Strategy for securing employment that considers job filling, separation, and productivity shocks

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Abstract
Securing employment is one of the most important issues for a firm's production. This study investigates labor demand dynamics in a situation where the firm faces job filling and turnover using numerical analysis. This study derives the relationship between labor input and strategic labor input target and introduces the relationship into a labor demand model. This relationship can be concave, convex, or linear, depending on the ratio of the job-filling rate to job-separation rate. The firm adjusts the labor input by choosing a strategic labor input target that incurs adjustment costs. The response of labor input to a shock in productivity increases with an increase in the ratio in the model with adjustment costs but does not change in the model without adjustment costs. The response of the strategic labor input target to the shock is increased or decreased by increasing the ratio in the model with or without adjustment costs. From the viewpoint of securing employment, a ratio that most easily secures employment exists when a shock occurs. Therefore, policies that increase this ratio may not necessarily facilitate securing employment if the ratio is high.
Keywords: Adjustment costs, Job filling, Job separation, Labor demand, Securing employment
Classification codes: D21, J23, J30

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1 Introduction

Securing employment is important for firms in their production activity. Labor demand dynamics has been studied using various shapes of the adjustment cost functions, focusing on the size of adjustment costs and speed of adjustment. In other words, emphasis has been placed on ease of adjustment. However, from the viewpoint of securing employment, it is important to study the issue of setting targets and hiring to secure the necessary employment in the face of job filling, job separation, and business cycles. This study investigates how job filling and separation rates affect hiring behavior. Additionally, it examines how the labor demand response to a shock in productivity is affected by adjustment costs.

In the labor demand literature, Bentolila and Saint-Paul (1994) have analyzed hiring and firing that derived by a shock in the revenue function. Cooper et al. (2015) have analyzed the response of employment and hours growth to a profitability shock. Nickell (1986) has discussed employment fluctuations, focusing on the role of adjustment costs when assuming a constant voluntary quitting rate and wage cycle. Goux et al. (2001) have studied the labor input dynamics that are changed by voluntary quits and a productivity shock. Cabo and Martín-Román (2019) have analyzed the effect of adjustment costs and voluntary quitting on collective bargaining between a trade union and a firm. Furthermore, Campbell and Fisher (2000) have examined the effects of a wage change on job flows in a situation where the plant's productivity is given by good or bad state probability.

This study derives the relationship between labor input and strategic labor input targets in a situation where job filling and separation exist. The strategic labor input target is one that a firm can control. The firm adjusts its labor input to maximize profit by choosing a strategic labor input target, whereas in the aforementioned studies, the firm chooses labor input or hiring. The strategic labor input target is regarded as a job vacancy if we assume that all employees leave their jobs at the end of a period. This study defines the target achievement rate as the ratio of labor input to the strategic labor input target. A high ratio indicates a situation in which it is easier for a firm to secure employment. In addition, an adjustment cost is assumed when the firm changes its strategic labor input target. The adjustment cost function is assumed to be quadratic, as discussed by Cabo and Martín-Román (2019), Cooper et al. (2015), and Nickell (1986). This study assumes a productivity shock similar to that in Goux et al. (2001). Using numerical analysis, this study examines how the responses of labor input, strategic labor input target, and target achievement rate to a productivity shock are affected by the ratio of the job-filling rate to the job separation rate.

The results show that an increase in the strategic labor input target increases labor input: the relationship between the two is concave when the ratio of the job filling rate to the job separation rate is between 0 and 1, convex when the ratio is greater than 1, and linear when the ratio equals 1. The responses of the labor input and strategic labor input targets are positive for a positive shock in productivity. Then, the response of labor input is increased by increasing the ratio of the job-filling rate to the job separation rate in the model with adjustment costs, which is similar to the effect of the decrease in adjustment costs in the standard adjustment cost model. Additionally, the response of labor input does not change with changes in the ratio in the model without adjustment costs, because the firm can adjust labor input by immediately adjusting the strategic labor input target. The response of the strategic labor input target changes depending on the ratio of the job-filling rate to the job separation rate in the model with or without adjustment costs. The response of the strategic labor input target changes from decreasing to increasing as the ratio of the job filling rate to the job separation rate increases.

The target achievement rate is affected by changes in the ratio of the job filling rate to the job separation rate. The response of the target achievement rate to the shock is negative when the ratio is less than 1 and positive when the ratio is greater than 1. This indicates that the firm is less likely to secure employment when the ratio is less than 1, whereas it is more likely to secure employment when the ratio is greater than 1. In addition, the positive response of the target achievement rate increases and then decreases with an increase in the ratio of the job-filling rate to the job separation rate. That is, there is a ratio of job-filling rate to job-separation rate that makes the target achievement rate the highest.

The contributions of this study are as follows. First, it derives the relationship between labor input and strategic labor input target in a situation where job filling and separation exist, which is consistent with the equation derived from one of the fundamental models of queuing theory. Second, this study finds how the ratio of job filling rate to job separation rate and adjustment costs affect the response of labor input, strategic labor input target, and target achievement rate.

The remainder of this paper is organized as follows. Section 2 derives the relationship between labor input and strategic labor input targets in situations in which firms face job filling and turnover. Section 3 analyzes firm optimization using a model with a strategic labor input target. Section 4 investigates the dynamics of the model using numerical analysis. Finally, Section 5 concludes the study.

2 Labor input and strategic labor input target

We consider the relationship between labor input and strategic labor input target and assume that the firm adjusts labor inputs by adjusting the strategic labor input target, considering job filling and turnover. The strategic labor input target can be regarded as a job vacancy if we assume that all employees leave their jobs at the end of the period. Fig. 1 illustrates the relationship between labor input and strategic labor input target. The firm makes S_t strategic labor input target and cannot employ more than this target, as indicated by the size of the square in the figure. The circles in the square represent labor input. We assume that workers are employed in a Poisson process at a rate $\lambda > 0$—the average hiring per unit of time, whereas workers leave in a Poisson process at a rate $\mu > 0$—the average leaving per unit of time. In this model, the first labor input is not necessarily the first to leave. The average labor input L_t is expressed as follows:

$$L_t = \sum_{n=0}^{S_t} nP_n, \qquad (1)$$

where $0 \leq n \leq S_t$ is labor input, and P_n is the steady-state probability of n labor input in the system.

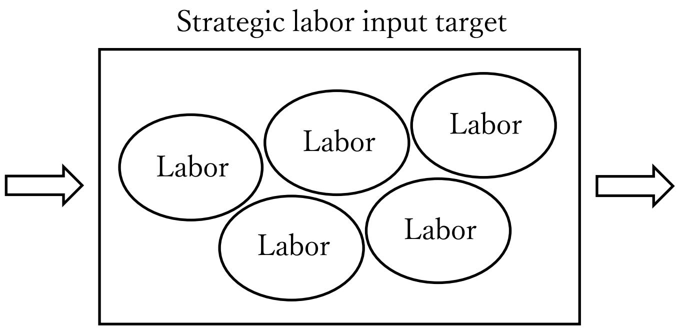

Fig. 1 Labor input and strategic labor input target

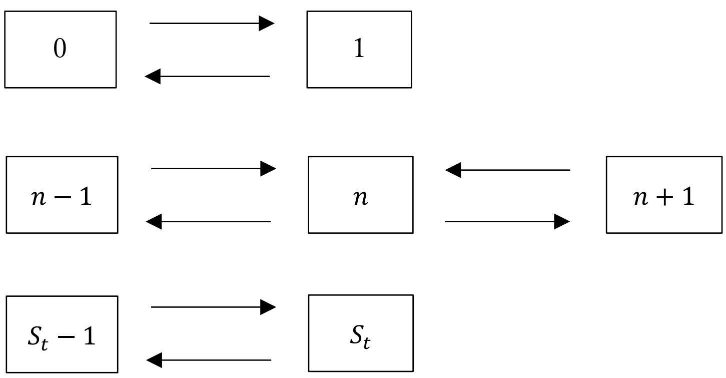

Fig. 2 Transition diagram of the system

Fig. 2 shows the change patterns in the labor input. Labor input is indicated by the numbers in the squares, and the arrows indicate increases or decreases in labor input. To obtain L_t in Equation (1), we derive P_n from balance equations for the cases $n = 0$, $1 \leq n \leq S_t - 1$, and $n = S_t$.

The upper transition in Fig. 2 corresponds to the case in which $n = 0$. The balance equation when $n = 0$ is given by

$$\lambda P_0 = \mu P_1, \tag{2}$$

which shows that the rate at which the labor input increases from 0 equals the rate at which it decreases from 1; that is, the rate at which the labor input changes from 0 equals the rate at which it changes to 0. The middle transition in Fig. 2 corresponds to the case $1 \leq n \leq S_t - 1$. The balance equation when in state $1 \leq n \leq S_t - 1$ is given by

$$(\lambda + \mu) P_n = \lambda P_{n-1} + \mu P_{n+1}, \tag{3}$$

which shows that the sum of the rates at which labor input increases and decreases from n equals the sum of the rates at which it increases from $n - 1$ and decreases from $n + 1$; that is, the rate at which labor input changes from n equals the rate at which it changes to n. The lower transition in Fig. 2 corresponds to the case $n = S_t$. The balance equation when in state $n = S_t$ is given by

$$\mu P_{S_t} = \lambda P_{S_t - 1}, \tag{4}$$

which shows that the rate at which labor input decreases from S_t equals the rate at which it increases from $S_t - 1$; that is, the rate at which labor input changes from S_t equals the rate at which it changes to S_t.

From Equations (2)–(4), the relationship between P_n and P_{n-1} is expressed as follows:

$$\mu P_n = \lambda P_{n-1}, \; n = 1, 2, \cdots, S_t. \tag{5}$$

Solving Equation (5) for P_0, we obtain the following:

$$\begin{aligned} P_1 &= \theta P_0, \\ P_2 &= \theta P_1 = \theta^2 P_0, \\ P_3 &= \theta P_2 = \theta^3 P_0, \\ &\vdots \\ P_n &= \theta P_{n-1} = \theta^n P_0, \end{aligned} \tag{6}$$

where $\theta = \lambda / \mu > 0$ is the relative imbalance between job filling and turnover in this study. Based on the assumption that the firm cannot employ more than the strategic labor input target, there is no possibility that the labor input increases infinitely. Then, it does not need to impose the condition that $\theta < 1$. By substituting Equation (6) into $\sum_{n=0}^{S_t} P_n = 1$, we obtain

$$1 = P_0 \sum_{n=0}^{S_t} \theta^n. \tag{7}$$

From $\sum_{n=0}^{S_t} \theta^n = (1-\theta^{1+S_t})/(1-\theta)$, Equation (7) is transformed as follows:

$$P_0 = \frac{1-\theta}{1-\theta^{1+S_t}}. \tag{8}$$

From Equations (6) and (8), we obtain $P_n = \theta^n(1-\theta)/(1-\theta^{1+S_t})$. As θ approaches 1, both $\theta^n(1-\theta)$ and $1-\theta^{1+S_t}$ approach 0. Using L'Hôpital's rule, we obtain the following equation:

$$\lim_{\theta\to 1} \frac{\theta^n(1-\theta)}{1-\theta^{1+S_t}} = \frac{1}{1+S_t}.$$

Then, the probability of n is obtained as:

$$P_n = \begin{cases} \frac{\theta^n(1-\theta)}{1-\theta^{1+S_t}} & \text{for} \quad \theta \neq 1, \\ \frac{1}{1+S_t} & \text{for} \quad \theta = 1, \end{cases} \tag{9}$$

where $n = 0,1,\cdots,S_t$.

From Equations (1) and (9), the average labor input is given by

$$L_t = \begin{cases} \frac{\theta\left[1-(1+S_t)\theta^{S_t}+S_t\theta^{1+S_t}\right]}{(1-\theta)(1-\theta^{1+S_t})} & \text{for} \quad \theta \neq 1, \\ S_t/2 & \text{for} \quad \theta = 1. \end{cases} \tag{10}$$

Equation (10) expresses the relationship between labor input and strategic labor input target, as shown in Fig. 3. The figure indicates that an increase in the strategic labor input target leads to an increase in labor input. At the same level of strategic target of labor input, the larger the θ, which is the case with a larger λ and/or smaller μ, the larger the labor input. In matching models, the Cobb–Douglas matching function is widely assumed, such as in Leduc and Liu (2016) and Zanetti (2019). The number of matches is a function of the numbers of unemployed workers and vacancies. In these studies, a large vacancy led to large employment via a matching function. The relationship between labor input and strategic labor input target in this study is similar to that between new hiring and vacancies, via a matching function.

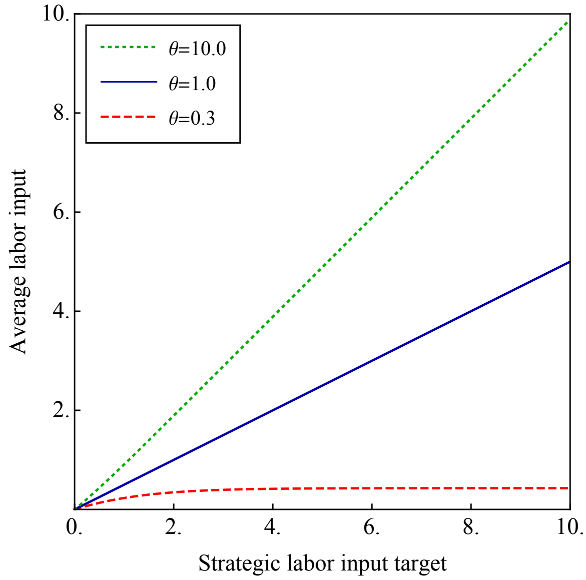

Fig. 3 Relationship between average labor input and the strategic labor input target

Note: The horizontal and vertical axes represent the strategic labor input targets and average labor input, respectively. The lines represent the relationship between strategic labor input target and average labor input with θ as 0.3, 1.0, and 10.0, respectively.

We also confirm the relationship between labor input and strategic labor input target analytically. From Equation (10), the first-order derivative with respect to S_t is obtained as follows:

$$\frac{dL_t}{dS_t} = \begin{cases} \frac{\theta^{1+S_t}\left[\theta^{1+S_t}-(1+S_t)\log\theta-1\right]}{(\theta^{1+S_t}-1)^2} > 0 & \text{for} \quad \theta \neq 1 \\ \frac{1}{2} & \text{for} \quad \theta = 1 \end{cases} \tag{11}$$

The sign of Equation (11) is discussed in Appendix A. From Equation (10), the second-order derivative with respect to S_t can be obtained as follows:

$$\frac{d^2L_t}{dS_t^2} = \begin{cases} \frac{\theta^{1+S_t}\left[2-2\theta^{1+S_t}+(1+S_t)(1+\theta^{1+S_t})\log\theta\right]\log\theta}{(\theta^{1+S_t}-1)^3} < 0 & \text{for} \quad 0 < \theta < 1 \\ \frac{\theta^{1+S_t}\left[2-2\theta^{1+S_t}+(1+S_t)(1+\theta^{1+S_t})\log\theta\right]\log\theta}{(\theta^{1+S_t}-1)^3} > 0 & \text{for} \quad \theta > 1 \\ 0 & \text{for} \quad \theta = 1 \end{cases} \tag{12}$$

The sign of Equation (12) is discussed in Appendix B.

This study defines the target achievement rate as the ratio of labor input to the strategic labor input target L_t/S_t, which indicates the ease of securing employment. A higher rate represents a situation in which a firm is more likely to secure employment. This study analyzes the effects of changes in the ratio of the job filling rate to the job separation rate on firms' decision-making.

The relationship between labor input and the strategic labor input target is consistent with the equation derived from one of the queuing theory's fundamental models. The system is classified as M/M/1/K, which is a stochastic process that describes the dynamics of a single-server queue with finite capacity. Ross (2023) has explained this system. In this study, the queue and finite capacity in the system correspond to the labor input and strategic labor input target, respectively. Some studies have investigated the labor market by applying a queuing system, unlike that used in this study. Deutsch and David (2020) assume that workers and jobs arrive at a system independently, and the job is assigned to a worker or discarded within a limited time. It analyzes the optimal choices of workers in the system. Feigin and Landsberger (1981) have constructed a model using an unemployment queue and discuss the stationary distribution of unemployment.

3 Labor demand model with strategic labor input target

We introduce the relationship between labor input and strategic labor input target into a labor demand model and analyze firm optimization. Assuming that capital stock is constant, production Y_t depends on the labor input:

$$Y_t = F(L_t; A_t),$$

where A_t is productivity, and the production function satisfies $F_L > 0$ and $F_{LL} < 0$. We assumed that productivity follows a first-order autoregressive process, that is:

$$logA_t = \rho logA_{t-1} + \varepsilon_t, \tag{13}$$

where $-1 < \rho < 1$ is the autoregressive parameter, and ε_t is the shock to productivity. As in Equation (10), we assume that L_t is a function of the strategic labor input target S_t.

The firm's profit is expressed as follows:

$$\sum_{t=0}^{\infty} \beta^t [F(L_t; A_t) - wL_t - C(S_t - S_{t-1})],$$

where $C(S_t - S_{t-1})$ is the adjustment cost of the strategic labor input target. In the literature, the adjustment costs of labor include advertising job positions, interviewing, training, disruption of production costs, and severance pay. In this study, we assume that firms incur costs by changing their strategic labor input targets. The costs include the disruption of production costs when considering the level of the strategic labor input target when they change the strategy. The adjustment cost functions are formulated in various forms. Campbell and Orszag (1998) and Galí and van Rens (2010) assume that it depends on hiring and firing. Cabo and Martín-Román (2019) assume that this depends on hiring, firing, and wage rates. Bloom (2009) and Fairise and Fève (2006) assume that this depends on hiring, turnover, and employment levels. Belo et al. (2014) assume that this depends on hiring, firing, employment level, and output. Finally, Ju et al. (2014)

assume that households pay an adjustment cost that depends on the difference between the labor supply and the steady-state level of labor. Appendix C discusses the standard adjustment cost model.

The firm chooses S_t to maximize profit, subject to Equation (10). The first-order conditions for the strategic labor input target are given by

$$F_L(L_t; A_t)L'(S_t) = wL'(S_t) + C'(S_t - S_{t-1}) - \beta C'(S_{t+1} - S_t). \tag{14}$$

The left-hand side of Equation (14) indicates the marginal product of labor and the right-hand side expresses the marginal cost of labor. The first term on the right side of Equation (14) is related to wages, and the second and third terms are related to adjustment costs. The second term expresses the cost of changing the strategic labor input target in the current period, and the third term indicates future cost savings.

4 Numerical experiments

We analyze the dynamics of the model with a strategic labor input target and investigate the effects of changes in the relative imbalance between job filling and turnover on the response to labor input, strategic labor input target, and target achievement rate when a productivity shock occurs in the model.

4.1 Dynamics of the model with strategic labor input target

We analyze the responses of labor input, strategic labor input target, and target achievement rate to productivity shocks using numerical experiments. To examine the responses, we specify the production function and the adjustment cost function. As in Elsby and Gottfries (2022), the production function is

$$Y_t = A_t L_t^{\alpha}, \tag{15}$$

where $0 < \alpha < 1$. The adjustment cost function is assumed as follows:

$$C(S_t - S_{t-1}) = \tau \frac{(S_t - S_{t-1})^2}{2}, \tag{16}$$

where $\tau \geq 0$ is the adjustment cost parameter. The firm faces a standard convex cost when it increases or decreases its strategic labor input target. The quadratic specification for the standard convex cost has the property that the larger the change in the strategic labor input target, the more the adjustment cost increases. Assuming such an adjustment cost function, the larger the adjustment cost, the smoother the change in the strategic labor input target. From Equations (14), (15), and (16), the first-order conditions for the strategic labor input target are given by

$$\alpha A_t [L(S_t)]^{\alpha-1} L'(S_t) = wL'(S_t) + \tau(S_t - S_{t-1}) - \beta\tau(S_{t+1} - S_t). \tag{17}$$

Assuming that a steady-state solution exists, it satisfies $S_t = S_{t-1} = S$ and $L_t =$

$L_{t-1} = L$. From Equation (13), we obtain $A = 1$ when $A_t = A_{t-1} = A$ and $\varepsilon_t = 0$. Then, we obtain S from Equation (17), L from Equation (10), and L/S from these values under the given parameters. If we assume that $\theta = 1$, then we obtain $S = 2(w/\alpha)^{-1/(1-\alpha)}$ and $L = (w/\alpha)^{-1/(1-\alpha)}$, which indicate that the raise in wage rate decreases S and L.

Table 1. Parameters in the model

Parameter		Value
α	Parameter in production	0.64
β	Discount factor	0.99
ρ	Autoregressive parameter	0.95
τ	Adjustment cost	0.1
w	Wage rate	1.0
λ	Job-filling rate	0.9594
μ	Job-separation rate	0.0985

The parameters in this section are listed in Table 1. We assume that the parameter in production function α is 0.64, which is the same value as that used in Elsby and Gottfries (2022). The discount factor $\beta = 0.99$ and the autoregressive parameter $\rho = 0.95$ are widely used in macroeconomic literature. The adjustment cost parameter τ is set to 0.1. Cabo and Martín-Román (2019) set the adjustment cost parameter, the coefficient of the square of employment adjustments, to the same value. In this study, the wage rate w is 1.0. The parameters λ and μ are set to 0.9594 and 0.0985 in this analysis. To set λ, we use the estimation results that the daily job filling rate is 0.052, which is estimated by Davis et al. (2013) using Job Openings and Labor Turnover Survey (JOLTS) data. Assuming that three month consists of 60 business days, the quarterly job filling rate is set to $1 - (1 - 0.052)^{60} \approx 0.9594$. Leduc and Liu (2020) set the monthly job separation rate to 0.034 by using JOLTS data. The quarterly job separation rate is set to $1 - (1 - 0.034)^3 \approx 0.0985$. Then, we have $\theta = \lambda/\mu \approx 9.7401$. Also, we assume that $\varepsilon_t = 0$ and $A_t = A_{t-1} = A$ in the steady state.

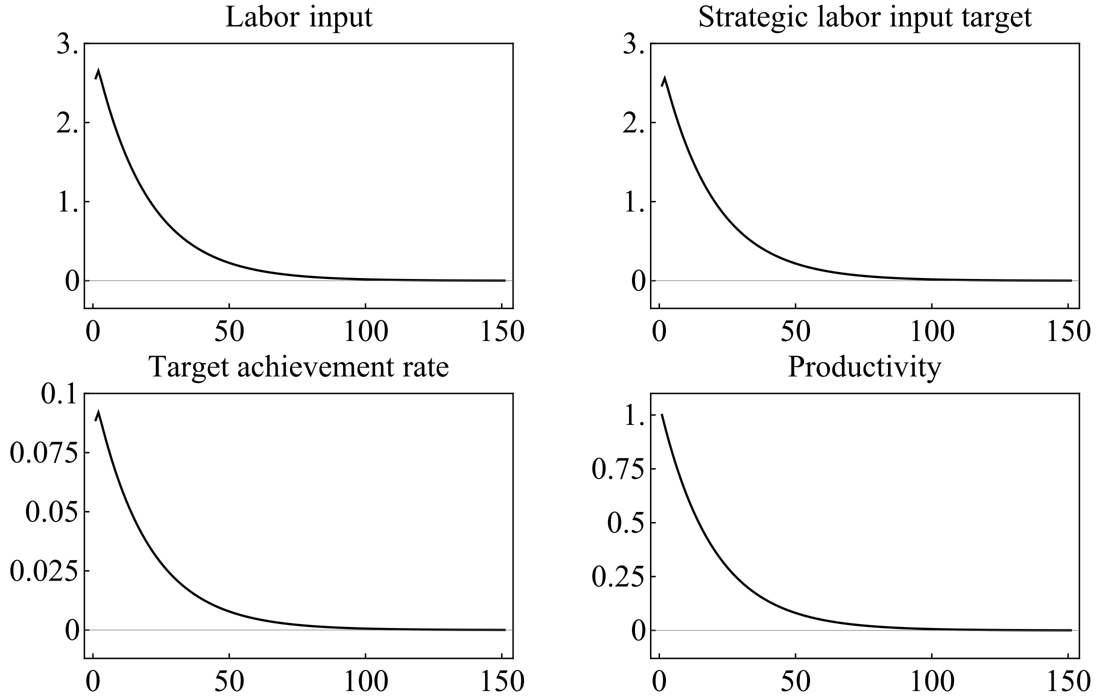

Fig. 4 The response of the model with strategic labor input target to the productivity shock

Note: The horizontal and vertical axes represent time and percentage, respectively. The lines represent the percentage deviation of the variables from their steady-state values when the shock occurs.

Fig. 4 presents responses to productivity shocks. According to Equation (13), productivity increases in period 0 and gradually returns to the steady state. The marginal products of labor increase because of a positive productivity shock. To increase labor input, the firm increases its strategic labor input target by considering filling and leaving jobs. Therefore, the responses of labor input $(L_t - L)/L$ and strategic labor input target $(S_t - S)/S$ are positive, where L and S are the steady-state values of labor input and strategic labor input target, respectively. The response of the target achievement rate, $(L_t/S_t - L/S)/(L/S)$, increases because the response of the labor input is larger than that of the strategic labor input target. This indicates that it is easier for firms to secure employment. The adjustment cost smoothens the responses; thus, the response peaks do not occur in the shock period.

4.2 Response of the model with adjustment cost

We investigate the effects of changes in θ on the labor input, strategic labor input target, and target achievement rate when the productivity shock occurs using the model with an adjustment cost. The parameters are the same as those in Section 4.1, except for θ. The numerical experiments show the cases where θ is 0.5, 2.3, and 10.0. We assume that a positive temporary productivity shock occurs and productivity increases by 1% in period 0.

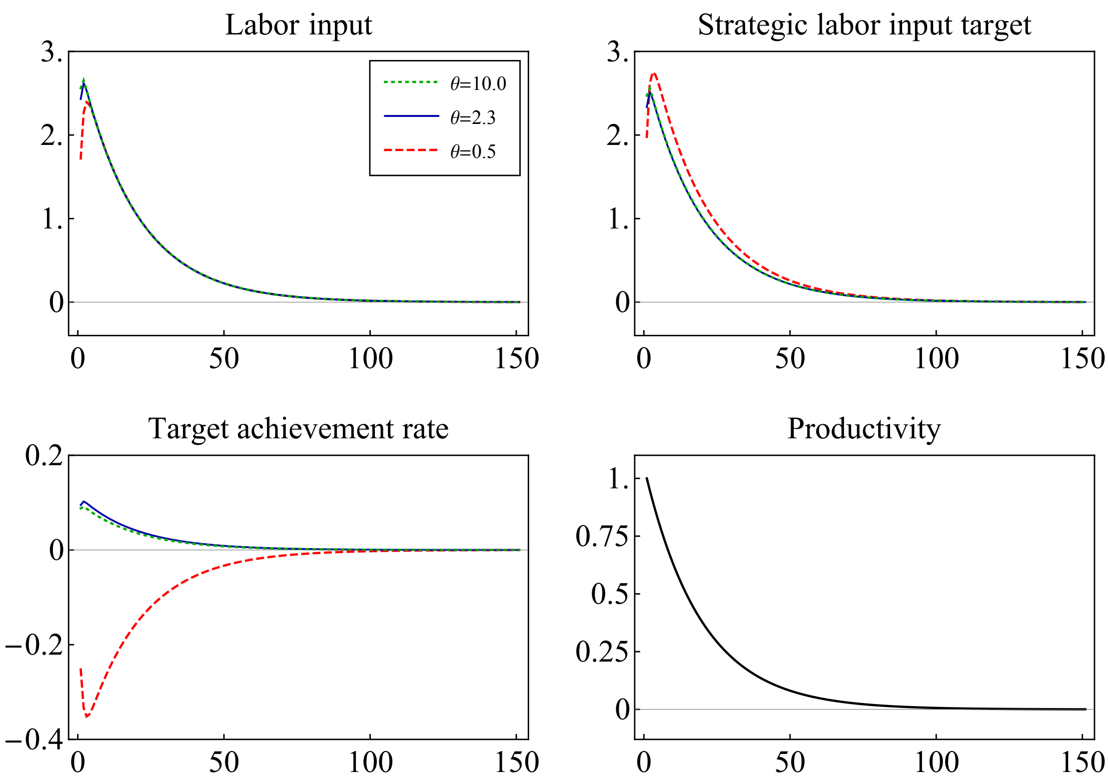

Fig. 5 The response of the model with adjustment cost to the productivity shock
Note: The horizontal and vertical axes represent time and percentage, respectively. The lines represent the percentage deviations of the variables from their steady-state values when the shock occurs.

Regardless of the level of θ, the labor input and strategic labor input target increase in response to the positive temporary productivity shock as shown in Fig. 5. The response of labor input is amplified by the raise in θ when the shock occurs. The response of strategic labor input target is ambiguous by the raise in θ. The positive and negative responses to the target achievement rate are shown in the figure.

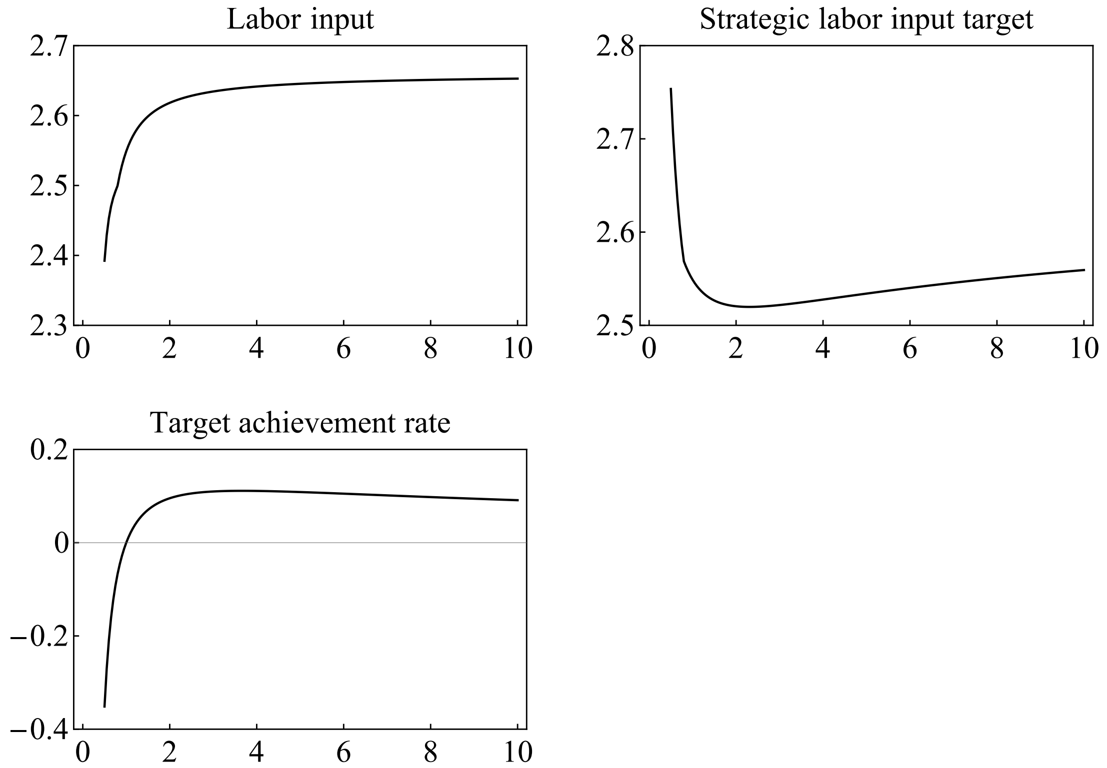

Fig. 6 The level of θ and response of the model with adjustment cost
Note: The horizontal and vertical axes represent the relative imbalance between job filling and turnovers θ and percentage, respectively. The lines represent the variables' peak percentage deviations from their steady-state values when the shock occurs.

Fig. 6 shows that the relationships between the level of θ and peak of the responses in the model with adjustment costs when the productivity shock occurs. The response peaks of labor input, strategic labor input target, and target achievement rate are plotted for simulations with θ ranging from 0.5 to 10.0 in steps of 0.05. The response peaks in these variables occur in period 3 when θ is 0.5 to 0.75 and in period 2 when θ is 0.8 to 10.0. The response of labor input is increased by a raise in θ. The response of strategic labor input target changes from decreasing to increasing as θ increases. The response of strategic labor input target is reduced by an increase in θ when $\theta < 2.3$, but is amplified when $\theta > 2.3$. The response of strategic labor input target takes a minimum of 2.51978% when θ is 2.3. The response of target achievement rate is decreasing, increasing, and decreasing with an increase in θ. The figure shows that the response of target achievement rate is negative when $\theta < 1.0$, positive when $\theta > 1.0$, and unchanged when $\theta = 1.0$. These responses indicate that the firm is less likely to secure employment when the ratio is less than 1, whereas it is more likely to secure employment when the ratio is greater than 1. The deviation from the target achievement rate is reduced by increasing in θ when $\theta < 1.0$; however, it is larger when $1.0 < \theta < 3.65$ and smaller when $3.65 > \theta$. The response of target achievement rate takes a maximum of 0.111232% when θ is 3.65.

From Equation (10), the increase in strategic labor input target makes it easy to increase labor input when θ is large, because a large θ reflects more jobs getting filled and/or fewer turnovers taking place. Therefore, the response of strategic labor input target is reduced when θ increases from a lower value, whereas the response of labor input is amplified. Nevertheless, the strategic labor input target response is amplified when θ increases from a higher value. Since the increase in strategic labor input target brings about a larger increase in labor input when $\theta > 1.0$, the firm increases the strategic labor input target larger to the shock. Consequently, the responses of the strategic labor input target and labor input are amplified. This model can express the change in labor input response by the change in adjustment costs and change in θ. The target achievement rate response is more influenced by strategic labor input target increases when $\theta < 1.0$; however, it is more influenced by labor input increases when $\theta > 1.0$. Therefore, the response of target achievement rate is negative when $\theta < 1.0$ and positive when $\theta > 1.0$. In other words, the firm is less likely to secure employment when $\theta < 1.0$ and more likely to secure employment when $\theta > 1.0$.

4.3 Response of the model without adjustment cost

We investigate the effects of changes in θ on labor input, strategic labor input target, and the target achievement rate when a productivity shock occurs using the model without adjustment costs. If we assume that $\tau = 0$, Equation (17) is transformed as follows:

$$\alpha A_t[L(S_t)]^{\alpha-1} = w. \tag{18}$$

The parameters are the same as those in Section 4.1, except for θ. The numerical experiments show the cases in which θ is 0.5, 3.6, and 10.0. We assume that the productivity increases by 1% in period 0 and gradually returns to the steady state. The responses of labor input, strategic labor input target, and the target achievement rate when $\tau = 0$ are shown in Fig. 7. Labor input and strategic labor input target increase significantly during the shock period, and then gradually decrease. The response of labor input is not affected by the raise in θ when the shock occurs, which is different from that of the model with adjustment costs. The response of strategic labor input target is ambiguous by the raise in θ. Positive and negative responses regarding the target achievement rate are observed. In the model without adjustment costs, these variables largely react during the shock period. The sign of the response for each variable in the model without adjustment costs is the same as that in the model with adjustment costs. In the model with adjustment costs, the responses of these variables are smaller, because the larger the change in the strategic labor input target, the larger the adjustment costs.

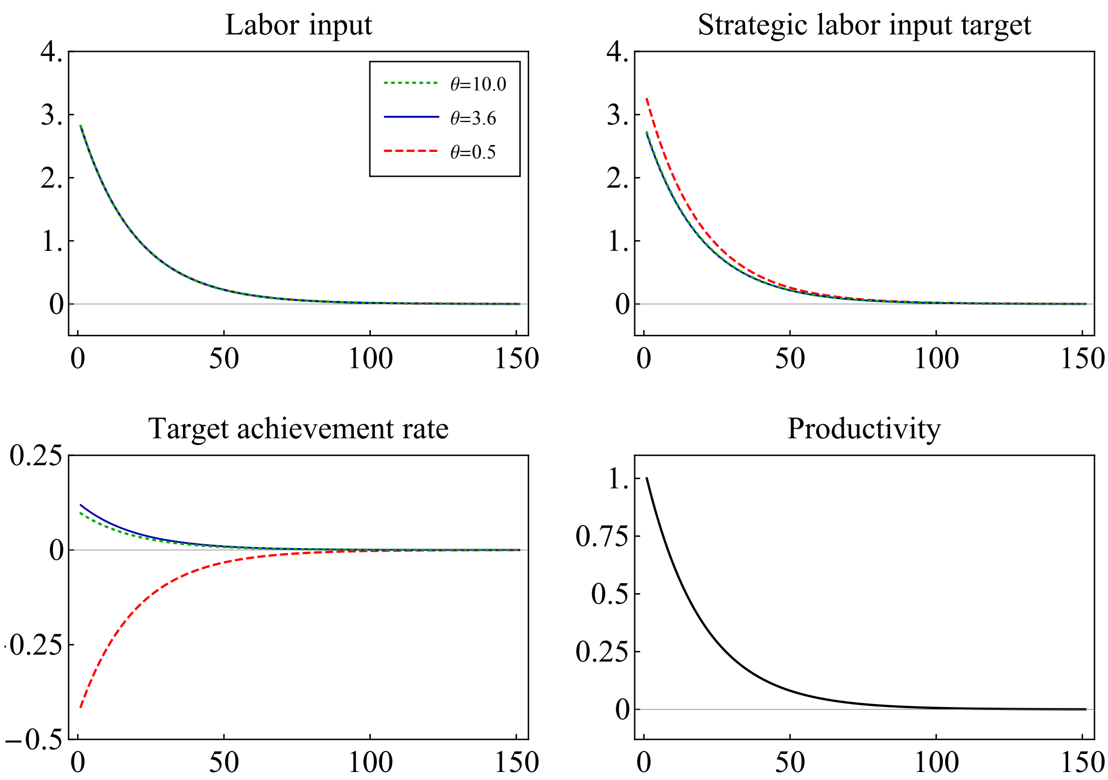

Fig. 7 The response of the model without adjustment cost to the productivity shock
Note: The horizontal and vertical axes represent time and percentage, respectively. The lines represent the percentage deviations of the variables from their steady-state values when the shock occurs.

In the model without adjustment costs, the response of labor input is the same in all cases of θ. The strategic labor input target is changed by immediately to employ the necessary labor inputs, whatever the level of θ, because there are no adjustment costs. From Equations (10) with $\theta = 1$ and (18), we obtain $\alpha A_t (S_t/2)^{\alpha-1} = w$. We log-linearize this equation and Equation (18) around the steady-state and obtain $(S_t - S)/S \approx [1/(1-\alpha)]\,(A_t - A)/A$ and $(L_t - L)/L \approx [1/(1-\alpha)]\,(A_t - A)/A$. Thus, the response of strategic labor input target is consistent with that of labor input when it is assumed that $\theta = 1$ and there are no adjustment costs.

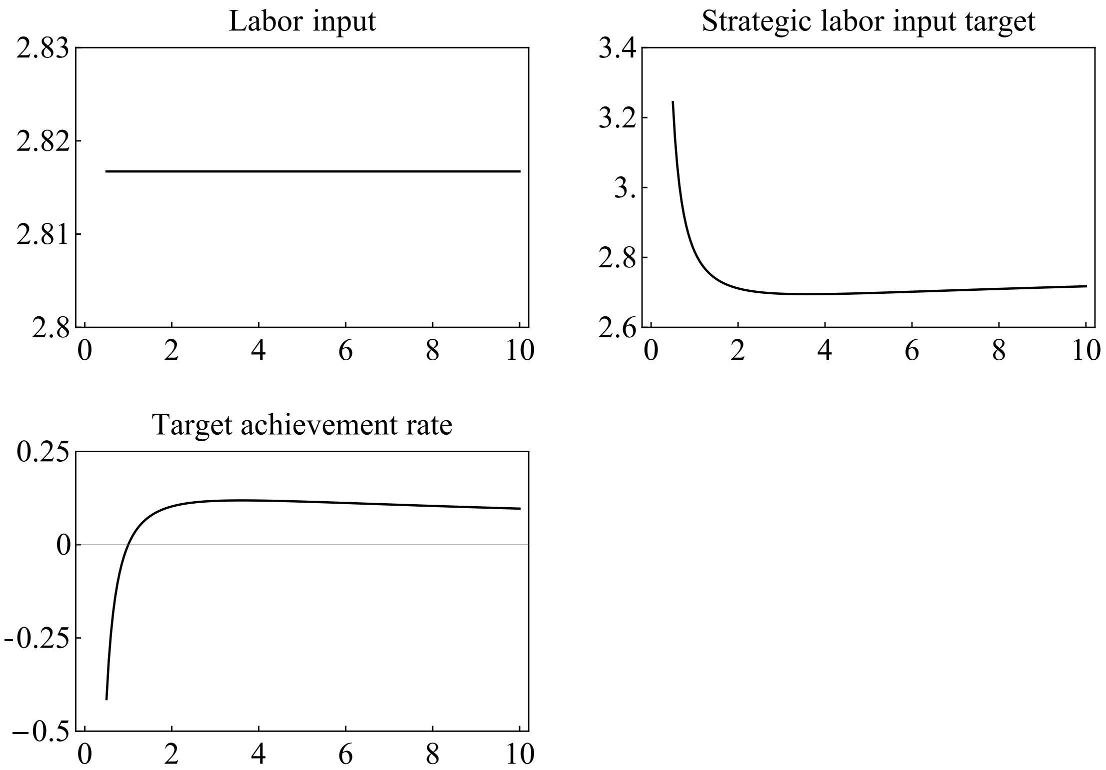

Fig. 8 The level of θ and response of the model without adjustment cost
Note: The horizontal and vertical axes represent the relative imbalance between job fillings and turnovers θ and percentage, respectively. The lines represent the variables' peak percentage deviations from their steady-state values when a shock occurs.

Fig. 8 shows the relationships between the level of θ and peak of the responses in the model without adjustment costs when the productivity shock occurs. The response peaks of labor input, strategic labor input target, and target achievement rate are plotted for simulations with θ ranging from 0.5 to 10.0 in steps of 0.05. Peaks in the responses to these variables occur during the shock period. The responses of labor input are not affected by the increase in θ. The response of strategic labor input target changes from decreasing to increasing as θ increases. The strategic labor input target response is reduced by increasing θ when $\theta < 3.6$, but amplified when $\theta > 3.6$. The strategic labor input target response takes a minimum of 2.69483% when θ is 3.6. The response of target achievement rate is decreasing, increasing, and then decreasing with an increase in θ. The figure shows that the target achievement rate response is negative for $\theta < 1$, positive for $\theta > 1$, and unchanged for $\theta = 1$. The deviation from the target achievement rate is reduced by increasing in θ when $\theta < 1.0$; it is larger when $1.0 < \theta < 3.6$ and smaller when $3.6 > \theta$. The peak of response takes the maximum of 0.118686% when θ

is 3.6.

The responses of labor input, strategic labor input target, and target achievement rate in the model without adjustment costs are larger than those in the model with adjustment costs. In the model without adjustment costs, the firm can instantly adjust the strategic labor input target so that labor input is at the optimal level regardless of the level of θ. Consequently, the response of labor input is unchanged by the level of θ. Similar to the model with adjustment costs, the response of strategic labor input target is reduced when θ increases from a lower value but amplified when θ increases from a higher value. The response of target achievement rate is more influenced by increases in the strategic labor input target when $\theta < 1.0$, whereas it is more influenced by increases in labor input when $\theta > 1.0$. Therefore, the response of target achievement rate is negative when $\theta < 1.0$ and positive when $\theta > 1.0$.

5 Conclusion

This study focuses on securing employment and examines how the ratio of the job filling rate to the job separation rate and adjustment costs affect labor demand dynamics. This study derives the relationship between labor input and strategic labor input target, which depends on the ratio of the job filling rate to the job separation rate. It is concave when the ratio is between 0 and 1, convex when it is greater than 1, and linear when it is 1. The response of labor input to a productivity shock increases with an increase in the ratio in the model with adjustment costs, whereas it does not change in the model without adjustment costs. The response of the strategic labor input target increases or decreases depending on the ratio, with or without adjustment costs. The response of target achievement rate to the positive productivity shock is negative when the ratio is less than 1, whereas it is positive when the ratio is greater than 1. In addition, the positive response of the target achievement rate increases and then decreases with an increase in the ratio of the job-filling rate to the job separation rate.

This study contributes to the literature on labor demand from the perspective of securing employment. The simulation analysis shows that the relationship between the response of the target achievement rate and the ratio of the job-filling rate to job separation rate is concave. A policy that increases this ratio is useful for securing employment if it is low. For example, changes in selection methods, regulations, and unemployment insurance can alter the ratio. However, there exists a ratio that maximizes the response to the target achievement rate. Therefore, the analysis indicates that attention needs to be paid to the ratio when discussing economic policies regarding securing employment.

This study assumes several ratios of the job filling rate to the job separation rate. Job filling and separation rates can be influenced by labor market institutions and economic policies. In addition to the productivity shock in this study, potentially deeper insights can be gained by assuming shocks to job-filling and separation rates. Moreover, the model can be extended to examine the effects of heterogeneous firm behaviors on labor market dynamics. Future studies should address these limitations.

Appendix A: The first-order derivative with respect to S_t in Equation (10)

From Equation (10), the first-order derivative with respect to S_t is obtained as follows:

$$\frac{dL_t}{dS_t} = \begin{cases} \frac{\theta^{1+S_t}[\theta^{1+S_t} - (1+S_t)\log\theta - 1]}{(\theta^{1+S_t}-1)^2} & \text{for} \quad \theta \neq 1 \\ \frac{1}{2} & \text{for} \quad \theta = 1 \end{cases} \tag{A1}$$

In Equation (A1) when $\theta \neq 1$, the denominator and θ^{1+S_t} in the numerator are positive, because it is assumed that $\theta > 0$ and $S_t \geq 0$. We examine the sign of $f(\theta) = g(\theta) - h(\theta) - 1$ in Equation (A1), where $g(\theta) = \theta^{1+S_t}$ and $h(\theta) = (1 + S_t)\log\theta$. Fig. 9 shows $g(\theta)$ and $h(\theta)$ with $S_t = 10.0$, which are increasing functions of θ. To check the sign of $f(\theta)$, we should examine $g'(\theta) - h'(\theta)$ when $0 < \theta < 1$ and $\theta > 1$, respectively.

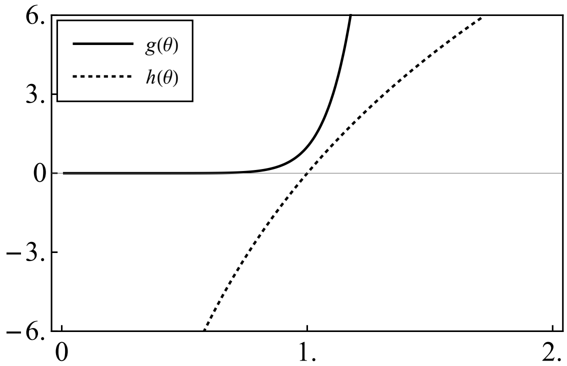

Fig. 9 Functions $g(\theta)$ and $h(\theta)$

Note: The horizontal axis represents the relative imbalance between the filling a job and turnovers θ, and the vertical axis represents $g(\theta)$ and $h(\theta)$. The solid and dotted lines represent $g(\theta)$ and $h(\theta)$ when $S_t = 10.0$, respectively.

We obtain $g'(\theta)$, $h'(\theta)$, and $g'(\theta) - h'(\theta)$ as follows:

$$g'(\theta) = (1 + S_t)\theta^{S_t},$$

$$h'(\theta) = \frac{1+S_t}{\theta},$$

$$g'(\theta) - h'(\theta) = \frac{(1+S_t)(\theta^{1+S_t}-1)}{\theta}.$$

We obtain $g(\theta) - h(\theta) = 1$ when $\theta = 1$. In the case of $0 < \theta < 1$, from $S_t \geq 0$, we have $\theta^{1+S_t} - 1 < 0$, and then $g'(\theta) - h'(\theta) < 0$. Therefore, we have $g(\theta) - h(\theta) > 1$, where $g(\theta) > 0$ and $h(\theta) < 0$. In the case of $\theta > 1$, from $S_t \geq 0$, we have $\theta^{1+S_t} - 1 > 0$, and then $g'(\theta) - h'(\theta) > 0$. Therefore, we have $g(\theta) - h(\theta) > 1$, where $g(\theta) > 0$ and $h(\theta) > 0$. We obtain $g(\theta) - h(\theta) > 1$ in both cases. It indicates that $f(\theta) > 0$, and then $dL_t/dS_t > 0$ for $\theta \neq 1$.

Appendix B: The second-order derivative with respect to S_t in Equation (10)

From Equation (10), the second-order derivative with respect to S_t is obtained as follows:

$$\frac{d^2L_t}{dS_t^2} = \begin{cases} \frac{\theta^{1+S_t}\left[2-2\theta^{1+S_t}+(1+S_t)(1+\theta^{1+S_t})\log\theta\right]\log\theta}{(\theta^{1+S_t}-1)^3} & \text{for} \quad 0 < \theta < 1 \text{ and } \theta > 1 \\ 0 & \text{for} \quad \theta = 1 \end{cases} \tag{B1}$$

We examine the sign of the denominator in Equation (B1) when $0 < \theta < 1$ and $\theta > 1$. In the case of $0 < \theta < 1$, we obtain $\theta^{1+S_t} - 1 < 0$, and then $(\theta^{1+S_t} - 1)^3 < 0$. In the case of $\theta > 1$, we obtain $\theta^{1+S_t} - 1 > 0$, and then $(\theta^{1+S_t} - 1)^3 > 0$.

The numerator in Equation (B1), if $0 < \theta < 1$ and $\theta > 1$, is transformed as follows:

$$-\theta^{1+S_t}\left[2\theta^{1+S_t} - (1+S_t)(1+\theta^{1+S_t})\log\theta - 2\right]\log\theta$$

In the case of $0 < \theta < 1$, we have $\log\theta < 0$. In the case of $\theta > 1$, we have $\log\theta > 0$. We examine the sign of $j(\theta) = k(\theta) - l(\theta) - 2$ in the numerator, where $k(\theta) = 2\theta^{1+S_t}$ and $l(\theta) = (1+S_t)(1+\theta^{1+S_t})\log\theta$. To check the sign of $j(\theta)$, we should examine $k'(\theta) - l'(\theta)$ when $0 < \theta < 1$ and $\theta > 0$, respectively. We obtain $k'(\theta)$, $l'(\theta)$, and $k'(\theta) - l'(\theta)$ as follows.

$$k'(\theta) = 2(1+S_t)\theta^{S_t},$$

$$l'(\theta) = \frac{(1+S_t)\left[1+\theta^{1+S_t}+(1+S_t)\theta^{1+S_t}\log\theta\right]}{\theta},$$

$$k'(\theta) - l'(\theta) = \frac{(1+S_t)\left[\theta^{1+S_t}-(1+S_t)\theta^{1+S_t}\log\theta-1\right]}{\theta}. \tag{B2}$$

We obtain $k(\theta) - l(\theta) = 2$ when $\theta = 1$. In the case of $0 < \theta < 1$, if we obtain $k'(\theta) - l'(\theta) > 0$, then we have $j(\theta) < 0$. If we obtain $k'(\theta) - l'(\theta) < 0$ when $0 < \theta < 1$, then we have $j(\theta) > 0$. In the case of $\theta > 1$, if we obtain $k'(\theta) - l'(\theta) > 0$, then we have $j(\theta) > 0$. If we obtain $k'(\theta) - l'(\theta) < 0$ when $\theta > 1$, then we have $j(\theta) < 0$.

In Equation (B2), we examine the sign of $g(\theta) - m(\theta) - 1$ when $0 < \theta < 1$ and $\theta > 1$, where $g(\theta) = \theta^{1+S_t}$ and $m(\theta) = (1+S_t)\theta^{1+S_t}\log\theta$. We obtain $g(\theta) - m(\theta) = 1$ when $\theta = 1$. In the case of $0 < \theta < 1$, if we obtain $g'(\theta) - m'(\theta) > 0$, then we have $g(\theta) - m(\theta) - 1 < 0$, and $k'(\theta) - l'(\theta) < 0$. If we obtain $g'(\theta) - m'(\theta) < 0$ when

$0 < \theta < 1$, then we have $g(\theta) - m(\theta) - 1 > 0$ and $k'(\theta) - l'(\theta) > 0$. In the case of $\theta > 1$, if we obtain $g'(\theta) - m'(\theta) > 0$, then we have $g(\theta) - m(\theta) - 1 > 0$ and $k'(\theta) - l'(\theta) > 0$. If we obtain $g'(\theta) - m'(\theta) < 0$ when $\theta > 1$, then we have $g(\theta) - m(\theta) - 1 < 0$ and $k'(\theta) - l'(\theta) < 0$. We examine the sign of $g'(\theta) - m'(\theta)$ when $0 < \theta < 1$ and $\theta > 1$, respectively, and obtain the following:

$$g'(\theta) - m'(\theta) = -(1 + S_t)^2 \theta^{S_t} \log \theta$$

In the case of $0 < \theta < 1$, we obtain $g'(\theta) - m'(\theta) > 0$. In the case of $\theta > 1$, we obtain $g'(\theta) - m'(\theta) < 0$.

Now, we can check the sign of Equation (B1) for $0 < \theta < 1$ and $\theta > 1$, respectively. In the case of $0 < \theta < 1$, we obtain $g'(\theta) - m'(\theta) > 0$, and then $k'(\theta) - l'(\theta) < 0$ and $j(\theta) > 0$. Then, the numerator in Equation (B1) for $0 < \theta < 1$ is positive. Moreover, the denominator in Equation (B1) for $0 < \theta < 1$ is negative. Therefore, we have $d^2 L_t / dS_t^2 < 0$ for $0 < \theta < 1$. In the case of $\theta > 1$, we obtain $g'(\theta) - m'(\theta) < 0$, and then $k'(\theta) - l'(\theta) < 0$ and $j(\theta) < 0$. Then, the numerator in Equation (B1) for $\theta > 1$ is positive. In addition, the denominator in (B1) for $\theta > 1$ is positive. Therefore, we have $d^2 L_t / dS_t^2 > 0$ for $\theta > 1$.

Appendix C: Standard adjustment cost model

We compare the proposed model with the standard adjustment cost model. The firm chooses new hiring h_t to maximize its profit.

$$\sum_{t=0}^{\infty} \beta^t [F(L_t; A_t) - wL_t - C(h_t)],$$

where $h_t = L_t - L_{t-1}$. The productivity follows a first-order autoregressive process:

$$logA_t = \rho logA_{t-1} + \varepsilon_t, \tag{C1}$$

The first-order conditions are given by

$$F_L(L_t; A_t) = w + C'(L_t - L_{t-1}) - \beta C'(L_{t+1} - L_t). \tag{C2}$$

The equation indicates that the marginal product of labor equals the marginal cost of labor. Similar to the model with strategic labor input target, the marginal cost of labor is constructed using the wage rate, current adjustment cost, and future adjustment cost.

To study the responses to a shock in productivity numerically, we specify the production function and the adjustment cost function. The production function is assumed as follows:

$$Y_t = A_t L_t^{\alpha}. \tag{C3}$$

The adjustment cost function is assumed as follows:

$$C(L_t - L_{t-1}) = \tau \frac{(L_t - L_{t-1})^2}{2} \tag{C4}$$

A firm faces convex costs when it increases or decreases employment. From Equations

(C2)–(C4), we obtain the first-order conditions of the standard adjustment cost model with a quadratic adjustment cost as follows:

$$\alpha A_t L_t^{\alpha-1} = w + \tau(L_t - L_{t-1}) - \beta\tau(L_{t+1} - L_t) \quad \text{(C5)}$$

Assuming that a steady-state solution exists, it satisfies $L_t = L_{t-1} = L$. From Equation (C1), we obtain $A = 1$ when $A_t = A_{t-1} = A$ and $\varepsilon_t = 0$. Then, we obtain $L = (w/\alpha)^{-1/(1-\alpha)}$ from Equation (C5). It is the same as the model with strategic labor input target when $\theta = 1$ as discussed in Section 4.1.

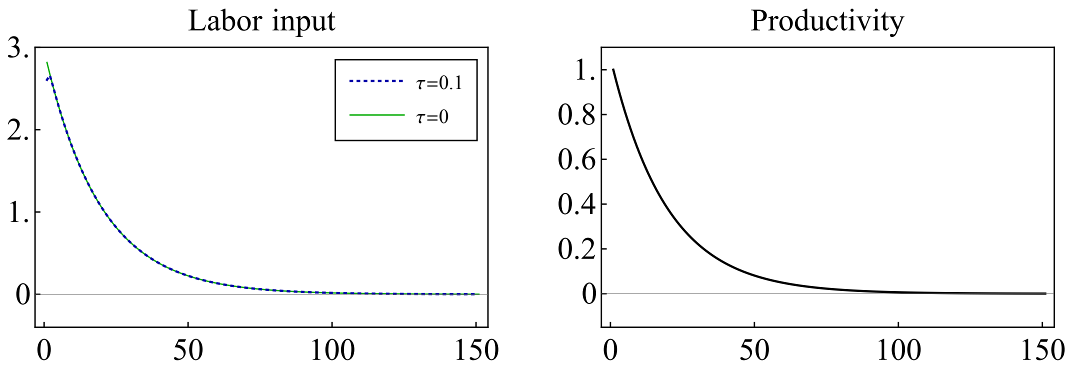

Fig. 10 The response of the standard adjustment cost model to the productivity shock
Note: The horizontal and vertical axes represent time and percentage, respectively. The lines represent the percentage deviations of the variables from their steady-state values when the shock occurs.

The reactions of the standard adjustment cost model to a productivity shock are shown in Fig. 10. The parameters are the same as those described in Section 4.1. Productivity increases by 1% in period 0 and gradually returns to the steady state, according to Equation (C1). The simulation results show cases with an adjustment cost of 0 and 0.1. The adjustment cost reduces the labor input response. Consider the log-linear approximation in Equation (C5) without the adjustment costs around the steady state, then we have $(L_t - L)/L \approx [1/(1-\alpha)]\,(A_t - A)/A$. This indicates that the response of labor input in the model is consistent with that of labor input and strategic labor input target discussed in Section 4.3.

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