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Abstract

First, with volatile income adjustment and the E-V rule of Markowitz (1952), a bond is embedded into the RBC economy under symmetric information conditions. Second, equipped with this way of involving credit, commercial bank and central bank are embedded into the flexible price economy with the monetary effectiveness of Huang (2021). Based on this monetary economy with banks, an enormous asset price bubble economy similar to the Great Depression was created and stabilized. Credit is a Pareto improvement to the original economies. The resource allocation in the unique equilibrium of the multiagent economy with taxes, money, and credit in this paper is Pareto optimal.

Keywords

Credit, Bond, Equity Premium, Financial Intermediary, Commercial Bank, Central Bank Regime, Asset Price Volatility Puzzle, Great Depression, Monetary Policy, Seigniorage, Neoclassical Monetary Economics, Pareto Optimality

E13, E3, E42, E43, E44, E5

1. Introduction

Although it is a historic topic in economics, it was only after the publishing of Kydland and Prescott (1982) and Long and Plosser (1983) that credit could be studied in a quantitative DSGE setting. In the literature, there are two main approaches involving credit in the quantitative DSGE economy. One approach is to endogenously deduce credit through a financial contract between the fund supplier and fund demander under the asymmetric information condition. Bernanke and Gertler (1989) were the first to involve credit in the macroeconomy in this way; they created credit through the costly state verification mechanism of Townsend (1979) in a dynamic economy and analyzed it qualitatively. Carlstrom and Fuerst (1997) implemented the treatment of Bernanke and Gertler (1989) quantitatively in an RBC economy. The other approach involving credit in the macroeconomy is to propose an ad hoc credit constraint. Kiyotaki and Moore (1997) adopted this approach and emphasized the role of asset price in amplifying economic shocks. Bernanke, Gertler, and Gilchrist (1999), based on the above works, advocated the amplification effect of the financial accelerator in a dynamic new Keynesian sticky price monetary economy. Later studies of this school, e.g., Gertler and Kiyotaki (2011), emphasized the importance of financial intermediaries in the amplification mechanism. However, Kocherlakota (2000), Krishnamurthy (2003), and Carlstrom, Fuerst, and Paustian (2016) raised doubts about the amplification effect of financial factors. Based on the above asymmetric information credit economy, Mendoza (2010), Brunnermeier and Sannikov (2014), Di Tella (2017), and He and Krishnamurthy (2019), among others, stressed that financial friction could lead to financial crisis when the economy moves far from its steady state. For comprehensive reviews of the origin of credit, its role in the macroeconomy,

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and financial crisis, please see Gertler (1988) and Brunnermeier, Eisenbach, and Sannikov (2017), among others.

In contrast to the above asymmetric information treatment, in this paper, credit is introduced into the RBC economy and the flexible price neoclassical monetary economy under symmetric information conditions. This task is accomplished with the help of two vehicles: volatile income adjustment, briefly VIA, and the E-V rule of Markowitz (1952). Using the RBC economy as an example, we divide the representative household in the RBC economy of King, Plosser, and Rebelo (1988) into two representative entities: the entrepreneur and the worker. The entrepreneur owns the firm and provides capital, and the worker provides labor. Because there is a technology shock in production, the future income of the entrepreneur is volatile. As a volatile income averser, the entrepreneur could be bettered if she issues a bond with the worker, that is, shares the income and volatility of the production with the worker. With respect to the worker, who is also a volatile income averser, she could also be bettered because she can enjoy the return from the bond when undertaking acceptable volatility inherent in the bond. Consequently, a bond is likely to be created endogenously if it benefits both the entrepreneur and the worker. In this case, the entrepreneur provides equity by herself and issues bonds to the worker; that is, the total capital is financed by both equity and bonds, and all the information is symmetric.

The volatile adjusted income mentioned above can be implemented with the following equation:

$$\tilde{I} = (1 - \Delta\sigma)I \qquad \text{E1.1}$$

where I is the expected income of the next period, σ is the standard deviation related to this income, which stands for its volatility, Δ is the volatility aversion parameter, and we have Δ > 0 when the entity is a volatile income averser, and a variable with an ~ over it is a volatility adjusted variable. In the economy of this paper, what the entities care about is the volatile adjusted income $\tilde{I}$ rather than the expected income I, and it is clear from E1.1 that the higher the value of Δ is, the more volatility aversing is the respective volatile income taker. In addition, it is evident that $0 < \Delta\sigma < 1$ is a reasonable requirement.

The other device needed in the origination of bonds in the symmetric information condition is the expected returns-variance of returns rule, briefly the E-V rule, of Markowitz (1952), which pointed out that, for a financial instrument to exist, its rate of return and volatility of return should lie on the efficient frontier. There could be many forms concerning this requirement; in this paper, we adopt the following simple one between bond, equity, and capital in the RBC economy:

$$\frac{R^K-(1-\delta)}{\sigma^{\mathrm{K}}} = \frac{R^E-(1-\delta)}{\sigma^E} = \frac{R^B-(1-\delta)}{\sigma^B} \qquad \text{E1.2}$$

where R^K, R^E, and R^B are the gross rates of return of capital, equity, and bonds, respectively; δ is the depreciation rate of capital; and σ^K, σ^E, and σ^B are the standard deviations of capital, equity, and bonds, respectively. E1.2 means that the ratios between the net rate of return of an asset and its respective volatility are all equal, which is one of the feasible forms of the E-V rule of Markowitz (1952). Note that E1.2 is also consistent with common sense about the relationship between the rate of return and risk.

With volatile income adjustment and the Markowitz rule, the readers will see in the main text that bonds and equity become an endogenous result of the economy, there is no financial friction or asymmetric information in this economy, credit is a utility improvement to credit participants, and the equity premium of Mehra and Prescott (1985) is a natural outcome because of the difference in volatility between equity and bonds. In addition, credit amplifies fluctuations in the economy to some extent, but this effect is limited and does not lead to a financial crisis.

Equipped with the above credit mechanism, we have the chance to embed the financial intermediary, the commercial bank, and the central bank regime into the macroeconomy of Huang (2021). Starting from the purchasing power of money and a group of simple transaction equations, Huang (2021) proposed a simple flexible price dynamic general equilibrium economy named seigniorage channeled monetary economy, briefly SCME, and demonstrated that monetary shocks impact the real economy effectively and persistently. The mechanism of effectiveness is that the variation of seigniorage in monetary operations leads to resource reallocation in the economy; in other words, money issuance activity involves taxation to the economy, and raising/cutting the money market interest rate actually raises/cutting taxes. SCME is an integration of the transaction, production, consumption, and investment processes; the pricing in SCME is interactive between the supply side, that is, the firm, and the demand side, that is, the household, SCME, nests the RBC economy as a special case, and the resource allocation in its unique equilibrium is Pareto optimal. In addition to the flexible price monetary effectiveness result, SCME in Huang (2021) has clearly explained some notable puzzles in empirical studies, such as the price puzzle, the origin of the money market interest rate, the negative movement of hours under a positive technology shock, the best inflation rate, the best tax rate. However, the monetary system of Huang (2021) was a consolidated system that combined the central bank and the commercial bank. In this work, with the help of the above new credit mechanism, we separate the central bank and commercial bank. In particular, the commercial bank plays two roles: one is to issue money, that is, demand deposits, to the firm and pay the seigniorage to the central bank; the other is to issue term deposits to the worker and make corresponding loans to the entrepreneur. The SCME with banks will be simulated, and we will obtain the monetary effectiveness result and the other results we obtained in Huang (2021) again.

Mathematically, the involvement of a bond and/or bank makes the worker and/or the bank the additional intertemporal optimizers, and economies of this type become multiagent problems, which is different from the single agent intertemporal optimization problems of the RBC economy in Stokey, Lucas, with Prescott (1989) and the flexible price monetary economy in Huang (2021). When the respective optimal curves of the agents are monotonous, the equilibrium of this type of multiagent macroeconomy is unique. In addition, the resource allocation in the unique equilibrium of this kind of multiagent economy is Pareto optimal, that is, the invisible hand conjecture of Adam Smith, which means that the resource allocation in the market economy is optimal and applies to the economies with money, taxes, and credit in this paper.

As an application of the SCME with credit we obtain above, the issue of what monetary policy can do when there is a bubble in asset price is studied in this paper, which was heatedly debated two decades ago. In that debate, there are two main schools of starkly opposite opinions. The first one, for example, Bernanke and Gertler (1999, 2001), suggested that monetary authority should be inactive in asset price inflation, and the other one, for example, Cecchetti, Genberg, Lipsky, Wadhwani (2000) and Cecchetti, Genberg, and Wadhwani (2002), suggested that the interest rate policy should respond directly and strongly to the asset price bubble.

To answer this question, we will first produce an enormous asset price bubble economy similar to the Great Depression of the 1920s-1930s based on the SCME with banks and then check whether monetary policy can do something to stabilize the enormous bubble economy. The following are the findings from the experiments of this paper: 1. The normal interest rate monetary policy, which means that an interest rate rule responds to the expected inflation rate gap and/or expected economic growth rate gap, is pro-bubble; that is, under normal monetary policy, output increases when there is a positive asset price shock. 2. The three shocks, that is, the production shock, the asset price shock, and the monetary policy shock, are all needed in producing the enormous asset price bubble economy, each with a different role: the main role of the production shock is to trigger the asset price bubble, the main role of the monetary shock is to intensify the inflation of asset price, and the asset price shock is the decisive factor of the whole bubble economy process. 3. Early monetary policy action is helpful in stabilizing the bubble economy. However, because the status of a bubble process is not easy to accurately detect, it is better for central banks to be inactive when they are unclear about the state of the asset price bubble or when it is already in the late periods of the booming stage of the bubble economy. In other words, we do not generally suggest that central banks use the bubble piercing strategy. Another finding of this study is that the long duration can explain the volatility puzzle of stock prices.

In summary, unlike the literature that involves credit in the macroeconomy under asymmetric information conditions, this paper embeds credit in the macroeconomy under symmetric information conditions, and an enormous asset price bubble economy similar to the Great Depression is produced and stabilized on the basis of SCME. The layout of the paper is as follows: bond and equity are embedded into the RBC economy in Section 2, the flexible price monetary economy with a commercial bank and the central bank regime is provided in Section 3, and as a byproduct, we obtain the national balance sheet in Section 3, the bubble economy and the related monetary policy actions are studied in Section 4, and Section 5 concludes the paper. The appendix provides the main models of this paper. The economy of a large bubble is simulated with Simulink.

2. Bond and Equity in an RBC Economy

2.1 The Model

Let us begin with the RBC economy of King, Plosser, and Rebelo (1988). We can separate the household of the RBC economy into two entities: the worker and the entrepreneur. Therefore, the new version of the RBC economy consists of three representative entities*: workers, entrepreneurs, and firms*. This separated RBC economy is depicted in Panel (b) of Figure 1. In this separated RBC economy, the firm rents labor, N, from the worker and rents capital, K, from the entrepreneur; that is, we have the following equations:

$$W_t^K K_{t-1} = Y_t^K \qquad \text{E2.1}$$

$$W_t^N N_t = Y_t^N \qquad \text{E2.2}$$

where W^K and W^N are the rent rates of capital and labor, respectively; Y^K and Y^N are the products obtained by the entrepreneur and the worker, respectively; the subscript is the time indicator; the product is the numeraire; and its price, P, is normalized to unity. The entrepreneur is the only intertemporal optimizer of this economy. The production function is the standard constant returns to scale Cobb–Douglass function, that is,

$$Y_t = Z_t^T K_{t-1}^{\alpha} (A_t N_t)^{1-\alpha} \qquad \text{E2.3}$$

In E2.3, α is the share of capital in production, and the growth rate of technology is exogenously given; that is, A_t/A_{t-1} is set to be a constant Γ, and the log form of the technology shock, Z^T, is a stationary first-order autoregressive process:

$$\ln Z_t^T = (1-\rho^T)\ln Z^T + \rho^T \ln Z_{t-1}^T \qquad \text{E2.4}$$

where $0<\rho^T<1$. The steady state of Z^T is set to unity. The white noise process $\varepsilon^T_t \sim N(0,{\sigma^T}^2)$ is added to the log-linear form of E2.4, so we have $\widehat{Z_t^T} = \rho^T \widehat{Z_{t-1}^T} + \varepsilon_t^T$, where a variable with a ^ above it represents the percentage deviation from its steady state. At the beginning of each period, ε^T is realized.

It is easy to obtain from the maximizing behavior of the firm that

$$W_t^K K_{t-1} = \alpha Y_t \qquad \text{E2.5}$$

$$W_t^N N_t = (1-\alpha) Y_t \qquad \text{E2.6}$$

and the profit of the firm, that is, $U^F_t = \max(Y_t - Y^K_t - Y^N_t) \equiv 0$.

It is evident that the separation treatment has not substantially changed the original economy.

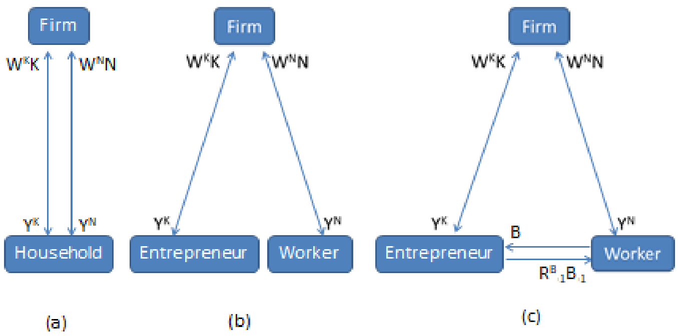


Figure1

Note that there is an important factor that is ignored in canonical RBC studies: the volatility in income. In the separated economy, the entrepreneur is the only undertaker of volatility in production. When the entrepreneur is volatility averse, it is evident that she prefers income without volatility to income with volatility of the same amount. Here, we quantify this point with the following expression:

$$\widetilde{E_t Y_{t+1}^K} = (1 - \Delta^{\mathrm{E}} E_t \sigma_{t+1}^T) E_t Y_{t+1}^K \qquad \text{E2.7}$$

where $\Delta^E>0$ is the volatility averse coefficient of the entrepreneur, E is the expectation operator, and $E_t \sigma_{t+1}^T$ is the expected volatility undertaken by the entrepreneur in the next period. The production volatility is known according to the settings of the RBC models, so E2.7 equals

$$\widetilde{E_t Y_{t+1}^K} = (1 - \Delta^{\mathrm{E}} \sigma^T) E_t Y_{t+1}^K \qquad \text{E2.8}$$

Note that, in this separated economy, the entrepreneur, as the only volatility undertaker, has no choice but to endure production volatility. Importantly, when $1-\Delta^{\mathrm{E}}\sigma^{T}<1$, which is held when the entrepreneur is a volatility averser, the volatile adjusted income, $\widetilde{E_t Y_{t+1}^K}$, is less than the income considered in the literature, that is, $E_t Y_{t+1}^K$.

This situation of the entrepreneur as the only production volatility undertaker can be improved if there is a volatility sharing mechanism. In particular, if the entrepreneur issues bond B to the worker and pays the corresponding interest in the next period and, in the meantime, allows the total production volatility $\boldsymbol{\sigma}^{\mathbf{T}}$ share by both the entrepreneur, as the equity provider, and the worker, as the bond holder, it is possible for both the entrepreneur and the worker to improve their respective utilities. Certainly, the volatility rate of the return relation meets the following form of the Markowitz rule (1952), which has been discussed in the introduction section:

$$\frac{R_t^E-(1-\delta)}{E_t\sigma_{t+1}^E}=\frac{R_t^B-(1-\delta)}{E_t\sigma_{t+1}^B}=\frac{R_t^K-(1-\delta)}{\sigma^T} \qquad \text{E2.9}$$

In addition, we have two additional relationships for this RBC economy: one is the balance sheet constraint, that is,

$$K_t=B_t+E_t \qquad \text{E2.10}$$

where B and E are bond and equity, respectively. E2.10 is evident from the balance sheet of the entrepreneur.

The other additional constraint is the equal return constraint, that is,

$$R_t^K K_t=R_t^B B_t+R_t^E E_t \qquad \text{E2.11}$$

E2.11 means that the returns of the two sides of the balance sheet of the entrepreneur should be equal, which is also evident.

Note that we obtain the expression of R^K in Huang (2021), which we replicate below as

$$R_t^K=\frac{\mathrm{Q_{t+1}(1-\delta)K_t+\alpha E_t Y_{t+1}}}{\mathrm{Q_t K_t}} \qquad \text{E2.12'}$$

Because the canonical simple capital formation equation,

$$K_t=(1-\delta)K_{t-1}+X_t \qquad \text{E2.13}$$

is adopted in this paper, the price of capital, Q, equals the price of output, P. Correspondingly, We have

$$R_t^K=\frac{\mathrm{(1-\delta)K_t+\alpha E_t Y_{t+1}}}{\mathrm{K_t}} \qquad \text{E2.12}$$

Note that we obtain E2.12 because $P_t \equiv 1$ in this case.

Now, let us consider the budget constraints in this RBC economy with bonds. The period t budgets of the entrepreneur and the worker are as follows:

$$\widetilde{C_t^{\mathrm{E}}}=(1-\Delta^{\mathrm{E}}E_{t-1}\sigma_t^{\mathrm{E}})(Y_t^K-(R_{t-1}^B-(1-\delta))B_{t-1})-(X_t-(B_t-(1-\delta)B_{t-1})) \qquad \text{E2.14}$$

$$\widetilde{C_t^N}=Y_t^N+(1-\Delta^N E_{t-1}\sigma_t^{\mathrm{B}})(R_{t-1}^B-(1-\delta))B_{t-1}-(B_t-(1-\delta)B_{t-1}) \qquad \text{E2.15}$$

where Δ^{N} is the volatility aversion coefficient of the worker.

The first term on the right side of E2.15 is the income from labor and is not connected with volatility. The second term, that is, $(1-\Delta^N E_{t-1}\sigma_t^{\mathrm{B}})(R_{t-1}^B-(1-\delta))B_{t-1}$, is the volatile income of workers by taking the bond. The third term is the investment of workers in period t. The first term on the right side of E2.14, that is, $(1-\Delta^{\mathrm{E}}E_{t-1}\sigma_t^{\mathrm{E}})(Y_t^K-(R_{t-1}^B-(1-\delta))B_{t-1})$, is the volatile income of the entrepreneur by taking the equity. With $K_t = B_t + E_t$, the second term on the right side of E2.14 equals $E_t-(1-\delta)E_{t-1}$, which is the investment of the entrepreneur in period t; that is, the investment of this economy, X_t, consists of two parts: investment of the worker, $B_t-(1-\delta)B_{t-1}$, and investment of the entrepreneur, $E_t-(1-\delta)E_{t-1}$. $\widetilde{C^E}$ and $\widetilde{C^N}$ are consumptions corresponding to the volatile income of the entrepreneur and the worker, respectively. We can obtain the respective budgets of period t+1 and beyond similarly. Notably, it is possible to embed the loss triggered by volatility into the above budgets. To make this simple, we do not implement it in this paper.

The permanent utilities of the entrepreneur and the worker are, respectively,

$$UU_t^{\mathrm{E}} = maxE_t \sum_{i=0}^{\infty}(\beta^{\mathrm{E}})^i\, U^{\mathrm{E}}(\widetilde{C_{t+\iota}^{\mathrm{E}}}) \qquad \text{E2.16}$$

$$UU_t^{N} = maxE_t \sum_{i=0}^{\infty}(\beta^{\mathrm{N}})^i\, U^{N}(\widetilde{C_{t+\iota}^{N}}, J_{t+i}) \qquad \text{E2.17}$$

where β^E and β^N are the respective subjective discount rates of the entrepreneur and the worker, $U^E(\cdot)$ and $U^N(\cdot)$ are the respective single-period utilities, J is the leisure of the worker, and we neglect the leisure of the entrepreneur in this paper.

Now, we have completed the construction of this RBC economy with bonds, which is depicted in Panel (c) of Figure 1. Because B and R^B are endogenous state variables, the worker becomes the other intertemporal utility maximizer in this economy, in addition to the entrepreneur. Both the entrepreneur and the worker maximize their respective utilities, E2.16 and E2.17, subject to four types of constraints, that is, the respective budget constraints, E2.14 and E2.15; the balance sheet constraint, E2.10; the equal return constraint, E2.11; and the constraint of the Markowitz rule, E2.9. We can obtain the respective value functions of the entrepreneur and the worker. Unlike the RBC economy of Stokey, Lucas, with Prescott (1989), which is a single-agent dynamic optimization problem, the involvement of bonds turns this new RBC economy into a multiagent dynamic optimization problem. According to Stokey, Lucas, and Prescott (1989), when the utility functions meet the required conditions, the entrepreneur and the worker can obtain their respective solutions to their dynamic problem; however, in this bond case, they can obtain only a monotonous line rather than a fixed point because each one cannot solve the bond problem independently. However, since the interests of the entrepreneur and worker are opposite in terms of the bond mechanism, which is obvious from their standpoint on the terms of the bond, these two lines are distinct. Correspondingly, the intersection point of these two lines is the only local fixed point of the economy. Because the two monotonous lines are the respective Pareto optimal solutions of the entrepreneur and the worker, the only fixed point obtained is the unique Pareto optimal equilibrium of the multiagent RBC economy with bonds. In Appendix A, we use a much simpler pure-credit economy to clearly explain this unique Pareto optimal equilibrium of the multiagent situation.

2.2 Performance of the Model

To simulate the model of the above subsection, we need a concrete form of the utility functions and values of the parameters. To ensure robustness, a well-accepted functional form and parameter values are adopted in this paper; the only two exceptions are the value of α in the production function and the value of δ, that is, the depreciation rate of capital, which are discussed below:

The entrepreneur–worker separation treatment of the above subsection leads to an evident change in the value of α. It is helpful to check this in a simple case. Specifically, let us compare the steady-state investment-output ratio of the traditional RBC economy, which is depicted in Panel (a) of Figure 1, and that of the entrepreneur–worker separated RBC economy, that is, Panel (b) of Figure 1, and it is helpful to introduce tax and public goods into the RBC economy to ensure that the steady state of the economy is close to that of the real-world economy, which provides a benchmark for choosing the parameter values. In this paper, the simple flat rate income tax is adopted, and we have

$$T_t = \tau Y_t \qquad \text{E2.18}$$

where τ is the tax rate. The form of the public goods production function adopted is also simple; that is,

$$G_t = T_t \qquad \text{E2.19}$$

E2.19 is an additional constraint undertaken by the household in the traditional RBC economy, and the entrepreneur and the worker will undertake similar constraints in the separated RBC economy.

Note that the government in the economy is not a utility maximizer; it just runs according to the given rules, that is, E2.18 and E2.19.

With the income tax, the budget constraints of the household of the traditional RBC economy and the entrepreneur and worker of the separated RBC economy are, respectively,

$$C_t^H = (1-\tau)Y_t - X_t \qquad \text{E2.20}$$

$$C_t^E = (1-\tau)Y_t^K - X_t \qquad \text{E2.21}$$

$$C_t^N = (1-\tau)Y_t^N \qquad \text{E2.22}$$

For the utility functions, the following period t forms are adopted by the household, the entrepreneur, and the worker:

$$\mathrm{U}_t^{\mathrm{H}} = \frac{((\frac{C_t^{\mathrm{H}}}{A_t})^{\chi H}(\frac{G_t}{A_t})^{1-\chi H})^{1-\eta H}}{1-\eta H} + \xi^H(1-N_t) \qquad \text{E2.23}$$

$$\mathrm{U}_t^{\mathrm{E}} = \frac{((\frac{C_t^{\mathrm{E}}}{A_t})^{\chi E}(\frac{G_t}{A_t})^{1-\chi E})^{1-\eta \mathrm{E}}}{1-\eta \mathrm{E}} \qquad \text{E2.24}$$

$$U_t^N = \frac{((\frac{C_t^{\mathrm{N}}}{A_t})^{\chi N}(\frac{G_t}{A_t})^{1-\chi N})^{1-\eta \mathrm{N}}}{1-\eta N} + \xi^N(1-N_t) \qquad \text{E2.25}$$

where η^H, η^E, and η^N are the coefficients used to ensure the existence and stability of the equilibrium and where ξ^H and ξ^N are the balance parameters used to obtain a reasonable steady-state value for hours in the equilibrium. The labor of the entrepreneur is omitted in this paper, as mentioned before.

From the above utility functions, budget constraints, and public goods constraints, it is not difficult to obtain the steady-state investment-output ratios of the traditional RBC economy and the separated RBC economy as follows:

$$\frac{X}{Y}^{traditional\ RBC} = \frac{(\Gamma-(1-\delta))\beta\alpha(1-\tau)\frac{1}{\chi}}{(\Gamma-\beta(1-\delta))+\beta\frac{1-\chi}{\chi}(\Gamma-(1-\delta))\alpha} \qquad \text{E2.26}$$

$$\frac{X}{Y}^{seperated\ RBC} = \frac{(\Gamma-(1-\delta))\beta\alpha^2(1-\tau)\frac{1}{\chi}}{(\Gamma-\beta(1-\delta))+\beta\frac{1-\chi}{\chi}(\Gamma-(1-\delta))\alpha} \qquad \text{E2.27}$$

In the comparison, the values of β^H and β^E, that is, the respective subjective discount rates, are the same, the values of χ^H and χ^E are also the same, and we use β and χ to represent them, respectively, in E2.26 and E2.27. Note that the only difference in the above two equations is that α in the numerator of E2.26 is replaced by α^2 in E2.27; thus, to ensure that the steady-state investment–output ratio of the separated RBC economy is close to that of the traditional RBC economy, the value of α in the separated RBC economy must be much larger than that of the traditional RBC economy. The steady-state values of G/Y and X/Y are both between 0.15 and 0.20, where we use the data from the USA as the benchmark, and from the well-accepted values of the parameters of economy (a), in which the value of α is approximately 0.35, a value of approximately 0.6 for α is acceptable in the separation case (b). The commonly accepted values of $\Gamma=1.005$, δ=0.025, and τ=0.15 are adopted in the calculation. For χ, we adopt the value of 0.75 from Huang (2021). With the new α value and the other commonly adopted parameter values, the steady-state investment-output ratio, government spending-output ratio, and consumption-output ratio in the separated RBC economy are all close to those in the real-world economy. Interestingly, the enlarged α value does not change the share of consumption in output in the separated RBC economy compared with that of the traditional RBC economy. Note that in the separated case, the total consumption equals that of C^E+C^N.

Now, let us turn to the economy with the bond of the above subsection, which is depicted in Panel (c) of Figure 1. To make it close to the real world economy, as we do above, tax and public goods are added to that economy, and briefly, we call it economy (c). The respective period t utility functions of economy (c) are the same as those of economy (b). Equipped with the credit side of the economy, we have more information and find that the traditional value of δ, which is approximately 0.025, is doubtful in the bond economy, which is discussed below:
First, from the equal return constraint E2.11, the balance sheet constraint E2.10, and the Markowitz rule E2.9, we have, in the steady state,

$$RK = (b + \sigma\, e)\, RB \qquad \text{E2.28}$$

where $RK=R^K-(1-\delta)$, $RB=R^B-(1-\delta)$, $\sigma=\sigma^E/\sigma^B$, $b=B/K$, and $e=E/K$.

Then, from E2.12, E2.28, and $\frac{X}{Y}=\frac{(\Gamma-(1-\delta))K}{\mathrm{Y}}$, we have

$$\alpha\frac{\Gamma-(1-\delta)}{\frac{X}{Y}} = (b + \sigma\, e)\, RB \qquad \text{E2.29}$$

From the steady-state equity premium, EP, and the Markowitz rule, we have

$$\mathrm{EP} = R^E - R^B = RE - RB = \left(\sigma - 1\right) RB \qquad \text{E2.30}$$

Combining E2.29 and E2.30, we have

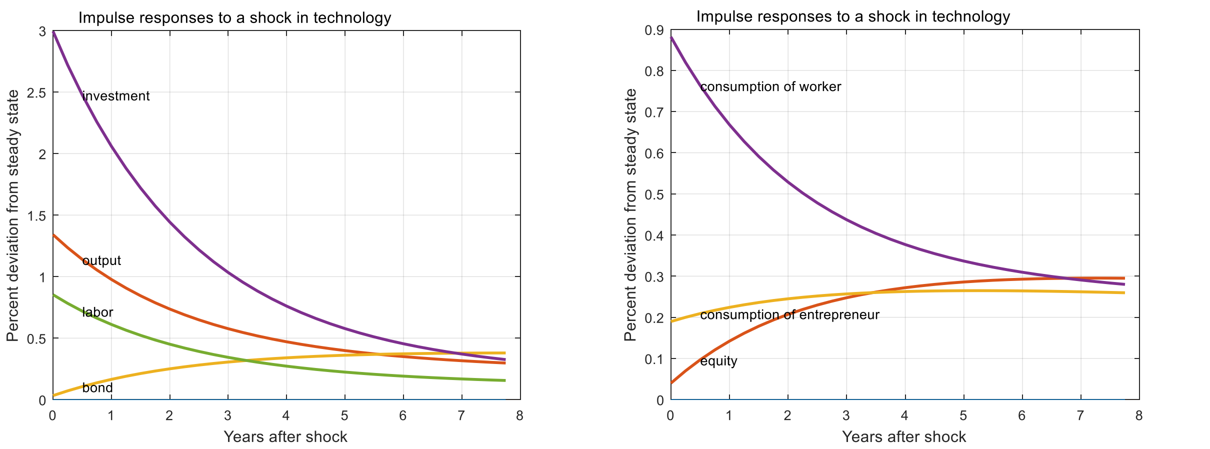
$$\alpha\frac{\Gamma-(1-\delta)}{\frac{X}{Y}} = R^B - (1+\delta) + EP\, e \qquad \text{E2.31}$$

Since we have the values of α≈0.6, $\Gamma\approx1.005$, X/Y≈0.17, b≈0.5, e=1-b, EP<0.05 (quarterly), and $R^B\approx1.015$ (quarterly) in the real-world economy, from E2.31, we find that the quarterly value of δ must be less than 0.01, which is much less than the commonly adopted value of 0.025 in the macroeconomic literature. δ=0.0075 is adopted in this paper. δ=0.0075 means that the usage period of capital is approximately 33 years, in contrast to the 10-year usage period when $\delta=0.025$. Because the usage period of buildings, one of the main types of production capital, is much longer than 33 years, the δ value we adopt is acceptable.

In summary, $\alpha=0.6$, $\Gamma=1.005$, $\delta=0.0075$, $\tau=0.15$, $\eta^E=\eta^N=0.5$, and $\chi^E=\chi^N=0.75$ are adopted in economy (c), and the value of ξ^N is set to ensure that the steady-state value of hours is 1/3. For the parameters of the shocks, $\rho^T=0.9$ and $\sigma^T=0.7\%$, which are extensively adopted in the literature. The values of the new parameters Δ^E and Δ^N are not easy to estimate. The strategy we adopt in this paper is to choose the values of b and σ, which are relatively easy to obtain, and then use these values to determine the values of Δ^E and Δ^N with the help of the other parameters and steady-state relations. When $b=0.4$ and $\sigma=5$, the values adopted in this paper, Δ^E and Δ^N are approximately 45 and 229, respectively, which shows that workers are much more sensitive to volatility than entrepreneurs are.

Under the above conditions, the consumption-output ratio, government spending-output ratio, and investment-output ratio of economy (c) in the steady state are 0.6715, 0.15, and 0.1785, respectively, which are all close to those in the real-world U.S. economy. The two volatility adjustment factors, that is, $\Delta^E\sigma^E$ and $\Delta^N\sigma^{B,}$ are both less than unity. The capital-output ratio, X/Y, is approximately 14. The whole system of this RBC economy with taxes and bonds is provided in Appendix B, and the absolute values of the characteristic roots of the system are all less than 1.

With respect to the impulse response of the economy under technology shock, Figure 2 shows the response of the respective variables under a one percentage positive technology shock, whose meaning is clear and for which we do not take space to explain it.



(a) (b)

Figure 2

Now, let us focus on the impact of the financial side on the economy. Figure 3 compares some key variables between the bond economy (c) and the economy without bonds, that is, economy (b), when se/sb, that is, σ, spans from 2 to 10, with the blue lines from economy (c) and the red lines from economy (b). Figure 3 shows that capital, output, consumption and utility during period t obviously increase in Economy (c) compared with those in Economy (b). Moreover, the volatility of the economy with bonds increases to a limited degree compared with that of the economy without bonds, as shown in Table 1.

The mechanism of these results is evident: By undertaking controllable volatility, credit, which is a return and volatility sharing device, makes it possible to involve more funds in the investment and, correspondingly, accumulate more capital in the production. Consequently, more output is produced, which leads to an increase in the respective steady-state consumption and utility of the participants.

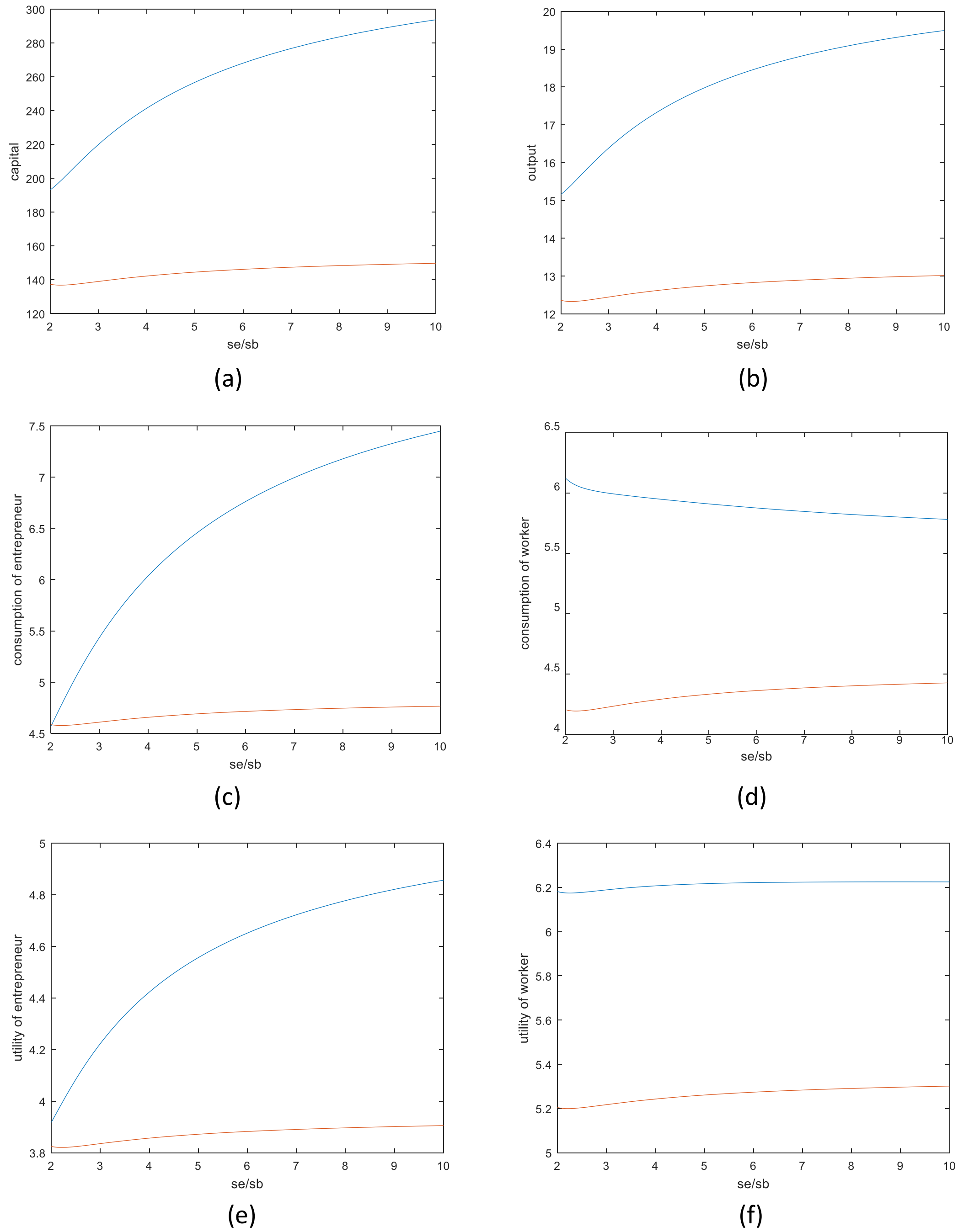


Figure 3

Table 1 Standard Deviation of Variables

	Economy (b)	Economy (c)
capital	0.14	0.15
output	1.11	1.19
consumption of entrepreneur	0.27	0.28
consumption of labor	1.11	0.82
investment	3.38	3.67
labor	0.56	0.74
tax	1.11	1.19
public goods	1.11	1.19

3. Flexible Price Monetary Economy with Commercial Bank and Central Bank

Based on the credit mechanism of the above section, we can obtain a monetary economy with commercial bank, brief bank, and central bank under symmetric information conditions. In particular, the credit mechanism is involved in the seigniorage channeled monetary economy of Huang (2021), and monetary effectiveness results in flexible price conditions and other results, such as the price puzzle under monetary shock and the negative movement of hours under technology shock, are obtained again.

The credit in Section 2 is direct, which means that the financial resource is transferred from the owner, that is, the worker, to the borrower, that is, the entrepreneur, directly. However, many studies emphasize the importance of financial intermediaries; that is, financial resources are not transferred directly from the owner to the ultimate user but through an intermediary. The literature focuses on asymmetric information when the origin of the financial intermediary is studied. For macroeconomic surroundings, see Gertler and Kiyotaki (2011) for an example. In contrast to the asymmetric information approach, with the bond mechanism of the above section, we have the chance to introduce intermediaries into the economy under symmetric information conditions. In particular, an entity with a value of Δ different from those of the entrepreneur and the worker, which shows her distinct attitude toward volatile income, could emerge as the financial intermediary between the entrepreneur and the worker by issuing a financial instrument to the worker and lending the corresponding fund to the entrepreneur. For brevity, in this paper, we omit the case of a simple intermediary and go directly to the case of a commercial bank, which undertakes two roles: the financial intermediary role and the money issuance role. Readers will soon see that, on the one hand, as a financial intermediary, the bank accepts term deposits from the worker and lends them to the entrepreneur as loans, and on the other hand, as the money issuer, the bank issues money, in the form of demand deposits, to the firm, collects the corresponding seigniorage, and in the meantime, the bank obtains high-power money from the central band and pays the corresponding seigniorage to the central bank. Note that there are two tiers of seigniorage in this economy: the seigniorage paid to the bank by the firm and the seigniorage paid to the central bank by the bank. This two-tier seigniorage mechanism is what happens in the modern central bank regime. In this economy, the commercial bank is an additional intertemporal utility maximizer, and the central bank, which is the same as the government of the above section, is not a utility maximizer; it runs according to the given rules.

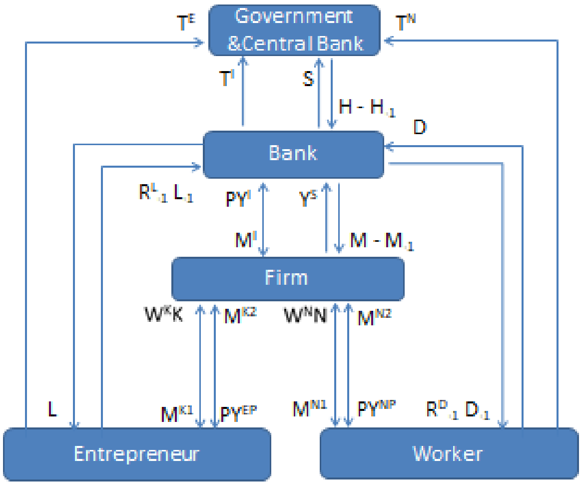


Figure4

To help illustrate the running of the economy, Figure 4 shows the main activities of the economy in period t. This economy is a combination of the monetary economy of Huang (2021) and the credit economy of the last section, except that the direct finance of the last section is expanded to the indirect finance of a commercial bank and the consolidated banking system of Huang (2021) is correspondingly divided into a commercial bank and a central bank. Because the main aspects of the model have been articulated in Huang (2021) and Section 2 of this paper, below, we review the economy as quickly as possible, and the meaning of Figure 4 will be clear when we review the economy.

3.1 Monetary, Transaction, and Supply Sides of the Economy

Like in Huang (2021), the interactive interest rate rule adopted here by the central bank is as follows:

$$R_t = Z_t^M R \, \Gamma^{\frac{\Gamma_t^e}{\Gamma}-1} \, \Pi^{\frac{\Pi_t^e}{\Pi}-1} \qquad \text{E3.1}$$

where R is the money market interest rate; Γ^e_t is the expected gross economic growth rate of period t+1; Π^e_t is the expected gross inflation rate of period t+1; R, Γ, and Π are the respective steady-state values; Γ and Π are the respective parameters in the policy rule; and Z^M is the monetary policy shock. Similar to the technology shock, we have

$$\ln Z_t^M = (1-\rho^M)\ln Z^M + \rho^M \ln Z_{t-1}^M \qquad \text{E3.2}$$

where $0<\rho^M<1$. The steady-state value of Z^M is set to unity. The white noise process $\varepsilon^M_t \sim N(0, {\sigma^M}^2)$ is added to the log-linear form of E3.2, and we have $\widehat{Z_t^M} = \rho^M \widehat{Z_{t-1}^M} + \varepsilon_t^M$. At the beginning of each period, ε^M is realized. ε^M and ε^T are independent of each other in this paper.

The corresponding seigniorage, S, of issuing high-power money is

$$S_t = \frac{H_t - H_{t-1}}{P_t} \quad \text{E3.3}$$

where H is the aggregate of the high-power money, which is also called the base money, and P is the product price. E3.3 means that the seigniorage equals the value of goods that can be purchased by the additional amount of high-power money of period t. With the approach of Huang (2021), we have

$$\mathrm{R_t} = \frac{\mathrm{E_t H_{t+1}}}{\mathrm{H_t}} \quad \text{E3.4}$$

Concerning Y^S, the seigniorage of money issuance, we have, from Huang (2021),

$$Y_t^S = \frac{M_t - M_{t-1}}{M_t} Y_t \quad \text{E3.5}$$

where M is the monetary aggregate. From Huang (2021), we have

$$R_t^M = \frac{\mathrm{E_t M_{t+1}}}{\mathrm{M_t}} \quad \text{E3.6}$$

where R^M is the interest rate of issuing money.

For the transaction equations, we have, similar to those in Huang (2021),

$$W_t^K K_{t-1} = M_t^{K1} \quad \text{E3.7}$$

$$W_t^N N_t = M_t^{N1} \quad \text{E3.8}$$

$$M_t^{K1} + M_t^{N1} = M_t \quad \text{E3.9}$$

$$M_t^{K2} = M_t^{K1} - R_{t-1}^{\mathrm{L}} \mathrm{L}_{t-1} + \mathrm{L}_t \quad \text{E3.10}$$

$$M_t^{N2} = M_t^{N1} + R_{t-1}^{\mathrm{D}} \mathrm{D}_{t-1} - \mathrm{D}_t \quad \text{E3.11}$$

$$M_t^I = R_{t-1}^{\mathrm{L}} \mathrm{L}_{t-1} - R_{t-1}^{\mathrm{D}} \mathrm{D}_{t-1} \quad \text{E3.12}$$

$$M_t^{K2} = P_t Y_t^{\mathrm{EP}} \quad \text{E3.13}$$

$$M_t^{N2} = P_t Y_t^{\mathrm{NP}} \quad \text{E3.14}$$

$$M_t^I = P_t Y_t^I \quad \text{E3.15}$$

where M^{K1} and M^{N1} are the money used in renting capital and labor hours by the firm, respectively; D and L are the term deposit and loan, respectively, and we have D=L; R^D and R^L are the gross interest rates of D and L, respectively; M^{K2}, M^{N2}, and M^I are the money used to purchase products by the entrepreneur, the worker, and the bank, respectively; Y^{EP} and Y^{NP} are the pseudo-income of the entrepreneur and the worker, respectively; and Y^I is the income of the bank. Because interest and investment are included in Y^{EP} and Y^{NP}, the name pseudo-income is used, which will be clear soon when we discuss E3.36 and E3.37. Money is the numeraire here, and the price of money, $P^M_{t,}$ is constant and normalized to unity. Again, as in Huang (2021), we do not scrutinize the velocity of money, ω, assume it to be constant, and normalize it to unity in this paper.

The production function is the same as that of the above section, that is,

$$Y_t = Z_t^T K_{t-1}^{\alpha} (A_t N_t)^{1-\alpha} \quad \text{E3.16}$$

The utility function of the firm is as follows:

$$U_t^F = \max(Y_t - Y_t^{\mathrm{EP}} - Y_t^{\mathrm{NP}} - Y_t^I) \quad \text{E3.17}$$

Similar to that in Huang (2021), from the transaction equations, the production function, and maximizing behavior of the firm, we can obtain

$$W_t^K K_{t-1} = \alpha P_t Y_t = M_t^{K1} \quad \text{E3.18}$$

$$W_t^N N_t = (1-\alpha) P_t Y_t = M_t^{N1} \quad \text{E3.19}$$

From E3.18, E3.19, and E3.9, we obtain the equation of exchange,

$$M_t = P_t Y_t \quad \text{E3.20}$$

E3.20 can be regarded as the supply function of this economy.

According to the reserve requirement mechanism of the central bank regime and from the interest of the bank, we have

$$\frac{H_t}{M_t} = \iota \qquad \text{E3.21}$$

where ι is the reserve requirement ratio. From E3.3, E3.20, E3.21, and E3.5, we obtain

$$S_t = \frac{M_t - M_{t-1}}{M_t} \iota Y_t = \iota Y_t^S \qquad \text{E3.22}$$

In addition, we have, from E3.6, E3.21, and E3.4,

$$R_t^M = \frac{\mathrm{E_t M_{t+1}}}{\mathrm{M_t}} = \frac{\frac{\mathrm{E_t H_{t+1}}}{\iota}}{\frac{\mathrm{H_t}}{\iota}} = \frac{\mathrm{E_t H_{t+1}}}{\mathrm{H_t}} = R_t \qquad \text{E3.23}$$

With E3.6, E3.20 and E3.23, the interest rate form of the equation of exchange is

$$R_t = \Pi_t^e \Gamma_t^e \qquad \text{E3.24}$$

From E3.24, we obtain the actual gross inflation rate as

$$\Pi_t^a = \frac{R_{t-1}}{\Gamma_t^a} \qquad \text{E3.25}$$

where $\Gamma_t^a = \frac{Y_t}{Y_{t-1}}$ is the actual output growth rate of period t. Note that $H_t = E_{t-1} H_t$ and $M_t = E_{t-1} M_t$ are adopted in obtaining E3.25, which means that the operation errors in the monetary policy implementation and money issuance operation are both ignored.

3.2 Financial Side of the Economy

In this economy, the worker deposits in the bank, and the bank transfers it to the firm as a loan; correspondingly, the Markowitz rule of this economy is as follows:

$$\frac{\frac{R_t^E}{\Pi_t^e} - 1}{E_t \sigma_{t+1}^E} = \frac{\frac{R_t^L}{\Pi_t^e} - 1}{E_t \sigma_{t+1}^L} = \frac{\frac{R_t^D}{\Pi_t^e} - 1}{E_t \sigma_{t+1}^D} = \frac{\frac{R_t^K}{\Pi_t^e} - 1}{\sigma^T} \qquad \text{E3.26}$$

where σ^L and σ^D are the volatility of the loan and the term deposit, respectively. E3.26 means that the ratios of the real net interest rate over the respective volatility are equal between financial instruments, and in the monetary case, the depreciation factor, δ, is discarded in the Markowitz rule. To prepare for later use, we restate the Markov rule as follows:

$$\frac{\frac{R_t^E}{\Pi_t^e} - 1}{E_t \sigma_{t+1}^E} = \frac{\frac{R_t^K}{\Pi_t^e} - 1}{\sigma^T} \qquad \text{E3.27}$$

$$\frac{\frac{R_t^L}{\Pi_t^e} - 1}{E_t \sigma_{t+1}^L} = \frac{\frac{R_t^K}{\Pi_t^e} - 1}{\sigma^T} \qquad \text{E3.28}$$

$$\frac{\frac{R_t^D}{\Pi_t^e} - 1}{E_t \sigma_{t+1}^D} = \frac{\frac{R_t^K}{\Pi_t^e} - 1}{\sigma^T} \qquad \text{E3.29}$$

Concerning the gross rate of return of capital, in this monetary economy, we have

$$R_t^K = \frac{\mathrm{E_t Q_{t+1}(1-\delta) K_t} + \mathrm{E_t} W_{t+1}^K \mathrm{K_t}}{\mathrm{Q_t K_t}} \qquad \text{E3.30}$$

where Q is the price of capital. With E3.18, E3.30 becomes

$$R_t^K = \frac{\mathrm{E_t Q_{t+1}(1-\delta)K_t + \alpha E_t P_{t+1} E_t Y_{t+1}}}{\mathrm{Q_t K_t}} \qquad \text{E3.31}$$

We adopt the simple capital formation equation in this paper, that is,

$$K_t = (1-\delta)K_{t-1} + X_t \qquad \text{E3.32}$$

and price of capital and the price of the product are equal; that is, $Q_t=P_t$. Correspondingly, we can obtain from E3.31,

$$\frac{R_t^K}{\Pi_t^e} - 1 = \alpha \frac{\mathrm{E_t Y_{t+1}}}{\mathrm{K_t}} - \delta \qquad \text{E3.33}$$

The balance sheet constraint and the equal return constraint of this economy are

$$Q_t K_t = \mathrm{L}_t + E_t \qquad \text{E3.34'}$$

$$R_t^K Q_t K_t = R_t^{\mathrm{L}} \mathrm{L}_t + R_t^E E_t \qquad \text{E3.35'}$$

Together with $Q_t=P_t$, E3.34' and E3.35' can be transformed to

$$K_t = \mathrm{l}_t + e_t \qquad \text{E3.34}$$

$$R_t^K K_t = R_t^{\mathrm{L}} \mathrm{l}_t + R_t^E e_t \qquad \text{E3.35}$$

where e=E/P and l=L/P are the respective real values.

Note that there are two kinds of deposits in the economy: one is the term deposit, D, briefly deposit, which obtains the gross interest rate of R^D, and the other is the demand deposit, M, briefly money, which obtains the benefit of settlement services and obtains no interest. The succinct balance sheets of the central bank and the commercial bank at the end of period t are, respectively, shown in Panel (a) and (b) of Figure 7. In this economy, the central bank issues high-power money H by loan L^H, and the bank issues money by loan L^M. Because the earnings of these two loans are the respective seigniorages, there is no risk/volatility inherent in them. In the real world economy, it is easy to obtain D and M, but it is not easy to differentiate L and L^M. The asset side of the balance sheet of the bank of the real-world economy is a mixture of L^M and L.

3.3 Demand Side of the Economy

With respect to the demand side of the economy, let us begin from the budgets of the entrepreneur, the worker, and the bank. From the transaction equations, we have

$$Y_t^{\mathrm{EP}} = \frac{M_t^{K2}}{P_t} = \frac{M_t^{K1} - R_{t-1}^{\mathrm{L}} \mathrm{L}_{t-1} + \mathrm{L}_t}{P_t} = \alpha Y_t - \left(\frac{R_{t-1}^{\mathrm{L}}}{\Pi_t^a} - 1\right) \mathrm{l}_{t-1} + (\mathrm{l}_t - \mathrm{l}_{t-1}) \qquad \text{E3.36}$$

$$Y_t^{\mathrm{NP}} = \frac{M_t^{N2}}{P_t} = \frac{M_t^{N1} + R_{t-1}^{\mathrm{D}} \mathrm{D}_{t-1} - \mathrm{D}_t}{P_t} = (1-\alpha) Y_t + \left(\frac{R_{t-1}^{\mathrm{D}}}{\Pi_t^a} - 1\right) \mathrm{d}_{t-1} - (\mathrm{d}_t - \mathrm{d}_{t-1}) \qquad \text{E3.37}$$

$$Y_t^{\mathrm{I}} = \frac{M_t^I}{P_t} = \frac{R_{t-1}^{\mathrm{L}} \mathrm{L}_{t-1} - R_{t-1}^{\mathrm{D}} \mathrm{D}_{t-1}}{P_t} = \left(\frac{R_{t-1}^L}{\Pi_t^a} - 1\right) l_{t-1} - \left(\frac{R_{t-1}^D}{\Pi_t^a} - 1\right) d_{t-1} \qquad \text{E3.38}$$

From E3.36, Y^{EP}_t consists of three parts, the income from renting capital, that is, αY_t, minus the net interest of the loan, that is, $\left(\frac{R_{t-1}^{\mathrm{L}}}{\Pi_t^a} - 1\right) \mathrm{l}_{t-1}$, and the investment of the bank, that is, $\mathrm{l}_t - \mathrm{l}_{t-1}$; from E3.37, Y^{NP}_t consists of three parts too, the income of renting labor, the net interest income of the term deposit, minus the worker's investment; E3.38 shows that the bank earns the net interest income from financial intermediary activity. With investment, income tax, and seigniorage, the budget constraints of the entrepreneur, the worker, and the bank in period t are, respectively,

$$\widetilde{C_t^{\mathrm{E}}} = (1-\tau)(1 - \Delta^{\mathrm{E}} E_{t-1} \sigma_t^{\mathrm{E}})\left(\alpha Y_t - \left(\frac{R_{t-1}^{\mathrm{L}}}{\Pi_t^a} - 1\right) \mathrm{l}_{t-1}\right) - X_t + (\mathrm{l}_t - \mathrm{l}_{t-1}) - Y_t^S \qquad \text{E3.39}$$

$$\widetilde{C_t^{\mathrm{N}}} = (1-\tau)(1-\alpha)Y_t + (1-\tau)(1-\Delta^N E_{t-1}\sigma_t^{\mathrm{D}})(\frac{R_{t-1}^{\mathrm{D}}}{\Pi_t^a} - 1)\mathrm{d}_{t-1} - (\mathrm{d}_t - \mathrm{d}_{t-1}) \qquad \text{E3.40}$$

$$\widetilde{C_t^{\mathrm{I}}} = (1-\tau)(1-\Delta^{\mathrm{I}} E_{t-1}\sigma_t^{\mathrm{L}})\left(\frac{R_{t-1}^L}{\Pi_t^a} - 1\right) l_{t-1} - (1-\tau)\left(\frac{R_{t-1}^D}{\Pi_t^a} - 1\right) d_{t-1} + Y_t^S - S_t \qquad \text{E3.41}$$

where Δ^{I} is the volatility aversion parameter of the bank. Note that the entrepreneur pays the seigniorage, Y^{S}, because she is the owner of the firm, to which the bank issues the money.
The respective income taxes of the entrepreneur, the worker, and the bank in this economy are

$$\mathrm{T_t^E} = \tau(\alpha Y_t - \left(\frac{R_{t-1}^{\mathrm{L}}}{\Pi_t^a} - 1)\mathrm{l}_{t-1}\right) \qquad \text{E3.42}$$

$$\mathrm{T_t^N} = \tau((1-\alpha)Y_t + (\frac{R_{t-1}^{\mathrm{D}}}{\Pi_t^a} - 1)\mathrm{d}_{t-1}) \qquad \text{E3.43}$$

$$\mathrm{T_t^I} = \tau\left(\left(\frac{R_{t-1}^L}{\Pi_t^a} - 1\right) l_{t-1} - \left(\frac{R_{t-1}^D}{\Pi_t^a} - 1\right) d_{t-1}\right) \qquad \text{E3.44}$$

from which we obtain the total income tax of this economy as

$$\mathrm{T_t} = \mathrm{T_t^E} + \mathrm{T_t^N} + \mathrm{T_t^I} = \tau \mathrm{Y_t} \qquad \text{E3.45}$$

With the simple public goods production function, as we adopt in the above section, we have the respective public goods of the entrepreneur and the worker as follows:

$$\mathrm{G_t} = \mathrm{T_t} + {Y^S}_t \qquad \text{E3.46}$$

and the public goods of the bank are

$$G_t^I = \mathrm{T_t} + \mathrm{S_t} \qquad \text{E3.47}$$

The respective period t utility functions are as follows:

$$\mathrm{U}_t^{\mathrm{E}} = \frac{((\frac{\widetilde{C_t^{\mathrm{E}}}}{A_t})^{\chi \mathrm{E}} (\frac{\mathrm{G_t}}{A_t})^{1-\chi \mathrm{E}})^{1-\eta \mathrm{E}}}{1-\eta \mathrm{E}} \qquad \text{E3.48}$$

$$U_t^N = \frac{((\frac{\widetilde{C_t^{\mathrm{N}}}}{A_t})^{\chi N} (\frac{\mathrm{G_t}}{A_t})^{1-\chi N})^{1-\eta N}}{1-\eta N} + \xi(1-N_t) \qquad \text{E3.49}$$

$$\mathrm{U}_t^{\mathrm{I}} = \frac{((\frac{\widetilde{C_t^{\mathrm{I}}}}{A_t})^{\chi \mathrm{I}} (\frac{G_t^{\mathrm{I}}}{A_t})^{1-\chi \mathrm{I}})^{1-\eta \mathrm{I}}}{1-\eta \mathrm{I}} \qquad \text{E3.50}$$

where we neglect the labor of the entrepreneur and the bank in E3.54 and E3.56, respectively.
The permanent utilities beginning from period **t** are, respectively,

$$UU_t^{\mathrm{E}} = max \sum_{i=0}^{\infty} (\beta^{\mathrm{E}})^i E_t U_{t+i}^{\mathrm{E}} \qquad \text{E3.51}$$

$$UU_t^{\mathrm{N}} = max \sum_{i=0}^{\infty} (\beta^{\mathrm{N}})^i E_t U_{t+i}^{\mathrm{N}} \qquad \text{E3.52}$$

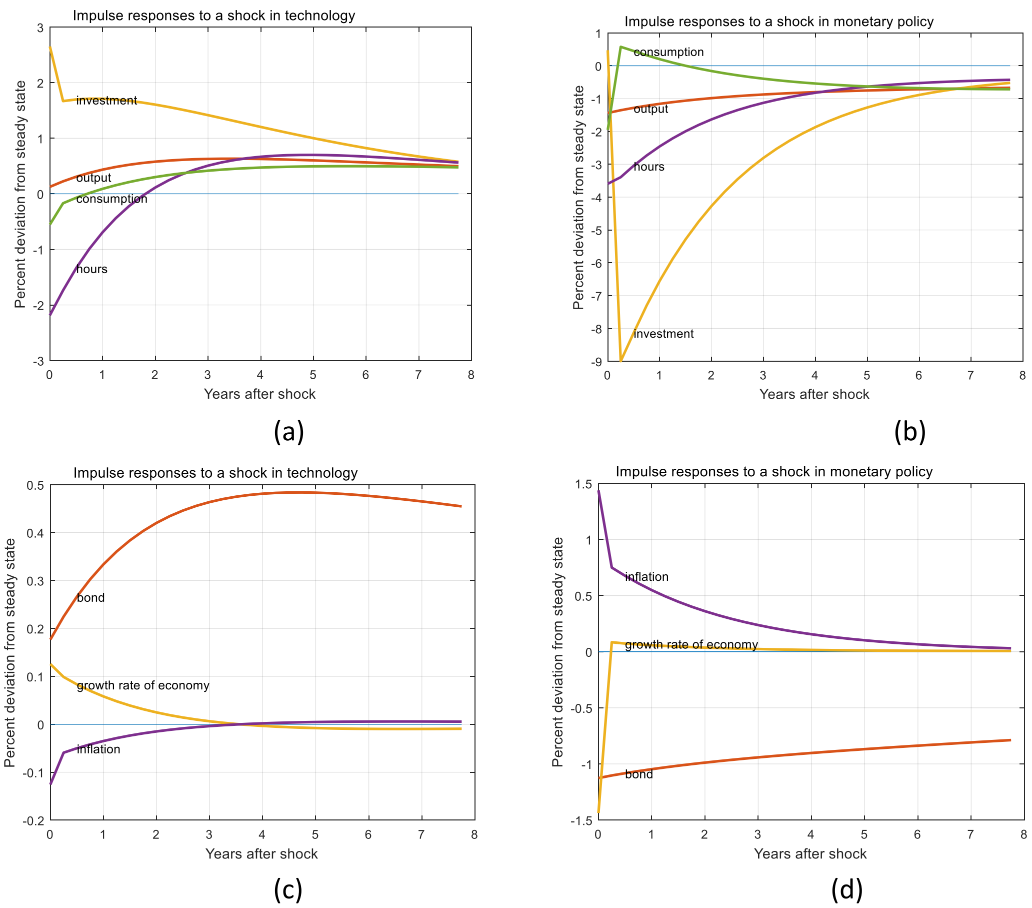
$$UU_t^{\mathrm{I}} = max \sum_{i=0}^{\infty} (\beta^{\mathrm{I}})^i E_t U_{t+i}^{\mathrm{I}} \qquad \text{E3.53}$$

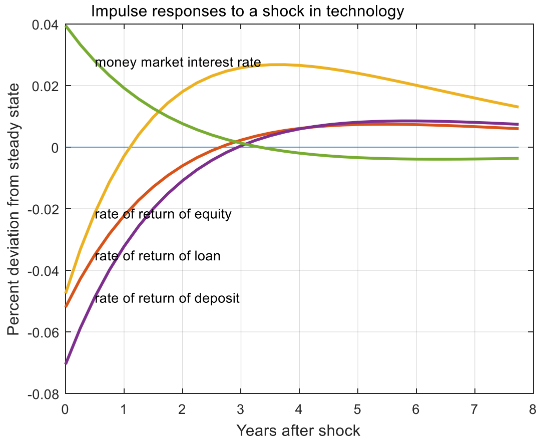
In this SCME with bank, the entrepreneur maximizes her utility E3.51 subject to the budget constraint E3.39, the public goods constraint E3.46, the equation of exchange constraint E3.24, the balance sheet constraint E3.34, the equal return constraint E3.35, the Markowitz rule constraint E3.27, the interest rate rule constraint E3.1, the worker maximizes her utility E3.52 subject to the budget constraint E3.40, the public goods constraint E3.46, the equation of exchange constraint E3.24, the Markowitz rule constraint E3.28, the interest rate rule constraint E3.1, the bank maximizes her utility E3.53 subject to the budget constraint E3.41, the public goods constraint E3.47, the balance sheet constraint E3.34, the equal return constraint E3.35, and the Markowitz rule constraints E3.28 and E3.29. We can obtain the unique local Pareto optimal equilibrium in the same way as we do in Section 2 and Appendix A.

3.4 Performance of the Economy

Let us discuss the parameter values first. The parameter values of this monetary economy with banks are almost the same as those of the RBC economy with bonds in Section 2. For the new parameters, R=1.01 quarterly, which is close to the average federal fund rate of the U.S. Correspondingly, from the interest rate form of the exchange equation, $\overline{\Pi}\Pi\approx1.005$, quarterly, when $\Gamma=1.005$, quarterly. In addition, follow Huang (2021), $\ln\overline{\mathbb{O}}=0.25$ and $\ln\underline{\mathbb{O}}=-0.25$ are adopted for the monetary policy rule, and $\eta^I=0.5$, $\chi^I=0.75$. For banking in the real-world economy, we adopt a quarterly net interest margin, that is, NIM, of 0.0075. With the above values, Δ^E, Δ^I, and Δ^N equal 26, 270, and 1356, respectively, which shows increasing sensitivity to volatility among the respective entities. For the parameters in the monetary shock, $\rho^M=0.9$ and $\sigma^M=0.7\%$.

With the above values, we have steady-state quarterly values of C/Y=0.6692, X/Y=0.1799, G/Y=0.1510, K/Y=14.4, $R^E=1.0555$, $R^L=1.0151$, and $R^D=1.0076$, which are all close to those in the everyday economy of the US. The three volatility adjustment factors in the model economy, that is, $\Delta^E\sigma^E$, $\Delta^I\sigma^L$, and $\Delta^N\sigma^D$, are all less than unity. The whole economy is provided in Appendix C, and the absolute values of the characteristic roots of the system are all less than 1. Figure 5 shows the impulse–response curves of the main economic variables under a one percentage positive technology shock and a one percentage contractionary monetary policy shock. Again, the meaning of the curves is clear, and we do not explain them. Note the negative movement of hours under positive technology shock in Panel (a) and the positive inflation under contractionary monetary policy shock in Panel (d).





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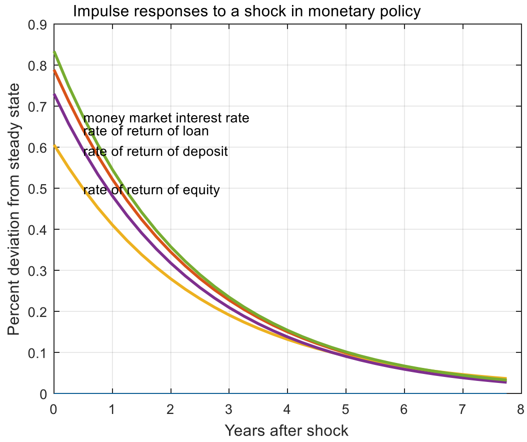


(f)

Figure 5

The standard deviation and cross-correlation of the main variables of this model economy are provided below in Table 2.

Table 2: Standard Deviation and Cross-correlation

	SD%	cross-correlation of output with:										
		−5	−4	−3	−2	−1	0	1	2	3	4	5
output	1.31	−0.02	0.10	0.27	0.47	0.71	1.00	0.71	0.47	0.27	0.10	−0.02
consumption	1.60	0.02	−0.06	−0.18	−0.33	−0.53	0.13	0.09	0.06	0.04	0.02	0.01
bond	1.04	0.04	0.17	0.32	0.51	0.73	0.99	0.68	0.43	0.22	0.05	−0.07
investment	8.41	−0.03	0.13	0.34	0.61	0.94	0.68	0.48	0.31	0.17	0.06	−0.02
capital	0.34	0.65	0.67	0.64	0.54	0.36	0.08	−0.13	−0.28	−0.38	−0.44	−0.47
hours	3.88	−0.03	0.07	0.21	0.38	0.59	0.81	0.56	0.35	0.18	0.05	−0.05
money market interest rate	0.75	0.13	0.01	−0.15	−0.36	−0.62	−0.95	−0.69	−0.47	−0.29	−0.14	−0.02
growth rate of economy	1.00	−0.17	−0.21	−0.26	−0.32	−0.38	0.38	0.32	0.26	0.21	0.17	0.12
inflation	1.02	0.17	0.10	−0.00	−0.14	−0.33	−0.88	−0.66	−0.47	−0.31	−0.18	−0.08

Figure 6 shows the dynamics of the outputs and inflations of the economy when there is a time lag in the transmission of a one percentage contractionary shock in monetary policy, with transmission periods lasting 1, 2, 3, and 4 quarters, from which we observe the humps.

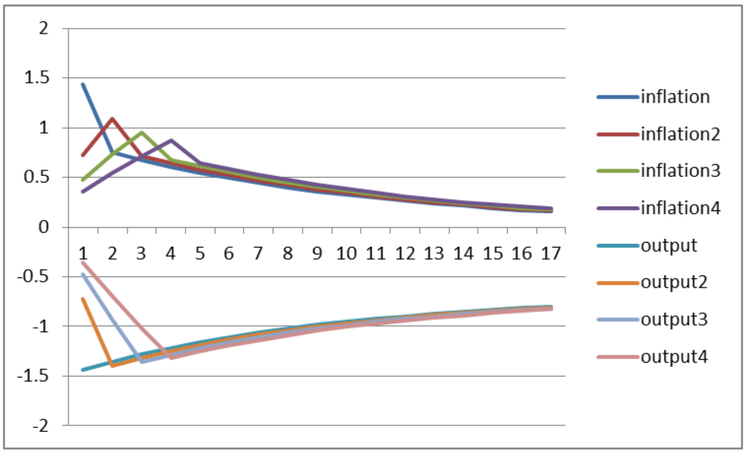


Figure 6

The above results are consistent with the empirical findings in the literature, which have been summarized in, for example, Ramey (2016).

A byproduct of this section is the balance sheet of the whole economy. The succinct balance sheets of the central bank, the commercial bank, the worker, the entrepreneur, and the firm are shown in Figure

7. The timing of the balance sheets is the end of period t. From all these balance sheets, we obtain the consolidated balance sheet of the whole economy, which is shown in Panel (f) of Figure 7. Note that the monetary and financial instruments are all offset in the consolidated balance sheet. In Figure 7, Q^N is the price of labor, and E and E^N are the net worth of the entrepreneur and worker, respectively.

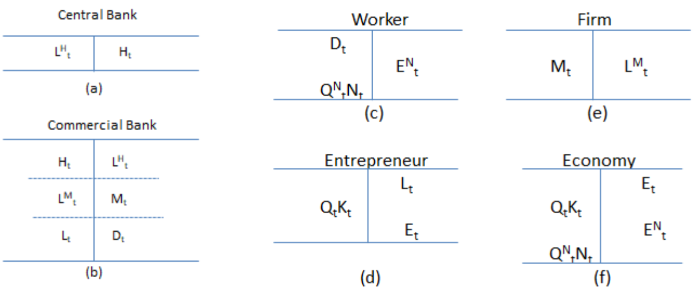


Figure 7

4. Monetary Policy in the Bubble Economy

As an application of the platform proposed in the last section, the role of monetary policy in the bubble economy is studied in this section. First, an enormous asset price bubble economy similar to the Great Depression of the 1920s-1930s was produced; then, the enormous bubble economy was stabilized by monetary policy.

4.1 Introducing an Asset Price Bubble into the Economy

An asset price bubble emerges when the subjective price of capital, Q^u, is different from its actual level, that is, Q_t, and we have

$$BU_t = \frac{Q_t^U}{Q_t} \qquad \text{E4.1}$$

The bubble is exogenous, and its evolution is given as

$$BU_t = Z_t^U BU \qquad \text{E4.2}$$

where BU is the steady-state value of the bubble and is set to unity. With respect to Z_t^U, that is, the asset price shock, we have, similar to the technology and monetary shocks,

$$\ln Z_t^U = (1-\rho^U)\ln Z^U + \rho^U \ln Z_{t-1}^U \qquad \text{E4.3}$$

where $0<\rho^U<1$. The steady-state value of Z^U is set to unity. The white noise process $\varepsilon^U_t \sim N(0, {\sigma^U}^2)$ is added to the log-linear form of E4.3, and we have, as before, $\widehat{Z_t^U} = \rho^U \widehat{Z_{t-1}^U} + \varepsilon_t^U$. At the beginning of each period, ε^U is realized. ε^M, ε^T, and ε^U are independent of each other.

In this paper, we study the case in which the subjective asset price is the common sense of the economy, that is, the entrepreneur, the worker, and the bank have the same opinion about Q^U. In the asset price bubble situation, the subjective asset price rather than the actual asset price matters in the respective decisions; correspondingly, the balance sheet constraint and the equal return constraint turn to, respectively,

$$Q_t^U K_t = \mathrm{L}_t + E_t \quad \text{E4.4'}$$

$$R_t^{KU} Q_t^U K_t = R_t^{\mathrm{LU}} \mathrm{L}_t + R_t^{EU} E_t \quad \text{E4.5'}$$

where R^{KU}, R^{LU}, and R^{EU} are the respective subjective rates of returns in bubble time. In real terms, these two equations turn out that, respectively,

$$BU_t K_t = \mathrm{l}_t + e_t \quad \text{E4.4}$$

$$R_t^{KU} BU_t K_t = R_t^{\mathrm{LU}} \mathrm{l}_t + R_t^{EU} e_t \quad \text{E4.5}$$

Note that $Q_t = P_t$ has been used in obtaining E4.4 and E4.5.
Similar to how we obtain R^K, we have

$$R_t^{KU} = \frac{\mathrm{E_t} Q_{t+1}^U (1-\delta)\mathrm{K_t} + \alpha \mathrm{E_t P_{t+1} E_t Y_{t+1}}}{Q_t^U \mathrm{K_t}} \quad \text{E4.6}$$

The Markovitz rule of the bubble case is as follows:

$$\frac{\frac{R_t^{EU}}{\Pi_t^e}-1}{E_t \sigma_{t+1}^E} = \frac{\frac{R_t^{LU}}{\Pi_t^e}-1}{E_t \sigma_{t+1}^L} = \frac{\frac{R_t^{DU}}{\Pi_t^e}-1}{E_t \sigma_{t+1}^D} = \frac{\frac{R_t^{KU}}{\Pi_t^e}-1}{\sigma^T} \quad \text{E4.7}$$

From E4.1, E4.6 and $Q_t = P_t$, we obtain

$$\frac{R_t^{KU}}{\Pi_t^e} - 1 = \frac{\mathrm{BU_{t+1}}}{\mathrm{BU_t}}(1-\delta) - 1 + \frac{1}{\mathrm{BU_t}} \frac{\alpha \mathrm{E_t Y_{t+1}}}{\mathrm{K_t}} \quad \text{E4.8}$$

Concerning the price of each share of equity, Q^E, we have, from the valuation method,

$$Q_t^E = \sum_{i=1}^{\infty} \frac{(R_t^E - 1) Q_t^E}{(RD^E)^i} \quad \text{E4.9}$$

where the numerator is the expected dividend of each share of the equity and RD^E in the denominator is the gross discount rate for equity.
Correspondingly, we have, in the bubble case,

$$Q_t^{EU} = \sum_{i=1}^{\infty} \frac{(R_t^{EU} - 1) Q_t^E}{(RD^E)^i} \quad \text{E4.10}$$

From E4.9 and E4.10, we have the equity price bubble, BU^E, as

$$BU_t^E = \frac{Q_t^{EU}}{Q_t^E} = \frac{R_t^{EU} - 1}{RD^E - 1} \quad \text{E4.11}$$

For the loan price, Q^L, since the duration of the loan is one period in this model economy, we have

$$Q_t^L = \frac{(R_t^L - 1) Q_t^L + Q_t^L}{RD^L} \quad \text{E4.12}$$

The numerator of the right side of E4.12 is the cash flow in period t+1, which consists of the principal and interest of the loan; RD^L is the gross discount rate of the loan.
Similarly, we have the loan price in the bubble economy as

$$Q_t^{LU} = \frac{(R_t^{LU} - 1) Q_t^L + Q_t^L}{RD^L} \quad \text{E4.13}$$

The loan price bubble BU^L is as follows:

$$BU_t^L = \frac{Q_t^{LU}}{Q_t^L} = \frac{R_t^{LU}}{RD^L} \quad \text{E4.14}$$

From E4.11 and E4.14, we can obtain the log-linear forms,

$$\widehat{BU_t^E} = \frac{R^{EU}}{RD^E - 1} \widehat{R_t^{\mathrm{EU}}} \quad \text{E4.15}$$

$$\widehat{BU_t^L} = \frac{R^{LU}}{RD^L} \widehat{R_t^{\mathrm{LU}}} \quad \text{E4.16}$$

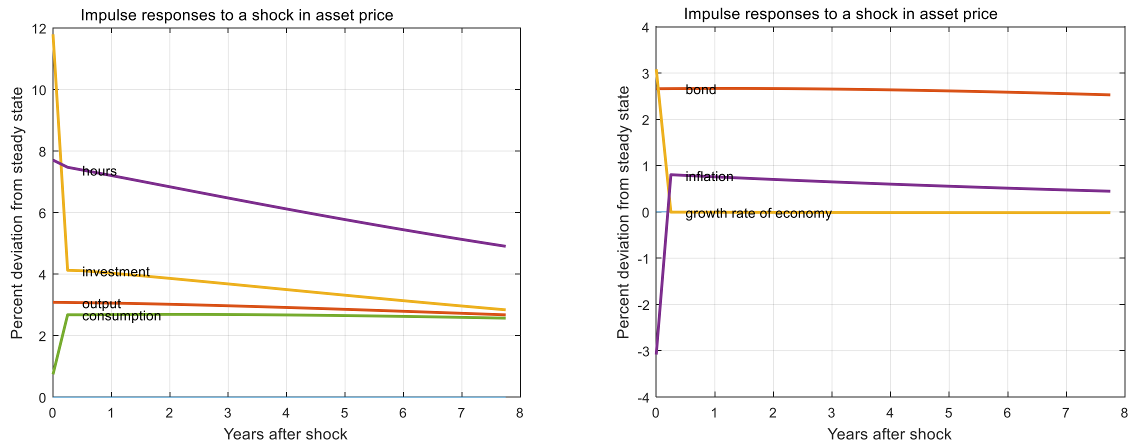
Compared with the bubble level of short-duration instruments, such as the BU^L of the loan, the long duration of equity makes its bubble level, that is, the BU^E, much more sensitive to the change in its rate of return, which is just the cause of the volatility puzzle of the stock price. This high volatility of stock prices in a very large bubble economy is shown in Panel (a) of Figure 9. Note that from the steady states of E4.9 and E4.13, we obtain $RD^E=R^E$ and $RD^L=R^L$, respectively.

The other parts of the economy are the same as those in Section 3, and we can obtain the respective first orders of the economy under asset price shock.

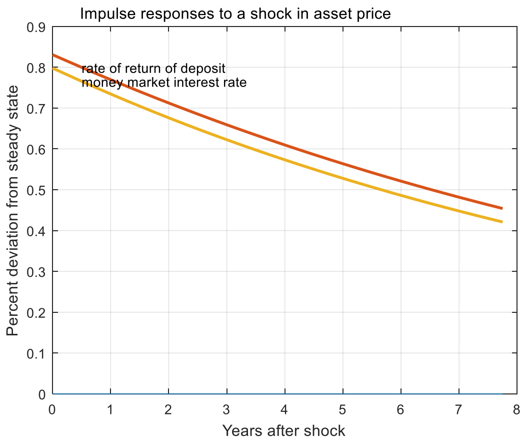
The values of the respective parameters of this model economy are the same as those of the economy of the last section. For the parameters of the asset price shock, $\rho^U=0.98$, and $\sigma^U=2$ is adopted. Because the asset price shock is exogenous, the impulse response of this economy under technology shock and monetary shock is the same as that in Section 3, as shown in Figure 5. For the impulse–response curves of the economy under asset price shock, we compare the economy with E3.1 and the economy with the following E4.17:

$$R_t = Z_t^M R\, \Gamma^{\frac{\Gamma_t^e}{\Gamma}-1}\, \Pi^{\frac{\Pi_t^e}{\Pi}-1}\, BU^{\frac{BU_t}{BU}-1} \qquad \text{E4.17}$$

where BU is the parameter of the asset price bubble in the monetary policy equation, and $\ln BU=1$ is adopted in the simulation. Experiments show that the two economies are both proactive under a one percentage positive asset price shock. The impulse response of the economy with E4.17 is more moderate than that of the economy with E3.1 is because of the increase in the money market interest rate under E4.17 when there is a positive bubble that is contractionary. We choose the economy with E4.17 as the basis of the study in the next two subsections. The impulse–response curves of this chosen economy under a one percentage positive asset price shock are shown in Figure 8. The meaning of Figure 8 is evident, and we do not take time to explain it. Note that the impulse–response curves under technology shock and monetary shock in economies with E3.1 and E4.17 are the same because E3.1 and E4.17 are the same when there is no asset price bubble.



(a) (b)



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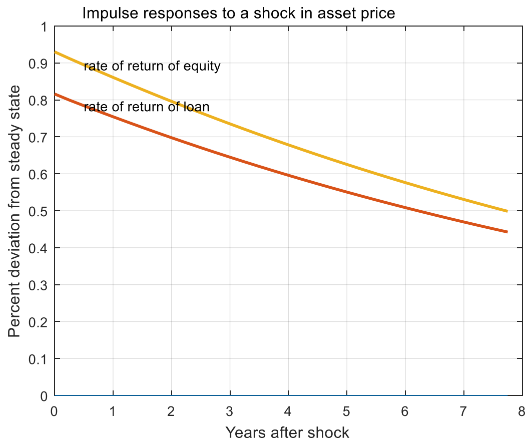


(d)

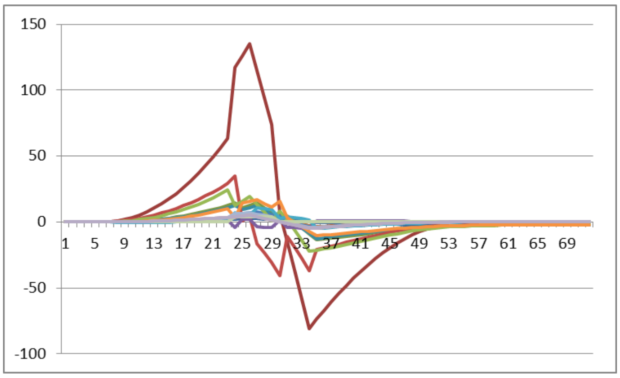
Figure 8

4.2 A large bubble economy similar to that of the Great Depression

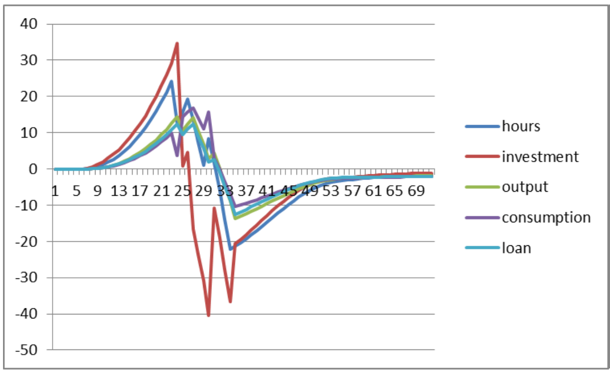
Now, it is time to create an enormous bubble economy similar to the Great Depression. According to Friedman and Shwartz (1963), Okina, Shirakawa, and Shiratsuka (2000), Bordo and Jeanne (2002), and many other studies, there are several common characteristics of enormous bubble economies, such as the Great Depression and the bubble economy of Japan in the 1980s-1990s, which we conclude below as follows:

1. There are roughly three stages in the whole process of the boom-bust of an enormous bubble economy, with the first stage, that is, the booming stage, lasting no less than 5 years; the third stage, that is, the depressing stage, lasting at least 5 years; and the second stage, that is, the collapsing stage, which lies between the first stage and the third stage, with a much shorter time period than the other two stages.
2. In the booming stage, the stock price climbs to a level much higher than its steady-state value, credit, the employment rate, consumption, and investment increase strongly, while the economic growth rate and the price level are relatively stable, sometimes and/or somewhere there is even deflation.
3. In the collapsing stage, the stock price falls sharply to a level much lower than its steady state, and credit, the employment rate, consumption, and investment decrease sharply.
4. In the depressing stage, unemployment pervades, and the recovery of the economy is very slow.

After many experiments, a very large asset price bubble economy based on the economy of the previous subsection is produced, as shown in Figure 9. The enormous bubble economy lasts more than twelve years, with the peak stock price increasing to more than 100% higher than its steady-state value, decreasing sharply in the collapsing stage, and remaining much lower than its steady-state value in the depressing stage. The output, consumption, investment, hours, and loan increase sharply in the booming stage, and the values of these variables are much lower than their respective steady-state values in the depressing stage. The movements of the economic growth rate and inflation rate are much milder than the changes in the above variables in the booming stage, and we even obtain little deflation in the booming stage in Panel (c). The respective interest rates are all procyclical. Panel (a) of Figure 9 shows all the main variables together, which provides a whole profile of the enormous bubble economy. The most prominent curve in Panel (a) of Figure 9 is the movement of the stock price, whose peak is about 130 percentages higher than its steady state value; the other panels of Figure 9 zoom in Panel (a) and show the movement of other variables more clearly. The movements of the variables in Figure 9 are closely consistent with the observations of Friedman and Shwartz (1963), among others.



(a)



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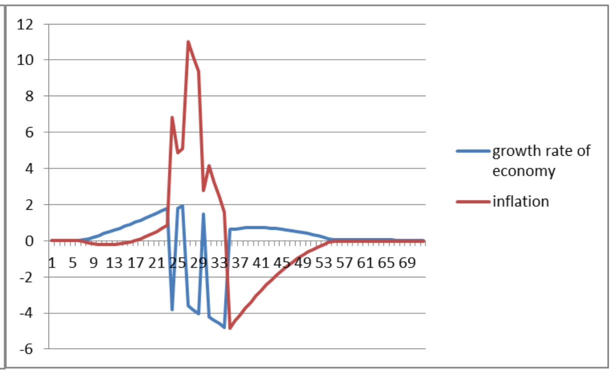

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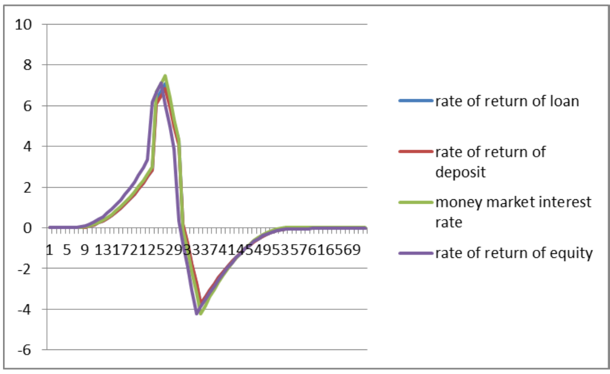

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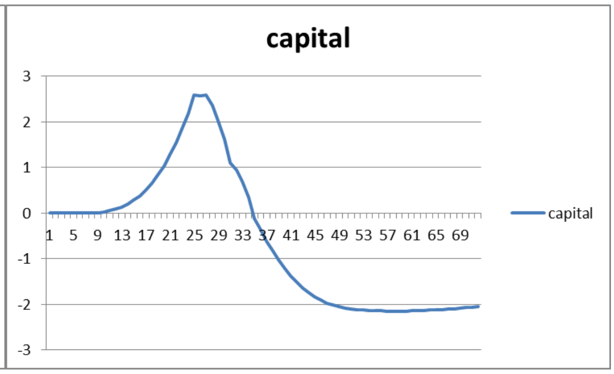

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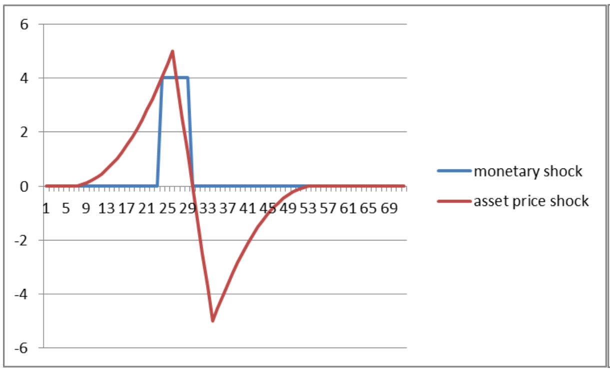

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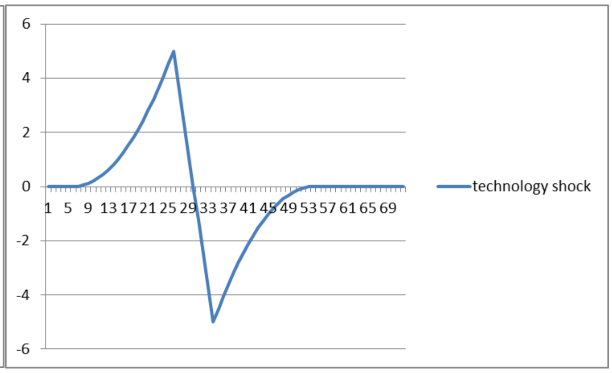


(g)

Figure 9

Before we illustrate the way we create this enormous bubble economy, let us discuss the distinct characteristics of the impacts of the asset price shock, the technology shock, and the monetary shock to the economy.

As shown in Figure 8, a positive asset price shock evidently increases output, hours, investment, consumption, and loan, and it increases the rate of return of equity, which increases the stock price according to E4.15.

The impact of the monetary shock is significantly different from that of the asset price shock. As shown in Figure 5, the contractionary monetary shock lowers the output, hours, investment, consumption, and bonds, but it raises the rate of return of equity as the positive asset price shock does.

A positive technology shock impacts the economy in another way. It increases investment, output, and consumption and lowers hours.

Another important point is that the strengths of the impacts of these three shocks differ substantially. By and large, the impact of a one percentage technology shock is much more tender than those of a one percentage asset price shock and/or a one percentage monetary shock.

Based on these observations, we exploit the exogeneity of the shocks and produce the above enormous bubble economy in the following way:

A 12-year, that is, 48 quarters, three-stage symmetric bubble process is designed, with the booming stage lasting for 5 years, the collapsing stage lasting for two years, and the depressing stage lasting for 5 years. The asset price bubble and the technology shock are set to be the same. The monetary policy action is triggered in the late periods of the booming stage when the stock price increases to more than 60% higher than its steady-state value, which reflects the fact that in real-world economies, the central bank is hesitant and tardiness in confronting asset bubbles. The particular processes of these three shocks are shown in E4.18, and these processes are shown in Panels (f) and (g) of Figure 9.

$$Z^U_t/Z^T_t = \begin{cases} (\frac{\sqrt{5}}{20}t - \frac{5\sqrt{5}}{20})^2 & 5 \le t \le 25 \\ -10t/8+290/8 & 26 \le t \le 33 \\ -(\frac{\sqrt{5}}{20}t - \frac{53\sqrt{5}}{20})^2 & 34 \le t \le 53 \\ 0 & \text{else} \end{cases}$$

$$Z^M_t = \begin{cases} 4 & 23 \le t \le 28 \\ 0 & \text{else} \end{cases} \qquad \text{E4.18}$$

E4.18 means that the asset price bubble, BU_t, and the technology shock increase gradually to 5 percentage points above its steady stage value in the first 20 quarters, collapse in the second stage, return to their respective steady states gradually in the third stage, and the monetary policy shock emerges in the late period of the booming stage and lasts for 6 periods. The sharp increase in the stock price in Panel (a) of Figure 9 comes just from the assistance of the contractionary monetary shock because it raises R^E, an effect we mentioned above.

From E4.18, the readers know that we can obtain various bubbles when we change the routes of the respective exogenous shocks. For example, we can actually create a similar enormous bubble economy even in the absence of a technology shock because the strength of the impact of the technology shock is not as strong as that of the asset price shock. However, the main role of the technology shock in the emergence of enormous bubble economies is to initiate market illusion and, correspondingly, trigger the asset price bubble. Therefore, the three shocks are all indispensable in the development of a very large bubble economy, and asset price shocks are the main cause of the bubble process.

4.3 Monetary Policy in Stabilizing the Enormous Bubble Economy

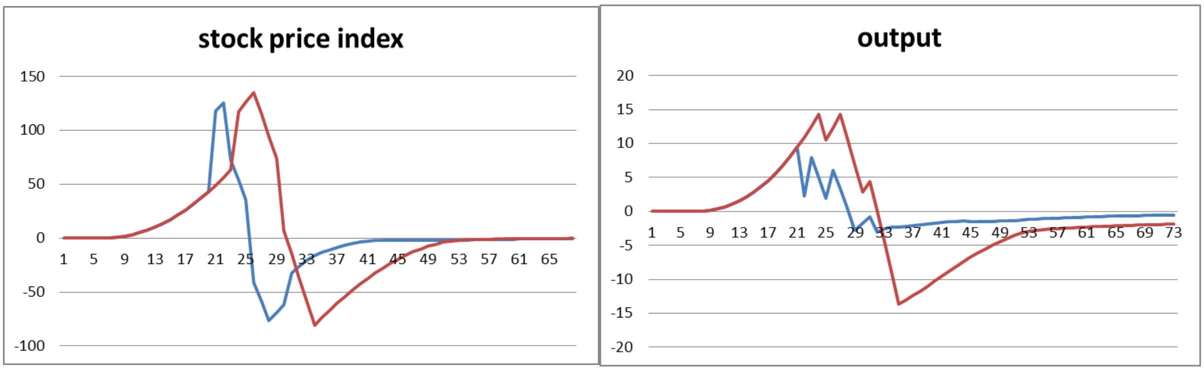
Now that monetary policy shocks can impact the economy substantially, it is natural to check whether monetary policy can help stabilize the enormous bubble economy created in the last subsection. We conduct the bubble fighting process through the following 3 steps:

Step 1: Assume that the central bank detects the bubble when the stock price is 40% higher than its steady-state value, which is the 14th quarter since the beginning of the bubble in this case, and initiates the anti-bubble policy, that is, initiates the contractionary monetary policy. In our example, we set Z^M_t=6 for 4 periods. In this step, assume that the process of the technology shock and the asset price bubble are not affected by the action of the central bank.

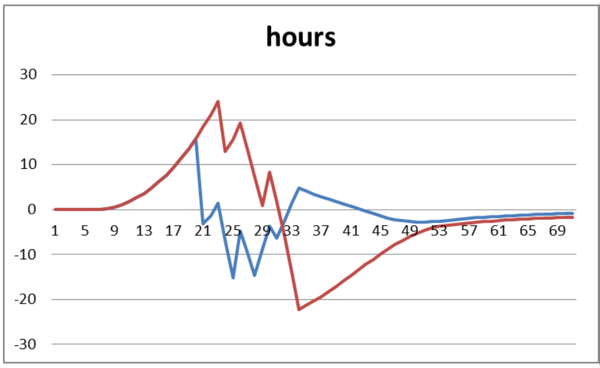
Step 2: A powerful anti-bubble monetary policy strongly impacts the economy, and both the investment rate and the employment rate quickly decrease to negative values, that is, to levels lower than their respective steady-state values. The asset price illusion is reversed by sharply changing economic conditions and, correspondingly, jumps down to a negative bubble. In our example, this change occurs in the 17th quarter, and the collapse of the asset price lasts 6 periods. Note that in our example, this new asset price bubble process is still symmetric, but it is shortened and weakened, which is caused by the intervention of monetary policy. The two bubble processes are shown in Panel (n) of Figure 10. We assume that the entire technology shock process is not affected by the change in circumstances; see Panel (p) of Figure 10. Again, the readers can modify these arrangements.

Step 3: The monetary authority cuts the money market interest rate for some periods to confront the collapse and depression of the economy triggered by the negative asset price bubble. The process of monetary policy shock is shown in Panel (o) of Figure 10.

With the above three steps, the economy of the large bubble clearly stabilizes. In Figure 10, we compare the performances of key economic variables of these two economies, with the red lines representing variables in the enormous bubble economy and the blue lines representing those of the stabilized economy. The meaning of the curves in Figure 10 is also evident, and we do not take time to explain it.



(a) (b)


hours
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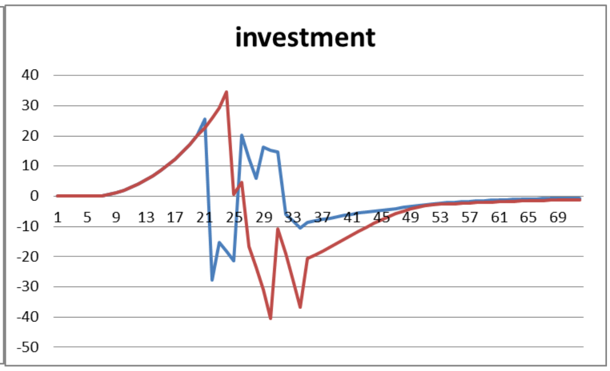
1 5 9 13 17 21 25 29 33 37 41 45 49 53 57 61 65 69

(c)


investment
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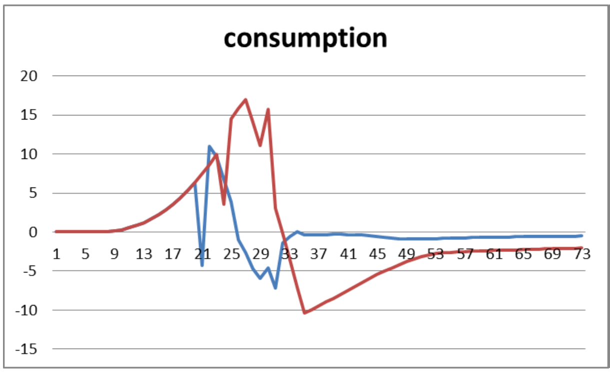
1 5 9 13 17 21 25 29 33 37 41 45 49 53 57 61 65 69

(d)


consumption
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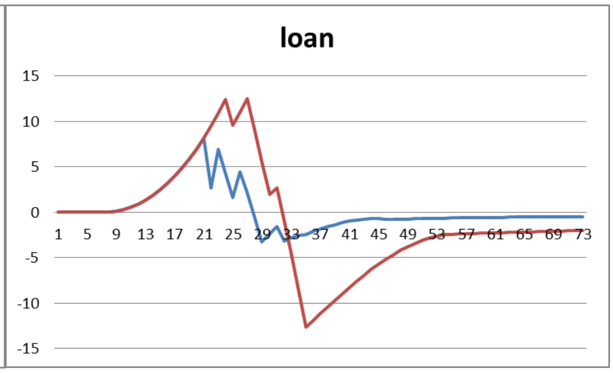
1 5 9 13 17 21 25 29 33 37 41 45 49 53 57 61 65 69 73

(e)


loan
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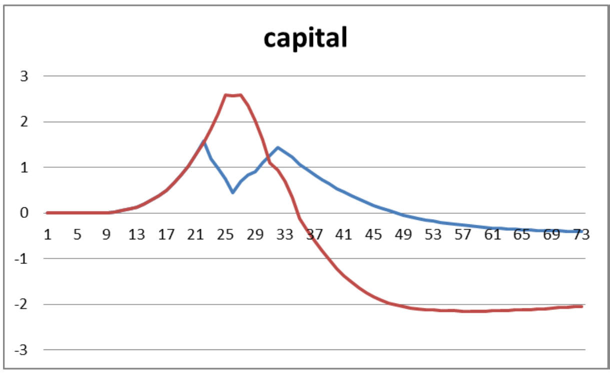
1 5 9 13 17 21 25 29 33 37 41 45 49 53 57 61 65 69 73

(f)


capital
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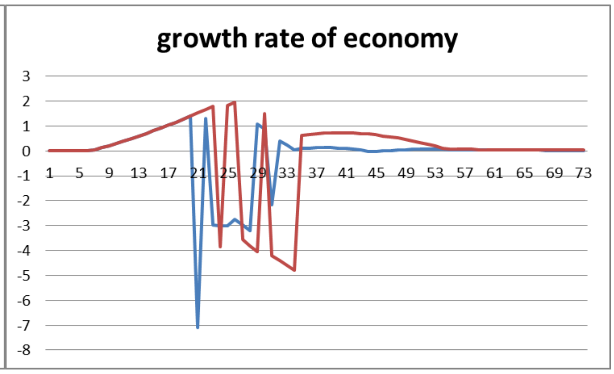
1 5 9 13 17 21 25 29 33 37 41 45 49 53 57 61 65 69 73

(g)


growth rate of economy
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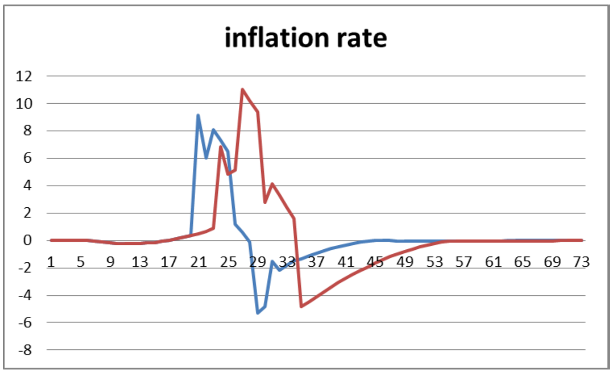
1 5 9 13 17 21 25 29 33 37 41 45 49 53 57 61 65 69 73

(h)


inflation rate
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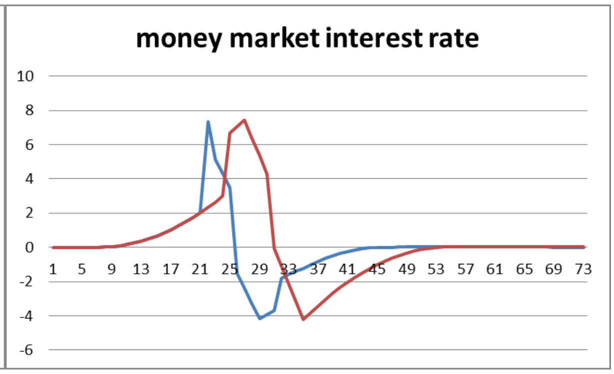
1 5 9 13 17 21 25 29 33 37 41 45 49 53 57 61 65 69 73

(i)


money market interest rate
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1 5 9 13 17 21 25 29 33 37 41 45 49 53 57 61 65 69 73

(j)

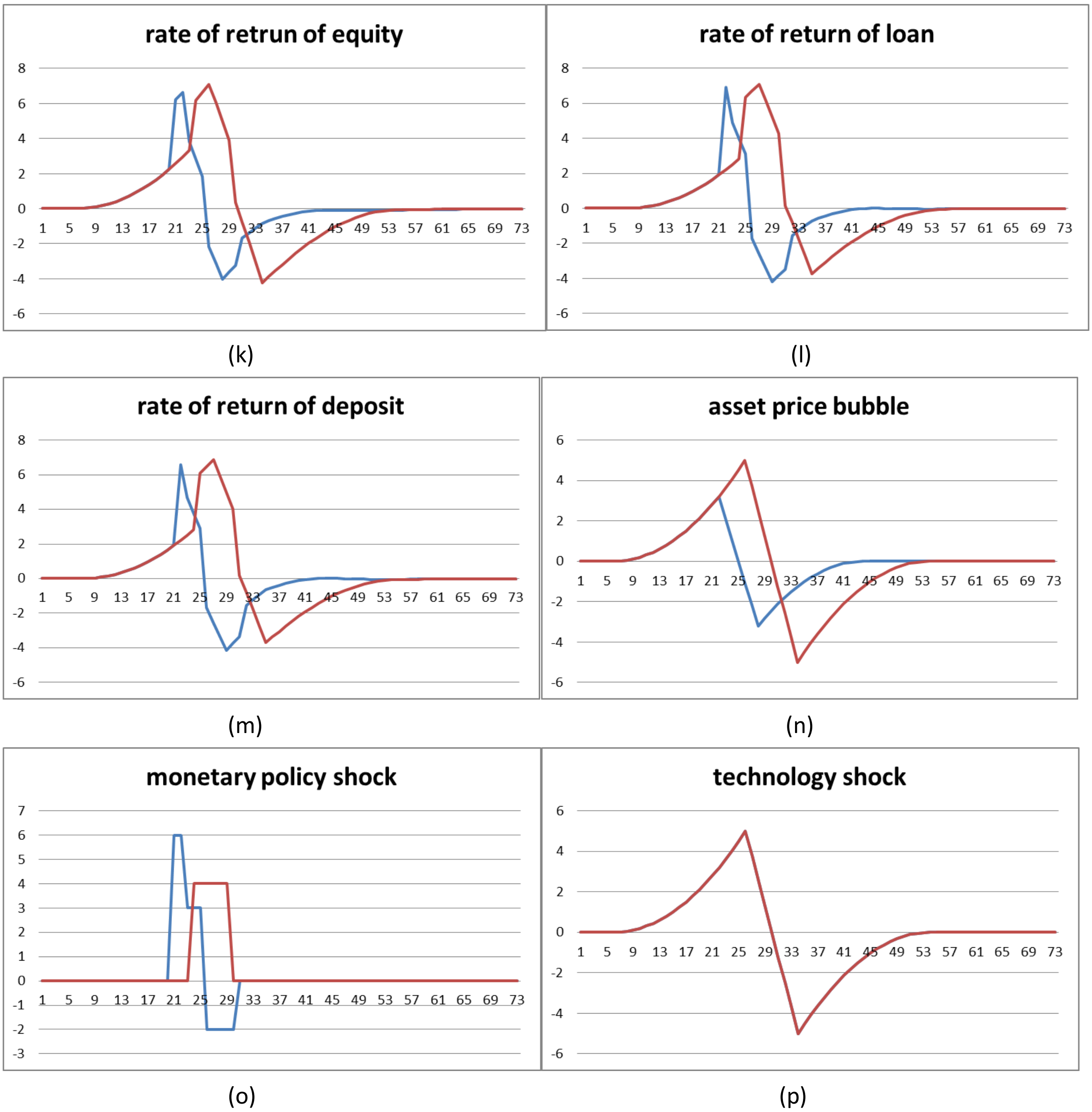

Figure 10

Although we have clearly shown the role of the anti-bubble policy, we do not generally recommend it to central banks because the existence and strength of an asset price bubble are both not easy to detect correctly. In addition, for those who have taken the bubble piercing strategy, the monetary policy should return to its normal level swiftly when the economy is recovering.

5. Conclusion

With the help of volatility income adjustment, VIA, and the E-V rule of Markowitz (1952), we accomplish three tasks in this paper:

1. Embedding credit, that is, bonds, equity, and banks, into the quantitative DGSE economy in symmetric information and flexible price conditions;

2. Realizing the modern central bank monetary regime in the seigniorage channeled monetary economy of Huang (2021);
3. An enormous financial crisis similar to the Great Depression in terms of symmetric information, flexible prices, and exogenous shock conditions was produced, and it was stabilized with monetary policy.

The findings of this paper are as follows:

a. Credit is a Pareto improvement to an RBC and/or flexible price monetary economy, where the welfare of the market participants improved. The resource allocation in the unique equilibrium of the multiagent economies of this paper is Pareto optimal.
b. The monetary effectiveness result of Huang (2021) is reobtained in SCME with the central bank regime, and other results of that paper, such as the price puzzle and the negative movement of hours under positive technology shock, reappear.
c. The difference in attitudes toward volatile income among different entities is the origin of both direct finance and indirect finance.
d. Normal monetary policy is pro-bubble, and in enormous bubble economies, early action anti-bubble monetary policy is helpful for the well-being of market participants.
e. The value of α in the worker-entrepreneur separation economy is much greater than commonly accepted, and the value of δ is much lower than commonly accepted.
f. The equity premium is naturally accommodated in the symmetric information models of this paper, and the long duration is the key to the origin of the asset price volatility puzzle.

Many works must be done to expand the SCME platform, which include but are not limited to,

i. The money demand in the tremendously increasing financial industry and the changeability of the velocity of money deserve special attention to ensure the accuracy of the equation of exchange, that is, E3.20 and E3.24, a crucial part of SCME.
ii. Because there is a net interest margin in the bank, it is possible to include nonperforming loans in the models of this paper.
iii. Foreign exchange and international trade are introduced into the model to make it an open economy.

Appendix[2]

A. A Pure-credit Economy and the Unique Equilibrium of a Multiagent Bond Economy

This is a much simpler version of the bond economy of Section 2, with which we show the existence of the unique equilibrium of this kind of multiagent economy more clearly. There are two agents in this economy, which are called K and N, and there are no firms and no production in this economy. Assume N obtains a fixed amount N of product each period, and K obtains a stochastic amount Z_tK of product each period where K is a fixed number and the log form of the shock, Z_t, is a stationary first-order autoregressive process:

$$\ln Z_t = (1-\rho)\ln Z + \rho \ln Z_{t-1} \qquad \text{A.A.1}$$

where 0<ρ<1. The steady state of Z is set to unity. The white noise process ε_t~N(0,σ^2) is added to the log-linear form of A.A.1, so we have $\widehat{Z_t} = \rho\widehat{Z_{t-1}} + \varepsilon_t$. At the beginning of each period, ε is realized.

Both K and N are risk aversions. Similar to the bond economy of subsection 2.1, it is possible for N to lend the amount B_t of the product to K in period t and to receive the amount R_tB_t of the product from K in period t+1, where R_t is the gross rate of return of B_t. In addition, similar to the Markowitz rule of subsection 2.1, assume that there is an income–volatility relation in this pure-credit economy, which is

$$\frac{E_tK_{t+1}}{\sigma} = \frac{R_tB_t}{E_t\sigma^B_{t+1}} = \frac{E_tK_{t+1}-R_tB_t}{E_t\sigma^{\mathrm{K}}_{t+1}} \qquad \text{A.A.2}$$

where σ^K and σ^B are volatility undertaken by K and N, respectively. A.A.2 means the respective ratios between a volatile income and its volatility are equal.

Similar to that of the economy of Section 2, the entities maximize their respective volatility adjusted consumptions, that is, we have,

$$UU^{\mathrm{K}}_t = maxE_t\sum_{i=0}^{\infty}(\beta^{\mathrm{K}})^i\, U^{\mathrm{K}}(\widetilde{C^{\mathrm{K}}_{t+\iota}}) \qquad \text{A.A.3}$$

$$UU^N_t = maxE_t\sum_{i=0}^{\infty}(\beta^{\mathrm{N}})^i\, U^N(\widetilde{C^{\mathrm{N}}_{t+\iota}}) \qquad \text{A.A.4}$$

$$\mathrm{U}^{\mathrm{K}}_t = \frac{(\widetilde{C^{\mathrm{K}}_t})^{1-\eta\mathrm{K}}}{1-\eta\mathrm{K}} \qquad \text{A.A.5}$$

$$U^N_t = \frac{(\widetilde{C^{\mathrm{N}}_t})^{1-\eta\mathrm{N}}}{1-\eta N} \qquad \text{A.A.6}$$

The respective period t budgets of K and N are as follows:

$$\widetilde{C^{\mathrm{K}}_t} = (1-\Delta^{\mathrm{K}}E_{t-1}\sigma^{\mathrm{K}}_t)(\mathrm{K}_t - R_{t-1}B_{t-1}) + B_t \qquad \text{A.A.7'}$$

$$\widetilde{C^{\mathrm{N}}_t} = N + \left(1-\Delta^{\mathrm{N}}E_{t-1}\sigma^{\mathrm{B}}_t\right)R_{t-1}B_{t-1} - B_t \qquad \text{A.A.8'}$$

where Δ^K and Δ^N are the volatility aversion coefficients of K and N, respectively. From A.A.2, we can obtain $E_t\sigma^{\mathrm{K}}_{t+1} = \sigma - E_t\sigma^{\mathrm{B}}_{t+1}$ and $E_t\sigma^B_{t+1} = \frac{R_tB_t}{E_tK_{t+1}}\sigma$; correspondingly, A.A.7' and A.A.8' turn to, respectively,

[2] Because the existence and uniqueness of the equilibrium of the multiagent economies has been obtained, according to Stokey, Lucas, with Prescott (1989) and Chow (1997), we use the Lagrange method to obtain the whole economic systems in this appendix and we use the toolkit of Uhlig (1999) to conduct the simulation of these economic systems.

$$\widetilde{C_t^{\mathrm{K}}} = (1 - \Delta^{\mathrm{K}}(1 - \frac{R_{t-1}B_{t-1}}{E_{t-1}K_t})\sigma)(K_t - R_{t-1}B_{t-1}) + B_t \qquad \text{A.A.7}$$

$$\widetilde{C_t^{\mathrm{N}}} = N + \left(1 - \Delta^{\mathrm{N}}\frac{R_{t-1}B_{t-1}}{E_{t-1}\mathrm{K}_t}\sigma\right)R_{t-1}B_{t-1} - B_t \qquad \text{A.A.8}$$

As in Stokey, Lucas, with Prescott (1989), we can obtain the value equations of K and N as follows:

$$v^{\mathrm{K}}(R_{t-1}, B_{t-1}, Z_t) =$$

$$\max_{\substack{B>0\\R>1}}\left\{\frac{((1-\Delta^{\mathrm{K}}(1-\frac{R_{t-1}B_{t-1}}{E_{t-1}Z_tK})\sigma)(Z_tK-R_{t-1}B_{t-1})+B_t)^{1-\eta\mathrm{K}}}{1-\eta\mathrm{K}} + \beta^{\mathrm{K}}\int v^{\mathrm{K}}(R_t, B_t, Z_{t+1})\lambda(dZ_{t+1})\right\} \qquad \text{A.A.9}$$

$$v^N(R_{t-1}, B_{t-1}, Z_t) = \max_{\substack{B>0\\R>1}}\left\{\frac{(N + \left(1 - \Delta^{\mathrm{N}}\frac{R_{t-1}B_{t-1}}{E_{t-1}\mathrm{Z}_tK}\sigma\right)R_{t-1}B_{t-1} - B_t)^{1-\eta\mathrm{N}}}{1 - \eta N} + \beta^N\int v^N(R_t, B_t, Z_{t+1})\lambda(dZ_{t+1})\right\}$$

A.A.10

Note that K and N make decisions in the same (R, B, Z) space in this pure-credit economy, which is evident from the above two value equations. The only difference between this dual economy and the single-agent economy of Stokey, Lucas, with Prescott (1989) is that the agents in this credit economy cannot locate the only fixed point independently because both K and N have only one equation for the two endogenous state variables, that is, R and B. What they can do is to provide their respective first order, which is also their respective Pareto optimal condition, and let the system decide the equilibrium. Because the credit condition is opposite to K and N and because the respective first-order conditions of both K and N are monotonous, the only fixed point, that is, the only equilibrium of this credit economy, can be obtained.

Specifically, let $\eta^{K}=\eta^{N}=0.5$, $\beta^{K}=\beta^{N}=0.98$, K=5, N=10, ρ=0.9, σ=0.01, $\Delta^{K}=10$, and $\Delta^{N}=20$. We can depict the steady-state Pareto optimal curves of K and N in Figure 11 below, where the blue line is K's curve, the red line is N's curve, and the equilibrium values of R and B are 1.1774 and 1.4156, respectively. Note that, unlike the models in the main text, this simple pure-credit economy is illustrative, and we do not match the data of this case with those of the real-world economy.

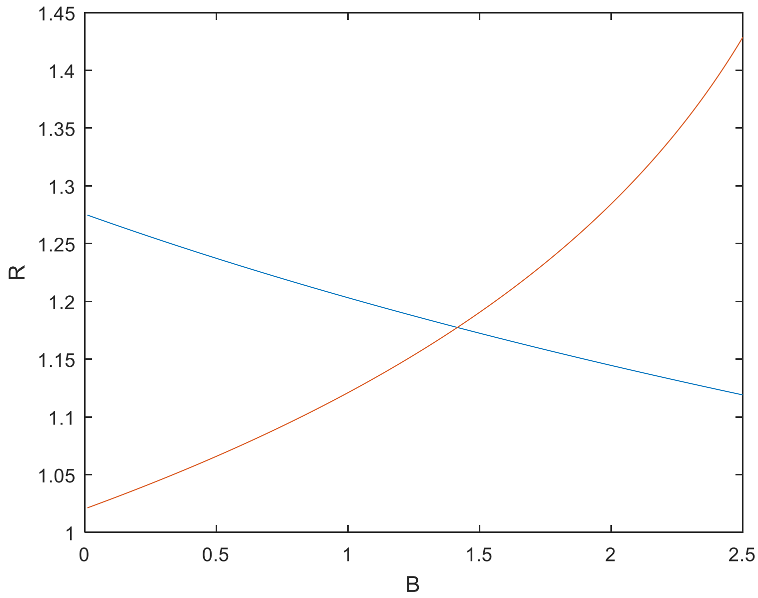

Figure 11

B. Detrend Form of the Economy of Section 2.2

$$UU_t^{\mathrm{E}} = maxE_t \sum_{i=0}^{\infty}\left(\beta^{\mathrm{E}}\right)^i U^{\mathrm{E}}(\widetilde{C_{t+\iota}^{\mathrm{E}}}, G_{t+i}) \qquad \text{A.B.1}$$

$$UU_t^{N} = maxE_t \sum_{i=0}^{\infty}\left(\beta^{\mathrm{N}}\right)^i U^{N}(\widetilde{C_{t+\iota}^{\mathrm{N}}}, G_{t+i}, J_{t+i}) \qquad \text{A.B.2}$$

$$\mathrm{U}_t^{\mathrm{E}} = \frac{(\widetilde{C_t^{\mathrm{E}}}^{\chi E} G_t^{1-\chi E})^{1-\eta \mathrm{E}}}{1-\eta \mathrm{E}} \qquad \text{A.B.3}$$

$$U_t^{N} = \frac{(\widetilde{C_t^{\mathrm{N}}}^{\chi N} G_t^{1-\chi N})^{1-\eta \mathrm{N}}}{1-\eta N} + \xi(1-N_t) \qquad \text{A.B.4}$$

$$Y_t = Z_t^T K_{t-1}^{\alpha} N_t^{1-\alpha} \qquad \text{A.B.5}$$

$$\Gamma K_t = (1-\delta)K_{t-1} + X_t \qquad \text{A.B.6}$$

$$\ln Z_t^T = (1-\rho^T)\ln Z^T + \rho^T \ln Z_{t-1}^T \qquad \text{A.B.7}$$

$$W_t^K K_{t-1} = Y_t^K \qquad \text{A.B.8}$$

$$W_t^N N_t = Y_t^N \qquad \text{A.B.9}$$

$$U_t^{\mathrm{F}} = \max(Y_t - Y_t^K - Y_t^N) \qquad \text{A.B.10}$$

$$W_t^K K_{t-1} = \alpha Y_t \qquad \text{A.B.11}$$

$$W_t^N N_t = (1-\alpha)Y_t \qquad \text{A.B.12}$$

$$Y_t^{EE} = Y_t^K - (R_{t-1}^B - (1-\delta))B_{t-1} \qquad \text{A.B.13}$$

$$Y_t^{NN} = (R_{t-1}^B - (1-\delta))B_{t-1} \qquad \text{A.B.14}$$

$$\widetilde{C_t^{\mathrm{E}}} = (1-\tau)(1-\Delta^{\mathrm{E}}E_{t-1}\sigma_t^{\mathrm{E}})(E_{t-1}Y_t^K - (R_{t-1}^B - (1-\delta))B_{t-1}) - X_t + (\Gamma B_t - (1-\delta)B_{t-1}) \qquad \text{A.B.15}$$

$$\widetilde{C_t^{\mathrm{N}}} = (1-\tau)Y_t^N + (1-\tau)(1-\Delta^N E_t \sigma_{t+1}^{\mathrm{B}})(R_t^B - (1-\delta))B_t - (\Gamma B_t - (1-\delta)B_{t-1}) \qquad \text{A.B.16}$$

$$T_t = \tau Y_t \qquad \text{A.B.17}$$

$$G_t = T_t \qquad \text{A.B.18}$$

$$\frac{R_t^E-(1-\delta)}{E_t\sigma_{t+1}^E}=\frac{R_t^B-(1-\delta)}{E_t\sigma_{t+1}^B}=\frac{R_t^K-(1-\delta)}{\sigma^T} \quad \text{A.B.19}$$

$$K_t = B_t + E_t \quad \text{A.B.20}$$

$$R_t^K K_t = R_t^B B_t + R_t^E E_t \quad \text{A.B.21}$$

$$R_t^K = \frac{(1-\delta)\mathrm{K_t}+\mathrm{E_t}W_{t+1}^K\mathrm{K_t}}{\mathrm{K_t}} \quad \text{A.B.22}$$

$$\Lambda_t^{\mathrm{E}} = \frac{\chi^E\widetilde{\mathrm{CEG_t}}}{\widetilde{\mathrm{C}_t^{\mathrm{E}}}} \quad \text{A.B.23}$$

$$\Lambda_t^{\mathrm{EG}} = \frac{(1-\chi^E)\widetilde{\mathrm{CEG_t}}}{G_t} \quad \text{A.B.24}$$

$$\widetilde{\mathrm{CEG_t}} = (\widetilde{\mathrm{C}_t^{\mathrm{E}}}^{\chi^E} G_t{}^{(1-\chi^E)})^{1-\eta E} \quad \text{A.B.25}$$

$$\Lambda_t^{\mathrm{E}}\Gamma = \beta^{\mathrm{E}}E_t\Lambda_{t+1}^{\mathrm{E}}\left((1-\Delta^{\mathrm{E}}E_t\sigma_{t+1}^{\mathrm{E}})(1-\tau)(\alpha\frac{\alpha E_tY_{t+1}}{K_t}-\frac{E_t\sigma_{t+1}^{\mathrm{E}}}{\sigma^T}\alpha\frac{(\alpha-1)E_tY_{t+1}}{K_t}\frac{B_t}{K_t})+(1-\delta)\right)+$$
$$\beta^{\mathrm{E}}E_t\Lambda_{t+1}^{\mathrm{EG}}\tau\frac{\alpha E_tY_{t+1}}{K_t}+\Lambda_t^{E\sigma}(\frac{E_t\sigma_{t+1}^{\mathrm{E}}}{\sigma^T}-1) \quad \text{A.B.26}$$

$$\Lambda_t^{\mathrm{E}}\Gamma-\beta^{\mathrm{E}}E_t\Lambda_{t+1}^{\mathrm{E}}\left((1-\Delta^{\mathrm{E}}E_t\sigma_{t+1}^{\mathrm{E}})(1-\tau)\left(R_t^B-(1-\delta)\right)+(1-\delta)\right)=\Lambda_t^{E\sigma}(\frac{E_t\sigma_{t+1}^{\mathrm{E}}}{\sigma^T}-\frac{E_t\sigma_{t+1}^{\mathrm{B}}}{\sigma^T})$$
$$\text{A.B.27}$$

$$\beta^{\mathrm{E}}E_t\Lambda_{t+1}^{\mathrm{E}}\Delta^{\mathrm{E}}(1-\tau)(\alpha E_tY_{t+1}-\left(R_t^B-(1-\delta)\right)B_t)=\Lambda_t^{E\sigma}\frac{1}{\sigma^T}E_t \quad \text{A.B.28}$$

$$\Lambda_t^{\mathrm{N}} = \frac{\chi^N\widetilde{\mathrm{CNG_t}}}{\widetilde{\mathrm{C}_t^{\mathrm{N}}}} \quad \text{A.B.29}$$

$$\Lambda_t^{\mathrm{NG}} = \frac{(1-\chi^N)\widetilde{\mathrm{CNG_t}}}{G_t} \quad \text{A.B.30}$$

$$\widetilde{\mathrm{CNG_t}} = (\widetilde{\mathrm{C}_t^{\mathrm{N}}}^{\chi^N} G_t{}^{(1-\chi^N)})^{1-\eta N} \quad \text{A.B.31}$$

$$\xi = \Lambda_t^N(1-\tau)(1-\alpha)(1-\alpha)\frac{Y_t}{N_t}+\Lambda_t^{NG}\tau(1-\alpha)\frac{Y_t}{N_t} \quad \text{A.B.32}$$

$$\Lambda_t^N\Gamma-\beta^NE_t\Lambda_{t+1}^N\left((1-\tau)(1-\alpha)\frac{\alpha E_tY_{t+1}}{K_t}+\left(1-\Delta^{\mathrm{N}}E_t\sigma_{t+1}^{\mathrm{B}}\right)(1-\tau)\left(\frac{E_t\sigma_{t+1}^{\mathrm{B}}}{\sigma^T}\alpha\frac{\alpha E_tY_{t+1}}{K_t}\frac{B_t}{K_t}+\right.\right.$$
$$\left.\left.\frac{E_t\sigma_{t+1}^{\mathrm{B}}}{\sigma^T}\frac{\alpha E_tY_{t+1}}{K_t}\right)+(1-\delta)\right)-\beta^{\mathrm{N}}E_t\Lambda_{t+1}^{\mathrm{NG}}\tau\frac{\alpha E_tY_{t+1}}{K_t}-\Lambda_t^{N\sigma}(\frac{E_t\sigma_{t+1}^{\mathrm{B}}}{\sigma^T}-1)=0 \quad \text{A.B.33}$$

$$\Lambda_t^{N\sigma}=\beta^NE_t\Lambda_{t+1}^N(2\Delta^{\mathrm{N}}E_t\sigma_{t+1}^{\mathrm{B}}-1)(1-\tau)\frac{\alpha E_tY_{t+1}}{K_t} \quad \text{A.B.34}$$

A.B.19, A.B.20, A.B.21, and A.B.22 can be combined and we have the consolidated financial constraint as

$$K_t = \frac{E_t\sigma_{t+1}^{\mathrm{B}}}{\sigma^T}B_t+\frac{E_t\sigma_{t+1}^{\mathrm{E}}}{\sigma^T}E_t \quad \text{A.B.35}$$

Λ^{E}, Λ^{EG}, and $\Lambda^{E\sigma}$ are Lagrange multipliers of the entrepreneur on her budget constraints, public goods constraints, and consolidated financial constraints, respectively. Λ^{N}, Λ^{NG}, and $\Lambda^{N\sigma}$ are the Lagrange multipliers of the worker on her budget constraints, public goods constraints, and consolidated financial constraints, respectively. A.B.23, A.B.24, A.B.26, A.B.27, and A.B.28 are first orders on $C^{E}{}_{t}$, G_t,

K_t, B_t, and $E_t\sigma^E_{t+1}$, respectively, of the entrepreneur. A.B.29, A.B.30, A.B.32, A.B.33, and A.B.34 are the first orders of the worker on C^N_t, G_t, N_t, B_t, and $E_t\sigma^B_{t+1}$, respectively.

C. Detrend Form of the Economy of Section 3

A.B.1 A.C.1

A.B.2 A.C.2

$$UU_t^{\mathrm{I}} = maxE_t \sum_{i=0}^{\infty} \left(\beta^{\mathrm{I}}\right)^i U^{\mathrm{I}}(\widetilde{C_{t+\iota}^{\mathrm{I}}}, G_{t+i}^{\mathrm{I}}) \qquad \text{A.C.3}$$

A.B.3 A.C.4

A.B.4 A.C.5

$$\mathrm{U}_t^{\mathrm{I}} = \frac{(\widetilde{C_t^{\mathrm{I}}}^{\chi I} {G_t^{\mathrm{I}}}^{1-\chi I})^{1-\eta \mathrm{I}}}{1-\eta \mathrm{I}} \qquad \text{A.C.6}$$

$$R_t = Z_t^M \overline{R}\, \Gamma^{\frac{\Gamma_t^{\mathrm{e}}}{\overline{\Gamma}}-1}\, \Pi^{\frac{\Pi_t^{\mathrm{e}}}{\overline{\Pi}}-1} \qquad \text{A.C.7}$$

$$\ln Z_t^{\mathrm{M}} = (1-\rho^{\mathrm{M}}) \ln Z^{\mathrm{M}} + \rho^{\mathrm{M}} \ln Z_{t-1}^{\mathrm{M}} \qquad \text{A.C.8}$$

$$S_t = \frac{H_t - H_{t-1}}{P_t} \qquad \text{A.C.9}$$

$$\mathrm{R_t} = \frac{\mathrm{E_t H_{t+1}}}{\mathrm{H_t}} \qquad \text{A.C.10}$$

$$Y_t^S = \frac{M_t - M_{t-1}}{M_t} Y_t \qquad \text{A.C.11}$$

$$R_t^M = \frac{\mathrm{E_t M_{t+1}}}{\mathrm{M_t}} \qquad \text{A.C.12}$$

$$\frac{H_t}{M_t} = \mathrm{hmr} \qquad \text{A.C.13}$$

$$R_t^M = \frac{\mathrm{E_t M_{t+1}}}{\mathrm{M_t}} = \frac{\frac{\mathrm{E_t H_{t+1}}}{hmr}}{\frac{\mathrm{H_t}}{hmr}} = \frac{\mathrm{E_t H_{t+1}}}{\mathrm{H_t}} = R_t \qquad \text{A.C.14}$$

A.B.5 A.C.15

A.B.6 A.C.16

A.B.7 A.C.17

$$W_t^K K_{t-1} = M_t^{K1} \qquad \text{A.C.18}$$

$$W_t^N N_t = M_t^{N1} \qquad \text{A.C.19}$$

$$M_t^{K1} + M_t^{N1} = M_t \qquad \text{A.C.20}$$

$$M_t^{K2} = M_t^{K1} - R_{t-1}^{\mathrm{L}} \mathrm{L}_{t-1} + \mathrm{L}_t \qquad \text{A.C.21}$$

$$M_t^{N2} = M_t^{N1} + R_{t-1}^{\mathrm{D}} \mathrm{D}_{t-1} - \mathrm{D}_t \qquad \text{A.C.22}$$

$$M_t^I = R_{t-1}^{\mathrm{L}} \mathrm{L}_{t-1} - R_{t-1}^{\mathrm{D}} \mathrm{D}_{t-1} \qquad \text{A.C.23}$$

$$M_t^{K2} = P_t Y_t^{\mathrm{EP}} \qquad \text{A.C.24}$$

$$M_t^{N2} = P_t Y_t^{\mathrm{NP}} \qquad \text{A.C.25}$$

$$M_t^I = P_t Y_t^I \qquad \text{A.C.26}$$

$$U_t^F = \max(Y_t - Y_t^{\mathrm{EP}} - Y_t^{NP} - Y_t^I) \qquad \text{A.C.27}$$

$$W_t^K K_{t-1} = \alpha P_t Y_t = M_t^{K1} \qquad \text{A.C.28}$$

$$W_t^N N_t = (1-\alpha) P_t Y_t = M_t^{N1} \qquad \text{A.C.29}$$

$$M_t = P_t Y_t \qquad \text{A.C.30}$$

$$R_t = \Pi_t^e \Gamma_t^e \quad \text{A.C.31}$$

$$\Pi_t^a = \frac{R_{t-1}}{\Gamma_t^a} \quad \text{A.C.32}$$

$$S_t = \frac{H_t - H_{t-1}}{P_t} = \frac{M_t - M_{t-1}}{M_t} hmrY_t = hmrY_t^S \quad \text{A.C.33}$$

$$\frac{\frac{R_t^E}{\Pi_t^e} - 1}{E_t \sigma_{t+1}^E} = \frac{\frac{R_t^L}{\Pi_t^e} - 1}{E_t \sigma_{t+1}^L} = \frac{\frac{R_t^D}{\Pi_t^e} - 1}{E_t \sigma_{t+1}^D} = \frac{\frac{R_t^K}{\Pi_t^e} - 1}{\sigma^T} \quad \text{A.C.34.}$$

$$R_t^K = \Pi_t^e((1-\delta) + \frac{\alpha \mathrm{E_t Y_{t+1}}}{\mathrm{K_t}}) \quad \text{A.C.35}$$

$$K_t = \mathrm{l}_t + e_t \quad \text{A.C.36}$$

$$R_t^K K_t = R_t^{\mathrm{L}} \mathrm{l}_t + R_t^E e_t \quad \text{A.C.37}$$

$$Y_t^{\mathrm{EP}} = \frac{M_t^{K2}}{P_t} = \frac{M_t^{K1} - R_{t-1}^{\mathrm{L}} \mathrm{L}_{t-1} + \mathrm{L}_t}{P_t} = \alpha Y_t - \left(\frac{R_{t-1}^{\mathrm{L}}}{\Pi_t^a} - 1\right) \mathrm{l}_{t-1} + (\mathit{\Gamma} \mathrm{l}_t - \mathrm{l}_{t-1}) \quad \text{A.C.38}$$

$$Y_t^{\mathrm{NP}} = \frac{M_t^{N2}}{P_t} = \frac{M_t^{N1} + R_{t-1}^{\mathrm{D}} \mathrm{D}_{t-1} - \mathrm{D}_t}{P_t} = (1-\alpha) Y_t + \left(\frac{R_{t-1}^{\mathrm{D}}}{\Pi_t^a} - 1\right) \mathrm{d}_{t-1} - (\mathit{\Gamma} \mathrm{d}_t - \mathrm{d}_{t-1}) \quad \text{A.C.39}$$

$$Y_t^{\mathrm{I}} = \frac{M_t^I}{P_t} = \frac{R_{t-1}^{\mathrm{L}} \mathrm{L}_{t-1} - R_{t-1}^{\mathrm{D}} \mathrm{D}_{t-1}}{P_t} = \left(\frac{R_{t-1}^L}{\Pi_t^a} - 1\right) l_{t-1} - \left(\frac{R_{t-1}^D}{\Pi_t^a} - 1\right) d_{t-1} \quad \text{A.C.40}$$

$$\widetilde{C_t^E} = (1-\tau)(1 - \Delta^{\mathrm{E}} E_{t-1} \sigma_t^{\mathrm{E}})\left(\alpha Y_t - \left(\frac{R_{t-1}^{\mathrm{L}}}{\Pi_t^a} - 1\right) \mathrm{l}_{t-1}\right) - X_t + (\mathit{\Gamma} \mathrm{l}_t - \mathrm{l}_{t-1}) - Y_t^S \quad \text{A.C.41}$$

$$\widetilde{C_t^N} = (1-\tau)(1-\alpha) Y_t + (1-\tau)(1 - \Delta^N E_{t-1} \sigma_t^{\mathrm{D}})\left(\frac{R_{t-1}^{\mathrm{D}}}{\Pi_t^a} - 1\right) \mathrm{d}_{t-1} - (\mathit{\Gamma} \mathrm{d}_t - \mathrm{d}_{t-1}) \quad \text{A.C.42}$$

$$\widetilde{C_t^I} = (1-\tau)(1 - \Delta^I E_{t-1} \sigma_t^L)\left(\frac{R_{t-1}^L}{\Pi_t^a} - 1\right) l_{t-1} - (1-\tau)\left(\frac{R_{t-1}^D}{\Pi_t^a} - 1\right) d_{t-1} + Y_t^S - S_t \quad \text{A.C.43}$$

$$\mathrm{T_t} = \tau \mathrm{Y_t} \quad \text{A.C.44}$$

$$\mathrm{G_t} = \mathrm{T_t} + Y_t^S \quad \text{A.C.45}$$

$$G_t^I = \mathrm{T_t} + \mathrm{S_t} \quad \text{A.C.46}$$

$$\Lambda_t^{\mathrm{E}} = \frac{\chi^E \widetilde{\mathrm{CEG_t}}}{\widetilde{\mathrm{C}_t^{\mathrm{E}}}} \quad \text{A.C.47}$$

$$\Lambda_t^{\mathrm{EG}} = \frac{(1-\chi^E) \widetilde{\mathrm{CEG_t}}}{G_t} \quad \text{A.C.48}$$

$$\widetilde{\mathrm{CEG_t}} = (\widetilde{\mathrm{C}_t^{\mathrm{E}}}^{\chi^E} G_t^{(1-\chi^E)})^{1-\eta E} \quad \text{A.C.49}$$

$$\Lambda_t^{\mathrm{E}} \Gamma = \beta^{\mathrm{E}} E_t \Lambda_{t+1}^{\mathrm{E}} \left((1 - \Delta^{\mathrm{E}} E_t \sigma_{t+1}^{\mathrm{E}})(1-\tau) \left(\alpha \frac{\alpha E_t Y_{t+1}}{K_t} - \frac{E_t \sigma_{t+1}^{\mathrm{E}}}{\sigma^T} \alpha \frac{(\alpha - 1) E_t Y_{t+1}}{K_t} \frac{\mathrm{l}_t}{K_t} \right) + (1-\delta) - \left(1 - \frac{1}{R_t}\right) \alpha \frac{E_t Y_{t+1}}{K_t} \right) + \beta^{\mathrm{E}} E_t \Lambda_{t+1}^{\mathrm{EG}} \left(\tau + 1 - \frac{1}{R_t}\right) \alpha \frac{E_t Y_{t+1}}{K_t} + \Lambda_t^{E\sigma} \left(\frac{E_t \sigma_{t+1}^{\mathrm{E}}}{\sigma^T} - 1\right) + \Lambda_t^{\mathrm{EP}} R_t \left(\ln \Phi_\Gamma \frac{\Gamma_t^{\mathrm{e}}}{\Gamma} - \ln \Phi_\Pi \frac{\Pi_t^{\mathrm{e}}}{\Pi}\right) \frac{\alpha}{K_t} - \beta^{\mathrm{E}} E_t \Lambda_{t+1}^{\mathrm{EP}} E_t R_{t+1} \left(\ln \Phi_\Gamma \frac{E_t \Gamma_{t+1}^{\mathrm{e}}}{\Gamma} - \ln \Phi_\Pi \frac{E_t \Pi_{t+1}^{\mathrm{e}}}{\Pi}\right) \frac{\alpha}{K_t} \quad \text{A.C.50}$$

$$\Lambda_t^{\mathrm{E}} \Gamma - \beta^{\mathrm{E}} E_t \Lambda_{t+1}^{\mathrm{E}} \left((1 - \Delta^{\mathrm{E}} E_t \sigma_{t+1}^{\mathrm{E}})(1-\tau) \left(\frac{R_t^{\mathrm{L}}}{\Pi_t^{\mathrm{e}}} - 1\right) + 1 \right) = \Lambda_t^{E\sigma} \left(\frac{E_t \sigma_{t+1}^{\mathrm{E}}}{\sigma^T} - \frac{E_t \sigma_{t+1}^{\mathrm{B}}}{\sigma^T}\right) \quad \text{A.C.51}$$

$$\beta^{\mathrm{E}} E_t \Lambda_{t+1}^{\mathrm{E}} \Delta^{\mathrm{E}} (1-\tau)\left(\alpha E_t Y_{t+1} - \left(\frac{R_t^B}{\Pi_t^{\mathrm{e}}} - 1\right) \mathrm{l}_t\right) = \Lambda_t^{E\sigma} \frac{1}{\sigma^T} \mathrm{e}_t \quad \text{A.C.52}$$

$$\Lambda_t^{\mathrm{EP}}(1-\ln\Phi_\Pi\frac{\Pi_t^{\mathrm{e}}}{\Pi}) = -\beta^{\mathrm{E}}E_t\Lambda_{t+1}^{\mathrm{E}}\frac{\alpha E_tY_{t+1}}{\mathrm{R}_t^2} + \beta^{\mathrm{E}}E_t\Lambda_{t+1}^{\mathrm{EG}}\frac{E_tY_{t+1}}{\mathrm{R}_t^2} \qquad \text{A.C.53}$$

$$\Lambda_t^{\mathrm{N}} = \frac{\chi^N\widetilde{\mathrm{CNG}}_{\mathrm{t}}}{\widetilde{\mathrm{C}}_t^{\mathrm{N}}} \qquad \text{A.C.54}$$

$$\Lambda_t^{\mathrm{NG}} = \frac{(1-\chi^N)\widetilde{\mathrm{CNG}}_{\mathrm{t}}}{G_t} \qquad \text{A.C.55}$$

$$\widetilde{\mathrm{CNG}}_{\mathrm{t}} = (\widetilde{\mathrm{C}}_t^{\mathrm{N}\,\chi^N} G_t{}^{(1-\chi^N)})^{1-\eta N} \qquad \text{A.C.56}$$

$$\xi = \Lambda_t^N\left((1-\tau)(1-\alpha)(1-\alpha)\frac{Y_t}{N_t} + (1-\tau)\frac{R_{t-1}^D}{\Pi_t^{\mathrm{a}}}\frac{1-\alpha}{N_t}\mathrm{l}_t\right) + \beta^{\mathrm{N}}E_t\Lambda_{t+1}^{\mathrm{N}}\left(1-\Delta^{\mathrm{N}}E_t\sigma_{t+1}^{\mathrm{D}}\right)(1-\tau)\frac{R_t^D}{\Pi_t^{\mathrm{e}}}\frac{1-\alpha}{N_t}\mathrm{l}_t + \Lambda_t^{NG}\left(\tau+1-\frac{1}{R_t}\right)(1-\alpha)\frac{Y_t}{N_t} + \Lambda_t^{\mathrm{N}\sigma}\frac{R_t^D}{\Pi_t^{\mathrm{e}}}\frac{1-\alpha}{N_t}\mathrm{l}_t - \Lambda_t^{\mathrm{NP}}R_t\left(\ln\Phi_\Gamma\frac{\Gamma_t^{\mathrm{e}}}{\Gamma} - \ln\Phi_\Pi\frac{\Pi_t^{\mathrm{e}}}{\Pi}\right)\frac{1-\alpha}{N_t} \qquad \text{A.C.57}$$

$$\Lambda_t^N\Gamma - \beta^NE_t\Lambda_{t+1}^N\left((1-\tau)(1-\alpha)\frac{\alpha E_tY_{t+1}}{K_t} + \left(1-\Delta^{\mathrm{N}}E_t\sigma_{t+1}^{\mathrm{D}}\right)(1-\tau)\left(\frac{R_t^D}{\Pi_t^{\mathrm{e}}} - 1 + \frac{R_t^D}{\Pi_t^{\mathrm{e}}}\frac{\alpha}{K_t}\mathrm{l}_t\right) + 1\right) + (\beta^N)^2E_t\Lambda_{t+2}^N\left(1-\Delta^{\mathrm{N}}E_t\sigma_{t+2}^{\mathrm{D}}\right)(1-\tau)\frac{E_t\mathrm{R}_{t+1}^D}{E_t\Pi_{t+1}^{\mathrm{e}}}\frac{\alpha}{K_t}E_t\mathrm{l}_{t+1} - \beta^{\mathrm{N}}E_t\Lambda_{t+1}^{\mathrm{NG}}\left(\tau+1-\frac{1}{R_t}\right)\frac{\alpha E_tY_{t+1}}{K_t} - \Lambda_t^{N\sigma}\left(\frac{E_t\sigma_{t+1}^{\mathrm{D}}}{\sigma^T}\alpha\frac{(\alpha-1)E_tY_{t+1}}{K_t}\frac{1}{K_t} - \frac{R_t^D}{\Pi_t^{\mathrm{e}}}\frac{\alpha}{K_t}\right) + \beta^NE_t\Lambda_{t+1}^{N\sigma}\frac{E_t\mathrm{R}_{t+1}^D}{E_t\Pi_{t+1}^{\mathrm{e}}}\frac{\alpha}{K_t} - \Lambda_t^{\mathrm{NP}}R_t\left(\ln\Phi_\Gamma\frac{\Gamma_t^{\mathrm{e}}}{\Gamma} - \ln\Phi_\Pi\frac{\Pi_t^{\mathrm{e}}}{\Pi}\right)\frac{\alpha}{K_t} + \beta^{\mathrm{N}}E_t\Lambda_{t+1}^{\mathrm{NP}}E_tR_{t+1}\left(\ln\Phi_\Gamma\frac{E_t\Gamma_{t+1}^{\mathrm{e}}}{\Gamma} - \ln\Phi_\Pi\frac{E_t\Pi_{t+1}^{\mathrm{e}}}{\Pi}\right)\frac{\alpha}{K_t} = 0 \qquad \text{A.C.58}$$

$$\Lambda_t^{N\sigma} = \beta^NE_t\Lambda_{t+1}^N\Delta^{\mathrm{N}}E_t\sigma_{t+1}^{\mathrm{B}}(1-\tau)\mathrm{l}_t \qquad \text{A.C.59}$$

$$\Lambda_t^{\mathrm{NP}}\left(1-\ln\Phi_\Pi\frac{\Pi_t^{\mathrm{e}}}{\Pi}\right) = -\beta^{\mathrm{N}}E_t\Lambda_{t+1}^{\mathrm{N}}\left(1-\Delta^{\mathrm{N}}E_t\sigma_{t+1}^{\mathrm{D}}\right)(1-\tau)\frac{R_t^D}{\Pi_t^{\mathrm{e}}}\frac{\mathrm{l}_t}{R_t} + \beta^{\mathrm{N}}E_t\Lambda_{t+1}^{\mathrm{NG}}\frac{E_tY_{t+1}}{\mathrm{R}_t^2} + \Lambda_t^{N\sigma}\frac{R_t^D}{\Pi_t^{\mathrm{e}}}\frac{1}{R_t} \qquad \text{A.C.60}$$

$$\Lambda_t^{\mathrm{I}} = \frac{\chi^{\mathrm{I}}\widetilde{\mathrm{CIG}}_{\mathrm{t}}}{\widetilde{\mathrm{C}}_t^{\mathrm{I}}} \qquad \text{A.C.61}$$

$$\Lambda_t^{\mathrm{IG}} = \frac{(1-\chi^{\mathrm{I}})\widetilde{\mathrm{CIG}}_{\mathrm{t}}}{\mathrm{G}_t^{\mathrm{I}}} \qquad \text{A.C.62}$$

$$\widetilde{\mathrm{CIG}}_{\mathrm{t}} = (\widetilde{\mathrm{C}}_t^{\mathrm{I}\,\chi^{\mathrm{I}}}\mathrm{G}_t^{\mathrm{I}\,(1-\chi^{\mathrm{I}})})^{1-\eta\mathrm{I}} \qquad \text{A.C.63}$$

$$\beta^{\mathrm{I}}E_t\Lambda_{t+1}^{\mathrm{I}}\left(\left(1-\Delta^{\mathrm{I}}E_t\sigma_{t+1}^{\mathrm{L}}\right)(1-\tau)\left(\left(\frac{R_t^{\mathrm{L}}}{\Pi_t^{\mathrm{e}}}-1\right) + \frac{E_t\sigma_{t+1}^{\mathrm{L}}}{\sigma^T}\alpha\frac{(\alpha-1)E_tY_{t+1}}{K_t}\frac{l_t}{K_t}\right) - (1-\tau)\left(\left(\frac{R_t^D}{\Pi_t^{\mathrm{e}}}-1\right) + \frac{E_t\sigma_{t+1}^{\mathrm{D}}}{\sigma^T}\alpha\frac{(\alpha-1)E_tY_{t+1}}{K_t}\frac{l_t}{K_t}\right) + \left(1-\frac{1}{R_t}\right)\alpha\frac{E_tY_{t+1}}{K_t}\right) + \Lambda_t^{I\sigma}\left(\frac{E_t\sigma_{t+1}^{\mathrm{D}}}{\sigma^T}-1\right) = 0 \qquad \text{A.C.64}$$

$$\Lambda_t^{I\sigma}\frac{E_t\sigma_{t+1}^{\mathrm{L}}}{\sigma^T} = \beta^IE_t\Lambda_{t+1}^I(1-\tau)\left(\frac{R_t^L}{\Pi_t^{\mathrm{e}}}-1\right)\left(2\Delta^{\mathrm{I}}E_t\sigma_{t+1}^{\mathrm{L}}-1\right) \qquad \text{A.C.65}$$

A.C.34, A.C.35, A.C.36, and A.C.37 can be combined and we have the consolidated financial constraint as

$$K_t = \frac{E_t \sigma^{\mathrm{L}}_{t+1}}{\sigma^T} \mathrm{l}_t + \frac{E_t \sigma^{\mathrm{E}}_{t+1}}{\sigma^T} \mathrm{e}_t \quad \text{A.C.66}$$

Λ^{E}, Λ^{EG}, $\Lambda^{E\sigma}$, and Λ^{EP} are Lagrange multipliers of the entrepreneur on her budget constraints, public goods constraints, consolidated financial constraints, and the monetary policy constraint, respectively. Λ^{N}, Λ^{NG}, $\Lambda^{N\sigma}$, and Λ^{NP} are the Lagrange multipliers of the worker on her budget constraints, public goods constraints, consolidated financial constraint, and the monetary policy constraint, respectively. Λ^{I}, Λ^{IG}, and $\Lambda^{I\sigma}$ are the Lagrange multipliers of the bank on her budget constraints, public goods constraints, and consolidated financial constraint, respectively. A.C.47, A.C.48, A.C.50, A.C.51, A.C.52, A.C.53 are first orders on C^{E}_{t}, G_t, K_t, L_t, $E_t\sigma^{E}_{t+1}$, R_t of the entrepreneur, respectively. A.C.54, A.C.55, A.C.57, A.C.58, A.C.59, A.C.60 are first orders on C^{N}_{t}, G_t, N_t, D_t, $E_t\sigma^{D}_{t+1}$, R_t of the worker, respectively. A.C.61, A.C.62, A.C.64, A.C.65 are first orders on C^{I}_{t}, G^{I}_{t}, L_t, $E_t\sigma^{L}_{t+1}$ of the bank, respectively.

References

1. Bernanke, Ben S., and Mark Gertler (1989) Agency Costs, Net Worth, and Business Fluctuations. The American Economic Review. Vol. 79 No.1.

2. Bernanke, Ben S., and Mark Gertler (1999) Monetary Policy and Asset Price Volatility. Economic Review, 4th quarter, 1999. Federal Rerserve Bank of Kansas City.

3. Bernanke, Ben S., and Mark Gertler (2001) Should Central Banks Respond to Movements in Asset Price? AER Papers and Preceedings, 91(2), May 2001.

4. Bernanke, Ben. S., Mark Gertler, and S. Gilchrist (1999) The Financial Accelerator in a Quantitative Business Cycle Framework. In Handbook of Macroeconomics, ed. J. B. Taylor and M. Woodford Vol. 1. Amsterdam: Elsevier.

5. Bordo, Micheal D., and Olivier Jeanne (2004) Boom-Bust in Asset Prices, Economic Instablity, and Monetary Policy. Deflation: Current and Historical Perspectives, Richard Burdekin and Pierre Siklos eds., Cambridge University Press, pp. 131-165.

6. Brunnermeier, Markus K., Thomas M. Eisenbach, and Yuliy Sannikov (2017) Macroeconomics with Financial Frictions: A Survey. Advances in Economics and Econometrics, Tenth World Congress of the Econometric Society, Vol. II: Applied Economics. Daron Acemoglu, Manuel Arellano and Eddie Dekel (eds.), Cambridge University Press, New York, 2013, pp. 4-94.

7. Brunnermeier, Markus K., and Yuliy Sannikov (2014) A Macroeconomical Model with a Financial Sector. American Economic Review. 104(2): 379-421.

8. Carlstrom, Charles T., and Timothy S. Fuerst (1997) Agency Costs, Net Worth, and Business Fluctuations: A Computable General Equilibrium Analysis. The American Economic Review. Vol. 87 No.5.

9. Carlstrom, Charles T., Timothy S. Fuerst, and Matthias Paustian (2016) Optimal Contracts, Aggregate Risk, and the Financial Accelerator. American Economic Journal: Macroeconomics 2016, 8(1): 119-147.

10. Cecchetti, Stephen G., Hans Genberg, and Sushil Wadhwani (2002) Asset Prices in a Flexible Inflation Targeting Framework.

11. Cecchetti, Stephen G., Hans Genberg, John Lipsky, and Sushil Wadhwani (2000) Asset Prices and Central Bank Policy. Geneva Reports on the World Economy 2, CEPR.

12. Chow, Gregory C. (1997) Dynamic Economics. Oxford University Press.

13. Di Tella, Sebastian (2017) Uncertainty Shocks and Balance Sheet. Journal of Political Economy, vol. 125, no. 6.

14. Friedman, Milton, and Anna J. Schwartz (1963) A Monetary History of the United States 1867-1960. Princeton University Press.

15. Gertler, Mark (1988) Financial Structure and Aggregate Economic Activity: An Overview. NBER working paper No. 2559.

16. Gertler, Mark, and Nobuhiro Kiyotaki (2011) Financial Intermediation and Credit Policy in Business Cycle Analysis. Handbook of Monetary Economics, Volume 3A.

17. He, Zhiguo, and Arvind Krishnamurthy (2019) A Macroeconomic Framework for Quantifying Systemic Risk. American Economic Journal: Macroeconomics 11(4):1-37.

18. Huang, Guangming (2021) Seigniorage Channel and Monetary Effectiveness in Flexible Price Economy. http://ideas.repec.org; http://www.ssrn.com.

19. King, Robert G., Charles I. Plosser, and Seigia T. Rebelo (1988) Production, Growth, and Business Cycles. Journal of Monetary Economics 21, pp 195-232. North-Holland.
20. Kiyotaki, Nobuhiro, and John Moore (1997) Credit Cycles. Journal of Political Economy, vol. 105, no.2, pp211-248.
21. Kocherlakota, Narayana R. (2000) Creating Business Cycles Through Credit Constraints. Quarterly Review. Federal Reserve Bank of Minneapolis.
22. Krishnamurthy, Arvind (2003) Collateral Constraints and the Amplification Mechanism. Journal of Economic Theory. 111:277-292.
23. Kydland, Finn E., and Edward C. Prescott (1982) Time to Build and Aggregate Fluctuations. Econometrica, Volume 50, Number 6.
24. Long, John B., Jr., and Charles I. Plosser (1983) Real Business Cycles. Journal of Political Economy 91:39-69
25. Markowitz, Harry (1952) Portfolio Selection. The Journal of Finance. Vol. 7, No.1, pp77-91.
26. Mehra, Rajnish, and Edward C. Prescott (1985) The Equity Premium: A Puzzle. Journal of Monetary Economics 15, pp 145-161. North-Holland.
27. Mendoza, Enrique G. (2010) Sudden Stops, Financial Crises, and Leverage. American Economic Review 100: 1941-1966.
28. Okina, Kunio, Masaaki Shirakawa, and Shigenori Shiratsuka (2000) The Asset Price Bubble and Monetary Policy: Japan’s experience in the late 1980s and the lessons. Discussion Paper No. 2000-E-12, Institute For Monetary And Economic Studies, Bank of Japan.
29. Stokey, Nancy L., Robert E. Lucas, with Edward C. Prescott (1989) Recursive Methods in Economic Dynamics. Harvard University Press.
30. Townsend, Robert M. (1979) Optimal Contracts and Competitive Markets with Costly State Verification. Journal of Economic Theory 21, pp265-293.
31. Uhlig, Harald (1999) A toolkit for analyzing nonlinear dynamic stochastic models easily. In Computational Methods for the Study of Dynamic Economies, edited by Ramon Marimon and Andrew Scott.