Unions, Growth and Inequality in a Schumpeterian Economy

Wang, Di and Wang, Xilin

China Center for Economic Studies, School of Economics, Fudan University, Shanghai, China, Department ofWorld Economy, School of Economics, Fudan University, Shanghai, China, Research Institute of Innovation and Digital Economy, Fudan University, Shanghai, China

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Di Wang Xilin Wang

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Abstract

This paper explores the dynamic effects of labor unions on economic growth and income inequality in a Schumpeterian growth model with heterogeneous households and endogenous market structure. A representative labor union bargains with a representative employer to determine the labor income share and employment level. We find that, in the short run, an increase in union bargaining power reduces economic growth and income inequality when the union is wage-oriented. In the long run, while stronger union bargaining power continues to reduce income inequality, it does not affect the steady-state growth rate due to endogenous market structure adjustments. To conduct a quantitative analysis, we calibrate the model using U.S. data. Our findings indicate that increasing union bargaining power from the 2016 level to the 1980 level would reduce the welfare of the top 30% of households, with significantly larger welfare losses for higher-income groups. Conversely, the bottom 70% of households would experience welfare gains, which are disproportionately larger for lower-income groups.

JEL: D30, J50, O30, O40

Keywords: Economic growth; Income inequality; Labor unions; Endogenous market structure

Wang: dwang22@m.fudan.edu.cn. China Center for Economic Studies, School of Economics, Fudan University, Shanghai, China.

Wang: xilinwang@fudan.edu.cn. Department of World Economy, School of Economics, Fudan University, Shanghai, China; Research Institute of Innovation and Digital Economy, Fudan University, Shanghai, China.

1 Introduction

The last four decades have witnessed a sharp rise in income inequality in the United States. Economists have extensively investigated the forces driving up income inequality. Stiglitz (2015) points out that a large part of rising inequality in the U.S. could not be attributed to the market forces discussed in Thomas Piketty's well-known book *Capital in the Twenty-First Century* (Piketty, 2014), but rather stems from institutional and policy differences that distinguish the U.S. from Europe. This paper explores the role of labor unions in shaping the dynamics of income inequality.

The secular decline of trade unions in the U.S. is prominent in the last four decades. In the early 1980s, union density in the U.S. was above 20%, and it declined steadily to around 10% in 2016; see Figure 1 (Panel A). Over the same period, the Gini coefficient of income in the U.S. rose from 0.43 in 1980 to 0.51 in 2016, and the top 10% income share increased from 34% to 45%; see Figure 1 (Panel B). The decline of labor unions and the rise of income inequality have been broad trends in many other developed economies over the last four decades. The contrasting trends of union density and income inequality indicate that the decline of unions could, to some extent, have contributed to the rise in income inequality.

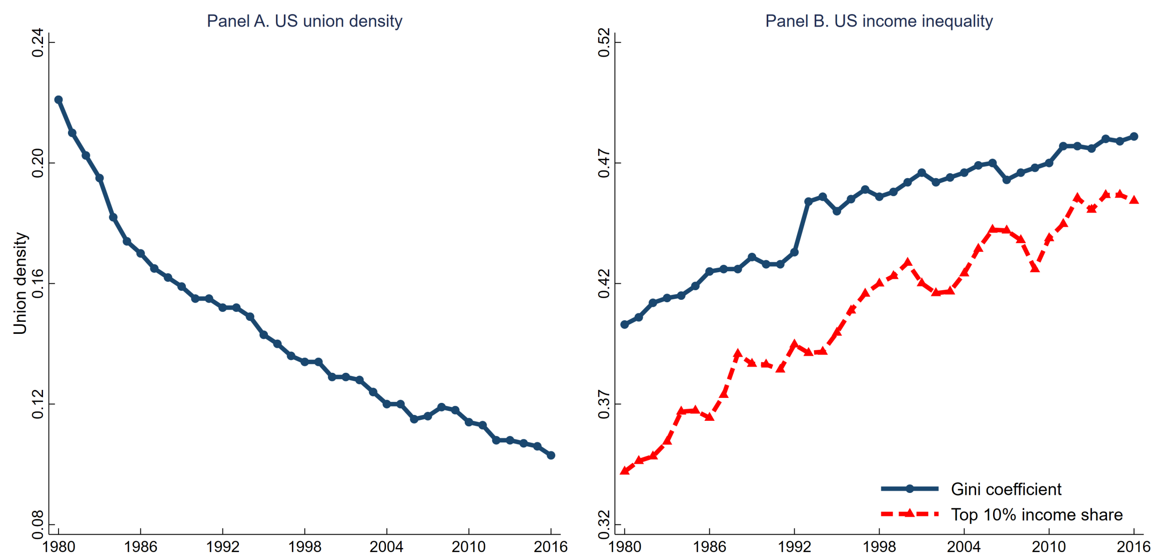

Figure 1. Union density and inequality in the U.S.

Data sources: OECD Statistics for union density, the Current Population Survey (CPS) for the Gini coefficient of income, and the World Inequality Database for the top 10% income share. The dataset covers the period from 1980 to 2016 for the United States.

This study provides a growth-theoretic analysis of the dynamic effects of labor unions on economic growth and income inequality. We incorporate labor unions, which bargain with employers over wages and employment, into a Schumpeterian growth model with heterogeneous households and endogenous market structure in Chu et al. (2021). Income inequality arises from the unequal distribution of intangible assets across households, which is documented by Koh et al. (2020), Piketty (2014), Piketty and Zucman (2014), Madsen (2019) and Madsen et al. (2021). The Schumpeterian growth model, based on Peretto (2007, 2011), features endogenous market structure, which drives the transition dynamics of the aggregate economy. The tractability of the model enables us to characterize analytically the dynamics of economic growth and the endogenous evolution of wealth and income distributions along the transition and balanced-growth paths, to clearly differentiate between the short-run and long-run effects of union bargaining power, and to evaluate the welfare effects of unions on heterogeneous households by considering both the transition and balanced growth paths.

Within this theoretical framework, we find that for a wage-oriented labor union, stronger union bargaining power leads to a lower output growth rate and a reduction in income inequality, as measured by the Gini coefficient and top income share, in the short run. In the long run, an increase in union bargaining power continues to reduce income inequality but does not affect the steady-state growth rate due to the endogenous adjustment of market structure. Intuitively, stronger union bargaining power reduces the equilibrium level of employment, which in turn reduces the market size of each firm. The shrinking market size dampens incentives for quality-improving innovation, leading to a lower output growth rate when the number of firms is fixed in the short run. However, the smaller market size also induces firm exit, which in turn increases firm size and the rate of return on quality-improving innovation gradually. In the long run, firm size returns to its initial level, thereby restoring the steady-state growth rate to its initial value.

The transition dynamics of the aggregate economy gives rise to the endogenous evolution of wealth and income distributions. Households own different amounts of assets, which drives income inequality. Asset income is determined by the rate of return on assets and their market value. In the short run, the lower growth rate of output caused by stronger union bargaining power reduces the real interest rate through the Euler equation. Additionally, a stronger union bargaining power increases the labor income share while reducing the profit share, leading to a decline in asset values. As a result, stronger union bargaining power lowers the degree of income inequality in the short run. In the long run, the negative interest-rate effect vanishes due to the scale-invariant steady-state growth rate, but the negative effect on asset values persists. Consequently, stronger union bargaining power continues to reduce income inequality in the long run.

We also calibrate the model to the U.S. economy to quantify the effects of union bargaining power on economic growth, income inequality and social welfare. Our numerical analysis shows that increasing union bargaining power from its 2016 level to its 1980 level leads to a 37.3% reduction in income inequality, as measured by the Gini coefficient, in the long run. The top 20% income share declines, while the bottom 80% income share increases. As a result, strengthening union bargaining power reduces the welfare of the top 30% of households, with disproportionately larger welfare losses for higher-income groups, while increasing the welfare of the bottom 70%, with significantly larger gains for lower-income groups. Specifically, the welfare loss for the top 10% of households is equivalent to a 27.3% reduction in annual consumption, whereas the welfare gain for the bottom 10% is equivalent to a 6.2% increase in annual consumption.

Empirical evidence supports the above theoretical results. Using a new source of microdata on union membership in the U.S., Farber et al. (2021) document consistent evidence that unions reduce inequality. Cross-country data provide further evidence, showing that the decline of union power has been a key driver of rising income inequality. Figure 2 illustrates the relationship between union density and the Gini coefficient of income (Panel A), as well as the relationship between union density and the top 10% income share (Panel B), using within-country variations. Both panels reveal a clear negative relationship.[1] In other words, countries experiencing a decline in union power tend to see increasing income inequality, whereas those maintaining stronger unions exhibit relatively stable income distributions. Jaumotte and Buitron (2020) conduct a cross-country empirical analysis documenting strong and causal evidence that the decline of union power has contributed significantly to the rise in top income shares. We also employ a panel vector autoregression (VAR) model to conduct a shock analysis to examine the dynamic effects of union density on economic growth.[2] Figure 3 presents the impulse response of GDP per capita growth rate to a positive union shock. The initial impact of a positive union shock on growth rate is negative, but the negative effect attenuates gradually and converges to zero after eight periods.

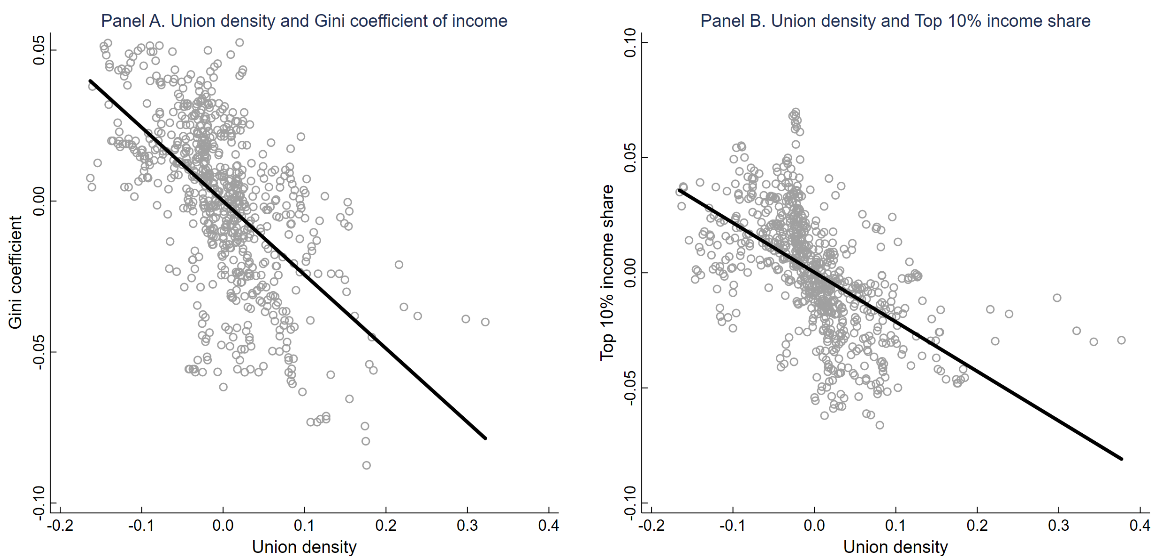

Figure 2. Union density and inequality in advanced economies

Data sources: OECD Statistics for union density, the Standardized World Income Inequality Database (SWIID) for the Gini coefficient of income, and the World Inequality Database for the top 10% income share. The dataset covers 20 developed countries from 1980 to 2016, including Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Ireland, Italy, Japan, South Korea, the Netherlands, New Zealand, Norway, Spain, Sweden, Switzerland, the United Kingdom, and the United States.

[1] The data represent within-country variations derived from the raw dataset, i.e., $\Delta x_{it} = x_{it} - \bar{x}_i$, where x_{it} denotes the value of variable x for country i in year t, and $\bar{x}_i$ denotes the mean of variable x for country i over the period 1980 to 2016.

[2] See Appendix B for a detailed discussion of the panel VAR analysis.

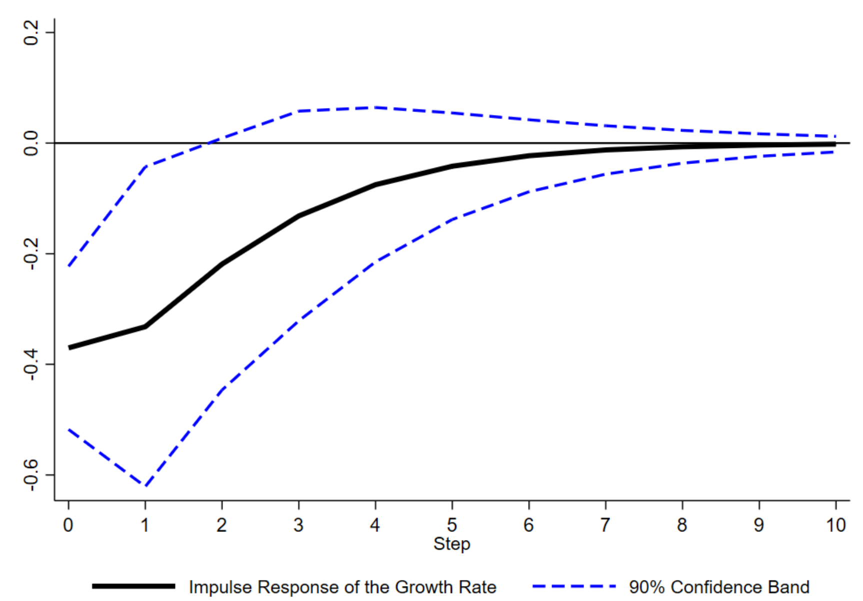

Figure 3. Impulse response of GDP per capita growth rate to a positive union shock

Data sources: See Appendix B.

This study relates to the literature on innovation and economic growth. Romer (1990) develops the seminal R&D-based endogenous growth model, in which economic growth is driven by horizontal innovation, i.e., the development of new products. Aghion and Howitt (1992) develop the creative-destruction Schumpeterian growth model, in which economic growth is driven by vertical innovation, i.e., the improvement in the quality of existing products; see also Grossman and Helpman (1991) and Segerstrom et al. (1990). Peretto (1998, 1999) and Smulders and van de Klundert (1995) integrate these two growth engines to develop the creative-accumulation endogenous growth model with endogenous market structure; see also Dinopoulos and Thompson (1998), Young (1998) and Howitt (1999) for creative-destruction versions of the theory. Cohen and Klepper (1996a, b), Laincz and Peretto (2006), Ha and Howitt (2007), Madsen (2008), and Ang and Madsen (2011) provide empirical evidence for this branch of growth models. Garcia-Macia et al. (2019) show that the main driver of the U.S. TFP growth in recent decades is the in-house R&D conducted by incumbents to improve their own products quality. Chu et al. (2021) develop a Schumpeterian growth model with heterogeneous households and endogenous market structure to investigate the dynamic effects of patent protection on economic growth and inequality. This paper contributes to the literature by incorporating labor unions in Chu et al. (2021) to explore the dynamic effects of labor unions on economic growth, income inequality and social welfare.

This paper also relates to the literature on labor unions. Flanagan (1999) reviews early studies on the impact of collective bargaining on macroeconomic performance in industrialized economies. Our paper most closely relates to the literature that investigates the effects of labor unions in modern growth theories. Palokangas (1996) is the seminal study that analyzes the effects of collective bargaining on innovation and economic growth in an R&D-based endogenous growth model. Subsequent studies explore the effects of managerial unions, whose objective functions depend on both membership size and wages, based on Pemberton (1988). These studies analyze the effects of unions in variants of endogenous growth models, including Chang et al. (2007) and Chang and Hung (2016) in the AK model, Chu et al. (2016) in an open-economy R&D-based growth model, Ji et al. (2016) in a Schumpeterian growth model with endogenous market structure, and Chu et al. (2018) in the Romer model. This paper contributes to the literature by investigating both the macroeconomic effects of labor unions on multiple dimensions of innovation and economic growth over different time horizons, and their microeconomic implications for income distribution and the welfare of heterogeneous households.

Finally, this paper relates to the literature on the relationship between innovation and inequality in R&D-based growth models. Chou and Talmain (1996), Zweimuller (2000), Foellmi and Zweimuller (2006), and Schetter et al. (2024) study how inequality affects innovation and economic growth. Meanwhile, Jones and Kim (2018) and Aghion et al. (2019) explore how innovation and creative destruction shape the dynamics of top income inequality. Chu and Peretto (2023) study the endogenous evolution of income inequality from stagnation to growth. Other studies focus on the role of policy instruments in shaping innovation and inequality, including patent policy (Chu, 2010; Kiedaisch, 2020; Chu et al., 2021), R&D subsidies and patent policy (Chu and Cozzi, 2018), monetary policy (Chu et al., 2019), and tax policy (Garcia-Penalosa and Wen, 2008; Chu et al., 2024). The study complements this literature by investigating how unions influence both innovation and income inequality.

The remainder of this paper is organized as follows. Section 2 presents the model. Section 3 analyzes the dynamics of the model. Section 4 explores the effects of labor unions qualitatively and quantitatively. Section 5 concludes.

2 The model

In this section, we introduce labor unions into the Schumpeterian growth model with heterogeneous households and endogenous market structure based on Chu et al. (2021). The model features two dimensions of innovation, i.e., variety-expanding innovation and quality-improving innovation. We analyze the effects of labor unions on economic growth and income inequality. A representative labor union bargains with a representative employer to determine the equilibrium employment and wages. This analysis derives a closed-form solution for economic growth and income distribution along both the transition and balanced-growth paths.

2.1 Households

There is a unit continuum of households indexed by $h \in [0, 1]$. Household h has the following lifetime utility function:

$$U(h) = \int_0^\infty e^{-\rho t} \ln c_t(h)\, dt, \tag{1}$$

where $\rho > 0$ is the subjective discount rate, and $c_t(h)$ denotes household consumption of final good (numeraire). Household h maximizes its lifetime utility subject to the following asset-accumulation equation:

$$\dot{a}_t(h) = r_t a_t(h) + w_t l_t(h) + b_t[L - l_t(h)] - \tau_t(h) - c_t(h). \tag{2}$$

$a_t(h)$ denotes the real value of assets held by the household, which includes equity shares of the representative final-good firm and monopolistic intermediate-good firms, and r_t is the real interest rate. $l_t(h)$ is labor supply, and w_t is the real wage rate. Each household inelastically supplies L units of labor, with unemployment given by $L - l_t(h)$. The government provides an unemployment benefit $b_t\,(< w_t)$, and levies a lump-sum tax $\tau_t(h)$. Dynamic optimization yields the following Euler equation:

$$\frac{\dot{c}_t(h)}{c_t(h)} = \frac{\dot{c}_t}{c_t} = r_t - \rho \tag{3}$$

for $h \in [0,1]$, where aggregate consumption is defined as $c_t \equiv \int_0^1 c_t(h)\,dh$. Therefore, due to the homothetic preference, the growth rate of consumption is uniform across all households.

2.2 Final good and labor union

A representative firm produces final good Y_t using the following production function:

$$Y_t = \int_0^{N_t} X_t^{\theta}(i)\left[Z_t^{\alpha}(i)\,Z_t^{1-\alpha}\left(\frac{l_t}{N_t}\right)^{\eta}\right]^{1-\theta} di, \tag{4}$$

where $\{\theta, \alpha, \eta\} \in (0,1)$. The mass of differentiated non-durable intermediate goods at time t is N_t, and $X_t(i)$ denotes the quantity of intermediate good $i \in [0, N_t]$. The productivity of each intermediate good i is determined by its quality $Z_t(i)$ and by the average quality of all intermediate goods $Z_t \equiv \left[\int_0^{N_t} Z_t(i)\,di\right]/N_t$ which captures technology spillovers across firms. Aggregate employment is l_t, and the term l_t/N_t implies that technology features a congestion effect of variety, removing the scale effect of labor size on long-run growth. The final-good production technology features decreasing returns to scale $\eta < 1$, ensuring that the representative final-good firm earns positive profits. This assumption can be justified by the presence of fixed factors, such as entrepreneurial skills possessed by the firm owner, which also facilitates bargaining between employers and labor unions; see Palokangas (1996), Chang et al. (2007), Chu et al. (2016) and Ji et al. (2016).

The profit function of the final-good firm is

$$\Pi_t = Y_t - w_t l_t - \int_0^{N_t} p_t(i)\,X_t(i)\,di, \tag{5}$$

where $p_t(i)$ is the price of intermediate good i, and the price of final good is normalized to one. Profit maximization yields the conditional demand function for $X_t(i)$:

$$X_t(i) = \left[\frac{\theta}{p_t(i)}\right]^{1/(1-\theta)} Z_t^{\alpha}(i)\,Z_t^{1-\alpha}\left(\frac{l_t}{N_t}\right)^{\eta}. \tag{6}$$

Therefore, the final-good firm pays $\theta Y_t = \int_0^{N_t} p_t(i) X_t(i) \, di$ for intermediate goods.

The asset-pricing equation for the final-good firm is

$$r_t = \frac{\Pi_t}{v_t} + \frac{\dot{v}_t}{v_t}, \tag{7}$$

where v_t represents the market value of the final-good firm. This equation states that the return on equity shares is determined by profit flow and capital gains.

Following the modelling approach of Pemberton (1988), we consider a managerial labor union whose objective is determined by both the union leaders' preference for a larger membership (employment level) and the union members' preference for a higher wage. The objective function of the labor union is given by the following Stone-Geary utility function:

$$O_t = (w_t - b_t)^{\omega} l_t. \tag{8}$$

The parameter $\omega > 0$ determines the relative importance of wage income net of unemployment benefit $w_t - b_t$ in the union's objective function. The union is wage-oriented when $\omega > 1$, and employment-oriented when $\omega < 1$. In the remainder of the paper, we focus on the more realistic case $\omega > 1$.

The labor union bargains with the employer federation to determine the equilibrium wage w_t and employment level l_t. The generalized Nash bargaining function is

$$\max_{w_t, l_t} B_t = O_t^{\gamma} \Pi_t^{1-\gamma} = \left[(w_t - b_t)^{\omega} l_t\right]^{\gamma} \left[(1-\theta) Y_t - w_t l_t\right]^{1-\gamma}. \tag{9}$$

The parameter $\gamma \in (0,1)$ represents the bargaining power of the labor union, while $1-\gamma$ represents the bargaining power of the employer. The first-order conditions for the Nash bargaining solutions are

$$\frac{\partial B_t}{\partial w_t} = 0 \Rightarrow \frac{(w_t - b_t) l_t}{\Pi_t} = \frac{\omega \gamma}{1-\gamma}, \tag{10}$$

$$\frac{\partial B_t}{\partial l_t} = 0 \Rightarrow \frac{w_t l_t - \eta (1-\theta) Y_t}{\Pi_t} = \frac{\gamma}{1-\gamma}. \tag{11}$$

Equations (10) and (11) jointly determine the equilibrium employment l_t and the Pareto-optimal allocation of labor income $w_t l_t$ and firm profit Π_t.

2.3 Intermediate goods and in-house R&D

The market structure of the intermediate goods sector is monopolistic competition, where each firm produces one type of differentiated intermediate good. A monopolistic firm uses $X_t(i)$ units of final goods to produce $X_t(i)$ units of intermediate goods for $i \in [0, N_t]$, such that the marginal cost of production is equal to one. The firm also incurs a fixed operating cost of $\phi Z_t^{\alpha}(i) Z_t^{1-\alpha}$ units of final goods. Additionally, the firm invests $R_t(i)$ units of final goods in in-house R&D to improve the quality of its product with the following innovation process:

$$\dot{Z}_t(i) = R_t(i). \tag{12}$$

The firm's profit flow before-R&D at time t is expressed as

$$\pi_t(i) = [p_t(i) - 1] X_t(i) - \phi Z_t^{\alpha}(i) Z_t^{1-\alpha}. \tag{13}$$

The market value of the firm is

$$V_t(i) = \int_t^{\infty} \exp\left(-\int_t^s r_v dv\right) [\pi_s(i) - R_s(i)] ds. \tag{14}$$

The firm maximizes its value (14) subject to the demand function (6), the innovation process (12), and the profit function (13). The unconstrained profit-maximizing price is $p_t(i) = 1/\theta$ (see Appendix A). Following Chu et al. (2020, 2021), knowledge diffuses from the monopolistic firm to imitators in each industry $i \in [0, N_t]$. However, imitative firms can only produce good i at a higher marginal cost $\mu \in (1, 1/\theta)$ with the same quality as the monopolistic firm due to the legal regulations such as patent laws. Therefore, the monopolistic firm sets its price at $p_t(i) = \mu$ to price out imitators from the market.

The initial quality of differentiated goods is identical across firms, with $Z_0(i) = Z_0$ for $i \in [0, N_0]$, which implies that symmetric equilibrium holds at any point in time across all industries, such that $Z_t(i) = Z_t$ and $X_t(i) = X_t$, and consequently $\pi_t(i) = \pi_t$, $R_t(i) = R_t$, and $V_t(i) = V_t$. By substituting $p_t(i) = \mu$ into the demand function (6) and imposing symmetry, the quality-adjusted firm size is derived as

$$\frac{X_t}{Z_t} = \left(\frac{\theta}{\mu}\right)^{1/(1-\theta)} \left(\frac{l_t}{N_t}\right)^{\eta},$$

which is determined by l_t/N_t. For analytical convenience, we define the following state variable:

$$x_t \equiv \frac{\mu^{1/(1-\theta)}}{l_t^{\eta}} \frac{X_t}{Z_t} = \frac{\theta^{1/(1-\theta)}}{N_t^{\eta}}, \tag{15}$$

which is determined by the quality-adjusted firm size X_t/Z_t. A change in the bargaining power of labor unions γ does not directly affect x_t through the market size l_t, but indirectly through the number of products N_t. In Appendix A, we derive the rate of return to quality-improving R&D r_t^q given by

$$r_t^q = \alpha \frac{\pi_t}{Z_t} = \alpha \left[\frac{\mu - 1}{\mu^{1/(1-\theta)}} x_t l_t^{\eta} - \phi\right], \tag{16}$$

which is increasing in both x_t and l_t.

2.4 Entrants

A new firm incurs an entry cost of βX_t (where $\beta > 0$) units of final goods to invent a new variety of intermediate goods with quality Z_t, ensuring symmetric equilibrium holds at any point in time. This specification of the entry cost implies that the setup cost for an entrant increases with the size of its initial output. The rate of return on assets is determined by its profit flow net of in-house R&D and the capital gain in firm value. The asset-pricing equation is

$$r_t = \frac{\pi_t - R_t}{V_t} + \frac{\dot{V}_t}{V_t}. \tag{17}$$

The free-entry condition is

$$V_t = \beta X_t. \tag{18}$$

Substituting (12), (13), (15), (18), and $p_t(i) = \mu$ into the asset-pricing equation (17) yields the return to entry as

$$r_t^e = \frac{\mu^{1/(1-\theta)}}{\beta}\left[\frac{\mu - 1}{\mu^{1/(1-\theta)}} - \frac{\phi + z_t}{x_t l_t^\eta}\right] + \frac{\dot{x}_t}{x_t} + z_t + \eta\frac{\dot{l}_t}{l_t}, \tag{19}$$

where $z_t \equiv \dot{Z}_t/Z_t$ is the growth rate of aggregate quality. The scrap value of exiting an industry is equal to the entry cost βX_t, such that $V_t(i) = \beta X_t$ always holds, which implies that $r_t^e = r_t$ at any point in time.

2.5 Government

The government provides unemployment benefits to households and levies a lump-sum tax on households to finance the benefits. The balanced-budget condition for the government is

$$\tau_t = b_t (L - l_t). \tag{20}$$

Assume that unemployment benefit b_t is proportional to output Y_t, such that $b_t = \bar{b}Y_t$ with $\bar{b} > 0$, ensuring balanced growth. The lump-sum tax $\tau_t(h)$ levied on household h is given by $\tau_t(h) = b_t[L - l_t(h)]$. This setup eliminates the effects of lump-sum taxes and unemployment benefits on inequality.

2.6 General equilibrium

The general equilibrium is a time path of allocations $\{a_t, c_t, Y_t, l_t, X_t(i), R_t(i)\}$, prices $\{r_t, w_t, p_t(i), v_t, V_t(i)\}$, and government policies $\{\tau_t, b_t\}$ such that the following conditions hold:

- households choose $c_t(h)$ to maximize their lifetime utility, taking $\{r_t, w_t, b_t, \tau_t(h)\}$ as given;
- the representative final-good firm produces Y_t to maximize its profit, taking $p_t(i)$ as given;
- a representative employer of final-good firm bargains with a representative labor union to determine $\{w_t, l_t\}$;
- monopolistic firms produce intermediate goods $X_t(i)$ and choose $\{p_t(i), R_t(i)\}$ to maximize their market value $V_t(i)$, taking r_t as given;
- entrants make entry decisions, taking V_t as given;

- the sum of the value of the representative final-good firm and the value of all existing monopolistic firms is equal to the value of households' financial assets: $v_t + N_t V_t = \int_0^1 a_t(h)\, dh \equiv a_t$;
- the government balances its budget such that $\tau_t = b_t (L - l_t)$;
- the market-clearing condition for labor holds such that $\int_0^1 l_t(h)\, dh = l_t$; and
- the market-clearing condition for final good holds such that $Y_t = c_t + N_t (X_t + \phi Z_t + R_t) + \dot{N}_t \beta X_t$.

2.6.1 Equilibrium employment

Substituting the profit of the final-good firm $\Pi_t = (1-\theta) Y_t - w_t l_t$ into (11) yields the labor share:

$$\frac{w_t l_t}{Y_t} = (1-\theta)\left[\eta + (1-\eta)\gamma\right], \tag{21}$$

which is increasing in the bargaining power of the labor union γ. Substituting this expression into the profit function $\Pi_t = (1-\theta) Y_t - w_t l_t$ yields the profit of the final-good firm as a share of final output:

$$\frac{\Pi_t}{Y_t} = (1-\theta)(1-\eta)(1-\gamma), \tag{22}$$

which is decreasing in γ. Substituting (21), (22) and $b_t = \bar{b} Y_t$ into (10) yields the equilibrium level of employment:[3]

$$l_t = (1-\theta)\frac{\eta + (1-\eta)(1-\omega)\gamma}{\bar{b}} \equiv l < L, \tag{23}$$

which is decreasing in the wage orientation of the labor union ω. When the labor union is wage-oriented $\omega > 1$, the equilibrium level of employment l is decreasing in its bargaining power γ. We summarize these results in Lemma 1.

Lemma 1 *Labor share is increasing in the union bargaining power γ, and the profit share of the final-good firm is decreasing in γ. Employment is decreasing in γ for a wage-oriented labor union $\omega > 1$.*

Proof. Proven in text. ■

2.6.2 Aggregation

Substituting the demand for $X_t(i)$ in (6) and $p_t(i) = \mu$ into the final-good production function in (4) and imposing symmetry yield the aggregate final-good production function as

$$Y_t = (\theta/\mu)^{\theta/(1-\theta)} N_t^{1-\eta} Z_t l^{\eta}. \tag{24}$$

[3] To ensure $l > 0$, we also impose the parameter restrictions $\eta + (1-\eta)(1-\omega)\gamma > 0$.

The growth rate of output is

$$g_t \equiv \frac{\dot{Y}_t}{Y_t} = (1-\eta)\, n_t + z_t, \tag{25}$$

where $n_t \equiv \dot{N}_t / N_t$ is the variety growth rate and z_t is the quality growth rate.

3 Dynamics

This section analyzes the dynamic properties of the model. Section 3.1 analyzes the dynamics of the aggregate economy, including the dynamic paths of x_t and g_t. Section 3.2 investigates the dynamics of wealth distribution, while Section 3.3 explores the dynamics of income distribution. Finally, Section 3.4 studies the dynamics of consumption distribution.

3.1 Dynamics of the aggregate economy

In this model, the dynamics of x_t determines the dynamics of the economy. To analyze the transition dynamics of x_t and g_t, we first show an important property of the model: the consumption-output ratio c_t/Y_t and the consumption-wealth ratio c_t/a_t remain constant along both the transition path and the balanced-growth path. Define the parameter $\Theta \equiv \beta\theta / [\mu (1-\theta)]$.

Lemma 2 *The consumption-output ratio jumps to its stable steady-state value:*

$$\frac{c_t}{Y_t} = (1-\theta)(1+\rho\Theta). \tag{26}$$

The consumption-wealth ratio also jumps to its stable steady-state value:

$$\frac{c_t}{a_t} = \frac{\rho(1+\rho\Theta)}{\rho\Theta + (1-\eta)(1-\gamma)}. \tag{27}$$

Proof. See Appendix A. ■

The stationarity of the consumption-output ratio c_t/Y_t implies that consumption and output grow at the same rate at any time t. Therefore, from the Euler equation in (3), the growth rate of output is given by

$$g_t \equiv \frac{\dot{Y}_t}{Y_t} = \frac{\dot{c}_t}{c_t} = r_t - \rho,$$

and substituting r_t^q in (16) into it yields the growth rate of output as

$$g_t = \alpha \left[\frac{\mu - 1}{\mu^{1/(1-\theta)}} x_t l^\eta - \phi \right] - \rho, \tag{28}$$

which is increasing in the quality-adjusted firm size $x_t l^\eta$. The growth rate of output g_t is positive if and only if:

$$x_t > \bar{x} \equiv \frac{\mu^{1/(1-\theta)}}{(\mu-1)\, l^\eta} \left(\frac{\rho}{\alpha} + \phi \right).$$

Since the market size of each intermediate good determines profit and R&D incentives, firm size must be large enough to provide sufficient incentives for innovation. For the rest of the paper, we assume that $x_t > \bar{x}$ always holds. The dynamics of x_t are summarized in the Proposition 1.

Proposition 1 *The dynamics of x_t follows the linearized autonomous differential equation:*

$$\dot{x}_t = \frac{\eta}{\beta}\mu^{1/(1-\theta)}\left[\frac{(1-\alpha)\phi-\rho}{l^\eta}\right] - \frac{\eta}{\beta}\left[(1-\alpha)(\mu-1)-\beta\rho\right]x_t. \tag{29}$$

Under the parameter restriction $\rho < \min\{(1-\alpha)\phi, (1-\alpha)(\mu-1)/\beta\}$, x_t is globally stable and gradually converges to its steady-state value:

$$x^* = \frac{\mu^{1/(1-\theta)}}{l^\eta}\frac{(1-\alpha)\phi-\rho}{(1-\alpha)(\mu-1)-\beta\rho} > \bar{x}. \tag{30}$$

The steady-state growth rate of output is

$$g^* = \alpha\left[(\mu-1)\frac{(1-\alpha)\phi-\rho}{(1-\alpha)(\mu-1)-\beta\rho} - \phi\right] - \rho > 0. \tag{31}$$

Proof. See Appendix A. ■

The dynamic paths of x_t and g_t can be illustrated as follows. Given an initial value $x_0 < x^*$ $(> x^*)$, x_t rises (declines) gradually and converges to its steady-state value x^* along the path described by $\dot{x}_t$. The output growth rate g_t rises (declines) gradually and converges to its steady-state value g^*. The number of differentiated products N_t adjusts according to $N_t = [\theta^{1/(1-\theta)}/x_t]^{1/\eta}$. Over time, N_t declines (rises) and converges to its steady-state value $N^* = (\theta^{1/(1-\theta)}/x^*)^{1/\eta}$.

3.2 Dynamics of the wealth distribution

From the Euler equation in (3), all households share the same consumption growth rate $\dot{c}_t(h)/c_t(h) = \dot{c}_t/c_t = r_t - \rho$. This implies that the consumption share of each household, defined as $s_{c,t}(h) \equiv c_t(h)/c_t$, remains constant and equal to its initial value $s_{c,0}(h) = c_0(h)/c_0$ for $h \in [0,1]$, which is endogenously determined, as shown in the Proof of Proposition 2.

The wealth share of household h is defined as $s_{a,t}(h) \equiv a_t(h)/a_t$, and its initial value $s_{a,0}(h) = a_0(h)/a_0$ is exogenously given at time 0. The growth rate of the wealth share $s_{a,t}(h)$ is given by[4]

$$\frac{\dot{s}_{a,t}(h)}{s_{a,t}(h)} = \frac{\dot{a}_t(h)}{a_t(h)} - \frac{\dot{a}_t}{a_t} = \frac{c_t - w_t l_t}{a_t} - \frac{s_{c,t}(h)c_t - w_t l_t}{a_t(h)}.$$

Proposition 2 shows that the wealth distribution among households remains stationary and is determined by its initial distribution such that $s_{a,t}(h) = s_{a,0}(h)$. In the Proof of Proposition 2, we derive the dynamics of $s_{a,t}(h)$ featured by the following one-dimensional linear differential equation:

$$\dot{s}_{a,t}(h) = \rho s_{a,t}(h) - \frac{c_t}{a_t}\left[s_{c,t}(h) - \frac{w_t l_t}{c_t}\right].$$

[4] We assume that employment is identical across all households. Given the unit continuum of households $h \in [0,1]$, it follows that $l_t(h) = l_t$ holds for all t.

We show that for any $t \geq 0$, the condition $\dot{s}_{a,t}(h) = 0$ must hold, and $s_{c,t}(h)$ must immediately jump to its steady-state value at $t = 0$ such that $s_{c,t}(h) = s_{c,0}(h)$. This ensures the stability of $s_{a,t}(h)$, resulting in a contant wealth share of household $h \in [0,1]$. The stationarity of wealth distribution along both the transition and balanced-growth paths arises from the stationary consumption-output ratio c_t/Y_t and consumption-wealth ratio c_t/a_t, despite the transition dynamics of the aggregate economy, which is governed by the law of motion of x_t.

Proposition 2 *The wealth share $s_{a,t}(h)$ of household $h \in [0,1]$ remains constant and is equal to its initial value at $t = 0$ such that*

$$s_{a,t}(h) = s_{a,0}(h). \tag{32}$$

Therefore, the wealth distribution is stationary along both the transition and balanced-growth paths.

Proof. See Appendix A. ■

Finally, we derive the Gini coefficient of wealth following Chu and Peretto (2023). We rank households $h \in [0,1]$ in ascending order of wealth, and define the Gini coefficient of wealth at time t as

$$\sigma_{a,t} \equiv 1 - 2\int_0^1 \mathcal{L}_{a,t}(h)\, dh,$$

where the Lorenz curve of wealth is given by

$$\mathcal{L}_{a,t}(h) \equiv \frac{\int_0^h a_t(\chi)\, d\chi}{\int_0^1 a_t(\chi)\, d\chi} = \frac{\int_0^h a_t(\chi)\, d\chi}{a_t} = \int_0^h s_{a,t}(\chi)\, d\chi = \int_0^h s_{a,0}(\chi)\, d\chi.$$

Since the Lorenz curve of wealth remains unchanged over time, $\mathcal{L}_{a,t}(h) = \mathcal{L}_{a,0}(h) = \mathcal{L}_a(h)$, the Gini coefficient of wealth remains constant $\sigma_{a,t} = \sigma_{a,0} = \sigma_a$ for all time, where $\sigma_{a,0}$ is the Gini coefficient of wealth at $t = 0$.

3.3 Dynamics of the income distribution

The income distribution features transition dynamics and is determined by both the unequal distribution of wealth and the ratio of asset income to labor income $r_t a_t / w_t l_t$. The evolution of income distribution is determined by the transition dynamics of the aggregate economy, specifically, the transition dynamics of the output growth rate g_t, which in turn determines the real interest rate r_t.

The total income received by household h consists of its asset income and wage income given by[5]

$$I_t(h) = r_t a_t(h) + w_t l_t.$$

[5] By assuming that all members in a household, including employed and unemployed individuals, share the household's total income, we focus on income inequality across households rather than within them; see also Chang and Hung (2016) and Ji et al. (2016).

Aggregating across all households $h \in [0,1]$, the aggregate income of the economy is given by

$$I_t = r_t a_t + w_t l_t.$$

The income share of household h is defined as $s_{I,t}(h) \equiv I_t(h)/I_t$, which, using $s_{a,t}(h) = s_{a,0}(h)$, can be expressed as

$$s_{I,t}(h) = \frac{r_t a_t(h) + w_t l_t}{r_t a_t + w_t l_t} = \frac{r_t a_t}{r_t a_t + w_t l_t} s_{a,0}(h) + \frac{w_t l_t}{r_t a_t + w_t l_t}, \tag{33}$$

Equation (33) shows that the income share of each household consists of two components, the labor income share which is identical across all households and equal to the aggregate labor income share, and the asset income share which varies across households and is determined by the wealth share they hold. If household h's wealth share $s_{a,t}(h)$ is larger than one (the average wealth share), an increase in the aggregate asset income share leads to an increase in its income share $s_{I,t}(h)$. Conversely, if household h's wealth share is smaller than one, an increase in the aggregate asset income share leads to a decrease in its income share.

Finally, we derive the Gini coefficient of income and the top income share. Households are sorted in ascending order of income, and the Gini coefficient of income at time t is given by

$$\sigma_{I,t} = \frac{r_t a_t}{r_t a_t + w_t l_t} \sigma_a. \tag{34}$$

Equation (34) shows that income inequality $\sigma_{I,t}$ is lower than wealth inequality σ_a because labor income is identical across all households. Income inequality $\sigma_{I,t}$ in this model is driven by wealth inequality σ_a, and it increases with the asset-to-wage income ratio $r_t a_t/(w_t l_t)$. Proposition 3 provides the analytical expression for the Gini coefficient of income $\sigma_{I,t}$, which features transition dynamics and is increasing in the output growth rate g_t, as a higher growth rate increases the real interest rate r_t through the Euler equation.

Proposition 3 *The degree of income inequality, measured by the Gini coefficient of income at any time t is given by*

$$\sigma_{I,t} = \left[1 + \frac{\rho}{\rho + g_t} \frac{\gamma + \eta/(1-\eta)}{1 + \rho\Theta/(1-\eta) - \gamma}\right]^{-1} \sigma_a. \tag{35}$$

Proof. See Appendix A. ■

The share of total income owned by the top ε households is given by

$$s_{I,t}^{\varepsilon} \equiv \int_{1-\varepsilon}^{1} s_{I,t}(h)\, dh = \frac{r_t a_t}{r_t a_t + w_t l_t} \int_{1-\varepsilon}^{1} s_{a,0}(h)\, dh + \frac{w_t l_t}{r_t a_t + w_t l_t} \varepsilon = \frac{\sigma_{I,t}}{\sigma_a} \left(s_{a,0}^{\varepsilon} - \varepsilon\right) + \varepsilon, \tag{36}$$

which uses (34) and the wealth share owned by the top ε households denoted by $s_{a,0}^{\varepsilon} = \int_{1-\varepsilon}^{1} s_{a,0}(h)\, dh$. Equation (36) indicates that the income share of the top ε households $s_{I,t}^{\varepsilon}$ is increasing in the Gini coefficient of income $\sigma_{I,t}$ if and only if the top ε households hold a wealth share greater than ε, i.e., $s_{a,0}^{\varepsilon} > \varepsilon$. Since the Gini coefficient of wealth σ_a is stationary, changes in income inequality arise solely from the evolution of the asset-to-wage income ratio $r_t a_t/(w_t l_t)$.

3.4 Dynamics of the consumption distribution

The household h's consumption is given by

$$c_t(h) = \left[r_t - \frac{\dot{a}_t(h)}{a_t(h)}\right] a_t(h) + w_t l_t(h) = \rho a_t(h) + w_t l_t,$$

where we have used the Euler equation in (3), $b_t[L - l_t(h)] = \tau_t(h)$, $l_t(h) = l_t$ and $\dot{a}_t(h)/a_t(h) = \dot{a}_t/a_t = \dot{c}_t/c_t$. This equation indicates that household consumption consists of labor income $w_t l_t$ and a constant fraction ρ of wealth $a_t(h)$. Aggregating across all households yields the aggregate consumption given by

$$c_t = \int_0^1 c_t(h)\, dh = \rho a_t + w_t l_t.$$

The share of consumption owned by household h at time t is

$$s_{c,t}(h) = \frac{\rho a_t(h) + w_t l_t}{\rho a_t + w_t l_t} = \frac{\rho a_t}{\rho a_t + w_t l_t} s_{a,0}(h) + \frac{w_t l_t}{\rho a_t + w_t l_t}. \tag{37}$$

The Gini coefficient of consumption is given by

$$\sigma_{c,t} = \frac{\rho a_t}{\rho a_t + w_t l_t} \sigma_a. \tag{38}$$

which can also be expressed as (see Appendix A)

$$\sigma_{c,t} = \left[1 + \frac{\gamma + \eta/(1-\eta)}{1 + \rho\Theta/(1-\eta) - \gamma}\right]^{-1} \sigma_a. \tag{39}$$

Equation (38) shows that consumption inequality is stationary $\sigma_{c,t} = \sigma_{c,0}$ and is lower than wealth inequality σ_a. Furthermore, comparing equations (35) and (39), we see that consumption inequality $\sigma_{c,t}$ is lower than income inequality $\sigma_{I,t}$ if and only if the growth rate g_t is positive.

The consumption share of the top ε households is

$$s_{c,t}^{\varepsilon} \equiv \int_{1-\varepsilon}^1 s_{c,t}(h)\, dh = \frac{\rho a_t}{\rho a_t + w_t l_t} \int_{1-\varepsilon}^1 s_{a,0}(h)\, dh + \frac{w_t l_t}{\rho a_t + w_t l_t} \varepsilon = \frac{\sigma_{c,t}}{\sigma_a}\left(s_{a,0}^{\varepsilon} - \varepsilon\right) + \varepsilon, \tag{40}$$

which uses (38). Equation (36) states that the consumption share of the top ε households $s_{c,t}^{\varepsilon}$ is increasing in the Gini coefficient of consumption $\sigma_{c,t}$ if and only if the wealth share of the top ε households is larger than ε, i.e., $s_{a,0}^{\varepsilon} > \varepsilon$. Since the Gini coefficient of wealth σ_a is stationary, the only source of change in consumption inequality is the evolution of the asset-to-wage income ratio $r_t a_t/(w_t l_t)$.

4 Dynamic effects of labor union

This section explores the dynamic effects of union bargaining power on economic growth, income inequality, and social welfare analytically and quantitatively. Section 4.1 presents the analytical results. Section 4.2 presents the quantitative results. Section 4.3 performs a welfare analysis.

4.1 Analytical results

We analyze the effects of stronger bargaining power of a wage-oriented labor union γ on growth and inequality. Equations (23) and (28) show that an increase in union bargaining power γ initially reduces the growth rate of output g_t. Equation (23) shows that stronger union bargaining power, a larger γ, leads to a permanent decline in employment l. This negative *market-size* effect reduces the incentives for monopolistic firms to do in-house R&D r_t^q, which reduces the growth rates of quality z_t and output g_t in the short run. However, the market structure is endogenous and the number of firms adjusts gradually. The smaller market size forces some firms to exit the market, which subsequently increases the firm size $x_t l^\eta$ and the rate of return on quality-improving innovation r_t^q. In the long run, the positive *entry* effect offsets the negative market-size effect such that the steady-state growth rate returns to its initial value because g^* is independent of γ. In summary, the endogenous market structure gives rise to distinct effects of γ on g_t at different time horizons; see Figure 4 for an illustration in which γ increases at time t.

The effect of union bargaining power on income inequality is determined by both the rate of return on assets r_t and the asset-to-wage income ratio $a_t/w_t l_t$. Equation (35) shows that an increase in γ reduces income inequality $\sigma_{I,t}$ in the short run. The short-run effect operates through two channels. A negative *interest-rate* effect arises because a larger γ lowers the growth rate g_t, which in turn reduces the real interest rate r_t via the Euler equation. There is also a negative *income-share* effect, captured by $a_t/w_t l_t$, which arises because a larger γ increases the labor income share and reduces the profit share and the ratio of assets to output a_t/Y_t. Since both effects work in the same direction, the short-run effect of a larger γ on $\sigma_{I,t}$ is negative. In the long run, the interest-rate effect vanishes due to the endogenous market structure, leading to a scale-invariant steady-state growth rate g^* and real interest rate r^*. However, the negative income-share effect persists, implying that an increase in union bargaining power results in a permanent reduction in income inequality σ_I^*; see Figure 5 for an illustration in which γ increases at time t. Proposition 4 summarizes the above results.[6]

Proposition 4 *A stronger bargaining power of a wage-oriented labor union has the following effects on economic growth and income inequality: (a) it reduces economic growth and income inequality in the short run; and (b) it reduces income inequality but does not affect the steady-state growth rate in the long run.*

Proof. Proven in text. ■

Equation (39) shows that a larger γ results in a one-time permanent decrease in consumption inequality $\sigma_{c,t}$ (see Figure 6), which is caused by the negative income-share effect, captured by $\rho a_t/w_t l_t$. Unlike the effect on income inequality, the interest-rate effect is absent because the log utility function ensures that consumption distribution remains stationary.

[6] For an employment-oriented union, i.e., $\omega < 1$, a stronger union bargaining power increases economic growth and has an ambiguous effect on income inequality in the short run, and it reduces income inequality but does not affect the steady-state growth rate in the long run. This is because a larger γ increases employment l and the output growth rate g_t in the short run. The higher growth rate leads to a positive interest-rate effect, which combined with the negative income-share effect, results in an ambiguous effect of γ on $\sigma_{I,t}$ in the short run.

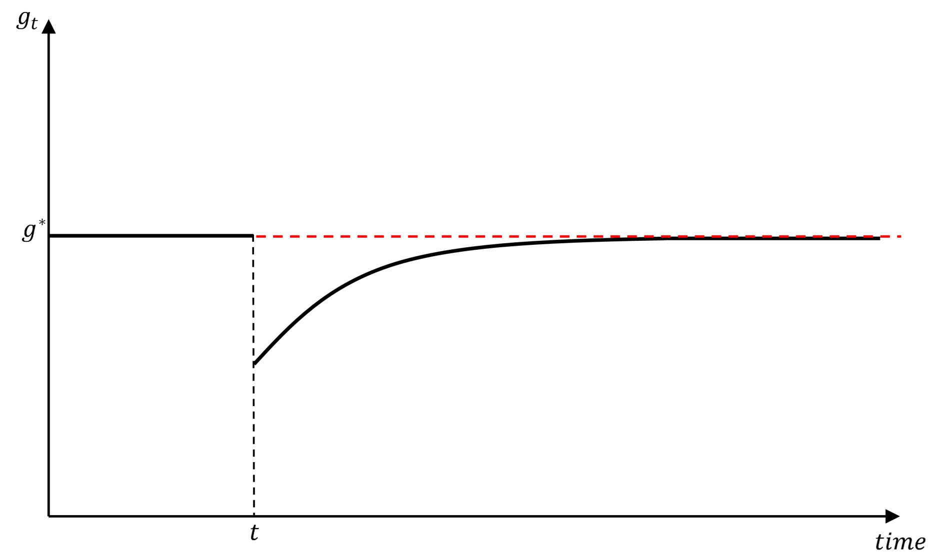

Figure 4. Dynamic effects of labor union on economic growth

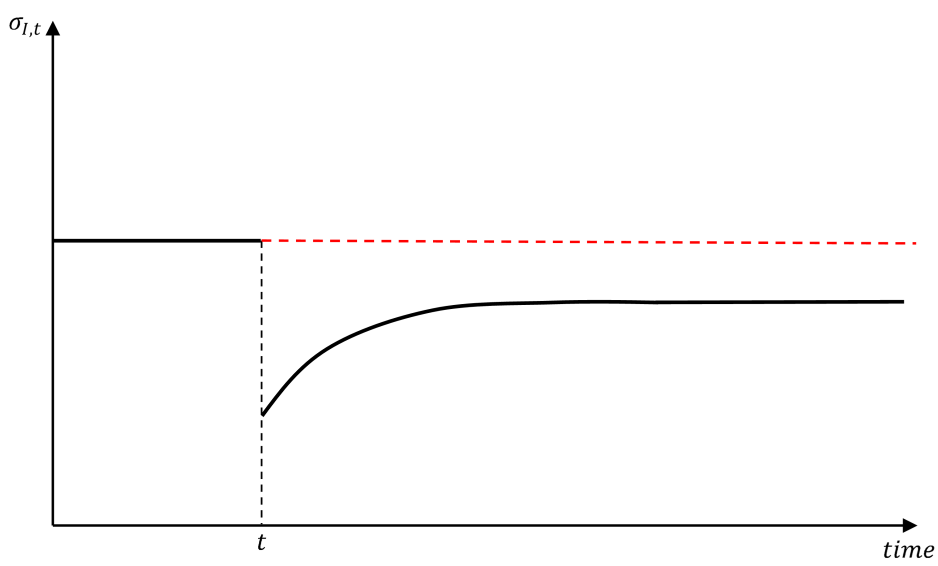

Figure 5. Dynamic effects of labor union on income inequality

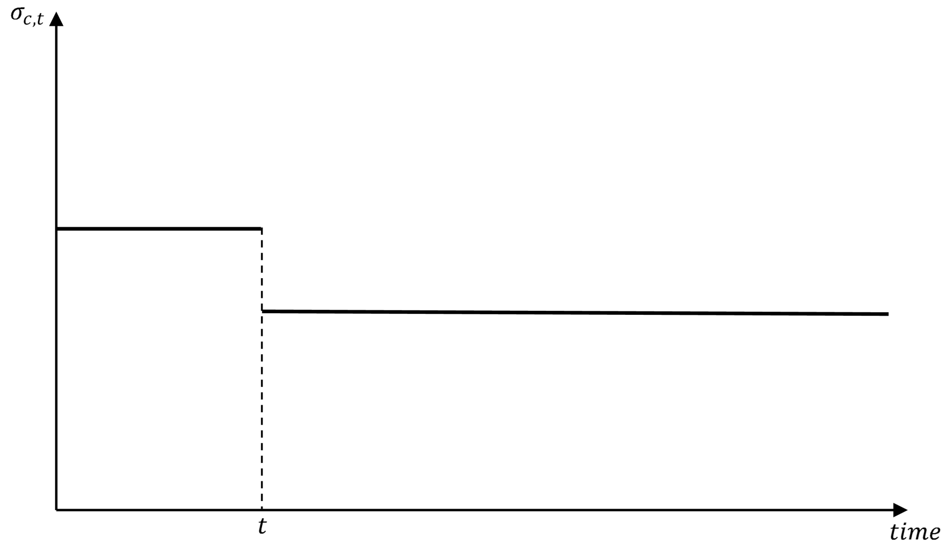

Figure 6. Dynamic effects of labor union on consumption inequality

4.2 Quantitative analysis

In this section, we calibrate the model using aggregate data from the U.S. to perform a quantitative analysis of the effects of union bargaining power on economic growth and income inequality. We assume that the economy is initially in a steady state in 1980 and then converges to a new steady state in 2016. The model features the following 10 parameters $\{\rho, \alpha, \theta, \eta, \beta, \phi, \mu, \omega, \bar{b}, \gamma\}$. In the baseline, we set the discount rate ρ to a conservative value of 0.03, and also consider $\rho \in \{0.04, 0.05\}$ in the welfare analysis; see Table 1C in Appendix C. The long-run growth rate of GDP per capita g^* is 2%, implying a long-run interest rate r^* of 5%. We follow Iacopetta et al. (2019) to set the private return to quality α to 1/6, which implies that the degree of technology spillovers $1 - \alpha$ is 5/6. Vollrath (2024) reports that the elasticity of output with respect to capital is between 0.21 and 0.39, and we set θ to the mean value of 0.3. We set η to 0.7, ensuring that the labor income share remains within a reasonable range of 0.49 to 0.7 for $\gamma \in (0, 1)$. We calibrate the parameters $\{\beta, \phi, \mu\}$ by targeting the following moments:[7] the consumption share of output is 72%, the growth rate of output per capita is 2%, and the investment rate is 22%.[8] We calibrate the union bargaining power γ by targeting the labor income share of output, which is 63.0% in 1980 and 56.9% in 2016. We calibrate the parameters $\{\omega, \bar{b}\}$ by targeting the unemployment rate,

[7] Data source: OECD Statistics for consumption share and investment rate, U.S. Bureau of Labor Statistics for labor income share (Giandrea and Sprague, 2017), World Bank Open Data for unemployment rate, U.S. Bureau of Economic Analysis for profit share, Current Population Survey for Gini coefficient of income, and World Inequality Database for wealth-income ratio, Gini coefficient of wealth, top 10% wealth share, and top 10% income share.

[8] Investment includes expenses on intermediate inputs and R&D.

which is 8.4% in 1980 and 6.2% in 2016.[9]

Table 1 Calibrated parameter values

ρ	α	θ	η	β	ϕ	μ	ω	$\bar{b}$	γ
0.030	0.167	0.300	0.700	3.050	0.840	1.373	1.183	0.507	0.667→0.376

Table 1 summarizes the calibrated parameter values. The markup μ is 1.373, which is within the range of markups estimated in the literature (De Loecker et al., 2020). The parameter ω is larger than 1, indicating that the labor union in the U.S. is wage-oriented. The parameter γ decreases from 0.667 in 1980 to 0.376 in 2016, indicating a decline in union bargaining power over this period. The model predicts that the profit share of output increases from 9.0% in 1980 to 15.1% in 2016, while the data are 9.0% in 1980 and 13.5% in 2016. The model predicts that the wealth-income ratio increases from 3.8 in 1980 to 6.1 in 2016, while the data are 3.9 in 1980 and 4.5 in 2016. Given that the Gini coefficient of wealth is 0.800 in 1980 and 0.878 in 2016, the model predicts that the Gini coefficient of income increases from 0.154 in 1980 to 0.269 in 2016, while the data are 0.404 in 1980 and 0.481 in 2016. This suggests that the model explains, on average, 48% of income inequality as measured by the Gini coefficient. The model also predicts that the Gini coefficient of consumption increases from 0.100 in 1980 to 0.184 in 2016, which is lower than the Gini coefficient of income. Given that the top 10% wealth share is 63.8% in 1980 and 74.1% in 2016, the model predicts that the top 10% income share increases from 20.3% in 1980 to 29.7% in 2016, while the data are 37.7% in 1980 and 47.9% in 2016. This numerical exercise shows that the model predicts a lower degree of income inequality, as measured by the Gini coefficient and top 10% income share, than the data observed in the U.S. economy. This is reasonable, as income inequality in the model is only driven by wealth inequality, but other factors such as wage inequality also influence income inequality. The magnitudes of the changes in income inequality predicted by the model are more consistent with data. The model predicts that the Gini coefficient of income increases by 0.115 and top 10% income share increases by 9.3% from 1980 to 2016, and the data show increases of 0.077 and 10.2% respectively.

Based on the these calibration results, we simulate the transition paths for firm size $x_t l^\eta$ (Figure 7(a)), growth rate g_t (Figure 7(b)), income inequality $\sigma_{I,t}$ (Figure 7(c)), and consumption inequality (Figure 7(d)) by raising the union bargaining power γ from 0.376 (the value in 2016) to 0.667 (the value in 1980). When the union bargaining power strengthens, employment l decreases permanently from 0.938 to 0.916, which leads to a rise in the unemployment rate from 6.2% to 8.4%. As a result, firm size $x_t l^\eta$ declines from 4.81 to 4.73, causing a contemporaneous decrease in the growth rate g_t from 2.0% to 1.69%. Meanwhile, the labor income share increases permanently from 56.9% to 63.0%, and consumption inequality $\sigma_{c,t}$ declines permanently by 40.4%. The higher labor income share and the lower growth rate lead to a reduction in income inequality $\sigma_{I,t}$ by 40.5%. As firms exit the market, firm size recovers gradually to its initial value of 4.81, and the growth rate increases back to

[9] We use the Hodrick-Prescott Filter to extract trends for labor income share, unemployment rate, profit share, wealth-income ratio, Gini coefficient of wealth, Gini coefficient of income, and top 10% income share and top 10% wealth share.

initial level of 2.0%, leading to a gradual rise in income inequality by 3.2%. Thus, the the long run reduction in income inequality is 37.3%.

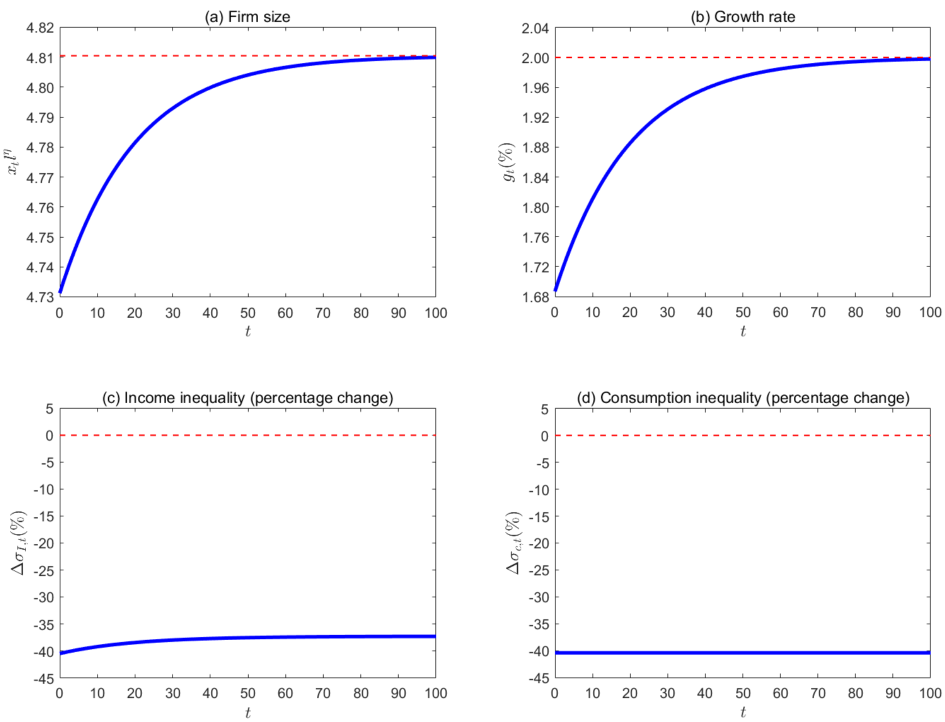

Figure 7. Simulated dynamic effects of labor union

To conclude this section, we calculate the income share of each decile based on the calibrated parameter values and the wealth share of each decile, using (36), and simulate the effects of raising γ on the steady-state income distribution; see Figure 8.[10] The red bars represent data in 2016, the blue bars represent the income shares predicted by the model, and the yellow bars represent the simulated steady-state income distribution after raising γ from 0.376 to 0.667 (the union bargaining power in 1980). This numerical exercise shows that the model generates a more equal income distribution than the data, which has a much heavier right tail. It is worth noting that a household belonging to a particular income decile also belongs to the same wealth decile, because income inequality in the model is solely driven by wealth inequality. When the union bargaining power strengthens, the shape of income distribution flattens in the new steady state. The top 10% income share (D10) decreases by 7.2%, while the bottom 50% income share (D1 to D5) increases by 5.8%. In summary,

[10] World Inequality Database (WID) provides the wealth and income shares of ten wealth and income deciles. In Figure 8, D1, D2,..., and D10 represent the first 10% income share, the second 10% income share,..., and the top 10% income share respectively.

a larger γ has a positive (negative) effect on the income shares for the deciles whose wealth shares are smaller (larger) than average $s^d_{a,0} < 0.1$ (> 0.1) as shown in (36).

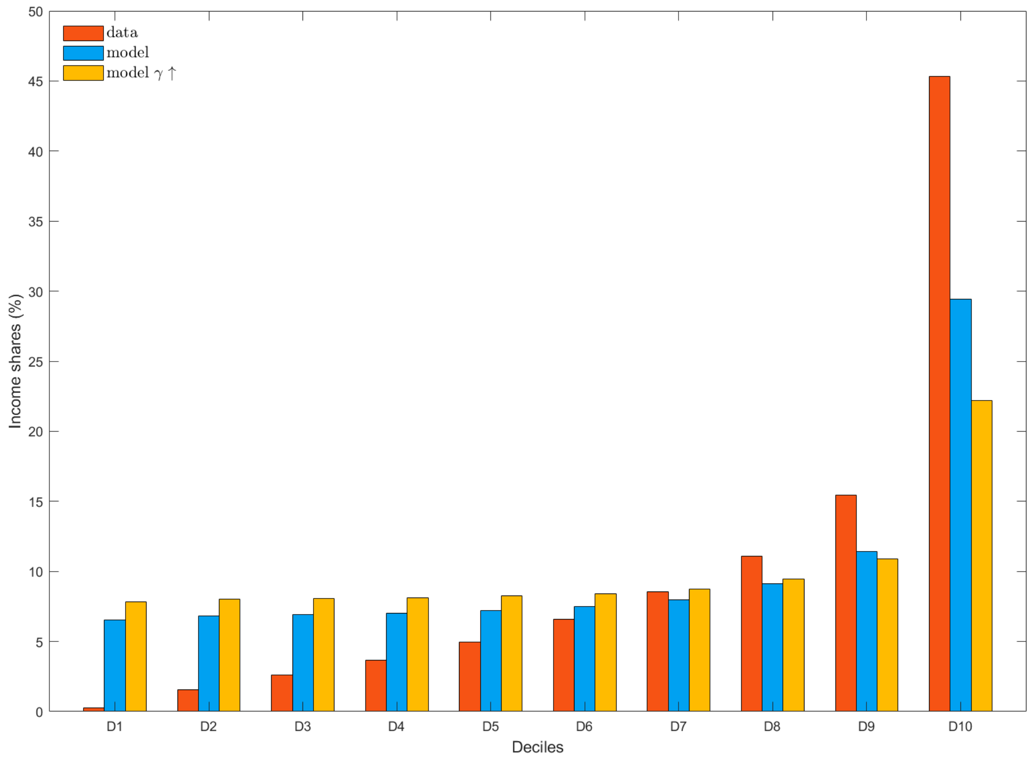

Figure 8. Simulated effects of labor union on income distribution

4.3 Welfare analysis

In this section, we conduct a welfare analysis to assess the effects of raising γ on social welfare. The model features a continuum of households, but the data provides wealth and income shares of different cohorts, such as quintiles and deciles. We use (40) to calculate the consumption share of each decile to simulate the effects of raising γ on the welfare of each decile as an approximation. When union bargaining power rises at $t = 0$, the change in lifetime utility for the d-th decile can be expressed as

$$\Delta U^d = \underbrace{\frac{\Delta \ln s^d_{c,0}}{\rho}}_{\substack{+\ (-)\ \text{for}\ s^d_{c,0}<0.1\ (>0.1) \\ \textit{consumption-share effect}}} + \underbrace{\frac{\eta \Delta \ln l}{\rho}}_{\substack{- \\ \textit{employment effect}}} + \underbrace{\int_0^\infty e^{-\rho t} \int_0^t \Delta g_s ds dt}_{\substack{- \\ \textit{growth effect}}}.$$

This expression consists of three components. The *consumption-share* effect is positive (negative) if the decile's wealth share is smaller (larger) than average $s^d_{a,0} < 0.1$ (> 0.1), as shown in (40). Both the *employment* effect and the *growth* effect are negative, because a larger γ

reduces employment l and transitional growth rate g_t. Then we compute the welfare effects of raising γ from 0.376 to 0.667 for each decile, measured in terms of consumption-equivalent welfare changes. Table 2 summarizes the results. If labor union bargaining power were restored to its 1980 level, the welfare loss for the top 10% would be equivalent to a 27.3% reduction in annual consumption, whereas the welfare gain for the bottom 10% would be equivalent to a 6.2% increase in annual consumption. We also report the welfare effects under $\rho \in \{0.04, 0.05\}$ in Table 2C(a) and Table 2C(b) in Appendix C, which demonstrate the robustness of our results. In summary, strengthening union bargaining power reduces welfare for the top 30% of households, with the losses being more pronounced for higher-income deciles, while it increases welfare for the bottom 70%, with larger gains for lower-income deciles. Notably, households in the 70%-80% income decile (D8) experience a welfare loss, as the negative employment and growth effects outweigh the positive consumption-share effect for this group.

Table 2 Effects of γ on welfare: consumption equivalent

Deciles	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10
$\Delta U^d(\%)$	6.21	4.96	4.56	4.27	3.39	2.35	0.60	-3.16	-8.96	-27.33

5 Conclusion

In this paper, we explore the dynamic effects of labor unions on economic growth, inequality, and welfare in a Schumpeterian growth model with heterogeneous households and endogenous market structure. A representatitve labor union bargains with a representative employer to determine the equilibrium employment and wages. Households own different amounts of wealth, leading to an unequal distribution of income. Our analysis shows that an increase in the bargaining power of a wage-oriented labor union has negative effects on economic growth and income inequality in the short run. In the long run, the increase in union bargaining power continues to have a negative effect on income inequality, but does not affect the steady-state growth rate. Calibrating the model to U.S. data, we find that restoring union bargaining power to its 1980 level would result in welfare losses for the top 30% of households, with significantly larger losses among higher-income groups. Conversely, the bottom 70% of households would experience welfare gains, which are disproportionately larger for lower-income groups.

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Appendix A: Proofs

Rate of return to quality-improving innovation. The dynamic optimization problem of monopolistic firm i is defined in the following current-value Hamiltonian:

$$H_t(i) = \pi_t(i) - R_t(i) + \lambda_t(i)\dot{Z}_t(i) + \xi_t(i)[\mu - p_t(i)], \tag{A1}$$

where $\lambda_t(i)$ is the co-state variable on (12), $\xi_t(i)$ is the multiplier on the constraint $p_t(i) \leq \mu$, and μ is the upper bound on price $p_t(i)$. By substituting (6), (12), and (13) into (A1), we derive

$$\frac{\partial H_t(i)}{\partial p_t(i)} = 0 \Rightarrow \frac{\partial \pi_t(i)}{\partial p_t(i)} = \xi_t(i), \tag{A2}$$

$$\frac{\partial H_t(i)}{\partial R_t(i)} = 0 \Rightarrow \lambda_t(i) = 1, \tag{A3}$$

$$\frac{\partial H_t(i)}{\partial Z_t(i)} = \alpha\left\{[p_t(i) - 1]\left[\frac{\theta}{p_t(i)}\right]^{1/(1-\theta)}\left(\frac{l_t}{N_t}\right)^{\eta} - \phi\right\} Z_t^{\alpha-1}(i)\, Z_t^{1-\alpha} = r_t\lambda_t(i) - \dot{\lambda}_t(i). \tag{A4}$$

If the constraint on $p_t(i)$ is not binding, i.e., $p_t(i) < \mu$, then $\xi_t(i) = 0$. In this case, $\partial\pi_t(i)/\partial p_t(i) = 0$ yields $p_t(i) = 1/\theta$. If the constraint on $p_t(i)$ is binding, then $\xi_t(i) > 0$. In this case, $p_t(i) = \mu$. Given that $\mu < 1/\theta$, the monopolistic firm sets its price at $p_t(i) = \mu$. Substituting (A3), (15), and $p_t(i) = \mu$ into (A4) and imposing symmetry, we obtain the rate of return to quality-improving innovation as given by (16).

Proof of Lemma 2. By substituting (18) and $\theta Y_t = N_t(\mu X_t)$ into the aggregate value of assets $a_t = v_t + N_t V_t$, we obtain

$$a_t = v_t + N_t\beta X_t = v_t + (\theta/\mu)\beta Y_t. \tag{A5}$$

Differentiating (A5) with respect to time t and substituting it and (A5) into (A14) yield

$$\dot{v}_t + (\theta/\mu)\beta\dot{Y}_t = \dot{a}_t = r_t[v_t + (\theta/\mu)\beta Y_t] + w_t l_t - c_t. \tag{A6}$$

Using (3) for r_t, $r_t v_t = \Pi_t + \dot{v}_t$, (21) and (22), (A6) can be rearranged as

$$\frac{\dot{c}_t}{c_t} - \frac{\dot{Y}_t}{Y_t} = \frac{\mu}{\beta\theta}\frac{c_t}{Y_t} - \left[\rho + \frac{\mu(1-\theta)}{\beta\theta}\right]. \tag{A7}$$

which implies that the consumption-output ratio c_t/Y_t must jump to its steady-state value in (26) to satisfy the transversality condition of households. Finally, substituting (3), (21) and (26) into (A14) yields

$$\frac{\dot{c}_t}{c_t} - \frac{\dot{a}_t}{a_t} = \frac{(1-\theta)(1-\eta)(1-\gamma) + \rho\beta\theta/\mu}{1 - \theta + \rho\beta\theta/\mu}\frac{c_t}{a_t} - \rho, \tag{A8}$$

which implies that the consumption-wealth ratio c_t/a_t must jump to its steady-state value in (27) to satisfy the transversality condition of households.

Proof of Proposition 1. Substituting (28) into (25), we obtain the quality growth rate given by

$$z_t = \alpha \left[\frac{\mu - 1}{\mu^{1/(1-\theta)}} x_t l^{\eta} - \phi \right] - \rho - (1 - \eta) n_t. \tag{A9}$$

Substituting (19) and (A9) into $(1 - \eta) n_t + z_t = g_t = r_t - \rho = r_t^e - \rho$, we derive the variety growth rate given by

$$n_t = \frac{[(1-\alpha)(\mu - 1) - \beta\rho] \left[x_t l^{\eta} / \mu^{1/(1-\theta)} \right] - (1-\alpha)\phi + \rho}{(\beta x_t l^{\eta}) / \mu^{1/(1-\theta)} - (1 - \eta)}, \tag{A10}$$

which also uses $\dot{l}_t = 0$ from (23). Then we substitute $\dot{x}_t / x_t = -\eta n_t$ from (15) into (A10) to obtain the dynamics of x_t in (29), where we approximate $(1 - \eta) \mu^{1/(1-\theta)} / (x_t l^{\eta}) \approx 0$. Rewriting (29) as $\dot{x}_t = d_1 - d_2 x_t$, the one-dimensional linear differential equation for x_t has a unique steady state that is globally stable if

$$d_1 \equiv \frac{\eta \mu^{1/(1-\theta)}}{\beta l^{\eta}} [(1-\alpha)\phi - \rho] > 0, \tag{A11}$$

$$d_2 \equiv \frac{\eta [(1-\alpha)(\mu - 1) - \beta\rho]}{\beta} > 0. \tag{A12}$$

Parameter restrictions $\rho < \min \{(1-\alpha)\phi, (1-\alpha)(\mu - 1)/\beta\}$ follow from (A11) and (A12). Finally, setting $\dot{x}_t = 0$ yields the steady-state value of firm size $x^* = d_1/d_2$ in (30), and substituting (30) into (28) gives the steady-state value of output growth rate g^* in (31).

Proof of Proposition 2. Lemma 2 demonstrates that the consumption-output ratio c_t/Y_t and the consumption-wealth ratio c_t/a_t are constant. This implies that the wealth-output ratio $a_t/Y_t = (c_t/Y_t) / (c_t/a_t)$ jumps to its steady-state value given by

$$\frac{a_t}{Y_t} = \frac{v_t + N_t V_t}{Y_t} = \frac{(1-\theta)(1-\eta)(1-\gamma)}{\rho} + \frac{\beta\theta}{\mu}. \tag{A13}$$

Additionally, the ratio of market value of final-good firm to final output jumps to its steady-state value $v_t/Y_t = (1-\theta)(1-\eta)(1-\gamma)/\rho$. Aggregating (2) across all households yields the aggregate asset-accumulation equation:

$$\dot{a}_t = r_t a_t + w_t l_t - c_t, \tag{A14}$$

which also uses the balanced-budget condition in (20). From (2), we can derive

$$\frac{\dot{a}_t(h)}{a_t(h)} = r_t + \frac{w_t l_t}{a_t(h)} - \frac{c_t(h)}{a_t(h)}, \tag{A15}$$

which uses $\tau_t(h) = b_t [L - l_t(h)]$ and $l_t(h) = l_t$. The growth rate of $s_{a,t}(h)$ is

$$\frac{\dot{s}_{a,t}(h)}{s_{a,t}(h)} = \frac{\dot{a}_t(h)}{a_t(h)} - \frac{\dot{a}_t}{a_t} = \frac{w_t l_t - c_t(h)}{a_t(h)} - \frac{w_t l_t - c_t}{a_t}, \tag{A16}$$

which uses (A14). Rearranging (A16) yields

$$\dot{s}_{a,t}(h) = \frac{c_t - w_t l_t}{a_t} s_{a,t}(h) - \frac{s_{c,t}(h) c_t - w_t l_t}{a_t}, \tag{A17}$$

where $s_{c,t}(h) = s_{c,0}(h)$ is constant for $h \in [0,1]$. Subtituting (21) for $w_t l_t$, (26) for c_t/Y_t, (27) for c_t/a_t, and (A13) for a_t/Y_t into (A17) yields

$$\dot{s}_{a,t}(h) = \rho s_{a,t}(h) - s_{c,0}(h) \frac{\rho(\rho\beta\theta/\mu + 1 - \theta)}{\rho\beta\theta/\mu + (1-\theta)(1-\eta)(1-\gamma)} + \frac{\rho(1-\theta)[\eta + (1-\eta)\gamma]}{\rho\beta\theta/\mu + (1-\theta)(1-\eta)(1-\gamma)}. \tag{A18}$$

The dynamics of $s_{a,t}(h)$ is determined by the above one-dimensional differential equation. $\dot{s}_{a,t}(h) = 0$ must hold for any $t \geq 0$ to ensure the stability of $s_{a,t}(h)$, because the coefficient of $s_{a,t}(h)$ is a positive constant. This condition can be satisfied if and only if $s_{c,0}(h)$ jumps to its steady-state value given by

$$s_{c,0}(h) = \frac{\eta + (1-\eta)\gamma}{\rho\Theta + 1} + \frac{\rho\Theta + (1-\eta)(1-\gamma)}{\rho\Theta + 1} s_{a,0}(h), \tag{A19}$$

where $\Theta \equiv \beta\theta / [\mu(1-\theta)]$.

Proof of Proposition 3. The Gini coefficient of income at time t is given by

$$\sigma_{I,t} = 1 - 2\int_0^1 \mathcal{L}_{I,t}(h)\, dh, \tag{A20}$$

where the Lorenz curve of income is defined as

$$\mathcal{L}_{I,t}(h) \equiv \frac{\int_0^h I_t(\chi)\, d\chi}{\int_0^1 I_t(\chi)\, d\chi} = \frac{r_t a_t \int_0^h s_{a,0}(\chi)\, d\chi + w_t l_t \int_0^h 1 d\chi}{r_t a_t + w_t l_t}. \tag{A21}$$

Substituting (A21) into (A20) yields

$$\sigma_{I,t} = 1 - \frac{2 r_t a_t}{r_t a_t + w_t l_t}\left[\int_0^1 \mathcal{L}_a(h)\, dh + \frac{w_t l_t}{r_t a_t}\int_0^1 h dh\right]. \tag{A22}$$

Substituting $\sigma_a = 1 - 2\int_0^1 \mathcal{L}_a(h)\, dh$ into (A22) yields the expression for the Gini coefficient of income in (34). Using (3) for r_t, (21) for $w_t l_t$, and (A13) for a_t, we obtain

$$\frac{r_t a_t}{w_t l_t} = \left[\frac{(1-\theta)(1-\eta)(1-\gamma)}{\rho} + \frac{\beta\theta}{\mu}\right] \frac{\rho + g_t}{(1-\theta)[\eta + (1-\eta)\gamma]}. \tag{A23}$$

Substituting (A23) into (34) yields (35).

Proof of Gini coefficient of consumption. The Gini coefficient of consumption at time t is given by

$$\sigma_{c,t} = 1 - 2\int_0^1 \mathcal{L}_{c,t}(h)\, dh, \tag{A24}$$

where the Lorenz curve of consumption is defined as

$$\mathcal{L}_{c,t}(h) \equiv \frac{\int_0^h c_t(\chi)\, d\chi}{\int_0^1 c_t(\chi)\, d\chi} = \frac{\rho a_t \int_0^h s_{a,0}(\chi)\, d\chi + w_t l_t \int_0^h 1 d\chi}{\rho a_t + w_t l_t}. \tag{A25}$$

Substituting (A25) into (A24) yields

$$\sigma_{c,t} = 1 - \frac{2\rho a_t}{\rho a_t + w_t l_t}\left[\int_0^1 \mathcal{L}_a(h)\, dh + \frac{w_t l_t}{\rho a_t}\int_0^1 h dh\right]. \tag{A26}$$

Substituting $\sigma_a = 1 - 2\int_0^1 \mathcal{L}_a(h)\, dh$ into (A26) yields the expression for the Gini coefficient of consumption in (38). Finally, substituting (21) and (A13) into (38) yields (39).

Appendix B: Panel VAR

In this appendix, we provide a detailed discussion on panel VAR analysis carried out in Introduction. We conduct the panel VAR analysis to investigate the dynamic relationship between economic growth and union density. The dataset comprises annual observations for 20 developed countries from 1980 to 2016. Economic growth is measured using annually growth rate of GDP per capita from the World Bank Open Data, while union density is sourced from the OECD Statistics. We present descriptive statistics in Table B1 and panel unit-root tests in Table B2. The panel unit-root tests are performed to examine the stationarity of the data, with results indicating that the time series for economic growth are stationary, but the time series for union density are non-stationary. So we difference the time series of union density, and the resulting first-order differenced union density time series (d_union_density) are stationary.

We estimate a recursive panel VAR model with a maximum of 2 lags to capture the dynamics in the data. Patent shocks are identified using Cholesky decomposition of the variance-covariance matrix of residuals. The model employs the GMM (Generalized Method of Moments) estimator, which is particularly effective in handling unobserved country heterogeneity. The variables are ordered as [d_union_density, growth], reflecting the theoretical assumption that union density is more exogenous than economic growth. A panel VAR-Granger causality Wald test confirms that union density is exogenous among the variables.

The primary objective is to track the response of economic growth to a union density shock. Figure 3 presents the impulse response functions, which show that a one standard deviation positive shock in union density leads to an initial decrease in economic growth, followed by a convergence to zero in the long run. The 90% confidence bands, derived from bootstrapping with 1000 draws, indicate the statistical significance of the responses. Figure B3 displays the eigenvalue stability condition graph, confirming that all eigenvalues lie within the unit circle, thus satisfying the stability condition for the panel VAR model.

Table B1 Descriptive statistics

variable	obs	mean	std. dev.	min	max
growth (%)	665	2.520	2.662	-8.074	25.16
d_union_density (%)	643	-0.405	0.933	-5.300	4.800

Table B2 Panel unit-root tests

variable	Inverse χ^2(x)-P	Modified Inverse χ^2(x)-Pm
growth (%)	259.299***	25.385***
d_union_density (%)	306.508***	30.800***

H0: the panel variable has a unit root; H1: the panel variable is stationary. The Fisher-type unit-root test, using Phillips-Perron tests, assesses the unit root null hypothesis against the stationary alternative. *, **, and *** denote significance at the 10%, 5%, and 1% levels, respectively.

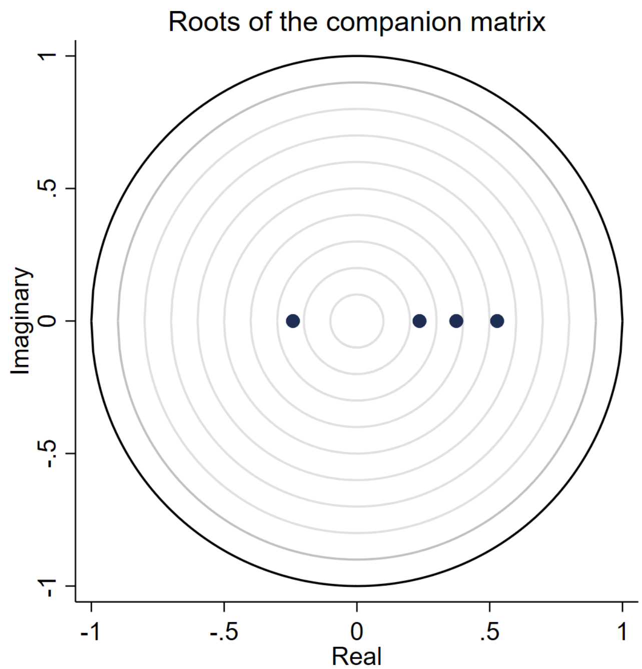

Figure B2 Eigenvalue stability condition

Appendix C: Robustness Checks

Table 1C Calibrated parameter values: Robustness check

ρ	α	θ	η	β	ϕ	μ	ω	$\bar{b}$	γ
0.040	0.167	0.300	0.700	2.285	1.020	1.371	1.183	0.507	0.667→0.376
0.050	0.167	0.300	0.700	1.826	1.200	1.370	1.183	0.507	0.667→0.376

Table 2C(a) Effects of γ on welfare: consumption equivalent ($\rho = 0.04$)

Deciles	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10
$\Delta U^d(\%)$	6.54	5.30	4.89	4.60	3.72	2.67	0.92	-2.86	-8.68	-27.10

Table 2C(b) Effects of γ on welfare: consumption equivalent ($\rho = 0.05$)

Deciles	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10
$\Delta U^d(\%)$	6.73	5.48	5.08	4.78	3.90	2.85	1.10	-2.69	-8.52	-26.97