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Public Goods, Trust, and Tax Policy: Shaping Economic Formalization

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Abstract

This paper develops a general equilibrium framework that integrates heterogeneous firms with both idiosyncratic productivity and subjective beliefs about public goods provision—specifically, confidence in institutional quality—alongside endogenous informality. We examine the impact of tax policy on the formalization process and highlight the crucial role of firms' trust in public institutions. Our findings reveal that when firms perceive the government as credible, an increase in both tax rates and tax revenues enhances public goods provision, fostering greater formalization. However, in environments with weak institutional trust, formalization policies may yield suboptimal economic outcomes—potentially even worsening conditions compared to scenarios with higher trust levels. This underscores how institutional confidence influences the productivity of formal firms and facilitates their transition into the formal sector. In the long run, effective tax policy can improve overall welfare, but its success is contingent on government credibility. Our research contributes to the literature on informality by providing novel insights for policymakers seeking to enhance formalization and economic welfare, particularly in settings where skepticism about government commitment and institutional capacity prevails.

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Key words: Informality, Optimal Tax Policy, Entrepreneurship, Tax Evasion.

*The opinions expressed are solely those of the authors and do not necessarily reflect the policy or official position of the institutions they represent.

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1. Introduction

Informality remains a pervasive structural feature of developing economies, profoundly shaping their labor markets and socio-economic dynamics. While it does not represent a market failure *per se*, informality is widely regarded as one of the most significant barriers to sustainable growth and prosperity in emerging and developing nations. According to the International Labour Organization (ILO), approximately 61% of the global employed population—amounting to around 2 billion people—engaged in informal employment in 2016. This encompasses both individuals operating within the informal sector and those in formal employment arrangements who lack access to employment-based social protections. Notably, the prevalence of informal employment varies substantially across regions and countries, with developing nations typically exhibiting much higher rates of informality compared to their developed counterparts.

Informal labor encompasses workers who are not covered by formal labor regulations and lack employment benefits, such as social security, paid leave, and protection from unfair dismissal. Similarly, informal productive units operate outside formal regulatory frameworks, including unregistered businesses that do not adhere to labor or environmental regulations. These two aspects of informality are deeply interconnected: informal workers often find employment within informal productive units or as self-employed individuals operating informally. Conversely, informal productive units frequently rely on informal labor as a cost-cutting measure to remain viable and competitive.

The concept of the informal economy is elusive and often overlaps with the notion of tax compliance. Several approaches have been followed over years to the analysis of *'tax compliance'* among citizens and firms ranging detection and punishment, the burden of taxation, public good provision and social norms.[1] All of elements have found empirical support in the literature, nonetheless, many of the theoretical models have focused only on the first two. In this paper we provide a framework flexible enough to encompass all of these factors, by focusing both on the provision of public goods and government trust. To our knowledge, we are the first ones to take this approach.

[1] See Alm et al. (1995) for a discussion of economic and noneconomic factors affecting tax compliance and the evidence found through experimental methods. Torgler (2007) provides a thorough analysis of all of these factors.

Levi (1988), through historical and comparative analysis, argues that voluntary tax compliance emerges primarily when taxpayers trust that (1) rulers will honor their commitments and (2) other taxpayers will also fulfill their obligations. Taxpayer behavior is thus shaped not only by individual motives and intentions but also by the actions and credibility of the government. Additionally, taxpayers' beliefs regarding the behavior of their peers significantly influence compliance decisions. We posit that these factors are intricately tied to the perceived quality of public goods that taxpayers expect to receive. This fundamental *quid pro quo* element of tax compliance is embedded in our framework.

Torgler (2007) underscores that trust in government is a critical determinant of tax compliance. Similarly, Richardson (2008) in a cross-national study involving 47 countries, demonstrates a positive correlation between trust in government and tax compliance. Appiah et al. (2024), examining small and medium enterprises in Ghana, also identifies a significant relationship between trust in government and tax compliance behavior.

These studies align with insights from cross-cultural psychology, suggesting that when taxpayers perceive public goods as misaligned with their preferences, disproportionately benefiting certain groups, or compromised by corruption and rent-seeking, the *quid pro quo* of taxation becomes less compelling. A breakdown in this reciprocity weakens voluntary compliance, linking it closely to the concept of political legitimacy. As Alt (1983) observes, "the legitimacy of a tax is that of the state that levies and collects it." A state perceived as lacking legitimacy—due to factors such as corruption, non-representativeness, or poor governance—risks a downward spiral of non-compliance, further eroding its ability to sustain public goods provision.

In the present paper, we bridge recent advances in modeling the endogenous decision of entrepreneurial informality (see Ulyssea (2018)) with the literature on the determinants of tax compliance, giving a role to both the provision of public goods and the trust that firms have in the quality and impact of these goods in their production function. Although the decision to be informal is complex, informal firms typically exhibit low productivity which, given the cost of becoming formal, leads them to self-select as informal. In turn, informality impacts their productivity through at least two channels.

First, directly, as they cannot fully access public goods and services. Second, indirectly, as lower tax collection by the government limits the provision of these goods and services. The latter effect

also generates a negative externality, since the decision to remain informal impacts—via foregone tax revenue—the supply of public goods and services, thereby reducing the productivity of other firms. Given the complexity of this dynamic, policymakers must carefully consider the trade-offs between increasing tax revenue and promoting formalization when designing tax policies.

To shed more light on this issue, we present an equilibrium model that incorporates the informal sector with heterogeneity across firms in their idiosyncratic productivity. Specifically, we investigate the effects of policies that increase the benefits of formalizing and the costs of operating in the informal sector on overall welfare. In the context of public goods and services, it is important to note that both formal and informal firms in the economy have access to these benefits provided by the government. However, informal firms typically have limited access, as highlighted by Loayza (1999). Formal firms, on the other hand, are responsible for financing these public goods and services through taxation.

This dynamic creates a trade-off for the government: imposing higher taxes on formal sector firms may discourage formalization and reduce the tax base, whereas imposing higher taxes on informal firms may further increase informality and reduce the government's capacity to provide public goods and services. The optimal tax policy depends on the relative importance of these trade-offs and the specific characteristics of the economy, such as the level of productivity heterogeneity and the costs of entering the formal sector.

The provision of public goods and services by the government to formal sector firms entails a trade-off involving higher tax burdens. This burden can have a dual impact on the level of formality in the economy. On the one hand, it may discourage firms from entering or remaining in the formal sector. On the other hand, increased tax collection could enable the government to finance public goods and services, benefiting both formal and informal firms. The overall effect of these measures on formalization depends on the extent to which the cost of financing public goods is offset by the benefits of their increased provision. To our knowledge, this study is the first to highlight this trade-off within a general equilibrium framework.

We analyze the mechanisms of this framework at the steady-state through comparative statics analysis. Specifically, we investigate how steady-state outcomes change under different values of key parameters, including the tax rate, formalization costs, the productivity of public capital, and the fraction of

access informal firms have to public goods and services. By conducting this analysis, we aim to illuminate the trade-offs inherent in the government's objectives of promoting formalization and financing public goods and services.

The nature of our model leads to multiple equilibria, as entrepreneurs' beliefs regarding the government's actions and the behavior of other entrepreneurs influence expectations about public goods provision and formalization decisions. Thus, the government's ability to make entrepreneurs *buy into* its plans can determine overall tax compliance and steady-state welfare. In this manner, both trust in the government and the provision of public goods emerge as critical determinants of overall tax compliance and informality rates.

Outline. The paper is organized as follows. Section 2 briefly reviews the literature. Section 3 presents the model. Section 4 discusses the methodology. Section 5 presents the results. Section 6 concludes. All proofs and details of the derivations are left for the appendix.

2. Literature Review

The literature on informal economies and taxation has grown considerably in recent decades. There is broad consensus that a higher tax rate can discourage firms from operating in the formal sector and can also deter new firms from entering the formal sector. However, the empirical evidence on the effects of taxation on informality is mixed, with some studies finding a negative correlation between tax rates and formality rates, while others find no significant effects.

Despite the mixed evidence, it is widely recognized that increasing tax rates can lead to greater tax revenue for the government, which can be used to finance public goods and services that benefit both formal and informal firms. This mechanism has the potential to increase the formality of productive units, and therefore, it is essential to consider the trade-offs between tax revenue and formalization when designing tax policies. Previous studies addressing this tradeoff faced by the government include Loayza (1999) and more recently, Bandaogo (2018).

The informal economy has been studied in the literature from three main perspectives: labor market informality, production unit informality, and credit market informality. Some studies have also focused on combining these perspectives, such as labor and credit informality, as shown in Wasmer and Weil

(2004), Cavalcanti and Villamil (2003), and De Paula and Scheinkman (2007). Others have examined labor and production unit informality, such as Boeri and Garibaldi (2005) and Frankel and Pissarides (2006). Batini et al. (2010) provides a comprehensive classification of these three modeling approaches up to the date of publication.

Most of the literature on the informal economy has focused on characterizing workers' endogenous decisions to work in the informal or formal sector. One way of modeling labor informality is by using the pioneering work of Todaro (1969) and Harris and Todaro (1970), who model urban to rural migration in a setup with two geographically distinct markets that are segmented, and two different wage equilibria prevail (wage duality). These papers capture a stylized fact of labor informality as higher wages in the formal sector. Another approach to modeling labor informality was developed by Lucas Jr (1978) and Rauch (1991), in which models insert informality as a cost-benefit analysis, where agents evaluate the tradeoffs of becoming formal or remaining informal. Building on these pioneering works, a third classification includes sophisticated models that incorporate the Mortensen-Pissarides search and matching mechanism in the Harris and Todaro (1970) model.

In the literature on informality, there is a second generation of models based on search and matching models. Batini et al. (2010) classifies these models into three categories: models with i) intersectoral margin for workers and firms, ii) intersectoral margin only for workers, and iii) with intra-firm margin, where both formal and informal labor contracts exist within the same firm. The works of Boeri and Garibaldi (2005) and Badaoui et al. (2006) fall into the first category, while those of Albrecht et al. (2008), Zenou (2008), Satchi and Temple (2009), and Kolm and Larsen (2003) fall into the second category. The researches of Bosch (2006) and Bosch (2007) belong to the third category.

In line with the focus of our research, we highlight the works of Castillo and Montoro (2012) and Batini et al. (2011), which not only model informality but also analyze its impact on the dynamics of inflation, monetary and fiscal policy. Batini et al. (2011) examine how the presence of informality in emerging economies affects the behavior of monetary and fiscal policy. To this end, they propose a New-Keynesian model with a closed economy that incorporates both the formal and informal sectors. The informal sector is more labor-intensive, is not taxed, has a classical labor market, faces strong credit constraints to finance investment, and is less visible in terms of observed output.

For welfare analysis, they compare the results of an optimal monetary policy, a discretionary policy, and a welfare-optimizing interest rate Taylor rule policy together with a balanced budget fiscal regime. They find that welfare losses are larger and significant in the discretionary policy case compared to the optimal Zero Lower Bound commitment policy. Also, the steady-state inflation rate needed to accommodate changes in interest rates rises considerably if policy is discretionary.

Castillo and Montoro (2012) analyze the effects of labor market informality on inflation dynamics and the transmission of aggregate demand and supply shocks. To do so, they modify the New-Keynesian model by incorporating labor market frictions as in the Diamond-Mortensen-Pissarides model. They find that the informal economy generates a dampening effect that decreases the pressure of demand shocks on inflation. This implies that in economies with large informal labor markets, changes in interest rates are more effective in stimulating real output and have less impact on inflation. Additionally, the model produces cyclical flows from informal to formal employment, consistent with the data.

The literature on informal credit markets is still very limited. However, research in this branch of the literature is also relevant, because, as Batini et al. (2010) point out, a large informal credit market could have an impact on the effectiveness of monetary policy. Ghosh et al. (2000) refers to the three strands with which the literature is approached, which are: adverse selection, moral hazard, and imperfect enforcement of contracts due to the characteristics of informal credit markets of imperfect information and limited enforcement capacity. Another way of classifying this branch of the literature is by means of partial equilibrium and general equilibrium papers. Among the general equilibrium models are the works by Bell (1990), Arnott and Stiglitz (1991), Guinnane et al. (1994), Sagrario Floro and Ray (1997), Hoff and Stiglitz (1997), Kochar (1997), Bose (1998), Dell'Ariccia and Garibaldi (2005), Antunes and Cavalcanti (2007), and Madestam (2014). While Dasgupta (2009) makes a general equilibrium analysis.

Regarding the analysis of firm informality, there has been limited research in the dynamic general equilibrium literature. Some notable contributions in this area include the works of Loayza (1999), Sarte (2000), Ihrig and Moe (2004), Allen and Schipper (2017) and Ulyssea (2018). The former two works develop non-stochastic general equilibrium models, while Ihrig and Moe (2004) presents a DSGE model. Ulyssea (2018) does not present a DSGE model; nonetheless, the model presented accounts for heterogeneity at the firms and workers level.

3. The Model

This paper develops a general equilibrium model with monopolistically competitive firms to examine the trade-offs governments face when setting tax rates that influence both firms' decisions to operate formally and the provision of public goods and services. The model explicitly accounts for the existence of an informal sector—an alternative operating environment where firms evade taxes but have limited access to public goods.

The model, building on the work of Loayza (1999) and Bandaogo (2018), incorporates a friction that emerges when formal firms view the informal sector as a viable alternative. Firms may choose to transition into the informal sector without considering the potential negative implications for public goods and services financing, which can diminish the aggregate productivity of the entire economy. Alternatively, firms may decide to remain formal, driven by the expectation of adequate public goods provision that supports their operations and enhances overall productivity.

This framework can also be employed to analyze the effects of higher taxes on formality. Formal firms might choose to transition to the informal sector if they lack trust in the government's capacity to deliver improved public goods in exchange for the higher tax burden. On the other hand, they may remain formal if they expect the increased tax revenues to translate into sufficient public goods and services that benefit their operations. Notably, such a policy could also encourage informal firms to formalize, depending on their expectations of improved public goods provision. Consequently, firms may experience varying levels of positive externalities—equal, greater, or reduced—shaped by their beliefs and confidence in the government's actions.

The equilibrium of the model is presented, serving as the foundation for conducting comparative scenario and static analyses. These scenarios are designed around two distinct types of beliefs, enabling an exploration of how differing expectations regarding government actions influence formality.

In summary, this paper contributes to the literature on informality by emphasizing the significance of firm beliefs regarding government actions and proposing tax policies that consider these beliefs. It explores the trade-offs between boosting tax revenue and fostering formality, offering valuable insights for policymakers to encourage formalization and enhance welfare amidst ongoing economic shocks.

3.1. Set up

This model builds on Melitz (2003) and Ulyssea (2018). The economy consists of three main agents: households, firms, and the government. Households, comprising individual members, provide labor to firms, which are ultimately owned by these households. Firms, in turn, operate with the objective of maximizing profits through a dynamic entrepreneurial process, which involves strategic decisions regarding their sector of operation, taxation, regulation, and public goods provision by the government.

- Potential entrants are identical and face unknown productivity levels. Therefore, these firms must incur a one-time pre-investment expenditure to discover their productivity. Based on this information, they decide whether to enter the market and subsequently choose to operate either in the formal or informal sector. In making this decision, firms take into account their own productivity levels as well as the institutional constraints associated with each sector. The selection criteria are based on their expected profits, which can be summarized as follows:

  (i) Firms enter the market only if the expected value of the firm and its anticipated profits are positive, given their productivity level.

  (ii) They choose to operate in the formal sector only if the profits are higher or equal to those of the informal sector counterfactual.

- To make this decision, firms will evaluate the structural parameters associated with each sector. For instance, they will assess the fixed costs associated with operating formally and informally, as outlined in Allen and Schipper (2017) and Sarte (2000). Additionally, formal firms must bear the tax burden on their sales (VAT), denoted by τ_t, following the framework in Allen and Schipper (2017). In contrast, informal firms face a probability of penalties, as described by Loayza (1999) and Ulyssea (2018).

- Furthermore, both formal and informal firms have access to capital provided by the government due to the non-excludable nature of public goods. We assume no restrictions ex-ante on the efficiency with which public goods are utilized. However, as a result of solving the model, it emerges as a necessary condition that formal firms are more efficient in utilizing these public goods compared to informal firms.

- Similar to Melitz (2003), there is a probability of an individual exogenous exit shock. This implies that, regardless of the firm's idiosyncratic productivity level upon entering the market, it may be forced to exit if this shock occurs with probability δ. Nonetheless, as long as the exit shock does not occur, the firm continues to operate with its current productivity level.

So, in the absence of short-term shocks, this economy is characterized by a long-term steady-state equilibrium between households, firms, and the government. However, to analyze the role of the government and firms' beliefs about it in determining formality, we consider long-term (or permanent) formal productivity shock.

3.2. Households

In this economy, different varieties Ω of the consumption good C are produced. During each period, families decide the consumption of each variety, $C(\omega)$, at a price $P(\omega)$, in such a way that they minimize the total expenditure.

$$\begin{aligned} &\min_{C(\omega)} PC = \int_{\omega\in\Omega} P(\omega)C(\omega)d\omega \\ &s.t. \\ &C = \left[\int_{\omega\in\Omega} C(\omega)^{\frac{\epsilon-1}{\epsilon}} d\omega\right]^{\frac{\epsilon}{\epsilon-1}} \end{aligned} \tag{1}$$

From this, the consumption demand for ω is derived.

$$P = \left[\int_{\omega\in\Omega} P(\omega)^{1-\epsilon} d\omega\right]^{\frac{1}{1-\epsilon}} \tag{2}$$

Moreover, in the absence of short-term shocks, households derive lifetime utility from consuming the good, C, which is primarily financed through three sources of income. First, labor income is earned by working in formal and informal firms, represented as $\int_{\omega\in\Omega} w(\omega)N(\omega)d\omega$, where $w(\omega)$ denotes wages and $N(\omega)$ the number of workers in firm ω. Second, income is generated during the pre-investment stage, N_e. Third, households receive profits as owners of firms in the economy, expressed as $\int_{\omega\in\Omega} \pi(\omega)d\omega$, where $\pi(\omega)$ represents the profit of firm ω.

$$\begin{aligned} &\max_{C} \left\{ \frac{1}{1-\beta} U(C) \right\} \\ &s.t. \\ &PC = \int_{\omega \in \Omega} w(\omega) N(\omega) d\omega + \int_{\omega \in \Omega} \pi(\omega) d\omega + N_e \end{aligned} \tag{3}$$

Similar to Melitz (2003), this study analyzes the equilibrium in the absence of short-term shocks. Under these conditions, consumption smoothing implies that $\beta^{-1} = (1 + r)$, where β represents the discount factor and r is the interest rate.

3.3. Government

The government collects taxes, T, from formal firms and receives formalization fees, f_f. These resources are then used to provide public goods, PK. Additionally, the government penalizes informal firms with a positive probability, ρ, which incurs an enforcement cost. (For simplicity, these costs and revenues do not affect the fiscal equilibrium.)

The government's behavior is described by the following equations for tax collection, T, penalties on informal firms, λ, and the allocation of resources to public goods, K. Assuming GDP is normalized to 1, we define $K = \hat{\kappa} \in (0, 1)$. Moreover, if firms' beliefs about public goods provision ($\tilde{\kappa}$) align with equilibrium outcomes, it follows that $\hat{\kappa} = \tilde{\kappa}$. Under this normalization, the level of public goods in the model can be interpreted as a percentage of GDP.

$$\begin{aligned} PT &= \int_{\omega \in \Omega_F} \tau P(\omega) q(\omega) d\omega \\ P\lambda &= \int_{\omega \in \Omega_I} \rho P(\omega) q(\omega) d\omega \\ P\hat{\kappa} &= PT + f_f M_f \end{aligned} \tag{4}$$

Where Ω_I and Ω_F represent the subsets of informal and formal varieties, respectively; τ denotes a sales tax and M_f corresponds to the total mass of formal firms.

3.4. Formal Sector Production

Production in the formal sector is characterized as follows: Each firm produces using labor, its own productivity φ, and public spending as a percentage of GDP, $\tilde{\kappa}$. Public spending generates positive externalities for firms (e.g., infrastructure such as ports, roads, security, etc.), with $\tilde{\kappa}$ determined ex-ante. Additionally, formal firms are influenced by long-term and aggregate productivity shocks, A_f.

In terms of costs, all formal firms face the same fixed cost of operating formally, f_f, the formal wage rate, w_f, and tax obligations, τ. The behavior of these firms is driven by the need to choose the optimal price for their variety, taking into account their demand function, $q^d(\varphi)$, and their production technology, $q^s(\varphi)$:

$$\begin{aligned} &\max_{p(\varphi)} \pi(\varphi) = (1-\tau)p(\varphi)q(\varphi) - w_f N(\varphi) - f_f \\ &s.t. \\ &q^s(\varphi) = \varphi A_f N(\varphi) F_f(\tilde{\kappa}) \\ &q^d(\varphi) = \left\{ \frac{p(\varphi)}{P} \right\}^{-\epsilon} Q \end{aligned} \tag{5}$$

3.5. Informal Sector Production

Firms in the informal sector produce using labor and their own productivity. Unlike formal firms, they are not affected by aggregate productivity shocks, evade taxes, face lower fixed costs (f_i), and pay lower informal wages (w_i). However, they operate under the risk of being identified and losing all their income with a probability ρ.

The behavior of these firms can be summarized by the following optimization problem:

$$\begin{aligned} &\max_{p(\varphi)} \pi(\varphi) = (1-\rho)p(\varphi)q(\varphi) - w_i N(\varphi) - f_i, \\ &\text{subject to:} \\ &q^s(\varphi) = \varphi N(\varphi) F_i(\tilde{\kappa}), \\ &q^d(\varphi) = \left\{ \frac{p(\varphi)}{P} \right\}^{-\epsilon} Q, \end{aligned} \tag{6}$$

3.6. Sectoral differences

Under these specifications, both sectors differ in their optimal prices:

$$\begin{aligned} &\text{Formal prices:} \\ &p(\varphi) = \frac{\epsilon}{\epsilon - 1} \frac{1}{\varphi} \frac{w_f}{(1-\tau) A_f F_f(\tilde{\kappa})} \\ &\text{Informal prices:} \\ &p(\varphi) = \frac{\epsilon}{\epsilon - 1} \frac{1}{\varphi} \frac{w_i}{(1-\rho) F_i(\tilde{\kappa})} \end{aligned} \tag{7}$$

Following the approach of Melitz (2003), the pricing rules derived in the previous equation yield identical production ratios, $\frac{q(\varphi_1)}{q(\varphi_2)} = \left(\frac{\varphi_1}{\varphi_2}\right)^{\epsilon}$, and revenue ratios, $\frac{r(\varphi_1)}{r(\varphi_2)} = \left(\frac{\varphi_1}{\varphi_2}\right)^{\epsilon-1}$, for firms within each sector. However, these ratios differ across sectors due to variations in costs, productivity shocks, and other sector-specific factors:

$$\begin{aligned} \frac{q(\varphi_f)}{q(\varphi_i)} &= \left\{ \frac{w_i}{w_f} \frac{(1-\tau)}{(1-\rho)} \frac{A_f F_f(\tilde{\kappa})}{F_i(\tilde{\kappa})} \right\}^{\epsilon} \left(\frac{\varphi_f}{\varphi_i} \right)^{\epsilon} \\ \frac{r(\varphi_f)}{r(\varphi_i)} &= \left\{ \frac{w_i}{w_f} \frac{(1-\tau)}{(1-\rho)} \frac{A_f F_f(\tilde{\kappa})}{F_i(\tilde{\kappa})} \right\}^{\epsilon-1} \left(\frac{\varphi_f}{\varphi_i} \right)^{\epsilon-1} \end{aligned} \tag{8}$$

As observed, equally productive firms (i.e., with the same φ) may differ in size across sectors. For instance, if two firms share the same productivity level but the externalities generated by public goods are greater for formal firms than for informal ones (due to institutional constraints, access limitations, or other factors), formal firms will tend to be larger than their informal counterparts.

Thus, beyond productivity, structural differences and the provision of public goods significantly contribute to the disparities in production and revenue between the two sectors. If firms form expectations about public goods as a percentage of GDP, these expectations become pivotal in their decision-making process—both in determining whether to enter the market and in choosing between the formal and informal sectors.

3.7. Aggregate

3.7.1. Productivity

Given the exogenous productivity distribution, $g(\cdot)$, the equilibrium will be characterized by:

- The number of entering firms, M_e, and the number of successfully entering firms, M.
- The equilibrium (ex-post) productivity distribution, $\mu(\cdot)$, over the range $(0, \infty)$.
- The clearing of the labor market, both pre-investment and post-entry, the clearing of the goods market, and the satisfaction of the government budget constraints.

The equilibrium ex-post productivity (conditioned on firms entering successfully, $\varphi^* \geq 0$) is determined by the ex-ante and exogenous productivity distribution, $g(\varphi)$:

$$\mu(\varphi) = \begin{cases} \dfrac{g(\varphi)}{1 - G(\varphi^*)}, \ \forall \varphi \geq \varphi^* \\ \\ 0, \text{ otherwise} \end{cases} \tag{9}$$

Then, the aggregate ex-post productivity of the economy can be expressed for each sector as follows:

$$\begin{aligned} &\text{Aggregate Informal Productivity} \\ &\tilde{\varphi}_I = \left[\int_{\varphi^*}^{\bar{\varphi}} \varphi^{\epsilon-1} \frac{g(\varphi)}{G(\bar{\varphi}) - G(\varphi^*)} d\varphi \right]^{\frac{1}{\epsilon-1}} \\ &\text{Aggregate Formal Productivity} \\ &\tilde{\varphi}_F = \left[\int_{\bar{\varphi}}^{\infty} \varphi^{\epsilon-1} \frac{g(\varphi)}{1 - G(\bar{\varphi})} d\varphi \right]^{\frac{1}{\epsilon-1}} \end{aligned} \tag{10}$$

Where, $\bar{\varphi}$, refers to the minimum productivity level at which a formal firm obtains positive profits that are greater than its opportunity cost of being informal. Furthermore, the average productivity, following Melitz (2003), in the presence of both sectors, $\tilde{\varphi} = \left[\int_0^{\infty} \varphi^{\epsilon-1} \mu(\varphi) \, d\varphi \right]^{\frac{1}{\epsilon-1}}$, can be expressed as follows:

$$\tilde{\varphi}^{\epsilon-1} = \left[\frac{w_i}{(1-\rho)F_i(\tilde{\kappa})}\right]^{1-\epsilon} \frac{G(\bar{\varphi}) - G(\varphi^*)}{1 - G(\varphi^*)} \tilde{\varphi}_I^{\epsilon-1} + \left[\frac{w_f}{(1-\tau)A_f F_f(\tilde{\kappa})}\right]^{1-\epsilon} \frac{1 - G(\bar{\varphi})}{1 - G(\varphi^*)} \varphi_F^{\epsilon-1} \tag{11}$$

Here, it can be observed that the aggregate productivity of the economy depends on the productivity of each sector and its impact on the overall economy. In other words, the aggregate productivity of a country with a strong bias towards informality will be primarily influenced by firms in that sector. Additionally, this aggregate productivity is also affected by public goods externalities. In this context, the government plays a crucial role in shaping the overall productivity of the economy.

On the other hand, the average productivity, adjusted for taxes, penalties, and exit probabilities δ_i and δ_f, is:

$$\tilde{\varphi}_m^{\epsilon-1} = \frac{(1-\rho)}{\delta_i}\left[\frac{w_i}{(1-\rho)F_i(\tilde{\kappa})}\right]^{1-\epsilon} \frac{G(\bar{\varphi}) - G(\varphi^*)}{1 - G(\varphi^*)} \tilde{\varphi}_I^{\epsilon-1} + \frac{(1-\tau)}{\delta_f}\left[\frac{w_f}{(1-\tau)F_f(\tilde{\kappa})}\right]^{1-\epsilon} \frac{1 - G(\bar{\varphi})}{1 - G(\varphi^*)} \varphi_F^{\epsilon-1} \tag{12}$$

3.7.2. Prices, Households, and Government

The aggregate price index is derived from the consumption demand equation of households for varieties, 2, 7, and 11:

$$P^{1-\epsilon} = MP(\tilde{\varphi})^{1-\epsilon}$$

$$\text{with:} \tag{13}$$

$$P(\tilde{\varphi}) = \frac{\epsilon}{\epsilon - 1}\frac{1}{\tilde{\varphi}}$$

On the household side, in equilibrium (replacing profits in equation 3), the total income or rents from the firm are allocated between consumption and covering the costs associated with informality. Consequently, informal firms experience an efficiency loss in resource allocation, as not all income, R, is fully allocated to consumption.

$$PC + \underbrace{f_i M_i + P\lambda}_{\text{informal costs}} = R + N_e$$
$$\text{with: } R = PQ \tag{14}$$

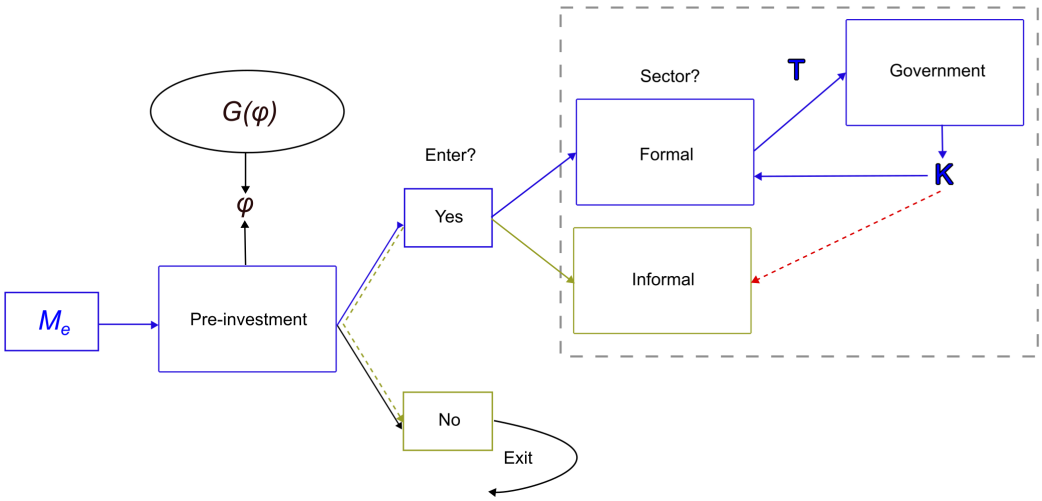
On the government side, in equilibrium, tax revenue, the penalty amount, and the fiscal balance will be determined by:

$$\begin{aligned} PT &= \tau \frac{\epsilon f_i}{1-\rho} \left\{ \frac{w_i}{w_f} \frac{(1-\tau)}{(1-\rho)} \frac{F_f(\tilde{\kappa})}{F_i(\tilde{\kappa})} \right\}^{\epsilon-1} \frac{1-G(\bar{\varphi})}{1-G(\varphi^*)} \left[\frac{\tilde{\varphi}_f}{\varphi^*} \right]^{\epsilon-1} M \\ P\lambda &= \rho \frac{\epsilon f_i}{1-\rho} \frac{G(\bar{\varphi}) - G(\varphi^*)}{1-G(\varphi^*)} \left[\frac{\tilde{\varphi}_i}{\varphi^*} \right]^{\epsilon-1} M \\ P\hat{\kappa} &= PT + \frac{1-G(\bar{\varphi})}{1-G(\varphi^*)} f_f M \end{aligned} \tag{15}$$

3.8. Firm entry

In Figure 1, we can observe the decision-making process for any firm. Initially, firms go through a pre-investment stage, followed by a decision on whether to enter or exit the market. If they choose to enter, they subsequently decide which sector—formal or informal—they will belong to.

Figure 1: Decision tree for entering firms



Note: The diagram illustrates the decision tree for firms. After observing their idiosyncratic productivity, agents choose between three options: abstaining from production, operating formally, or operating informally. T stands for taxes, K represents the public capital and $G(\varphi)$ reflects the ex-ante productivity distribution.

First, ex-ante, all firms (M_e) face the same productivity distribution, $G(\varphi)$, and, following Melitz

(2003), they face an exogenous shock that leads to firm exit, $\delta = \{\delta_i, \delta_f\}$; so, in equilibrium, entering firms replace the exiting firms, $(1-G(\varphi^*))M_e = \delta_i M_i + \delta_f M_f$. Thus, the ex-ante productivity distribution is not affected by the outcome of the economy, and the equilibrium productivity (ex-post), $\mu(\varphi)$, is only conditional on the successfully entering firms.

Second, from the decision tree, a firm with productivity φ will choose to enter the market only if its per-period profit satisfies $\pi(\varphi) \geq 0$. Conversely, if the firm's profit is negative, it will opt to exit the market. Since there are no short-term shocks in this model, the value function for a firm will be:

$$\upsilon(\varphi) = \max\{0, \sum_{t\geq 0}(1-\delta)^t \pi(\varphi)\} = \max\{0, \frac{1}{\delta}\pi(\varphi)\} \tag{16}$$

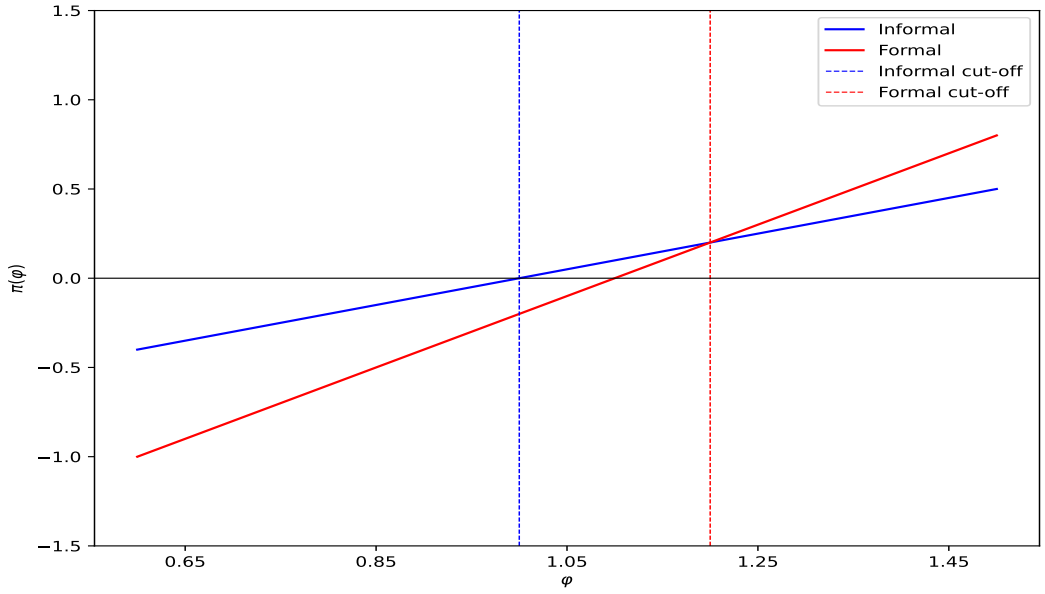
On the other hand, since $\pi'(\varphi) > 0$ and each firm faces a fixed production cost, for a productivity level $\varphi = 0$, the firm's profit will be $\pi(0) = -f_s$, with $s = \{i, f\}$. Therefore, there exists a productivity level, φ^*, such that $\pi(\varphi^*) = 0$. This is the zero-profit condition. Thus, for $\forall \varphi < \varphi^*$, the firm will choose to exit the market and will not produce, whereas if $\varphi \geq \varphi^*$, it will enter and produce in one of the sectors.

This decision is influenced by the fact that entering the formal sector requires a higher level of productivity, denoted as $\overline{\varphi}$, to generate positive profits due to the higher costs associated with formalization. A firm will choose to operate formally if its productivity satisfies $\varphi \geq \overline{\varphi}$, where $\overline{\varphi}$ represents the threshold productivity level at which more productive firms gain greater benefits from formalization compared to remaining informal.

3.9. Zero cut-off profit condition (ZCP)

The literature on the informal sector highlights several key advantages that informal firms enjoy compared to formal firms. These include lower fixed costs (e.g., avoiding registration and compliance costs), significantly lower average wages (due to bypassing labor compensations and competitive wage standards), and a reduced probability of detection by authorities (e.g., fiscalization efforts by the tax authority). As a result, the profit function for informal firms lies above that of formal firms at lower productivity levels. However, as productivity increases, this advantage diminishes, and the profit function of informal firms is eventually dominated by that of formal firms:

Figure 2: Example of profit functions



As demonstrated below, the smallest firm in the economy will operate informally due to the higher costs associated with formality. This smallest informal firm earns zero profits at the minimum productivity level. Specifically, this condition implies that the firm's income is precisely equal to the cost of remaining informal, given by $r_i(\varphi^*) = \epsilon \frac{f_i}{1-\rho}$.

Also, the minimum required productivity to be formal, derived from equations (5) and (6), $\bar{\varphi}$, will occur when both profit functions intersect, $\pi_i(\bar{\varphi}) = \pi_f(\bar{\varphi})$:

$$\begin{aligned}
\bar{\varphi}^{\epsilon-1} &= \frac{\epsilon P^{-\epsilon}}{Q}\left[\frac{\epsilon}{\epsilon-1}\right]^{\epsilon-1}\frac{1}{1-\rho}\left[\frac{w_i}{(1-\rho)F_i(\tilde{\kappa})}\right]^{\epsilon-1}\frac{(f_f-f_i)}{\frac{1-\tau}{1-\rho}\left\{\frac{w_i}{w_f}\frac{(1-\tau)}{(1-\rho)}\frac{A_f F_f(\tilde{\kappa})}{F_i(\tilde{\kappa})}\right\}^{\epsilon-1}-1} \\
\bar{\varphi}^{\epsilon-1} &= (\frac{f_f}{f_i}-1)\left[\frac{1-\tau}{1-\rho}\left\{\frac{w_i}{w_f}\frac{(1-\tau)}{(1-\rho)}\frac{A_f F_f(\tilde{\kappa})}{F_i(\tilde{\kappa})}\right\}^{\epsilon-1}-1\right]^{-1}\varphi^{*\epsilon-1}
\end{aligned} \tag{17}$$

With this segmentation of firms, the average income of the economy, $\bar{r} = \frac{R}{M}$, is determined by:

$$
\begin{aligned}
R &= M \int_{\varphi^*}^{\infty} r(\varphi)\mu(\varphi)d\varphi \\
\bar{r} &= \frac{R}{M} = \int_{\varphi^*}^{\infty} r(\varphi)\mu(\varphi)d\varphi \\
\bar{r} &= \left[\frac{w_i}{(1-\rho)F_i(\tilde{\kappa})}\right]^{\epsilon-1} (\frac{\tilde{\varphi}}{\varphi^*})^{\epsilon-1} r_i(\varphi^*)
\end{aligned} \tag{18}
$$

Thus, the zero-profit condition implies that the average income of firms depends on the informal cut-off, φ^*:

$$
\bar{r} = \left[\frac{w_i}{(1-\rho)F_i(\tilde{\kappa})}\right]^{\epsilon-1} (\frac{\tilde{\varphi}}{\varphi^*})^{\epsilon-1} \epsilon \frac{f_i}{1-\rho} \tag{19}
$$

Thus, the formal cut-off, equation (17), can be written as:

$$
\begin{gathered}
\bar{\varphi} = H\varphi^* \\
\text{where:} \\
H^{\epsilon-1} = (\frac{f_f}{f_i} - 1)\left[\frac{1-\tau}{1-\rho}\left\{\frac{w_i}{w_f}\frac{(1-\tau)}{(1-\rho)}\frac{A_f F_f(\tilde{\kappa})}{F_i(\tilde{\kappa})}\right\}^{\epsilon-1} - 1\right]^{-1}
\end{gathered} \tag{20}
$$

Note that, if $H \geq 1$ firms will decide to be formal if $\varphi \geq \overline{\varphi}$, and informal otherwise. Then, H measures how difficult it is to be formal versus informal. Moreover, note that, if $F_f(\tilde{\kappa}) >> F_i(\tilde{\kappa})$, government actions could promote greater formality.

3.10. Free entry (FE) and Value of firms

Since $\forall\varphi > \varphi^*, \pi(\varphi) > 0$, the average profit of the economy is greater than zero; therefore, firms are obtaining positive profits.

This induces a significant influx of firms willing to enter the economy. However, before doing so, they must make a pre-investment to determine their productivity level. Following this, they will decide whether to enter or exit the market. If they choose to enter, they will subsequently decide which sector—formal or informal—they will operate in.

As Melitz (2003) highlights, the only reason a firm would be willing to incur this entry cost, f_e, is the expectation of positive average profits. This necessitates that the firm's value, v_e, is non-negative.

$$v_e = (1 - G(\varphi^*))\bar{\pi} - f_e \tag{21}$$

According to Melitz (2003) and Allen and Schipper (2017), these firms will invest only if the average value of successfully entering firms is equal to or greater than the entry cost, f_e. Moreover, thanks to Allen and Schipper (2017) assumptions, this condition can be expressed in terms of average income, which in this research is given by:

$$\bar{r} = \epsilon \left[\frac{\tilde{\varphi}}{\tilde{\varphi}_m}\right]^{\epsilon-1} \left[\frac{f_e}{1 - G(\varphi^*)} + \frac{G(\bar{\varphi}) - G(\varphi^*)}{1 - G(\varphi^*)} f_i + \frac{1 - G(\bar{\varphi})}{1 - G(\varphi^*)} f_f\right] \tag{22}$$

3.11. Equilibria

First, for each $\tilde{\kappa}$, conditions (23, ZCP) and (24, FE) are two expressions that link the production cut-off with average income and determine the equilibrium in firm entry:

$$\bar{r}_{zcp} = \left[\frac{w_i}{(1-\rho)F_i(\tilde{\kappa})}\right]^{\epsilon-1} (\frac{\tilde{\varphi}}{\varphi^*})^{\epsilon-1} \epsilon \frac{f_i}{1-\rho} \tag{23}$$

$$\bar{r}_{fe} = \epsilon \left[\frac{\tilde{\varphi}}{\tilde{\varphi}_m}\right]^{\epsilon-1} \left[\frac{f_e}{1 - G(\varphi^*)} + \frac{G(\bar{\varphi}) - G(\varphi^*)}{1 - G(\varphi^*)} f_i + \frac{1 - G(\bar{\varphi})}{1 - G(\varphi^*)} f_f\right] \tag{24}$$

Thus, as seen, in equilibria the informal cut-off, φ^*, is a function of $\tilde{\kappa}$, and consequently, the formal cut-off is a function of the informal cut-off for each $\tilde{\kappa}$, $\bar{\varphi}(\varphi^*, \tilde{\kappa})$. Therefore, the aggregated productivities are a function of this cut-off: $\tilde{\varphi}_f(\varphi^*, \tilde{\kappa})$, $\tilde{\varphi}_i(\varphi^*, \tilde{\kappa})$, $\tilde{\varphi}(\varphi^*, \tilde{\kappa})$, as well as the average productivity, $\tilde{\varphi}_m(\varphi^*, \tilde{\kappa})$. In conclusion, all aggregated variables (also ex-post public goods $\hat{\kappa}$) depend on the informal cut-off and $\tilde{\kappa}$, being unique for each $\tilde{\kappa}$.

Second, the equilibrium in the labor market in the pre-investment stage, $N_e = f_e M_e$, leads to the equilibrium number of entering firms that must exactly offset the number of exiting firms: $(1 - G(\varphi^*))M_e = \delta_i M_i + \delta_f M_f$. Therefore, given the average value function, $v_e = (1 - G(\varphi^*))\bar{\pi} - f_e$, the number of firms in the economy is determined by:

$$
\begin{aligned}
f_e M_e &= f_e \frac{\delta_i M_i + \delta_f M_f}{1 - G(\varphi^*)} = \bar{\pi}(\delta_i M_i + \delta_f M_f) \\
f_e M_e &= \bar{\pi} M (\delta_i \frac{G(\bar{\varphi}) - G(\varphi^*)}{1 - G(\varphi^*)} + \delta_f \frac{1 - G(\bar{\varphi})}{1 - G(\varphi^*)}) \\
\bar{\pi} M &= f_e M_e \{\delta_i \frac{G(\bar{\varphi}) - G(\varphi^*)}{1 - G(\varphi^*)} + \delta_f \frac{1 - G(\bar{\varphi})}{1 - G(\varphi^*)}\}^{-1}
\end{aligned} \tag{25}
$$

Third, from the equilibrium in the ex-post labor market (inelastic supply): $\int_{\omega \in \Omega} w(\omega) N(\omega) d\omega$, it is determined as:

$$
\begin{aligned}
W &= M \left(\frac{\epsilon}{\epsilon - 1}\right)^{\epsilon - 1} \left(\frac{\epsilon f_i}{1 - \rho}\right) \left[\frac{w_i}{(1 - \rho) F_i(\tilde{\kappa})}\right]^{\epsilon - 1} \tilde{\varphi}_n^{\epsilon - 1} \\
\tilde{\varphi}_n^{\epsilon - 1} &= \frac{w_i}{F_i(\tilde{\kappa})} \left[\frac{\epsilon}{\epsilon - 1} \frac{w_i}{(1 - \rho) F_i(\tilde{\kappa})}\right]^{1 - \epsilon} \frac{G(\bar{\varphi}) - G(\varphi^*)}{1 - G(\varphi^*)} \tilde{\varphi}_I^{\epsilon - 1} + \\
&\quad \frac{w_f}{A_f F_f(\tilde{\kappa})} \left[\frac{\epsilon}{\epsilon - 1} \frac{w_f}{(1 - \tau) A_f F_f(\tilde{\kappa})}\right]^{1 - \epsilon} \frac{1 - G(\bar{\varphi})}{1 - G(\varphi^*)} \varphi_F^{\epsilon - 1}
\end{aligned} \tag{26}
$$

With this and by $\Pi = \bar{\pi} M$, the equilibrium in the goods market, equation (3), can be expressed as:

$$
\begin{aligned}
PC &= \int_{\omega \in \Omega} w(\omega) N(\omega) d\omega + \int_{\omega \in \Omega} \pi(\omega) d\omega + N_e = W + \Pi + f_e M_e \\
PC &= W + \left[\left(\frac{G(\bar{\varphi}) - G(\varphi^*)}{1 - G(\varphi^*)} \delta_i + \frac{1 - G(\bar{\varphi})}{1 - G(\varphi^*)} \delta_f\right)^{-1} + 1\right] f_e M_e
\end{aligned} \tag{27}
$$

The prices is given by:

$$
P^{1 - \epsilon} = M \left\{\frac{\epsilon}{\epsilon - 1} \frac{1}{\tilde{\varphi}}\right\}^{1 - \epsilon} \tag{28}
$$

Also, the fiscal balance is given by:

$$
\begin{aligned}
PT &= \tau \frac{\epsilon f_i}{1-\rho} \left\{ \frac{w_i}{w_f} \frac{(1-\tau)}{(1-\rho)} \frac{F_f(\tilde{\kappa})}{F_i(\tilde{\kappa})} \right\}^{\epsilon-1} \frac{1-G(\bar{\varphi})}{1-G(\varphi^*)} \left[\frac{\tilde{\varphi}_f}{\varphi^*} \right]^{\epsilon-1} M \\
P\hat{\kappa} &= PT + \frac{1-G(\bar{\varphi})}{1-G(\varphi^*)} f_f M
\end{aligned}
\tag{29}
$$

4. The Existence of Equilibrium in a Dual Market

Consider $A_f = 1$ and let be $\Theta_0 = \{w_f, w_i, f_f, f_i, f_e, \delta_f, \delta_i, \epsilon, \tau, \rho, M_e\}$, such that $\Theta_0 \subseteq \mathbb{R}^{11}_{++}$ and consider the subset: $\underline{\Theta} := \{\theta \in \Theta_0 | \exists! \varphi^* : \bar{r}_{zcp} = \bar{r}_{fe}\}$ (i.e equations 23 and 24 hold). Moreover, taking the following statements for dual market $\tau \geq \rho\,, w_f \geq w_i\,, f_f \geq f_i\,,$ and $\delta_i > \delta_f$, let $\underline{\Theta}' := \{\theta \in \underline{\Theta} | \tau \geq \rho\,, w_f \geq w_i\,, f_f \geq f_i\,,$ and $\delta_i > \delta_f\}$. Then $\underline{\Theta}' \subseteq \underline{\Theta}$, and for each $\tilde{\kappa}$, if the following conditions are satisfied, it follows that $\underline{\Theta}' \neq \varnothing$ so $\underline{\Theta} \neq \varnothing$:

(i) The externality functions must satisfy $F_f(\tilde{\kappa})^{\epsilon-1} \geq B F_i(\tilde{\kappa})^{\epsilon-1}$, with $B \geq \left(\frac{1-\rho}{1-\tau}\right)^{\epsilon} \left(\frac{w_f}{w_i}\right)^{\epsilon-1}$.

(ii) Due to the higher productivity demands in the formal sector, $\frac{f_f}{f_i} \geq \left[\frac{F_f(\tilde{\kappa})}{F_i(\tilde{\kappa})}\right]^{\epsilon-1} \ \forall \tilde{\kappa} > 0$

(iii) For higher formal survival, we have $(1-\delta_f) \geq (1-\delta_i)$ but it must be $\delta_i \geq \left(\frac{1-\rho}{1-\tau}\right) \delta_f$

(iv) Finally, given $G(\varphi)$ it must satisfy: $f_e \geq (f_f - f_i) \frac{g(\bar{\varphi})}{g(\varphi)} H$

In this sense, if conditions (a)-(e) are satisfied, a dual market exists and being formal requires higher productivity. The following consequences then apply:

(i) $\Phi_I^{\epsilon-1} = \frac{G(\bar{\varphi})-G(\varphi^*)}{1-G(\varphi^*)} \tilde{\varphi}_I^{\epsilon-1}$ and $\Phi'_I(\varphi^*) \geq 0$

(ii) $\Phi_F^{\epsilon-1} = \frac{1-G(\bar{\varphi})}{1-G(\varphi^*)} \tilde{\varphi}_F^{\epsilon-1}$ and $\Phi'_F(\varphi^*) \leq 0$

(iii) $\tilde{\varphi}'(\varphi^*) \leq 0$ and $\tilde{\varphi}'_m(\varphi^*) \leq 0$

(iv) $\bar{r}'_{zcp}(\varphi^*) \leq 0$ and $\bar{r}'_{fe}(\varphi^*) \geq 0$

It guarantees the existence of a φ^* for each $\tilde{\kappa}$, because $\bar{r}_{zcp}$ is decreasing over $(0, \infty)$ and $\bar{r}_{fe}$ is increasing over $(0, \infty)$. Therefore, there exists a φ^* such that $\bar{r}_{zcp}(\varphi^*) = \bar{r}_{fe}(\varphi^*)$.

5. The Ex-post consistency

So, for some $\theta \in \underline{\Theta}'$ and using $\overline{\pi}(\varphi) = \frac{f_e}{1-G(\varphi)}$, it implies that:

(i) $\overline{\pi}'(\varphi) \geq 0$

(ii) $M'(\varphi) \leq 0$, $P'(\varphi) \geq 0$, $(PT)'(\varphi) \leq 0$, $i'(\varphi) \geq 0$, with $i(\varphi^*) = \frac{G(\bar{\varphi})-G(\varphi^*)}{1-G(\varphi^*)}$, then

(iii) $\hat{\kappa}'(\varphi) \leq 0$

This indicates that there exists a region of $\tilde{\kappa}$ in which adjustments to the $\bar{r}_{zcp}(\varphi^*)$, and $\bar{r}_{fe}(\varphi^*)$ reallocate the firms in the economy. Let the region of convergence for $\tilde{\kappa}$ be denoted by $K \subseteq \mathbb{R}_{++}$, representing the subset of positive real numbers where convergence can be achieved through the application of this algorithm to φ:

(i) For each $\tilde{\kappa}$, guess an initial value φ_0 and compute $\hat{\kappa}(\varphi_0)$. Compare $\tilde{\kappa}$ and $\hat{\kappa}(\varphi_0)$. If $\hat{\kappa}(\varphi_0) \leq \tilde{\kappa}$, update $\varphi_1 \leq \varphi_0$ since $\hat{\kappa}(\varphi_1) \geq k_g(\varphi_0)$.

(ii) Repeat this process iteratively until $|\tilde{\kappa} - \hat{\kappa}| < \Delta$, where $\Delta > 0$ is an arbitrarily small positive tolerance.

So, there exists a non-empty set $\overline{\Theta} := \{\theta \in \underline{\Theta}' | \, \exists! \varphi^* : \bar{r}_{zcp} = \bar{r}_{fe} \wedge |\tilde{\kappa} - \hat{\kappa}| < \Delta \, , \text{for some } \kappa \in K, \forall \Delta > 0\}$, such that $\overline{\Theta} \subseteq \underline{\Theta}' \subseteq \underline{\Theta}$.

6. The Importance of Beliefs in the Formalization Process

For some $\theta \in \overline{\Theta}$, there exists a set of beliefs, $\tilde{\kappa}$, that are consistent with the macro-equilibrium, $\hat{\kappa}$. However, if a permanent shock occurs, such as $A_f \neq 1$, and this shock is only observable after firms have entered the market, it is possible for the micro-equilibrium to deviate from the macroeconomic solution.

To illustrate this, let $\hat{\kappa}_0$ represent the initial ex-post macro-equilibrium and $\tilde{\kappa}_0$ the ex-ante belief that was consistent with this initial aggregate equilibrium. Now, consider $\hat{\kappa}_a$ as the new macro-equilibrium resulting from the shock A_f. Depending on the firms' beliefs about the shock and the subsequent adjustment process, multiple scenarios may unfold. We highlight two possible cases:

(a) If the formal firms decide to become informal without considering the effects on $\hat{\kappa}$, on $\frac{F_f(\hat{\kappa})}{F_i(\hat{\kappa})}$, and consequently on $H(\hat{\kappa})$, they might assume that $\tilde{\kappa}$ will remain at its initial level and not adjust to the new macro-equilibrium $\hat{\kappa}_a$. This implies that their belief is: "*Formalization is not necessary to maintain public goods.*"

$$H^{\epsilon-1} = (\frac{f_f}{f_i} - 1)\left[\frac{1-\tau}{1-\rho}\left\{\frac{w_i}{w_f}\frac{(1-\tau)}{(1-\rho)}\frac{A_f F_f(\hat{\kappa}_0)}{F_i(\hat{\kappa}_0)}\right\}^{\epsilon-1} - 1\right]^{-1} \tag{30}$$

(b) On the other hand, they may consider that their decisions will indeed impact $\hat{\kappa}$, and consequently $H(\hat{\kappa})$. In this case, their belief would be: "*The government will not act effectively without formal firms.*"

$$H^{\epsilon-1} = (\frac{f_f}{f_i} - 1)\left[\frac{1-\tau}{1-\rho}\left\{\frac{w_i}{w_f}\frac{(1-\tau)}{(1-\rho)}\frac{A_f F_f(\hat{\kappa}_a)}{F_i(\hat{\kappa}_a)}\right\}^{\epsilon-1} - 1\right]^{-1} \tag{31}$$

In other words, if the first scenario occurs, there will be a new macro-equilibrium, $\hat{\kappa}'_a (< \hat{\kappa}_a)$, implying that the decisions of formal firms impact other firms in the economy. The main mechanism lies in $\frac{F_f(\hat{\kappa}'_a)}{F_i(\hat{\kappa}'_a)}$, especially if $F_{i,f}(\hat{\kappa}'_a)$ is a strictly increasing function over K. This is because, with a negative aggregate formal productivity shock, formal entry becomes more challenging, potentially reducing the aggregate formal productivity. This reduction could, in turn, diminish the public goods externality, resulting in $H(\hat{\kappa}'_a) \gg H(\hat{\kappa}_a)$. This means that formal firms, under this scenario, fail to consider the negative (or positive) consequences of their actions on their own benefits and the overall economy.

In contrast, under the second scenario, firms understand that their decisions impact $H(\tilde{\kappa}_a)$. They incorporate these changes into their decision-making process, leading to $\tilde{\kappa}_a = \hat{\kappa}_a$. In this case, firms consider the broader implications of their actions, including their effects on other firms, aggregate formal productivity, and the overall economy.

It follows that all $\theta \in \overline{\Theta}$ satisfy the conditions for the existence of equilibrium under this second rational scenario. However, in the first scenario, θ may belong to $\underline{\Theta} - \overline{\Theta}$, where the macro-equilibrium conditions might not hold. Thus, the formation of expectations is a central determinant in the formalization process and the achievement of economic equilibrium.

7. A numerical exercise: Pareto distribution

To analyze the effect of beliefs on the formalization process under a negative shock, $A_f < 1$, we start from some $\theta_0 \in \overline{\Theta}$ and $\tilde{\kappa}_0 \in K$. We assume that idiosyncratic productivity follows a Pareto distribution with a shape parameter α and a scale parameter φ_0. The corresponding probability density function is given by:

$$g(\varphi) = \frac{\alpha \varphi_0^{\alpha}}{\varphi^{\alpha+1}}.$$

Furthermore, we define the externality function as a strictly increasing function, $F_{i,f}(\tilde{\kappa}) = (\kappa_0 \tilde{\kappa})^{\alpha_{i,f}}$, with homogeneity degree $\alpha_{\{i,f\}}$, where the subscripts $\{i, f\}$ denote the informal and formal sectors, respectively.

First, we examine the set of solutions for two scenarios in a dual market with a Pareto distribution and the functional form of $F_{i,f}(\cdot)$, under the given assumptions and initial parameters θ_0 such that $\tilde{\kappa}_0 = 0.2$. Figures 3 and 4 illustrate both formal and informal costs, as well as the homogeneity degree of formal and informal externality functions, under varying wage conditions and mortality rates, respectively.

As shown, a solution exists in a dual market comprising both formal and informal firms, i.e., $\overline{\Theta} \neq \varnothing$. To satisfy the assumptions, the fixed cost of formal firms must be higher than that of informal firms. Similarly, the homogeneity degree of the externality function must reflect a greater macro-equilibrium impact of public goods on formal firms compared to informal ones. These conditions suggest that, for a range of parameters, an equilibrium exists that satisfies the aforementioned requirements.

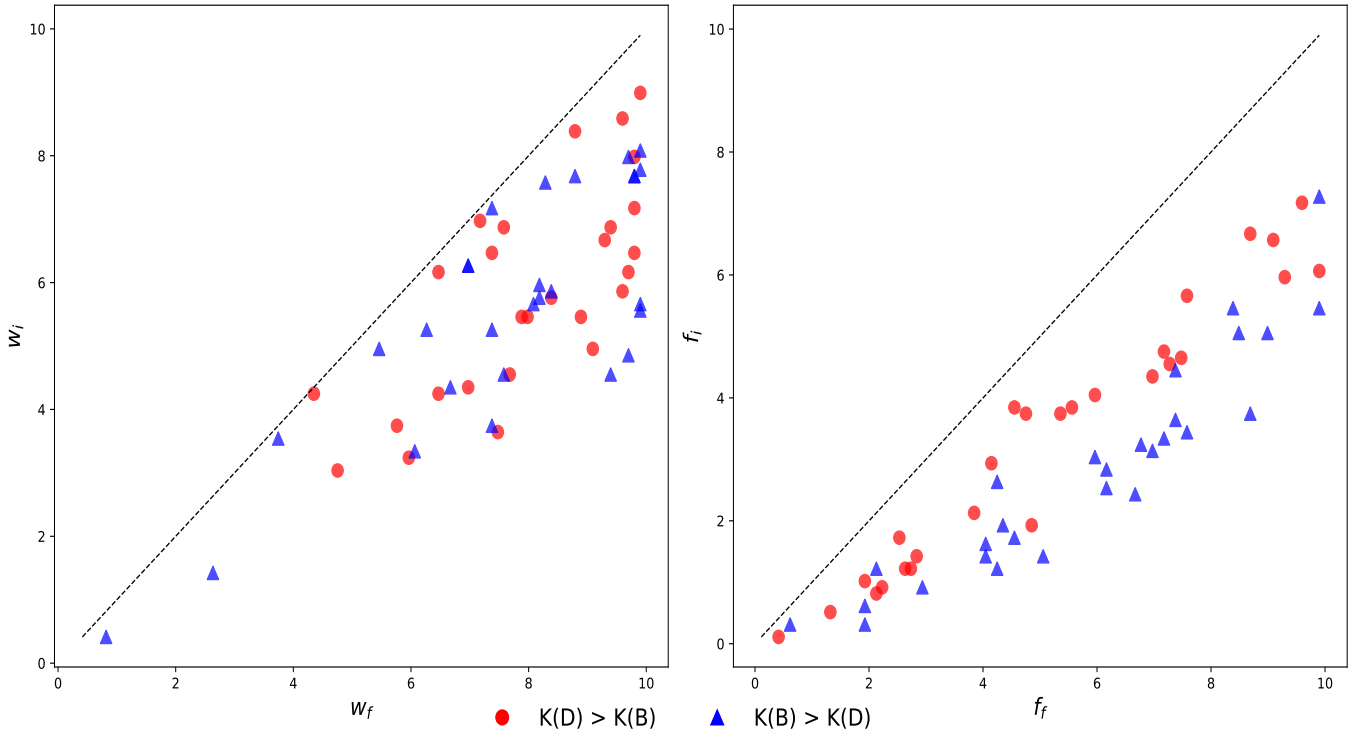
As a result, there exist subsets where macro public goods in scenario "b" (denoted as B in the figures) exceed those in scenario "a" (denoted as D in the figures).[2] This occurs when firms in scenario b perceive that transitioning to the informal sector increases the difficulty of re-entering formality. Such perceptions may stem from the belief that the government allocates all tax revenue to public goods or from trust in the government's actions. In these cases, transitioning to the informal sector could reduce overall formal productivity, shrink both the tax revenue and formal income base, and ultimately diminish the provision of public goods and their positive externalities on production. Consequently,

[2] P denotes a scenario where public goods maximize the utility function.

firms might inadvertently make a risky or suboptimal decision by opting for informality.

When firms trust that the government allocates tax revenue efficiently toward productive public goods, they are more likely to coordinate toward a favorable equilibrium characterized by higher aggregate productivity, greater public goods provision, increased formality, and higher consumption. However, if this trust is absent—due to doubts about the government's efficiency or perceptions of weak public institutions—the mechanism collapses, resulting in an equilibrium with lower aggregate productivity, reduced public goods provision, decreased formality, and diminished consumption. Thus, the stylized fact that "higher taxes lead to higher informality" emerges as a potential outcome of the model under scenario a.

Figure 3: A Space of Convergence for Wages and Fixed Costs

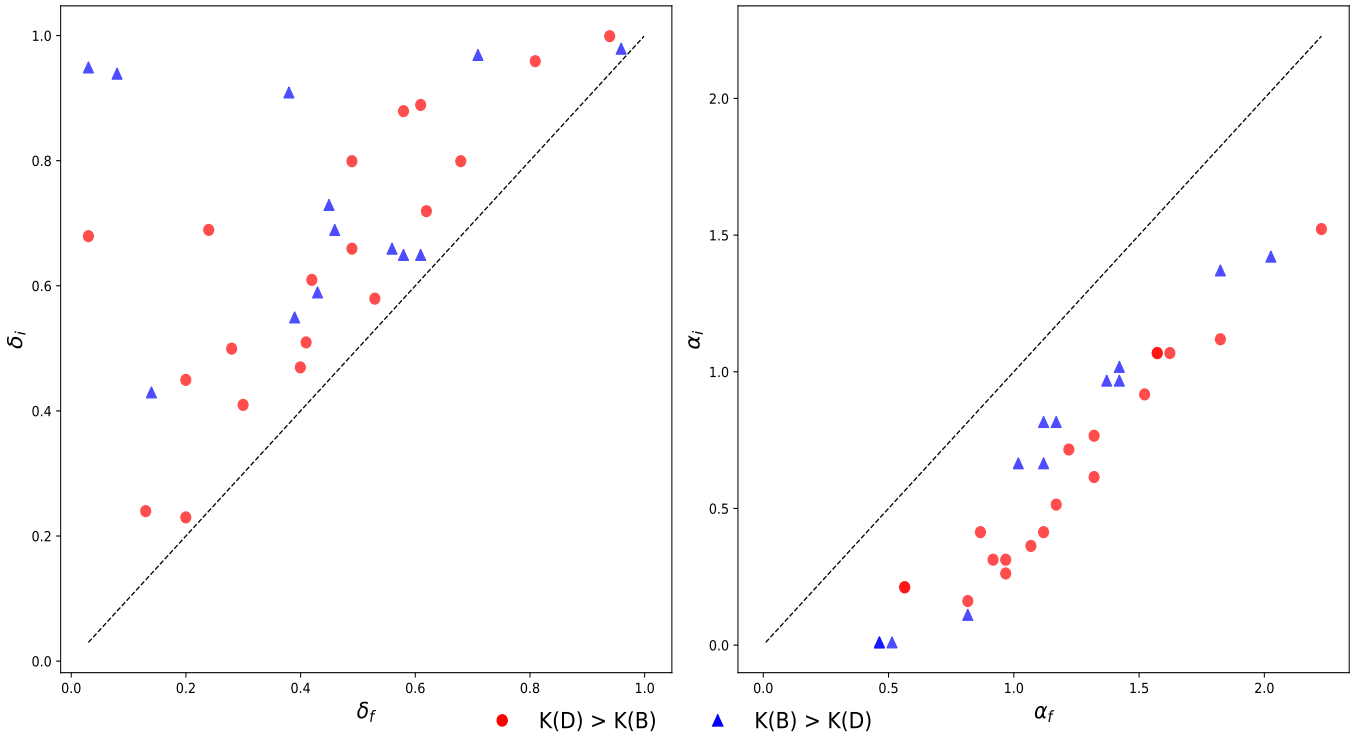


Note: The subplots illustrate the existence of equilibrium under Dual Market conditions, where $w_f > w_i$ and $f_f > f_i$. They show the presence of two types of equilibria, $K(D)$ and $K(B)$. Specifically, $K(D)$ represents the equilibrium level of public goods in Scenario (a), where firms are disinterested, while $K(B)$ corresponds to the public goods provision in Scenario (b), where firms act benevolently.

This mechanism can be applied to answer how both the tax rate and the probability of punishment influence the macro-equilibrium. The outcome depends on the credibility of public institutions and the government's commitment to allocating all revenue toward productivity public goods. Ultimately, it

hinges on how firms form expectations about $\tilde{\kappa}$ - the belief of public good that reflects the government's efficiency and trustworthiness in managing public spending.[3]

Figure 4: A Space of Convergence for Mortality rates and Homogeneity degrees

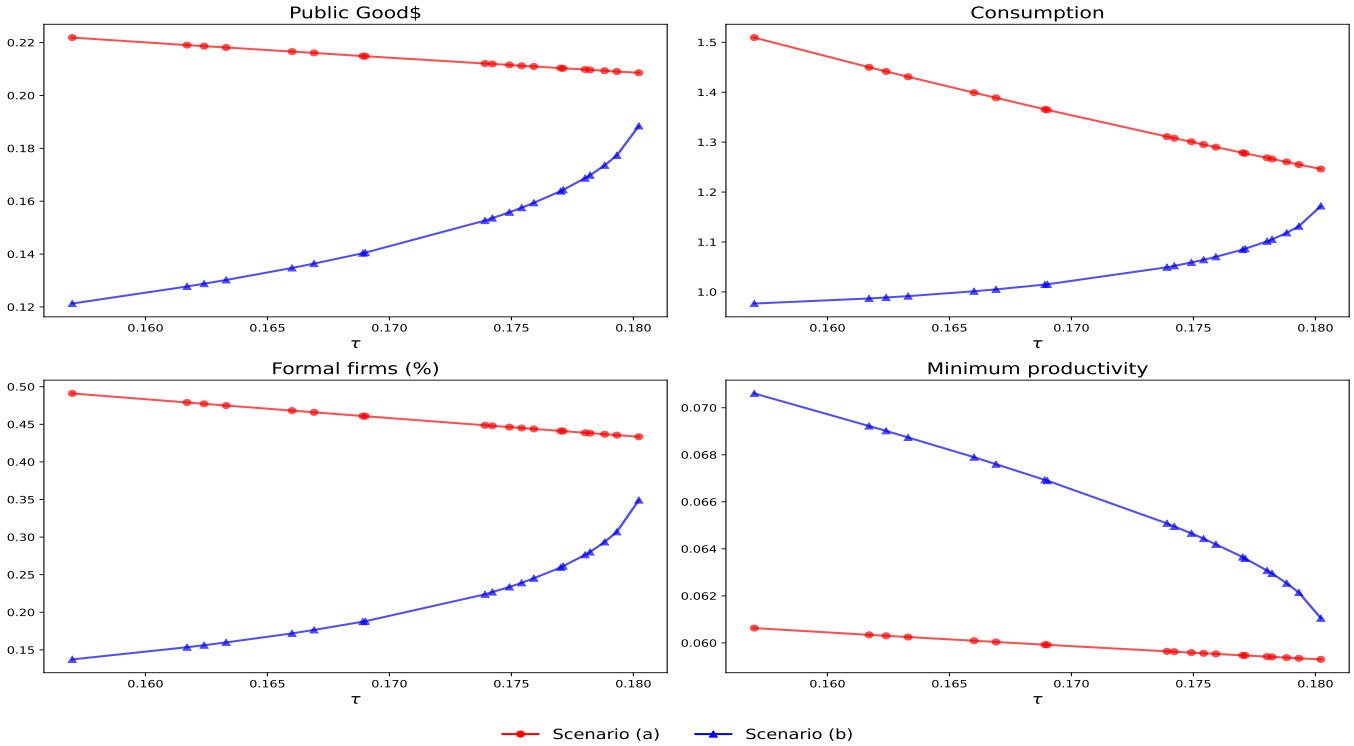


Note: The subplots illustrate the existence of equilibrium under Dual Market conditions, i.e $\delta_i > \delta_f$ and $\alpha_f > \alpha_i$. They show the presence of two types of equilibria, $K(D)$ and $K(B)$. Specifically, $K(D)$ represents the equilibrium level of public goods in Scenario (a), where firms are disinterested, while $K(B)$ corresponds to the public goods provision in Scenario (b), where firms act benevolently.

For this purpose, a broad range of tax rates can be applied to the model in both scenarios, offering valuable insights into the key differences in their responses to the same shock. Figure 5 illustrates the impact on variables such as informal cutoff productivity, public goods provision, formality rate, and household consumption.

[3]In our setup, these considerations are important only for firms at the margin between formality and informality, i.e., firms near the cut-off productivity levels. Evidence from experimental economics, such as Benhassine et al. (2018), shows that formalization efforts yield better results when targeted at firms that, prior to intervention, *look more like formal firms.*

Figure 5: Impact of tax across different scenarios



Note: The plot depicts both equilibria as a function of variations in the tax rate under both scenarios. Scenario (a) represents the case where firms are disinterested, whereas Scenario (b) corresponds to firms acting benevolently.

The results indicate that higher taxes can promote the formalization process, but only if firms trust that increased taxation will translate into more productive public goods that benefit them directly. In scenario "b", the response is strictly increasing with the tax rate, in contrast to scenario "a", which shows a negative slope. This suggests that the most favorable outcomes are achieved when the government's actions are trustworthy.

The main source of these differences lies in the externality of productive public goods, which enhance idiosyncratic productivity. If this effect is properly understood, it represents a shift in the production function, leading to an endogenous increase that will be internalized in the firm's beliefs. Consequently, this becomes an important variable of interest for the firms themselves.

Figure 6: Tax and informal cutoff

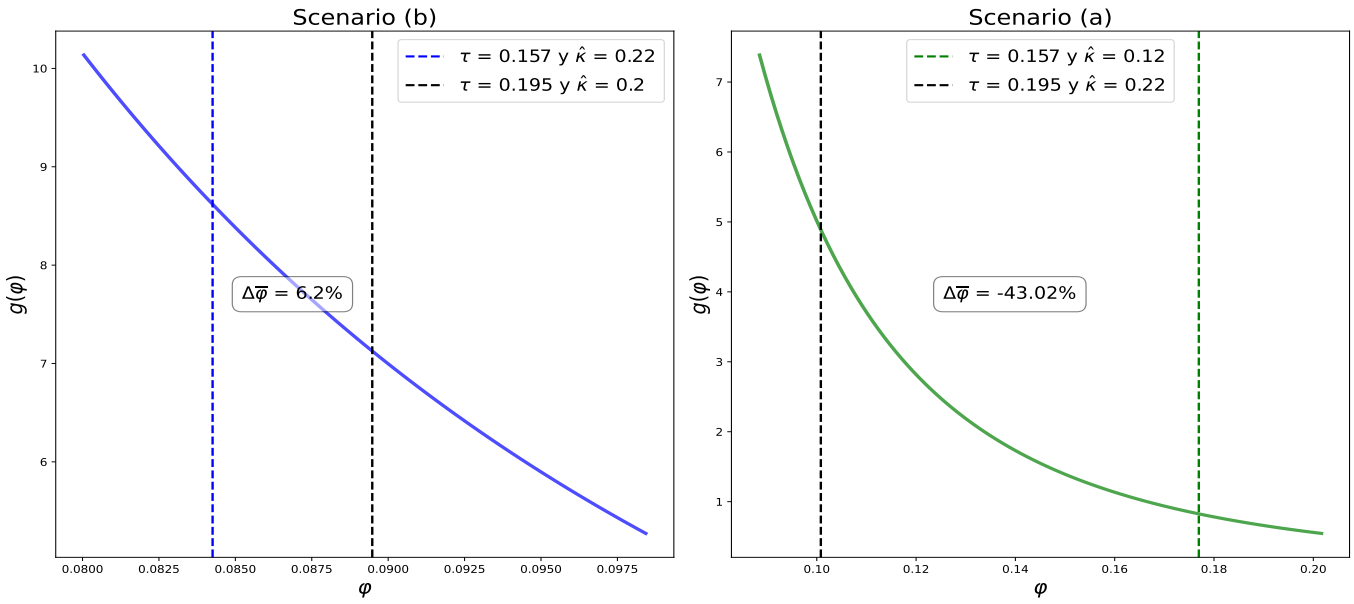


Figure 6 shows that the formal productive cutoff decreases as the tax rate increases in Scenario (b), while in Scenario (a), it increases. Thus, firms face a trade-off when deciding to operate informally in response to an external shock, as internalizing the effects of this decision will influence both the economy and their own productive capacity.

8. Conclusions

In this paper, we have presented a macroeconomic equilibrium model that examines the interactions between formal and informal sectors, emphasizing the role of public goods and institutional trust in shaping economic outcomes. Our model builds on the findings in experimental economics regarding tax compliance and the growing recognition of the importance of institutional quality in shaping economic behavior. By incorporating productivity distributions, tax policies, and the dynamics of formalization, the model provides insights into how firms' expectations and beliefs influence their decisions to enter or remain in the formal sector. Our analysis highlights the delicate balance between incentivizing formalization and managing the efficiency of public spending, revealing the critical trade-offs that policymakers face. The results demonstrate that trust in public institutions and the effective use of public resources are key factors in fostering a stable and productive economy.

The interplay between formal and informal sectors has significant implications for both individual firms and the broader economy. Firms' trust in public institutions plays a pivotal role in the success of formalization policies. When firms perceive that public institutions are credible and efficiently allocate resources to public goods, they are more likely to transition into the formal sector. This trust fosters an environment where formalization is seen as beneficial, thereby encouraging higher levels of formality.

Moreover, the model highlights the trade-offs associated with tax policies. Higher tax rates can serve as both an incentive and a deterrent to formalization. If firms believe that the additional tax revenues will be effectively used to enhance public goods that directly benefit them, formalization becomes more appealing. Conversely, if public goods are perceived as inadequately managed, higher taxes may discourage formal entry.

The informal sector, being a substantial part of many economies, plays a crucial role in shaping aggregate productivity. A high level of informality tends to reduce tax revenues and restrict the provision of essential public goods, which are critical for economic growth. Consequently, addressing the challenges posed by informality is essential for enhancing both productivity and social welfare.

Finally, the relationship between formal and informal sectors is intertwined with public spending and its effectiveness. Changes in formalization affect not only tax revenues but also the capacity of public expenditures to foster positive interactions between private and public sectors. By strategically aligning

formalization efforts with efficient public spending, economies can achieve higher aggregate productivity and a more sustainable growth trajectory.

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