Numerical Simulation of Economic Inequality Widened by the Persistent Effects of Temporary Rent Income

Harashima, Taiji

Syouei Economic Research Institute

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HARASHIMA Taiji*

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Abstract

There has been a deep-rooted view that economic rents are the main cause of high levels of economic inequality, but if economic rents are temporary, they may not be the cause. By employing numerical simulations, I show that even if economic rents are temporary, they can generate a high level of economic inequality that persists over a long period. Temporary economic rent incomes have two properties that can generate a high level of persistent economic inequality: (1) they follow a random walk process and (2) they gradually decrease. The numerical simulations employed use (1) a simulation method created on the basis of the concept of maximum degree of comfortability and (2) a newly created method that focuses only on the property of gradual decreases. The results show that these properties can increase economic inequality persistently and eventually generates extreme economic inequality. The origin of this temporary rent driven economic inequality is heterogeneity in the timings of obtaining randomly given temporary rent incomes among households. The simulation results strongly suggest that a government should intervene to restrain economic inequality from considerably widening even if rent incomes are only temporary.

JEL Classification: C53, C63, D63, E17, I30

Keywords: Economic rents; Government intervention; Inequality; Simulation: Temporary rent

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*Correspondence: HARASHIMA (Family name) Taiji (First name), Syouei Economic Research Institute, Kanazawa, Ishikawa, Japan. Email: t-harashima@mve.biglobe.ne.jp.

1 INTRODUCTION

Economic inequality has long been one of the central issues that economics has to solve, and recently it again has drawn wide attention because many empirical studies have shown that economic inequality has increased in developed countries since the 1980s (Piketty 2003, 2013; Piketty and Saez 2003; Atkinson et al. 2011; Parker 2014; Saez and Zucman 2016). Various kinds of explanations for the origin of inequality have been presented (e.g., Kuznets 1955; Boix 2010; Pickety 2013; Milanovic 2016), and to uncover the mechanism of recent increases in economic inequality, several explanations have been proposed (Katz and Murphy 1992; Autor et al. 1998, 2003; Card and DiNardo 2002; Leamer 1998; Goldberg and Pavcnik 2007; Helpman 2016; Piketty 2013), although no consensus has yet been formed.

Nevertheless, there has been a deep-rooted view that wealthy people, from the start, have exclusionary sources of wealth (i.e., economic rents), and these rents are foremost among the origins of high levels of economic inequality (Stiglitz 2015a, 2015b, 2015c, 2015d). At present, however, classical economic rents such as monopoly and natural resource rents may be less important economically than they were in the past (e.g., because they are currently strictly regulated). Nevertheless, considering existing high levels of economic inequality, Stiglitz (2015d) emphasized the importance of "exploitation rents", which are another type of economic rent that contribute to inequality, although his arguments are narrative and remain suggestive. On the other hand, Harashima (2016[1]) and Harashima (2020a[2]) showed different types of economic rent that had not been discussed previously: monopoly profits (rents) derived from people's ranking preferences and those derived from "mistakes" in business deals.

Harashima (2020c[3]) theoretically showed a mechanism through which economic rents can greatly widen economic inequality and examined the role of government in preventing it on the basis of the concept of "sustainable heterogeneity" (SH) (Harashima (2010[4], 2012[5], 2014). In this context, heterogeneity is defined as being sustainable if all optimality conditions of all heterogeneous households are satisfied indefinitely. In Harashima (2020c), the distinction between temporary and persistent economic rents is emphasized because persistent rents generate a persistent economic inequality, but temporary ones do not in the sense that the probability of obtaining rents is identical among households.

However, even if temporary rents do not generate persistent economic inequality

[1] Harashima (2016) is also available in Japanese as Harashima (2018b).
[2] Harashima (2020a) is also available in Japanese as Harashima (2023d).
[3] Harashima (2020c) is also available in Japanese as Harashima (2021d).
[4] Harashima (2010) is also available in Japanese as Harashima (2017b).
[5] Harashima (2012) is also available in Japanese as Harashima (2020b).

in the above sense, they may generate it in other ways. There are many measures of economic inequality, and each person will feel the level of economic inequality differently from others. The purpose of this paper is to examine the relation between temporary rent incomes and persistent economic inequality from different points of view that incorporate the probability of obtaining rents.

Following the examinations in Harashima (2020c, 2021a), Harashima (2023a) simulated the effect of economic rents obtained heterogeneously on economic inequality among households on the basis of the simulation method created in Harashima (2022c). In addition, employing the same simulation method, Harashima (2023b) numerically examined the mechanism underlying why economic inequality can increase in democratic countries following the theoretical examinations in Harashima (2021c).

The method used in these simulations was completely new, and with it, a numerical simulation of reaching the path to a steady state can easily be conducted. The method employs the concept of the "maximum degree of comfortability" (MDC), where MDC indicates the state at which a household feels most comfortable with its combination of income and assets. Usually, it is difficult to simulate the path to reach a steady state by assuming that households behave by generating their own rational expectations, but Harashima (2018a[6]) showed an alternative procedure for households to reach a steady state (i.e., the MDC-based procedure). Under the MDC-based procedure, households maintain their capital-wage ratio (CWR) at MDC, and their behavior is equivalent to that of households who base their behavior on rational expectations (i.e., their behaviors under the rate of time preference [RTP]-based procedure) (Harashima 2018a, 2021a, 2022a[7]). By assuming that households behave under the MDC-based procedure, it becomes very easy to simulate the path to reach a steady state because households are not required to do anything equivalent to computing a complex, large-scale economic model on a daily basis.

This simulation method was employed to study not only economic inequality but also for the numerical simulation of (1) the path to a steady state without generating any rational expectations (2022c), (2) endogenously growing economies and their balanced growth path (Harashima 2024a), and (3) economic depression (Harashima 2024b). In this paper, I use the same simulation method to study whether temporary rent incomes have persistent effects on economic inequality.

I first theoretically examine the likely factors that can make temporary rent incomes have persistent effects on economic inequality. Because temporary rent incomes have a random walk property, their variances increase as time passes, which will make them have a persistent nature. In addition, temporary rent incomes generally will not be

6 Harashima (2018a) is also available in Japanese as Harashima (2019).
7 Harashima (2022a) is also available in Japanese as Harashima (2022b).

spent all at once but continue to remain for many periods; that is, they will decrease gradually over a long period of time.

I then simulate the paths of 10 identical economies to examine the effect of the random walk process by using the simulation method created in Harashima (2022c) and show that this property can make economic inequality persistently increase. Next, I create a new simple simulation method to examine the effects of the gradual decrease property and use this simple method to simulate the capital paths of many households. The results show that this property can make economic inequality increase persistently and eventually become extreme. The origin of this kind of economic inequality is heterogeneity in timings of obtaining randomly given temporary rent incomes among households. Finally, I discuss the need for government intervention to restrain economic inequality from considerably widening even if rent incomes are only temporary.

2 PERSISTENT EFFECTS OF TEMPORARY RENTS

2.1 Temporary and persistent rent incomes

2.1.1 Kinds of economic rents

Monopoly and natural resource have been regarded as the main source of economic rents, but in developed countries, monopolies are strictly regulated, and oil and other natural resource rents may no longer play an important role in the degree of inequality within a country, at least within a developed country. On the other hand, Harashima (2016, 2018c) showed the existence of a type of economic rent that had not been discussed previously: monopoly profits (rents) derived from people's ranking preferences. These rents enable some individuals to be superstars in the world of sport, art, or music (Harashima 2016, 2018c), and enable some corporate executives to earn extremely high compensation (Harashima 2018d) because ranking preference is an important element in product differentiation that allows companies to accrue large amounts of monopoly rent (Harashima 2017a). In addition, another important kind of economic rent arises from heterogeneity in making "mistakes" in business deals (Harashima 2020a), and Harashima (2023c) showed that large amounts of economic rents are always generated in the process of consumption because there is the optimal level of lawful disinformation in advertisements to induce consumers' mistakes.

2.1.2 "Net" and "core" economic rents

Economic rents are obtained when a person's revenues from a factor of production exceed the cost to utilize that factor. This means that people transfer economic resources equivalent to the economic rent to the person controlling that factor of production. In principle, the amount of economic rents obtained and the total amount of economic resources extracted to finance them are equal.

In this paper, I call these extracted economic resources economic rents but give them negative values. A household can obtain positive and negative amounts of economic rents at the same time. That is, it may obtain positive economic rent, but at the same time, some of its economic resources may be extracted as negative economic rents.

In the following discussion, a household's "rent income" means its net economic rent income, that is, the sum of its positive and negative amounts of economic rent. To avoid confusion, I call positive economic rent incomes that do not include any element of negative economic rent income "core" rent incomes.

2.1.3 Definitions of temporary and persistent rent incomes

I define temporary and persistent rent incomes in this paper as follows. If the probability of obtaining rent incomes is identical for any household, the rent incomes are temporary; otherwise, they are persistent, where a household is infinitely living or represents a family line. This definition is equivalent to the following: if the mean rent incomes obtained for indefinitely long periods of time is zero for any household (family line), the rent incomes are temporary; if not, they are persistent.

2.1.4 Distinguishing between temporary and persistent rents

In accordance with local customs and for various other reasons, many people marry within the same or a similar group, which indicates that many family lines consist of households that share similar traits because they have members descended from common ancestors. This means that abilities such as those related to obtaining rent incomes will be highly likely unevenly distributed among family lines. As a result, some family lines will persistently obtain positive rent incomes, whereas other family lines will persistently obtain negative rent incomes (Harashima 2020c, 2020d, 2020e, 2021a). These rent incomes are persistent by definition.

Nevertheless, in practice, it is difficult to distinguish temporary and persistent rent incomes. For example, suppose that a person in a family line becomes an executive of a large company and obtains a huge amount of rent income by the mechanism shown in Harashima (2018d). Is this person's rent income temporary or persistent? If members of the person's family line become executives in large companies more often than average in the long run, the person's rent income will be judged to be persistent. However, if the probability of obtaining such positions is almost identical for most family lines in the long run, the rent income will be judged to be temporary. The same is true for rent incomes obtained by very popular artists or superstars in professional sports. However, it is difficult to know how often members in a specific family line become executives, very popular artists, or superstars in professional sports.

Even though some rent incomes are persistent according to the above definition, they may have to be treated practically as temporary if core rent incomes can be obtained only very sporadically for each individual person and thus only a few persons can obtain them during their lives. That is, considering the lifespan of human beings, many rent incomes may have to be treated practically as temporary although they may actually be persistent.

2.2 Factors that give persistency to temporary rent incomes

2.2.1 Random walk process

Temporary rent incomes as defined in Section 2.1.3 follow random walk processes by their nature. Suppose that there are N households that are identical, and in every period, one and only one of the N households randomly obtains core rent incomes (T) that are constant and the same for any household. It is assumed that the amounts extracted from the other $N-1$ households due to T are identical for any household that belongs to the $N-1$ households; that is, the amount is equally $\frac{T}{N-1}$ for any of these households.

Let R_t be a household's balance of (positive and negative) rent incomes in period t. R_t follows a random walk process such that

$$R_t = R_{t-1} + u_t \qquad (1)$$

where u_t is a random variable with mean 0 and variance σ^2 such that

$$u_t = T \qquad \text{at probability } \frac{1}{N}$$
$$= -\frac{T}{N-1} \qquad \text{at probability } 1 - \frac{1}{N}$$

and $R_0 = 0$. Therefore the mean is $E(R_t) = 0$ (i.e., the rent incomes are temporary), and the variance is

$$V(R_t) = t\sigma^2 \,. \qquad (2)$$

As equation (2) indicates, the variance of R_t ($t\sigma^2$) increases as time (t) passes. The variance of R_t indicates the possible range of dispersion of capital owned by households (i.e., the possible level of economic inequality). Hence, equation (2) means that temporary rent incomes can make the level of economic inequality increase as time passes (i.e., they can have persistent effects on economic inequality). Only because of very existence of temporary rent incomes, therefore, the level of economic inequality can continue to increase.

2.2.2 Gradual decreases in capital

The other element that can give temporary rent incomes is persistency. If a person luckily and suddenly becomes rich, it seems likely that they will spend larger amounts of money than before, but the money spent in each period will be less than the obtained core rent income, much of which be unspent and remain for many periods. That is, core rent incomes will be spent gradually over a long period. Of course, some people who obtain a large rent income may spend it all at once, but on average, core rent incomes will be spent gradually. This property gives temporary rent incomes a persistent nature.

This property is consistent with household behavior under the MDC-based procedure. Under this procedure, as a household's CWR deviates more largely downwards from its CWR at MDC, its consumption becomes more largely adjusted (i.e., increases in this case) to recover the state where its CWR is equal to its CWR at MDC. As a result, capital decreases more or increases less than before. Usually, this consumption adjustment process will not be completed in one period but will continue for many periods because households are risk averse and thus dislike large fluctuations of consumption.

3 SIMULATION I: THE RANDOM WALK PROCESS

First, I employ simulations to examine the effect of the random walk process. The simulation method is basically the same as that used in Harashima (2022c, 2023a, 2023b, 2024a, 2024b), which is briefly explained in this section and summarized in greater detail in Appendix 1. This simulation method was created on the basis of the MDC-based procedure shown in Harashima (2018a, 2021a, 2022a) and the concept of SH presented in Harashima (2010, 2012, 2014). These concepts are briefly summarized in Appendixes 2 and 3.

3.1 Simulation method

No technological progress and capital depreciation are assumed, and all values are expressed in real and per capita terms. It is assumed that there are H economies in a country, the number of households in each of economy is identical, and households within each economy are identical. The production function of Economy i ($1 \leq i \leq H$) is

$$y_{i,t} = \omega_i A_t^{\alpha} k_{i,t}^{1-\alpha} \quad , \tag{3}$$

where $y_{i,t}$ and $k_{i,t}$ are the production and capital of a household in Economy i in period t, respectively; ω_i is the productivity of a household in Economy i; A_t is technology in period t; and α ($0 < \alpha < 1$) is a constant and indicates the labor share. All variables are expressed in per capita terms. In simulations, I set $\alpha = 0.65$, $A_t = 1$, and $\omega_i = 1$ for any t and i. The initial capital a household owns is set at 1 for any household.

By equation (3), the production of a household in Economy i in period t ($y_{i,t}$) is calculated, for any i, by

$$y_{i,t} = k_{i,t}^{1-\alpha} \ .$$

The amount of capital used (not owned) by each household (i.e., $k_{i,t}$) is kept identical among households although the amount of capital owned (not used) by each household can be heterogeneous. For any i,

$$k_{i,t} = \frac{\sum_{i=1}^{H} \breve{k}_{i,t}}{H} \ ,$$

where $\breve{k}_{i,t}$ is the amount of capital a household in Economy i owns (not uses).

The capital income of a household in Economy i in period t ($x_{K,t}$) is calculated by

$$x_{K,i,t} = r_t \breve{k}_{i,t} \ ,$$

where r_t is the real interest rate in period t and

$$r_t = \frac{\partial k_{i,t}}{\partial y_{i,t}} \ .$$

The labor income of a household in Economy i in period t ($x_{L,i,t}$) is calculated by extracting its capital income from its production such that

$$x_{L,i,t} = y_{i,t} - r_t k_{i,t} = y_{i,t} - r_t \frac{\sum_{i=1}^{H} \breve{k}_{i,t}}{H} \ .$$

Household savings in Economy i in period t ($s_{i,t}$) are calculated by

$$s_{i,t} = x_{L,i,t} + x_{K,i,t} - c_{i,t} \ ,$$

where $c_{i,t}$ is the consumption of a household in Economy i in period t. In period t + 1, these savings ($s_{i,t}$) are added to the capital the household owns, and therefore,

$$\check{k}_{i,t+1} = \check{k}_{i,t} + s_{i,t} \ .$$

The following simple consumption formula is used.

Consumption formula 1: The consumption of a household in Economy i in period t is

$$c_{i,t} = \left(x_{L,i,t} + x_{K,i,t}\right)\left(\frac{\Gamma(\tilde{s}_i)}{\Gamma_{i,t}}\right)^{\gamma} \ ,$$

and equivalently

$$c_{i,t} = \left(x_{L,i,t} + x_{K,i,t}\right)\left(\frac{\theta_i}{\Gamma_{i,t}\frac{1-\alpha}{\alpha}}\right)^{\gamma} \ ,$$

where $\Gamma_{i,t}$ is the capital-wage ratio (CWR) of a household in Economy i in period t, $\Gamma(\tilde{s}_i)$ is $\Gamma_{i,t}$ of a household in Economy i in period t when the household is at its MDC, and γ is a parameter. In this paper, I set the value of γ to be 0.5. It is assumed that the intrinsic $\Gamma(\tilde{s}_i)$ (i.e., CWR at MDC) of a household is identical across households and economies, and I set this common $\Gamma(\tilde{s}_i)$ to be $0.04 \times 0.65/(1-0.65) = 0.0743$, which corresponds to an RTP of 0.04.

In a heterogeneous population, Consumption formula 1 should be modified to Consumption formula 2. Let $\Gamma_{R,i,t}$ be the adjusted value of $\Gamma_{i,t}$ of a household in Economy i in period t in a heterogeneous population, and $\Gamma(S_t)$ be the CWR of the country (i.e., the aggregate CWR).

Consumption formula 2: In a heterogeneous population, the consumption of a household in Economy i in period t is

$$c_{i,t} = \left(x_{L,i,t} + x_{K,i,t}\right)\left(\frac{\Gamma(\tilde{s}_i)}{\Gamma_{R,i,t}}\right)^{\gamma}$$

$$= \left(x_{L,i,t} + x_{K,i,t}\right)\left(\frac{\Gamma(\tilde{s}_i)}{r_t \frac{\alpha}{1-\alpha}}\right)^{\gamma} = \left(x_{L,i,t} + x_{K,i,t}\right)\left(\frac{\Gamma(\tilde{s}_i)\frac{1-\alpha}{\alpha}}{r_t}\right)^{\gamma},$$

and equivalently,

$$c_{i,t} = \left(x_{L,i,t} + x_{K,i,t}\right)\left(\frac{\theta_i}{r_t}\right)^{\gamma} \ .$$

If a household in Economy i is assumed to obtain economic rents, these rents are set to be added to the capital it owns. Let $\rho_{i,t}$ be the amount of economic rents a household in Economy i obtains in period t. Consequently, the capital that a household in the other $H-1$ economies owns is set to decrease by $\frac{\rho_{i,t}}{H-1}$. The amount of economic rents each household in Economy i obtains is identical, and the amount of capital decrease in each household in the other $H-1$ economies is also identical. Economic rents may be obtained either each period or intermittently, and they may be obtained either deterministically or stochastically.

Hence,

$$\breve{k}_{i,t} = \breve{k}_{i,t-1} + s_{i,t-1} + \rho_{i,t}$$

and for any $j(\neq i)$,

$$\breve{k}_{j,t} = \breve{k}_{j,t-1} + s_{j,t-1} - \frac{\rho_{i,t}}{H-1} \ .$$

Let κ_i be the $\breve{k}_{i,t}$ that a government aims for in order to induce a household in Economy i to own capital at a steady state (i.e., κ_i is the target value set by the government). Under these conditions, the bang-bang (two-step) control rule of government transfers is set as follows.

Transfer rule: The amount of government transfers from a household in Economy i to a household in Economy $i + 1$ in period t is T_{low} if $\breve{k}_{i,t}$ is lower than κ_i, and T_{high} if $\breve{k}_{i,t}$ is higher than κ_i, where T_{low} and T_{high} are constant amounts of capital predetermined by the government, and if $i = H$, $i + 1$ is replaced with 1.

In the simulations, T_{low} is set to be -1.5 and T_{high} to be 3. The value of κ_i is varied in each simulation depending on what steady state the government aims to achieve.

3.2 Results of simulations

3.2.1 Setup

It is assumed for simplicity that all households are identical, but the result is basically the same even if a heterogeneous population is assumed because all households are linked by SH in a heterogeneous population, as shown in Appendix 3.

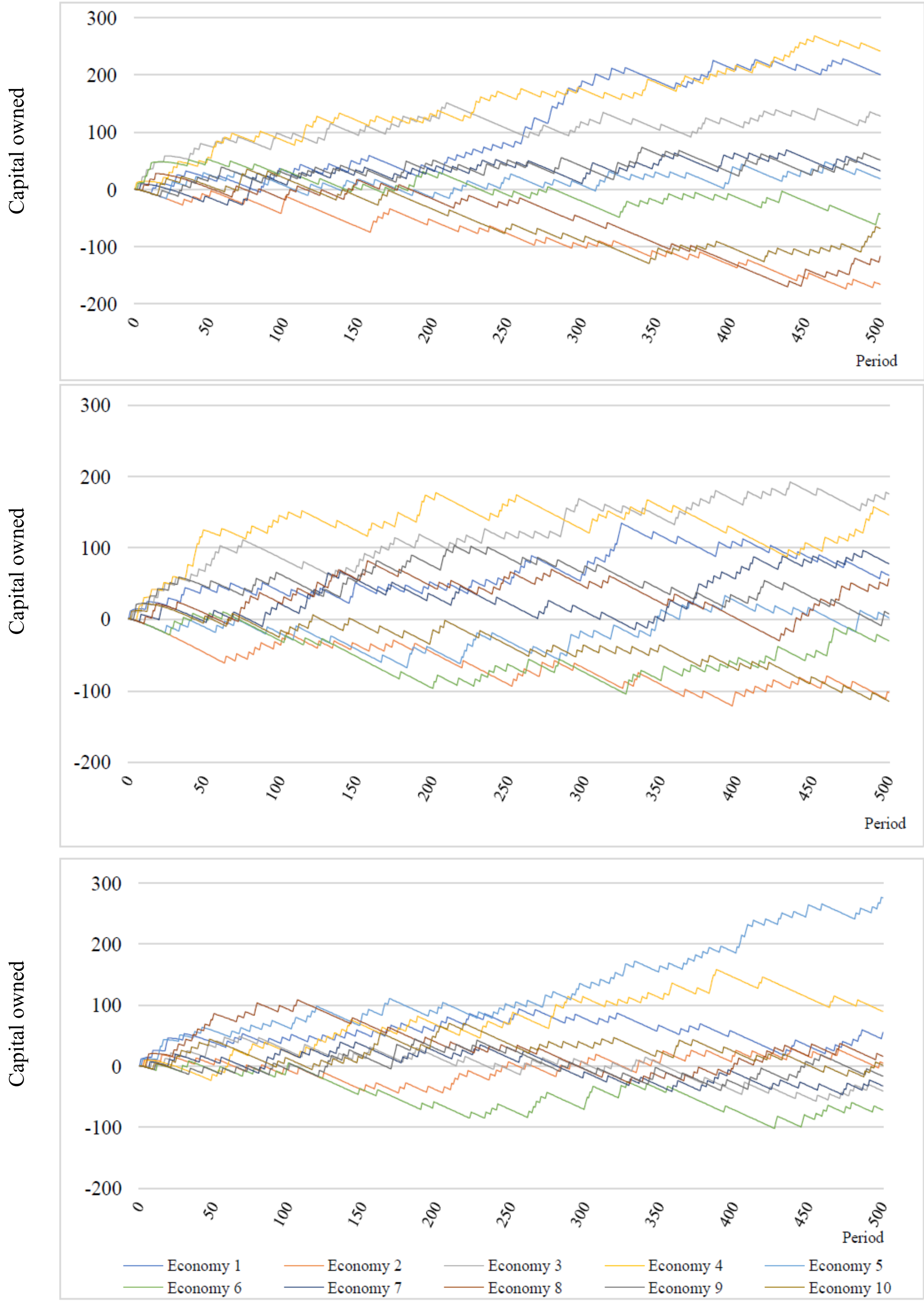
I simulate 10 identical economies and assume for simplicity that each economy consists of only one household. Because of assumed homogeneity, these 10 households in 10 economies are identical. Any household can equally and randomly obtain core rent incomes (I_{Temp}) that are constant and the same for any household. Households do not obtain any other kind of core rent incomes. The probability of obtaining I_{Temp} (I_{Prob}) is constant and identical for any household (i.e., the rent incomes are temporary), but in each period, one and only one of the 10 households obtains them; thus, I_{Prob} = 0.1. In each period, the amount of $\frac{I_{Temp}}{9}$ (i.e., I_{Temp} divided by the number of the other households that do not obtain them) is extracted from the capital owned by each of the 9 households that do not obtain I_{Temp} in that period.

I_{Temp} was set at 10. Because I_{Temp} is given randomly to households, the results are different in each simulation.

3.2.2 Without government intervention

I first simulate the case that the government does nothing to achieve SH. The paths of capital owned by each household in the 10 economies are shown in Figure 1. As mentioned above, results differ by simulation, and six typical results are shown in Figure 1.

First, Figure 1 indicates that, as predicted by equation (2), the variances of the 10 economies generally increase as time passes. This result is expected because of the random walk nature and the lack of government intervention. Secondly, Figure 1 indicates that the richest and poorest households differ randomly by simulation where "rich" means owning relatively larger amounts of capital and "poor" is owning relatively less capital. This occurs because the probability of obtaining I_{Temp} is identical. Finally, Figure 1 indicates that in general, once a household becomes relatively richer or poorer, it remains so for a long period (e.g., compare economy 4 to economy 2 in the first panel); that is, the rich and the poor are fixed for a long period even if the probability of obtaining I_{Temp} is identical.


Capital owned
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Economy 10

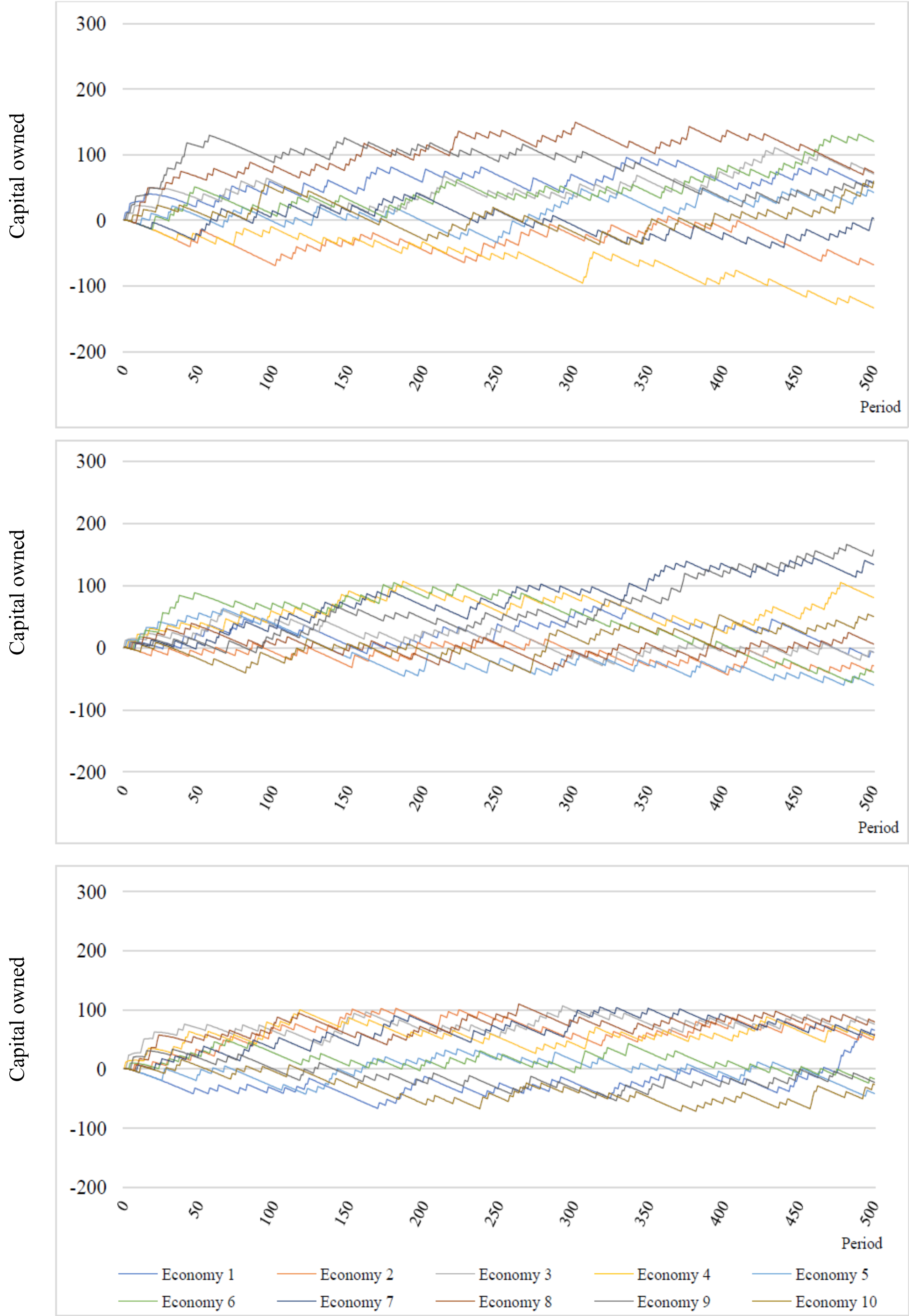

Figure 1: Six typical results of paths of capital owned by households in 10 economies in the absence of government intervention

3.2.3 With government intervention to achieve a SH

I next simulate the case in which a government appropriately intervenes to achieve SH. I set $\kappa_i = 28.13476 = \left(\frac{0.04}{0.35}\right)^{-\frac{1}{0.65}}$ for any i (= 1, 2, 3, …, 10). Note that this value of κ_i is equal to the amount of capital at the steady state in the case that there is no rent income.

The simulated paths of capital owned by each of 10 economies are shown in Figure 2. Although results differ by simulation, they are generally similar to the ones shown in Figure 2. Note that because a simple bang-bang (two-step) control is adopted as the government's transfer rule (Section 3.1), the simulated paths are not smooth. The paths of all 10 economies equally proceed around 28, which is around the level of capital at the steady state in the case when there is no rent income. The variance is small and changes little over time. This means that high levels of economic inequality and the persistent effect of temporary rent incomes on economic inequality are almost eliminated by government intervention.

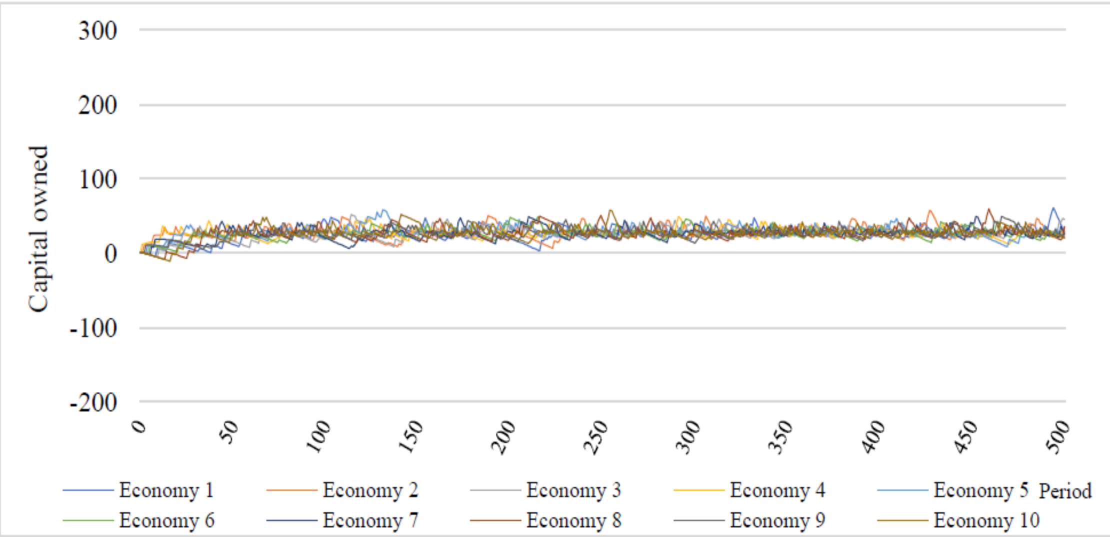

Figure 2: Typical paths of capital owned by households in 10 economies when the government appropriately intervenes

3.2.4 Different amounts of I_{Temp}

The variance will change if I_{Temp} changes, so I simulate three cases with different values of I_{Temp} (I_{Temp} = 5, 10, and 20) without government intervention. Figure 3 shows typical results for these three cases. Because of randomness of temporary rent incomes, results differ by simulation, but Figure 3 indicates that in general, as I_{Temp} increases, the variance of the 10 economies increases, as expected because of equation (2).

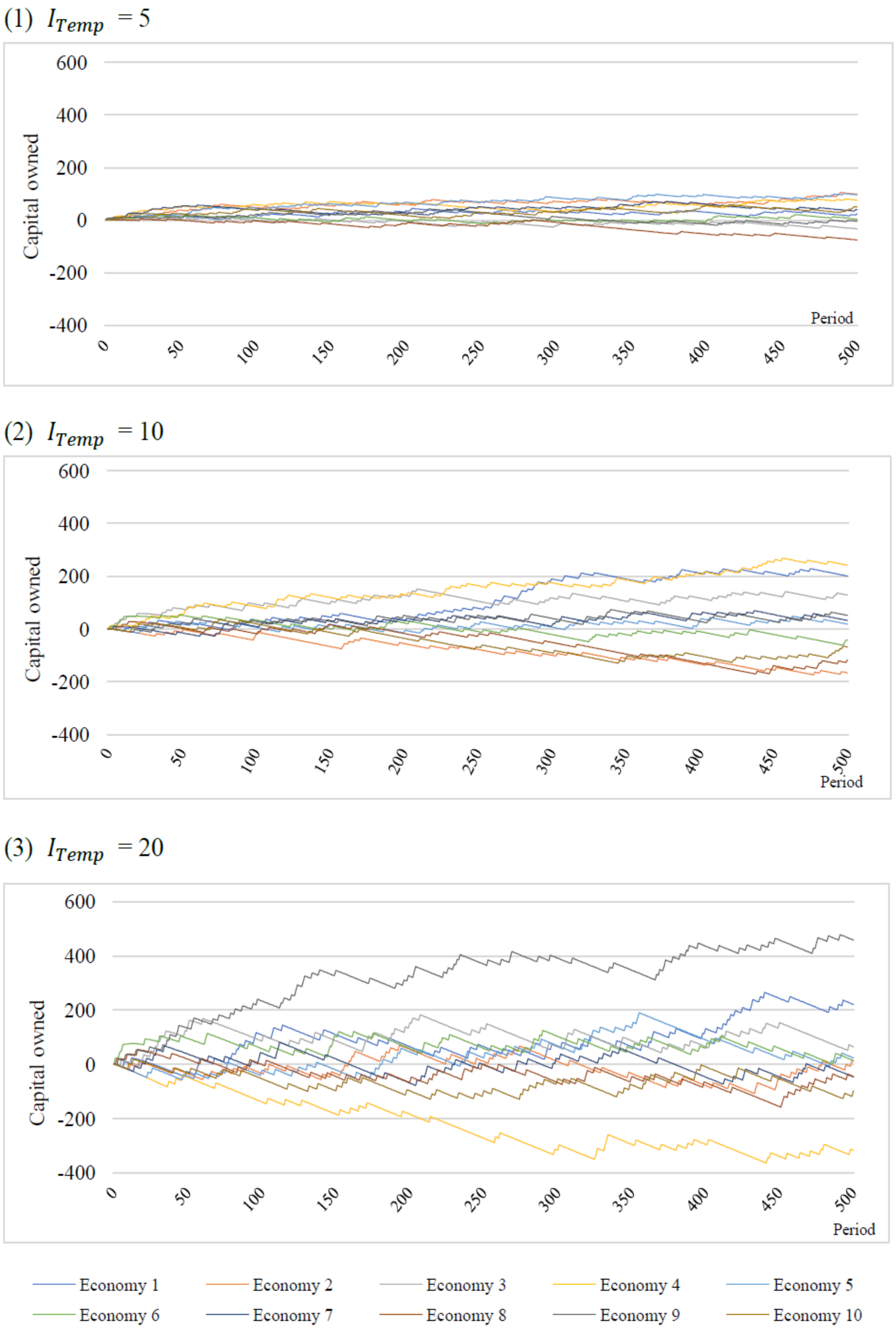

Figure 3: Paths of capital owned by households in 10 economies when the government does not intervene in the case of I_{Temp} = 5, 10, and 20, respectively

4 SIMULATION II: GRADUAL DECREASES

Next, I examine the effect of the property of gradual decreases by employing a simulation method that is completely different from that used in Section 3.

4.1 Simulation method

4.1.1 The method

The simulation method is simplified to focus only on the effect of gradual decrease such that production, consumption, investments, and technology are all assumed to be exogenously given, constant, and implicit. The only explicit variable is capital owned by households, and a household's capital changes only when it is transferred among households in connection with rent incomes. It is assumed for simplicity that all households are identical and equally divided into 100 groups, and each group consists of only one household. In addition, it is assumed that the total amount of capital owned by all 100 households is constant and set to be unity, which implicitly means that the real interest rate is kept constant. Initially (i.e., in period 0), each of the 100 households equally owns the same amount of capital (i.e., 0.01). Because the total amount of capital is unity, the amount of capital each household owns also indicates its share in the economy.

The household in one and only one of the 100 groups randomly obtains core rent incomes (I_{Temp}) in turn every 5 periods. The probability of obtaining I_{Temp} is identical for any household in the long run, and therefore, the rent incomes are temporary. However, once a household obtains I_{Temp}, it cannot obtain I_{Temp} again until all of the other 99 households obtain I_{Temp}. In the period when a household obtains I_{Temp}, the obtained I_{Temp} is added to the capital of the household. At the same time, if a household obtains I_{Temp} in a period, the capital owned by the other 99 households is equally decreased (extracted) by $\frac{I_{Temp}}{99}$ in that period.

The capital owned by a household that once obtained I_{Temp} decreases in the periods after obtaining I_{Temp} at a constant rate (D_{Rate}) every 5 periods where $0 < D_{Rate} < 1$. At the same time, the capital owned by the other 99 households equally increases by this amount of decrease divided by 99 because the total amount of capital in the economy and thus the real interest rate are kept constant.

4.1.2 Commentary

First, the assumption $D_{Rate} < 1$ indicates that I_{Temp} is not fully spent during the 5 periods after it is obtained (i.e., part of I_{Temp} remains for more than 5 periods), which means that temporary rent incomes can be persistent. Secondly, because a household can obtain I_{Temp} again only after all of the other 99 households obtain I_{Temp}, the richest household changes every 5 periods. In other words, the richest is always a newcomer and not a household that inherits a huge amount of money.

Thirdly, the path of capital owned by a household is not a random walk process. Although the probability of obtaining I_{Temp} is identical for any household in the long run, a household's instantaneous probability of obtaining I_{Temp} changes temporally because once a household obtains I_{Temp}, it cannot obtain I_{Temp} again until all the other 99 households obtain I_{Temp}. That is, in the first 5 periods, the instantaneous probability is equally 0.01 for any household, but after that, the instantaneous probability of households that did not obtain I_{Temp} increases as time passes, and after 500 periods, the instantaneous probability of one household is 1 in turn and the instantaneous probability of the other 99 households is 0 in any period after 500 periods.

Nevertheless, the focal point of Simulation II is not the effect of the random walk process but that of a gradual decrease of capital owned by a household. In addition, although each household's chance of obtaining I_{Temp} is once in 500 periods, I only consider the first 85 periods in the simulations. Even though each household's capital path does not exactly follow a random walk process, core rent incomes seem to be randomly given approximately to all households if only the first 85 periods are considered, which is why I use this simulation method.

Note that for simplicity, the decrease rate of capital (D_{Rate}) is assumed to be constant, but in actuality, this rate will highly likely decrease as the amount of capital a household owns decreases. Nevertheless, if the rate of the decrease decreases in this manner, the persistent nature of temporary rent incomes will be amplified because larger amounts of core rent incomes remain for many more periods.

4.2 Results of simulations

4.2.1 Base case

I set the parameter values in the base case such that I_{Temp} = 0.15 (i.e., I_{Temp} is 15% of the total capital existing in the economy), D_{Rate} = 0.2 (i.e., the decrease rate is 20% every 5 periods). The estimated Gini coefficient in the first 85 periods is shown in Figure 4; it indicates that the level of economic inequality increases as time passes. In the first several dozen periods, the Gini coefficient increases rapidly from about 0.15 to about 0.5 but slows down after that. The Gini coefficient eventually reaches about 0.6. Therefore, even though the only difference among households is the timing of obtaining I_{Temp}, a high level of economic inequality can be generated.

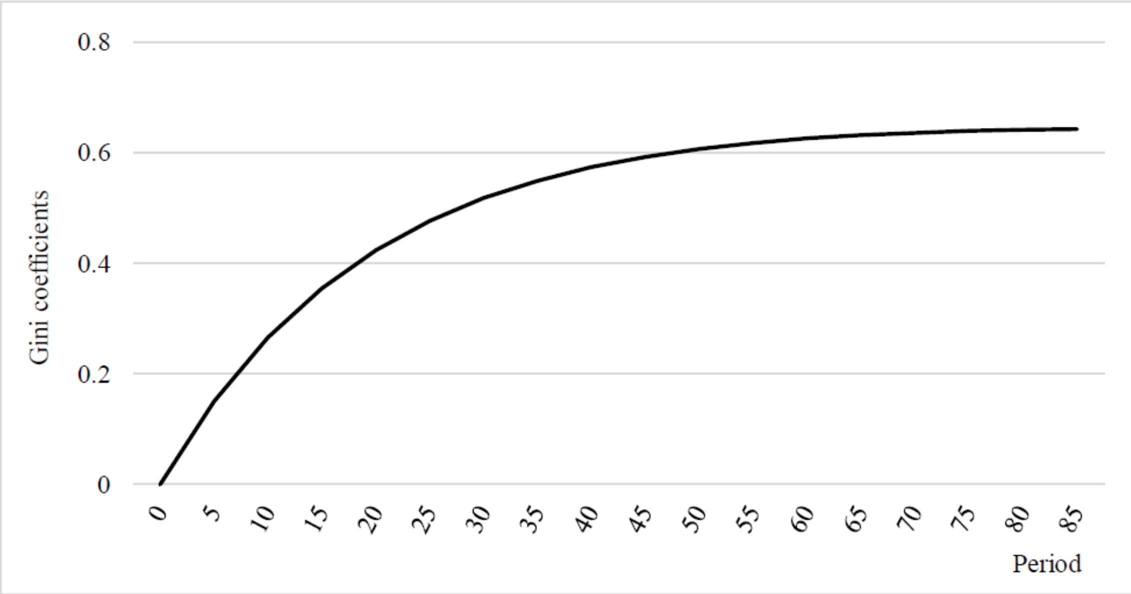

Figure 4: Gini coefficients in the base case

The ratio of the combined capital owned by the 10 richest households to that owned by the 10 poorest ones in each period is shown in Figure 5, and the capital ratio of the richest household to the poorest one is shown in Figure 6. Both indicators of economic inequality equally indicate rapid increases in economic inequality during the first several dozen periods, but the pace of increase slows down after that.

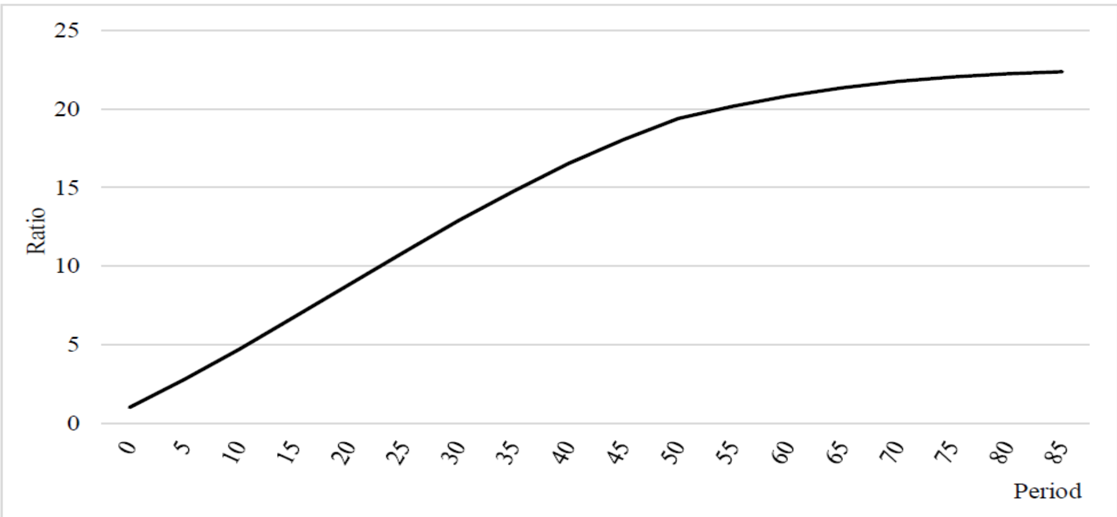

Figure 5: Ratio of capital owned by the 10 richest households relative to that of the 10 poorest ones in the base case

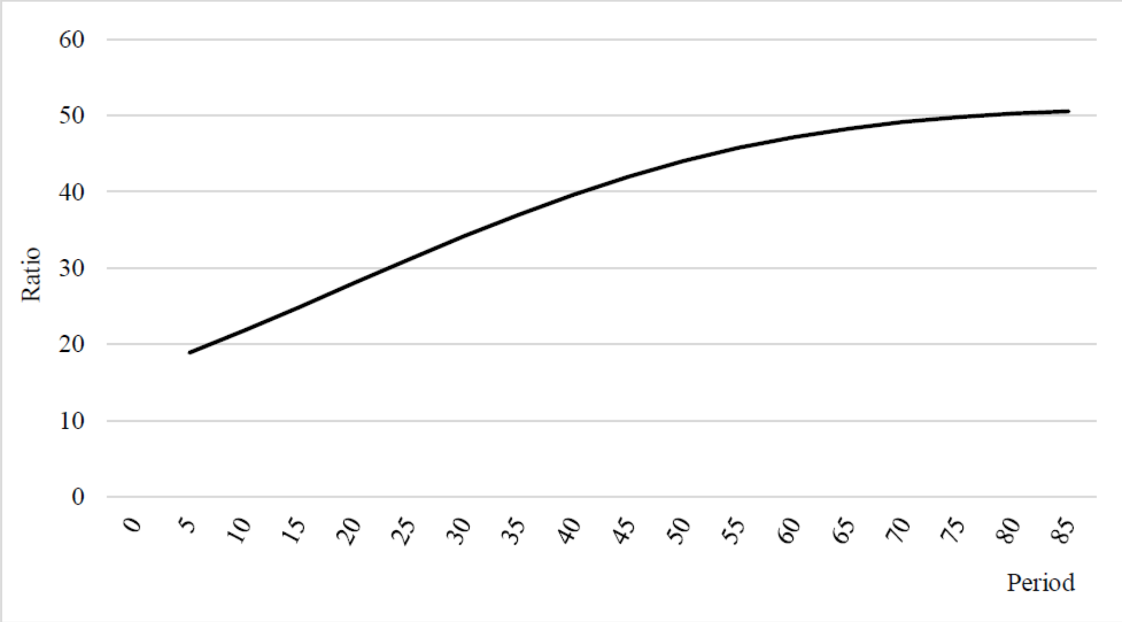

Figure 6: Ratio of capital owned by the richest household relative to that of the poorest one in the base case

The distribution of capital in period 85 for all 100 households is shown in Figure 7. Note that, as indicated in equation (1), R_{85} in Figure 7 means the balance of (positive and negative) rent incomes in period 85. It shows that a wide disparity among households is generated in period 85, even though their capital was initially equal.

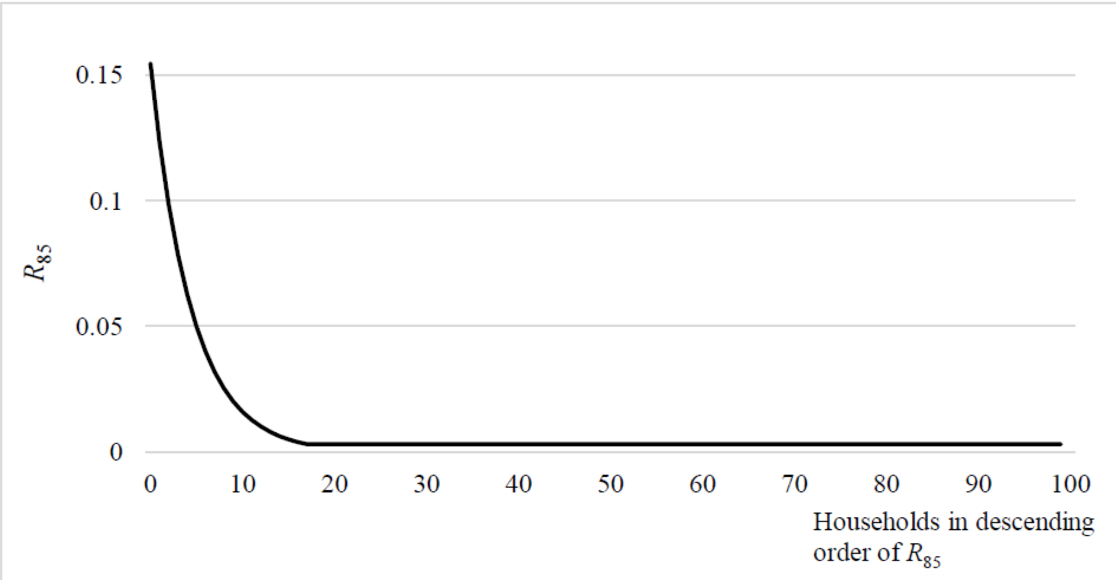

Figure 7: Distribution of capital owned in period 85 in the base case

Figure 8 shows the capital of the "middle-class" household (equivalently, the share of the middle-class household), where the "middle-class" household is the 50th richest (equivalently, 51st poorest) one in each period. The capital of the middle-class household is shown to continue to decrease gradually. That is, as time passes, the middle-class household becomes relatively poorer while some households become richer.

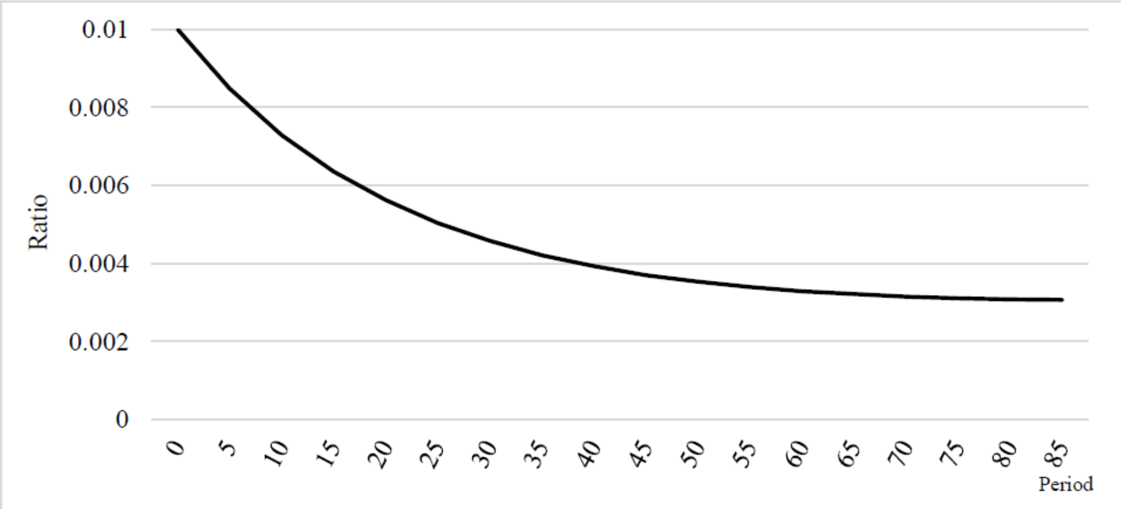

Figure 8: Ratio of capital owned by the middle-class household (the 50th richest household among 100 households) to the total capitals in the economy in the base case

The results in Simulation II indicate that a high level of economic inequality can be generated and that that inequality can continue to increase even if households' probabilities to obtain rent incomes are equal and the only difference among households is the timing of obtaining them.

4.2.2 Effects of differences in scale and decrease rate

If the values of I_{Temp} and D_{Rate} change, the results of the simulations will change, so I also simulate economies with different values of I_{Temp} and D_{Rate}. First, I simulate economies that have different values of I_{Temp} (i.e., I_{Temp} = 0.1, 0.15, and 0.2) with a common D_{Rate} (0.2). Note that the economy with I_{Temp} = 0.15 is the same as that in the base case in Section 4.2.1. Figure 9 shows the estimated Gini coefficients for these cases and indicates that, as the value of I_{Temp} increases, the level of inequality increases. In the case of I_{Temp} = 0.1, the Gini coefficient seems to converge at about 0.4, but in the case of I_{Temp} = 0.2, it exceeds 0.8, a value that will never be socially acceptable.

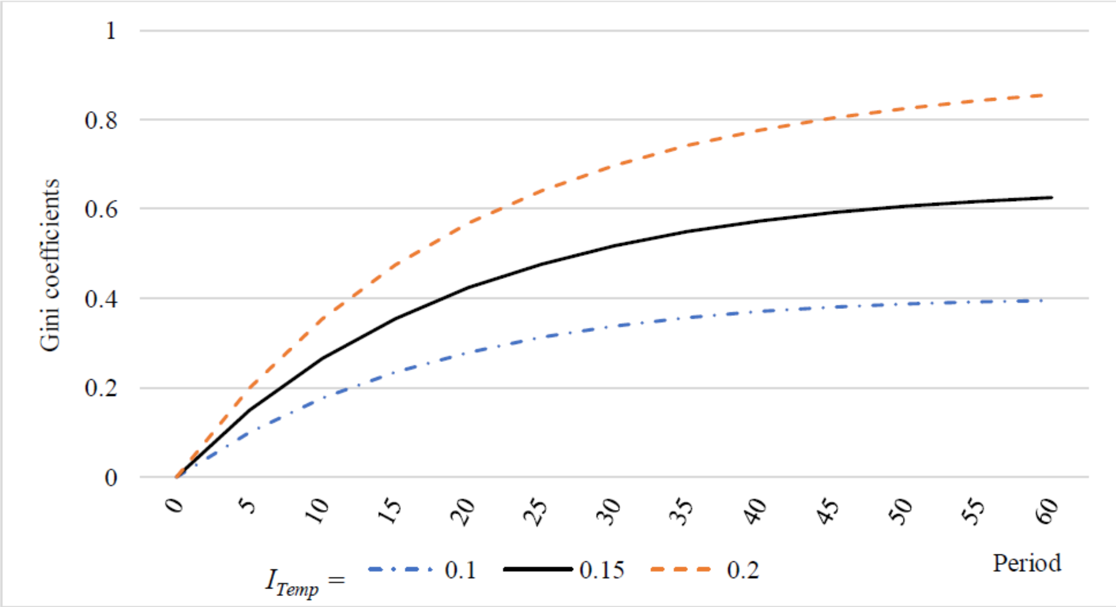

Figure 9: Gini coefficients in the cases of I_{Temp} = 0.1, 0.15, and 0.2

Figure 10 shows the ratio of the combined capital owned by the 10 richest households to that owned by the 10 poorest ones, and Figure 11 shows the capital ratio for the richest household to the poorest one. Both indicators of economic inequality commonly show that, as the value of I_{Temp} increases, the level of inequality increases. In the case of I_{Temp} = 0.2, the levels of inequality measured by these indicators no longer converge at a finite value. That is, extreme economic inequality can be generated if the amount of core rent incomes is large even though the rent incomes are only temporary.

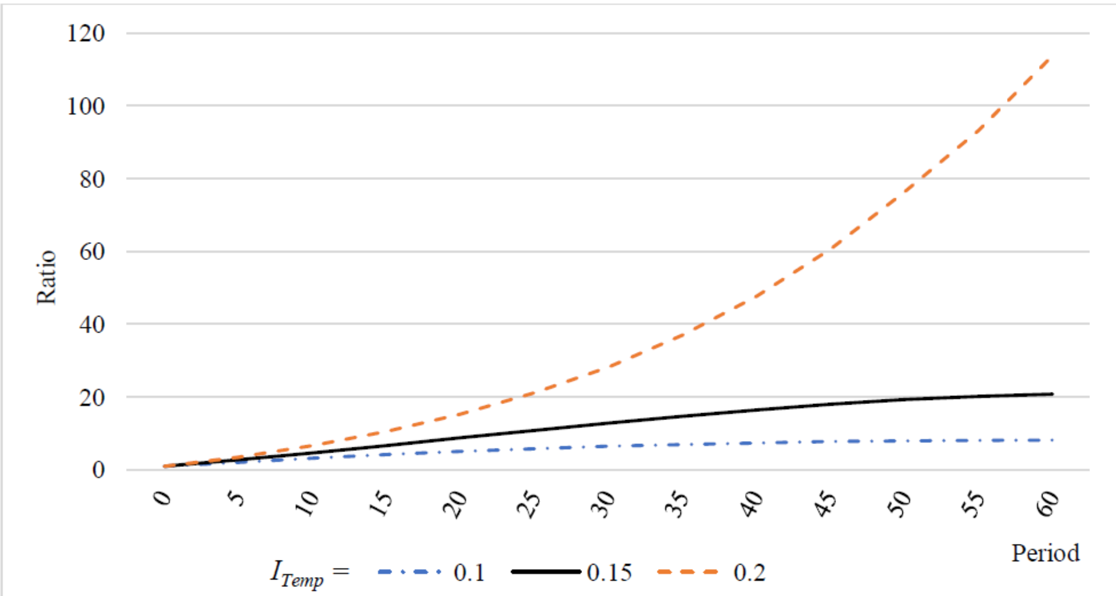

Figure 10: Ratios of capital owned by the 10 richest households to that owned by the 10 poorest ones in the cases of I_{Temp} = 0.1, 0.15, and 0.2

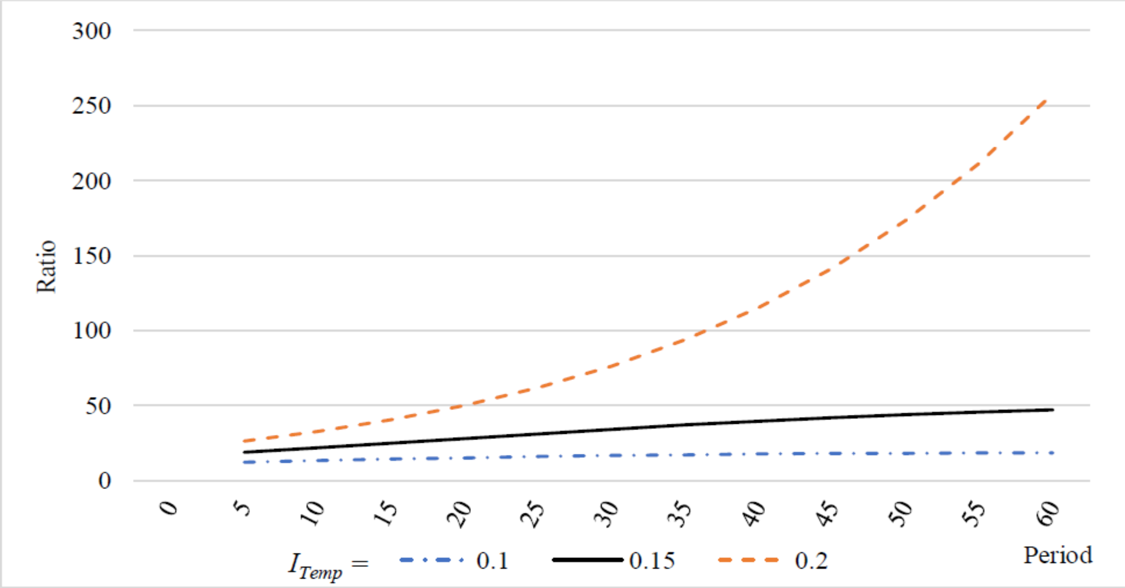

Figure 11: Ratios of capital owned by the richest household to that owned by the poorest one in the cases of I_{Temp} = 0.1, 0.15, and 0.2

Next, I simulate economies with different values of D_{Rate} (i.e., D_{Rate} = 0.25, 0.2, and 0.15), where I_{Temp} = 0.15. Note that the economy with D_{Rate} = 0.2 is the same as that in the base case in Section 4.2.1. Figure 12 shows the estimated Gini coefficients and indicates that, as the value of D_{Rate} decreases, the level of inequality increases. In the case of D_{Rate} = 0.25, the Gini coefficient seems to converge at around 0.5, and in the case of D_{Rate} = 0.15, it reaches nearly 0.8, which will never be socially acceptable.

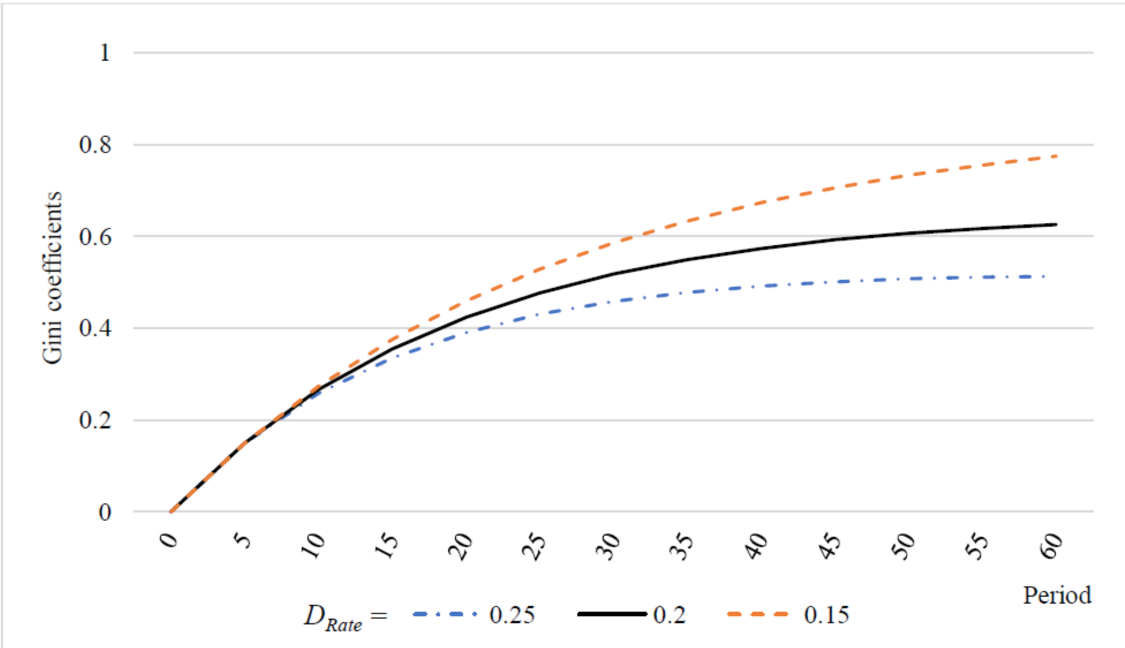

Figure 12: Gini coefficients in the cases of D_{Rate} = 0.25, 0.2, and 0.15

Figure 13 indicates the ratio of the combined capital owned by the 10 richest households to that owned by the 10 poorest ones, and Figure 14 indicates the capital ratio of the richest household to the poorest one. Both indicators of economic inequality commonly show that, as the value of D_{Rate} decreases, the level of inequality increases. In the case of I_{Temp} = 0.15, the levels of inequality measured by these indicators do not seem to converge at a finite level, similar to the case of large values of I_{Temp}. That is, extreme economic inequality can be generated if the decrease rate is small even though rent incomes are only temporary because larger amounts of core rent incomes are not spent and persist.

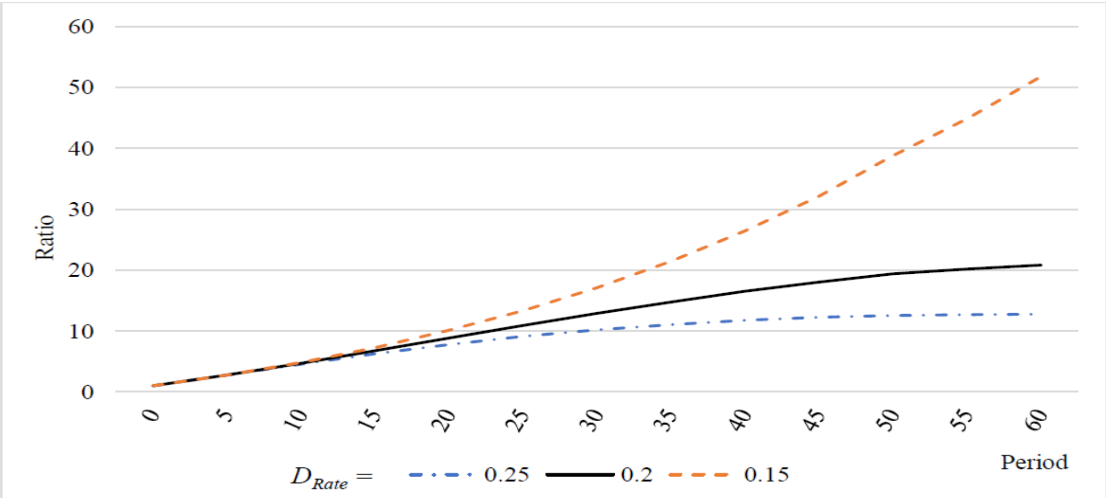

Figure 13: Ratios of capital owned by the 10 richest households to that owned by the 10 poorest ones in the cases of D_{Rate} = 0.25, 0.2, and 0.15

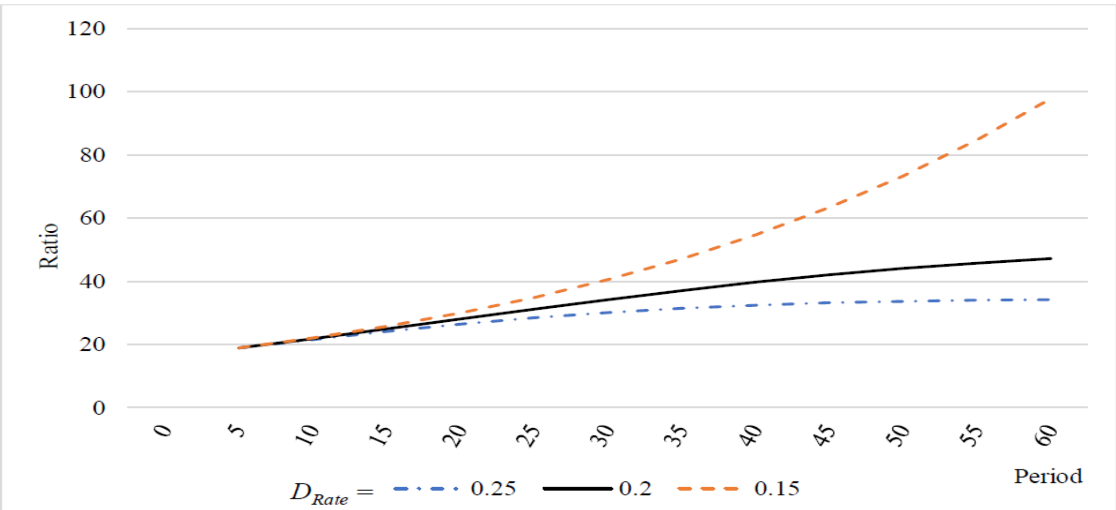

Figure 14: Ratios of capital owned by the richest household to that owned by the poorest one in the cases of D_{Rate} = 0.25, 0.2, and 0.15

5 TEMPORARY RENTS AND GOVERNMENT INTERVENTION

This paper shows that temporary rent incomes can generate persistent and high levels of economic inequality. The origin of this kind of economic inequality is heterogeneity in the timings of obtaining randomly given temporary rent incomes among households. However, should a government intervene to redistribute temporary rent incomes, as is true in the case of persistent rent incomes? From the point of view of SH, no government intervention is necessary if rent incomes are temporary.

In the case of persistent rent incomes and also heterogeneous preferences, the probability to enjoy benefits can largely differ among households (Becker 1980; Harashima 2010, 2012, 2014, 2020c, 2021a). Hence, in these cases, government interventions can be justified because the system is unfair. However, this justification cannot be applied to temporary rent incomes because the probability of obtaining them is identical for everybody.

Even so, many people may support government intervention for temporary rent incomes because they can generate extreme economic inequality. The random walk process implies that high levels of economic inequality can be inherited from generation to generation, as shown in Simulation I. In addition, the gradual decrease of capital can result in an extreme economic inequality. The possibility of extreme economic inequality may sufficiently justify government interventions to stop increasing economic inequality beyond some socially acceptable level.

Furthermore, even without extreme economic inequality, government interventions may be justified because one person's chance to obtain a very large amount of core rent income and become very rich will be very low even if chances are equal for everybody. In Simulation II, this probability is once every 500 periods. If one period is interpreted to be one year, this would mean the chance occurs once every 500 years for a household. On the other hand, once a lucky household obtains the rents, it and its later generations can continue to enjoy them for several decades, possibly for over a century. Considering the lifespan of human beings, 500 years will be an intolerably long period to wait. Hence, it may be justifiable to share a household's very large core rent income among many households simultaneously through government intervention in every period. Note that rare but very large risks are often shared by many people through insurance. Similarly, rare but very large instances of good luck (i.e., obtaining a very large amount of core rent income) may have to be shared by many people through government intervention, for example, by utilizing a progressive income tax.

6 CONCLUDING REMARKS

Economic inequality has long been one of the central issues that economics has to solve, and it continues to draw wide attention. There has been a deep-rooted view that wealthy persons, from the start, have exclusionary sources of wealth (i.e., rent incomes), and these rents are foremost among the origins of high levels of economic inequality. Harashima (2020c, 2021a) theoretically showed a mechanism whereby economic rents can greatly widen economic inequality as well as the role a government can play to prevent it.

In Harashima (2020c, 2021a), temporary and persistent economic rents are distinguished from the point of view of heterogeneity in probabilities of obtaining rents. Harashima shows that the former does not generate persistent economic inequality but that the latter does. However, it seems unlikely that temporary rents have no role in persistent economic inequality from any point of view. In this paper, I examined the relation between temporary rents and persistent economic inequality from various points of views.

I first theoretically showed that temporary rent incomes can have persistent effects on economic inequality because they have two fundamental properties: (1) the random walk process and (2) a gradual decrease in capital. Next, I simulated the paths of 10 identical economies to examine the effect of the random walk property using the simulation method created in Harashima (2022c) and showed that it can make economic inequality increase persistently. In addition, I simulated the capital paths of many households to examine the effects of the gradual decrease property using a newly created simulation method. The results show that this property can increase economic inequality persistently and eventually generate extreme economic inequality. The origin of this kind of economic inequality is heterogeneity in the timings of obtaining randomly given temporary rent incomes among households. The simulation results strongly suggest that a government should intervene to restrain economic inequality from considerably widening even if rent incomes are only temporary.

APPENDIX 1: Simulation method

A1.1 Simulation assumptions

A1.1.1 Environment

No technological progress and capital depreciation are assumed, and all values are expressed in real and per capita terms. It is assumed that there are H economies in a country, the number of households in each of economy is identical, and households within each economy are identical.

A1.1.2 Production

The production function of Economy i ($1 \leq i \leq H$) is

$$y_{i,t} = \omega_i A_t^{\alpha} k_{i,t}^{1-\alpha} \quad , \tag{A1.1}$$

where ω_i is the productivity of a household in Economy i. Because α indicates the labor share, I set $\alpha = 0.65$. In addition, I set $A_t = 1$ and $\omega_i = 1$ for any t and i. The initial capital a household owns is set at 1 for any household.

With $A_t = 1$ and $\omega_i = 1$, by equation (A1.1), the production of a household in Economy i in period t ($y_{i,t}$) is calculated, for any i, by

$$y_{i,t} = k_{i,t}^{1-\alpha} \ . \tag{A1.2}$$

A1.1.3 Capitals

Because the marginal productivity is kept equal across economies within the country through arbitrage in markets, the amount of capital used (not owned) by each household (i.e., $k_{i,t}$) is kept identical among households in all economies in any period; that is, $k_{i,t}$ is identical for any i although the amount of capital each household owns (not uses) can be heterogeneous. Hence, by equation (A1.2), the amount of production ($y_{i,t}$) is always identical across households and economies regardless of how much capital a household in Economy i owns, when $\omega_i = 1$. In addition, for any i,

$$k_{i,t} = \frac{\sum_{i=1}^{H} \breve{k}_{i,t}}{H} \quad ,$$

where $\breve{k}_{i,t}$ is the amount of capital a household in Economy i owns (not uses). As shown above, I set the initial capital of a household owns to be 1 (i.e., $\breve{k}_{i,0} = 1$ for any i)

throughout simulations in this paper.

A1.1.4 Incomes

The capital income of a household in Economy *i* in period *t* ($x_{K,t}$) is calculated by

$$x_{K,i,t} = r_t \breve{k}_{i,t} \ ,$$

where r_t is the real interest rate in period *t* and

$$r_t = \frac{\partial k_{i,t}}{\partial y_{i,t}} \ . \qquad \text{(A1.3)}$$

Hence, by equations (A1.1) and (A1.3), the real interest rate r_t is calculated by

$$r_t = (1-\alpha)k_{i,t}^{-\alpha} = (1-\alpha)\left(\frac{\sum_{i=1}^{H}\breve{k}_{i,t}}{H}\right)^{-\alpha} \ .$$

The labor income of a household in Economy *i* in period *t* ($x_{L,i,t}$) is calculated by extracting its capital income from its production such that

$$x_{L,i,t} = y_{i,t} - r_t k_{i,t} = y_{i,t} - r_t \frac{\sum_{i=1}^{H}\breve{k}_{i,t}}{H} \ .$$

Because the amount of capital used and the amount of labor inputted by a household is identical for any household in any economy when $\omega_i = 1$, household labor income is identical across economies. Note that if productivity ($\omega_{i.t}$) is heterogeneous among economies, production and labor income differ in proportion to their productivities. Note also that in a homogeneous population, the labor income becomes equal to $\alpha y_{i,t}$ for any household.

A1.1.5 Savings

Household savings in Economy *i* in period *t* ($s_{i,t}$) are calculated by

$$s_{i,t} = x_{L,i,t} + x_{K,i,t} - c_{i,t} \ .$$

In period *t* + 1, these savings ($s_{i,t}$) are added to the capital the household owns, and therefore,

$$\breve{k}_{i,t+1} = \breve{k}_{i,t} + s_{i,t} \ .$$

A1.2 Cconsumption formula

A1.2.1 Consumption formula in a homogeneous population

For a simulation to be implemented, the consumption formula that describes how a household adjusts its consumptions needs to be set beforehand. However, under the MDC-based procedure, there is no strict consumption formula for households. A household just has to behave roughly feeling and guessing (i.e., not exactly calculating) its CWR and CWR at MDC in each period. It increases its consumption somewhat if it feels that $\varGamma(\tilde{s}_i)$ is larger than $\varGamma_{i,t}$ and decreases its consumption somewhat if it feels the opposite way. The amount of the increase/decrease will differ by period. In this sense, the actual formula of consumption under the MDC-based procedure is lax and vague; therefore, it is difficult to set a strict consumption formula with a mathematical functional form.

Nevertheless, if we consider the average consumption over some periods (i.e., moving averages), it will be possible to describe a mathematical form of the consumption formula because households will behave in a similar manner on average. Considering this nature, I introduce the following simple consumption formula because it seems to simply but correctly capture the behavior of households under the MDC-based procedure on average. Please note that that this consumption formula is not the only possible choice. Other, possibly more complex and subtle, functional forms could be chosen.

Consumption formula 1: The consumption of a household in Economy *i* in period *t* is

$$c_{i,t} = \left(x_{L,i,t} + x_{K,i,t}\right)\left(\frac{\varGamma(\tilde{s}_i)}{\varGamma_{i,t}}\right)^{\gamma} \ , \tag{A1.4}$$

where $\varGamma_{i,t}$ is the CWR of household in Economy *i* in period *t* and γ is a parameter.

Because

$$\theta_i = \left(\frac{1-\alpha}{\alpha}\right)\varGamma(\tilde{s}_i) \ , \tag{A1.5}$$

as shown in Harashima (2018a, 2021a, 2022a), by equation (A1.5), equation (A1.4) is equal to

$$c_{i,t} = (x_{L,i,t} + x_{K,i,t}) \left(\frac{\theta_i}{\Gamma_{i,t} \frac{1-\alpha}{\alpha}} \right)^{\gamma} .$$

Athough a household is set to precisely follow equation (A1.4) in the simulations, in reality, they do not behave by calculating equation (A1.4). Furthermore, they are not even aware of Consumption formula 1 itself and cannot know the exact numerical value of each $\Gamma(\tilde{s}_i) = \theta_i \alpha/(1-\alpha)$. Instead, households feel and guess whether they should increase or decrease consumption considering their income and wealth.

That is, Consumption formula 1 is set only for the convenience of calculation in the simulation. It seems to well capture the essence of household behavior in that it increases or decreases consumption depending on a household's feelings with regard to $\Gamma_{i,t}$ and $\Gamma(\tilde{s}_i)$. In this context, the value of parameter γ represents the average adjustment velocity of increase or decrease in consumption.

Consumption formula 1 means that a household's consumption is roughly equal to the sum of its incomes $(x_{L,i,t} + x_{K,i,t})$. The reason for this equality is that there is no technological progress and capital depreciation, so savings stay around zero at the stabilized (steady) state. As mentioned above, the adjustment velocity of consumption in each period is determined by the value of γ in equation (A1.4). As the value of γ is larger, a stabilized (steady) state can be achieved more quickly (if it can be achieved). In this paper, I set the value of γ to be 0.5.

A1.2.2 Consumption formula in a heterogeneous population

As shown in Harashima (2018a, 2021a, 2022a), in a heterogeneous population, a household behaving under the MDC-based procedure does not use its CWR ($\Gamma_{i,t}$) to make decisions about its consumption. Instead, it uses an adjusted value of CWR considering the behaviors of other heterogeneous households and the government because the entire economic state of the country depends on these heterogeneous behaviors in a heterogeneous population. Accordingly, in a heterogeneous population, Consumption formula 1 has to be modified to accommodate the adjusted CWR. Let $\Gamma_{R,i,t}$ be the adjusted value of $\Gamma_{i,t}$ of a household in Economy i in period t and $\Gamma(S_t)$ be the CWR of the country (i.e., the aggregate capital-wage ratio).

A1.2.2.1 Consumption formula 2

Unilateral behavior implies that a household behaves supposing that other households must behave in the same manner as it does. In other words, it assumes that other households' preferences are almost identical to its preferences, or at least, its preferences are not exceptional but roughly the same as the preferences of the average household

(Harashima 2018a). If all households behaved in the same manner as a household in Economy i did, the real interest rate (r_t) would be equal to the household's $\Gamma_{R,i,t}(1-\alpha)/\alpha$ and eventually converge at its $\Gamma(\tilde{s}_i)(1-\alpha)/\alpha$. Hence, if a household in Economy i behaves unilaterally in a heterogeneous population, it feels and guesses that its $\Gamma_{R,i,t}$ $(1-\alpha)/\alpha$ is roughly identical to the real interest rate (r_t). That is, the real interest rate will be used as $\Gamma_{R,i,t}(1-\alpha)/\alpha$, and $r_t\alpha/(1-\alpha)$ will be used as its adjusted CWR ($\Gamma_{R,i,t}$).

Therefore, even if a unilaterally behaving household's raw (unadjusted) CWR is accidentally equal to its CWR at MDC, the household does not feel that it is at its MDC unless at the same time r_t is accidentally equal to its $\Gamma(\tilde{s}_i)(1-\alpha)/\alpha$. The household will instead feel that the value of r_t will soon change, and accordingly, its raw (unadjusted) CWR will also change soon. That is, it feels and guesses that the entire economic state of the country is not yet stabilized because r_t is not equal to its $\Gamma(\tilde{s}_i)(1-\alpha)/\alpha$. As a result, the household will still continue to change its consumption to accumulate or diminish capital (see Lemma 2 in Harashima, 2018a).

Considering the above-shown nature of the adjusted CWR, Consumption formula 1 can be modified to Consumption formula 2 to use in simulations with a heterogeneous population.

Consumption formula 2: In a heterogeneous population, the consumption of a household in Economy i in period t is

$$c_{i,t} = \left(x_{L,i,t} + x_{K,i,t}\right)\left(\frac{\Gamma(\tilde{s}_i)}{\Gamma_{R,i,t}}\right)^{\gamma}$$
$$= \left(x_{L,i,t} + x_{K,i,t}\right)\left(\frac{\Gamma(\tilde{s}_i)}{r_t\dfrac{\alpha}{1-\alpha}}\right)^{\gamma} = \left(x_{L,i,t} + x_{K,i,t}\right)\left(\frac{\Gamma(\tilde{s}_i)\dfrac{1-\alpha}{\alpha}}{r_t}\right)^{\gamma} \qquad \text{(A1.6)}$$

and equivalently, by equations (A1.5) and (A1.6),

$$c_{i,t} = \left(x_{L,i,t} + x_{K,i,t}\right)\left(\frac{\theta_i}{r_t}\right)^{\gamma} .$$

As with $\Gamma_{i,t}$ in Consumption formula 1, the use of r_t in equation (A1.6) does not mean that households always actually behave by paying attention to r_t. What Consumption formula 2 means is that, on average, unilaterally behaving households will feel and guess that r_t represents their adjusted CWRs.

Under the RTP-based procedure, a household changes its consumption according to

$$\frac{\dot{c}_{i,t}}{c_{i,t}}=\varepsilon^{-1}(r_t-\theta_i)\ ,$$

where ε is the degree of relative risk aversion. That is, a household changes its consumption by comparing r_t and its $\theta_i=\Gamma(\tilde{s}_i)(1-\alpha)/\alpha$. The household changes consumption as r_t increasingly differs from $\theta_i=\Gamma(\tilde{s}_i)(1-\alpha)/\alpha$. This household's behavior under the RTP-based procedure is very similar to that according to Consumption formula 2, which means that the formula is basically consistent with a household's behavior under the RTP-based procedure.

In addition, in a homogeneous population, r_t is always equal to a homogenous household's $\Gamma_{i,t}(1-\alpha)/\alpha$ because all households behave in the same manner. Hence, equation (A1.4) is practically identical to equation (A1.6) (i.e., Consumption formula 1 is practically identical to Consumption formula 2) because $\Gamma_{i,t}$ in equation (A1.4) can be replaced with $r_t\frac{\alpha}{1-\alpha}$.

A1.2.2.2 Consumption formula 2-a

In Consumption formula 2, a household is supposed to feel that its preferences are not exceptional and almost the same as the preferences of the average household, but it may not actually feel that way. It may instead feel that its preferences are different from those of the average household. In this case, the household will not only feel its preferences are different, but it will also have to guess how far its preferences are from the average (i.e., by how much its adjusted CWR is different from the real interest rate).

For example, a household in Economy *i* may feel and guess that its adjusted CWR is

$$\Gamma_{R,i,t}=\frac{\alpha}{1-\alpha}\ (r_t+\chi_i) \qquad \text{(A1.7)}$$

instead of $\Gamma_{R,i,t}=r_t\frac{\alpha}{1-\alpha}$ in Consumption formula 2, where χ_i is a constant and $\chi_i\neq\chi_j$ for any *i* and *j*. χ_i represents the magnitude of how much a household in Economy *i* feels it is different from the average household. I refer to a modified version of Consumption formula 2 in which $r_t\frac{\alpha}{1-\alpha}$ is replaced with $\frac{\alpha}{1-\alpha}\ (r_t+\chi_i)$ shown in equation (A1.7) as Consumption formula 2-a. In this case, a household in Economy *i* behaves feeling that

$$\Gamma_{R,i,t}=\frac{\alpha}{1-\alpha}\ (r_t+\chi_i)=\Gamma_{i,t} \qquad \text{(A1.8)}$$

holds at a stabilized (steady) state that will be realized at some point in the future.

A1.2.2.3 Consumption formula 2-b

In both Consumption formulae 2 and 2-a, the raw (unadjusted) CWR is not included and therefore plays no role. Nevertheless, a household may utilize a piece of information derived from its raw (unadjusted) CWR because past behaviors may contain some useful information for guiding future behavior. As indicated in Section A1.2.2.2, χ_i is a parameter that indicates how far a household is from the average household. In general, the value of the parameter should be adjusted if households obtain any new and additional pieces of information. This implies that a piece of information derived from the raw (unadjusted) CWR may be used to adjust the value of parameter χ_i.

For example, a household in Economy *i* may use its raw (unadjusted) CWR ($\Gamma_{i,t}$) to adjust the value of χ_i such that

$$\chi_{i,t} = \chi_{i,t-1} + \zeta_i \left(\Gamma_{i,t} \frac{1-\alpha}{\alpha} - r_{t-1} - \chi_{i,t-1} \right) , \tag{A1.9}$$

where $\chi_{i,t}$ is χ_i in period *t*, and ζ_i is a positive constant and its value is close to zero. Equation (A1.9) means that a household in Economy *i* increases the value of $\chi_{i,t}$ a little if its raw (unadjusted) CWR is higher than its adjusted CWR ($r_{t-1} + \chi_{i,t-1}$) in the previous period and vice versa. It fine-tunes $\chi_{i,t}$ in this manner because it feels that equation (A1.8) will eventually hold at some point in the future, as shown in Section A1.2.2.2. The value of ζ_i is close to zero because $\Gamma_{i,t}$ is highly likely to be almost equal to $\Gamma_{i,t-1}$, and therefore, the guess of $\chi_{i,t}$ in period *t* will not change largely from that of $\chi_{i,t-1}$ in period $t-1$. I refer to the modified version of Consumption formula 2-a in which χ_i is replaced with $\chi_{i,t}$ shown in equation (A1.9) as Consumption formula 2-b.

A1.3 Rule of government transfer

Although governments implement transfers among households in complex and subtle manners, a simple bang-bang (two-step) control is adopted in simulations in this paper as the rule of government transfer for simplicity. In addition, government transfers in each period are assumed to be added to or extracted from the capital of each relevant household in the next period.

In simulations with government transfers, it is assumed for simplicity that there are two economies (Economies 1 and 2) in a country, the economies are identical except for each $\Gamma(\tilde{s}_i)(1-\alpha)/\alpha = \theta_i$, and all households in each economy are identical. Let κ be the $\check{k}_{1,t}$ that a government aims for to force a household in Economy 1 to own capital at a stabilized (steady) state (i.e., κ is the target value set by the government). Under these conditions, the bang-bang (two-step) control of government transfers is set as follows.

Transfer rule: The amount of government transfers from a household in Economy 1 to a household in Economy 2 in period t is T_{low} if $\check{k}_{1,t}$ is lower than κ and T_{high} if $\check{k}_{1,t}$ is higher than κ, where T_{low} and T_{high} are constant amounts of capital predetermined by the government.

In the simulations, I set T_{low} to be -0.1 and T_{high} to be 0.5. The value of κ is varied in each simulation depending on what stabilized (steady) state the government is aiming to achieve. Note that because of the discontinuous control signal in bang-bang (two-step) control, flow variables may show discontinuous zigzag paths but stock variables can move relatively smoothly. These zigzag paths may look unnatural, but they are generated only because of the bang-bang (two-step) control method that is adopted for simplicity.

Even if a household knows about the existence of government transfers, it still behaves based on Consumption formula 2 (or 2-a and 2-b) with no government transfer. That is, a household uses $x_{L,i,t} + x_{K,i,t}$, not $x_{L,i,t} + x_{K,i,t}$ + government transfers (T_{low} or T_{high}), as the "base" consumption in determining whether it should increase or decrease its consumption. This behavior superficially may mean that a household does not consider government transfers in the process of adjusting its CWR. However, it is implicitly assumed that a household knows that government transfers exist and that they are an exogenous factor. Therefore, the household feels that the transfers should be removed from the elements that it can change or control freely. Furthermore, it is implicitly assumed that a household correctly knows the exact amount of government transfers.

However, these assumptions may be oversimplifications, and they can be relaxed to allow for incorrect guesses on the amount of government transfers. This relaxation enables a household to use $x_{L,i,t} + x_{K,i,t}$ + government transfers (T_{low} or T_{high}) instead of $x_{L,i,t} + x_{K,i,t}$ in determining its consumption.

APPENDIX 2: The MDC-based procedure

A2.1 "Comfortability" of CWR

Let k_t and w_t be per capita capital and wage (labor income), respectively, in period t. Under the MDC-based procedure, a household should first subjectively evaluate the value of $\frac{\tilde{w}_t}{\tilde{k}_t}$ where $\tilde{k}_t$ and $\tilde{w}_t$ are household k_t and w_t, respectively. Let Γ be the subjective valuation of $\frac{\tilde{w}_t}{\tilde{k}_t}$ by a household and Γ_i be the value of $\frac{\tilde{w}_t}{\tilde{k}_t}$ of household i ($i = 1, 2, 3, \ldots$, *M*). Each household assesses whether it feels comfortable with its current Γ (i.e., its combination of income and capital expressed by CWR). "Comfortable" in this context means "at ease," "not anxious," and other similar feelings.

Let the "degree of comfortability" (DOC) represent how comfortable a household feels with its Γ. The higher the value of DOC, the more a household feels comfortable with its Γ. For each household, there will be a most comfortable CWR value because the household will feel less comfortable if CWR is either too high or too low. That is, for each household, a maximum DOC exists. Let $\tilde{s}$ be a household's state at which its DOC is the maximum (MDC). MDC therefore indicates the state at which the combination of revenues and assets is felt most comfortable. Let $\Gamma(\tilde{s})$ be a household's Γ when it is at $\tilde{s}$. $\Gamma(\tilde{s})$ indicates the Γ that gives a household its MDC, and $\Gamma(\tilde{s}_i)$ is household *i*'s Γ_i when it is at $\tilde{s}_i$.

A2.2 Homogeneous population

I first examine the behavior of households in a homogeneous population (i.e., all households are assumed to be identical).

A2.2.1 Rules

Household *i* should act according to the following rules:

Rule 1-1: If household *i* feels that the current Γ_i is equal to $\Gamma(\tilde{s}_i)$, it maintains the same level of consumption for any *i*.
Rule 1-2: If household *i* feels that the current Γ_i is not equal to $\Gamma(\tilde{s}_i)$, it adjusts its level of consumption until it feels that Γ_i is equal to $\Gamma(\tilde{s}_i)$ for any *i*.

A2.2.2 Steady state

Households can reach a steady state even if they behave only according to Rules 1-1 and 1-2. Let S_t be the state of the entire economy in period *t* and $\Gamma(S_t)$ be the value of $\frac{w_t}{k_t}$ of the entire economy at S_t (i.e., the economy's average CWR). In addition, let $\tilde{S}_{MDC}$ be the steady state at which MDC is achieved and kept constant by all households, and $\Gamma(\tilde{S}_{MDC})$ be $\Gamma(S_t)$ for $S_t = \tilde{S}_{MDC}$. Let also $\tilde{S}_{RTP}$ be the steady state under the RTP-based procedure; that is, it is the steady state in a Ramsey-type growth model in which households behave based on rational expectations generated by discounting utilities by θ, where θ (> 0) is the RTP of a household. In addition, let $\Gamma(\tilde{S}_{RTP})$ be $\Gamma(S_t)$ for $S_t = \tilde{S}_{RTP}$.

Proposition 1: If households behave according to Rules 1-1 and 1-2, and if the value of

θ that is calculated from the values of variables at $\tilde{S}_{MDC}$ is used as the value of θ under the RTP-based procedure in an economy where θ is identical for all households, then $\Gamma(\tilde{S}_{MDC}) = \Gamma(\tilde{S}_{RTP})$.
Proof: See Harashima (2018a).

Proposition 1 indicates that we can interpret $\tilde{S}_{MDC}$ to be equivalent to $\tilde{S}_{RTP}$. This means that both the MDC-based and RTP-based procedures can function equivalently and that CWR at MDC can be substituted for RTP as a guide for household behavior.

A2.3 Heterogeneous population

In actuality, however, households are not identical—they are heterogeneous—and if heterogeneous households behave unilaterally, there is no guarantee that a steady state other than corner solutions exists (Becker 1980; Harashima 2010, 2012). However, Harashima (2010, 2012) has shown that SH exists under the RTP-based procedure. In addition, Harashima (2018a) has shown that SH also exists under the MDC-based procedure, although Rules 1-1 and 1-2 have to be revised, and a rule for the government should be added in a heterogeneous population.

Suppose that households are identical except for their MDCs (i.e., their values of $\Gamma(\tilde{s})$). Let $\tilde{S}_{MDC,SH}$ be the steady state at which MDC is achieved and kept constant by any household (i.e., SH in a heterogeneous population under the MDC-based procedure), and let $\Gamma(\tilde{S}_{MDC,SH})$ be $\Gamma(S_t)$ for $S_t = \tilde{S}_{MDC,SH}$. In addition, let Γ_R be a household's numerically adjusted value of Γ for SH based on its estimated value of $\Gamma(\tilde{S}_{MDC,SH})$ and several other related values. Specifically, let $\Gamma_{R,i}$ be Γ_R of household i, T be the net transfer that a household receives from the government with regard to SH, and T_i be the net transfer that household i receives ($i = 1,2,3, \dots , M$).

A2.3.1 Revised and additional rules

Household i should act according to the following rules in a heterogeneous population:

Rule 2-1: If household i feels that the current $\Gamma_{R,i}$ is equal to $\Gamma(\tilde{s}_i)$, it maintains the same level of consumption as before for any i.
Rule 2-2: If household i feels that the current $\Gamma_{R,i}$ is not equal to $\Gamma(\tilde{s}_i)$, it adjusts its level of consumption or revises its estimated value of $\Gamma(\tilde{S}_{MDC,SH})$ so that it perceives that $\Gamma_{R,i}$ is equal to $\Gamma(\tilde{s}_i)$ for any i.

At the same time, the government should act according to the following rule:

Rule 3: The government adjusts T_i for some i if necessary so as to make the number of votes cast in elections in response to increases in the level of economic inequality equivalent to the number cast in response to decreases.

A2.3.2 Steady state

Even if households and the government behave according to Rules 2-1, 2-2, and 3, there is no guarantee that the economy can reach $\tilde{S}_{MDC,SH}$. However, thanks to the government's intervention, SH can be approximately achieved. Let $\tilde{S}_{MDC,SH,ap}$ be the state at which $\tilde{S}_{MDC,SH}$ is approximately achieved (an approximate SH), and $\Gamma(\tilde{S}_{MDC,SH,ap})$ be $\Gamma(S_t)$ at $\tilde{S}_{MDC,SH,ap}$ on average. Here, let $\tilde{S}_{RTP,SH}$ be the steady state that satisfies SH under the RTP-based procedure, that is, in a Ramsey-type growth model in which households that are identical except for their θs behave generating rational expectations by discounting utilities by their θs. Furthermore, let $\Gamma(\tilde{S}_{RTP,SH})$ be $\Gamma(S_t)$ for $S_t = \tilde{S}_{RTP,SH}$.

Proposition 2: If households are identical except for their values of $\Gamma(\tilde{s})$ and behave unilaterally according to Rules 2-1 and 2-2, if the government behaves according to Rule 3, and if the value of θ_i that is calculated back from the values of variables at $\tilde{S}_{MDC,SH,ap}$ is used as the value of θ_i for any i under the RTP-based procedure in an economy where households are identical except for their θs, then $\Gamma(\tilde{S}_{MDC,SH,ap}) = \Gamma(\tilde{S}_{RTP,SH})$.
Proof: See Harashima (2018a).

Proposition 2 indicates that we can interpret $\tilde{S}_{MDC,SH,ap}$ as being equivalent to $\tilde{S}_{RTP,SH}$. No matter what values of T, Γ_R, and $\Gamma(\tilde{S}_{MDC,SH})$ are estimated by households, any $\tilde{S}_{MDC,SH,ap}$ can be interpreted as the objectively correct and true steady state. In addition, a government need not necessarily provide the objectively correct T_i for $\tilde{S}_{MDC,SH,ap}$ even though the $\tilde{S}_{MDC,SH,ap}$ is interpreted as objectively correct and true.

APPENDIX 3: Sustainable heterogeneity

A3.1 SH

Here, three heterogeneities—RTP, degree of risk aversion (DRA), and productivity—are considered. Suppose that there are two economies (Economy 1 and Economy 2) that are identical except for RTP, DRA, and productivity. Each economy is interpreted as representing a group of identical households, and the population in each economy is constant and sufficiently large. The economies are fully open to each other, and goods, services, and capital are freely transacted between them, but labor is immobilized in each economy. Households also provide laborers whose abilities are one of the factors that determine the productivity of each economy. Each economy can be interpreted as representing either a country or a group of identical households in a country. Usually, the concept of the balance of payments is used only for international transactions, but in this paper, this concept and the associated terminology are used even if each economy represents a group of identical households in a country.

The production function of Economy i (= 1, 2) is

$$y_{i,t} = A_t^{\alpha} k_{i,t}^{1-\alpha} \ ,$$

where $y_{i,t}$ and $k_{i,t}$ are the production and capital of Economy i in period t, respectively; A_t is technology in period t; and α ($0 < \alpha < 1$) is a constant and indicates the labor share. All variables are expressed in per capita terms. The current account balance in Economy 1 is τ_t and that in Economy 2 is $-\tau_t$. The accumulated current account balance

$$\int_0^t \tau_s ds$$

mirrors capital flows between the two economies. The economy with current account surpluses invests them in the other economy. Since $\frac{\partial y_{1,t}}{\partial k_{1,t}} \left(= \frac{\partial y_{2,t}}{\partial k_{2,t}} \right)$ is returns on investments,

$$\frac{\partial y_{1,t}}{\partial k_{1,t}} \int_0^t \tau_s ds \ \text{ and } \ \frac{\partial y_{2,t}}{\partial k_{2,t}} \int_0^t \tau_s ds$$

represent income receipts or payments on the assets that an economy owns in the other economy. Hence,

$$\tau_t - \frac{\partial y_{2,t}}{\partial k_{2,t}} \int_0^t \tau_s ds$$

is the balance on goods and services of Economy 1, and

$$\frac{\partial y_{1,t}}{\partial k_{1,t}} \int_0^t \tau_s ds - \tau_t$$

is that of Economy 2. Because the current account balance mirrors capital flows between the economies, the balance is a function of capital in both economies such that

$$\tau_t = \kappa(k_{1,t}, k_{2,t}) .$$

This two-economy model can be easily extended to a multi-economy model. Suppose that a country consists of *H* economies that are identical except for RTP, DRA, and productivity (Economy 1, Economy 2, … , Economy *H*). Households within each economy are identical. $c_{i,t}$, $k_{i,t}$, and $y_{i,t}$ are the per capita consumption, capital, and output of Economy *i* in period *t*, respectively; and θ_i, $\varepsilon_q = -\frac{c_{1,t}u_i''}{u_i'}$, ω_i, and u_i are the RTP, DRA, productivity, and utility function of a household in Economy *i*, respectively (*i* = 1, 2, …, *H*). The production function of Economy *i* is

$$y_{i,t} = \omega_i A_t^{\alpha} k_{i,t}^{1-\alpha} .$$

In addition, $\tau_{i,j,t}$ is the current account balance of Economy *i* with Economy *j*, where *i*, *j* = 1, 2, … , *H* and $i \neq j$.

Harashima (2010) showed that if, and only if,

$$\lim_{t\to\infty} \frac{\dot{c}_{i,t}}{c_{i,t}} = \left(\frac{\sum_{q=1}^{H} \varepsilon_q \omega_q}{\sum_{q=1}^{H} \omega_q}\right)^{-1} \left\{\left[\frac{\varpi\alpha \sum_{q=1}^{H} \omega_q}{Hmv(1-\alpha)}\right]^{\alpha} - \frac{\sum_{q=1}^{H} \theta_q \omega_q}{\sum_{q=1}^{H} \omega_q}\right\} \tag{A3.1}$$

for any *i* (= 1, 2, … , *H*), all the optimality conditions of all heterogeneous economies are satisfied, where *m*, *v*, and ϖ are positive constants. Furthermore, if, and only if, equation (A3.1) holds,

$$\lim_{t\to\infty} \frac{\dot{c}_{i,t}}{c_{i,t}} = \lim_{t\to\infty} \frac{\dot{k}_{i,t}}{k_{i,t}} = \lim_{t\to\infty} \frac{\dot{y}_{i,t}}{y_{i,t}} = \lim_{t\to\infty} \frac{\dot{A}_t}{A_t} = \lim_{t\to\infty} \frac{\dot{\tau}_{i,j,t}}{\tau_{i,j,t}} = \lim_{t\to\infty} \frac{\frac{d\int_0^t \tau_{i,j,s} ds}{dt}}{\int_0^t \tau_{i,j,s} ds}$$

is satisfied for any *i* and *j* ($i \neq j$). Because all the optimality conditions of all heterogeneous economies are satisfied, the state at which equation (A3.1) holds is SH by definition.

A3.2 SH with government intervention

As shown above, SH is not necessarily naturally achieved, but if the government properly transfers money or other types of economic resources from some economies to other economies, SH is achieved.

Let Economy 1+2+…+ $(H-1)$ be the combined economy consisting of Economies 1, 2, …, and $(H-1)$. The population of Economy 1+2+… + $(H-1)$ is therefore $(H-1)$ times that of Economy i (= 1, 2, 3, …, H). $k_{1+2+\cdots+(H-1),t}$ indicates the capital of a household in Economy 1+2+…+ $(H-1)$ in period t. Let g_t be the amount of government transfers from a household in Economy 1+2+…+ $(H-1)$ to households in Economy H, and $\bar{g}_t$ be the ratio of g_t to $k_{1+2+\cdots+(H-1),t}$ in period t to achieve SH. That is,

$$g_t = \bar{g}_t k_{1+2+\cdots,+(H-1),t} \ .$$

$\bar{g}_t$ is solely determined by the government and therefore is an exogenous variable for households.

Harashima (2010) showed that if

$$\lim_{t\to\infty} \bar{g}_t = \left(\frac{\sum_{q=1}^{H}\varepsilon_q\omega_q}{\omega_H}\right)^{-1} \left\{\frac{\varepsilon_H\sum_{q=1}^{H}\omega_q - \sum_{q=1}^{H}\varepsilon_q\omega_q}{\sum_{q=1}^{H-1}\omega_q}\left[\frac{\varpi\alpha\sum_{q=1}^{H}\omega_q}{Hmv(1-\alpha)}\right]^{\alpha} - \frac{\varepsilon_H\sum_{q=1}^{H}\theta_q\omega_q - \theta_H\sum_{q=1}^{H}\varepsilon_q\omega_q}{\sum_{q=1}^{H-1}\omega_q}\right\}$$

is satisfied for any i (= 1, 2, …, H) in the case that Economy H is replaced with Economy i, then equation (A3.1) is satisfied (i.e., SH is achieved by government interventions even if households behave unilaterally). Because SH indicates a steady state, $\lim_{t\to\infty}\bar{g}_t$= constant.

Note that the amount of government transfers from households in Economy 1+2+ … + $(H-1)$ to a household in Economy H at SH is

$$(H-1)g_t = (H-1)\ k_{1+2+\cdots+(H-1),t}\lim_{t\to\infty}\bar{g}_t \ .$$

Note also that a negative value of g_t indicates that a positive amount of money or other type of economic resource is transferred from Economy H to Economy 1+2+ · · · + $(H-1)$ and vice versa.

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