Monetary Theory of Macro Accounting for Supply Chain Finance

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Abstract

We present a monetary theory where money is taken primarily as a medium of debt repayment and not as a medium of exchange. Money and products are exchanged in reciprocal contracts of disposals of property rights within two-sided obligation contracts. Money demand arises because production takes time and producers need to pay suppliers of resources before they are paid at markets for their products. Accordingly, money is part of monetary systems of macro accounting for supply chain finance where producers are exchanging their products for money in order to repay loans of investments. We take advantage of two legal principles of separation and abstraction in order to clarify the concepts of obligations, debts, claims, disposals, property rights or money. Monetary systems exist to organise the division of labour at the micro level of economies. At the meso level of banking money helps to organise the sharing of risks from investments. At the macro level monetary systems help to distribute the product of the common productive effort (GDP). We analyse macro (aka quadruple accounting) systems composed as parallel bookings in one or more micro (aka double) accounting systems of the agents exchanging products and money in networks of obligations created by contracts. We use the Bill of Exchange (BoE) as the financial instrument to unify views on paper, gold and fractional monetary systems and to understand how to book money creation at central banks. We propose to keep track of invariances of accounting systems over micro, meso and macro levels of economies by sheaf theory and homology theory to detect and resolve inconsistencies as the mathematical foundation of monetary policy. We discuss open games as an implementation technology for monetary macro accounting (MoMa) systems in reduced form (Markov) models and for the analysis of data for a structural analysis by models of belief formation or multi-agent systems. We also discuss industrial applications of our monetary theory.

Keywords: monetary theory, non-walrasian, institutional economics

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1 Introduction

It has been said that money makes the world go round. It touches upon many aspects of human life in most societies ever since and yet economics and monetary institutions are still confused about money as an admittedly rather complex and abstract real world phenomenon. As a phenomenon it is documented by tally sticks being some of the earliest human artefacts dating back about 40.000 years. Tally sticks have been used for debt recording in UK until 1876 [35]. Ever since the starting point of economics in "The Wealth of Nations" by Adam Smith in 1776 [30], up to modern times in "The Challenge of Monetary Theory" by Martin Hellwig in 1993 [9], economists ask the question: What is money?

> *In general terms, the problem is to find appropriate conceptual foundations for monetary economics. I believe that we do not, as yet, have a suitable theoretical framework for studying the functioning of a monetary system. The main obstacle to the development of such a framework is our habit of thinking in terms of frictionless, organised, i.e. Walrasian markets. The problem is partly a problem of language. The central concepts of economics, demand, supply, and price, are concepts that presuppose a system of centralized Walrasian markets. We are used to thinking in terms of these concepts, and we tend to apply them rather automatically to the 'market' for money, frequently without realizing that the 'market' for money must be a rather non-Walrasian 'market', if indeed it can be called a 'market' at all.*

Economic theory defines a Walrasian market as a process where a central auctioneer calculates a market clearing price from demand and supply functions of prices of interacting agents. The agents trade at the clearing price simultaneously without any frictions, in perfect competition, without transaction costs or informational asymmetries.

Since Hellwig's challenge, we have seen search theoretic models of money [11] addressing decentralised trade and the endogenous emergence of money as a medium of exchange. In *Money is Memory* [13] the idea is explored that money substitutes for imperfect record keeping in economies with frictions like limited enforcement or anonymity. Heterodox approaches like Post-Keynesianism and institutional views of money as in [39], discuss the role of states and central banks in defining and managing money, focusing away from a single market for money. Recent developments in *Modern Monetary Theory* [10] reexamine the state's central role in defining money, taxation, and public spending. Dynamic Stochastic General Equilibrium models [6], within a theory of Walrasian markets put money in the utility function with *cash-in-advance* constrains, acknowledging that the frictionless Walrasian markets can neither capture financial crises nor the essential role of monetary policy in those crises. Central banks and other institutions, notably the Bank of England in [16], have published more explicit explanations of how money is created within modern economies. This practical perspective resonates with Hellwig's demand for a framework that recognises the real world mechanisms of money and banking rather than seeing money as a mere nominal veil over the real production and consumption in economies. The rise of cryptocurrencies with Bitcoin's launch in 2009, followed by many others, reignited discussions on the definition and nature of money. Crypto currencies and decentralised finance (DeFi) highlight the frictions and record keeping issues of *Money is Memory* [13]. Scholarly work about digital currencies can be found in [14]. The Bank for International Settlements (BIS) and International Monetary Fund (IMF) also publish papers on digital currency, blockchain technology, and central bank digital currencies and often get back [31] to Hellwig's fundamental question of *what is money?* If we strip away the layers of institutional scaffolding, can we identify the core functionality of money in any public or private, centralised or decentralised economic system?

We develop a monetary theory where the core functionality of money is to help to organise labour sharing and commodities distribution in supply chains:[1]

[1]Menéndez has worked out the principle of money for supply chains in his doctoral thesis in 1989. In [17]

Money is needed because production takes time and therefore producers need to get indebted for paying suppliers of resources and employees for work before they in turn are being paid for their products by their customers.

Money is accordingly not primarily useful as a medium of exchange but as a medium of payment in obligation and disposal contracts (about debt releations and property rights, respectively) in labour sharing economies.

In our monetary theory we focus to understand how debts and obligations interoperate with money being the means of payment. Money is a financial technology for the organisation of a labor sharing production, exemplified in Adam Smith's story about pins. It solves by that an *intertemporal* problem of liquidity rather than the follow up problem of *intratemporal* exchanges and the double coincidence of wants in the exchange of products, like in the Adam Smith's *natural* theory of relative prices of beavers and deer. Relative prices are recorded as agreements in contracts and we take them as given for our theory in the accounting systems which drive the economies.

We do not take as our goal for our monetary theory to justify any of societies' decisions on relative prices for the exchange of products or values of money. Our theory starts from recordings of the contractual value based judgements of the agents involved in any exchange. Money accordingly exists to record and settle debts from contracts of any type with value based judgements of any type and by that money does not need to have any value as an item that is used in the settlement of obligations which are related to the commodities exchanged as part of the underlying contract.

Our theory (MoMaT) of monetary macro accounting (MoMa) systems takes advantage of a view on the national accounting systems where in any economic exchange two double accounting systems are involved. These systems are sometimes called quadruple accounting systems which we rather prefer to call macro accounting and the usual double accounting systems are accordingly called micro accounting.[2] The invariances of micro accounting and their macro variants in the sums of receivables, liabilities, expenditures and revenues are of paramount importance in our monetary theory.

To us economic monetary theories of price and value theories of money have their merits in explaining the maximising behaviour of individuals but miss the point of what monetary systems were designed for to solve. A value theory is detrimental and a dead end for tackling the requirements of supply chains in need of a monetary theory. We believe that we need to start from debt relations as they are accounted without any theory of value or judgement. The challenges of monetary economics are accordingly at the basic level of debt accounting as implemented since the medieval times by the Bill of Exchange (BoE) as the contractual record of networks of debts and obligations. Our theory of monetary macro accounting for monetary policy, banking sectors, national and macro accounting systems should be functional for any kind of value theory, economic theory, theory of justice or distributional goals for the GDP. Like double accounting

he implemented the simplest possible national accounting simulation following the monetary principle. Winschel joined research in 2018 and related the principle to invariances of macro and quadruple accounting as in SNA ("System of National Accounts 2008" [18]) and implemented the invariances of micro and double accounting as algebras of [24]. He implemented the model of [17] as a macro accounting system, both in Julia and category theoretical libraries for dynamic systems and identified the mathematics of sheafs for hierarchical consistency conditions in MoMa. He co-developed compositional game theory [7] which can be used to implement macro accounting systems and data analysis by structural econometric models for monetary policy and applications.

[2]The standard system of national accounts (SNA) is called quadruple accounting because one booking in a quadruple accounting usually consists of two double accounting bookings. We prefer the name macro accounting since one macro booking may consist of one, when paper money is created at the central bank, or more than two micro bookings for contracts and events with more than two parties involved. *Micro* and *macro* emphasise that double or micro and quadruple or national accounting systems are about micro and macro economics, respectively. However, macro and micro are also more like relational rather than absolute concepts, like whole and parts, since a macro accounting system, say national accounting, may be seen as a micro accounting in an international setup for example in nominal exchange rate regime policies, like pegging, floating or monetary unions.

systems, MoMa systems should work for the network of debts starting from the legal contracts of the agents involved in the property right transfers of monies and products.

Beyond the micro level of labour sharing and payments in networks of debts, we take a look at the meso and macro levels of economies as well. At the meso level the risk of failing investment projects is to be distributed by banks among the whole debt community in clearing and settlement processes of debts. At the macro level of the national product the distribution of the GDP and the socialisation of risks is to be understood as well as the accounting mechanics of the creation of money at central banks.

At the meso level of economies banks share risks in portfolios of investments. Investments may succeed or fail and in order to balance these risks banks calculate interest rates which we conceptualise primarily as insurance premiums before interest income can convert into profit income. At this meso level invariances of macro accounting systems arise which are of paramount importance for any monetary theory. For debts, since every receivable must correspond to a liability, the sum of receivables must be equal to the sum of liabilities. And for payments, the sum of payments must be equal to the sum of revenues.

In the macro accounting of economies central banks issue money as the means of payment and means of repaying debts in any exchange. As a consequence of clearings and settlements of the network of debts at the meso level of banks, it is the GDP that is distributed among the members of economies at the macro level. We take this as the third phenomenon to be organised by money: at the micro level labour is shared, at the meso level risk is shared and at the macro level the GDP is shared among the members of an economy, i.e. among the money users.

The core functionality of money as a means of debt repayment is conceptually simple but understanding the whole of monetary systems and the accounting involved in MoMa is not that simple. For that we blend concepts of various disciplines like economics, law, mathematics and computer science. As a most important tool of MoMaT, we take advantage of the legal *principles of separation and abstraction* in order to disentangle the various conceptual difficulties in monetary theory about what are debts or payments following the question of *What is money?*

The unifying financial principle and instrument used in this paper in order to understand paper and gold based monetary systems over the hierarchical levels of economies, is the Bill of Exchange (BoE). The BoE is a real world instantiation of the informal *I owe you* contract which may also be a beer coaster in a bar. At the micro level the BoE legalises and operationalises the creation, exchange and deletion of claims by money as receivables and liabilities of producers and consumers in supply chains. In general, the BoE can define any content as its means of fulfilment. At the meso level financial BoEs allow banks to trade liquidity and gold to track the clearing and settlement processes. At the macro level the BoE can highlight how paper money in fiat systems and bank notes in gold standards are created at central banks' double accounting systems. Before central banks emerged, the BoE was used at banks to issue many different bank notes.

The BoE is not only of principle interest in monetary theory but it is of practical interest in industrial applications. Central banks settle claims and liabilities by digital Promissory Notes (PN), which are some form of a BoE, at the Bank of International Settlement (BIS) [19]. We argue in this paper that BoE, Sola (actually called Solawechsel, which is similar to PN and BoE) and credit loans (gold denominated Sola) span the most important dimensions of our MoMaT.

Our MoMaT starts locally at micro accounting in networks of debt relations and extends to invariances of macro accounting. For that sheaf theory is suited to keep track of consistencies in layered banking systems over the micro, meso and macro levels of accounting systems. Sheafs are concerned with the transition from local to global structures in mathematics. They allow to construct and study complex models as systems of equations and to combine bits of local information into a consistent whole [26]. Homology theory can be used to resolve local inconsistencies into a consistent global behaviour. In MoMaT we can evaluate local and global consistency conditions to supply liquidity and money for matching demand and supply as part of monetary policy. In fractional systems inconsistencies arise as needs of clearing or settling debts, restitute debt cuts

or redistribute wealth like in bail outs of bad banks.

We propose open games as the domain specific language for a compositional agent based technology to implement MoMa systems. Open games can represent bidirectional computational structures in time, i.e. into and from the future, but also relations, accounting systems, constraint propagation, smart contracts or bookings in micro and macro accounting systems. Open games can also represent structural economic and multi-agent models of forward looking belief formation and goal oriented strategic decisions. Back propagation are open games as well, useful for learning econometric, machine learning, AI or process models from data.

Open games generalise decision network and influence diagrams of multi-agent AI research [12] and the process languages in industry like business process model and notation (BPMN) and decision model and notation (DMN) standards [33]. Given the formal semantics of open games by category theory we can formulate formal semantics for economic models as a semantics of a programming language for economics. Open games come with a formal semantics of a visual programming language (aka string diagrams), making properties of economic models and accounting system implementations provable, if formulated as open games. The invariances of micro and macro accounting systems are fundamental properties to proof, others are for example technical properties of security protocols, economic properties of monetary policy, regulations of banking and their compliance, properties of exchange rate regimes, properties of the GDP like distributional goals or theories of justice to agree upon in societies and politics. Just like multi-agent AI systems share the structural econometrics goals of generating explainable learnings from data, the provability of properties of code will develop into a discipline of compliance, for example in applications where life and wealth is at risk.

The applications of our MoMaT are monetary policy, supply chains, trade finance, liquidity and treasury management of holdings, banking, liquidity or any other platforms, energy and environmental systems with MoMa providing a value free approach to their financial systems. Since open games arose from an interdisciplinary research on quantum computing programming languages economic theories formulated in open games are quantum ready.[3] Therefore transactions of MoMa systems with open games as smart contracts can be about quantum secured assignments and transfers of property rights with non-deletable data with free write and read access on blockchains.

2 Microeconomics

The MoMa is an accounting system that records networks of debts characterising a modern labour sharing economy.

> *According to MoMaT money exists for debt contracts because production takes time and producers sign contracts for paying money to suppliers of resources and the labour services of their employees. Later producers can turn their commodities into money in sales contracts with their customers. Money demand arises as what producers want as exchange for their commodities. Producers do not need their products for consumption but to get money to repay their outlays of the investments.*

This approach does not need anything like money in the utility function, nor money to have any intrinsic value motivating its use and demand. The paper itself, as a thing, is useless but the not the functionality it provides. We need to book debts in money, liquidity exchange, in securitisation of gold or claims on deposits (aka checking accounts).

To disentangle the network of debts we want to use two foundational legal **Principles of Separation and Abstraction** ("Trennungs- und Abstraktionsprinzip") developed in the German

[3]The paper on coalgebraic games [2] has been a precursor to open games [7] heavily inspired by the work [1] on quantum protocols.

Civil Code ("Bürgerliches Gesetzbuch", BGB). We want to clarify what is a claim, receivable, debt, liability, money, payment, repayment, purchase, loan or investment. Some lawyers say about themselves that *Juris non calculat* and economists should not need to admit that *Oeconomi non iura cogitant* (economists do not think in legal principles) from now on.

The first **Separation Principle** states that the **obligation contract**, creating an obligation ("Verpflichtungsgeschäft", "schuldrechtliches Geschäft", debt transaction) and the **disposition contract** ("Verfügungsgeschäft", "dingliches Geschäft", disposition transaction on things) about transferring property or other rights, are **separate legal transactions**. Transactions and contracts are the same for our purposes, a transaction is based on an underlying contract. Obligation contracts concern two agents, like purchase contracts, lease agreements, or contracts on work and services. Two mutual disposition contracts about the items exchanged are associated to each obligation contract. The loan contract accompanies the purchase contract as the second important contract in MoMaT.

We have to note, that payments are executed not only through a physical handout of the means of payment. With the help of another contract, on bank deposits, the physical handout of money can be substituted by transferring an ownership right about money to the payee. This will be a minor complication but it is important to keep in mind that deposits at banks are, once made available by banks to customers, not money but an option right for customers to command his bank a payout of money or to command the bank to transfer this option right. It is not that once an obligation contract about deposits is signed banks are to be asked to transfer money. Instead they have to, as their duty from the obligation contract, to fulfil the disposal contract by waiting for their customers' commands to hand over money. This is the exchange for the bank's client's duty from the obligation contract to repay the deposit or loan in money as their disposal contact. If deposits are used by customers to deposit money first of all then the difference between money and commands for money from deposits is more clear cut. We will call the transfer rights on money by deposits as "money" or as *command for money*.

An **obligation contract** transfers, modifies, or terminates a right, like the property right of a product or some money in a two-sided purchase or loan transaction. The **disposition contract** serves to **fulfil a prior obligation contract**, which is why the disposal transaction is also called a **fulfilment transaction** ("Erfüllungsgeschäft"). An exchange of two things, product for product, product for money or money for money in loans, consists of one **obligation contract and two disposition contracts**. The catchy summary is **dispositions fulfil obligations** or more concrete **things fulfil debts** while a thing may be a product or money.

The second **Abstraction Principle** states that the validity of the obligation contract is **independent of the validity** of the disposition contracts, and vice versa. As a consequence, if the contract creating an obligation is invalid, the transfers of ownership executed through the contract effecting a disposition remains legally effective, the same if any of the disposition contracts is invalid. This leads to an imbalance, as the buyer or seller has acquired ownership without a valid underlying contractual basis. To address such imbalances, § 812 BGB provides a **restitution claim** based on unjust enrichment ("ungerechtfertigte Bereicherung"). This provision ensures that when one transaction is valid while the others are not, the agent who has received a benefit without a legal basis must return it. In practice, this allows for example the seller to reclaim ownership or demand compensation, if the purchase agreement is void but ownership was already transferred or because the buyer is not fulfilling his duty. § 812 BGB functions as a corrective mechanism, ensuring that any unintended shifts in rights or assets due to the Abstraction Principle are rectified through restitution.

It might seem overly complicated to establish an abstraction principle and the restitution claims. But, especially in supply chains, this comes in as a very practical legal pattern. It might help to think about this pattern as a *legal recursion for decomposing failed chains of contracts.*

Applying the principles to **purchase a product for money (§ 433 BGB)**:

1. Obligation contract ("Verpflichtungsgeschäft"): The purchase agreement commits to deliver

a product by the seller and money by the buyer.

2. Disposition contracts ("Verfügungsgeschäft"): Two transfers of ownership (§ 929 BGB) of
   (a) the product from seller to buyer
   (b) and money from buyer to seller.
3. If one of the contracts is invalid a restitution claim (§ 812 BGB) rectifies unjust enrichment.

Note that the obligation to hand over the contracted item also comes with the obligation to accept this item as fulfilment and to take control of it. Failures to accept the agreed item also break the contract and may lead to restitution claims.

Applying the principles to **a loan contract (§ 488 BGB)**:

1. Obligation contract ("Verpflichtungsgeschäft"): A loan contract commits the bank to grant access to money and the debtor to repay it. The bank fulfils the loan agreement by issuing a credit note to the borrower. This means that the borrower is given an option right to dispose of and give instructions to the bank over the agreed amount of money. This is normally referred to as a deposit.
2. Disposition contracts ("Verfügungsgeschäft"): Two transactions
   (a) the bank grants by the deposit to transfers money on debtor's command and carries out the transfers of central bank money commanded by the borrower.
   (b) and debtor repays the loan.
3. If one of the contracts is invalid a restitution claim (§ 812 BGB) rectifies unjust enrichment.

The confusions about the concepts of monetary theory are often due to a neglect to apply these two principles. We use them to sort out the economic concepts being discussed in the following. For example, money is taken as debt in concepts like *Kreditgeld* (German word for *loan money*) where we would think that a *loan* is a two-sided obligation contract whereas *money* is a means for fulfilling an obligation arising from an obligation contract.

The monetary debt in the obligation contract is measured in monetary units like [Euro]. Paper money is an asset that is posted in the asset side of the financial accounting of a central bank when it is issued, the counter entry on the liability side consists of the issued volumes of IDs of papers representing money in the database called "Circulating Money". In gold standards it was gold or alternatively a command for gold against the central bank documented by a bank note which was used to have an unconditional claim on gold safely deposited at the vault at the central bank.

There are clearing operations related to debts and the means of payment and repayment. These are so to say means of clearing (aka claims) being able to be netted out before they are settled by a means of payment. Claims can be initiated by hand shakes, beer coaster, deposits or other financial assets that can be netted and settled by money. Deposits of customers like companies, persons, or administrative bodies at a bank are commands for paper money but not money in the sense of means of payment. Banks are entitled to use their deposits at the central bank for claiming paper money from the central bank. The customer's right from a deposit is an option right to command the bank to transfer central bank money in the disposal contract within an obligation contract with the bank. Customers of the central bank are banks and banks have companies and households as customers to hand over paper money. The content of a deposit is measured in monetary units [Euro] booked in the deposit at a bank but these accounts are an expression of the indebtedness of the bank and not a means of payment.

To be utterly careful in the realm of monetary theory, it is helpful to look at some related concepts to train detecting confusions in money theory. We say that Anna is not "Anna". She is

a person while the second is a reference or the name "Anna" of Anna. Anna's name is "Anna". With Anna and "Anna" it is rather strange to confuse the person and her name. Also "Anna" and "Betti" cannot be netted out and then settled by Anna or Betti. But with money, commands for money or deposits matters are different. First of all the claims can be netted out and the remaining difference can be settled by money. Money and "money" are not of the same type of things and more like what is used to settle (money) and what can be used to command for money ("money"). If we mix these types up, we get a categorical mismatch, expressions that do not type check as computer scientists say and we get into monetary crises first of all and most of all again and again.

A severe error of this type in economic monetary theory is that banks are said to be able to create money. This confusion goes on in M1, adding money as cash, the content of the claims from deposits at central bank with another type being the content of the claims from deposits at banks. For some purposes it might be useful to calculate with the disjoined union type "money, central bank or bank deposits" but we get rather confused for purposes of answering *what is money*? However, type mismatch errors are more likely in fiat systems, as matters are more abstract and confusing than in gold standards and accordingly we have to be pedantic on the types of money.

At the micro level of production during the time lag from production to sales, the producers have to make sure to pay wages since workers and entrepreneurs differ by risk preferences. The workers are being paid independent of market success while *the entrepreneur rides to success on debt*, as Schumpeter's views are often summarised. It is a risky affair to reach success in competitive and evolutionary market economies. Markets are selecting well adapted entrepreneurs, companies, banks and investors, who are able to appropriately produce commodities meeting demand and balancing costs and revenues. Banks have to calculate, offer and secure interest payments that balance the returns with losses from investments. The interest rate is by that primarily an insurance premium rather than about a time preference which is a secondary property. Sharing and mitigating risk across micro and macro levels is the realm of banking, mostly at the meso level. At the macro level central banks are lenders of last resort in gold based monetary systems, which must operate under a fractional reserve regime due to the lack of sufficient means of payment while they are creators and lenders of first liquidity for fiat systems. In both cases the central bank is designed to mitigate the risk of illiquidity for the connected banks although in the gold system case the central bank itself is subject to liquidity risk while in the paper systems liquidity risk does not exist due to the central bank's right to issue infinitely the legal means of payment.

2.1 Supply Chains and Pins

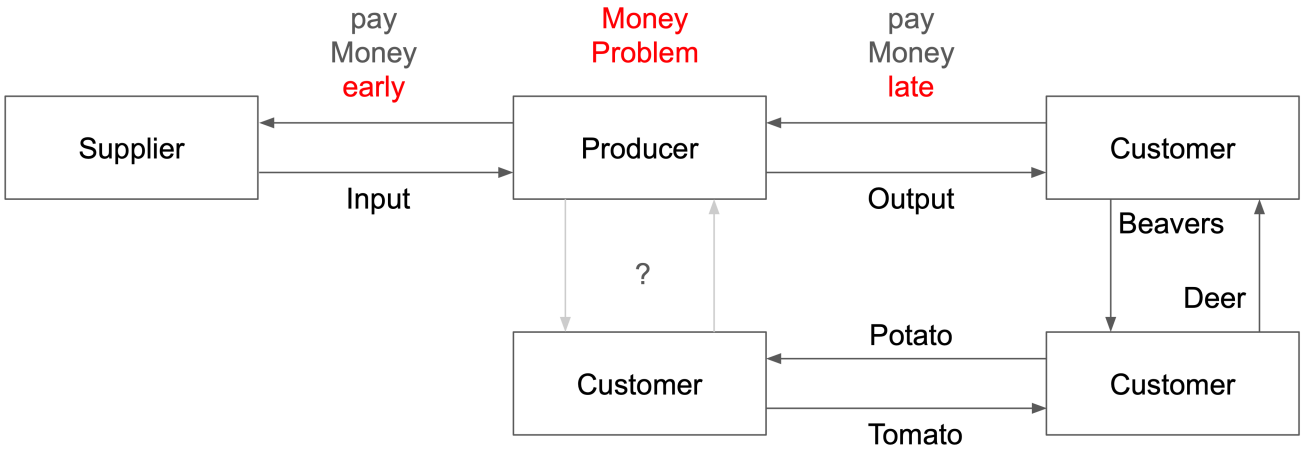


Figure 1: Money Demand in Supply Chains

Figure 1 shows the operational needs of monetary systems to provide a technology for integrating

various aspects of interpersonal obligations to make supply chains work. Money exists because the producer needs to pay the supplier of resources before he is paid by the customer. There is a demand for money for the purpose of investment, and another demand for money in order to service the debt resulting from an investment. These systemic interconnections do not only highlight another purpose of money. Instead, we need to contrast the isolationist conception of money, underlying the idea of *money as a medium of exchange*, with the evolved system design of supply chain finance. We see that the double coincidence of wants story for consumer goods will miss commodities as products of the producers that are to be integrated in the interpretation of how and for what monetary systems operate.

The same idea can be found in standard textbooks on double accounting [23], describing how to book a BoE.

> *When buying goods, two different interests meet. The supplier of inputs of production wants to receive his money as early as possible, since he has made payments in advance in various ways - he has produced, processed or procured the goods at his own expense. The producer wants to pay as late as possible, as he still has to sell the goods and only receives the money from his customers later.* (own translation, "buyer" replaced by "producer")

Both ideas can be traced back to the very first sentence of "The Wealth of Nations" by Adam Smith [30] in chapter 1, followed by the famous example of the production of pins.

> *"The greatest improvement in the productive powers of labour, and the greater part of the skill, dexterity, and judgment with which it is any where directed, or applied, seem to have been the effects of the division of labour."[...]*
> *"One man draws out the wire, another straights it, a third cuts it, a fourth points it, a fifth grinds it at the top for receiving the head; to make the head requires two or three distinct operations; to put it on, is a peculiar business, to whiten the pins is another; it is even a trade by itself to put them into the paper; and the important business of making a pin is, in this manner, divided into about eighteen distinct operations, which, in some manufactories, are all performed by distinct hands."*

Labour sharing economies are sequences of decentralised processes of resource transformations into products spread over companies, consumers, money, claims and liabilities, raw materials, intermediate and consumption goods.[4] The indebted producer usually does not want to consume the product he is producing and he also needs to buy other products for consumption. He can solve both problems by selling his product for money to buy other products. He can then repay his debts at the investor, his workers, resource owners like landlords or suppliers of resources in general. The workers are not indebted but use money for consumption or non-consumption, aka saving.

[4] In Figure 1, the accounting a BoE and the MoMaT we argue about supply chains among companies engaged in labour sharing. The story on pins by Adam Smith is about labour sharing within a company. MoMaT can be applied inside a company as well. There, the BoE is about transfer pricing: at what prices is one department delivering to another one? How are these input or output units and contributions related to the overall output of the company? How is this to be related to the distribution of the resources used within the company? How to account for all that? Monetary theory can be about internal accounting for nations of money users or it is about using money in companies or holdings. The crypto community tells a story about tokens, cost accountants about cost centres, theoretical accountants about multi-dimensional accounting, computer scientist about types and economist about money, prices or external effects. However, treasury management for example is like liquidity exchange and cash pools among subsidiaries within a holding. Accordingly, MoMaT can be used in DeFi for treasury management as a service. These are credit communities, compound of companies which are creating, accepting and using legally sound BoEs. By that BoE exchange claims and payments among each member of the community, just like subsidiaries exchange receivables and liabilities using holding contracts.

We may want to use two special words for products to highlight this distinction: From the perspective of an entrepreneur products are commodities as means of attraction of money. From the perspective of a consumer products are goods as a means to benefit and receive utility from consumption.

But beware, it only looks like products and money flow around in circles. Since they are timed, they are like vortices in space times of futures and past. The papers of money circulate but the paper stays the same paper while used for different obligation and disposition contracts. The important point is that we are interested in debt contracts and how they start, are cleared and end. The papers circulate but they repay different debts by being used in different disposition contracts to fulfil different obligation contracts.

It is also important to note that BoEs have a finite run time with a maturity date. Once the producer has sold his commodities, he is forced to settle the BoE and the debt relation ceases to exist and with it the reason for the existence of money. This contrasts with the idea of (closed) *monetary circles*, instead the vertices of debt relations are created and deleted all the time.

As denoted in Figure 1 by the question mark and the greyed arrows between producers and the customer below him, theories of money as a means of exchange miss the actually important difference between *commodities* and *goods*. This difference is important because it clarifies that these demands for products (commodities and goods) and money match in their opposing directions between producer and consumer and make theories of some value of money obsolete. Even more so, in the dynamics of producers creating wage payment from their investments is the key to understand that money does not need money to be created and the interest payments are not created from interest payments, instead the wage payments of the producer create demand for their commodities. This was supposedly understood by Henry Ford, who answered to the question: "why are you paying that high wages to your employees" by "because they are supposed to buy my cars." Actually, this insight is at the core of MoMaT in that it tells us that monetary systems can be stable, self-contained and non-contradictory, if we were to properly understand, design and maintain them.

We will clarify this strange and the world confusing rather than world go round phenomenon. The important point to take away is that producers demand money in exchange for their commodities to repay debts while consumer get goods for their money and since both match, we can do macro accounting without being subject to a value theory of money. We need to have a look on how receivables, liabilities and money record, clear and settle in the network of debts.

2.2 Prices and Beavers and Deer

The right hand side and lower part of Figure 1 shows another usual starting point of monetary theories: some products are to be exchanged, here beavers and deer originating again in Adam Smith's "Wealth of Nations" [30], this time in the first paragraph of chapter 6:

> *If among a nation of hunters, for example, it usually costs twice the labour to kill a beaver which it does to kill a deer, one beaver should naturally exchange for or be worth two deer. It is natural that what is usually the produce of two days' or two hours' labour, should be worth double of what is usually the produce of one day's or one hour's labour.*

If there was no money, the number of beavers exchanged for the number of deer would represent the agreed upon relative value. The *natural* theory argues that the relative price should depend on the relative amount of labor used to produce the products. The cause of the existence of money is then to argue that money enables exchanges even so for example owners of deer do not want to exchange them for beavers or tomatoes for potatoes. This overly restricted requirement for exchanges is called the double coincidence of wants. Money relieves this requirement and enables former impossible or computationally rather intensive matches of exchanges. Money is

conceptualised as a neutral commodity for exchange, called numeraire. The value of money is then, as one possible argument goes, given by the welfare increase from the exchanges enabled which otherwise would have not taken place. Price theories of goods and value theories of money and the arguments why money exists are tightly interacting in such argumentation lines which to us are dead ends for a monetary theory.

Another way to tell the story of beavers and deer is by MoMaT. If the owner B of beavers b, in units of b denoted by [b], and the owner D of deer d in [d] want to exchange 1 [b] for 1 [d], they need to record the difference in value, say at an agreed upon price of 2 [b] for 1 [d], i.e. 2/1 [b/d]. This might have been the birth of the marks, measured in [b], on tally sticks, documenting the claim of D to B. If D has trained his son in hunting d at no costs for D, he can be taken as a child abusing entrepreneur, who made a profit from a markup over his costs. The next day D gets one [b] from B, the debt relation ceases to exist, and the tally stick can be thrown away, used as a tooth brush or for burning down the British parliament, as in 1876 [35].

In a gold standard we express product prices in gold [kg of gold/b] and via the idea of money as a numeraire [kg of gold/$] and we get a money price [$/b]. In fiat systems we measure prices directly in monetary units [$/b] without the need of a price of a numeraire. We do not care whether we call the money and its units Dollar, Pound, Stirling or FunBaseCryptoToken, as long as it is a legally defined means of payment which allows us in disposal contracts to fulfil the debts from obligation contracts measured in units of the means of payment. We are hopefully becoming more and more comfortable by the legal abstraction and separation principles without getting confused in these cross sections of legal, economic, type and measure theoretic complications of monetary concepts.

The legal separation principle comes to our rescue: money is the thing to be transferred in the disposal contract in order to fulfil the debt from an obligation contract where the height of debt is measured in monetary units. These things with monetary units can be anything as long as it is the thing that is the legal means of payment or the means of debt repayment like a registered piece of paper called paper money. Cash is king or equivalently a deposit at the central bank as a claim in which the central bank is legally committed to pay out cash on command just as in former times payouts were guaranteed to be gold. Debt arises from an obligation contract with its height being an agreed upon amount of monetary units. Debts are fulfilled in disposal contracts by the purchased item and money with as many units as agreed upon in the debt contract. These units can be anything, like [Euro], where a Euro coin is a money with the debt canceling capability of 1 [Euro]. From here at the micro level, we need an MoMoT, institutions and rules of conduct, like MoMa, to arrive at the distribution of the GDP via the meso level of sharing of risk, taking into account investments in time taking productions with inherently insecure futures.

As will see in Figure 6 about the vortex of money in a macro economic context, money can be deleted once the investment is repaid and resources are transformed into commodities and by that there are no debt relations for which money is needed in the real processes at hand. Then money is useless not because it has no intrinsic value but because there are no debt contracts from exchanges. There is no need for infinite horizons to argue for an intrinsic value of money. An investment is an obligation contract between a producer and an investor or bank with a time series of a payout of money in the beginning and repayments by money until the end of the investment. The many parallel and sequential investments imply money demand as a demand for the means of debt repayment, effected by producers who want to exchange their commodities being products as goods for consumers in exchange for their money. Hence, the debt relations are the paramount important building blocks of monetary systems.

Money does not need to have an intrinsic value for the entrepreneur's usage, either. They calculate by monetary units the value of input costs in relation to the value of output prices and add a markup for a profit. Again the monetary unit cancels in this ratio. Actually, an intrinsic value of money would be detrimental, obfuscating the ratio. A value for money smells like a Russell paradox which led to type theory in mathematics and computer science. A type theory

is possibly a good idea for a starting point for a theory of money measuring debts in units (aka types) of accounts.

To summarise: the quest for a value of money reduces in MoMaT to money being able to represent (A) what is to be paid out at the beginning, 100.000 [$], and repaid at the end of an investment, 110.000 [$], including an interest payment as an risk insurance premium, (B) a price of a product, 6.59 [$/BigMac], (C) a relative price of two products, 2.45 [BigMac/Cheeseburger] $\approx$ 6.59 [$/BigMac] / 2.69 [$/Cheeseburger], and (D) the ratio of costs to output prices for production and profit calculations via some markup pricing, like $\pi = p - c = mc - c$ where $\pi :=$ profit [$/product], $p :=$ price [$/product], $m :=$ markup [$\mathbb{R}, > 1.0$], $c :=$ costs [$/product].

There is a nice quote of Simmel from 1907 that might shed some more light on our point on the ability of money to measure value without having some own value.

> *Since there is a constant ratio between the quanta of the one factor and those of the other, the magnitudes of the one determine the relative magnitudes of the other, without requiring any qualitative relationship or equality between them. This breaks the logical principle that seemed to make the ability of money to measure value dependent on the fact of its own value.*

3 Mesoeconomics

At the meso level of economies, banks cope with the question of the inherently risky nature of investment, whether the credited amount of money is being paid back and whether clearing and settlement is properly timed, sized and whether risk is properly accounted for. Here, MoMa becomes MoMaT as believes about financial and monetary data. In a stochastic setup[5], the meso level is a systematic approach to payment processing of clearing and settlement in wholesale financial markets. If monetary theories of financial intermediaries, politicians or central bankers are misdirected then debt cuts, bail outs of systemically relevant bad banks is an expectable result. The core principle is that failures of investments are to be booked as consumption expenditures which reduce the net wealth of the investor, who gains returns from successful investments and who has to ultimately settle the losses from investments. However, we usually conceptualise a three layered system of risk absorption: the entrepreneur is the first to neutralise failed investments, then banks (or investors with a portfolio of investments) jump in and distribute these risks among all debt contracts via an insurance premium calculation by the interest rate. The third and final layer is the ultimate risk taker which are central banks with an unlimited ability of risk absorption.

For the purpose to properly account for the returns and losses from investments, it is of paramount importance to understand that banks are not allowed to produce paper money but only references to money or commands for money which are known as deposits characterised as non securitised debt relations. Banks can instruct the central bank to pay out paper money by using their option right to command the central bank to do so. Banks are indebted at central banks in order to begin to produce loans or investments. Intuitively, banks produce solvency for companies as their products from money as their input. The central bank is a supplier of means of payment to banks and the supply chain is central bank, bank, company.

However, the last paragraph's informal way to describe matters, is a hinderance in economic theory to understand the proper legal process of using deposits. There is nothing *in* or *on* a deposit, because while a debt obligation does indeed define a medium of settlement, it is not itself that medium. It is important to understand that deposits are not turned into money, an obligation does not transform into the means of payment but the obligation is instead extinguished upon fulfilment. So, what happens is that the option rights (deposits) are transformed into claims

[5]Open games have been extended to stochastic games including games on asymmetric information in [4] using Giry monads and other categorical machinery for games of asymmetric information.

(unilateral declarations of intent requiring notice), which amounts to calling the debt. As a result, the claim is fulfilled—or the debt obligation is discharged—by paying out paper money.

The problem of a central bank is that it is central, no way around that for its monopolistic authority to issue money as a means of payment or fulfilment of obligation contracts. Concretely, the problem being central is that the central bank lacks information about the abilities of the decentralised entrepreneurs which a bank has to access for a forecast of the investment's success and to set the appropriate price or insurance markup for the investment. However, the central bank coordinates a whole sale liquidity market called the interbanking market with a given minimal bound as a standard for credit ratings, making banks and investors to compete for the best method to forecast the success of investments and for the most profitable investment process. Money and liquidity is traded at these quite Walrasian markets but money is not created at markets, but from thin air, which is totally fine if economists and the monetary institutions knew what money is.

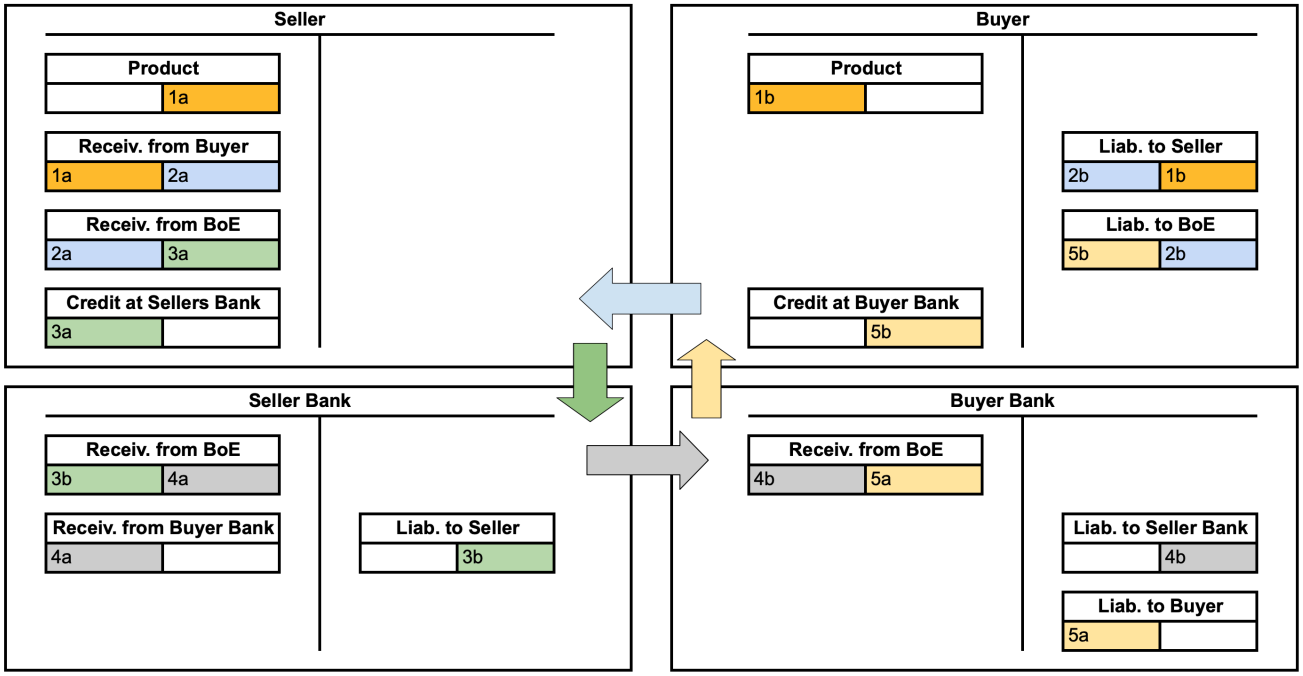
Whether the biggest wine barrel in the world in Heidelberg castle qualifies as a central bank, the king's finance ministry's treasury to tax and to pay vassals by wine or as a bank with some means of payment is a complicated question. What is rather simple to classify are modern liberal societies of citizens with obligation and disposal contracts for any transfer of property rights in any exchange. There are 1. central banks producing the means to fulfil the monetary sides of the obligation contracts in disposal contracts about money and products, 2. banks financing investments by issuing loan arrangements by using the means of payment of central banks, 3. companies using the credited means of payments to buy resources and labour to produce over time and pay some insurance premiums for the temporarily right to use the means pf payment, 4. consumers to consume products produced from their resources, with consumers as households owning the resources and companies, 5. a database of IDs of paper money at the central bank with printed amounts of monetary units for the ultimate repayment of debts. The complications come as the need to run the monetary systems without a monetary theory which is a big challenge to politicians, institutions, economists, banks and central banks, let alone a big challenge to citizens, who are to be made to understand, that monetary systems are understood, while they are not.

3.1 Bill of Exchange

A BoE is a written unconditional order or disposal contract as part of an obligation contract by one agent (the drawer) to another agent (the drawee) to pay a specified amount of money to a third agent (the payee, often also the drawer) at a specified time or on demand (then often at a discount by an interest rate, the discount rate). These are the roles of agents in a BoE contract:

1. Drawer: The agent who issues the BoE and unconditionally orders the payment.
2. Drawee: The agent on whom the BoE is drawn and who is ordered to make the payment.
3. Acceptor: The drawee becomes an acceptor by signing the BoE as the obligation to pay.
4. Payee: The agent to whom the payment is to be made.

The BoE is usually negotiable, meaning that it can be transferred to others by endorsement. BoEs are used in trade and commercial transactions to provide credit and to facilitate payments between buyers and sellers. According to §1 WG ("Wechselgesetz", German law on the BoE), for any piece of paper to become a legal document of a BoE, it has to contain at least eight pieces of information: the written word "Wechsel" (German word for BoE), the amount of money to transfer, the name of the drawee, the maturity date, the place of payment, the name of the payee, the date and place of issuance and the signature of the drawer. Analogous legal requirements are to be applied to digital versions of the BoE, with the place of issuance might be unclear for a digital BoE.



	Macro Accounting			
	Micro Accounting		Micro Accounting	
Exchange	**Booking**	**Comment**	**Booking**	**Comment**
1. Delivery of Product	1a	Seller sells Product to Buyer	1b	Buyer buys Product from Seller
2. Creation of BoE	2a	Seller issues BoE to Buyer	2b	Buyer accepts BoE from Seller
3. Cash for BoE	3a	Seller sells BoE to Seller Bank	3b	Seller Bank buys BoE from Seller
4. BoE for Liab.	4a	Seller Bank sells BoE to Buyer Bank	4b	Buyer Bank buys BoE from Seller Bank
5. BoE for Cash	5a	Buyer Bank submits BoE to Buyer	5b	Buyer pays BoE

Figure 2: Booking Bill of Exchange

Figure 2 shows the macro and micro bookings of a purchase of a product "paid" by a BoE to be later cleared by bank transfers or settled by money. The BoE is designed to shift payments in time into the future while the product is already delivered today. In times of gold standards, credibility and settlement possibilities were accounted by the credit loan to shift in time also the delivery of a good, mostly gold. In the bookings of the BoE there are four micro accounting systems of Seller, Buyer, Seller Bank and Buyer Bank involved. There are five events and five macro bookings each consisting of two micro bookings, making up ten micro bookings. We have coloured each two micro bookings of one macro booking in the same colour.

The MoMa in Figure 2 does not show the actual numbers being booked, they have to be discounted for the time involved. Bookings are time indexed towards the full amount of the contract at maturity date. The geometry of debts in the MoMa in Figure 2 is denoted by the blue, green, grey and yellow arrows: debt is first recorded in bookings 1, due to the agreement of the seller to allow the buyer to pay the product later on. Debt is established between buyer (debtor) to seller (creditor) against the direction of the transfer of the product. In bookings 2 receivables and liabilities are prepared to be transferred by the BoE. This is done by creating a BoE. Since a BoE is a negotiable instrument, the claims of receivables and liabilities can be transferred by the BoE. In both double accountings, the bookings 2 are mere changes of the types from general receivables and liabilities to a specialised subtype, the receivables and liabilities from a BoE. The BoE then is transfered from seller to seller's bank where seller stays creditor, now in the form of credits at seller's bank and not in the form of the BoE, while seller's bank is now having more liabilities to seller but also more receivables from the BoE. Then the seller's bank transfers the BoE in bookings 3 to buyer's bank and they swap liabilities and receivables. Finally,

in bookings 4 buyer's bank presents the BoE to buyer to be settled by being exchanged for credits of buyer at buyer's bank. The obligation contract of the purchase and the debt cycle ends. The BoE as a paper documenting the disposal contract to transfer "money" can be deleted. The debt cycle is actually a vortex in time shown in Figure 6 in a MoMa of a macro economy.

We have not called the credits of seller and buyer at their bank to be money but "money" because there is another step involved in the MoMa before money is actually transferred. This is the case when the banks clear their mutual receivables and liabilities in their accounts at the central bank possibly followed by a settlement in paper money or more likely by a settlement in their deposits at the central bank which is guaranteed to be exchangeable to paper money which then is a paperless or non-physical money transfer. Macro accounts are involved in these events and bookings are to be compliant with macro invariances. Figure 6 of the MoMa vortex will highlight that money does exist first of all for debts being repaid.

An interesting feature of the BoE is that it can also be used as a *substitute* for payments by money. For that we take a look at a special kind of a BoE called Sola. A usual BoE is created by the drawer (like a seller, who is usually also a payee; a creditor). He sends the BoE to the drawee (like a buyer; a debtor). He signs it, by that accepts the debt (becoming an acceptant) and sends the BoE back to the drawer. The Sola simplifies this sending around back and forth. The drawee creates the BoE, signs it and sends it to the payee, that's it. This BoE with a simplified form of the hand shake is called a Solawechsel, or simply Sola or financial BoE.

There are two types of Solas (same for BoEs): with and without endorsement. An endorsement list is used when the original drawer (and payee) is transferring the Sola (by being a payee) for example for paying his supplier (becoming a payee and creditor). The seller must sign on the endorsement list which may contain more endorsers (and debtors like the original drawee). An endorsement occurs when the payee of the BoE transfers the right to collect the payment to another agent. This is done by signing (endorsing) at the back of the BoE. At maturity date, if the drawee does not pay and the "Wechselprotest" process is announced to some court, the holder of the Sola can claim money from anyone on the endorsement list or the original drawee, they are jointly liable. If one endorser is asked to pay the money, he can claim it back from another one before on the list. Ultimately, since the original drawee has to pay, if he can not, the first who has accepted the BoE for a payment has to size assets of the drawee. Hence, the credibility of the BoE increases with the length of the list. This is why the endorsement list can be called the *blockchain of the renaissance*, which is about the time when the BoE was developed into its modern form. Without endorsement the transfer of the Sola is anonymous like a means of payment and only the original debtor, the drawee, is liable.

The endorsement list is another example of a chain of contracts which can be disentangled in a recursive way by the principles of separation and abstraction once restitution claims are due in a protest situation of a failed BoE. Actually, the disentanglement of restitution claims in case of bad banks pose similar type of problems. A bank is not that simply closed down like a usual bankrupt company since a bank is situated within a network of debts and restitution claims are accordingly based on chains of debt contracts which are difficult and complex to be restituted.

The anonymous usage of a Sola is a way to think about money in gold and fiat standards, as we will see in the section on central banking. A buyer could use a Sola without endorsement to pay his suppliers, if they agreed on such a contract. The BoEs and Solas can be freely created, exchanged for money, cleared with other claims like other BoEs or Solas and finally destroyed, once the debt is repaid.

3.2 Schumpeter Entrepreneur

The basic building block of MoMaT is an investment, defined by Kruschwitz as a time series of payments, starting with a payout of the investor to the producer who pays the sellers of the resources and labor, followed by an agreed upon, expected repayment scheme in the next periods

until the end of the complete repayment of the investment loan. *Money is not memory* as in [13] but by the separation principle, the memory are the contracts about loans and the history of the investment and repayment processes. Figure 3 shows an investment on the left hand side as the red dotted line of expected repayments. Depending on the actual, not expected, success of the producer at the markets in selling his products and commodities, he may or may not match the repay needs: expected needs as red lines and actual repayments as green lines. This creates additional liquidity and money demand or supply. *The entrepreneurs ride to the risky success on debt*, is a saying that we have already attributed to Schumpeter.

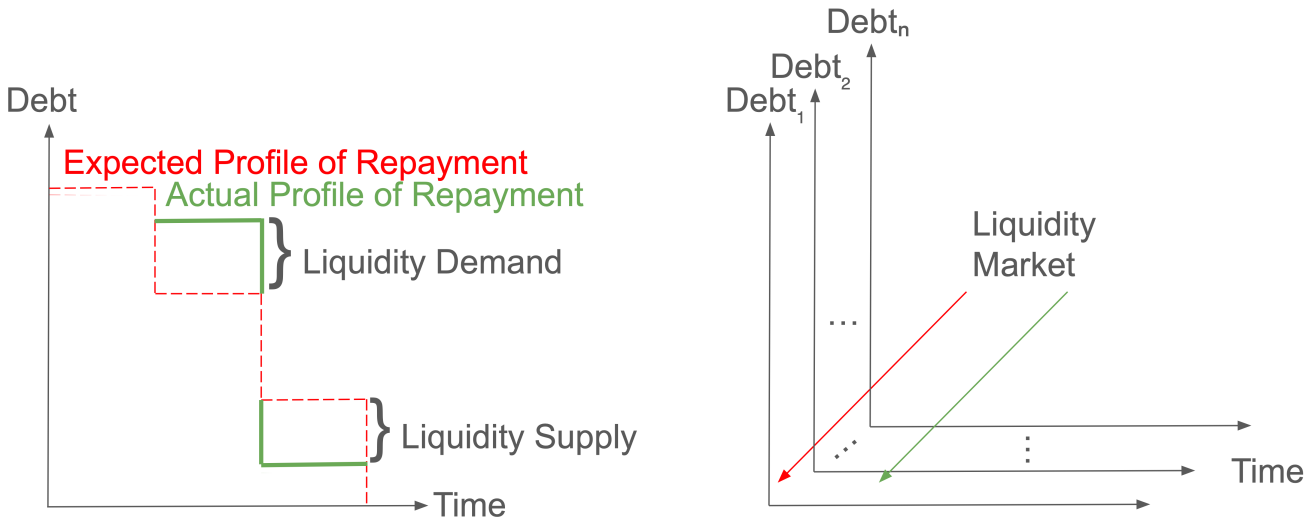


Figure 3: Kruschwitz Investment of an Schumpeter Entrepreneur

Since there are many over and under expected actual repayment events, liquidity markets emerge to match demand and supply. The right hand side of Figure 3 may be taken as the daily closing interbanking wholesale liquidity market. It is quite a Walrasian market with an auctioneer. The discussions in prevailing money theories, as in the quote of Hellwig in the introduction, is according to MoMaT not whether money is traded at markets - it is - but whether money is created at markets - which is not the case. The central banks can create money, not by some market but by free will, as much as they want to. However, a central bank can create money but banks need to be willing to get indebted at the central bank in order for the money to be actually used. This happened in the aftermath of the 2008 financial crises. Central banks created money but banks were not willing to use it for investments. Also, it should be a suitable amount of money created, which in MoMaT, as in usual monetary theory depends on the activities in the economy, the banking sector and supply chains where liquidity demand and supply is data in enterprise resource planing (ERP, like R/3 of SAP) and accounting systems.

3.3 Banking

In fiat systems banks cannot create money as the ultimate means of settlement and payment just like central banks can neither freely create gold in gold standards. Since discoveries of new gold are more or less random and most of all do not respond to the needs of monetary policy, new forms of claims like the BoE or deposits at banks, created by banks, have been invented. A useful process of modelling towards the MoMaT was *economic archeology*: we go back in history and look at monetary functionalities implemented with the then prevailing technologies. This functionality is to be abstracted and transferred into modern times with the core question of *how to implement this functionality* with the help of modern technology, possibly in a better way.

The role of banks can be seen in Figure 2, as the simplest case of investment in supply chain finance. When seller's bank bought the BoE, it actually did a risky investment. Until the BoE is

sold to the buyer's bank, seller's bank is in risk because if the buyer go bankrupt the BoE goes to protest.[6] Seller's bank is actually in risk until the obligation contract is fulfilled with the BoE of the disposal contract being paid by the buyer. Until the obligation contract of the product delivery and the fulfilment by the disposal contract of the BoE, by payment in the future, the seller's bank is jointly liable with all endorsers and the drawee. The loss from the investment in the BoE would need to be disentangled by restitution claims among the endorsers and drawee. All risky investments of a bank taken together must balance the returns and losses. If the bank cannot repay its debts at the central bank in terms of money, in which the bank is indebted, then the bank goes illiquid which for banks means bankruptcy. The role of the interest rate is an insurance premium which first and foremost has to balance the losses and revenues from investments before interest payments turn into profit.

For the case of keeping the macro invariances and the network of debt relations consistent we need to make sure that losses from investments are booked appropriately as consumption (costs without returns, negative profits). These bookings are usually done by money, clearing of claims or deposits at banks by realising a loss in the profits and losses of the financial statement. The profit and loss account is the first buffer that preserves the global identity of receivables and liabilities in the risk sharing hierarchy. As soon as the risk absorption capacity of a company is no longer sufficient to cover the write-offs of losses, which is equivalent to bankruptcy, the second safety buffer comes into play, whereby the banks have to write off the loan volumes of the bankrupt company that cannot be recovered despite collateralisation and thus have to record a negative entry in their profit and loss account.

However, whether these claims and money payments in between the hierarchical levels of micro, meso and macro agents and their accounting systems are in line with macro invariances of debts and payments is a fundamental question of monetary theory and specification of MoMa in MoMaT. The central macro invariance of debt relations is the equality of the sum of receivables and the sum of liabilities. The macro invariance of payments is the equality of the sum of expenditures and the sum of revenues. These invariances must be consistent in that the invariance of debt is about state variables while the macro invariance of payments is about flow variables. These are invariances because they must be preserved under MoMaT in MoMa systems. It is not a behavioural or empirical theory but instead a theoretical necessity for a practical and consistent distributed accounting system.

At the micro level of production, claims and debts are created and exchanged and cleared and settled with money and other claims. At the meso level of banking, receivables and liabilities are created in between banks, they are cleared and settled at the central bank or by direct payment. At the macro level of central banking, money, commands for money like deposits of banks (and sometimes companies) are created. Central banks may also buy, i.e. refinance or invest in BoEs of companies. Deposits at central banks, are commands for money and cleared against each other. Central banks create money, exclusively by their defining and monopolistic authority. Money creation is booked in the central bank's micro accounting but money is unfolding its functionality as a means of payment in the macro accounting among the money users and their micro accountings.

3.4 Supply Chain Finance

Money exists, according to MoMaT, as a functionality for SCF, like in the simplest one in Figure 1. Accordingly, economic applications of MoMaT apart from monetary policy are financial services in supply chains like liquidity and treasury management. In section 4.1 we will list the infrastructures being public goods which central banks need to provide for SCF.

[6]In former times, a BoE which was not paid in due time by a company, meant that the company had to be closed. Not paying in time was considered a deadly sin. Therefore, usually a liability to BoE that the drawee was not able to serve, had to be refinanced by a loan with a prolonged maturity date.

A digital BoE can be used in supply chains as follows. For example, Volkswagen (VW) offers to its supplier of tires, let's say Goodyear (GY), a flexible way to be paid but at latest after 30 days. Before the digital age, GY had the choice to be paid by VW in the first week of the 30 days by a price reduced by a 3% discount. Alternatively, GY could sell their invoices to factoring companies whenever they like, to be paid immediately. The factoring company goes for the money from VW, at the maturity date in 30 days, or later, if VW happened not to pay in due time. In both cases, the factoring company or the bank is paid by a discount of the command for money as the content of the invoice being sold by BoE or GY to the factoring company.

A digital BoE based service called Dynamic Discounting (DD) of a SCF platform[7] may orchestrate the processes involved, offering these services in a more flexible and efficient way. GY gets a button in their ERP system, popping up next to the invoices to VW. Once GY decides to be paid, an accountant presses the button and "money" (commands for money) is transferred from VW's to GY's bank account. The processes in the background involve several automated digital processes. The request of GY is transferred to a digital depository which creates a suitable BoE. The BoE is bought by a bank and the dynamically discounted amount of money is transferred to VW which pays GY. The *dynamic* in Dynamic Discounting (DD) means that not a 3% fixed discount, only in the first week, is calculated to reduce the amount to be paid, but the discount is calculated at any point in time, taking into account the remaining time to maturity date. The bank then asks VW at maturity day of the BoE to be repaid the full, not discounted amount of money of the invoice. The bank earns the discount as a distribution channel for their products, loans, and the BoE is deleted in the digital depository since the debt of the obligation contract about the delivery of tires is fulfilled by the disposal contract on the BoE and on the bank deposits for the "money" transfer.

The are various advantages for VW to finance the early payment not by a general loan or via a factoriser to GY but via the BoE based DD: 1. a reputational advantage for VW, GY will not see that VW lacks liquidity by being paid by VW and not some bank. 2. The BoE involves a standardised document handling instead of loans, factoring contracts or the like. 3. The BoE can be exchanged by the bank with other banks, again in a standard process, without the need to recur to the underlying exchange of products, even so it may be witnessed by the attachable invoice. 4. The bank can use the BoE to pay its own debts from obligation contracts since the BoE is also a substitutional means of payment. 5. The BoE, if transferred to GY, could also be used by GY to repay debts, say at a supplier of GY, if this supplier accepts a BoE of GY, which is likely the case since the BoE inherits the credibility of VW, since the supplier of GY can claim the money from VW, if GY were insolvent at maturity date. 6. But most importantly, the BoE has a lower risk and discount rate compared to the interest rate of a general loan.

The reason for the lower risk is that the BoE originates from a known, even attachable invoice to document the delivery of the product bought. However, the claim in the BoE is an unconditional payment duty, and by that it is not subject to the so called *verity risk*. This is the risk from verifying, possibly to the bad, whether the payment is claimable by checking the underlying obligation and reciprocal disposal contract, for example whether the product has been delivered or not, with or without an intact obligation contract (by the abstraction principle). The abstraction principle comes in handy in supply chains by decomposing a disposal contract over a chain into a chain of disposal contracts and restitutions.

However, the payment duty of a BoE is unconditionally due at maturity day since the BoE is an abstract financial instrument, independent of the status of the contracts to deliver the product. Hence, it is said that the BoE is documenting an abstract claim. Of course, the payment can be restituted by §812 BGB but at maturity date the payment is save which is of paramount importance for liquidity management. A failed BoE can be turned in at the court within a

[7]This section on SCF very much profited from many discussions with Markus Wohlgeschaffen, the product manager of the company Traxpay in Frankfurt. They are pioneers in the establishment of products and business models around digital versions of the BoE in SCF.

standard legally defined process ("Wechselprotest" by the WG and even BGB). A judge can then issue an enforcement title within minutes. On behalf of such a title the creditor can force the bank of the debtor to transfer the "money" immediately, as commands for money by deposits. This certainty of the process for claiming the money at maturity date results in a reduced risk of a BoE and a lower interest rate for discounting. These properties and processes have been developed in centuries of banking practise in SCF. It is the unique feature of the BoE being by that the ideal instrument for SCF, even more so in its digital form, currently being developed to be used in industry with central banks expected to follow. See section 4.1 for the infrastructure needed from central banks for SCF.

The DD for early payments can be accompanied by another, in a sense dual service, called Post Maturity Finance (PMF), which is similar to the offers of factorising companies. For example: VW lacks liquidity to pay GY at maturity day, 30 days after the delivery of tires. At this day, VW as a drawee and acceptor of a BoE sings it. The contract on the BoE can be done with VW's or any other bank. The bank supplying liquidity is the drawer. This contract extends VW's payment for another 40 days. The BoE, signed and accepted by VW returns to the bank. The bank transfers "money" (commands for money by deposits) to VW, VW pays GY at the original due day of the invoice (30 days after the delivery of tires) and VW repays the BoE to the bank 40 days later, i.e. 70 days after the delivery of tires.

A (digital) BoE can be used not only for liquidity demand but also for liquidity supply, which in DD and PMF was supplied by a bank. The liquidity can also be supplied by some company down the supply chain, for example by a retailer of VW, if they happen to have unused money and issue a BoE. These are the kind of services of a two-sided SCF platform with a macro accounting system (of double accounts of the customers or members of the platform) calculating optimal aggregation, risk and insurance premiums and timings of BoEs. These are the usual banking functionalities according to economic theory: banking is the business of transforming the quantity, risk and timings of the liabilities into those of the assets. For example small, not risky deposits of the bank's customers on the liabilities side of the balance sheet, due at any time are transformed into large, risky investments due in the future on the assets side of the balance sheet. Without such a platform each member of a supply chain needs a bank to process the liquidity exchange. With a platform only one bank is needed for the excess demand or supply of liquidity of the members of the platform who exchange liquidity among themselves. The SCF platform is like a bank with the strategy to cut-out-the-middle-man being the banks of the members of the supply chain.

Such a SCF platform is a service of DeFi using the BoE as a symmetric instrument being capable to support the direct exchange of liquidity among demanders and suppliers without intermediate banks. Actually this was the original use case of the BoEs since centuries at money and liquidity markets with transaction costs also encompassing shoe leather costs, meaning that a money trader was running from one market in Tuscany to another with a paper and pencil noting excess liquidity demand and supply of companies. These papers and the money traders were lying and sometimes sitting at benches, called Bank in German or banco in Italian, calculating the matching of demand and supply.

Another functionality of a digital BoE is in Deep Tier Supply Chain Finance (DTSCF). Small, not well known suppliers of known companies with known products like Apple and iPhones (or the supplier of GY in the example with VW), profit from the credibility of Apple by an improved access to loans. The supplier, so to say imports the credibility of Apple when holding a BoE or an invoice payable by Apple. A digital BoE automatically transfers credibility along the supply chain by the endorsement list, i.e. from the end product to the beginnings of raw materials. Credibility was transferred in the gold standard by credit loans (also called gold backed credit instruments, see Figure 5) which technically transfers ownership of gold over time. It is worth considering what innovative services a digital version of these instruments could bring about economically, i.e. in terms of producer and consumer rents.

Digitalising real entities may also affect the financial layer. A digital Bill of Loading (BoL) can

document the subdivision of the loading volume of ships transporting products in international trade. Digital BoLs simplify the verification that the volume of a ship is not subdivided beyond 100% (which may happen quite easily in modern harbours). This reduces the risk and interest rate for financing the load over the shipping time. Given that world trade volume was around $\$20\times10^{12}$ (trillions) in 2022 where about 2/3 are open account transactions, i.e. subject to delayed payments, even small reductions of interest rates and simplifications of document handling result in gigantic economic rents for consumers and producers, enabled by innovative uses of digital BoEs and BoLs.

The opportunity for services around digital records of documents, including a digital BoE, arose lately in UK adopting the Model Law of Electronic Transferable Records (MLETR) of UNCITRAL (the trade division of the UN) by the Electronic Trade Documents Act (ETDA) in 2023.[8]

4 Macroeconomics

Marx's major argumentation line against capitalism implodes in MoMaT: *money is not created from money*. Similarly, the confusion of *interest payments for interest payments* vanishes once interest rates are accepted to be insurance premiums calculated from cross sections of parallel investments. We have already hinted several time at this issue and we are now ready to see the full macro economic picture of it.

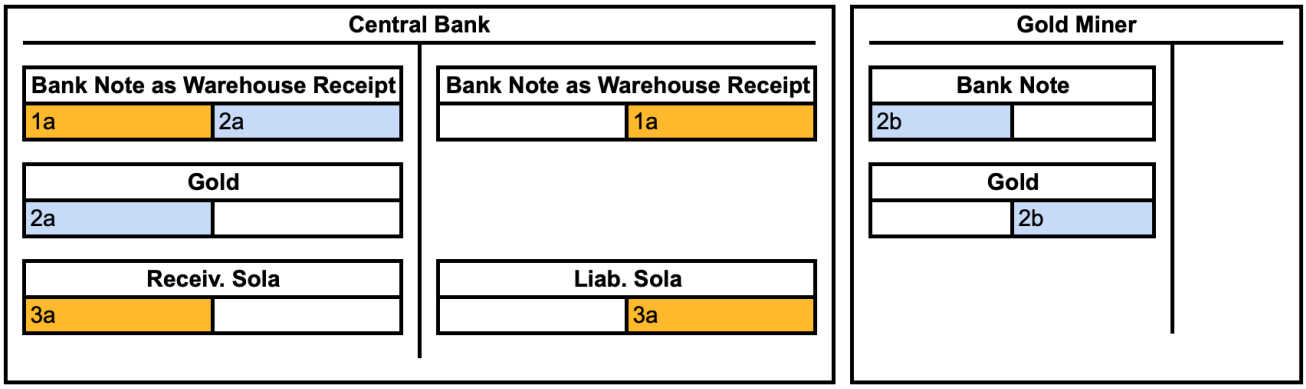
The misconceptions arise because there is only seemingly a money circulation while the really important dynamics of debts as receivables and liabilities are more appropriately taken to form a vortex rather than a circle. A piece of paper of money is used in different debt relations over time which can erroneously be taken to suggest that circulation of money is important. The confusion arises due to not taking into account the legal principle of separation. We really care about the dynamics of debts and do not want to confuse it with circulation of money in networks, the means of payment or the means of debt repayments. The geometric point is that a vortex with a start and an end at different points in time looks like a circle, from the perspective along the time line, projected on some area like a paper of money.

Copernicus world view on our solar system with a static sun is fine in order to contrast it with the world view of Ptolemy. In case of money or Marx, a similar misconception was devastating, leading to communism as a theory of the distribution of the common product of labour and capital. Looked at the concept of the circulation of money through the glasses of political economics, communism provides a perfect argumentation line to conceal the redistribution of benefits to special interest groups, members of the one and only party or intellectuals with an argumentation line on high moral grounds to pretend fighting for the working poor. Another view is that without a monetary theory, special interest groups may let society bail out bad banks with accounted losses from investments while benefiting from returns on successful investments. Without a revolutionary tone: without a money theory we lack control on distributional policies for the GDP, bad banks or monetary crises. All kind of conspiracy theories emerge from the mysticism of money lacking a theory while all what money is actually about are obligation and disposal contracts informing us about the financial state of an economy. We want to clarify these issues by first looking at the creation of money in the gold and fiat standards.

4.1 Central Banking

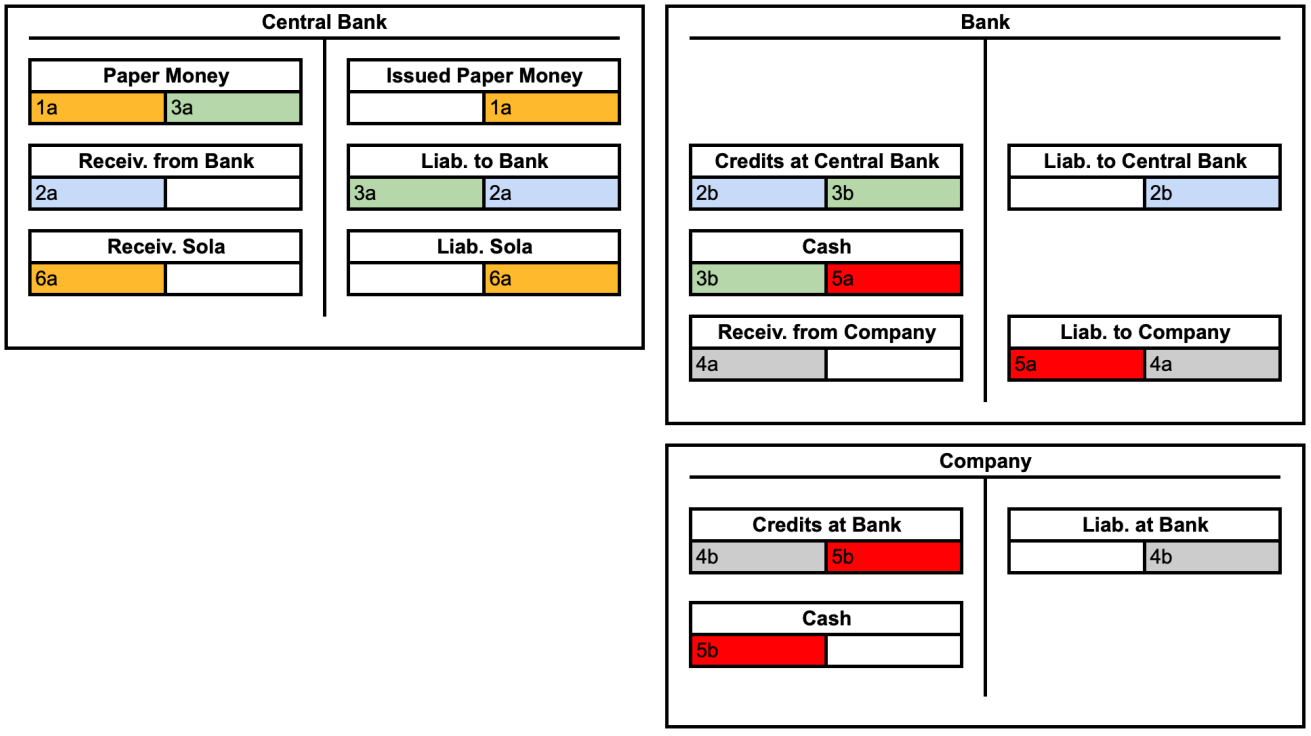
Before central banks emerged in the 17th century, banks based their bank notes on gold, silver or similar valuable resources. This made bank notes into a debt instrument, meaning that the bank owes gold to the holder of a bank note. The Wisselbank emerged ("Wissel" is the Dutch word

[8]See the Bills of Exchange Act of 1882 [20], the German Wechsel Law of 1933 [25], the MLETR of 2017 of UNCITRAL [32] and its UK adaptation ETDA [21].



	Macro Accounting			
	Micro Accounting		Micro Accounting	
Exchange	Booking	Comment	Booking	Comment
1. Issue Bank Note as Warehouse Receipt	1a	Prepare issuing Bank Note		
2. Delivery of Gold	2a	Central Bank buys Gold from Gold Miner	2b	Gold Miner receives Bank Note
3. Issue Sola	3a	Like Issuing Sola = 1a		

(a) Issuing Bank Notes in Gold Standard



	Macro Accounting			
	Micro Accounting		Micro Accounting	
Exchange	Booking	Comment	Booking	Comment
1. Issue Paper Money	1a	Central Bank can pay out Paper Money		
2. Central Bank gives loan to Bank	2a	Central Bank increases Receivables from and Liabilities to Bank	2b	Bank increases Credits at and Liabilities to Central Bank
3. Centra Bank pays out Cash to Bank	3a	Central Bank reduces Liabilities to Bank for Paper Money	3b	Bank reduces Credits at Central Bank for Paper Money
4. Bank gives loan to Company	4a	Bank increases Receivables from and Liabilities to Company	4b	Company increases Credits at Bank and Liabilities at Bank
5. Bank pays out Cash to Company	5a	Bank reduces Liabilities to Company for Paper Money	5b	Company reduces Credits at Bank for Paper Money
6. Sola creation	6a	Sola creation is like Paper Money creation = 1a		

(b) Issuing Paper Money in Fiat Standard

Figure 4: Money Creation at Central Banks

for "Wechsel", the German word for BoE) in Amsterdam in 1609 [36]. It is considered as the first central bank by providing the first paper less transfer of cash like the transfers of deposits at the central bank which is as good as paper money transfers but without paper money. We have seen that we have to be utterly careful what we are talking about with money, paper money, cash, transfers by deposits or "money". We have also seen that the abstraction and separation principles help to sort out the types of the involved concepts and financial instruments.

The Wisselbank's role was in international payments to, in and from the European colonies being a supply chain with huge investments in the international shipping projects. Resources had to be paid, the shipping times to the eastern colonies were long, the risk of failures was high and the revenues from sales of the imported goods have been divided among the investors. The Wisselbank issued something like an international reserve currency, the bank guilder. From this bank, central banks emerged like the Bank of England a century later which managed national currencies, like the Pound based in Sterling silver. The central banks were indebted in gold (or silver) documented by the bank notes. In 1971, the gold standard was abandoned by Nixon, such that there was only a paper left, the Dollar, without any claims on gold at the US central bank. This caused major issues in monetary theory and practical confusions, for example how to book paper money creation at the central bank's double accounting system, whether we should return to a gold standard again, base money on crypto currencies or what crypto currencies are first of all. Cryptos brought us blockchains which are digital simulations by distributed databases of secured physical property right transfers. In digital computers with copy-and-paste for free, it is really complicated to make sure that data is not corrupted. Quantum computers do not need to simulate the property right transfers digitally but they secure the transfers by quantum physics.[9] Blockchains also implement non-deletable data and free entry to them for reading and writing data. The applications to take advantage of these properties is what the economics and business models of blockchain products are about.

Figure 4 shows money creation in gold and fiat standards. Figure 4a is about a central bank issuing bank notes in a gold standard. The legal means of payment is gold and in order to not transferring gold in each payment, bank notes were invented to be used instead. The central bank prepares to add more means of payments, gold, to its vault in booking 1a by creating and booking a warehouse receipt. We have denoted the warehouse receipt in booking 1a already as a bank note which is what it actually is. The central bank then buys gold in 2a from a gold miner in 2b. The warehouse receipt in 2a is a bank note in the booking 2b of the gold miner. The bank note is a claim of the gold miner, as the holder of the bank note, to the central bank on gold, it is actually a right to command the bank to hand out gold. We can also tell a much simpler story: the gold miner brought gold to a warehouse, the bank handed over a warehouse receipt - actually a bank note. The bank note can be transferred in purchases as the content of one disposal contract with the other disposal contract being about the product, with both disposal contracts being parts of an obligation contract of the purchase. The gold miner or any other holder of a bank note, like the seller of a product, can claim gold from the central bank.

Nixon abandoned the gold standard in 1971 by stopping to guarantee the exchange of bank notes of Dollars for gold. The fiat standard came into existence with paper money being what contracts are about since then. An important point is that bank notes represent claims against a central bank documenting a debt of the central bank. Paper money does not represent a debt of the central bank because paper money does not have a claim content. As a consequence the opposite booking entry "Issued Paper Money" or "Money in Circulation" can not represent an

[9]Quantum computers are built on *linear*, aka *resource aware logic*. It means that in linear logic sentences are used up in proofs. For example, A is a fact in the set of available resources to reason with, like $\{A, A \to B, A \to C\}$ where arrows denote modus ponens, stating that from A we can conclude B and C. We are not allowed in linear logic to use A twice to conclude B and C. Instead, A needs to be explicitly copied before using it to proof C (with the cloned A) after having proved B (with the original A). In linear logic, logical resources vanish after being used up in proofs, just like what happens in production in economies.

obligation either: without a claim there is no obligation. These are the two main models of monetary systems with the complication of a fractional banking system.

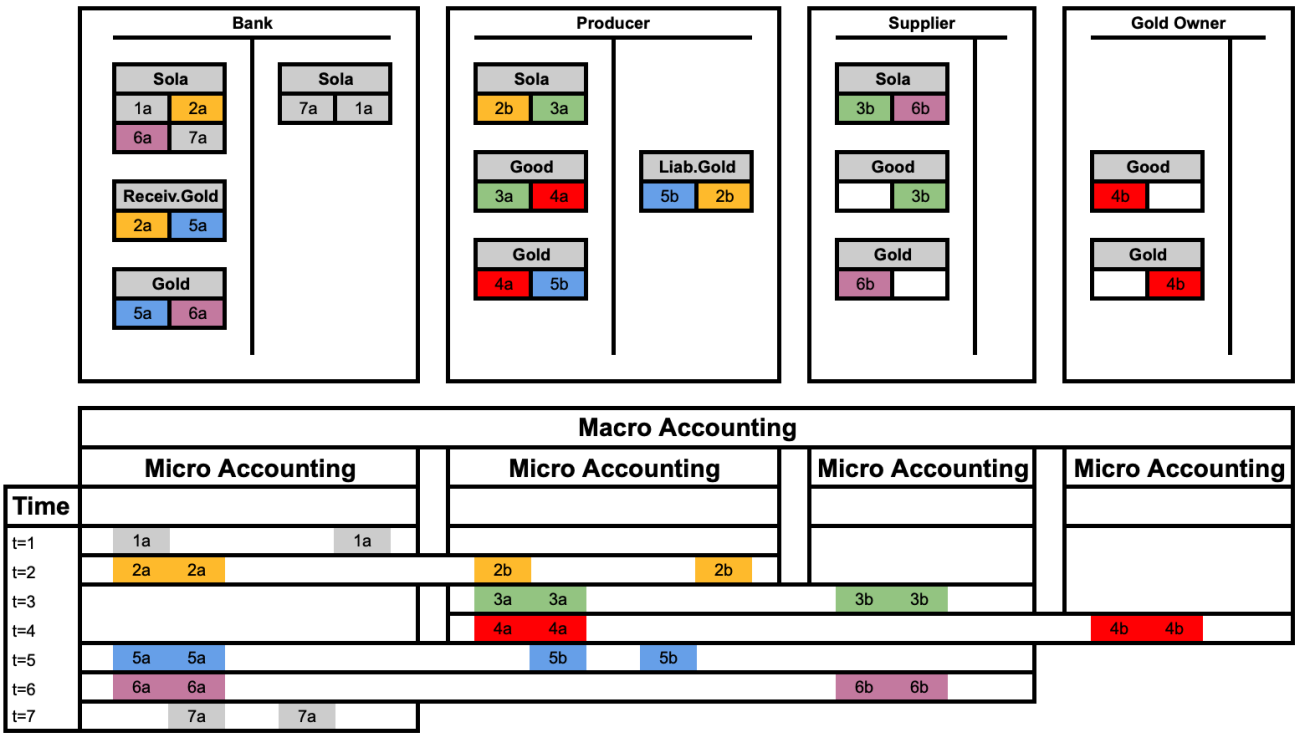
How to not be confused about all that? Mathematicians and computer scientists invented type theory for deconfusation of the Russell paradox: "Who shaves the barber, who shaves all those who do not shave themselves?" So, we ask: "Who provides money to the bank which provides money to those banks who cannot provide money to themselves?" It is the legal authority provided by a consensus of money users to the one and only, by that central, bank. This is similar to the way mathematicians have solved the puzzle of set containing alls sets which, if they allowed to include themselves, leads to severe contradictions in the foundations of mathematics (like in the Russell paradox). This simplest way to solve or actually to avoid the puzzle is by declaring the existence of a hierarchy of ever encompassing sets. In the same way, we lifted the central bank into the unique position and allowed it to be the only one to issue money. The problem seems to be solved, which is not the case since the same question arises again at the next level: What is creating units of accounts, like Euros and what is creating the central bank which is managing the new currency? Do we need to introduce another layer, like a bank or ministry of central banks? What issues arise on the next level, if there is any? What type is the creation of types? What are exchange rate regimes among currencies? It seems that we need a largest fixed point of this hierarchy just like the non-well founded sets approach to the Russell paradox.

The economic origin of central banking under the gold standard, was a control instance of a community which insures their members against illiquidity in gold. Before, each bank was allowed to create bank notes. Then they agreed to have one central bank, to issue only one bank note, with the central bank managing the pool of gold. Banks were getting liquidity from this central bank. The same thought was underlying the IMF, as an assistance during liquidity problems. The same is with BoE which in Germany is called the instrument for liquidity assistance ("Liquiditätsbeistand") among companies.

A central bank is akin to a simple solution of the Russell paradox by hierarchies of sets that may contain all sets but the one on the next meta level forming a sequences of universes of sets of sets of sets etc. Until we have a currency theory for that as a proper type theory, for mathematically understanding the type of creating types as the units of account, we can look at the Sola as a BoE principle to understand what is going on in a central bank when it creates money, by looking on the balance sheets and in the financial accounting. It is important to understand that money is not created in the balance sheet but in the financial accounting within a period of time. This then results in an increased "Circulation of M" that is reported in the balance sheet of the central bank.

In Figure 4a we see that creating a bank note is like a warehouse receipt in booking 1a and both can be conceptually thought as issuing a Sola as in booking 3a. This Sola would document a command on gold against the central bank. In Figure 4b we see how a central bank issues money in a fiat standard. In booking 1a the central bank uses its right to create infinite amounts of paper money. This booking documents the event that some printed paper money with an ID is registered by saving the ID in a database called "Issued Paper Money" which is the legal event making a printed paper with an ID into money, i.e. a legal means of payment. In booking 2 the central bank grants a loan to a bank and books it in booking 2a and the bank books it in 2b. The bank has now a deposit at the central bank. In booking 3 the bank withdraws some cash from the central bank, booked in 3a and by 3b at the bank. Now the bank is ready to do its job and invests some money in some company. For that it grants a loan in booking 4, by creating some credits in booking 4a at the deposit of the company, which books it in booking 4b. The company is now ready to withdraw some money in booking 5, booked in 5a at the bank and 5b at the company. The company can now pay its workers, for example. In booking 6a, we see that creating paper money at the central bank in booking 1a is similar to a Sola.

The Sola in 6a is only meant to clarify that issuing paper money can be though as if it was a Sola. There is a caveat to be aware of. The claim of the bank note, like a warehouse receipt



	Macro Accounting		
		Micro Accounting	Micro Accounting
Time	Exchange	Comment	Comment
t=1	Sola	Bank issues Sola	
t=2	Sola Gold	Bank Sola to Producer, has Receiv.Gold	Producer gets Sola, has Liab.Gold
t=3	Good Sola	Producer buys Good with Sola	Supplier delivers Good for Sola
t=4	Good Gold	Producer buys Gold with Good	Gold Owner sells Gold for Good
t=5	Gold Gold	Bank gets Gold from Producer, cancels Receiv.Gold	Producer delivers Gold to Bank, cancels Liab.Gold
t=6	Gold Sola	Producer delivers Gold for Sola	Supplier gets Gold for Sola
t=7	Sola	Bank cancels Sola	

Figure 5: Credit Loan

(or Sola), has a content, it is gold in the gold standard. In the fiat standard there is nothing to claim from paper money. Therefore, such a Sola as in booking 6a in the fiat system, would in fact not be a usual Sola which has a content to claim on. Accordingly, the central bank in the gold standard was indebted in gold and with the backuped gold, the overall debt was zero. Or put it this way: the liabilities are backuped or securitised by gold. Gold is not a claim or a liability but an element in the chemical periodic system. This property of gold, to be neither a claim nor a liability, is shared with paper money. The paper does not document a claim or command on anything. Paper money does not need to be backup by anything and only needs to be declared to be the legal means of payment.

The central bank in the fiat standard is not indebted to begin with. Just like in the gold standard the central bank is not indebted, netting bank notes and gold. The paper issuing central bank also does not need to go into debt since money is a means of payment for settling debt but it is not a debt itself. Accordingly, since Nixon, central banks cannot go bankrupt in terms of their own money in fiat systems. This is not a bug but a feature, gold is not needed neither any intrinsic value of money.

We can speculate that central banks emerged from insurances against illiquidity under the gold standard. For that *credit loans* (also called gold backed credit instruments) were used as the financial instruments for shifting in time ownership of gold and by that credibility. In Figure 5 we see the macro accounting bookings of a credit loan. In the usual BoE the payment is due in

the future based on the delivery of some other product today. In the credit loan the underlying product is also delivered in the future, here gold. The credit loan is probably better called *credibility* loan since the bank actually lends its credibility, here witnessed by gold, to the users of the credit loan. So, economically the credit loan is a financial instrument or model for booking, outsourcing, making tradable and shifting around credibility. This is dual to the BoE of shifting payments among buyers and sellers into the future.

Let's have a look at the bookings involved with the credit loan. The bank creates a Sola in booking 1a. In booking 2 the Sola is transferred to the producer who is using the Sola to buy a good, say a resource, from a supplier. Just as in the case of the usual BoE. In booking 4 the good of the producer is a commodity transformed from the good being a resource in booking 3. The commodity can be used to buy the gold for product at the gold owner. The producer can now deliver the gold in booking 5 to the bank as agreed upon in booking 2. In booking 6 the gold can be delivered to the supplier for the Sola. And in booking 7 finally the Sola can be deleted as the contracts are all served.

This instrument was used to shift the backup of gold in gold standard times. Nowadays, we can use this instrument to make sure that not too many financial liabilities, financial BoEs or Sola, are given away by the bank and to insure that the financial liabilities can be ultimately settled. The credit loan is by that a tool to maintain the books of to be settlementable. It calculates the predicate whether liabilities are backuped by gold, i.e. whether settlements and clearings are consistent over time, giving rise to applications of modal temporal logic.

In the gold standard the amount of means of payment was controlled by the amount of gold, silver or any precious metal. In the worst case this was influenced by new discoveries like new silver mines in South America. Later, fractional systems were developed starting from the credit loan instrument where banks were allowed to create substitutes for the means of payment, like deposits. A major risk of fractional banking systems are bank runs and not properly managed banks, the bad banks that have to be bailed out, ultimately by the central bank. However, paper monetary system are not subject to a risk of bank runs since paper money can be issued infinitely.

What is the one bank to book the losses from failed investments in central banks when creating currencies, units of account and book failed central banks? This is presumably a bank of central banks, then becoming the central bank of (former central) banks, creating and booking events in nominal exchange rate regimes. Today the European central bank is doing experiments with money issuing, national, former central banks and at the same time is wondering about popping up Target 2 accounts. This all suggests that MoMaT and MoMa needs some more developments before it may be used to ask questions about nominal exchange rate regimes like the European monetary union.

The amount of money and loans created should depend on the productive activity in the economy in need of debt in investments and networks of debt relations. MoMaT spells out explicitly the needs in SCF as the important variables to base money policy and supply on. In digital times data on these variables is readily available from the volumes of the receivables and liabilities of current invoices in the ERP and accounting systems of companies. Monetary policy as well as liquidity exchange on platforms will need to take liquidity demand and supply into (macro) account. Refinancing of BoEs has been monetary policy until the 1990s at the Bundesbank, the German central bank and the discount rate was a monetary instrument. It fell out of favour and instead the interest rate at the wholesale liquidity market called the interbanking market where banks trade liquidity was used as a monetary instrument instead. As of today, central banks also directly intervene at asset markets as a monetary policy.

In MoMaT the central bank's policy is threefold: 1. define the means of payment as money and issue it by loans to banks as credits at the deposits at the central bank exchangeable to money, 2. define a minimal credibility rating standard as a requirement for the loans in 1., 3. run the payment system and a macro accounting service with regulatory laws for financial intermediaries. A principle of monetary policy by MoMaT is in line with Bagehot: *in crises we*

need loans without bounds but at expensive conditions, while inflation is for the income policy of governments. Accordingly, there is no need to artificially produce scarcity in MoMaT as opposed to the monetary policy recommendations of usual monetary economics. The necessary volume of money in MoMaT depends on the level of settlement, i.e. how much claims and liabilities can not be cleared and settled by existing money supplied. Monetary policy under MoMaT is to refinances all BoEs of companies and banks in line with a credibility rating standard for SCF like the German Bundesbank which used to refinance BoEs of blue chip companies. The calculations in these markets might be taken to be homologies which detect holes in topological spaces where a hole might be taken to be a lack of liquidity. We will relate to the mathematics of homologies and sheafs for invariances in section 5. The homological calculations can be taken as the matching technology for this market of clearing claims and liabilities with the balance being money supply. Homologies might be the computational devices or models of the interbanking market, matching demand and supply of money and liquidity. Homology theory may be extended into an approach for automating monetary policy based on macro accounting within the old monetary economics topic of rule based versus discretionary monetary policies.

The central banks have been the lender of last resort in the gold standard. In the fiat standard central banks are lenders of first liquidity. The last resort has not been enough, when central bank themselves were short of gold, which is why Nixon had to abandon the gold standard. Instead, in the fiat standard, the central banks cannot run out of liquidity since they can infinitely issue money. If black swan events cannot be insured and bad banks occur then central banks can provide insurances as bail outs. These can be thought as discounting loss carry forwards and by that smoothing out investment losses as consumption, as costs without prospect for a return on the investment. The bail outs can be shifted to the highest possible level of the whole society as the top level risk sharing community running the central bank.

Central banks can provide the following infrastructures for SCF as part of projects like the infrastructure of the digital Euro [3]: 1. Payment systems for secure transactions, possibly quantum secured and anonymous. 2. Digital depositories for digital BoEs and other negotiable or formally not negotiable financial instruments. 3. Refinancing digital BoEs given some acceptable credibility threshold for the debtors. 4. Digital courts for failing BoEs for fast issuing of titles for an access to debtors' assets. These public goods are naturally to be supplied by central banks. The design of private SCF products on the other side, like the private goods of DD or PMF has to remain in the realm of private companies and should be left for a competitive evolution for the best demanded products.

4.2 Monetary Vortex

In Figure 6 we finally show that *vortices of debts* and not *circulating money* is the important phenomenon to take care in MoMa and MoMaT. We see balance sheets and bookings of macro accounting of a Central Bank and the Bank, Company, and Household sectors. Resources and products, liabilities and money are used, created, exchanged and deleted during the productive exchange dynamics of economies.[10]

At t=1.1.2001, in the very right column, the balance sheet of Household shows the resources of the economy. Without liabilities, the assets are equal to wealth or equity E:=A-0. Then money is created in t=2001 out of nothing at the Central Bank. This is a singular booking in the micro accounting of the central bank as there is only the central bank's micro (double) accounting involved in this sovereign act. It is as if a central bank in a gold standard created gold from a costless alchemistic process. But we do not need gold or any value attached to money. Paper is

[10]In Figure 6 points in time are denoted for example by t=1.1.2001. We list balance sheets which are records of state variables at a point in time. The bookings listed under the balance sheets are bookings in financial statements about flow variables recording changes in state variables and are related to a period, like 2001. In the text we will also use t=2001 to denote the period of the year 2001 from 1.1.2001 to 31.12.2001.

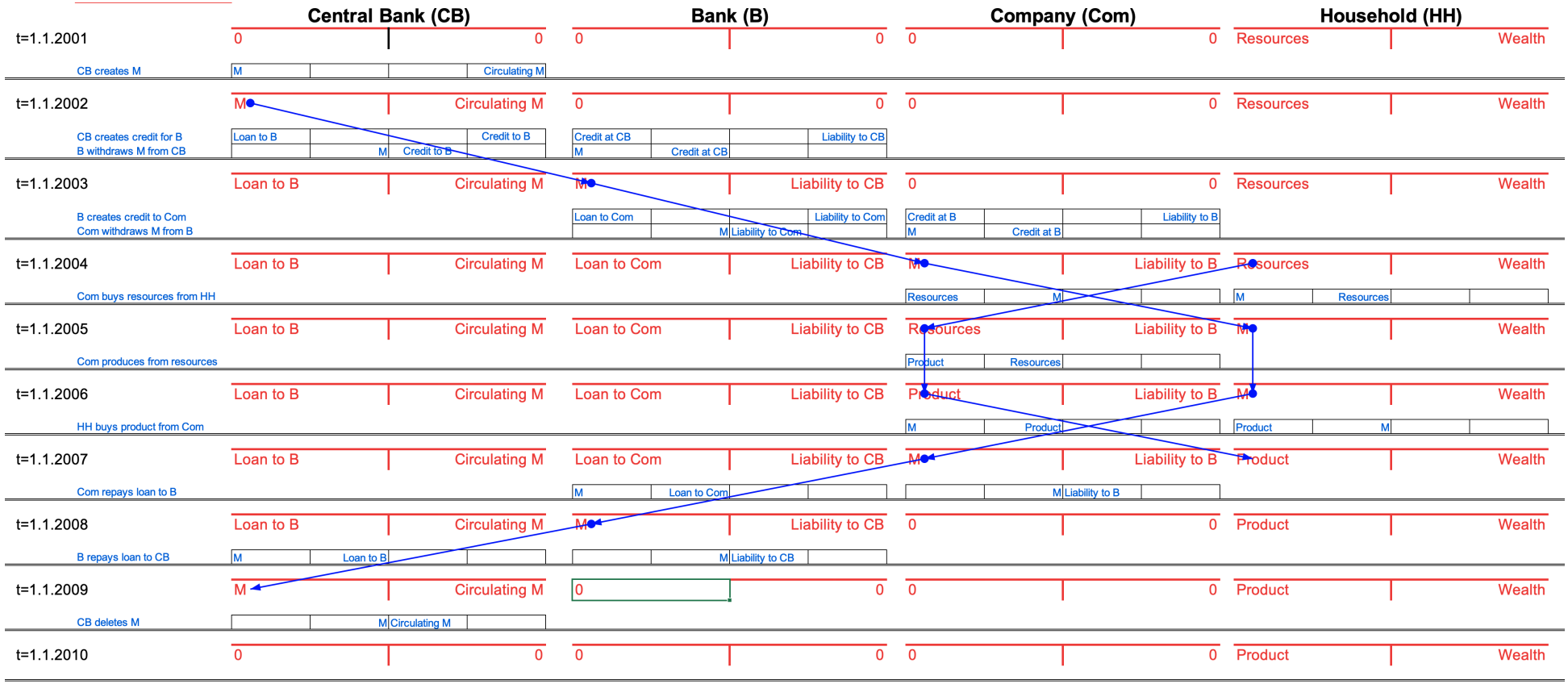


Figure 6: Monetary Vortex

good enough for representing amounts of debts measured in some arbitrary monetary units.

The money creation results in a balance sheet at t=1.1.2002 where cash M is booked on the active side and "Circulating (money) M" on the liability side. "Circulating M" is essentially the numerical amount of paper money registered with its ID in a database. The central bank is ready to operate, i.e. granting credits as loans. In t=2002 the Central Bank creates credits for Bank and books it as a loan. Since the central bank has already created cash, the Bank can withdraw cash from its deposit at the central bank. With the paper money at hand, the Bank can credit a loan in t=2003 to the Company, withdrawn as cash M. In t=2004 the Company buys resources from Household for cash M and produces the product in t=2005. In t=2006 the Household buys the product for cash M from the Company. In t=2007 the Company repays its loan by cash M at the Bank. The Bank repays in t=2008 its loan at the Central Bank as well with cash M. In t=2009 the Central Bank destroys the paper money and in t=2010 assets and liabilities of Central Bank, Bank and Company are back to zero. The difference between t=2001 and t=2010 is that resources have been transformed by companies into into commodities, the GDP, at the Household. This is a labour sharing economy at work over time with the aid of vortices of money and debt repayments in MoMa.

The monetary vortex in Figure 6 is tracing the cash M in the balance sheets from creation to deletion at the central bank. The physical vortex is tracing the transformation of resources into products in the opposing direction of the transfer direction of money. If we had a look along the time direction from top to down in Figure 6, we would see debts, money, resources and products floating in circles, like in Figure 1.

Money and debt bridge the time gap of production and help to transform resources into products and to swap ownership of resources and products of the GDP. The swap might look like a circulation of money and products but is actually a vortex of resources and products intertwined with the monetary vortex being the important one for monetary theory. What is important is that since all obligation contracts are fulfilled after the debt has been repaid in t=2009 money can be deleted. If the pieces of paper of money are used in the next debt processes it looks as if money circulates.

If money papers circulate over parallel debt processes being partly repaid, intervened with swapping of deposits or BoEs then keeping track of consistency conditions over hierarchical layers and agents becomes a complicated affair. Legally, we are well prepared to cope with the conceptual confusions of monetary theory by the principles of separation and abstraction in obligation and disposal contracts with property transfers. Economically, we need to cope with *the challenge of monetary theory* [9] as attempted by MoMaT. Mathematically, we need to keep track of consistencies and resolutions of inconsistencies in between local and global structures proposed to be modelled by sheaves and homologies. Computer scientifically, we need a software architecture for a potentially gigantic world wide web of debts and wealth in MoMa systems which we propose to be implemented by open games.

Depending on how risk and failures of investments are allocated in the banking sector and how resources, work, products, investments and insurances are paid from t=2002 to t=2009, we get some distribution of the wealth and GDP in the household sector in t=2010. We see how money is a technology for organising the division of labour and for the distribution of risk and the products of the community using the monetary system. For the allocation of risk, we need to model stochastic processes, as the investment processes, with some probabilities for failures of investments. Subsequently, a systematic accounting of the losses of investments as consumption or costs without returns by the investors. We should not book failed investments into intransparent bad banks, without a theory and ultimately bailing them out by the taxpayers or their ultimate bank, the central bank, without an explaining politics.

We can now give an answer by MoMaT to *what is a currency?* Why do we want to have more than one money on earth: *Currencies are a way to modularise the sharing of risk and GDP.* People in Ghana want to distribute their GDP according to the monetary policy of Ghana and

not according to the ideas of the central bank of the USA, if the Dollar was used in Ghana. At the top level of the world, the MoMaT is for managing the distribution of the world GDP, risk and its relation to a world reserve currency which does account for the world debts and payments intertwined with the supply chain of international trade for exchanging resources and products.

The dynamics of debts and money and resources and products of the GDP as vortices is akin to our solar system's planets in Figure 7, see the movie at [40]. The planets are not really circulating

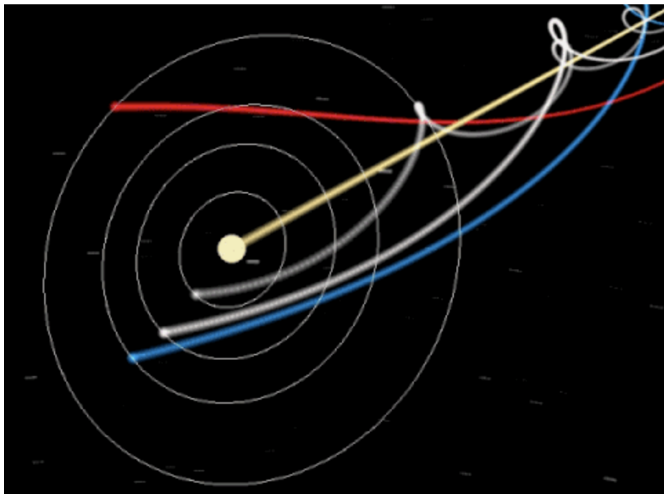


Figure 7: Planetary Vortices

or ellipsing around the sun, but actually move in vortices, following the sun on her movement at a speed of about 70,000km/h around the center of our galaxy. The dynamics of the planets look like a circulation around the sun, if the planetary paths are projected on a 2D plane through the sun, i.e. if the space and time direction, where the sun is moving in, is neglected.

This is similar to debts if projected on the concrete paper of money reused for repayments of different debts over time. We need to formally connect receivables and liabilities with expenditures and revenues in macro economic invariances over the hierarchical layers of economies and type the financial contracts appropriately in order to meet the challenges of monetary theory and policy.

5 Mathematics

The mathematical difficulty of monetary theory and policy is in formalising a MoMa like in Figure 6. It is a Markov model of the network of debt contracts. MoMa models these parallel, sequential, hierarchical and overlapping processes of debts, for all banks, companies, households and administrative bodies or agents in national accounting systems. MoMaT is needed for the appropriate money policy of money supply to support the exchange of receivable and liabilities by expenditures and revenues in all supply chains.

Macro accounting (MoMa) systems can be implemented by reduced form or Markov models. Smart contracts like BoE and the invariances involved can be modelled by open games and their bidirectional nature, discussed below. They can be smart as simple contracts in reduced form models taking into account the past only or very smart contracts as structural econometric models, like in [38], in MoMaT with forwards looking decisions.

The MoMaT macro invariances can be formulated within sheaf theory over automata of micro accounting, as in [24], being placed in a network of contracts among the agents involved. The analysis of the economics of the MoMa system is then about monetary policy and management by MoMaT. Questions to theory are system stability properties in parameter spaces, separations of the real (production) and nominal (monetary) sphere (*money is a veil*) or distributional effects of interest of monetary policies and institutions. In industrial applications cascades of clearing and settlement processes about risk and dynamics in supply chains with BoEs are to be implemented.

We propose open games [7], in Figure 8, with a probabilistic extension in [4], as a programming

Figure 8: Open Game

language MoMaL for implementing reduced form (or Markov) models of MoMa with MoMaT as data driven decision making for monetary policy to be analysed by structural econometric models, also implementable by open games.

The essence of open games is the bidirectional notion of time, from the past to the future from left to right and from the future to the past from right to left, shown in Figure 8. Decisions are one or more person games, composed from smaller games of n-person games into bigger n+1-person games by sequential and parallel combinators. In category theory these combinators are function composition and a tensor product with general categorical morphisms. Concatenations and juxtaposition are composing parallel and sequential processes in the visual 2D string diagrams of Figure 8. Hierarchical composition is by equational substitution as white and black boxing[11] as the quintessence of compositionality. Compositionality is achieved by functional, aka process oriented approaches. For that we can use category theory as the meta mathematics being a process oriented functional approach to mathematics itself [28]. Figure 8 shows a string diagram with the box meaning to be functions, processes or morphism in category theory and function types like open games of type

$$G : D \times C^* \to A \times U^*$$

The superscript * denotes contra variance of teleology, purpose, goal, utility, believes, forward lookingness or anticipation with arrows to the left whereas usual time flows from left to right. String diagrams are not mere representations but mathematical objects with a formal semantics usable for proofs in equations of visual formulas. Open games define a formal semantics for economic programs as a visual programming language for any forward looking model in economics. Hence, open game are runnable software with a formal semantics for provable properties of the software.

Since open games also capture back propagation, see [5], we can also represent parameter estimation and data science algorithms as open games. The modern implementation [12] of multi-agent AI systems are structural econometric models and can be represented as open games as well.

A compositional semantics for a programming language that implements MoMa has to show how the behaviour of the macro system arises from the interconnected behaviours at the meso level of the micro systems. Constraints arising from micro and macro invariances are to be booked in a transparent way. The behaviour is to be described as state transitions in initial algebras or behaviourally as infinite time series in the final coalgebras for the bookings of smart (aka algorithmic) contracts in macro accounting. Category theory is to be used for the semantics of the programming language MoMaL for MoMaT and MoMa. Category theory also reorganises mathematics [28] as a module system by its pure form, as a theory of structure and contextualisation. It also gives economics a semantics in coalgebras [29] or other forms of semantics.[12]

[11] *White boxing* means *making explicit* by showing the content of a *black box*. We can use the *white box* $a = b + c$ to show in $d = a + e$ the *black box* a as $d = a + e = b + c + e$.

[12] Open games have been a side effect of the search for a mathematics suitable for the Lucas' critique and for the general reflexivity of social sciences [34]. The work of Paul Graham [8] on the meta circular interpreter ended the second author's search for a suitable mathematical theory for reflexive structures in economics in about 2010. Graham implemented in Common Lisp the original work of McCarthy [15] implementing lambda calculus as Lisp, the traditional programming language of AI. Lambda calculus, higher order functions, universal Turing machines,

MoMaT proposes the economic concepts and structures for the semantics of an economic specification and programming language. The software implements debt and payment processes of monetary systems. A compositional theory of monetary macro accounting (MoMaT) needs to zoom in and out [37] at different behavioural levels of the systems. Systems are decomposed and composed while keeping the levels looked at in a consistent state. Global and local consistencies and inconsistencies of behaviours interact and we need to understand how local goals and obstacles can be resolved into a consistently behaving global whole.

The compositionality of open games arises because the components Data, Action, Context and Utility are open Games as well. Composed open games are again open games. Algebraic substitution of equals for equals is a first step and for monetary systems we need hierarchical transitions from local to global structures as modeled by the consistency conditions of sheafs. We need sheaves instead of "=" in simple equational substitution. Sheaves are generalising algebraic substitutions for behaviours of stochastic, infinite streams and approximations while preserving invariances over hierarchical layers.

The mathematical theory of sheafs allows to construct and study complex models as systems of equations and to combine bits of local information into a consistent whole, applications are ubiquitous [26], [27]. Two (local) eyes combine two fields of view into one (global) picture along consistent data in the overlaps. A (global) globe is a 3D map of the earth composed from (local) 2D sheets of paper in an atlas consistently spliced at the overlapping edges of each sheet. A system of equations has solutions to some variables in each (local) equation, which together form the solution of the whole (global) system of equations for all variables. Approximations of a (global) function are spliced at their overlaps together from pieces of (local) functions. A joint density, representing empirical models as Bayesian networks updates to observations by the Bayes formula in (local) subnetworks which communicate among each other until the whole (global) network is data updated.

A MoMa system takes into account agents like resource owners, consumers, producers, banks, central banks or the bank of central banks. They maintain local data in their micro accountings about the debt and payment relations among each other. We can form a network of these agents in the nodes and the edges being debt and payment relations appropriately spliced at their borders, the receivables and liabilities accounts being touched in contracts among the agents. The network of debt relations is then combined into the global macro accounting guided by macro invariances that form the consistency conditions. In sheaf theory the space where the global solutions live where all consistency conditions are fulfilled are called global sections. Accordingly, monetary policy has to take place in these global sections. In situations of crises outside of global sections monetary policy is about returning into the global section of usual conduct.

The name of a *sheaf* for these mathematical structures denotes an analogy to agricultural sheaves in Figure 9. The stalks are the data structures of micro accounting on top of the topology or network of the local agents while the sheaf of macro accounting is the global structure with the tying thread holding all together by the consistency conditions of macro and micro invariances. The ability to stand is an emergent property of the sheaf which is not present in the stalks.

Homology theory allows to calculate the obstacles to a global solution and macro invariances if problems and inconsistencies arise at some level or in some part of the system. In MoMaT this could be liquidity supplies and demands of local agents which are to be supplied or absorbed by calculating some suitable BoEs or other financial instruments at the micro, meso and macro level. This may open a path to a decentralised and automated monetary policy. Automated monetary policy is an old debate for example via so called *Taylor functions* which calculate optimal central bank policies as an interest rate given inflation or some level of activity like the current phase of

computability and programability theories, formal semantic models in theoretical computer science are the reflexive structures developed in mathematics [22]. Social sciences and economics contribute rich epistemological situations for example as in MoMaT in the econometrics of monetary systems where category theory, programming language semantics, type theory, sheafs, homologies or coalgebras can be applied to the benefits of the societies.

Figure 9: Sheaf

the business cycle.

6 Conclusion

We can now answer what money is: Money is a technology which is part of the macro accounting in labour sharing economies. Macro (aka national or quadruple) accounting is for organising the sharing of labor, risk and GDP at the micro, meso and macro levels of economies, respectively.

At the micro level money is solving the problem that production takes time and money is needed by producers to pay suppliers of resources before they are being paid money by customers for their products. To bridge this time gap from purchases of resources to sales of the products, producers need to get indebted at an investor or bank. The loan is to be repaid by the money earned from customers.

The legal base of money is that any exchange is based on contracts which create obligations to be paid by money. An exchange is a purchase of a product for money or a loan of money today for money in the future. The disposal contracts associated to the obligation contracts are about canceling the obligations by a transfer of the ownership of the exchanged products or money. The contractually fixed amounts of product units and monetary units exchanged do not need an economic theory which most of all is not needed for understanding monetary systems or for setting up functional monetary macro accounting systems. Neither the money representing the agreed upon amounts of exchanged monetary units in contracts needs to have an intrinsic value. Money amounts are measured in arbitrarily units for the fulfilment of two-sided obligation contracts. The only need is that money and its monetary unit are the legal means of payment capable to legally cancel the obligations from contracts. Hence, money pays obligations in purchase contracts and repays debts in loan contracts in networks of exchanges of products, monies, loans and investments. By that money is of paramount importance for labour sharing economies in need of exchanging resources, products and money.

At the meso level banks balance losses and returns from pooled loans as investments by interest rates as insurance premiums. Since the success of the production and sales process is risky, bank provide loans as a risk sharing activity. For that money is transferred as loans of banks by deposits at central banks to loans of companies by deposits at banks.

At the macro level, central banks exclusively create money as the legal means of payment and provide it to banks for their investment activities. Central banks socialise non-insurable risks as lenders of first liquidity in fiat systems and as lenders of last resort in fractional gold systems.

We have proposed sheaves to specify data of networks and topologies of debts and payments and to model the consistency needs over hierarchical micro, meso and macro levels. The consistency is described by invariances in macro accounting systems as equalities on sums of receivables,

liabilities, payments and revenues. Homology theory can calculate global corrections of local inconsistencies and may form a base for automated monetary policy. Open games are conceptual building blocks for a domain specific programming language MoMaL for implementing MoMa systems from automata of micro (double) accounting systems. Open games can also represent any forward looking economic or multi-agent model and back propagation for the econometric analysis of data in MoMa systems.

The content of MoMaT boils down to the core function of (paper) money: money is capable of paying off monetary obligations. This capability gives reason to attribute further but secondary functions of money traditionally denoted as being a unit of account, means of exchange and means of storage of value. It is important to realise that money is not only a thing to be exchanged, like in the conceptualisation of it being a numeraire. Instead money gets its core functionality in two-sided contracts and relations of obligations and debts.

7 Literature, Links, Abbreviations

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Abbreviations

Abbreviation	Description
AI	Artifical Intelligence
BGB	Bürgerliches Gesetzbuch ("Civil Code", German)
BIS	Bank for International Settlements
BoE	Bill of Exchange
BoL	Bill of Loading
BPMN	Business Process Model and Notation (see also DMN)
DD	Dynamic Discounting (a SCF product)
DeFi	Decentralised Finance
DLT	Distributed Ledger Technology
DMN	Decision Model and Notation (see also BPMN)
DTSCF	Deep-Tier Supply Chain Finance
ETDA	Electronic Trade Documents Act from 2023 adapting MLETR
eWpG	Gesetz über elektronische Wertpapiere ("Law on Electronic Assets", German)
GDP	Gross Domestic Product
GY	Goodyear (producer of tires for cars)
IMF	International Monetary Fund
MLETR	Model Law of Electronic Transferable Records from 2017, UNCITRAL
MoMa	Monetary Macro Accounting by MoMaT (Reduced Form Model)
MoMaL	Monetary Macro Accounting Language (Structural and Reduced Form)
MoMaT	Monetary Macro Accounting Theory (Structural Form Model)
OECD	Organisation for Economic Co-operation and Development
PMF	Post Maturity Finance (a SCF product)
PN	Promissory Note
SCF	Supply Chain Finance
SNA	System of National Accounts 2009
Sola	financial BoE ("Solawechsel" in German)
UK	United Kingdom
UN	United Nations
UNCITRAL	United Nations Commission on International Trade Law
USA	United States of America
WB	World Bank
VW	Volkswagen (producer of cars)
WG	Wechselgesetz ("Law on Bill of Exchange", German)