The Effect of Market Information on Market Prices

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Abstract

Empirical studies on the effect of internet on market prices report that market prices have not always reduced in response to increased competition that is induced by the easily and relatively costlessly available market information. In this paper, we provide an explanation for why prices of all goods may not reduce, and in fact, price of some goods may even increase in presence of more market information. Market information not only induces stiffer competition amongst sellers but also makes for better matches between consumers and producers. While the former feature has a tendency to reduce prices, the latter feature may in fact cause prices to rise. The direction in which prices change as more information becomes available depends on the balance of these forces. We analyse this in context of a differentiated market, and characterise how prices change in response to freely available market information.

JEL Classification: D43, L130

Keywords: price competition, product match, information, differentiated market

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1 Introduction

The introduction of technologies such as the internet, and associated platforms, which enable consumers to almost costlessly obtain information about products and prices offered by various sellers, and compare them, is usually expected to bring about a reduction in prices. This is because access to such costless information eliminates the search costs that consumers may have to incur in identifying the seller offering the lowest price; since it is well known that informational frictions imposed by search costs may lead to higher prices, the elimination of search costs ought to lower prices. However, empirical studies on prices after the introduction of the internet and e-commerce paint a more mixed picture – while prices of some goods have reduced, the prices of other goods have witnessed an increase. The objective of this paper is to examine this ambiguity in light of another facet of technologies such as the internet, which, when taken into consideration, helps in explaining why price may not always decrease – the internet not only facilitates price comparisons but, by providing more information, also enables better product-matches between sellers and buyers. It is this feature of enabling better product-matches between the two sides of the market that may lead to price increases even in the face of increased competition amongst sellers that stems from greater price transparency. We construct a simple model to explore how the interplay between these two competing forces affects market prices.

We consider a horizontally differentiated duopoly that we represent by a Salop circle. There are two firms located diametrically opposite to each other. Consumers are distributed uniformly along the circumference of the circle, and each consumer prefers the product of the firm that is closer to him.

In the initial situation, consumers are only partially informed about the market. Specifically, each consumer knows only about one particular firm, and this firm may not be the one whose product better aligns with the preferences of the consumer. Thus, partial market information may potentially lead to a poor match between consumers and firms, and this affects both sides of the market adversely. One the one hand, the consumers may be constrained to consume a product that is not a good match with their preferences. On the other hand, the firms may be compelled to operate in a market segment whose consumers do not have a high valuation for its product. At the same time, partial market information confers to the firm monopoly power over the consumers who are informed only about its product. When the consumers and firms happen to be reasonable well matched, this monopoly power is fortuitous for the firm; however, in the case where the match is poor, the firm may find itself unable to satisfactorily exploit its monopoly power.

We use the term "internet" as a moniker for technologies or phenomenon due to which market information becomes freely available to the market participants. Now, the consumers are informed about both firms. This has two effects. Firstly, consumers may come to know of the product that is more suited to their preference. Secondly, firms lose their lack of information based monopoly power, and may now have to compete with each other for consumers. Intuitively, one expects the first factor to increase prices as consumers may now demand a product that they have a higher valuation for, and the firms are also able to sell to consumers who have a higher valuation for their product. According to conventional wisdom, the second factor is expected to have a deflating effect on the price owing to the induced stiffer competition.

We examine how the balance of these two countervailing forces affects the market price. We find that the direction of the price change depends on both the strength of the consumers' preference for the product of the firm closer to them, and also on the degree of mismatch between firms and consumers in the initial partial market information situation. Intuitively, one expects that, with complete market information, the price increases when the consumers' preference for the product of the firm that is closer to it is stronger, and when the extent of mismatch between consumers and the firms is more severe. This is because stronger consumer preferences for a particular firm impedes inter-firm competition in the complete market information situation, and thereby stymies the price-reducing effect of competition. On the other hand, if the mismatch between the consumers is severe, then complete market information remedies this to a greater extent, and this may result in prices rising to a greater degree. We find that this intuition holds when the degree of mismatch between consumers and firms exceeds a 'threshold'. For milder degrees of mismatch, we obtain the counter-intuitive result that stronger firm-specific preferences of the consumers does not lead to higher prices in the complete market information situation. We postpone a fuller discussion of these results and the underlying intuition to the results section as it is best presented after a complete description of the model.

In related literature, it is usually held that technologies such as the internet enabled search engines and platforms, which provide more market information, and promote market transparency, will lead to reduction in prices. The reasoning is that, since search costs may result in higher prices (see, for instance, the seminal papers by Diamond 1971 and Stahl 1989), greater market information/tranparency, which is thought to eliminate the search costs that consumers may have to incur in identifying seller with lower prices, should result in price falls. However, Ellison and Ellison (2005, 2018) state that this has not always been the case, and cite evidence that the price of used books have increased with introduction of the internet. Similarly, Nagaraj and Reimers (2023) show digitization increased the sales of physical books which could not otherwise be found in the absence of internet. These papers attribute this to the matching functions of the internet, and Ellison and Ellison (2018) construct a dynamic model with consumer arrivals to explain the effect of better generation of matches on the market price and price dispersion. In contrast to Ellison and Ellison (2018), we present an arguably simpler static model. Our our primary objective is to examine the tension between the matching effect and the competition inducing effect of more market information.

2 Model

We consider a horizontally differentiated product market, which we represent by a Salop circle of unit circumference. There are two firms namely, $F1$ and $F2$, that are located on the circumference of the Salop circle such that the distance between them is the same in both the clockwise direction and the anti-clockwise direction. There are no fixed costs of production, and the constant marginal cost of production of each firm is normalised to zero. Consumers are distributed uniformly along the circumference of the unit circle. A consumer who is located at a distance of x from a particular firm obtains a gross utility of $v - \tau x$, where $\tau > 0$, from consuming the good produced by that firm. Here, by the term "distance", we refer to the closest distance between a consumer and a firm in the clockwise direction or the anti-clockwise direction. This distance between the consumer and the firm represents the extent of mismatch between the consumer's ideal product on the one hand, and the product that is produced by the firm on the other hand, and τx is the disutility of the mismatch. Thus, the feature of horizontal product differentiation is captured by the fact that each consumer prefers the product of the firm that is closer to him, and that one-half of the unit mass of consumers is closer to each firm and hence prefers that firm over the other firm. Furthermore, τ, in addition to representing the disutility a consumer receives from not being able to consume his ideal product type, also measures the degree of market differentiation; a higher value of τ amplifies the difference in gross utility obtained by each consumer from the two firms, and hence reflects a higher market differentiation.

The firms set prices simultaneously. Each consumer either purchases exactly one unit of the good from one of the two firms, or abstains from consuming the good. A consumer's net utility obtained from purchasing one unit of the good from a firm that sets a price of p, and is at a distance of x from him, is $v - \tau x - p$. The utility from not consuming the good at all is normalised to zero. The price set by the firms $F1$ and $F2$ is denoted by p_1 and p_2, and we will now describe how the firms' prices determine their demand. We discuss this in two different contexts. We begin with the *partial market information* case, or the "pre-internet" era, where consumers' have partial information about the product offerings in the market. We model this by each consumer only being aware of one particular product. Hence, a consumer may not be informed about the firm that is a better match for him, and as a result, may not be able to purchase the product that is a better match with his preferences. Next, we consider the *complete market information* case, or the "post-internet" era, where consumers costlessly obtain information about the products in the market. In our model, this implies that each consumer is informed about both firms, and this enables him to purchase the product that is aligns best with his preferences.

In each case, the consumers' objective is to maximise his own net utility. So, in the partial market information case, each consumer compares the net utility (i.e. gross utility less price) obtained from purchasing from the firm that is aware of with the net utility of zero from abstaining from consumption, and chooses the option which gives a higher net utility. In the case of complete market information, where the consumer is informed about the products of both firms, the consumer compares the net utility obtained from purchasing from each firm as well as the net utility of zero from abstaining from consumption, and, again, chooses the option that maximises net utility.

On the other hand, each firm's objective is to set its price so as to maximise its profit. In the partial market information case, each firm's demand can only come from the consumer segment that is aware of its product – so each firm is a monopolist over the consumer segment that is informed of its product. In the case of complete market information, all consumers know about both products – now firms may have to compete with each other for consumers.

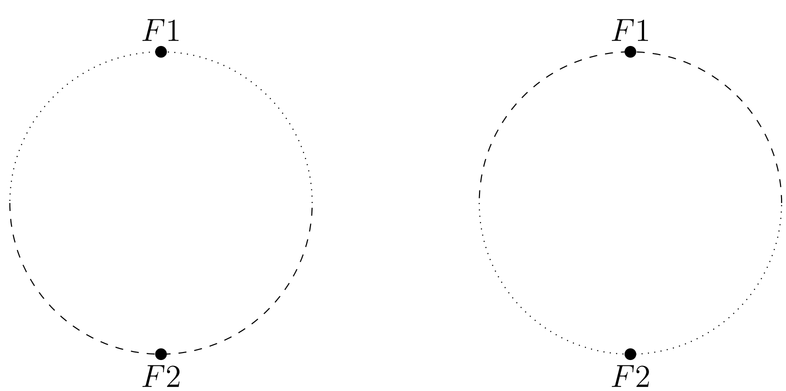
Thus, the prices set by the firms determines the consumers' choices, and hence their demand; and the consumers and firms make their choice in order to maximise net utility and profit, respectively. We discuss this more precisely in the next two subsections.

2.1 Partial Market Information

We begin with a situation where each consumer is aware of only one firm's product – this firm may not be the one that produces the good that gives the consumer a higher gross utility. We model this by a contiguous mass of consumers of measure one-half being aware of the product of only one particular firm, and the complementary contiguous mass of consumers of measure one-half being aware of the product of the other firm. The extent of mismatch between what consumers may purchase on the one hand, and the product that is a better match with their preferences on the other hand, is given by the distance between this contiguous mass of consumers who are aware of the product of a firm and the firm itself. The most extreme case of product mismatch arises when the *entire* mass of consumers who are aware of the product of one particular firm lie closer to the other firm - we refer to this particular case as the partial market information situation with *extreme mismatch* between consumers and firms/product. We further elaborate and clarify this by first, presenting two specific cases of partial market information – one where consumers happen to be informed about the firm that is the better match with their preferences, and the other where there is extreme mismatch (in the sense explained above) between consumers and the firms – and then, presenting the general model of partial market information.

2.1.1 Partial Market Information: Two specific cases

In the figure below, we depict the Salop circle, and the two firms $F1$ and $F2$, in two panels side by side. As mentioned earlier, the consumers are distributed uniformly along the circumference. The part of the circumference that has been etched using the dotted line represents the mass of consumers who are informed only about the product of firm $F1$ while the part of the circumference that has been etched using the dashed line represents the mass of consumers who are informed only about the product of firm $F2$.



(a) Perfect match (b) Extreme mismatch

Figure 1: Two specific instances of partial market information

In the left hand panel, the consumers who are informed only about the product of firm $F1$ lie closer to $F1$ than to $F2$, and a similar statement holds about consumers who are informed only about the product of firm $F2$. Here, each consumer is informed about the product that is a better match to his preference in the following sense – if, hypothetically, the consumers would have been aware of both firms, then a higher gross utility is obtained from the firm that is more proximate to it, and in this case, each consumer is only aware of the more proximate firm's product. One may think of this as the partial market information situation with *perfect match* between consumers and firms. We underline that even though each consumer is matched to firm that is better aligned with his preferences, this still represents one particular partial market information scenario simply because each consumer is informed of only one firm's product offering.

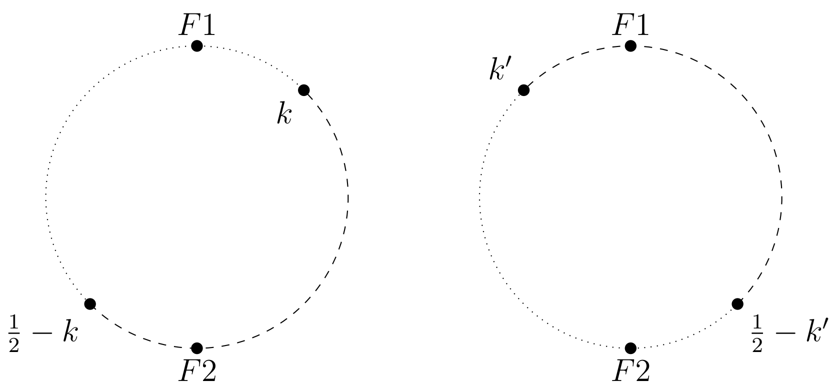
This may be contrasted with the right hand panel where the mass of consumers who are only aware of the product of $F1$ actually closer to $F2$, and vice-versa. This is the most extreme situation of mismatch between the consumers' preferences and the product type – each consumer who is currently matched to firm 1 obtains a higher gross utility from firm 2's product, and vice-versa.

Thus, these two situations represent the two extreme possibilities when consumers are partially informed about the products in the market – consumers are perfectly matched on the left hand panel, and there is extreme mismatch in the right hand panel.

2.1.2 Partial Market Information: The General Model

In the diagram below, we depict the general case of partial market information. We split this into two cases – the left hand panel and the right hand panel show the situations of mild mismatch and severe mismatch, respectively. In each panel, as before, the dotted arc denotes the consumer segment that is informed only about firm 1 while the dashed arc denotes the consumer segment that is informed only about firm 2.

In the left hand panel, the distance $k \in [0, \frac{1}{4}]$ denotes the extent of firm 1's market to its right while the distance $\frac{1}{2} - k$ denotes the extent of firm 1's market to its left; an analogous statement holds for firm 2. Importantly, k also represents an *inverse* index of mismatch. In order to see this, suppose that $k = \frac{1}{4}$ – then, the left hand panel above corresponds to the left hand panel of Figure 1; here, the consumers are perfectly matched. On the other hand, if $k = 0$, then one half of firm 1's market comprises of one half of the consumers who derive a higher gross utility from its product while the other half of firm 1's market comprises of one half of the consumers who derive a higher gross utility



(a) Mild mismatch: $k \in [0, \frac{1}{4}]$ (b) Severe mismatch: $k' \in [0, \frac{1}{4}]$

Figure 2: Partial market information – illustration of the general case

from firm 2's product. A similar statement holds for firm 2. So, the mismatch between consumers and firms is greater when $k = 0$ than when $k = \frac{1}{4}$. Furthermore, it may be seen from the diagram that the extent of mismatch is continuously deceasing in k.

In contrast, in the right hand panel, firm 1's market on its left-hand side starts from a distance $k' \in [0, \frac{1}{4}]$ from it while, on its right-hand side, it starts from a distance of $\frac{1}{2} - k$. Now, k' is a direct measure of mismatch. In order to see this, suppose that $k' = \frac{1}{4}$ – then, the right hand panel above corresponds to the right hand panel of Figure 1; here, there is extreme mismatch between the consumers and the firms. On the other hand, if $k' = 0$, then firm 1's market comprises of one half of the consumers who derive a higher gross utility from its product and one half of the consumers who derive a higher gross utility from firm 2's product. A similar statement holds for firm 2 as well. So, the mismatch between consumers and firms is greater when $k' = \frac{1}{4}$ than when $k' = 0$. And, it may be seen from the diagram that the extent of mismatch is continuously increasing in k'.

It follows that least degree of mismatch in the right hand panel, which occurs when $k' = 0$, corresponds to the highest degree of mismatch in the left hand panel, which occurs when $k = \frac{1}{4}$. Hence, the degree of mismatch in the left hand panel is milder compared to the degree of mismatch in the right hand panel where it is more severe.

Here, we emphasise that, in the case of mild mismatch and severe mismatch, the measure of consumers who are informed only about firm 1's product and lie to the left of it equals $\frac{1}{2} - k$ and $\frac{1}{2} - k'$, respectively. And, the measure of consumers who are informed only about firm 1's product and lie to the right of it equals k and k', respectively. Now, $k, k' \in [0, \frac{1}{4}]$ implies that, in cases of both mild mismatch and sever mismatch, firm 1's consumer segment on its left is at least as large as its consumer segment on the right. The difference, however, is that in case of mild mismatch, the consumer segment begins from the location of firm 1 and extends to a distance of $\frac{1}{2} - k$ on the left and k on the right, whereas, in case of severe mismatch, it begins from a distance of k' on the left and $\frac{1}{2} - k'$ on the right. In fact, it is this difference which creates the disparity in the extent of mismatch in these two cases, and an analogous statement holds for firm 2 as well.

The fact that the consumers are informed about only one particular firm's product carries an implication for the firms as well. Each firm's objective is to choose its price so as to maximise its profit, and it knows that its demand arises only from the consumers that are informed about its product; so, a firm is essentially a monopolist over that consumer segment. However, not all such monopolies are created equally. For instance, if one compares the situation of extreme mismatch to the situation of perfect match between consumers and firms, then, clearly, each firm prefers to be a monopolist when all consumers are perfectly matched. The reason is that, in case of a perfect match between consumers and firms, each firm is a monopolist over the consumer segment that has a higher gross valuation for its product – this enables a firm to set a higher price than it would be able to in the case of extreme mismatch where it is compelled to serve the consumers who have a low gross utility for its product.

We will now specify the demand $d_1(p_1)$ faced by firm 1, which is a monopolist on the dotted segment, when it sets a price p_1. Due to symmetry of the situation, it suffices to only examine the demand and the consequent monopoly price of one particular firm. A consumer in the dotted segment of the market who is located at distance of x from firm 1 will purchase from firm 1 at price p_1 if and only if he obtains non-negative utility from doing so, i.e. if and only if $v - \tau\, x - p_1 \geq 0$. Recall that a consumer in the dotted segment cannot purchase from firm 2 as he is not aware of firm 2's product.

In case of mild mismatch (i.e. the left-hand panel of Figure 2), where $k \in [0, \frac{1}{4}]$ is the inverse index of mismatch between consumers and firms, firm 1's demand is zero when its price $p_1 > v$. So, we only consider prices $p_1 \leq v$. Then, the demand function of firm 1 is:

$$d(p_1) = \begin{cases} 2\left(\frac{v-p_1}{\tau}\right) & \text{if } p_1 \in (v - \tau\, k, v] \\ k + \frac{v-p_1}{\tau} & \text{if } p_1 \in (v - \tau(\frac{1}{2} - k), v - \tau\, k] \\ \frac{1}{2} & \text{if } p_1 \leq v - \tau(\frac{1}{2} - k) \end{cases}$$

The explanation of the demand function of firm 1 is as follows. The consumers who demand firm 1′s product are those who receive a net utility of at least zero on purchasing the product. Suppose that this consumer is located at a distance of $\bar{x}$ from firm 1. Then,

$v - \tau\,\bar{x} - p_1 = 0$, or $\bar{x} = \frac{v-p_1}{\tau}$. In fact, there may exist two such consumers – one on the left of the firm, and other on the right. Consumers who are closer to (similarly, farther away from) firm 1 than this consumer also receive non-negative net utility (similarly, negative net utility) from firm 1, and so demand (similarly, do not demand) from firm 1.

Now, if $p_1 \in (v - \tau\,k, v]$, then $\bar{x} = \frac{v-p_1}{\tau} < k$ holds – in this case, some consumers but not all consumers from each side of firm 1 purchase from the firm; in fact, $\bar{x} = \frac{v-p_1}{\tau}$ mass of consumers on each side of firm 1 demand from the firm. So, when $p_1 \in (v - \tau\,k, v]$, then $d(p_1) = 2(\frac{v-p_1}{\tau})$. Next, if $p_1 \in (v - \tau(\frac{1}{2} - k), v - \tau\,k]$, then $\bar{x} \in [k, \frac{1}{2} - k)$. Here, all the consumers on the right of firm 1, who are at a distance of at most k from the firm, where $k \leq \bar{x}$, obtain non-negative utility, and demand from the firm. On the other hand, there are $\frac{1}{2} - k$ mass of consumers on the left of the firm, and $\bar{x} < \frac{1}{2} - k$ implies that only $\bar{x}$ mass of these consumers – and not all consumers on the left of the firm – demand from the firm. Hence, when $p_1 \in (v - \tau(\frac{1}{2} - k), v - \tau\,k]$, then $d(p_1) = k + \frac{v-p_1}{\tau}$. Finally, when $p_1 \leq v - \tau(\frac{1}{2} - k)$, then $\bar{x} \geq \frac{1}{2} - k$. So, all the consumers obtain non-negative net utility from firm 1, and demand from its product, thus implying that, in this case, $d(p_1) = \frac{1}{2}$.

On the other hand, in case of severe mismatch (i.e. the right-hand panel of Figure 2), where $k' \in [0, \frac{1}{4}]$ is the direct measure of mismatch between consumers and the firms, firm 1's demand is zero when its price $p_1 > v - \tau\,k'$. So, we only consider $p_1 \leq v - \tau\,k'$. Then, the demand function of firm 1 is:

$$d(p_1) = \begin{cases} \frac{v-p_1}{\tau} - k' & \text{if } p_1 \in (v - \tau(\frac{1}{2} - k'), v - \tau\,k'] \\ 2(\frac{v-p_1}{\tau}) - \frac{1}{2} & \text{if } p_1 \in (v - \frac{\tau}{2}, v - \tau(\frac{1}{2} - k')] \\ \frac{1}{2} & \text{if } p_1 \leq v - \frac{\tau}{2} \end{cases}$$

The explanation of the demand function follows a similar reasoning. Firstly, when $p_1 \in (v - \tau(\frac{1}{2} - k'), v - \tau\,k']$, then $\bar{x} \in (k', \frac{1}{2} - k')$. This implies that the consumer closest to firm 1 on its right, who is at a distance of $\frac{1}{2} - k'$ from the firm, does not obtain non-negative utility from the firm; the same holds for the other consumers on the right who are even farther away from the firm. So, none of the consumers on the right of the firm demand from the firm. On the other hand, the mass of consumers on the left of the firm who demand from the firm equals $\bar{x} - k'$. The reason is that since the firm's market to the left of it starts from a distance of k' from it, the consumers on the left who obtain non-negative utility from the firm – and hence demand from the firm – equals $\bar{x} - k' = \frac{v-p_1}{\tau} - k'$. As a result, $d(p_1) = \frac{v-p_1}{\tau} - k'$. Next, when $p_1 \in (v - \frac{\tau}{2}, v - \tau(\frac{1}{2} - k')]$, then $\bar{x} \in (\frac{1}{2} - k', \frac{1}{2})$. Now, because firm 1's market on the left (similarly, right) starts from a distance of k' (similarly, $\frac{1}{2} - k'$) from it, the mass of consumers on the left (similarly, right) who receive non-negative utility from the firm, and hence demand from the firm, equals $\bar{x} - k'$ (similarly, $\bar{x} - (\frac{1}{2} - k')$). Consequently, $d(p_1) = [\bar{x} - k'] + [\bar{x} - (\frac{1}{2} - k')] = 2\bar{x} = 2(\frac{v-p_1}{\tau})$. Finally, when $p_1 \leq v - \frac{\tau}{2}$, then $\bar{x} \geq \frac{1}{2}$ – now, all consumers obtain non-negative utility from firm 1, and so $d(p_1) = \frac{1}{2}$.

In the partial market information situation, the market outcome is described by each firm choosing its price in order to maximise its own profit, and we examine this Section 3.

2.2 Complete market information

Now, with the introduction of technologies such as the internet, all the consumers are informed about the product of both firms. This reduces the situation to the standard model of price competition on the Salop circle. The two firms choose their respective prices simultaneously, and after observing both prices, each consumer chooses from the firm that gives him a higher net utility subject to it being higher than the abstinence utility of zero. Thus, not only does the internet facilitate price comparisons, but it also facilitates a better match between consumers and the products. We depict this situation of complete market information in the diagram below – the solid line used to etch the circumference of circle is meant to denote that the all consumers are informed about the product of both firms. Thus, firms are no longer monopolists over a particular part of the consumer segment, and so, may need to compete for consumers.

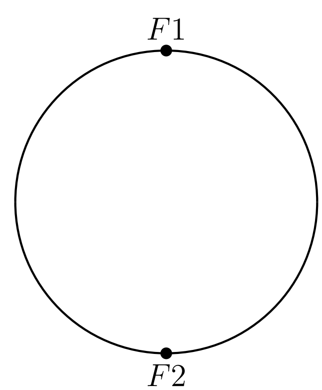


Figure 3: Partial market information: the general case

Now, a consumer who is located at a distance of x from firm 1 obtains a net utility of $v - \tau x - p_1$ on purchasing from firm 1, and a net utility of $v - (\frac{1}{2} - x)\tau - p_2$ on purchasing from firm 2. The consumer purchases a product if at least one of these utilities is non-negative, in which case, it purchases from firm 1 if and only if $v - \tau x - p_1 \geq v - (\frac{1}{2} - x)\tau - p_2$, and from firm 2 otherwise. With this is mind, we specify $d_i(p_i, p_j)$, the demand of firm i when firm i and firm j set prices of p_i and p_j, respectively, where $i, j = 1, 2$ and $i \neq j$. We focus on the case where both prices are less than the parameter v.

In order to obtain the demand of a firm, say firm 1, we note that if the consumer, whose location coincides with the location of firm 1, obtains a higher net utility from firm 2, then all consumers in the market obtain a higher net utility from firm 2 – this happens when $v - p_1 > v - \frac{\tau}{2} - p_2$, i.e. when $p_1 - p_2 > \frac{\tau}{2}$. Here, firm $1's$ demand is zero while firm 2 serves the entire market. The converse holds when $p_2 - p_1 > \frac{\tau}{2}$. The firms share the market when none of these two possibilities occur, i.e. when $\mid p_1 - p_2 \mid \leq \frac{\tau}{2}$.

In the situation where $\mid p_1 - p_2 \mid \leq \frac{\tau}{2}$, in order to determine the demand of firm 1, one has to obtain the location of the marginal consumer of firm 1 – the marginal consumer of firm 1 is one who receives non-negative utility from firm 1, and, in addition, is either indifferent between purchasing from firm 1 and not purchasing at all, or indifferent between purchasing from any of the two firms. In the former case, firm 1's marginal consumer obtains negative utility from firm 2, whereas in the latter case, firm 1's marginal consumer obtains an equal non-negative utility from firm 2 as well. In both of these cases, the demand of firm 1 comes from all the consumers who are at least as close to it as its marginal consumer.

Let firm $1's$ marginal consumer be located at a distance of $\hat{x}$ from firm 1, and hence, at a distance of $\frac{1}{2} - \hat{x}$ from firm 2. In the case where the marginal consumer obtains negative utility from firm 2, the following two relations must hold: $v - \tau\hat{x} - p_1 = 0$ and $v - \tau(\frac{1}{2} - \hat{x}) - p_2 < 0$. This gives $\hat{x} = \frac{v - p_1}{\tau}$ from the first equation, which, when substituted in the second inequality yields $p_2 > 2v - p_1 - \frac{\tau}{2}$. Since there are two such marginal consumers for firm 1, the demand of firm 1 is $2(\frac{v-p_1}{\tau})$. Similarly, the demand of firm 2 is $2(\frac{v-p_2}{\tau})$.

On the other hand, if $p_2 \leq 2v - p_1 - \frac{\tau}{2}$, then firm $1's$ marginal consumer also obtains a non-negative utility of equal magnitude from both firms. If $\hat{x}$ denotes the distance of this marginal consumer from firm 1, then $v - \tau\hat{x} - p_1 = v - \tau(\frac{1}{2} - \hat{x}) - p_2$, or $\hat{x} = \frac{1}{4} + \frac{p_2 - p_1}{2\tau}$. All consumers who are closer to firm 1 than the consumer at $\hat{x}$ consume from firm 1 while the other consumers consume from firm 2. Since there are two such indifferent consumers – one on each side of firm 1 – the demand of firm 1 equals $2\hat{x}$. The demand of firm 2 comes from the complementary mass of consumers. This specifies the demand function of a firm.

$$d_i(p_i,p_j) = \begin{cases} 2(\frac{v-p_1}{\tau}) & \text{if } \mid p_j - p_i \mid \leq \frac{\tau}{2} \text{ and } p_j > 2v - p_i - \frac{\tau}{2} \\ \frac{1}{2} + \frac{p_j - p_i}{\tau} & \text{if } \mid p_j - p_i \mid \leq \frac{\tau}{2} \text{ and } p_j \leq 2v - p_i - \frac{\tau}{2} \\ 1 & \text{if } p_j - p_i > \frac{\tau}{2} \\ 0 & \text{if } p_i - p_j > \frac{\tau}{2} \end{cases}$$

In this case of complete market information, the firms set prices simultaneously, and this determines each firm's demand. Each firm's objective is to choose its price in order to maximise its own profit. The market equilibrium is described by the Nash equilibrium of this price setting game between the two firms, and we analyse this in the next section.

3 Results

We analyse how the market evolves as one moves from the "pre-internet" partial market information paradigm to the "post-internet" complete market information paradigm. There are two salient forces which shape this transition. Firstly, complete market information permits the consumers to choose the product that is a better match with their preference. So, consumers, who, in the partial information situation, may have been constrained to purchase a product for which they have low gross utility may now switch to the product for which they have a higher gross valuation. This migration is expected to be beneficial for the firms as they can now cater to consumers who have a higher valuation for their product, and, intuitively, this ought to make the firms' operations more profitable. At the same time, the fact that consumers are now informed of both products induces price competition between the firms, and one expects this to be to the detriment of the firms. The net effect depends on the balance of these two forces, and we examine this in what follows next. We describe and discuss the nature of the market equilibrium, first for the partial market information case, and then for the complete market information case, and finally examine the change in the market equilibrium that is brought about by the two above mentioned countervailing forces that arise from complete market information.

3.1 Partial market information

In the two subsections that follow, we present and discus the market equilibrium in the two subcases of partial market information – first, for the situation of mild mismatch, and the second for the situation of severe mismatch between the consumers and the firms. The formal proofs of the propositions are presented in the appendix – here, after presenting each proposition, we discuss the intuition and primary features of the equilibrium.

3.1.1 Mild mismatch

The proposition below presents the equilibrium for the case of mild mismatch between the consumers and the firms (illustrated in the left hand panel of Figure 2), where $k \in [0, \frac{1}{4}]$ is the inverse index of mismatch between the consumers and the firms.

Proposition 1. (i) *Suppose that* $\tau \geq 2v$. *Then:*

(a) *the equilibrium price equals* $\frac{v+\tau k}{2}$ *when* $k \in [0, \frac{v}{3\tau}]$

(b) *the equilibrium price equals* $v - \tau k$ *when* $k \in [\frac{v}{3\tau}, \frac{v}{2\tau}]$

(c) *the equilibrium price equals* $\frac{v}{2}$ *when* $k \in [\frac{v}{2\tau}, \frac{1}{4}]$

(ii) *Suppose that* $\tau \in (\frac{4v}{3}, 2v)$. *Then:*

(a) *the equilibrium price equals* $\frac{v+\tau k}{2}$ *when* $k \in [0, \frac{v}{3\tau}]$

(b) *the equilibrium price equals* $v - \tau k$ *when* $k \in [\frac{v}{3\tau}, \frac{1}{4}]$

(iii) *Suppose that* $\tau \in (0, \frac{4v}{3}]$. *Then:*

(a) *the equilibrium price equals* $\frac{v+\tau k}{2}$ *when* $k \in [0, \frac{\tau - v}{\tau}]$

(b) *the equilibrium price equals* $v - \tau(\frac{1}{2} - k)$ *when* $k \in [\frac{\tau - v}{\tau}, \frac{1}{4}]$.

We find that the equilibrium depends on the parameters τ and k. A higher value of τ implies that each consumer obtains a lower gross utility from the firm while a higher value of k indicates a lower degree of mismatch. So, lower values of τ but higher values of k are more beneficial for the firm.

In order to discuss the results, we begin by recalling that, in case of mild mismatch, the firm's market extends from the location of the firm itself to a distance of $\frac{1}{2} - k$ on its left, and a distance of k on its right. And, $k \in [0, \frac{1}{4}]$ implies that the $\frac{1}{2} - k$ measure of consumers on the left is at least as much as the k measure of consumers on the right. So, the measure k of consumers immediately on the left of the firm, and the measure $\frac{1}{4}$ of consumers immediately to the right of the firm, are in fact appropriately matched to the firm. That is, the market segment to the right of the firm comprises only of appropriately matched consumers whereas the market segment to the left comprises of a mix of all the consumers who have a preference for the firm as well as some consumers who have a preference for firm 2. Furthermore, the consumers on right of the firm who are matched to the firm obtain a higher gross utility from firm 1 compared to the consumers on the right of the firm who also obtain a higher gross utility from firm 1 but are matched to firm 2. Thus, the mismatch stems from the fact that consumers who lie at a distance of $(\frac{1}{4}, \frac{1}{2} - k]$ from the firm on the left are matched to the firm when they should not be, and that consumers who lie at a distance of $(k, \frac{1}{4}]$ from the firm on the right are not matched to the firm when they should be.

Now, the firm, in order to maximise profit, may do one of three things. Firstly, it may only cater to a strict subset of consumers on each side; it follows from the paragraph above that all of these consumers happen to be fact appropriately matched to firm 1, and, in addition, these are measure of consumers who obtain the highest gross utility from firm 1's product. In this case, the profit maximising price is $\frac{v}{2}$. Secondly, it may cater to all consumers on its right, which implies, by symmetry, that it caters to at least k measure of consumers on its right as well. Here, the profit-maximising price is $v - \tau\, k$ when it also serves exactly k measure of consumers on its right while it equals $\frac{v+\tau\, k}{2}$ when the firm serves more than k measure of consumers but not all the consumers on its left. In this latter case, some of the mismatched consumers, who are at a distance of more than $\frac{1}{4}$ on the left of the firm, also purchase the good. Thirdly, it may serve all consumers, in which case the profit-maximising price is $v - \tau\,(\frac{1}{2} - k)$ – in this case, the firm also serves the mismatched consumers. These are the prices which appear in the proposition.

In order to convey the intuition behind the equilibrium prices, first suppose that $\tau \geq 2v$. Here, consumers derive relatively low gross utility from the firm.

(a) When consumers are poorly matched to the firms (i.e. $k \leq \frac{v}{3\tau}$), then it is profit maximising for the firm to serve all its consumers to its right. The reason is that, since these consumers are located immediately next to the firm, they derive relatively high gross utility from the firm – so it is profitable for the firm to serve these consumers. This implies that k measure of consumers on the firm's left are also served. Now, since the value of k is low, the mass of mismatched consumers, all of whom are on the left of the firm, is so substantial that the firm can ill-afford to ignore this consumer segment. So, it serves some, but not all, of these consumers as serving the substantial mass of mismatched consumers in its entirety can only come at a substantial reduction in both price and profit. The profit maximising price in this case equals $\frac{v+\tau\, k}{2}$

(b) When k increases and takes on more moderate values (i.e. $k \in [\frac{v}{3\tau}, \frac{v}{2\tau}]$), the match between consumers and firms improves, and the mass of mismatched consumers decreases. As a result, the firm can now afford to ignore the mismatched consumers who anyway required relatively lower prices to be enticed to purchase. The firm focuses, instead, only on the appropriately matched consumers as they form a large enough market. Here, the

firm sets the price $v - \tau k$, and extracts all the surplus from the marginal appropriately matched consumer on the right. We note that this price excludes the appropriately matched consumers on its left who are at a distance of $(\frac{1}{4}, \frac{1}{2} - k)$ from the firm.
(c) When k increases even more (i.e. $k \geq \frac{v}{2\tau}$), causing the match between the consumers and firms to improve further, the firm can even afford to raise it price and exclude some of the appropriately matched consumers on its right. Here, it is profit-maximising for the firm to price as if it were a monopolist on the entire Salop circle – it sets the price $\frac{v}{2}$, which is higher than the price $v - \tau k$ that extracted all the surplus from the marginal appropriately matched consumer on its right, thereby excluding some of the appropriately matched consumers on both sides.

Next, if the consumers' disutility parameter τ decreases to a more moderate value, i.e. $\tau \in (\frac{4v}{3}, 2v)$, then the gross utility obtained by the consumers from the firm is relatively higher. Then, for the same reason outlined above, when k takes on low values (i.e. $k \leq \frac{v}{3\tau}$), the firm sets a price of $\frac{v+\tau k}{2}$ and caters to all the appropriately matched consumers on either side along with some of the mismatched consumers on its left. It is also for the same reason explained above that a higher value of k (i.e. $k \geq \frac{v}{3\tau}$) results in the firm serving only the appropriately matched consumers, and choosing the price $v - \tau k$ that extracts all the surplus from the marginal appropriately matched consumer on its right. However, in this case where $\tau \in (\frac{4v}{3}, 2v)$, in contrast to the case where $\tau \geq 2v$, it is not profit maximising for the firm to choose the price $\frac{v}{2}$ that excludes some of the appropriately matched consumers on its right. This is because the relatively lower value of τ results in all consumers obtaining a higher gross utility – so, the firm finds it more profitable to extract the additional gross utility (due to the lower value of τ) of the marginal appropriately matched consumer on its right than to exclude some of the appropriately matched consumers on the right.

Finally, consider the case where τ is low (i.e. $\tau \leq \frac{4v}{3}$). When consumers are not as well matched to the firms (i.e. $k \leq \frac{\tau - v}{\tau}$), then, for reasons elaborated above, it is profit maximising for the firm to serve all the appropriately matched consumers on either side along with some of the mismatched consumers on its left. This results in an equilibrium price of $\frac{v+\tau k}{2}$. When k increases, and the match between the consumers and the firms improves, the firm sets the price $v - \tau(\frac{1}{2} - k)$ which serves all consumers. The reason behind not excluding even the mismatched consumers is firstly, that a high value of k implies there even the most mismatched consumer has a higher gross utility (compared to the case where k takes on lower values), and secondly, due to low values of τ, all consumers – even the mismatched consumers – obtain a higher gross utility. Hence, it is profit maximising for the firm to serve all consumers.

We close this subsection by drawing attention to a couple of counter-intuitive comparative statics.

Firstly, since higher values of k indicate a better match between consumers and firms, one may expect the equilibrium price to be non-decreasing in k. However, when the values assumed by τ and k are not too low (i.e. either $\tau \geq 2v$ and $k \in [\frac{v}{3\tau}, \frac{v}{2\tau}]$, or $\tau \in (\frac{4v}{3}, 2v)$ and $k \in [\frac{v}{3\tau}, \frac{1}{4}]$), then the equilibrium price $v - \tau\, k$ is decreasing in k. The intuition is that, in these cases, for reasons elaborated above, it is profit maximising for the firm to extract the surplus of the marginal appropriately matched consumer on its right. As k increases, the marginal appropriately matched consumer on the firm's right moves farther away from the firm implying that the gross utility received by the marginal appropriately matched consumer on its right decreases as k increases. So, if the firm has to extract surplus from this consumer, the price must decrease.

Secondly, since higher values of τ imply that consumers obtain lower gross utility from the firm, one may anticipate that an increase in τ ought to reduce the equilibrium price. However, when the quality of the match between the consumers and the firms is low (i.e. k assumes low values), then the equilibrium price $\frac{v+\tau\, k}{2}$ is increasing in τ – we recall that in these cases, the firm serves all the appropriately matched consumers on both sides along with a strict subset of the mismatched consumers. The reason for this counter-intuitive comparative static is that, when τ increases, if the firm maintains the same price, then some of the mismatched consumers exit the market but all the appropriately matched consumers continue to demand from the firm. Since the appropriately matched consumers derive a higher gross utility than the mismatched consumers, not only is a reduction in price to attract the latter profit deteriorating, but it is profit-maximising to increase the price in order to extract more surplus from the appropriately matched consumers and a smaller subset of the mismatched consumers who have a relatively higher gross utility from the product than the other mismatched consumers.

3.1.2 Severe mismatch

The next proposition describes the equilibrium price when there is severe mismatch between the consumers and the firms (illustrated in the right hand panel of Figure 2), where $k' \in [0, \frac{1}{4}]$ is a direct measure of the extent of mismatch between the consumers and the firms.

Proposition 2. (i) *Suppose that* $\tau \geq 4v$. *Then, the equilibrium price equals* $\frac{v-\tau\, k'}{2}$.

(ii) *Suppose that* $\tau \in [\frac{4v}{3}, 4v)$. *Then:*

(a) *the equilibrium price equals* $\frac{v-\tau k'}{2}$ *when* $k' \in [0, \frac{1}{2\sqrt{2}} - (\sqrt{2}-1)\frac{v}{\tau}]$

(b) *the equilibrium price equals* $\frac{v}{2} - \frac{\tau}{8}$ *when* $k' \in [\frac{1}{2\sqrt{2}} - (\sqrt{2}-1)\frac{v}{\tau}, \frac{1}{4}]$

(iii) *Suppose that* $\tau \in [v, \frac{4v}{3})$. *Then:*

(a) *the equilibrium price equals* $\frac{v-\tau k'}{2}$ *when* $k' \in [0, \frac{v}{\tau} - \sqrt{\frac{2v-\tau}{\tau}}]$

(b) *the equilibrium price equals* $v - \frac{\tau}{2}$ *when* $k' \in [\frac{v}{\tau} - \sqrt{\frac{2v-\tau}{\tau}}, \frac{1}{4}]$

(iv) *Suppose that* $\tau \in (0, v)$. *Then, the equilibrium price equals* $v - \frac{\tau}{2}$.

We find, as before, that the equilibrium depends on the parameters τ and k. In order to elaborate on the nature of the equilibrium, we recall that, in case of severe mismatch, the firm's market starts from a distance of k' on its left and extends to a distance of $\frac{1}{2}$, while it starts from a distance of $\frac{1}{2} - k'$ on its right and extends to a distance of $\frac{1}{2} - k'$. And, $k' \in [0, \frac{1}{4}]$ implies that the $\frac{1}{2} - k'$ measure of consumers on the left is at least as much as the k' measure of consumers on the right. Furthermore, all the consumers to the firm's right are mismatched whereas the $\frac{1}{4} - k$ mass of consumers who are located at a distance of $[k, \frac{1}{4}]$ from the firm on its left are appropriately matched to the firm. As a result, the firm finds the market segment on its left more attractive than the market segment on its right. However, even the consumers on the left who are appropriately matched obtain a lower gross utility from firm 1 than the measure k of the mismatched consumers who are located immediately to firm $1'$s left but are matched to firm 2.

Now, the firm, in order to maximise profit, may do one of three things. Firstly, it may only cater to a strict subset of consumers on its left but not to any consumers on its right. In this case, the profit maximising price equals $\frac{v-\tau k'}{2}$. Secondly, it may cater to a strict subset of its market segment on each side, in which case, the profit maximising price is $\frac{v}{2} - \frac{\tau}{8}$. Thirdly, it may serve all consumers, in which case the profit-maximising price is $v - \frac{\tau}{2}$. These are the prices which define the equilibrium.

In order to explain the intuition behind the equilibrium prices, we first suppose that $\tau \geq 4v$. Here, consumers derive low gross utility from the firm. As a result, the low prices that the firm has to set if it wishes to serve the relatively less attractive market segment on its right substantially erodes into its profit – it is more profitable for the firm to only serve the more attractive market segment on the left, in which case, the profit maximising price is $\frac{v-\tau k'}{2}$.

Next, suppose that $\tau \in [\frac{4v}{3}, 4v]$. Here, each consumer's gross utility for the product is relatively higher. Nonetheless, due to the reason above, when the degree of mismatch is sufficiently low (i.e. $k' \leq \frac{1}{2\sqrt{2}} - (\sqrt{2}-1)\frac{v}{\tau}$) while still remaining within the confines of the severe mismatch situation, it is still profit maximising for the firm to only serve a subset of the market segment on its left. Correspondingly, the profit maximising price is $\frac{v-\tau k'}{2}$. However, as k' increases, and the match between the consumers and the firms worsens (i.e. $k' \in [\frac{1}{2\sqrt{2}} - (\sqrt{2}-1)\frac{v}{\tau}, \frac{1}{4}]$), the market segment to the right of the firm becomes relatively more favourable, both in terms of size and gross valuations of the consumers in that segment. The converse holds for the market segment on the left; as k' increases, the consumers in this market segment, starting with those with highest gross utilities, recede away from the firm leaving behind a market segment that is less attractive, both in terms of size and gross utilities of the consumers in this segment. Now, the firm can no longer afford to cater only to the market segment on its left, and must attract consumers from both market segments. At the same time, the gross valuations of the consumers is not high enough (i.e. τ is not low enough) that the firm will find it profitable to serve all consumers. Thus, a strict subset of both market segments are served, and the profit-maximising price is $\frac{v}{2} - \frac{\tau}{8}$.

Now, as τ decreases in magnitude (i.e. $\tau \in [v, \frac{4v}{3})$), and each consumer's gross utility is relatively higher, it is profit-maximising for the firm to serve only the left market segment, and set a price of $\frac{v-\tau k'}{2}$, when the mismatch between the consumers and firms is relatively low (i.e. $k' \leq \frac{v}{\tau} - \sqrt{\frac{2v-\tau}{\tau}}$). As in the previous paragraph, as k' increases and when the mismatch worsens (i.e. $k' \geq [\frac{v}{\tau} - \sqrt{\frac{2v-\tau}{\tau}}, \frac{1}{4}]$), it is more profitable for the firm to start catering to both market segments. In fact, in this case, the low value of τ, or equivalently, the relatively high gross utilities of the consumers, implies that it is, in fact, profitable for the firm to serve all consumers. Since the consumer who is farthest away from the firm is at a distance of $\frac{1}{2}$ from the firm, the profit-maximising price such that all consumers consume is $v - \frac{\tau}{2}$.

This logic carries over to the case where $\tau < v$. Compared to the situations discussed above, now, each consumer gross utility is even higher So, it is profit-maximising for the firm to serve all consumers. As a result, the profit-maximising price is $v - \frac{\tau}{2}$.

Finally, the comparative statics in this case are rather intuitive. The equilibrium price is non-increasing in both the degree of mismatch between consumers and firms (i.e. non-increasing in k') and the gross valuation of each consumer (i.e. τ).

3.2 Complete market information

The proposition below presents the equilibrium for the complete market information case. This corresponds to Nash equilibrium of the standard price competition game between the firms on the Salop circle. Nonetheless, for completeness, we present its proof in the appendix, and briefly discuss the nature of the equilibrium.

Proposition 3. (i) *Suppose that $\tau > 2v$. Then, in the unique symmetric pure strategy Nash equilibrium, each firm chooses price equal to $\frac{v}{2}$.*
(ii) *Suppose that $\tau \in [\frac{4v}{3}, 2v]$. Then, in the unique symmetric pure strategy Nash equilibrium, each firm chooses price equal to $v - \frac{\tau}{4}$.*
(iii) *Suppose that $\tau \in (0, \frac{4v}{3}]$. Then, in the unique symmetric pure strategy Nash equilibrium, each firm chooses price equal to $\frac{\tau}{2}$.*

The equilibrium price depends on the extent of differentiation in the market. When the market is highly differentiated (i.e. $\tau > 2v$), the equilibrium price $\frac{v}{2}$ set by each firm corresponds to the price that a firm would choose if it were a monopolist in the Salop circle. The reason is that, high market differentiation results in consumers being relatively more unwilling to consume from the firm that is farther away. This gives the firms a high degree of market power over the consumers located near it. Furthermore, a firm would have to set an unprofitably low price to attract consumers of the other firm. This results in the market equilibrium corresponding to the monopoly case. Next, when the market is moderately differentiated (i.e. $\tau \in [\frac{4v}{3}, 2v]$), then the equilibrium price $v - \frac{\tau}{4}$ is such that the firms extract all the surplus from the marginal consumer. Finally, when the market differentiation is low (i.e. $\tau < \frac{4v}{3}$), then the firms compete more actively for the consumer, and this results in an equilibrium where each consumer receives positive net utility.

3.3 Partial Market Information versus Complete Market Information

We will now compare the equilibrium in the partial market information situation and the complete market information situation in order to understand how the equilibrium price changes when consumers have more information. In the proposition below, we focus on the conditions that are both necessary and sufficient for the price to increase with complete market information. The proof of the proposition is in the appendix, and we devote this subsection to uncovering the intuition underlying the results.

Proposition 4. **(i)** *In the situation of mild mismatch between consumers are firms, the equilibrium price in the complete market information situation is higher if and only if:*
(a) $\tau \in (\frac{4v}{3}, 2v)$ *and* $k < \frac{2v-\tau}{2\tau}$*, or* (b) $\tau \in (0, \frac{4v}{3}]$ *and* $k < \frac{\tau - v}{\tau}$.
(ii) *In the situation of severe mismatch between consumers are firms, the equilibrium price in the complete market information situation is higher if and only if:*
(a) $\tau \geq 2v$ *and* $k' > 0$*, or* (b) $\tau \in [\frac{4v}{3}, 2v]$ *and* $k' > \frac{1}{2} - \frac{v}{\tau}$*, or* (c) $\tau \in (v, \frac{4v}{3})$ *and* $k' > \frac{v-\tau}{\tau}$.

Intuitively, one may expect that the market equilibrium price will be higher in the complete market information situation when the disutility/market differentiation parameter τ is high, and the degree of mismatch between consumers and firms is high. This is because the consumers' firm-preference is not as pronounced when τ takes on low values, and, when consumers are completely informed about both firms in the complete market information case, this may lead to more intense price competition between firms thus resulting in lower market prices. On the other hand, if the consumers and firms are poorly matched in the partial market information situation, then the rectification of this in the complete market information situation is expected to boost the equilibrium price.

We find that, in the case of mild mismatch between consumers and the firms, this intuition holds up partly – while the necessity of low values of the inverse index of mismatch k holds, the necessity of high values of τ does not. In fact, when τ is sufficiently high (i.e. $\tau > 2v$), then, irrespective of the extent of mismatch between consumers and firms, the equilibrium price never increases with complete market information. This is because, when τ takes on high values in the partial market information situation with mild mismatch, the firm simply caters to the consumers who are appropriately matched to it. And, these consumers, on account of being located immediately next to the firm, are the ones who derive relatively high gross utility from the firm. Interestingly, these are also exactly the consumers that the firm serves when there is complete market information – recall that with high values of τ and complete market information, the firms do not actively compete for consumers but rather price as if they are monopolists. As a result, when τ is high, the market equilibrium price does not decrease when the consumers are informed about both firms. Thus, in this case, the market price is higher with complete market information only when τ is not very high; at the same time, this, by itself is not sufficient; for reasons outlined earlier, the moderated values of τ must be coupled with a sufficiently poor match between consumers and firms for the market price is higher with complete market information

On the other hand, when the mismatch in the partial information situation is severe, then the intuition alluded to above holds. The market price is higher in the complete market information situation whenever τ is sufficiently high (i.e. $\tau \geq 2v$), and this is irrespective of the degree of mismatch. However, as τ decreases, the degree of mismatch must be sufficiently poor (i.e. k' must be sufficiently high) for the market price to be higher in the complete market information. This reasoning extends to the case where τ is very low. Here, irrespective of how severe the mismatch between consumers and firms may be, the competition inducing effect of low values of τ dominates, and results in lower equilibrium prices in the complete market information situation.

4 Conclusion

In this paper, we present a stylised model to examine the effect of market information on market outcomes. This paper is motivated by empirical studies which report that, even though internet and associated platforms have enabled consumers to almost frictionlessly obtain information about products available on the market, and compare their prices, contrary to conventional wisdom, this has not always been accompanied by the decrease in market prices that is expected because of stiffer competition amongst sellers that is induced by the easily available market information.

Our explanation for the empirical finding that prices of some products may fall, but the price of other products may not, is that more easily accessible market information has another facet beyond simply inducing more intense competition amongst sellers. Easily available market information may also result in consumers finding out about products which are more aligned with their tastes and preferences. In the absence of such information, consumers, due to lack of knowledge about better alternatives, may be compelled to consume products that are not as good a match. The fact that more market information can facilitate better matches between consumers and products may, in fact, lead to an increase in prices. We explore this in the context of a differentiated market, and analyse how the balance between these two forces – namely, better matches between consumers and firms on the one hand, and stiffer competition between sellers on the other hand – that are brought to life by more market information affects market prices.

Appendix

Proof of Proposition 1. A firm's demand function $d(p)$ is:

$$d(p) = \begin{cases} 2\,(\frac{v-p}{\tau}) & \text{if } p \in (v-\tau\,k, v] \\ k + \frac{v-p}{\tau} & \text{if } p \in (v-\tau(\frac{1}{2}-k), v-\tau\,k] \\ \frac{1}{2} & \text{if } p \leq v-\tau(\frac{1}{2}-k) \end{cases}$$

Now, conditional on $p \in [v-\tau\,k, v]$ so that $d(p) = 2\,(\frac{v-p}{\tau})$, the profit maximising price is $\frac{v}{2}$, and this price lies in the relevant interval $[v-\tau\,k, v]$ whenever $k \geq \frac{v}{2\tau}$. Otherwise, i.e. if $p \in [v-\tau\,k, v]$ but $k < \frac{v}{2\tau}$, it follows from the concavity of profit function that is continuous in p that the profit-maximising price is $v-\tau\,k$. Thus, conditional on $p \in [v-\tau\,k, v]$, the maximum profit attainable is:

$$\tilde{\pi} = \begin{cases} \frac{v^2}{2\tau} & \text{if } k \geq \frac{v}{2\tau} \\ 2k(v-\tau\,k) & \text{if } k < \frac{v}{2\tau} \end{cases}$$

Next, conditional on $p \in [v-\tau(\frac{1}{2}-k), v-\tau\,k]$ so that $d(p) = k + \frac{v-p}{\tau}$, the profit maximising price is $\frac{k\tau}{2} + \frac{v}{2}$, and this price lies in the relevant interval $[v-\tau(\frac{1}{2}-k), v-\tau\,k]$ whenever $k \leq \min\{\frac{v}{3\tau}, \frac{\tau-v}{\tau}\}$. On the other hand, if $p \in [v-\tau(\frac{1}{2}-k), v-\tau\,k]$ but the above price $\frac{k\tau}{2} + \frac{v}{2}$ exceeds $v-\tau\,k$ (similarly, is less than $v-\tau(\frac{1}{2}-k)$) or equivalently when $k > \frac{v}{3\tau}$ (similarly, equivalently $k > \frac{\tau-v}{\tau}$), then it follows from concavity of the profit function that is continuous in p that the profit-maximising price is $v-\tau\,k$ (similarly, $v-\tau(\frac{1}{2}-k)$). Thus, conditional on $p \in [v-\tau(\frac{1}{2}-k), v-\tau\,k]$, the maximum profit attainable is:

$$\bar{\pi} = \begin{cases} (\frac{k}{2} + \frac{v}{2\tau})(\frac{\tau\,k+v}{2}) = \tau(\frac{k}{2} + \frac{v}{2\tau})^2 & \text{if } k \leq \min\{\frac{v}{3\tau}, \frac{\tau-v}{\tau}\} \\ 2k(v-k\tau) & \text{if } k > \frac{v}{3\tau} \text{ and } k \leq \frac{\tau-v}{\tau} \\ \frac{1}{2}[v-\tau(\frac{1}{2}-k)] & \text{if } k > \frac{\tau-v}{\tau} \text{ and } k \leq \frac{v}{3\tau} \end{cases}$$

Finally, conditional on $p \leq v-\tau(\frac{1}{2}-k)$ so that $d(p) = \frac{1}{2}$, it is obvious that the profit-maximising price is $v-\tau(\frac{1}{2}-k)$. In this case, the maximum attainable profit is $\hat{\pi} = \frac{1}{2}[v-\tau(\frac{1}{2}-k)]$.

The proposition is established by a comparison of the profit levels $\tilde{\pi}$, $\bar{\pi}$, and $\hat{\pi}$ under the various possible values of k.

Firstly, suppose that the market differentiation is such that $\tau \geq 2v$. Then, $\frac{v}{2\tau} \leq \frac{1}{4}$ and $\frac{\tau-v}{\tau} > \frac{1}{4}$ hold thus implying that $k \leq \frac{\tau-v}{\tau}$ must *always* hold. As a result, we need to consider three cases: $k \in [0, \frac{v}{3\tau}]$, $k \in [\frac{v}{3\tau}, \frac{v}{2\tau}]$, and $k \in [\frac{v}{2\tau}, \frac{1}{4}]$.

Case I: Suppose $k \in [\frac{v}{2\tau}, \frac{1}{4}]$. This implies $k \leq \frac{v}{3\tau}$ cannot hold. So, the profit $\tilde{\pi} = \frac{v^2}{2\tau}$ that is obtained by setting the price $\frac{v}{2}$ has to be compared with the profit $\bar{\pi} = 2k(v - k\tau)$ that is obtained by setting the price $v - k\tau$, and the profit $\hat{\pi} = \frac{1}{2}[v - \tau(\frac{1}{2} - x)]$ that is obtained by setting the price $v - \tau(\frac{1}{2} - x)$. Then, the profit maximising price is $\frac{v}{2}$ because it can be verified that $\tilde{\pi} = \frac{v^2}{2\tau} \geq \bar{\pi} = 2k(v - k\tau)$ and $\tilde{\pi} = \frac{v^2}{2\tau} \geq \hat{\pi} = \frac{1}{2}[v - \tau(\frac{1}{2} - x)]$. Furthermore, $\bar{\pi} = \tilde{\pi}$ (similarly, $\hat{\pi} = \tilde{\pi}$) if and only if $k = \frac{v}{2\tau}$ (similarly, $\tau = 2v$ and $k = \frac{1}{4}$), in which case the two corresponding prices $v - k\tau$ and $\frac{v}{2}$ (similarly, $v - \tau(\frac{1}{2} - x)$ and $\frac{v}{2}$) are equal. Hence, the profit-maximising price is $\frac{v}{2}$.

Case II: Suppose $k \in [\frac{v}{3\tau}, \frac{v}{2\tau}]$. Then $\frac{\tau - v}{\tau} > \frac{v}{2\tau}$ holds, which implies that $k \geq \frac{\tau - v}{\tau}$ can never hold. So, $\tilde{\pi}$ and $\bar{\pi}$ give an identical profit of $2k(v - \tau\, k)$ as the price in both cases corresponds to $v - \tau\, k$. It follows that when $k \in [\frac{v}{3\tau}, \frac{v}{2\tau}]$, the profit-maximising price is $v - \tau\, k$ because it can be verified that $\hat{\pi} < \bar{\pi} = \tilde{\pi}$ holds.

Case III: Suppose $k \in [0, \frac{v}{3\tau}]$. Then, since $k \leq \frac{\tau - v}{\tau}$ always holds (because $\frac{v}{3\tau} < \frac{\tau - v}{\tau}$ holds whenever, as in this case, $\tau > 2v$), the profit-maximising price is $\frac{v + k\tau}{2}$ as it can be verified that $\bar{\pi} = \tau(\frac{k}{2} + \frac{v}{2\tau})^2 \geq \tilde{\pi} = 2k(v - k\tau)$ and $\bar{\pi} = \tau(\frac{k}{2} + \frac{v}{2\tau})^2 \geq \hat{\pi} = \frac{1}{2}[v - \tau(\frac{1}{2} - k)]$. Furthermore, $\bar{\pi} = \tilde{\pi}$ (similarly, $\hat{\pi} = \bar{\pi}$) if and only if $k = \frac{v}{3\tau}$ (similarly, $k = \frac{\tau - v}{\tau}$), in which case the two corresponding prices $v - k\tau$ and $\frac{v + k\tau}{2}$ (similarly, $v - \tau(\frac{1}{2} - x)$ and $\frac{v + k\tau}{2}$) are equal. Hence, the profit-maximising price is $\frac{v + k\tau}{2}$.

Secondly, suppose that the market differentiation is such that $\tau \in (\frac{4v}{3}, 2v)$. Then, $\frac{v}{3\tau} < \frac{1}{4}$, $\frac{v}{2\tau} > \frac{1}{4}$, and $\frac{\tau - v}{\tau} > \frac{1}{4}$ hold thus implying that $k \leq \frac{v}{2\tau}$ and $k \leq \frac{\tau - v}{\tau}$ must *always* hold. In this situation, we need to consider two cases: $k \in [0, \frac{v}{3\tau}]$ and $k \in (\frac{v}{3\tau}, \frac{1}{4}]$.

Case I: Suppose $k \in [0, \frac{v}{3\tau}]$. Then, the profit-maximising price is $\frac{v + k\tau}{2}$ as it can be verified that $\bar{\pi} = \tau(\frac{k}{2} + \frac{v}{2\tau})^2 \geq \tilde{\pi} = 2k(v - \tau\, k)$ and $\bar{\pi} = \tau(\frac{k}{2} + \frac{v}{2\tau})^2 > \hat{\pi} = \frac{1}{2}[v - \tau(\frac{1}{2} - k)]$. Furthermore, $\bar{\pi} = \tilde{\pi}$ only if $k = \frac{v}{3\tau}$, in which case the two corresponding prices $v - \tau\, k$ and $\frac{v + k\tau}{2}$ are equal. Hence, the profit-maximising price is $\frac{v + k\tau}{2}$.

Case II: Suppose $k \in (\frac{v}{3\tau}, \frac{1}{4}]$. Then $\bar{\pi} = \tilde{\pi} = 2k(v - \tau\, k)$ as the price equals $v - \tau\, k$ in both cases. Here, the profit-maximising price is $v - \tau\, k$ because it can be verified that $\bar{\pi} = \tilde{\pi} \geq \hat{\pi}$. Furthermore, $\bar{\pi} = \tilde{\pi} = \hat{\pi}$ only if $k = \frac{1}{4}$, in which case the two corresponding prices $v - \tau\, k$ and $v - \tau(\frac{1}{2} - k)$ are equal. Hence, the profit-maximising price is $v - \tau\, k$.

Finally, suppose that the market differentiation is such that $\tau \in (0, \frac{4v}{3}]$. Then, $\frac{v}{3\tau} \geq \frac{1}{4}$ holds thus implying, due to $k \in [0, \frac{1}{4}]$, that $k \leq \frac{v}{3\tau}$, and hence $k < \frac{v}{2\tau}$, must *always* hold; further, $\tau \leq \frac{4v}{3}$ also implies $\frac{\tau - v}{\tau} \leq \frac{1}{4}$. As a result, we need to consider two cases: $k \in [0, \frac{\tau - v}{\tau}]$ and $k \in (\frac{\tau - v}{\tau}, \frac{1}{4}]$.

Case I: Suppose $k \in [0, \frac{\tau - v}{\tau}]$. Then, in view of the fact that $k \leq \frac{v}{3\tau}$ holds, the profit-maximising price is $\frac{v + k\tau}{2}$ as it can be verified that $\bar{\pi} = \tau(\frac{k}{2} + \frac{v}{2\tau})^2 \geq \tilde{\pi} = 2k(v - \tau\, k)$ and

$\bar{\pi} = \tau(\frac{k}{2} + \frac{v}{2\tau})^2 \geq \hat{\pi} = \frac{1}{2}[v - \tau(\frac{1}{2} - k)]$. Furthermore, $\bar{\pi} = \tilde{\pi}$ (similarly, $\bar{\pi} = \hat{\pi}$) if and only if $k = \frac{1}{4}$ and $\tau = \frac{4v}{3}$ (similarly, $k = \frac{\tau - v}{\tau}$), in which case the two corresponding prices $v - \tau\, k$ and $\frac{v+k\tau}{2}$ (similarly, $v - \tau(\frac{1}{2} - k)$ and $\frac{v+k\tau}{2}$) are equal. Hence, the profit-maximising price is $\frac{v+k\tau}{2}$.

Case II: Suppose $k \in (\frac{\tau-v}{\tau}, \frac{1}{4}]$. Here, $\bar{\pi} = \hat{\pi} = \frac{1}{2}(v - \tau(\frac{1}{2} - k))$ because the corresponding price equals $v - \tau(\frac{1}{2} - k)$ in both cases. The price $v - \tau(\frac{1}{2} - k)$ is also the profit maximising price as it can be verified that $\bar{\pi} = \hat{\pi} = \frac{1}{2}(v - \tau(\frac{1}{2} - k)) \geq \tilde{\pi} = 2k(v - \tau\, k)$. Furthermore, $\bar{\pi} = \hat{\pi} = \tilde{\pi} =$ if and only if $k = \frac{1}{4}$, in which case, the two corresponding prices $v - \tau(\frac{1}{2} - k)$ and $v - \tau\, k$ are equal. Hence, the profit-maximising price is $v - \tau(\frac{1}{2} - k)$. ■

Proof of Proposition 2. A firm's demand function $d(p)$ is:

$$d(p) = \begin{cases} \frac{v-p}{\tau} - k' & \text{if } p \in (v - \tau(\frac{1}{2} - k'), v - \tau\, k'] \\ 2(\frac{v-p}{\tau}) - \frac{1}{2} & \text{if } p \in (v - \frac{\tau}{2}, v - \tau(\frac{1}{2} - k')] \\ \frac{1}{2} & \text{if } p \leq v - \frac{\tau}{2} \end{cases}$$

Now, conditional on choosing a price $p \in [v - \tau(\frac{1}{2} - k'), v - \tau\, k']$, in which case the demand function is $d(p) = \frac{v-p}{\tau} - k'$, the profit maximising price is $\frac{v - \tau\, k'}{2}$ which lies in the relevant interval $[v - \tau(\frac{1}{2} - k'), v - \tau\, k']$ whenever $k' \leq \frac{\tau - v}{3\tau}$. Otherwise, i.e. if $k' > \frac{\tau - v}{3\tau}$, then, it follows from the concavity of profit function that is continuous in p, that the profit-maximising price in the interval $[v - \tau(\frac{1}{2} - k'), v - \tau\, k']$ is $v - \tau(\frac{1}{2} - k')$. Thus, conditional on the firm choosing $p \in [v - \tau(\frac{1}{2} - k'), v - \tau\, k']$, the maximum profit attainable is:

$$\tilde{\pi} = \begin{cases} \frac{1}{\tau}(\frac{v - \tau\, k'}{2})^2 & \text{if } k' \leq \frac{\tau - v}{3\tau} \\ (v - \tau(\frac{1}{2} - k'))(\frac{1}{2} - 2k') & \text{if } k' > \frac{\tau - v}{3\tau} \end{cases}$$

Next, conditional on the firm choosing a price $p \in [v - \frac{\tau}{2}, v - \tau(\frac{1}{2} - k')]$, where the demand function is $d(p) = 2(\frac{v-p}{\tau}) - \frac{1}{2}$, the profit maximising price is $\frac{v}{2} - \frac{\tau}{8}$, and this lies in the relevant interval $[v - \frac{\tau}{2}, v - \tau(\frac{1}{2} - k')]$ whenever $\tau \geq \frac{4v}{3}$ and $k' \geq \frac{3\tau - 4v}{8\tau}$. On the other hand, it follows from the concavity of the profit function that is continuous in p that, if $\tau < \frac{4v}{3}$, then the profit maximising price is $v - \frac{\tau}{2}$, and if $k' < \frac{3\tau - 4v}{8\tau}$, the profit maximising price is $v - \tau(\frac{1}{2} - k')$. Thus, conditional on the firm choosing a price $p \in [v - \frac{\tau}{2}, v - \tau(\frac{1}{2} - k')]$, the maximum profit attainable is:

$$\bar{\pi} = \begin{cases} (\frac{v}{\tau} - \frac{1}{4})(\frac{v}{2} - \frac{\tau}{8}) & \text{if } \tau \geq \frac{4v}{3} \text{ and } k' \geq \frac{3\tau - 4v}{8\tau} \\ (v - \tau(\frac{1}{2} - k'))(\frac{1}{2} - 2k') & \text{if } k' < \frac{3\tau - 4v}{8\tau} \text{ (which, due to } k' \geq 0\text{, implies } \tau \geq \frac{4v}{3}) \\ \frac{1}{2}[v - \frac{\tau}{2}] & \text{if } \tau < \frac{4v}{3} \text{ (which, due to } k' \geq 0\text{, implies } k' \geq \frac{3\tau - 4v}{8\tau}) \end{cases}$$

Finally, conditional on the firm choosing a price $p \leq v - \frac{\tau}{2}$, where the demand function

$d(p) = \frac{1}{2}$, it is obvious that the profit-maximising price is $v - \frac{\tau}{2}$, and the maximum attainable profit is $\hat{\pi} = \frac{1}{2}[v - \frac{\tau}{2}]$.

The proposition is established by a comparison of the profit levels $\tilde{\pi}$, $\bar{\pi}$, and $\hat{\pi}$ under the various possible values of k'.

Firstly, suppose that the market differentiation is such that $\tau \geq 4v$. Then, the inequalities $0 < \frac{1}{4} \leq \frac{\tau - v}{3\tau} \leq \frac{3\tau - 4v}{8\tau}$ hold thereby implying, because of $k' \in [0, \frac{1}{4}]$, that $k' \leq \frac{\tau - v}{3\tau}$ and $k' \leq \frac{3\tau - 4v}{8\tau}$ must always hold. Then, the profit-maximising price, for all $k' \in [0, \frac{1}{4}]$, equals $\frac{v - \tau k'}{2}$ because it can be verified that $\tilde{\pi} = \frac{1}{\tau}(\frac{v - \tau k'}{2})^2 \geq \bar{\pi} = (v - \tau(\frac{1}{2} - k'))(\frac{1}{2} - 2k')$ and $\tilde{\pi} = \frac{1}{\tau}(\frac{v - \tau k'}{2})^2 > \hat{\pi} = \frac{1}{2}[v - \frac{\tau}{2}]$. Furthermore, $\tilde{\pi} = \bar{\pi}$ if and only if $k' = \frac{1}{4}$ and $\tau = 4v$, in which case the two corresponding prices $v - \tau(\frac{1}{2} - k')$ and $\frac{v - \tau k'}{2}$ are equal. Hence, the profit-maximising price is $\frac{v - \tau k'}{2}$.

Secondly, suppose that the market differentiation is such that $\tau \in [\frac{4v}{3}, 4v)$. Then, the inequalities $0 \leq \frac{3\tau - 4v}{8\tau} < \frac{\tau - v}{3\tau} < \frac{1}{4}$ hold thereby implying, because of $k' \in [0, \frac{1}{4}]$, that we have to consider the three cases: $k' \in [0, \frac{3\tau - 4v}{8\tau})$, $k' \in [\frac{3\tau - 4v}{8\tau}, \frac{\tau - v}{3\tau}]$, and $k' \in (\frac{\tau - v}{3\tau}, \frac{1}{4}]$.
Case I: Suppose $k' \in [0, \frac{3\tau - 4v}{8\tau})$. Then, the profit-maximising price is $\frac{v - \tau k'}{2}$ because it can be verified that $\tilde{\pi} = \frac{1}{\tau}(\frac{v - \tau k'}{2})^2 > \bar{\pi} = (v - \tau(\frac{1}{2} - k'))(\frac{1}{2} - 2k')$ and $\tilde{\pi} = \frac{1}{\tau}(\frac{v - \tau k'}{2})^2 > \hat{\pi} = \frac{1}{2}[v - \frac{\tau}{2}]$.
Case II: Suppose $k' \in [\frac{3\tau - 4v}{8\tau}, \frac{\tau - v}{3\tau}]$. Then, it can be verified that $\hat{\pi} = \frac{1}{2}[v - \frac{\tau}{2}] < \bar{\pi} = (\frac{v}{\tau} - \frac{1}{4})(\frac{v}{2} - \frac{\tau}{8})$. Now, a comparison of $\tilde{\pi} = \frac{1}{\tau}(\frac{v - \tau k'}{2})^2$ and $\bar{\pi} = (\frac{v}{\tau} - \frac{1}{4})(\frac{v}{2} - \frac{\tau}{8})$ reveals that the profit-maximising price equals $\frac{v - \tau k'}{2}$ when $k' \in [\frac{3\tau - 4v}{8\tau}, \frac{1}{2\sqrt{2}} - (\sqrt{2} - 1)\frac{v}{\tau}]$ (because, in this case, $\tilde{\pi} \geq \bar{\pi}$) and $\frac{v}{2} - \frac{\tau}{8}$ when $k' \in [\frac{1}{2\sqrt{2}} - (\sqrt{2} - 1)\frac{v}{\tau}, \frac{\tau - v}{3\tau}]$ (because, in this case, $\tilde{\pi} \leq \bar{\pi}$).
Case III: Suppose $k' \in (\frac{\tau - v}{3\tau}, \frac{1}{4}]$. Then, the profit-maximising price is $\frac{v}{2} - \frac{\tau}{8}$ because it can be verified that $\bar{\pi} = (\frac{v}{\tau} - \frac{1}{4})(\frac{v}{2} - \frac{\tau}{8}) > \hat{\pi}$ and $\bar{\pi} = (\frac{v}{\tau} - \frac{1}{4})(\frac{v}{2} - \frac{\tau}{8}) > \tilde{\pi} = (v - \tau(\frac{1}{2} - k'))(\frac{1}{2} - 2k')$.

The results obtained above can be combined and expressed more conveniently as: when $\tau \in (\frac{4v}{3}, 4v)$, the profit-maximising price equals $\frac{v - \tau k'}{2}$ when $k' \in [0, \frac{1}{2\sqrt{2}} - (\sqrt{2} - 1)\frac{v}{\tau}]$ and $\frac{v}{2} - \frac{\tau}{8}$ when $k' \in [\frac{1}{2\sqrt{2}} - (\sqrt{2} - 1)\frac{v}{\tau}, \frac{1}{4}]$.

Thirdly, suppose that the market differentiation is such that $\tau \in [v, \frac{4v}{3})$. Then, the inequalities $\frac{3\tau - 4v}{8\tau} < 0 \leq \frac{\tau - v}{3\tau} < \frac{1}{4}$ hold thereby implying, because of $k' \in [0, \frac{1}{4}]$, that $k' < \frac{3\tau - 4v}{8\tau}$ can never be satisfied. So, we have to consider the two cases $k' \in [0, \frac{\tau - v}{3\tau}]$ and $k' \in (\frac{\tau - v}{3\tau}, \frac{1}{4}]$.
Case I: Suppose $k' \in [0, \frac{\tau - v}{3\tau}]$. Then, since $k \geq \frac{3\tau - 4v}{8\tau}$ always holds, $\hat{\pi}$ and $\bar{\pi}$ yield the same profit as the corresponding prices and demand are the same. So, comparing $\tilde{\pi}$ with $\bar{\pi} = \hat{\pi}$, the profit-maximising price is $\frac{v - \tau k'}{2}$ when $k' \in [0, \frac{v}{\tau} - \sqrt{\frac{2v - \tau}{\tau}}]$ as it can be verified that $\tilde{\pi} \geq \bar{\pi} = \hat{\pi}$ in this case, and it equals $v - \frac{\tau}{2}$ when $k' \in [\frac{v}{\tau} - \sqrt{\frac{2v - \tau}{\tau}}, \frac{\tau - v}{3\tau}]$ as it can be verified that $\bar{\pi} = \hat{\pi} \geq \tilde{\pi}$ in this case.

Case II: Suppose $k' \in (\frac{\tau-v}{3\tau}, \frac{1}{4}]$. Then, keeping in mind that since $k \geq \frac{3\tau-4v}{8\tau}$ always holds, $\hat{\pi}$ and $\bar{\pi}$ yield the same profit as the corresponding prices and demand are the same, the profit-maximising price is $v - \frac{\tau}{2}$ because it can be verified that $\bar{\pi} = \hat{\pi} = \frac{1}{2}(v - \frac{\tau}{2}) > \tilde{\pi} = (v - \tau(\frac{1}{2} - k'))(\frac{1}{2} - 2k')$.

The results above can be re-written in the following manner: when $\tau \in [v, \frac{4v}{3}]$, the profit-maximising price equals $\frac{v-\tau k'}{2}$ when $k' \in [0, \frac{v}{\tau} - \sqrt{\frac{2v-\tau}{\tau}}]$ and $v - \frac{\tau}{2}$ when $k' \in [\frac{v}{\tau} - \sqrt{\frac{2v-\tau}{\tau}}, \frac{1}{4}]$.

Finally, suppose that the market differentiation is such that $\tau \in (0, v)$. Then, the inequalities $\frac{3\tau-4v}{8\tau} < \frac{\tau-v}{3\tau} < 0 < \frac{1}{4}$ hold, thereby implying, because of $k' \in [0, \frac{1}{4}]$, that $k' < \frac{3\tau-4v}{8\tau}$ and $k' < \frac{\tau-v}{3\tau}$ can never be satisfied. So, since $k \geq \frac{3\tau-4v}{8\tau}$ always holds, $\hat{\pi}$ and $\bar{\pi}$ yield the same profit as the corresponding prices and demand are the same. Then, when $\tau \in (0, v)$, the profit-maximising price for all $k' \in [0, \frac{1}{4}]$ equals $v - \frac{\tau}{2}$ because it can be verified that $\hat{\pi} = \bar{\pi} = \frac{1}{2}[v - \frac{\tau}{2}] > \tilde{\pi} = (v - \tau(\frac{1}{2} - k'))(\frac{1}{2} - 2k')$. ■

Proof of Proposition 3. In equilibrium, both firms must have positive demand; for if not, a firm with zero demand and hence zero profit has a profitable unilateral deviation whereby it chooses a price lower than the other firm's price thus obtaining positive demand and positive profit. We define a firm's marginal consumer as one who is indifferent between purchasing from the firm on the one hand, and either nor purchasing at all or purchasing from the other firm on the other hand. Then, there are two mutually exclusive and exhaustive cases. Firstly, the firms do not compete for the marginal consumer, i.e. the price set by the firms is such that the marginal consumer of each firm obtains non-negative net utility from it but does not obtain a non-negative net utility from the other firm. Secondly, the firms compete for the marginal consumer, i.e. the price set by the firms is such that the marginal consumer of each firm obtains a non-negative net utility from it but does not receive a higher net utility from the other firm.

Step 1. We begin by taking the case where, in equilibrium, the firms do not compete for the marginal consumer. Let the marginal consumer who purchases from firm 1 be located at a distance of $\hat{x}$ from firm 1. Clearly, there are two such marginal consumers, one on each side of firm 1. Furthermore, this consumer, who does not correspond to firm $2'$s marginal consumer, is located at a distance of $\frac{1}{2} - \hat{x}$ from firm 2. Then, it must be that firm 1's marginal consumer derives a net utility of exactly zero from firm 1 but negative net utility from firm 2. That is, one must have $0 = v - \tau\hat{x} - p_1 > v - \tau(\frac{1}{2} - \hat{x}) - p_2$, implying $\hat{x} = \frac{v-p_1}{\tau}$ and $p_2 - p_1 > 2\tau\hat{x} - \frac{\tau}{2}$. Then, the demand of firm 1 is $d_1(p_1, p_2) = 2\hat{x} = 2\,\frac{v-p_1}{\tau}$. The corresponding profit function is $p_1\, d_1(p_1, p_2) = 2\, p_1 \frac{v-p_1}{\tau}$, and we note that this also corresponds to the profit function for the case where there is only one firm (i.e.

a monopolist) in the market. Solving the profit maximisation problem gives $p_1^* = \frac{v}{2}$. By symmetry, $p_2^* = \frac{v}{2}$. However, these prices must also satisfy condition $p_2 - p_1 > 2\tau\hat{x} - \frac{\tau}{2}$ stated above. This gives $\hat{x} < \frac{1}{4}$, i.e. $\frac{v-p_1^*}{\tau} < \frac{1}{4}$, or $\tau > 2v$. Thus, when $\tau > 2v$, in equilibrium, firms choose price equal to $\frac{v}{2}$. Finally, since this situation is identical to what one would have if, hypothetically, there was only one firm (i.e. a monopoly) in the market, the price $\frac{v}{2}$ also represents the monopoly price and hence gives the highest possible profit; so, this is also the unique equilibrium corresponding to this case $\tau > 2v$.

Step 2. Next, take the case where, in equilibrium, the firms compete for the marginal consumer, *but* the marginal consumer obtains an identical net utility of zero from each firm. We will argue that in the unique pure strategy equilibrium, the two firms set a price equal to $v - \frac{\tau}{4}$ when $\tau \in (\frac{4v}{3}, 2v)$. We note that $\tau > 2v$ implies the our claimed equilibrium price $v - \frac{\tau}{4}$ is higher than the price $\frac{v}{2}$ obtained in the previous step. In order to show that this is an equilibrium, we will argue that firm 1 (without loss of generality) does not have a profitable unilateral deviation. When both firms choose the price $v - \frac{\tau}{4}$, the marginal consumer is located at a distance of $\tilde{x} = \frac{1}{4}$ from firm 1 and obtains a net utility of zero from each firm – so, an infinitesimally small price reduction (increase) by a firm results in the firms competing (not competing) for the marginal consumer.

First, suppose firm 1 unilaterally deviates with a higher price $p_1' > v - \frac{\tau}{4} > \frac{v}{2}$. Then, the firms do not compete for the marginal consumer. It follows from Step 1 that firm $1'$s profit function is $2\,(\frac{v-p_1'}{\tau})p_1'$, and this is decreasing in p_1' whenever $p_1' > \frac{v}{2}$. Hence, the price increase from $v - \frac{\tau}{4}$ to p_1' reduces firm $1'$s profit, and this is not a profit-improving unilateral deviation. Furthermore, this also implies that both firms choosing a price $p \geq v - \frac{\tau}{4}$ (so that they do not compete for the marginal consumer) cannot be an equilibrium as a firm has a profitable unilateral deviation by choosing a slightly lower price such that there is still no competition for the marginal consumer.

Next, suppose firm 1 unilaterally deviates with a lower price $p_1' < v - \frac{\tau}{4}$. Now, the firms compete for the marginal consumer, and so, firm 1's profit function is $p_1'\,d_1(p_1, p_2) = p_1'[2(\frac{1}{4} + \frac{p_2 - p_1'}{2\tau})] = p_1'(\frac{1}{2} + \frac{2v - \frac{\tau}{2} - 2p_1'}{2\tau})$, where the last equality follows from $p_2 = v - \frac{\tau}{4}$. This profit function is increasing in p_1' whenever $p_1' < \frac{v}{2} + \frac{\tau}{8}$. Now, $v - \frac{\tau}{4} < \frac{v}{2} + \frac{\tau}{8}$ holds whenever $\tau > \frac{4v}{3}$. So, when firm 1 reduces its price from $v - \frac{\tau}{4}$ to $p_1' < v - \frac{\tau}{4}$, it experiences a lower profit. Hence, firm 1 cannot profitably unilaterally deviate by reducing the price. Furthermore, this also implies both firms setting a price $p \leq v - \frac{\tau}{4}$, so that the firms compete for the marginal consumer, cannot be an equilibrium as a firm has a profitable unilateral deviation by choosing a slightly higher price such that there is still competition for the marginal consumer.

This implies that there is no profitable unilateral deviation from the situation where both firms set price equal to $v - \frac{\tau}{4}$. Finally, the last line in each of the two preceding paragraphs establishes that this is the only equilibrium when $\tau \in (\frac{4v}{3}, 2v)$.

Step 3. Next, we consider the case where, in equilibrium, the two firms compete for the marginal consumer, and the marginal consumer obtains an identical positive net utility from both firms. Let this marginal consumer be located at a distance of $\bar{x}$ from firm 1. Since this consumer derives the same positive net utility from both firms, one obtains $v - \tau\hat{x} - p_1 = v - \tau(\frac{1}{2} - \hat{x}) - p_2 > 0$, so that $\bar{x} = \frac{1}{4} + \frac{p_2 - p_1}{2\tau}$. Since there are two such marginal consumers for firm 1, firm 1's demand function and its profit function are $d_1(p_1, p_2) = 2\bar{x} = 2\,(\frac{1}{4} + \frac{p_2 - p_1}{2\tau})$ and $p_1 d_1(p_1, p_2) = p_1[2\,(\frac{1}{4} + \frac{p_2 - p_1}{2\tau})]$. Firm 2's demand function and profit function are $d_2(p_1, p_2) = 1 - 2\bar{x} = 2\,(\frac{1}{4} + \frac{p_1 - p_2}{2\tau})$ and $p_2 d_2(p_1, p_2) = p_2[2\,(\frac{1}{4} + \frac{p_1 - p_2}{2\tau})]$. Each firm chooses its price to maximise its own profit. Solving the two first order conditions simultaneously gives unique prices $p_1^* = p_2^* = \frac{\tau}{2}$. This implies that the marginal consumer $\bar{x} = \frac{1}{4}$. However, if the firms compete for the marginal consumer at this set of prices, then this marginal consumer must obtain the same non-negative utility from each firm. That is, $v - p_1^* - \tau\bar{x} = v - p_2^* - \tau\bar{x} > 0$, and this, along with $\bar{x} = \frac{1}{4}$, gives $\tau \in (0, \frac{4v}{3})$. Finally, note that this is the only equilibrium corresponding to this case. ■

Proof of Proposition 4. First, consider the case where there is mild mismatch between the consumers and the firms. We take each of the subcases one by one:
(i) Suppose $\tau \geq 2v$. In the partial market information situation, the equilibrium price is $\frac{v+\tau k}{2}$ when $k \leq \frac{v}{3\tau}$, $v - \tau k$ when $k \in [\frac{v}{3\tau}, \frac{v}{2\tau}]$ and $\frac{v}{2}$ when $k > \frac{v}{2\tau}$ The equilibrium price in the complete market information situation is $\frac{v}{2}$. Since $\frac{v+\tau k}{2} \geq \frac{v}{2}$, the equilibrium price under complete market information is higher if and only if $v - \tau k < \frac{v}{2}$ and $k \in [\frac{v}{3\tau}, \frac{v}{2\tau}]$ hold together. However, this is not possible as $v - \tau k < \frac{v}{2}$ implies $k > \frac{v}{2\tau}$. So, when $\tau \geq 2v$, the equilibrium price in the complete market information situation is never higher.
(ii) Suppose $\tau \in (\frac{4v}{3}, 2v)$. In the partial market information situation, the equilibrium price is $\frac{v+\tau k}{2}$ when $k \leq \frac{v}{3\tau}$, and $v - \tau k$ when $k > \frac{v}{3\tau}$. The equilibrium price in the complete market information situation is $v - \frac{\tau}{4}$. So, the equilibrium price under complete market information is higher when $k \leq \frac{v}{3\tau}$ if and only if $\frac{v+\tau k}{2} < v - \frac{\tau}{4}$, and it is higher when $k > \frac{v}{3\tau}$ if and only if $v - \tau k < v - \frac{\tau}{4}$. Now, $\frac{v+\tau k}{2} < v - \frac{\tau}{4} \Leftrightarrow k < \frac{2v-\tau}{2\tau} > 0$. Furthermore, $\tau \in (\frac{4v}{3}, 2v)$ implies $\frac{2v-\tau}{2\tau} \in (0, \frac{v}{3\tau})$. On the other hand, $v - \tau k < v - \frac{\tau}{4}$ can never hold when $k \leq \frac{1}{4}$. Hence, when $\tau \in (\frac{4v}{3}, 2v)$, the equilibrium price under complete market information is higher if and only if $k < \frac{2v-\tau}{2\tau}$.

(iii) Suppose $\tau \leq \frac{4v}{3}$. In the partial market information situation, the equilibrium price is $\frac{v+\tau k}{2}$ when $k \leq \frac{\tau - v}{\tau}$, and $v - \tau(\frac{1}{2} - k)$ when $k > \frac{\tau - v}{\tau}$. The equilibrium price in the complete market information situation is $\frac{\tau}{2}$. So, the equilibrium price in the complete market information situation is higher when $k \leq \frac{\tau-v}{\tau}$ if and only if $\frac{v+\tau k}{2} < \frac{\tau}{2}$, and it is higher when $k > \frac{\tau-v}{\tau}$ if and only if $v - \tau(\frac{1}{2} - k) < \frac{\tau}{2}$. Now, $\frac{v+\tau k}{2} < \frac{\tau}{2} \Leftrightarrow k < \frac{\tau-v}{\tau}$ and $v - \tau(\frac{1}{2} - k) < \frac{\tau}{2} \Leftrightarrow k < \frac{\tau-v}{\tau}$. Thus, $\frac{v+\tau k}{2} < \frac{\tau}{2}$ and $k \leq \frac{\tau-v}{\tau}$ always hold together but $v - \tau(\frac{1}{2} - k) < \frac{\tau}{2}$ and $k > \frac{\tau-v}{\tau}$ never hold together. Hence, when $\tau \leq \frac{4v}{3}$, the equilibrium price in the complete market information situation is higher if and only if $k < \frac{\tau-v}{\tau}$.

Next, consider the case where there is mild mismatch between the consumers and the firms. Again, we take each of the subcases one by one:

(i) Suppose $\tau \geq 4v$. The equilibrium price in the partial market information situation and the complete market information situation is $\frac{v-\tau k'}{2}$ and $\frac{v}{2}$, respectively. Since $\frac{v-\tau k'}{2} < \frac{v}{2}$ whenever $k > 0$, the equilibrium price is higher in the complete market information case whenever $k > 0$.

(ii) Suppose that $\tau \in (2v, 4v)$. The equilibrium price in the partial market information situation is $\frac{v-\tau k'}{2}$ when $k' \in [0, \frac{1}{2\sqrt{2}} - (\sqrt{2}-1)\frac{v}{\tau}]$, and $\frac{v}{2} - \frac{\tau}{8}$ when $k' \in [\frac{1}{2\sqrt{2}} - (\sqrt{2}-1)\frac{v}{\tau}, \frac{1}{4}]$, while it equals $\frac{v}{2}$ under the complete market information situation. So, the price in the complete market information case is higher when $k' \in [0, \frac{1}{2\sqrt{2}} - (\sqrt{2}-1)\frac{v}{\tau}]$ if and only if $\frac{v-\tau k'}{2} < \frac{v}{2}$, and it is higher when $k' \in [0, \frac{1}{2\sqrt{2}} - (\sqrt{2}-1)\frac{v}{\tau}]$ if and only if $\frac{v}{2} - \frac{\tau}{8} < \frac{v}{2}$. Now, $\frac{v-\tau k'}{2} < \frac{v}{2}$ whenever $k' > 0$, while $\frac{v}{2} - \frac{\tau}{8} < \frac{v}{2}$ always holds. Hence, when $\tau \in (2v, 4v)$, the equilibrium price is higher under complete market information if and only if $k' > 0$.

(iii) Suppose $\tau \in [\frac{4v}{3}, 2v]$. The equilibrium price in the partial market information situation is $\frac{v-\tau k'}{2}$ when $k' \in [0, \frac{1}{2\sqrt{2}} - (\sqrt{2}-1)\frac{v}{\tau}]$, and $\frac{v}{2} - \frac{\tau}{8}$ when $k' \in [\frac{1}{2\sqrt{2}} - (\sqrt{2}-1)\frac{v}{\tau}, \frac{1}{4}]$, while it equals $v - \frac{\tau}{4}$ under the complete market information situation. So, the price in the complete market information case is higher when $k' \in [0, \frac{1}{2\sqrt{2}} - (\sqrt{2}-1)\frac{v}{\tau}]$ if and only if $\frac{v-\tau k'}{2} < v - \frac{\tau}{4}$, and it is higher when $k' \in [0, \frac{1}{2\sqrt{2}} - (\sqrt{2}-1)\frac{v}{\tau}]$ if and only if $\frac{v}{2} - \frac{\tau}{8} < v - \frac{\tau}{4}$. Now, $\frac{v-\tau k'}{2} < v - \frac{\tau}{4} \Leftrightarrow k > \frac{1}{2} - \frac{v}{\tau}$, and since $t < 4v$, $k > \frac{1}{2} - \frac{v}{\tau} \Leftrightarrow \frac{1}{2} - \frac{v}{\tau} < \frac{1}{2\sqrt{2}} - (\sqrt{2}-1)\frac{v}{\tau}$. Hence, when $k' \in (\frac{1}{2} - \frac{v}{\tau}, \frac{1}{2\sqrt{2}} - (\sqrt{2}-1)\frac{v}{\tau}]$, the equilibrium price under complete market information is higher. On the other hand, since $t < 4v$, the inequality $\frac{v}{2} - \frac{\tau}{8} < v - \frac{\tau}{4}$ holds as well. Hence, the equilibrium price under complete market information is higher whenever $k' \in [\frac{1}{2\sqrt{2}} - (\sqrt{2}-1)\frac{v}{\tau}, \frac{1}{4}]$. Combining both of these, when $\tau \in [\frac{4v}{3}, 2v]$, the equilibrium price under complete market information is higher whenever $k' \in (\frac{1}{2} - \frac{v}{\tau}, \frac{1}{4}]$.

(iv) Suppose $\tau \in [v, \frac{4v}{3})$. The equilibrium price in the partial market information situation equals $\frac{v-\tau k'}{2}$ when $k' \in [0, \frac{v}{\tau} - \sqrt{\frac{2v-\tau}{\tau}}]$, and $v - \frac{\tau}{2}$ when $k' \in [\frac{v}{\tau} - \sqrt{\frac{2v-\tau}{\tau}}, \frac{1}{4}]$, while it equals $\frac{\tau}{2}$

in the complete market information case. So, the price in the complete market information case is higher when $k' \in [0, \frac{v}{\tau} - \sqrt{\frac{2v-\tau}{\tau}}]$ if and only if $\frac{v-\tau k'}{2} < \frac{\tau}{2}$, and it is higher when $k' \in [\frac{v}{\tau} - \sqrt{\frac{2v-\tau}{\tau}}, \frac{1}{4}]$ if and only if $v - \frac{\tau}{2} < \frac{\tau}{2}$. Now, $\frac{v-\tau k'}{2} < \frac{\tau}{2} \Leftrightarrow k' > \frac{v-\tau}{\tau}$, and $\tau > v \Leftrightarrow \frac{v-\tau}{\tau} < \frac{v}{\tau} - \sqrt{\frac{2v-\tau}{\tau}}$. Hence, the equilibrium price under the complete market information situation is higher when $k' \in [\frac{v-\tau}{\tau}, \frac{v}{\tau} - \sqrt{\frac{2v-\tau}{\tau}}]$. On the other hand, $\tau > v \Leftrightarrow v - \frac{\tau}{2} < \frac{\tau}{2}$. So, when $k' \in [\frac{v}{\tau} - \sqrt{\frac{2v-\tau}{\tau}}, \frac{1}{4}]$ the equilibrium price under complete market information is always higher. Combining both of the above, when $\tau \in (v, \frac{4v}{3})$, the equilibrium price under complete market information is higher if and only if $k' \in (\frac{v-\tau}{\tau}, \frac{1}{4}]$.
(iv) Suppose $\tau < v$. The equilibrium price in the partial market information situation and the complete market information situation equals $v - \frac{\tau}{2}$ and $\frac{\tau}{2}$, respectively. Now, since $\tau < v \Leftrightarrow v - \frac{\tau}{2} > \frac{\tau}{2}$, the equilibrium price under the complete market information situation is never higher. ■

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