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Abstract. Incumbent firms may acquire start-ups to eliminate potential competition without intending to develop new technology (killer acquisitions). We develop a model to examine the incentives and welfare implications of killer acquisitions under different market structures: vertical separation and integration. Our model focuses on the competition between an upstream incumbent firm and a start-up with the potential to develop superior technology, where the incumbent has the option to acquire the start-up and decide whether to continue the development of the superior technology. We find that killer acquisitions are more likely when the cost of developing the superior technology is moderate under both vertical separation and integration. However, these acquisitions lead to a welfare loss only when the development cost is relatively low. Comparing vertical integration to separation, the probability of killer acquisition is higher (lower) when the incumbent firm has a greater (smaller) chance of successfully developing the superior technology.

Keywords: innovation incentive, killer acquisitions, vertical integration.

JEL Classification Number: D8, L1

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1 Introduction

Recently, much attention have paid to the acquisitions by large incumbent firms. One concern is that an incumbent firm may engage in so-called killer acquisition by purchasing start-ups with the sole purpose of eliminating potential competition without intending to develop the new product (Cunningham et al., 2021; Motta and Peitz, 2021). Existing literature mostly focuses on the study of killer acquisition in horizontal industries (Cabral, 2018, 2021; Federico et al., 2018; Bryan and Hovenkamp, 2020; Cunningham et al., 2021). However, many start-up acquisitions occur in vertically related industries. For example, *Illumina*, an American biotechnology company, has been a near monopolist in next-generation DNA sequencing (NGS), a critical input in multi-cancer early detection tests.[1] Many biotechnology companies use NGS to offer diagnostic services. In 2018, *Illumina* proposed to acquire an innovative NGS firm, *Pacific Bioscience*. Though *Pacific Bioscience* had a very small market share (2%-3%) it employed long-read sequencing technology which has significant technological advantages over the shorted-read sequencing technology adopted by *Illumina*.[2] The acquisition was challenged by FTC in December 2019 for the concern that "the acquisition constituted unlawful maintenance of Illumina' monopoly in the U.S. market for next-generation DNA sequencing systems, by extinguishing *Pacific Biosciences* as a nascent competitive threat." [3]

In light of the above observations, we aim to examine the following research questions. Under which circumstances do killer acquisitions tend to occur in vertically related industries? What motivates an upstream incumbent firm to engage in killer acquisitions? In what situations may killer acquisitions result in a welfare loss? Does vertical integration, in comparison to vertical separation, increase or decrease the likelihood of killer acquisitions? How does market structure impact the incentives and welfare implications of killer acquisitions?

[1] See "How did Illumina dominate the sequencing market?" https://frontlinegenomics.com/how-did-illumina-monopolize-the-sequencing-market/

[2] See "Illumina to Acquire Pacific Biosciences for Approximately $1.2 Billion, Broadening Access to Long-Read Sequencing and Accelerating Scientific Discovery". https://www.illumina.com/company/news-center/press-releases/2018/2374913.html

[3] FTC Challenges Illumina's proposed acquisition of PacBio. https://www.ftc.gov/news-events/news/press-releases/2019/12/ftc-challenges-illuminas-proposed-acquisition-pacbio

To address these questions, we develop a model of innovation in vertically related industries to examine an upstream incumbent's motivation to acquire an upstream start-up in two distinct market structures: vertical separation and vertical integration. Initially, downstream firms produce differentiated products utilizing an input provided by the upstream incumbent. This input is sold through a two-part tariff contract. A start-up with a more advanced, albeit immature, technology emerges. If the start-up enters the upstream market and successfully develops the new technology, the upstream incumbent would be forced out of the market. The incumbent firm has the choice to acquire the start-up and can subsequently opt to either continue the development of the newly acquired technology or to terminate it altogether.

We first conduct the equilibrium analysis under vertical separation where the upstream incumbent is independent of the downstream firms. As our first result, we show that the incumbent firm has a weaker incentive than the start-up to develop the more advanced technology. This is because, upon successful innovation, the incumbent firm merely replaces itself and thus earns lower expected profits than the start-up. As a consequence of this incentive gap, there is a possibility that the incumbent firm may seek to acquire the start-up without intending to develop the new technology. On the other hand, there is a strong (weak) synergy effect resulting from acquisition if the incumbent firm's chance of successful innovation is relatively high (low) compared to the start-up. We show that killer acquisitions occur when the investment cost is moderate and the synergy effect is weak. The reasoning behind this is as follows: the incumbent firm weighs the benefits against the costs of three options: acquire-to-develop, acquire-to-kill, and no acquisition strategies. If the benefit from the synergy effect outweighs the development cost, the incumbent firm will choose to develop the advanced technology through acquisition. By contrast, if the benefit from synergy effect is less significant compare to the development cost, the incumbent firm will not continue the development of the new technology. In this case, for a low development cost the incumbent would need to pay an excessively high acquisition price to the start-up to buy the innovation idea, so it would forgo acquiring the start-up. However, when the development cost is at a moderate level, an intriguing equilibrium arises in which the incumbent firm acquires the start-up but refrains from developing the new technology.

Next, we study the welfare implication of killer acquisitions under vertical separation. While killer acquisitions may decrease welfare, the underlying reasons for this outcome can be different. If the incumbent firm possesses a higher probability of successful innovation compared to the start-up, deciding not to continue the technology development post-acquisition would result in a loss of welfare. This occurs because the incumbent fails to internalize the positive externality to consumers stemming from technological progress. On the other hand, if the start-up has a greater likelihood of successful innovation, a killer acquisition by the incumbent also leads to a welfare loss since the more advanced technology ideally should have been developed by the start-up.

We also conduct analysis for the scenario of vertical integration in which the upstream incumbent firm is vertically integrated with a downstream firm and find that the equilibrium and welfare results are similar to those under vertical separation.

We further study the impact of market structure on equilibrium outcomes and welfare implications. Note that in our model where two-part tariff contracting is employed, the upstream incumbent firm can capture the entire industry profit and thus earn higher profits under vertical separation. Consequently, vertical integration reduces the profitability of the incumbent as an upstream supplier. Hence, when the incumbent upstream firm has a low likelihood of successful innovation and thus lacks the motivation to develop the new technology, the probability of a killer acquisition is lower under vertical integration because the current upstream supplier has lower incentive to maintain its incumbent position. Conversely, if the incumbent upstream firm has a high probability of successful innovation and thus a strong incentive of developing the new technology, the killer acquisition zone will be larger under vertical integration. For welfare results, we show that, in the case of linear market demand and quantity competition, total welfare rises as the killer acquisition zone shrinks. By contrast, total welfare decreases as the killer acquisition zone expands provided that the start-up's probability of successful innovation is not too low or that the number of downstream firms is large.

Previous research has primarily focused on killer acquisitions in horizontal market. Notably, Cunningham et al. (2021) and its subsequent extensions have extensively examined the concept of killer acquisitions. Letina et al. (2021) build upon Cunningham et al. (2021)'s work by considering ex-ante innovation and policy implications on innovation strategies. In the context of a differentiated oligopoly model, Bryan and Hovenkamp (2020) demonstrate that the absence of limitations on start-up acquisitions can lead to distortions in efficient innovation. Cabral (2021) suggests that when markets for technology are imperfect, incumbent firms may opt for acquisitions as the simplest means to acquire the technology developed by a start-up entrant. Additionally, several studies explore the potential strategic reactions of both acquiring firms and non-targeted firms with respect to their investment strategies (Katz, 2021; Motta and Peitz, 2021; Teh et al., 2022; Motta and Shelegia, 2022; Cabral, 2024; Letina et al., 2024; Dijk et al., 2024). Unlike these existing studies, which primarily focus on start-up acquisitions in horizontal markets, our paper specifically examines the motivations and welfare effects of start-up acquisitions in vertical markets.[4]

Several recent studies have examined the impact of mergers on R&D incentives in vertically related industries. For example, Chen and Sappington (2010) explore the effects of vertical structure on upstream process innovation. They show that vertical integration generally enhances innovation when downstream competition follows a Cournot fashion. However, innovation may be reduced when downstream firms engage in Bertrand competition. Loertscher and Riordan (2019) analyze a model in which integration grants a downstream firm the option to source internally or from an independent supplier. They find that vertical integration tends to discourage independent suppliers from making cost-reducing innovations, while the integrated firm increases its innovation efforts. However, none of these studies have specifically investigated how the vertical market structure influences a firm's incentive to engage in killer acquisitions. This highlights a gap in the existing literature, which this paper aims to address by examining the motivations and welfare effects of killer acquisitions in a vertical market setting.

In terms of model settings, our paper is closely aligned with Lin et al. (2020), which also examines a monopoly upstream supplier providing inputs to multiple downstream firms. Both studies analyze market performance across various market structures, yet they emphasize

[4] Cao and Lin (2023) establish a synergy effect test for evaluating the competitive impact of start-up acquisitions.

different aspects. Lin et al. (2020) focus on the innovation incentives of a downstream firm that may consider entering the upstream market, whereas our research investigates the acquisition and innovation incentives of the upstream incumbent firm in light of a potential upstream entrant.

The remaining sections of the paper are organized as follows. In Section 2, we present the model that serves as the foundation for our analysis. Section 3 delves into the analysis under the scenario of vertical separation. In Section 4, we study the effects of vertical integration and compare the outcomes to those obtained under vertical separation. Section 5 draw the conclusions.

2 The Model

Consider a model comprising two vertically related industries: a downstream industry and an upstream industry. In the downstream industry, there are n downstream firms, denoted as $D_1, D_2, ..., D_n$, which produce horizontally differentiated final products. The demand function for the product of firm D_i is represented by $p_i(q_i, q_{-i})$. The production of these final products requires input goods supplied initially by a monopolistic upstream firm U, which incurs a constant marginal cost of production, c. One unit of the final product necessitates precisely one unit of the input good. The costs associated with the transformation of the input good into the final product are normalized to zero.

We consider two types of initial market structures: vertical separation (VS), where all firms operate independently, and vertical integration (VI), where the upstream firm U and one of the downstream firms, specifically D_1, are vertically integrated. The integrated firm resulting from vertical integration is denoted as UD. The input good is sold to each independent downstream firm through a two-part tariff contract, namely $T_i + \omega_i q_i$. In this contract, T_i represents the lump-sum fee that firm D_i must pay, and ω_i denotes the marginal cost associated with obtaining the input. For simplicity, we assume that contracts between upstream firms and non-integrated downstream firms are publicly observable.[5]

In the market, there exists a start-up or potential entrant labeled as E. This start-up

[5] See Rey and Vergé (2004) for a study on contracts that are only privately observed.

possesses an innovative idea or research project that has the potential to be transformed into an innovation at a development cost denoted as k. Specifically, if E incurs the cost k, with probability $\rho^E \in (0, 1)$, it generates a new input capable of replacing the input used by the incumbent firm. The new input exhibits a higher level of quality and the quality advantage can be translated into a cost advantage, whereby the new input can be produced at a lower effective marginal cost denoted as c' with $c' < c$.

The incumbent supplier U, can make a take-it-or-leave-it offer, represented by b, to acquire the start-up E before any development decision. If the acquisition is successful, U has the choice to independently develop the innovation. Specifically, if U invests k in the development of the new input, with probability $\rho^U \in (0, 1)$, the new input will be successfully invented. In line with Cunningham et al. (2021), we allow for the possibility of differential capabilities in project development. If $\rho^U \geq \rho^E$, the incumbent supplier possesses an advantage over the start-up in developing the new input. This advantage can stem from synergies with the incumbent's existing input development. In contrast, if $\rho^U < \rho^E$, it is more likely for the start-up to successfully develop the new product than that is for U. This can be attributed to factors such as key personnel in technology leaving the start-up following acquisitions. Additionally, we normalize the liquidation value of the research project to 0 if it discontinues.

The game unfolds in the following sequence of stages. In stage 1, the upstream incumbent, represented as U or UD, makes a decision regarding the acquisition of E by presenting a price offer b. E determines whether to accept the acquisition offer. Moving on to stage 2, if the acquisition occurs, the upstream incumbent firm then chooses whether to proceed with the R&D project by incurring an investment cost of k or to abandon the project entirely (referred to as a killer acquisition). If no acquisition occurs, the start-up decides whether to pursue the R&D project by investing k. Finally, in stage 3, firms engage in two-part tariff contracting as described above, and the outcome of the innovation is realized, and the corresponding payoffs for the relevant parties are determined.

We solve for the Subgame Perfect Nash Equilibrium via the backward induction procedure. Denote the possible equilibrium outcomes, respectively, as follows:

$(A, D) - U$ acquires E and develops the innovation;

$(A, ND) -$ U acquires E but does not develop the innovation;

$(NA, D) -$ No acquisition occurs and E develops the innovation; and

$(NA, ND) -$ No acquisition occurs and E does not develop the innovation.

We next turn to the analysis under vertical separation.

3 Analysis under vertical separation

In this section, we analyze the scenario of vertical separation by employing backward induction to solve the model. Our analysis consists of three steps. Firstly, we examine the competition in the product market under various scenarios and derive the reduced-form profits for all relevant parties involved. Secondly, we delve into the innovation incentives of both the incumbent and the start-up, exploring the conditions under which these parties are motivated to engage in innovation development. Finally, we investigate the acquisition motivations of the incumbent, considering their strategic rationale for acquiring the start-up, and subsequently characterize the market equilibrium.

3.1 Product market competition

The analysis for product market competition is similar to that in Lin et al. (2020). In particular, under vertical separation, the upstream firm U initially make a public two-part tariff offer to each downstream firm. We denote the two-part tariff contract as (ω_i, T_i) or $T_i + \omega_i q$, where the total payment for supplying q units of the input is determined. U's profit-maximization problem in this context is as follows:

$$Max_{(\omega_i,T_i)} \quad \sum_{i=1}^{n} \left[(\omega_i - c)\, q\left(\omega_i, \omega_{-i}\right) + T_i \right].$$

For any given values of ω_i and ω_{-i}, the optimal choice of T_i is designed to extract all profits from each individual downstream firm: $T_i = \pi(\omega_i, \omega_{-i}) = [p\left(\omega_i, \omega_{-i}\right) - \omega_i] q\left(\omega_i, \omega_{-i}\right)$. Substituting the expression of optimal T_i into U's profit-maximization problem, U's objective function becomes

$$Max_{\omega_i} \sum_{i=1}^{n} [p(\omega_i, \omega_{-i}) - c] q\left(\omega_i, \omega_{-i}\right). \tag{1}$$

We look at the symmetric equilibrium. Absence of entry by E, the upstream firm U charges the same price, denoted as $\omega^S(c)$, to all downstream firms, where $\omega^S(c) > c$. Since tariffs offered by U are observed by all downstream firms, U can credibly commit to an input price $\omega^S(c)$ such that each the downstream firms produces a quantity Q^m/n where Q^m is the monopoly output level in the downstream market. As a result, the downstream firms each earn zero profit since U utilizes the lump-sum fee T_i to extract the monopoly profits generated in the downstream market. The resulting monopoly profit for U is denoted as $\Pi^S(c)$. In that case, U can fully exert its monopoly power and get the entire monopoly profit.

In the presence of potential entry by firm E, which occurs after E successfully developing its superior input, U can choose to acquire E's innovative project before E conducts its R&D project. If U acquires E and successfully develops the innovation, U will maintain its position as the sole supplier of the more advanced input. In this case, U's optimization problem remains similar to the previous formulation (1), with the only difference being the input cost, now denoted as $c' < c$. As a result, U's resulting profit is represented as $\Pi^S(c')$, and the equilibrium input price is determined as $\omega^S(c')$. If U successfully acquires E but fails to develop the innovation or if U did not develop the innovation in the first place, U will still maintain its position as the exclusive supplier of the old input. In this particular scenario, U's optimization problem remains exactly the same as described in (1).

In the scenario where no acquisition takes place and E successfully innovates, E becomes the more efficient upstream supplier. To provide a clear illustration of the economic forces, we assume that the innovation is drastic: if E achieves a successful innovation, it will replace U and effectively become the sole upstream supplier, producing the input at a cost of c' and earning profits denoted as $\Pi^S(c')$.[6] If no acquisition occurs and E does not achieve a successful

[6] One possible micro foundation is as follows. U and E simultaneously offer two-part tariff contracts to a downstream firm D_i. Upstream competition in this case is similar to standard Bertrand competition, except that each competing supplier has two instruments: the unit price and the lump-sum fee. In equilibrium, the less efficient supplier, U, will be priced out and hence earn zero profit with a contract $(\omega_U, T_U) = (c, 0)$. The more efficient supplier, E, solves a problem similar to (1) with marginal cost c' instead of c, subject to the constraint that its contract must win the competing contract $(c, 0)$ to supply D_i.Using the notation introduced above, the solution to the unconstrained optimization problem is $\omega^S(c')$. If $\omega^S(c') \leq c$, which will happen if c' is sufficiently small, then the presence of U does not constrain E's

innovation, U will retain its position as the sole supplier of the old input. In this case, U's optimization problem is identical to (1).

3.2 Innovation decisions

Next, we consider the innovation decisions made by U and E in two distinctive cases, respectively: (i) the innovation decision by U following its acquisition of E, and (ii) the innovation decision by E in the absence of acquisition by U.[7] In each case, we derive the condition under which it is profitable for the project owner to develop the new input by incurring the fixed cost, k.

We summarize the payoffs for U and E in the following table.

Outcomes	U	E
(A, D)	$\rho^U \Pi^S(c') + (1-\rho^U)\Pi^S(c) - k - b$	b
(A, ND)	$\Pi^S(c) - b$	b
(NA, D)	$(1-\rho^E)\Pi^S(c)$	$\rho^E \Pi^S(c') - k$
(NA, ND)	$\Pi^S(c)$	0

Table 1: Payoffs under Vertical Separation

Consider the scenario where U has acquired E and now faces the decision of whether to incur the innovation cost k to develop the innovation that would subsequently reduce the production cost to c'. If U decides to develop the innovation, it will obtain a profit of $\Pi^S(c')$ with a probability of ρ^U. However, there is also a probability of $1-\rho^U$ where U will continue to earn a profit of $\Pi^S(c)$ when the development is unsuccessful. Hence, the expected gain from developing the new input for U is $\rho^U \left[\Pi^S(c') - \Pi^S(c)\right]$. Thus, after acquiring E, U will

choice, in which case, E's payoff equals $\Pi^S(c')$, and D_i's payoff is zero.

[7] Note that our analysis in this section differs from that in Lin et al. (2020). Specifically, they examine the innovation incentives of a downstream firm that may consider entering the upstream market, while we investigate the acquisition and innovation incentives of the upstream incumbent firm in response to a potential upstream entrant.

develop the new input if $k \leq k^U$ but choose not to do so if $k > k^U$ where

$$k^U = \rho^U \left[\Pi^S \left(c'\right) - \Pi^S \left(c\right)\right]. \tag{2}$$

Consider the case where E is not acquired by U and make the decision of whether to develop the new technology. If E chooses to develop the new technology, it will earn an expected profit of $\rho^E \Pi^S (c')$. On the other hand, if E decides not to develop the new technology, its payoff would be zero. Thus, E will choose to develop the new technology if $k \leq k^E$ where

$$k^E = \rho^E \Pi^S \left(c'\right). \tag{3}$$

By comparing (2) to (3), we have $k^U \leq k^E$ if and only if $\frac{\rho^U}{\rho^E} \leq \hat{\rho}$, where

$$\hat{\rho} \equiv \frac{\Pi^S \left(c'\right)}{\Pi^S \left(c'\right) - \Pi^S \left(c\right)}. \tag{4}$$

The above discussions lead to the following lemma.

Lemma 1 *(i) Suppose that U acquires E. There exists a cutoff k^U such that U will develop the new technology if $k \leq k^U$; (ii) Suppose that E is not acquired by U. There exists a cutoff k^E such that E will develop the new input if $k \leq k^E$; and (iii) $k^U \leq k^E$ if and only if $\frac{\rho^U}{\rho^E} \leq \hat{\rho}$.*

According to Lemma 1, U may have a higher or a lower incentive than E to develop the innovation, depending on the relative innovation probability, represented by $\frac{\rho^U}{\rho^E}$. It is worth noting that when U and E have the same probability of achieving a successful innovation ($\frac{\rho^U}{\rho^E} = 1$), the incentive for U to develop the innovation is lower compared to E ($k^U \leq k^E$). This is because U, as an incumbent firm, would replace itself after a successful innovation and consequently earn a lower expected net profit, which is simply the standard "replacement effect" first studied by Arrow (1962).

A key parameter in our analysis throughout the paper is the incumbent supplier U's likelihood of developing the new input after acquiring E, namely ρ^U. By Lemma 1, the incumbent firm may have greater capability of developing the new input, relative to that of E, due to such factors as a larger R&D team, greater resources in product promotion, etc. We say that an acquisition of E by incumbent U generates a "synergy effect" in innovation if $\hat{\rho} > 1$.

3.3 Acquisition decision

We now analyze the acquisition decision made by the upstream incumbent U. In evaluating the acquisition decision, U compares the expected profit gained through acquisition with the potential costs incurred in the absence of acquisition. Note that these costs include the loss of profits resulting from being replaced by E if it successfully develops the new input. The expected profit from an acquisition will depend on whether U decides to develop the new technology after the acquisition.

To compensate E for the acquisition, U must pay E an acquisition price equal to the higher of two values: the (net) expected profit from innovation or the liquidation value. Specifically,

$$b = \begin{cases} \rho^E \Pi^S(c') - k & \text{if } k \leq k^E \\ 0 & \text{if } k > k^E \end{cases}. \tag{5}$$

We proceed with the analysis by considering the following cases.

(1) $k > k^E$. In this case, if E is not acquired by U, E will not develop the innovation and will accept an acquisition price $b = 0$. Anticipating this outcome, U will only proceed with the acquisition of E if it decides to develop the new technology post-acquisition. It follows that, by Lemma 1, U will acquire E and develop the innovation if $k \leq k^U$.

(2) $k \leq k^E$. In this scenario, if E is not acquired by U, E will proceed with developing the innovation. As a result, U would need to pay an acquisition price of $b = \rho^E \Pi^S(c') - k$ to compensate E. If the innovation cost k is less than or equal to U's innovation cost threshold k^U ($k \leq k^U$), then U will also develop the innovation following the acquisition. U will decide to make the acquisition if the expected profit from the acquisition and development outweighs the potential profit loss under the scenario of no acquisition.

$$\rho^U \Pi^S\left(c'\right) + \left(1 - \rho^U\right) \Pi^S(c) - k - b \geq \left(1 - \rho^E\right) \Pi^S(c) \tag{6}$$

or equivalently

$$\frac{\rho^U}{\rho^E} \geq 1$$

since $\Pi^S(c') - \Pi^S(c) \geq 0$.

On the contrary, if the innovation cost k exceeds U's innovation cost threshold k^U ($k > k^U$), U will decide not to develop the innovation following the acquisition. In other words, if

U proceeds with the acquisition, it will not undertake the innovation development (killer acquisition). U will make the acquisition if the expected profit from the "killer acquisition" outweighs the potential profit loss that would occur under the scenario of no acquisition.

$$\Pi^S(c) - b \geq \left(1 - \rho^E\right) \Pi^S(c)$$

or

$$k \geq \hat{k} = \rho^E \left[\Pi^S\left(c'\right) - \Pi^S(c)\right]. \tag{7}$$

Comparing (3) with (7), we have $\hat{k} < k^E$. Moreover, comparing (2) with (7), we have $\hat{k} < k^U$ if and only if $\frac{\rho^U}{\rho^E} \geq 1$. We are now in a position to characterize the equilibrium under vertical separation.

Proposition 1 *Under vertical separation, the equilibrium outcomes are:*

$$\begin{cases} (A, D) & if \quad k \leq k^U \; and \; \frac{\rho^U}{\rho^E} \geq 1 \\ (A, ND) & if \quad \max\left\{k^U, \hat{k}\right\} < k < k^E \\ (NA, ND) & if \quad k > \max\left\{k^U, k^E\right\} \\ (NA, D) & if \quad k < \hat{k} \; and \; \frac{\rho^U}{\rho^E} < 1 \end{cases}.$$

Proof. See Appendix. ■

The equilibrium outcomes are displayed in Figure 1. Intuitively, when the innovation cost k is less than or equal to U's innovation cost threshold $k^U (k \leq k^U)$, by Lemma 1, U will decide to develop the innovation upon processing the innovation idea (after acquiring E). Therefore, if the acquisition price is relatively low compared to the expected benefit, particularly when there is a high synergy effect (i.e. $\frac{\rho^U}{\rho^E} \geq 1$), U will make the optimal choice to acquire E and develop the more advanced technology. This situation is depicted in the dark gray area in Figure 1.

If the innovation cost k exceeds U's innovation cost threshold k^U $(k > k^U)$, the incumbent firm U will not choose to develop the innovation. However, in the context of the acquisition decision, U needs to consider the potential loss of incumbent profits if E successfully innovates. When the innovation cost is high $(k > k^E)$ to the extent that E itself does not find it profitable to develop the innovation, U will opt not to make the acquisition. This situation is depicted in

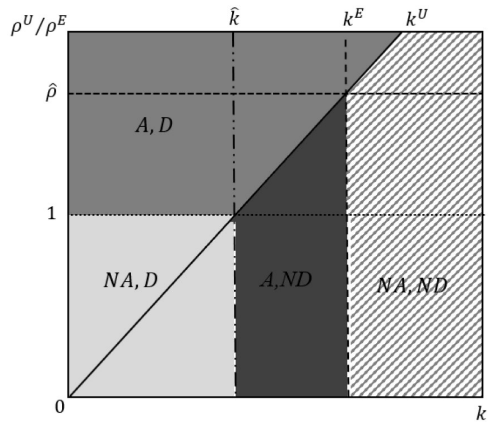

Figure 1: Equilibrium Outcomes under Vertical Separation.

the shadow area in Figure 1. On the other hand, if the innovation cost falls within a medium range ($\hat{k} < k < k^E$) where E deems it profitable to develop the innovation, U will choose to make the acquisition but not actively develop the innovation. This "acquire-to-kill" strategy ensures that U is protected from the potential innovation by E and can continue to enjoy its incumbency. This situation is represented in the black area in Figure 1.

In the final scenario, if the synergy effect is weak (i.e., $\frac{\rho^U}{\rho^E} < 1$) and E can develop the innovation at a low cost ($k < \hat{k}$), U perceives the acquisition of E as too costly. The acquisition price b is deemed too high compared to the potential benefits U would gain either from developing the innovation or maintaining its incumbency. Consequently, in equilibrium, no acquisition takes place, and E will choose to independently develop the innovation. This situation is represented in the light gray area in Figure 1.

Corollary 1 *Under vertical separation, killer acquisition occurs when innovation cost is at a medium level.*

Interestingly, killer acquisition tends to occur when the investment cost is at a medium level. This is because when the investment cost is small, the cost of acquiring the technology becomes disproportionately high for the incumbent firm. In such cases, the entrant can potentially earn a high profit by developing the innovation independently, making the acquisition less attractive for the incumbent.

3.4 Welfare implication of killer acquisitions

We have demonstrated the possibility of killer acquisition as an equilibrium outcome. This raises the question of whether such acquisitions result in welfare loss. From the perspective of a social planner, the criterion for developing an innovation is whether the cost of development is lower than the expected increase in welfare resulting from the innovation. Furthermore, if the decision is made to develop the innovation, it is optimal for the firm with a higher probability of successful innovation to undertake the development.

To address the question of whether killer acquisitions lead to welfare loss, we begin by deriving the first-best outcome. Subsequently, we compare the market equilibrium outcome with this first-best outcome. To this end, we consider two distinct cases based on the strength of U's synergy effect in developing innovation. If the synergy effect is strong, meaning that $\frac{\rho^U}{\rho^E} \geq 1$, it is more favorable for U to be the innovator and develop the innovation compared to E. However, it is not always socially optimal for U to choose to develop the innovation, especially when the cost of innovation is excessively high. Therefore, when the synergy effect is strong, the first-best outcome is (A, D) but subject to the condition that the innovation cost k does not exceed k^{U^o}

$$k \leq k^{U^o} = \rho^U \left[W^S\left(c'\right) - W^S\left(c\right)\right]$$

where $W^S(c)$ and $W^S(c')$ represent the total welfare when the old input and the new input, respectively, are sold in the downstream market under vertical separation.

If the synergy effect is weak, meaning that $\frac{\rho^U}{\rho^E} < 1$, it is more preferable for E to be the innovator and develop the innovation compared to U. Likewise, it is not always socially optimal for E to develop the new technology when the innovation cost is high. Consequently, when the synergy effect is weak, (NA, D) is the socially optimal outcome, but the optimality should be subject to the condition that k is not higher than k^{E^o} $(k < k^{E^o})$ where

$$k^{E^o} = \rho^E \left[W^S\left(c'\right) - W^S\left(c\right)\right].$$

Note that $k^{U^o} > k^U$. This is because U does not fully internalize the consumer benefit when deciding whether to innovate or not. As a result, U may underestimate the overall welfare gains associated with innovation, leading to a lower perceived innovation cost threshold (k^U) compared to the assessment of the social planner (k^{U^o}). However, E's private innovation incentive may exceed that of the social planner ($k^{U^o} < k^E$) since E does not internalize the negative externality imposed on U. Comparing the market equilibrium outcome with the first-best outcome, we can establish the following proposition.

Proposition 2 *Under vertical separation, killer acquisition leads to welfare loss if* $k < k^{U^o}$ *for* $\frac{\rho^U}{\rho^E} \geq 1$ *and* $k < k^{E^o}$ *for* $\frac{\rho^U}{\rho^E} < 1$.

Proof. See Appendix. ■

The results in Proposition 2 are illustrated in Figure 2. Killer acquisitions lead to efficiency loss when the innovation cost (k) is at an intermediate level, but for different reasons. When $\frac{\rho^U}{\rho^E} \geq 1$, the social optimal outcome suggests that U should acquire E and develop the technology. However, in the market equilibrium, U chooses not to develop the technology if the innovation cost is not sufficiently small because U fails to consider the positive externality to consumers resulting from technology development. On the other hand, when $\frac{\rho^U}{\rho^E} < 1$, it is socially more efficient for E to develop the superior technology, provided that the innovation cost is not too high. However, if the innovation cost is not small, U can acquire E at a lower price since E would earn a low profit if it chooses to innovate. Consequently, killer acquisition occurs in equilibrium, resulting in a loss of welfare.

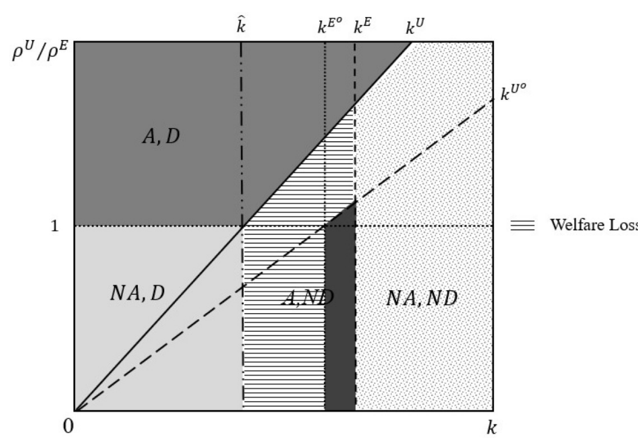

Figure 2: Welfare under Vertical Separation.

Note that for $k \in (k^{E^o}, k^E) \cap (k^{U^o}, +\infty)$ (the black area in Figure 2), killer acquisition does not lead to welfare loss because it is socially too costly to develop the new technology. However, E would choose to develop the new technology if U would not make the acquisition. Hence, a policy that forbids U's acquisition of E would lead to welfare decrease in this case.

4 The effects of vertical integration

4.1 Analysis under vertical integration

In this section, we undertake the analysis considering vertical integration and subsequently compare the outcomes to those obtained under vertical separation. Under the framework of vertical integration, we follow Lin et al. (2020) and assume that the integrated entity UD provides the input to its own downstream firm D_1 at the internal cost c, while offering a two-part tariff contract (ω_j, T_j) to other downstream firms. One justification for this assumption is that when an integrated firm produces its inputs in-house, it has an incentive to lower the input price for its own downstream unit, given other downstream firms' outputs. Additionally, because the contract with the integrated firm is likely confidential, this may further weaken the integrated firm's credibility in committing to a high input price for its downstream unit. Therefore, it is reasonable to assume that the integrated firm supplies inputs to its downstream unit at an internal cost c.

The optimization problem of the integrated entity UD is

$$Max_{(\omega_j, T_j)} \quad \left\{ [p\left(c, \omega_j\right) - c] q\left(c, \omega_j\right) \quad + \sum_{j=2}^{n} \left(\omega_j - c\right) q\left(\omega_j, c\right) + T_j \right\}.$$

For any given ω_j, the optimal T_j is to extract all the profit from each downstream firm. Thus, $T_j = \pi(\omega_j, c) = [p\left(\omega_j, c\right) - \omega_j] q\left(\omega_j, c\right)$. Then the objective function becomes

$$Max_{\omega_j} \left\{ [p\left(c, \omega_j\right) - c] q\left(c, \omega_j\right) + \sum_{j=2}^{n} [p(\omega_j, c) - c] q\left(\omega_j, c\right) \right\}. \tag{8}$$

We also focus on symmetric equilibrium: The integrated entity UD charges the same price, denote as $\omega_j = \omega^I(c)$, to all unintegrated downstream firms. The resulting profit for UD is denoted as $\Pi^I\left(c\right)$.

If UD acquires E and develops the innovation successfully, it will maintain its position as the exclusive input supplier. In this scenario, UD faces an optimization problem similar to the one represented by equation (8), with the only difference being that the input cost is now denoted as c'. The equilibrium input price is denoted as $\omega^I(c')$, and $\Pi^I(c')$ represents the profit earned by UD under the new input cost of c'. On the other hand, provided that U does not acquire E, E will emerge as the sole upstream supplier, if it successfully carries out the innovation process. In this case, under drastic innovation, E solves an optimization problem that is similar to (1) and receives a profit $\Pi^S(c')$.[8]

The following table summarizes the payoffs for UD and E under various outcomes.

Outcomes	UD	E
(A, D)	$\rho^U \Pi^I(c') + (1-\rho^U)\Pi^I(c) - k - b$	b
(A, ND)	$\Pi^I(c) - b$	b
(NA, D)	$(1-\rho^E)\Pi^I(c)$	$\rho^E \Pi^S(c') - k$
(NA, ND)	$\Pi^I(c)$	0

Table 2: Payoffs under Vertical Integration

For the decision regarding innovation development, we have the following result.

Lemma 2 *Under vertical integration, (i) suppose that UD acquires E. There exists a cutoff*

$$k_I^U = \rho^U \left[\Pi^I\left(c'\right) - \Pi^I(c)\right] \tag{9}$$

such that UD will develop the new technology if $k \leq k_I^U$; (ii) Suppose that E is not acquired by UD. There exists a cutoff

$$k_I^E = \rho^E \Pi^S\left(c'\right) = k^E \tag{10}$$

[8] It's important to note that under drastic innovation (e.g., when c' is significantly lower than c), UD, even with a downstream unit, cannot impose a constraint on the contracts that E will offer to downstream firms. This occurs because, under such drastic innovation, UD will be unable to sell its final products even at price c, as other downstream firms will opt to purchase the more efficient input product from E.

such that E will develop the new input if $k \leq k_I^E$; and (iii) $k_I^U \leq k_I^E$ if and only if

$$\frac{\rho^U}{\rho^E} \leq \hat{\rho}_I \equiv \frac{\Pi^S(c')}{\Pi^I(c') - \Pi^I(c)}.$$

Proof. See Appendix. ■

It is worth noting that the incentive for E to develop the innovation remains the same under both vertical separation and integration. This is because, upon a successful innovation, E would become the exclusive upstream supplier without any downstream unit, regardless of the vertical structure. As a result, the equilibrium acquisition price under vertical integration is identical to that under vertical separation, which is described as in (5).

The analysis of the acquisition decision by the upstream incumbent U under vertical integration is similar to the scenario of vertical separation: U will evaluate the potential benefits of an acquisition against the costs it may face if it decides not to acquire. We have the following result.

Proposition 3 *Under vertical integration, the equilibrium outcomes are:*

$$\begin{cases} (A, D) & if \quad k \leq k_I^U \text{ and } \frac{\rho^U}{\rho^E} \geq \delta \\ (A, ND) & if \quad \max\{k_I^U, \hat{k}_I\} < k < k_I^E \\ (NA, ND) & if \quad k > \max\{k_I^E, k_I^U\} \\ (NA, D) & if \quad k < \hat{k}_I \text{ and } \frac{\rho^U}{\rho^E} < \delta \end{cases}$$

where

$$\delta = \frac{\Pi^S(c') - \Pi^I(c)}{\Pi^I(c') - \Pi^I(c)}.$$

Proof. See Appendix. ■

The results in Proposition 3 are illustrated in Figure 3. To assess the potential welfare loss of killer acquisitions under vertical integration, it is helpful to define, from the social perspective, the cutoff values for UD and E to develop the innovations respectively:

$$k_I^{U^o} = \rho^U[W^I(c') - W^I(c)]$$

and

$$k_I^{E^o} = \rho^E[W^S(c') - W^I(c)]$$

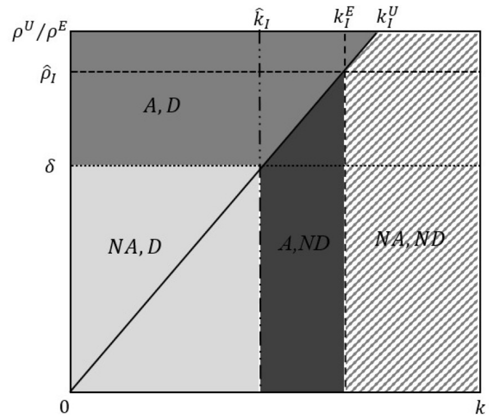

Figure 3: Equilibrium Outcomes under Vertical Integration.

where $W^I(c)$ and $W^I(c')$ represent the respective total welfare derived from selling the old input and the new input to the downstream firms under the vertical integration structure.

Proposition 4 *Under vertical integration, killer acquisition leads to welfare loss if* $k < k_I^{U^o}$ *for* $\frac{\rho^U}{\rho^E} \geq \delta^o$*, and* $k < k_I^{E^o}$ *for* $\frac{\rho^U}{\rho^E} < \delta^o$*, where*

$$\delta^o = \frac{W^S(c') - W^I(c)}{W^I(c') - W^I(c)}.$$

Proof. See Appendix. ■

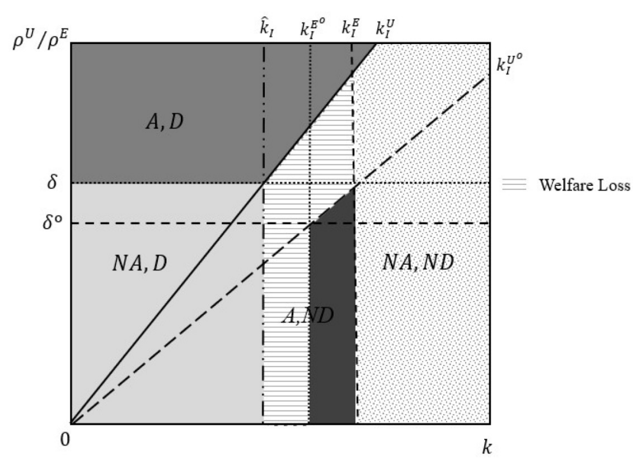

Figure 4: Welfare under Vertical Integration.

The reasons why killer acquisition may lead to welfare loss under vertical integration is as follows. In situations where $\left(\frac{\rho^U}{\rho^E}\right)$ is large, from social perspective, it would be beneficial for

U to acquire E and invest in technology development. However, in the market equilibrium, U may choose not to pursue technology development if the cost of innovation is too high because U does not take into account the positive externalities that arise from technological advancements and benefit consumers. In the situation where $\left(\frac{\rho^U}{\rho^E}\right)$ is small, it is socially more efficient for E to invest in developing superior technology, as long as the cost of innovation is not too high. However, if the innovation cost is significant, E would earn a low profit by choosing to innovate and thus U has the advantage of acquiring E at a low price. Consequently, in the market equilibrium, "killer acquisition" can take place and lead to a loss of overall welfare.

4.2 Vertical separation vs. integration

We are now in the position to compare market performance and welfare implications between vertical separation and integration. First note that in our model, as formally shown in Lin et al. (2020), for any given cost of producing the input, c, we have

$$\Pi^I(c) \leq \Pi^S(c)$$

with equality only when the final products are perfect substitutes or completely independent.[9] To understand this result, note that under vertical separation and with two-part tariff contracts, U is able to achieve the industry monopoly outcome. In contrast, under vertical integration, the upstream division of the integrated firm supplies its downstream division at the marginal cost of c, while the non-integrated firms purchase the required input at the marginal input price, w^I, set by UD. This constraint restricts UD from effectively "coordinating" downstream firms in making decisions, as it could do in the case of vertical separation. Consequently, the total profit obtained by UD is lower under vertical integration compared to what it could achieve under vertical separation.

Comparing the equilibrium outcomes and social welfare under vertical separation with integration, we have the following results.

Proposition 5 *Comparing to vertical separation, vertical integration decreases the probability of "killer acquisition zone" when* $\frac{\rho^U}{\rho^E} \leq \hat{\rho}^{SI}$ *where* $\hat{\rho}^{SI} = \frac{\Pi^S(c')-\Pi^I(c)}{\Pi^S(c')-\Pi^S(c)}$ *but increases the likelihood*

9 Chen and Ross (2003) also obtain similar results in their models.

of the "killer acquisition zone" when $\frac{\rho^U}{\rho^E} > \hat{\rho}^{SI}$.

Proof. See Appendix. ■

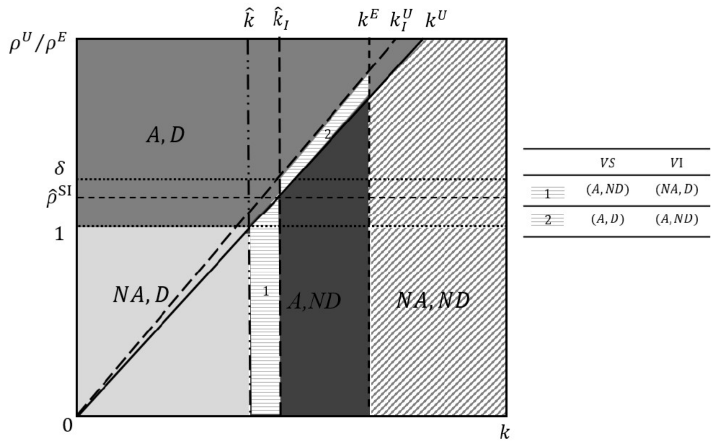

Figure 5: Comparison of Killer Acquisition Zones under Vertical Separation and Vertical Integration.

The intuition behind these statements can be explained as follows: If the probability of success for U is low ($\rho^U \leq \hat{\rho}^{SI}$), U will not choose to develop the superior technology and thus face the decision between "acquire to kill" or "no acquisition". From vertical separation to integration, the acquisition price b remains the same, but U experiences a reduction in profit from maintaining its position as the upstream supplier. As a result, the incentive for U to acquire E with the intention of abandoning the technology decreases. Therefore, the killer acquisition zone shrinks under vertical integration if $\rho^U \leq \hat{\rho}^{SI}$, as shown in area 1 of Figure 5.

On the other hand, if ρ^U is high ($\rho^U > \hat{\rho}^{SI}$), U has a strong incentive to develop the technology itself. In this case, U faces the choice between "acquire to develop" and "acquire to kill". The switch from vertical separation to integration diminishes U's incentive for "acquire to kill". However, vertical integration also reduces U's profit from developing the technology, and the impact is greater when ρ^U is higher. Consequently, vertical integration results in a larger profit decrease for U if it chooses to develop the technology compared to adopting the "acquire to kill" strategy. Thus, the killer acquisition zone expands under vertical integration if $\rho^U > \hat{\rho}^{SI}$, as shown in area 2 of Figure 5.

For welfare implications, we have the following results.

Proposition 6 *If the market demand is linear and downstream firms compete in Cournot fashion, welfare increases as the probability of the "killer acquisition zone" decreases. Conversely, welfare decreases as the probability of the "killer acquisition zone" increases if*

$$\rho^E \geq \frac{(1-\gamma)(2+(n-1)\gamma)^2(4+(n-1)\gamma)}{(1+(n-1)\gamma)[-\gamma^3(n-1)^2-4\gamma^2(2n+1)(n-1)+4\gamma(3n^2-6n-1)+16n]} \in (0, 0.5)$$

which is more likely to hold if ρ^E is high and the number of downstream firms is larger.

Proof. See Appendix. ■

If a change in market structure from vertical separation to integration leads to a shrink of killer acquisition zone, it brings benefits to society by increasing the likelihood of innovation materializing and leading to lower prices for the final product. Moreover, the shift in market structure from vertical separation to integration also leads to an increase in welfare. Consequently, if the killer acquisition zone decreases which is caused by a market structure change, total welfare increases. However, in the event that a shift in market structure from vertical separation to integration results in the expansion of the killer acquisition zone, it does not necessarily guarantee a decrease in total welfare since the market structure change also directly enhances welfare. Nevertheless, our analysis demonstrates that if the entrant has a high probability of a successful innovation or if the number of downstream firms is larger, the negative impact stemming from the enlarged killer acquisition zone becomes dominant. As a result, welfare declines as the killer acquisition zone expands.

5 Conclusion

In recent years, there has been a growing focus on killer acquisitions. While many acquisitions of start-ups have involved industries that are vertically related, there have been few studies conducted on this topic. Our research is among the first to examine the incentives of upstream incumbent firms to engage in killer acquisitions, as well as the implications for overall welfare. We have found that incumbent firms have less incentive to invest in developing new, advanced technologies, and thus may instead opt for an acquire-to-kill strategy when the investment costs are moderate and the synergy effects are weak. In addition, engaging in killer acquisitions can lead to a decrease in overall welfare, as incumbent firms fail to internalize the positive externalities that result from technological progress or terminate the development of more advanced technology that ideally should be developed by start-ups. When comparing vertical integration with separation, killer acquisition is more probable if the synergy effect is strong while the opposite is true if the synergy effect is weak. Our analysis highlights the importance of considering the vertical structure of an industry when studying the incentives of incumbent firms to kill advanced technologies of acquired entrants.

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Appendix

Proof of Proposition 1. We determine the conditions that lead to the four equilibrium outcomes by using the values of both the parameter k and the relative magnitudes of ρ^U and ρ^E.

(1) From section 3.3, equilibrium outcome (A, D) occurs in both $k > k^E$ and $k \leq k^E$ scenarios: (i) If $k > k^E$, then (A, D) is the equilibrium outcome if and only if $k \leq k^U$; (ii) if $k \leq k^E$, then (A, D) is the equilibrium outcome if and only if $k \leq k^U$ and $\frac{\rho^U}{\rho^E} \geq 1$.

From Lemma 1, $k^U \leq k^E$ if and only if $\frac{\rho^U}{\rho^E} \leq \hat{\rho}$. From (4), we have $\hat{\rho} \geq 1$. This also implies that (A, D) will occur only if $\frac{\rho^U}{\rho^E} \geq 1$, so we focus on $\frac{\rho^U}{\rho^E} \geq 1$ and further divide the value of $\frac{\rho^U}{\rho^E}$ into the following two cases: (a) If $1 \leq \frac{\rho^U}{\rho^E} \leq \hat{\rho}$, then $k^U \leq k^E$. Thus, in this case, $(k^E, +\infty) \cap (0, k^U] = \emptyset$, scenario (i) is not applicable. For scenario (ii), $k \leq k^E$ is not binding as $k^U \leq k^E$. Thus, (A, D) is the equilibrium outcome if $k \leq k^U$. (b) If $\frac{\rho^U}{\rho^E} > \hat{\rho}$, then $k^U > k^E$. Thus, in this case, for scenario (i), (A, D) is the equilibrium outcome if $k^E < k \leq k^U$. For scenario (ii), since $k^U > k^E$ and $\hat{\rho} > 1$, then (A, D) is the equilibrium outcome if $k \leq k^E$. Overall, (A, D) is the equilibrium outcome if $k \leq k^U$.

To summarize, for both $1 \leq \frac{\rho^U}{\rho^E} \leq \hat{\rho}$ and $\frac{\rho^U}{\rho^E} > \hat{\rho}$, we concluded that (A, D) is the equilibrium outcome if $k \leq k^U$. Therefore, (A, D) is the equilibrium outcome if and only if $k \leq k^U$ and $\frac{\rho^U}{\rho^E} \geq 1$.

(2) From section 3.3, equilibrium outcome (A, ND) occurs only if $k \leq k^E$. Also, from (7), we have that (A, ND) is the optimal strategy if $k > k^U$ and $k \geq \hat{k}$.

From (2) and (7), we have $k^U \geq \hat{k}$ if and only if $\frac{\rho^U}{\rho^E} \geq 1$. Also, $k^U \leq k^E$ if and only if $\frac{\rho^U}{\rho^E} \leq \hat{\rho}$. So, from (3), if $\frac{\rho^U}{\rho^E} < 1$, then $\max\{\hat{k}, k^U\} = \hat{k} < k^E$. If $1 \leq \frac{\rho^U}{\rho^E} < \hat{\rho}$, then $\max\{\hat{k}, k^U\} = k^U < k^E$. Hence, if $\frac{\rho^U}{\rho^E} < \hat{\rho}$, then the set $(\max\{\hat{k}, k^U\}, k^E) \neq \emptyset$. Therefore, (A, ND) is the equilibrium outcome if and only if $\max\{\hat{k}, k^U\} < k < k^E$.

(3) Equilibrium outcome (NA, ND) occurs only if $k > k^E$, as E will choose to develop if $k \leq k^E$.

If $\frac{\rho^A}{\rho^E} < \hat{\rho}$, then $k^E > k^U$, and thus (NA, ND) is the equilibrium outcome if $k > k^E$; If $\frac{\rho^A}{\rho^E} \geq \hat{\rho}$, then $k^E \leq k^U$. In this case, from (6), (A, D) is the equilibrium outcome if $k^E < k < k^U$, so (NA, ND) is the equilibrium outcome if $k > k^U$. Overall, (NA, ND) is the equilibrium outcome if $k > \max\{k^U, k^E\}$.

(4) Equilibrium outcome (NA, D) occurs only if $k \leq k^E$ since otherwise E will choose not to develop.

For $k \leq k^E$, (NA, D) may be the equilibrium outcome for both $k \leq k^U$ and $k > k^U$ cases. If $k \leq k^U$, then (NA, D) is the equilibrium outcome if (6) fails to hold, which is true when $\frac{\rho^U}{\rho^E} < 1$; if $k > k^U$, then (NA, D) is the equilibrium outcome if (7) fails to hold, which is true when $k < \hat{k}$. Overall, (NA, D) is the equilibrium outcome if $\frac{\rho^U}{\rho^E} < 1$ and $k < \hat{k}$. From (7), the set $k < \hat{k}$ is not empty as long as $\rho^E > 0$. ∎

Proof of Proposition 2. We first show that the first-best outcomes are as follows: (i) if $\frac{\rho^U}{\rho^E} \geq 1$

$$\begin{cases} (A, D) & if \quad k < k^{U^o} \\ (NA, ND) & if \quad k > k^{U^o} \end{cases}$$

and (ii) if $\frac{\rho^U}{\rho^E} < 1$

$$\begin{cases} (NA, D) & if \quad k < k^{E^o} \\ (NA, ND) & if \quad k > k^{E^o} \end{cases}.$$

To see this, consider the welfare under (A, D). In this case, U, as the developer, has a probability of ρ^U to successfully develop the new input at a fixed cost of k. Hence, the producer surplus (under vertical separation) is given by

$$PS^S(A, D) = \rho^U \Pi^S(c') + (1 - \rho^U)\Pi^S(c) - k$$

and consumer welfare (under vertical separation) is given by

$$CS^S(A, D) = \rho^U CS^S(c') + (1 - \rho^U) CS^S(c)$$

where $CS^S(c)$ and $CS^S(c')$ represent the consumer welfare when the old input and the new input, respectively, are sold in the downstream market under vertical separation. Then, $W^S(c) = \Pi^S(c) + CS^S(c)$ and $W^S(c') = \Pi^S(c') + CS^S(c')$, clearly, $W^S(c') > W^S(c)$. The total welfare under (A, D) is

$$W^S(A, D) = \rho^U W^S(c') + (1 - \rho^U) W^S(c) - k. \tag{11}$$

Second, we consider the total welfare of equilibrium outcome (NA, D). In this case, with probability ρ^E, E develops the new input successfully, replaces U as the upstream monopolist and earns $\Pi^S(c')$. With probability $1-\rho^E$, E does not succeed and U remains to be the only supplier in the upstream market supplying the old input. Thus, producer surplus is

$$PS^S(NA, D) = \rho^E \Pi^S(c') + (1-\rho^E)\Pi^S(c) - k$$

and consumer welfare is

$$CS^S(NA, D) = \rho^E CS^S(c') + (1-\rho^E) CS^S(c)$$

and total welfare is

$$W^S(NA, D) = \rho^E W^S(c') + (1-\rho^E) W^S(c) - k. \tag{12}$$

Third, in both (A, ND) and (NA, ND), the new input will never be developed and thus U continues supplying old inputs and obtains $\Pi^S(c)$ and consumer welfare is $CS^S(c)$. The total welfare is denoted as $W^S(c)$.

Comparing (11) with (12), we have $W^S(A, D) \geq W^S(NA, D)$ if and only if $\rho^U \geq \rho^E$. Besides, $W^S(A, D) \geq W^S(c)$ if and only if

$$k \leq k^{U^o} \equiv \rho^U[W^S(c') - W^S(c)].$$

Note that for any given ρ^U, we have $k^{U^o} > k^U \equiv \rho^U[\Pi^S(c') - \Pi^S(c)]$. Hence, if $\rho^U \geq \rho^E$ and $k \leq k^{U^o}$, then $W^S(A, D)$ is higher than $W^S(NA, D)$ and $W^S(c)$, thus the first-best outcome is (A, D); if $\rho^U \geq \rho^E$ and $k > k^{U^o}$, then $W^S(c) > W^S(A, D) \geq W^S(NA, D)$, the first-best outcome is (A, ND) or (NA, ND).

From (11) and (12), if $\rho^U < \rho^E$, then $W^S(A, D) < W^S(NA, D)$. Besides, $W^S(NA, D) \geq W^S(c)$ if and only if

$$k \leq k^{E^o} \equiv \rho^E[W^S(c') - W^S(c)].$$

Notice that $k^{E^o} > \hat{k} = \rho^E[\Pi^S(c') - \Pi^S(c)]$. Thus, (NA, D) is the first-best outcome if $\rho^U < \rho^E$ and $k \leq k^{E^o}$. However, if $\rho^U < \rho^E$ and $k > k^{E^o}$, then $W^S(A, D) < W^S(NA, D) < W^S(c)$. Thus, the first-best outcome is (A, ND) or (NA, ND).

To see whether there are welfare loss caused by killer acquisitions, notice that if $\rho^U \geq \rho^E$ and $k^U < k < k^{U^o}$, the equilibrium outcome is either (A, ND) or (NA, ND). However, the first-best outcome is (A, D); If $\rho^U < \rho^E$ and $\hat{k} < k < k^{E^o}$, the equilibrium outcome is (A, ND). However, the first-best outcome is (NA, D). Thus, given that the equilibrium outcome is (A, ND), the social inefficiency occurs when $k < k^{U^o}$ for $\rho^U \geq \rho^E$ and $k < k^{E^o}$ for $\rho^U < \rho^E$. ■

Proof of Lemma 2. (i) Suppose that UD acquires E, then UD chooses to develop the new technology if and only if

$$\rho^U \Pi^I(c') + (1 - \rho^U)\Pi^I(c) - k \geq \Pi^I(c)$$

which holds if $k \leq k_I^U = \rho^U[\Pi^I(c') - \Pi^I(c)]$.

(ii) Suppose that E is not acquired by UD, then E chooses to develop the new technology if and only if

$$\rho^E \Pi^S(c') - k \geq 0$$

which holds if $k \leq k_I^E = \rho^E \Pi^S(c')$.

(iii) $k_I^U \leq k_I^E$ if and only if

$$\rho^U \left[\Pi^I\left(c'\right) - \Pi^I\left(c\right)\right] \leq \rho^E \Pi^S(c')$$

which holds if and only if

$$\frac{\rho^U}{\rho^E} \leq \hat{\rho}_I \equiv \frac{\Pi^S\left(c'\right)}{\Pi^I\left(c'\right) - \Pi^I\left(c\right)}.$$

■

Proof of Proposition 3. The characterization of equilibrium under vertical integration follows a similar approach to that of vertical separation.

(1) If $k > k_I^E$, UD will acquire E and develop the innovation if $k \leq k_I^U$. (2) If $k \leq k_I^E$, we also consider the following two cases: (a) if $k \leq k_I^U$, then U will develop the innovation given that the acquisition has already happened. Then, U will proceed the acquisition if and only if

$$\rho^U \Pi^I(c') + (1 - \rho^U)\Pi^I(c) - k - b \geq (1 - \rho^E)\Pi^I(c) \tag{13}$$

where b is the same as that in (5). As a result, (13) holds if and only if

$$\frac{\rho^U}{\rho^E} \geq \frac{\Pi^S(c') - \Pi^I(c)}{\Pi^I(c') - \Pi^I(c)} \equiv \delta.$$

Hence, (A, D) is the equilibrium outcome if and only if $k \leq k_I^U$ and $\frac{\rho^U}{\rho^E} \geq \delta$. In contrast, if (13) does not hold, then UD will find such acquisition is unprofitable and thus (NA, D) is the equilibrium outcome.

(b) If $k > k_I^U$, then U will not develop the innovation after an acquisition. In this case, U will conduct such killer acquisition if and only if

$$\Pi^I(c) - b \geq (1 - \rho^E)\Pi^I(c) \tag{14}$$

which hold if $k \geq \hat{k}_I$, where

$$\hat{k}_I = \rho^E[\Pi^S(c') - \Pi^I(c)]. \tag{15}$$

Hence, (A, ND) is the equilibrium outcome if both $k > k_I^U$ and $k \geq \hat{k}_I$ hold. Notice that if $\frac{\rho^U}{\rho^E} < \delta$, then $k_I^U < \hat{k}_I < k_I^E$; and if $\frac{\rho^U}{\rho^E} \geq \delta$, then $k_I^U \geq \hat{k}_I < k_I^E$. Hence, (A, ND) is the equilibrium outcome if and only if $\max\{k_I^U, \hat{k}_I\} < k \leq k_I^E$. In contrast, if (14) does not hold, then UD will find killer acquisition is unprofitable and thus (NA, D) is the equilibrium outcome.

Thus, (NA, D) only occurs when $k \leq k_I^E$. It is the equilibrium outcome if $k \leq k_I^U$ and $\frac{\rho^U}{\rho^E} < \delta$, or $k > k_I^U$ and $k < \hat{k}_I$. Thus, (NA, D) is the equilibrium outcome if $k < \hat{k}_I$ and $\frac{\rho^U}{\rho^E} < \delta$.

Moreover, (NA, ND) is the equilibrium outcome if $k > k_I^E$ and $k > k_I^U$. From Lemma 2, we have $k_I^U \leq k_I^E$ if $\frac{\rho^U}{\rho^E} \leq \hat{\rho}_I$, and $k_I^U > k_I^E$ if $\frac{\rho^U}{\rho^E} > \hat{\rho}_I$. Hence, (NA, ND) is the equilibrium outcome if $k > \max\{k_I^E, k_I^U\}$. ■

Proof of Proposition 4. We first show that the first-best outcomes under vertical integration are as follows: (i) if $\frac{\rho^U}{\rho^E} \geq \delta^o$, then

$$\begin{cases} (A, D) & if \quad k < k_I^{U^o} \\ (NA, ND) & if \quad k > k_I^{U^o} \end{cases}$$

and (ii) if $\frac{\rho^U}{\rho^E} < \delta^o$

$$\begin{cases} (NA, D) & if \quad k < k_I^{E^o} \\ (NA, ND) & if \quad k > k_I^{E^o} \end{cases}.$$

To see this, consider the total welfare when the equilibrium outcome is (A, D) is

$$W^I(A, D) = \rho^U W^I(c') + (1 - \rho^U)W^I(c) - k \tag{16}$$

and total welfare when the equilibrium outcome is (NA, D) is

$$W^I(NA, D) = \rho^E W^S(c') + (1 - \rho^E)W^I(c) - k. \tag{17}$$

Similar to the vertical separation case, we can show total welfare under equilibrium outcomes (A, ND) and (NA, ND) are $W^I(c)$ since the new technology will not be developed. By comparing (16) and (17), we have $W^I(A, D) \geq W^I(NA, D)$ if and only if

$$\frac{\rho^U}{\rho^E} \geq \frac{W^S(c') - W^I(c)}{W^I(c') - W^I(c)} \equiv \delta^o$$

also, $W^I(A, D) \geq W^I(c)$ if and only if

$$k \leq \rho^U[W^I(c') - W^I(c)] \equiv k_I^{U^o}.$$

Note that for any ρ^U, we have $k_I^{U^o} > k_I^U \equiv \rho^U[\Pi^I(c') - \Pi^I(c)]$. Moreover, $W^I(NA, D) \geq W^I(c)$ if and only if

$$k \leq \rho^E[W^S(c') - W^I(c)] \equiv k_I^{E^o}.$$

Notice that $k_I^{U^o} = k_I^{E^o}$ when $\frac{\rho^U}{\rho^E} = \delta^o$. If $\frac{\rho^U}{\rho^E} \geq \delta^o$, then $k_I^{U^o} \geq k_I^{E^o}$ and if $\frac{\rho^U}{\rho^E} < \delta^o$, then $k_I^{U^o} < k_I^{E^o}$. Hence, if $\frac{\rho^U}{\rho^E} \geq \delta^o$, then the first-best outcome is (A, D) if $k \leq k_I^{U^o}$, while the first-best outcome is (A, ND) or (NA, ND) if $k > k_I^{U^o}$. On the contrary, if $\frac{\rho^U}{\rho^E} < \delta^o$, then the first-best outcome is (NA, D) if $k \leq k_I^{E^o}$, while the first-best outcome is (A, ND) or (NA, ND) if $k > k_I^{E^o}$.

Therefore, the equilibrium outcome (A, ND), which occurs when $\max\{k_I^U, \hat{k}_I\} < k < k_I^E$, leads to welfare loss if $k \leq k_I^{U^o}$ for $\frac{\rho^U}{\rho^E} \geq \delta^o$ and $k \leq k_I^{E^o}$ if $\frac{\rho^U}{\rho^E} < \delta^o$. ■

Proof of Proposition 5. (1) First, we show how the equilibrium change from VS to VI under $\frac{\rho^U}{\rho^E} \leq \hat{\rho}^{SI}$. From (3), (7), and (15), we have

$$\hat{k} = \rho^E[\Pi^S(c') - \Pi^S(c)] < \hat{k}_I = \rho^E[\Pi^S(c') - \Pi^I(c)] < k^E.$$

Since $k^U = \rho^U[\Pi^S(c') - \Pi^S(c)]$ and $\Pi^S(c) > \Pi^I(c)$, we have $\hat{k}_I = k^U$ when $\frac{\rho^U}{\rho^E} = \hat{\rho}^{SI}$, where

$$\hat{\rho}^{SI} = \frac{\Pi^S(c') - \Pi^I(c)}{\Pi^S(c') - \Pi^S(c)} \in (1, \delta).$$

If $\frac{\rho^U}{\rho^E} < \hat{\rho}^{SI}$, then $\hat{k}_I > k^U$. From Proposition 1, since $\hat{k} < \hat{k}_I < k^E$, then if $\max\{k^U, \hat{k}\} < k < \hat{k}_I$, the equilibrium outcome is (A, ND) under VS. From Proposition 3, if $k < \hat{k}_I$ and $\frac{\rho^U}{\rho^E} < \hat{\rho}^{SI} < \delta$, then the equilibrium outcome is (NA, D) under VS. Therefore, $\max\{k^U, \hat{k}\} < k < \hat{k}_I$ is a "killer acquisition zone" under VS but not under VI.

(2) Second, we show how the equilibrium outcomes change from VS to VI under $\frac{\rho^U}{\rho^E} > \hat{\rho}^{SI}$. If $\frac{\rho^U}{\rho^E} > \hat{\rho}^{SI}$, then $\hat{k}_I < k^U$. In this case, from Propositions 1 and 3, if $\hat{\rho}^{SI} < \frac{\rho^U}{\rho^E} < \delta$ and $\hat{k}_I \leq k \leq k^U$, the equilibrium outcome is (A, D) under VS and is (A, ND) under VI; if $\frac{\rho^U}{\rho^E} \geq \delta$ and $k_I^U < k < \max\{k^U, k^E\}$, then the equilibrium outcome is (A, D) under VS and is (A, ND) under VI. Therefore, if $\frac{\rho^U}{\rho^E} > \hat{\rho}^{SI}$, the parameter area $\max\{\hat{k}_I, k_I^U\} \leq k \leq \min\{k^E, k^U\}$ is a "killer acquisition zone" under VI but not under VS. ■

Proof of Proposition 6. To examine the effect of market structure on welfare, we consider a setting with linear consumer demand functions and Cournot competition. Specifically, the demand function for firm i is $p_i = a - q_i - \gamma \sum_{i \neq j}^n q_j$. Under vertical separation (VS), given the input price ω set by U, the downstream competition lead to the symmetric Cournot equilibrium

$$q_i = \frac{a - \omega}{2 + \gamma(n-1)} \tag{18}$$

and

$$p_i = \frac{a + [1 + \gamma(n-1)]\omega}{2 + \gamma(n-1)}. \tag{19}$$

Given $q_i(\omega)$ and $p_i(\omega)$, U chooses the input price ω to maximize $\Pi^S(\omega) = \sum_{i=1}^n (p_i(w) - c)q_i(w)$, which results in the following optimal input price

$$\omega^S(c) = \frac{\gamma(n-1)(a-c)}{2(1 + \gamma(n-1))} + c$$

and also equilibrium quantities and prices of final products

$$q_i^S(c) = \frac{a - c}{2(1 + \gamma(n-1))} \qquad \text{and} \qquad p_i^S(c) = \frac{a + c}{2}.$$

Thus, the industry profit under VS is

$$\Pi^S(c) = \frac{n(a-c)^2}{4(1+\gamma(n-1))}. \tag{20}$$

Hsu and Wang (2005) derive expressions for consumer surplus under differentiated goods oligopoly (Also used by Cunningham et al.(2021)). Using their expressions:

$$CS = \frac{1-\gamma}{2}\sum_{i=1}^{n} q_i^2 + \frac{\gamma}{2}\Big(\sum_{i=1}^{n} q_i\Big)^2, \tag{21}$$

we obtain

$$CS^S(c) = \frac{n(a-c)^2}{8(1+\gamma(n-1))}. \tag{22}$$

Summing (20) and (22) gives the total welfare under vertical separation:

$$W^S(c) = \frac{3n(a-c)^2}{8(1+\gamma(n-1))}. \tag{23}$$

Under vertical integration (VI), the upstream firm UD supplies its downstream subsidiary at c while supplying the other non-integrated downstream firms at ω. UD chooses ω to maximize $\Pi^I(c,\omega) = (p_1(c,\omega)-c)q_1(c,\omega) + \sum_{i\neq 1}^{n}(p_i(\omega,c)-c)q_i(\omega,c)$, and the solution is

$$\omega^I(c) = \frac{(a-c)(n-1)(2-\gamma)^2\gamma}{8+8(n-2)\gamma-6(n-1)\gamma^2} + c.$$

The equilibrium quantities and prices are given by

$$\begin{cases} q_1^I(c) = \frac{(a-c)[4-\gamma(6-n(2-\gamma)-\gamma)]}{8+8(n-2)\gamma-6(n-1)\gamma^2} \\ q_i^I(c) = \frac{2(a-c)(1-\gamma)}{4-\gamma[8-3\gamma-n(4-3\gamma)]} \end{cases}$$

and

$$\begin{cases} p_1^I(c) = \frac{(a-c)[4-\gamma(6-n(2-\gamma)-\gamma)]}{8+8(n-2)\gamma-6(n-1)\gamma^2} + c \\ p_2^I(c) = \frac{(a-c)[4+\gamma(n(2-\gamma)^2+(4-\gamma)\gamma-8)]}{8+8(n-2)\gamma-6(n-1)\gamma^2} \end{cases}.$$

Thus, the industry profit under VI is

$$\Pi^I(c) = \frac{(a-c)^2[n(2-\gamma)^2-\gamma^2]}{4[4-\gamma(8-3\gamma+n(3\gamma-4))]} \tag{24}$$

also, consumer surplus under VI is given by

$$CS^I(c) = \frac{(a-c)^2[4n-4\gamma-3(n-1)\gamma^2]}{8[4-\gamma(8-3\gamma+n(3\gamma-4))]}. \tag{25}$$

Summing (24) and (25) gives the total welfare under vertical integration:

$$W^I(c) = \frac{(a-c)^2[12n - (4-\gamma)\gamma - n\gamma(8+\gamma)]}{8[4-\gamma(8-3\gamma+n(3\gamma-4))]}. \tag{26}$$

The innovation is drastic implies that $\omega^S(c-d) < c$, if and only if

$$d > \frac{\gamma(a-c)(n-1)}{2+(n-1)\gamma} = \hat{d}. \tag{27}$$

Next, we first show how the decrease in the probability of "killer acquisition zone" affect social welfare. If $\frac{\rho^U}{\rho^E} \leq \hat{\rho}^{SI}$, the equilibrium outcome is (A, ND) under VS while becoming (NA, D) under VI. In this case, the total welfare is $W^S(c)$ under equilibrium (A, ND), and the total welfare is $\rho^E W^S(c') + (1-\rho^E)W^I(c) - k$ under equilibrium (NA, D). Since $k < \hat{k}_I$ and $\hat{k}_I < k_I^{E^o}$, then $k < k_I^{E^o} = \rho^E[W^S(c') - W^I(c)]$. Thus, $\rho^E W^S(c') + (1-\rho^E)W^I(c) - k > W^I(c)$. From (23) and (26), for any $\gamma \in (0,1)$, we have

$$W^I(c) - W^S(c) = \frac{(a-c)^2[4+(n-1)\gamma](1-\gamma)\gamma(n-1)}{8[1+(n-1)\gamma][4(n-2)\gamma - 3(n-1)\gamma^2 + 4]}.$$

Since $\gamma \in (0,1)$ and $n \geq 2$, then $4(n-2)\gamma - 3(n-1)\gamma^2 + 4 > 4(n-2)\gamma - 3(n-1)\gamma + 4 = (n-5)\gamma + 4 > 0$. Hence, $W^I(c) - W^S(c) > 0$. It follows that $\rho^E W^S(c') + (1-\rho^E)W^I(c) - k > W^I(c) > W^S(c)$. The equilibrium change from (A, ND) to (NA, D) increases total welfare.

Second, we show how the increase in the probability of "killer acquisition zone" affect social welfare. If $\frac{\rho^U}{\rho^E} > \hat{\rho}^{SI}$, the equilibrium outcome is (A, D) under VS while becoming (A, ND) under VI. In this case, the total welfare is $\rho^U W^S(c') + (1-\rho^U)W^S(c) - k$ under (A, D) and is $W^I(c)$ under (A, ND). The former one is higher if and only if

$$k < \rho^U[W^S(c') - W^S(c)] + W^S(c) - W^I(c). \tag{28}$$

The case we consider satisfies $k < k^U = \rho^U[\Pi^S(c') - \Pi^S(c)]$. If the RHS in (28) is larger than k^U, then (28) will hold. This situation is true when

$$\rho^U[CS^S(c') - CS^S(c)] > W^I(c) - W^S(c). \tag{29}$$

Since $\frac{\rho^U}{\rho^E} > \hat{\rho}^{SI}$, then if $\rho^E\hat{\rho}^{SI} > \frac{W^I(c)-W^S(c)}{CS^S(c')-CS^S(c)}$, we have $\rho^U > \rho^E\hat{\rho}^{SI} > \frac{W^I(c)-W^S(c)}{CS^S(c')-CS^S(c)}$ and thus (29) will hold.

From (20), (22), (23), (24) and (26), $\rho^E \hat{\rho}^{SI} > \frac{W^I(c)-W^S(c)}{CS^S(c')-CS^S(c)}$ can be rewritten as

$$\rho^E > \frac{(a-c)^2(1-\gamma)[4+(n-1)\gamma]\gamma(n-1)}{nd[2(a-c)+d][4(n-2)\gamma-3(n-1)\gamma^2+4]+(a-c)^2(n-1)^2\gamma^2(1-\gamma)}. \quad (30)$$

Denote the RHS of (30) as $\Delta(d)$. Since $4(n-2)\gamma - 3(n-1)\gamma^2 + 4 > 0$, then $\frac{\partial \Delta(d)}{\partial d} < 0$. Hence, under drastic innovation ($d > \hat{d}$), if $\rho^E > \Delta(\hat{d})$, then (30) will hold, where

$$\Delta(\hat{d}) = \frac{(1-\gamma)(2+(n-1)\gamma)^2(4+(n-1)\gamma)}{(1+(n-1)\gamma)[-\gamma^3(n-1)^2-4\gamma^2(2n+1)(n-1)+4\gamma(3n^2-6n-1)+16n]}.$$

We can show that $\Delta(\hat{d}) > 0$. It is easy to see that all other terms in the numerator and denominator in the above expression is positive except for $-\gamma^3(n-1)^2-4\gamma^2(2n+1)(n-1)+4\gamma(3n^2-6n-1)+16n$. Notice that $-\gamma^3(n-1)^2-4\gamma^2(2n+1)(n-1)+4\gamma(3n^2-6n-1)+16n > -\gamma(n-1)^2-4\gamma(2n+1)(n-1)+4\gamma(3n^2-6n-1)+16n = 16n+[3(n-6)n-1]\gamma$. Since $16n+[3(n-6)n-1]\gamma$ increases in n and thus the minimum value of it is $32-25\gamma > 0$ when $n = 2$. Thus, we have $-\gamma^3(n-1)^2-4\gamma^2(2n+1)(n-1)+4\gamma(3n^2-6n-1)+16n > 32-25\gamma > 0$. Hence, $\Delta(\hat{d}) > 0$.

We can also show that $\Delta(\hat{d}) \leq 0.5$. First, when $n = 2$, we have $\Delta(\hat{d})$ decreases in γ, then

$$\Delta(\hat{d}; n=2) = \frac{(1-\gamma)(2+\gamma)^2(4+\gamma)}{(1+\gamma)[32-\gamma(4+\gamma(20+\gamma))]} \leq \Delta(\hat{d}; n=2, \gamma=0) = 0.5.$$

Next, we show that as n increases, $\Delta(\hat{d})$ decreases. To achieve this, we differentiate $\Delta(\hat{d})$ with respect to n.

$$\frac{\partial \Delta(\hat{d})}{\partial n} = -\frac{(1-\gamma)[2+(n-1)\gamma][128+\gamma(\tau(\gamma)+5(n-1)^3\gamma^4)]}{[1+(n-1)\gamma]^2[(2-\gamma)^2\gamma-2n(2-\gamma)(4-\gamma(4+\gamma))-n^2\gamma(12-\gamma(8+\gamma))]^2} \quad (31)$$

where

$$\tau(\gamma) = 2(n-1)^2(31-28n)\gamma^3+4(n-1)[65+n(17n-94)]\gamma^2+8[61+3n(13n-36)]\gamma+384n-416.$$

It is straightforward to see that except for $\tau(\gamma)$, all other terms in the numerator and denominator of (31) are positive. Hence, in order to demonstrate that $\frac{\partial \Delta(\hat{d})}{\partial n} < 0$, it suffices to show that $\tau(\gamma) > 0$. To accomplish this, we proceed to show the following: $\tau(0) > 0$, $\tau(1) > 0$, and that $\tau(\gamma)$ initially increases and then decreases in $\gamma \in (0,1)$.

First, we can show that for any $n \geq 2$, $\tau(0) = 384n - 416 > 0$ and $\tau(1) = 6(3+n)(2n^2+n-7) > 0$. Second, we examine the monotonicity of $\tau(\gamma)$ for the following two cases: (1) If

$2 \le n \le 4.7$, then $\tau'(0) = 8[61+3n(13n-36)] > 0$ and $\tau'(1) = 154-2n[66+n(27+16n)] < 0$, and also $\tau''(\gamma) \le 0$ for any $\gamma \in (0,1)$. It follows that $\tau(\gamma)$ initially increases and then decreases in $\gamma \in (0,1)$; (2) If $n > 4.7$, we still have $\tau'(0) > 0$ and $\tau'(1) < 0$. But $\tau''(\gamma) > 0$ when γ is small and $\tau''(\gamma) < 0$ when γ is large, and also $\tau'''(\gamma) = -12(n-1)^2(28n-31) < 0$ for any $n \ge 2$, implying that $\tau'(\gamma)$ first increases and then decreases in γ. Since $\tau'(0) > 0$ and $\tau'(1) < 0$, we still have that $\tau(\gamma)$ initially increases and then decreases in $\gamma \in (0,1)$.

Therefore, for any $n \ge 2$, we have $\tau(\gamma) > 0$ for $\gamma \in (0,1)$. Hence, $\frac{\partial \Delta(\hat{d})}{\partial n} < 0$. Since $\Delta(\hat{d})$ decreases in n and $\Delta(\hat{d}) \le 0.5$ if $n = 2$, then $\Delta(\hat{d}) < 0.5$ if $n > 2$. Therefore, for $n \ge 2$, we have $0 < \Delta(\hat{d}) \le 0.5$.

Since $\Delta(\hat{d})$ decreases in n, then $\rho^E > \Delta(\hat{d})$ is more likely to hold as n increases and also (28) is more likely to be satisfied. Hence, the social welfare is lower under VI if $\rho^E > \Delta(\hat{d})$ which is more likely to hold if n is larger. ■