



Munich Personal RePEc Archive

# Foreign Exchange Interventions in the New-Keynesian Model: Policy, Transmission, and Welfare

Yakhin, Yossi

Bank of Israel

12 December 2024

Online at <https://mpra.ub.uni-muenchen.de/123609/>  
MPRA Paper No. 123609, posted 11 Feb 2025 16:48 UTC

# Foreign Exchange Interventions in the New-Keynesian Model: Policy, Transmission, and Welfare\*

Yossi Yakhin<sup>†</sup>

December 2024

## Abstract

The paper introduces foreign exchange interventions (FXIs) into a standard New-Keynesian small open economy model. It solves for the optimal FXI policy, suggests an implementable policy rule, and studies the transmission mechanism of FXIs. Relying on the portfolio balance channel, deviations from the uncovered interest rate parity (UIP) reflect financial inefficiencies. Therefore, a policy rule that stabilizes the UIP premium moves the economy toward its optimal allocation, regardless of the type of shocks it faces. Augmenting the rule with foreign reserves smoothing further improves welfare. The paper discusses the conditions under which strict targeting of the UIP premium is optimal. FXIs are transmitted by affecting the UIP premium. Purchasing foreign reserves increases the UIP premium, thereby raising the effective return home agents face and depreciating the domestic currency. Consequently, domestic demand contracts and export expands. The results are robust to a variety of modeling alternatives for the financial sector.

JEL classification: E44, E58, F30, F31, F41, G15.

Keywords: Foreign Exchange Interventions, Policy Rule, UIP Premium, Monetary Policy, Open Economy Macroeconomics.

---

\* I thank Eliezer Borenstein, Louphou Coulibaly, Zvi Hercowitz, Eran Hoffmann, Alexey Khazanov, Vincent Sterk, and Amir Yaron for helpful comments and suggestions. I also thank seminar and conference participants at the Bank of Israel, the Hebrew University of Jerusalem, the Israel Economic Association, and the Central Bank Research Association (CEBRA). The views expressed in this paper are solely my own and they do not necessarily reflect those of the Bank of Israel.

<sup>†</sup> Research Department, Bank of Israel. Email: [Yossi.Yakhin@boi.org.il](mailto:Yossi.Yakhin@boi.org.il)

# 1 Introduction

About three-quarters of the IMF’s inflation-targeting members engage in some form of foreign exchange intervention (FXI).<sup>1</sup> This paper seeks to offer policy guidance on the implementation of FXIs for these countries.

The paper utilizes a standard New-Keynesian small open economy model, along the lines of Galí and Monacelli (2005), to analyze sterilized FXIs as an additional policy tool of the central bank, alongside the monetary interest rate.<sup>2,3</sup> It examines the transmission mechanism of FXIs, studies their role as a macroeconomic stabilizer, solves for the optimal FXI policy, and proposes an implementable policy rule. The paper then quantifies the potential welfare gains of using FXIs by calibrating the model to the Israeli economy.

Building on the portfolio balance channel, FXIs affect the economy by altering the uncovered interest rate parity (UIP) premium, thereby affecting effective returns and the exchange rate. These trigger intertemporal substitution in demand and intratemporal substitution between domestic and foreign goods. Exploiting this mechanism, the model suggests that FXIs should stabilize the UIP premium, thereby stabilizing the effective returns agents face and smoothing demand. This policy insulates the economy from the effects of financial shocks. It is also optimal against real shocks when export demand is perfectly elastic, provided that monetary policy can perfectly counteract the effects of nominal rigidities in the model; otherwise, tradeoffs emerge. The potential welfare gains of following optimal FXIs are modest but economically meaningful. All results are robust to a variety of modeling strategies regarding the microstructure of the financial markets.

The main contributions of the paper include analyzing the intertemporal channel of FXI transmission mechanism; demonstrating that stabilizing the UIP premium results in a near-optimal equilibrium regardless of the type of shocks the economy faces; and highlighting the role of FXIs as a macroeconomic stabilizer rather than merely a shield against carry trade costs.

Before describing the results in more detail, it is important to clarify why sterilized FXIs may affect the exchange rate and other equilibrium outcomes. More broadly, this question is related to the conditions under which the size and composition of the central bank balance sheet may matter for equilibrium allocations. Generally, the literature suggests that they do not matter, unless the assets traded by the central bank offer

---

<sup>1</sup> IMF (2023). See definitions therein for the classification of exchange rate arrangements.

<sup>2</sup> FXIs are sterilized in the sense that the interest rate is set as an independent policy tool.

<sup>3</sup> Throughout the paper, I refer to the central bank as the agency that decides on FXI policy, although in practice this is not always the case. While FXIs are typically executed by monetary agencies, in some countries they are directed by the treasury, e.g. the US and Japan, and in others the central bank is solely responsible for FXI policy, e.g. Israel and Switzerland. In this paper, the fiscal and monetary authorities are fully consolidated; therefore, the identity of the agency deciding on FXIs is irrelevant to the analysis.

benefits beyond their pecuniary return, or if different agents face different prices for these assets.<sup>4</sup> In the context of sterilized FXIs this means that one must introduce a financial friction so as to deviate from the UIP condition; otherwise, agents are indifferent between holding home and foreign assets, and sterilized FXIs are deemed ineffective.

Recent contributions have revived the argument for sterilized FXIs, e.g. [Benes et al. \(2015\)](#), [Cavallino \(2019\)](#), [Alla et al. \(2020\)](#), [Fanelli and Straub \(2021\)](#), [Faltermeier et al. \(2022\)](#), [Itskhoki and Mukhin \(2023\)](#). To make interventions effective, this literature builds on the portfolio balance channel. That is, in these models agents are willing to change the composition of their financial portfolio for a premium, giving rise to deviations from the UIP. While the details of the financial friction supporting this channel differ from one contribution to another, they arrive at similar UIP specifications. In that vein, [Yakhin \(2022\)](#) shows that, to a first order approximation, a simple reduced-form portfolio adjustment cost, as in [Schmitt-Grohé and Uribe \(2003\)](#), is isomorphic to more elaborate modeling strategies that attempt to capture the microfoundations of the financial friction.<sup>5</sup> Generating UIP deviations using a simple portfolio adjustment cost is therefore robust to a variety of interpretations regarding the underlying microstructure of the financial markets; hence, I adopt it in this paper.<sup>6</sup>

**Policy.** In the model, the UIP is an efficiency condition in the international financial markets; hence, deviations from the UIP entail welfare costs. Central banks should therefore restore efficiency by adopting an FXI policy rule that stabilizes the UIP premium. The advantage of using the UIP premium as a policy target is that it does not require knowledge about the shocks affecting the economy.<sup>7</sup> Optimal policies in [Cavallino](#)

---

<sup>4</sup> [Wallace \(1981\)](#) shows that under complete financial markets open market operations are irrelevant for equilibrium outcomes. [Backus and Kehoe \(1989\)](#) argue for the inefficacy of sterilized FXIs even under incomplete financial markets, provided that the central bank faces the same market incompleteness as other agents. [Cúrdia and Woodford \(2011\)](#) demonstrate that the central bank balance sheet has no role in equilibrium determination unless financial markets are "sufficiently impaired", in their language.

<sup>5</sup> [Yakhin \(2022\)](#) demonstrates that the simple portfolio adjustment cost is isomorphic, up to a first order approximation, to the financial frictions in [Gabaix and Maggiori \(2015\)](#) and in [Fanelli and Straub \(2021\)](#). In [Gabaix and Maggiori \(2015\)](#) the UIP premium arises due to limited commitment of financial intermediaries to honor their liabilities. In [Fanelli and Straub \(2021\)](#) regulatory exposure limits coupled with participation cost in the international financial markets drive a wedge in the UIP. [Uribe and Yue \(2006\)](#) provide microfoundations for the portfolio adjustment cost as operational costs of the financial sector. In [Itskhoki and Mukhin \(2021, 2023\)](#) risk aversion of financial intermediaries gives rise to a UIP premium. Under standard first order approximation their model is also isomorphic to the simple portfolio adjustment cost (see [Appendix A](#)). That said, the welfare function is approximated to second order and therefore it is not obvious that the different models result in identical normative implications. Nevertheless, assuming that the financial sector is owned entirely by domestic agents, as I assume throughout most of the paper, guarantees identical welfare criteria across models.

<sup>6</sup> Aside from allowing a theoretical discussion on sterilized FXIs, these frictions have empirical relevance as well. They help reconcile many of the long-standing exchange rate puzzles: the exchange rate disconnect, the sensitivity of exchange rates to financial flows, the profitability of carry trades and the forward premium puzzle, [Gabaix and Maggiori \(2015\)](#) and [Itskhoki and Mukhin \(2021\)](#).

<sup>7</sup> The UIP premium is not directly observed in the data, therefore using it as a policy target requires

(2019), Basu et al. (2020, 2023) and Itskhoki and Mukhin (2023) also support stabilizing the financial wedge in their models.

That said, full stabilization of the UIP premium is not necessarily optimal. That depends on the structure of the economy and the type of shocks to which the economy is subject. In the model, when the economy is hit by financial shocks, capital flow or risk premium shocks, FXIs are able to completely insulate the economy from their effect. Itskhoki and Mukhin (2023) report a similar result. When the economy is exposed to other shocks, e.g. productivity or demand shocks, strict targeting of the UIP premium is welfare improving, relative to the case of fixed foreign reserves, but it is not necessarily optimal. Its optimality depends on the market imperfections the central bank faces. One imperfection is clearly the financial friction that generates the UIP premium, while another imperfection may result from export demand. When global demand for home exports is downward sloping, the home economy possesses market power in the global goods market. If domestic exporters do not internalize the monopolistic power of the economy, then the central bank has an incentive to manipulate the terms of trade in their favor.<sup>8</sup> As a result, the central bank faces a tradeoff between stabilizing the UIP premium and exploiting the economy's market power. When export demand is perfectly elastic, strict UIP premium targeting turns optimal, provided that monetary policy faces no additional tradeoffs.

Augmenting the policy rule with foreign reserves smoothing also improves welfare. When agents only trade risk-free assets, their asset position exhibits unit root dynamics, even if shocks are stationary. For example, following a positive temporary productivity shock, agents permanently increase their asset holdings and use the additional return to increase consumption for perpetuity. A policy rule incorporating foreign reserves smoothing raises their persistence, and brings equilibrium closer to the optimal allocation.

A few comments on the policy analysis in relation to the literature are in order. First, UIP deviations provide carry trade opportunities, and therefore are costly for the home economy when exploited by foreigners, e.g. Cavallino (2019), Amador et al. (2020)<sup>9</sup>, Basu et al. (2020, 2023), Fanelli and Straub (2021). Stabilizing the UIP premium reduces carry trade opportunities, and hence reduces, on average, the loss of resources for the home economy. In this paper I assume that the financial sector is owned entirely by domestic agents, thereby abstracting from welfare gains resulting from this channel. This assumption focuses attention on the role of FXIs as a macroeconomic stabilizer, rather than a means of stripping carry trade profits from foreigners.

---

estimation. See Kalemli-Özcan and Varela (2023) for recent measurement of UIP deviations and documentation of their properties in emerging and advanced economies.

<sup>8</sup> This incentive is emphasized by Corsetti and Pesenti (2001).

<sup>9</sup> Amador et al. (2020) study *covered* interest rate parity deviations, but the argument is the same.

Second, since the financial markets are central to the transmission and efficacy of FXIs, some contributions focus solely on policy response to financial shocks, [Cavallino \(2019\)](#), [Alla et al. \(2020\)](#), [Chen et al. \(2023\)](#). The results of this paper justify a special focus on financial shocks, as FXIs can perfectly insulate the economy from their effect. However, as mentioned, the paper finds that FXIs are useful for stabilizing the economy from the effect of real shocks as well.

Finally, while numerous papers suggest that FXIs should aim to stabilize inflation, the output gap or the exchange rate, e.g. [Faltermeier et al. \(2022\)](#), [Benes et al. \(2015\)](#), [Chen et al. \(2023\)](#), the proposal to target the UIP premium is not new, [Basu et al. \(2020, 2023\)](#), [Adrian et al. \(2021\)](#) and [Itskhoki and Mukhin \(2023\)](#). However, augmenting the policy rule with foreign reserves smoothing and demonstrating that such a rule is welfare-improving regardless of the type of shocks hitting the economy is, to the best of my knowledge, a novel result.

**Transmission.** In the model, an exogenous rise in foreign reserves is partially financed by a reduction in the private sector holdings of foreign assets. This raises the UIP premium, which, in turn, increases the effective return home agents face on foreign assets. The higher return contracts domestic demand on impact. At the same time, the rise in foreign reserves increases demand for foreign currency and depreciates the value of the home currency; this reduces the terms of trade, i.e. home goods become cheaper relative to foreign goods. Both effects reduce consumption of imported goods, while cheaper home goods stimulates export demand. Overall net exports rises, which is the other source of financing for the rise in foreign reserves.

On the production side, the effect is ambiguous. A rise in foreign reserves expands equilibrium labor if the wealth effect on labor supply is sufficiently large. Total production then follows the same path as labor.

**Welfare.** I compare welfare in an economy with fixed foreign reserves to one where the central bank conducts optimal FXIs. In both cases, monetary policy sets the interest rate optimally. Hence, this comparison evaluates the role of FXIs over and above that of traditional monetary policy, as it exhausts any potential welfare gains from monetary policy before resorting to FXIs. *Lifetime* welfare gains amount to 2.4% of annual steady state consumption. That is, a representative household living in the fixed-reserves economy would be willing to pay a one-time amount of up to 2.4% of its annual steady state consumption to move to the optimal FXI economy. Comparing to an economy where the central bank follows a policy rule that stabilizes the UIP premium, this value is reduced to 0.8%. Augmenting the policy rule with foreign reserves smoothing, the welfare gains fall to merely 0.1%.<sup>10</sup> These results imply that the suggested policy rule brings the equilibrium allocation close to the optimal one.

---

<sup>10</sup>In these calculations, the model's parameters are set to match the characteristics of the Israeli economy.

As mentioned, the paper assumes that the financial sector is owned entirely by home agents. In the model, welfare declines as the proportion of foreign ownership rises. When foreigners own the entire financial sector the welfare loss amounts to 1.6% of annual steady state consumption.<sup>11</sup> While this is not a negligible figure, it is smaller than the stabilization benefits of following optimal FXIs when the financial sector is owned solely by home agents, highlighting their role as a macroeconomic stabilizer.

The rest of the paper is organized as follows. The next section presents the model. Section 3 develops the welfare criterion of a utilitarian social planner. Section 4 sets parameter values based on the characteristics of the Israeli economy. Section 5 studies the transmission mechanism of FXIs. Section 6 analyzes the optimal FXI response to various shocks and explores the conditions under which strict targeting of the UIP premium is optimal. Section 7 suggests a policy rule for FXIs. Section 8 conducts welfare analysis, and Section 9 concludes.

## 2 The Model

The model is a variant of Galí and Monacelli (2005). The world economy is composed of a continuum of small open economies, represented by the unit interval  $[0, 1]$ . The home economy is identified as country 0. All economies share the same preferences, technology and market structure. Foreign countries face identical realization of shocks. This assumption facilitates easier aggregation of quantities of foreign origin. Each economy consists of producers, households, employment agencies and a government.

Production is organized in three layers. In the first layer, monopolistically competitive producers use labor to produce differentiated intermediate goods. In the second layer, perfectly competitive assembly lines aggregate these intermediate goods into a homogeneous domestic product. The domestic good is used for government consumption, exports, and as an input in the production of a final good. Producers in the third layer, also operating in perfect competition, use the domestic good together with imported goods to compose the final good, which is used for private consumption.<sup>12</sup>

Households consume the final good, trade home and foreign bonds and supply dif-

---

<sup>11</sup>This exercise assumes optimal monetary and FXI policies, and compares welfare under full home ownership of the financial sector to the case where only foreigners own it.

<sup>12</sup>This setting deviates from Galí and Monacelli (2005) by introducing competitive domestic good producers, whereas in their model each monopolistic intermediate good producer takes account of the demand function for its product in every country. However, this setting results in the same equilibrium allocation as the setting of Galí and Monacelli (2005). The advantage of this approach is its simplicity and transparency in assuming that producers do not internalize the monopolistic power of their economy. Notice that in Galí and Monacelli (2005), intermediate good producers charge a markup based only on the elasticity of substitution between domestically produced goods, without considering the demand elasticity for their country's exports.

ferentiated labor skills. Employment agencies aggregate these skills into homogeneous labor services and supply them to the intermediate goods producers. The government consumes the domestic good, sets the domestic interest rate and conducts FXIs.

The business cycle is driven by productivity shocks, demand shocks (households' preferences, government expenditure, and world trade shocks), and financial shocks (capital flows, and "risk premium" shocks). The law of one price holds. Foreign inflation is constant at its steady state level. The world gross real interest rate is constant at  $\beta^{-1}$ , where  $\beta$  is the households' discount factor.<sup>13</sup> In the initial period the economy starts from an internationally symmetric steady state. Households are indexed by  $h$ , firms by  $f$  and countries by  $c$ . The exposition below focuses on the home economy.

## 2.1 Home, Foreign and Final Goods

Let  $y_t(f)$  denote domestic production of intermediate good  $f$  in the home economy. Total production of the home good,  $Y_t^H$ , is a constant elasticity of substitution (CES) aggregate of  $y_t(f)$ ,  $f \in [0, 1]$ :

$$Y_t^H = \left( \int_0^1 y_t(f)^{\frac{\varepsilon^L - 1}{\varepsilon^L}} df \right)^{\frac{\varepsilon^L}{\varepsilon^L - 1}} \quad (1)$$

$Y_t^H$  is used as input in the production of the final good,  $d_t^H$ , for government consumption,  $G_t$ , and for exports,  $EX_t$ :

$$Y_t^H = d_t^H + G_t + EX_t \quad (2)$$

Producers of  $Y_t^H$  are price takers. Given the price of intermediate  $f$ ,  $P_t^H(f)$ , the demand for each intermediate,  $y_t^d(f)$ , and the price index of the home good,  $P_t^H$ , are given by:

$$y_t^d(f) = \left( \frac{P_t^H(f)}{P_t^H} \right)^{-\varepsilon^L} Y_t^H \quad f \in [0, 1] \quad (3)$$

$$P_t^H = \left[ \int_0^1 P_t^H(f)^{1-\varepsilon^L} df \right]^{\frac{1}{1-\varepsilon^L}} \quad (4)$$

Producers of the final good are price takers. The final good is a CES aggregate of home inputs,  $d_t^H$ , and imported goods,  $IM_t$ , and it is only used for private consumption,  $C_t$ :

$$C_t = \left[ (1 - \lambda)^{\frac{1}{\varepsilon}} (d_t^H)^{\frac{\varepsilon - 1}{\varepsilon}} + \lambda^{\frac{1}{\varepsilon}} (IM_t)^{\frac{\varepsilon - 1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon - 1}} \quad (5)$$

---

<sup>13</sup>For sake of exposition, the derivation below includes foreign inflation and interest rate explicitly.



$\lambda \in [0, 1]$  is a measure of the openness of the economy. Imports are a CES aggregate of goods from all foreign country,  $IM_t(c)$  for  $c \in (0, 1]$ <sup>14</sup>:

$$IM_t = \left( \int_{0^+}^1 IM_t(c)^{\frac{\varepsilon^* - 1}{\varepsilon^*}} dc \right)^{\frac{\varepsilon^*}{\varepsilon^* - 1}}$$

Letting  $P_t^F(c)$  denote the home-currency price of imports from country  $c$ , demand for  $IM_t(c)$  and the price index of total imports,  $P_t^F$ , are given by:

$$IM_t(c) = \left( \frac{P_t^F(c)}{P_t^F} \right)^{-\varepsilon^*} IM_t \quad c \in (0, 1] \quad (6)$$

$$P_t^F = \left[ \int_{0^+}^1 P_t^F(c)^{1-\varepsilon^*} dc \right]^{\frac{1}{1-\varepsilon^*}} \quad (7)$$

Finally, demand for  $d_t^H$  and  $IM_t$ , and the consumer price index (CPI),  $P_t$ , are given by:

$$d_t^H = (1 - \lambda) \left( \frac{P_t^H}{P_t} \right)^{-\varepsilon} C_t \quad (8)$$

$$IM_t = \lambda \left( \frac{P_t^F}{P_t} \right)^{-\varepsilon} C_t \quad (9)$$

$$P_t = \left[ (1 - \lambda) (P_t^H)^{1-\varepsilon} + \lambda (P_t^F)^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}} \quad (10)$$

## 2.2 The Law of One Price, Terms of Trade, and Export Demand

Let  $S_t$  denote the nominal effective exchange rate of the home currency, that is the price of a basket of the foreign currencies in terms of the home currency.<sup>15</sup> Let  $P_t^{F*}$  denote the price of imports,  $IM_t$ , in foreign effective terms.  $P_t^{F*}$  is exogenous to the home economy. Assuming the law of one price holds:  $P_t^F = S_t P_t^{F*}$ . The foreign analog of equation (6) gives the demand for the home good by an arbitrary foreign country. Aggregating foreign demand and using the law of one price gives the global demand for home exports:

$$EX_t = TOT_t^{-\varepsilon^*} WT_t \quad (11)$$

where  $WT_t$  is world trade, which arises from aggregation of imports across all countries, and  $TOT_t$  is the terms of trade:  $TOT_t \equiv \frac{P_t^H}{P_t^F}$ .

For future reference, define prices relative to consumption price:  $p_t^H \equiv P_t^H/P_t$  and

<sup>14</sup>Each foreign good,  $IM_t(c)$ , is by itself a CES aggregate of country  $c$ 's intermediate goods with an elasticity of substitution of  $\varepsilon^L$ , just as home exports,  $EX_t$ , are composed of domestic intermediates.

<sup>15</sup>Formally,  $S_t \equiv \exp \left( \int_{0^+}^1 \log(S_t^c) dc \right)$ , where  $S_t^c$  is the exchange rate between the home currency and the currency of country  $c$ . Under the assumption that foreign countries are identical  $S_t^c = S_t, \forall c \in (0, 1]$ .

$p_t^F \equiv P_t^F/P_t$ . Note that  $p_t^F$  is the CPI-based real exchange rate.

### 2.3 Intermediate Goods Producers

Each intermediate good producer operates two departments, production and sales. The production department is a price taker. Given factor prices it operates efficiently to satisfy demand at the on-going prices. The sales department sets the price of the good.

**The Production Department.** The production function of firm  $f$  is given by:

$$y_t(f) = A_t n_t(f)^\alpha \quad 0 < \alpha \leq 1 \quad (12)$$

where  $n_t(f)$  is the firm's labor input, and  $A_t$  is an aggregate, country-specific, productivity shock. Total production,  $y_t(f)$ , is determined by demand, equation (3).

The government subsidizes labor at rate  $\tau_w$ .<sup>16</sup> Letting  $W_t$  denote the wage level, the real marginal cost of production, in terms of the home good, is given by:

$$RMC_t^H(f) = \frac{1 - \tau_w}{\alpha} \frac{W_t}{P_t^H} A_t^{-\frac{1}{\alpha}} y_t(f)^{\frac{1-\alpha}{\alpha}} \quad (13)$$

**The Sales Department.** The sales department sets the price of its good,  $P_t^H(f)$ . However, price setting is staggered across firms, à la Calvo (1983). The probability of price adjustment is  $1 - \xi_p$ . Whenever a firm is unable to freely adjust its price, the price is automatically scaled by the steady state gross inflation rate,  $\pi_{ss}$ . Otherwise, the firm maximizes the present discounted value of its expected profits under the new price. The standard solution applies. Optimal price setting results in:

$$\frac{P_{t/t}^H}{P_t} = \frac{\varepsilon^L}{\varepsilon^L - 1} \frac{E_t \sum_{s=0}^{\infty} \Lambda_{t,t+s} \xi_p^s \left( \frac{\pi_{t,t+s}}{\pi_{ss}^s} \right)^{\varepsilon^L} Y_{t+s}^H (p_{t+s}^H)^{1+\varepsilon^L} RMC_{t+s/t}^H}{E_t \sum_{s=0}^{\infty} \Lambda_{t,t+s} \xi_p^s \left( \frac{\pi_{t,t+s}}{\pi_{ss}^s} \right)^{\varepsilon^L - 1} Y_{t+s}^H (p_{t+s}^H)^{\varepsilon^L}} \quad (14)$$

where  $P_{t/t}^H$  is date  $t$  price of firms that reoptimize their prices on that date,  $RMC_{t+s/t}^H$  is date  $t+s$  real marginal cost of firms that last reoptimized on date  $t$ , and  $\pi_{t,t+s} \equiv P_{t+s}/P_t$ .  $\Lambda_{t,t+s}$  is the stochastic discount factor between time  $t$  and  $t+s$ . Firms discount future payoffs in accordance with the preferences of their shareholders—the households, that is  $\Lambda_{t,t+s} = \beta \frac{U_{C,t+s}}{U_{C,t}}$ , where  $U_{C,t}$  is the households' marginal utility of consumption. Taking first order approximation, gives rise to the standard New-Keynesian Phillips curve:

$$\widetilde{\pi}_t^H \cong \beta E_t (\widetilde{\pi}_{t+1}^H) + \frac{(1 - \xi_p)(1 - \beta \xi_p)}{\xi_p} \frac{\alpha}{\alpha + (1 - \alpha) \varepsilon^L} \widetilde{RMC}_t^H \quad (15)$$

---

<sup>16</sup>The role of the labor subsidy is discussed in section 3.

where tilded variables denote log-deviations from deterministic steady state,  $\pi_t^H \equiv P_t^H / P_{t-1}^H$ , and  $RM C_t^H$  is the average real marginal cost in the economy.

## 2.4 Employment Agencies and the Wage Index

Employment agencies are price takers. They aggregate households' differentiated labor efforts,  $n_t(h)$   $h \in [0, 1]$ , to construct a homogeneous labor input,  $N_t$ :

$$N_t = \left[ \int_0^1 n_t(h)^{\frac{\varepsilon^N - 1}{\varepsilon^N}} dh \right]^{\frac{\varepsilon^N}{\varepsilon^N - 1}} \quad (16)$$

$N_t$  is then supplied to the domestic intermediate goods producers.

Given the wage of each labor skill,  $W_t(h)$ , cost minimization results in the demand for each skill and the aggregate wage index:

$$n_t(h) = \left( \frac{W_t(h)}{W_t} \right)^{-\varepsilon^N} N_t \quad (17)$$

$$W_t = \left[ \int_0^1 W_t(h)^{1-\varepsilon^N} dh \right]^{\frac{1}{1-\varepsilon^N}} \quad (18)$$

## 2.5 Households

Households consume the final good, supply labor, and trade risk-free home and foreign nominal bonds.

Domestic (foreign) bonds cost one unit of the domestic (effective foreign) currency at date  $t$  and pay  $1 + i_t$  ( $1 + i_t^*$ ) units in  $t + 1$ .<sup>17</sup> Let  $B_t^{HH}(h)$  and  $B_t^{*,HH}(h)$  denote the quantities of home and foreign bonds, respectively, held by household  $h$ , and define:

$$b_t^{*,HH}(h) \equiv \frac{B_t^{*,HH}(h)}{P_t^{F*}} \quad , \quad \widehat{b}_t^{*,HH}(h) \equiv \frac{b_t^{*,HH}(h)}{TOT_{ss} Y_{ss}^{H,An.}}$$

$b_t^{*,HH}(h)$  is the foreign asset position of household  $h$  in units of foreign goods, and  $\widehat{b}_t^{*,HH}(h)$  is that position relative to annual steady state (per-capita) GDP,  $Y_{ss}^{H,An.}$ .

Trading in the international asset markets is costly. Households face a portfolio adjustment cost of  $\Theta(\widehat{b}_t^{*,HH}(h) - \widehat{\theta}_t^*)$ , measured in units of foreign goods, where  $\widehat{\theta}_t^*$  is an exogenous aggregate shock. The function  $\Theta(\cdot)$  satisfies  $\Theta(\cdot) \geq 0$ ,  $\Theta''(\cdot) > 0$ ,  $\Theta(0) = \Theta'(0) = 0$ . That is, households incur a cost whenever their foreign asset position deviates from some benchmark level,  $\widehat{\theta}_t^*$ .  $\widehat{\theta}_t^*$  is interpreted as a risk-premium shock, in the sense that a rise in  $\widehat{\theta}_t^*$  requires households to hold a higher level of  $\widehat{b}_t^{*,HH}(h)$  in order

---

<sup>17</sup>Formally, the foreign bond is an aggregate of bonds from all foreign countries and  $i_t^*$  is their effective return. Under the assumption that foreign countries are identical, it is safe to treat them as one entity.

to avoid the cost. The cost  $\Theta(\cdot)$  is interpreted as resources captured by the financial sector. Domestic households own a fraction  $\vartheta$  of the financial sector, and are rebated this fraction of the aggregate cost through dividend distribution of the financial firms. However, the households do not internalize this effect when choosing their asset position.

Introducing a financial friction is necessary because otherwise the UIP holds, and FXIs are deemed ineffective. The choice of a simple portfolio adjustment cost is motivated by [Yakhin \(2022\)](#), who demonstrates that to a first-order approximation this modelling strategy is isomorphic to models with richer microfoundations such as [Gabaix and Maggiori \(2015\)](#) and [Fanelli and Straub \(2021\)](#). [Appendix A](#) extends the result to the model of [Itskhoki and Mukhin \(2021, 2023\)](#) as well. Hence, the ad hoc portfolio adjustment cost is robust to different interpretations regarding the underlying microstructure of the financial markets.

Each household is endowed with a differentiated labor skill,  $n_t(h)$ , and holds monopolistic power over supplying it to the employment agencies. Wage setting is staggered à la [Calvo \(1983\)](#). The probability of wage adjustment is  $1 - \xi_w$ . When a household cannot freely adjust its wage, it is scaled by the steady state gross inflation rate,  $\pi_{ss}$ . The role of wage rigidity in the model is to generate policy tradeoffs (see discussion in [section 6.3](#)).

Household  $h$  periodical budget constraint is given by:

$$c_t(h) + \frac{S_t P_t^{F*} b_t^{*,HH}(h)}{P_t} + \frac{B_t^{HH}(h)}{P_t} = \frac{W_t(h)}{P_t} n_t(h) + \frac{(1 + i_{t-1}) B_{t-1}^{HH}(h)}{P_t} + \frac{\Pi_t + T_t}{P_t} + \frac{S_t P_t^{F*}}{P_t} \left[ \frac{1 + i_{t-1}^*}{\pi_t^{F*}} b_{t-1}^{*,HH}(h) - \Theta(\widehat{b}_t^{*,HH}(h) - \widehat{\theta}_t^*) \right]$$

where  $c_t(h)$  denotes consumption of household  $h$ ,  $\pi_t^{F*}$  is the foreign gross inflation rate  $\pi_t^{F*} \equiv P_t^{F*}/P_{t-1}^{F*}$ ,  $\Pi_t$  is firms' profits (including the rebate from domestically owned financial firms), and  $T_t$  denotes government lump-sum transfers. Households rank allocations of consumption and labor effort using utility function,  $U[c_t(h), n_t(h); \eta_t]$ , which satisfies the standard properties, where  $\eta_t$  is an aggregate preference shock. The exact formulation of the households' problem is spelled out in [Appendix B](#). The formulation assumes perfect insurance against the idiosyncratic wage risk. The households' optimality conditions are described below.

### 2.5.1 Households' Euler Equations and the UIP

Full insurance against idiosyncratic wage risk, coupled with equal asset endowment across households in the initial period, results in equal marginal utilities of consumption across households and equal foreign asset positions. Therefore, we can omit the household index

from these variables, and the Euler equations for home and foreign bonds read:

$$U_{C_t} = \beta (1 + i_t) E_t \left\{ U_{C_{t+1}} \frac{1}{\pi_{t+1}} \right\} \quad (19)$$

$$U_{C_t} \left[ 1 + \frac{\Theta' \left( \widehat{b}_t^{*,HH} - \widehat{\theta}_t^* \right)}{TOT_{ss} Y_{ss}^{H,An.}} \right] = \beta (1 + i_t^*) E_t \left\{ U_{C_{t+1}} \frac{\sigma_{t+1}}{\pi_{t+1}} \right\} \quad (20)$$

$$\text{where } \pi_{t+1} \equiv \frac{P_{t+1}}{P_t}, \quad \sigma_{t+1} \equiv \frac{S_{t+1}}{S_t}$$

Combining (19) and (20) yields the modified UIP. After log-linearization it reads:

$$\widetilde{(1 + i_t)} \cong \widetilde{(1 + i_t^*)} + E_t \{ \widetilde{\sigma}_{t+1} \} - \underbrace{\frac{\Theta''(0)}{TOT_{ss} Y_{ss}^{H,An.}} \left( \widehat{b}_t^{*,HH} - \widehat{\theta}_t^* \right)}_{\text{UIP premium}} \quad (21)$$

From this representation it is clear that the convexity of the portfolio adjustment cost introduces a wedge to the UIP condition. With  $\Theta''(0) = 0$  the UIP holds, exchange rate dynamics are governed by interest rate differentials, and sterilized FXIs are ineffective. As demonstrated below, FXIs work by altering the private sector holdings of foreign assets,  $\widehat{b}_t^{*,HH}$ , and hence the UIP premium.

### 2.5.2 Optimal Wage Setting

Optimal wage setting results in:

$$\frac{W_{t/t}}{P_t} = - \frac{\varepsilon^N}{\varepsilon^N - 1} \frac{\sum_{s=0}^{\infty} \xi_w^s \beta^s E_t \left\{ \left( \frac{W_{t+s}}{\pi_{ss}^s} \right)^{\varepsilon^N} N_{t+s} U_{n_{t+s/t}} \right\}}{\sum_{s=0}^{\infty} \xi_w^s \beta^s E_t \left\{ \left( \frac{W_{t+s}}{\pi_{ss}^s} \right)^{\varepsilon^N} N_{t+s} \frac{U_{C_{t+s}}}{\pi_{ss}^{t,t+s}} \right\}} \quad (22)$$

where  $W_{t/t}$  is date  $t$  nominal wage of households that reoptimize on that date, and  $U_{n_{t+s/t}} < 0$  is date  $t + s$  marginal utility of labor of households that last reoptimized on date  $t$ .

Define the real wage  $w_t \equiv W_t/P_t$ , wage inflation  $\pi_t^w \equiv W_t/W_{t-1}$ , and let  $MRS_t$  denote the average marginal rate of substitution between labor and consumption across households:  $MRS_t = - \sum_{s=0}^{\infty} (1 - \xi_w) \xi_w^s U_{n_{t/t-s}}/U_{C_t}$ . Using these definitions, taking first order approximation to (22) gives rise to the following wage inflation dynamics:

$$\widetilde{\pi}_t^w \cong \beta E_t \left( \widetilde{\pi}_{t+1}^w \right) - \frac{(1 - \xi_w \beta) (1 - \xi_w)}{\xi_w} \frac{1}{1 + \left( \gamma_{nn} - \frac{\gamma_{nc} \gamma_{cn}}{\gamma_{cc}} \right) \varepsilon^N} \left( \widetilde{w}_t - \widetilde{MRS}_t \right) \quad (23)$$

where tilded variables denote log-deviations from deterministic steady state, and  $\gamma_{xy} \equiv \frac{U_{xy}}{U_x} y_{ss}$  is the elasticity of the marginal utility of variable  $x$  with respect to variable  $y$

evaluated in steady state.  $\gamma_{nn} - \frac{\gamma_{nc}\gamma_{cn}}{\gamma_{cc}}$  is the *inverse* of the steady state Frisch elasticity of labor supply (see Appendix C). For an additive separable utility function, i.e. for  $\gamma_{nc} = \gamma_{cn} = 0$ , equation (23) takes its familiar form, e.g. Galí (2015) chapter 6.

## 2.6 The Government

The government operates in two arms: fiscal (treasury) and monetary (the central bank). The fiscal arm consumes the domestic good,  $G_t$ , subsidizes labor at a rate of  $\tau_w$ , and provides lump-sum transfers,  $T_t$ , to households. The monetary arm sets the nominal interest rate,  $i_t$ , and manages foreign reserves,  $FX_t$ .

Let  $B_t^{*,CB}$  denote foreign bonds held by the central bank; these constitute the stock of foreign reserves. Foreign reserves,  $FX_t$ , are measured in units of foreign goods:  $FX_t \equiv \frac{B_t^{*,CB}}{P_t^{F*}}$ . Similarly to households, the central bank faces a portfolio adjustment cost,  $\Theta^{CB}(FX_t)$ , also measured in units of foreign goods.

The function  $\Theta^{CB}(\cdot)$  satisfies  $\Theta^{CB}(\cdot) \geq 0$ ,  $\Theta^{CB''}(\cdot) > 0$ ,  $\Theta^{CB}(FX_{ss}) = \Theta^{CB'}(FX_{ss}) = 0$ . This adjustment cost is required for imposing stationarity on the linearized system when solving for the optimal FXI policy.<sup>18</sup> In the calibration below, I assume the central bank faces only minor adjustment costs.

Finally, let  $B_t^{ROW}$  and  $B_t^{HH}$  denote domestic bonds held by the rest of the world and by domestic households, respectively. Using these notations, the consolidated government budget constraint is given by:

$$\begin{aligned} & S_t P_t^{F*} \frac{1 + i_{t-1}^*}{\pi_t^{F*}} FX_{t-1} + (B_t^{ROW} + B_t^{HH}) \\ &= P_t^H G_t + \tau_w \int_0^1 W_t n_t(f) df + T_t + (1 + i_{t-1}) (B_{t-1}^{ROW} + B_{t-1}^{HH}) + S_t P_t^{F*} [FX_t + \Theta^{CB}(FX_t)] \end{aligned} \quad (24)$$

### 2.6.1 The Central Banks' Policy Tools

The central bank uses two instruments: the domestic nominal interest rate and FXIs.

Foreign reserves,  $FX_t$ , evolve according to:

$$FX_t = \frac{1 + i_{t-1}^*}{\pi_t^{F*}} FX_{t-1} + TOT_{ss} Y_{ss}^{H,An} \widehat{\phi}_t \quad (25)$$

where  $\widehat{\phi}_t$  denotes the purchase of foreign reserves relative to annual GDP.

The analysis below explores different FXI policies. For convenience, I use  $FX_t$  as the policy instrument rather than the interventions themselves,  $\widehat{\phi}_t$ . Throughout I assume

---

<sup>18</sup>See the optimality condition for foreign reserves,  $FX_t$ , equation (E.15) in Appendix E. This issue is akin to the one of closing small open economy models, as studied in Schmitt-Grohé and Uribe (2003), but instead of having unit root dynamics in the marginal utility of consumption of households it arises in the Lagrange multiplier of the balance of payments.

that the interest rate is set optimally.

## 2.7 Aggregate Technology and the Balance of Payments

**Aggregate Technology.** Aggregate labor input,  $N_t$ , is given by:

$$\int_0^1 n_t(f) df = N_t = \left[ \int_0^1 n_t(h) \frac{\varepsilon^N - 1}{\varepsilon^N} dh \right]^{\frac{\varepsilon^N}{\varepsilon^N - 1}}$$

Aggregating production of intermediate goods, equation (12), and using their demand functions (3), results in:

$$Y_t^H = A_t \left( \frac{N_t}{pd_t} \right)^\alpha \quad (26)$$

where  $pd_t \equiv \int_0^1 \left( \frac{P_t^H(f)}{P_t^H} \right)^{-\frac{\varepsilon^L}{\alpha}} df$  is a measure of price dispersion in the economy, which is second-order.

**The Balance of Payments.** To derive the balance of payments note that aggregate firms' profits,  $\Pi_t$ , are given by:

$$\Pi_t = P_t^H Y_t^H - (1 - \tau_w) W_t N_t + \vartheta S_t P_t^{F*} \left[ \Theta \left( \widehat{b}_t^{*,HH} - \widehat{\theta}_t^* \right) + \Theta^{CB} (FX_t) \right]$$

where the first two terms are the profits of the intermediate goods producers, and the last term is the rebate of portfolio adjustment costs. Consolidating the households' budget constraints and combining the result with the government's budget constraint and aggregate profits results in the balance of payments identity:

$$\begin{aligned} FX_t + TOT_{ss} Y_{ss}^{H,An} \widehat{b}_t^{*,HH} &= \frac{1 + i_{t-1}^*}{\pi_t^{F*}} \left( FX_{t-1} + TOT_{ss} Y_{ss}^{H,An} \widehat{b}_{t-1}^{*,HH} \right) \quad (27) \\ &- (1 - \vartheta) \left[ \Theta^{CB} (FX_t) + \Theta \left( \widehat{b}_t^{*,HH} - \widehat{\theta}_t^* \right) \right] \\ &+ TOT_{ss} Y_{ss}^{H,An} \widehat{\phi}_t^* + TOT_t EX_t - IM_t \end{aligned}$$

where  $\widehat{\phi}_t^*$  is capital inflows to the home economy relative to annual GDP.<sup>19</sup>  $\widehat{\phi}_t^*$  is exogenous.

## 2.8 Characterizing Equilibrium

This section spells out the linearized system of equations that characterize equilibrium in the model economy. I consider a symmetric global steady state, in which trade is balanced, prices of home and foreign goods are equal, households hold zero foreign assets

---

<sup>19</sup>The derivation of (27) uses the law of motion for the accumulation of domestic bonds by foreigners:  $b_t^{ROW} = \frac{1+i_{t-1}^*}{\sigma_t \pi_t^{F*}} b_{t-1}^{ROW} + TOT_{ss} Y_{ss}^{H,An} \widehat{\phi}_t^*$ , where  $b_t^{ROW} \equiv \frac{B_t^{ROW}}{S_t P_t^{F*}}$ , and  $\widehat{\phi}_t^*$  is the purchase of domestic bonds by foreigners (e.g. FXIs of foreign central banks) as a percent of annual domestic GDP.

position, and inflation rates are equal across countries, that is:

$$TOT_{ss}EX_{ss} = IM_{ss} \quad , \quad \widehat{b}_{ss}^{*,HH} = \widehat{\theta}_{ss}^* = 0 \quad , \quad TOT_{ss} = p_{ss}^H = p_{ss}^F = 1 \quad , \quad \pi_{ss}^H = \pi_{ss}^{F*}$$

The steady state inflation rates are set at an arbitrary level. Foreign reserves are held at an exogenous target level,  $FX^T$ . Under these conditions:

$$FX_{ss} = FX^T \quad , \quad \sigma_{ss} = 1 \quad , \quad \widehat{\phi}_{ss}^* = -\frac{1-\beta}{\beta} \frac{FX^T}{Y_{ss}^{H,An.}} \quad , \quad IM_{ss} = WT_{ss}$$

Note that although world trade is exogenous from the point of view of each economy, it is endogenous in the model and is pinned down by import demand. Also notice that capital inflows are negative in steady state, as they reflect interest payments to foreign central banks for their holdings of domestic bonds, which are part of their foreign reserves.

The following system of equations characterizes the equilibrium in the model. Tilded variables denote log deviations from deterministic steady state.

Optimal wage and price setting:

$$\widetilde{\pi}_t^w \cong \beta E_t(\widetilde{\pi}_{t+1}^w) - \frac{(1-\xi_w/\beta)(1-\xi_w)}{\xi_w} \frac{1}{1 + \left(\gamma_{nn} - \frac{\gamma_{nc}\gamma_{cn}}{\gamma_{cc}}\right) \varepsilon^N} \left(\widetilde{w}_t - \widetilde{U}_{N_t} + \widetilde{U}_{C_t}\right) \quad (28)$$

$$\widetilde{\pi}_t^H \cong \beta E_t(\widetilde{\pi}_{t+1}^H) + \frac{(1-\xi_p)(1-\beta\xi_p)}{\xi_p} \frac{\alpha}{\alpha + (1-\alpha)\varepsilon^L} \begin{bmatrix} \widetilde{w}_t - \widetilde{p}_t^H \\ -\widetilde{A}_t - (\alpha-1)\widetilde{N}_t \end{bmatrix} \quad (29)$$

The Euler equations:

$$\widetilde{U}_{C_t} \cong \widetilde{(1+i_t)} + E_t\{\widetilde{U}_{C_{t+1}}\} - E_t\{\pi_{t+1}\} \quad (30)$$

$$\widetilde{U}_{C_t} + \frac{\Theta''(0)}{Y_{ss}^{H,An.}} \left(\widehat{b}_t^{*,HH} - \widehat{\theta}_t^*\right) \cong \widetilde{(1+i_t^*)} + E_t\{\widetilde{U}_{C_{t+1}}\} + E_t\{\widetilde{\sigma}_{t+1}\} - E_t\{\widetilde{\pi}_{t+1}\} \quad (31)$$

Consumption and its composition:

$$\widetilde{C}_t \cong (1-\lambda)\widetilde{d}_t^H + \lambda\widetilde{IM}_t \quad (32)$$

$$\widetilde{d}_t^H \cong \widetilde{C}_t - \varepsilon\widetilde{p}_t^H \quad (33)$$

$$\widetilde{IM}_t \cong \widetilde{C}_t - \varepsilon\widetilde{p}_t^F \quad (34)$$

Export demand:

$$\widetilde{EX}_t \cong -\varepsilon^* \widetilde{TOT}_t + \widetilde{WT}_t \quad (35)$$



Technology, the resource constraint and the balance of payments:

$$\tilde{Y}_t^H \cong \tilde{A}_t + \alpha \tilde{N}_t \quad (36)$$

$$\tilde{Y}_t^H \cong (1 - \lambda) \frac{C_{ss}}{Y_{ss}^H} \tilde{d}_t^H + \frac{G_{ss}}{Y_{ss}^H} \tilde{G}_t + \lambda \frac{C_{ss}}{Y_{ss}^H} \widetilde{EX}_t \quad (37)$$

$$\begin{aligned} \frac{FX_{ss}}{Y_{ss}^{H,An.}} \widetilde{FX}_t + \widehat{b}_t^{*,HH} &\cong \beta^{-1} \left( \frac{FX_{ss}}{Y_{ss}^{H,An.}} \widetilde{FX}_{t-1} + \widehat{b}_{t-1}^{*,HH} \right) + \left( \widehat{\phi}_t^* - \widehat{\phi}_{ss}^* \right) \\ &+ \beta^{-1} \frac{FX_{ss}}{Y_{ss}^{H,An.}} \left[ \widetilde{(1 + i_{t-1}^*)} - \widetilde{\pi}_t^{F*} \right] + \frac{\lambda C_{ss}}{Y_{ss}^{H,An.}} \left( \widetilde{TOT}_t + \widetilde{EX}_t - \widetilde{IM}_t \right) \end{aligned} \quad (38)$$

Definitions and identities:

$$\tilde{U}_{N_t} \cong \gamma_{nc} \tilde{C}_t + \gamma_{nn} \tilde{N}_t + \gamma_{n\eta} \tilde{\eta}_t \quad (39)$$

$$\tilde{U}_{C_t} \cong \gamma_{cc} \tilde{C}_t + \gamma_{cn} \tilde{N}_t + \gamma_{c\eta} \tilde{\eta}_t \quad (40)$$

$$\tilde{w}_t - \tilde{w}_{t-1} \cong \tilde{\pi}_t^w - \tilde{\pi}_t \quad (41)$$

$$\tilde{p}_t^H - \tilde{p}_{t-1}^H \cong \tilde{\pi}_t^H - \tilde{\pi}_t \quad (42)$$

$$\tilde{p}_t^F - \tilde{p}_{t-1}^F \cong \tilde{\sigma}_t + \tilde{\pi}_t^{F*} - \tilde{\pi}_t \quad (43)$$

$$\widetilde{TOT}_t \cong \tilde{p}_t^H - \tilde{p}_t^F \quad (44)$$

This gives a system of 17 equations in 19 endogenous variables:  $\tilde{U}_{N_t}$ ,  $\tilde{U}_{C_t}$ ,  $\tilde{C}_t$ ,  $\tilde{N}_t$ ,  $\tilde{w}_t$ ,  $\tilde{\pi}_t^w$ ,  $\tilde{\pi}_t$ ,  $\tilde{Y}_t^H$ ,  $\tilde{p}_t^H$ ,  $\tilde{\pi}_t^H$ ,  $\widetilde{(1 + i_t)}$ ,  $\widehat{b}_t^{*,HH}$ ,  $\tilde{\sigma}_t$ ,  $\tilde{d}_t^H$ ,  $\widetilde{IM}_t$ ,  $\tilde{p}_t^F$ ,  $\widetilde{TOT}_t$ ,  $\widetilde{EX}_t$ ,  $\widetilde{FX}_t$ . The model is closed by specifying how the central bank sets the interest rate and foreign reserves.

### 3 The Welfare Criterion

This section obtains the labor subsidy that supports efficiency in a decentralized steady state, and then presents a second-order approximation to the welfare function of a utilitarian policymaker. Centering the economy around an efficient steady state is required for deriving a second-order approximation of the welfare criterion that: (1) can be used as an objective function in a linear-quadratic optimization problem whose solution approximates the solution of the exact problem; and (2) correctly ranks alternative equilibrium allocations that are approximated to first order. See [Benigno and Woodford \(2012\)](#).

#### 3.1 The Optimal Labor Subsidy

To solve for the efficient labor subsidy, one must first characterize the efficient steady state. To that end, consider a utilitarian social planner aiming to maximize aggregate utility subject to technological constraints and equilibrium conditions. Focusing on the

steady state reduces the optimization to a static problem:

$$\begin{aligned}
& \underset{\{C_{ss}, N_{ss}, IM_{ss}, d_{ss}^H, TOT_{ss}\}}{\text{Max}} && \frac{1}{1-\beta} U[C_{ss}, N_{ss}; \eta_{ss}] \\
& \text{s.t.} && C_{ss} = \left[ (1-\lambda)^{\frac{1}{\varepsilon}} (d_{ss}^H)^{\frac{\varepsilon-1}{\varepsilon}} + \lambda^{\frac{1}{\varepsilon}} (IM_{ss})^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}} \\
& && A_{ss} N_{ss}^\alpha = d_{ss}^H + G_{ss} + TOT_{ss}^{-\varepsilon^*} WT_{ss} \\
& && 0 = \frac{1-\beta}{\beta} (FX_{ss} + b_{ss}^{*,HH}) + \phi_{ss}^* + TOT_{ss}^{1-\varepsilon^*} WT_{ss} - IM_{ss} \\
& && \frac{d_{ss}^H}{IM_{ss}} = \frac{1-\lambda}{\lambda} TOT_{ss}^{-\varepsilon}
\end{aligned}$$

where the first two constraints are dictated by technology and the resource constraint. The third constraint is the balance of payments, and the last is an equilibrium condition for the composition of consumption, which is derived from (8) and (9).

Notice that  $b_{ss}^{*,HH}$  and  $FX_{ss}$  are not part of the choice variables of the planner.  $b_{ss}^{*,HH}$  is determined by the households' Euler equation for foreign bonds, which is a constraint the planner must obey.  $FX_{ss}$  is indeterminate, but its level does not affect the optimal allocation. Higher  $FX_{ss}$  implies higher government debt to foreigners, which in turn raises steady state capital outflows due to higher debt service. In the balance of payments, the rise in debt service exactly offsets the return on higher reserves. The considerations for the appropriate level of foreign reserves are related to the type of risks the economy faces, which are irrelevant for the deterministic steady state allocation.

After imposing symmetry across countries, as described in Section 2.8, the solution to the planner's problem is characterized by:

$$-\frac{U_{N_{ss}}}{U_{C_{ss}} C_{ss}^{\frac{1}{\varepsilon}} (1-\lambda)^{\frac{1}{\varepsilon}} (d_{ss}^H)^{-\frac{1}{\varepsilon}}} = \frac{(1-\lambda)\varepsilon + \varepsilon^* - 1}{(1-\lambda)\varepsilon + \varepsilon^* - (1-\lambda)} \alpha A_{ss} N_{ss}^{\alpha-1} \quad (45)$$

and the four constraints above.

In a decentralized economy, equilibrium conditions dictate:

$$-\frac{U_{N_{ss}}}{U_{C_{ss}} C_{ss}^{\frac{1}{\varepsilon}} (1-\lambda)^{\frac{1}{\varepsilon}} (d_{ss}^H)^{-\frac{1}{\varepsilon}}} = \frac{1}{1-\tau_w} \frac{\varepsilon^L - 1}{\varepsilon^L} \frac{\varepsilon^N - 1}{\varepsilon^N} \alpha A_{ss} N_{ss}^{\alpha-1} \quad (46)$$

which uses equations (8), (12), (13), (14), and (22).

Comparing (45) to (46), it is clear that the social planner can support the efficient steady state as a decentralized equilibrium by setting:

$$1 - \tau_w = \frac{\varepsilon^L - 1}{\varepsilon^L} \frac{\varepsilon^N - 1}{\varepsilon^N} \frac{(1-\lambda)\varepsilon + \varepsilon^* - (1-\lambda)}{(1-\lambda)\varepsilon + \varepsilon^* - 1} \quad (47)$$

Note that (47) generalizes the formulation in Galí and Monacelli (2005). In their case

$\varepsilon^N \rightarrow \infty$  and  $\varepsilon = \varepsilon^* = 1$ , suggesting  $1 - \tau_w = \frac{\varepsilon^L - 1}{\varepsilon^L} \frac{1}{1 - \lambda}$ . In a closed economy  $\lambda = 0$ , and we get  $1 - \tau_w = \frac{\varepsilon^L - 1}{\varepsilon^L} \frac{\varepsilon^N - 1}{\varepsilon^N}$ , which completely offsets the monopolistic distortions. Finally, under perfectly elastic export demand, i.e.  $\varepsilon^* \rightarrow \infty$ , we get  $1 - \tau_w = \frac{\varepsilon^L - 1}{\varepsilon^L} \frac{\varepsilon^N - 1}{\varepsilon^N}$ , as in a closed economy.

To shed light on these results, notice that  $\frac{(1-\lambda)\varepsilon + \varepsilon^* - (1-\lambda)}{(1-\lambda)\varepsilon + \varepsilon^* - 1} > 1$ , suggesting that the subsidy in (47) does not fully offset the monopolistic distortions; as a result, steady state production is lower than its competitive level. The reason is that the planner internalizes the monopolistic power of the economy in the international goods market, as the economy faces a downward sloping demand for its exports, equation (11). Therefore, the social planner faces a tradeoff between labor market efficiency<sup>20</sup> and exploiting the monopolistic power of the economy. This tradeoff is optimally balanced in (47).<sup>21</sup> In a closed economy only labor market efficiency matters, and the planner fully offsets the monopolistic distortions. When  $\varepsilon^* \rightarrow \infty$ , the economy has no monopolistic power, and the planner is left with restoring efficiency in the labor market, as in a closed economy.

### 3.2 Second-Order Approximation to the Welfare Criterion

A utilitarian policymaker seeks to maximize welfare in the economy as measured by the aggregate expected discounted utility of domestic households, that is:

$$\mathbb{W} \equiv E_0 \sum_{t=0}^{\infty} \beta^t \int_0^1 U(c_t(h), n_t(h); \eta_t) dh$$

---

<sup>20</sup>By labor market efficiency I mean equating the marginal product of labor to the households' marginal rate of substitution between labor and consumption of the home good.

<sup>21</sup>This formulation corresponds to the standard approach in the literature, e.g. [Galí and Monacelli \(2005\)](#), [De Paoli \(2009\)](#) and [Cavallino \(2019\)](#). However, it is not clear why the social planner should be constrained by equilibrium conditions that can be altered by taxation. The fact that the optimal subsidy in the text maintains some of the monopolistic power of the economy reflects the social planner's incentive to manipulate the terms of trade in favor of domestic agents, as highlighted by [Corsetti and Pesenti \(2001\)](#). However, one could introduce, in addition to the labor subsidy, a subsidy that directly alters the terms of trade. For example, consider a subsidy,  $\tau_H$ , to domestic consumption of the home good,  $d^H$ . This subsidy discriminates between domestic agents and foreigners, as the latter pay the full price for the same good. In this case the effective terms of trade domestic agents face is  $(1 - \tau_H) P_t^H / P_t^F$ , and the optimal subsidies are given by  $1 - \tau_H = \frac{\varepsilon^* - 1}{\varepsilon^*}$  and  $1 - \tau_w = \frac{\varepsilon^N - 1}{\varepsilon^N} \frac{\varepsilon^L - 1}{\varepsilon^L} \frac{\varepsilon^*}{\varepsilon^* - 1}$ . These subsidies suggest that the planner fully exploits the monopolistic power of the economy while maintaining efficiency in the labor market. In the text I restrict  $\tau_H$  to zero. Keeping in mind that the model is symmetric across countries, I interpret the subsidy system in the text as an internationally cooperative system that forbids protective tariffs, e.g. a system that is supported by trade agreements.

After taking second order approximation, substituting for equilibrium conditions approximated to second-order and using the optimal subsidy, the welfare criterion reads:

$$\begin{aligned} \frac{\mathbb{W} - \mathbb{W}_{ss}}{U_C C_{ss}} &= E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{1}{2} y'_{1,t} \Omega_{11} y_{1,t} + \frac{1}{2} y'_{2,t} \Omega_{22} y_{2,t} + x'_t \Omega_{x1} y_{1,t} \right\} \\ &+ E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \begin{aligned} &\frac{1}{2} \frac{\varepsilon^L}{\alpha} \left[ (1 - \varepsilon^L) + \frac{\varepsilon^L}{\alpha} \right] \frac{U_{N_{ss}} N_{ss}}{U_{C_{ss}} C_{ss}} \frac{\xi_p}{1 - \xi_p} \frac{1}{1 - \beta \xi_p} (\tilde{\pi}_t^H)^2 \\ &+ \frac{1}{2} \varepsilon^N \frac{U_N N_{ss}}{U_C C_{ss}} \left[ 1 + \left( \gamma_{nn} - \frac{\gamma_{nc} \gamma_{cn}}{\gamma_{cc}} \right) \varepsilon^N \right] \frac{\xi_w}{1 - \xi_w} \frac{1}{1 - \beta \xi_w} (\tilde{\pi}_t^w)^2 \end{aligned} \right\} \\ &+ t.i.p. + \mathcal{O}(\|\cdot\|^3) \end{aligned} \quad (48)$$

where *t.i.p.* stands for terms independent of policy, and:

$$y_{1,t} \equiv \begin{bmatrix} \tilde{C}_t \\ \tilde{N}_t \\ \widetilde{TOT}_t \end{bmatrix}, \quad y_{2,t} \equiv \begin{bmatrix} \widehat{b}_t^{*,HH} - \widehat{\theta}_t^* \\ \widetilde{FX}_t \end{bmatrix}, \quad x_t \equiv \begin{bmatrix} \tilde{\eta}_t \\ \tilde{A}_t \\ \widetilde{WT}_t \end{bmatrix}'$$

$$\begin{aligned} \Omega_{11} &= \begin{bmatrix} \gamma_{cc} & \gamma_{cn} & \frac{\lambda \varepsilon (1 - \lambda)}{(1 - \varepsilon)(1 - \lambda) - \varepsilon^*} \\ \gamma_{cn} & \frac{U_{N_{ss}} N_{ss}}{U_{C_{ss}} C_{ss}} (\gamma_{nn} + 1 - \alpha) & 0 \\ \frac{\lambda \varepsilon (1 - \lambda)}{(1 - \varepsilon)(1 - \lambda) - \varepsilon^*} & 0 & \left\{ \begin{aligned} &(2 - 3\lambda) \varepsilon - (2 - \lambda) \\ &+ 3\varepsilon^* - \frac{\varepsilon^*(1 - \varepsilon^*)}{\varepsilon(1 - \lambda)} \end{aligned} \right\} \end{bmatrix} \\ \Omega_{22} &= (1 - \vartheta) \frac{1}{C_{ss}} \frac{\varepsilon^* + \varepsilon(1 - \lambda)}{(1 - \varepsilon)(1 - \lambda) - \varepsilon^*} \begin{bmatrix} \Theta''(0) & 0 \\ 0 & \Theta^{CB''}(FX_{ss}) FX_{ss}^2 \end{bmatrix} \\ \Omega_{x1} &= \begin{bmatrix} \gamma_{cn} & \frac{U_{N_{ss}} N_{ss}}{U_{C_{ss}} C_{ss}} \gamma_{nn} & 0 \\ 0 & -\frac{U_{N_{ss}} N_{ss}}{U_{C_{ss}} C_{ss}} & 0 \\ 0 & 0 & \frac{\varepsilon \lambda (1 - \lambda)}{(\varepsilon - 1)(1 - \lambda) + \varepsilon^*} \end{bmatrix} \end{aligned}$$

Appendix D details the derivation of (48). Optimal policies are derived by maximizing (48) while taking linearized equilibrium conditions, equations (28) through (44), as constraints. See Appendix E.

## 4 Parameter Values

This section presents the baseline parameterization. Details of the calibration, estimation, and data are provided in Appendix F. Table 1 summarizes the choice of parameter values.

Parameter values are chosen based on the characteristics of the Israeli economy. A period in the model corresponds to one quarter. Values are mostly adopted from the Bank of Israel DSGE model, Argov et al. (2012). The calibration assumes a symmetric steady state across countries, as described in Section 2.8. The key parameter is the financial friction parameter,  $\Theta''(0)$ , as it governs the efficacy of FXIs. Parameters for

Table 1: Steady State and Parameter Values, Baseline Parameterization

**Panel A: Steady State**

Terms of trade	$TOT_{ss}$	1
Private sector foreign asset position	$b_{ss}^{*,HH}$	0
Inflation	$\pi_{ss}$	$1.02^{1/4}$
Productivity	$A_{ss}$	1
Labor input	$N_{ss}$	0.32
Share of government expenditure in GDP	$\frac{G_{ss}}{Y_{ss}^H}$	0.3
Shares of exports and imports in GDP	$\frac{EX_{ss}^*}{Y_{ss}^H}, \frac{IM_{ss}}{TOT_{ss} Y_{ss}^H}$	0.33
Target level of reserves (30 percent of annual GDP)	$\frac{FX^T}{TOT_{ss} Y_{ss}^{H,An.}}$	0.3
Preference shock	$\eta_{ss}$	1
Risk premium shock	$\hat{\theta}_{ss}^*$	0

**Panel B: Calibrated Parameters**

Elasticity of domestic output with respect to labor	$\alpha$	0.67
Subjective discount factor	$\beta$	$1.025^{-1/4}$
EoS between home and foreign goods	$\varepsilon$	1.1
EoS between differentiated labor skills	$\varepsilon^N$	13/3
EoS between intermediate goods of the same country	$\varepsilon^L$	13/3
EoS between goods of different countries	$\varepsilon^*$	13/3
Probability of price adjustment	$1 - \xi_p$	1/3
Probability of wage adjustment	$1 - \xi_w$	0.25
Frisch elasticity of labor supply	$\nu^{-1}$	2
Intertemporal EoS	$\gamma^{-1}$	1/3
Domestic ownership share of the financial sector	$\vartheta$	0.999
2nd derivative of the CB's portfolio adjustment cost	$\Theta_{ss}^{CB''}$	0.1

**Panel C: Estimated Parameters (Mode Posteriors)**

2nd derivative of the HHS' portfolio adjustment cost	$\Theta''(0)$	2.569
<b>Exogenous Shocks</b>	<b>Persistence</b>	<b>STD</b>
Productivity, $\tilde{A}_t$	0.640	0.010
Preference shock, $\tilde{\eta}_t$	0.657	0.020
Government expenditure, $\tilde{G}_t$	0.578	0.007
World trade, $\tilde{WT}_t$	0.832	0.009
Risk premium, $\tilde{\theta}_t^*$	0.858	0.006
Capital inflows, $\tilde{\phi}_t^* - \hat{\phi}_{ss}^*$	0.150	0.006

Note: EoS = Elasticity of Substitution. Calibrated values are mostly adopted from the parameterization of the Bank of Israel DSGE model, as reported in [Argov et al. \(2012\)](#). Parameters in Panel C are mode posteriors of Bayesian estimation. For details on the calibration, estimation and data see [Appendix F](#).

the stochastic processes of the exogenous variables are also important, as they directly affect second moments, and hence welfare, equation (48). Exogenous variables follow a first-order auto-regressive process:  $X_t = \rho_X X_{t-1} + \epsilon_t^X$  where  $\epsilon_t^X \stackrel{iid}{\sim} N(0, \sigma_X^2)$  and  $X_t \in \{\tilde{A}_t, \tilde{\eta}_t, \tilde{G}_t, \widetilde{WT}_t, \tilde{\theta}_t^*, \tilde{\phi}_t^* - \tilde{\phi}_{ss}^*\}$ . I use Bayesian estimation to evaluate  $\Theta''(0)$  and the parameters of the auto-regressive processes. The prior for  $\Theta''(0)$  relies on estimations for the effect of FXIs in Israel on the New Israeli Shekel nominal effective exchange rate, Ribon (2017), Hertrich and Nathan (2022) and Caspi et al. (2022). Priors for parameters of the auto-regressive processes are based on Argov et al. (2012) and self estimation.

Finally, I consider a standard additive separable utility function:  $U(C_t, N_t; \eta_t) = \eta_t \left[ \frac{C_t^{1-\gamma} - 1}{1-\gamma} - \psi \frac{N_t^{1+\nu}}{1+\nu} \right]$ . With this specification, the Frisch elasticity of labor supply is given by  $\nu^{-1}$ , and the intertemporal elasticity of substitution is  $\gamma^{-1}$ .

## 5 The Transmission Mechanism of FXIs

To study the transmission mechanism of FXIs, it is useful to focus on an economy without nominal rigidities. While nominal rigidities affect equilibrium outcomes quantitatively, they are not crucial for understanding the transmission mechanism. To further simplify the analysis, assume that foreign reserves are white noise,  $\widetilde{FX}_t \sim WN$ .

Considering exogenous foreign reserves allows analyzing their impact without concern for feedback effects from the economy to policy. Endogenizing the policy response is the subject of the next section. Assuming a white noise process reveals the persistence the model generates endogenously. The transmission of FXIs is summarized by the impulse response functions in Figure 1. Before analyzing the transmission mechanism, it is useful to establish the following result.

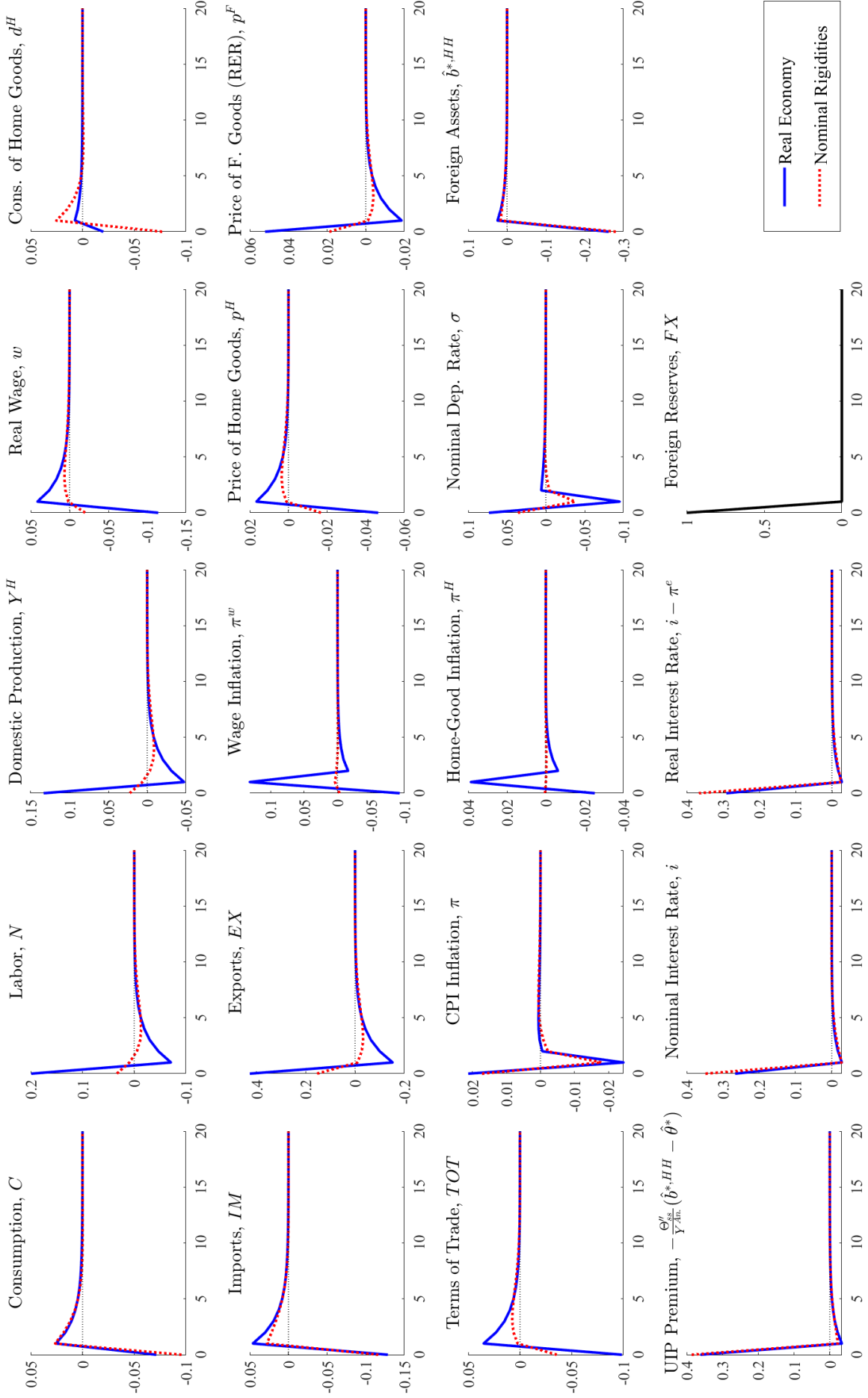
**Lemma 1** *Assuming  $\xi_p = \xi_w = 0$ , and preferences satisfying  $\gamma_{mn} - \gamma_{cn} \geq 0$  and  $\gamma_{nc} - \gamma_{cc} \geq 0$ , consumption,  $\tilde{C}_t$ , the terms of trade,  $\widetilde{TOT}_t$ , and imports,  $\widetilde{IM}_t$ , comove positively in response to variation in foreign reserves,  $\widetilde{FX}_t$ .*

**Proof.** See Appendix G. ■

The condition on preferences in Lemma 1 is a sufficient condition. It holds for additive-separable preferences in consumption and labor ( $\gamma_{cn} = \gamma_{nc} = 0$ ) and other standard utility functions, e.g. Cobb-Douglas and the utility function in Greenwood et al. (1988), GHH hereinafter. The proof of Lemma 1 relies on establishing that, holding other exogenous variables fixed, consumption rises with the terms of trade. This is achieved by combining the resource constraint with the labor market equilibrium condition. Then, the comovement with imports follows immediately from import demand.

We are now ready to evaluate the economy's response to a temporary rise in foreign reserves (solid blue lines in Figure 1). To that end, observe that after substituting for

Figure 1: The Transmission of FXIs. Response to a 1 Percent Rise in Foreign Reserves ( $100 \times \log$  points)



Note: Under the real economy  $\zeta_p = 0.01$  and  $\zeta_w = 0$ . A slight deviation from purely flexible prices is required in order to support a unique equilibrium path for the nominal variables.

export demand (35), the balance of payments, equation (38), reads:

$$\frac{FX_{ss}}{Y_{ss}^{H,An.}} \widetilde{FX}_t + \widehat{b}_t^{*,HH} \cong \frac{\lambda C_{ss}}{Y_{ss}^{H,An.}} \underbrace{\left[ \widetilde{WT}_t - (\varepsilon^* - 1) \widetilde{TOT}_t - \widetilde{IM}_t \right]}_{\text{Net Exports}} + EXOG_t \quad (49)$$

where  $EXOG_t$  summarizes exogenous and predetermined variables. Equation (49) makes clear that the resources for raising foreign reserves can come from the private sector's financial portfolio,  $\widehat{b}_t^{*,HH}$ , and/or from increasing net exports.

Although in the model all variables move simultaneously, it is convenient to think of  $\widehat{b}_t^{*,HH}$  as moving first. Specifically, households must be on the other side of the transaction for increasing foreign reserves, suggesting that  $\widehat{b}_t^{*,HH}$  falls. The fall in  $\widehat{b}_t^{*,HH}$  affects the Euler equation for foreign bonds, equation (31). Combined with (34) and (43) it reads:

$$\widetilde{U}_{IM_t} \cong \widetilde{(1 + i_t^*)} - E_t \{ \widetilde{\pi}_{t+1}^{F*} \} - \frac{\Theta''(0)}{Y_{ss}^{H,An.}} \left( \widehat{b}_t^{*,HH} - \widehat{\theta}_t^* \right) + E_t \left\{ \widetilde{U}_{IM_{t+1}} \right\} \quad (50)$$

where  $U_{IM_t}$  is the marginal utility of imported goods,  $U_{IM_t} = U_{C_t} \frac{\partial C_t}{\partial IM_t}$ . Since  $\Theta''(\cdot) > 0$ , the fall in  $\widehat{b}_t^{*,HH}$  raises the effective return on foreign assets that households face. This triggers an intertemporal substitution in imports, reducing  $\widetilde{IM}_t$  in the present. By Lemma 1, the fall in imports must be accompanied by a reduction in the terms of trade,  $\widetilde{TOT}_t$ , and in consumption,  $\widetilde{C}_t$ . The fall in  $\widetilde{TOT}_t$  triggers an intratemporal substitution from imported goods to home goods, further reducing  $\widetilde{IM}_t$ . Lower  $\widetilde{TOT}_t$  also stimulates exports, equation (35), and net exports rise.

The effect on consumption of home goods,  $\widetilde{d}_t^H$ , depends on the elasticity of substitution  $\varepsilon$ . For a sufficiently large  $\varepsilon$ ,  $\widetilde{d}_t^H$  and  $\widetilde{IM}_t$  move in opposite directions, and  $\widetilde{d}_t^H$  may rise despite the fall in  $\widetilde{C}_t$ , equation (33). In our case,  $\varepsilon$  is close to unity, and  $\widetilde{d}_t^H$  falls.

In the labor market, the fall in  $\widetilde{TOT}_t$  reduces labor demand because the value of the marginal product, measured in consumption units, falls. At the same time, the fall in  $\widetilde{C}_t$  raises labor supply. However, this effect hinges on the specification of the utility function; with GHH preferences, for example, labor supply remains unchanged. Consequently, the real wage must fall, but the effect on labor effort is ambiguous. With additive-separable utility, as is the case here, the rise in labor supply dominates, increasing labor effort. As a result, domestic production rises as well. However, with GHH preferences, labor effort and output fall slightly (not shown).<sup>22</sup>

In the period immediately after the shock, all effects reverse as  $\widetilde{FX}_t$  returns to its original level. The model generates modest persistence, with effects dissipating after

<sup>22</sup>Under GHH preferences:  $U(C_t, N_t; \eta_t) = \frac{\eta_t}{1-\gamma} \left[ \left( C_t - \psi \frac{N_t^{1+\nu}}{1+\nu} \right)^{1-\gamma} - 1 \right]$ . Parameter values are chosen to match the Frisch elasticity of labor supply and intertemporal elasticity of substitution as in Table 1.  $\psi$  is then pinned down by the steady state equilibrium condition in the labor market.



about 6 quarters. The persistence is due to the gradual adjustment of  $\widehat{b}_t^{*,HH}$ , as agents smooth the portfolio adjustment cost over time.

Finally, although without nominal rigidities nominal quantities are irrelevant for the real allocation, notice that the rise in foreign reserves depreciates the domestic currency, i.e.  $\sigma$  rises, as it raises demand for foreign currency. Introducing nominal rigidities does not qualitatively change any of the impulses (dotted red lines in Figure 1).<sup>23</sup>

## 6 Optimal FXIs: Response to Shocks

This section shows that optimal FXIs seek to stabilize the UIP premium. It demonstrates that full stabilization is always optimal under financial shocks but may not be under real shocks. Throughout the analysis, monetary policy sets the interest rate optimally. The presentation below focuses on the reaction of the UIP premium and policy instruments. Appendix E characterizes the optimal allocations.

### 6.1 Financial Shocks

Capital inflow shocks,  $\widehat{\phi}_t^*$ , and the risk premium shocks,  $\widehat{\theta}_t^*$ , are indistinguishable from exogenous fluctuations in foreign reserves, except for the properties of their stochastic processes. For capital flows, this is evident by substituting for  $\widetilde{FX}_t$  using its approximated law of motion, equation (25), in the balance of payments, equation (38):

$$\left(\widehat{\phi}_t - \widehat{\phi}_{ss}\right) + \left(\widehat{b}_t^{*,HH} - \beta^{-1}\widehat{b}_{t-1}^{*,HH}\right) \cong \left(\widehat{\phi}_t^* - \widehat{\phi}_{ss}^*\right) + \frac{\lambda C_{ss}}{Y_{ss}^{H,An.}} \left(\widetilde{TOT}_t + \widetilde{EX}_t - \widetilde{IM}_t\right)$$

From this formulation it is clear that a shock to capital inflows,  $\widehat{\phi}_t^*$ , is equivalent to a shock (in the opposite direction) to FXIs,  $\widehat{\phi}_t$ , as both enter the system only through the balance of payments.

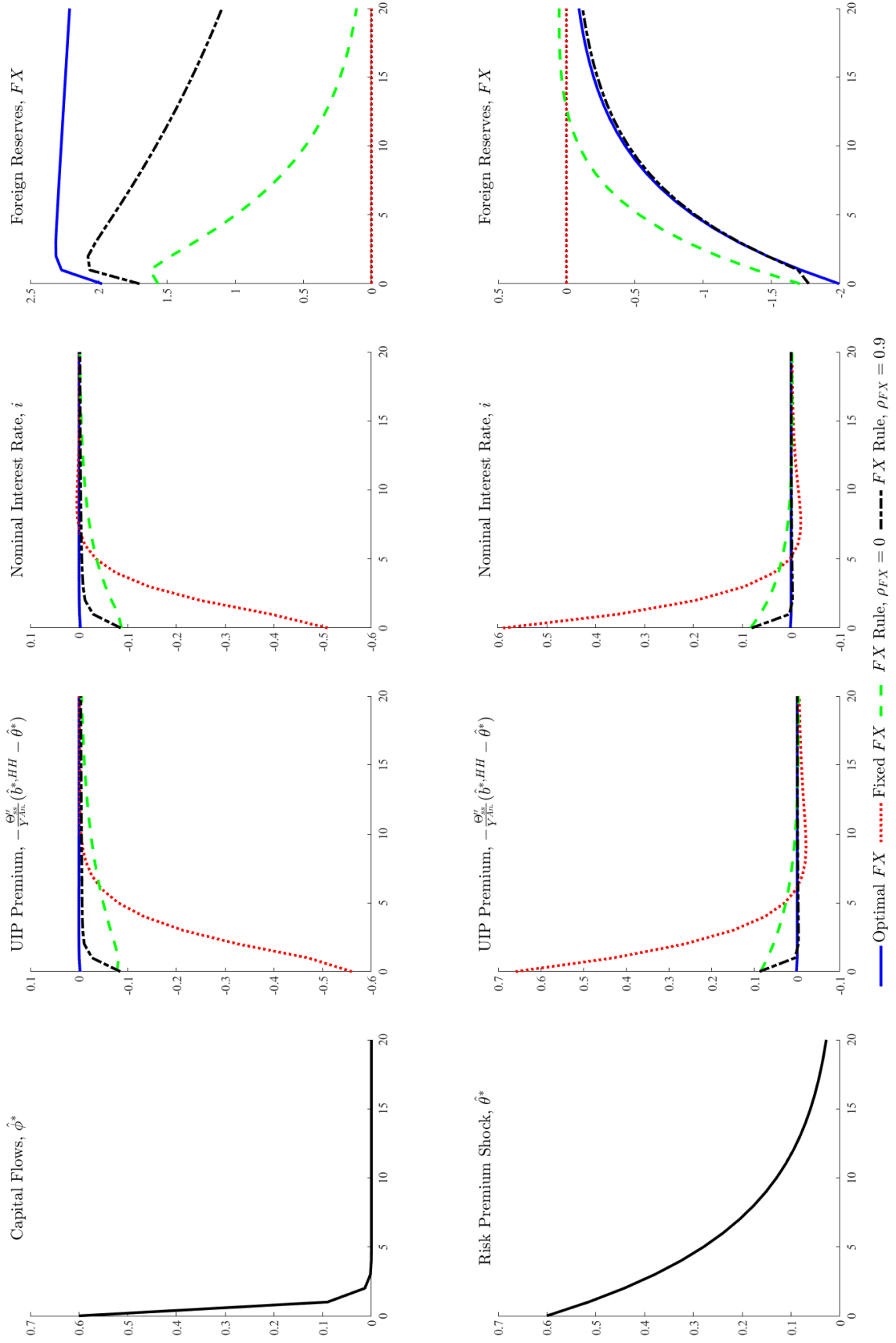
For the risk premium shock, note that from the Euler equations, (30) and (31), we get  $\widehat{b}_t^{*,HH} \cong \widehat{\theta}_t^* + \frac{Y_{ss}^{H,An.}}{\Theta''(0)} \left[ E_t \{ \widetilde{\sigma}_{t+1} \} + \widetilde{(1+i_t^*)} - \widetilde{(1+i_t)} \right]$ . Substituting  $\widehat{b}_t^{*,HH}$  in the balance of payments, equation (38), suggests that  $\widehat{\theta}_t^*$  and  $\widetilde{FX}_t$  are indistinguishable, up to their stochastic properties.

Two conclusions emerge. First, when foreign reserves are fixed or exogenous, the system's response to both financial shocks is similar to its response to foreign reserve shocks, as analyzed in Section 5 (with the opposite sign in the case of capital flows,  $\widehat{\phi}_t^*$ ). The impulse response functions are depicted in Figure 2 (dotted red lines for the case of fixed foreign reserves). Second, optimal FXI policy fully neutralizes their effects (solid

---

<sup>23</sup>In both cases, with and without nominal rigidities, the impulses assume the central bank follows optimal interest rate policy.

Figure 2: Response to Financial Shocks (1 standard deviation shocks,  $100 \times \log$  points)



blue lines in Figure 2). FXIs absorb capital flows, thereby fully stabilizing  $\widehat{b}_t^{*,HH}$  and the UIP premium, and halting their transmission to the economy. After a risk premium shock, the central bank maintains a stable return on foreign assets by selling foreign reserves, thereby raising  $\widehat{b}_t^{*,HH}$  just enough to offset the effect of the shock on the UIP premium, thus disabling its transmission to the economy.

## 6.2 Real Shocks

Under real shocks, optimal FXIs reduce variation in the UIP premium but do not necessarily fully stabilize it. Figure 3 presents the impulse response functions to real shocks.

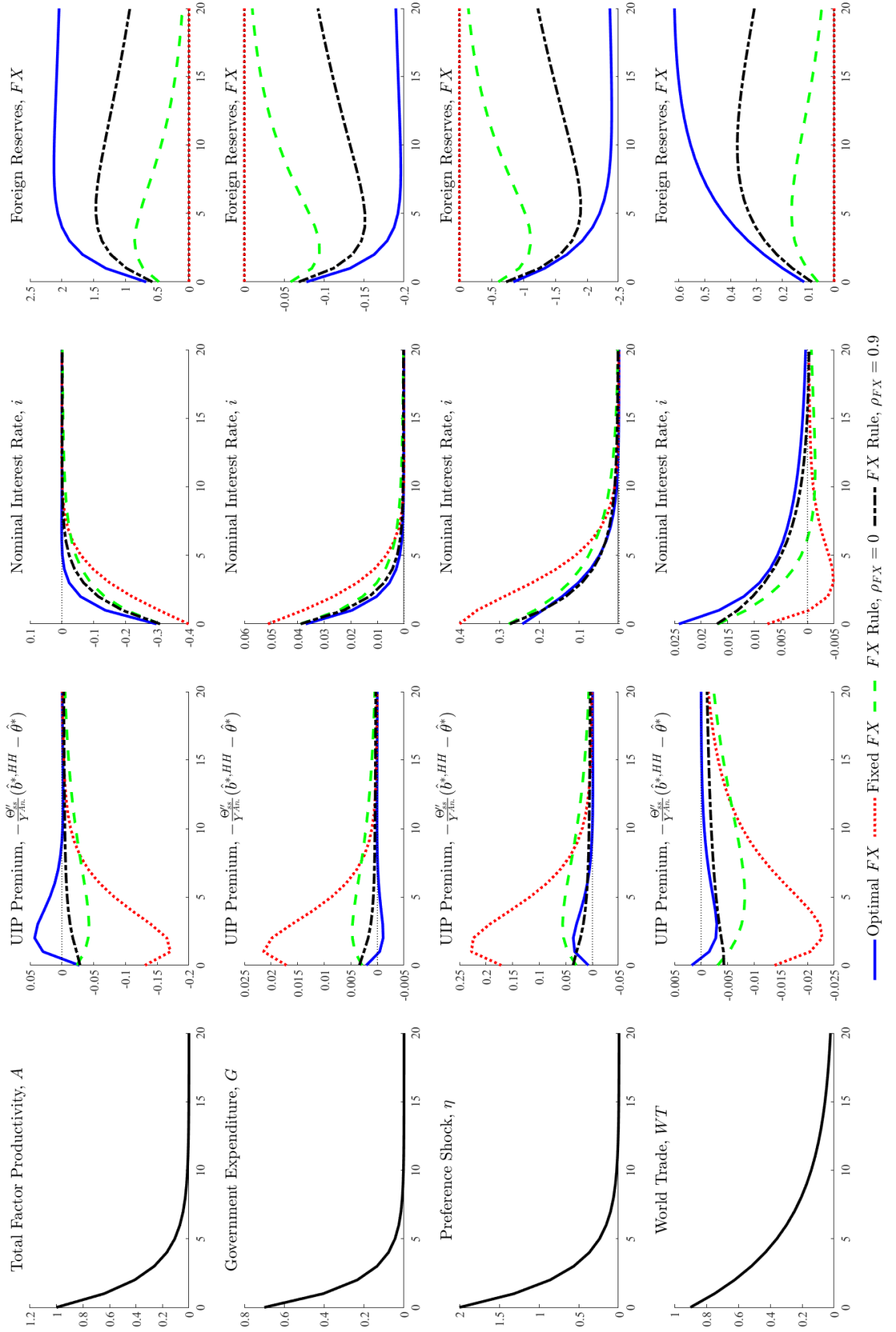
**Productivity,  $A_t$ .** A rise in productivity raises the supply of home goods and reduces their price. Optimal monetary policy lowers the interest rate to close the "output gap". To smooth consumption, the higher productivity raises savings. With fixed foreign reserves the only way to save is by increasing  $\widehat{b}_t^{*,HH}$ , which, in turn, reduces the UIP premium. When FXIs are available, the central bank helps raising the economy's savings by purchasing foreign reserves. This stabilizes the movement in  $\widehat{b}_t^{*,HH}$  and in the UIP premium. The monetary expansion is less aggressive in this case, suggesting that optimal monetary and FXI policies work in tandem.

**Government Expenditure,  $G_t$ .** A rise in  $G_t$  increases demand for home goods, raising their price and stimulating domestic production,  $Y_t^H$ , while crowding out exports,  $EX_t$ , and domestic consumption,  $d_t^H$ . When foreign reserves are fixed, households smooth consumption by reducing  $\widehat{b}_t^{*,HH}$ , which raises the UIP premium. Monetary policy curbs excess demand by raising the interest rate. When FXIs are available, the central bank sells foreign reserves, which stabilizes  $\widehat{b}_t^{*,HH}$  and the UIP premium. Monetary policy is less contractionary in this case, as more of the adjustment occurs through the external sector: sharper currency appreciation and a larger export decline absorb the rise in  $G_t$ .

**Preference Shock,  $\eta_t$ .** A rise in  $\eta_t$  increases current marginal utility relative to the future, shifting consumption demand to the present, thereby raising  $C_t$  and contracting labor supply, which suppresses production,  $Y_t^H$ . With higher demand and fewer resources, savings fall. Under fixed foreign reserves, this occurs by reducing  $\widehat{b}_t^{*,HH}$ , which raises the UIP premium. Monetary policy raises the interest rate to curb demand. When FXIs are available, the central bank sells foreign reserves, moderating the fall in  $\widehat{b}_t^{*,HH}$  and stabilizing the UIP premium. Monetary policy is less aggressive in this case as well.

**World Trade,  $WT_t$ .** A rise in  $WT_t$  raises demand for exports,  $EX_t$ , increasing the terms of trade,  $TOT_t$ , and generating a trade surplus, which must be matched with higher savings. Under fixed foreign reserves,  $\widehat{b}_t^{*,HH}$  rises and the UIP premium falls. The interest rate is hardly changed, as the endogenous rise in the terms of trade absorbs most of the shock. When FXIs are available, the central bank helps raise savings by purchasing foreign reserves, thereby stabilizing  $\widehat{b}_t^{*,HH}$  and the UIP premium.

Figure 3: Response to Real Shocks (1 standard deviation shocks,  $100 \times \log$  points)



### 6.3 When Is Full Stabilization of the UIP Premium Optimal?

As discussed above, by perfectly stabilizing the UIP premium, FXIs fully insulate the economy from the effect of financial shocks (Figure 2). Against real shocks, optimal FXIs reduce but do not eliminate variation in the UIP premium (Figure 3).

Under real shocks, the central bank faces tradeoffs, determined by the inefficiencies in the economy. These are<sup>24</sup>: (1) price rigidity, limiting firms ability to adjust production optimally; (2) wage rigidity, which similarly constrains labor supply; (3) a financial friction, distorting asset pricing; and (4) a downward-sloping export demand, endowing the economy with monopolistic power, while exporters are price takers.

Given that the central bank has only two tools—the interest rate and FXIs—generally it cannot fully offset all distortions simultaneously, resulting in tradeoffs. Shutting down at least two of the frictions may support strict targeting of the UIP premium as optimal. However, to ensure FXI efficacy, the financial friction must be kept. That is, shutting down one nominal rigidity ( $\xi_p = 0$  or  $\xi_w = 0$ ) and assuming perfectly elastic export demand ( $\varepsilon^* \rightarrow \infty$ ) is expected to eliminate the tradeoffs. By the same reasoning, with  $\varepsilon^* \rightarrow \infty$ , maintaining price rigidity while setting  $\xi_w = 0$ , is expected to give rise to strict targeting of domestic price inflation,  $\pi^H$ ; while maintaining wage rigidity, when  $\xi_p = 0$ , is expected to result in strict targeting of wage inflation,  $\pi^w$ .

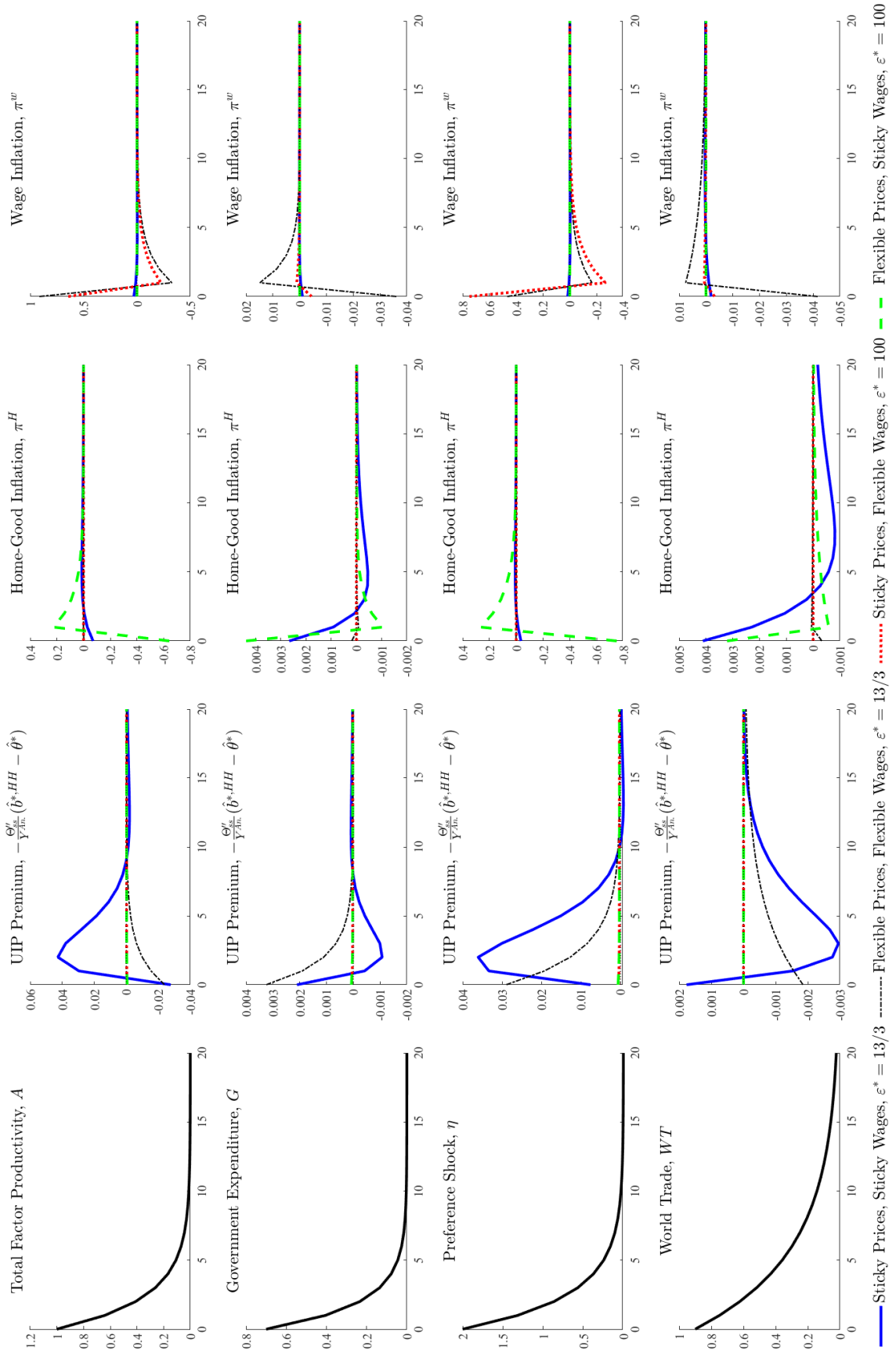
Figure 4 demonstrates this point. It displays the response to real shocks of the UIP premium,  $\pi^H$  and  $\pi^w$ , under optimal monetary and FXI policies in four cases: (1) the baseline parameterization with all frictions; (2) no nominal rigidities while maintaining a downward-sloping export demand; (3) sticky prices, flexible wages and (almost) perfectly elastic export demand ( $\varepsilon^* = 100$ ); and (4) flexible prices, sticky wages and  $\varepsilon^* = 100$ .

Under the baseline parameterization (case 1, blue solid lines) optimal FXIs do not fully stabilize the UIP premium. This is also the case in the model with no nominal rigidities (case 2, black dash-dotted lines). Monetary policy is neutral in this case, leaving FXIs with a tradeoff between counteracting the effect of the financial friction and internalizing the monopolistic power of the economy. With one nominal rigidity and no monopolistic power (cases 3 and 4, dotted red and dashed green lines, respectively), monetary policy addresses the nominal rigidity and FXIs address the financial friction, resulting in full stabilization of the UIP premium. Moreover, strict targeting of either domestic price inflation,  $\pi^H$ , or wage inflation,  $\pi^w$ , turns optimal. These results are simply a manifestation of the Tinbergen rule.

---

<sup>24</sup>Monopolistic competition in the goods and labor markets also results in inefficient equilibrium. However, the optimal labor subsidy, equation (47), accounts for that inefficiency. Although the subsidy is fixed, when facing one nominal rigidity, it is effective in offsetting the monopolistic distortion along the business cycle because the desired mark-ups in the model are constant.

Figure 4: Optimal Response of Target Variables to Real Shocks (1 standard deviation shocks,  $100 \times \log$  points)



## 7 FXI Policy Rule

The optimal policy tailors the best FXI response to each shock. However, central banks do not observe the shocks as they hit the economy. This section proposes an implementable FXI policy rule that aims to support an equilibrium allocation that is close to the optimal one, regardless of the type of shocks the economy faces.

Two features of optimal FXI policy emerge from the analysis in the previous section: (1) the optimal policy stabilizes the UIP premium, either fully or partially; and (2) foreign reserves,  $FX_t$ , are highly persistent, as evident in their response to all shocks (blue solid lines in figures 2 and 3). In fact, reserves do not follow a random walk only because they are restricted to be stationary.<sup>25</sup> Following a temporary rise in productivity, for example, a social planner would choose to permanently raise reserves and use the additional return to increase consumption in perpetuity.

A natural suggestion for a policy rule is therefore to use FXIs to stabilize the UIP premium while smoothing the path of foreign reserves. Specifically, consider:

$$\frac{FX_t}{FX^T} = \left( 1 + \frac{\Theta'(\widehat{b}_t^{*,HH} - \widehat{\theta}_t^*)}{TOT_{ss} Y_{ss}^{H,An.}} \right)^\Xi \left( \frac{FX_{t-1}}{FX^T} \right)^{\rho_{FX}} \quad \Xi > 0 \quad , \quad 0 \leq \rho_{FX} < 1 \quad (51)$$

The first term is the inverse of the gross UIP premium. Since  $\Theta''(\cdot) > 0$  and since raising  $FX_t$  crowds out  $\widehat{b}_t^{*,HH}$ , to stabilize the UIP premium the parameter  $\Xi$  must be positive. Note that although the premium is not directly observed, it can be estimated by evaluating deviations from the UIP condition, e.g. [Kalemli-Özcan and Varela \(2023\)](#). The second term in (51) controls the persistence of foreign reserves.

Setting  $\Xi = \rho_{FX} = 0$  fixes foreign reserves. As  $\Xi \rightarrow \infty$  policy strictly targets the UIP premium. Technically, however, we cannot fully stabilize the premium, as that reintroduces unit root dynamics to the model's solution, [Schmitt-Grohé and Uribe \(2003\)](#). For the same reason,  $\rho_{FX}$  must be strictly smaller than 1. I simulate the model with the (arbitrary) value  $\Xi = 20$ , and experiment with  $\rho_{FX} = 0$  and  $\rho_{FX} = 0.9$ .

The impulse response functions are displayed in figures 2 and 3 ( $\rho_{FX} = 0$  in green dashed lines;  $\rho_{FX} = 0.9$  in black dash-dotted lines). The response of  $FX_t$  lies between zero and its optimal path (blue solid lines), indicating the rule pushes policy toward optimality. Notably, reserves smoothing brings the response functions closer to the optimal path compared to the case of  $\rho_{FX} = 0$ . These results support adopting policy rule (51).

---

<sup>25</sup>Recall that the central bank incurs a minor adjustment cost when foreign reserves deviate from their target level.

## 8 Welfare Evaluation

### 8.1 Optimal FXIs vs Alternative Policies

This section evaluates the welfare gains from adopting optimal FXIs against alternative policies: fixed foreign reserves and the FXI rule, equation (51). In all cases, the interest rate is set optimally. Table 2 summarizes the results. The table presents the *lifetime* welfare gains, expressed as a percentage of annual steady state consumption.<sup>26</sup>

**Fixed Foreign Reserves.** Panel A of Table 2 shows welfare gains from optimal policy compared to fixed foreign reserves. Under the benchmark model (Column 1) welfare gains are 2.4% of annual steady state consumption. Columns 2 and 3 show that, as expected, welfare gains fall as the financial friction lessens. Column 4 examines a real economy, without nominal rigidities. Welfare gains are similar to the benchmark model, suggesting monetary policy does little to alleviate the effect of the financial friction; FXIs appear better suited for this job. Finally, notice that quantitatively FXIs play an important role not only against the financial shocks, but also against productivity and preference shocks.

**FXI Policy Rule.** Panels B and C of Table 2 display the welfare gains relative to policy rule (51) with  $\rho_{FX} = 0$  and  $\rho_{FX} = 0.9$ , respectively. Regardless of nominal rigidities, financial friction intensity, or the type of shocks, the policy rule with  $\rho_{FX} = 0$  always improves welfare over fixed reserves (Panel B vs A), and reserves smoothing,  $\rho_{FX} = 0.9$ , further enhances welfare (Panel C vs B). Overall, under the benchmark parameterization (Column 1), the welfare gains from optimal policy over the policy rule with  $\rho_{FX} = 0.9$  are merely 0.1%, indicating it results in near-optimal outcomes.

### 8.2 Welfare Gains from Owning the Financial Sector

Importing financial intermediation services is costly for the economy, as UIP deviations provide foreign financiers with profit opportunities, e.g. [Cavallino \(2019\)](#) and [Fanelli and Straub \(2021\)](#).<sup>27</sup> The planner's welfare criterion, equation (48), accounts for this cost by multiplying the variance of the UIP premium by the share of foreign ownership of the financial sector,  $1 - \vartheta$ . By stabilizing the UIP premium, the central bank reduces carry trade opportunities and, consequently, the cost to the economy. Nevertheless, the analysis thus far has abstracted from this cost by setting  $\vartheta \rightarrow 1$ . Therefore, the welfare gains in Table 2 indicate benefits of using FXIs as a macroeconomic stabilizer, rather than a means of stripping intermediation profits from foreigners.

---

<sup>26</sup>Recall that the model is calibrated to match the characteristics of the Israeli economy; thus, quantitative results may not apply to other countries.

<sup>27</sup>[Amador et al. \(2020\)](#) raise a similar argument regarding deviations from the covered interest parity.



Table 2: Lifetime Welfare Gains from Adopting Optimal FXI Policy  
Percent of Annual Steady State Consumption

	(1)	(2)	(3)	(4)
	Nominal Rigidities			Real
$\Theta''(0)$ , % of Benchmark Value:	100%	10%	1%	100%
<b>Panel A: Welfare Gains Relative to Fixed Foreign Reserves</b>				
Productivity, $A$	0.56	0.24	0.05	0.40
Preference shock, $\eta$	0.64	0.24	0.04	0.70
Government expenditure, $G$	0.01	< 0.01	< 0.01	< 0.01
World trade, $WT$	0.03	0.01	< 0.01	0.03
Risk premium, $\hat{\theta}^*$	0.34	0.03	< 0.01	0.40
Capital inflows, $\hat{\phi}^*$	0.87	0.26	0.04	0.91
All shocks	2.44	0.77	0.13	2.44
<b>Panel B: Welfare Gains Relative to FX Rule without Persistence, <math>\rho_{FX} = 0</math></b>				
Productivity, $A$	0.25	0.17	0.04	0.16
Preference shock, $\eta$	0.25	0.16	0.03	0.28
Government expenditure, $G$	< 0.01	< 0.01	< 0.01	< 0.01
World trade, $WT$	0.01	0.01	< 0.01	0.01
Risk premium, $\hat{\theta}^*$	0.03	0.01	< 0.01	0.04
Capital inflows, $\hat{\phi}^*$	0.27	0.18	0.04	0.27
All shocks	0.81	0.54	0.12	0.77
<b>Panel C: Welfare Gains Relative to FX Rule with Persistence, <math>\rho_{FX} = 0.9</math></b>				
Productivity, $A$	0.05	0.05	0.02	0.02
Preference shock, $\eta$	0.03	0.03	0.01	0.04
Government expenditure, $G$	< 0.01	< 0.01	< 0.01	< 0.01
World trade, $WT$	< 0.01	< 0.01	< 0.01	< 0.01
Risk premium, $\hat{\theta}^*$	< 0.01	< 0.01	< 0.01	< 0.01
Capital inflows, $\hat{\phi}^*$	0.03	0.03	0.01	0.04
All shocks	0.12	0.12	0.05	0.10

Note: The table presents the lifetime welfare gains from using optimal FXIs compared to fixed foreign reserves (Panel A), and compared to policy rule (51) with  $\rho_{FX} = 0$  (Panel B), and  $\rho_{FX} = 0.9$  (Panel C). In all cases, monetary policy sets the interest rate optimally. Welfare gains are expressed as a percentage of annual steady state consumption. These gains represent the maximum amount that an agent living in an economy with a sub-optimal FXI policy would be willing to pay to move to an identical economy with optimal FXIs. Nominal rigidities in columns (1) through (3); real economy in column (4). The portfolio adjustment cost parameter,  $\Theta''(0)$ , takes its benchmark value, 2.569, in columns (1) and (4), 10% of that value in column (2), and 1% of the benchmark value in column (3).

Table 3 presents welfare differentials between an economy that owns the entire financial sector ( $\vartheta \rightarrow 1$ ) and identical economies that only differ in their ownership share. In all cases, monetary and FXI policies are set optimally. Welfare clearly falls as foreigners own larger portions of the financial sector, reaching 1.65% of annual steady state consumption

Table 3: Welfare Gains from Domestic Ownership of the Financial Sector  
Percent of Annual Steady State Consumption

	(1)	(2)	(3)
Domestic ownership share, $\vartheta$ :	90%	50%	0%
Productivity, $A$	0.13	0.29	0.37
Preference shock, $\eta$	0.16	0.35	0.46
Government expenditure, $G$	< 0.01	< 0.01	< 0.01
World trade, $WT$	0.01	0.02	0.02
Risk premium, $\hat{\theta}^*$	0.03	0.12	0.21
Capital inflows, $\hat{\phi}^*$	0.17	0.42	0.58
All shocks	0.50	1.19	1.65

Note: The table presents the lifetime welfare gains resulting from full domestic ownership of the financial sector,  $\vartheta \rightarrow 1$ , relative to partial ownership,  $\vartheta < 1$ . All other parameters take their benchmark values. The central bank follows optimal monetary and FXI policies. Gains are expressed as a percentage of annual steady state consumption. These gains represent the maximum amount an agent living in an economy with partial ownership would be willing to pay to move to an identical economy with full domestic ownership of the financial sector. The ownership share,  $\vartheta$ , takes the value, 0.9, in columns (1), 0.5 in column (2), and 0 in column (3).

when foreigners own the entire financial sector ( $\vartheta = 0$ , Column 3). While this cost is not negligible, it is smaller than the potential benefits in Table 2 of following optimal FXIs when  $\vartheta \rightarrow 1$ . This result supports the role of FXIs as a macroeconomic stabilizer, and implies it is at least as important as shielding the economy from carry trade costs.

## 9 Conclusion

The paper incorporates FXIs into an otherwise standard New-Keynesian small open economy model. Relying on the portfolio balance channel, FXIs affect the UIP premium, thereby influencing effective returns and the exchange rate. These trigger intertemporal substitution in demand and intratemporal substitution between its domestic and foreign components. The paper demonstrates that FXIs can perfectly insulate the economy from the effect of financial shocks, and that they may be useful against real shocks as well. Since in the model the UIP premium reflects market inefficiency, policy should generally seek to stabilize it. Nevertheless, strict UIP premium targeting is not always optimal; that depends on the number of inefficiencies in the economy relative to the number of effective policy tools, adhering to the Tinbergen rule.

In this paper, as in other contributions that rely on the portfolio balance channel, the

financial friction is the only source of UIP deviations, e.g. [Benes et al. \(2015\)](#), [Cavallino \(2019\)](#), [Alla et al. \(2020\)](#), [Fanelli and Straub \(2021\)](#), [Faltermeier et al. \(2022\)](#) and [Itskhoki and Mukhin \(2023\)](#). Nevertheless, deviations from the UIP may potentially reflect other factors; for example, the pricing of sovereign default risk. In that case, the risk is driven by fiscal factors and FXIs can probably do little to affect it. Moreover, efficient markets would price that risk properly, and it is not clear whether central banks should attempt to restore the UIP condition in this situation. In this light, further research is needed to refine the policy recommendations of the paper. The research agenda should aim to decompose the UIP premium into components that the central bank should stabilize and those that should be allowed to fluctuate freely. Techniques for estimating these components should be developed as well.

## A Appendix: Model Equivalence

This appendix demonstrates that the equivalence result of [Yakhin \(2022\)](#) is robust to introducing foreign reserves, capital flows and risk premium shocks to the model, and extends the result to the model of [Itskhoki and Mukhin \(2021, 2023\)](#) as well. That is, modeling the financial friction using a simple, reduced-form, portfolio adjustment cost, as in the main text, is isomorphic, up to a first-order approximation, to the microfounded modeling strategy of [Gabaix and Maggiori \(2015\)](#), [Fanelli and Straub \(2021\)](#) and [Itskhoki and Mukhin \(2021, 2023\)](#), GM, FS and IM, respectively, hereinafter. Below I only focus on GM and IM, as the extension to FS is immediate<sup>28</sup>, and I strip the model from anything that is unrelated to the financial friction. There is no production, differentiated goods, or nominal rigidities. These abstractions do not affect the result.

### A.1 The Basic Settings

Consider a small open economy populated by a unit mass of households, a government and a financial sector. The economy is perfectly integrated in the world's goods market. There is one perishable good in the world economy and two currencies, home and foreign. Each period, households in the home economy are endowed with a random allocation of the good,  $Y_t$ . Prices are flexible, and law of one price holds. Foreign prices are normalized to 1. Generally, variables are denoted using the same symbols as in the main text. Any deviation is noted explicitly.

The central bank issues domestic risk-free bonds and controls their return,  $i_t$ . Let  $B_t^G$  denote the government holdings of these bonds. Domestic households hold  $B_t^{HH}$

---

<sup>28</sup>With linear participation cost in FS, their financial friction turns identical to that of GM. Any non-linearity in the cost function is washed away in the first-order approximation. See [Yakhin \(2022\)](#).

units of the bonds, and foreigners hold  $B_t^{ROW}$ . Capital inflows,  $\phi_t^*$ , are exogenous, they are measured in foreign currency, and relate to the foreign holdings of domestic bonds,  $B_t^{ROW}$ , by:

$$\phi_t^* = \frac{B_t^{ROW}}{S_t} - \frac{1 + i_{t-1}}{\sigma_t} \frac{B_{t-1}^{ROW}}{S_{t-1}} \quad (\text{A.1})$$

The central bank holds foreign reserves,  $FX_t$ . Foreign reserves pay the foreign risk-free interest rate,  $i_t^*$ .<sup>29</sup> In steady state  $1 + i_{ss}^* = \frac{1+i_{ss}}{\sigma_{ss}} = \beta^{-1}$ .

The consolidated government (monetary and fiscal authorities) budget constraint is given by:

$$(1 + i_{t-1}) B_{t-1}^G + S_t (1 + i_{t-1}^*) FX_{t-1} = B_t^G + S_t FX_t + T_t \quad (\text{A.2})$$

where  $T_t$  is lump-sum transfers to the households.

## A.2 The Portfolio Adjustment Cost Model

Domestic households have access to the international financial markets, but face a convex adjustment cost whenever the level of their foreign asset position,  $b_t^{*,HH}$ , deviates from some long run target level,  $\bar{b}^{*,HH}$ , plus a zero-mean noise,  $\theta_t^*$ . A fraction  $\vartheta$  of the cost is rebated to the households.

**Households** The households maximize their expected lifetime utility,  $E_0 \sum_{t=0}^{\infty} \beta^t U(C_t)$ , subject to the flow budget constraint:

$$\begin{aligned} & S_t C_t + B_t^{HH} + S_t b_t^{*,HH} + S_t \Theta \left( b_t^{*,HH} - \bar{b}^{*,HH} - \theta_t^* \right) \\ & \leq S_t Y_t + (1 + i_{t-1}) B_{t-1}^{HH} + S_t (1 + i_{t-1}^*) b_{t-1}^{*,HH} + \vartheta S_t \Pi_t + T_t \end{aligned}$$

where  $\Theta(\cdot)$  is a convex cost function that satisfies:

$$\Theta(\cdot) \geq 0 \quad , \quad \Theta(0) = 0 \quad , \quad \Theta'(0) = 0 \quad , \quad \Theta''(\cdot) > 0$$

$\Pi_t$  is the average adjustment cost in the economy and each household is rebated a portion  $\vartheta$  of that cost. Since the rebate is a function of the economy's average cost, households do not internalize the effect of their choice of  $b_t^{*,HH}$  on  $\Pi_t$ .

The first order conditions of households:

$$U_{C,t} = \beta (1 + i_t) E_t \left( \frac{U_{C,t+1}}{\sigma_{t+1}} \right) \quad (\text{A.3})$$

$$U_{C,t} \left[ 1 + \Theta' \left( b_t^{*,HH} - \bar{b}^{*,HH} - \theta_t^* \right) \right] = \beta (1 + i_t^*) E_t (U_{C,t+1}) \quad (\text{A.4})$$

---

<sup>29</sup>Note that here foreign reserves are expressed in units of foreign currency rather than units of foreign goods, as in the text.

Combining the two equations gives the modified UIP:

$$(1 + i_t) E_t \left( \frac{U_{C,t+1}}{\sigma_{t+1}} \right) \left[ 1 + \Theta' \left( b_t^{*,HH} - \bar{b}^{*,HH} - \theta_t^* \right) \right] = (1 + i_t^*) E_t (U_{C,t+1}) \quad (\text{A.5})$$

**Market Clearing and the BOP** In the financial markets:

$$B_t^G + B_t^{HH} + B_t^{ROW} = 0$$

The BOP identity is derived by consolidating the government budget constraint and the households' budget constraint together with the market clearing condition above, while taking into account that a portion  $\vartheta$  of the portfolio adjustment cost is rebated to the households. This results in:

$$\begin{aligned} FX_t + b_t^{*,HH} &= (1 + i_{t-1}^*) \left( FX_{t-1} + b_{t-1}^{*,HH} \right) + \phi_t^* \\ &+ Y_t - C_t - (1 - \vartheta) \Theta \left( b_t^{*,HH} - \bar{b}^{*,HH} - \theta_t^* \right) \end{aligned} \quad (\text{A.6})$$

where  $\phi_t^*$  is defined in (A.1).

**Closing the Model** The households' optimality conditions, equations (A.3) and (A.5), together with the BOP, equation (A.6), result in a system of 3 equations in 5 endogenous variables:  $C_t$ ,  $i_t$ ,  $\sigma_t$ ,  $b_t^{*,HH}$  and  $FX_t$ .  $Y_t$ ,  $i_t^*$ ,  $\phi_t^*$  and  $\theta_t^*$  are exogenous. The model is closed by specifying a policy rule for the nominal interest rate,  $i_t$ , and for foreign reserves,  $FX_t$ .

**Log-Linearized Equations** Log-linearizing equations (A.3), (A.5) and (A.6), the approximated model is characterized by:

$$\gamma_{cc} \widetilde{C}_t \cong \widetilde{(1 + i_t)} + \gamma_{cc} E_t \left( \widetilde{C}_{t+1} \right) - E_t \left( \widetilde{\sigma}_{t+1} \right) \quad (\text{A.7})$$

$$\begin{aligned} E_t \left( \widetilde{\sigma}_{t+1} \right) &\cong \widetilde{(1 + i_t)} - \widetilde{(1 + i_t^*)} \\ &+ \Theta''(0) \left[ \left( b_t^{*,HH} - \bar{b}^{*,HH} \right) - \theta_t^* \right] \end{aligned} \quad (\text{A.8})$$

$$\begin{aligned} \frac{FX_{ss}}{Y_{ss}} \widetilde{FX}_t + \frac{b_t^{*,HH} - \bar{b}^{*,HH}}{Y_{ss}} &\cong \widetilde{Y}_t - \frac{C_{ss}}{Y_{ss}} \widetilde{C}_t + \beta^{-1} \frac{FX_{ss} + \bar{b}^{*,HH}}{Y_{ss}} \widetilde{(1 + i_t^*)} \\ &+ \beta^{-1} \left[ \frac{FX_{ss}}{Y_{ss}} \widetilde{FX}_{t-1} + \frac{b_{t-1}^{*,HH} - \bar{b}^{*,HH}}{Y_{ss}} \right] + \frac{\phi_t^* - \phi_{ss}^*}{Y_{ss}} \end{aligned} \quad (\text{A.9})$$

### A.3 The GM Model

This section builds on Gabaix and Maggiori (2015). In this model, households only hold domestic risk-free bonds, as they do not have access to the international financial markets. Financial arbitrageurs absorb domestic saving imbalances for a premium. Limited

commitment generates deviations from the UIP. Domestic households own a fraction  $\vartheta$  of the financial firms.

**Households** The households maximize their expected lifetime utility,  $E_0 \sum_{t=0}^{\infty} \beta^t U(C_t)$ , subject to the flow budget constraint:

$$S_t C_t + B_t^{HH} \leq S_t Y_t + (1 + i_{t-1}) B_{t-1}^{HH} + \vartheta S_t \Pi_t + T_t$$

where here  $\Pi_t$  represents the dividends from the financiers'. The first order conditions of households is given by:

$$U_{C,t} = \beta (1 + i_t) E_t \left( \frac{U_{C,t+1}}{\sigma_{t+1}} \right) \quad (\text{A.10})$$

which is identical to (A.3).

**Financiers** Agents are selected at random to operate the financial firms for a single period. The selection process is memoryless. Financiers start each period with no liabilities and a net worth of  $\bar{\mathcal{B}}^* + \theta_t^*/\vartheta$ , denominated in foreign currency, which is held in foreign bonds.  $\theta_t^*$  is a zero-mean random shock. They maintain this position through their dividend distribution policy. The quantity  $\bar{\mathcal{B}}^* + \theta_t^*/\vartheta$  is interpreted as the financiers' preferred asset position, as they require a premium for deviating from it in order to absorb excess domestic savings.

Let  $Q_t$  denote the financiers' holdings of domestic bonds, which can be either positive or negative. The absolute value of  $Q_t$  reflects the scale of financial intermediation in the economy. When domestic agents require excess resources, the financiers borrow from abroad in foreign currency and extend a loan of the same value in domestic currency to domestic agents ( $Q_t > 0$ ). When domestic agents wish to save, they lend the financiers in domestic currency ( $Q_t < 0$ ) and the financiers convert these funds into foreign bonds. The asset portfolio of the financial sector is therefore composed of  $Q_t$  units of domestic bonds and  $\bar{\mathcal{B}}^* + \theta_t^*/\vartheta - \frac{Q_t}{S_t}$  units of foreign bonds.

The financiers' pre-dividend domestic-currency value at the end of their one period term is given by  $(1 + i_t) Q_t + S_{t+1} (1 + i_t^*) \left( \bar{\mathcal{B}}^* + \theta_t^*/\vartheta - \frac{Q_t}{S_t} \right)$ , and they seek to maximize its expected discounted value, which can be written as:

$$V_t = \left[ 1 - \frac{1 + i_t^*}{1 + i_t} E_t(\sigma_{t+1}) \right] Q_t + E_t(S_{t+1}) \frac{1 + i_t^*}{1 + i_t} \left( \bar{\mathcal{B}}^* + \frac{\theta_t^*}{\vartheta} \right) \quad (\text{A.11})$$

Financiers are unable to perfectly commit to repay their creditors, and before the end of period  $t$ , i.e. before  $S_{t+1}$  is realized, they can divert a portion  $\Gamma \left| \frac{Q_t}{S_t} \right|$  of their liabilities,  $\Gamma > 0$ . Since creditors correctly anticipate the incentives of the financiers, the latter are

subject to a credit constraint of the form:

$$V_t \geq E_t(S_{t+1}) \frac{1+i_t^*}{1+i_t} \left( \bar{\mathcal{B}}^* + \frac{\theta_t^*}{\vartheta} \right) + \Gamma \left| \frac{Q_t}{S_t} \right| |Q_t| = E_t(S_{t+1}) \frac{1+i_t^*}{1+i_t} \left( \bar{\mathcal{B}}^* + \frac{\theta_t^*}{\vartheta} \right) + \Gamma \frac{Q_t^2}{S_t} \quad (\text{A.12})$$

The financiers' problem is therefore to choose  $Q_t$  so as to maximize  $V_t$ , as presented in (A.11), subject to (A.12). Since the objective function is linear in  $Q_t$  while the constraint is convex, at the optimum the constraint always binds, and the financiers' demand for foreign assets, in excess of their base position  $\bar{\mathcal{B}}^* + \theta_t^*/\vartheta$ , is given by:

$$-\frac{Q_t}{S_t} = \frac{1}{\Gamma} \left[ \frac{1+i_t^*}{1+i_t} E_t(\sigma_{t+1}) - 1 \right] \quad (\text{A.13})$$

This is the modified UIP equation in the GM model. I will now express it in terms of quantities comparable to those of the portfolio adjustment cost model. Let  $b_t^{*,HH}$  denote the value of assets, in units of foreign currency, that domestic households hold through financial intermediaries. These assets are composed of  $-Q_t$  home-currency deposits, and a claim to a fraction  $\vartheta$  of the financiers' net worth, suggesting:

$$\begin{aligned} b_t^{*,HH} &= -\frac{Q_t}{S_t} + \bar{b}^{*,HH} + \theta_t^* \\ \text{where } \bar{b}^{*,HH} &\equiv \vartheta \bar{\mathcal{B}}^* \end{aligned}$$

Substituting for  $-\frac{Q_t}{S_t}$  in (A.13) and rearranging, the modified UIP reads:

$$E_t(\sigma_{t+1}) = \frac{1+i_t}{1+i_t^*} \left[ 1 + \Gamma \left( b_t^{*,HH} - \bar{b}^{*,HH} - \theta_t^* \right) \right] \quad (\text{A.14})$$

Finally, The financiers' distributed dividends are given by:

$$\Pi_t = (1+i_{t-1}^*) \left( \bar{\mathcal{B}}^* + \frac{\theta_{t-1}^*}{\vartheta} - \frac{Q_{t-1}}{S_{t-1}} \right) + \frac{1+i_{t-1}}{\sigma_t} \frac{Q_{t-1}}{S_{t-1}} - \left( \bar{\mathcal{B}}^* + \frac{\theta_t^*}{\vartheta} \right)$$

where the first two terms on the right-hand sides are the gross return on the previous period's holdings of foreign and domestic bonds, and the last term subtracts the financier's net worth that is carried over to the current period.

**Market Clearing and the BOP** In the financial markets:

$$B_t^G + B_t^{HH} + Q_t + B_t^{ROW} = 0$$

The BOP identity is derived by consolidating the government budget constraint and the households' budget constraint together with the market clearing condition above, while taking into account that a portion  $\vartheta$  of the financiers' dividends are distributed to

domestic households. This results in:

$$\begin{aligned}
FX_t + b_t^{*,HH} &= (1 + i_{t-1}^*) FX_{t-1} + \phi_t^* + Y_t - C_t \\
&+ \left[ (1 - \vartheta) \frac{1 + i_{t-1}}{\sigma_t} + \vartheta (1 + i_{t-1}^*) \right] b_{t-1}^{*,HH} \\
&+ (1 - \vartheta) \left[ (1 + i_{t-1}^*) - \frac{1 + i_{t-1}}{\sigma_t} \right] (\bar{b}^{*,HH} + \theta_{t-1}^*)
\end{aligned} \tag{A.15}$$

where  $\phi_t^*$  is defined in (A.1).

**Closing the Model** The households' optimality condition, equation (A.10), the modified UIP, equation (A.14), together with the BOP, equation (A.15), result in a system of 3 equations in 5 endogenous variables:  $C_t$ ,  $i_t$ ,  $\sigma_t$ ,  $b_t^{*,HH}$  and  $FX_t$ .  $Y_t$ ,  $i_t^*$ ,  $\phi_t^*$  and  $\theta_t^*$  are exogenous. The model is closed by specifying a policy rule for the nominal interest rate,  $i_t$ , and for foreign reserves,  $FX_t$ .

**Log-Linearized Equations** Log-linearizing equations (A.10), (A.14) and (A.15), the approximated GM model is characterized by:

$$\gamma_{cc} \widetilde{C}_t \cong \widetilde{(1 + i_t)} + \gamma_{cc} E_t \left( \widetilde{C}_{t+1} \right) - E_t \left( \widetilde{\sigma}_{t+1} \right) \tag{A.16}$$

$$\begin{aligned}
E_t \left( \widetilde{\sigma}_{t+1} \right) &\cong \widetilde{(1 + i_t)} - \widetilde{(1 + i_t^*)} \\
&+ \Gamma \left[ \left( b_t^{*,HH} - \bar{b}^{*,HH} \right) - \theta_t^* \right]
\end{aligned} \tag{A.17}$$

$$\begin{aligned}
\frac{FX_{ss}}{Y_{ss}} \widetilde{FX}_t + \frac{b_t^{*,HH} - \bar{b}^{*,HH}}{Y_{ss}} &\cong \widetilde{Y}_t - \frac{C_{ss}}{Y_{ss}} \widetilde{C}_t + \beta^{-1} \frac{FX_{ss} + \bar{b}^{*,HH}}{Y_{ss}} \widetilde{(1 + i_{t-1}^*)} \\
&+ \beta^{-1} \left[ \frac{FX_{ss}}{Y_{ss}} \widetilde{FX}_{t-1} + \frac{b_{t-1}^{*,HH} - \bar{b}^{*,HH}}{Y_{ss}} \right] + \frac{\phi_t^* - \phi_{ss}^*}{Y_{ss}}
\end{aligned} \tag{A.18}$$

## A.4 The IM Model

This section adopts the financial structure of [Itskhoki and Mukhin \(2021\)](#). The derivation below builds on Appendix A.4 of their paper. In their model, risk aversion of financial intermediaries generates deviations from the UIP.<sup>30</sup> The households' problem is identical to that of the GM model, so I start with the description of the financial sector.

---

<sup>30</sup>[Itskhoki and Mukhin \(2023\)](#) adopt a slightly different modelling of the financial sector and resort to a novel approximation technique that leaves their UIP equation non-linear. Nevertheless, under standard first order approximation of variables around their deterministic steady state, coupled with the assumption that as the variance of exchange rate movements falls, the financiers' risk aversion rises proportionally (see [Itskhoki and Mukhin \(2021\)](#) and below), it is immediate to show that the simple portfolio adjustment cost is isomorphic to the model of [Itskhoki and Mukhin \(2023\)](#) as well.



**Financiers** As in the GM model, financiers start each period with no liabilities and a net worth of  $\bar{B}^* + \theta_t^*/\vartheta$ , denominated in foreign currency, and held in foreign bonds. They maintain this position through their dividend distribution policy. Let  $Q_t$  denote the financiers' holdings of domestic bonds. The asset portfolio of the financial sector is composed of  $Q_t$  units of domestic bonds and  $\bar{B}^* + \theta_t^*/\vartheta - \frac{Q_t}{S_t}$  units of foreign bonds.

Letting  $q_t \equiv -\frac{Q_t}{S_t}$ , the present discounted value of the financiers' pre-dividend portfolio, denominated in foreign currency, is given by:

$$V_t = \left[ 1 - \frac{1+i_t}{1+i_t^*} \frac{1}{\sigma_{t+1}} \right] q_t + \bar{B}^* + \frac{\theta_t^*}{\vartheta} \quad (\text{A.19})$$

Financial intermediaries optimally choose  $q_t$  by maximizing the expected value of a CARA utility,  $U(V_t) = -\frac{1}{\omega} \exp(-\omega V_t)$ . Note that:

$$E_t U(V_t) = -\frac{1}{\omega} E_t \exp \left\{ -\omega \left[ 1 - \frac{1+i_t}{1+i_t^*} \frac{1}{\sigma_{t+1}} \right] q_t \right\} \exp \left\{ -\omega \left( \bar{B}^* + \frac{\theta_t^*}{\vartheta} \right) \right\}$$

and since  $\exp \left\{ -\omega \left( \bar{B}^* + \frac{\theta_t^*}{\vartheta} \right) \right\}$  is positive and known at the time of the portfolio choice, it does not affect the financiers' decision and can be dropped from the objective function.

Letting:

$$x_{t+1} \equiv \log(1+i_t) - \log(1+i_t^*) - \log(\sigma_{t+1})$$

The financiers' problem can be written as:

$$\underset{q_t}{Max} \quad -\frac{1}{\omega} E_t \exp[-\omega(1 - e^{x_{t+1}}) q_t] \quad (\text{A.20})$$

At this stage [Itskhoki and Mukhin \(2021\)](#) approximate the problem to its continuous time counterpart. When time periods are short  $x_{t+1}$  corresponds to increments of a normal diffusion process  $d\mathcal{X}_t$  with time-varying drift  $\mu_t = \log(1+i_t) - \log(1+i_t^*) - E_t[\log(\sigma_{t+1})]$  and time-invariant conditional variance  $\sigma_s^2 = \text{var}_t[\log(\sigma_{t+1})]$ :

$$d\mathcal{X}_t = \mu_t dt + \sigma_s^2 dB_t \quad (\text{A.21})$$

where  $B_t$  is a Brownian motion. With short time periods, the solution to [\(A.20\)](#) is equivalent to:

$$\underset{q_t}{Max} \quad -\frac{1}{\omega} E_t \exp[-\omega(1 - e^{d\mathcal{X}_t}) q_t]$$

where  $d\mathcal{X}_t$  follows [\(A.21\)](#). Using Ito's lemma the financiers' problem can be written as:

$$\underset{q_t}{Max} \quad -\frac{1}{\omega} E_t \exp \left[ \omega \left( \mu_t + \frac{1}{2} \sigma_s^2 \right) q_t + \frac{\omega^2 \sigma_s^2}{2} q_t^2 \right] \quad (\text{A.22})$$

Taking first order condition and rearranging:

$$q_t = -\frac{\mu_t + \frac{1}{2}\sigma_s^2}{\omega\sigma_s^2}$$

Substituting for  $\mu_t$  and  $q_t$  results in:

$$-\frac{Q_t}{S_t} = -\frac{\log(1+i_t) - \log(1+i_t^*) - E_t[\log(\sigma_{t+1})] + \frac{1}{2}\sigma_s^2}{\omega\sigma_s^2} \quad (\text{A.23})$$

This is the modified UIP equation in the IM model. I will now express it in terms of quantities comparable to those of the portfolio adjustment cost model. Let  $b_t^{*,HH}$  denote the value of assets, in units of foreign currency, that domestic households hold through financial intermediaries. These assets are composed of  $-Q_t$  home-currency deposits, and a claim to a fraction  $\vartheta$  of the financiers' net worth, suggesting:

$$\begin{aligned} b_t^{*,HH} &= -\frac{Q_t}{S_t} + \bar{b}^{*,HH} + \theta_t^* \\ \text{where } \bar{b}^{*,HH} &\equiv \vartheta\bar{B}^* \end{aligned}$$

Substituting for  $-\frac{Q_t}{S_t}$  in (A.23) and rearranging, the modified UIP reads:

$$E_t[\log(\sigma_{t+1})] = \log(1+i_t) - \log(1+i_t^*) + \omega\sigma_s^2 \left( b_t^{*,HH} - \bar{b}^{*,HH} - \theta_t^* \right) - \frac{1}{2}\sigma_s^2 \quad (\text{A.24})$$

Itskhoki and Mukhin (2021) assume that as  $\sigma_s^2$  shrinks, i.e. exchange rate risk falls, the financiers' risk aversion,  $\omega$ , rises proportionally leaving the product  $\omega\sigma_s^2$  constant and nonzero in the limit. This assumption guarantees that the risk premium in (A.24) is first order, and does not wash into the approximation error in the log-linearized system.

Finally, the financiers' distributed dividends are given by:

$$\Pi_t = (1+i_{t-1}^*) \left( \bar{B}^* + \theta_{t-1}^*/\vartheta - \frac{Q_{t-1}}{S_{t-1}} \right) + \frac{1+i_{t-1}}{\sigma_t} \frac{Q_{t-1}}{S_{t-1}} - \left( \bar{B}^* + \theta_t^*/\vartheta \right)$$

which is the same as in the GM model.

**Market Clearing and the BOP** In the financial markets:

$$B_t^G + B_t^{HH} + Q_t + B_t^{ROW} = 0$$

The BOP identity is derived by consolidating the government budget constraint and the households' budget constraint together with the market clearing condition above, while taking into account that a portion  $\vartheta$  of the financiers' dividends are distributed to

domestic households. This results in:

$$\begin{aligned}
FX_t + b_t^{*,HH} &= (1 + i_{t-1}^*) FX_{t-1} + \phi_t^* + Y_t - C_t \\
&+ \left[ \vartheta (1 + i_{t-1}^*) + (1 - \vartheta) \frac{1 + i_{t-1}}{\sigma_t} \right] b_{t-1}^{*,HH} \\
&+ (1 - \vartheta) \left[ (1 + i_{t-1}^*) - \frac{1 + i_{t-1}}{\sigma_t} \right] (\bar{b}^{*,HH} + \theta_{t-1}^*)
\end{aligned} \tag{A.25}$$

which is the same as the BOP in the GM model, equation (A.15).

**Closing the Model** The households' optimality condition is the same as in the GM model, equation (A.10), together with the modified UIP, equation (A.24), and the BOP, equation (A.25), result in a system of 3 equations in 5 endogenous variables:  $C_t$ ,  $i_t$ ,  $\sigma_t$ ,  $b_t^{*,HH}$  and  $FX_t$ .  $Y_t$ ,  $i_t^*$ ,  $\phi_t^*$  and  $\theta_t^*$  are exogenous. The model is closed by specifying a policy rule for the nominal interest rate,  $i_t$ , and for foreign reserves,  $FX_t$ .

**Log-Linearized Equations** Log-linearizing equations (A.10), (A.24) and (A.25), the approximated IM model is characterized by:

$$\gamma_{cc} \widetilde{C}_t \cong \widetilde{(1 + i_t)} + \gamma_{cc} E_t \left( \widetilde{C}_{t+1} \right) - E_t \left( \widetilde{\sigma}_{t+1} \right) \tag{A.26}$$

$$\begin{aligned}
E_t \left( \widetilde{\sigma}_{t+1} \right) &\cong \widetilde{(1 + i_t)} - \widetilde{(1 + i_t^*)} \\
&+ \omega \sigma_s^2 \left[ \left( b_t^{*,HH} - \bar{b}^{*,HH} \right) - \theta_t^* \right]
\end{aligned} \tag{A.27}$$

$$\begin{aligned}
\frac{FX_{ss}}{Y_{ss}} \widetilde{FX}_t + \frac{b_t^{*,HH} - \bar{b}^{*,HH}}{Y_{ss}} &\cong \widetilde{Y}_t - \frac{C_{ss}}{Y_{ss}} \widetilde{C}_t + \beta^{-1} \frac{FX_{ss} + \bar{b}^{*,HH}}{Y_{ss}} \widetilde{(1 + i_{t-1}^*)} \\
&+ \beta^{-1} \left[ \frac{FX_{ss}}{Y_{ss}} \widetilde{FX}_{t-1} + \frac{b_{t-1}^{*,HH} - \bar{b}^{*,HH}}{Y_{ss}} \right] + \frac{\phi_t^* - \phi_{ss}^*}{Y_{ss}}
\end{aligned} \tag{A.28}$$

## A.5 Model Equivalence

Equations (A.16) and (A.26) are identical to (A.7), equations (A.18) and (A.28) are identical to (A.9), and for  $\Gamma = \Theta''(0) = \omega \sigma_s^2$  equations (A.17) and (A.27) are identical to (A.8), suggesting the portfolio adjustment cost is isomorphic, up to a first-order approximation, to the GM and IM models.

## B Appendix: The Households' Problem

This appendix models explicitly the insurance market against the households' idiosyncratic risk, and sets up the their problem. The optimality conditions of this problem are displayed in the main text.

Date  $t$  aggregate exogenous events are denoted by  $s_t$ , and  $s^t$  denotes the history of events from date zero to date  $t$ , that is  $s^t = (s_0, s_1, \dots, s_t)$ .

## B.1 The Insurance Market Against Idiosyncratic Risk

Households receive a binary idiosyncratic shock,  $\Upsilon_t(h)$ , that signals whether they are able to reset their wage. When  $\Upsilon_t(h) = 1$  household  $h$  is allowed to adjust its nominal wage, otherwise  $\Upsilon_t(h) = 0$  and  $W_t(h) = \pi_{ss} W_{t-1}(h)$ . The probability of wage adjustment is  $1 - \xi_w$ .

Insurance companies operate in a perfectly competitive market. Every period households and insurance companies meet to sign state-contingent wage insurance contracts against next period's idiosyncratic shocks. Under each contract household  $h$  is obliged to pay the insurance company one unit of the domestic currency in period  $t+1$  if  $\Upsilon_{t+1}(h) = 1$ , otherwise  $\Upsilon_{t+1}(h) = 0$  and the household receives  $\psi_t$  units. Let  $b_t(s^t, s_{t+1}, h)$  denote the quantity of such contracts associated with household  $h$ . Notice that the time index of both  $\psi$  and  $b$  highlights that they are determined at date  $t$  when the contract is signed.

Zero profits for any history of aggregate events,  $(s^t, s_{t+1})$ , requires:

$$\int_0^1 \Upsilon_{t+1}(h) b_t(s^t, s_{t+1}, h) dh = \psi_t(s^t, s_{t+1}) \int_0^1 [1 - \Upsilon_{t+1}(h)] b_t(s^t, s_{t+1}, h) dh$$

Taking expectations conditional on date  $t$  information and using  $E_t[\Upsilon_{t+1}(h)] = 1 - \xi_w$ , pins down  $\psi_t$ :

$$\psi_t(s^t, s_{t+1}) = \frac{1 - \xi_w}{\xi_w} \quad (\text{B.1})$$

which reflects actuarially fair pricing.

## B.2 Households

Households consume the final good, trade risk-free home and foreign nominal bonds, supply labor, and trade wage insurance contracts.

Domestic bonds,  $B_t$ , cost one unit of the domestic currency at date  $t$  and pay  $1 + i_t$  units in  $t+1$ . Foreign bonds,  $B_t^*$ , cost one unit of the effective foreign currency and pay  $1 + i_t^*$  units in  $t+1$ .<sup>31</sup> Let  $B_t^{HH}(h)$  and  $B_t^{*,HH}(h)$  denote the corresponding quantities held by household  $h$ , and define its foreign asset position in units of foreign goods as:

$$b_t^{*,HH}(h) \equiv \frac{B_t^{*,HH}(h)}{P_t^{F*}}$$

---

<sup>31</sup>  $B_t^*$  is an aggregate of bonds from all foreign countries and  $i_t^*$  is their effective return. By symmetry across foreign countries and assuming they face the same shocks, it is safe to treat the rest of the world as one entity.

Trading in the international asset markets is costly. Households face a portfolio adjustment cost of  $\Theta \left( b_t^{*,HH} (h), \theta_t^* \right)$ , measured in units of foreign goods, where  $\theta_t^*$  is an exogenous aggregate financial shock. The function  $\Theta (\cdot)$  satisfies:

$$\Theta (\cdot) \geq 0 \quad , \quad \Theta'' (\cdot) > 0 \quad , \quad \Theta (0) = \Theta' (0) = 0$$

That is, a household incurs a cost whenever its foreign asset position,  $b_t^{*,HH} (h)$ , deviates from some benchmark,  $\theta_t^*$ . A fraction  $\vartheta$  of the *aggregate* portfolio adjustment costs is rebated to domestic households. The households do not internalize this effect when they choose their asset position.

Each household is endowed with a differentiated labor skill,  $n_t (h)$ , and holds a monopolistic power over supplying it to the employment agencies. Wage setting is staggered à la Calvo (1983), with parameters as described above in Section B.1.

Consumption of household  $h$  is denoted by  $c_t (h)$ . Households rank allocations of consumption and labor using utility function,  $U [c_t (h), n_t (h); \eta_t]$ , that satisfies the standard properties, where  $\eta_t$  is an aggregate preferences shock. Finally, let  $\Pi_t$  denote firms' profits, and  $T_t$  denote government lump-sum transfers.

Household  $h$  solves:

$$\begin{aligned} & V_{1,t} \left( s^t, \Upsilon_t (h) = 1, B_{t-1}^{HH} (h), b_{t-1}^{*,HH} (h), b_{t-1} (s^t, h) \right) \\ = & \underset{c_t (h), B_t^{HH} (h), b_t^{*,HH} (h), b_t (s^{t+1}, h), W_t (h)}{Max} \left\{ \begin{array}{l} U [c_t (h), n_t (h); \eta_t] \\ + (1 - \xi_w) \beta E_t \{ V_{1,t+1} (\cdot) / s^t, \Upsilon_{t+1} (h) = 1 \} \\ + \xi_w \beta E_t \{ V_{0,t+1} (\cdot, \bar{W}_{t+1} (h)) / s^t, \Upsilon_{t+1} (h) = 0 \} \end{array} \right\} \\ \\ s.t. & \quad n_t (h) = \left( \frac{W_t (h)}{W_t} \right)^{-\varepsilon^N} N_t \\ & \quad \bar{W}_{t+1} (h) = \pi_{ss} W_t (h) \\ c_t (h) + & \frac{S_t P_t^{F*} b_t^{*,HH} (h)}{P_t} + \frac{B_t^{HH} (h)}{P_t} = \frac{W_t (h)}{P_t} n_t (h) \\ & \quad + \frac{S_t (1 + i_{t-1}^*) P_{t-1}^{F*} b_{t-1}^{*,HH} (h)}{P_t} - \frac{S_t P_t^{F*} \Theta \left( b_t^{*,HH} (h), \theta_t^* \right)}{P_t} \\ & \quad + \frac{(1 + i_{t-1}) B_{t-1}^{HH} (h)}{P_t} + \frac{\Pi_t + T_t - b_{t-1} (s^t, h)}{P_t} \end{aligned}$$

and:

$$\begin{aligned}
& V_{0,t} \left( s^t, \Upsilon_t(h) = 0, B_{t-1}(h), b_{t-1}^{*,HH}(h), b_{t-1}(s^t, h), \bar{W}_t(h) \right) \\
&= \underset{c_t(h), B_t^{HH}(h), b_t^{*,HH}(h), b_t(s^{t+1}, h)}{\text{Max}} \left\{ \begin{array}{l} U[c_t(h), n_t(h); \eta_t] \\ + (1 - \xi_w) \beta E_t \{ V_{1,t+1}(\cdot) / s^t, \Upsilon_{t+1}(h) = 1 \} \\ + \xi_w \beta E_t \{ V_{0,t+1}(\cdot, \bar{W}_{t+1}(h)) / s^t, \Upsilon_{t+1}(h) = 0 \} \end{array} \right\} \\
& \text{s.t.} \quad n_t(h) = \left( \frac{\bar{W}_t(h)}{W_t} \right)^{-\varepsilon^N} N_t \\
& \quad \bar{W}_{t+1}(h) = \pi_{ss} \bar{W}_t(h) \\
& c_t(h) + \frac{S_t P_t^{F*} b_t^{*,HH}(h)}{P_t} + \frac{B_t^{HH}(h)}{P_t} = \frac{\bar{W}_t(h)}{P_t} n_t(h) \\
& \quad + \frac{S_t (1 + i_{t-1}^*) P_{t-1}^{F*} b_{t-1}^{*,HH}(h)}{P_t} - \frac{S_t P_t^{F*} \Theta(b_t^{*,HH}(h), \theta_t^*)}{P_t} \\
& \quad + \frac{(1 + i_{t-1}) B_{t-1}^{HH}(h)}{P_t} + \frac{\Pi_t + T_t + \psi_{t-1}(s^t) b_{t-1}(s^t, h)}{P_t}
\end{aligned}$$

Where  $\bar{W}_t(h)$  is the wage of  $n_t(h)$  whenever  $\Upsilon_t(h) = 0$ . Under  $V_1$  wage is a choice variable, while under  $V_0$  it is part of the state variables. Also notice that the budget constraints differ in the payment to/from the insurance companies.

## C Appendix: The Frisch Elasticity of Labor Supply

After deriving the dynamics of wage inflation, equation (23) in the text, we noted that the expression  $\gamma_{nn} - \frac{\gamma_{nc} \gamma_{cn}}{\gamma_{cc}}$  is the inverse of the Frisch elasticity of labor supply evaluated in steady state. This appendix shows, more generally, that this expression corresponds to the inverse of the Frisch elasticity of labor supply under *flexible wages*.

**Proposition C.1** *Under flexible wages, i.e. as  $\xi_w \rightarrow 0$ , the Frisch elasticity of labor supply is given by:*

$$\begin{aligned}
\frac{\partial N_t}{\partial \omega_t} \Big|_{\lambda_t} \frac{\omega_t}{N_t} &= \left( \gamma_{nn,t} - \frac{\gamma_{nc,t} \gamma_{cn,t}}{\gamma_{cc,t}} \right)^{-1} \\
\text{where} \quad \gamma_{cc,t} &\equiv \frac{U_{cc,t} C_t}{U_{c,t}} \quad , \quad \gamma_{nn,t} \equiv \frac{U_{nn,t} N_t}{U_{n,t}} \\
\gamma_{cn,t} &\equiv \frac{U_{cn,t} N_t}{U_{c,t}} \quad , \quad \gamma_{nc,t} \equiv \frac{U_{nc,t} C_t}{U_{n,t}}
\end{aligned}$$

$\omega_t \equiv \frac{W_t}{P_t}$  is real wage and  $\lambda_t$  is the Lagrange multiplier on the households' intertemporal budget constraint.

**Proof.** Under flexible wages, the households' optimality conditions are given by:

$$U_{c,t} = \lambda_t \quad (\text{C.1})$$

$$-\frac{\varepsilon^N}{\varepsilon^N - 1} U_{n,t} = \lambda_t \omega_t \quad (\text{C.2})$$

where we have suppressed the household index, as they are identical under flexible wages. Partially differentiating with respect to the real wage while holding  $\lambda_t$  constant results in:

$$U_{cc,t} \left. \frac{\partial C_t}{\partial \omega_t} \right|_{\lambda_t} + U_{cn,t} \left. \frac{\partial N_t}{\partial \omega_t} \right|_{\lambda_t} = 0 \quad (\text{C.3})$$

$$-\frac{\varepsilon^N}{\varepsilon^N - 1} U_{nc,t} \left. \frac{\partial C_t}{\partial \omega_t} \right|_{\lambda_t} - \frac{\varepsilon^N}{\varepsilon^N - 1} U_{nn,t} \left. \frac{\partial N_t}{\partial \omega_t} \right|_{\lambda_t} = \lambda_t \quad (\text{C.4})$$

By (C.3):

$$\left. \frac{\partial C_t}{\partial \omega_t} \right|_{\lambda_t} = - \frac{U_{cn,t}}{U_{cc,t}} \left. \frac{\partial N_t}{\partial \omega_t} \right|_{\lambda_t}$$

and by the optimality condition for wages, equation (C.2):

$$\lambda_t = - \frac{\varepsilon^N}{\varepsilon^N - 1} \frac{U_{n,t}}{\omega_t}$$

Substituting the results into (C.4) gives:

$$\frac{U_{nc,t} U_{cn,t}}{U_{cc,t}} \left. \frac{\partial N_t}{\partial \omega_t} \right|_{\lambda_t} - U_{nn,t} \left. \frac{\partial N_t}{\partial \omega_t} \right|_{\lambda_t} = - \frac{U_{n,t}}{\omega_t}$$

Rearrange and get the Frisch elasticity of labor supply:

$$\left. \frac{\partial N_t}{\partial \omega_t} \right|_{\lambda_t} \frac{\omega_t}{N_t} = \frac{U_{n,t}}{N_t \left( U_{nn,t} - \frac{U_{nc,t} U_{cn,t}}{U_{cc,t}} \right)}$$

We now rewrite this expression in terms of the elasticities of the marginal utilities,  $U_{c,t}$  and  $U_{n,t}$ , with respect to consumption and labor,  $C_t$  and  $N_t$ :

$$\left. \frac{\partial N_t}{\partial \omega_t} \right|_{\lambda_t} \frac{\omega_t}{N_t} = \frac{1}{\frac{U_{nn,t}}{U_{n,t}} N_t - \frac{\frac{U_{nc,t}}{U_{n,t}} C_t \frac{U_{cn,t}}{U_{c,t}} N_t}{\frac{U_{cc,t}}{U_{c,t}} C_t}}$$

Suggesting:

$$\left. \frac{\partial N_t}{\partial \omega_t} \right|_{\lambda_t} \frac{\omega_t}{N_t} = \left( \gamma_{nn,t} - \frac{\gamma_{nc,t} \gamma_{cn,t}}{\gamma_{cc,t}} \right)^{-1}$$

■

## D Appendix: Second-Order Approximation of the Welfare Function

A utilitarian policymaker seeks to maximize welfare in the economy as measured by the aggregate expected discounted utility of domestic households, that is:

$$\mathbb{W} \equiv E_0 \sum_{t=0}^{\infty} \beta^t \int_0^1 U(c_t(h), n_t(h); \eta_t) dh$$

Taking second order approximation results in:

$$\begin{aligned} \frac{\mathbb{W} - \mathbb{W}_{ss}}{U_C C_{ss}} &= E_0 \sum_{t=0}^{\infty} \beta^t \left( \tilde{C}_t + \frac{U_{N_{ss}} N_{ss}}{U_{C_{ss}} C_{ss}} \tilde{N}_t \right) \\ &+ \frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t \begin{bmatrix} \tilde{C}_t \\ \tilde{N}_t \end{bmatrix}' \begin{bmatrix} \gamma_{cc} + 1 & \gamma_{cn} \\ \gamma_{cn} & \frac{U_{NN} N_{ss}}{U_C C_{ss}} (\gamma_{nn} + 1) \end{bmatrix} \begin{bmatrix} \tilde{C}_t \\ \tilde{N}_t \end{bmatrix} \\ &+ E_0 \sum_{t=0}^{\infty} \beta^t \left[ \gamma_{cn} \tilde{\eta}_t \tilde{C}_t + \frac{U_{N_{ss}} N_{ss}}{U_{C_{ss}} C_{ss}} \gamma_{nn} \tilde{\eta}_t \tilde{N}_t \right] \\ &+ E_0 \sum_{t=0}^{\infty} \beta^t \begin{bmatrix} \frac{1}{2} \gamma_{cc} Var_h [\tilde{c}_t(h)] + \frac{1}{2} \frac{U_{N_{ss}} N_{ss}}{U_{C_{ss}} C_{ss}} (\gamma_{nn} + \frac{1}{\varepsilon^N}) Var_h [\tilde{n}_t(h)] \\ + \gamma_{cn} Cov_h [\tilde{c}_t(h), \tilde{n}_t(h)] \end{bmatrix} \\ &+ t.i.p. + \mathcal{O}(\|\cdot\|^3) \end{aligned}$$

where:

$$\begin{aligned} Var_h [\tilde{c}_t(h)] &\equiv \int_0^1 (\tilde{c}_t(h) - E_h [\tilde{c}_t(h)])^2 dh \quad , \quad E_h [\tilde{c}_t(h)] \equiv \int_0^1 \tilde{c}_t(h) dh = \tilde{C}_t \\ Var_h [\tilde{n}_t(h)] &\equiv \int_0^1 (\tilde{n}_t(h) - E_h [\tilde{n}_t(h)])^2 dh \quad , \quad E_h [\tilde{n}_t(h)] \equiv \int_0^1 \tilde{n}_t(h) dh = \tilde{N}_t \end{aligned}$$

$$Cov_h [\tilde{c}_t(h), \tilde{n}_t(h)] \equiv \int_0^1 (\tilde{c}_t(h) - E_h [\tilde{c}_t(h)]) (\tilde{n}_t(h) - E_h [\tilde{n}_t(h)]) dh$$

Equating marginal utilities of consumption across households, yields:

$$\tilde{c}_t(h) - E_h [\tilde{c}_t(h)] = -\frac{\gamma_{cn}}{\gamma_{cc}} [\tilde{n}_t(h) - E_h [\tilde{n}_t(h)]] + \mathcal{O}(\|\cdot\|^2)$$

Suggesting:

$$\begin{aligned} Var_h [\tilde{c}_t(h)] &= \left( \frac{\gamma_{cn}}{\gamma_{cc}} \right)^2 Var_h [\tilde{n}_t(h)] + \mathcal{O}(\|\cdot\|^3) \\ Cov_h [\tilde{c}_t(h), \tilde{n}_t(h)] &= -\frac{\gamma_{cn}}{\gamma_{cc}} Var_h [\tilde{n}_t(h)] + \mathcal{O}(\|\cdot\|^3) \end{aligned}$$



and using demand for labor skill  $h$ , equation (17) in the text, we get:

$$Var_h [\tilde{n}_t(h)] = (\varepsilon^N)^2 Var_h [\tilde{w}_t(h)]$$

Substituting the results into the approximated welfare function, gives:

$$\begin{aligned} \frac{\mathbb{W} - \mathbb{W}_{ss}}{U_C C_{ss}} &= E_0 \sum_{t=0}^{\infty} \beta^t \left( \tilde{C}_t + \frac{U_{N_{ss}} N_{ss}}{U_{C_{ss}} C_{ss}} \tilde{N}_t \right) \\ &+ \frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t \begin{bmatrix} \tilde{C}_t \\ \tilde{N}_t \end{bmatrix}' \begin{bmatrix} \gamma_{cc} + 1 & \gamma_{cn} \\ \gamma_{cn} & \frac{U_N N_{ss}}{U_C C_{ss}} (\gamma_{nn} + 1) \end{bmatrix} \begin{bmatrix} \tilde{C}_t \\ \tilde{N}_t \end{bmatrix} \\ &+ E_0 \sum_{t=0}^{\infty} \beta^t \left[ \gamma_{c\eta} \tilde{\eta}_t \tilde{C}_t + \frac{U_{N_{ss}} N_{ss}}{U_{C_{ss}} C_{ss}} \gamma_{n\eta} \tilde{\eta}_t \tilde{N}_t \right] \\ &+ \frac{1}{2} \varepsilon^N \frac{U_N N_{ss}}{U_C C_{ss}} \left[ 1 + \left( \gamma_{nn} - \frac{\gamma_{nc} \gamma_{cn}}{\gamma_{cc}} \right) \varepsilon^N \right] E_0 \sum_{t=0}^{\infty} \beta^t Var_h [\tilde{w}_t(h)] \\ &+ t.i.p. + \mathcal{O}(\|\cdot\|^3) \end{aligned} \quad (\text{D.1})$$

In order to solve for optimal policies, we will seek to maximize the approximated welfare criterion subject to linearized equilibrium conditions. However, [Benigno and Woodford \(2012\)](#) show that for the solution of such a problem to approximate the solution of the exact optimization problem, all endogenous variables in the objective function must be second order. Furthermore, this condition is also required for the approximated welfare criterion to correctly rank alternative equilibrium allocations that are approximated to first order. Hence, in order to derive a valid welfare criterion we must express the linear term in (D.1) as:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left( \tilde{C}_t + \frac{U_{N_{ss}} N_{ss}}{U_{C_{ss}} C_{ss}} \tilde{N}_t \right) = t.i.p. + \mathcal{O}(\|\cdot\|^2) \quad (\text{D.2})$$

This can be achieved by choosing the subsidy rate  $\tau_w$  to support an efficient steady state, and by substituting for the linear term using second order approximation to the balance of payments and the resource constraint of the economy.

Rolling forward the balance of payments, equation (27), setting gross foreign real interest rate to  $\beta^{-1}$ , substituting for  $EX_t$ , using (11) and for  $IM_t$ , using (9) and (10), we get the intertemporal budget constraint of the economy:

$$\begin{aligned} &\beta^{-1} \left( Y_{ss}^A \hat{b}_{-1}^{*,HH} + FX_{-1} \right) \\ &= \sum_{t=0}^{\infty} \beta^t \left\{ (1 - \vartheta) \left[ \Theta \left( \hat{b}_t^{*,HH} - \hat{\theta}_t^* \right) + \Theta^{CB} (FX_t) \right] - Y_{ss}^A \hat{\phi}_t^* \right\} \\ &\quad \left\{ + \lambda \left[ (1 - \lambda) TOT_t^{1-\varepsilon} + \lambda \right]^{\frac{\varepsilon}{1-\varepsilon}} C_t - TOT_t^{1-\varepsilon} WT_t \right\} \end{aligned}$$

The resource constraint, equation (2), after substituting for  $Y_t^H$  using (26), for  $d_t^H$  using

(8) and (10), and for  $EX_t$  using (11), reads:

$$A_t \left( \frac{N_t}{pd_t} \right)^\alpha = (1 - \lambda) \left[ (1 - \lambda) + \lambda TOT_t^{\varepsilon-1} \right]^{\frac{\varepsilon}{1-\varepsilon}} C_t + G_t + TOT_t^{-\varepsilon^*} WT_t$$

Taking second order approximation to both, and combining the results by substituting for  $\widetilde{TOT}_t$ , we get:

$$\begin{aligned} & \sum_{t=0}^{\infty} \beta^t \left\{ \tilde{C}_t + \frac{\varepsilon(1-\lambda) + \varepsilon^* - 1}{\varepsilon(1-\lambda) + \varepsilon^* - (1-\lambda)} \frac{1}{\Phi} \frac{U_{N_{ss}} N_{ss}}{U_{C_{ss}} C_{ss}} \tilde{N}_t \right\} \\ = & \sum_{t=0}^{\infty} \beta^t \left\{ \begin{array}{l} \frac{1}{2} y'_{1,t} \Psi_{11} y_{1,t} + x'_t \Psi_{x1} y_{1,t} + \frac{1}{2} y'_{2,t} \Psi_{22} y_{2,t} \\ - \frac{1}{2} \lambda \phi_1 \phi_2 \frac{\varepsilon^L [\alpha(1-\varepsilon^L) + \varepsilon^L]}{\alpha} \frac{A_{ss} N_{ss}^\alpha}{IM_{ss}} Var_f [\tilde{p}_t^H(f)] \end{array} \right\} \\ & + t.i.p. + \mathcal{O}(\|\cdot\|^3) \end{aligned} \quad (D.3)$$

where:

$$\begin{aligned} y_{1,t} &\equiv \left[ \tilde{C}_t \quad \tilde{N}_t \quad \widetilde{TOT}_t \right]' & \Phi &\equiv \frac{1}{1-\tau_w} \frac{\varepsilon^N - 1}{\varepsilon^N} \frac{\varepsilon^L - 1}{\varepsilon^L} \\ y_{2,t} &\equiv \left[ \hat{\vartheta}_t^{*,HH} - \hat{\theta}_t^* \quad \widetilde{FX}_t \right]' & \phi_1 &\equiv \frac{(1-\lambda)\varepsilon + \varepsilon^*}{(1-\lambda)\varepsilon + \varepsilon^* - (1-\lambda)} \\ x_t &\equiv \left[ \tilde{\eta}_t \quad \tilde{A}_t \quad \widetilde{WT}_t \right]' & \phi_2 &\equiv \frac{(1-\lambda)\varepsilon + \varepsilon^* - 1}{(1-\lambda)\varepsilon + \varepsilon^*} \end{aligned}$$

and:

$$\begin{aligned} \Psi_{11} &\equiv -\lambda \phi_1 \left[ \begin{array}{ccc} 1 + \phi_2 \frac{1-\lambda}{\lambda} & 0 & \frac{(1-\lambda)\varepsilon}{(1-\lambda)\varepsilon + \varepsilon^*} \\ 0 & -\phi_2 \alpha^2 \frac{A_{ss} N_{ss}^\alpha}{IM_{ss}} & 0 \\ \frac{(1-\lambda)\varepsilon}{(1-\lambda)\varepsilon + \varepsilon^*} & 0 & \varepsilon(1-\lambda) \left[ 1 - \frac{(1-\lambda)(1-\varepsilon) + \lambda\varepsilon}{(1-\lambda)\varepsilon + \varepsilon^*} \right] \\ & & - (1-\varepsilon^*)^2 + \phi_2 (\varepsilon^*)^2 \end{array} \right] \\ \Psi_{x1} &\equiv \lambda \phi_1 \left[ \begin{array}{ccc} 0 & 0 & 0 \\ 0 & \phi_2 \alpha \frac{A_{ss} N_{ss}^\alpha}{IM_{ss}} & 0 \\ 0 & 0 & (1-\varepsilon^*) + \phi_2 \varepsilon^* \end{array} \right] \\ \Psi_{22} &\equiv -\frac{\lambda}{IM_{ss}} \phi_1 \left[ \begin{array}{cc} (1-\vartheta) \Theta''(0) & 0 \\ 0 & (1-\vartheta) \Theta^{CB''}(FX_{ss}) FX_{ss}^2 \end{array} \right] \end{aligned}$$

Comparing (D.3) to (D.2), it follows that the condition for a valid welfare criterion is satisfied if:

$$1 - \tau_w = \frac{\varepsilon^N - 1}{\varepsilon^N} \frac{\varepsilon^L - 1}{\varepsilon^L} \frac{(1-\lambda)\varepsilon + \varepsilon^* - (1-\lambda)}{(1-\lambda)\varepsilon + \varepsilon^* - 1}$$

which is exactly the optimal subsidy, equation (47) in the text, that supports the efficient equilibrium in steady state. Under this subsidy we can now use (D.3) to substitute for  $\sum_{t=0}^{\infty} \beta^t \left( \tilde{C}_t + \frac{U_N N_{ss}}{U_C C_{ss}} \tilde{N}_t \right)$  in (D.1).

The last step is to move from dispersion of wages and prices,  $Var_h[\tilde{w}_t(h)]$  and  $Var_f[\tilde{p}_t^H(f)]$ , to wage inflation and home-good inflation,  $\tilde{\pi}_t^w$  and  $\tilde{\pi}_t^H$ . Using proposi-

tion 6.3 in [Woodford \(2003\)](#), we get:

$$\begin{aligned}\sum_{t=0}^{\infty} \beta^t Var_f [\tilde{p}_t^H(f)] &= \frac{\xi_p}{(1-\xi_p)(1-\beta\xi_p)} \sum_{t=0}^{\infty} \beta^t (\tilde{\pi}_t^H)^2 \\ \sum_{t=0}^{\infty} \beta^t Var_h [\tilde{w}_t(h)] &= \frac{\xi_w}{(1-\xi_w)(1-\beta\xi_w)} \sum_{t=0}^{\infty} \beta^t (\tilde{\pi}_t^w)^2\end{aligned}$$

Following these steps, and using steady state equilibrium relations to simplify coefficients, we get the approximated welfare function as presented in equation (48) in the text.

## E Appendix: Characterizing the Optimal Allocation

This appendix characterizes the equilibrium allocations under optimal policies. I consider three cases. First is the fully optimal allocation, where the central bank uses both its tools, FXI and the interest rate, optimally. Second, consider the case where the central bank uses an optimal interest rate policy while holding foreign reserves fixed. In the third case, the interest rate is set optimally while FXIs follow a predetermined policy rule.

### E.1 Optimal FXI and Optimal Interest Rate Policy

Before solving for the optimal allocation, I first reduce the system of equilibrium conditions by substituting for  $\tilde{Y}_t^H$ ,  $\tilde{d}_t^H$ ,  $\tilde{IM}_t$ ,  $\tilde{EX}_t$ ,  $\tilde{p}_t^H$ ,  $\tilde{p}_t^F$  and  $\tilde{\sigma}_t$ , to get the following set of constraints.

Wage inflation dynamics, equation (28), and the change in real wage, equation (41), are:

$$\xi_w \tilde{\pi}_t^w \cong \xi_w \beta E_t (\tilde{\pi}_{t+1}^w) - \frac{(1-\xi_w\beta)(1-\xi_w)}{1 + \left(\gamma_{nm} - \frac{\gamma_{ne}\gamma_{cn}}{\gamma_{cc}}\right) \varepsilon^N} (\tilde{w}_t - \tilde{U}_{N_t} + \tilde{U}_{C_t}) \quad (\text{E.1})$$

$$\tilde{w}_t - \tilde{w}_{t-1} \cong \tilde{\pi}_t^w - \tilde{\pi}_t \quad (\text{E.2})$$

Home inflation dynamics, equation (29), and after substituting  $\tilde{p}_t^H \cong \lambda \widetilde{TOT}_t$  into equation (42), we have:

$$\begin{aligned}\xi_p \tilde{\pi}_t^H &\cong \xi_p \beta E_t (\tilde{\pi}_{t+1}^H) \\ &+ \frac{(1-\xi_p\beta)(1-\xi_p)\alpha}{\alpha + (1-\alpha)\varepsilon^L} \left[ \tilde{w}_t - \lambda \widetilde{TOT}_t - \tilde{A}_t - (\alpha-1)\tilde{N}_t \right]\end{aligned} \quad (\text{E.3})$$

$$\lambda \widetilde{TOT}_t - \lambda \widetilde{TOT}_{t-1} \cong \tilde{\pi}_t^H - \tilde{\pi}_t \quad (\text{E.4})$$

The Euler equation for domestic bonds, equation (30), is:

$$\tilde{U}_{C_t} \cong \widetilde{(1 + i_t)} + E_t \left\{ \tilde{U}_{C_{t+1}} \right\} - E_t \{ \pi_{t+1} \} \quad (\text{E.5})$$

Using  $\tilde{p}_t^F \cong -(1 - \lambda) \widetilde{TOT}_t$  and equation (43) to substitute for  $\tilde{\sigma}_{t+1}$  in the Euler equation for foreign bonds, equation (31), it reads:

$$\begin{aligned} \tilde{U}_{C_t} + \frac{\Theta''(0)}{Y_{ss}^{H,An.}} \left( \widehat{b}_t^{*,HH} - \widehat{\theta}_t^* \right) - (1 - \lambda) \widetilde{TOT}_t \\ \cong \widetilde{(1 + i_t^*)} - E_t \{ \tilde{\pi}_{t+1}^{F*} \} + E_t \left\{ \tilde{U}_{C_{t+1}} \right\} - (1 - \lambda) E_t \left\{ \widetilde{TOT}_{t+1} \right\} \end{aligned} \quad (\text{E.6})$$

Substituting for technology, exports and demand for home goods, the resources constraint, equation (37), reads:

$$\tilde{A}_t + \alpha \tilde{N}_t \cong (1 - \lambda) \frac{C_{ss}}{Y_{ss}^H} \tilde{C}_t + \frac{G_{ss}}{Y_{ss}^H} \tilde{G}_t + \lambda \frac{C_{ss}}{Y_{ss}^H} \widetilde{WT}_t - \lambda \frac{C_{ss}}{Y_{ss}^H} [(1 - \lambda) \varepsilon + \varepsilon^*] \widetilde{TOT}_t \quad (\text{E.7})$$

Substituting for exports and imports demand, the balance of payments, equation (38), is given by:

$$\begin{aligned} FX_{ss} \widetilde{FX}_t + Y_{ss}^{H,An.} \widehat{b}_t^{*,HH} &\cong \frac{1}{\beta} \left( FX_{ss} \widetilde{FX}_{t-1} + Y_{ss}^{H,An.} \widehat{b}_{t-1}^{*,HH} \right) \\ &+ \frac{1}{\beta} FX_{ss} \left[ \widetilde{(1 + i_{t-1}^*)} - \tilde{\pi}_t^{F*} \right] \\ &- \lambda C_{ss} \tilde{C}_t + \lambda C_{ss} [1 - \varepsilon^* - (1 - \lambda) \varepsilon] \widetilde{TOT}_t \\ &+ Y_{ss}^{H,An.} \left( \widehat{\phi}_t^* - \phi_{ss}^* \right) + \lambda C_{ss} \widetilde{WT}_t \end{aligned} \quad (\text{E.8})$$

This gives a system of 8 periodical equations in 10 endogenous variables:  $\tilde{C}_t$ ,  $\tilde{N}_t$ ,  $\tilde{w}_t$ ,  $\tilde{\pi}_t^w$ ,  $\tilde{\pi}_t$ ,  $\tilde{\pi}_t^H$ ,  $\widetilde{TOT}_t$ ,  $\widetilde{(1 + i_t)}$ ,  $\widetilde{FX}_t$ , and  $\widehat{b}_t^{*,HH}$ ; where we have 2 definitions:

$$\tilde{U}_{N,t} \cong \gamma_{nc} \tilde{C}_t + \gamma_{nm} \tilde{N}_t + \gamma_{n\eta} \tilde{\eta}_t \quad (\text{E.9})$$

$$\tilde{U}_{C,t} \cong \gamma_{cc} \tilde{C}_t + \gamma_{cn} \tilde{N}_t + \gamma_{c\eta} \tilde{\eta}_t \quad (\text{E.10})$$

To solve for the optimal allocation, set a Lagrangian using the objective function (48), and the constraints (E.1) to (E.8), and differentiate with respect to each of the endogenous variables. The first order conditions are presented below.

First order condition with respect to consumption,  $\widetilde{C}_t$ :

$$\begin{aligned}
& \widetilde{U}_{C_t} + \frac{\lambda \varepsilon (1 - \lambda)}{(1 - \varepsilon)(1 - \lambda) - \varepsilon^*} \widetilde{TOT}_t \\
& - \frac{(1 - \xi_w \beta)(1 - \xi_w)}{1 + \left( \gamma_{nn} - \frac{\gamma_{nc} \gamma_{cn}}{\gamma_{cc}} \right) \varepsilon^N} (\gamma_{nc} - \gamma_{cc}) \phi_{wInf,t} \\
& - (1 - \lambda) \frac{C_{ss}}{Y_{ss}^H} \phi_{RC,t} + \lambda C_{ss} \phi_{BOP,t} + \gamma_{cc} (\phi_{hEuler,t} + \phi_{fEuler,t}) \\
& = \frac{\gamma_{cc}}{\beta} (\phi_{hEuler,t-1} + \phi_{fEuler,t-1})
\end{aligned} \tag{E.11}$$

First order condition with respect to labor,  $\widetilde{N}_t$ :

$$\begin{aligned}
& \frac{U_{N_{ss}} N_{ss}}{U_{C_{ss}} C_{ss}} \left[ \widetilde{U}_{N_t} + (1 - \alpha) \widetilde{N}_t - \widetilde{A}_t \right] + \gamma_{cn} (\phi_{hEuler,t} + \phi_{fEuler,t}) + \alpha \phi_{RC,t} \\
& - \frac{(1 - \xi_w \beta)(1 - \xi_w)}{1 + \left( \gamma_{nn} - \frac{\gamma_{nc} \gamma_{cn}}{\gamma_{cc}} \right) \varepsilon^N} (\gamma_{nn} - \gamma_{cn}) \phi_{wInf,t} - \frac{(1 - \xi_p \beta)(1 - \xi_p) \alpha}{\alpha + (1 - \alpha) \varepsilon^L} (1 - \alpha) \phi_{hInf,t} \\
& = \frac{\gamma_{cn}}{\beta} (\phi_{hEuler,t-1} + \phi_{fEuler,t-1})
\end{aligned} \tag{E.12}$$

First order condition with respect to the terms of trade,  $\widetilde{TOT}_t$ :

$$\begin{aligned}
& \frac{\lambda \varepsilon (1 - \lambda)}{(1 - \varepsilon)(1 - \lambda) - \varepsilon^*} \left\{ \left[ \begin{array}{c} 3\varepsilon^* - \frac{\varepsilon^*(1 - \varepsilon^*)}{\varepsilon(1 - \lambda)} \\ + (2 - 3\lambda) \varepsilon - (2 - \lambda) \end{array} \right] \widetilde{TOT}_t + \widetilde{C}_t - \widetilde{WT}_t \right\} \\
& + \lambda \phi_{TOT,t} + \frac{(1 - \xi_p \beta)(1 - \xi_p) \alpha}{\alpha + (1 - \alpha) \varepsilon^L} \lambda \phi_{hInf,t} - (1 - \lambda) \phi_{fEuler,t} - \beta \lambda E_t (\phi_{TOT,t+1}) \\
& + \lambda \frac{C_{ss}}{Y_{ss}^H} [(1 - \lambda) \varepsilon + \varepsilon^*] \phi_{RC,t} - \lambda C_{ss} [1 - \varepsilon^* - (1 - \lambda) \varepsilon] \phi_{BOP,t} \\
& = - \frac{1 - \lambda}{\beta} \phi_{fEuler,t-1}
\end{aligned} \tag{E.13}$$

First order condition with respect to the households' foreign assets position,  $\widehat{b}_t^{*,HH}$ :

$$\begin{aligned}
& \frac{1 - \vartheta}{C_{ss}} \frac{\varepsilon^* + \varepsilon(1 - \lambda)}{(1 - \varepsilon)(1 - \lambda) - \varepsilon^*} \Theta''(0) \left( \widehat{b}_t^{*,HH} - \widehat{\theta}_t^* \right) \\
& + \frac{\Theta''(0)}{Y_{ss}^{H,An.}} \phi_{fEuler,t} + Y_{ss}^{H,An.} \phi_{BOP,t} \\
& = Y_{ss}^{H,An.} E_t \{ \phi_{BOP,t+1} \}
\end{aligned} \tag{E.14}$$

First order condition with respect to foreign reserves,  $\widetilde{FX}_t$ :

$$\frac{1 - \vartheta}{C_{ss}} \frac{\varepsilon^* + \varepsilon(1 - \lambda)}{(1 - \varepsilon)(1 - \lambda) - \varepsilon^*} \Theta^{CB''} (FX_{ss}) FX_{ss} \widetilde{FX}_t + \phi_{BOP,t} = E_t \{ \phi_{BOP,t+1} \} \tag{E.15}$$

First order condition with respect to home price inflation,  $\tilde{\pi}_t^H$ :

$$\frac{\varepsilon^L}{\alpha} \left[ (1 - \varepsilon^L) + \frac{\varepsilon^L}{\alpha} \right] \frac{U_{N_{ss}} N_{ss}}{U_{C_{ss}} C_{ss}} \frac{\xi_p}{1 - \xi_p} \frac{1}{1 - \xi_p \beta} \tilde{\pi}_t^H - \phi_{TOT,t} + \xi_p \phi_{hInf,t} = \xi_p \phi_{hInf,t-1} \quad (\text{E.16})$$

First order condition with respect to wage inflation,  $\tilde{\pi}_t^w$ :

$$\varepsilon^N \frac{U_N N_{ss}}{U_C C_{ss}} \left[ 1 + \left( \gamma_{nn} - \frac{\gamma_{nc} \gamma_{cn}}{\gamma_{cc}} \right) \varepsilon^N \right] \frac{\xi_w}{1 - \xi_w} \frac{1}{1 - \xi_w \beta} \tilde{\pi}_t^w - \phi_{wDef,t} + \xi_w \phi_{wInf,t} = \xi_w \phi_{wInf,t-1} \quad (\text{E.17})$$

First order condition with respect to real wage,  $\tilde{w}_t$ :

$$\phi_{wDef,t} + \frac{(1 - \xi_w \beta)(1 - \xi_w)}{1 + \left( \gamma_{nn} - \frac{\gamma_{nc} \gamma_{cn}}{\gamma_{cc}} \right) \varepsilon^N} \phi_{wInf,t} - \frac{(1 - \xi_p \beta)(1 - \xi_p) \alpha}{\alpha + (1 - \alpha) \varepsilon^L} \phi_{hInf,t} = \beta E_t \{ \phi_{wDef,t+1} \} \quad (\text{E.18})$$

First order condition with respect to CPI inflation,  $\tilde{\pi}_t$ :

$$\phi_{wDef,t} + \phi_{TOT,t} + \frac{1}{\beta} \phi_{hEuler,t-1} = 0 \quad (\text{E.19})$$

And the first order condition with respect to the interest rate,  $(1 + \tilde{i}_t)$ :

$$\phi_{hEuler,t} = 0 \quad (\text{E.20})$$

where  $\phi_{wInf,t}$  is the Lagrange multiplier of wage inflation dynamics, equation (E.1);  $\phi_{wDef,t}$  is the Lagrange multiplier of the change in real wage, equation (E.2);  $\phi_{hInf,t}$  is the Lagrange multiplier of home inflation dynamics, equation (E.3);  $\phi_{TOT,t}$  is the Lagrange multiplier of the change in the terms of trade, equation (E.4);  $\phi_{hEuler,t}$  is the Lagrange multiplier of the Euler condition for domestic bonds, equation (E.5);  $\phi_{fEuler,t}$  is the Lagrange multiplier of the Euler condition for foreign bonds, equation (E.6);  $\phi_{RC,t}$  is the Lagrange multiplier of the resource constraint, equation (E.7); and  $\phi_{BOP,t}$  is the Lagrange multiplier of the balance of payments, equation (E.8).

Equations (E.1) through (E.20) characterize the optimal allocation for  $\tilde{C}_t$ ,  $\tilde{N}_t$ ,  $\tilde{w}_t$ ,  $\tilde{\pi}_t^w$ ,  $\tilde{\pi}_t$ ,  $\tilde{\pi}_t^H$ ,  $\tilde{TOT}_t$ ,  $(1 + \tilde{i}_t)$ ,  $\tilde{FX}_t$ ,  $\tilde{b}_t^{*,HH}$ ,  $\tilde{U}_{N,t}$  and  $\tilde{U}_{C,t}$ , together with the Lagrange multipliers  $\phi_{wInf,t}$ ,  $\phi_{wDef,t}$ ,  $\phi_{hInf,t}$ ,  $\phi_{TOT,t}$ ,  $\phi_{hEuler,t}$ ,  $\phi_{fEuler,t}$ ,  $\phi_{RC,t}$  and  $\phi_{BOP,t}$ .

Other variables are pinned down using:

$$\begin{aligned}
\tilde{p}_t^H &\cong \lambda \widetilde{TOT}_t \\
\tilde{p}_t^F &\cong -(1-\lambda) \widetilde{TOT}_t \\
\tilde{Y}_t^H &\cong \tilde{A}_t + \alpha \tilde{N}_t \\
\tilde{d}_t^H &\cong \tilde{C}_t - \varepsilon \tilde{p}_t^H \\
\widetilde{IM}_t &\cong \tilde{C}_t - \varepsilon \tilde{p}_t^F \\
\widetilde{EX}_t &\cong -\varepsilon^* \widetilde{TOT}_t + \widetilde{WT}_t \\
\tilde{\sigma}_t &\cong \tilde{p}_t^F - \tilde{p}_{t-1}^F + \tilde{\pi}_t - \tilde{\pi}_t^{F*}
\end{aligned}$$

To end this section, a remark on the optimality condition for foreign reserves, equation (E.15), is in order. Notice that if either  $\vartheta = 1$  or  $\Theta^{CB''}(FX_{ss}) = 0$ , the Lagrange multiplier of the balance of payments would follow a random walk. Therefore, in order to impose stationarity on the system we have to deviate from these values. This condition is similar to the requirement of a portfolio adjustment cost in order to impose stationarity on the marginal utility of consumption, see [Schmitt-Grohé and Uribe \(2003\)](#). The difference here is that we also have to deviate from full ownership of the financial sector, because, unlike households, the social planner internalizes the fact that the adjustment costs are rebated to the households. Hence, from the standpoint of the social planner, full ownership, i.e.  $\vartheta = 1$ , is equivalent to no adjustment costs on foreign reserves.

## E.2 Optimal Interest Rate Policy and Fixed Foreign Reserves

Now consider the case of optimal interest rate policy with fixed foreign reserves. In that case the equilibrium allocation is characterized by equations (E.1) through (E.20), where the optimality condition with respect to foreign reserves,  $\widetilde{FX}_t$ , is replaced by:

$$\widetilde{FX}_t = 0 \tag{E.21}$$

Note that formally we should add  $\widetilde{FX}_t = 0$  as a constraint, introduce an additional Lagrange multiplier associated with the new constraint, and then solve for the optimal allocation. In this case, all optimality conditions are the same as those in Section E.1, except the one with respect to  $\widetilde{FX}_t$ , equation (E.15), which is modified slightly as it now contains the new Lagrange multiplier. However, since this is the only equation where the new multiplier shows up and since we are not interested in the multiplier itself, we can drop from the system both the optimality condition with respect to  $\widetilde{FX}_t$  and the new multiplier. In other words, to solve for the equilibrium allocation in this case, simply replace the optimality condition (E.15) with the constraint (E.21).

### E.3 Optimal Interest Rate Policy and Predetermined FXI Rule

Finally, consider the case where monetary policy is set optimally while FXIs follow a predetermined rule:

$$\frac{FX_t}{FX^T} = \left( 1 + \frac{\Theta'(\widehat{b}_t^{*,HH} - \widehat{\theta}_t^*)}{TOT_{ss} Y_{ss}^{H,An.}} \right)^\Xi \left( \frac{FX_{t-1}}{FX^T} \right)^{\rho_{FX}}$$

where  $\Xi \gg 0$  ,  $0 \leq \rho_{FX} < 1$

Taking first order approximation, the policy rule reads:

$$\widetilde{FX}_t \cong \Xi \frac{\Theta''(0)}{TOT_{ss} Y_{ss}^{H,An.}} \left( \widehat{b}_t^{*,HH} - \widehat{\theta}_t^* \right) + \rho_{FX} \widetilde{FX}_{t-1} \quad (\text{E.22})$$

This rule seeks to stabilize the UIP premium, while smoothing the path of foreign reserves. Note that strict targeting of the UIP premium, i.e.  $\Xi \rightarrow \infty$ , introduces unit root dynamics in the approximated system through the households' Euler equation for foreign bonds. However, to substantially stabilize the UIP premium, it is sufficient to set  $\Xi$  to a value large enough. I use:

$$\Xi = 20$$

The optimization problem is the same as before, except that (E.22) is added as a constraint. All optimality conditions of section E.1 are the same as before, except those of  $\widetilde{FX}_t$  and  $\widehat{b}_t^{*,HH}$  - the endogenous variables in (E.22).

The first order condition with respect to foreign reserves,  $\widetilde{FX}_t$ , now reads:

$$\begin{aligned} & \frac{1 - \vartheta}{C_{ss}} \frac{\varepsilon^* + \varepsilon(1 - \lambda)}{(1 - \varepsilon)(1 - \lambda) - \varepsilon^*} \Theta^{CB''}(FX_{ss}) FX_{ss} \widetilde{FX}_t + \frac{\phi_{FXRule,t}}{FX_{ss}} + \phi_{BOP,t} \\ & = E_t \{ \phi_{BOP,t+1} \} + \frac{\beta \rho_{FX}}{FX_{ss}} E_t \{ \phi_{FXRule,t+1} \} \end{aligned} \quad (\text{E.23})$$

And the first order condition with respect to the households' holdings of foreign assets,  $\widehat{b}_t^{*,HH}$ , is given by:

$$\begin{aligned} & \frac{1 - \vartheta}{C_{ss}} \frac{\varepsilon^* + \varepsilon(1 - \lambda)}{(1 - \varepsilon)(1 - \lambda) - \varepsilon^*} \Theta''(0) \left( \widehat{b}_t^{*,HH} - \widehat{\theta}_t^* \right) \\ & + \frac{\Theta''(0)}{Y_{ss}^{H,An.}} \left( \phi_{fEu,t} - \Xi \phi_{FXRule,t} \right) + Y_{ss}^{H,An.} \phi_{BOP,t} \\ & = Y_{ss}^{H,An.} E_t \{ \phi_{BOP,t+1} \} \end{aligned} \quad (\text{E.24})$$

where  $\phi_{FXRule,t}$  is the Lagrange multiplier of the FXI policy rule, equation (E.22).

The equilibrium allocation under optimal monetary policy and the FXI rule is charac-



terized by equation (E.22) together with equations (E.1) through (E.20), where equation (E.23) replaces (E.15), and equation (E.24) replaces (E.14).

## F Appendix: Parameter Values: Calibration, Estimation and Data

Parameter values are chosen based on the characteristics of the Israeli economy. A period in the model corresponds to one quarter. Values are mostly adopted from the parameterization of the Bank of Israel DSGE model, as reported in Argov et al. (2012). The calibration considers a symmetric steady state across countries, as described in Section 2.8 in the text. Table 4 summarizes the values of calibrated parameters.

Most important is the financial friction parameter,  $\Theta''(0)$ , as it governs the efficacy of FXIs. Also important are the parameters of the stochastic processes of the exogenous shocks, as they directly affect the second moments of the endogenous variables and hence welfare. I use Bayesian estimation to evaluate their values.

The sample period for calibration and estimation is the decade after the global financial crisis and prior to the COVID-19 crisis, 2010-2019.<sup>32</sup>

### F.1 Calibration

**Production Function.**  $\alpha$ , the elasticity of output with respect to labor, is set to 0.67, as in Argov et al. (2012). Steady state productivity,  $A_{ss}$ , is normalized to unity.  $N_{ss}$  is calibrated to match the percent of time individuals allocate to work. During the sample period hours worked per employee averaged 36.1 hours per week. Assuming time allocation of 16 hours per day, this implies  $N_{ss} = 0.32$ . Using the aggregate production function these values determine  $Y_{ss}^H$ .

**Great Ratios.** The government expenditure share,  $\frac{G_{ss}}{Y_{ss}^H}$ , is set to 0.3, and trade shares,  $\frac{EX_{ss}}{Y_{ss}^H}$  and  $\frac{IM_{ss}}{TOT_{ss}Y_{ss}^H}$ , to 0.33. Since the model abstracts from capital formation,  $Y^H$  is interpreted as GDP net of investment. The values above approximately match the sample averages after adjusting for investment. Given these values and  $Y_{ss}^H$ , we can pin down  $G_{ss}$ ,  $IM_{ss}$  (using  $TOT_{ss} = 1$ ) and  $EX_{ss}$ .  $d_{ss}^H$  is then pinned down from the domestic resource constraint.

**Openness.** The openness parameter,  $\lambda$ , is pinned down from the relative demand of  $d_{ss}^H$  to  $IM_{ss}$ ; these equations suggest  $\lambda = \frac{IM_{ss}}{IM_{ss} + d_{ss}^H}$ , which yields  $\lambda \cong 0.47$ . Given  $\lambda$  and  $IM_{ss}$ ,  $C_{ss} = IM_{ss}/\lambda$ .

---

<sup>32</sup>I focus on a relatively recent period due to significant structural changes the Israeli economy has gone through over the years. These include transitioning toward a market-based economy, disinflation during the 1990s, absorbing massive immigration after the fall of the Soviet Union, and liberalizing the current account and financial sector, among others.

Table 4: Calibrated Parameters and Steady State Values, Baseline Parameterization

Panel A: Steady State		
Terms of trade	$TOT_{ss}$	1
Private sector net foreign asset position	$b_{ss}^{*,HH}$	0
Inflation	$\pi_{ss}$	$1.02^{1/4}$
Productivity	$A_{ss}$	1
Labor input	$N_{ss}$	0.32
Share of government expenditures in domestic output	$\frac{G_{ss}}{Y_{ss}^H}$	0.3
Shares of exports and imports in domestic output	$\frac{E_{ss}^X}{Y_{ss}^H}, \frac{IM_{ss}}{TOT_{ss} Y_{ss}^H}$	0.33
Target level of reserves (30 percent of annual GDP)	$\frac{FX^T}{TOT_{ss} Y_{ss}^{H,An.}}$	0.3
Preference shock	$\eta_{ss}$	1
Risk premium shock	$\theta_{ss}^*$	0
Panel B: Parameters		
Elasticity of domestic output with respect to labor	$\alpha$	0.67
Subjective discount factor	$\beta$	$1.025^{-1/4}$
EoS between home and foreign goods	$\varepsilon$	1.1
EoS between differentiated labor skills	$\varepsilon^N$	13/3
EoS between intermediate goods of the same country	$\varepsilon^L$	13/3
EoS between goods of different countries	$\varepsilon^*$	13/3
Probability of price adjustment	$1 - \xi_p$	1/3
Probability of wage adjustment	$1 - \xi_w$	0.25
Frisch elasticity of labor supply	$\nu^{-1}$	2
Intertemporal EoS	$\gamma^{-1}$	1/3
Share of domestic ownership of the financial sector	$\vartheta$	0.999
2nd derivative of the CB portfolio adjustment cost	$\Theta_{ss}^{CB''}$	0.1
Interest rate rule: interest smoothing coefficient	$\theta_i$	0.814
Interest rate rule: inflation coefficient	$\theta_\pi$	2.538
Interest rate rule: output coefficient	$\theta_y$	0.204

**Discount Factor and Inflation.** The subjective discount factor,  $\beta$ , takes the value  $1.025^{-1/4}$ , which corresponds to an annual steady state real interest rate of 2.5 percent. This value matches the average 10-15 years forward rate of CPI-indexed government bonds during the sample period. Steady state inflation is set at 2 percent, the mid-range of the Bank of Israel's inflation target. Home and foreign nominal interest rates are pinned down by the Fisher equation. Since real prices are constant in steady state:  $\pi_{ss} = \pi_{ss}^H = \pi_{ss}^F = \pi_{ss}^w = \pi_{ss}^{F*}$ .

**Elasticities of Substitution (EoS).** I adopt elasticities of substitution from [Argov et al. \(2012\)](#). The EoS between home and foreign goods,  $\varepsilon$ , is set to 1.1, and the EoS between labor skills,  $\varepsilon^N$ , intermediate goods,  $\varepsilon^L$ , and goods of different countries,  $\varepsilon^*$ , are set to  $\frac{13}{3}$ . This suggests a markup of 30 percent for intermediate goods producers and labor suppliers. The labor subsidy is set as suggested by equations (47) in the text, and

real wage is pinned down using labor demand:  $w_{ss} = \frac{1}{1-\tau_w} \frac{\varepsilon^L-1}{\varepsilon^L} \alpha p_{ss}^H A_{ss} N_{ss}^{\alpha-1}$ .

**Price and Wage Stickiness.** In a micro-level study on the frequency of price adjustments in Israel, [Ribon and Sayag \(2013\)](#) report an average price duration of 9.3 months. I calibrate firms' probability of price adjustment to match an average duration of 9 months, suggesting  $1 - \xi_p = \frac{1}{3}$ . Using macro data, [Argov et al. \(2012\)](#) estimate this probability at 0.394. Wages typically adjust more slowly than prices. I assume an average mean wage duration of one year, suggesting  $1 - \xi_w = 0.25$ . This value deviates substantially from [Argov et al. \(2012\)](#), as they report an estimate of 0.544, which suggests that wages are more flexible than prices. My choice of wage adjustment probability is close to that of [Smets and Wouters \(2007\)](#), who estimate it at 0.27 for the American economy, and to [Smets and Wouters \(2003\)](#), who estimate it at 0.263 for Europe.

**Utility Parameters.** I consider a standard additive separable utility function:

$$U(C, N; \eta) = \eta \left[ \frac{C^{1-\gamma} - 1}{1-\gamma} - \psi \frac{N^{1+\nu}}{1+\nu} \right]$$

The Frisch elasticity of labor supply is given by  $\nu^{-1}$ . Following [Argov et al. \(2012\)](#), it is set to 2, suggesting  $\nu = 0.5$ . The steady state value of the preference shock,  $\eta_{ss}$ , is set to unity. The intertemporal elasticity of substitution (IES) is given by  $\gamma^{-1}$ . [Havránek \(2015\)](#) conducts a meta-analysis on reported estimates of the IES in 169 published papers. He concludes that the best calibrated value for the IES is around 0.3-0.4. I use an elasticity of 1/3, that is  $\gamma = 3$ . Finally,  $\psi$  is pinned down using labor supply:  $\psi = \frac{\varepsilon^N-1}{\varepsilon^N} w_{ss} N_{ss}^{-\nu} C_{ss}^\gamma$ .

**Foreign Reserves.** The target level of foreign reserves,  $FX^T$ , is set at 30 percent of annual GDP, which roughly equals its level in Israel during the decade preceding the COVID-19 crisis. This pins down the scale of foreign exchange interventions in steady state, and, by symmetry across countries, the size of capital inflows:  $\hat{\phi}_{ss}^* = -\frac{1-\beta}{\beta} \frac{FX^T}{Y_{ss}^{H.An.}}$ .

**The Central Bank's Adjustment Cost and Domestic Ownership of the Financial Sector.** I assume the central bank faces a minor adjustment cost when operating in the foreign exchange markets, and set  $\Theta^{CB''}(FX_{ss}) = 0.1$ . I also assume that the financial sector is owned entirely by domestic agents, suggesting  $\vartheta$  approaches unity. Specifically I set  $\vartheta = 0.999$ .<sup>33</sup>

**Interest Rate Rule.** The main analysis in this paper assumes that monetary policy sets the interest rate optimally. Nevertheless, for the purpose of estimation it is useful to rely on an empirically relevant rule, regardless of whether it reflects optimal policy

---

<sup>33</sup>Under optimal FXI policy, setting  $\vartheta = 1$  and/or  $\Theta^{CB''}(FX_{ss}) = 0$  gives rise to unit root dynamics in the social planner's Lagrange multiplier of the balance of payments. See equation (E.15) in Appendix E.

reaction. To that end I adopt the specification of [Argov et al. \(2012\)](#)<sup>34</sup>:

$$\frac{1 + i_t}{1 + i_{ss}} = \left( \frac{1 + i_{t-1}}{1 + i_{ss}} \right)^{\theta_i} \left[ \left( \frac{\pi_{t-2} + \pi_{t-1} + \pi_t + E_t(\pi_{t+1})}{4\pi_{ss}} \right)^{\theta_\pi} \left( \frac{Y_t^H}{Y_{ss}^H} \right)^{\theta_y} \right]^{1-\theta_i} \quad (\text{F.25})$$

Following [Argov et al. \(2012\)](#),  $\theta_i = 0.814$ ,  $\theta_\pi = 2.538$  and  $\theta_y = 0.204$ .

## F.2 Bayesian Estimation: The Adjustment Cost, Exogenous Processes, and Measurement Errors

This section employs Bayesian estimation to evaluate the financial friction parameter,  $\Theta''(0)$ , and the parameters governing the stochastic processes of the exogenous variables. For the estimation I assume that the interest rate follows policy rule (F.25), and that foreign reserves follow an exogenous auto-regressive process. This enables the estimation to rely on empirically relevant processes, without the presumption that policy was conducted optimally during the sample period.

The section describes the choice of prior distributions, and presents the estimation results in Table 5. I use posterior modes as parameter values for the analysis in the text.

The sample period is the first quarter of 2010 until the fourth quarter of 2019. Toward the end of 2009 the Bank of Israel changed its FXI policy, and moved from purchasing pre-announced quantities to discretionary interventions that respond to market conditions, [Bank of Israel \(2010\)](#). The latter better reflects the role of FXIs in the model. I therefore start the sample at the beginning of 2010. The sample ends just before the COVID-19 crisis, which aside from introducing unprecedented economic volatility, also triggered a large pre-announced program to purchase foreign reserves, [Bank of Israel \(2022\)](#).

The estimation was carried out using Dynare version 5.2 and [Sims \(1999\)](#) *csmmwel* optimizer. The Markov Chain Monte Carlo (MCMC) Metropolis-Hastings algorithm employed 5 parallel chains with 2 million draws per chain. The first 40 percent of the draws were used as burn-in.

Figures F.1 through F.3 display the prior and posterior distributions for all estimated parameters of the model.

### F.2.1 Exogenous Processes

The exogenous variables in the model are productivity,  $\tilde{A}_t$ , government expenditure,  $\tilde{G}_t$ , the preference shock,  $\tilde{\eta}_t$ , world trade,  $\tilde{WT}_t$ , capital inflows,  $\tilde{\phi}_t^* - \tilde{\phi}_{ss}^*$ , and the risk premium shock,  $\tilde{\theta}_t^*$ . For the purpose of estimation, foreign reserves,  $\tilde{FX}_t$ , also follow an exogenous

---

<sup>34</sup>In [Argov et al. \(2012\)](#) the interest rate also reacts to the nominal exchange rate, though with a small coefficient. I omit it from the specification of (F.25) because, here, policy reaction to the external sector takes a central role through FXIs, which are absent from [Argov et al. \(2012\)](#).

Table 5: Prior and Posterior Distributions of Estimated Parameters<sup>(1)</sup>

	Prior Distribution				Posterior Distribution				
	Type	Mode	STD	Implied Mean	Mode	STD	Mean	HPD Interval <sup>(2)</sup>	
								5%	95%
<b>Panel A:</b> Portfolio Adjustment Cost									
2nd derivative of PAC function, $\Theta''(0)$	Inv. $\Gamma$	6.350	3.175	8.057	2.569	0.683	2.834	1.7763	3.8435
<b>Panel B:</b> Autocorrelation of exogenous variables									
Productivity, $\rho_A$	Beta	0.645	0.146	0.616	0.640	0.137	0.618	0.3992	0.8379
Preference shock, $\rho_\eta$	Beta	0.782	0.241	0.602	0.657	0.187	0.585	0.2818	0.8737
Government exp., $\rho_G$	Beta	0.274	0.148	0.324	0.578	0.093	0.571	0.4179	0.7247
World trade, $\rho_{WT}$	Beta	0.723	0.095	0.703	0.832	0.053	0.824	0.7380	0.9116
Risk premium, $\rho_{\hat{\theta}^*}$	Beta	0.582	0.105	0.574	0.858	0.055	0.834	0.7475	0.9220
Capital inflows, $\rho_{\hat{\phi}^*}$	Beta	0.319	0.144	0.355	0.150	0.066	0.169	0.0604	0.2733
Foreign Reserves, $\rho_{FX}$	Beta	0.913	0.068	0.876	0.868	0.042	0.863	0.7941	0.9329
<b>Panel C:</b> Standard deviation of exogenous shocks <sup>(3)</sup>									
Productivity, $\sigma_A$	Inv. $\Gamma$	0.011	0.001	0.011	0.010	0.001	0.010	0.0086	0.0115
Preference shock, $\sigma_\eta$	Inv. $\Gamma$	0.012	0.006	0.015	0.020	0.004	0.019	0.0126	0.0257
Government exp., $\sigma_G$	Inv. $\Gamma$	0.007	0.001	0.007	0.007	0.001	0.007	0.0065	0.0084
World trade, $\sigma_{WT}$	Inv. $\Gamma$	0.008	0.001	0.009	0.009	0.001	0.009	0.0079	0.0102
Risk premium, $\sigma_{\hat{\theta}^*}$	Inv. $\Gamma$	0.002	0.0004	0.003	0.006	0.001	0.006	0.0053	0.0077
Capital inflows, $\sigma_{\hat{\phi}^*}$	Inv. $\Gamma$	0.005	0.001	0.005	0.006	0.0004	0.006	0.0049	0.0063
Foreign Reserves, $\sigma_{FX}$	Inv. $\Gamma$	0.018	0.002	0.018	0.018	0.001	0.018	0.0158	0.0204

Notes: (1) Sample period 2010-2019. (2) HPD = Highest Posterior Density. (3) Under the prior the variance of each shock follows an inverse gamma distribution, not its standard deviation.

process. All exogenous variables follow a first-order auto-regressive process of the form:

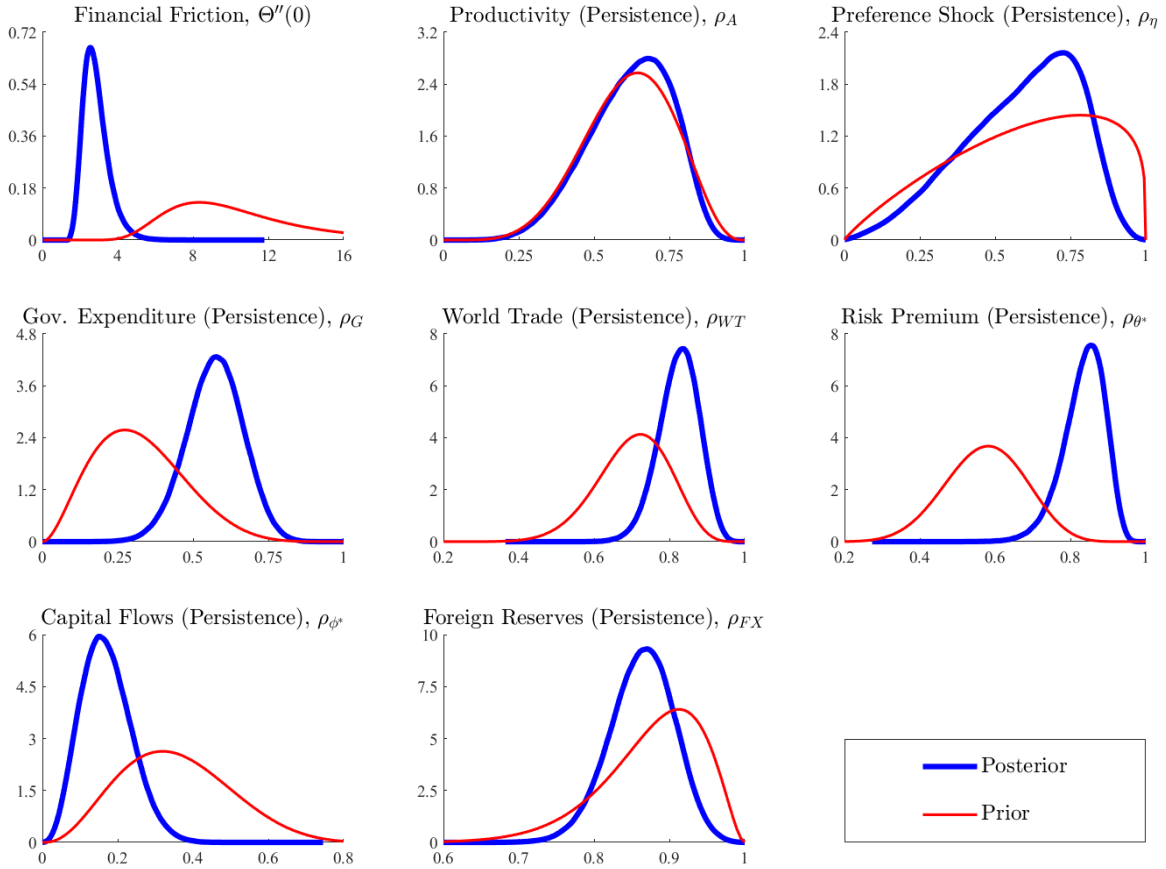
$$X_t = \rho_X X_{t-1} + \epsilon_t^X \quad , \quad \epsilon_t^X \stackrel{iid}{\sim} N(0, \sigma_X^2)$$

where  $X_t \in \{\tilde{A}_t, \tilde{G}_t, \tilde{\eta}_t, \tilde{WT}_t, \hat{\phi}_t^* - \hat{\phi}_{ss}^*, \hat{\theta}_t^*, \tilde{FX}_t\}$

I use the beta distribution as prior for the persistence parameters,  $\rho_X$ , and the inverse gamma distribution for the variance of the shocks,  $\sigma_X^2$ . To form priors, I estimate an auto-regressive process for (detrended) productivity<sup>35</sup>,  $\tilde{A}_t$ , government expenditure,  $\tilde{G}_t$ , world trade,  $\tilde{WT}_t$ , capital inflows,  $\hat{\phi}_t^* - \hat{\phi}_{ss}^*$ , and foreign reserves,  $\tilde{FX}_t$ , using quarterly data for the period 2010 – 2019. The point estimates and their standard deviation are used as modes and standard deviations of each prior distribution, respectively. For the

<sup>35</sup>Productivity is measured as  $\log(GDP_t) - \alpha \log(N_t)$ , where  $GDP_t$  is gross domestic product in fixed prices and  $N_t$  is total hours worked, both seasonally adjusted.

Figure F.1: Prior and Posterior Distributions of **Model's Parameters**



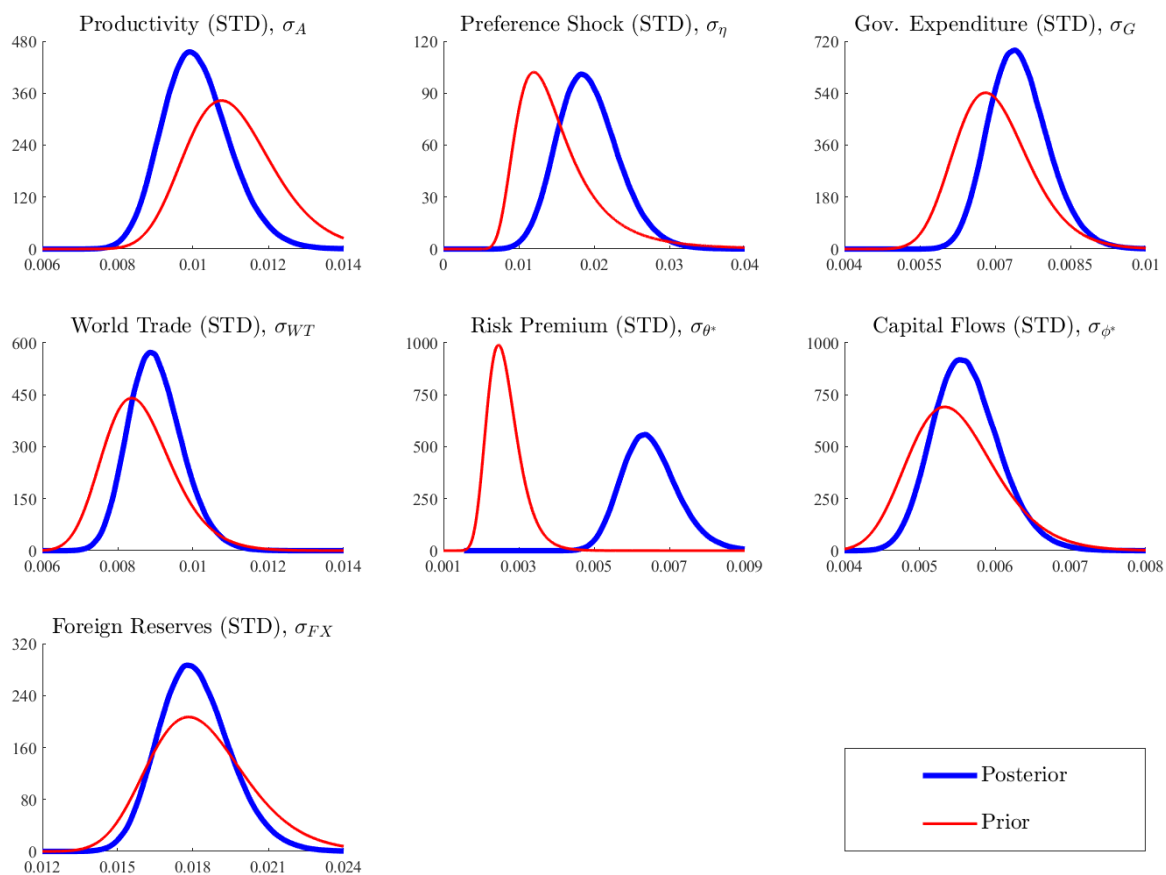
unobserved processes of the preference shock,  $\tilde{\eta}_t$  and the risk premium shock,  $\tilde{\theta}_t^*$ , I adopt as prior the estimation results of [Argov et al. \(2012\)](#).<sup>36</sup> Table 5 summarizes the results.

## F.2.2 The Portfolio Adjustment Cost

To shape a prior for the distribution of the portfolio adjustment cost parameter,  $\Theta''(0)$ , I rely on estimates for the effect of the Bank of Israel's FXIs on the New Israeli Shekel (NIS) nominal effective exchange rate. [Ribon \(2017\)](#) uses data in monthly frequency and employs various instrumental variables to estimate the effect. She finds that a purchase of \$1 billion by the Bank of Israel is associated with a depreciation of about 0.72 percent of the NIS exchange rate, with little evidence for the erosion of the effect over time. [Hertrich and Nathan \(2022\)](#) use data in daily frequency and estimate the effect using

<sup>36</sup>The standard deviation of the risk premium shock is divided by  $\Theta''(0) \frac{1}{Y_{ss}^{H,An}}$  in order to account for the coefficient multiplying it in the UIP, equation (21) in the text, as in [Argov et al. \(2012\)](#) that coefficient is 1.

Figure F.2: Prior and Posterior Distributions of the **Standard Deviations of the Exogenous Shocks**



instrumental variables in GMM. They find that a purchase of \$1 billion by the Bank of Israel is associated with a depreciation of about 0.82 percent of the NIS. While the authors report that the point estimate of the effect remains stable over time, it is statistically significant for only 5 trading days. [Caspi et al. \(2022\)](#) estimate the effect of unexpected FXIs on the exchange rate. They identify policy shocks using intraday data, and then employ local projection methods to estimate the effect of these shocks on the exchange rate in daily frequency. They find that a typical daily policy surprise depreciates the NIS by approximately 0.4 percent, and that the effect remains significant for 40 – 60 days. Their estimates suggest that a purchase of \$1 billion is associated, on average, with a depreciation of about 0.94 percent of the NIS exchange rate.<sup>37</sup>

Converting these results to the units of the model, I assess that the effect of a one standard deviation shock to foreign reserves on the exchange rate is about 1.4 percent

<sup>37</sup>I thank the authors for providing the information necessary for converting their results to units of exchange rate movement per \$1 billion of intervention.

Table 6: Prior and Posterior Distributions of the Standard Deviations of the Measurement Errors<sup>(1)</sup>

	Prior Distribution				Posterior Distribution				
	Type	Mode	STD	Implied Mean	Mode	STD	Mean	HPD Interval <sup>(2)</sup>	
								5%	95%
GDP	Inv. $\Gamma$	0.003	0.002	0.004	0.005	0.001	0.005	0.0032	0.0065
Private Consumption	Inv. $\Gamma$	0.005	0.002	0.006	0.004	0.001	0.005	0.0031	0.0065
Exports	Inv. $\Gamma$	0.016	0.008	0.021	0.024	0.003	0.025	0.0196	0.0298
Imports	Inv. $\Gamma$	0.015	0.008	0.019	0.023	0.003	0.024	0.0192	0.0277
Hours worked	Inv. $\Gamma$	0.010	0.005	0.013	0.011	0.003	0.012	0.0071	0.0160
Nominal interest rate	Inv. $\Gamma$	0.0003	0.0002	0.0004	0.0004	0.0001	0.0005	0.0003	0.0006
CPI inflation	Inv. $\Gamma$	0.003	0.001	0.003	0.005	0.001	0.005	0.0038	0.0058
Nominal depreciation	Inv. $\Gamma$	0.015	0.007	0.019	0.022	0.003	0.023	0.0188	0.0269
Terms of trade	Inv. $\Gamma$	0.010	0.005	0.012	0.016	0.002	0.016	0.0131	0.0187
Private sector net foreign assets	Inv. $\Gamma$	0.013	0.007	0.017	0.022	0.002	0.023	0.0190	0.0270

Notes: (1) Under the prior the variance of each shock follows an inverse gamma distribution, not its standard deviation. (2) HPD = Highest Posterior Density.

in [Hertrich and Nathan \(2022\)](#) and in [Caspi et al. \(2022\)](#), and about 1.0 percent in [Ribon \(2017\)](#).<sup>38</sup> That said, given that the effects in [Hertrich and Nathan \(2022\)](#) and [Caspi et al. \(2022\)](#) lose statistical significance during the quarter, their estimates may overstate the effect of FXIs in quarterly frequency. As a benchmark value, I assume that a typical intervention generates a 1.0 percent movement in the exchange rate. To get a sense of the magnitude of  $\Theta''(0)$ , I search for its value such that a one standard deviation shock to foreign reserves in the model generates a 1.0 percent depreciation on impact. The resulting value, given the parameterization under the prior modes of the exogenous processes, is approximately 6.35. I use this value as the prior mode for the distribution of  $\Theta''(0)$ , a value half that size for its standard deviation, and the inverse gamma as the prior distribution. These choices and estimation results are summarized in [Table 5](#).

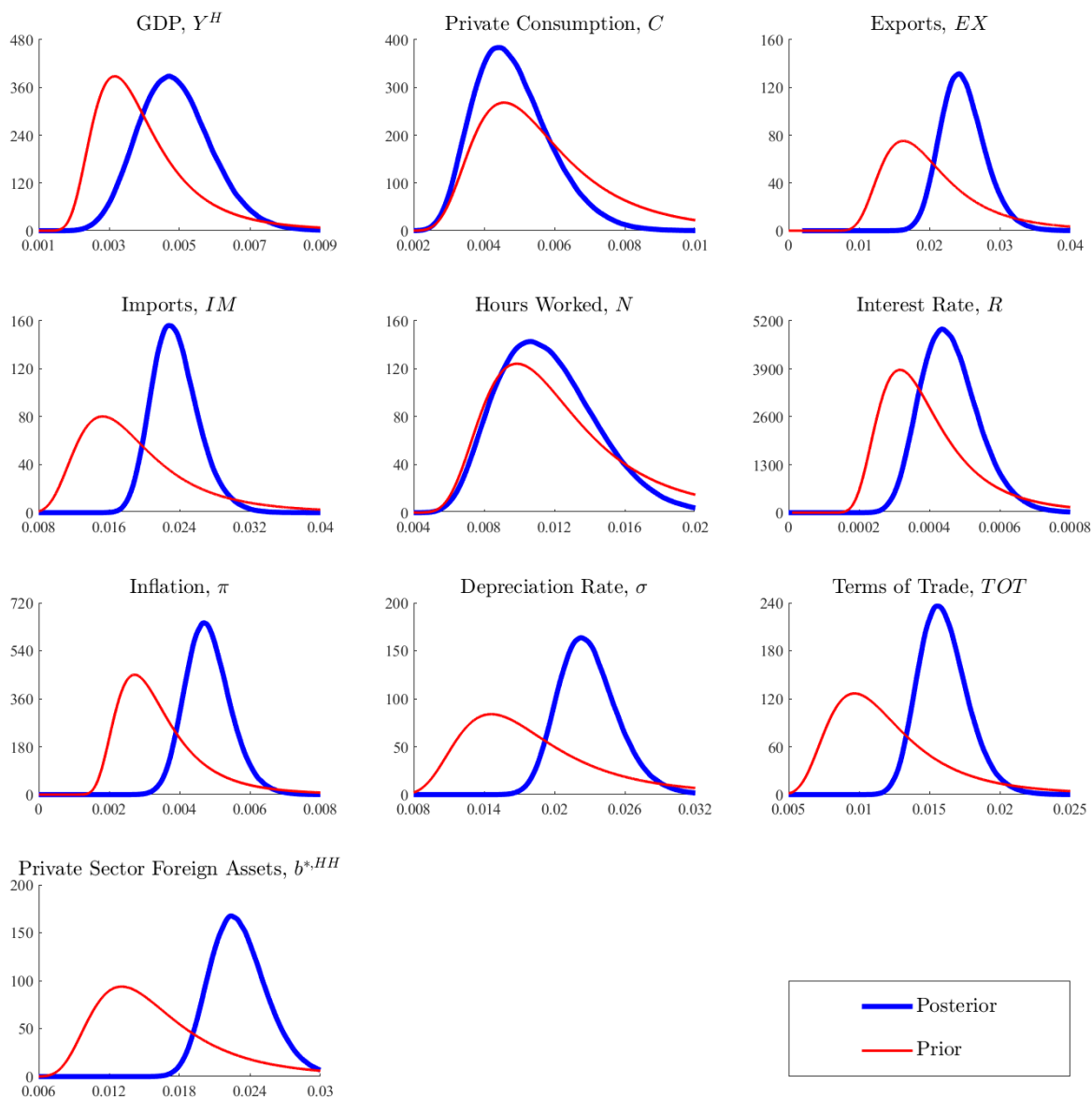
### F.2.3 Measurement Errors

As observable variables I use data on GDP (log), private consumption (log), government consumption (log), exports (log), imports (log), total hours worked (log), the return on Bank of Israel 3-month unindexed bill ("Makam", quarterly average), CPI inflation rate (quarter average over quarter average), nominal effective depreciation rate (quarter

<sup>38</sup>In the sample of [Hertrich and Nathan \(2022\)](#) \$1 billion is about 1.0 percent of foreign reserves, in [Caspi et al. \(2022\)](#) it is about 1.2 percent, and in [Ribon \(2017\)](#) it is 1.3 percent. Under the prior, I estimate the standard deviation of the shock to  $FX_t$  at about 1.8 percent ([Table 5](#)).



Figure F.3: Prior and Posterior Distributions of the **Standard Deviations of the Measurement Errors**



average over quarter average), the terms of trade (log), net private holdings of foreign assets (relative to trend GDP), foreign reserves (log), world trade (log), and capital inflow to public-sector financial instruments (relative to trend GDP). The exact definition and data source of each variable are detailed below.

All series are first-differenced and demeaned. Measurement errors are assigned to all endogenous variables. For the prior distributions of the standard deviations of the measurement errors, I assume that their variance follows the inverse gamma distribution and that they account for one-third of the variation in the data. The exogenous variables ( $\tilde{A}_t$ ,  $\tilde{G}_t$ ,  $\tilde{WT}_t$ ,  $\hat{\phi}_t^*$  and  $\tilde{FX}_t$ ) are not assigned measurement errors, as these may gener-

ate weak identification of the standard deviations of the shocks in their auto-regressive process. Table 6 summarizes the estimation results for the standard deviations of the measurement errors.

### F.3 Data Description

The estimation uses data on GDP, private consumption, government consumption, exports, imports, total hours worked, the return on Bank of Israel 3-month unindexed bill ("Makam"), CPI inflation, the NIS nominal effective exchange rate, the terms of trade, net private holdings of foreign assets, foreign reserves, world trade, and capital inflow to public-sector financial instruments. Series are in quarterly frequency. The sample period is 2010:Q1 - 2019:Q4. Following is a description of each variable, by categories:

**National Accounts** Source: Israel Central Bureau of Statistics.

- Gross domestic product. Fixed prices, seasonally adjusted, demeaned log first-difference.
- Total private consumption. Fixed prices, seasonally adjusted, demeaned log first-difference.
- Government consumption, excluding imported defense. Fixed prices, seasonally adjusted, demeaned log first-difference.
- Exports of goods and services, excluding startups and diamonds. Fixed prices, seasonally adjusted, demeaned log first-difference.
- Imports of goods and services, excluding imported defense, ships and aircraft, and diamonds. Fixed prices, seasonally adjusted, demeaned log first-difference.
- Terms of trade: calculated as the ratio of export prices (excluding startups and diamonds) to import prices (excluding imported defense, ships and aircraft, and diamonds). Demeaned log first-difference.

**Labor Market Data** Source: Israel Central Bureau of Statistics.

- Total labor input (hours) per week. Seasonally adjusted, demeaned log first-difference.

**Nominal Variables** Source: Israel Central Bureau of Statistics (ICBS) and Bank of Israel (BoI).

- CPI inflation rate. Seasonally adjusted, quarter average over quarter average, demeaned first difference. (Source: ICBS)
- Nominal 3-month return on Bank of Israel unindexed bill ("Makam"). Average, demeaned first difference. (Source: BoI)
- Nominal effective depreciation rate. Quarter average over quarter average, demeaned first difference. (Source: BoI)

**International Investment Position** Source: Bank of Israel.

- Foreign reserves held by the Bank of Israel, expressed in terms of imported goods: calculated by multiplying the quarterly average foreign reserves (in dollars) by the quarterly average ILS/USD exchange rate and dividing by import prices (excluding imported defense, ships and aircraft, and diamonds). Demeaned log first-difference.
- Net private-sector (excluding banks) holdings of foreign assets relative to trend GDP, both expressed in terms of imported goods. Quarterly average net assets (in dollars) are multiplied by the quarterly average ILS/USD exchange rate and then divided by import prices (excluding imported defense, ships and aircraft, and diamonds). Trend GDP is calculated as the linear trend of (log) nominal GDP divided by import prices (excluding imported defense, ships and aircraft, and diamonds). Demeaned first difference.
- Capital inflow to public-sector financial instruments relative to trend GDP, both expressed in terms of imported goods. Capital inflow in dollars is measured using financial investment in public-sector tradable securities. Transformation to units of imported goods and measurement of trend GDP are the same as in net private-sector holdings of foreign assets above. Demeaned first difference.

**World Trade** Source: OECD.

- World trade: Total imports of goods and services by OECD countries. Volume index seasonally adjusted (VIXOBSA), demeaned log first-difference.

## G Appendix: Proof of Lemma 1

**Lemma G.2 (The comovement of consumption, the terms of trade, and imports)**

*With no nominal rigidities,  $\xi_w = \xi_p = 0$ , and preferences satisfying  $\gamma_{nn} - \gamma_{cn} \geq 0$  and  $\gamma_{nc} - \gamma_{cc} \geq 0$ , consumption,  $\widetilde{C}_t$ , the terms of trade,  $\widetilde{TOT}_t$ , and imports,  $\widetilde{IM}_t$ , comove positively in response to variation in foreign reserves,  $\widetilde{FX}_t$ .*

**Proof.** First, notice that by the consumption aggregator, demand functions for  $\tilde{d}_t^H$  and  $\widetilde{IM}_t$ , and the definition of the terms of trade, equations (32), (33), (34), and (44), respectively in the text, we get:

$$\tilde{p}_t^H \cong \lambda \widetilde{TOT}_t \quad (\text{G.26})$$

$$\tilde{p}_t^F \cong -(1 - \lambda) \widetilde{TOT}_t \quad (\text{G.27})$$

Using (G.26), together with (33), (35), and (36) in the text, the resource constraint, equation (37), reads:

$$\tilde{A}_t + \alpha \tilde{N}_t \cong (1 - \lambda) \frac{C_{ss}}{Y_{ss}^H} \tilde{C}_t - [(1 - \lambda) \varepsilon + \varepsilon^*] \lambda \frac{C_{ss}}{Y_{ss}^H} \widetilde{TOT}_t + \frac{G_{ss}}{Y_{ss}^H} \tilde{G}_t + \lambda \frac{C_{ss}}{Y_{ss}^H} \widetilde{WT}_t$$

And by setting  $\xi_w = \xi_p = 0$  in equations (28) and (29), labor market equilibrium reads:

$$(\gamma_{nc} - \gamma_{cc}) \tilde{C}_t + (\gamma_{nn} - \gamma_{cn}) \tilde{N}_t + (\gamma_{n\eta} - \gamma_{c\eta}) \tilde{\eta}_t \cong \lambda \widetilde{TOT}_t + \tilde{A}_t + (\alpha - 1) \tilde{N}_t$$

Putting the two together, by substituting for  $\tilde{N}_t$ , we get:

$$\begin{aligned} \Psi_{TOT} \widetilde{TOT}_t &\cong \Psi_C \tilde{C}_t + EXOG_t & (\text{G.28}) \\ \Psi_{TOT} &= \alpha \frac{\lambda}{\gamma_{nn} - \gamma_{cn} + 1 - \alpha} + [(1 - \lambda) \varepsilon + \varepsilon^*] \lambda \frac{C_{ss}}{Y_{ss}^H} \\ \Psi_C &= \alpha \frac{\gamma_{nc} - \gamma_{cc}}{\gamma_{nn} - \gamma_{cn} + 1 - \alpha} + (1 - \lambda) \frac{C_{ss}}{Y_{ss}^H} \end{aligned}$$

where  $EXOG_t$  summarizes the exogenous variables in both equations ( $\tilde{A}_t$ ,  $\tilde{G}_t$ ,  $\widetilde{WT}_t$ ,  $\tilde{\eta}_t$ ). Assuming  $\gamma_{nn} - \gamma_{cn} \geq 0$  and  $\gamma_{nc} - \gamma_{cc} \geq 0$  guarantees that the coefficients  $\Psi_{TOT}$  and  $\Psi_C$  are positive; though this is a sufficient condition, and it can be relaxed. Since  $EXOG_t$  does not react to FXIs, and since  $\Psi_{TOT}$ ,  $\Psi_C > 0$ , equation (G.28) dictates that if  $\widetilde{TOT}_t$  and  $\tilde{C}_t$  respond to FXIs they must move in the same direction.

Finally, using (G.27), import demand, equation (34) in the text, reads:

$$\widetilde{IM}_t \cong \tilde{C}_t + \varepsilon (1 - \lambda) \widetilde{TOT}_t$$

Suggesting that imports,  $\widetilde{IM}_t$ , comoves positively with  $\tilde{C}_t$  and  $\widetilde{TOT}_t$ . ■

## Bibliography

- Adrian, T., C. Erceg, M. Kolasa, J. Linde, and J. Zabczyk (2021). A Quantitative Microfounded Model for the Integrated Policy Framework. International Monetary Fund, Working Paper WP/21/292.
- Alla, Z., R. A. Espinoza, and A. R. Gosh (2020). FX Intervention in the New Keynesian Model. *Journal of Money, Credit and Banking* 52(7), 1755–1791.

- Amador, M., J. Bianchi, L. Bocola, and F. Perri (2020). Exchange Rate Policies at the Zero Lower Bound. *The Review of Economic Studies* 87(4), 1605–1645.
- Argov, E., E. Barnea, A. Binyamini, E. Borenstein, D. Elkayam, and I. Rozenshtrom (2012). MOISE: A DSGE Model for the Israeli Economy. Bank of Israel, Research Department, Discussion Paper 2012.06.
- Backus, D. K. and P. J. Kehoe (1989). On the Denomination of Government Debt: A Critique of the Portfolio Balance Approach. *Journal of Monetary Economics* 23(3), 359–376.
- Bank of Israel (2010). Bank of Israel Annual Report – 2009. Bank of Israel.
- Bank of Israel (2022). Bank of Israel Annual Report – 2021. Bank of Israel.
- Basu, S., E. Boz, G. Gopinath, F. Roch, and F. Unsal (2020). A Conceptual Model for the Integrated Policy Framework. International Monetary Fund, Working Paper WP/20/121.
- Basu, S., E. Boz, G. Gopinath, F. Roch, and F. Unsal (2023). Integrated Monetary and Financial Policies for Small Open Economies. International Monetary Fund, Working Paper WP/23/161.
- Benes, J., A. Berg, R. A. Portillo, and D. Vavra (2015). Modeling Sterilized Interventions and Balance Sheet Effects of Monetary Policy in a New-Keynesian Framework. *Open Economies Review* 26(1), 81–108.
- Benigno, P. and M. Woodford (2012). Linear-Quadratic Approximation of Optimal Policy Problems. *Journal of Economic Theory* 147(1), 1–42.
- Calvo, G. A. (1983). Staggered Prices in a Utility-Maximizing Framework. *Journal of Monetary Economics* 12(3), 383–398.
- Caspi, I., A. Friedman, and S. Ribon (2022). The Immediate Impact and Persistent Effect of FX Purchases on the Exchange Rate. *International Journal of Central Banking* 18(5), 165–195.
- Cavallino, P. (2019). Capital Flows and Foreign Exchange Intervention. *American Economic Journal: Macroeconomics* 11(2), 127–170.
- Chen, K., M. Kolasa, J. Linde, P. Wang, H. abd Zabczyk, and J. Zhou (2023). An Estimated DSGE Model for Integrated Policy Analysis. International Monetary Fund, Working Paper WP/23/135.
- Corsetti, G. and P. Pesenti (2001). Welfare and Macroeconomic Interdependence. *The Quarterly Journal of Economics* 116(2), 421–445.
- Cúrdia, V. and M. Woodford (2011). The Central-Bank Balance Sheet As An Instrument of Monetary Policy. *Journal of Monetary Economics* 58(1), 54–79.
- De Paoli, B. (2009). Monetary Policy under Alternative Asset Market Structures: The Case of a Small Open Economy. *Journal of Money, Credit and Banking* 41(7), 1301–1330.
- Faltermeier, J., R. Lama, and J. P. Medina (2022). Foreign Exchange Intervention for Commodity Booms and Busts. *European Economic Review* 143, 104018.
- Fanelli, S. and L. Straub (2021). A Theory of Foreign Exchange Interventions. *The Review of Economic Studies* 88(6), 2857–2885.
- Gabaix, X. and M. Maggiori (2015). International Liquidity and Exchange Rate Dynamics. *The Quarterly Journal of Economics* 130(3), 1369–1420.
- Gali, J. (2015). *Monetary Policy, Inflation, and the Business Cycle: An Introduction to the New Keynesian Framework and Its Applications* (Second ed.). Princeton University Press (Princeton, NJ).

- Galí, J. and T. Monacelli (2005). Monetary Policy and Exchange Rate Volatility in a Small Open Economy. *Review of Economic Studies* 72(3), 707–734.
- Greenwood, J., Z. Hercowitz, and G. W. Huffman (1988). Investment, Capacity Utilization, and the Real Business Cycle. *American Economic Review* 78(3), 402–417.
- Havránek, T. (2015). Measuring Intertemporal Substitution: The Importance of Method Choices and Selective Reporting. *Journal of the European Economic Association* 13(6), 1180–1204.
- Hertrich, M. and D. Nathan (2022). Foreign Exchange Interventions and Their Impact on Expectations: Evidence from the USD/ILS Options Market. Bank of Israel, Research Department, Discussion Paper 2022.10.
- IMF (2023). Annual Report on Exchange Arrangements and Exchange Restrictions 2022. International Monetary Fund: Washington, DC.
- Itskhoki, O. and D. Mukhin (2021). Exchange Rate Disconnect in General Equilibrium. *Journal of Political Economy* 129(8), 2183–2232.
- Itskhoki, O. and D. Mukhin (2023). Optimal Exchange Rate Policy. National Bureau of Economic Research, Working Paper 31933.
- Kalemli-Özcan, S. and L. Varela (2023). Five Facts about the UIP Premium. National Bureau of Economic Research, Working Paper 28923.
- Ribon, S. (2017). Why the Bank of Israel Intervenes in the Foreign Exchange Market, and What Happens to the Exchange Rate. Bank of Israel, Research Department, Discussion Paper 2017.04.
- Ribon, S. and D. Sayag (2013). Price Setting Behavior in Israel – An Empirical Analysis Using Microdata. Bank of Israel, Research Department, Discussion Paper 2013.07.
- Schmitt-Grohé, S. and M. Uribe (2003). Closing Small Open Economy Models. *Journal of International Economics* 61(1), 163–185.
- Sims, C. (1999). Matlab Optimization Software. *Quantitative Macroeconomics and Real Business Cycles*, QM&RBC Codes 13.
- Smets, F. and R. Wouters (2003). An Estimated Dynamic Stochastic General Equilibrium Model of the Euro Area. *Journal of the European Economic Association* 1(5), 1123–1175.
- Smets, F. and R. Wouters (2007). Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach. *American Economic Review* 97(3), 586–606.
- Uribe, M. and V. Z. Yue (2006). Country Spreads and Emerging Countries: Who Drives Whom? *Journal of International Economics* 69(1), 6–36.
- Wallace, N. (1981). A Modigliani-Miller Theorem for Open-Market Operations. *American Economic Review* 71(3), 267–274.
- Woodford, M. (2003). *Interest and Prices: Foundations of a Theory of Monetary Policy*. Princeton University Press (Princeton, NJ).
- Yakhin, Y. (2022). Breaking the UIP: A Model-Equivalence Result. *Journal of Money, Credit and Banking* 54(6), 1889–1904.