Productive Public Spending, Knowledge Spillovers and Convergence: A Multi-Country Analysis

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Productive Public Spending, Knowledge Spillovers and Convergence: A Multi-Country Analysis

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Abstract: This paper develops a multi-country AK model of endogenous growth with international knowledge transmission to analyze the impact of productive public expenditure on growth and convergence. A leader economy drives knowledge advancement, while follower countries benefit from spillovers if their knowledge exceeds a threshold. The findings suggest that public expenditure helps laggard followers acquire foreign technology, but does not enhance long-term growth. Empirical analysis using a dynamic panel model confirms the threshold effect: public investment boosts growth only in countries far from the technological frontier, while it is ineffective for those at or beyond the threshold.

JEL classification: O10, O33, O47

Keywords: AK growth, Knowledge spillovers, Development thresholds, Dynamic heterogeneous panels , Cross-Sectional dependence.

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1 Introduction

Recent research indicates that cross-country income inequality has declined since the mid-1980s (Roy *et al.* 2016; Kremer *et al.* 2021; Patel *et al.* 2021). Many economists and commentators attribute this convergence process to trade liberalization and institutional reforms, driven by the global dissemination of ICT technology and the adoption of Western governance standards (Kremer *et al.* 2021). However, less attention has been drawn to globalization-related factors such as knowledge transmission and technology transfer.

This paper examines the role of productive public expenditure in enabling low-income countries to converge toward higher income levels by facilitating the acquisition of foreign-produced technical knowledge. To achieve this, the paper develops a multi-country AK model of endogenous growth with productive public expenditure *à la* Barro (1990) and learning-by-investing externalities *à la* Frankel (1962) and Romer (1986), in which a single frontier country (the leader) sets the global growth trend, while a finite set of non-frontier countries (the followers) benefits from international knowledge spillovers generated by the leader.

To differentiate among follower countries, we build on Parello and Venturini (2025) and focus on the ratio of the follower's capital stock per capita to that of the leader as a statistic for measuring technological distance.[1] In this sense, a country is considered capable of absorbing foreign technology if its relative capital stock per capita exceeds a specified threshold.

The paper finds that two distinct equilibrium growth regimes can emerge for follower countries: one for *capital-abundant* followers, who converge toward the leader due to international knowledge spillovers, and one for *capital-poor* followers, whose initial technological gap is too large to allow convergence. The implications of this result are that only *capital-abundant* economies are able to catch up with the leader, while *capital-poor* economies are condemned to fall further behind. However, in our multi-country framework, where increasing productive public spending accelerates capital accumulation and growth, the policy implications suggest that the government of a *capital-poor* follower country can effectively help it reach the threshold, provided that it remains sufficiently resilient over time to allow the economy to bridge the knowledge gap with the leader. If this occurs, the follower economy sheds its status as a diverging *capital-poor* economy and

[1] According to Romer (1986) and Rebelo (1991), the aggregate stock of capital indirectly represents the total knowledge base available in the economy, as each firm's investment in capital generates external benefits (e.g., improved production techniques, shared innovations, etc.). Consequently, a natural measure of an economy's distance from the knowledge frontier is its capital stock relative to that of a frontier economy.

adopts that of a *capital-abundant* economy, starting to benefit from knowledge externalities from the leader.

Once presented the model, in the second part of the paper we take the model to data to test whether the implications of the model are either validated or rejected by the data. Using a sample of 110 countries, primarily non-developed nations, over the period 1970–2019, and employing a heterogeneous dynamic panel data model that accounts for cross-sectional dependence, we find evidence of a technological distance threshold. This threshold, measured as the ratio of a follower country's capital per capita to that of the leader, namely the United States, is estimated at 0.2. Below this level, the effects of technology spillovers are minimal. However, once this threshold is surpassed, follower countries can effectively leverage these spillovers to accelerate growth. Additionally, we find that (i) the impact of spillovers diminishes as an economy matures and its capital stock approaches that of the leading country, and (ii) productive public investments promote faster growth only in countries that remain far from the technological frontier and have yet to reach the threshold.

The paper draws from and contributes to multiple strands of economic literature. The first strand of literature this paper is related to is that exploring the connections between public spending and economic growth. This research stream dates back to Arrow and Kurz (1970), who highlight the critical role of public investment in economic growth through a neoclassical framework, but gained full consideration into the endogenous growth literature after the seminal contribution of Barro (1990), showing that government spending can effectively spur long-run growth, while utility-type spending ultimately decreases it. [2] After Barro's contribution, other studies have investigated the relationship between specific categories of public spending and their long-term growth effects, including public infrastructure (Turnovsky and Fisher 1995; Devarajan *et al.* 1996; Agénor 2008, Agénor 2010), public education (Eckstein and Zilcha 1994; Glomm and Ravikumar 1997; Blankenau and Simpson (2004); Blankenau (2005); Blankenau *et al.* 2007; Chu *et al.*, 2024), and public health (Aísa and Pueyo 2006; Agénor 2010, Agénor 2012).

This paper contributes to this literature by providing a multi-country extension of Barro's (1990) model that incorporates serendipitous technology transmission, which generates a catching-up process in income per capita similar to the findings reported by Kremer *et al.* (2021) and Patel *et al.* (2021).

[2] Futagami *et al.* (1993) build on Barro (1990) to include both public and private capital, identifying a unique and stable transitional path. They also determine an optimal tax rate that is lower than the growth-maximizing rate, underscoring the complexity of public capital's role in fostering economic growth.

This paper also connects to the literature on cross-country technological interdependence and long-term growth in multi-country frameworks. The literature began with the seminal work of Nelson and Phelps (1966), which examined an economy's capacity to absorb foreign technology. Since then, several contributions have enriched the field—especially in the 2000s with the spread of globalization.

Arguably, the most influential paper advancing the discussion on technological knowledge transmission for per capita income convergence and growth is Howitt (2000), which emphasizes the critical roles of innovation and institutions in sustaining economic growth and convergence. Later, Klenow and Rodriguez-Clare (2005) documented stylized facts on global interdependence and explored how cross-country externalities influence economic performance, while Acemoglu *et al.* (2006) examines how a country's proximity to the global technology frontier affects its growth strategies. Specifically, in Acemoglu et al.'s (2006) work, countries far from the frontier benefit by adopting existing technologies through long-term, stable investments and larger, established firms, whereas those near the frontier drive growth by innovating with short-term, agile strategies and high-skill management.

In the aforementioned studies, the theoretical frameworks focus on a single representative economy to extend results to the global level. Subsequently, other research has investigated the effects of technology transmission through fully fledged multi-country analytical frameworks. For example, Moll (2008) explores the implications of involuntary international externalities arising from capital accumulation within a multi-country neoclassical growth model, establishing conditions for the uniqueness and stability of steady states.[3] Moreover, Ertur and Koch (2007, 2011) analyze technological interdependence and spatial externalities in endogenous growth models by proposing a spatial econometric approach to assess the impact of knowledge spillovers and R&D on long-term growth.

More recently, a new wave of papers has explored channels of technological transmission beyond innovation and R&D. For instance, Alvarez (2017) integrates capital accumulation into a trade model, analyzing the effects of tariffs and steady-state dynamics while comparing static and dynamic welfare models. Buera and Oberfield (2020) out-

[3]Building on Moll's (2008) framework, Jin and Zhou (2022) develops a multi-country Ramsey growth model with technological interdependence that explains diverse growth rates, convergence dynamics, and persistent income divergences driven by structural differences and initial conditions. However, unlike Moll (2008), these authors adopt diffusion curves similar to those identified by Comin and Hobijn (2010) and highlight how interdependence fosters long-run convergence in growth rates while transitional dynamics capture the "advantages of backwardness."

lines a framework for innovation diffusion, emphasizing the role of trade on a country's knowledge frontier and its contributions to TFP growth over time. Kleinman *et al.* (2023) generalizes the open-economy neoclassical growth model to account for frictions in trade and international capital mobility, highlighting how these factors influence shock adjustments and convergence to steady state, particularly in the context of U.S.-China relations. Hsieh *et al.* (2023) presents a model linking trade liberalization to creative destruction, describing how international flows of ideas maintain technological parity and influence export dynamics. Finally, Gross and Klein (2024) develops an endogenous growth model focused on innovation that captures global spillovers while also replicating declines in research productivity.

Our paper contributes to this literature in three ways. First, our study focuses on the role of productive public expenditures in stimulating growth. Second, while most of these studies concentrate on either assessing the welfare and growth effects of potential de-globalization or providing the optimal policy mix to maximize net benefits from externalities, our paper aims to reconcile recent evidence of per capita income convergence in global growth over the past forty years. Finally, while these studies assume that all countries contribute equally to the global stock of knowledge, our framework posits that a single country drives the knowledge frontier forward, with all others functioning as follower economies.[4]

Among this body of research, the study by Parello and Venturini (2025) is most closely related to our work, as it investigates international knowledge spillovers and their impact on economic convergence within a multi-country, leader–follower endogenous growth framework that incorporates learning-by-investing externalities. Although both studies explore income convergence and knowledge diffusion, Parello and Venturini (2025) primarily emphasizes capital intensity and international spillovers, whereas our analysis also considers the role of government spending in shaping growth trajectories.

Although both studies compute a knowledge gap threshold—defined as the relative capital intensity between laggard economies and the leader—they adopt distinct econometric methodologies. In particular, while Parello and Venturini (2025) estimates the threshold using a panel dynamic regression framework (Kremer *et al.*, 2013), our analysis employs a heterogeneous dynamic panel model that accounts for cross-sectional depen-

[4] Debarsy and Ertur (2019) provide evidence for the existence of three distinct *local* technological leaders—Germany, Japan, and the United States. For analytical simplicity, our theoretical framework assumes a single *global* technological leader. Extending the model to include multiple leader economies is straightforward but adds complexity without significant theoretical insights.

dence (Chudik *et al.*, 2017). Furthermore, whereas the threshold in Parello and Venturini (2025) is driven solely by countries' investments in capital accumulation, our approach also considers the influence of productive public expenditure on a country's ability to absorb technology. Remarkably, our estimates yield a threshold value that closely aligns with that of Parello and Venturini (2025), thereby confirming the robustness of the threshold concept and underscoring the complementary roles of capital investment and public spending in driving technology absorption and economic convergence.

Finally, another branch of literature this paper would like to contribute to is that empirically gauging the impact of productive public spending and growth. This literature is vast and large, and subject of ongoing debate as empirical findings have yielded mixed results. For instance, Devarajan *et al.* (1996) find that, in developing countries, an increase in the share of current expenditure has a positive effect on growth, whereas the relationship between the capital component of public expenditure and per-capita growth is negative. In contrast, Gupta *et al.* (2005) observe that developing countries focusing spending on wages tend to experience lower growth, whereas those allocating a greater share to capital and non-wage goods and services achieve faster output expansion. Kneller *et al.* (1999), using a panel of OECD countries and distinguishing between productive and non-productive government expenditure as well as distortionary and non-distortionary taxation, found that productive government spending fosters growth, whereas non-productive expenditure does not.

On the taxation side, only distortionary taxes were found to negatively impact GDP per capita growth. More recently, Afonso and Jalles (2014) analyze a large panel of 155 developed and developing countries and find that public wages, interest payments, subsidies, and government consumption negatively affect growth. Morozumi and Veiga (2016) explore how institutions influence the relationship between public spending and economic growth. They find that institutions that foster accountability among political officeholders play an important role in generating the growth effects of capital spending, while no growth-promoting effect is observed for current spending. When governments are held accountable, capital spending significantly promotes growth, regardless of whether it is financed by reallocating current spending, increasing revenue, or expanding the budget deficit.

This paper is organized as follows. Section 2 develops the multi-country AK model with knowledge spillovers. Section 3 explores the dynamic equilibrium of the model. Section 4 theoretically examines the dynamics of income per capita inequalities across countries. Section 5 presents descriptive evidence and a robust econometric analysis to validate the stylized facts of the theoretical model. Finally, Section 6 concludes the discussion.

2 The model

2.1 Overview of the model

The world economy consists of a single frontier (or leader) country, indexed by ℓ, and a set $\mathcal{S}$ of non-frontier (or follower) countries, with cardinality $m \geq 1$. For simplicity, throughout this paper, we assume that all economies share the same preference structure and production technology, but differ in terms of technical knowledge and macroeconomic parameters. Specifically, we assume the following:

1. Each country is characterized by its own parameter space, capturing the distinctive features of its macroeconomic environment;

2. Each non-frontier economy $s \in \mathcal{S}$ exhibits a distinct degree of technological backwardness relative to the leader.

To incorporate endogenous growth, the model assumes that each country's TFP benefits from productivity spillovers arising from both private gross investment in physical capital, as in Romer (1986), and productive government expenditure, as in Barro (1990). However, due to differences in each country's capacity to absorb knowledge from the leader, the model predicts a cross-country scenario where only a subset of follower countries can catch up with the leader.

Time is set in continuous time. However, for ease of exposition, we will suppress the time variable t where no confusion arises.

2.2 Preferences and consumption

Every country in the world economy is inhabited by a large number of households, each of which consists of a continuum of family members of measure L_s. The representative household of country $s \in \ell \cup \mathcal{S}$ seeks to maximize the present discounted value of its lifetime utility:

$$\mathcal{U}_s = \int_0^\infty e^{-(\rho_s - \nu_s)t} \log c_s \mathrm{d}t, \quad \rho_s > \nu_s, \tag{1}$$

where $\rho_s > 0$ and $\nu_s \geq 0$ are, respectively, the subjective discount rate and the (constant) demographic growth rate of the representative household of country s, and c_s is per capita consumption.

Households use their after-tax income to accumulate financial assets, denoted by a_s. We abstract from capital mobility, in the sense that domestic households can borrow and lend only in the domestic capital market at the endogenously determined interest rate r_s. Their objective is therefore to choose the time path of consumption, $\{c_s\}_{t\in[0,\infty)}$, to maximize (1) subject to the following flow budget constraint:

$$\dot{a}_s = (r_s - \nu_s)\, a_s + (1 - \tau_s^w)\, w_s - c_s, \tag{2}$$

where w_s is the wage rate currently paid in country s at time t and $\tau_s^w \in (0,1)$ is the tax rate on labor income. Solving this dynamic optimization problem yields the Euler equation and the transversality condition:

$$\dot{c}_s = (r_s - \rho_s)\, c_s \tag{3}$$

$$\lim_{t\to\infty} \left\{ e^{-\int_0^t [r_s(z)-\nu_s]\mathrm{d}z} \frac{a_s\,(t)}{c_s\,(t)} \right\} = 0. \tag{4}$$

2.3 Technologies and production

In each country $s \in \ell \cup \mathcal{S}$, production is carried out by a fringe of competitive firms, each producing a homogeneous commodity using capital and labor. The production technology for a typical firm j in country s is described by:

$$Y_{j,s} = \left(\mathcal{A}_s N_{j,s}\right)^{1-\alpha} K_{j,s}^{\alpha}, \tag{5}$$

where $\alpha \in (0,1)$ is the output elasticity of capital, $Y_{j,s}$ is the output flow of the commodity produced by the firm, $N_{j,s}$ is its employment level, $K_{j,s}$ is its capital stock, and $\mathcal{A}_s$ is a country-specific labor-augmenting productivity index, whose composition will be detailed in Section 2.4.

Each firm takes $\mathcal{A}_s$, w_s, and r_s as given and chooses $N_{j,s}$ and $K_{j,s}$ to maximize profits, subject to (5). From necessary and sufficient conditions, we obtain:

$$r_s = \frac{\alpha Y_{j,s}}{K_{j,s}} - \delta_s, \tag{6}$$

$$w_s = \frac{(1-\alpha)\, Y_{j,s}}{N_{j,s}}, \tag{7}$$

where $\delta_s \in [0,1)$ represents the depreciation rate of the capital stock in country s. From (6) and (7), it follows that every firm in country s optimally chooses the same capital-to-worker ratio:

$$k_s \equiv \frac{K_{j,s}}{N_{j,s}} = \frac{\alpha w_s}{(1-\alpha)\,(r_s + \delta_s)}.$$

Therefore, in the remainder of this paper, we focus on the symmetric equilibrium, where each country's capital intensity is determined by k_s.

2.4 Aggregate output and cross-country externalities

Aggregating (5) over firms, the production technology of the economy s can be written as:

$$Y_s = (\mathcal{A}_s L_s)^{1-\alpha} K_s^{\alpha}, \tag{8}$$

where Y_s is country s's final good (henceforth *aggregate output*), and $L_s = \int_0^1 N_{j,s} \mathrm{d}j$ and $K_s = \int_0^1 K_{j,s} \mathrm{d}j$ are aggregate employment and capital respectively.

To account for technology transfer across countries, the technology parameters are assumed to be country-specific and given by:

$$\mathcal{A}_\ell = A_\ell \left(\frac{G_\ell}{L_\ell} \right)^{1-\xi} k_\ell^{\xi} \tag{9}$$

for the leader, and

$$\mathcal{A}_s = \begin{cases} A_s \left(\frac{G_s}{L_s} \right)^{1-\xi} \left(k_s \kappa_s^{-\psi} \right)^{\xi} & \text{if} \quad \tilde{\kappa} \leq \kappa_s \\ A_s \left(\frac{G_s}{L_s} \right)^{1-\xi} k_s^{\xi}, & \text{if} \quad \tilde{\kappa} > \kappa_s \end{cases}, \tag{10}$$

for each follower $s \in \mathcal{S}$, where $A_\ell > 0$ and $A_s > 0$ (with $A_\ell > A_s$) are efficiency parameters that capture the effectiveness with which each economy generates knowledge improvements from gross investment, G_ℓ and G_s represent the government purchases of goods and services in economies ℓ and s, respectively, which contribute to the productivity index as public goods (Barro, 1990), $\kappa_s \equiv k_s / k_\ell$ is the inverse of the relative capital intensity of the non-frontier economy s, and $\tilde{\kappa}$ is a threshold level for κ_s. In the remainder of the paper, κ_s will serve as a metric to measure the knowledge gap between countries, while $\tilde{\kappa}$ will be used as a threshold value to identify the minimum level of technical knowledge the non-frontier economy must have in order to benefit from the productivity externality of the frontier economy.

In equations (9) and (10), the parameters ξ and ψ quantify the influence of knowledge spillovers on domestic productivity. Specifically, $\xi \in [0, 1)$ measures the extent to which changes in government spending and private investment affect the productivity of the domestic economy. On the other hand, $\psi \in [0, 1)$ captures the strength with which capital accumulation in the frontier economy stimulates productivity growth in the non-frontier economy. When $\tilde{\kappa} > \kappa_s$, the knowledge gap is too large for the follower country s to effectively absorb and apply the frontier knowledge produced by the leader, thus preventing the follower from achieving the same level of production efficiency as the leader. In equation (10), this scenario is represented by setting $\psi = 0$, indicating that the leader and the follower are isolated technological "islands", with no knowledge diffusion between them.

Conversely, when the knowledge gap is not too large, i.e., $\tilde{\kappa} \leq \kappa_s$, the parameter $\psi \in (0, 1)$ allows for cross-country knowledge diffusion. In this case, the leader and the follower are intertwined through technology transfer, with the leader economy growing according to an AK model and the follower growing according to a 'neoclassical' model.

2.5 Government

At each point in time, the government of country $s \in \ell \cup \mathcal{S}$ adjusts the labor income tax rate, τ_s^w, to balance its budget. This results in the following balanced-budget condition:

$$G_s = \tau_s^w w_s L_s. \tag{11}$$

In equation (11), the left-hand side represents total government expenditures, while the right-hand side denotes total tax revenue. To solve for the steady state, we assume throughout the paper that each government follows the spending rule $G_s = \zeta_s Y_s$, where ζ_s is the ratio of public expenditure to final output in economy s at time t. For the remainder of the paper, we assume $\zeta_s \in (0, 1-\alpha)$ for all $s \in \ell \cup S$ and treat ζ_s as a policy instrument.[5]

2.6 Cross-country inequality

The ratio between the incomes per capita represents a natural index for measuring cross-country inequality. Let $y_s \equiv Y_s / L_s$ denote the level of income per capita of economy s at time t. Substituting from (9) and (10) into (8) yields:

$$y_\ell = \left(A_\ell \zeta_\ell^{1-\xi} \right)^{(1-\alpha)/[\alpha+(1-\alpha)\xi]} k_\ell \tag{12}$$

$$y_s = \left(A_s \zeta_s^{1-\xi} \right)^{(1-\alpha)/[\alpha+(1-\alpha)\xi]} \kappa_s^{-\xi\psi(1-\alpha)/[\alpha+(1-\alpha)\xi]} k_s, \tag{13}$$

[5]The assumption $\zeta \in (0, 1-\alpha)$ is imposed to ensure the tractability of the stability analysis and to maintain the dynamic properties of the model not too far from those of Romer (1986) and Barro (1990).

for all $s \in \mathcal{S}$. Hence, dividing (13) by (12), the degree of income inequality between the leader and the typical follower s can be measured by the following expression:

$$I_s \equiv \frac{y_s}{y_\ell} = \left(\frac{A_s}{A_\ell}\right)^{(1-\alpha)/[\alpha+(1-\alpha)\xi]} \mathcal{G}_s^{(1-\xi)(1-\alpha)/[\alpha+(1-\alpha)\xi]} \kappa_s^{1-\xi\psi(1-\alpha)/[\alpha+(1-\alpha)\xi]}, \tag{14}$$

where $\mathcal{G}_s \equiv \zeta_s/\zeta_\ell$ captures the leader-follower distance in government spending.

According to (14), increases (decreases) in I_s correspond to reductions (increases) in cross-country inequality in income per capita. Specifically, smaller values of $\mathcal{G}_s$ and κ_s, i.e., larger gaps in government spending and technical knowledge, leads to greater gaps in income per capita between countries, as reflected in I_s.

2.7 Factor prices

Factor markets are perfectly competitive. Substituting (12) into (6) and (7), we find the following expressions for the rental rate and wage of the leader economy:

$$r_\ell = \alpha \left(A_\ell \zeta_\ell^{1-\xi}\right)^{(1-\alpha)/[\alpha+(1-\alpha)\xi]} - \delta_\ell \tag{15}$$

$$w_\ell = (1-\alpha)\left(A_\ell \zeta_\ell^{1-\xi}\right)^{(1-\alpha)/[\alpha+(1-\alpha)\xi]} k_\ell. \tag{16}$$

Similarly, by substituting (13) into (6) and (7), it follows that country s's rental rate and wage can be written as:

$$r_s = \alpha \left(A_s \zeta_s^{1-\xi}\right)^{(1-\alpha)/[\alpha+(1-\alpha)\xi]} \kappa_s^{-\xi\psi(1-\alpha)/[\alpha+(1-\alpha)\xi]} - \delta_s \tag{17}$$

$$w_s = (1-\alpha)\left(A_s \zeta_s^{1-\xi}\right)^{(1-\alpha)/[\alpha+(1-\alpha)\xi]} \kappa_s^{-\xi\psi(1-\alpha)/[\alpha+(1-\alpha)\xi]} k_s. \tag{18}$$

From (15) and (17), it is easy to see that while the equilibrium interest rate of the leader economy remains constant over time, that of a typical follower economy decreases with κ_s. This implies that the leader country exhibits the same dynamic properties as AK growth models, whereas the typical follower country follows the dynamics of neoclassical growth models, provided that $\psi > 0$.

3 General equilibrium

3.1 Characterization of the equilibrium

At each moment of time, the absence of capital mobility across countries implies that the value of the financial assets per inhabitant, A_s, must equal the value of the capital stock per worker, k_s. Consequently, the perfect-foresight equilibrium of the model can be characterized as follows:

Definition 1. *A dynamic equilibrium for the entire world economy can be defined as a set of time paths for consumption per capita $\{c_\ell \cup \{c_s\}_{s\in\mathcal{S}}\}_{t\in[0,\infty)}$ and capital per worker $\{k_\ell \cup \{k_s\}_{s\in\mathcal{S}}\}_{t\in[0,\infty)}$ that: (i) satisfies equations (2)-(3); (ii) does not violate the balanced-budget rule of governments (11); (iii) fulfills the inequality constraints $c_\ell \geq 0$, $\{c_s \geq 0\}_{s\in\mathcal{S}}$, $k_\ell \geq 0$, $\{k_s \geq 0\}_{s\in\mathcal{S}}$; (iv) satisfies the transversality condition (4).*

To study how each economy evolves over time, we proceed in two steps. First, we determine the dynamic equations governing the time evolution of the leader economy. To do this, we substitute from (11), (15) and (16) into (2) and (3) to obtain:

$$\frac{\dot{k}_\ell}{k_\ell} = (1-\zeta_\ell)\left(A_\ell \zeta_\ell^{1-\xi}\right)^{(1-\alpha)/[\alpha+(1-\alpha)\xi]} - \frac{c_\ell}{k_\ell} - \delta_\ell - \nu_\ell \tag{19}$$

$$\frac{\dot{c}_\ell}{c_\ell} = \alpha\left(A_\ell \zeta_\ell^{1-\xi}\right)^{(1-\alpha)/[\alpha+(1-\alpha)\xi]} - \rho_\ell - \delta_\ell \tag{20}$$

Next, we determine the dynamic equations governing the time paths of each follower economy $s \in \mathcal{S}$ by substituting from (11), (17) and (18) into (2) and (3). This gives:

$$\frac{\dot{k}_s}{k_s} = (1-\zeta_s)\left(A_s \zeta_s^{1-\xi} \kappa_s^{-\xi\psi}\right)^{(1-\alpha)/[\alpha+(1-\alpha)\xi]} - \frac{c_s}{k_s} - \delta_s - \nu_s \tag{21}$$

$$\frac{\dot{c}_s}{c_s} = \alpha\left(A_s \zeta_s^{1-\xi} \kappa_s^{-\xi\psi}\right)^{(1-\alpha)/[\alpha+(1-\alpha)\xi]} - \rho_s - \delta_s. \tag{22}$$

Overall, system (19)-(22) consists of $2m+2$ differential equations in $2m+2$ unknowns: $m+1$ capital stocks, $k_\ell, k_{f_1}, \ldots, k_{f_m}$, which act as predetermined/state variables; and $m+1$ per capita consumption expenditures, $c_\ell, c_{f_1}, \ldots, c_{f_m}$, which act as non-predetermined/control variables. These equations, alongside with transversality conditions:

$$\lim_{t\to\infty} e^{-\int_0^t \left\{\alpha\left(A_\ell \zeta_\ell^{1-\xi}\right)^{(1-\alpha)/[\alpha+(1-\alpha)\xi]} - \delta_\ell - \nu_\ell\right\} \mathrm{d}z} \frac{k_\ell(t)}{c_\ell(t)} = 0 \tag{23}$$

$$\lim_{t\to\infty} e^{-\int_0^t \left\{\alpha\left[A_s \zeta_s^{1-\xi} \kappa_s(z)^{-\xi\psi}\right]^{(1-\alpha)/[\alpha+(1-\alpha)\xi]} - \delta_s - \nu_s\right\} \mathrm{d}z} \frac{k_s(t)}{c_s(t)} = 0, \tag{24}$$

complete the characterization of the reduced-form of the model.

To solve the model, we focus on the following re-scaled variables: the consumption-to-capital ratio, denoted by $x_s \equiv c_s/k_s$, and the relative capital intensity (knowledge gap), denoted by $\kappa_s \equiv k_s/k_\ell$. By log-differentiating x_ℓ with respect to t and substituting from (20), we obtain:

$$\frac{\dot{x}_\ell}{x_\ell} = x_\ell - (1-\alpha-\zeta_\ell)\left(A_\ell \zeta_\ell^{1-\xi}\right)^{(1-\alpha)/[\alpha+(1-\alpha)\xi]} - (\rho_\ell - \nu_\ell)\,, \tag{25}$$

Similarly, for each country $s \in \mathcal{S}_c$, log-differentiating x_s and κ_s with respect to t, and substituting (22), (19), and (21) into the resulting expressions, yields:

$$\frac{\dot{x}_s}{x_s} = x_s - (1-\alpha-\zeta_s)\left(A_s \zeta_s^{1-\xi}\kappa_s^{-\xi\psi}\right)^{(1-\alpha)/[\alpha+(1-\alpha)\xi]} - (\rho_s - \nu_s) \tag{26}$$

$$\begin{aligned}\frac{\dot{\kappa}_s}{\kappa_s} &= x_\ell - x_s - (1-\zeta_\ell)\left(A_\ell \zeta_\ell^{1-\xi}\right)^{(1-\alpha)/[\alpha+(1-\alpha)\xi]} + \\ &\quad + (1-\zeta_s)\left(A_s \zeta_s^{1-\xi}\kappa_s^{-\xi\psi}\right)^{(1-\alpha)/[\alpha+(1-\alpha)\xi]} - (\delta_s - \delta_\ell) - (\nu_s - \nu_\ell)\,.\end{aligned} \tag{27}$$

Dynamic system (25)-(27), along with the initial condition $\kappa_s(0)$, describes the equilibrium dynamics of the entire world economy. However, since the dynamics of each follower economy strongly depend on the size of its knowledge gap, the next section will examine the equilibrium paths of each type of follower separately.

3.2 Determination of the equilibrium growth paths

At all moments of time, the world economy consists of one leader country and a set of m follower countries, each of which is characterized by a specific level of backwardness with respect to the leader. Let $\mathcal{S}_d \equiv \{s \in \mathcal{S} : \kappa_s < \tilde{\kappa}\}$ and $\mathcal{S}_c \equiv \{s \in \mathcal{S} : \tilde{\kappa} \leq \kappa_s\}$ be a partition of $\mathcal{S}$. The set $\mathcal{S}_d$ can be interpreted as the set of *capital-poor* follower countries, whose degree of backwardness prevents them from benefiting from knowledge spillovers from the leader, while $\mathcal{S}_c$ represents the set of *capital-abundant* follower countries that do enjoy knowledge spillovers from the leader.

For the leader country, the dynamic properties of its economy are the same as those of a Barro (1990) AK economy and can be summarized as follows.

Proposition 1. *If $\zeta_\ell < 1-\alpha$, there exists a unique steady-state growth path along which: (i) capital and output per capita grow at the same constant rate:*

$$g_\ell = \alpha\left(A_\ell \zeta_\ell^{1-\xi}\right)^{(1-\alpha)/[\alpha+(1-\alpha)\xi]} - \rho_\ell - \delta_\ell;$$

(ii) The consumption-to-capital ratio, x_ℓ, remains constant over time and is given by:

$$\hat{x}_\ell = \rho_\ell - \nu_\ell + (1-\alpha-\zeta_\ell)\left(A_\ell \zeta_\ell^{1-\xi}\right)^{(1-\alpha)/[\alpha+(1-\alpha)\xi]};$$

(iii) The economy exhibits no transitional dynamics.

Proof. See Appendix A ■

Similarly to the leader country, for each *capital-poor* follower $s \in \mathcal{S}_d$ the relative stock of knowledge capital is too low to induce its economy to absorb foreign-produced new technology. Consequently, its dynamic behavior follows the paths described by (26) and (27) when $\psi = 0$, and can be summarized by the following proposition.

Proposition 2. *If $\zeta_s < 1 - \alpha$ holds for all $s \in \mathcal{S}_d$, there exists a unique steady-state growth path for each capital-poor follower along which: (i) the long-run growth rate in income per capita may differ from that of the leader economy and is given by:*

$$\hat{g}_s = \alpha \left(A_s \zeta_s^{1-\xi} \right)^{(1-\alpha)/[\alpha+(1-\alpha)\xi]} - \rho_s - \delta_s;$$

(ii) The consumption-to-capital ratio, $\hat{x}_s$, is constant over time and equal to:

$$\hat{x}_s = \rho_s - \nu_s + (1 - \alpha - \zeta_s) \left(A_s \zeta_s^{1-\xi} \right)^{(1-\alpha)/[\alpha+(1-\alpha)\xi]};$$

(iii) The knowledge gap with respect to the leader, κ_s, can either increase or decrease over time depending on whether $\hat{g}_s$ is either lower or larger than $\hat{g}_\ell$; (iv) The economy admits no adjustment dynamics.

Proof. See Appendix B ■

From Propositions 1 and 2, we have that the growth rates of the leader and the *capital-poor* countries are determined by their own productive public spendings. Moreover, regardless of their initial capital stocks, $k_\ell(0)$ and $k_s(0)$, these economies undergo no gradual adjustment process in response to real shocks, but instead transitions instantaneously (i.e. jumps) to the new growth path determined by the updated parameters.

For *capital-abundant* follower countries $s \in \mathcal{S}_c$, the situation differs as their productivity index is closely linked to that of the leader. Specifically, every advancement in technical knowledge by the frontier economy spills over to these countries as knowledge externalities, temporarily accelerating their growth. Consequently, the equilibrium dynamics of income per capita for each *capital-abundant* follower depend critically on the leader's trajectory and are fully described by the two-dimensional system (26)-(27) when $\psi > 0$. This leads to the results summarized in the following proposition.

Proposition 3. *If $\zeta_s < 1-\alpha$ holds for all $s \in \mathcal{S}_c$, there exists a unique steady-state growth path for each capital-abundant follower characterized by the following properties: (i) Each capital-abundant follower economy $s \in S_c$ grows at same rate as the leader:*

$$g_\ell = \alpha \left(A_\ell \zeta_\ell^{1-\xi} \right)^{(1-\alpha)/[\alpha+(1-\alpha)\xi]} - \rho_\ell - \delta_\ell;$$

(ii) The consumption-to-capital ratio of each capital-abundant economy s, x_s, is constant over time and given by:

$$\hat{x}_s = \rho_s - \nu_s + (1-\alpha-\zeta_s)\left[\left(A_\ell \zeta_\ell^{1-\xi} \right)^{(1-\alpha)/[\alpha+(1-\alpha)\xi]} + \frac{(\rho_s+\delta_s-\nu_s)-(\rho_\ell+\delta_\ell-\nu_s)}{\alpha} \right];$$

(iii) The knowledge gap of country s relative to the leader, κ_s, stabilizes at:

$$\kappa_s = \left[\left(\frac{A_\ell G_\ell^{1-\xi}}{A_s} \right)^{(1-\alpha)/[\alpha+(1-\alpha)\xi]} + \frac{(\rho_s+\delta_s-\nu_s)-(\rho_\ell+\delta_\ell-\nu_s)}{\alpha \left(A_s \zeta_s^{1-\xi} \right)^{(1-\alpha)/[\alpha+(1-\alpha)\xi]}} \right]^{-[\alpha+(1-\alpha)\xi]/[\xi\psi(1-\alpha)]};$$

(iv) The steady-state growth path is (asymptotically) saddle-path stable.

Proof. See Appendix C ■

From Propositions 2 and 3, we can conclude that the growth trajectory of a typical follower economy s may converge or diverge from that of the leader economy depending on its initial condition $\kappa_s(0)$. Specifically, if at $t=0$ the follower economy has $\kappa_s(0) > \tilde{\kappa}$, its production system can absorb foreign-produced knowledge, enabling it to initially grow faster than the leader economy and eventually converge to the leader's growth rate in the long run. On the other hand, if the follower begins with $\kappa_s(0) \leq \tilde{\kappa}$, it will be unable to absorb knowledge from the leader, and its growth rate will deviate from that of the leader, potentially leading its economy onto a diverging path.

4 Equilibrium Inequality and convergence

In equilibrium, the dynamics of the inequality index, I_s, vary across countries depending on whether the economy is *capital-abundant* or *capital-poor*.

Consider first a *capital-abundant* economy $s \in \mathcal{S}_s$, which begins with a technology gap relative to the leader, $\kappa_s(0) > \hat{\kappa}_s$, and with an initial level of per capita income inequality equal to:

$$\hat{I}_s(0) = \left(\frac{A_s}{A_\ell} \right)^{(1-\alpha)/[\alpha+(1-\alpha)\xi]} \mathcal{G}_s^{(1-\xi)(1-\alpha)/[\alpha+(1-\alpha)\xi]} \kappa_s(0)^{1-\xi\psi(1-\alpha)/[\alpha+(1-\alpha)\xi]}. \quad (28)$$

According to Proposition 3, such an economy benefits from knowledge spillovers and experiences, all else equal, a temporary period of faster growth than the leader. During this transitory phase, the inequality index between economy s and the leader increases over time, eventually stabilizing at a new equilibrium level given by:

$$\hat{I}_s = \left(\frac{A_s}{A_\ell}\right)^{(1-\alpha)/[\alpha+(1-\alpha)\xi]} \mathcal{G}_s^{(1-\xi)(1-\alpha)/[\alpha+(1-\alpha)\xi]} \hat{\kappa}_s^{1-\xi\psi(1-\alpha)/[\alpha+(1-\alpha)\xi]}. \tag{29}$$

As a result, when the follower country is *capital-abundant*, international knowledge transmission can effectively reduce per capita income inequality across countries.

Consider now a *capital-poor* economy $s \in \mathcal{S}_d$, characterized by both an initial technology gap $\kappa_s(0) \leq \hat{\kappa}_s$ and an initial level of income inequality still given by equation (28). Based on Proposition 2, such an economy fails to absorb knowledge from the leader, resulting in a long-run equilibrium trajectory of income per capita that can potentially diverge from that of the leader depending on the sign of the growth differential $\hat{g}_s - \hat{g}_\ell$ (see item (iii) of Proposition 2). Specifically, the model predicts that the inequality index I_s will show a decreasing trend if the leader economy's growth rate exceeds that of the *capital-poor* economy (leading to increased inequality), or an increasing trend in the opposite case (leading to decreased inequality).

In the latter scenario, where the *capital-poor* country intensively accumulates knowledge capital on its own, the relative capital stock κ_s will increase over time, indicating that the economy is narrowing its technology gap with the leader. If this phase of sustained growth continues, κ_s will eventually reach the threshold value $\tilde{\kappa}$. At this point, country s will shed its status of *capital-poor* economy and enter the leader country's basin of attraction, joining other *capital-abundant* economies. Conversely, if this growth phase does not persist long enough to facilitate the transition from *capital-poor* to *capital-abundant*, κ_s will begin to decline and possibly approach zero if the growth differential between the economy s and the leader ℓ persists over time.

All these considerations lead us to the natural questions: for a diverging *capital-poor* country, how can the government help the economy enter the converging basin toward the leader? Is it possible for the government to design a policy package capable of transferring the country from the set of *capital-poor* economies to that of *capital-abundant* economies?

In line with Barro (1990), increasing productive public spending can lead to faster growth. However, our model introduces an additional mechanism: the propelling effect induced by an increase in ζ_s also enhances the absorbing capacity of the production sector. Indeed, if we interpret physical capital K as a shorthand for "total capital", encompassing both physical and human capital, then the core result of Barro (1990) (i.e., that higher public expenditure is beneficial for capital accumulation and growth) can be extended to include the implementation capability of foreign-produced technology. In other words, for low-income emerging-market economies, productive public spending can serve as an effective policy tool to facilitate catch-up with wealthier countries.

In this sense, thus, the question is to understand to what extent the predictions of our multi-country growth model, particularly those concerning the result that government's productive expenditure is pivotal for technology acquisition, can explain the reduction in cross-country income inequality that has characterized the last forty/forty-five years. To do this, in the next section, we will take the model to data and test whether public expenditure has effectively played a role in bridging income gaps across countries.

5 Empirical Analysis

In this section, we empirically assess the theoretical model. The model's key predictions are as follows: (i) a sustained increase in the share of productive public spending relative to GDP raises the steady-state growth rate of GDP per capita in *capital-poor* economies but has no significant effect in *capital-abundant* economies; (ii) overseas knowledge spillovers contribute to the convergence of developing countries only if they are *capital-abundant* economies; and (iii) the magnitude of cross-country learning-by-investing externalities decreases with economic development, meaning their long-term contribution to growth is inversely related to the knowledge gap.

Section 5.1 describes the data sources and provides descriptive statistics for the key variables. Section 5.2 examines their time series properties. Section 5.3 outlines the econometric model, and Section 5.4 discusses the results.

5.1 The Data and Descriptive Statistics

The data for this study are sourced from the following:

1. The *IMF Investment and Capital Stock Dataset* (2021), which provides information on GDP, public, private, and public-private partnership (PPP) investments, as well as the aggregate capital stock. All values are expressed in constant 2017 international dollars.

2. The *United Nations' World Population Prospects* (2024), which offers population size estimates and is used to express macroeconomic variables in per capita terms.

Our analysis focuses on countries with a population of at least one million in 2019, excluding those where oil extraction constitutes a significant share of GDP. This exclusion is necessary because, in these economies, a substantial fraction of GDP reflects the extraction of existing resources rather than value-added production.[6]

To proxy productive public spending, we use the public investment-to-GDP ratio, which captures the share of national output allocated to government-funded investments. This variable is constructed using data from the IMF dataset, which provides information on public, private, and Public-Private Partnership (PPP) investments. However, PPP investments are relatively small compared to public and private investments. Furthermore, in most countries, the dataset records a nonzero PPP investment value only in the final years of our sample. Given these considerations, we sum half of PPP investments into the IMF's public investment figure.

The resulting dataset forms a balanced panel covering 110 countries over the period from 1970 to 2019. Tables 6 and 7 in Appendix D list all the countries in the sample and provide summary statistics for the main variables used in the empirical specification, respectively. Notably, only 27 countries, including the United States (the country leader in our analysis), are classified as advanced economies as of 2019, according to the IMF World Economic Outlook. This emphasizes that the panel primarily consists of low-income and emerging economies.

Our primary variable of interest is the relative physical capital stock per capita, defined as the ratio of a follower country's capital stock per capita to that of the United States. As reported in Figure 1, while physical capital has increased in most countries in our sample over the period 1970 to 2019, cross-country differences in capital stocks persist (see Figure 2).

[6]The excluded countries are Bahrain, Gabon, Iran, Iraq, Kuwait, Oman, Saudi Arabia, and the United Arab Emirates.

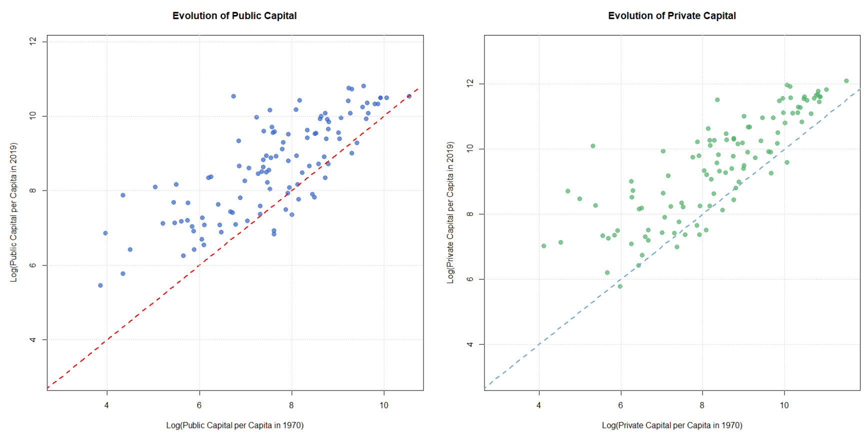

Figure 1: Changes in Public and Private Capital Over 50 Years (1970–2019)

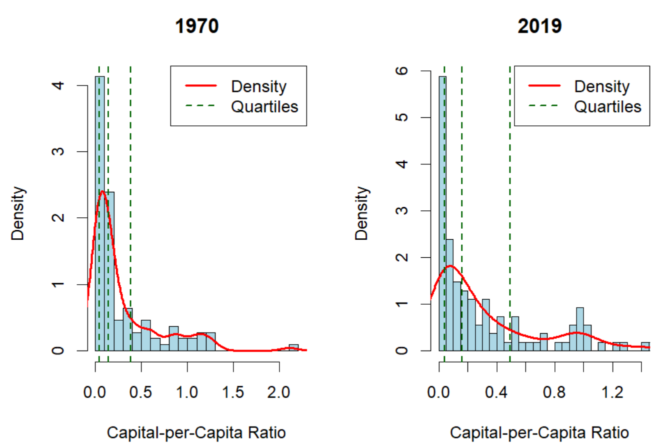

Figure 2: Evolution of capital differentials over time

In 1970, the sampled countries exhibited relative capital levels averaging only 0.3 of the U.S.. This figure markedly exceeds the median value of 0.14, emphasizing the heavily right-skewed nature of the distribution. The dispersion is substantial, with a standard deviation of 0.39 and values ranging from as low as 0.003 to as high as 2.11. Notably, 75% of the sampled countries had relative capital levels below 38% of the U.S., while the lowest quartile fell below just 4%. By 2019, some improvements in the distribution were evident, particularly in the upper-middle segment. The 75th percentile increased from 0.38 to 0.49 of U.S. levels, reflecting significant relative capital accumulation among these countries. However, disparities persist, as the lowest quartile showes no improvement. The top decile remains the only segment nearing or exceeding U.S. levels.

The trends shown in figure 2 underscore little improvements in the levels of capital per worker across the distribution, with the most pronounced gains observed in the upper-middle tier of countries. Similarly, when examining the evolution of relative income per capita - defined as the ratio of a follower country's income per capita to that of the leader - we observe a persistent right-skewed distribution in both 1970 and 2019, as shown in Figure 3. However, some improvements occurred over time, particularly in the median and 75th percentile, which increase from 0.14 and 0.36 in 1970 to 0.17 and 0.47 in 2019, respectively.

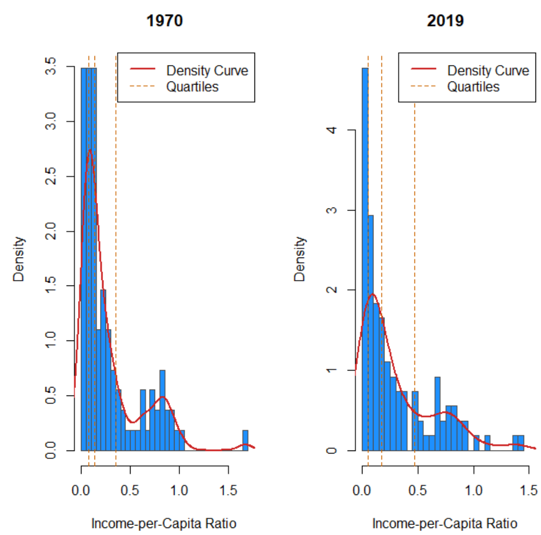

Figure 3: Evolution of income differentials over time

In summary, despite improvements in capital accumulation and income distribution over time, significant disparities persist, especially among lower-income countries. With these descriptive insights in mind, we now turn to examining the time-series properties of our key variables.

5.2 Time Series Properties

We now provide a discussion of the time-series properties of our main variables of interest. These include the log of GDP per capita for country i at time t, denoted by y_{it}, where $i = 1, \dots, N$; the log-ratio of the follower's capital stock per capita to that of the leader, denoted by κ_{it}; and the log-ratio of public investments to GDP, represented by ζ_{it}, which serves as our chosen statistic to proxy productive public expenditures.

Before conducting unit root tests, we first assess whether our variables exhibit cross-sectional dependence. As is well established, first-generation panel unit root tests suffer from size distortions and low power when cross-sectional dependence is present (Strauss and Yigit, 2003). In panel data models, such dependence may arise from unobserved common factors that influence all cross-sectional units to varying degrees. To investigate this, we estimate the number of common factors in our variables using the eigenvalue ratio (ER) and growth ratio (GR) estimators proposed by Ahn and Horenstein (2013), along with the estimator proposed by Gagliardini *et al.* (2019) (GOS).[7]

Table 1: Estimated Number of Common Factors

	Levels			First Differences		
Variable	ER	GR	GOS	ER	GR	GOS
y_{it}	1	2	1	0	2	0
ζ_{it}	1	1	0	0	0	2
κ_{it}	3	3	2	2	2	2

Notes: The variance of each cross-section is standardized to 1 as required by the GOS estimator, and individual fixed effects are removed.

Table 1 presents the estimated number of common factors. For κ_{it}, for instance, the criteria by Ahn and Horenstein (2013) suggest the presence of three common factors, whereas the estimator by Gagliardini *et al.* (2019) indicates two. Overall, our findings suggest that all variables are influenced by at least one common factor.

After identifying a common factor structure in the observed variables, we test for weak cross-sectional dependence using Pesaran's (2015) Cross-Sectional Dependence (CD) statistic. This test evaluates the null hypothesis of weak cross-sectional dependence against the alternative of strong cross-sectional dependence.[8]

[7]For a comprehensive discussion on estimators for the number of common factors in panel data models, see Ditzen and Reese (2023).

[8]For a detailed discussion on weak and strong cross-sectional dependence, see Chudik *et al.* (2011).

Table 2: CD Test for Cross-Sectional Dependence

Variable	Levels (CD Statistic)	First Differences (CD Statistic)
y_{it}	245.19***	40.82***
ζ_{it}	13.35***	14.86***
κ_{it}	25.19***	100.30***

Notes: $^{***}p < 0.01$. All variables are expressed in logs.

H_0: weak cross-sectional dependence vs. H_1: strong cross-sectional dependence.

Table 2 reports the results of the CD test on y_{it}, ζ_{it} and κ_{it}. According to the table, the null hypothesis of weak cross-sectional dependence is rejected at 1% significance level, implying that there is evidence for strong cross-sectional dependence for all variables. Accordingly, it becomes necessary to employ a second-generation panel unit root test that accounts for cross-sectional dependence. To this end, we perform the Cross-sectionally augmented Im-Pesaran-Shin (CIPS) test, to check whether our time series variables contains unit roots (Pesaran, 2007).

Table 3: CIPS Test Results

	Levels		First Differences	
Variable	Constant	Constant & Trend	None	Constant
y_{it}	-1.82	-2.90^{***}	-3.72^{***}	-4.18^{***}
ζ_{it}	-2.09^{**}	-3.07^{***}	-5.82^{***}	-5.88^{***}
κ_{it}	-0.62	-1.91	-1.61^{**}	-2.40^{***}

Notes: ***, **, * denote significance at 1%, 5%, and 10% respectively.

All variables are expressed in logs.

H_0: Unit root for all cross-sectional units vs. H_1: Stationarity for at least some cross-sectional units.

Table 3 reports the results of the CIPS test. Under the null hypothesis that all cross-sectional units follow a unit root process, our results indicate that y_{it} and ζ_{it} are stationary when a constant and a deterministic trend are included in the specification, whereas κ_{it} is integrated of order one, i.e., I(1).

Thus, our time-series variables are, at most, integrated of order one. To properly account for the different orders of integration among them, we rely on the Auto-Regressive Distributed Lag (ARDL) methodology. Furthermore, to address the presence of strong cross-sectional dependence, we augment our ARDL specification with cross-sectional averages of the variables, as detailed in the next subsection.

5.3 Econometric Framework

To empirically validate our theoretical framework, we model the long-run equilibrium growth rate of GDP per capita as follows:

$$\Delta y_{it} = \eta_i + \theta_{i1}\zeta_{it} + \theta_{i2}\kappa_{it} + \gamma_i' f_t + u_{it}, \tag{30}$$

where $i = 1, \ldots, 109$ represents countries and $t = 1971, \ldots, 2019$ denotes time. In this specification, η_i captures country-specific fixed effects, while θ_{i1} and θ_{i2} represent the long-run coefficients. The term u_{it} denotes the error term, and slope heterogeneity is explicitly allowed. The vector f_t represents unobserved common factors with country-specific factor loadings, γ_i. These unobserved factors may potentially influence both the unobserved determinants of the dependent variable and the observed regressors, ζ_{it} and κ_{it}. As a result, the ordinary least squares estimates of the long-run coefficients may be biased and inconsistent (see Eberhardt and Teal (2011)).

To estimate the long-run effects of a permanent increase in the share of public investments in GDP on the steady-state GDP per capita growth rate path, as well as the role of technology diffusion, proxied in this context by κ_{it}, we employ an ARDL specification of Eq. (30). As highlighted by Pesaran and Smith (1995), the Mean Group (MG) estimates of the long-run coefficients are consistent as long as the errors are cross-sectionally independent. Furthermore, as noted by Pesaran and Smith (1995) and Pesaran *et al.* (1995), the ARDL approach is robust to endogeneity among regressors and remains valid whether the time series variables are $I(1)$ or $I(0)$.

In this context, we recognize that cross-sectional dependence is a critical issue. Therefore, we begin by providing evidence of strong cross-sectional dependence through the CD Statistic (Pesaran (2015)), calculated from the average pair-wise correlation of residuals derived from the Mean Group (MG) estimates of the long-run coefficients in Eq. (30).

To address strong cross-sectional dependence, Chudik and Pesaran (2015) suggest applying the Pesaran (2006) Common Correlated Effects approach in panel ARDL models.

Following this, we augment our ARDL specification with cross-sectional averages of the dependent and independent variables, adopting the Cross-Sectionally augmented ARDL (CS-ARDL) methodology (Chudik *et al.* (2016)). The empirical model estimated using the CS-ARDL (p_y, p_w) approach is:

$$\Delta y_{it} = \mu_i + \sum_{\ell=1}^{p_y} \lambda_{i,\ell} \Delta y_{i,t-\ell} + \sum_{\ell=0}^{p_w} \beta'_{i,\ell} w_{i,t-\ell} + \sum_{\ell=0}^{p_T} \varphi'_{i,\ell} \bar{\mathbf{z}}_{t-\ell} + \varepsilon_{it}, \tag{31}$$

where $w_{it} = (\zeta_{it}, \kappa_{it})'$, and $\bar{\mathbf{z}}_t = (\Delta\bar{y}_t, \bar{\zeta}_t, \bar{\kappa}_t)'$, with $\bar{y}_t = \frac{1}{N}\sum_{i=1}^{N} y_{it}$, $\bar{\zeta}_t = \frac{1}{N}\sum_{i=1}^{N} \zeta_{it}$, and $\bar{\kappa}_t = \frac{1}{N}\sum_{i=1}^{N} \kappa_{it}$ denoting the cross-sectional averages of the dependent and independent variables. As suggested by Chudik and Pesaran (2015), the CS-ARDL estimator gains consistency if the floor of $p_T = \lfloor T^{1/3} \rfloor$ lags of the cross-section averages is added for both the dependent and independent variables. It turns out that $p_T = 3$ is sufficient to control for cross-sectional dependence. For the lag structure p_y and p_w, based on the Akaike Information Criterion (AIC)[9], we select $p_y = p_w = 2$. The long-run coefficients are estimated using the MG estimator.

To assess the robustness of the long-run coefficients obtained from equation (31), we estimate the coefficients of equation (30) directly, without relying on short-run coefficients. This method, known as the cross-sectionally augmented distributed lag (CS-DL) approach, was proposed by Chudik *et al.* (2016). The DL approach is advantageous due to its robustness against serial correlation, structural breaks, and dynamic mispecifications. However, it does not account for feedback effects from the dependent variable onto the regressors. Nonetheless, Chudik *et al.* (2016) have shown that the DL approach often outperforms the ARDL approach when the time dimension is not too large. Consequently, the CS-DL empirical specification reads:

$$\Delta y_{it} = \mu_i + \theta'_i w_{it} + \sum_{\ell=0}^{p_w-1} \beta'_{i,\ell} \Delta w_{i,t-\ell} + \varphi_{y,i} \Delta\bar{y}_t + \sum_{\ell=0}^{p_T} \varphi'_{w,i,\ell} \bar{w}_{t-\ell} + e_{it}, \tag{32}$$

where $\bar{y}_t$ and $\bar{w}_{t-\ell}$ represent the cross-sectional averages. In our analysis, we set $p_w = p_T = 3$.

Our final objective is to identify whether a threshold exists for κ beyond which its effect on GDP per capita growth changes. To achieve this, we employ a test for threshold

[9] The literature does not provide a general method for determining the optimal lag length structure of an ARDL model in a panel setting. Our approach is to first determine the optimal lag length for each country separately using the AIC criterion, with a maximum common lag length of 3 for each variable. We then compute the averages of the optimal lag lengths found for each country, obtaining an estimated average of 1.28 for p_y, 0.39 for ζ, and 0.44 for κ. Since we set $p_y = p_w$, we assign a value of 2 to both.

effects within the framework of large dynamic heterogeneous panel data models with cross-sectionally dependent errors, as proposed by Chudik *et al.* (2017). Accordingly, the new CS-ARDL specification is expressed as follows:

$$\Delta y_{it} = \mu_i + \pi I(\kappa_{it} > \tilde{\kappa}) + \sum_{\ell=1}^{p_y} \lambda_{i,\ell} \Delta y_{i,t-\ell} + \sum_{\ell=0}^{p_w} \beta'_{i,\ell} \Delta w_{i,t-\ell} + \sum_{\ell=0}^{p_T} \varphi'_{i,\ell} \bar{\mathbf{h}}_{t-\ell} + \chi_i I(\bar{\kappa}_t > \tilde{\kappa}) + \varepsilon_{it}, \tag{33}$$

where $\bar{h}_t = (\Delta\bar{y}_t, \Delta\bar{w}_t)'$, and $I(A)$ is an indicator variable that equals 1 if event A occurs, and 0 otherwise. Chudik *et al.* (2017) recommend using the same lag structure for the dependent and independent variables, as well as for the cross-sectional averages. To fully account for cross-sectional dependence and short-run dynamics, we set $p_y = p_w = p_T = 3$. Subsequently, by defining a set of admissible values for the threshold $\tilde{\kappa}$, a filtered pooled estimator for π is obtained for each admissible value of $\tilde{\kappa}$. The threshold $\tilde{\kappa}$ is then estimated using a grid search method (Chudik *et al.*, 2017; Anderson and Raissi, 2018).

5.4 Regression Results

Table 4 presents the MG estimates for the long-run coefficients from Eq. (31)[10]. When interpreting the long-run estimates of relative capital, it is essential to note that we defined κ_s as the ratio of the capital per capita of the follower country to that of the leader. Consequently, a decrease in κ_s may indicate an increase in the knowledge gap between the leader and the follower.

Based on our model, in *capital-poor* economies - those that do not catch up with the leader - the knowledge gap has no positive (or marginal) effect on growth, whereas national productive spending does. Econometrically, this implies that the coefficient of κ_s should be either insignificant or significant, but negligible in size, while the coefficient of ζ_s should be significantly positive. In contrast, in *capital-abundant* economies, both the knowledge gap and productive spending may influence growth. However, it is also possible that only the knowledge gap matters, as private innovation may be substitute for productive spending in advanced countries.

Consequently, according to our model, follower countries that converge to the threshold $\tilde{\kappa}$ can benefit from knowledge spillovers originating from the leader, thereby experiencing faster growth in GDP per capita. This effect should be reflected in a negative

[10] In all regressions, the parameter measuring the speed of adjustment towards the long-run cointegration equilibrium is both negative and statistically significant, indicating the presence of a stable relationship (stochastic trend) governing the dynamics of the variables.

and statistically significant coefficient for the variable κ_s. For countries with capital levels below $\tilde{\kappa}$, limited effects from knowledge spillovers may occur but are expected to remain modest.

Table 4: Mean Group estimates of long-run effects (ARDL, CS-ARDL and CS-DL)

	ARDL	CS-ARDL			CS-DL		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Public Inv/GDP	0.033***	0.052**	0.071**	−0.039**	0.026***	0.031***	−0.036*
	(0.010)	(0.023)	(0.035)	(0.019)	(0.008)	(0.008)	(0.020)
Relative Capital	−0.064***	−0.031*	−0.050***	-0.038	−0.031**	−0.052***	-0.005
	(0.010)	(0.018)	(0.019)	(0.036)	(0.013)	(0.014)	(0.053)
CD	33.71	1.85	0.06	-0.38	0.35	-0.79	-0.41
Obs	5,123	5,014	4,370	644	5,123	4,465	658
Countries	109	109	95	14	109	95	14

Notes: ***$p < 0.01$, **$p < 0.05$, *$p < 0.1$. CD is CD Statistic.

Sample: (1) ARDL entire sample; (2)-(4) CS-ARDL estimates: (2) entire sample, (3) countries with $\kappa < 1$, (4) countries with $\kappa > 1$; (5)-(7) CS-DL estimates: (5) entire sample, (6) countries with $\kappa < 1$, (7) countries with $\kappa > 1$.

Table 4 shows the results of the regressions. Columns (1) and (2) present the MG estimates of the long-run coefficients for the ARDL and CS-ARDL specifications, respectively, using the entire sample. As expected, the CD test statistics for the CS-ARDL specification indicates a substantial decline in the average pairwise correlation of residuals following the cross-sectional augmentation of the ARDL model. Specifically, results in column (2) suggest that a permanent increase in public investment is expected, on average, to generate a higher long-run growth rate. Conversely, the negative coefficient for relative capital is smaller in magnitude and indicates the existence of positive knowledge spillovers, albeit statistically significant only at the 10% level. To ensure the robustness of our findings, column (5) reports the long-run estimates for the CS-DL specification. The results confirm the positive contribution of public investment to growth, although the magnitude of the effect is reduced by half. Additionally, the coefficient for knowledge spillovers remains unchanged but is now statistically significant at the 5% level.

Overall, at the aggregate level, there is robust evidence supporting the positive effects of public investment on long-run growth across follower countries. However, evidence that all follower countries universally benefit from knowledge spillovers remains weak. This result can be interpreted by saying that the strength with which knowledge externalities tend to contribute to growth tends to fade away as long as the country's development level gets close to that of the leader.

To test this hypothesis, in columns (3) and (4), we re-estimate the model by distinguishing between those countries whose capital levels never exceeded that of the leader country (i.e., the U.S.) throughout the entire sample period (i.e. $\kappa_s < 1$ for all t), and those countries whose capital levels surpassed that of the leader capital level for one or more years (i.e. $\kappa_s > 1$ for some t)

Column (3) indicates that both productive public spending and cross-country knowledge spillovers are significant drivers of economic growth in countries where $\kappa_s < 1$. In particular, we find that a 1% increase in knowledge gap, due to a 1% reduction in κ_s, leads to an approximate 0.05 percentage point increase in long-run GDP per capita growth. Similarly, a 1% permanent increase in public investment as a share of GDP is expected to raise long-run growth by approximately 0.071 percentage points. These CS-ARDL results align with the CS-DL estimates from column (6), although the long-run effects of public investment are notably smaller, with the estimated impact more than halved.

On the contrary, results in column (4), referred to those economies whose relative capital stock is close to that of the U.S ($\kappa_s \approx 1$), deliver no evidence of spillover effects, as the MG estimate for relative capital is negative but not statistically significant. Notably, increasing public investment appears to slow long-run growth, with a negative MG estimate of -0.039, statistically significant at the 5% level. However, under the CS-DL specification provided in column (7), the point estimate remains largely unchanged but is now statistically significant at the 10% level. This latter result may be interpreted as an evidence for the existence of more than one leader, as they share the same knowledge stock and share very similar macroeconomic fundamentals and institutions. [11]

Finally, to further validate the model's results, we now test the existence of a development threshold that promotes cross-country knowledge spillovers ($\tilde{\kappa}$). According to our theoretical model, once this threshold is reached, follower countries should be able to benefit from knowledge spillovers from the leading country.

Table 5 presents the estimation results for the threshold relative capital, $\tilde{\kappa}$, along with the regression results of the CS-ARDL and CS-DL models for *capital-poor* economies.

[11] Indeed, according to our analysis, of the 14 countries with relative capital stocks close to that of the U.S., 12—namely Austria, Belgium, Denmark, Finland, France, Germany, Ireland, Italy, the Netherlands, Norway, Sweden, and Switzerland—are Western countries. The remaining two, Japan and Singapore, have institutions closely tied to those of the U.S.

Table 5: Threshold Test and Mean Group Estimates of Long-Run Effects

	Threshold Test	CS-ARDL		CS-DL	
	(1)	(2)	(3)	(4)	(5)
$\tilde{\kappa}$	0.20***				
SupF	5.21				
AveF	2.39				
Public Inv/GDP		0.035	0.043***	0.007	0.043***
		(0.033)	(0.01)	(0.024)	(0.008)
Relative Capital		−0.189**	−0.059**	−0.124***	−0.048**
		(0.075)	(0.023)	(0.047)	(0.02)
CD	0.39	-1.24	-1.30	-1.21	-2.19
Obs	5,014	690	2,576	705	2,632
Countries	109	15	56	15	56

Notes: ***$p < 0.01$, **$p < 0.05$, *$p < 0.1$. CD is CD Statistic.
SupF (*AveF*) refers to the largest (average) test statistic associated with the threshold coefficient, π. These test statistics test the null hypothesis that $\pi = 0$.
Sample: CS-ARDL estimates: (2) *capital-poor* converging, (3) *capital-poor* diverging; CS-DL estimates: (4) *capital-poor* converging, (5) *capital-poor* diverging.

Our estimations confirm the existence of threshold level for κ of 0.2 (i.e., $\tilde{\kappa} \approx 0.2$), with statistical significance at the 1% level, as shown in column (1) of Table 5. These findings are consistent with Parello and Venturini (2025), who estimate a threshold of 0.23[12] Consequently, in columns (2) and (3), we estimate the model separately for *capital-poor* economies that either transition to being *capital-abundant* or do not. We refer to the former as *capital-poor* converging economies - those that began the sample period (1970–1974) with an average κ_s below 0.2 and have since reached or exceeded this threshold, on average, in the last five years of the sample. In contrast, we refer to the latter as *capital-poor* diverging economies - those that started with the same initial conditions but have remained below the 0.2 threshold, on average, in the last five years of the sample.

Figure 4 in Appendix D provides a graphical representation of κ_s over the entire sample for diverging and converging capital-poor economies. Our classification of converging and diverging economies appears to be appropriate, as all identified converging coun-

[12] In fact, Parello and Venturini (2025) define the development threshold differently as $k_s = k_\ell / k_s$ and estimate a threshold level of $\tilde{\kappa} \approx 4.3$. In our model, this corresponds to a threshold of approximately 0.23.

tries exhibit an upward trend even after surpassing the threshold. In contrast, the diverging economies consistently remain below the threshold, with the exception of four countries: Afghanistan, Jordan, Nigeria, and Paraguay. These countries temporarily surpass the threshold but subsequently experience significant divergence.

The results presented in columns (2) and (3) indicate that a higher share of public investment in GDP fosters long-run economic growth in both converging and diverging *capital-poor* economies. However, in line with the predictions of our theoretical framework, the impact of productive public spending is statistically significant only for diverging *capital-poor* countries.

Regarding knowledge spillovers, we find positive effects for both converging and diverging *capital-poor* economies (see column (2)). Specifically, a 1% increase in the knowledge gap between countries, measured as a 1% reduction in κ_s, results in an increase in the long-run GDP per capita growth rate by approximately 0.189 percentage points for the converging economies and by approximately 0.059 percentage points for the diverging economies, with statistical significance at the 5% level. The CS-ARDL estimates in columns (2) and (3) closely align with the CS-DL estimates in columns (4) and (5), suggesting that cross-country spillovers tend to increase with a country's level of development. Specifically, the MG coefficient is estimated at -0.124 for converging economies and -0.048 for diverging economies, and it is statistically significant at the 1% and 5% levels, respectively.

These results, together with those reported in Table 4, confirm the model's prediction that knowledge spillovers play a limited role in driving growth at early stages of development. However, they also suggest the presence of an inverted U-shaped relationship between cross-country knowledge spillovers and growth. This implies that spillovers are weak when follower countries have relatively low capital compared to the leader economy, strengthen as these countries transition to *capital-abundant* economies, and eventually fade once they complete their catching-up process with the leader economy.

6 Conclusions

This paper develops a multi-country AK model of endogenous growth with international knowledge transmission to analyze the role of productive public expenditure in fostering economic growth and convergence. Our theoretical framework highlights the existence of a technological threshold, beyond which follower countries can effectively leverage knowledge spillovers from the global leader economy. Countries below this threshold face persistent growth limitations unless targeted public investments enhance their absorptive capacity.

Our empirical analysis, based on a dynamic heterogeneous panel model covering 110 countries over the period 1970–2019, provides robust evidence supporting these theoretical predictions. We find that productive public spending significantly enhances growth for economies far from the technological frontier but loses its effectiveness once countries surpass the threshold. Moreover, knowledge spillovers play a pivotal role in the growth process, but their impact is conditional on the relative capital stock of follower countries.

These findings underscore important policy implications. While productive public investment can accelerate capital accumulation and facilitate technological catch-up for lagging economies, its role in sustaining long-term growth diminishes as countries approach the frontier. Policymakers in developing economies should prioritize investments that build absorptive capacity, enabling their economies to transition from divergence to convergence. At the same time, mature economies may need to shift their focus from public-led investment strategies to fostering innovation and private-sector-driven growth.

Future research could build on our findings in several ways. First, extending the theoretical framework to endogenize the development threshold would provide deeper insights into the factors that determine a country's ability to absorb foreign knowledge and transition to higher growth paths. One possible approach is to introduce nonlinearity in the propagation mechanisms driven by the knowledge gap, as suggested by our empirical finding of an inverted U-shaped relationship between the magnitude of cross-country knowledge spillovers and the level of economic development. Second, investigating the heterogeneity in public investment effectiveness across different institutional contexts could help identify the specific conditions under which government spending most effectively fosters economic growth. Finally, incorporating alternative channels of knowledge transmission—such as innovation networks and patent flows—into the model would enable a more comprehensive analysis of how technological diffusion influences cross-country convergence dynamics.

References

Acemoglu, D., Aghion, P. and Zilibotti, F. (2006). 'Distance to frontier, selection, and economic growth', *Journal of the European Economic Association*, vol. 4(1), pp. 37–74, doi: 10.1162/jeea.2006.4.1.37.

Afonso, A. and Jalles, J.T. (2014). 'Fiscal composition and long-term growth', *Applied Economics*, vol. 46(3), pp. 349–358.

Agénor, P.R. (2008). 'Fiscal policy and endogenous growth with public infrastructure', *Oxford Economic Papers*, vol. 60, pp. 57–87.

Agénor, P.R. (2010). 'A theory of infrastructure-led development', *Journal of Economic Dynamics and Control*, vol. 34, pp. 932–950.

Agénor, P.R. (2012). *Public Capital, Growth and Welfare: Analytical Foundations for Public Policy*, Princeton and Oxford: Princeton University Press.

Ahn, S.C. and Horenstein, A.R. (2013). 'Eigenvalue ratio test for the number of factors', *Econometrica*, vol. 81(3), pp. 1203–1227.

Alvarez, F. (2017). 'Capital accumulation and international trade', *Journal of Monetary Economics*, vol. 91, pp. 1–18, doi:10.1016/j.jmoneco.2017.08.002.

Anderson, G. and Raissi, M.M. (2018). *Corporate indebtedness and low productivity growth of Italian firms*, International Monetary Fund.

Arrow, K. and Kurz, M. (1970). *Public investment, the rate of return and optimal fiscal policy*, Baltimore, MD: Johns Hopkins University.

Aísa, R. and Pueyo, F. (2006). 'Government health spending and growth in a model of endogenous longevity', *Economics Letters*, vol. 90, pp. 249–253.

Barro, R. (1990). 'Government spending in a simple model of endogenous growth', *Journal of Political Economy*, vol. 98(5), pp. S103–26.

Blanchard, O.J. and Kahn, C.M. (1980). 'The solution of linear difference models under rational expectations', *Econometrica*, vol. 48(5), pp. 1305–1311, doi:10.2307/1912186.

Blankenau, W.F. (2005). 'Public schooling, college subsidies and growth', *Journal of Economic Dynamics and Control*, vol. 29, pp. 487–507.

Blankenau, W.F. and Simpson, N.B. (2004). 'Public education expenditures and growth', *Journal of Development Economics*, vol. 73, pp. 583–605.

Blankenau, W.F., Simpson, N.B. and Tomljanovich, M. (2007). 'Public education expenditures, taxation, and growth: Linking data to theory', *The American Economic Review*, vol. 97, pp. 393–397.

Buera, F. and Oberfield, E. (2020). 'The global diffusion of ideas', *Econometrica*, vol. 88(1), pp. 83–114.

Chu, A.C., Peretto, P. and Wang, X. (2024). 'Government spending and industrialization in a schumpeterian economy', MPRA Paper 120797, University Library of Munich, Germany.

Chudik, A., Mohaddes, K., Pesaran, M.H. and Raissi, M. (2016). 'Long-run effects in large heterogeneous panel data models with cross-sectionally correlated errors', in (*Essays in Honor of Aman Ullah*pp. 85–135, Emerald Group Publishing Limited.

Chudik, A., Mohaddes, K., Pesaran, M.H. and Raissi, M. (2017). 'Is there a debt-threshold effect on output growth?', *Review of Economics and Statistics*, vol. 99(1), pp. 135–150.

Chudik, A. and Pesaran, M.H. (2015). 'Common correlated effects estimation of heterogeneous dynamic panel data models with weakly exogenous regressors', *Journal of econometrics*, vol. 188(2), pp. 393–420.

Chudik, A., Pesaran, M.H. and Tosetti, E. (2011). 'Weak and strong cross-section dependence and estimation of large panels', *The Econometrics Journal*, vol. 14(1), pp. C45–C90.

Comin, D. and Hobijn, B. (2010). 'An exploration of technology diffusion', *American Economic Review*, vol. 100(5), pp. 2031–2059, doi:10.1257/aer.100.5.2031.

Debarsy, N. and Ertur, C. (2019). 'Interaction matrix selection in spatial autoregressive models with an application to growth theory', *Regional Science and Urban Economics*, vol. 75, pp. 49–69, doi:10.1016/j.regsciurbeco.2019.01.005.

Devarajan, S., Swaroop, V. and Zou, H.f. (1996). 'The composition of public expenditure and economic growth', *Journal of monetary economics*, vol. 37(2), pp. 313–344.

Ditzen, J. and Reese, S. (2023). 'xtnumfac: A battery of estimators for the number of common factors in time series and panel-data models', *The Stata Journal*, vol. 23(2), pp. 438–454.

Eberhardt, M. and Teal, F. (2011). 'Econometrics for grumblers: a new look at the literature on cross-country growth empirics', *Journal of economic Surveys*, vol. 25(1), pp. 109–155.

Eckstein, Z. and Zilcha, I. (1994). 'The effects of compulsory schooling on growth, income distribution and welfare', *Journal of Public Economics*, vol. 53, pp. 339–359.

Ertur, C. and Koch, W. (2007). 'Growth, technological interdependence and spatial externalities: Theory and evidence', *Journal of Applied Econometrics*, vol. 22(6), pp. 1033–1062.

Ertur, C. and Koch, W. (2011). 'A contribution to the theory and empirics of schumpeterian growth with worldwide interactions', *Journal of Economic Growth*, vol. 16(3), pp. 215–255.

Frankel, M. (1962). 'The production function in allocation and growth: A synthesis', *American Economic Review*, vol. 52, pp. 995–1022.

Futagami, K., Morita, Y. and Shibata, A. (1993). 'Dynamic analysis of an endogenous growth model with public capital', *Scandinavian Journal of Economics*, vol. 95, pp. 607–625.

Gagliardini, P., Ossola, E. and Scaillet, O. (2019). 'A diagnostic criterion for approximate factor structure', *Journal of Econometrics*, vol. 212(2), pp. 503–521.

Glomm, G. and Ravikumar, B. (1997). 'Productive government expenditures and long-run growth', *Journal of Economic Dynamics and Control*, vol. 21, pp. 183–204.

Gross, T. and Klein, P. (2024). 'Innovation-driven growth in a multi-country world', SSRN Working Paper.

Gupta, S., Clements, B., Baldacci, E. and Mulas-Granados, C. (2005). 'Fiscal policy, expenditure composition, and growth in low-income countries', *Journal of International Money and Finance*, vol. 24(3), pp. 441–463.

Horn, R.A. and Johnson, C.R. (2013). *Matrix Analysis*, Cambridge: Cambridge University Press, 2nd edn., doi:10.1017/CBO9781139020411.

Howitt, P. (2000). 'Endogenous Growth and Cross-Country Income Differences', *American Economic Review*, vol. 90(4), pp. 829–846.

Hsieh, C.T., Klenow, P.J. and Nath, I. (2023). 'A global view of creative destruction', *Journal of Political Economy: Macroeconomics*, vol. 1, pp. 243–275.

Jin, W. and Zhou, Y. (2022). 'Growth and convergence through technological interdependence', *Macroeconomic Dynamics*, vol. 26(5), pp. 1338–1374, doi:10.1017/S1365100520000589.

Kleinman, B., Liu, E., Redding, S.J. and Yogo, M. (2023). 'Neoclassical growth in an interdependent world', National Bureau of Economic Research, doi:10.3386/w31951.

Klenow, P.J. and Rodriguez-Clare, A. (2005). 'Externalities and growth', in (P. Aghion and S. Durlauf, eds.), *Handbook of Economic Growth*, pp. 817–861, vol. 1 of *Handbook of Economic Growth*, chap. 11, Elsevier.

Kneller, R., Bleaney, M.F. and Gemmell, N. (1999). 'Fiscal policy and growth: evidence from oecd countries', *Journal of public economics*, vol. 74(2), pp. 171–190.

Kremer, M., Willis, J. and You, Y. (2021). 'Converging to convergence', in (*NBER Macroeconomics Annual*pp. 337–412, vol. 36, chap. 11, National Bureau of Economic Research, Inc.

Kremer, S., Bick, A. and Dieter, N. (2013). 'Inflation and growth: New evidence from a dynamic panel threshold analysis', *Empirical Economics*, vol. 44, pp. 861–878.

Moll, B. (2008). 'Capital accumulation with interdependent countries', Harvard University, unpublished.

Morozumi, A. and Veiga, F.J. (2016). 'Public spending and growth: The role of government accountability', *European Economic Review*, vol. 89, pp. 148–171.

Nelson, R.R. and Phelps, E.S. (1966). 'Investment in humans, technological diffusion, and economic growth', *The American Economic Review*, vol. 56, pp. 69–75.

Parello, C.P. and Venturini, F. (2025). 'Knowledge gaps, convergence and growth', Sapienza University of Rome, unpublished.

Patel, D., Sandefur, J. and Subramanian, A. (2021). 'The new era of unconditional convergence', *Journal of Development Economics*, vol. 152(102687), p. 102687.

Pesaran, M.H. (2006). 'Estimation and inference in large heterogeneous panels with a multifactor error structure', *Econometrica*, vol. 74(4), pp. 967–1012.

Pesaran, M.H. (2007). 'A simple panel unit root test in the presence of cross-section dependence', *Journal of applied econometrics*, vol. 22(2), pp. 265–312.

Pesaran, M.H. (2015). 'Testing weak cross-sectional dependence in large panels', *Econometric reviews*, vol. 34(6-10), pp. 1089–1117.

Pesaran, M.H., Shin, Y. *et al.* (1995). *An autoregressive distributed lag modelling approach to cointegration analysis*, vol. 9514, Department of Applied Economics, University of Cambridge Cambridge, UK.

Pesaran, M.H. and Smith, R. (1995). 'Estimating long-run relationships from dynamic heterogeneous panels', *Journal of econometrics*, vol. 68(1), pp. 79–113.

Rebelo, S. (1991). 'Long-run policy analysis and long-run growth', *Journal of Political Economy*, vol. 99, pp. 500–521.

Romer, P.M. (1986). 'Increasing returns and long-run growth', *Journal of Political Economy*, vol. 94(5), pp. 1002–1037.

Roy, S., Kessler, M. and Subramanian, A. (2016). 'Glimpsing the end of economic history? Unconditional convergence and the missing middle income trap', Center for Global Development.

Strauss, J. and Yigit, T. (2003). 'Shortfalls of panel unit root testing', *Economics Letters*, vol. 81(3), pp. 309–313.

Turnovsky, S.J. and Fisher, W.H. (1995). 'The composition of government and its consequences for macroeconomic performance', *Journal of Economic Dynamics and Control*, vol. 19, pp. 747–786.

Appendix

A Proof of Proposition 1

To establish items (i) and (ii) of Proposition 1, we express system (19) and (20) as:

$$g_\ell = (1-\zeta_\ell)\left(A_\ell \zeta_\ell^{1-\xi}\right)^{(1-\alpha)/[\alpha+(1-\alpha)\xi]} - x_\ell - \delta_\ell - \nu_\ell \tag{A.1}$$

$$g_\ell = \alpha\left(A_\ell \zeta_\ell^{1-\xi}\right)^{(1-\alpha)/[\alpha+(1-\alpha)\xi]} - \rho_\ell - \delta_\ell, \tag{A.2}$$

where $g_\ell \equiv \dot{k}_\ell/k_\ell = \dot{c}_\ell/c_\ell$ represents the equilibrium growth rate, and $x_\ell \equiv c_\ell/k_\ell$ denotes the consumption-to-capital ratio of the leader country. Solving (A.1) and (A.2) gives:

$$\hat{g}_\ell = \alpha\left(A_\ell \zeta_\ell^{1-\xi}\right)^{(1-\alpha)/[\alpha+(1-\alpha)\xi]} - \rho_\ell - \delta_\ell \tag{A.3}$$

$$\hat{x}_\ell = \rho_\ell - \nu_\ell + (1-\alpha-\zeta_\ell)\left(A_\ell \zeta_\ell^{1-\xi}\right)^{(1-\alpha)/[\alpha+(1-\alpha)\xi]}, \tag{A.4}$$

confirming the existence and uniqueness of the steady-state growth path for the leader economy.

To prove item (iii), we reformulate system (19) and (20) as follows:

$$\dot{k}_\ell = \left[(1-\zeta_\ell)\left(\frac{\rho_\ell + \hat{g}_\ell + \delta_\ell}{\alpha}\right) - \delta_\ell - \nu_\ell\right] k_\ell - c_\ell \tag{A.5}$$

$$\dot{c}_\ell = g_\ell c_\ell, \tag{A.6}$$

Solving the differential equation in (A.6) and substituting the solution into (A.5) yields:

$$\dot{k}_\ell - \left[(1-\zeta_\ell)\left(\frac{\rho_\ell + \hat{g}_\ell + \delta_\ell}{\alpha}\right) - \delta_\ell - \nu_\ell\right] k_\ell = -c_\ell(0)\, e^{\hat{g}_\ell t}.$$

Solving forward between 0 and t yields:

$$k_\ell(t) = e^{\vartheta_\ell t}\left[k_\ell(0) - \frac{c_\ell(0)}{\vartheta_\ell - \hat{g}_\ell}\right] + e^{\hat{g}_\ell t}\frac{c_\ell(0)}{\vartheta_\ell - \hat{g}_\ell}. \tag{A.7}$$

where $\vartheta_\ell \equiv (1-\zeta_\ell)(\rho_\ell + \hat{g}_\ell + \delta_\ell)/\alpha - \delta_\ell - \nu_\ell > 0$ is a collection of given parameters.

For (A.7) to be a valid solution, it must satisfy the transversality condition (4). Substituting (15) for r_ℓ in (4) and solving the resulting integral, we obtain

$$\lim_{t\to\infty} e^{-(\rho_\ell + \hat{g}_\ell - \nu_\ell)t} k_\ell(t) = 0.$$

Substituting from (A.7) into the above expression, we obtain:

$$\lim_{t\to\infty} e^{-(\rho_\ell+\hat{g}_\ell-\nu_\ell-\vartheta_\ell)t}\left[k_\ell(0)-\frac{c_\ell(0)}{\vartheta_\ell-\hat{g}_\ell}\right]+\lim_{t\to\infty} e^{-(\rho_\ell-\nu_\ell)t}\frac{c_\ell(0)}{\vartheta_\ell-\hat{g}_\ell}=0. \tag{A.8}$$

Since $\rho_\ell - \nu_\ell > 0$, the second limit on the left-hand side of (A.8) vanishes as $t \to \infty$. Thus, the transversality condition holds if and only if the first limit in (A.8) also tends to zero. Given that $\rho_\ell + \hat{g}_\ell - \nu_\ell - \vartheta_\ell < 0$ when $\zeta_\ell < 1 - \alpha$, the transversality condition requires the initial level of consumption $c_\ell(0)$ to jump to:

$$c_\ell(0) = (\vartheta_\ell - \hat{g}_\ell)\, k_\ell(0),$$

so that the term appearing in the first limit vanishes, and the proof of the proposition is done.

B Proof of Proposition 2

For each country $s \in \mathcal{S}_d$, we have $\psi = 0$. Consequently, from (21) and (22), the dynamic system governing its intertemporal evolution is described by the following pair of linear differential equations:

$$\frac{\dot{k}_s}{k_s} = (1-\zeta_s)\left(A_s\zeta_s^{1-\xi}\right)^{(1-\alpha)/[\alpha+(1-\alpha)\xi]} - \frac{c_s}{k_s} - \delta_s - \nu_s$$

$$\frac{\dot{c}_s}{c_s} = \alpha\left(A_s\zeta_s^{1-\xi}\right)^{(1-\alpha)/[\alpha+(1-\alpha)\xi]} - \rho_s - \delta_s.$$

Letting $g_s \equiv \dot{k}_s/k_s = \dot{c}_s/c_s$ and $x_s \equiv c_s/k_s$ represent, respectively, the equilibrium growth rate and the consumption-to-capital ratio of the generic *capital-poor* follower country s, the proof of items (i), (ii), and (iv) of Proposition 2 follows similarly to the proof of items (i), (ii), and (iii) of Proposition 1. This yields the following results:

$$\hat{g}_s = \alpha\left(A_s\zeta_s^{1-\xi}\right)^{(1-\alpha)/[\alpha+(1-\alpha)\xi]} - \rho_s - \delta_s$$

$$\hat{x}_s = \rho_s - \nu_s + (1-\alpha-\zeta_s)\left(A_s\zeta_s^{1-\xi}\right)^{(1-\alpha)/[\alpha+(1-\alpha)\xi]}$$

$$\lim_{t\to\infty} e^{-(\rho_s+\hat{g}_s-\nu_s-\vartheta_s)t}\left[k_s(0)-\frac{c_s(0)}{\vartheta_s-\hat{g}_s}\right]=0 \quad \text{iff} \quad c_s(0)=(\vartheta_s-\hat{g}_s)\,k_s(0),$$

where $\vartheta_s \equiv (1-\zeta_s)(\rho + \hat{g}_s + \delta_s)/\alpha - \delta_s - \nu_s > 0$ is given by exogenous parameters.

To establish item (iii) of Proposition 2, we log-differentiate $\kappa_s \equiv k_s/k_\ell$ with respect to time to obtain:

$$\frac{\dot{\kappa}_s}{\kappa_s} = \hat{g}_s - \hat{g}_\ell = \begin{cases} > 0 & \text{iff} \quad \hat{g}_s > \hat{g}_\ell \\ = 0 & \text{iff} \quad \hat{g}_s = \hat{g}_\ell \\ < 0 & \text{iff} \quad \hat{g}_s < \hat{g}_\ell \end{cases} .$$

This completes the proof of Proposition 2.

C Proof of Proposition 3

We begin by demonstrating items (i), (ii) and (iii) of the proposition. To do this, define the cardinality of the set S_d by m_d. In steady state, $\dot{\kappa}_s = \dot{x}_s = 0$ hold simultaneously for all $s \in \mathcal{S}_c$. This implies the following steady-state relations:

$$\hat{x}_s - (1 - \alpha - \zeta_s) \left(A_s \zeta_s^{1-\xi} \hat{\kappa}_s^{-\xi\psi} \right)^{(1-\alpha)/[\alpha+(1-\alpha)\xi]} = \rho_s - \nu_s \tag{C.1}$$

$$\hat{x}_s - \hat{x}_\ell + (1 - \zeta_\ell) \left(A_\ell \zeta_\ell^{1-\xi} \right)^{(1-\alpha)/[\alpha+(1-\alpha)\xi]} = \tag{C.2}$$
$$= (1 - \zeta_s) \left(A_s \zeta_s^{1-\xi} \hat{\kappa}_s^{-\xi\psi} \right)^{(1-\alpha)/[\alpha+(1-\alpha)\xi]} - (\delta_s - \delta_\ell) - (\nu_s - \nu_\ell) ,$$

where $\hat{x}_\ell$ is given by (A.4) and where $s = 1, 2 ... m_d$. Solving (C.1) and (C.2) simultaneously for $\hat{x}_s$ and $\hat{\kappa}_s$ gives:

$$\hat{x}_s = \rho_s - \nu_s + (1 - \alpha - \zeta_s) \left[\frac{\alpha \left(A_\ell \zeta_\ell^{1-\xi} \right)^{(1-\alpha)/[\alpha+(1-\alpha)\xi]} + (\rho_s + \delta_s) - (\rho_\ell + \delta_\ell)}{\alpha} \right] \tag{C.3}$$

$$\hat{\kappa}_s = \left[\left(\frac{A_\ell}{A_s \mathcal{G}_s^{1-\xi}} \right)^{(1-\alpha)/[\alpha+(1-\alpha)\xi]} + \frac{(\rho_s + \delta_s) - (\rho_\ell + \delta_\ell)}{\alpha \left(A_s \zeta_s^{1-\xi} \right)^{(1-\alpha)/[\alpha+(1-\alpha)\xi]}} \right]^{-[\alpha+(1-\alpha)\xi]/[\xi\psi(1-\alpha)]} \tag{C.4}$$

for all $s \in \mathcal{S}_c$. This concludes the demonstration of items (ii) and (iii) of the Proposition.

To demonstrate the result in item (i), it suffices to substitute from (C.4) into (22) to get:

$$\hat{g}_s = \alpha \left(A_\ell \zeta_\ell^{1-\xi} \right)^{(1-\alpha)/[\alpha+(1-\alpha)\xi]} - \rho_\ell - \delta_\ell = \hat{g}_\ell,$$

which implies that, in the steady state, each follower economy s grows at the same growth rate of the leader economy.

Finally, to demonstrate the item (iv) of the proposition, we express (26) and (27) as:

$$\frac{\dot{\kappa}_s}{\kappa_s} = \hat{x}_\ell - x_s - (1-\zeta_\ell)\left(A_\ell \zeta_\ell^{1-\xi}\right)^{(1-\alpha)/[\alpha+(1-\alpha)\xi]} - (\delta_s - \delta_\ell)$$
$$+ (1-\zeta_s)\left(A_s \zeta_s^{1-\xi} \kappa_s^{-\xi\psi}\right)^{(1-\alpha)/[\alpha+(1-\alpha)\xi]} - (\nu_s - \nu_\ell) \quad \text{(C.5)}$$

$$\frac{\dot{x}_s}{x_s} = x_s - (1-\alpha-\zeta_s)\left(A_s \zeta_s^{1-\xi} \kappa_s^{-\xi\psi}\right)^{(1-\alpha)/[\alpha+(1-\alpha)\xi]} - (\rho_s - \nu_s), \quad \text{(C.6)}$$

where $\hat{x}_\ell$ is given and constant of time. Dynamic system (C.5)-(C.6) presents a 'pairwise' structure, meaning that it can be decomposed into $\#S_s$ independent sub-systems of dimension 2, where each subsystem characterizes the transitional dynamics of each pair of endogenous variables $\langle \kappa_s(t), x_s(t) \rangle$. Solving the above 2×2 system for the steady state, it follows that each economy s has an own steady-state growth path characterized by the stationary quantities:

$$\hat{\kappa}_s(\hat{x}_\ell) \equiv \left\{ \frac{\alpha \left(A_s \zeta_s^{1-\xi}\right)^{(1-\alpha)/[\alpha+(1-\alpha)\xi]}}{\rho_s + (1-\zeta_\ell)\left(A_\ell \zeta_\ell^{1-\xi}\right)^{(1-\alpha)/[\alpha+(1-\alpha)\xi]} - \hat{x}_\ell - (\delta_\ell - \delta_s) - \nu_s} \right\}^{[\alpha+(1-\alpha)\xi]/[(1-\alpha)\xi\psi]} \quad \text{(C.7)}$$

$$\hat{x}_s(\hat{x}_\ell) \equiv \frac{(1-\alpha-\zeta_s)\left[(1-\zeta_\ell)\left(A_\ell \zeta_\ell^{1-\xi}\right)^{(1-\alpha)/[\alpha+(1-\alpha)\xi]} + \delta_s - \hat{x}_\ell - \delta_\ell\right]}{\alpha} +$$
$$+ \frac{(1-\zeta_s)}{\alpha}(\rho_s - \nu_s). \quad \text{(C.8)}$$

Taylor-expanding system (C.5)-(C.6) around the stationary solution:

$$< \hat{\kappa}_s(\hat{x}_\ell), \hat{x}_s(\hat{x}_\ell) : s = 1, 2, ...m_d >$$

yields:

$$\begin{bmatrix} \dot{\kappa}_1 \\ \dot{x}_1 \\ \vdots \\ \dot{\kappa}_{m_d} \\ \dot{x}_{m_d} \end{bmatrix} = \begin{bmatrix} J_1 & 0 & \cdots & 0 \\ 0 & J_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & J_{m_d} \end{bmatrix} \cdot \begin{bmatrix} \kappa_1 - \hat{\kappa}_1 \\ x_1 - \hat{x}_1 \\ \vdots \\ \kappa_{m_d} - \hat{\kappa}_{m_d} \\ x_{m_d} - \hat{x}_{m_d} \end{bmatrix},$$

where the coefficient matrix:

$$J \equiv \begin{bmatrix} J_1 & 0 & \cdots & 0 \\ 0 & J_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & J_{m_d} \end{bmatrix} = J_1 \oplus J_2 \oplus ... \oplus J_{m_d}$$

is a diagonal block matrix where the off-diagonal blocks are 2×2 zero matrices and the diagonal blocks are 2×2 coefficient matrices, with the representative block given by:

$$J_s = \begin{bmatrix} -(1-\zeta_s) \cdot \Omega_s & -1 \\ (1-\alpha-\zeta_s) \cdot \Omega_s & 1 \end{bmatrix}, \tag{34}$$

where

$$\Omega_s \equiv \frac{\xi\psi(1-\alpha)}{\alpha+(1-\alpha)\xi} \left(A_s \zeta_s^{1-\xi}\right)^{(1-\alpha)/[\alpha+(1-\alpha)\xi]} \hat{\kappa}_s^{-[\xi\psi(1-\alpha)]/[\alpha+(1-\alpha)\xi]-1} > 0$$

is exogenously given. From Matrix Analysis, we have that the spectrum of J is the union of the spectra of its diagonal blocks, i.e. $\sigma(J) = \bigcup_{s=1}^{m_d} \sigma(J_s)$ (Horn and Johnson, 2013) Consequently, the rest point generated by the $m_d \times m_d$ system of steady-state relations (C.1) and (C.2) turns out the be saddle-path stable iff each $\sigma(J_s)$ is formed by one eigenvalue with positive real part and one eigenvalue with negative real part (Blanchard and Kahn, 1980). For this to happen, it must be that $\det J_s < 0$. From (34), it follows that

$$\det J_s = -\alpha\Omega_s,$$

which turns out to be negative as Ω_s is positive. This completes the proof of Proposition 3.

D Empirical Analysis

Table 6: List of Countries by Continental Area

Continent	Countries
Africa	Algeria, Angola, Benin, Botswana, Burkina Faso, Burundi, Cameroon, Central African Republic, Chad, Cote d'Ivoire, Democratic Republic of the Congo, Djibouti, Egypt, Equatorial Guinea, Eswatini, Ethiopia, Gambia, Ghana, Guinea, Guinea-Bissau, Kenya, Lesotho, Liberia, Madagascar, Malawi, Mali, Mauritania, Mauritius, Morocco, Mozambique, Namibia, Niger, Nigeria, Republic of Congo, Rwanda, Senegal, Sierra Leone, South Africa, Tanzania, Togo, Tunisia, Uganda, Zambia
Asia	Afghanistan, Bangladesh, Cambodia, China, Hong Kong SAR, India, Indonesia, Israel, Japan, Jordan, Korea, Lebanon, Malaysia, Mongolia, Myanmar, Nepal, Pakistan, Philippines, Singapore, Sri Lanka, Thailand, Turkey, Vietnam
Europe	Albania, Austria, Belgium, Bulgaria, Denmark, Finland, France, Germany, Greece, Hungary, Ireland, Italy, Netherlands, Norway, Poland, Portugal, Romania, Spain, Sweden, Switzerland, United Kingdom
North America	Canada, Costa Rica, Dominican Republic, El Salvador, Guatemala, Haiti, Honduras, Mexico, Nicaragua, Panama, United States
South America	Argentina, Bolivia, Brazil, Chile, Colombia, Ecuador, Paraguay, Peru, Uruguay, Venezuela
Oceania	Australia, New Zealand

Table 7: Summary Statistics for the Data used in the Regressions

Variable	Mean	Std. Dev.	Min	Country	Max	Country
GDP per Capita	12,315	14,588	230	Venezuela	88,133	Ireland
GDP per Capita Growth	1.69%	5.57	-85.6%	Lebanon	64.6%	Liberia
Public Investments	4.74%	4.34	0 %	D.R. Congo	53%	Venezuela
Relative Capital	30.8%	39.7	0.31%	Myanmar	224%	Switzerland

Notes: All variables are measured in real terms. Real GDP per capita is expressed in constant 2017 international dollars. For clarity of presentation, the values of the remaining variables have been multiplied by 100%. However, these variables are used in decimals (not multiplied by 100) in the regressions. Public investments represent the ratio of public investment to GDP, while relative capital refers to the ratio of per capita capital in the follower country to that in the leader country.

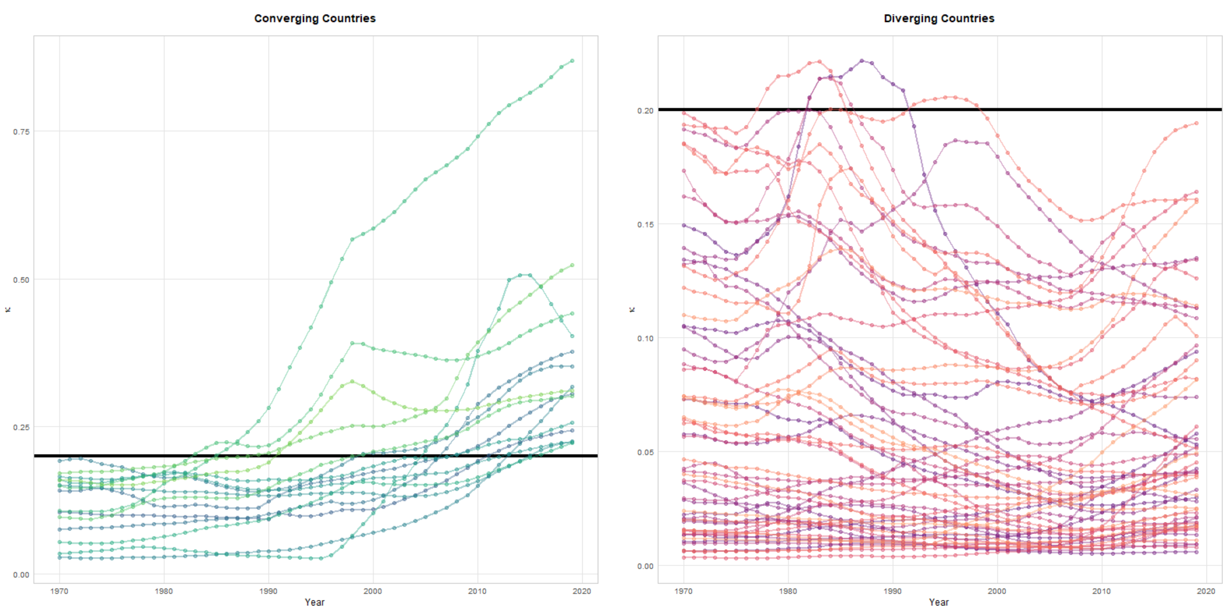

Figure 4: Evolution of κ_s over time for capital-poor converging and diverging economies. The horizontal line is drawn at $\kappa_s = \tilde{\kappa}_s = 0.2$.