-q'(p)}{q(p)}\right]$$

and

$$\begin{aligned}\frac{\partial \mathcal{L}(p,c,s)}{\partial c} &= \frac{\pi(\overline{p};c)}{\pi(p;c)}\left[\frac{1}{\pi(\overline{p};c)}\frac{\partial \pi(\overline{p},c)}{\partial c} + \frac{\partial \overline{p}}{\partial c}\frac{\pi'(\overline{p};c)}{\pi(\overline{p};c)} - \frac{1}{\pi(p;c)}\frac{\partial \pi(p,c)}{\partial c}\right] \\ &= \frac{\pi(\overline{p};c)}{\pi(p;c)}\left[\frac{1}{p-c} - \frac{1}{\overline{p}-c} + \frac{\partial \overline{p}}{\partial c}\left(\frac{1}{\overline{p}-c} - \frac{-q'(\overline{p})}{q(\overline{p})}\right)\right] .\end{aligned}$$

Then note that if $\overline{p} = p^m(c)$, (17) is identical to (9) from the Lerner index, but it differs when $\overline{p} = p^r(c,n,s) < p^m(c)$.

Now consider unit demand, where consumers have a common (and known) willingness to pay of $v > c$ and $q(p) = q > 0\ \forall p \leq v$, such that $p^m(c) = v$ and $\varepsilon(p) = 0$. Substituting into (17) yields $\frac{\partial p^*(z,c,\sigma,n,s)}{\partial c} = \left[1 - \frac{p-c}{\overline{p}-c}\left(1 - \frac{\partial \overline{p}}{\partial c}\right)\right]_{p=p^*(z,c,\sigma,n,s)}$. Thus, to prove that $\frac{\partial p^*(z,c,\sigma,n,s)}{\partial c} = 1$ $\forall z$, it remains to show that $\frac{\partial \overline{p}}{\partial c} = \frac{\partial p^r(c,n,s)}{\partial c} = 1$. This follows from Janssen et al. (2011) who demonstrated that $p^r(c,n,s) = c + \frac{s}{1-\beta}$ where β is a parameter that is independent of c. Hence, $\frac{\partial p^r(c,n,s)}{\partial c} = 1$ such that $\frac{\partial p^*(z,c,\sigma,n,s)}{\partial c} = 1\ \forall z$. □

Supplementary Appendix

In this appendix, we generate explicit solutions for $\frac{\partial p^*(z,c,\sigma,n)}{\partial c}$ for the two specific functional forms that are used in Section 7: linear demand and isoelastic demand. Recall that $p^*(z,c,\sigma,n)$ is defined as the level of p that satisfies

$$1 - F(p) = z \Longrightarrow (p-c)q(p) - \frac{1}{1+\frac{\sigma}{1-\sigma}nz^{n-1}}\pi(p^m(c);c) = 0 \tag{39}$$

where $F(p)$ is defined in (4). For convenience, let $\Gamma(z) \equiv \frac{1}{1+\frac{\sigma}{1-\sigma}nz^{n-1}} \in [0,1]$ where $\frac{\partial \Gamma(z)}{\partial z} = -\frac{\sigma(1-\sigma)n(n-1)z^{n-1}}{(1-\sigma+\sigma n z^{n-1})^2} < 0$.

Linear demand

Suppose $q(p) = \frac{a-p}{b}$ for some $a > c$ and $b > 0$, where $q(p) > 0$ for all $p < a = \hat{p}$. The curvature of demand is $\xi(p) \equiv \frac{q(p)q''(p)}{q'(p)^2} = 0 < 1$ for all p, so demand is log-concave. Furthermore, $\varepsilon(p) = \frac{p}{a-p}$ and $\left(\frac{p-c}{p}\varepsilon(p)\right)'' = \frac{2(a-c)}{(a-p)^3} > 0$ for all $p < a$. Substituting into (39) and rearranging yields

$$p^2 - (a+c)p + ca + b\Gamma(z)\pi(p^m(c);c) = 0$$

where $p^m(c) = \frac{a+c}{2}$ and $\pi(p^m(c);c) = \frac{1}{b}\left(\frac{a-c}{2}\right)^2$. Hence,

$$p^*(z,c,\sigma,n) = p^m(c) - \sqrt{b\pi(p^m(c);c)\left(1-\Gamma(z)\right)} = \frac{a+c}{2} - \left(\frac{a-c}{2}\right)\sqrt{1-\Gamma(z)}$$

such that

$$\frac{\partial p^*(z,c,\sigma,n)}{\partial c} = \frac{1}{2} + \frac{1}{2}\sqrt{1-\Gamma(z)}$$

In the paper, we show that $\frac{\partial p^*(z,c,\sigma,n)}{\partial z} < 0 \; \forall z > 0$ and that log-concave demand has $\frac{\partial^2 p^*(z,c,\sigma,n)}{\partial c \partial z} > 0$ and $\frac{\partial^2 p^*(z,c,\sigma,n)}{\partial c \partial \sigma} > 0 \; \forall z > 0$. To see that here, note that

$$\frac{\partial p^*(z,c,\sigma,n)}{\partial z} = \left(\frac{a-c}{4}\right)\left(1-\Gamma(z)\right)^{-\frac{1}{2}} . \frac{\partial \Gamma(z)}{\partial z}$$

and

$$\frac{\partial^2 p^*(z,c,\sigma,n)}{\partial c \partial z} = -\frac{1}{4}\left(1-\Gamma(z)\right)^{-\frac{1}{2}} . \frac{\partial \Gamma(z)}{\partial z}$$

where $\frac{\partial p^*(z,c,\sigma,n)}{\partial z} < 0$ and $\frac{\partial^2 p^*(z,c,\sigma,n)}{\partial c \partial z} > 0 \; \forall z > 0$ given $\frac{\partial \Gamma(z)}{\partial z} < 0$.

Isoelastic demand (with $\varepsilon = 2$)

Suppose $q(p) = \frac{\nu}{p^2}$ for some $\nu > 0$, where $q(p) > 0$ for all $p < \infty$. The curvature of demand is $\xi(p) \equiv \frac{q(p)q''(p)}{q'(p)^2} = \frac{3}{2} > 1$ for all p, so demand is log-convex. Furthermore, $\varepsilon(p) = 2$ and $\left(\frac{p-c}{p}\varepsilon(p)\right)'' = -\frac{4c}{p^3} < 0$. Substituting into (39) and rearranging yields

$$\Gamma(z)\pi(p^m(c); c)p^2 - \nu p + \nu c = 0$$

where $p^m(c) = 2c$ and $\pi(p^m(c); c) = \frac{\nu}{4c}$. Hence,

$$p^*(z, c, \sigma, n) = \frac{2c\left[1 - \sqrt{1 - \Gamma(z)}\right]}{\Gamma(z)}$$

such that

$$\frac{\partial p^*(z, c, \sigma, n)}{\partial c} = \frac{2\left[1 - \sqrt{1 - \Gamma(z)}\right]}{\Gamma(z)}$$

In the paper, we show that $\frac{\partial p^*(z,c,\sigma,n)}{\partial z} < 0 \ \forall z > 0$ and that log-convex demand has $\frac{\partial^2 p^*(z,c,\sigma,n)}{\partial c \partial z} < 0 \ \forall z > 0$. To see that here, note that

$$\frac{\partial p^*(z, c, \sigma, n)}{\partial z} = -\frac{2c}{\Gamma(z)^2}\frac{\partial \Gamma(z)}{\partial z}\left[1 - \sqrt{1 - \Gamma(z)} - \frac{\Gamma(z)}{2}(1 - \Gamma(z))^{-\frac{1}{2}}\right]$$

and

$$\frac{\partial^2 p^*(z, c, \sigma, n)}{\partial c \partial z} = -\frac{2}{\Gamma(z)^2}\frac{\partial \Gamma(z)}{\partial z}\left[1 - \sqrt{1 - \Gamma(z)} - \frac{\Gamma(z)}{2}(1 - \Gamma(z))^{-\frac{1}{2}}\right]$$

Both of these expressions will be strictly negative $\forall z > 0$ if the term in square brackets is strictly negative. To prove this, first note that

$$\lim_{\sigma \to 1}\left[1 - \sqrt{1 - \Gamma(z)} - \frac{\Gamma(z)}{2}(1 - \Gamma(z))^{-\frac{1}{2}}\right] = 0$$

because $\lim_{\sigma \to 1}\Gamma(z) = 0$. Then differentiating the term in square brackets with respect to σ yields

$$\begin{aligned} &= -\left(\tfrac{1}{2}(1 - \Gamma(z))^{-\frac{1}{2}} . -\tfrac{\partial \Gamma(z)}{\partial \sigma}\right) - \left(\frac{1}{2}\frac{\partial \Gamma(z)}{\partial \sigma}(1 - \Gamma(z))^{-\frac{1}{2}}\right) - \frac{\Gamma(z)}{4}(1 - \Gamma(z))^{-\frac{3}{2}} . \frac{\partial \Gamma(z)}{\partial \sigma} \\ &= -\tfrac{\Gamma(z)}{4}(1 - \Gamma(z))^{-\frac{3}{2}} . \tfrac{\partial \Gamma(z)}{\partial \sigma} \qquad > 0 \end{aligned}$$

given $\frac{\partial \Gamma(z)}{\partial \sigma} = -\frac{nz^{n-1}}{1-\sigma+\sigma n z^{n-1}} < 0 \ \forall z > 0$. This implies that $\frac{\partial p^*(z,c,\sigma,n)}{\partial z} < 0$ and $\frac{\partial^2 p^*(z,c,\sigma,n)}{\partial c \partial z} < 0$ $\forall z > 0$, because the expression in square brackets is strictly negative for all $\sigma < 1$.