Capital Income Tax, RD Technology, and Economic Growth

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Abstract

This paper shows that, in a R&D-based growth model in which vertical and horizontal innovations occur simultaneously, increasing the capital income tax leads to faster the productivity growth and an welfare growth. For this result to hold, the production function for both vertical and horizontal innovations must have constant marginal labor productivity. Furthermore, the paper investigates whether subsidies for both R&D accelerate or deteriorate economic growth and welfare. The government gives more subsidies to the vertical sector with more productivity, leading to economic growth and welfare increases. Conversely, when the government provides subsidies that exceed the threshold ratio in low-productivity sectors, economic growth and welfare are impeded.

Keywords: Endogenous growth, Capital income tax, Vertical innovation, Horizontal innovation, Scale effect, Subsidy.

JEL Classification: O31, O40, H20, J22.

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1 Introduction

The effects of capital income taxation on economic growth is an important topic for not only economists but also policymakers. A substantial body of literature concludes that taxing capital income is bad for growth [see, e.g., Judd (1985); Chamley (1986); Lucas (1990); and Peretto (2003)]. However, some studies cast doubt on this view. For example, Uhilg and Yanagawa (1996), De Hek (2006), and Chen and Lu (2013) show that higher capital income taxes may lead to faster growth. Conesa, Kitao, and Kruger (2009), Hiraguchi and Shibata (2015), and Annicchiarico, Antonarol, and Pelloni (2022) have emphasized that the optimal tax rate on capital is positive. Peretto (2007), Abel (2007) and Anagnostopoulous, Cárceles-Poved, and Lin (2012) show that taxation of dividends and retained earnings do not have the same effects on economic growth and that shifting the corporate tax burden from the latter to the former can boost growth rates. ten Kate and Milionis (2019) investigate theoretically and empirically the relationship between capital taxation and economic growth. They find empirically that capital taxation and growth rates tend to be positively related for developed countries, but for developing countries. Whether a government should tax capital income remains an open question.

From a theoretical point of view, it is crucial to consider what kind of function form to assume. Jones, Manuelli, and Rossi (1993) state, "[...] in models with endogenous government spending, the limiting capital tax rate depends critically on the specification of the production technology" (p.511). Lansing (1999) considered a special case of the setup in Judd (1985) with the intertemporal elasticity of substitution being one, and found that positive long-run capital taxes are possible. Straub and Werning (2020) proved that, in the model based on Judd (1985), the long-run tax on capital is positive and significant whenever the intertemporal elasticity of substitution is below one. The results obtained by such subtle changes in the specification of functions can vary.

The present paper contributes to the literature supporting non-zero capital income taxation in an endogenous growth model. The analysis is closely related to the analyses in Young (1998), Dinopoulos and Thompson (1998), Peretto (1998, 2003, 2007), Chu, Cozzi, and Gali (2012), Chu, Furukawa, and Ji (2016) and Niwa (2016). They developed R&D-based growth models with vertical and horizontal innovations. The model in the present paper is based on that of Peretto (2003). He assumes that vertical innovation's production (knowledge accumulation) function has decreasing returns to labor inputs and shows that an increase in the tax rate on capital income induces a decline in the long-run growth rate. However, according to Chu, Cozzi, and Gali (2012) and Niwa (2016), the present paper assumes that the production function has constant returns to labor inputs. In this situation, the present paper shows that the economic system is saddle-stable and that the linear production function concerning labor inputs leads to an opposite result; i.e., an increase in capital income tax positively affects the growth rate.

In addition to the above analysis, the present paper investigates whether subsidies for both R&D sectors accelerate or deteriorate economic growth and welfare. In the R&D growth literature, a subsidy policy for R&D is well known for driving down the marginal cost of innovation. The decrease in the marginal cost makes more profit for R&D and leads to more economic growth. Chu, Furukawa, and Ji (2016) use the two dimensions of the technological progress model, i.e., a Schumpeterian growth model with endogenous market structure, to investigate how patent breadth and R&D subsidies affect economic growth and endogenous market structure. They showed that R&D subsidies increase economic growth. However, they consider only one sector of subsidies: the government subsidizes only the vertical innovation sector. Furthermore, they assume that firms use final goods to improve R&D productivity. In the present model, labor is an input to enhance the quality of firms' products.

The present paper obtains the results as follows. When the subsidy rate for the more productive sector, i.e., the vertical R&D sector, is higher or equal to that for the less productive sector, i.e., the horizontal R&D sector, the government's subsidy policy accelerates economic growth and welfare. When the reverse situation occurs, there are three possibilities for the results. Suppose the ratio of the subsidy rate for the more productive sector to the one for the less productive sector exceeds a threshold value. In that case, the subsidy policy derives the same positive result. The subsidy policy does not affect economic growth and welfare when the ratio corresponds to the threshold. In the final case, where the ratio sinks below the threshold value, economic growth and welfare are impeded. This implies that providing extremely more subsidies for the less productive sector than the more productive sector is not desirable for economic growth and welfare.

The remainder of the present paper is organized as follows. Section 2 introduces the model. Section 3 considers the market equilibrium dynamics and derives the main result. Section 4 investigates whether subsidies for both R&D accelerate or deteriorate economic growth and welfare. Section 5 is conclusion.

2 The Model

The model draws on work by Peretto (2003). It allows individuals to allocate time to labor supply and leisure, and consists of two types of innovation sector: vertical innovation and horizontal innovation. A government taxes consumption and labor, capital, and corporate incomes to provide public goods and lump-sum transfers.

2.1 The Households

I consider the closed economy populated by identical individuals who supply labor services and consumption loans in competitive labor and assets markets. The population at time t is represented as $L_t = L_0 e^{\lambda t}$, where L_0 is the initial population and

λ is the rate of population growth. The lifetime utility is

$$U_t = \int_t^{\infty} L_0 e^{-(\rho-\lambda)(\tau-t)} \log u_\tau d\tau, \qquad \rho > \lambda \geq 0, \tag{1}$$

where ρ is the individual discount rate. Instantaneous utility at time t is

$$\log u_t = \log C_t + \gamma \log(1 - l_t) + \mu \log G_t, \qquad \gamma, \mu > 0, \tag{2}$$

where C_t is a consumption index, l_t is the fraction of time allocated to labor supply [so that $(1-l_t)$ is leisure], and G_t represents public goods supplied by the government. Constant parameters, γ and μ, are the elasticity of instantaneous utility with respect to leisure and public goods, respectively. The consumption index is symmetric over a continuum of differentiated goods,

$$C_t = \left[\int_0^{N_t} (c_{it})^{(\varepsilon-1)/\varepsilon} \, di \right]^{\varepsilon/(\varepsilon-1)}, \qquad \varepsilon > 1, \tag{3}$$

where ε is the elasticity of differentiated product substitution, c_{it} is the demand for each differentiated good, and N_t is the number of goods (firms). Individuals face the flow budget constraint

$$\dot{A}_t = [r_t(1 - t_A) - \lambda] A_t + (1 - t_L) W_t l_t - (1 + t_E) E_t + T_t. \tag{4}$$

All variables are in per capita terms. A_t is financial wealth, r_t is the rate of return on capital, and W_t is the wage rate. $E_t = \int_0^{N_t} P_{it} c_{it} di$ is consumption expenditure, where P_{it} is good i's price. The wage rate is the numéraire, $W \equiv 1$. The government taxes labor income at rate t_L, capital income at rate t_A, and consumption at rate t_E, and pays lump-sum transfers T_t. All tax rates imposed by the government are $0 \leq t_i < 1$, $i = L, A, E, \pi$.

Individuals maximize (1) subject to equations (2)–(4). The optimal condition for the problem is obtained as follows.

$$\frac{\dot{E}_t}{E_t} = r_t(1 - t_A) - \rho \tag{5}$$

$$L_t l_t = L_t \left[1 - \frac{1 + t_E}{1 - t_L}\gamma E_t\right] \tag{6}$$

Equation (5) is a Euler equation, and equation (6) is the aggregate labor supply.

Furthermore, at each time, individuals decide how they consume each differentiated good to maximize (3), given the expenditure E_t. Solving the well-known static problem yields the aggregate consumption of good i,

$$X_{it} = L_t c_{it} = L_t E_t \frac{P_{it}^{-\varepsilon}}{\int_0^{N_t} P_{jt}^{1-\varepsilon} dj}. \tag{7}$$

2.2 The Manufacturing Firms

The firm with a patent supplies its differentiated good exclusively with the technology

$$X_{it} = Z_{it}^{\theta}(L_{X_{it}} - \phi), \qquad 0 < \theta < 1, \quad \phi > 0, \tag{8}$$

where X_{it} is output, $L_{X_{it}}$ is labor employment, and ϕ is a fixed management cost. Z_{it}^{θ} is labor productivity, which is a function of the firm's accumulated stock of innovations, Z_{it}, with elasticity θ.

2.3 The Vertical Innovation Firms: Corporate R&D

The firm can increase its productivity by innovation, which occurs according to

$$\dot{Z}_{it} = \alpha K_t L_{Z_{it}}, \qquad \alpha > 0, \tag{9}$$

where $\dot{Z}_{it}$ is the flow of innovations generated by employing $L_{Z_{it}}$ units of labor in R&D for an interval of time dt, and αK_t is the productivity of labor in R&D, as determined by the exogenous parameter α and the stock of public knowledge,

$K_t = Z_t \equiv \frac{1}{N_t}\int_0^{N_t} Z_{it}di$. The level of public knowledge is determined by the average productivity among each firm; thus, (9) is rewritten as

$$\dot{Z}_{it} = \alpha Z_t L_{Z_{it}}. \tag{10}$$

According to Chu, Cozzi, and Gali (2012) and Niwa (2016), the function is assumed to be linear for labor inputs. It means that both vertical and horizontal innovation sectors have the same technology for labor inputs [See equation (15)]. However, it is natural that considering an externality effect, both sectors have different technologies all in all. On the other hand, Peretto (2003) assumes the decreasing returns to labor inputs; nevertheless, he uses the linear function to labor inputs. The present paper shows that this change generates the opposite effect of capital income tax on the growth rate of productivity.

The present discounted value of after-tax profit for the firm that has a patent on the differentiated good i is

$$V_{it} = \int_t^\infty e^{-\int_t^\tau r_s ds}(1-t_\pi)\Pi_{i\tau}d\tau,$$

where t_π is the tax rate on profit, and pre-tax profit is $\Pi_{it} = P_{it}X_{it} - L_{X_{it}} - L_{Z_{it}}$.

At any time, t, the firm chooses price to maximize the pre-tax profit subject to the demand (7), the output of the differentiated good (8), and the given stock of public knowledge Z_{it}. The optimal price for good i is

$$P_{it} = \frac{\varepsilon}{\varepsilon - 1} Z_{it}^{-\theta}. \tag{11}$$

Given this price, the demand for each good i is obtained as follows:

$$X_{it} = \frac{\varepsilon - 1}{\varepsilon}\frac{Z_{it}^{\theta\varepsilon}}{\int_0^{N_t} Z_{jt}^{\theta(\varepsilon-1)}dj}E_t L_t.$$

Substituting these into pre-tax profit yields the maximized profit

$$\Pi_{it} = \frac{Z_{it}^{\theta(\varepsilon-1)}}{\int_0^{N_t} Z_{jt}^{\theta(\varepsilon-1)} dj} \frac{E_t L_t}{\varepsilon} - \phi - L_{Z_{it}}. \tag{12}$$

Before proceeding to the dynamic problem, I impose the following assumption.

Assumption 1. $\theta(\varepsilon - 1) < 1$.

This condition guarantees that the second order condition of the R&D decision problem is satisfied and is a necessary condition that the interest rate is positive as discussed below.

Each firm chooses R&D strategies to maximize the present discounted value of after-tax profit, into which the maximized profit is substituted, subject to the innovation technology (10) and other firms' strategies.

Since R&D follows constant returns to scale technology, the equilibrium condition for finite R&D to occur is

$$q_{it} = \frac{1 - t_\pi}{\alpha Z_t}, \tag{13}$$

where q_{it} is the co-state variable, which is the marginal value of productivity Z_{it}. Equation (13) implies that the marginal value is equal to its marginal cost. Since, in the present model, the marginal cost is independent of the number of labor employed in the vertical innovation firms, the marginal value of productivity is not directly affected by the input L, and an optimal R&D level is not yet determined. As discussed below, it is determined by such as no arbitrage condition in the capital market.

The return for vertical innovation must satisfy the following.

$$r_t = (1 - t_\pi)\theta(\varepsilon - 1) \frac{Z_{it}^{\theta(\varepsilon-1)-1}}{\int_0^{N_t} Z_{jt}^{\theta(\varepsilon-1)} dj} \frac{E_t L_t}{\varepsilon q_{it}} + \frac{\dot{q}_{it}}{q_{it}}. \tag{14}$$

The transversality condition is $\lim_{\tau \to \infty} e^{-\int_t^\tau \tau_s ds} q_{i\tau} Z_{i\tau} = 0$.

2.4 The Horizontal Innovation Firms: Entrepreneurial R&D

The main objective of entrepreneurial R&D is the creation of new goods. Entrepreneurs can create new goods and enter the industry by using only labor inputs.

$$\dot{N}_t = \beta L_{Nt}, \qquad \alpha > \beta > 0, \tag{15}$$

where β is the productivity of labor in entry, and L_{Nt} is the amount of employment required to create $\dot{N}_t$ new firms for an interval of time dt. The productivity of entrepreneurs is equal to the average productivity among incumbent firms, $\frac{1}{N_t}\int_0^{N_t} Z_{jt}dj$, and incumbent firms are assumed to be symmetric. This implies that entrant firms are also symmetric with respect to productivity. Therefore, the values for new firms are always the same as those for symmetric incumbent firms.

Entrepreneurs may enter freely into variety-expanding R&D and finance the product development costs by issuing equity. The after-tax profit for them[1] is

$$(1 - t_\pi)\pi_t^{R\&D}dt = (1 - t_\pi)(V_t dN_t - L_{Nt}dt) = (1 - t_\pi)(V_t\beta - 1)L_{Nt}dt.$$

Imposing the free entry condition on this implies that, if $L_{Nt} > 0$, the following condition is satisfied,

$$V_t = \frac{1}{\beta}. \tag{16}$$

Entry is positive if the value of the firm is equal to its start-up cost. The profit that accrues to an entrepreneur is given by the expression derived for incumbents. Thus, the market value of a firm's shares satisfies the arbitrage condition: $r_t = (1 - t_\pi)\frac{\Pi_{it}}{V_t} + \frac{\dot{V}_t}{V_t}$. Note that the second term in the right-hand side is always zero, because V_t is constant over time. Imposing symmetry on the pre-tax profit for

[1] The wage rate is the numéraire.

production firm i, (12), I obtain the following:

$$\Pi_t = \Pi_{it} = \Pi_{jt} = \frac{E_t L_t}{\varepsilon N_t} - \phi - L_{Z_t}, \quad \text{for all } j \neq i. \tag{17}$$

Substituting this and (16) into the arbitrage condition yields the rate of return on entrepreneurial R&D

$$r_t = (1 - t_\pi)\beta \left[\frac{E_t L_t}{\varepsilon N_t} - \phi - L_{Z_t} \right]. \tag{18}$$

2.5 The Government

The government taxes consumption expenditure, labor income, capital income, and corporate profit. These tax rates are constant over time. The government produces public goods, hiring labor at $W_t \equiv 1$. The production function is $G_t = L_{G_t}$, where L_{G_t} is public employment at time t. The government cannot borrow and allocates fraction g of tax revenues to the provision of public goods and fraction $1 - g$ to lump-sum transfers to individuals. This satisfies the flow budget constraint of the government: $t_L L_t l_t + t_\pi \int_0^{N_t} \Pi_{it} di + t_E E_t L_t + t_A r_t A_t L_t = L_{G_t} + T_t L_t$, where $L_{G_t} = g \left[t_L L_t l_t + t_\pi \int_0^{N_t} \Pi_{it} di + t_E E_t L_t + t_A r_t A_t L_t \right]$.

2.6 The Labor Market

There are four sources of labor demand. First, the production sector employs $\int_0^{N_t} L_{X_{it}} di$ units of labor to produce differentiated goods. Second, in the corporate R&D sector, $\int_0^{N_t} L_{Z_{it}} di$ units of labor are employed. Third, employment in the entrepreneurial R&D sector is L_{N_t}. Fourth, L_{G_t} units of labor are employed to provide public goods. Equating units of labor to the aggregate labor supply L_t gives the labor market clearing condition: $L_t l_t = \int_0^{N_t} (L_{X_{it}} + L_{Z_{it}}) di + L_{N_t} + L_{G_t}$.

3 The Market Equilibrium Dynamics

3.1 Equilibrium Values and Dynamic Equations

The assumption that firm's productivity Z_{it} is symmetric causes price P_{it} and output X_{it} to be symmetric. That is, for all i, $P_t = P_{it} = \frac{\varepsilon}{\varepsilon-1} Z_t^{-\theta}$, and $X_t = X_{it} = \frac{\varepsilon-1}{\varepsilon}\frac{E_t L_t}{N_t} Z_t^{\theta}$. Substituting the latter into (8) yields

$$L_{X_t} = \frac{\varepsilon-1}{\varepsilon}\frac{E_t L_t}{N_t} + \phi. \tag{19}$$

In what follows, I focus on an internal equilibrium, where both corporate and entrepreneurial R&D occur.[2] In this situation, equalization of the returns to vertical innovation and horizontal innovation is required. In the capital market, this is called no arbitrage condition. Since, under the homogeneous productivity Z_t, plugging (10) and (13) into (14), equation (14) can be rewritten as

$$r_t = \alpha\theta(\varepsilon-1)\frac{E_t L_t}{\varepsilon N_t} - \alpha L_{Z_t}. \tag{20}$$

This equation implies that the return to vertical innovation does not depend on the labor force increase rate. This is due to the specification of the R&D technology for the vertical sector, which makes the equilibrium dynamics simple.

The no-arbitrage condition requires that the two rates of return just derived be equal.

$$\alpha\left[\theta(\varepsilon-1)\frac{E_t L_t}{\varepsilon N_t} - L_{Z_t}\right] = (1-t_\pi)\beta\left[\frac{E_t L_t}{\varepsilon N_t} - \phi - L_{Z_t}\right]. \tag{21}$$

This equation holds at all moments in time and characterizes equilibrium.

Before proceeding to analysis of economic dynamics, I impose the following assumption. It guarantees the stability of an internal equilibrium, in which two kinds

[2] In the present model, since R&D functions (9) and (15) are linear functions of labor input, it is possible that one of the two R&Ds is not implemented. In other words, a corner solution may occur. For the aim of this paper, however, the internal solution is assumed.

of R&D are implemented.

Assumption 2. $\alpha\theta(\varepsilon - 1) > (1 - t_\pi)\beta$.

Under Assumptions 1 and 2, the level of corporate R&D is determined so that equation (21) can be satisfied all times. Solving (21) for L_{Z_t} in the corporate R&D sector yields

$$L_{Z_t} = \frac{\alpha\theta(\varepsilon - 1) - (1 - t_\pi)\beta}{\alpha - (1 - t_\pi)\beta} \frac{E_t L_t}{\varepsilon N_t} + \frac{(1 - t_\pi)\beta}{\alpha - (1 - t_\pi)\beta}\phi. \tag{22}$$

The interest rate is simultaneously determined,

$$r_t = \frac{\alpha(1 - t_\pi)\beta}{\alpha - (1 - t_\pi)\beta} \left\{ [1 - \theta(\varepsilon - 1)] \frac{E_t L_t}{\varepsilon N_t} - \phi \right\}. \tag{23}$$

These are illustrated as the following figure.

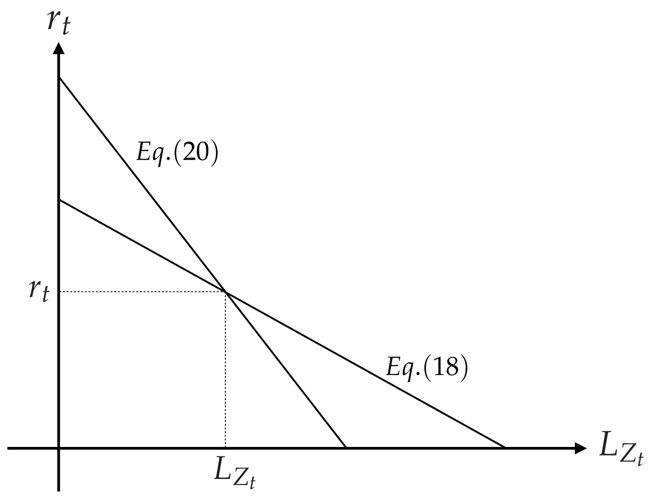

Figure 1: Equilibrium on vertical and horizontal R&D

The after-tax rate of return to investment is indeed the rate of return to saving since, in this economy, the only financial asset available to individuals is ownership shares of firms (stocks). In particular, the capital market clears when $A_t L_t = N_t V_t$. Using this condition, the arbitrage condition, $r_t = (1 - t_\pi)\frac{\Pi_t}{V_t}$, and (17), one can rewrite public employment as

$$L_{G_t} = g\left\{t_L L_t l_t + [t_\pi + t_A(1-t_\pi)]\left(\frac{E_t L_t}{\varepsilon N_t} - \phi - L_{Z_t}\right)N_t + t_E E_t L_t\right\}. \tag{24}$$

The market equilibrium dynamics can be described by the Euler equation and the growth rate of the number of goods per capita, $n_t \equiv \frac{N_t}{L_t}$. Using (33), the Euler equation can be written as

$$\frac{\dot{E}_t}{E_t} = \frac{\alpha(1-t_A)(1-t_\pi)\beta}{\alpha-(1-t_\pi)\beta}\left\{[1-\theta(\varepsilon-1)]\frac{E_t}{\varepsilon n_t} - \phi\right\} - \rho. \tag{25}$$

The labor market clearing condition in the symmetric situation reads $L_t l_t = N_t(L_{X_t} + L_{Z_t}) + L_{N_t} + L_{G_t}$. Using (6), (15), (19), (22), and (24), this can be rewritten

$$\begin{aligned}\frac{\dot{n}_t}{n_t} = \beta\Bigg[(1-gt_L)\frac{1}{n_t} - \frac{1}{\alpha-(1-t_\pi)\beta}\left\{\Gamma\frac{E_t}{\varepsilon n_t} + (1-g\tilde{\tau})\alpha\phi\right\} \\ - \left\{\frac{1-gt_L(1+t_E)\gamma}{1-t_L} + gt_E\right\}\frac{E_t}{n_t}\Bigg] - \lambda, \end{aligned} \tag{26}$$

where $\Gamma \equiv \alpha[(1+\theta)(\varepsilon-1)+g\tilde{\tau}(1-\theta(\varepsilon-1))] - \varepsilon(1-t_\pi)\beta$, and $\tilde{\tau} \equiv t_\pi + t_A(1-t_\pi)$. As shown below this system has a unique steady state that can be shown to be saddle stable under Assumptions 1 and 2.

3.2 Steady-State Analysis

In this subsection, I analyze the steady-state of the system and prove the system to be saddle stable. Let me consider the steady-state condition at first. Set $\dot{E}_t = 0$ to obtain

$$E_t = \frac{\varepsilon}{1-\theta(\varepsilon-1)}\left[\frac{\alpha-(1-t_\pi)\beta}{\alpha(1-t_A)(1-t_\pi)\beta}\rho + \phi\right]n_t, \tag{27}$$

and set $\dot{n}_t = 0$ to obtain

$$E_t = -\frac{1}{\Psi}\left[\frac{\lambda}{\beta} + \frac{\alpha\phi}{\alpha-(1-t_\pi)\beta}(1-g\tilde{\tau})\right]n_t + \frac{1-gt_L}{\Psi}, \tag{28}$$

where $\Psi = \frac{(1-gt_L)(1+t_E)}{1-t_L} + gt_E + \frac{\Gamma}{\varepsilon(\alpha-(1-t_\pi)\beta)}$. Under Assumptions 1 and 2, the $\dot{E}_t = 0$ is a positive slope line through the origin, and the $\dot{n}_t = 0$ line has a negative slope. In Appendix A, the signs of the terms Γ and $1 - g\tilde{\tau}$ are proved to be positive and non-negative, respectively. Therefore, Ψ and terms in the square brackets are confirmed to be positive. The intercept of equation (28) is also positive because $g \in [0, 1]$ and $t_L \in [0, 1)$.

The intersection in (n_t, E_t) space of equations (27) and (28) determines the steady state values of consumption expenditure and the number of goods per capita, as illustrated in Figure 2. The steady state values are represented as n^* and E^*.

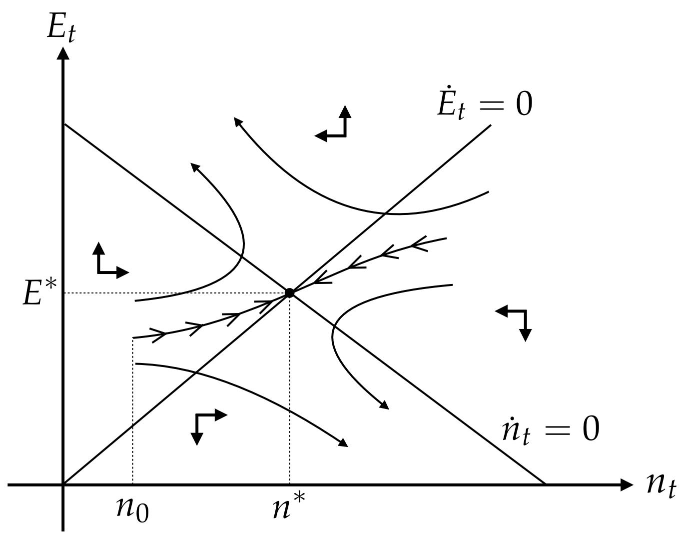

Figure 2: The Phase Diagram on E_t and n_t

Next, I study the stability of the present model and prove it to be saddle stable. The system of differential equations that characterize the present model is given from equations (25) and (26). I take first-order Taylor expansions of these equations around the steady-state values, E^* and n^*.

$$\begin{bmatrix} E_t \dot{-} E^* \\ n_t \dot{-} n^* \end{bmatrix} = \begin{bmatrix} \frac{\alpha(1-t_A)(1-t_\pi)\beta(1-\theta(\varepsilon-1))}{\alpha-(1-t_\pi)\beta} \frac{E^*}{\varepsilon n^*} & -\frac{\alpha(1-t_A)(1-t_\pi)\beta(1-\theta(\varepsilon-1))}{\alpha-(1-t_\pi)\beta} \frac{(E^*)^2}{\varepsilon(n^*)^2} \\ -\beta\Psi & -\left[\frac{\beta(1-g\tilde{\tau})\alpha\phi}{\alpha-(1-t_\pi)\beta} + \lambda\right] \end{bmatrix} \begin{bmatrix} E_t - E^* \\ n_t - n^* \end{bmatrix}$$

I represent the above 2×2 matrix as A.

$$A \equiv \begin{bmatrix} \frac{\alpha(1-t_A)(1-t_\pi)\beta(1-\theta(\varepsilon-1))}{\alpha-(1-t_\pi)\beta}\frac{E^*}{\varepsilon n^*} & -\frac{\alpha(1-t_A)(1-t_\pi)\beta(1-\theta(\varepsilon-1))}{\alpha-(1-t_\pi)\beta}\frac{(E^*)^2}{\varepsilon(n^*)^2} \\ -\beta\Psi & -\left[\frac{\beta(1-g\tilde{\tau})\alpha\phi}{\alpha-(1-t_\pi)\beta}+\lambda\right] \end{bmatrix} = \begin{bmatrix} a_{11} & -a_{12} \\ -a_{21} & -a_{22} \end{bmatrix},$$

where $a_{11} = \frac{\alpha(1-t_A)(1-t_\pi)\beta(1-\theta(\varepsilon-1))}{\alpha-(1-t_\pi)\beta}\frac{E^*}{\varepsilon n^*} > 0$, $a_{12} = \frac{\alpha(1-t_A)(1-t_\pi)\beta(1-\theta(\varepsilon-1))}{\alpha-(1-t_\pi)\beta}\frac{(E^*)^2}{\varepsilon(n^*)^2} > 0$, $a_{21} = \beta\Psi > 0$, and $a_{22} = \left[\frac{\beta(1-g\tilde{\tau})\alpha\phi}{\alpha-(1-t_\pi)\beta}+\lambda\right] > 0$.

Using this expression, the determinant of A is obtained:

$$\det A = -\frac{\alpha(1-t_A)(1-t_\pi)\beta(1-\theta(\varepsilon-1))}{\alpha-(1-t_\pi)\beta}\frac{E^*}{\varepsilon n^*}\left[\frac{\beta(1-g\tilde{\tau})\alpha\phi}{\alpha-(1-t_\pi)\beta}+\lambda\right] - \frac{\alpha(1-t_A)(1-t_\pi)\beta(1-\theta(\varepsilon-1))}{\alpha-(1-t_\pi)\beta}\frac{(E^*)^2}{\varepsilon(n^*)^2}\beta\Psi < 0.$$

The determinant of A is negative because all the matrix elements a_{ij}, $(i, j = 1, 2)$, are positive.

To compute the eigenvalues, denoted by ν, I use the condition $|A - \nu I| = 0$:

$$\begin{vmatrix} a_{11}-\nu & -a_{12} \\ -a_{21} & -a_{22}-\nu \end{vmatrix} = 0,$$

where I is identity matrix. This condition corresponds to a quadratic equation in ν:

$$\nu^2 + (a_{22} - a_{11})\nu - a_{11}a_{22} - a_{12}a_{21} = 0$$

When ν is zero, the value of the equation is $\det A = -a_{11}a_{22} - a_{12}a_{21}$ and negative. The discriminant D is $D = (a_{22} - a_{11})^2 - 4(-a_{11}a_{22} - a_{12}a_{21})$ and positive. That means that the condition has two different real solutions, and each root has a different sign: one is positive and another is negative. Therefore, the dynamic system is saddle stable, and the steady-state point is (n^*, E^*), which is illustrated in Figure 2.

Figure 2 states that, in the case where the initial number of goods per capita,

n_0, is relative low, specifically $n_0 < n^*$, the number of goods per capita, n_t, and the consumption expenditure, E_t, both increase toward the steady state. In addition, one can confirm that the ratio $\frac{E_t}{n_t}$ gradually decreases. The amount of the input into corporate R&D, L_{Z_t}, decreases as the economy approaches the steady state.

The growth rate of productivity, L_Z^*, and the interest rate, r^*, in the long-run are determined to equalize the rate of returns on two kinds of R&D.[3]

$$L_Z^* = \frac{\alpha\theta(\varepsilon-1)-(1-t_\pi)\beta}{\alpha-(1-t_\pi)\beta}\frac{E^*}{\varepsilon n^*} + \frac{(1-t_\pi)\beta}{\alpha-(1-t_\pi)\beta}\phi \tag{29}$$

$$r^* = \frac{\alpha(1-t_\pi)\beta}{\alpha-(1-t_\pi)\beta}\left\{[1-\theta(\varepsilon-1)]\frac{E^*}{\varepsilon n^*} - \phi\right\} \tag{30}$$

The steady-state rate of return, r^*, is dependent on the consumption expenditure per good, $\frac{E^*}{n^*}$. On the balanced growth path, however, consumption expenditure, E_t, is constant[4] and, hence, r^* is determined to satisfy the condition that the after-tax interest rate is equal to the discount rate:

$$(1-t_A)r^* = \rho. \tag{31}$$

From this condition, one can confirm that the vales of $\frac{E^*}{n^*}$ and L_Z^* are determined irrespective of the labor market equilibrium. This means that the growth rate of productivity is independent of the population L_t and, specifically, that there is no scale effect in the present model.[5]

These three equations (29)–(31) yield the following important result of this paper.

Proposition 1. *Capital income tax has a positive effect on the growth rate of productivity, L_Z^*.*

Proof. See Appendix B. □

[3] In the present model, the growth rate of productivity is given as $\frac{\dot{Z}_t}{Z_t} = \alpha L_{Z_t}$, which depends on the labor employment in corporate R&D. Thus, one can express the productivity growth as L_{Z_t}.

[4] See the Euler equation (5).

[5] The labor market equilibrium is achieved by the adjustment of the number of firms per capita, n^*.

The intuition of the proposition is as follows. The introduction of and/or increase in capital income tax leads to a higher rate of return on R&D [see equation (31)]. The higher rate of return stimulates the consumption expenditure per good [see equation (30)], which increases the growth rate of productivity [see equation (29)]. By contrast, in Peretto (2003), a rise in the consumption expenditure per good is relatively low, with the result that firms must reduce the number of employees for the higher rate of return to hold. This leads to a decline in the productivity growth.

Effects of other fiscal variables on the productivity growth are the same as those obtained in Peretto (2003). Corporate income tax has a positive effect on productivity growth, but labor income and consumption taxes have no effect.[6]

The growth rate of an individual's utility is derived as follows.[7]

$$\frac{\dot{u}_t}{u_t} = \theta \frac{\dot{Z}_t}{Z_t} + \left(\frac{1}{\varepsilon - 1} + \mu \right) \lambda.$$

One can confirm that the growth rate of an individual's utility is independent of the population scale and increases as capital income taxes increase. The discussion so far is summarized as the following proposition.

Proposition 2. *The steady-state equilibrium of the economy is saddle stable. The growth rate of an individual's utility bears a positive proportionate relationship to the growth rate of the productivity,* L_Z.

4 Subsidies to Vertical and Horizontal Innovation Sectors

The R&D growth literature usually assumes a subsidy to drive down the marginal cost of innovation. The decrease in the marginal cost makes more profit for R&D and leads to more economic growth. This section extends the model by adding

[6]See Appendix C for the proof of this.

[7]See Appendix D for a detailed derivation.

the same subsidy policy to vertical and horizontal innovation sectors and analyzes whether the subsidy model below leads to the same result.

There are two innovation sectors with different R&D productivity in the present model. Under Assumptions 1 and 2, parameter α in the vertical innovation sector is more extensive than parameter β in the horizontal innovation sector. In addition, the vertical innovation sector has an externality effect. For this reason, one can confirm that the productivity in the vertical innovation sector is more prominent than that in the horizontal innovation sector. In this situation, how do subsidies to both innovation sectors promote economic growth in the economy?

4.1 Vertical and Horizontal Innovation Firms under Subsidies

The paper solves the baseline model under the assumption that the government covers a proportion $s_i \in [0, 1)$, $(i = V, H)$, of R&D cost. s_V represents the subsidy rate of the vertical innovation firm. The pre-tax profit, then, is rewritten as

$$\Pi_{it} = P_{it}X_{it} - L_{X_{it}} - (1 - s_V)L_{Z_{it}}.$$

s_H represents the subsidy rate of the horizontal innovation sector, and then the after-tax profit for the horizontal innovation firm is as follows.

$$(1 - t_\pi)\pi_t^{R\&D} = (1 - t_\pi)(V_t dN - (1 - s_H)L_{N_t} dt).$$

The subsidy policy in the present model does not directly affect households' problems, the output of differentiated goods, L_{X_t}, the labor market clearing condition, or the capital market clearing condition.

Given the above expression, the equilibrium condition for finite R&D to occur, (13), is now replaced by

$$q_{it} = \frac{(1-t_\pi)(1-s_V)}{\alpha Z_t}, \tag{32}$$

and the free entry condition (16) is given by

$$V_t = \frac{1-s_H}{\beta}.$$

The marginal value of productivity q_{it} decreases by $1-s_V$, and the free entry condition, if L_{N_t} is positive, also decreases by $1-s_H$. Combining these equations with the arbitrage condition yields the modified rate of return on entrepreneurial R&D:

$$r_t = \frac{(1-t_\pi)\beta}{1-s_H}\left[\frac{E_t}{\varepsilon n_t} - \phi - (1-s_V)L_{Z_t}\right]. \tag{33}$$

To derive the return on vertical R&D, I substitute (9) and (32) into (14), and then obtain the following expression.

$$r_t = \alpha\theta(\varepsilon-1)\frac{E_t}{(1-s_V)\varepsilon n_t} - \alpha L_{Z_t}. \tag{34}$$

Suppose that the consumption expenditure per good ratio $\frac{E_t}{n_t}$ and labor supply L_{Z_t} be constant and not changed. The subsidy to horizontal innovation firms increases the return on entrepreneurial R&D. The subsidy to vertical innovation firms increases both innovation firms' returns. It is the direct effect of the subsidy. However, the subsidy can affect the consumption expenditure per good ratio and labor supply, so this impact should be considered. I will analyze this point later.

4.2 The Government Budget Constraint under the Subsidy Policy

The government's tax policy is the same as the baseline model explained in section 2.5. The government uses the tax revenue to provide public goods and subsidies. In the baseline model, the only subsidy is a lump-sum transfer to consumers, but the model discussed here includes the subsidies for vertical and horizontal innovation firms. So, the government has four channels for its expenditure. Consumption, labor income, capital income, and corporate profit tax rates are constant over time.

The flow budget constraint of the government is now

$$t_L L_t l_t + t_\pi \int_0^{N_t} \Pi_{it} di + t_E E_t L_t + t_A r_t A_t L_t = L_{G_t} + L_t T + \int_0^{N_t} s_V L_{Z_{it}} di + s_H L_{N_t}.$$

Given the tax rates, the government divides the tax revenue into the supply of the public good, the transfer to consumers, and the subsidy policy. The proportion for public goods provision is assumed to be the same as that in the baseline model. Therefore, the government allocates the fraction of $1-g$ into the transfers and the subsidy.

$$G_t = L_{G_t} = g\left[t_L L_t l_t + t_\pi \int_0^{N_t} \Pi_{it} di + t_E E_t L_t + t_A r_t A L_t\right]. \tag{35}$$

This equation is not changed.

4.3 Equilibrium Values and Dynamical Analysis under the Subsidy Policy

In this subsection, the paper derives the equilibrium values and analyzes the dynamics of the subsidy policy model. The subsidy policy model also focuses on symmetric equilibrium. As in the baseline model, I assume the following condition, which guarantees the two kinds of R&D are implemented.

Assumption 3. $\dfrac{1-s_V}{1-s_H} < \dfrac{\alpha\theta(\varepsilon-1)}{(1-t_\pi)\beta}$.

In the equilibrium, returns of horizontal and vertical innovation firms coincide due to the no-arbitrage condition, so from (33) and (34), the following condition is satisfied:

$$\frac{(1-t_\pi)\beta}{1-s_H}\left[\frac{E_t}{\varepsilon n_t}-\phi-(1-s_V)L_{Z_t}\right]=\alpha\theta(\varepsilon-1)\frac{E_t}{(1-s_V)\varepsilon n_t}-\alpha L_{Z_t}.$$

Solving this for L_{Z_t} yields the equilibrium labor demand in the corporate R&D sector:

$$L_{Z_t}=\frac{\alpha\theta(\varepsilon-1)(1-s_H)-(1-t_\pi)(1-s_V)\beta}{(1-s_V)\{\alpha(1-s_H)-(1-t_\pi)(1-s_V)\beta\}}\frac{E_t}{\varepsilon n_t}+\frac{(1-t_\pi)\beta\phi}{\alpha(1-s_H)-(1-t_\pi)(1-s_V)\beta}. \tag{36}$$

The paper substitutes this into (33) and then obtains the equilibrium rate of return.

$$r_t=\frac{\alpha(1-t_\pi)\beta}{\alpha(1-s_H)-(1-t_\pi)(1-s_V)\beta}\left[(1-\theta(\varepsilon-1))\frac{E_t}{\varepsilon n_t}-\phi\right]. \tag{37}$$

When $s_H \leq s_V$, Assumptions 1 and 2 assure that $\alpha\theta(\varepsilon-1)(1-s_H)-(1-s_V)(1-t_\pi\beta$ and $\alpha(1-s_H)-(1-s_V)(1-t_\pi)\beta$ are positive. Even if $s_H > s_V$, under Assumption 3, these should be still positive.

Using (33) and (34), one can also express the vertical and horizontal innovation equilibrium like Figure 1. Although the shapes of the graphs resemble those in Figure 1, the values of slopes and intercepts might be changed. Suppose again that the consumption expenditure per good ratio $\frac{E_t}{n_t}$ and labor supply L_{Z_t} are constant and the same as that without subsidies. If $s_H \leq s_V$, both intercepts become larger than in Figure 1. If s_V is larger than s_H, the slope of equation (33) is more moderate. The slope of equation (34) does not change because of independent of the subsidy. It implies that under subsidies, L_{Z_t} can be more prominent.

If $s_H > s_V$, both intercepts increase similarly, but the equation (33) is steeper than without subsidy. Because of this, one cannot immediately confirm how the graphs' correct positional relation was illustrated.

As mentioned earlier, since the subsidy can affect the consumption expenditure per good ratio and labor supply, one cannot analyze the subsidies' effect correctly.

After discussing the steady-state analysis, let me go back to this point.

The Euler equation (5) and the growth rate of the number of goods per capita can also explain the market equilibrium dynamics. The dynamical system is obtained the same way as in the previous section.

$$\frac{\dot{E}_t}{E_t} = \frac{\alpha(1-t_A)(1-t_\pi)\beta}{\alpha(1-s_H)-(1-t_\pi)(1-s_V)\beta}\left\{[1-\theta(\varepsilon-1)]\frac{E_t}{\varepsilon n_t}-\phi\right\}-\rho$$

$$\frac{\dot{n}_t}{n_t} = \beta\left[(1-gt_L)\left(1-\frac{1+t_E}{1-t_L}\gamma E_t\right)\frac{1}{n_t}-\frac{\Gamma_s\frac{E_t}{\varepsilon n_t}+\omega_s\phi}{\alpha(1-s_H)-(1-t_\pi)(1-s_V)\beta}-gt_E\frac{E_t}{n_t}\right]-\lambda,$$

where

$$\Gamma_s = \alpha(1-s_H)\left\{g\tilde{\tau}(1-\theta(\varepsilon-1))+(\varepsilon-1)\left(1+\frac{\theta}{1-s_V}\right)\right\}-(1-t_\pi)[\varepsilon(1-s_V)+s_V]\beta,$$

$$\omega_s = (1-g\tilde{\tau})\alpha(1-s_H)+(1-t_\pi)\beta s_V.$$

Setting $\dot{E}_t = 0$ and $\dot{n}_t = 0$ yields the following steady-state lines for the dynamics.

$$E_t = \frac{\varepsilon}{1-\theta(\varepsilon-1)}\left[\frac{\alpha(1-s_H)-(1-t_\pi)(1-s_V)\beta}{\alpha(1-t_A)(1-t_\pi)\beta}\rho+\phi\right]n_t, \tag{38}$$

$$E_t = -\frac{1}{\Psi_s}\left[\frac{\lambda}{\beta}+\frac{(1-g\tilde{\tau})\alpha(1-s_H)+(1-t_\pi)\beta s_V}{\alpha(1-s_H)-(1-t_\pi)(1-s_V)\beta}\phi\right]n_t+\frac{1-gt_L}{\Psi_s}, \tag{39}$$

where $\Psi = \frac{\Gamma_s}{\varepsilon\{\alpha(1-s_H)-(1-t_\pi)(1-s_V)\beta\}}+\frac{(1-gt_L)(1+t_E)\gamma}{1-t_L}+gt_E$. Equation (38) is the $\dot{E}_t = 0$, and equation (39) is the $\dot{n}_t = 0$ line. The term Ψ_s, Γ_s, the coefficient of ρ in (38), and the coefficient of ϕ in (39) are different from those in the model without subsidies. The paper must discuss how the phase diagram is illustrated and what has changed compared to the model without subsidies. However, since the expressions with subsidies are complicated, the paper will examine the slope of the $\dot{E}_t = 0$ line, which is relatively easy to analyze.

The only difference between (27) and (38) is numerator values in the coefficient of ρ. Whether the slope in the model with subsidies is larger depends on the relationship between these values. If $s_V = s_H = s$, the numerator in equation (38) is arranged as $(1-s)\{\alpha-(1-t_\pi)\beta\}$ and is smaller than $\alpha-(1-t_\pi)\beta$. If $s_V > s_H$, the relationship depends on the relative scale of $\frac{s_V}{s_H}$. The difference

$$\alpha-(1-t_\pi)\beta-\{\alpha(1-s_H)-(1-t_\pi)(1-s_V)\beta\}=\alpha s_H - s_V(1-t_\pi)\beta,$$

is positive if $\frac{\alpha}{(1-t_\pi)\beta} > \frac{s_V}{s_H} > 1$, zero if $\frac{\alpha}{(1-t_\pi)\beta} = \frac{s_V}{s_H}$, and negative if $1 < \frac{\alpha}{(1-t_\pi)\beta} < \frac{s_V}{s_H}$. If $s_V < s_H$, $\frac{\alpha}{(1-t_\pi)\beta} > 1 > \frac{s_V}{s_H}$ and the difference is positive.

In the present paper, The condition $\alpha > (1-t_\pi)\beta$ is assumed: the productivity in the vertical innovation sector is higher than that in the horizontal innovation sector. Intuitively, if the government imposed a relatively higher subsidy rate on the vertical innovation sector, the vertical innovation firm would produce more output, and the steady-state ratio $\frac{E}{n}$ would become relatively large. On the other hand, if the government imposed a relatively higher subsidy rate on the horizontal innovation sector, the horizontal innovation firm would produce more varieties of goods, and $\frac{E}{n}$ would become relatively small.

One can confirm this point by compareing the return rates with and without subsidies. Remark that under the model with subsidies, the steady-state rate of return continues to be determined by the equation (30) because the Euler equation does not change. That is, the steady-state value r^s,

$$r^s = \frac{\alpha(1-t_\pi)\beta}{\alpha(1-s_H)-(1-t_\pi)(1-s_V)\beta}\left\{[1-\theta(\varepsilon-1)]\frac{E^s}{\varepsilon n^s}-\phi\right\}, \tag{40}$$

is equivalent to r^*. The variables with superscript s represent the steady-state values under the model with subsidies (e.g., r^s, E^s, and n^s).

The difference points between (30) and (40) are the denominator of the coefficient and the consumption expenditure per good ratio $\frac{E}{n}$. Subtracting the denominator of (40) from that of (30), the following expression is obtained.

$$\frac{1}{\alpha-(1-t_\pi)\beta}-\frac{1}{\alpha(1-s_H)-(1-t_\pi)(1-s_V)\beta}$$
$$=\frac{-\alpha s_H+(1-t_\pi)s_V\beta}{\{\alpha-(1-t_\pi)\beta\}\{\alpha(1-s_H)-(1-t_\pi)(1-s_V)\beta\}}$$

The sign of this expression depends on that of the numerator, $-\alpha s_H + (1-t_\pi)s_V\beta$. If this is positive, that is, $\frac{s_V}{s_H} > \frac{\alpha}{(1-t_\pi)\beta} > 1$, the coefficient of (40) is smaller than that of (30). However, since the steady-state return is invariant under the subsidy model, the consumption expenditure per good ratio must expand for $r^* = r^s$ to be satisfied. If $\frac{s_V}{s_H} = \frac{\alpha}{(1-t_\pi)\beta}$, both coefficients are the same, and then $\frac{E^*}{n^*}$ and $\frac{E^s}{n^s}$ also coincide. Finally, if $\frac{s_V}{s_H} < \frac{\alpha}{(1-t_\pi)\beta}$, the coefficient of (40) is more significant than that of (30). For $r^* = r^s$ to be satisfied, the consumption expenditure per good ratio must diminish. Summarizing the discussion about subsidies so far yields the following lemma.

Lemma 1. *The steady-state value of the consumption expenditure per good ratio $\frac{E^s}{n^s}$ varies as follows.*

(i) If $1 < \frac{\alpha}{(1-t_\pi)\beta} < \frac{s_V}{s_H}$, the consumption expenditure per good ratio in the subsidy policy model becomes large relative to that in the baseline model, $\frac{E^*}{n^*}$;

(ii) If $1 < \frac{\alpha}{(1-t_\pi)\beta} = \frac{s_V}{s_H}$, the consumption expenditure per good ratio in the subsidy policy model is equivalent to that in the baseline model;

(iii) If $\frac{\alpha}{(1-t_\pi)\beta} > \frac{s_V}{s_H}$, the consumption expenditure per good ratio in the subsidy policy model becomes small.

The situation that the productivity of the vertical innovation sector is higher than that of the horizontal innovation sector, that is $s_V > s_H$, includes all the cases (i)-(iii) of lemma 1. In case (iii), the condition is $\frac{\alpha}{(1-t_\pi)\beta} > \frac{s_V}{s_H} > 1$. On the other hand, the situation $s_V \leq s_H$ is satisfied in only case (iii). In this situation, $\frac{\alpha}{(1-t_\pi)\beta} > 1 \geq \frac{s_V}{s_H}$, and the equal sign holds when $s_V = s_H$.

Next, let me consider the steady-state stability in the subsidy model. As the baseline model, the dynamical system is always saddle-stable under Assumptions 1-3. The detailed discussion is provided in Appendix E.

Finally, the present paper investigates how the growth rate of productivity L_{Z_t} is affected by the subsidy policy. The steady-state value L_Z^s is obtained from (36):

$$L_Z^s = \frac{\alpha\theta(\varepsilon-1)(1-s_H)-(1-t_\pi)(1-s_V)\beta}{(1-s_V)\{\alpha(1-s_H)-(1-t_\pi)(1-s_V)\beta\}}\frac{E^s}{\varepsilon n^s}+\frac{(1-t_\pi)\beta\phi}{\alpha(1-s_H)-(1-t_\pi)(1-s_V)\beta} \tag{41}$$

Using this expression and equation (29), I compare the growth rates of productivity. The result is summarized as Proposition 3.

Proposition 3. *For the subsidy policy model,*

(i) if $s_H \leq s_V$, the subsidy policy accelerates the productivity and economic growth;

(ii) if $s_H > s_V$ and $\frac{s_V}{s_H} > \frac{1}{1+\frac{(1-t_\pi)(1-t_A)\beta\phi}{\rho}}$, the subsidy policy ameliorates both productivity and economic growth;

(iii) if $s_H > s_V$ and $\frac{s_V}{s_H} = \frac{1}{1+\frac{(1-t_\pi)(1-t_A)\beta\phi}{\rho}}$, the subsidy policy remains them;

(iv) if $s_H > s_V$ and $\frac{s_V}{s_H} < \frac{1}{1+\frac{(1-t_\pi)(1-t_A)\beta\phi}{\rho}}$, diminishes them.

Proof. See Appendix F. □

Proposition 3 says that if the subsidy rate of the vertical innovation sector s_V is larger or equal to that of the horizontal innovation sector, the subsidy policy promotes the growth rate of productivity. Under the subsidy policy model, the growth rates $\frac{\dot{Z}_t}{Z_t}$ and $\frac{\dot{u}_t}{u_t}$ are the same as those in the baseline model, leading to an accomplishment in higher economic growth and welfare improvement. However, if the government gives relatively more subsidies for horizontal innovation firms, the results may not be what the government wants. At $\frac{s_V}{s_H} = \frac{1}{1+\frac{(1-t_\pi)(1-t_A)\beta\phi}{\rho}}$, the subsidy policy for both R&D sectors will no longer affect productivity and economic growth. Furthermore, if the government provides relatively large subsidies to horizontal R&D

firms to the extent that they exceed the critical value, it will increase labor for product development and significantly reduce labor for R&D to improve productivity. As a result, it will hinder economic growth. These results are summarized in Figure 3.

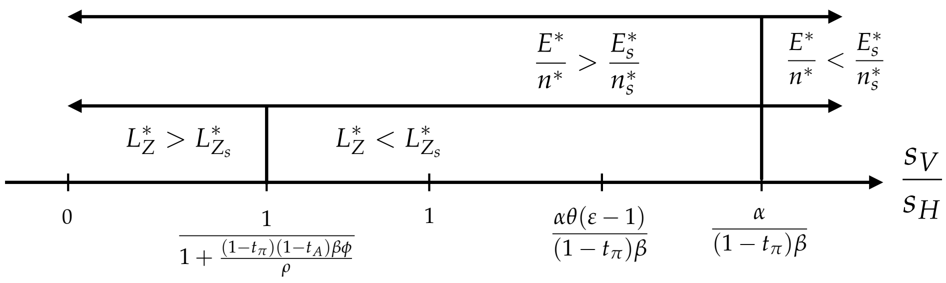

Figure 3: The relationship among $\frac{s_V}{s_H}$, $\frac{E}{n}$, and L_Z

When the ratio of vertical innovation subsidies to horizontal innovation subsidies exceeds the critical value, the subsidy policy increases productivity and economic growth. Otherwise, the policy reduces them. Figure 4 illustrates some typical cases of the result of proposition 3.

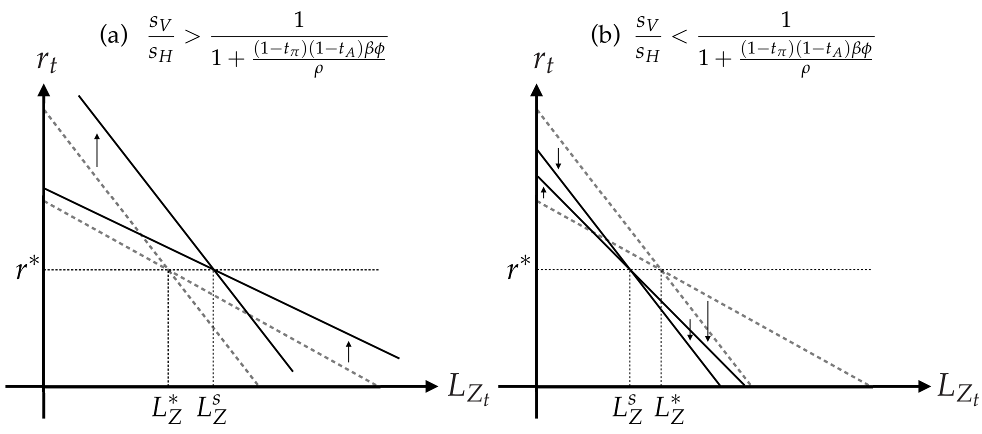

Figure 4: Equilibrium on vertical and horizontal R&D under the subsidy policy

From Lemma 1, when s_V is reasonably large in comparison to s_H, that is $\frac{\alpha}{(1-t_\pi)\beta} < \frac{s_V}{s_H}$, not only the direct effect of subsidies but also the indirect effect through an increase in $\frac{E}{n}$ positively pushes up the labor demand for vertical innovation. When the subsidy gap between s_V and s_H is mild, $\frac{1}{1+\frac{(1-t_\pi)(1-t_A)\beta\phi}{\rho}} < \frac{s_V}{s_H} < \frac{\alpha}{(1-t_\pi)\beta}$, the consumption expenditure per good decreases. However, the direct effect will exceed the indirect effect, so the labor demand for vertical innovation will still increase, leading to economic growth. These situations are illustrated in Figure 4(a).

As s_H is considerably larger than s_V, that is $\frac{s_V}{s_H} < \frac{1}{1+\frac{(1-t_\pi)(1-t_A)\beta\phi}{\rho}}$, a degree of decrease in $\frac{E^s}{n^s}$ becomes large so that the line expressing equation (20) shifts downward. In other words, the shift of labor from the vertical innovation sector to the horizontal innovation sector deteriorates the productivity growth $\frac{\dot{Z}_t}{Z_t}$. Although subsidies directly increase the intercepts of the graphs (18) and (20), a decrease in $\frac{E}{n}$ due to relatively high s_V exceeds this direct effect. When $s_H > s_H$, the slope of the equation (18) becomes steeper [See equation (33)], but the intercept is not immediately determined. Figure 4(b) shows a graph where the intercept is still increasing, but the labor demand for vertical innovation is decreasing.

When the government gives importance to the more productive sector, i.e., the vertical innovation sector, and implements a higher subsidy rate than that in the less productive sector, i.e., the horizontal innovation sector, the subsidy policy accelerates productivity and economic growth. Similarly, even if the government sets a higher subsidy rate for the horizontal innovation sector to protect the industry, the subsidy policy will expand the economy unless the subsidy rate gap is not so large, whereas it will reduce the degree of productivity improvement. In contrast, if the government sets a considerably high subsidy rate for the horizontal innovation sector that exceeds a critical value, the demand for labor in the vertical innovation sector will decrease, impeding economic growth.

5 Conclusion

The present paper has used a R&D-based growth model which vertical and horizontal innovations simultaneously occur. Individuals allocate time to labor supply and leisure and the government taxes consumption and labor, capital, and corporate incomes to provide public goods and lump-sum transfers. This paper has shown that under constant return to labor inputs R&D technology in both vertical and horizontal innovation sectors taxing capital income leads to an increase in the rate of economic growth. This result is in accordance with the recent literature on supporting non-zero income taxation. In the case where the R&D technology in vertical innovation is decreasing returns to labor inputs the result is opposite. The difference between these results may be due to the extent of an increase in the consumption expenditure per good, not consumption expenditure and the number of goods, respectively.

Next, the present paper has investigated the situation where the government can use the tax revenue as R&D subsidies. The subsidy policy does not affect the result obtained in the baseline model: capital income tax continues to affect the growth rate of productivity positively. However, subsidies can boost or suppress economic growth and welfare. When the subsidy rate for the vertical R&D sector is higher or equal to that for the horizontal R&D sector, the government's subsidy policy accelerates economic growth and welfare. When the ratio of the subsidy rate for the more productive sector to the one for the less productive sector exceeds a threshold value, the subsidy policy derives the same positive result. In the case, where the ratio sinks below the threshold value, economic growth and welfare are impeded.

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Appendix

A The sign of the terms in equation (28)

In this section, it is confirmed that the terms, Γ, and $1 - g\tilde{\tau}$ are positive. First, I check the sign of Γ. This is arranged as follows.

$$\begin{aligned}
\Gamma &= \alpha[(1+\theta)(\varepsilon-1) + g\tilde{\tau}(1-\theta(\varepsilon-1))] - \varepsilon(1-t_\pi)\beta \\
&= \alpha(\varepsilon-1) + \alpha\theta(\varepsilon-1) - \varepsilon(1-t_\pi)\beta + \alpha g\tilde{\tau}(1-\theta(\varepsilon-1)) \\
&= \alpha(\varepsilon-1) + \alpha\theta(\varepsilon-1) - \varepsilon(1-t_\pi)\beta + (1-t_\pi)\beta - (1-t_\pi)\beta + \alpha g\tilde{\tau}(1-\theta(\varepsilon-1)) \\
&= \alpha(\varepsilon-1) + \alpha\theta(\varepsilon-1) - (\varepsilon-1)(1-t_\pi)\beta - (1-t_\pi)\beta \\
&= \underbrace{(\varepsilon-1)(\alpha-(1-t_\pi)\beta)}_{+} + \underbrace{\alpha\theta(\varepsilon-1)-(1-t_\pi)\beta}_{+} + \underbrace{\alpha g\tilde{\tau}(1-\theta(\varepsilon-1))}_{+}.
\end{aligned}$$

The arranged equation is divided into three terms. The third term is positive, due to Assumption 1, and the second term is positive, due to Assumption 2. Combining Assumptions 1 and 2 assures that α is larger than $(1-t_\pi)\beta$, which means the first term is positive. Therefore, the term Γ is positive.

Next, I check the sign of the term $1 - g\tilde{\tau}$. Suppose that $\tilde{\tau} = t_\pi + t_A(1-t_\pi)$ is larger than one.

$$t_\pi + t_A(1-t_\pi) > 1 \quad \Leftrightarrow \quad t_A(1-t_\pi) > 1 - t_\pi$$

When $t_\pi = 1$, the above equation is $0 > 0$, which is a contradiction. When $t_\pi \in [0,1)$, the equation is arranged to satisfy $t_A > 1$. This contradicts the assumption, $t_A \in [0,1)$. Suppose also $\tilde{\tau}$ is one. This leads to $1-t_\pi > 1-t_\pi$, which is a contradiction. Therefore, $\tilde{\tau} < 1$. In addition, both tax rates are assumed that $t_\pi, t_A \in [0,1)$, so that τ is larger than or equal to zero. Therefore, $0 \leq \tilde{\tau} < 1$.

Since $g \in [0,1]$, the condition, $0 < 1 - g\tilde{\tau} \leq 1$, obtained and positive.

B Proof of Proposition 1

I investigate how capital income tax affects the growth rate of productivity, L_Z^*. The interest rate in the steady state can be written as

$$r^* = \frac{\rho}{1-t_A}.$$

Differentiating this with respect to t_A yields

$$\frac{\partial r^*}{\partial t_A} = \frac{\rho}{(1-t_A)^2} > 0.$$

Thus, capital income tax increases the rate of return on R&D. The interest rate affects the consumption expenditure per good, $\frac{E^*}{n^*}$. To investigate this effect, I rearrange equation (30) as follows.

$$\frac{E^*}{n^*} = \frac{\varepsilon}{1-\theta(\varepsilon-1)}\left[\frac{\alpha-(1-t_\pi)\beta}{\alpha(1-t_\pi)\beta}r^* + \phi\right]. \tag{42}$$

Differentiating this with respect to r^* yields

$$\frac{d\left(E^*/n^*\right)}{dr^*} = \frac{\varepsilon}{1-\theta(\varepsilon-1)}\left[\frac{\alpha-(1-t_\pi)\beta}{\alpha(1-t_\pi)\beta}\right] > 0.$$

This implies that the consumption expenditure per good is increasing along with the interest rate. Differentiating equation (29) with respect to $\frac{E^*}{n^*}$, one can easily confirm that the growth rate of productivity, L_Z^*, is an increasing function of the consumption expenditure per good.

$$\frac{dL_Z^*}{d\left(E^*/n^*\right)} = \frac{\alpha\theta(\varepsilon-1)-(1-t_\pi)\beta}{\alpha-(1-t_\pi)\beta}\frac{1}{\varepsilon} > 0.$$

Therefore, the capital income tax has a positive effect on the growth rate of productivity, $(dL_Z^*/dt_A) > 0$.

C Effects of Corporate Income, Labor Income, and Consumption Taxes

Firstly, I investigate how corporate income tax affects the growth rate of productivity, L_Z^*. There are two effects of the tax on L_Z^*: a direct effect and an indirect effect. The indirect effect is through a change in the consumption expenditure per good. Since this tax has no effect on the interest rate, I differentiate (42) with respect to t_π, which yields

$$\frac{d\left(E^*/n^*\right)}{dt_\pi} = \frac{\varepsilon}{1-\theta(\varepsilon-1)} \frac{\alpha^2\beta}{\{\alpha(1-t_\pi)\beta\}^2} r^* > 0.$$

Thus, corporate income tax increases the consumption expenditure per good. Considering the indirect effect, I differentiate (29) with respect to t_π.

$$\begin{aligned}\frac{\partial L_Z^*}{\partial t_\pi} = \frac{\alpha\beta}{\{\alpha-(1-t_\pi)\beta\}^2} &\left[\{1-\theta(\varepsilon-1)\}\frac{E^*}{\varepsilon n^*} - \phi\right] \\ &+ \frac{\alpha\theta(\varepsilon-1)-(1-t_\pi)\beta}{\{\alpha-(1-t_\pi)\beta\}\varepsilon} \frac{\partial\left(E^*/n^*\right)}{\partial t_\pi} > 0.\end{aligned}$$

Therefore, the corporate income tax has a positive effect on the growth rate of productivity.

Secondly, it is clear that labor income tax and consumption tax have no effect on the growth rate of productivity, L_Z^*, because equations (29)–(31) are independent of these tax parameters.

D Derivation of the Growth Rate of an Individual's Utility

In the symmetric case, the equilibrium consumption index can be written as follows.

$$C_t = \left[\int_0^{N_t} (c_t)^{(\varepsilon-1)/\varepsilon} di\right]^{\varepsilon/(\varepsilon-1)} = \left[N_t (c_t)^{(\varepsilon-1)/\varepsilon}\right]^{\varepsilon/(\varepsilon-1)} = N_t^{\varepsilon/(\varepsilon-1)} c_t,$$

where $c_t = c_{it} = c_{jt}$, for all $j \neq i$. Using the aggregate consumption of each differentiated good, obtained in equation (7), and the production function, (8), one can rearrange this expression:

$$C_t = \frac{N_t^{\varepsilon/(\varepsilon-1)}}{L_t} X_t = \frac{N_t^{\varepsilon/(\varepsilon-1)}}{L_t} Z_t^\theta (L_{X_t} - \phi).$$

In the steady state, the labor employment in the production sector is constant and, thus,

$$C_t = \frac{N_t^{\varepsilon/(\varepsilon-1)}}{L_t} Z_t^\theta (L_X^* - \phi). \tag{43}$$

Substituting (19) into this, I obtain

$$C_t = \frac{N_t^{\varepsilon/(\varepsilon-1)}}{L_t} Z_t^\theta \left(\frac{\varepsilon-1}{\varepsilon}\frac{E^*}{n^*} + \phi - \phi\right) = \frac{N_t^{\varepsilon/(\varepsilon-1)}}{L_t} Z_t^\theta \left(\frac{\varepsilon-1}{\varepsilon}\right)\frac{E^*}{n^*}.$$

Differentiating this with respect to t yields

$$\dot{C}_t = \frac{N_t^{\varepsilon/(\varepsilon-1)}}{L_t} Z_t^\theta \left(\frac{\varepsilon-1}{\varepsilon}\right)\frac{E^*}{n^*}\left[\frac{\varepsilon}{\varepsilon-1}\frac{\dot{N}_t}{N_t} - \frac{\dot{L}_t}{L_t} + \theta\frac{\dot{Z}_t}{Z_t}\right]. \tag{44}$$

The growth rate of the consumption index is, therefore,

$$\frac{\dot{C}_t}{C_t} = \frac{\varepsilon}{\varepsilon-1}\frac{\dot{N}_t}{N_t} - \frac{\dot{L}_t}{L_t} + \theta\frac{\dot{Z}_t}{Z_t}.$$

Now, I consider the relationship between the growth rates of N_t and L_t. The definition of the number of firms per capita is $n_t \equiv \frac{N_t}{L_t}$. Differentiating this with respect to t yields

$$\frac{\dot{n}_t}{n_t} = \frac{\dot{N}_t}{N_t} - \frac{\dot{L}_t}{L_t}.$$

In the steady state, $\dot{n}_t$ is zero, which means that the growth rate of the number of goods is equal to that of the population, $\frac{\dot{N}_t}{N_t} = \frac{\dot{L}_t}{L_t} = \lambda$. Hence,

$$\frac{\dot{C}_t}{C_t} = \theta \frac{\dot{Z}_t}{Z_t} + \frac{1}{\varepsilon - 1}\lambda, \tag{45}$$

where the growth rate of productivity is

$$\frac{\dot{Z}_t}{Z_t} = \alpha L_Z^* = \alpha \left[\frac{\alpha\theta(\varepsilon - 1) - (1 - t_\pi)\beta}{\alpha - (1 - t_\pi)\beta} \frac{E^*}{\varepsilon n^*} + \frac{(1 - t_\pi)\beta}{\alpha - (1 - t_\pi)\beta}\phi \right]. \tag{46}$$

I derive the growth rate of an individual's utility. The instantaneous utility function is defined as

$$\log u_t = \log C_t + \gamma \log(1 - l_t) + \mu \log G_t, \qquad \gamma, \mu > 0.$$

Firstly, C_t is the consumption index; its long-run growth rate is calculated in equation (45). Secondly, the fraction of time allocated to labor supply is represented as l_t; its optimal value is obtained in equation (6). The time-dependent variable for this is only the consumption expenditure, E_t. This variable converges to E^* in the long-run, which implies that l_t is constant in the long-run,

$$l^* = 1 - \frac{1 + t_E}{1 - t_L}\gamma E^*.$$

Thus, the growth rate of the fraction of time allocated to labor supply is zero. Thirdly, G_t represents public goods supplied by the government. The production function is $G_t = L_{G_t}$. The steady state value of L_{G_t} is

$$L_{G_t} = g\left\{t_L L_t\left(1 - \frac{1+t_E}{1-t_L}\gamma E^*\right) + [t_\pi + t_A(1-t_\pi)]\left(\frac{E^*}{\varepsilon n^*} - \phi - L_Z^*\right)n^* L_t + t_E E^* L_t\right\}.$$

The growth rate of public goods is, therefore,

$$\begin{aligned}\frac{\dot{G}_t}{G_t} &= \frac{\dot{L}_{G_t}}{L_{G_t}}\\ &= \frac{g\dot{L}_t\left\{t_L\left(1 - \frac{1+t_E}{1-t_L}\gamma E^*\right) + [t_\pi + t_A(1-t_\pi)]\left(\frac{E^*}{\varepsilon n^*} - \phi - L_Z^*\right)n^* + t_E E^*\right\}}{gL_t\left\{t_L\left(1 - \frac{1+t_E}{1-t_L}\gamma E^*\right) + [t_\pi + t_A(1-t_\pi)]\left(\frac{E^*}{\varepsilon n^*} - \phi - L_Z^*\right)n^* + t_E E^*\right\}}\\ &= \lambda.\end{aligned}$$

Combining these results yields the growth rate of an individual's utility.

$$\frac{\dot{u}_t}{u_t} = \frac{\dot{C}_t}{C_t} + \mu\frac{\dot{G}_t}{G_t} = \theta\frac{\dot{Z}_t}{Z_t} + \left(\frac{1}{\varepsilon-1} + \mu\right)\lambda.$$

This is not affected by the population scale but is endogenously determined by parameters such as preference and fiscal variables.

E The Stability for the Subsidy Policy Model

The system of differential equations that characterize the subsidy model is given from

$$\frac{\dot{E}_t}{E_t} = \frac{\alpha(1-t_A)(1-t_\pi)\beta}{\alpha(1-s_H)-(1-t_\pi)(1-s_V)\beta}\left\{[1-\theta(\varepsilon-1)]\frac{E_t}{\varepsilon n_t} - \phi\right\} - \rho$$

$$\frac{\dot{n}_t}{n_t} = \beta\left[(1-gt_L)\frac{1}{n_t} - \frac{1}{\alpha(1-s_H)-(1-t_\pi)(1-s_V)\beta}\left\{\Gamma_s\frac{E_t}{\varepsilon n_t} + \beta\omega_s\phi\right\} - gt_E\frac{E_t}{n_t}\right] - \lambda,$$

where

$$\Gamma_s = \alpha(1-s_H)\left\{g\tilde{\tau}(1-\theta(\varepsilon-1)) + (\varepsilon-1)\left(1+\frac{\theta}{1-s_V}\right)\right\} - (1-t_\pi)[\varepsilon(1-s_V)+s_V]\beta,$$

$$\omega_s = (1-g\tilde{\tau})\alpha(1-s_H) + (1-t_\pi)\beta s_V.$$

Before proceeding to the next step, I will consider the signs of Γ_s and ω_s.

$$\begin{aligned}
\Gamma_s &= \alpha(1-s_H)\left\{g\tilde{\tau}(1-\theta(\varepsilon-1)) + (\varepsilon-1)\left(1+\frac{\theta}{1-s_V}\right)\right\} - (1-t_\pi)[\varepsilon(1-s_V)+s_V]\beta \\
&= \frac{1}{1-s_V}\Big[\alpha(1-s_H)\{(1-s_V)g\tilde{\tau}(1-\theta(\varepsilon-1)) + (\varepsilon-1)(1-s_V+\theta)\} \\
&\qquad - (1-s_V)(1-t_\pi)\{\varepsilon-(\varepsilon-1)s_V\}\Big]\beta \\
&= \frac{1}{1-s_V}\Big[\alpha(1-s_H)(1-s_V)g\tilde{\tau}(1-\theta(\varepsilon-1)) + \alpha(1-s_H)(\varepsilon-1) + \alpha\theta(1-s_H)(\varepsilon-1) \\
&\qquad - \alpha(1-s_H)(\varepsilon-1)s_V - (1-s_V)(1-t_\pi)\varepsilon\beta + (1-t_\pi)(1-s_V)(\varepsilon-1)s_V\beta\Big] \\
&= \frac{1}{1-s_V}\Big[\alpha(1-s_H)(1-s_V)g\tilde{\tau}(1-\theta(\varepsilon-1)) + \alpha(1-s_H)(\varepsilon-1) + \alpha\theta(1-s_H)(\varepsilon-1) \\
&\qquad - \alpha(1-s_H)(\varepsilon-1)s_V - (\varepsilon-1)(1-s_V)(1-t_\pi)\beta - (1-t_\pi)(1-s_V)\beta \\
&\qquad + (1-t_\pi)(1-s_V)(\varepsilon-1)s_V\beta\Big] \\
&= \frac{1}{1-s_V}\Big[\underbrace{\alpha(1-s_H)(1-s_V)g\tilde{\tau}(1-\theta(\varepsilon-1))}_{+} + \underbrace{\alpha\theta(\varepsilon-1)(1-s_H) - (1-t_\pi)(1-s_V)\beta}_{+} \\
&\qquad + \underbrace{(\varepsilon-1)(1-s_V)[\alpha(1-s_H)-(1-t_\pi)(1-s_V)\beta]}_{+}\Big] > 0.
\end{aligned}$$

The second and third terms in the brackets are positive under Assumption 3. The sign of ω_s is also positive because $(1-g\tilde{\tau}) \in (0,1]$.

I take first-order Taylor expansions of these differential equations around the steady-state values, E^s and n^s.

$$\begin{bmatrix} E_t \dot{-} E^s \\ n_t \dot{-} n^s \end{bmatrix} = \begin{bmatrix} \frac{\alpha(1-t_A)(1-t_\pi)\beta(1-\theta(\varepsilon-1))}{\alpha(1-s_H)-(1-t_\pi)(1-s_V)\beta}\frac{E^s}{\varepsilon n^s} & -\frac{\alpha(1-t_A)(1-t_\pi)\beta(1-\theta(\varepsilon-1))}{\alpha(1-s_H)-(1-t_\pi)(1-s_V)\beta}\frac{(E^s)^2}{\varepsilon(n^s)^2} \\ -\beta\Psi_s & -\left[\frac{\omega_s}{\alpha(1-s_H)-(1-t_\pi)(1-s_V)\beta}\beta\phi+\lambda\right] \end{bmatrix} \cdot \begin{bmatrix} E_t - E^s \\ n_t - n^s \end{bmatrix},$$

where

$$\Psi_s = \left[\frac{\beta \Gamma_s}{\varepsilon\{\alpha(1-s_H)-(1-t_\pi)(1-s_V)\beta\}} + \frac{(1-gt_L)(1+t_E)}{1-t_L}\gamma + \beta g t_E \right].$$

I represent the above 2×2 matrix as A_s.

$$\begin{aligned} A_s &\equiv \begin{bmatrix} \frac{\alpha(1-t_A)(1-t_\pi)\beta(1-\theta(\varepsilon-1))}{\alpha(1-s_H)-(1-t_\pi)(1-s_V)\beta}\frac{E^s}{\varepsilon n^s} & -\frac{\alpha(1-t_A)(1-t_\pi)\beta(1-\theta(\varepsilon-1))}{\alpha(1-s_H)-(1-t_\pi)(1-s_V)\beta}\frac{(E^s)^2}{\varepsilon (n^s)^2} \\ -\beta\Psi_s & -\left[\frac{\omega_s}{\alpha(1-s_H)-(1-t_\pi)(1-s_V)\beta}\beta\phi + \lambda\right] \end{bmatrix} \\ &= \begin{bmatrix} b_{11} & -b_{12} \\ -b_{21} & -b_{22} \end{bmatrix}, \end{aligned}$$

where $b_{11} = \frac{\alpha(1-t_A)(1-t_\pi)\beta(1-\theta(\varepsilon-1))}{\alpha(1-s_H)-(1-t_\pi)(1-s_V)\beta}\frac{E^s}{\varepsilon n^s} > 0$, $b_{12} = \frac{\alpha(1-t_A)(1-t_\pi)\beta(1-\theta(\varepsilon-1))}{\alpha(1-s_H)-(1-t_\pi)(1-s_V)\beta}\frac{(E^s)^2}{\varepsilon (n^s)^2} > 0$, $b_{21} = \beta\Psi_s > 0$, and $b_{22} = \left[\frac{\omega_s}{\alpha(1-s_H)-(1-t_\pi)(1-s_V)\beta}\beta\phi + \lambda\right] > 0$.

Using the matrix A_s, the determinant of A_s is obtained:

$$\det A_s = -b_{11}b_{22} - b_{12}b_{21} < 0.$$

The determinant of A is from the above discussion.

To compute the eigenvalues, denoted by ν_s, I use the condition $|A_s - \nu_s I| = 0$:

$$\begin{vmatrix} b_{11} - \nu_s & -b_{12} \\ -b_{21} & -b_{22} - \nu_s \end{vmatrix} = 0,$$

where I is identity matrix. This condition corresponds to a quadratic equation in ν_s:

$$\mu_s^2 + (b_{22} - b_{11})\nu_s - b_{11}b_{22} - b_{12}b_{21} = 0$$

When ν_s is zero, the value of the equation is $\det A_s = -b_{11}b_{22} - b_{12}b_{21} < 0$. The discriminant D_s is $D_s = (b_{22} - b_{11})^2 - 4(-b_{11}b_{22} - b_{12}b_{21}) > 0$. That means that the condition has two different real solutions, and each root has a different sign. Therefore, the dynamic system is saddle stable, and the steady-state point is (n^s, E^s).

F Proof of Proposition 3

In the present model, the Euler equation in the steady-state (31) determines the rate of return. It does not depend on whether the model includes the subsidy policy, implying that $r^* = r^s$ holds. The present paper represents (29) and (41) using r^*. Substituting (30) into (29) yields

$$L_Z^* = \frac{\alpha\theta(\varepsilon-1) - (1-t_\pi)\beta}{\alpha(1-t_\pi)\beta[1-\theta(\varepsilon-1)]} r^* + \frac{\theta(\varepsilon-1)}{1-\theta(\varepsilon-1)}\phi,$$

and substituting (40) into (41) yields

$$L_Z^s = \frac{\alpha\theta(\varepsilon-1)(1-s_H) - (1-t_\pi)(1-s_V)\beta}{(1-s_V)\alpha(1-t_\pi)\beta[1-\theta(\varepsilon-1)]} r^* + \frac{\theta(\varepsilon-1)}{(1-s_V)[1-\theta(\varepsilon-1)]}\phi.$$

Subtracting the latter expression from the former, one can compare the magnitude relationship.

$$L_Z^* - L_Z^s = \frac{\theta(\varepsilon-1)}{(1-s_V)[1-\theta(\varepsilon-1)]}\left[\frac{s_H - s_V}{(1-t_\pi)\beta} r^* - s_V\phi\right].$$

If $s_H < s_V$, the first term in the square brackets is negative so that $L_Z^* < L_Z^s$ holds. If $s_H = s_V$, the first term is zero so that $L_Z^* < L_Z^s$ holds. If $s_H > s_V$, one cannot confirm the sign immediately. For this to be negative, the sum of the terms in the square brackets must be positive, that is, the condition

$$\frac{s_H - s_V}{(1-t_\pi)\beta} r^* - s_V\phi < 0 \quad\Longleftrightarrow\quad \frac{s_V}{s_H} > \frac{1}{1+\frac{(1-t_\pi)(1-t_A)\beta\phi}{\rho}}.$$

must hold. Conversely, if $\frac{s_V}{s_H} < \frac{1}{1+\frac{(1-t_\pi)(1-t_A)\beta\phi}{\rho}}$, the employing units of labor in the vertical innovation firms decreases and deteriorate productivity by innovation. This leads to a decrease in economic growth.