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Abstract: This paper develops a growth framework with international knowledge spillovers driven by learning-by-investing externalities that is able to replicate most stylized facts on income convergence and economic growth. The model predicts that knowledge spillovers from the frontier enhance the relative levels of productivity and income of the laggards, but materialise only if the gap in capital intensity between the frontier and lagging economies is not too wide. We bring the model to the data and, for a global sample of countries, observe that relative capital does mostly shape their growth pattern, in line with the predictions of our theory. We also document that differentials in income growth are driven by the gaps in capital intensity with respect to the frontier economy (the United States) only below an identified threshold. This effect is independent of knowledge inputs such as human capital and innovation. Our findings suggest that the absorption of knowledge through learning processes and embodied technological change remains a crucial driver of income growth for a significant number of lagging economies worldwide.

JEL classification: F43, F63, O11, O41, O47

Keywords: Multi-Country AK Model, Cross-Country Knowledge Transmission, Inequality, Convergence

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1 Introduction

One of the mostly debated issues in macroeconomic literature is the global trend to income convergence, i.e. whether less developed countries are catching up with developed economies and international differences in income per capita are shrinking (so-called 'convergence hypothesis').[1] The debate has attracted the attention of academics and policymakers interested in explaining international gaps in living standards and identifying the best modeling of the modern growth processes (Johnson and Papageorgiou, 2020). According to the neoclassical growth theory, cross-country differences in initial conditions would show up in different (diminishing) rates of marginal product of capital and these would explain income growth gaps along the stage of dynamic transition towards the equilibrium path. Conversely, endogenous growth models based on capital or knowledge externalities, human capital and ideas generation, contend that differences in income per capita would be persistent over time because of either disparities in internal conditions or as a result of the individuals' decision to access foreign markets or invest in innovation and human capital accumulation.

One key result in the earlier empirical literature was that poor countries tend to either lag behind rich countries (Barro, 1991; Pritchett, 1997), or at best 'conditionally' converge to their own steady state (Barro and Sala-i Martin, 1991; Barro and Sala-i Martin, 1992; Mankiw *et al.*, 1992). Another stream of studies has looked at productivity dynamics finding, for instance, 'unconditional' convergence in the manufacturing sector (Madsen and Timol, 2011; Rodrik, 2012). However, at least since the 1970s, if one considers the global sample of countries, there is evidence of within-country efficiency convergence but productivity divergence with respect to the frontier economy (the US), making the exception of East Asian countries (Battisti *et al.*, 2022).

Recently, a new stream of studies has provided evidence for 'unconditional' convergence since the mid-1990s (Kremer *et al.*, 2021), also showing that: (i) emerging-market and middle-income economies are growing faster than advanced-frontier economies (henceforth, 'Wilde'-convergence after Roy *et al.*, 2016); and (ii) low-income countries are catching up slowly with the rich at an estimated rate of 0.7 percent per year (Patel *et al.*, 2021). The latter result is particularly important for the literature as it contrasts the so-called Barro's 'iron law' of 2 percent (Barro, 2012). Patel *et al.* (2021) also document that the global pattern of income convergence would be inverted-U shaped, with middle-income economies growing faster than any other group of countries, and more regular over time, with less frequent growth spells and decelerations.

This new wave of empirical studies naturally raises the question of what has driven income

[1] In growth literature, the convergence hypothesis refers to the process by which poorer economies grow faster than richer ones, thereby closing the income gap over time. There are two key concepts of convergence: 'conditional' convergence and 'unconditional' (or 'absolute') convergence. Conditional convergence suggests that economies will converge to parallel growth paths in the long term if they do not share the same macroeconomic fundamentals but have access to the same technology. In such a case, poorer economies are expected to converge to the same growth rate as richer ones, but not to the same levels of income per capita. In contrast, absolute convergence posits that all economies will eventually converge to the same growth path regardless of their initial conditions, structural characteristics, or policies. This implies that poorer economies are expected to converge to the same growth rate as richer ones and the same levels of income per capita. For a deeper discussion about the distinction between conditional and unconditional convergence, interested readers are referred to Barro and Sala-i Martin (2005).

convergence over the past four decades and which theoretical framework best explains it. Despite numerous studies using regression analysis to identify factors influencing income growth, the empirical question of which specific factors most significantly contribute remains unresolved. One possible explanation for this prolonged period of economic convergence lies in the tendency of developing countries to adopt Western institutions, macroeconomic practices, and cultural features—a process facilitated by the global spread of ICT technologies (Kremer *et al.*, 2021).

From a theoretical perspective, this explanation aligns with the neoclassical view that cross-country income convergence results from adjustment dynamics following shifts in macroeconomic fundamentals, implying that any policy measures can have only temporary effects on growth. An alternative explanation for the emergence of unconditional convergence is rooted in endogenous growth theory, which allows lagging countries to absorb technology from advanced economies (Barro and i Martin, 1997; Howitt, 2000). This concept of technology diffusion draws upon Gerschenkron's idea of 'the advantage of backwardness', which posits that latecomers to industrialization can leapfrog stages of development by learning from the experiences of early industrializers (Gerschenkron, 1962).

Building on this second research line, this paper seeks to explain the recent "new era" of convergence by developing an econometrically grounded multi-country endogenous growth model with external effects *à la* Arrow (1962), Frankel (1962), and Romer (1986). Kremer *et al.* (2021) express skepticism about the ability of AK models to explain convergence, arguing that the observed trend toward unconditional convergence since 1990 aligns with neoclassical growth models and rejects a class of endogenous growth models that predict divergence, such as the AK models of Frankel (1962) and Romer (1986). This paper demonstrates that an extended AK growth model incorporating international knowledge externalities can effectively describe the convergence trends of the past forty years while also providing an econometric framework to identify development thresholds beyond which countries begin to benefit from knowledge acquisition.

Following Romer (1986) and Greiner and Semmler (1996), our framework assumes that the capital stock proxies the economy's stock of technical knowledge. This assumption implies that capital accumulation not only expands the physical capital base but also embeds technological advancements that enhance productivity.[2] To account for different stages of economic development, we postulate the existence of a single frontier (leader) economy, capable of advancing the global technology frontier, alongside a set of non-frontier (follower) economies. The pivotal points in our framework are: (i) productivity gains from knowledge improvements are not limited to the leader economy, but spill over to follower economies through international external effects; (ii) only those countries with a stock of capital per worker exceeding a certain threshold are able to implement foreign-produced technical knowledge and thus catch up with the leader.

The analysis is developed in several steps. As a first step, we present a two-country version of our multi-country model where the knowledge gap between economies is so large that it pre-

[2] In learning-by-investing growth models of Frankel (1962), Romer (1986) and Rebelo (1991), the concept of capital encompasses both physical and human capital. This broader definition reflects the idea that investments in physical infrastructure and equipment, as well as in education, skills, and knowledge production, contribute to the productive capacity of the economy and drive long-term growth.

vents the non-frontier (poor) economy from tracking the state of technology in use in the frontier (rich) economy. In such a scenario, each country operates as an independent technology 'island', resulting in the model predicting only 'unconditional' divergence (see Barro, 1991; Pritchett, 1997).

Next, we consider the alternative case in which the technology gap between countries is not so large as to prevent the poor economy from absorbing and implementing part of the foreign-produced technical knowledge. In this context, we find that if the poorer economy has access to the same pool of technical knowledge as the richer, the model predicts 'conditional' convergence; i.e., laggard economies converge towards parallel growth paths (Barro and Sala-i Martin, 1992; Mankiw *et al.*, 1992).

Furthermore, if the non-frontier country shares the same macroeconomic fundamentals and institutions as the frontier country, then our two-country AK model can predict both the standard 'absolute' convergence described by Patel *et al.* (2021) and Kremer *et al.* (2021), and the 'Wilde' convergence documented by Roy *et al.* (2016).

Since the size of the knowledge gap is key in establishing whether a laggard country can close its gap with respect to the frontier, we bring the model to the data and evaluate econometrically the main predictions of our theory. Using information for a global sample of countries (i.e., 179 economies with different levels of development observed from 1950 to 2019), we perform a panel dynamic regression documenting the soundness of the model. Specifically, we show that: (i) differences in the capital stocks between frontier and laggard economies can explain income growth disparities in the long run; (ii) there is a threshold over which the propulsive growth effect of relative capital endowment vanishes; and (iii) the income growth effect generated by the creation of knowledge through investments in capital equipment does not overlap with that of disembodied knowledge, which is sourced from R&D expenditure and innovation.

This study relates to several streams of economic research. First, our paper contributes to the extensive literature on endogenous growth models with knowledge spillovers (Romer, 1986; Lucas, 1988; Tamura, 1991; Stokey, 1988, 1991; Greiner and Semmler, 1996; Romer, 1990; Aghion and Howitt, 1992; Howitt, 1999). Particularly related to our study are the papers that examine the convergence implications of these theoretical frameworks (Howitt, 2000; Acemoglu and Ventura, 2002; Howitt and Mayer-Foulkes, 2005).

We contribute to this line of research in three ways. First, we extend the literature by developing a multi-country version of the Frankel (1962)-Romer (1986) model with learning by investing, where positive productivity externalities can spill over beyond the frontier country. Second, through empirical testing of the theoretical results, we demonstrate that extending the AK growth framework to include international knowledge transfers effectively explains the convergence trends observed in the data over the last forty years. Third, our econometric analysis confirms the existence of thresholds in technology gaps that hinder low-income countries from catching up and initiating a process of sustained economic development.

Relatedly, our study is also related to the literature on cross-country technological interdependence. This literature includes contributions by Rivera-Batiz and Romer (1991), Eaton and Kortum (1999), Klenow and Rodriguez-Clare (2005), Acemoglu *et al.* (2006), Ertur and Koch (2007, 2011), Moll (2008), Alvarez (2017), Jin and Zhou (2022), and Kleinman *et al.* (2023). With respect to this literature, our paper contributes by addressing the pivotal question of how technological interdependence shapes worldwide income distribution and how knowledge gaps between leading and following economies exacerbate per capita income inequality.

Our study is particularly connected with Ertur and Koch (2007), as it extends Ertur and Koch's framework in two directions. First, instead of assuming that all countries are capable of producing new knowledge, we distinguish between frontier and non-frontier economies, where the former drives technical advancements that create opportunities for knowledge absorption and positive externalities for the latter. Second, we analyze both the equilibrium and transitional dynamics of the model, as well as its capacity to generate either convergence or divergence in income per capita. This contrasts with Ertur and Koch (2007), who primarily focus on (and test) the steady-state characteristics of their framework.

Finally, our study contributes to the empirical literature inspired by the latest development of growth theories and that has sought to test which model is closer to the data (Madsen, 2008; Ang and Madsen, 2011; Venturini, 2012). In this connection, we propose an endogenous growth model which is consistent with several growth trends that arise when the technological knowledge is still concentrated on a bunch of richer countries (e.g. the OECD member states), but can spread out worldwide. Our findings confirm the existence of a global convergence process in relative income per capita, driven by capital investment and the absorption of embodied knowledge through learning-by-investing. However, this mechanism is effective only for countries whose capital intensity is not significantly lower than that of the technological leader. Economies that fall too far below this threshold—quantified as a per capita capital stock at least four times lower than that of the US—are unable to leverage this transmission channel and struggle to match the economic growth rates and living standards of advanced economies. We further demonstrate that these results are robust to several potential concerns. In particular, they hold when accounting for the absorption of disembodied knowledge from the technological frontier, remain unaffected by the unstable growth trajectories (growth spells) of less developed countries, and exhibit consistency across the income distribution.

The paper is organized as follows. Section 2 develops the two-country AK model with cross-country knowledge externalities, characterizes the dynamic equilibrium and solve the model for the steady-state growth path. Section 3 investigates, both theoretically and numerically, about the dynamic properties of the steady state and shows under which conditions the two-country AK model can generate 'conditional' and 'unconditional' convergence. Section 4 conducts a regression analysis on a global sample of countries to validate the idea that learning-by-investing externalities enabled by the absorption of knowledge embodied in capital goods, developed at home and abroad, can fuel convergence and promote long-run growth. Section 5 examines potential applications of our study, discusses the policy implications of the model, and addresses its limitations. Finally, Section 6 concludes.

2 The two-country framework

Consider an asymmetric world economy made up of two countries: a technology-frontier (or *leader*) country, indexed by ℓ, and a non-technology-frontier (or *follower*) country, indexed by f. In each country, households supply labor inelastically and accumulate capital assets, while firms produce by assembling physical capital and labor services.

To incorporate endogenous growth, we assume throughout the paper that productivity increases as a result of learning-by-investing externalities due to knowledge externalities, due to knowledge externalities, which fuel the rate of economic growth through learning-by-investing (Arrow, 1962; Romer, 1986 Greiner and Semmler, 1996). However, we differentiate between the frontier and the non-frontier economy and assume the existence of two types of knowledge spillovers: (i) localized knowledge spillovers, through which increases in each country's stock of knowledge results into more economic growth; (ii) cross-country knowledge spillovers, through which the advances in labor productivity made by the frontier country are transmitted to the non-frontier country.

The model is set in continuous time. For simplicity, we abstract from money and other nominal assets, focusing on only real quantities.

2.1 Preferences and consumption

Each country $i = \ell, f$ is populated by an infinitely-lived representative household, comprising a continuum L_i of identical individuals who provide labor services in exchange for wages. The size of each household grows over time at a constant rate ν, and all individuals globally share the same rate of time preference, ρ.

The representative household in country i maximizes the discounted flow of lifetime utility:

$$\mathcal{U}_i = \int_0^{\infty} e^{-(\rho-\nu)t} \log c_i \mathrm{d}t, \quad \rho > \nu, \tag{1}$$

subject to the flow budget constraint

$$\dot{k}_i = (r_i - \nu) k_i + w_i - c_i, \tag{2}$$

where k_i is the household i's capital stock, r_i is the rental rate of capital, w_i is the wage rate, and c_i is consumption expenditure. The necessary and sufficient conditions for an interior optimum are:

$$\dot{c}_i = (r_i - \rho) c_i \tag{3}$$

$$\lim_{t\to\infty} \left\{ e^{-\int_0^t [r_i(z)-\nu]\mathrm{d}z} \frac{k_i(t)}{c_i(t)} \right\} = 0, \tag{4}$$

for all $i = \ell, f$.

Eq. (3) is the consumption Euler equation, derived from individual utility maximization. It states that per capita consumption grows smoothly over time if and only if the rental rate of capital, r_i, exceeds the subjective discount rate, ρ. Eq. (4) is the transversality (or boundary) condition, ensuring that the solution remains optimal over an infinite time horizon.

2.2 Technologies and production

The production sector of each country i comprises a unit continuum of identical firms, each producing a unique homogeneous commodity, denoted by Y_i. The representative firm j of country i produces with the following technology:

$$Y_{j,i} = \left(\mathcal{A}_i N_{j,i}\right)^{1-\alpha} K_{j,i}^{\alpha}, \tag{5}$$

where $\alpha \in (0, 1)$ is the output elasticity of capital, supposed to be the same across countries, $N_{j,i}$ and $K_{j,i}$ are, respectively, the firm-specific levels of employment and physical capital, and $\mathcal{A}_i$ is a country-specific labor-augmenting technology parameter, whose analytical properties will be described later on in Section 2.3.

Perfect competition implies that the rental rates of capital and wages are determined by the usual marginal product conditions:

$$r_i = \alpha \mathcal{A}_i^{1-\alpha} k_{j,i}^{\alpha-1} - \delta \tag{6}$$

$$w_i = (1-\alpha)\, \mathcal{A}_i^{1-\alpha} k_{j,i}^{\alpha}, \tag{7}$$

where $k_{j,i} := K_{j,i}/N_{j,i}$ is the capital-to-labor ratio of the firm and $\delta \in (0, 1)$ is the depreciation rate of capital (supposed identical for both countries).

In the symmetric equilibrium, each firm residing in the same country finds it optimal to employ the same capital-to-labor ratio, k_i. Consequently, aggregating (5) over production units, the economy-wide production function can be written as:

$$Y_i = \left(\mathcal{A}_i L_i\right)^{1-\alpha} K_i^{\alpha}, \tag{8}$$

where $L_i = \int_0^1 N_{j,i} \mathrm{d}j$ and $K_i = \int_0^1 K_{j,i} \mathrm{d}j$ are aggregate employment and capital respectively.

2.3 Learning-by-investing and cross-country knowledge externalities

In our setting, productivity $\mathcal{A}_i$ depends on learning-by-investing externalities that capital accumulation generates on the aggregate economy (Arrow, 1962; Romer, 1986). For the developing economy f, this is also the result of successful assimilation of frontier technology through international knowledge transmission (Parente and Prescott, 1994, 2002; Alfaro *et al.*, 2008). Formally, we assume

$$\mathcal{A}_\ell = A_\ell^{1/(1-\alpha)} k_\ell \qquad \mathcal{A}_f = \begin{cases} A_f^{1/(1-\alpha)} k_f & \text{if } \kappa_f > \tilde{\kappa}_f \\ \\ A_f^{1/(1-\alpha)} \kappa_f^{\psi} k_f & \text{if } \kappa_f \leq \tilde{\kappa}_f \end{cases}, \tag{9}$$

where $A_i > 0$ is a country-specific efficiency parameter that captures the effectiveness with which each country is able to generate knowledge improvements from gross investment, $\kappa_f := k_\ell / k_f$ is the relative capital stock of the frontier economy, which measures the potential knowledge gap between countries, and $\psi \in [0, 1]$ is a given parameter capturing the portion of the leader's knowledge that is successfully assimilated by the follower. [3]

In Eq. (9), $\tilde{\kappa}_f$ is a threshold (inversely) identifying the minimum level of capital intensity (in ratio to the technology leader) required to acquire technology from abroad. When $\kappa_f > \tilde{\kappa}_f$ holds, the knowledge gap with respect to the frontier is so large that the follower country cannot assimilate any foreign-developed knowledge. This implies $\psi = 0$, and thus no knowledge transfers from the leader to the follower. On the contrary, when $\kappa_f \leq \tilde{\kappa}_f$, the knowledge gap with respect to the frontier is not so wide as to prevent the follower from absorbing foreign knowledge. In this case, the non-frontier economy can benefit from cross-country learning-by-investing externalities at an intensity which is governed by the parameter $\psi > 0$.[4]

By letting $y_i = Y_i / L_i$ denote the level of income per capita of country i at time t, and recalling the symmetric equilibrium condition $k_{j,i} = k_i$, each country's level of per capita income can be written as:[5]

$$y_\ell = A_\ell k_\ell \tag{10}$$

$$y_f = A_f \kappa_f^{\psi(1-\alpha)} k_f. \tag{11}$$

In Eq. (11), the term $\psi (1 - \alpha)$ represents the output elasticity of relative capital intensity, κf, i.e., a measure of how responsive the output of the non-frontier economy is to changes in the knowledge gap. As long as $\kappa f < \tilde{\kappa}_f$, technical advances made by the frontier economy tend to enlarge the knowledge base of the non-frontier economy, thereby raising its per capita income level.

[3]the parameter A_i can be interpreted as a catch-all parameter capturing several country-specific features such as, for instance, the quality of institutions and human capital, the level of development of the financial system, the degree of protection of property rights, etc. All things being equal, the higher A_i, the higher the ability of the country to turn gross investment into additional technological knowledge.

[4]More generally, the term $k_f \kappa_\ell^{\psi}$ appearing on the right-hand side of the productivity index $\mathcal{A}_f$, can be thought of as a special case of a Weighted Generalized Mean, or Weighted Power Mean or *Hölder* Mean, over knowledge stocks. *Hölder* Means are a family of weighted means generating functions that can include all the most important means as special cases. In the simpler case of only two countries, ℓ and f, the weighted *Hölder* mean with exponent $p \geq 0$ and weights $\psi_f > 0$ and $\psi_\ell > 0$ is: $\mu_p \left(k_\ell, k_f\right) = \left(\psi_\ell k_\ell^p + \psi_f k_f^p\right)^{1/p} / \left(\psi_\ell + \psi_f\right)$. It can be shown that when $p \to 0$, the Weighted *Hölder* mean boils down to the weighted geometric mean: $\mu_0 \left(k_\ell, k_f\right) = k_\ell^{\psi_\ell/(\psi_\ell+\psi_f)} k_f^{\psi_f/(\psi_\ell+\psi_f)}$. Consequently, the term $k_f \kappa_\ell^{\psi}$ can be seen as a special case of $\mu_p \left(k_\ell, k_f\right)$ when $p \to 0$ and $\psi_\ell + \psi_f = 1$.

[5]To obtain the expressions in Eqs. (10) and (11), we manipulate (8) to obtain: $y_i = \mathcal{A}_i^{1-\alpha} k_i^{\alpha}$. Then, we use (9) to get rid of $\mathcal{A}_i$.

2.4 Equilibrium dynamics

2.4.1 Derivation of the equilibrium system

The dynamic system of the model can be constructed as follows. Firstly, we determine the equilibrium factor prices in each country by substituting Eq. (9) into Eqs. (6) and (7); this yields:

$$r_\ell = \alpha A_\ell - \delta, \qquad r_f = \alpha A_f \kappa_f^{\psi(1-\alpha)} - \delta \tag{12}$$

$$w_\ell = (1-\alpha) A_\ell k_\ell, \qquad w_f = (1-\alpha) A_f \kappa_f^{\psi(1-\alpha)} k_f. \tag{13}$$

Then, we plug Eqs. (12)-(13) into Eqs. (2)-(3) to obtain the following 4×4 system of differential equations:

$$\frac{\dot{k}_\ell}{k_\ell} = A_\ell - \frac{c_\ell}{k_\ell} - \nu - \delta \tag{14}$$

$$\frac{\dot{k}_f}{k_f} = A_f \kappa_f^{\psi(1-\alpha)} - \frac{c_f}{k_f} - \nu - \delta \tag{15}$$

$$\frac{\dot{c}_\ell}{c_\ell} = \alpha A_\ell - (\rho + \delta) \tag{16}$$

$$\frac{\dot{c}_f}{c_f} = \alpha A_f \kappa_f^{\psi(1-\alpha)} - (\rho + \delta). \tag{17}$$

Definition 1. *A dynamic equilibrium for the two-country AK model with international knowledge transmission can be defined as a set of infinite sequences of allocations* $\{c_\ell, c_f, k_\ell, k_f\}_{t\in[0,\infty)}$ *that:* (i) *satisfies Eqs. (14)-(17);* (ii) *fulfills the inequality constraints* $c_f \geq 0$, $c_\ell \geq 0$, $k_f \geq 0$, $k_\ell \geq 0$*; and* (iii) *satisfies the transversality condition in Eq. (4).*

Eqs. (15) and (17) show that the knowledge gap influences the equilibrium paths of c_f and k_f. Consequently, to solve the model for the long-run equilibrium, it is convenient to reduce the dynamic system to one dimension and focus on the following rescaled variables: the consumption-to-capital ratio of the frontier economy, $x_\ell := c_\ell/k_\ell$, the consumption-to-capital ratio of the non-frontier economy, $x_f := c_f/k_f$, and the cross-country knowledge gap, $\kappa_f := k_\ell/k_f$. Hence, combining Eqs. (14)-(17) yields the following 3×3 dynamic system in κ_f, x_f, and x_ℓ:

$$\frac{\dot{\kappa}_f}{\kappa_f} = x_f - x_\ell + A_\ell - A_f \kappa_f^{\psi(1-\alpha)} \tag{18}$$

$$\frac{\dot{x}_f}{x_f} = x_f - (1-\alpha) A_f \kappa_f^{\psi(1-\alpha)} - (\rho - \nu) \tag{19}$$

$$\frac{\dot{x}_\ell}{x_\ell} = x_\ell - (1-\alpha) A_\ell - (\rho - \nu) \tag{20}$$

In the system (18)-(20), the variables x_ℓ and x_f are the non-predetermined (jump) variables, while κ_f is the predetermined (state) variable. Therefore, for any given $\kappa_f(0) \leq \tilde{\kappa}_f$, the dynamic system (18)-(20) and the transversality condition (4) completely characterize the transitional dynamics of our two-country AK model with international knowledge transmission.

2.4.2 Characterization of the steady state

Let the asterisk ("*") indicate steady-state quantities for the endogenous variables.

Definition 2. *A steady-state equilibrium for the two-country world economy consists of a set of infinite sequences for the allocations* $\{c_\ell, c_f, k_\ell, k_f\}_{t\in[0,\infty)}$ *satisfying Definition 1 such that: (i) variables* x_ℓ*,* x_f *and* κ_f *are constant over time; (ii) individual consumption expenditures,* c_ℓ *and* c_f*, and capital stocks,* k_ℓ *and* k_f*, grow at the same constant rate* g*.*

The steady state can be determined by setting the equations in system (18)-(19) equal to zero and then solving the resulting 3×3 static system for the three endogenous variables κ_f, x_f and x_ℓ. This gives the following:

Proposition 1. *If* $\kappa_f(0) \leq \tilde{\kappa}_f$ *and* $A_\ell > A_f$*, then the model predicts a unique steady-state equilibrium where: (i) The consumption-to-capital ratios,* x_ℓ^* *and* x_f^**, are the same for both countries and equal to:*

$$x_\ell^* = x_f^* = \rho - \nu + (1-\alpha)\, A_\ell; \tag{21}$$

(ii) The knowledge gap, κ_f^**, is strictly larger than one and equal to:*

$$\kappa_f^* = \left(\frac{A_\ell}{A_f}\right)^{1/[\psi(1-\alpha)]}; \tag{22}$$

(iii) The unique steady-state growth path for the entire world economy is given by Eq. (16) and equates:

$$g^* = \alpha A_\ell - \rho - \delta; \tag{23}$$

(iv) The steady-state growth path is (asymptotically) saddle-path stable.

Proof. See Appendix A ■

From Proposition 1, we can extract the following key results. In the long run, the efficiency level of the frontier economy, A_ℓ, is crucial for having sustained growth (item *iii*). The persistence over time of knowledge gaps between economies are due to the existence of a sizeable disparity in the efficiency parameters, $A_\ell/A_f \neq 1$ (item *ii*). Looking at the income ratio, y_ℓ/y_f, we can establish that the rising of income inequality across countries can also be explained by efficiency differences according to[6]:

$$\frac{y_\ell^*}{y_f^*} = \left(\frac{A_\ell}{A_f}\right)^{1/[\psi(1-\alpha)]} = \kappa_f^*. \tag{24}$$

[6] To obtain (24), it suffices to divide (10) by (11), and then substitute from (22) into the resulting equation.

Finally, the world economy follows a unique stable adjustment path given by (item *iv*):[7]

$$\log \kappa_f(t) = \log \kappa_f^* + \left[\log \kappa_f(0) - \log \kappa_f^*\right] e^{\hat{\lambda} t} \tag{25}$$

$$\log x_f(t) = \log x_f^* + \vartheta \left[\log \kappa_f(0) - \log \kappa_f^*\right] e^{\hat{\lambda} t} \tag{26}$$

$$\log x_\ell(t) = \log x_\ell^*, \tag{27}$$

where

$$\hat{\lambda} := \frac{\zeta - (1-\alpha)\psi A_\ell - \left\{4(1-\alpha)\alpha\zeta\psi A_\ell + [\zeta - (1-\alpha)\psi A_\ell]^2\right\}^{1/2}}{2} < 0 \tag{28}$$

is the stable eigenvalue of the coefficient matrix of the log-linearized system, and

$$\vartheta := \frac{2(1-\alpha)^2\psi A_\ell}{\zeta + (1-\alpha)\psi A_\ell + \left\{4(1-\alpha)\alpha\zeta\psi A_\ell + [\zeta - (1-\alpha)\psi A_\ell]^2\right\}^{1/2}} > 0 \tag{29}$$

$$\zeta := \rho - \nu + (1-\alpha) A_\ell$$

are two collections of exogenous parameters.

From Eqs. (25)-(27), we can conclude that the frontier economy is always in its steady-state growth path. In contrast, the non-frontier economy may temporarily deviate from its steady-state path but adjusts smoothly over time at a rate determined by magnitude of the stable eigenvalue, $\hat{\lambda}$. Since the size of $\hat{\lambda}$ depends on the magnitude of the absorption parameter ψ, we can establish the following:

Proposition 2. *There is an inverse relationship between the adjustment speed of the non-frontier economy and the size of the absorption parameter,* ψ*.*

Proof. This result comes straightforwardly by differentiating Eq. (28) with respect to ψ. ■

In conclusion, while standard AK models predict that the economy expands at a constant rate, g^*, without transitional dynamics, our two-country extension incorporating cross-country knowledge externalities predicts the existence of a unique, asymptotically stable, steady-state growth path for the global economy resembling that of the neoclassical growth model. However, whereas the long-run growth trend in neoclassical models is exogenous, our framework endogenously determines the global growth trend through knowledge spillovers facilitated by capital accumulation.

Given the importance of Proposition 2 in explaining the ”catching-up” dynamics of the model, the next section examines the transitional dynamics in detail, outlining the conditions under which the model can reproduce the convergence patterns observed in empirical studies - namely, divergence, ’conditional’ convergence, and ’unconditional’ convergence.

[7] The analytical details of the derivation of the adjustment path (25)-(27) are collected in Appendix B.

3 Selecting convergence patterns

In our two-country setting, cross-country convergence (divergence) in income per capita can be seen as a permanent reduction (increase) in the income ratio, $y_\ell(t)/y_f(t)$. More formally, given an initial value for the income ratio, $y_\ell(0)/y_f(0) \neq y_\ell^*/y_f^*$, cross-country convergence (divergence) between time 0 and t' occurs if and only if $y_\ell(t')/y_f(t') < y_\ell(0)/y_f(0)$ (resp.: $y_\ell(t')/y_f(t') > y_\ell(0)/y_f(0)$).

To show under what conditions a specific convergence pattern can emerge in the long run, let us first consider the standard case without cross-country knowledge transmission, i.e., $\kappa_f > \tilde{\kappa}_f$ and/or $\psi = 0$. When $\psi \to 0$, from Eqs. (28) and (29) we have that $\hat{\lambda} \to 0$ and $\vartheta \to 0$, implying that each country in the world economy operates as an isolated technology 'island'. Hence, starting from a pair of initial conditions for capital stocks, $k_\ell(0)$ and $k_f(0)$, each economy will instantly jump onto its own steady-state growth path and will stay there forever. Notice that when $\psi \to 0$, from (24) we have that $y_\ell/y_f \to \infty$. This is the 'unconditional' divergence result predicted by standard AK models *à la* Romer (1986), and econometrically supported by Barro (1991) and Pritchett (1997) for the decades before the 1990s.[8] However, as shown by Barro and Sala-i Martin (1991, 1992) and Mankiw *et al.* (1992), the main prediction of these models is rejected using both region- and country-level data, in favor of 'conditional' convergence, i.e., a long-run equilibrium scenario where countries converge towards parallel growth paths.

To check whether the model can also predict 'conditional' convergence, we now consider the more general case of $\kappa_f \leq \tilde{\kappa}_f$, such that $\psi > 0$. In this case, we have both $\hat{\lambda} < 0$ and $\vartheta > 0$, implying that countries are tethered together due to knowledge transmission. In such an alternative scenario, if the world economy starts with a plausible relative capital intensity such that $\kappa_f(0) \leq \tilde{\kappa}$, then the ratio y_ℓ/y_f is no longer explosive over time and monotonically approaches its long-run value shown in Eq. (24). The theoretical prediction of the model is thus convergence in growth rates, but not in per capita income levels as predicted by the 'conditional' convergence literature.

Yet, since in our model cross-country differentials along different macroeconomic dimensions reflect the size of the efficiency parameter A_i, the follower country can bridge the gap with the leader by introducing an appropriate 'policy device' that offsets the gap $A_f - A_\ell$. This would turn 'conditional' convergence into 'absolute' convergence. Kremer *et al.* (2021) refers to it as the 'converging-to-convergence' hypothesis; i.e., a medium-term scenario where 'institutional homogenization' can propel convergence of development-friendly policies alternative to those of income.

To assess to what extent the 'converging-to-convergence' hypothesis can be responsible for the low convergence speed estimated by Patel *et al.* (2021), in the rest of this section we calibrate our two-country model using the following set of exogenous parameters based on data for the US

[8] The reason why divergence appears when $\psi = 0$ can be easily explained by recalling that, without cross-country productivity externalities, the two-country AK model ends up describing the case of a world economy characterized by two independent economies, each of which grows over time according to its own growth rate, which is equal to $g_i^* = \alpha A_i - \rho - \delta$, with $g_\ell^* > g_f^*$ since we assumed that $A_\ell > A_f$.

economy:

$$\langle \nu = 0, \alpha = 0.33, \rho = 0.04, A_\ell = 0.2589, A_f = 0.2303, \psi = 0.0687, \delta = 0.0354 \rangle .$$

To set the rate of time preference, ρ, and the output elasticity of capital, α, we follow the bulk of growth literature and assume $\rho = 0.04$ and $\alpha = 0.33$. Moreover, since demographic growth is not key in shaping the transition paths of the model, for simplicity we set $\nu = 0$. To calibrate the efficiency parameters A_ℓ and A_f, we set $A_\ell = 0.2589$ so as to get the same average growth rate in PPPs GDP per worker of 1% shown by the US economy between 1960 and 2010 (source: Penn World Tables, PWT 10.0), and $A_f = 0.2695$ to get a steady-state value for the inequality index, y_ℓ/y_f, of about 13 in equilibrium. As for the depreciation rate, we fix $\delta = 0.0354$ so as to get an annual depreciation rate close to that characterizing (on average) the US economy in the period 1960-2010.

Finally, to calibrate the size of the absorption parameter, we focus on the new wave of convergence studies and set $\psi = 0.0589$, so as to get an elasticity of κ_f with respect to the knowledge gap κ_f of around 4%.[9] The steady-state equilibrium is characterized by the following quantities:

variable	κ_f^*	x_f^*	x_ℓ^*	g^*
value	12.71	0.2135	0.2135	0.010

Table 1: Steady-state quantities under the benchmark parametrization.

Suppose that, at a certain moment of time, the non-frontier countries manages to increase the size of its efficiency parameter to match that of the frontier economy: $A_f \to A_\ell = 0.2589$. The adjustment dynamics generated by the income ratio y_ℓ/y_f are portrayed in the left chart in Figure 1.

As is easy to see, a permanent increase in A_f leads to an initial drop in the relative income per capita of the leader and to a smooth adjustment dynamics towards the new long-run equilibrium value of 1 afterwards.[10] During the whole transitional dynamics, the growth rate of the frontier economy remains constant, $g^* = 0.01$, while that of the non-frontier economy dramatically deviates from its long-run equilibrium value of 1% (See the right chart in Figure 1). This implies the following further corollary results: (i) the frontier country is always in its own equilibrium path; (ii) during the adjustment dynamics, the non-frontier economy always grows faster than the frontier economy ('Wilde'-convergence).

Finally, to quantify the convergence speed predicted by the model, we log-linearize Eqs. (18)-(20) around the steady state and calculate the so-called half-time. Our simulation confirms the

[9] Indeed, with $\psi = 0.0589$, the elasticity y_f to κ_f equates $\psi(1-\alpha) = 0.039463$ (i.e., $\approx 4\%$). This means that a 1% increase in the knowledge gap of the non-frontier country, κ_f, leads to approximately a 0.04% increase in its output level, y_f.

[10] For simplicity, throughout the section we suppose that the transmission of knowledge from the leader to the follower is so quick to make A_f exhibits a 'quantic' jump from 0.2303, to 0.2589. Results do not change qualitatively if we assume that the baseline knowledge gap bridges gradually over time according to an adjustment function of the form: $Z\left(A_\ell, A_f\right) = A_f e^{-\vartheta t} + A_\ell\left(1 - e^{-\vartheta t}\right)$, with $\vartheta > 0$.

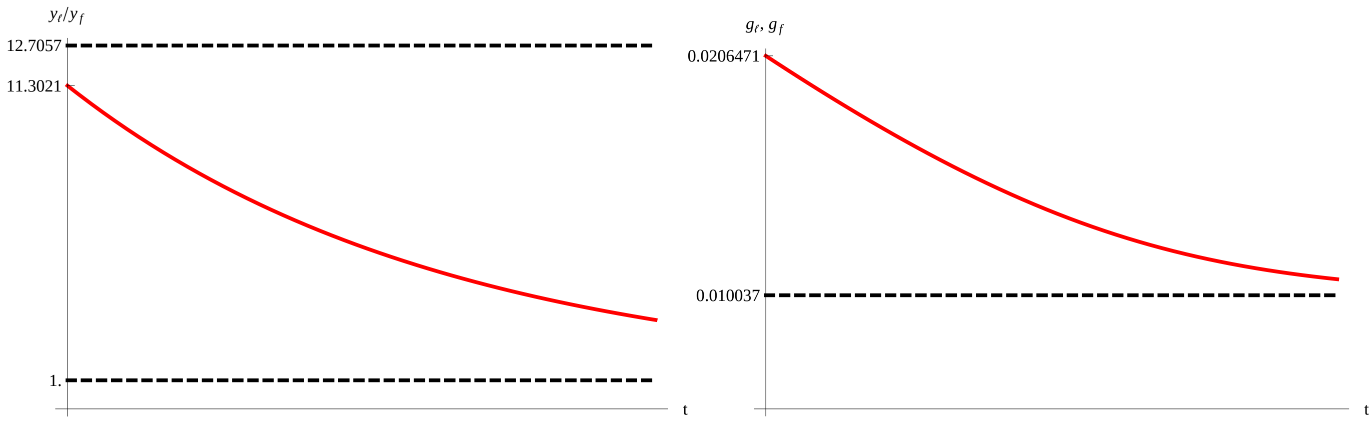

Figure 1: Adjustment paths of income ratio y_n/y_s (left) and growth rates (right). The 'dotted' line in the diagram on the right indicates the growth rate of the frontier economy and the 'thick' line the growth rate of the non-frontier economy.

existence of one stable (negative) eigenvalue and two unstable (positive) eigenvalues (see Table 2), and thus the saddle-path stability of the steady state. However, as the modulus of the negative eigenvalue is very close to zero, the implied half-life is very large and adds to 169.7 years. This result lines up with Patel *et al.* (2021) and Kremer *et al.* (2021) finding that poor economies tend to catch-up with rich only slowly over time.

	λ_1	λ_2	λ_3
Eigenvalue	-0.0041	0.2056	0.2135
Eigenvectors	$\begin{bmatrix} -1 \\ -0.001 \\ 0 \end{bmatrix}$	$\begin{bmatrix} -1 \\ -0.0171 \\ 0 \end{bmatrix}$	$\begin{bmatrix} 0 \\ 0.7071 \\ 0.7071 \end{bmatrix}$

Table 2: Eigenvalues of the Jacobian matrix of the linearized system.

4 Empirical analysis

Now, we conduct an econometric test to validate the main predictions of our theoretical framework by performing a dynamic panel regression analysis. Our model indicates that, in the long run, the growth rate of per capita income is positively related to the gap in capital endowment between the lagging and the frontier economy. This gap is expressed as the inverse ratio of the the laggard's capital intensity compared to the frontier economy (κ).

According to our model, the relative capital intensity should facilitate knowledge spillovers through learning by investing. This, in turn, would enhance efficiency levels and ultimately lead to an increase in the rate of growth of GDP per capita. Accordingly, as an economy develops and its income level rises, the role of relative capital in driving growth should diminish. This is because the scope of knowledge spillovers tends to shrink as the economy expands. Therefore, if our model holds true, when we account for differences in productivity and income levels, the relationship between relative capital and the rate of income growth should weaken.

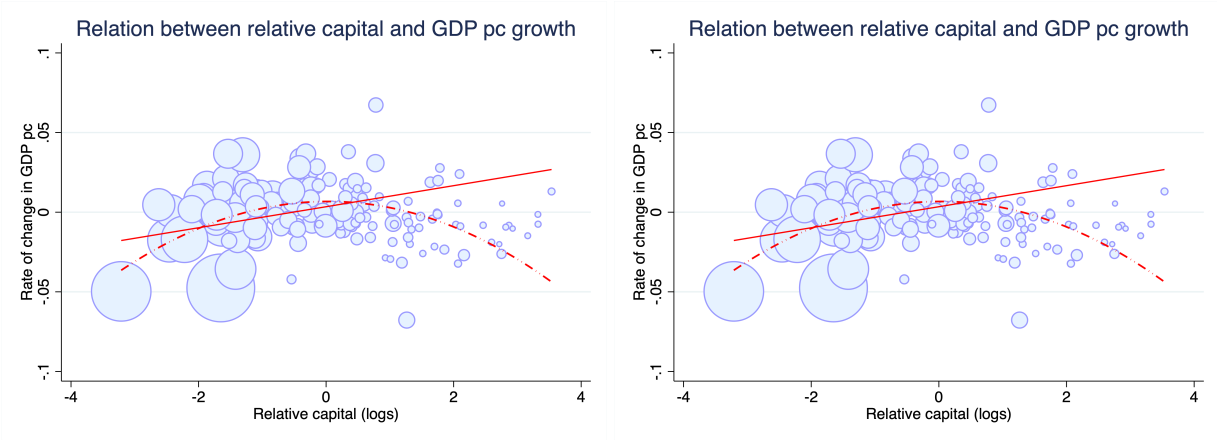

Figure 2: Linear and quadratic fit between GDP p.c. (per capita) growth, relative capital and initial income (country means using GDP p.c. as weights, 1950-2019)

In this section, we aim to validate these predictions by presenting various pieces of evidence. We utilize data from the PWT 10.0 and primarily focus on measuring per capita income as GDP in 2017 million US dollars (in millions of chained PPP) per inhabitant. Relative capital endowment refers to the ratio between the per person capital stock of the US, which serves as our frontier economy, and the same measure of capital intensity in countries that are lagging behind (in millions of chained US PPP). The time frame considered in this analysis spans from 1950 to 2019, during which data is available for a sample of 179 countries.

4.1 Descriptive evidence

As first evidence, in Figure 2 we report the scatter plots between the growth rate of GDP per capita and relative capital (left-hand side), and between the growth rate of GDP per capita and initial value of income per capita taken in logs (right-hand side). In all charts, observations are obtained as averages at country level over the full time period, and are based on size weights reflecting the levels of income, so to mitigate the impact of outlying behaviour of poorer countries. In both graphs, the red solid line identifies the linear fit, whilst the red dotted line is obtained from a quadratic regression. The figures point to a positive correlation between relative capital endowment and income growth ($y = 0.003 + 0.007\times$), and a negative correlation between income growth and its initial level ($y = 0.12 - 0.013\times$). As expected, richer countries –which are denoted by larger circles– benefit from a weaker effect of relative capital and have a slower convergence process. Not less relevantly, there is a group of poor countries, identified by the small circles, that falls well below the linear fit, explaining why the the quadratic interpolation may provide a better approximation for these relationships. This issue has been pointed out, among others, by Patel *et al.* (2021) in studying global income convergence.

Evidence in Figure 3 is aimed to validate the idea that the relationship between income growth and relative capital weakens once that cross-country differentials in productivity and income are taken into account. The figure displays the year-by-year coefficient associated with relative capital used as explanatory variable in a regression having the annual rate of change of GDP per capita as

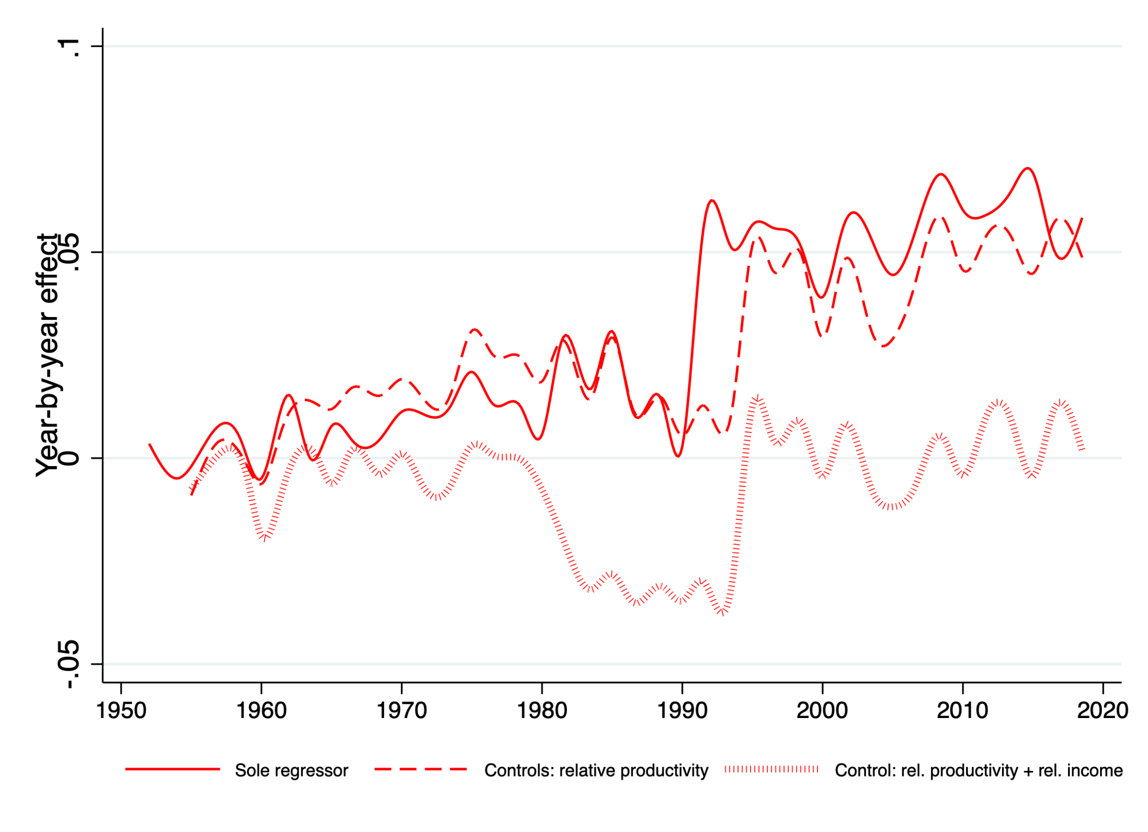

Figure 3: Impact of Relative capital on GDP per capita growth (year-by-year effect, 1950-2019).

Notes: Year-by-year coefficients of Relative capital are estimated with the Mean Observational OLS regression, including country fixed effects and common time dummies (see Keane and Neal, 2020).

dependent variable. The regression also include country fixed effects, common time dummies and is estimated with the Mean Observational OLS regression (see Keane and Neal, 2020). The solid line collects annual coefficients associated with relative capital used as sole regressor. The dashed line and dotted lines are obtained adding, in order, relative productivity and relative income to the baseline regression. Relative productivity corresponds to the logarithmic difference (or log-ratio) between the total factor productivity of the United States and that of less developed economies. Relative productivity represents the standard measure of the distance to the frontier used in the literature. Relative income is the log-difference between GDP p.c. (per capita) of the United States and that of the laggard economies. In the figure, the growth impact of relative capital can be seen as (vertical) decomposition of two effects. The effect of relative productivity (through learning-by-investing) would correspond to the movement from the solid to the dashed line. The effect of relative income (via capital accumulation) would correspond to the movement from the dashed to the dotted line.

Figure 3 demonstrates that when considered as sole regressor, the impact of relative capital is positive and increasing over time. However, since relative capital is expected to enhance relative productivity, we observe a downward shift in the curve of year-by-year coefficients of our main regressor when we account for the distance to frontier. When we control for relative income, the curve moves further downs and ends to fluctuate around the horizontal axis (i.e. around zero). The graph suggests that the income growth effect of relative capital shows up entirely through the channels of income convergence driven by decreasing marginal returns and technologies transfers driven by embodied knowledge spillovers. This evidence hence lends credit to the main mechanism designed by our model for the global economy and that we seek to identify below through an econometric analysis.

4.2 Benchmark regressions

We support more rigorously our theory by developing a panel dynamic regression analysis. The stochastic version of our (long-run) equilibrium relationship is defined as:

$$\Delta \log y_{it} = a_{i0} + b \log \kappa_{it} + c \log X_{it} + F_t + \epsilon_{it} \tag{30}$$

where i denotes countries ($i = 1, ..., 179$), t years ($t = 1950, ..., 2019$). X is a vector including various controls that we introduce below, whilst F_t captures the effect of un-observable factors affecting income dynamics, that we primarily model with common time dummies.[11] a_{i0} are country fixed effects and ϵ_{it} spherical error terms.

Table 3 presents the estimates for the long-run relationship between relative capital and income growth. These estimates are derived from the short-run coefficients obtained through the implementation of an Auto-Regressive Distributed Lags (ARDL) model. This procedure is robust to reverse causality and other important econometric issues (dynamic adjustment, heterogeneity, the integration order of the variables, etc.).[12]

	(1)	(2)	(3)	(4)	(5)	(6)
Relative capital	0.013***	0.009**	0.011***	0.012***	0.038***	0.017**
	(0.004)	(0.004)	(0.003)	(0.003)	(0.012)	(0.007)
Income per person	GDP p.c.	GDP p.w.	GDP p.c.	GDP p.c.	GDP p.c.	GDP p.c.
Population	No limit	No limit	>0.35 M	>1 M	>1 M	>1 M
Slope Heterogeneity	No	No	No	No	Yes	Yes
Un-observable shocks	TD	TD	TD	TD	TD	CCE
Obs.	9,350	8,503	8,545	7,804	7,804	7,216
R-squared	0.981	0.978	0.971	0.957	0.681	0.584
No. countries	179	177	162	147	147	147

Table 3: Long-run estimation of the relationship between Relative capital and GDP per capita growth (full sample)

Notes: Long-run estimates derived from an ARDL specification including country fixed effects and controls for unobservable shocks, namely time dummies (TD) or common correlated effects (CCE). The dependent variable is the annual rate of change in Gross Domestic Product per capita (GDP p.c.) or in Gross Domestic Product per worker (GDP p.w.). All right-hand side variables are in logs. Relative capital is the ratio between the US per person capital stock and that of the follower country. Population thresholds: 0.35 M (average population, 350 thousand inhabitants); 1 M (average population, 1 million inhabitants). The lag order of the variables is set to four ($= T^{1/3}$, T is the number of time points of the analysis). Column (6) reports the mean group estimates robust to the outliers. Standard errors robust to heteroskedasticity and first-order autocorrelation in parentheses. **, **, and * significant at 1, 5 an 10%

[11] The effect of dummies are modeled by expressing all variables as deviations from their cross-sectional mean.

[12] In all regressions, the parameter that measures the speed of adjustment towards the long-run cointegration equilibrium is negative and significant indicating that there is a stable relationship (stochastic trend) governing the dynamics of the variables.

Our baseline regression in column (1) considers the annual rate of change of GDP per inhabitant as dependent variable whilst in column (2) we utilize GDP per worker as an alternative measure of income. In column (3), we narrow down the regression sample to countries with an average population greater than 0.350 million inhabitants and to those with at least one million inhabitants in column (4), as in Johnson and Papageorgiou (2020). In columns (1) through (4), we assume homogeneity across countries in the impact of relative capital. In column (5), we relax this assumption and allow the coefficient of the explanatory variable to vary across the units of our panel sample. To estimate the parameters for each panel unit, we employ the Mean Group estimator, i.e., we run the regression country by country and calculate the cross-sectional mean of the parameter b of Eq. (30) (see Pesaran and Smith, 1995). Further, we introduce heterogeneity in the impact of un-observable shocks across countries in column (6). We approximate these with the cross-sectional means of all the variables in the model (so-called Common Correlated Effects, CCEs) and, contrarily to when using dummy dummies, we allow the impact of these factors to change across economies.[13]

Our long-term regression analysis largely confirms the positive relationship between relative capital and income growth. Based on our benchmark estimates in column (4), a ten-percentage (one-off) difference (in logs) in the capital intensity between leader and follower would generate a 0.12% faster rate of income growth for the latter. Notably, our benchmark estimates are conservative with respect to the results of the regressions that account for heterogeneity in slope coefficients (columns (5) and (6)). The results in column (6) are particularly important as they are obtained as means robust to outliers (see Bond *et al.*, 2010), and hence mitigate the effect of countries with unusually large coefficients. The simple mean of individual coefficients would be 0.180 (S.E. 0.007), while the median value 0.019 (S.E. 0.008). Furthermore, it is important to observe that the regression in column (6) corresponds to an open-economy specification as it incorporates the cross-country (unweighted) means of both the dependent variable and the regressor (i.e., the CCE terms) to account for the effect of cross-sectional dependence. These additional terms capture the average disparity in rates of economic growth and capital accumulation between frontier and laggards, ensuring thus that the coefficient of relative capital does indeed reflect the impact of learning-by-investing spillovers.

In Table 4 we examine how the results vary based on the capital intensity of the different groupings of countries. Regression in column (1) is our benchmark estimation shown above. Firstly, we exclude from this estimation those countries with a capital stock per person higher than the United States for all the time horizon, $\kappa < 1$ (10 countries; column (2)). According to our model, these countries should not experience significant benefits from knowledge spillovers. The estimates in column (2) support this prediction, as they do not differ significantly from those in column (1) based on the larger sample. Secondly, we analyze the impact of relative capital on economies that have had capital intensity lower than the United States all along the time interval between 1950 and 2019, $\kappa > 1$ (116 countries; column (3)). These countries have not completed the catching-up process towards the frontier and hence can be considered as "ongoing followers". For these coun-

[13]The estimator used in column (6) corresponds to the Cross-Sectionally augmented ARDL approach studied by Chudik and Pesaran (2015). The coefficients of CCEs are omitted from the table for the sake of brevity.

tries, relative capital has an effect in line with our previous estimates. Thirdly, we narrow down the regression analysis to economies that, starting from a lower input intensity, have achieved (or even surpassed) the level of capital stock per person of the leading economy during the sample period, $\kappa \rightarrow 1$ (21 countries; column (4)). For this group, hereinafter labelled as "completed catching-up" countries, the impact of relative capital is more pronounced, and this may explain their ability to catch up and reach the capital intensity of the United States. Following the anecdotal evidence presented in Figure (2), in column (5) we acknowledge the possibility of a non-linear relationship between relative capital and income growth. Our objective is to explore whether there exists a threshold for relative capital beyond which the effect of the factor on the growth rate of GDP per capita changes. We identify endogenously this threshold with the procedure developed by Kremer *et al.* (2013) for panel dynamic regressions.[14] This estimation confirms the expectation of a positive relation between relative capital and GDP per capita growth only for countries with a relative capital intensity not too high, i.e., with a gap in logged capital stock p.c. lower than the estimated value of 4.3. Beyond this threshold, no significant impact is found for our key regressor. This finding suggests that the difference in capital intensity between laggard and frontier economies should not be too wide, as otherwise the former would not be able to benefit from knowledge spillovers embedded in capital inputs. Below the identified threshold, relative capital is estimated with a long-run effect of 0.039, which is higher than what we run in the regression based on the full sample of countries assuming homogeneous slope parameters (columns (1)-(4)).

	(1)	(2)	(3)	(4)	(5)
Relative capital	0.012***	0.012***	0.011***	0.020**	
	(0.003)	(0.003)	(0.003)	(0.008)	
Below threshold					0.038***
					(0.008)
Above threshold					0.012
					(0.013)
Threshold					4.3
Lower bound					3.4
Upper bound					5.8
Countries	All	Catching-up	Ongoing Catching-up	Completed catching-up	All
Obs.	7,804	7,294	6,073	1,221	7,808
R-squared	0.957	0.955	0.958	0.902	-
No. Countries	147	137	116	21	157

Table 4: Long-run relationship between Relative capital and GDP per capita growth by follower status

Notes: Long-run estimates derived from an ARDL specification including country fixed effects and time dummies. The dependent variable is the annual rate of change in real Gross Domestic Product per capita (GDP p.c.). All right-hand side variables are in logs. Relative capital is the ratio between the US capital stock per capita and that of the follower country. Slope parameters are assumed homogeneous. The lag order of the variables is set to four ($= T^{1/3}$, where T is number of time points of the series). Estimates consider countries with an average population greater than one million inhabitants. Standard errors robust to heteroskedasticity and autocorrelation in parentheses. **, **, and * significant at 1, 5 an 10%.

[14] Kremer *et al.* (2013) apply the procedure devised by Hansen (1999) to dynamic panel data using forward orthogonal deviation transformations to remove fixed effects and preserve the distributional properties required for applying the original methodology, that was devised by static regressions.

To evaluate the plausibility of the estimated threshold, in Figure 4, we illustrate the quadratic interpolation of the country-specific coefficients derived from Mean Group regression underlying estimates in column (5) of Table 3. The (non-linear) interpolation fit is plotted against the distribution of relative capital along our sample of countries. In order to provide a comprehensive analysis, we consider the distribution against minimum values (red), the distribution against mean values (green), and that against the maximum values (blue). The graph restricts the sample to countries with relative capital intensity, κ_f, smaller or equal to 10, to exclude the possibility that those lying at the very bottom-end of the distribution, that may have an anomalous performance, affect the results of our analysis. The shaded area in the graph represents the 95% confidence interval for the quadratic fit over the mean value of the relative capital (green line). Notably, in all the three scenarios, the interpolation curve exhibits a bell-shaped pattern. Furthermore, the parameter distribution, conditionally to the mean, reaches its maximum around a value relative close to the estimated threshold of 4.3. Consistent with the predictions of our model, relative capital does not exhibit a statistically significant effect for those economies with a capital intensity similar to the United States ($\kappa \sim 1$). In line with our estimates above, the parameter value of relative capital falls between 0.04 and 0.05 just below the threshold ($\kappa < 4.3$), and turns again insignificant for values of capital intensity exceeding the threshold ($\kappa > 4.3$).

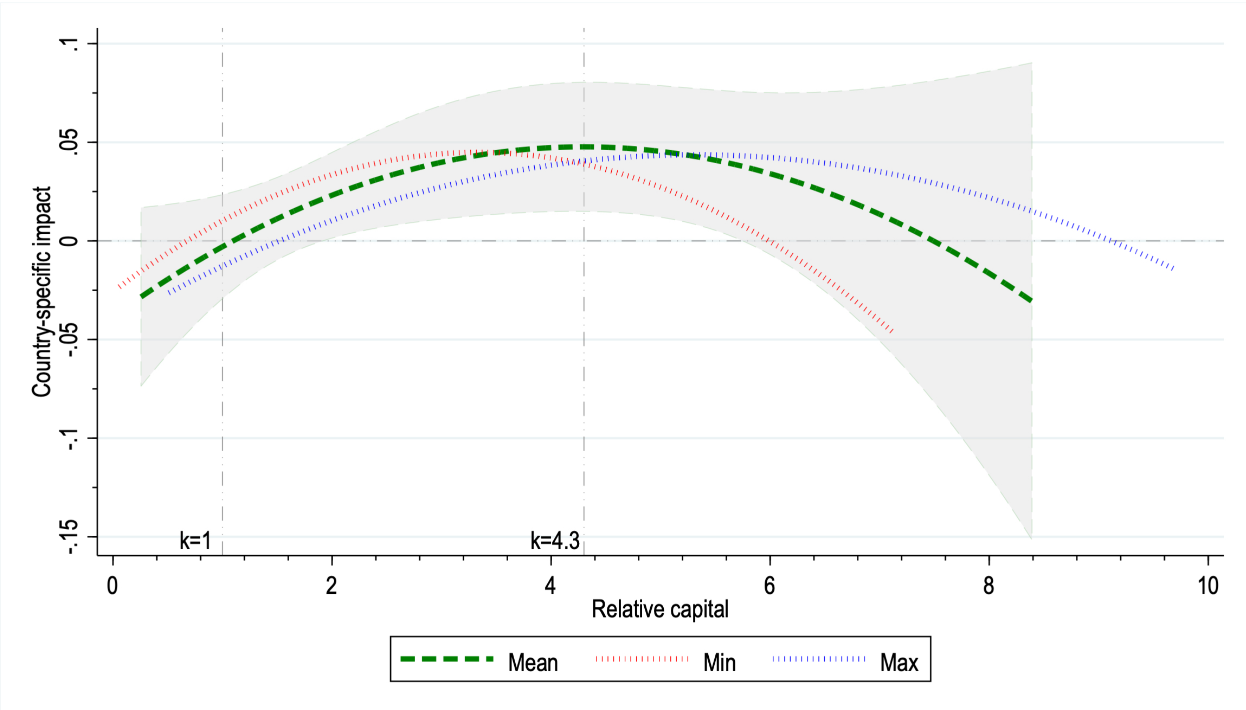

Figure 4: Growth effect of relative capital along the distribution and the threshold

Notes: Quadratic regression fit of the country-specific (long-run) coefficients yielded by the regression robust-to-outliers reported in col. (5), Table 3. The green line interpolates the long-run coefficients against the mean value of relative capital of each country. The red line uses the distribution of the minimum level of relative capital, whist the red line uses the maximum level. The shadowed area identifies the 95% confidence interval for the quadratic fit over the mean value of relative capital (green line). The graph restricts the sample to countries with a capital intensity smaller or equal to 10.

In Table 5, we look at the mechanisms by which relative capital accelerates the growth rate of GDP per capita. Our theory posits that disparities in capital intensity relative to the frontier

(κ 'not much' larger than 1) should lead to higher levels of total factor productivity and income. Building on this premise, we re-evaluate our model by incorporating some proxies for these factors. Column (1) presents our benchmark estimates as mentioned earlier. To ensure consistency, column (2) replicates this regression, but narrows down the sample to countries with consistently available productivity data (110 countries in the PWT). In column (3), we consider the productivity value relatively to the frontier's levels. Moving on to column (4), we introduce the lagged level of productivity of each country as a control, following equation (23) of the model. Columns (5) and (6) incorporate relative income (namely, GDP p.c.) and the lagged level of GDP p.c. In column (7), we combine both relative measures of productivity and income. Finally, the last two columns present the results for the baseline regression and the regression with controls (namely, estimates in columns (2) and (7)) for countries with a capital intensity not 'too far' from the US levels (i.e., below the threshold of 4.3 for κ). Taken as a whole, the impact of relative capital weakens when controls are introduced, resulting insignificant in the regression focused on countries lying above the threshold ($\kappa < 4.3$). The coefficient size of relative productivity that emerges from these regressions is not far from estimates in earlier works (see, among others, Kneller and Stevens, 2006, Madsen *et al.*, 2010). Estimates in the last two columns clearly indicate that differences in capital endowment translate into a positive growth effect by closing the gap in income and productivity with the frontier. The former of these effects would be due to convergence via a differential rate of capital accumulation; the latter would be due to technology transfers via learning-by-investing of capital-embodied knowledge. This is in line with the evidence in Figure 3.

	(1)	(2)	(3)	(5)	(4)	(6)	(7)	(8)	(9)
								Below the threshold ($\kappa < 4.3$)	
Relative capital	0.012***	0.013***	0.005***	0.009***	-0.024***	-0.039***	-0.018***	0.027***	-0.006
	(0.003)	(0.004)	(0.002)	(0.003)	(0.005)	(0.005)	(0.002)	(0.008)	(0.004)
Relative productivity (DTF)			0.009*				0.054***		0.036***
			(0.005)				(0.006)		(0.007)
Productivity level (1-yr lag)				0.022***					
				(0.006)					
Relative income					0.060***		0.052***		0.053***
					(0.005)		(0.003)		(0.005)
Income level (1-yr lag)						-0.089***			
						(0.006)			
Obs.	7,804	5,509	5,509	5,509	5,509	5,509	5,509	3,231	3,155
R-squared	0.957	0.942	0.260	0.624	0.251	0.916	0.251	0.960	0.237
No. of Countries	147	110	110	110	110	110	110	73	68

Table 5: Long-run estimation with controls, below and above the threshold ($\kappa = 4.3$)

Notes: Long-run estimates derived from an ARDL specification including country fixed effects and time dummies. The dependent variable is the annual rate of change in real Gross Domestic Product per capita (GDP p.c.). All right-hand side variables are in logs. Relative capital (or TFP or GDP p.c.) is the ratio between the US level of per capita stock (or productivity or GDP p.c.) and the corresponding value of the follower country. DTF denotes distance to frontier (or relative productivity with respect to the US). Slope parameters are assumed as homogeneous. The lag order of the variables is set to four ($= T^{1/3}$, where T is number of time points of the series). Estimates consider countries with an average population greater than one million inhabitants. Standard errors robust to heteroskedasticity and autocorrelation in parentheses. **, **, and * significant at 1, 5 an 10%.

4.3 Robustness regressions

One might wonder whether the long-term (equilibrium) parameters of relative productivity and relative income encompass the entire impact of relative capital on income growth, or they collect the effect of omitted variables. For instance, there could be some forms of disembodied knowledge, not incorporated in capital equipment, that spills over across economies and affect the rate of income growth, but that our regression model is not able to control for.

In Table 6, we address these issues for the group of countries that fall below the threshold ($\kappa < 4.3$). In columns (2) and (3), we interact relative capital, respectively, with relative productivity and relative income. These two additional terms are not statistically significant, indicating that there are no un-accounted knowledge transfers, channeled by learning-by-investing processes between the leader and follower economies. In columns (4)-(7), we include proxy measures for human capital and innovation capabilities (in logs), as well as the interaction between these variables and relative productivity. Since the latter is measured as the Distance To the Frontier (i.e., the US), it is hereinafter denoted as DTF for brevity. The index of human capital is sourced by PWT and reflects the years of primary, secondary and tertiary education across different population cohorts. Innovation capabilities is measured in terms of patent propensity, defined as ratio between patent applications, provided by the World Intellectual Property Office (WIPO), and the population level. Both measures of human capital and innovation are expressed as the inverse ratio to the levels observed in the United States. The main effect of these variables should capture transfers from the frontier of knowledge (so-called disembodied) which depends on cross-country differentials in educational levels and technology base. Conversely, the interaction between these measures of disembodied knowledge with the DTF should reveal whether technology transfers are easier for countries with higher or lower levels of production efficiency (relative TFP). The results indicate that the gap in the endowment of knowledge inputs (human capital and patenting) does not translate into an income growth premium. Rather, in line with endogenous growth theories (Lucas, 1988; Romer, 1990; Aghion and Howitt, 1992), a greater difference in these inputs is found to widen the gap in the growth rate of GDP per capita between the leader and follower economy. Additionally, the interaction terms between knowledge inputs and the DTF are insignificant, leaving unchanged the pattern of long-run effects associated with relative productivity and income.

Further, we investigate whether countries experience different gains from relative capital, depending on their structural differences. Some countries may not share the same technology with the frontier and have a persistently high capital intensity ratio, κ. To address this, we replicate our benchmark regression (column (1)) by dividing countries into two groups based on their average relative capital: those below and above the median value of κ (column (8) and (9)). The idea is that countries with a capital intensity closer to that of the US (i.e., below the median value of κ) may benefit more from knowledge spillovers, leading to faster income growth and convergence. The results of Table 6 mildly support this expectation, as the coefficient sizes of our key regressors are slightly larger for the former group of countries. Note that our estimates remain robust even after controlling for trade openness (export/GDP), returns to capital (internal interest rate), and capital obsolescence (average depreciation rate; source: PWT 10.0). These additional checks are not included here for the sake of brevity, but are available from the authors upon request.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
								Below median κ	Above median κ
Relative capital	-0.006	-0.012***	-0.005	-0.011**	-0.009***	-0.008*	-0.006**	-0.015**	-0.002
	(0.004)	(0.004)	(0.007)	(0.005)	(0.003)	(0.005)	(0.003)	(0.007)	(0.006)
Relative productivity (DTF)	0.036***	0.043***	0.034***	0.046***	0.046***	0.035***	0.031***	0.046***	0.035***
	(0.007)	(0.013)	(0.007	(0.009)	(0.008)	(0.007)	(0.005)	(0.010)	(0.010)
Relative income	0.053***	0.057***	0.048***	0.057***	0.057***	0.059***	0.060***	0.068***	0.045***
	(0.005)	(0.006)	(0.007)	(0.006)	(0.004)	(0.006)	(0.004)	(0.009)	(0.006)
Relative capital × DTF		-0.005							
		(0.007)							
Relative income × Relative income			0.005						
			(0.007)						
Relative human capital				-0.048***	-0.051***				
				(0.017)	(0.011)				
Relative human capital × DTF					0.006				
					(0.019)				
Relative patenting						-0.003***	-0.003***		
						(0.001)	(0.001)		
Relative patenting × DTF							0.001		
							(0.002)		
Obs.	3,155	2,841	2,841	2,841	2,755	2,805	2,719	1,680	1,475
R-squared	0.237	0.212	0.222	0.206	0.223	0.211	0.233	0.251	0.222
No. of countries	68	54	54	54	51	53	50	34	34

Table 6: Long-run estimation controlling for disembodied knowledge spillovers, below the threshold ($\kappa < 4.3$)

Notes: Long-run estimates derived from an ARDL specification including country fixed effects and time dummies. The dependent variable is the annual rate of change in real Gross Domestic Product per capita (GDP p.c.). All right-hand side variables are in logs. Relative capital (or TFP or GDP p.c. or human capital or patenting) is the ratio between the US level of per capita stock (or productivity or GDP p.c. or human capital or patents) and the corresponding value of the follower country. DTF= Distance to frontier (relative productivity). Slope parameters are assumed as homogeneous. The lag order of the variables is set to four ($= T^{1/3}$, where T is number of time points of the series). Estimates consider countries with an average population greater than one million inhabitants and an average relative capital intensity lower than the threshold ($\kappa < 4.3$). Standard errors robust to heteroskedasticity and autocorrelation in parentheses. **, **, and * significant at 1, 5 an 10%.

One may wonder whether our results are robust to, or can help explain, other growth patterns recently identified in the literature. In particular, significant attention has been given to growth spells or accelerations –episodes in which GDP growth rates deviate from their long-term trends (Pritchett, 2000). The high volatility of in the rate of change of GDP per capita in these economies reflects underlying instability in their fundamentals, which, in turn, reduces the effectiveness of standard growth drivers. Growth spells, which are common to most least-developed countries, tend to be recurrent in these economies, making their duration the primary factor contributing to their negative impact (Hausmann *et al.*, 2005). In the context of our analysis, it is crucial to determine whether the learning-by-investing mechanism and the capital threshold we have identified reflect the intrinsic characteristics of economies with unstable growth. These economies are likely to be concentrated at the lower end of the capital (income) distribution, where such instability may reinforce persistent underdevelopment.

In Table 7, we examine whether the effect of relative capital on income growth is influenced by growth spells. Specifically, we re-estimate the main regressions from the previous table, using a measure of growth spells as the dependent variable. Following Pritchett *et al.* (2016), we define growth spells as absolute deviations from the world's average rate of change in GDP per capita. We distinguish between temporary and persistent deviations, with the latter defined as those lasting at least five consecutive years. This analysis is conducted separately for countries below and above the capital threshold κ to determine whether the threshold emerges as a consequence of income growth instability, whether conversely growth spells arise due to a lack of international knowledge transfer, or whether these processes are unrelated. Our main finding—that relative capital drives increases in relative income, likely through embodied knowledge spillovers—is largely confirmed. However, these regressions offer additional insights. First, below the threshold, technology transfers from the frontier appear unrelated to long-lasting growth spells, as indicated by the insignificance of the relative productivity parameter in column (4). Above the threshold, the factors driving income growth (column (6)) and growth spells (columns (6) and (7)) are similar. Overall, this evidence supports the validity of the threshold in distinguishing different development regimes, identifying the range of countries where investment and learning-by-investing remain crucial drivers of growth.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	Below the threshold ($\kappa < 4.3$)				Above the threshold ($\kappa > 4.3$)			
Dep. Variable	Income growth		Growth spells - All	Growth spells - > 5 years	Income growth		Growth spells - All	Growth spells - > 5 years
Relative capital	0.028***	-0.006	-0.008*	-0.000	0.001	-0.026***	-0.029***	-0.030***
	(0.008)	(0.004)	(0.004)	(0.004)	(0.003)	(0.005)	(0.004)	(0.005)
Relative productivity		0.036***	0.037***	0.001		0.077***	0.087***	0.075***
		(0.007)	(0.007)	(0.007)		(0.012)	(0.012)	(0.013)
Relative income		0.053***	0.058***	0.039***		0.054***	0.059***	0.058***
		(0.005)	(0.005)	(0.005)		(0.007)	(0.007)	(0.008)
Obs	3,830	3,155	3,100	3,155	3,885	2,017	1,961	2,017
R-squared	0.967	0.237	0.489	0.618	0.947	0.275	0.570	0.692
N. countries	87	68	68	68	84	44	43	44

Table 7: Long-run estimation of income growth and growth spells

Notes: Long-run estimates derived from an ARDL specification including country fixed effects and time dummies. The dependent variable is the annual rate of change in real Gross Domestic Product per capita (GDP p.c.) in column (1)-(2), and (5)-(6), and the excess of GDP p.c growth over the annual world average in columns (3)-(4) and (7)-(8). Regressions in column (4) and (8) considers episodes of growth spells of more than 5 years. All right-hand side variables are in logs. Relative capital (or TFP or GDP p.c. or human capital or patenting) is the ratio between the US level of per capita stock (or productivity or GDP p.c. or human capital or patents) and the corresponding value of the follower country. Slope parameters are assumed as homogeneous. The lag order of the variables is set to four,= $T^{1/3}$), where T is number of time points of the series. Estimates consider countries with an average population greater than one million inhabitants, an average relative capital intensity lower than the threshold ($\kappa < 4.3$) in columns (1)-(4), and relative capital intensity higher than the threshold ($\kappa > 4.3$) in columns (5)-(8). Standard errors robust to heteroskedasticity and autocorrelation in parentheses. **, **, and * significant at 1, 5 an 10%.

Another well-established finding in the literature is that income distribution exhibits a degenerate pattern, with heterogeneous convergence dynamics across groups of countries. This has historically fueled two main lines of research: one on convergence clubs and another on the distributional properties of global income growth (Quah, 1997, Canova, 2004). These studies examine whether countries with similar economic or institutional fundamentals tend to self-select and converge toward the same equilibrium, as well as which distribution best characterizes global income growth. Depending on a country's position within the distribution, the forces driving economic growth may vary.

Building on these premises, the final part of our analysis explores the role of investment and learning-by-investing in income convergence across the income growth distribution. To do so, we estimate a panel dynamic quantile fixed-effects regression using the minimum distance criterion to obtain parameter estimates at different deciles (Melly and Pons, 2024). Table 8 reports the long-run coefficients obtained through this procedure, comparing them with the conditional mean estimates from the previous analysis. We run this regression for both the baseline specification and the model with controls, focusing exclusively on countries below the capital threshold κ. The results confirm that relative capital is a key driver of income growth, promoting relative income convergence for countries at the lower end of the income growth distribution: these countries likely have lower GDP per capita and remain well below the threshold κ. This pattern is consistent with the findings in Table 5 (see columns (8)-(9)). In the estimations with controls, however, relative capital becomes insignificant across the entire GDP per capita growth distribution. Notably, relative productivity –expected to capture technology transfers– exhibits a negative sign at the lower tail and a positive sign at the upper tail of the distribution. This suggests that the scope for technology transfers from the frontier (e.g., the US) is greater for fast-growing economies. Also, for these economies, the convergence process, as reflected in the coefficient of relative income, is undeniably faster.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
	OLS					Quantile				
	Mean	Q10	Q20	Q30	Q40	Q50	Q60	Q70	Q80	Q90
Baseline estimation										
Relative capital	0.028***	0.164	0.103*	0.089**	0.075**	0.062**	0.050*	0.047	0.026	0.021
	(0.008)	(0.122)	(0.054)	(0.045)	(0.035)	(0.031)	(0.030)	(0.032)	(0.034)	(0.042)
Estimation with controls										
Relative capital	-0.006	-0.008	-0.006	-0.008	-0.006	-0.007	-0.004	-0.007	-0.002	-0.001
	(0.004)	(0.009)	(0.009)	(0.009)	(0.009)	(0.008)	(0.008)	(0.007)	(0.008)	(0.008)
Relative productivity	0.036***	-0.026***	-0.021***	-0.014**	-0.015**	-0.01	-0.005	-0.001	0.010**	0.013**
	(0.007)	(0.008)	(0.008)	(0.007)	(0.006)	(0.006)	(0.007)	(0.006)	(0.005)	(0.006)
Relative income	0.053***	0.002	0.005	0.008	0.009	0.009	0.009*	0.007	0.010**	0.010*
	(0.005)	(0.007)	(0.007)	(0.007)	(0.006)	(0.006)	(0.005)	(0.005)	(0.005)	(0.005)

Table 8: Conditional mean and quantile dynamic estimation

Notes: Long-run estimates derived from an ARDL specification including country fixed effects and time dummies obtained with OLS regression in column (1) and Quantile regression in columns (2)-(10). The dependent variable is the annual rate of change in real Gross Domestic Product per capita (GDP p.c.). All right-hand side variables are in logs. Relative capital (or TFP or GDP p.c. or human capital or patenting) is the ratio between the US level of per capita stock (or productivity or GDP p.c. or human capital or patents) and the corresponding value of the follower country. Slope parameters are assumed as homogeneous. Estimates consider countries with an average population greater than one million inhabitants and an average relative capital intensity lower than the threshold ($\kappa < 4.3$). Standard errors robust to heteroskedasticity and autocorrelation in parentheses. **, **, and * significant at 1, 5 an 10%.

5 Discussion

The main contribution of this paper is the identification of a relative capital threshold that distinguishes economies capable of leveraging international knowledge spillovers through capital learning-by-investing from those with insufficient capital intensity to benefit from this growth mechanism. In this section, we first highlight the relevance of our findings in explaining the leapfrogging phases in the development trajectories of different country groups. We then discuss policy interventions derived from our analysis to foster economic growth. Finally, we acknowledge the study's limitations and suggest promising directions for future research.

5.1 Technology acquisition and convergence: some country examples

The analysis above has classified countries into distinct groups based on their growth patterns and, specifically, on the relationship between their capital intensity and the frontier. We identified a group of countries whose capital intensity remained consistently higher than that of the US throughout the entire period, keeping them permanently below the threshold ($\kappa < 1$ in all years). These countries were excluded from the development of the analysis because they were not the focus of our paper. Next, we identified a group of diverging economies characterized by persistently low capital intensity over the past half-century, consistently remaining above the identified threshold ($\kappa = 4.3$ or, equivalently, 1.45 if expressed in logs). These economies were denoted as 'no-catching-up'. We then distinguished a group of 'ongoing catching-up' countries, which initially had capital intensity slightly above the threshold but managed to reduce most, though not all, of their income gap with the frontier over time. Finally, the remaining group comprised countries that started below the frontier in terms of capital intensity but successfully completed the catching-up process.

In this section, we provide concrete examples of the growth experiences of various country groups, to support the connection between the magnitude of technology gap - proxied by the relative capital stock per worker, κ_f -, and the emergence of a country's convergence process. Figure 5 illustrates the dynamics of the log of the relative capital stock per worker with respect to the U.S., κ_f, alongside the log of the relative income per capita, $\log(y_l/y_f)$, for four selected countries: Pakistan ('no-catching-up or diverging'), China ('ongoing catchinp-up'), Japan and South Korea ('completed catchinp-up'). Figure 5 shows that each country has followed a distinct development trajectory. While Pakistan exhibits no signs of economic convergence, China, Japan, and South Korea followed a path characterized by prolonged periods of strong technological acquisition and sustained catch-up growth lasting for decades. For Pakistan, the lack of a decline in either (inverse) measure of relative capital or income throughout the analyzed time interval suggests that the country failed to implement any effective measure to reduce its technology gap by absorbing capital-embodied knowledge. For the other countries, while the convergence process for Japan and South Korea can be considered as complete, China appears to be at an advanced stage, nearing completion.

A crucial finding of our analysis is that the evolution of the income gap mirrors that of relative capital and, most importantly, that income convergence process concludes, with relative income becoming stationary, once relative capital consistently and stably falls below the threshold. However, each examined country crossed κ at different points in time: Japan in the early 1960s, South Korea in the mid-1980s, and China in the early 2010s. From that moment onward, these countries began developing their own leading technologies in key sectors, with the role of government

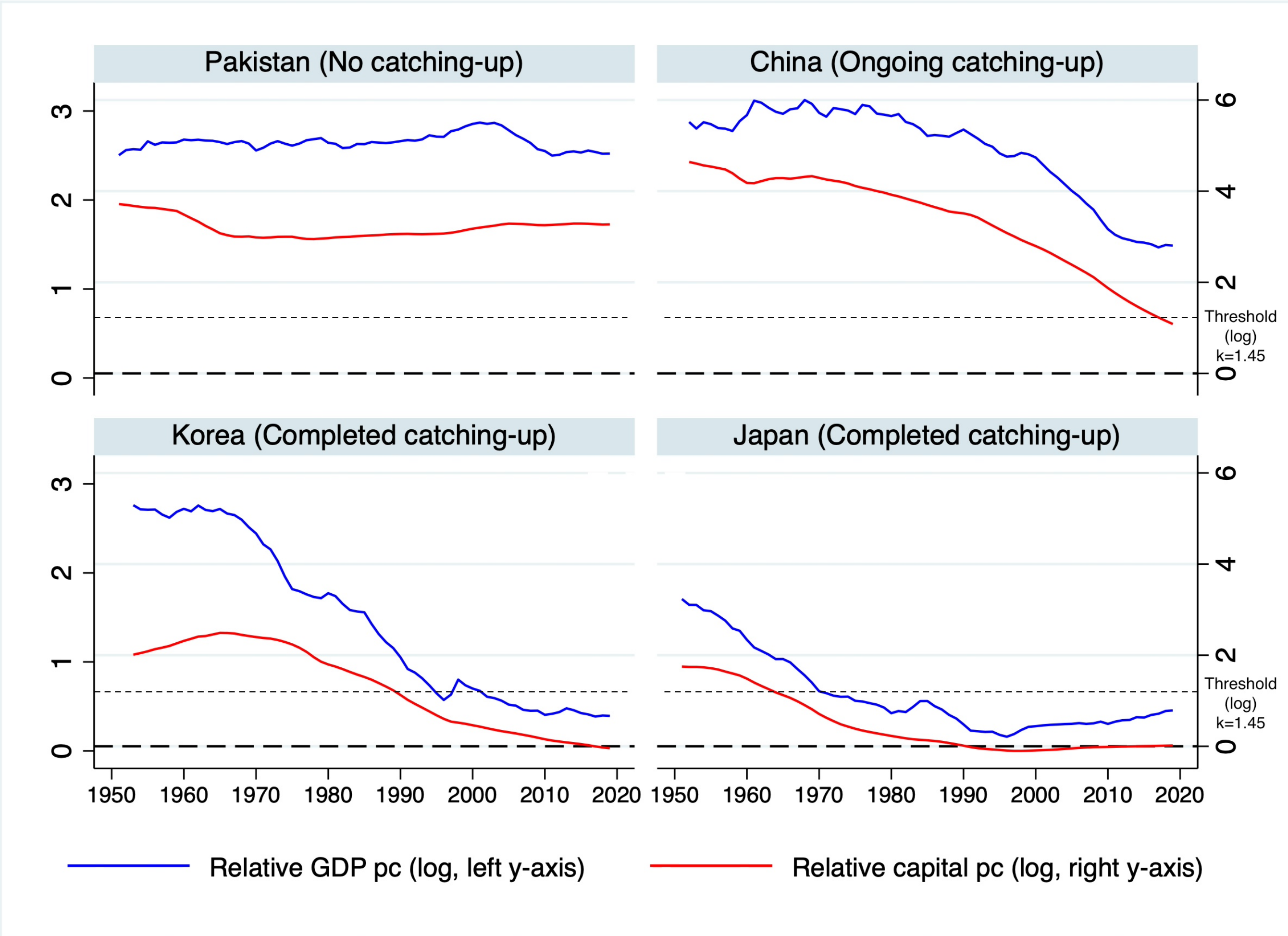

Figure 5: Relative capital stock per worker dynamics in China, Japan, South Korea, and Taiwan. Data for κ are displayed in logarithmic scale, with $log(\kappa_f) = 0$ indicating completed convergence. Relative capital threshold =1.45 (in logs); 4.3 in absolute terms

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gradually diminishing and private firms and corporations emerging as the primary drivers of development.

Historically, Japan emerged as a global leader in several key industries over the decades of the 1960s, 1970s and 1980s, after a long phase of overseas knowledge acquisition (Odagiri and Goto, 1996). In the 1960s, Japan established its dominance in the consumer electronics market, with companies like Sony and Panasonic leading the charge with groundbreaking products. The 1970s and 1980s saw Japan's automotive industry rise to global prominence, with Toyota, Honda, and Nissan becoming renowned for their reliable and fuel-efficient vehicles (Watanabe and Honda, 1992). During the 1980s, Japan also became a leader in robotics and automation, driven by its emphasis on precision manufacturing and the philosophy of continuous improvement, known as 'Kaizen' (Kumaresan and Miyazaki, 1999; Dekle, 2020). These achievements have cemented Japan's reputation as a powerhouse in high-tech technology and innovation.

Similarly, South Korea's economic takeoff, often referred to as the 'Miracle on the Han River,' is a remarkable tale of rapid industrialization and growth that began in the aftermath of the Korean War. The country first gained prominence in the 1980s in shipbuilding, with companies like Hyundai Heavy Industries and Samsung Heavy Industries helping establish South Korea as a top global player (Kim, 1991). In the 1990s, South Korea expanded its dominance to electronics and automobiles, driven by technological advancements and robust export growth, with Samsung and Hyundai at the forefront. This period marked a significant shift, with South Korea becoming a global leader in these industries. The 2000s saw continued success in shipbuilding, electronics, and automotive industries, fueled by innovation and global competitiveness. South Korean companies maintained their edge in technology and production capacity, solidifying their positions in key markets worldwide. By the 2010s, South Korea had confirmed its leadership in semiconductors and display technology, with Samsung and LG excelling in OLED panels and semiconductor manufacturing. Hyundai and Kia also continued their global expansion, further cementing South Korea's position in the automotive sector (Soh *et al.*, 2023).

Finally, China has emerged as a global leader in several key industries over the past two decades. In the 2010s, China established itself as a global leader in e-commerce, with platforms like Alibaba and JD.com revolutionizing online retail and digital payments through Alipay and WeChat Pay. The country also became the world's largest producer and installer of solar panels, driving down costs and accelerating the global transition to renewable energy. Additionally, China expanded its dominance in high-speed rail infrastructure, constructing the largest and most advanced high-speed rail network in the world (Gaida *et al.*, 2024). In the 2020s, China has also become a leader in 5G technology, with companies like Huawei and ZTE spearheading significant advancements and widespread deployment of 5G networks across the country and beyond (Harwit, 2023). The country is also at the forefront of biotechnology and pharmaceuticals, playing a crucial role in vaccine production, gene editing, and biopharmaceutical research (Gaida *et al.*, 2024). Currently, China is competing with Western countries to become a leader in the production of Electric Vehicles (EV) and Artificial Intelligence, AI (Bazavan and Huidumac-Petrescu, 2023). Companies like BYD, NIO, and XPeng are challenging established automakers with cutting-edge

EV technology, while China dominates the global supply chain for lithium-ion batteries through firms like CATL and BYD (Allison *et al.*, 2021;Atkinson, 2024). In AI, China is rapidly advancing in machine learning, facial recognition, and automation, with companies like Baidu, Tencent, and Alibaba leading research and development. As a result, China is not only competing with but also shaping the future of technology, energy, and digital transformation on a global scale (Allison *et al.*, 2021).

5.2 Policy implications

Our analysis provides valuable insights into growth policies that can be pursued to raise income levels and how their effectiveness varies across different stages of a country's development. Our model highlights the critical role of sustained investment in physical capital—whether domestically produced or imported—and the absorption of embodied knowledge as key drivers of growth.[15] This supports three main types of policy interventions: (i) public incentives for capital formation or reduced barriers to investment; (ii) openness to capital goods imports; and (iii) educational policies aimed at enhancing technical competencies, promoting vocational training, and fostering lifelong learning.

Governments can strategically combine these three policies based on their international positioning. For example, a country with relatively low capital intensity—i.e., well above the threshold—can accelerate economic growth through a temporary tax incentive for investment, complemented by a reduction in trade barriers. However, if trade openness has already reached its maximum level or cannot be further increased, such fiscal measures may be insufficient. In this case, tax incentives for investment should be sustained over the long term to support continued growth. In either scenario, the ability to exploit knowledge through learning-by-investing processes depends on the workforce's capacity to adapt to and implement new equipment and technologies. A country's absorptive capacity, particularly at the middle and lower ends of the income distribution, hinges on the technical skills of workers—developed through secondary education and specialized graduate university programs.

For countries whose relative capital stock falls short of the threshold $\tilde{\kappa}$, introducing growth-enhancing policies to accelerate the production of domestic technical knowledge through investment is essential to bridge the technological gap with the leader and thus narrow welfare gaps with respect to richer economies. For those countries that have surpassed the threshold, policymakers should shift their focus from enhancing domestic knowledge to reforming markets and institutions. This is crucial because GDP per capita no longer lags due to a lack of knowledge, but because of inadequate institutions, regulations, and market incentives that causes the domestic macroeconomic fundamentals to align with those of the leader economies ($A_f \to A_\ell$).

[15] To explore the growth-enhancing effects of fiscal policy in AK growth models, readers can refer to Aghion and Howitt (1998), Barro and i Martin (2004), and Jones and Manuelli (2005) for analyses based on closed-economy frameworks, and to Turnovsky (2011) for an analysis based on small open economies.

5.3 Limitations and possible extensions

Despite its simplicity and analytical tractability, our study has several limitations that future research could address. First, the model distinguishes between "diverging" and "converging" countries based on a threshold, $\tilde{\kappa}$, which is assumed to be exogenous. This assumption ensures the mathematical tractability of the steady-state equilibrium and transitional dynamics, while making the theoretical model more amenable to econometric analysis. A natural extension would be to endogenize $\tilde{\kappa}$ so as to identify the economic incentives and policies that allow a country to absorb foreign technology initially and eventually develop an effective national innovation system.

Another simplifying assumption that can be seen as a limitation of our analysis is the existence of a single leading country. Using a trade-flow-based interaction matrix, Debarsy and Ertur (2019) identify one global technological leader, the USA, along with two local leaders, Germany and Japan. A valuable extension of our study would therefore be to consider multiple technological leaders shaping the frontier, alongside a broader set of follower countries striving to keep pace with technological advancements.

Similarly, the interaction between technological leaders and followers is overly simplified in this study, whereas a more comprehensive analysis would require a deep look at the complex network of technological linkages. Addressing this calls for a larger dynamic modeling framework, the application of numerical methods to analyze the economy's properties and, empirically, the availability of measures capturing the evolving patterns of technological development across countries. Early evidence in this direction suggests that technological interdependence between countries have been changing over time, influencing both the magnitude and patterns of international knowledge spillovers (Fronzetti Colladon *et al.*, 2024).

Finally, another limitation of this study is that it does not account for the evolving composition of capital assets, particularly the growing role of intangible investments (Corrado *et al.*, 2022). While it is clear that these new forms of fixed investment play different roles across countries depending on their level of development and degree of specialization, it remains uncertain whether they can serve as a conduit for embodied knowledge and what factors might facilitate such transfers. Intangibles entail large sunk costs and significant scale effects, making them highly firm-specific and scarcely reversible (Haskel and Westlake, 2018). These characteristics, along with their intangible nature, raise doubts about their ability to facilitate knowledge transfers through learning-by-investing. Investigating this further presents a valuable avenue for future research.

6 Conclusions

In this paper, we have developed a growth framework that incorporates endogenous cross-country knowledge diffusion through learning-by-investing. This framework successfully replicates the majority of long-run growth patterns and income convergence observed in recent literature. To test the validity of our model, we have analyzed data from a global sample of countries, including industrialized, developing, and underdeveloped economies. Our findings demonstrate that disparities in income growth can be attributed to differentials in capital intensity. These differences facilitate the transmission of embodied knowledge and offer a pathway to narrow income gaps and increase economic growth. However, this effect is significant only when the cross-country gap in capital endowment, relative to the frontier, is not excessively pronounced. We have identified a relative capital threshold beyond which knowledge transfers are very unlikely to show up. Furthermore, our extensive section of robustness checks reveals that technology transfers are not primarily driven by disparities in the endowment of knowledge inputs, such as human capital and innovation. This suggests that the absorption of knowledge through learning processes and embodied technological change remains the primary driver of income growth for a significant portion of countries that are lagging behind (but not much) the technological frontier.

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Appendix

A. Proof of Proposition 1

Results in items (i) and (ii) come straightforwardly by solving the dynamic system (18)-(20) for the steady-state equilibrium. The result in item (iii) can be obtained by observing that in the steady state all endogenous variables grow at the same constant rate g^*, and then by applying this result to (16). Finally, to demonstrate the saddle-path stability of the steady state - Item (iv) of the proposition -, it suffices to show that the 3×3 Jacobian matrix of the linearized system shows one eigenvalue with negative real part and two eigenvalues with positive real part. However, this means showing that the trace of the Jacobian is positive and the determinant negative (Blanchard and Kahn, 1980).

First-order approximation of system (18)-(20) about the rest point $(x_\ell^*, x_f^*, \kappa_f^*)$ yields

$$\begin{bmatrix} \mathrm{d}\left(\log \kappa_f\right)/\mathrm{d}t \\ \mathrm{d}\left(\log x_f\right)/\mathrm{d}t \\ \mathrm{d}\left(\log x_\ell\right)/\mathrm{d}t \end{bmatrix} = \begin{bmatrix} -\psi\left(1-\alpha\right)A_\ell & \zeta & -\zeta \\ -\psi\left(1-\alpha\right)^2 A_\ell & \zeta & 0 \\ 0 & 0 & \zeta \end{bmatrix} \begin{bmatrix} \log \kappa_f - \log \kappa_f^* \\ \log x_f - \log x_f^* \\ \log x_\ell - \log x_\ell^* \end{bmatrix}, \tag{A.1}$$

where $\zeta := \rho - \nu + (1-\alpha)\,A_\ell$ is a collection of given parameters. Let

$$J^* := \begin{bmatrix} -\psi\left(1-\alpha\right)A_\ell & \zeta & -\zeta \\ -\psi\left(1-\alpha\right)^2 A_\ell & \zeta & 0 \\ 0 & 0 & \zeta \end{bmatrix}$$

denote the Jacobian matrix of the linearized system. Straightforward computations give:

$$\mathrm{Tr}J^* = 2\zeta - \psi\left(1-\alpha\right)A_\ell > 0, \mathrm{Det}J^* = -(1-\alpha)\alpha\zeta^2\psi A_\ell < 0,$$

thereby confirming that the characteristic polynomial of the Jacobian presents one stable real root and two unstable real roots. This result completes the demonstration of Proposition 1.

B. Determination of the equilibrium paths

Let λ_j denote the jth eigenvalue of the Jacobian matrix, J^*. From Appendix A, it follows that the general solution of the linearized system (A.1) can be written as:

$$\begin{bmatrix} \log \kappa_f(t) \\ \log x_f(t) \\ \log x_\ell(t) \end{bmatrix} = \begin{bmatrix} \log \kappa_f^* \\ \log x_f^* \\ \log x_\ell^* \end{bmatrix} + \sum_{j=1}^{3} B_j \begin{bmatrix} \Lambda_{j1} \\ \Lambda_{j2} \\ \Lambda_{j3} \end{bmatrix} e^{\lambda_j t},$$

where B_j is a constant of integration and Λ_{ji}, with $j = (1, 2, 3)$, is the eigenvector associated to the jth eigenvalue.

The characteristic polynomial of the matrix J^* can be written as:

$$\mathcal{P}(\lambda) = \lambda^3 - [2\zeta - \psi(1-\alpha) A_\ell] \lambda^2 + \zeta \left[(1-\alpha^2)\psi A_\ell - \zeta\right] \lambda + (1-\alpha)\alpha\zeta^2\psi A_\ell$$

Solving $\mathcal{P}(\lambda) = 0$ for λ yields:

$$\lambda_1 = \frac{\zeta - (1-\alpha)\psi A_\ell - \left\{4\zeta(1-\alpha)\alpha\psi A_\ell + [\zeta - (1-\alpha)\psi A_\ell]^2\right\}^{1/2}}{2} < 0$$

$$\lambda_2 = \zeta > 0$$

$$\lambda_3 = \frac{\zeta - (1-\alpha)\psi A_\ell + \left\{4\zeta(1-\alpha)\alpha\psi A_\ell + [\zeta - (1-\alpha)\psi A_\ell]^2\right\}^{1/2}}{2} > 0.$$

Transversality (terminal) condition (4) implies $B_2 = B_3 = 0$. Consequently, the general solution of the log-linearized dynamic system (A.1) can be rewritten as:

$$\begin{bmatrix} \log\kappa_f(t) \\ \log x_f(t) \\ \log x_\ell(t) \end{bmatrix} = \begin{bmatrix} \log\kappa_f^* \\ \log x_f^* \\ \log x_\ell^* \end{bmatrix} + B_1 \begin{bmatrix} \Lambda_{11} \\ \Lambda_{12} \\ \Lambda_{13} \end{bmatrix} e^{\lambda_1 t},$$

where Λ_{1i}, with $i = 1, 2, 3$, are the components the of eigenvector associated to the stable eigenvalue, λ_1.

Setting $t = 0$, the value of B_1 can be obtained jointly with the initial value of the two jump variables $\log x_f(0)$ and $\log x_\ell(0)$ as solution of the following static system:

$$\begin{aligned} \Lambda_{11}B_1 &= -\log\kappa_f^* + \log\kappa_f(0) \\ -\log x_f(0) + \Lambda_{12}B_1 &= -\log x_f^* \\ -\log x_\ell(0) + \Lambda_{13}B_1 &= -\log x_\ell^* \end{aligned} \tag{31}$$

where $\log\kappa_f(0)$ is given as the value of $\kappa_f(0)$ is predetermined. using the substitution method to solve the above system yields:

$$B_1 = \frac{\log\kappa_f(0) - \log\kappa_f^*}{\Lambda_{11}}$$

$$\log x_f(0) = \log x_f^* + \frac{\Lambda_{12}}{\Lambda_{11}}\left[\log\kappa_f(0) - \log\kappa_f^*\right]$$

$$\log x_\ell(0) = \log x_\ell^* + \frac{\Lambda_{13}}{\Lambda_{11}}\left[\log\kappa_f(0) - \log\kappa_f^*\right].$$

Thus, using these results to substitute for B_1, $\log x_f(0)$ and $\log x_\ell(0)$ in the general solution, we obtain the following triple of particular-solution paths:

$$\log \kappa_f(t) = \log \kappa_f^* + \left[\log \kappa_f(0) - \log \kappa_f^*\right] e^{\lambda_1 t} \tag{B.1}$$

$$\log x_f(t) = \log x_f^* + \frac{\Lambda_{12}\left[\log \kappa_f(0) - \log \kappa_f^*\right]}{\Lambda_{11}} e^{\lambda_1 t} \tag{B.2}$$

$$\log x_\ell(t) = \log x_\ell^* + \frac{\Lambda_{13}\left[\log \kappa_f(0) - \log \kappa_f^*\right]}{\Lambda_{11}} e^{\lambda_1 t}. \tag{B.3}$$

Finally, to get Λ_{11}, Λ_{12} and Λ_{13}, we solve

$$\begin{bmatrix} -\psi(1-\alpha)A_\ell - \lambda_1 & \zeta & -\zeta \\ -\psi(1-\alpha)^2 A_\ell & \zeta - \lambda_1 & 0 \\ 0 & 0 & \zeta - \lambda_1 \end{bmatrix} \begin{bmatrix} \Lambda_{11} \\ \Lambda_{12} \\ \Lambda_{13} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix},$$

to obtain:

$$\begin{aligned} \Lambda_{11} &= \frac{\zeta + (1-\alpha)\psi A_\ell + \sqrt{4(1-\alpha)\alpha\zeta\psi A_\ell + [\zeta - (1-\alpha)\psi A_\ell]^2}}{2(1-\alpha)^2\psi A_\ell} \\ \Lambda_{12} &= 1 \\ \Lambda_{13} &= 0. \end{aligned}$$

Consequently, the unique stable adjustment path of the model can be written as

$$\log \kappa_f(t) = \log \kappa_f^* + \left[\log \kappa_f(0) - \log \kappa_f^*\right] e^{\lambda_1 t}$$

$$\log x_f(t) = \log x_f^* + \vartheta \left[\log \kappa_f(0) - \log \kappa_f^*\right] e^{\lambda_1 t}$$

$$\log x_\ell(t) = \log x_\ell^*,$$

where

$$\vartheta := \frac{2(1-\alpha)^2\psi A_\ell}{\zeta + (1-\alpha)\psi A_\ell + \sqrt{4(1-\alpha)\alpha\zeta\psi A_\ell + [\zeta - (1-\zeta)\psi A_\ell]^2}}$$

is another collection of exogenous parameters.

C. Quantitative analysis

Under the parametrization:

$$\left\langle \nu = 0, \alpha = 0.33, \rho = 0.04, A_\ell = 0.2589, A_f = 0.2303, \psi = 0.0687, \delta = 0.0354 \right\rangle,$$

the steady-state solution for the dynamic system (18)-(20) consists of the following 3×1 vector of quantities:

$$X^* := \begin{bmatrix} \kappa_f^* \\ x_f^* \\ x_\ell^* \end{bmatrix} = \begin{bmatrix} 12.7057 \\ 0.2134 \\ 0.2134 \end{bmatrix}.$$

which, in turn, gives a steady-state growth rate equal to $g^* = 0.01$.

Linearizing system (18)-(20) around X^* yields the Jacobian matrix:

$$J^* = \begin{bmatrix} -0.0119 & 12.7057 & -12.7057 \\ -0.0001 & 0.2135 & 0 \\ 0 & 0 & 0.2135 \end{bmatrix},$$

whose characteristic polynomial writes:

$$\mathcal{P}(\lambda) = -0.00018 - 0.04218\lambda + 0.41501\lambda^2 - \lambda^3$$

Solving $\mathcal{P}(\lambda) = 0$ for λ, it is possible to write the following system of eigenvalues and associated eigenvectors:

$$\begin{array}{cccc} & 1 & 2 & 3 \\ \text{eigenvalue, } \lambda & -0.00408 & 0.20563 & 0.21346 \\ \text{eigenvector, } \Lambda & \begin{bmatrix} -1 \\ -0.0006 \\ 0 \end{bmatrix} & \begin{bmatrix} -1 \\ -0.0171 \\ 0 \end{bmatrix} & \begin{bmatrix} 0 \\ 0.70711 \\ 0.70711 \end{bmatrix} \end{array}.$$

Finally, using these numerical results to substitute in the particular solution (B.1)-(B.3), and recalling that for (4) to converge to zero both B_2 and B_3 must be set to equal 0, it is possible to write:

$$\log \kappa_f(t) = 2.54203 + \left[\log \kappa_f(0) - 2.54203\right] e^{-0.00408t}$$

$$\log x_f(t) = -1.54459 + 0.0078 \left[\log \kappa_f(0) - 2.54203\right] e^{-0.00408t}$$

$$\log x_\ell(t) = -1.54459.$$