An Empirical Study of the Fisher Effect and the Dynamic Relation Between Nominal Interest Rate and Inflation in Singapore

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Abstract

The Fisher effect postulated that real interest rate is constant, and that nominal interest rate and expected inflation move one-for-one together. This paper employs Johansen’s method to investigate for the existence of a long-run Fisher effect in the Singapore economy over the period 1976 to 2006, and finds evidence of a positive relationship between nominal interest rate and inflation rate while rejecting the notion of a full Fisher Effect. The dynamic relationship between nominal interest rate and inflation rate is also examined from the error-correction models derived, and the analysis is extended to investigate the impulse response functions of inflation and nominal interest rates where we discover the presence of the Price Puzzle in the Singapore market.

JEL Classification: E40, E43, E31

Keywords: Fisher effect, Price puzzle, Singapore, interest rate, inflation, cointegration, impulse response function

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1. Introduction

The Fisher effect has, since its proposition, been widely accepted in theory. In essence, it postulates that the real interest rate on a financial asset is constant over time. As such, changes in nominal interest rate fully reflect changes in expected inflation.

However despite the general acceptance for the theory of the Fisher Effect, empirical support for its existence in the real world has been rather mixed. Many studies including Fama and Gibbon (1982), Huizinga and Mishkin (1986) and Kandel et al. (1996) have found the estimated slope coefficients in regressions of nominal interest rates on various measures of expected inflation to be substantially less than the hypothesized value of one, implying that real interest rates are negatively associated with expected inflation. This is particularly puzzling as Darby (1975) has demonstrated that the response of nominal rates should in fact be much greater than one due to the taxation on interest income. Several explanations have been proposed for the inability of these studies to detect a full Fisher effect. For example, the existence of a Tobin effect where investors re-balance their portfolios in favour of real assets during times of high expected, or money illusion (Modigliani and Cohn, 1979; Tanzi, 1980; Summers, 1983) by financial markets and investors would have prevented a full Fisher Effect. In addition, Evans and Lewis (1995) also highlighted how possible peso problems in the nominal debt market could lead to an inaccurate conclusion that agents behave irrationally, while Fried and Howitt (1983) argued that because financial assets feature a liquidity premium that increases with expected inflation, therefore the response of nominal interest rates to changes in inflation will depend on
the riskiness of the bonds. The lack of a full Fisher Effect has also been put down to a lack of consideration for the time series properties of both interest rates and inflation, which necessarily led to misleading results.

In recent years, the development of new and more powerful econometric methods has rekindled empirical work into the validity of Fisher effect. Mishkin (1992) employed the Engle-Granger error-correction mechanism to test for the effect in US, while Hawtrey (1997) and Daniels et al (1996) have separately applied Johansen’s method to Australian data. Both latter works have yielded support for the existence of the Fisher effect. Payne and Ewing (1997) applied a similar approach to the data of nine less developed countries (LDCs) and their results showed that five of the countries display no sign of a long run Fisher effect, while three of them provided convincing evidence of it.

The motivation for this paper is twofold:

Firstly, an investigation into the Fisher Effect is immensely important because the Fisher Effect forms the cornerstone of many theoretical models that generate monetary neutrality, and therefore has important policy implications for the behaviour of interest rates and the efficiency of financial markets. For example, standard theoretical consumption-based asset pricing models depend critically on the necessary condition of stationary ex-ante real interest rates which are implied by the notion of the super-neutrality of money in the Fisher effect. This critical assumption of a constant real interest rate in steady state is also found in neo-classical growth theory that is based on the dynamic optimisation of the representative economic agent. In addition, the existence of a stationary real interest rate has important practical policy implications. Besides its implication on the continuing debate between active and passive policy-making, the implication on monetary policy for small, open economies that are reliant on exports and international trade is particularly pertinent as the competitiveness of these small, open economies, and hence its trade and capital flows, are governed by the real exchange rate which is ultimately determined by the real interest rate. Also, the popular use of short-term
interest rate as a leading economic indicator assumes that movements in short-term interest rate will primarily reflect fluctuations in expected inflation. An investigation into the Fisher Effect is therefore also an investigation into the appropriateness of interest rate as an indicator of monetary policy.

The second motivation for this paper stems from the fact that while there exists a large body of literature studying the Fisher Effect in US, Canada and European countries, little work has been done for Asian countries, particularly for small, open economies like Singapore. Also, studies adopting vector error-correction techniques for investigating the existence of the Fisher Effect have frequently stopped at the proof of existence (or lack of it) of the Fisher Effect without further analysis into the dynamic relationships between the variables of nominal interest rate and inflation rate as well as a study of the corresponding impulse response functions.

This paper therefore employs Johansen’s cointegration method to investigate for the existence of a long-run Fisher effect in the Singapore economy over the period 1976 to 2006. While it finds evidence of a positive relationship between interest rate and inflation rate, the notion of a full Fisher Effect is rejected. The error-correction models are derived and the dynamic relationship between interest rate and inflation rate examined before analysis is extended to investigate the impulse response functions of the inflation and interest rates where we discover the existence of the Price Puzzle in the Singapore market.

The paper is structured as follows: Section 2 gives a theoretical framework for the test procedures employed, and more importantly, provides the rationale for the choice of certain methods or specifications. Section 3 gives a description of the data used. This is followed by the empirical results and their interpretations in Section 4. Section 5 provides the conclusions of the paper.

This paper primarily uses the EasyReg package by Herman Bierens for its econometric analysis.
2. Test Procedure

2.1 Theoretical Framework: The Fisher Effect

Suppose a dollar invested in the beginning of period $t$ yields a nominal interest $i_t$ at the end of it, and assume that inflation in one period to be $\pi_t$. In that case, the real end-of-period investment value is given by

$$1 + r_t = (1 + i_t)/(1 + \pi_t) \quad (1)$$

$$i_t = r_t + \pi_t + (r_t \cdot \pi_t) \approx r_t + \pi_t \quad (2)$$

Since decisions are made at the beginning of the period, and nominal interest rates are contracted in advance, therefore the equation above is modified to be

$$i_t = r_t + \pi_t^e \quad (3)$$

This forms the basis for Fisher’s postulation that in an efficient market, the ex ante nominal interest rate, $i_t$, is the sum of the ex-ante real interest rate, $r_t$, and the expected rate if inflation, $\pi_t^e$.

Fisher’s neutrality hypothesis further states that real interest rate is constant over time and only determined by real factors. Therefore a possible regression equation for testing his hypothesis is

$$i_t = \alpha + \beta \pi_t^e + \varepsilon_t \quad (4)$$

where $\varepsilon_t$ is the normally distributed disturbance term.

Expectations are, however, unobservable. Employing the rational expectations hypothesis that expectations are
correct in the long run, the realised future inflation rate, $\lambda_t$, can be written as

$$\lambda_t^e = E_t(\lambda_t)$$  \hspace{1cm} (5)$$

$$= \lambda_t + \eta_t$$  \hspace{1cm} (6)$$

where $\eta_t$ is the forecast error of inflation which is orthogonal to $i_t$, and $E_t(.)$ is the expectation condition on all info available at time $t$.

This then yields the regression equation

$$i_t = \alpha + \beta \lambda_t + u_t$$  \hspace{1cm} (7)$$

where $u_t = \epsilon_t + \eta_t$ is a disturbance term.

Note that any test of the Fisher effect is therefore a test of the joint hypothesis that both the Fisher effect and Rational Expectations hold.

### 2.2 Unit root Tests

Conventionally, the Fisher equation is tested by running a simple Ordinary Least Squares (OLS) regression on the above equation, followed by a t-test on the coefficient $\beta = 1$. However this is likely to be unsatisfactory because of the possibility of $i_t$ and $\lambda_t$ containing stochastic trends/unit roots in their time series processes. If one or both of the series are non-stationary, the standard OLS approach will produce a spurious
regression, thus rendering standard testing techniques invalid.

This paper will be employing the Augmented Dickey-Fuller (ADF) procedure to test for stationarity and the order of integration for both $i_t$ and $\lambda_t$. This approach permits sufficient dynamics (in the form of lagged differences) to approximate the ARIMA process in the error term, thereby eliminating autocorrelation.

The ADF test with trend is based on the following equation

$$\Delta X_t = a_0 + (1-m)a_0 T_t - mX_{t-1} + \sum_{i=1}^{m} \gamma_i \Delta X_{t-i} + u_t$$  \hspace{1cm} (8)

where $\Delta X_t = X_t - X_{t-1}$, and $m$ is the order of augmentation of the test. The test without trend is similar, except that now, $T_t = 0$ and $a_0$ is replaced by $a_0(1-m)$. The performance of a test with a trend in addition to a test without trend is theoretically unnecessary since trends in interest rates and inflation seemed unlikely. However, if real interest rate is related to economic and population growth as proposed by the Golden Rule of Capital Accumulation, then real interest rate may be trended if GDP and population growth are trended. This will then impart trend properties onto $i_t$. In particular, since Singapore has shown consistently increasing and high economic growth of 6 - 8% over the past thirty years, it is highly likely for $r_t$ and hence $i_t$ to be trended.

The tests are performed for 0 to 20 lags i.e. five years, with the choice of the number of lagged differenced terms (in the ADF) chosen based on the Hannan-Quinn Criteria (HQC) and Schwarz Bayesian Criteria (SBC). The null hypothesis is that of non-stationarity ie $H_0 : \rho = 0$. The most appropriate order of augmentation is then chosen.

The Hannan-Quinn Information Criterion (HQC) is defined as
\[ HQC = \ln(\theta) - p.\ln(\ln(n)) \tag{9} \]

The Schwarz Bayesian Criterion (SBC) is defined by

\[ SBC = \ln(\theta) - p.(\ln(n))/2 \tag{10} \]

where \( \ln(\theta) \) is the maximised value of the log-likelihood function,

\( \theta \) is the maximum likelihood estimate

\( p \) is the number of freely estimated parameters

If non-stationarity is not rejected, the variable is differenced once and the ADF test performed. This is repeated until stationarity is achieved. The number of differences taken before the series becomes stationary is then the order of integration i.e. \( I(d) \).

2.3 Johansen’s Cointegration Method

When two series are integrated of the same order, they are said to be cointegrated i.e. \( CI(d,b) \), if a linear combination of the two series is integrated of order \( d-b \). If, say, \( i_t \) and \( \lambda_t \) are both integrated of order one, then a unit root test on \( u_t \) in the cointegrating regression

\[ i_t = \alpha + \beta\lambda_t + u_t \tag{12} \]

estimated by OLS will also be a test for cointegration. Alternatively, a Cointegrating Regression Durbin-
Watson test can be performed. This simply involves comparing the d-statistic of the equation with the critical values of 0.511, 0.386 and 0.322 at 1%, 5% and 10% significance levels respectively. The null hypothesis of cointegration i.e. $d=0$ is rejected if $d$ is less than the critical value.

Although both these methods are fairly simple and intuitive, there are certain disadvantages to them. One fundamental problem with them is that they may have low power i.e. their ability to distinguish between the two alternatives of cointegration and no cointegration is limited. Also, reliable hypotheses tests cannot be performed on the point estimates of the cointegrating relation because the standard errors are misleading. This paper therefore pursues Johansen’s method which not only has higher power, but also allows various restrictions on the point estimates of the cointegrating vector to be tested using likelihood-ratio tests.

In this current application, Johansen’s procedure is based on the following error-correction forms:

$$
\Delta i_t = \mu_1 + \sum_{j=1}^{k-1} \Gamma_{1j} \Delta i_{t-j} + \sum_{j=1}^{k-1} \Gamma_{12(j)} \Delta \lambda_{t-j} + \Pi_{11} i_{t-k} + \Pi_{12} \lambda_{t-k}
$$

(13)

$$
\Delta \lambda_t = \mu_2 + \sum_{j=1}^{k-1} \Gamma_{21(j)} \Delta i_{t-j} + \sum_{j=1}^{k-1} \Gamma_{22(j)} \Delta \lambda_{t-j} + \Pi_{21} i_{t-k} + \Pi_{22} \lambda_{t-k}
$$

(14)

where the matrix $\Gamma$ represents the short run dynamics of the relationship between $i_t$ and $\lambda_t$, and the matrix $\Pi$ captures the long run information in the data.

Johansen’s cointegration test then involves determining the rank of matrix $\Pi$ (denoted by $r$) which reflects the number of cointegrating vectors in the process governing movements of $i_t$ and $\lambda_t$. In this application, there are three possible ranks of $\Pi$. If $r = 2$ (i.e. $\Pi$ is full rank), that means that both $i_t$ and $\lambda_t$ are stationary processes. This however contradicts the earlier findings that they are both integrated of order one. If $r = 0$, then there is no cointegration between the two variables, and no stationary long run relationship exists i.e. Fisher effect is rejected. When $r = 1$, there is a single cointegrating vector between $i_t$ and $\lambda_t$ such that $\Pi =$
$\alpha\beta'$, where $\alpha$ contains the cointegrating vector and $\beta$ is the corresponding error-correction coefficients. This supports the presence of a Fisher effect.

To determine between these three cases, this paper uses the trace test and the maximal eigenvalue test, based on the classical likelihood ratio test, developed by Johansen. The critical values are given by EasyReg.

In Johansen’s method, the specification of a constant and a trend (either restricted or unrestricted) is very important. The inclusion of a constant term is unambiguously supported by economic theory since Fisher hypothesizes a real interest rate represented by the constant. The dilemma lies in whether this constant should be restricted and forced to lie in the cointegration space, or unrestricted to allow for non-zero drift in the unit-root processes. Theoretically, interest rates should have no possibility of inherent drift or trending. As such, the test would involve a restricted constant with no trends.

The order (lag length) of the Vector Autoregressive (VAR) form is chosen by selecting the VAR(p) model based on its HQC and SBC values. Lags of 0 to 20 quarters are tested.

Likelihood ratio tests can then be used to test restrictions on $\alpha, \beta$ as postulated by the Fisher effect. The short-run and long-run dynamics will also be examined. If the restriction of Fisher effect holds, I will proceed to generate two error-correction models of the following forms with changes in inflation and changes in nominal interest rates as the dependent variables:

$$\Delta i_t = k + A_{11}(L)\Delta i_{t-1} + A_{12}(L)\Delta i_{t-1} + \gamma(i_t - \lambda_t) + \epsilon_{1t} \quad (15)$$

$$\Delta \lambda_t = k + A_{21}(L)\Delta \lambda_{t-1} + A_{22}(L)\Delta i_{t-1} + \delta(i_t - \lambda_t) + \epsilon_{2t} \quad (16)$$
The terms in the brackets refer to the cointegrating relation used in the models.

### 2.4 Impulse Response Function

Using the VECM system estimated, the analysis will be extended to generate impulse response functions. A shock to the \( i \)th variable impacts both the \( i \)th variable as well as the other endogenous variables through the dynamic (lag) structure of the VECM. An impulse response function therefore traces the effect of a one-time unit shock to one of the innovations on current and future values of the endogenous variables. This is derived by first expressing a VAR in a vector MA(\( \infty \)) such as

\[
y_t = \mu + \Psi_1 \epsilon_{t-1} + \Psi_2 \epsilon_{t-2} + \Psi_3 \epsilon_{t-3} + \ldots
\]

Where matrix \( \Psi_s = \partial y_{ts} / \partial \epsilon_i \) is a vector such that row \( i \) column \( j \) of \( \Psi_s \) identifies the consequences of a one-unit increase in the \( j \)th variable’s innovation at date \( t \), holding all the other innovations constant.

### 3. Data Sample

Singapore is a relatively young country, with the monetary authority installed only in September 1970. Its exchange rate was floated in 1973 and gradual liberalisation of investment and exchange rate controls followed. The data sample chosen is therefore quarterly data from 1976Q1 to 2006 Q4.

Singapore’s monetary policy has mainly centered on managing the Singapore dollar against a basket of currencies of her main trading partners. Coupled with the virtual non-existence of exchange controls on inflows and outflows of foreign currency funds i.e. openness of Singapore’s capital accounts, domestic interest
rates are largely left to be determined by market forces. The 3-month domestic inter-bank rate, \( i_t \) (in percent per annum) is therefore used as the variable representing interest rates. The measure of inflation used is the quarterly Consumer Price Index change annualised, \( \pi_t \) (in percent per annum). Both data are obtained from the *IMF Financial Statistics*.

Ideally, the government Treasury-bill rate should have been used instead of the interbank rate, since it is risk-free. However, as mentioned before, Singapore’s relatively short history means that 3-month T-bills are only available from 1988 while the 2-years and 7-years T-bills are only issued as recent as January 1997. The sample size is therefore not sufficiently large enough for any meaningful investigation to be done, especially when the Fisher effect is a long-run relation. As such, implicit in the selection of \( i \) is the assumption of constant risk premiums. Historically, there is no reason to suspect any change in risk premiums.

4. Empirical Results

Graph 1 shows the time series plot of \( i \) and \( \pi \). It can be seen that both variables have tended to move together in a co-trending fashion and displays no time-trending properties.

Table 1 then shows the tests for non-stationarity using ADF tests with (1) a constant term, and (2) a constant term with trend. The tests for unit root and unit root with drift for variables \( i \) and \( \pi \) are not rejected at 10% significance level while the ADF tests for the variables in first difference, \( \Delta i \) and \( \Delta \pi \), are rejected at 5% significance level. The results therefore show that both variables \( i \) and \( \pi \) are integrated of order one i.e. first-difference stationary.

In addition, with both tests (with and without trend term) yielding the same conclusions, it hints at a lack of existence of a time trend which is supported by a visual examination of the time series plot in Graph 1.
This supports the theoretical reasoning for the adoption of a model with a restricted intercept and no time trend in the Johansen’s cointegration test that will follow.

After confirming that both processes are $I(1)$, we then proceed to perform cointegration tests. Before performing Johansen’s test, however, the order of the VAR model is chosen by running an unrestricted VAR from 0 to 20 lags. Table 2 shows the computations for HQC and SBC. While the HQC suggests a VAR of order 6, the SBC suggests a VAR of order 1. We chose a VAR of order 6 as a VAR of order 1 would have implied no existence of short-run dynamics.

Table 3 shows the results of the Johansen’s cointegration tests for a model with restricted intercept and no time trend. Both the $\lambda$-test and the trace test reject the null hypothesis of no cointegrating relation at 10% level although they fail to reject the null hypothesis at 5% level. Both tests also fail to reject the null hypothesis of one cointegrating vector at 20% level. There is therefore strong evidence of the presence of one cointegrating relation between $i_t$ and $\lambda_t$.

Table 4 shows the estimated cointegrating vector (with the coefficient of $\pi_t$ normalised) and the error correction model. Note that the estimated eigenvectors are reported as they would appear on one side of the estimated equation. The estimated vector therefore represents the regression equation $0.43 = 0.56i_t - \lambda_t$ which follows the form of the Fisher Hypothesis $r = i_t - \lambda_t$. It can be observed that the signs of the coefficients are exactly as hypothesised. The coefficient of $i_t$ for the VAR is however less than one. In addition we tested for the full Fisher Effect by applying the restriction of $\beta = [1 -1]$ and recalculating the estimated eigenvector by the modified Newton-Raphson iterative algorithm with a damping factor of 0.01. Table 5 shows the new estimated restricted cointegrating vectors, and the relevant LR statistic which is distributed as a $\chi^2$ variate with one degree of freedom (given by the total number of restrictions i.e. two, less the number of just-identifying restriction i.e. one). The LR statistic is significant at 5% level, therefore the restriction is rejected. This implies
that while a positive relationship exists in the long-run between nominal interest rate and inflation rate in Singapore, the full Fisher Effect of a one-for-one change in expected inflation and nominal interest rate is invalid, thus rejecting the notion of super neutrality of money. This result is similar to the findings in other studies, and indicates the likely existence of a Tobin effect, money illusion, liquidity premium in Singaporean financial assets, or a peso problem in the Singapore nominal debt market.

Table 4 also shows the error-correction models. To evaluate the long-run dynamics, the coefficients of the error-correction terms are inspected. Besides having the correct signs, the coefficients of the error-correction terms are also statistically significant at 5% level. The magnitude is fairly significant, implying that it will take a moderate amount of time for the equations to return to their equilibrium after a shock occurs. In particular for \( \Delta i_t \), a coefficient of 0.184 means that interest rate moves to eliminate 18.4% of the long-run disequilibrium within one quarter, while a coefficient of -0.402 for \( \Delta P_t \) means that inflation moves to eliminate 40.2% of the long-run disequilibrium within a quarter. This is indicative of a fairly efficient Singaporean financial market. The direction of movement is given by the sign of the coefficients, with respect to the cointegrating relation. Another observation is that the coefficients of the error-correction terms for inflation equations are generally larger than the corresponding nominal interest rate equations. This seems inconsistent with standard rational expectations-macroeconomic models which assume efficient financial markets and sticky goods markets. Interest rates, being more flexible, should be faster adjusting than prices, which are sticky. The reason for this could lie with the country’s choice of exchange rate as a monetary policy instrument. With the relative openness of Singapore’s economy and its heavy reliance on both imports and exports due to its lack of natural resources, Singapore’s exchange rate policy is conducted so as to maintain, among other things, the competitiveness of its exports. As such prices probably do not need to adjust as much given its new equilibrium value is likely not too far from the original one, unlike interest rates.

To examine the short-run dynamics, the short-run coefficients are inspected. It is observed that the
signs reflecting the short-run adjustment process for both $\Delta i_t$ and $\Delta \pi_t$ are as hypothesized by theory over the different lagged periods. In addition, most of the coefficients are significant. Most other studies by various econometricians using data of different countries have yielded results showing insignificant short-run coefficients. While Mishkin have clarified that the non-existence of a short-run Fisher effect does not rule out the possibility that there is a long-run Fisher effect in which inflation and interest rates share a common trend when they exhibit cointegrating trends, our results show very strong short-run dynamics exist as indicated by the high and statistically significant magnitudes of the short-run coefficients.

Having derived the error-correction matrix, we then investigate the impulse response functions of inflation rate and nominal interest rate to unit shocks in the variables. We are particularly interested in the dynamic relation between inflation and interest rate to surprise shocks in the corresponding variable respectively.

Graph 2 shows the impulse response function to $i$ to a unit shock in $\pi$ over 20 quarters i.e. five years. It can be seen that a shock increase in inflation leads to a permanent increase in nominal interest rates.

Graph 3 shows the impulse response function to $\pi$ to a unit shock in $i$ over 20 quarters i.e. five years. It can be seen in this VAR that inflation rises for the first 3 quarters after a shock in interest rate. The finding is counter-intuitive as standard monetary theory would suggest that inflation falls for a given shock increase in interest rate. This phenomenon of an initial positive response to a contractionary monetary policy is called the Price Puzzle, and is a stylised fact of most empirical studies measuring the effects of monetary policy on the aggregate economy. This behaviour is often referred to as ‘puzzling’ because macroeconomic models either cannot explain it theoretically (eg a standard sticky-price model) or, are unable to produce a positive price response even if they can explain it in principle (eg models of the cost channel transmission of monetary policy). The discovery of the existence of a price puzzle has important policy implications as it casts serious...
doubts on the possibility of correctly identifying a monetary policy shock through the use of nominal interest rate.

5. Conclusion

The Fisher effect postulated that real interest rate is constant, and that nominal interest rate and expected inflation move one-for-one together. This paper employs Johansen’s method to investigate for the existence of a long-run Fisher effect in the Singapore economy over the period 1976 to 2006, and finds evidence of a positive relationship between nominal interest rate and inflation rate while rejecting the notion of a full Fisher Effect. The dynamic relationship between nominal interest rate and inflation rate is also examined from the error-correction models derived, with the analysis extended to investigate the impulse response functions of inflation and nominal interest rates where we discover the existence of the Price Puzzle in the Singapore market.
References:


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Graph 1: Time Series Plot of 3-month Interbank Rate versus Inflation Rate

![Graph 1](image)

Table 1: Unit Root Test

<table>
<thead>
<tr>
<th>Variables</th>
<th>$H_0$: Unit Root</th>
<th>$H_0$: Unit Root with Drift</th>
<th>$H_1$: Stationary Process</th>
<th>$H_1$: Trend Stationary</th>
<th>Conclusion</th>
</tr>
</thead>
<tbody>
<tr>
<td>$i$</td>
<td>-1.5561 (4)</td>
<td>-3.0630 (4)</td>
<td></td>
<td></td>
<td>Not rejected at 10% sig level</td>
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<tr>
<td>$\pi$</td>
<td>-2.3101 (5)</td>
<td>-2.7648 (5)</td>
<td></td>
<td></td>
<td>Not rejected at 10% sig level</td>
</tr>
<tr>
<td>$\Delta i$</td>
<td>-4.6897 (4)</td>
<td>-4.6751 (4)</td>
<td></td>
<td></td>
<td>$I(1)$</td>
</tr>
<tr>
<td>$\Delta \pi$</td>
<td>-6.1482 (6)</td>
<td>-6.1157 (6)</td>
<td></td>
<td></td>
<td>$I(1)$</td>
</tr>
<tr>
<td>Critical Values</td>
<td></td>
<td></td>
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<tr>
<td>5%</td>
<td>-2.890</td>
<td>-3.400</td>
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<tr>
<td>10%</td>
<td>-2.580</td>
<td>-3.130</td>
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</tr>
</tbody>
</table>

$ADF = \text{Augmented Dickey-Fuller test. \text{`c'} and \text{`c+t'} refer to the inclusion of a constant or both a constant and a time trend in the unit root equation for ADF. The number in the parenthesis denotes the number of lagged differenced terms (in the ADF) as chosen based on the Akaike Information Criteria, Hannan-Quinn Criteria and Schwarz Bayesian Criteria}$
## Table 2: Choosing Rank of \( \text{Var}(p) \)

<table>
<thead>
<tr>
<th>Variables</th>
<th>( i, \pi )</th>
<th>HQC</th>
<th>SBC</th>
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<td>2.47095</td>
<td>3.44544</td>
<td></td>
</tr>
<tr>
<td>17</td>
<td>2.55497</td>
<td>3.59470</td>
<td></td>
</tr>
<tr>
<td>18</td>
<td>2.62277</td>
<td>3.72853</td>
<td></td>
</tr>
<tr>
<td>19</td>
<td>2.53575</td>
<td>3.70837</td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>2.43777</td>
<td>3.67808</td>
<td></td>
</tr>
</tbody>
</table>

Optimal \( p \) | 6 | 1

## Table 3: Johansen's Cointegration Analysis

### Test for the cointegration rank

<table>
<thead>
<tr>
<th>( H_0 )</th>
<th>( H_1 )</th>
<th>Test stat</th>
<th>Critical Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \lambda )-Max test</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( r = 0 )</td>
<td>( r = 1 )</td>
<td>15.3</td>
<td>15.8</td>
</tr>
<tr>
<td>( r \leq 1 )</td>
<td>( r = 2 )</td>
<td>4.1</td>
<td>9.1</td>
</tr>
<tr>
<td>Trace test</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( r = 0 )</td>
<td>( r \geq 1 )</td>
<td>19.4</td>
<td>20.2</td>
</tr>
<tr>
<td>( r \leq 1 )</td>
<td>( r \geq 2 )</td>
<td>4.1</td>
<td>9.1</td>
</tr>
</tbody>
</table>

Number of cointegrating relations: 1

\( r \) is the number of cointegrating relations
Table 4: Vector Error Correction Matrix

**Restricted Intercept and No Trend**

*Estimated Cointegrated Vector and Error Correction Term in Johansen Estimation*

<table>
<thead>
<tr>
<th>Cointegrating vector</th>
<th>( \pi )</th>
<th>constant</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.56</td>
<td>1.00</td>
<td>0.43</td>
</tr>
</tbody>
</table>

**Error Correction Model**

<table>
<thead>
<tr>
<th></th>
<th>( \Delta i )</th>
<th>( \Delta \pi )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>0.054 (0.547)</td>
<td>-0.244 (0.246)</td>
</tr>
<tr>
<td>ECM(_t)</td>
<td>0.184 (0.003)</td>
<td>-0.402 (0.006)</td>
</tr>
<tr>
<td>( \Delta i_{t-1} )</td>
<td>-0.221 (0.016)</td>
<td>0.723 (0.001)</td>
</tr>
<tr>
<td>( \Delta i_{t-2} )</td>
<td>-0.266 (0.004)</td>
<td>0.585 (0.007)</td>
</tr>
<tr>
<td>( \Delta i_{t-3} )</td>
<td>-0.093 (0.334)</td>
<td>0.295 (0.195)</td>
</tr>
<tr>
<td>( \Delta i_{t-4} )</td>
<td>-0.244 (0.010)</td>
<td>0.365 (0.103)</td>
</tr>
<tr>
<td>( \Delta i_{t-5} )</td>
<td>-0.062 (0.515)</td>
<td>0.135 (0.546)</td>
</tr>
<tr>
<td>( \Delta \pi_{t-1} )</td>
<td>0.117 (0.001)</td>
<td>-0.43 (0.000)</td>
</tr>
<tr>
<td>( \Delta \pi_{t-2} )</td>
<td>0.158 (0.000)</td>
<td>-0.526 (0.000)</td>
</tr>
<tr>
<td>( \Delta \pi_{t-3} )</td>
<td>0.125 (0.007)</td>
<td>-0.731 (0.000)</td>
</tr>
<tr>
<td>( \Delta \pi_{t-4} )</td>
<td>0.157 (0.003)</td>
<td>-0.459 (0.000)</td>
</tr>
<tr>
<td>( \Delta \pi_{t-5} )</td>
<td>0.198 (0.001)</td>
<td>-0.585 (0.000)</td>
</tr>
</tbody>
</table>

**Summary Statistics**

<table>
<thead>
<tr>
<th>Std Error</th>
<th>R-Square</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.986</td>
<td>0.211</td>
</tr>
<tr>
<td>2.322</td>
<td>0.450</td>
</tr>
</tbody>
</table>

**\( H_0 \): Parameters are jointly zero**

<table>
<thead>
<tr>
<th>Wald test</th>
<th>p-value</th>
<th>Critical Values*</th>
</tr>
</thead>
<tbody>
<tr>
<td>31.54</td>
<td>0.00</td>
<td>18.55 21.03</td>
</tr>
</tbody>
</table>

### Critical Values

<table>
<thead>
<tr>
<th></th>
<th>5%</th>
<th>10%</th>
<th>5%</th>
<th>10%</th>
</tr>
</thead>
<tbody>
<tr>
<td>18.55</td>
<td>21.03</td>
<td></td>
<td>18.55</td>
<td>21.03</td>
</tr>
</tbody>
</table>

*Asymptotic null distribution: Chi-square(12)

The p-values are shown in parentheses.
Graph 2: Impulse Response Function of $i$ to a Unit Shock in $\pi$

Graph 3: Impulse Response Function of $\pi$ to a Unit Shock in $i$
Graph 4: Impulse Response Function of $i$ to a Unit Shock in $i$

Graph 5: Impulse Response Function of $\pi$ to a Unit Shock in $\pi$