Dynamic portfolio rebalancing with safe-haven assets

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Abstract

The main objective of this dissertation is to evaluate the role of safe-haven assets (SHAs) in enhancing the performance of an international portfolio (IP) during financial crises. The study used daily price data from 2007 to 2024 and focused on three major financial crises: The Global Financial Crisis (GFC), the Covid-19 pandemic, and the Russia-Ukraine (RU) war. The methodology combined a Conditional Value at Risk (CVaR) optimisation with a periodic rebalancing strategy to assess the impact of including gold as the SHA on portfolio performance. The results showed that including the SHA, like gold, significantly improved portfolio performance during all three crises. Notably, portfolio performance decreased by -0.741% during the GFC, and portfolio performance decreased by -0.179% during COVID-19 when gold was included in the IP. In contrast, during the RU war, the highest portfolio value increased by 2.99% when gold was included in the IP. During the GFC, the portfolio that included gold experienced a reduced volatility of 20.589%, compared to that of a portfolio without gold, which had a volatility of 25.290%. The IP with SHAs showed a maximum drawdown, a critical measure of downside risk, of -0.38069, compared to the -0.55929 maximum drawdown of the IP without SHAs. Similar improvements were observed during the COVID-19 pandemic, where portfolio volatility decreased to 11.436%, and the highest Sharpe ratio was 0.301. The policy recommendation is that institutional investors incorporate SHAs such as gold into their portfolios. This strategy enhances portfolio resilience and reduces downside risks, ensuring better risk-adjusted returns.

1. Introduction

Portfolio optimisation, a cornerstone of financial management, focuses on constructing portfolios that maximise returns for a given risk level or minimise risk for a desired return. Grounded in Modern Portfolio Theory (MPT) by Markowitz (1952), it employs mathematical models to identify the optimal asset allocation that achieves the best risk-return trade-off. By analysing historical returns, variances, and covariances, investors can determine the efficient frontier, representing portfolios with the highest returns for a given risk (Markowitz, 1959). The aim is to build a diversified portfolio that enhances financial performance by balancing expected returns.

Maintaining an optimised portfolio necessitates ongoing adjustments through rebalancing. Market fluctuations can cause deviations from the optimal allocation, necessitating periodic realignments to meet risk and return objectives (Binsbergen & Brandt, 2007). Rebalancing strategies can be time-based or triggered by significant deviations from target weights, preserving the benefits of optimisation while preventing over-concentration in specific assets (Daryanani, 2008).

An international portfolio (IP) comprising investments across diverse markets introduces additional rebalancing complexities due to currency risk, geopolitical events, and market-specific factors. These portfolios offer enhanced diversification benefits but require a more dynamic rebalancing approach to manage the challenges posed by global markets (Solnik, 1974; De Santis & Gerald, 1997). International diversification impacts risk-return dynamics and rebalancing frequency, necessitating strategies aligning with long-term investment goals (Errunza, 1977).

Safe-haven assets (SHAs) are critical in IPs, particularly during financial crises. SHAs, such as gold, government bonds, and select currencies, retain or increase value during market turmoil, offering stability amidst uncertainty (Baur & McDermott, 2010). However, their effectiveness varies based on the nature of the crisis and market exposures (Beckmann et al., 2015). SHAs can mitigate increased correlations among traditional assets during crises, preserving portfolio stability (Longin & Solnik, 2001). Identifying the appropriate SHA for an IP requires significant analysis, as their protective qualities are context-dependent.

Previous studies have analysed the role of a safe haven in a dynamic IP (see Yae & Tian, 2024; Hoque et al., 2023; Kocaarslan et al., 2017; Robiyanto et al., 2020; Chevallier, 2023). However, few studies have investigated the impact of SHAs assets on portfolio rebalancing. For example, Belhassine and Karamti (2021) employed the DCC-GARCH to rebalance a portfolio of the Chinese stock market with commodity markets during COVID-19. Their results showed that gold lost its SHA property, and oil and Bitcoin acted as SHAs. A study by Hung et al. (2024), contrary to Belhassine and Karamti's study, showed that gold acts as a SHA when Bitcoin destroys value when employing the volatility timing rebalancing strategy on US markets during COVID-19. A study by Karamti and Belhassine (2022) employed a wavelet coherency rebalancing strategy to investigate the impact of gold and cryptocurrencies on US portfolios during COVID-19. Results showed that gold and cryptocurrencies serve as SHAs. On the other hand, a study by Thampanya et al. (2020) concluded that adding gold or cryptocurrency (Bitcoin) to a stock portfolio does not enhance the risk-adjusted return, and thus, they do not act as SHAs. A study by Yousfi et al. (2024) employed the DCC-GARCH to rebalance equity markets, commodities, and cryptocurrencies during the Russia-Ukraine (RU) war. It showed that gold and Bitcoin are safe long-term investments, while oil is the least effective SHA. Rubbaniy et al. (2024) employed the Time-Varying Parameter Vector Auto Regressive (TVP-VAR) to hedge and rebalance a portfolio during COVID-19 and the RU war. They concluded that including commodities such as crude oil, gas, gold, and biofuel increased portfolio performance during the crises. At the same time, Alshammari and Obeid (2023) studied the hedging and rebalancing effects of gold, silver, and oil using range-based DCC and return-based DCC. They concluded that portfolio performance improved when using the range-based DCC. Diaz et al. (2022) investigated the impact of bonds, gold, oil, bitcoin, and treasury bonds on ESG and traditional equity portfolios by employing different rebalancing strategies, AR-GARCH and DCC skew-Student copulas, during COVID-19. They revealed that gold and oil improve portfolio performance.

Portfolio rebalancing during financial crises presents a critical challenge for managing international portfolios. Crises such as the 2008 Global Financial Crisis (GFC) and the 2020 COVID-19 pandemic disrupt markets, increase volatility, and alter asset correlations, diminishing the effectiveness of traditional diversification. These periods underscore the need for dynamic rebalancing strategies that adapt to changing market conditions (Baur & McDermott, 2010; Fahlenbrach et al., 2021). SHA, including gold, US Treasury bonds, and Swiss franc, are integral to mitigating risks during market turbulence.

Gold, for instance, is often viewed as the universal safe haven, mainly owing to its long-standing role as a store of value and its independence from any single government or economy. However, its performance as a safe haven can fluctuate depending on the type of crisis. During broad-based economic crises or periods of currency instability, investors often flock to gold as a hedge against inflation or currency devaluation. However, gold may not provide the same stability in crises rooted in commodity price shocks or sector-specific downturns. For instance, during a commodity sector crisis, gold prices might experience volatility as demand patterns shift, reducing its effectiveness as a safe haven.

Similarly, in a debt-driven crisis, US Treasury bonds may outperform gold as the safer option, benefiting from the US government's and the dollar's perceived stability. Alternatively, during regional crises, such as those affecting the Eurozone, certain currencies, such as the Swiss franc, might be preferred as safe havens because investors view the country, such as Switzerland, as politically neutral and economically stable. Thus, while assets like gold are generally stable, their reliability as safe havens can be context-dependent, shaped by the nature and scope of each crisis (see Tronzano, 2020; Thuy et al., 2023; Arneric & Paic, 2023).

Another issue in portfolio conservation during a crisis period is that static asset allocations may fail to account for rapid changes in asset correlations and market conditions during financial crises, highlighting the need for more dynamic rebalancing approaches (Forbes & Rigobon, 2002). In this context, this study contributes to the literature on IP diversification by examining how the role of gold as a SHA may vary depending on the financial or economic crisis. Furthermore, to enhance portfolio allocation strategies, the study extends the application from static to dynamic portfolio allocation, focusing on portfolio rebalancing as a means to determine the optimal dynamic allocation of an IP that includes the SHA, particularly gold. The study proposes introducing portfolio rebalancing based on CVaR.

While there is extensive research on portfolio rebalancing and risk management strategies, fewer studies focus on the performance of these strategies during financial crises, particularly in the context of IPs exposed to various geopolitical, economic, and currency risks (for example, Bohn & Tesar, 1996; Gyntelberg et al., 2014; Camanho et al., 2022). Periodic rebalancing is a common strategy used to maintain target asset allocations. Its combination with advanced risk measures like Conditional Value at Risk (CVaR) has not been explored in crisis contexts. This study contributes by introducing a tactical rebalancing trigger based on CVaR breaches. Instead of pure periodic (daily, monthly, quarterly, annually) rebalancing, the portfolio could be rebalanced dynamically when the portfolio CVaR exceeds a pre-specified risk threshold. This would create a hybrid rebalancing approach that combines both time-based and event-driven strategies, leading to more proactive risk management during periods of heightened uncertainty. The CVaR provides a more robust risk management tool focusing on loss distributions' tail-end. This offers practical guidance for institutional investors managing large globally diversified portfolios on how to manage portfolio stability, control downside risk, and optimise performance. According to our knowledge, this is the first study to do this.

This study investigates the interplay between financial crises, SHAs, and portfolio rebalancing by addressing whether SHAs, such as gold, behave consistently across different crises, how CVaR-based portfolio rebalancing can be formulated, and how such dynamic strategies compare to equally weighted portfolios. It hypothesizes that SHA effectiveness varies by type, CVaR-based rebalancing enhances risk management by mitigating extreme losses, and portfolios employing CVaR-based weights outperform equally weighted portfolios regarding risk-adjusted returns. This study highlights the limitations of static allocation methods, like equally weighted portfolios, in managing downside risk during market disruptions. By investigating dynamic rebalancing strategies using CVaR, this study aims to demonstrate how adaptive approaches and SHAs can enhance portfolio resilience and performance.

The findings offer investors, portfolio managers, and policymakers insights into optimising portfolios to withstand financial crises.

The rest of the study presents the literature review of previous studies, the data and methodology used, the results obtained, and the conclusion and pointers for future research.

2. Literature Review

This literature review explores on portfolio rebalancing and SHAs. A broad mix of studies exists on portfolio rebalancing and SHAs under different financial crises, with various methods employed to analyse this relationship. The literature reviewed is grouped according to the objectives and similar results for each study in this section. The literature first discusses the papers on portfolio rebalancing, SHAs and IPs, and later discusses the papers on portfolio rebalancing and SHAs.

Kocaarslan et al. (2017) examine how portfolio rebalancing impacts time-varying conditional correlations between BRICS and US stock markets affected by predictions of volatility in gold, oil, currencies, and the US stock market. The paper used VAR Asymmetric DCC-EGARCH and VAR-DCC-EGARCH models on data spanning from 2011 to 2015. Their results highlight that there are few chances of diversification during high correlation periods, global investors view gold as a SHA while dynamic correlations typically increase during strong correlations after an increase in oil volatility. A study by Karampti and Belhassaine (2022) using a wavelet coherence rebalancing strategy to examine the relationship between main major financial markets: S&P500, CAC40, DAX, Nikkei 225, FTSE and Shanghai SE, and the COVID-19 pandemic. They discovered that investing in cryptocurrency, gold, and SSE is safer. The primary aim of the study by Robiyanto et al. (2020) was to evaluate the effectiveness of put replication, gold, and oil in hedging equities within the five countries (Indonesia, Malaysia, Singapore, Thailand, and the Philippines) from 2007 to 2018. The study employs a protective put strategy, DCC-GARCH models, and Markowitz optimisation to assess hedge effectiveness and risk-adjusted returns. The results indicate that gold is a more cost-effective SHA while oil is less appealing than gold.

In contrast to these papers, Thampanya et al. (2020) study the asymmetric long- and short-term impacts of portfolio rebalancing on cryptocurrency and gold returns on Thai stock markets using daily data ranging from 2000 to 2019 for gold prices and 2013 to 2019 for Bitcoin. The method applied is the non-linear Autoregressive Distributed Lag (ARDL) framework. Their findings implied that neither cryptocurrencies nor gold are suitable as SHAs, suggesting that holding gold or cryptocurrency does not increase the portfolio's risk-adjusted return. Boubaker and Larbi (2022) use the AFRIMA-FIAPARCH to rebalance the asymmetric volatility spillovers between BRICS and crude oil prices during various financial crises, including the Asian crisis, the burst dot.com bubble, the 2008 GFC, the decline in oil prices and COVID-19. Shocks to crude oil prices negatively affect the shock-oil hedge's portfolio performance, demonstrating that the oil market can support the stock market by acting as a hedge in a portfolio. For equity investments in G7 economies under various market scenarios, Tarchella et al. (2024) examine the safe-haven, hedging, and diversification qualities of oil, gold, and two cryptocurrencies, Bitcoin and Ethereum. The DCC-GARCH, Asymmetric DCC-GARCH, and GO-GARCH models, were used to rebalance daily data from before and after COVID-19. While Ethereum is the most robust hedging tool for the United States and Canada, Bitcoin offers the best protection for the G7 European and UK equities during stressful times. On the other hand, oil is better at hedging Japanese stocks in all market scenarios.

Belhassine and Karamti (2021) examined the effects of rebalancing on COVID-19 on the interconnectivity between the Chinese stock market and key financial and commodity markets, including gold, silver, Bitcoin, WTI, S&P500, and EuroSTOXX 50. The study used a wavelet power spectrum and DCC-GARCH in daily data from 2018 to 2021. Results showed that gold and silver lost their safe-haven qualities while WTI and Bitcoin acted as safe-havens from Shanghai SE markets. Another study by Ahad et al. (2024) investigated the effectiveness of ESG indexes as a safe haven against the extremely volatile European oil and gas sectors using the cross-quantilogram. They revealed that the approach for rebalancing a portfolio indicates that a safe haven portfolio with a one-month investing horizon produces returns during a war. Therefore, this study recommended that investors use ESG indices as crisis-SHAs instead of European oil and gas.

Hung et al. (2024) examined the financial benefits of including Bitcoin in a traditional portfolio rather than gold from a volatility timing and DCC-RGACRH rebalancing framework perspective. The authors use four futures: Bitcoin, gold, E-mini S&P 500, and 10-year T-note from 2018 to 2022. This sample period includes both the COVID-19 pandemic and the ongoing RU war. The authors discover that when the monetary policy is neutral, Bitcoin adds more value to the portfolio than gold. However, in times of severe rate increases, Bitcoin devalues, whereas gold provides protection and diversification advantages. A study by Yousfi et al. (2024) evaluated the portfolio implications and time-varying connectivity between commodities, cryptocurrency, and equities markets. The paper used a time-frequency framework and DCC-GARCH to model daily data from January 2018 to October 2023. Their findings indicated that while Bitcoin and gold both lose their safe-haven qualities over the short and medium terms, gold is still a solid long-term investment, while oil is the least profitable safe-haven. These results indicate that investors should frequently rebalance the portfolio structure instead of using a static strategy.

In light of the COVID-19 pandemic and the RU conflict, Rubbaniy et al. (2024) use a Time-Varying Parameter Vector Auto Regressive (TVP-VAR) approach to examine the interconnectedness of portfolio performance and hedging effectiveness between FTSE 100, S&P 500 and MSCI GCC, gold and energy commodities including crude oil, natural gas, and biofuel. The authors also included clean energy equity indices: S&P/TSX, technology index, NASDAQ OMX, global wind energy, and Stoxx global solar power from 2019 to 2023. Analysis showed that portfolio techniques show a minimal variance in the portfolio's managed portfolio weights. The hydrogen economy equities grew from 7% to 12% during COVID-19, dropping by 6% during the RU war. A study by Dias et al. (2023) examines portfolio rebalancing in times of stress to investigate the interconnections between the capital markets of AEX, CAC 40, DAX 30, FTSE100, FTSE MIB, IBEX 35 and IMOEX, and spot prices of crude oil, silver, gold, and platinum over the period from January 1, 2018, to December 31, 2022. The authors employ the VAR Granger Causality/Block Exogeneity Wald Tests model to assess the significance of causal relationships. The results indicate that during the events of 2020 and 2022, there was a notable increase in co-movements among the capital markets and commodities, suggesting that alternative markets such as oil, silver, gold, and platinum do not act as SHAs.

A study by Asih et al. (2024) assessed the role of Indonesia's ESG-based equities, represented by the ETF-SRI-KEHATI, in achieving optimal portfolio diversification across various asset classes: equities, bonds, gold, crude oil, and Bitcoin. The analysis uses daily return data for each asset from January 2018 to December 2023, covering periods before, during, and after the COVID-19 pandemic. The methodology uses the Pruned Exact Linear Time algorithm, the DCC-GACRH model, and quadratic programming optimisation. The findings suggest that a portfolio that combines ESG-based stocks with gold yields a higher mean return of 5.3% compared to a strategy focused solely on gold and demonstrates a reduction in associated risk of -9.0%. The combined portfolio achieves a 3.1% improvement in risk-adjusted returns. One study by Khaki et al. (2023) explores the portfolio diversification potential of cryptocurrencies: gold, exchange rate, bonds, Bitcoin, Ethereum, Litecoin, Chainlink, Ripple, EOS, NEO, and IOTA categorised by their leadership during the Covid-19 pandemic. The study uses mean-variance and higher-order movement optimisation methods to assess the diversification capabilities of cryptocurrencies across various frameworks. Results from the mean variance suggest that it is advantageous for investors to consider periodic rebalancing to enhance diversification benefits through cryptocurrencies. The findings indicate a significant alignment with a risk-averse portfolio, achieving a Sharpe ratio of 6.90%, particularly with substantial allocations to Bitcoin and Chainlink.

Another study by Dias et al. (2023) explores the relationship between energy metals ECO, SPGTCLEN, and CEXX, and precious metals gold, silver, platinum, aluminum, nickel, and copper to evaluate their potential as SHAs in clean energy investment portfolios. The study confirms that energy and precious metals are essential to clean energy portfolios, offering robust support during market turbulence. Closest to our study is a paper by Diaz et al. (2022) that examines the impact of treasury bonds, gold, crude oil, and cryptocurrencies on ETFs and how they rebalance and diversify throughout the COVID-19 pandemic. The authors use data from January 2019 to December 2021 to create minimum variance portfolios by limiting short positions and varying rebalancing frequencies. Two methods are used: DCC-skew and AR-GARCH models to fit the marginal distributions of individual assets. Results showed that Treasury bonds, gold, crude oil, and Bitcoin significantly contributed to portfolios consisting of traditional stocks regarding diversification and financial performance. Nonetheless, these studies analyse only one financial crisis, thus, failing to assess whether all financial crises influence the behavior of SHAs in the same way. Moreover, the study mentioned above also fails to provide the optimisation of portfolio rebalancing based on CVaR-optimised portfolio weights. CVaR technique based on loss function should be relevant for portfolio rebalancing, especially when analysing portfolio behavior during crisis periods.

3. Methodology

The purpose of this section is to present the analytical framework used to rebalance the IP in this study. This section first explains the definition of CVaR as a systemic risk measure and continues to define the periodic rebalancing strategy.

3.1 Conditional Value at Risk

Rockafeller (1999), offers a coherent and convex risk approach to risk management. CVaR, also known as expected shortfall, is the average of losses exceeding the Value of Risk (VaR) at a given confidence level and addresses the limitations of VaR, which is not a coherent risk measure and may lead to non-convex optimisation challenges. By leveraging a convex formulation of CVaR as a weighted average, this methodology allows for linear programming techniques to optimise portfolio weights, accommodating non-normal distributions. This methodology outlines a periodic portfolio rebalancing strategy using CVaR-optimised initial weights, where the primary objective is to minimise risk while maintaining a target return. The primary objective of the rebalancing strategy is to minimise CVaR, which captures the average of the worst-case losses beyond a given confidence level α. The portfolio optimisation problem can be formulated as follows:

$$\min_{w} CVaR_{\alpha}(w) \tag{1}$$

Where $w = (w_1, w_2, \dots, w_n)$ are the portfolio weights for n assets and $CVaR_{\alpha}(w)$ is the CVaR at confidence level α. The VaR risk measure is defined as:

$$VaR_{\alpha} = inf\{l \in \mathbb{R} : P(L > l) \leq 1 - \alpha\} = inf\{l \in \mathbb{R} : F_L(l) \geq \alpha\} \tag{2}$$

Where VaR_{α} is the threshold loss level such that the probability of losses exceeding this threshold is at most $1 - \alpha$. l is a natural number representing a specific loss. The goal is to find the minimum value of l such that the probability of losses exceeding l is less than or equal to $1 - \alpha$. $P(L - l)$ is the probability that the loss L exceeds the value l, and this is the tail probability associated with losses. $\alpha = 0.95$ is the confidence level ranging from 0 to 1. F_L is the cumulative distribution function of the losses L, and $\mathbb{R}$ is the set of real numbers. Once the CVaR risk measure is established, it is possible to formulate the objective function. While the existence of a continuous multivariate distribution for portfolio returns, denoted by $p(r|w)$, and consequently for losses, is not essential, it is assumed for convenience in notation. In general, the loss function associated with portfolios can be represented as $f(w,r)$, where w has a dimension of N and r has a dimension of M. The dimension difference reflects the presence of non-random returns, which are considered fixed and thus risk-free. The loss function maps from $\mathbb{R}^N * \mathbb{R}^M \to \mathbb{R}$, producing a real-valued output. The probability of losses being below a certain threshold z can then be expressed through a probability function:

$$\psi(w,z) = \int_{f(w,r)\leq z}^{\cdot} p(r|w)dr \tag{3}$$

Where $z_\alpha = \phi^{-1}(\alpha)$ denotes the quantile of the loss distribution at confidence level α, and w represents the portfolio weights. If one substitutes z for VaR for a given confidence level α and introduces a performance function for CVaR as $\frac{1}{1-\alpha}\boldsymbol{\Phi}$**,**

$$\boldsymbol{\Phi}(w) = \int_{f(w,r)\geq VaR(w,\alpha)}^{\cdot} f(w,r)p(r|wdr \tag{4}$$

Then, the minimisation of the excess loss function $\frac{1}{1-\alpha}\boldsymbol{\Phi}$ can be reduced to the minimisation of:

$$F(w,z) = (1-\alpha)z + \int_{f(w,r)\leq z}^{\cdot} (f(w,r) - z)p(r|w)dr \tag{5}$$

The optimal value determined by z corresponds to the VaR; therefore, by optimising equation 5, one can derive a solution for portfolio weights that minimises CVaR.:

$$CVaR_\alpha(w) = \frac{1}{1-\alpha}\int_\alpha^1 VaR_\beta(w)\, d\beta \tag{6}$$

The portfolio optimisation is subject to the following constraints, ensuring the risk minimisation and target return objectives are met. The budget constraint is given by:

$$\sum_{i=1}^{n} w_i = 1 \tag{7}$$

This constraint ensures the entire investment is allocated across the assets without borrowing or leverage. This also ensures that all weights allocated to each asset do not exceed 1. The target constraint is therefore given by:

$$\sum_{i=1}^{n} w_i \mu_i \geq R_{target} \tag{8}$$

Where μ_i is the expected return of asset i and R_{target} is the desired target return. This constraint ensures that the portfolio achieves at least the specified target return, preventing it from becoming overly conservative in its risk minimisation. The non-negativity constraint is expressed as:

$$w_i \geq 0 \tag{9}$$

This constraint ensures that no short-selling occurs, meaning all weights are non-negative, aligning with a long-only investment strategy. Therefore, the portfolio problem that yields a minimum CVaR in discrete terms is often used. To demonstrate how portfolio optimisation on minimising CVaR can be formulated as a linear program (Rockafellar & Uryasev, 2000), the objective function can be expressed as follows:

$$\min_{w,\xi,z} z + \frac{1}{(1-\alpha)T} \sum_{t=1}^{T} \xi_t \tag{10}$$

Subject to:

$$\xi_t \geq 0, \xi_t \geq z - w^T r_t \tag{11}$$

Where ξ_t captures the losses beyond VaR, r_t is the return vector at time t, and T is the number of return scenarios. The objective function $z + \frac{1}{(1-\alpha)T} \sum_{t=1}^{T} \xi_t$ minimises the tail-end losses while respecting the budget and return constraints.

After determining the initial CVaR-optimised weights w^*, the portfolio is rebalanced periodically at the end of each month to restore these weights. If the asset weights of the portfolio deviate significantly from the target allocation or the optimised weights by CVaR, rebalancing is triggered to restore the intended distribution. The portfolio is adjusted to maintain the optimal risk-return trade-off by realigning the weights to their initial levels w^* every month (Fabozzi et al., (2010). The rebalancing rule is formulated as follows:

$$w_{t+1} = w^* \tag{12}$$

Where w_{t+1} are the weights at the start of the month, $t + 1$ and w^* are the initial CVaR-optimised weights at the start of the investment period.

3.2 Periodic Rebalancing

Periodic rebalancing is a portfolio management strategy where asset allocations are adjusted back to their target weights at regular intervals, such as monthly, quarterly, or annually (Daryanani, 2008). This method helps maintain the desired risk-return profile of a portfolio by encountering the effects of market movements that can cause individual asset weights to drift from their initial target proportions (Arnott & Lovell, 1993). The rebalancing frequency is typically determined based on factors like risk tolerance and the stability of asset returns. The portfolio value at any time t is calculated as the sum of the current market values of all assets in the portfolio:

$$P_0 = \sum_{i=1}^{n} x_{i,0} * p_{i,0} \tag{13}$$

Where P_0 is the initial total portfolio value at time t, $x_{i,0}$ is the initial number of shares of asset i held at time t, and p_0 is the initial price of asset i at time t. The initial weights of each asset for an equally weighted portfolio at time $t = 0$ are:

$$w_{i,0} = \frac{x_{i,0} * p_{i,0}}{P_0} \tag{14}$$

The initial weights for each asset for a CVaR-optimised portfolio are expressed in Equation 6. As time progresses, asset prices change, causing the actual portfolio weights to deviate from the target weights. At any time t, the portfolio value P_t becomes:

$$P_t = \sum_{i=1}^{n} x_{i,t} * p_{i,t} \tag{15}$$

Where $x_{i,t}$ remains constant, that is, no rebalancing occurs, but $p_{i,t}$ has changed based on the market movements. The actual weight of each asset i at time t (before rebalancing) is:

$$w_{i,t} = \frac{x_{i,t} * p_{i,t}}{P_t} \tag{16}$$

As these asset prices fluctuate, $w_{i,t}$ will vary, moving away from ${w_{i,t}}^{target}$, being the equal or CVaR target weights. At each rebalancing date, that is, at the end of each month, we adjust $x_{i,t}$ to bring the weights back to the target proportions. Let $t = T$ represent a rebalancing date. The total portfolio value at T is P_T, calculated as:

$$P_T = \sum_{i=1}^{n} x_{i,T} * p_{i,T} \tag{17}$$

To achieve the target weights ${w_i}^{target}$ for each asset i, the desired dollar amount allocated to asset i after rebalancing is:

$$Target\ Value_i = {w_i}^{target} * P_T \tag{18}$$

Thus, the new number of shares $x_{i,T}$ for each asset i is:

$$x_{i,T} = \frac{Target\ Value_i}{P_{i,T}} \tag{19}$$

Following each rebalancing, the portfolio weights are reset to target weights but will drift again as asset prices fluctuate. This cycle continues at each rebalancing date, with weights returning to target proportions. This periodic rebalancing ensures that the portfolio respects the initial risk profile and target return objectives, minimising the risk drift caused by market fluctuations. We analyse this rebalancing process's effect on portfolio performance over time by simulating it. The monthly rebalancing schedule provides a practical method for investors to make timely adjustments in response to changing market conditions.

4. Data estimation, Results and Discussion

4.1 Dataset

The international equity-based portfolio examined includes indices from emerging markets-NIFTY50 (India), FTSE/JSE All Share-ALSI (South Africa), and SHANGHAI SE Composite-SSE (China), selected for their potential to enhance returns through faster economic growth and diversification benefits, given their unsynchronised economic cycles with developed markets (Harvey, 1995; Bekaert & Harvey, 1997). Developed markets indices, including FTSE100 (UK), NASDAQ100 (US), and TOPIX (Japan), are included for their size, liquidity, and performance (Kane & Marcus, 2020). Gold is incorporated as the safe-haven asset (SHA), recognised for preserving value during economic uncertainty (Baur & McDermott, 2010). Other SHAs are excluded to focus on gold's established role in mitigating severe market downturns (Ji et al., 2020; Raatikainen, 2023). The study covers three major financial crises-GFC, COVID-19, and the RU war, using daily closing prices from 9th August 2007 to 30th August 2024, obtained from the Thomson Reuters Database. Crisis periods are defined as 9th August 2007 to 30th June 2009 (GFC), 32st December 2019 to 5th May 2023 (COVID-19), and 24th February 2022 to 30th August 2024 (RU war) (Reinhart & Rogoff, 2009; Wu & McGoogan, 2020; Ochim & Ahmed, 2023). The initial portfolio capital is set at R1 000 000 for the rebalancing analysis to ensure a practical and scalable analysis, and a sufficient base to illustrate diversification effects.

4.2 Results and Discussion

Figure 1 Daily Global Prices

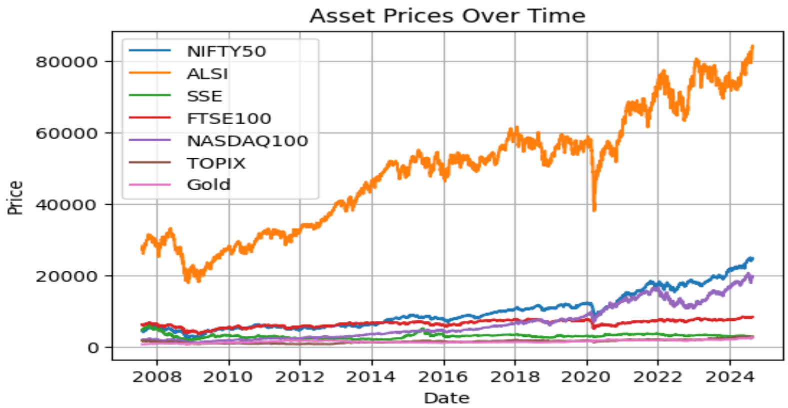


Figure 1 shows the price evolution of seven global assets, NIFTY50, ALSI, SSE, FTSE100, NASDAQ100, TOPIX, and Gold, from 2007 to 2024. ALSI exhibits the most striking growth, particularly after 2015, reaching a peak of around 80,000 in 2024, representing rapid growth in the South African market. In contrast, NIFTY50 and NASDAQ100 show a more modest upward trend, peaking just above 20,000 by 2024, reflecting steady growth in US information technology and tech-heavy industries. Other assets, such as the FTSE100, SSE, and TOPIX, demonstrate more gradual but consistent growth over the same period, indicating a stable long-term performance in the respective markets. Gold remains relatively stable compared to equities, serving as a traditional SHA during market turbulence. The overall trend highlights the divergent paths of these assets, with equities generally experiencing more pronounced growth than gold's steadiness. This diverse set of asset performances illustrates the varying growth dynamics across global markets.

Table 1 below presents the descriptive statistics, the skewness and kurtosis, and the autocorrelation of the variables employed in this study. This table provides a background of the patterns and nature of the data.

Table 1 Descriptive Statistics

	Nifty50	**ALSI**	**SSE**	**FTSE100**	**Nasdaq100**	**TOPIX**	**Gold**
Mean	0.0412	0.0261	-0.0121	0.0067	0.0552	0.0111	0.0318
Median	0.0739	0.0145	0.0000	0.0230	0.0932	0.0000	0.0444
Minimum	-13.90	-10.22	-8.87	-11.51	-13.00	-13.04	-8.87
Maximum	16.334	9.048	9.034	9.384	11.849	12.864	10.391
1st Quantile	-0.533	-0.566	-0.588	-0.482	-0.515	-0.597	-0.501
3rd Quantile	0.6712	0.6575	0.6355	0.5586	0.7425	0.6879	0.5869
Stand. Dev.	1.3368	1.2319	1.4622	1.1805	1.4577	1.3626	1.0930
Skewness	-0.308	-0.159	-0.504	-0.450	-0.289	-0.507	-0.230
Kurtosis	14.613	6.2101	5.8692	10.388	8.0671	9.1787	6.0234
Serial Corr.	0.0313	0.0071	0.0183	-0.0208	-0.1071	0.0124	-0.002

Table 1 provides a detailed summary of the statistical characteristics of the returns of the assets. By evaluating these metrics, we can understand how each asset behaves individually. This is essential for portfolio construction because the portfolio's performance will depend on the behavior of its underlying assets. The mean and median returns indicate each asset's central tendency. NASDAQ100 stands out with the highest mean return of 5.52%. NIFTY50 follows with a relatively high mean return of 4.12%, suggesting good growth in the Indian market. In contrast, SSE has a negative mean of -1.21%. Indicating an overall decline in the Chinese market. The median values show a similar pattern, with NASDAQ100 AND NIFTY50 having high medians and SSE and TOPIX having zero medians. In terms of volatility, measured by standard deviation, SSE and NASDAQ100 are the most volatile, with standard deviations of 1.4622 and 1.4577, respectively. This suggests that while these assets may offer higher returns, they also offer higher risk, with more significant price fluctuations. Conversely, Gold has the lowest volatility of 1.0930, reinforcing its reputation as a SHA. The minimum and maximum values illustrate extreme fluctuations in assets like NIFTY50, with the highest maximum return of 16.334 and the lowest minimum return of -13.90. The high kurtosis values of NIFTY50 and NSADAQ100 suggest that these assets experience extreme price movements more frequently than a normal distribution. Finally, the skewness for most assets suggests that significant negative returns are more likely than large positive returns, which is consistent with the tendency for markets to experience sudden downturns more often than rapid gains.

Table 2 below presents the unit root tests for the variables used in this study. The two tests used are the Augmented Dickey-Fuller test (ADF) and the Kwiatkowski-Phillips-Schmidt-Shin (KPSS) test.

Table 2 Unit Root Tests

	Nifty50	***ALSI***	***SSE***	***FTSE100***	***NASDAQ100***	***TOPIX***	***Gold***
ADF stat	-13.993	-18.628	-11.957	-15.269	-72.158	-13.05	-27.8
ADF p-value	(0.0)	(0.0)	(0.0)	(0.0)	(0.0)	(0.0)	(0.0)
KPSS stat	0.0747	0.0359	0.1409	0.0529	0.1255	0.3659	0.149
KPSS p-value	(0.1)	(0.1)	(0.1)	(0.1)	(0.1)	(0.09)	(0.1)

The results from Table 2 for ADF show highly negative statistics corresponding to a p-value of 0.0, indicating a solid rejection of the null hypothesis that the assets are unit root or non-stationary, concluding that all assets are stationary. The p-values also indicate that these test statistics are significant at all levels. On the other hand, the KPSS statistics, with p-values of 0.1, indicate the failure to reject the null hypothesis that all assets are stationary. The p-values of 0.1 and 0.09 all indicate significance at 10%, concluding that all the assets are stationary.

4.3 Results and Discussion from CVaR-Periodic Rebalancing

After conducting all the necessary tests on the data collected, we employed CVaR-periodic rebalancing as a strategy that evaluates the impact of the SHA on the equity-based IP. The IP is analysed with and without the SHA. Firstly, we analyse the performance of a portfolio with equal weights and periodic rebalancing for the entire sample period. We then analyse the performance of a portfolio with CVaR-periodic rebalancing for the entire sample period to compare the differences in portfolio performance. Lastly, we evaluate the performance of each portfolio during a specific crisis to analyse the impact of gold as the SHA.

Table 3 Initial Equal Weights of International Portfolio

	NIFTY50	**ALSI**	**SSE**	**FTSE100**	**NASDAQ100**	**TOPIX**	**Gold**
IP	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	
w. SHA	0.1429	0.1429	0.1429	0.1429	0.1429	0.1429	0.1429

Table 3 presents the initial allocation of weights for an IP with and without a SHA under an equally weighted portfolio rebalancing strategy. The equal weights approach assigns each asset an identical allocation of approximately 16.67%, resulting in a balanced IP. The second column presents the equal weights of an IP with a SHA. The weights decrease from 16.67% to 14.29% for all respective assets as a SHA is added to the portfolio. By distributing investments equally among the assets, the portfolio seeks to minimise concentration and enhance exposure to different geographic and economic environments. This method reflects a straightforward diversification strategy and is often preferred by investors who seek to avoid biases toward particular assets.

Figure 2 Equal Weights Portfolio Value

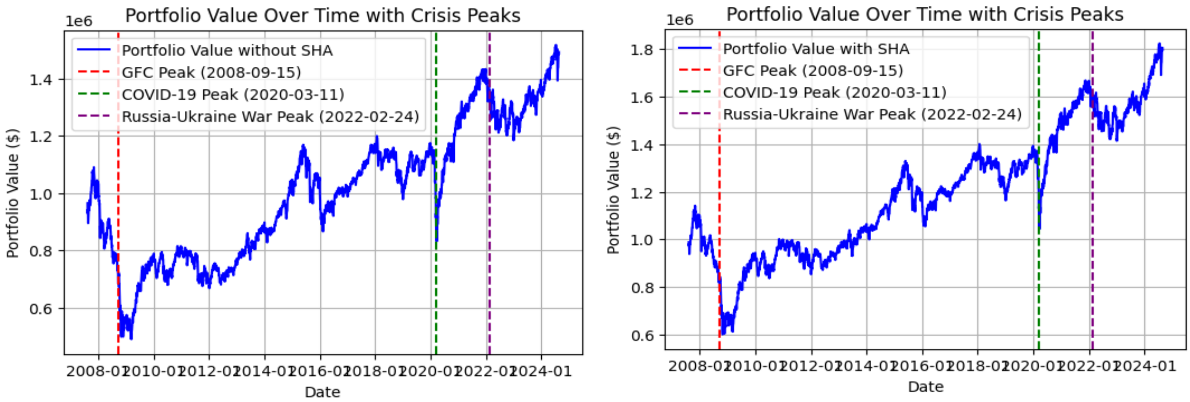


Figure 2 illustrates the value of an international portfolio (IP) without and with a safe-haven asset (SHA) under an equally weighted rebalancing strategy. The IP without a SHA shows consistent growth but with significant volatility during key financial crises, including the Global Financial Crisis (GFC), Chinese stock market turbulence, COVID-19 pandemic, and Russia-Ukraine (RU) war. Conversely, the IP with a SHA demonstrates lower volatility during downturns, indicating that incorporating gold enhanced portfolio stability and growth.

Critical peaks in the figure correspond to pivotal crisis dates. During the GFC, the collapse of Lehman Brothers on September 15, 2008, marked a significant market event (Reinhart & Rogoff, 2009). The COVID-19 peak corresponds to March 11, 2020, when WHO declared the pandemic (Baker et al., 2020). For the RU war, February 24, 2022, marks Russia's invasion of Ukraine (UN Security Council, 2022). Portfolio values at these peaks reveal the impact of including gold. During the GFC, the IP without a SHA had a value of R716,440.94, compared to R814,956.94 for the IP with a SHA, reflecting a 13.75% improvement. During COVID-19, the portfolio with gold achieved R1,208,211.51, outperforming the portfolio without gold (R984,107.98) by 22.77%. Similarly, during the RU war, the portfolio with a SHA reached R1,553,810.41, a 19.29% increase over the portfolio without a SHA (R1,302,595.41).

These findings highlight gold's varying effectiveness across crises. During the GFC, gold provided moderate protection, with a 13.75% increase in performance, reflecting the constraints of systemic financial instability. In contrast, the COVID-19 pandemic and RU war demonstrated more pronounced benefits, with performance increases of 22.77% and 19.29%, respectively. These differences underscore that financial crises influence the behavior of SHAs differently. Under equal-weighted rebalancing, gold's effectiveness was more constrained during the GFC compared to its stronger performance in the COVID-19 and RU war periods, reaffirming the crisis-specific nature of SHA behavior.

Table 4 Initial CVaR Weights for International Portfolio

	NIFTY50	ALSI	SSE	FTSE100	NASDAQ100	TOPIX	Gold
IP	0.14172	0.1559	0.1882	0.14105	0.21509	0.1580	
w. SHA	0.07666	0.0716	0.1017	0.08027	0.12821	0.1489	0.3925

Table 4 presents the initial allocation of weights for an IP with and without a SHA under a CVaR-weighted portfolio rebalancing strategy. We apply our optimisation, as explained in the methodology in Equations 10 and 11, to find the equivalent weights. Looking at the IP, we observe that the initial allocation of weights across the barrios' assets differs. The NASDAQ100 receives the highest allocation of 21.51%, followed by the SSE at 18.82% and ALSI at 15.59%. The NIFTY50, FTSE100 and TOPIX have relatively lower weights of 14.17%, 14.11% and 15.80%. This distribution suggests NASDAQ100 as a key contributor to overall performance. Looking at the IP with SHA, the weights of NASDAQ100 are reduced to 12.82%, SSE to 10.17% and ALSI to 7.16%. The NFITY50, FTSE100, and TOPIX weights have also reduced to 7.76%, 8.03%, and 14.89%, respectively. The gold allocation takes up a substantial 39.25% of the overall portfolio, suggesting it is the key contributor to performance.

Figure 3 CVaR Weights Portfolio Value

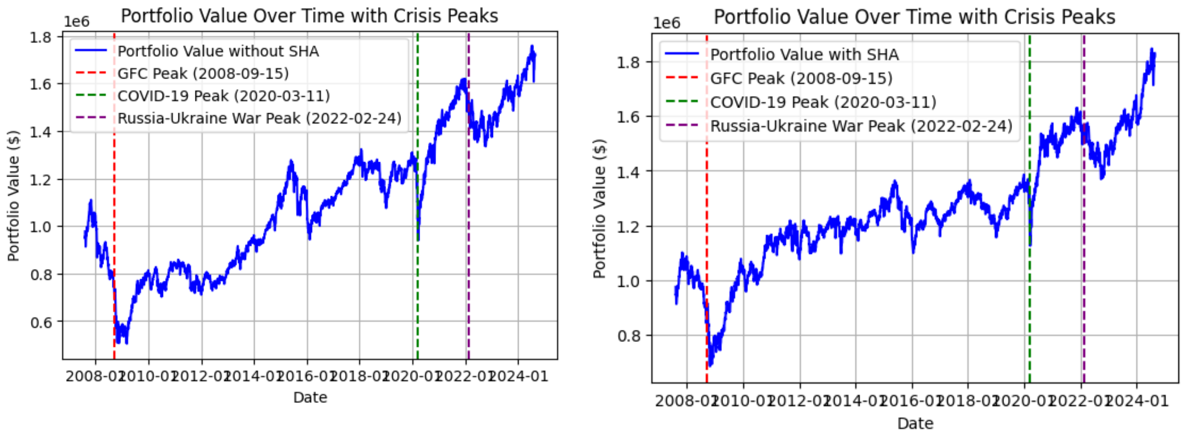


Figure 3 illustrates the performance of an international portfolio (IP) with and without a safe-haven asset (SHA) under a CVaR-optimised rebalancing strategy. The IP without a SHA shows consistent growth, reflecting positive returns from a diversified equity-based portfolio. However, the IP with a SHA demonstrates more stable growth and greater resilience, highlighting the benefits of including gold for risk-averse investors or those seeking capital preservation during volatile periods.

During the GFC peak, the IP with a SHA achieved R854,531.06, outperforming the portfolio without a SHA (R730,554.38) by 16.97%. Similarly, during the COVID-19 pandemic, the IP with a SHA reached R1,261,676.00, an improvement of 14.32% over the portfolio without a SHA (R1,103,641.26). In contrast, during the RU war peak, the performance increase was smaller, with the IP with a SHA achieving R1,524,818.59, just 4.12% higher than the IP without a SHA (R1,464,460.77).

These results highlight gold's varying effectiveness as a SHA across crises. During the GFC, gold significantly mitigated severe market disruptions, leading to the highest relative improvement (16.97%). During the COVID-19 pandemic, gold also provided substantial benefits amid extreme volatility, improving performance by 14.32%. However, the RU war demonstrated gold's limitations, with only a modest improvement of 4.12%. This variation confirms that SHAs respond differently depending on the nature of the crisis.

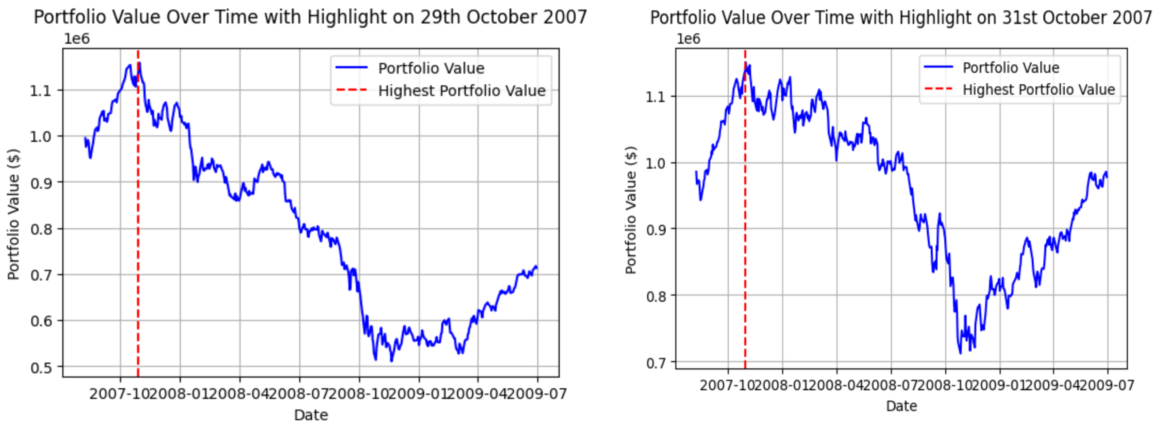
Comparing rebalancing strategies, the CVaR-optimised portfolio consistently outperformed the equally weighted portfolio during the GFC and COVID-19 pandemic, with performance improvements of 4.86% and 4.42%, respectively. However, during the RU war, the equally weighted portfolio outperformed the CVaR-optimised portfolio by 1.9%. These findings emphasize the importance of tailoring rebalancing strategies to the specific characteristics of each crisis to maximise portfolio performance.

Table 5 Initial CVaR Asset Weights for Global Financial Crisis

	NIFTY50	ALSI	SSE	FTSE100	NASDAQ100	TOPIX	Gold
IP	2.707-19	0.2545	0.3722	0.07596	0.27080	0.0265	
w. SHA	0.0071	0.1149	0.2357	0.0355	0.1567	0.0176	0.4321

Table 5 gives an outline of the initial portfolio weights before rebalancing. The table shows that ALSI, SSE and NASDAQ100 received the highest allocations of 25%, 37% and 27% respectively. NIFTY50 received approximately zero weights during GFC, while FTSE100 and TOPIX received 7.60% and 2.65% for IP. The IP with SHA weights show that gold received the most significant weight of 43.21%, reinforcing its status as a SHA during economic downturns. This high allocation underscores a strategic shift towards capital preservation, reflecting an understanding that gold can provide stability and protection. The initial weights of NIFTY50, ALSI, SSE, FTSE100, NASDAQ100, and TOPIX were 0.71%, 11.49%, 23.57%, 3.55%, 15.67% and 1.76% respectively.

Figure 4 CVaR Global Financial Crisis Portfolio Value



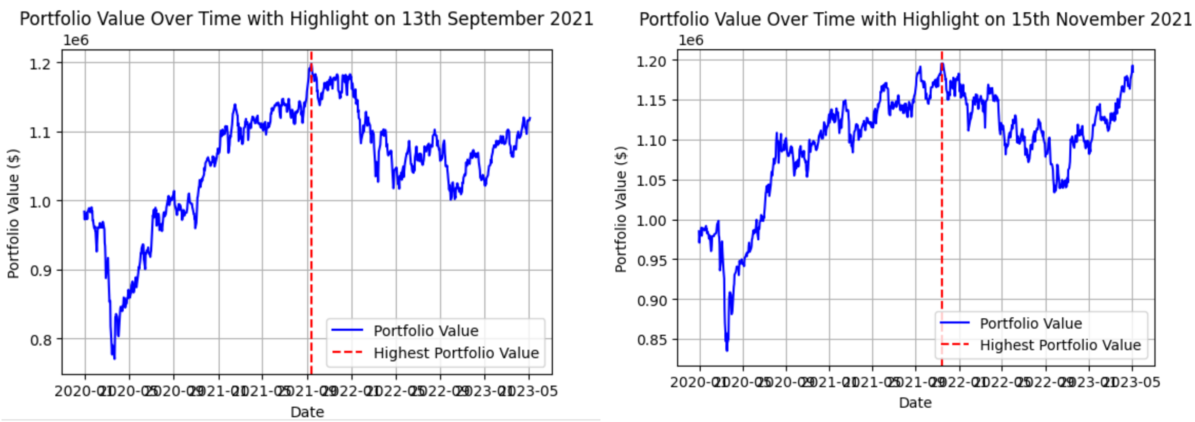
In the precedent cases, we assume that the initial rebalancing portfolio started on the 9th of August 2007 and ended on the 30th of August 2024. However, we want to analyse a portfolio rebalancing that starts at the beginning and ends when the crisis is finished. Figure 4 illustrates the performance of a portfolio during the GFC from 2007 to 2009. It shows the negatively sloping trend of the portfolio value without the SHA over the specified period. The market decrease underscored the heightened volatility caused by adverse market conditions. The highest observed portfolio value amounted to R1 158 330 on the 29th of October 2007. This value was given by 0, 0.247285, 0.389626, 0.070842, 0.269620 and 0.022628 weights of the respective assets. This suggests that, during the GFC, NIFTY50 was not an asset to invest in. The GFC with SHA then illustrates how the portfolio value changes as the weights change when rebalancing is applied. The most significant value that the portfolio experienced is R1 149 737.68. The value was produced on the 31st of October 2007 by 0.005132 weights of NIFTY50, 0.109016 weights of ALSI, 0.248603 weights of SSE, 0.029231 weights of FTSE100, 0.155791 weights of NASDAQ100, 0.014091 weights of TOPIX and 0.438136 weights of gold. Portfolio performance decreased by -0.741% during the GFC.

Table 6 Initial CVaR Asset Weights for COVID-19

	NIFTY50	ALSI	SSE	FTSE100	NASDAQ100	TOPIX	Gold
IP	0.0	0.0	0.3833	0.1274	0.1095	0.3784	
w. SHA	0.0535	0.0	0.1846	0.0487	0.0572	0.2584	0.3975

Table 6, IP column, shows that NIFTY50 and ALSI were assigned zero weights, indicating a complete avoidance of these markets before periodic rebalancing. At the same time, SSE and TOPIX received the highest weights of 38.33% and 37.84%, respectively, and FTSE100 and NASDAQ100 received the smallest weights of 12.74% and 10.95%, respectively. While in the IP with SHA column, we observe that ALSI has no allocation at all, with FTSE100, NIFTY50, and NASDAQ100 receiving smaller weights of 4.87%, 5.35%, and 5.72%, respectively. SSE and TOPIX were allocated higher weights of 18.46% and 25.84% respectively. Gold receives the highest allocation of 39.75%.

Figure 5 CVaR COVID-19 Portfolio Value



Following the COVID-19 crisis, Figure 5 shows the portfolio value from 2020 to 2023. The graph shows a significant decline in the IP value in the early onset of the pandemic, showing the volatility and market turbulence experienced during the initial stages. However, the results show R1 197 746.21 as the highest value this portfolio achieved during the crisis, corresponding to 0, 0, 0.366032, 0.12393, 0.115977, and 0.394061 asset weights of NIFTY50, ALSI, SSE, FTSE100, NASDAQ100, and TOPIX, respectively on the 13 September 2021. The IP with SHA shows the portfolio's performance after rebalancing, and it reveals that the trend is relatively stable, highlighting the success of incorporating gold to mitigate risks. The highest value observed was on the 15th of November 2021 of R1 195 597.02, produced by 0.060587, 0, 0.177316, 0.049193, 0.067703, 0.277569 and 0.367633 of the respective assets. The portfolio performance also decreased during the COVID-19 pandemic by -0.179%.

Table 7 Initial CVaR Asset Weights for Russia-Ukraine War

	NIFTY50	ALSI	SSE	FTSE100	NASDAQ100	TOPIX	Gold
IP	0.2383	0.0	0.3330	0.2094	0.1290	0.0904	
w. SHA	0.2135	0.0	0.2029	0.1400	0.0565	0.0501	0.3369

Lastly, Table 7 presents the first-day weights of the rebalancing period of respective assets. It shows that the IP during the RU war, ALSI received no allocation, and NIFTY50, SSE, and FTSE100 received an allocation of 23.83%, 33.30%, and 20.94%, respectively. NASDAQ100 and TOPIX received the smallest weights of 12.90% and 9.04%. The IP with SHA shows that NIFTY50 is assigned 21.35%, ALSI still receives no allocation, SSE receives 20.29%, FTSE100 receives 14.00%, NASDAQ100 receives 5.65%, TOPIX receives 5.01%, and gold receives 33.69%. The gold weight continues to underscore its critical role in the portfolio as a protective measure against market volatility and economic uncertainty.

Figure 6 CVaR Russia-Ukraine War Portfolio Value

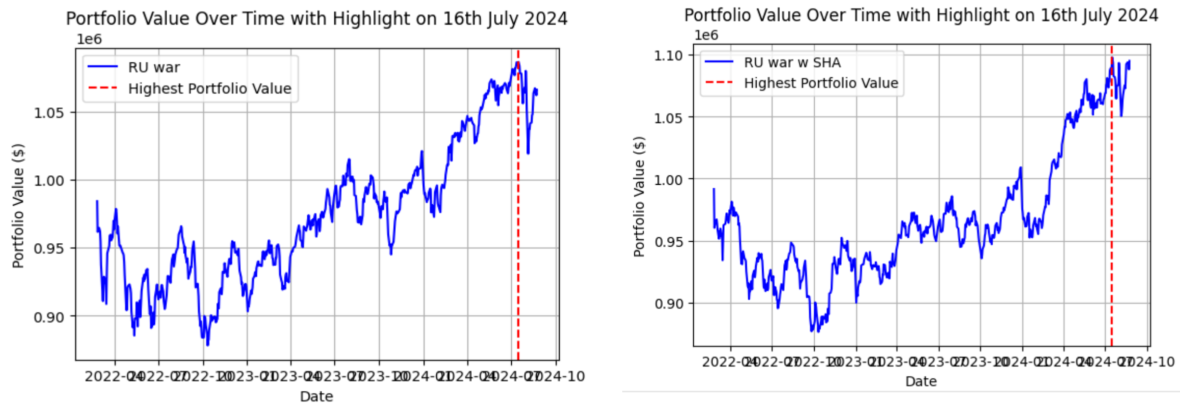


Figure 6 shows a general upward trend of the IP performance from 2022 to 2024, with some fluctuations observed. The portfolio starts with a value of R984 126.96 and gradually increases, reaching a value of R1 066 052.78 at the end of the observed period. The highest portfolio value observed during the ongoing RU war is R1 086 559.83, generated by 0.249174, 0, 0.312268, 0.202890, 0.131415 and

0.104251 weights of the respective assets on the 16 July 2024. These weights also show the insignificance of including ALSI in an IP. The IP with SHA shows that the portfolio is increasing in value with less volatility, supporting the incorporation of gold into the IP. The portfolio's most significant value is R1 097 910.22, produced on the 16th of July 2024. The weights corresponding to this value are 0.201762 of NIFTY50, 0 of ALSI, 0.18434 of SSE, 0.133862 of FTSE100, 0.055738 of NASDAQ100, 0.055555 of TOPIX and 0.368742 of gold. During the RU war, the portfolio improved its performance by 2.99%.

4.4 Performance Metrics

This section analyses the performance metrics of a periodic rebalancing strategy during different crisis periods, focusing on average returns, volatility, maximum drawdown, Sharpe ratio, Calmar ratio, skewness, kurtosis, and daily VaR. These metrics are assessed based on final portfolio weights at the end of each period to evaluate performance and risk under varying market conditions. These metrics provide a comprehensive view of portfolio performance and resilience across different market environments.

Table 8 Final Equal Weights Performance Metrics

	International Portfolio	***International Portfolio with SHA***
Average Returns	0.02008	0.03034
	(0.05058)	(0.05924)
Volatility	0.15210	0.13278
	(0.11193)	(0.10150)
Max Drawdown	-0.45548	-0.34899
	(-0.16416)	(-0.13929)
Sharpe Ratio	0.132	0.229
	(0.452)	(0.584)
Calmar Ratio	0.044	0.087
	(0.308)	(0.425)
Skew	-0.856	-0.844
	(-1.711)	(-1.821)
Kurtosis	7.826	7.864
	(14.148)	(16.296)
Daily VaR	-3.170	-2.541
	(-2.126)	(-1.751)

Note: Out-of-sample performance metrics are indicated in brackets.

Table 8 compares the performance metrics of an international portfolio (IP) without a safe-haven asset (SHA) and an IP with a SHA under equally weighted rebalancing. The in-sample metrics assess the optimised investment strategy, while out-of-sample metrics evaluate its robustness. The data was split 80-20% for the analysis. The portfolio with a SHA consistently outperformed the portfolio without a SHA. The average return for the IP with a SHA was 3.034%, compared to 2.008% for the IP without a SHA, indicating enhanced overall returns. Risk, measured by standard deviation, was lower for the portfolio with a SHA (13.28%) than for the portfolio without a SHA (15.21%), showing that including gold reduced volatility. Similarly, the maximum drawdown was smaller for the portfolio with a SHA (-34.90%) compared to the portfolio without a SHA (-45.55%).

Risk-adjusted performance metrics also favored the portfolio with a SHA. The Sharpe ratio (0.229) and Calmar ratio (0.087) were higher than those of the portfolio without a SHA (0.132 and 0.044, respectively), indicating improved returns per unit of risk. Both portfolios displayed negative skewness, reflecting a higher likelihood of downside outcomes, and kurtosis values indicative of "fat tails," suggesting a greater probability of extreme events. Daily Value at Risk (VaR) was lower for the portfolio with a SHA (-2.54%) than for the portfolio without a SHA (-3.17%), highlighting reduced exposure to downside risk. The out-of-sample metrics mirrored these results, with the portfolio with a SHA continuing to show higher returns, lower volatility, smaller drawdowns, and superior risk-adjusted returns. This consistency across both in-sample and out-of-sample periods underscores the robustness of including a SHA like gold in an equally weighted portfolio.

Table 9 Final CVaR Weights Performance Metrics

	International Portfolio	***International Portfolio with SHA***
Average Returns	0.02790	0.03063
	(0.05867)	(0.06645)
Volatility	15.130	12.373
	(10.898)	(9.901)
Max Drawdown	-0.40687	-0.26484
	(-0.16204)	(-0.14115)
Sharpe Ratio	0.184	0.248
	(0.538)	(0.671)
Calmar Ratio	0.069	0.116
	(0.362)	(0.471)
Skew	-0.820	-1.030
	(-0.651)	(-1.319)
Kurtosis	7.728	8.850
	(3.166)	(8.087)
Daily VaR	-3.176	-2.593
	(-2.014)	(-1.636)

Note: Out-of-sample metrics are indicated in brackets.

Table 9 offers a comparative analysis of the in-sample and out-sample performance measures. The in-sample average returns for IP with SHA are higher at 3.06% compared to IP, suggesting that including a SHA enhances overall returns. Similarly, the out-sample returns are also higher for IP with SHA. The IP with SHA also demonstrates lower in-sample and out-of-sample volatility measures of 12.373 and 9.901, respectively, compared to 15.130 and 10.898 for IP. The in-sample and out-of-sample maximum drawdowns are significantly lower at -0.26484 and -0.14115 for IP with SHA, indicating better protection during periods of market stress. The Sharpe and Calmar ratio in and out-of-sample for IP with SHA exhibits a higher value than IP, implying that the incorporation of gold has enhanced the portfolio's risk-return. Finally, the skewness and kurtosis suggest that the IP with SHA exhibits more negative skewness and higher kurtosis, implying a slightly higher probability of extreme negative returns. However, the reduction in overall volatility and maximum drawdown may outweigh this slight increase in tail risk for risk-averse investors. The daily VaR of the IP with SHA in-sample and out-of-sample daily also show higher values than IPs. Therefore, the potential benefits of incorporating a safe-have asset into an IP include improved returns, reduced volatility, better downside protection, and enhanced risk-adjusted performance.

Table 10 CVaR Performance Metrics for International Portfolio without SHA

	Global Financial Crisis	Covid-19	Russia-Ukraine War
Average Returns	-0.35363	0.02745	0.01399
	(0.50094)	(0.08626)	(0.10811)
Volatility	0.25290	0.14005	0.10030
	(0.22116)	(0.09207)	(0.09170)
Max Drawdown	-0.55929	-0.18348	-0.10389
	(-0.10619)	(-0.07655)	(-0.06211)
Sharpe Ratio	-1.398	0.196	0.139
	(2.265)	(0.937)	(1.179)
Calmar Ratio	-0.632	0.150	0.135
	(4.718)	(1.127)	(1.741)
Skew	-0.161	-0.448	-0.494
	(0.544)	(-0.071)	(-1.558)
Kurtosis	2.311	5.029	1.693
	(1.003)	(0.603)	(7.986)
Daily VaR	-4.174	-2.407	-1.990
	(-2.672)	(-1.382)	(-1.689)

Note: Out-of-sample metrics are indicated in brackets

Table 10 above presents the comprehensive IP analysis across three significant market events: The GFC, the COVID-19 pandemic, and the RU war. During the GFC, the portfolio experienced substantial negative average returns of -35.36%, indicating significant losses, with a very high level of volatility (25.29%), reflecting the extreme uncertainty and market turbulence during this period. The maximum drawdown, a measure of the most significant peak-to-trough decline, was steel at -0.55929, signifying severe portfolio losses. Additionally, the Sharpe ratio, which measures risk-adjusted returns, was highly negative at -1.398, showing that the portfolio underperformed relative to its risk. The Calmar ratio was similarly negative at -0.632, indicating poor performance during the GFC. The negative skewness (-0.161) indicates a higher probability of extreme negative returns, while the kurtosis of 2.311 suggests a relatively higher risk tail. The daily VaR, at -4.174, signals significant potential losses on any given day. However, out-of-sample performance appears more favorable, particularly the Sharpe and Calmar ratios, suggesting better performance than anticipated.

While challenging, the performance during the Covid-19 pandemic was markedly better than during the GFC. The portfolio exhibited positive returns of 2.75%. However, the associated volatility was still elevated at 14.005%. The maximum drawdown of -0.18348 indicates a less severe decline compared to the GFC, and the Sharpe was slightly positive at 0.196, reflecting improved risk-adjusted returns. The Calmar ratio of 0.150 also reflects better performance relative to the drawdown. Notably, skewness (-0.448) remained negative, implying the continued downside risk, and kurtosis spiked to 5.029, indicating fat tails and an increased likelihood of extreme events during the pandemic. The daily VaR of -2407 suggests less extreme potential losses than the GFC. The out-of-sample performance, particularly in Sharpe and Calmar ratios, indicates that the portfolio may have outperformed expectations, although downside risks remained high, as shown by skewness and kurtosis.

During the RU war, portfolio average returns were positive at 1.40%, although lower than during the Covid-19 pandemic. Volatility was the lowest of the three crises at 10.03%, suggesting more stable market conditions. The maximum drawdown of -0.10389 was also relatively mild, indicating that the portfolio avoided large losses during this period. The Sharpe ratio of 0.139 points to moderate risk-adjusted returns, while the Calmar ratio of 0.135 aligns with the lower drawdown observed. However, skewness became more negative (-0.494), suggesting increased downside risk, and a kurtosis of 1.693 indicates a reduction in the likelihood of extreme events. The daily VaR of -1.990 shows a further decline in potential daily losses. Out-of-sample results, especially the skewness (-1.558) and kurtosis (7.986), reveal increased tail risk and the probability of extreme negative returns during this period.

Table 11 CVaR Performance Metrics for International Portfolio with SHA

	Global Financial Crisis	Covid-19	Russia-Ukraine War
Average Returns	-0.09157	0.03447	-0.0749
	(0.34149)	(0.14294)	(0.23297)
Volatility	20.589	11.436	9.425
	(15.424)	(9.342)	(9.111)
Max Drawdown	-0.38069	-0.13640	-0.11419
	(-0.07550)	(-0.05767)	(-0.04320)
Sharpe Ratio	-0.445	0.301	-0.080
	(2.214)	(1.530)	(2.557)
Calmar Ratio	-0.241	0.253	-0.066
	(4.523)	(2.479)	(5.393)
Skew	-0.268	-0.619	-1.412
	(-0.037)	(0.096)	(-0.733)
Kurtosis	2.646	4.576	7.434
	(0.230)	(0.548)	(2.947)
Daily VaR	-3.513	-2.250	-1.781
	(-2.259)	(-1.211)	(-1.405)

Note: Out-of-sample metrics are indicated in brackets

Table 11 analyses an IP's performance metrics, including the SHA during the GFC, the Covid-19 pandemic, and the RU war. During the GFC, including a SHA significantly improved the portfolio's performance. Returns were less negative at -9.16% compared to her portfolio without the SHA, showing that the portfolio was better shielded from severe losses. The volatility decreased to 20.589%, indicating reduced market fluctuations and overall risk. The maximum drawdown of -0.38069 reflects a significant improvement compared to the portfolio without a SHA, signaling lower peak-to-trough declines. The Sharpe ratio of -0.445 is still negative but represents better risk-adjusted returns. Similarly, the Calmar ratio improved to -0.241, indicating less adverse performance than the drawdown. Skewness remained negative (-0.268), implying some downside risk, while kurtosis rose to 2.646, indicating higher tail risk than typical market conditions but lower than the FC without the SHA. The daily VaR of -3.513 shows a reduction in potential daily losses. The out-of-sample results show a Sharpe and Calmar ratio of 2.214 and 4.523, respectively, suggesting that the portfolio performed better than anticipated by including a SHA, reducing extreme losses.

In the Covid-19 pandemic, adding a SHA continued to boost portfolio performance. Average returns increased to 3.45%, higher than the portfolio with a SHA, and volatility was reduced to 11.436%, reflecting lower overall risk. The maximum drawdown improved to -0.13640, showing a smaller decline during market downturns. The Sharpe ratio of 0.301 indicates a stronger risk-adjusted return, outperforming the previous portfolio. The Calmar ratio also improved to 0.253, reflecting better returns than the drawdown. While skewness remained negative at -0.619, the kurtosis value of 4.576 indicated a relatively higher likelihood of extreme events, although lower than in the case of an IP without SHA. The daily VaR decreased to -2.250, reflecting smaller potential daily losses. The out-of-sample results show better performance, especially regarding risk-adjusted returns and drawdown resilience.

During the RU war, including a SHA resulted in further improvement. Average returns rose to 7.49%, outperforming the previous portfolio. Volatility dropped to 9.43%, indicating a more stable performance. The maximum drawdown increased to -0.11419 from -10389 in IP, suggesting higher possible losses during this period of instability. The Sharpe ratio decreased to -0.080, highlighting inferior risk-adjusted returns than the IP without SHA. The Calmar ratio of -0.066 reflects inferior performance relative to the maximum drawdown. However, skewness became more negative (-1.412), signaling an increased downside risk, while kurtosis rose to 7.434, indicating greater tail risk, though still lower than in the Covid-19 period. The daily VaR of -1.781 was the lowest in all three crises, suggesting limited potential daily losses. Out-of-sample results, especially the Sharpe ratio (2.557) and Calmar ratio (5.393), indicate a significant outperformance, highlighting the portfolio's robustness with a SHA during market turbulence.

Importantly, this study confirms that including a SHA in a CVaR-optimised portfolio consistently improved performance across all crises. Portfolios achieved higher returns, lower volatility, and reduced drawdowns, with significantly better risk-adjusted performance, as indicated by Sharpe and Calmar ratios. While some downside risks persisted, reflected by skewness and kurtosis, SHAs like gold mitigated losses and enhanced portfolio stability during market stress, supporting findings by Kocaarslan et al. (2017) and Kang and Lee (2019).

5. Conclusions and Recommendations

The analysis of portfolio rebalancing and the role of SHAs during the financial crises presents insights for investors and policymakers. This dissertation has illustrated the complexities of managing IPs amid extreme market volatility, particularly during crises such as the 2008 GFC and the COVID-19 pandemic. The central objective was determining whether SHAs like gold could enhance portfolio resilience and performance through portfolio rebalancing during various financial crises.

The study reveals that portfolios incorporating safe-haven assets (SHAs) consistently outperform those without SHAs, demonstrating lower volatility, smaller drawdowns, and improved risk-adjusted returns. Portfolios rebalanced using CVaR-optimised weights significantly outperformed those rebalanced with equal weights during the Global Financial Crisis (GFC) and the COVID-19 pandemic by 4.86% and 4.42%, respectively. However, the equally weighted portfolio outperformed by 1.9% during the Russia-Ukraine war. These findings highlight the necessity of dynamic rebalancing strategies tailored to changing market conditions, as static allocations prove inadequate in mitigating financial shocks.

The study's key findings reveal that including gold as the SHA proved highly effective in stabilising portfolios during crises. For instance, during the GFC, the portfolio that included gold experienced a reduced volatility of 20.589% compared to that of a portfolio without gold, which had a volatility of 25.290%. The IP with SHA showed a maximum drawdown, a critical measure of downside risk, of -0.38069 compared to -0.55929 of the IP without SHA. Similar improvements were observed during the COVID-19 pandemic, where portfolio volatility decreased to 11.436%, and the Sharpe ratio improved to 0.301, indicating better risk-adjusted returns. Although less severe in its impact than the other crises, the RU war showed similar trends, with the portfolio experiencing lower volatility and better drawdown protection. Most importantly, the overall results showed that including gold in a portfolio improved performance by 5.01%. During the GFC, portfolio performance decreased by -0.741%, while during the COVID-19 pandemic, the portfolio performance decreased by -0.179% when gold was included in the IP, while in contrast, during the RU war, the highest portfolio value increased by 2.99%, respectively, when gold was included in the IP. The optimised weights that produced the highest portfolio values are 0.005 weights of NIFTY50, 0.109 weights of ALSI, 0.248 weights of SSE, 0.029 weights of FTSE100, 0.155 weights of NASDAQ100, 0.014 weights of TOPIX and 0.438 weights of gold for the GFC, 0.060, 0, 0.177, 0.049, 0.067, 0.277, and 0.367 for the pandemic, respective to NIFTY50, ALSI, SSE, FTSE100, NASDAQ100, and Gold, and 0.201 of NIFTY50, 0 of ALSI, 0.184 of SSE, 0.133 of FTSE100, 0.055 of NASDAQ100, 0.055 of TOPIX and 0.368 of gold for RU-war. The COVID-19 pandemic and the RU war showed insignificance in including the FTSE/JSE All Share index in the IP, where the CVaR optimised weights returned zero for ALSI.

The study has critical implications for institutional investors and regulators. Institutional investors should prioritize SHAs during periods of heightened market volatility to buffer against severe drawdowns and achieve better risk-adjusted returns. Regulators are encouraged to promote sophisticated risk management strategies, such as CVaR, to manage tail risk effectively during financial crises. These strategies can contribute to financial stability by mitigating extreme market stress. However, the study faced limitations, including the exclusion of Bitcoin due to insufficient data during the GFC and the omission of oil, which is closely tied to economic cycles. The focus on gold as the primary SHA also limits the analysis of alternative SHAs such as U.S. Treasury bonds or silver. This constrained scope may restrict the generalizability of conclusions regarding asset behavior during crises.

Future research should explore the role of other SHAs and alternative rebalancing strategies to enhance portfolio performance. Additionally, expanding the analysis to include more financial crises and regional variations in SHA performance could provide a more comprehensive understanding of their behavior across diverse market environments. This would offer valuable insights for investors and policymakers in managing portfolio risks during turbulent times.

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