Price Stickiness in a Dual-Channel Supply Chain

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Abstract. In this paper, we study price stickiness in a dual-channel supply chain where a single manufacturer sells its product through an online channel and a retailer. We construct a noncooperative game where the manufacturer and the retailer decide on whether or not to costlessly adjust their prices after a demand shock. If the demand shock is positive, then the Nash equilibrium is always unique and non-sticky. If the demand shock is negative, then there exist Nash equilibria where some prices are sticky. Moreover, no Nash equilibrium is always Pareto optimal, pointing to the possibility of the Prisoner's Dilemma.

Keywords: Supply chain; price adjustment; price stickiness.

1 Introduction

In this paper, we study price stickiness in a dual-channel supply chain facing demand shocks. Many empirical works show that the prices in many industries involving online retailing, consumer products, restaurants, and newspapers are sticky for periods ranging from a few months to more than a year (see, for example, Arbatskaya and Baye, 2004; Bils and Klenow, 2004; Kauffman and Lee, 2010; Klenow and Kryvtsov, 2008; Knotek, 2008; MacDonald and Aaronson, 2006). According to the economic theory, possible reasons for price stickiness involve the cost of price adjustment (menu costs, managerial costs, synchronization, and staggering), market structure (industry concentration, coordination failure), asymmetric information (price as signal of quality, search and kinked demand curve, psychological price points), demand-based factors (procyclical elasticity of demand, inventories, non-price competition), and contracts (explicit or implicit).[1] Among these reasons, we focus on the market structure in this paper, studying the effects of price competition under a vertical market structure on price stickiness. The effects of price competition were earlier derived under a horizontal market structure –for a differentiated duopoly– by Hansen et al. (1996). Their main findings showed that when price adjustment costs are absent, an equilibrium of a simultaneous-move game in price competition arises only when at least one of the firms in the duopoly adjusts its price to a demand shock. In particular, they found that both firms always adjust their prices if the demand shock is either positive or sufficiently large and negative, whereas only one of the firms adjusts its price if the demand shock is sufficiently small and negative.

In this paper, we ask whether, or to what extent, the results of Hansen et al. (1996) can arise when the industry structure involves firms that are both vertically and horizontally related. To answer this question, we consider a dual-channel supply chain model where a single manufacturer sells its product directly through an online channel and indirectly through an imperfectly substitutable retailing channel where a single retailer operates. Whereas the introduction of price adjustment to a one-period dual-channel supply chain model is novel to the best of our knowledge, the model itself has been extensively studied in the operations research literature. (See Balasubramanian, 1998; Chiang et al., 2003; Kurata et al., 2007; Yan and Pei, 2009;

[1] See Kauffman and Lee (2010) for a short review of the literature on price stickiness in economics and marketing.

Rodríguez and Aydın, 2015; Matsui, 2016, He et al. (2023), and Zhang et al. (2023), among others.)

In both operations research and economic theory, price stickiness has been usually treated in dynamic models where prices are assumed to incompletely adjust due to the difference between their current and notional levels (see, for example, Simaan and Takayama, 1978; Piga, 2000; Cellini and Lambertini, 2007; Wiszniewska-Matyszkiel et al., 2015; Liu et al, 2017; Lu et al., 2019, Chen et al., 2020, among others.) In these models, the adjustment of prices takes time and the rate of adjustment is usually exogenously given. However, our model endogenizes the adjustment of prices as an equilibrium outcome of a non-cooperative strategic game. In more detail, our model considers two strategic games like in Hansen et al. (1996). One of the games (the price-setting game) is played by the members of the supply chain before the realization of a demand shock and the other game (the price adjustment game) is played after they learn about this shock. In the price-setting game, prices are set in two stages. In the first stage, the manufacturer and the retailer cooperatively determine the wholesale price and in the second stage they engage in simultaneous-move price competition to determine the retailing price and the online price non-cooperatively. After the manufacturer and the retailer calculate and announce the equilibrium prices as a solution to the price-setting game, they find themselves in a situation where they both learn that the demand curves in the retailing and online channels are affected by an unanticipated common shock. To predict their pricing behavior under this new situation, we consider, as we said earlier, a price adjustment game where the manufacturer and the retailer simultaneously decide on whether to costlessly adjust their prices or keep them unadjusted after they learn the sign and size of a demand shock.

The theoretical and computational results we obtain by solving the price adjustment game show similarities with those of Hansen et al. (1996) but also some additional (and noncomparable) elements because of the vertical relationship present in our model but not in theirs. In more detail, our results show that the dominant equilibrium never arises in the price adjustment game but a (pure strategy) Nash equilibrium always exists. If the demand shock is positive, then the Nash equilibrium is always unique and fully non-sticky, like in Hansen et al. (1996), i.e., the equilibrium arises only when all prices are adjusted. If the demand shock is negative, then we can also observe, like in Hansen et al. (1996), Nash equilibria in which only some prices (either some of the prices of the manufacturer or the price of the retailer) are sticky. However, a Nash equilibrium where the retailer's price is sticky cannot arise unless the manufacturer adjusts both the wholesale price and the online price. The equilibrium of price adjustment is also affected by the relative powers of the manufacturer and the retailer when they bargain over the wholesale price. We show that the price adjustment game can have multiple equilibria (where either the manufacturer or the retailer, but not both, make price adjustments) only if the relative bargaining power of the manufacturer is extremely small. Also, an equilibrium where the only unadjusted price is the online price of the manufacturer can arise only if the relative bargaining power of the manufacturer is extremely high. On the other hand, the size of the substitutability between the products sold in the online and retail markets turns out to affect the equilibrium of the price adjustment game mostly if the relative bargaining power of the manufacturer is moderate and the demand shock is negative (and moderate). Moreover, we show that no Nash equilibrium is always Pareto optimal, implying that the supply chain is likely to be trapped in the Prisoner's Dilemma, though with small probability.

Our work studying price stickiness in a one-period dual-channel supply chain can also be related to a recent paper by He et al. (2023), who comprehensively study optimal pricing strategies in a two-period dual-channel supply chain under market changes. In their model, the manufacturer and the retailer can adopt a different or same price strategy over two periods in their online and offline markets respectively, thus there may arise four different strategy profiles unlike the eight strategy profiles in our paper. This difference is caused by the fact that in their model the retailer's decision to use the same or different retail prices in two periods must always be aligned with the manufacturer's implicit decision to use the same or different wholesale prices in two periods. Apart from this, their model and ours has two substantial differences: First, the prices in their model are determined using a sequential (Stackelberg) game, where the manufacturer first chooses the wholesale price and the online price for two periods and next the retailer chooses its retail prices for two periods. In contrast, in our model the manufacturer and the retailer first determine the wholesale price cooperatively using the generalized Nash bargaining and then they determine their online and retail prices using a simultaneous-move price competition game. Another substantial difference in their paper and ours is that they assume that the manufacturer and the retailer can choose, among the four possible strategy profiles, the one that yields the highest profit to each of them, and they call this solution notion as the strategy equilibrium. In contrast, our model predicts the equilibrium profile among the possible eight strategy profiles of the manufacturer and the retailer using the Nash equilibrium concept according to which each firm should decide whether or not to change its price(s), in the wake of an unanticapated demand shock, as a best response to the other firm's decision. Since the Nash equilibrium outcome does not have to be Pareto efficient, let alone being Pareto superior, we obtain results where the firms in the supply chain may be trapped in the Prisoner's Dilemma, unlike in the results of He et al. (2023).

The remainder of the paper is organized as follows. Section 2 presents the basic structures, Sections 3 and 4 give theoretical and computational results respectively, and finally Section 5 concludes.

2 Basic Structures

We consider a dual-channel supply chain involving a retailing channel and an online channel. A single manufacturer sells its product to a set of homogenous consumers directly through the online channel and indirectly through a single retailer in the retailing channel. The manufacturer produces its product at a constant marginal cost that is normalized to zero. The demands of the retailing and online channels are as follows:

$$D_R(p_R, p_O) = a - p_R + bp_O, \tag{1}$$

$$D_O(p_R, p_O) = 1 - a - p_O + bp_R. \tag{2}$$

Above, the retailing and online channels are assumed to share one unit of maximal demand for the product. The parameter a denotes the share of the retailing channel in this maximal demand, while the rest, $1 - a$, is the share of the online channel. The variables p_R and p_O denote the product prices in the retailing channel and the online channel respectively, while the parameter b denotes the cross-price elasticity coefficient between the retailing and online channels. We assume that $b \in (0, 1)$, which reflects that consumers consider two channels as imperfect substitutes. Also, we assume away any showrooming effect (or cost) of the retailing channel.

We let ω denote the wholesale price charged by the manufacturer to the retailer in the retailing channel. We assume that the prices (ω, p_R, p_O) are determined in a two-stage game. In the first stage of this game, the manufacturer and the retailer cooperatively determine the wholesale price ω using the generalized Nash bargaining process (Nash, 1950; Harsanyi and Selten, 1972). For this process, we denote the relative bargaining powers of the manufacturer and the retailer by $\lambda_M \in [0,1]$ and $1-\lambda_M$ respectively and normalize their disagreement payoffs to zero. In the second stage, knowing the value of ω chosen in the previous stage, the retailer and the manufacturer engage in simultaneous-move price competition to non-cooperatively determine the retailing price p_R and the online price p_O.

We assume that both the manufacturer and the retailer are risk-neutral i.e., they only care about maximizing their profits. Given a price vector (ω, p_R, p_O), we can calculate the profits of the manufacturer and the retailer in the retailing channel as

$$\pi_M(\omega, p_R, p_O) = \omega D_R(p_R, p_O) \tag{3}$$

and

$$\pi_R(\omega, p_R, p_O) = (p_R - \omega) D_R(p_R, p_O) \tag{4}$$

respectively. Similarly, we define the manufacturer's profit in the online channel as

$$\pi_O(p_R, p_O) \equiv p_O D_O(p_R, p_O). \tag{5}$$

Finally, we assume that all elements of the model described above, involving their cooperative or non-cooperative strategy choices in each stage of the game they are playing, are common knowledge.

3 Theoretical Results

Below, we will first solve the two-stage price-setting game described in Section 2. Recall that this game involves perfect information as both the manufacturer and the retailer are assumed to know the value of wholesale price ω determined in the first stage before starting the simultaneous-move price competition in the second stage. Therefore, we can solve this game starting from the second stage backward. In the second stage of this game, the manufacturer and the retailer will choose their prices p_O and p_R to maximize, for any given value of $\omega \geq 0$, their profits

$$\max_{p_O \geq 0} \ \pi_O(p_R, p_O) \tag{6}$$

and

$$\max_{p_R \geq 0} \ \pi_R(\omega, p_R, p_O) \tag{7}$$

respectively. Simultaneously solving these two problems will yield the Bertrand-Nash equilibrium plans $p_O(\omega)$ and $p_R(\omega)$ as a function of ω. Given these plans, the manufacturer and the retailer can solve in the first stage the bargaining problem

$$\max_{\omega \geq 0} \left[\pi_M(\omega, p_R(\omega), p_O(\omega))\right]^{\lambda_M} \left[\pi_R(\omega, p_R(\omega), p_O(\omega))\right]^{1-\lambda_M}. \tag{8}$$

The solution to this problem will be ω^* that will induce the equilibrium prices $p_O^* \equiv p_O(\omega^*)$ and $p_R^* \equiv p_R(\omega^*)$.

Proposition 1. *In the absence of any unanticipated demand shock, the two-stage price-setting game played by the supply chain results in the equilibrium prices (ω^*, p_O^*, p_R^*) given by*

$$\omega^* = \frac{\lambda_M \left(2a + b(1-a)\right)}{4 - 2b^2} \tag{9}$$

$$p_O^* = \frac{1}{4-b^2}\left(2(1-a) + ba + \frac{b\lambda_M(2a + b(1-a))}{4-2b^2}\right) \tag{10}$$

$$p_R^* = \frac{2a + b(1-a)}{4-b^2}\left(1 + \frac{2\lambda_M}{4-2b^2}\right). \tag{11}$$

We relegate the proofs of all propositions to Appendix B. The equilibrium prices in Proposition 1 will be instrumental for the rest of our theoretical results. (One can easily check that the prices ω^*, p_O^*, and p_R^* are all increasing in the manufacturer's bargaining power λ_M and the cross-price coefficient b. Also, ω^* and p_R^* are increasing in a, the demand share parameter of the retailing channel, and p_O^* is decreasing.)

Now, suppose that immediately after the price-setting game is strategically over and the manufacturer and the retailer announce to the public the equilibrium prices ω^*, p_O^*, and p_R^* in accordance with the demand curves in (1) and (2), the manufacturer and the retailer learn that the demand curves in the retailing and online channels were affected by a previously unanticipated common shock ϵ. That is, the manufacturer and the retailer become ex-post aware about the existence and the realization of ϵ. Consequently, they become ex-post aware about the actual demand curves in the retailing and online channels, satisfying

$$D'_R(p_R, p_O) = a(1+\epsilon) - p_R + bp_O, \tag{12}$$

$$D'_O(p_R, p_O) = (1-a)(1+\epsilon) - p_O + bp_R \tag{13}$$

where ϵ is a commonly known constant lying in the interval $(-1, \infty)$. (Thus, we allow negative as well as positive shocks.)

Given the newly acquired information about the demand functions, the manufacturer and the retailer have to decide whether to adjust their prices or not. If any of them chooses to adjust its price(s), it has to incur a lump-sum cost $z \geq 0$ for each price adjustment. We model the decision problems of the manufacturer and the retailer with the help of a simultaneous-move decision game where each player (echelon) has two strategies for each of its price(s), namely 'Adjust (A)' or 'Do not Adjust (D)'. In this game, we assume that the manufacturer decides whether to adjust ω^* and/or p^*_O and the retailer decides whether to adjust p^*_R. Thus, the manufacturer has four strategies, indexed by $s_{MO} \in S_{MO} = \{AA, AD, DA, DD\}$. For example, the strategy AD means that the manufacturer adjusts ω^* and does not adjust p^*_O. On the other hand, the retailer has only two strategies, indexed by $s_R \in S_R = \{A, D\}$, for adjusting p^*_R. Thus, we denote a strategy profile in the described decision game by $(s_{MO}, s_R) \in S_{MO} \times S_R$.

We assume that the manufacturer and the retailer simultaneously announce their price adjustment decisions represented by a strategy profile (s_{MO}, s_R), and then following this profile, they play the two-stage game described in Section 2 under the new demand functions (12) and (13). So, given a profile (s_{MO}, s_R) and the induced two-stage game, let $\pi_M(s_{MO}, s_R)$ and $\pi_O(s_{MO}, s_R)$ denote the resulting profits of the manufacturer in the retailing and online channels respectively and let $\pi_R(s_{MO}, s_R)$ denote the resulting profit of the retailer in the retailing channel. Also, let us denote by $\pi_{MO}(s_{MO}, s_R)$ the total profit of the manufacturer in the two channels, i.e., $\pi_{MO}(s_{MO}, s_R) = \pi_M(s_{MO}, s_R) + \pi_O(s_{MO}, s_R)$ for all (s_{MO}, s_R). (Here, all profit calculations take into account the cost of price adjustments, as well.)

The players M and R, their strategy spaces S_{MO} and S_R, and their profit functions $\pi_{MO}(s_{MO}, s_R)$ and $\pi_R(s_{MO}, s_R)$ defined for each $(s_{MO}, s_R) \in S_{MO} \times S_R$ constitute a single-shot noncooperative decision game for price adjustments, which we will simply call 'the price adjustment game'. In Table 1 we list all possible strategy profiles in this game, and in Table 2 we show the two players' payoffs at each strategy profile.

Table 1. The strategy profiles in the price adjustment game

Strategy Profile	**Manufacturer**		**Retailer**
Acronym	**Wholesale Price**	**Online Price**	**Retail Price**
DDD	Do Not Adjust	Do Not Adjust	Do Not Adjust
DDA	Do Not Adjust	Do Not Adjust	Adjust
DAD	Do Not Adjust	Adjust	Do Not Adjust
DAA	Do Not Adjust	Adjust	Adjust
ADD	Adjust	Do Not Adjust	Do Not Adjust
ADA	Adjust	Do Not Adjust	Adjust
AAD	Adjust	Adjust	Do Not Adjust
AAA	Adjust	Adjust	Adjust

Given the price adjustment game, we say that a strategy $s_{MO} \in S_{MO}$ is a dominant strategy for the manufacturer if

$$\pi_{MO}(s_{MO}, s_R) \;\; \geq \;\; \pi_{MO}(s_{MO}, s_R) \;\text{ for all }\; s_R \in S_R. \tag{14}$$

Likewise, we say that a strategy $s_R \in S_R$ is a dominant strategy for the retailer if

$$\pi_R(s_{MO}, s_R) \;\; \geq \;\; \pi_R(s_{MO}, s_R) \;\text{ for all }\; s_{MO} \in S_{MO}. \tag{15}$$

Then, we say that a strategy profile $(s_{MO}, s_R) \in S_{MO} \times S_R$ is a dominant equilibrium if s_{MO} and s_R are dominant strategies for the manufacturer and the retailer respectively. As the dominant equilibrium is extremely demanding, it rarely exist in normal-form games. Thus, we will also be considering the Nash (1950) equilibrium, as a weaker concept. A strategy profile (s^*_{MO}, s^*_R) is a (pure-strategy) Nash equilibrium of the price adjustment game if

$$\pi_{MO}(s^*_{MO}, s^*_R) \;\; \geq \;\; \pi_{MO}(s_{MO}, s^*_R) \;\text{ for all }\; s_{MO} \in S_{MO} \;\text{ and} \tag{16}$$

$$\pi_R(s^*_{MO}, s^*_R) \;\; \geq \;\; \pi_R(s^*_{MO}, s_R) \;\text{ for all }\; s_R \in S_R. \tag{17}$$

We will be concerned whether the dominant or Nash equilibrium, whenever exists, is also efficient, i.e., Pareto optimal. We say that a strategy profile $(s_{MO}, s_R) \in S_{MO} \times S_R$ is said to be Pareto optimal if there exists no other strategy profile $(s'_{MO}, s'_R) \in$

$S_{MO} \times S_R$ such that $\pi_i(s'_{MO}, s'_R) \geq \pi_i(s_{MO}, s_R)$ for all $i \in \{MO, R\}$ and $\pi_i(s'_{MO}, s'_R) > \pi_i(s_{MO}, s_R)$ for some $i \in \{MO, R\}$. If a strategy profile is not Pareto optimal, we say it is Pareto non-optimal.

Table 2. The price adjustment game

Manufacturer \ Retailer	D	A
DD	$\pi_{MO}(DD, D), \pi_R(DD, D)$	$\pi_{MO}(DD, A), \pi_R(DD, A)$
DA	$\pi_{MO}(DA, D), \pi_R(DA, D)$	$\pi_{MO}(DA, A), \pi_R(DA, A)$
AD	$\pi_{MO}(AD, D), \pi_R(AD, D)$	$\pi_{MO}(AD, A), \pi_R(AD, A)$
AA	$\pi_{MO}(AA, D), \pi_R(AA, D)$	$\pi_{MO}(AA, A), \pi_R(AA, A)$

To explore whether the price adjustment game admits dominant and/or Nash equilibria, we have to first calculate the profits of the manufacturer and the retailer at each possible strategy profile. We make these calculations in Appendix A. Trivially, if the cost of price adjustment z is sufficiently high, then (DD, D) would always be the unique equilibrium; i.e., it would be optimal for each player not to adjust its price(s). In the rest of this paper, we set the adjustment cost parameter z to zero and seek an answer to the following non-trivial question: Can the strategy of not adjusting a price can be the dominant, or a best-response, strategy for any player whenever the cost of adjustment is zero? To answer this question, we compare the profits calculated in Appendix A, and obtain several theoretical results.

Proposition 2. *The strategy profile (DD, D) cannot be a Nash equilibrium for any $\epsilon \in (-1, \infty) \setminus \{0\}$.*

Proposition 2 says that an equilibrium where no price in the supply chain is adjusted can never arise. The reason is that in a situation where the players are recommended to play according to the profile (DD, D), the retailer would always find it optimal to play the profile (DD, A) by adjusting its price unilaterally.

Proposition 3. *The strategy profile* (DA, D) *cannot be a Nash equilibrium whenever* $\epsilon > 0$ *or* $-1 < \epsilon < \max\left\{-1, -2b^2\left(1 + \frac{\lambda_M}{4-2b^2}\right)\right\}$.

The above result says that the manufacturer's adjusting only its online price and the retailer's not adjusting its price cannot constitute a Nash equilibrium whenever the demand shock is positive or it is negative but sufficiently large in the absolute value. The reason is that at such values of the demand shock, the retailer would find it optimal to deviate from the profile (DA, D) where it keeps its price unadjusted to the profile (DA, A) where it adjusts its price.

Proposition 4. *Neither the strategy profile* (AD, D) *nor the strategy profile* (DD, A) *can be a Nash equilibrium whenever* $\epsilon > 0$.

Proposition 4 says that whenever the demand shock is positive, an equilibrium where the only adjusted price in the supply chain is either the manufacturer's wholesale price or the retailer's price can never arise. The reason is that the manufacturer would find it optimal to adjust its online price, by deviating from the profile (AD, D) to the profile (AA, D) and by deviating from the profile (DD, A) to the profile (DA, A). The complexity of some profit comparisons prevents us from making further inferences theoretically. In the following section, we will obtain these inferences computationally.

4 Computational Results

We perform our computations with the help of MATLAB, Release 2023b. (The source code and the resulting data are available from the author upon request.) In our model, the parameters a, b, and λ_M are all confined to the unit interval $(0, 1)$. For our computations, we vary each of these parameters in the set $\{0.025, \ldots, 0.975\}$ with increments of 0.050. On the other hand, the demand shock parameter ϵ is confined in our model to the interval $(-1, \infty)$. To make ϵ bounded in our computations, we vary it in the set $\{-0.99, \ldots, 0.99\}$ with increments of 0.02. Given the above specifications about a, b, λ_m, and ϵ, we will be looking for the equilibria of the price adjustment game under a total number of 800,000 ($= 20^3 \times 100$) distinct parameter settings. We start our analysis by searching for the dominant equilibrium.

Result 1. *In the domain of our computations, there exists no vector of parameters* *under which either the price adjustment game has a dominant equilibrium or any player in this game has a dominant strategy.*

The above result implies that the price adjustment game is not strategically trivial for any player. Given the above result, we immediately turn our attention to the Nash equilibrium. In Table 3, we report the frequency each strategy profile arises in a Nash equilibrium during our computations. These computations yield the following results.

Result 2. *For each vector of parameters in the domain of our computations, the price adjustment game has a Nash equilibrium in pure strategies.*

Result 2 implies that the best responses of the manufacturer and the retailer always intersect at some strategy profiles, leading to a Nash equilibrium in pure strategies. Thus, the players never have to appeal to mixed strategies to find and coordinate on a Nash equilibrium.

Table 3. The frequency of each strategy profile to be played in a Nash equilibrium under 800,000 parameter settings

Profile	**Frequency**
(DD,D)	0
(DA,D)	0
(AD,D)	0
(AA,D)*	3,880
(DD,A)	133,914
(DA,A)	172,932
(AD,A)	656
(AA,A)	492,498

* Under any parameter setting where (AA, D) is an equilibrium, (DD, A) is also an equilibrium. Under all other parameter settings, the equilibrium is unique.

Result 3. *None of the strategy profiles* $(DD, D), (DA, D)$, *and* (AD, D) *ever arises as a Nash equilibrium, while each of the remaining strategy profiles can be a Nash equilibrium at some vectors of parameters in the domain of our computations.*

Result 3 implies that a strategy profile where the retailer chooses not to adjust its price cannot arise as a Nash equilibrium unless the manufacturer chooses to adjust its prices in both the online and retailing channels. This result is in line with the result in Proposition 2, while it also complements the sufficiency results in Propositions 3 and 4 where (DA, D) or (AD, D) are found to be non-equilibrium profiles only under certain restrictions on ϵ.

Result 4. *The strategy profile (AA, A) can arise as a Nash equilibrium at each value of ϵ, whether negative or positive. Moreover, if ϵ is positive, then (AA, A) is always the unique Nash equilibrium.*

The above result says that no Nash equilibrium other than (AA, A) can arise when the demand shock ϵ is positive. Results 3 and 4 together imply that a Nash equilibrium in which some of the prices in the supply chain are not adjusted can arise only when the demand shock is negative. Next, we consider the equilibrium that appeared with the smallest frequency in our computations.

Result 5. *The strategy profile (AD, A) can arise as a Nash equilibrium only if λ_M is sufficiently high, i.e., $\lambda_M \in \{0.925, 0.975\}$.*

Notice that the above (extremely restrictive) condition on λ_M is not sufficient for the profile (AD, A) to be a Nash equilibrium. We already know from Result 4 that ϵ must be negative for (AD, A) to be an equilibrium, and even that is not sufficient. In our computations, in 400,000 parameter settings the parameter ϵ is negative-valued and only 656 of them (1.64%) admit the profile (AD, A) to be a Nash equilibrium, as reported in Table 3.

Our next result establishes that the Nash equilibrium is always unique unless the bargaining power of the manufacturer is sufficiently low.

Result 6. *The price adjustment game can have multiple equilibria only if λ_M is sufficiently small, i.e., $\lambda_M \in \{0.025, 0.075\}$.*

Although multiple equilibria may exist, not all pairs of strategy profiles can co-exist in equilibrium.

Result 7. *Whenever the price adjustment game has multiple equilibria, these equilibria are always (DD, A) and (AA, D). Moreover, (AA, D) is a Nash equilibrium only if*

(DD, A) is also a Nash equilibrium whereas the converse is not true. Furthermore, the manufacturer always prefers to be in the equilibrium (AA, D) while the retailer prefers to be in (DD, A).

The above result says that if a strategic situation where the only unadjusted price in the supply chain is the retailer's price comprises an equilibrium, then a strategic situation where the only adjusted price is the retailer's price must also be an equilibrium, while the converse of this statement is not true. Moreover, none of these two equilibria Pareto dominates the other. Each player prefers to be in an equilibrium play where it adjusts its price(s) while the other player does not. Nonetheless, this result does not imply that any of these two equilibria should be Pareto optimal.

Now, we will deal with the Pareto optimality of each equilibrium. Table 4 reports the percentage of parameter settings in our computational domain at which a given Nash equilibrium is Pareto optimal. Table 4 shows that the equilibrium profile (AD, A) and especially the equilibrium profile (AA, D) are almost always Pareto optimal. Also, the likelihood that any of the two equilibrium profiles (DD, A) and (DA, A) will be Pareto optimal under randomly chosen parameter settings is quite high (around 90%). On the other hand, our results show that the profile (AA, A), the most frequently occurring equilibrium of the price adjustment game, is Pareto optimal only under approximately 73% of the parameter settings.

Table 4. The percentage of parameter settings at which a given Nash equilibrium is Pareto optimal

Equilibrium	**Percentage**
(AA,D)	99.95
(DD,A)	89.59
(DA,A)	89.76
(AD,A)	97.41
(AA,A)	72.98

Result 8. *For each equilibrium of the price adjustment game, there exists a nonempty set of parameter settings at which the given equilibrium is not Pareto optimal. However, the measure of this set is quite narrow for most of the equilibria.*

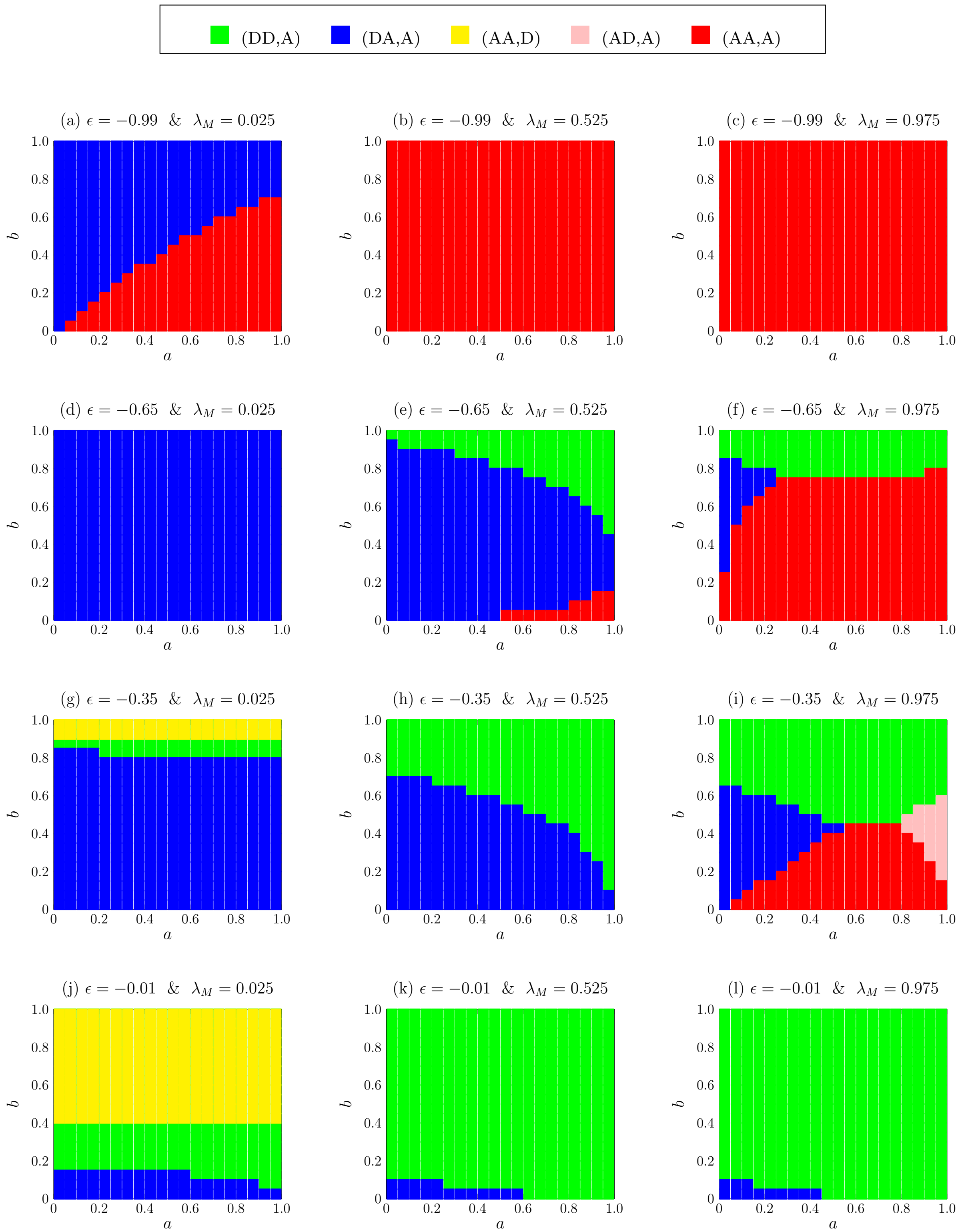
Overall, our computations reveal that for approximately 20 percent (a total of 160,863) of all 800,000 parameter settings in our computational domain, the resulting equilibrium (or equilibria if there is more than one) is not Pareto optimal. Thus, the members of the supply chain may occasionally face an annoying situation similar to what is already known as the Prisoner's Dilemma.[2]

Finally, we will investigate how the model parameters $(a, b, \lambda_M, \epsilon)$ will affect the identification of the equilibria of the price adjustment game. We already know from Result 4 that whenever $\epsilon > 0$, the equilibrium is unique and equal to (AA, A) independent of the values of the other parameters. Thus, we will restrict our attention to the case where $\epsilon < 0$. Under this setting, we will vary ϵ in the set of values $\{-0.99, -0.65, -0.35, -0.01\}$ and for each value of ϵ we will vary λ_M in the set $\{0.025, 0.525, 0.975\}$. Thus, we will have 12 pairs of (ϵ, λ_M) and for each of them we will vary each of the parameters a and b in the set $\{0.025, \ldots, 0.975\}$ with increments of 0.050 and depict the corresponding Nash equilibria in the 12 panels of Figure 1 using a color map.

Our results in panels (a)-(c) show that if the negative demand shock is extremely large ($\epsilon = -0.99$), then the equilibrium is always uniquely equal to (AA, A) unless the manufacturer's bargaining power is extremely small ($\lambda_M = 0.025$). In that extreme case, the profile (DA, A) can become the unique equilibrium if and only if the substitutability of retailing and online channels, b, is above a critical threshold that is generally increasing with a. Whenever b is below that threshold, (AA, A) remains to be the unique equilibrium. When the magnitude of demand shock becomes smaller, i.e., $\epsilon \in \{-0.65, -0.35\}$, as assumed in panels (d)-(i) of Figure 1, we observe that the frequency of the equilibrium (AA, A) becomes smaller, and even becomes zero whenever the manufacturer's bargaining power, λ_M, is extremely small. Moreover, we start to observe the equilibrium (DD, A) when ϵ is sufficiently small or when λ_M is sufficiently high.

[2]However, unlike in the well-known game of Prisoner's Dilemma, in the price adjustment game no Nash equilibrium is ever dominant equilibrium.

Figure 1. The effects of $(a, b, \lambda_M, \epsilon)$ on the identification of equilibria



We also notice that the frequency of the equilibrium (DA, A) becomes smaller and the frequency of the equilibrium (AA, A) becomes higher as λ_M becomes higher. When ϵ is as small as -0.35 and λ_M reaches its extreme level of 0.975 in our computational domain, we can even observe the rare equilibrium profile (AD, A) if a is large and b is intermediate. Moreover, whenever λ_M is as small as 0.025, if ϵ is either equal to -0.35 as in panels (g)-(i) or equal to -0.01 as in panels (j)-(l), then we observe multiple equilibria involving (AA, D) and (DD, A) provided that the channel substitution parameter b is sufficiently high. The frequency of observing multiple equilibria is higher if ϵ is lower. The last three panels also reveal that neither the frequent equilibrium (AA, A) nor the rare equilibrium (AD, A) is ever observed when ϵ is extremely small. Of the remaining three possible equilibria, (AA, D) ceases to appear if λ is intermediate or large. The typical equilibrium is always (DD, A) when ϵ is extremely small, while we also observe (DA, A) when the parameter b is sufficiently small.

5 Conclusion

In this paper, we studied the possibility of sticky prices in a dual-channel supply chain where a single manufacturer sells its product directly through the online channel and indirectly through a single retailer in the retailing channel. We assumed that the online and retailing channels which are imperfectly substitutable engage in a simultaneous-move price competition while the wholesale price charged to the retailer is determined by a Nash bargaining process between the manufacturer and the retailer. Using this model, we constructed a price adjustment game where the manufacturer and the retailer simultaneously decide on whether to adjust their prices or keep them unadjusted in response to a (positive or negative) demand shock when the cost of price adjustment is zero.

Theoretically calculating the possible profits in the supply chain under each possible decision profile and making several numerical computations, we established that the price adjustment game always has a (Nash) equilibrium in pure strategies and the likelihood that this equilibrium will be unique is very high. If the demand shock is positive, then the price adjustment game will always have a unique equilibrium; moreover, in this equilibrium all prices are adjusted. On the other hand, if the demand shock is negative, then in any equilibrium of the price adjustment game some prices are left unadjusted even when the cost of adjustment is zero. However, there exists no equilibrium where all prices are left unadjusted.

An equilibrium where the retailer does not adjust its price can arise only if the manufacturer adjusts both of its prices. We observe this particular equilibrium quite infrequently and only if the manufacturer's bargaining power is extremely low and the substitutability between the retailing and online channels is sufficiently high. Besides, this equilibrium arises only if the price adjustment game has another equilibrium where only the retailer adjusts its price.

Moreover, our results show that no equilibrium is always Pareto optimal; thus, the supply chain is always likely (though with low probability) to be trapped in a situation resembling the Prisoner's Dilemma. A paper by Lu et al. (2019) shows that an infinitely-lived supply chain that operates under incomplete and exogenously given price adjustment may use cooperative advertising to escape from the Prisoner's Dilemma in a strategic game where each member chooses to be either myopic or far-sighted in their dynamic calculations. Future research may study whether cooperative advertising may also be a solution to prevent the appearance of the Prisoner's Dilemma in our static model where the decision of price (un)adjustment is endogenously obtained as the equilibrium outcome of a strategic game.

References

Arbatskaya M, Baye MR (2004) Are prices "sticky" online? Market structure effects and asymmetric responses to cost shocks in online mortgage markets. International Journal of Industrial Organization, 22:10, 1443–1462.

Balasubramanian S (1998) Mail versus mall: A strategic analysis of competition between direct marketers and conventional retailers. Marketing Science, 17:3, 181–195.

Bils M, Klenow PJ (2004) Some evidence on the importance of sticky prices. Journal of Political Economy, 112:5, 947–985.

Cellini R, Lambertini L (2007) A differential oligopoly game with differentiated goods and sticky prices. European Journal of Operational Research, 176:2, 1131–1144.

Chen Z, Tian C, Zhang D, Chen D (2000) Dynamic model of a supply chain network with sticky price. Operational Research (An International Journal), 20, 649–670.

Chiang WYK, Chhajed D, Hess JD (2003) Direct marketing, indirect profits: A strategic analysis of dual-channel supply-chain design. Management Science, 49:1, 1–20.

Hansen PS, Møllgaard HP, Overgaard PB, Sørensen JR (1996) Asymmetric adjustment in symmetric duopoly. Economics Letters, 53, 183–188.

Harsanyi JC, Selten R (1972) A generalized Nash solution for two-person bargaining games with incomplete information. Management Science, 18, 80–106.

He P, Zhang G, Wang TY, Si Y (2023) Optimal two-period pricing strategies in a dual-channel supply chain considering market change. Computers & Industrial Engineering, 179, 109193.

Kauffman RJ, Lee D (2010) A multi-level theory approach to understanding price rigidity in internet retailing. Journal of the Association for Information Systems, 11:6, 303–338.

Klenow PJ, Kryvtsov O (2008) State-dependent or time-dependent pricing: does it matter for recent US inflation? Quarterly Journal of Economics 123:3, 863–904.

Knotek ES (2008) Convenient prices, currency, and nominal rigidity: Theory with evidence from newspaper prices. Journal of Monetary Economics, 55:7, 1303–1316.

Kurata H, Yao D, Liu JJ (2007) Pricing policies under direct vs. indirect channel competition and national vs. store brand competition. European Journal of Operational Research, 180:1, 262–281.

Liu Y, Zhang J, Zhang S, Liu G (2017) Prisoner's dilemma on behavioral choices in the presence of sticky prices: Farsightedness vs. myopia. International Journal of Production Economics, 191, 128–142.

Lu F, Tang W, Liu G, Zhang J (2019) Cooperative advertising: A way escaping from the prisoner's dilemma in a supply chain with sticky price. Omega, 86, 87–106.

MacDonald JM, Aaronson D (2006) How firms construct price changes: Evidence from restaurant responses to increased minimum wages. American Journal of Agricultural Economics, 88:2, 292–307.

Matsui K (2016) Asymmetric product distribution between symmetric manufacturers using dual-channel supply chains. European Journal of Operational Research, 248: 2, 646–657.

Nash JF (1950) The bargaining problem. Econometrica, 18, 155–162.

Piga C (2000) Competition in a duopoly with sticky price and advertising. International Journal of Industrial Organization, 18, 595–614.

Rodríguez B, Aydın G (2015) Pricing and assortment decisions for a manufacturer selling through dual channels. European Journal of Operational Research, 242:3, 901–909.

Simaan M, Takayama T (1978) Game theory applied to dynamic duopoly problems with production constraints. Automatica 14, 161–166.

Wiszniewska-Matyszkiel A, Bodnar M, Mirota F (2015) Dynamic oligopoly with sticky prices: Off-steady-state analysis. Dynamic Games and Applications, 5:4, 568–98.

Yan R, Pei Z (2009) Retail services and firm profit in a dual-channel market. Journal of Retailing and Consumer Services, 16:4, 306–314.

Zhang Y, Zhong W, Mei S (2023) Supply chain coordination by manufacturer reward contract. Operations Research Letters, 51, 432–438.

Appendix A (Price and Profit Calculations)

Here, we will calculate the profits of the manufacturer and the retailer at each strategy profile in the price adjustment game under the assumption that all price adjustment costs are zero.

For each strategy $s_{MO} \in S_{MO}$ of the manufacturer and for each strategy $s_R \in S_R$ of the retailer, let us denote the corresponding prices of the manufacturer in the retailing and online channels by $\omega(s_{MO}, s_R)$ and $p_O(s_{MO}, s_R)$ and the corresponding price of the retailer in the retailing channel by $p_R(s_{MO}, s_R)$. Below, we calculate these prices referring to the prices ω^*, p_O^*, and p_R^* calculated in the absence of any demand shock and using these new prices we will calculate the induced profits of the manufacturer and the retailer, $\pi_{MO}(s_{MO}, s_R)$ and $\pi_R(s_{MO}, s_R)$, respectively.

$(\boldsymbol{s_{MO}}, \boldsymbol{s_R}) = (\boldsymbol{DD}, \boldsymbol{D})$:

$$\omega(DD, D) = \omega^* \tag{18}$$

$$p_O(DD, D) = p_O^* \tag{19}$$

$$p_R(DD, D) = p_R^* \tag{20}$$

$$\pi_M(DD, D) = \omega^*(D_R^* + a\epsilon) = \omega^*(p_R^* - \omega^* + a\epsilon) \tag{21}$$

$$\begin{aligned} \pi_O(DD, D) &= p_O^* D_O(DD, D) = p_O^*(D_O^* + (1-a)\epsilon) \\ &= (p_O^*)^2 + (1-a)\epsilon p_O^* \end{aligned} \tag{22}$$

$$\pi_{MO}(DD, D) = \omega^*(p_R^* - \omega^* + a\epsilon) + (p_O^*)^2 + (1-a)\epsilon p_O^* \tag{23}$$

$$\pi_R(DD, D) = (p_R^* - \omega^*)(D_R^* + a\epsilon) = (p_R^* - \omega^*)(p_R^* - \omega^* + a\epsilon) \tag{24}$$

$(\boldsymbol{s_{MO}}, \boldsymbol{s_R}) = (\boldsymbol{DD}, \boldsymbol{A})$:

$$\omega(DD, A) = \omega^* \tag{25}$$

$$p_O(DD, A) = p_O^* \tag{26}$$

$$p_R(DD, A) = \frac{a(1+\epsilon) + bP_O^* + \omega^*}{2} = p_R^* + \frac{a\epsilon}{2} \tag{27}$$

$$\pi_M(DD, A) = \omega^*(D_R^* + a\epsilon + (p_R^* - p_R(DD, A))) = \omega^* \left(p_R^* - \omega^* + \frac{a\epsilon}{2}\right) \tag{28}$$

$$\begin{aligned} \pi_O(DD, A) &= p_O^*(D_O^* + (1-a)\epsilon - b(p_R^* - p_R(DD, A))) \\ &= (p_O^*)^2 + \left((1-a)\epsilon + \frac{ba\epsilon}{2}\right) p_O^* \end{aligned} \tag{29}$$

$$\pi_{MO}(DD, A) = \omega^* \left(p_R^* - \omega^* + \frac{a\epsilon}{2}\right) + (p_O^*)^2 + \left((1-a)\epsilon + \frac{ba\epsilon}{2}\right) p_O^* \tag{30}$$

$$\begin{aligned} \pi_R(DD, A) &= (p_R(DD, A) - \omega^*)(D_R^* + a\epsilon + (p_R^* - p_R(DD, A))) \\ &= \left(p_R^* - \omega^* + \frac{a\epsilon}{2}\right)^2 \end{aligned} \tag{31}$$

$(\boldsymbol{s_{MO}}, \boldsymbol{s_R}) = (\boldsymbol{DA}, \boldsymbol{D})$ **:**

$$\omega(DA, D) = \omega^* \tag{32}$$

$$p_O(DA, D) = \frac{(1-a)(1+\epsilon) + bP_R^*}{2} = p_O^* + \frac{(1-a)\epsilon}{2} \tag{33}$$

$$p_R(DA, D) = p_R^* \tag{34}$$

$$\begin{aligned} \pi_M(DA, D) &= \omega^*(D_R^* + a\epsilon + b(p_O(DA, D) - p_O^*)) \\ &= \omega^* \left(p_R^* - \omega^* + a\epsilon + \frac{b(1-a)\epsilon}{2}\right) \end{aligned} \tag{35}$$

$$\begin{aligned} \pi_O(DA, D) &= p_O(DA, D)(D_O^* + (1-a)\epsilon + p_O^* - p_O(DA, D)) \\ &= \left(p_O^* + \frac{(1-a)\epsilon}{2}\right)^2 \end{aligned} \tag{36}$$

$$\pi_{MO}(DA, D) = \omega^* \left(p_R^* - \omega^* + a\epsilon + \frac{b(1-a)\epsilon}{2}\right) + \left(p_O^* + \frac{(1-a)\epsilon}{2}\right)^2 \tag{37}$$

$$\begin{aligned} \pi_R(DA, D) &= (p_R^* - \omega^*)(D_R^* + a\epsilon + b(p_O(DA, D) - p_O^*)) \\ &= (p_R^* - \omega^*) \left(p_R^* - \omega^* + a\epsilon + \frac{b(1-a)\epsilon}{2}\right) \end{aligned} \tag{38}$$

$(\boldsymbol{s_{MO}}, \boldsymbol{s_R}) = (\boldsymbol{DA}, \boldsymbol{A})$ **:**

$$\omega(DA, A) = \omega^* \tag{39}$$

$$p_O(DA, A) = p_O^* + \frac{2(1-a)\epsilon + ba\epsilon}{4-b^2} \tag{40}$$

$$p_R(DA, A) = p_R^* + \frac{2a\epsilon + b(1-a)\epsilon}{4-b^2} \tag{41}$$

$$\begin{aligned} \pi_M(DA, A) &= \omega^*(a(1+\epsilon) - p_R(DA, A) + bp_O(DA, A)) \\ &= \omega^*(p_R(DA, A) - \omega^*) \end{aligned} \tag{42}$$

$$\begin{aligned}
\pi_O(DA,A) &= p_O(DA,A)((1-a)(1+\epsilon) - p_O(DA,A) + bp_R(DA,A)) \\
&= (p_O(DA,A))^2 \qquad (43)
\end{aligned}$$

$$\pi_{MO}(DA,A) = \omega^*(p_R(DA,A) - \omega^*) + (p_O(DA,A))^2 \qquad (44)$$

$$\pi_R(DA,A) = (p_R(DA,A) - \omega^*)^2 \qquad (45)$$

$(\boldsymbol{s_{MO}}, \boldsymbol{s_R}) = (\boldsymbol{AD}, \boldsymbol{D})$ **:**

$$\omega(AD,D) = \lambda_M p_R^* \qquad (46)$$

$$p_O(AD,D) = p_O^* \qquad (47)$$

$$p_R(AD,D) = p_R^* \qquad (48)$$

$$\pi_M(AD,D) = \omega(AD,D)(D_R^* + a\epsilon) = \omega(AD,D)\,(p_R^* - \omega^* + a\epsilon) \qquad (49)$$

$$\pi_O(AD,D) = p_O^*(D_O^* + (1-a)\epsilon) = (p_O^*)^2 + (1-a)\epsilon p_O^* \qquad (50)$$

$$\pi_{MO}(AD,D) = \omega(AD,D)\,(p_R^* - \omega^* + a\epsilon) + (p_O^*)^2 + (1-a)\epsilon p_O^* \qquad (51)$$

$$(52)$$

$$\begin{aligned}
\pi_R(AD,D) &= (p_R^* - \omega(AD,D))(D_R^* + a\epsilon) \\
&= (p_R^* - \omega(AD,D))\,(p_R^* - \omega^* + a\epsilon) \qquad (53)
\end{aligned}$$

$(\boldsymbol{s_{MO}}, \boldsymbol{s_R}) = (\boldsymbol{AD}, \boldsymbol{A})$ **:**

$$\begin{aligned}
\omega(AD,A) &= \left(\frac{\lambda_M(4-b^2)}{2\lambda_M + 4 - b^2}\right) p_R(AD,A) \\
&= \frac{\lambda_M\left(a(1+\epsilon)(4-b^2) + b\left[2(1-a) + ba + \frac{b\lambda_M(2a+b(1-a))}{4-2b^2}\right]\right)}{8-(2-\lambda_M)b^2} \qquad (54)
\end{aligned}$$

$$p_O(AD,A) = p_O^* \qquad (55)$$

$$
\begin{aligned}
p_R(AD,A) &= \frac{a(1+\epsilon)+bP_O^*+\omega(AD,A)}{2} \\
&= \frac{(2\lambda_M+4-b^2)\left(a(1+\epsilon)+\frac{b}{4-b^2}\left[2(1-a)+ba+\frac{b\lambda_M(2a+b(1-a))}{4-2b^2}\right]\right)}{8-(2-\lambda_M)b^2} \quad (56)
\end{aligned}
$$

$$\pi_M(AD,A) = \omega(AD,A)(p_R^*-\omega^*+a\epsilon+p_R^*-p_R(AD,A)) \quad (57)$$

$$\pi_O(AD,A) = p_O^*(p_O^*+(1-a)\epsilon+b(p_R(AD,A)-p_R^*)) \quad (58)$$

$$
\begin{aligned}
\pi_{MO}(AD,A) &= \omega(AD,A)(p_R^*-\omega^*+a\epsilon+p_R^*-p_R(AD,A)) \\
&\quad + p_O^*(p_O^*+(1-a)\epsilon+b(p_R(AD,A)-p_R^*)) \quad (59)
\end{aligned}
$$

$$\pi_R(AD,A) = (p_R(AD,A)-\omega(AD,A))(p_R^*-\omega^*+a\epsilon+p_R^*-p_R(AD,A)) \quad (60)$$

$\boldsymbol{(s_{MO}, s_R) = (AA, D):}$

$$\omega(AA,D) = \frac{-(1-\varphi p_R^*)+\sqrt{(1-\varphi p_R^*)^2+4\varphi\lambda_M p_R^*}}{2\Gamma} \quad (61)$$

$$\varphi = \frac{b^2}{(4-b^2)[p_R^*-\omega^*+a\epsilon+b(1-a)\frac{\epsilon}{2}]} \quad (62)$$

$$p_O(AA,D) = \frac{(1-a)(1+\epsilon)+bP_R^*}{2} = p_O^*+\frac{(1-a)\epsilon}{2} \quad (63)$$

$$p_R(AA,D) = p_R^* \quad (64)$$

$$
\begin{aligned}
\pi_M(AA,D) &= \omega(AA,D)(a(1+\epsilon)-p_R^*+bp_O(AA,D)) \\
&= \omega(AA,D)\left(p_R^*-\omega^*+a\epsilon+\frac{b(1-a)\epsilon}{2}\right) \quad (65)
\end{aligned}
$$

$$
\begin{aligned}
\pi_O(AA,D) &= p_O(AA,D)((1-a)(1+\epsilon)-p_O(AA,D)+bp_R^*) \\
&= \left(p_O^*+\frac{(1-a)\epsilon}{2}\right)^2 \quad (66)
\end{aligned}
$$

$$\pi_{MO}(AA,D) = \omega(AA,D)\left(p_R^*-\omega^*+a\epsilon+\frac{b(1-a)\epsilon}{2}\right)+\left(p_O^*+\frac{(1-a)\epsilon}{2}\right)^2 \quad (67)$$

$$\pi_R(AA,D) = (p_R^*-\omega(AA,D))\left(p_R^*-\omega^*+a\epsilon+\frac{b(1-a)\epsilon}{2}\right) \quad (68)$$

$\boldsymbol{(s_{MO}, s_R) = (AA, A):}$

$$\omega(AA, A) = (1+\epsilon)\omega^* \tag{69}$$

$$p_O(AA, A) = (1+\epsilon)p_O^* \tag{70}$$

$$p(AA, A) = (1+\epsilon)p_R^* \tag{71}$$

$$\pi_M(AA, A) = (1+\epsilon)^2\omega^*(p_R^* - \omega^*) \tag{72}$$

$$\pi_O(AA, A) = (1+\epsilon)^2(p_O^*)^2 \tag{73}$$

$$\pi_{MO}(AA, A) = (1+\epsilon)^2\left(\omega^*(p_R^* - \omega^*) + (p_O^*)^2\right) \tag{74}$$

$$\pi_R(AA, A) = (1+\epsilon)^2(p_R^* - \omega^*)^2 \tag{75}$$

Appendix B (Proofs of Propositions)

Proof of Proposition 1. We will first simultaneously solve the optimization problems in (6) and (7) to find the Bertrand-Nash equilibrium plans $p_O(\omega)$ and $p_R(\omega)$ as a function of ω. The first-order conditions for (6) and (7) yields the best-response functions

$$p_R(p_O) = \frac{a + bP_O + \omega}{2} \tag{76}$$

$$p_O(p_R) = \frac{1 - a + bP_R}{2}. \tag{77}$$

Solving the above functions together yield the Bertrand-Nash equilibrium plans

$$p_R(\omega) = \frac{2a + b(1-a) + 2\omega}{4 - b^2} \tag{78}$$

$$p_O(\omega) = \frac{2(1-a) + ba + b\omega}{4 - b^2}. \tag{79}$$

On the other hand, the first-order condition for the bargaining problem (8) can be calculated as

$$\frac{\lambda_M}{\omega} + \frac{1-\lambda_M}{p_R - \omega}\left(\frac{\partial p_R(\omega)}{\partial \omega} - 1\right) + \frac{1}{a - p_R + bp_O}\left(-\frac{\partial p_R(\omega)}{\partial \omega} + b\frac{\partial p_O(\omega)}{\partial \omega}\right) = 0. \tag{80}$$

Using (78) and (79), we calculate

$$\frac{\partial p_R(\omega)}{\partial \omega} = \frac{2}{4-b^2} \tag{81}$$

$$\frac{\partial p_O(\omega)}{\partial \omega} = \frac{b}{4-b^2}. \tag{82}$$

Inserting (78), (79), (81), and (82), into (80), we obtain the solution ω^* as in (9). Finally, inserting (9) into (78) and (79) yields (10) and (11). ■

Proof of Proposition 2. Using (24) and (31), we can calculate

$$\begin{aligned}\pi_R(DD,A)-\pi_R(DD,D) &= \left(p_R^*-\omega^*+\frac{a\epsilon}{2}\right)^2-(p_R^*-\omega^*)(p_R^*-\omega^*+a\epsilon)\\ &= \frac{a^2\epsilon^2}{4}\end{aligned} \tag{83}$$

which is always non-negative. Thus, in the face of any demand shock, the retailer always would find it optimal to unilaterally deviate from the strategy profile (DD,D) whenever the adjustment costs are zero (or sufficiently small), implying that (DD,D) cannot be a Nash equilibrium. ■

Proof of Proposition 3. Using (38) and (45), we can calculate

$$\begin{aligned}\pi_R(DA,A)-\pi_R(DA,D) &= \left(p_R^*+\frac{2a\epsilon+b(1-a)\epsilon}{4-b^2}-\omega^*\right)^2\\ &\quad -(p_R^*-\omega^*)\left(p_R^*-\omega^*+a\epsilon+\frac{b(1-a)\epsilon}{2}\right)\\ &= (p_R^*-\omega^*)(4a+2b(1-a))\left(\frac{b^2}{4-b^2}\right)\epsilon\\ &\quad +\left(\frac{2a\epsilon+b(1-a)\epsilon}{4-b^2}\right)^2\\ &= \left(\frac{2a+b(1-a)}{4-b^2}\right)^2\left(\left(1+\frac{\lambda_M}{4-2b^2}\right)2b^2\epsilon+\epsilon^2\right)\end{aligned} \tag{84}$$

which is negative if and only if

$$\max\left\{-1,-2b^2\left(1+\frac{\lambda_M}{4-2b^2}\right)\right\}<\epsilon<0. \tag{85}$$

Thus, whenever the above inequality does not hold, the retailer always would find it optimal to unilaterally deviate from the strategy profile (DA, D) to (DA, A) whenever the adjustment costs are zero (or sufficiently small), implying that (DA, D) cannot be a Nash equilibrium. ■

Proof of Proposition 4. We should notice that

$$\frac{\lambda_M}{\omega(AA,D)} + \frac{1-\lambda_M}{p_R^* - \omega(AA,D)}(-1) + \frac{1}{p_R^* - \omega^* + a\epsilon + b(1-a)\frac{\epsilon}{2}}\left(\frac{b^2}{4-b^2}\right) = 0 \quad (86)$$

and

$$\frac{\lambda_M}{\omega(AD,D)} + \frac{1-\lambda_M}{p_R^* - \omega(AD,D)}(-1) = 0. \quad (87)$$

Therefore,

$$\frac{\lambda_M}{\omega(AA,D)} + \frac{1-\lambda_M}{p_R^* - \omega(AA,D)}(-1) < \frac{\lambda_M}{\omega(AD,D)} + \frac{1-\lambda_M}{p_R^* - \omega(AD,D)}(-1), \quad (88)$$

implying $\omega(AA, D) > \omega(AD, D)$. Using (51) and (67), we can now calculate

$$\begin{aligned}\pi_{MO}(AA,D) - \pi_{MO}(AD,D) &= [\omega(AA,D) - \omega(AD,D)](p_R^* - \omega^* + a\epsilon) \\ &\quad + \omega(AA,D)\frac{b(1-a)\epsilon}{2} + \frac{(1-a)^2\epsilon^2}{4}, \end{aligned} \quad (89)$$

which is always positive if $\epsilon > 0$. Thus, whenever the demand shock is positive, the manufacturer would always find it optimal to unilaterally deviate from the strategy profile (AD, D) to (AA, D) whenever the adjustment costs are zero (or sufficiently small), implying that (AD, D) cannot be a Nash equilibrium. ■

Proof of Proposition 5. Using (30) and (44), we can calculate

$$\begin{aligned}\pi_{MO}(DA,A) - \pi_{MO}(DD,A) &= \omega^*\left(\frac{[2a + b(1-a)]}{4-b^2} - \frac{a}{2}\right)\epsilon \\ &\quad + p_O^*\left(\frac{[4(1-a) + 2ba]}{4-b^2} - (1-a) - \frac{ba}{2}\right)\epsilon \\ &\quad + \left(\frac{[2(1-a) + ba]}{4-b^2}\right)^2\epsilon^2\end{aligned}$$

which is positive if $\epsilon > 0$. Thus, whenever ϵ is positive, the manufacturer would find it optimal to unilaterally deviate from the strategy profile (DD, A) to (DA, A) whenever the adjustment costs are zero (or sufficiently small), implying that (DD, A) cannot be a Nash equilibrium. ■