Leveraging Firm Entry Policy to Drive Innovation, Growth, and Reduce Income Inequality, in the Presence of Entry Threats and Rent-Seeking

R S, Vaidyanathan and Keswani Mehra, Meeta

Centre for International Trade and Development, School of International Studies, Jawaharlal Nehru University, New Delhi

20 March 2025

Leveraging Firm Entry Policy to Drive Innovation, Growth, and Reduce Income Inequality, in the Presence of Entry Threats and Rent-Seeking*

R S Vaidyanathan † Meeta Keswani Mehra ‡

Abstract

This paper presents an analytical model that investigates the dynamics of rent-seeking, innovation, and entry policies in a two-sector economy characterized by skilled and unskilled labor. The model explores how incumbent firms in an intermediate goods sector react to the threat of new entrants and how rent-seeking behavior influences innovation and economic productivity. A key feature of the model is the role of a policymaker who sets firm entry policies and responds to bribes offered by incumbent firms seeking to restrict market entry.

The analysis distinguishes between advanced and backward incumbent firms. Advanced firms, which operate at the frontier of technological productivity, choose to innovate to retain their competitive position in response to entry threats. In contrast, backward firms face higher barriers to innovation and are more likely to bribe policymakers to deter new competition. The magnitude of the bribes depends on the difference in profits with and without entry threats, as well as the costs of innovation.

*Comments received at the "Research Scholars Workshop" hosted by the Department of Economics of Calcutta University have been invaluable in developing this paper.

†PhD Scholar, Centre for International Trade and Development, School of International Studies, Jawaharlal Nehru University, New Delhi, India. Email: rsvaidya59@gmail.com

‡Professor of Economics, Centre for International Trade and Development, School of International Studies, Jawaharlal Nehru University, New Delhi, India. Email: meetakm@jnu.ac.in

The model highlights how rent-seeking by backward firms distorts market competition, leading to suboptimal innovation and lower aggregate productivity in the skilled sector. Policymakers, balancing between maximizing bribes and addressing wage inequality, face conflicting incentives. If a policymaker prioritizes welfare, they may restrict entry to reduce wage inequality, thereby lowering competitive pressures and innovation. Alternatively, a policymaker focused on maximizing bribes may encourage higher entry threats, fostering innovation but exacerbating income inequality.

This paper contributes to the literature on rent-seeking and economic growth by providing a nuanced understanding of how firm behavior, entry policies, and innovation are interlinked, with important implications for labor markets and income inequality. The model provides insights into the broader economic consequences of rent-seeking behavior and entry regulation, emphasizing the need for balanced policies that encourage innovation while minimizing economic distortions caused by rent-seeking activities.

Keywords: Lobbying, Market Structure, R&D Investment, Growth, Welfare

JEL Classification: H11, I31, O32, O40, P00

1 Introduction

The relationship between economic growth, income inequality, and institutional dynamics remains a cornerstone of modern economic research. While a vast body of literature has explored these themes, much of it has evolved along two distinct but interconnected strands. The first strand focuses on growth and income inequality within the framework of Schumpeterian growth, emphasizing how creative destruction drives innovation and long-term growth. The second strand examines the political economy of growth, highlighting the role of institutional structures, such as democracy and rent-seeking behavior, in shaping economic trajectories. While each strand has significantly deepened our understanding of growth dynamics, few studies have integrated them in a way that also considers the distributional consequences of innovation. This paper aims to bridge this gap by developing a unified framework where income inequality and political economy considerations jointly determine innovation incentives and growth outcomes in a dual-sector economy.

The Schumpeterian growth literature, rooted in Joseph Schumpeter's concept of creative destruction [Schumpeter (1942)], places innovation at the heart of economic progress. In these models, new technologies and products replace outdated ones, with incumbent firms displaced by more innovative entrants. This process ensures dynamic efficiency and long-term growth but simultaneously creates winners and losers, leading to distributional consequences. Grossman and Helpman (1991) were among the first to highlight the role of knowledge spillovers, showing that innovations by frontier firms reduce the cost of future innovations. However, these spillovers are often unevenly distributed, especially when backward sectors lack the absorptive capacity to benefit from them. This sectoral divergence can exacerbate income inequality, particularly between skilled and unskilled labor markets.

Despite its emphasis on innovation, traditional Schumpeterian theory often overlooked the distributional consequences of growth. However, recent contributions have addressed this gap. For example, Xu (2013) explores the growth-inequality nexus in dual-sector economies, highlighting how urban-rural divides—analogous to skilled-unskilled labor divides—lead to persistent income disparities. Xu demonstrates that innovation asymmetries between sectors can reinforce inequality, as backward sectors lag in adopting new technologies. This observation raises critical questions: How do policymakers balance the growth benefits of frontier innovation with the inequality costs of sectoral divergence? What role do market entry policies play in moderating these effects?

Similarly, Kaufman (2018) argues that Schumpeterian innovation cycles can endogenously generate income inequality, particularly when unskilled sectors fail to adapt to technological advancements. In his macro-political economy model, inequality emerges not just from market forces but also from institutional responses that may exacerbate disparities. Kaufman's findings underscore the need for growth models that incorporate both sectoral heterogeneity and distributional concerns, particularly when innovation incentives are influenced by institutional settings. Addressing this gap requires integrating insights from the political economy literature, which examines how institutions and policy choices shape growth trajectories.

While Schumpeterian models have increasingly addressed distributional issues, the political economy literature has focused on how institutions mediate the relationship between growth and innovation, particularly in contexts where rent-seeking behavior by incumbents threatens to stifle competition. A central theme in this literature is the role of democracy in shaping policy choices that affect market entry and innovation incentives.

Barro (1999) provides an empirical examination of the determinants of democracy, arguing that while democracy is correlated with higher income levels, the causal relationship between democracy and growth remains unclear. Barro's findings suggest that democracy often emerges endogenously as economies develop, implying that institutional structures cannot be treated as exogenous when analyzing growth. This perspective is critical for understanding how policymaker preferences, shaped by income distribution concerns, influence decisions about market entry regulation and innovation incentives.

Building on this theme, Aghion et al. (2004) develop a theoretical model of endogenous political institutions, showing how majoritarian systems are more likely to arise in societies with high inequality due to their capacity for redistribution. Conversely, proportional systems emerge in more equal societies, where redistribution is less of a concern. This distinction is critical for understanding the institutional foundations of growth because it implies that institutional choices are not merely responses to economic development but also determinants of how growth unfolds. In dual-sector economies, where skilled and unskilled labor markets create divergent pressures, the potential for institutional conflict is heightened. Policymakers in such contexts must navigate trade-offs between promoting innovation and addressing inequality—trade-offs that this paper explicitly models.

Acemoglu et al. (2005) add complexity by showing that the positive correlation between democracy and income disappears when country fixed effects are accounted for. This suggests that the relationship between democracy and growth is shaped by historical and institutional factors that persist over time. The implication is that rent-seeking behavior by incumbents can become institutionalized, leading to persistent entry barriers and innovation blockages. These historical institutional arrangements complicate efforts to design growth-friendly policies, especially when income inequality remains a pressing concern.

However, even when democratic institutions are in place, they do not guarantee growth-enhancing policies. Mulligan et al. (2004) show that democracies are not inherently better at promoting growth, as they often adopt policies that are uncorrelated with innovation, such as education spending or corporate taxation. This raises a critical question: What drives policymakers to adopt pro-innovation policies when faced with rent-seeking pressures from incumbent firms? The answer, this paper argues, lies in understanding how policymaker preferences are shaped by the interaction between inequality concerns and institutional constraints.

This view aligns with Acemoglu (2008), who distinguishes between oligarchic and democratic societies, arguing that oligarchies tend to block innovation to protect incumbent rents, thereby slowing growth. Democracies, by contrast, impose checks and balances that limit the ability of incumbents to block entry, fostering innovation. However, even in democracies, rent-seeking behavior can persist if policymakers face incentives to prioritize redistribution over growth. This paper extends Acemoglu's analysis by showing how inequality concerns, particularly in dual-sector economies, can influence entry policies in ways that either promote or hinder innovation.

Acemoglu and Robinson (2006) further highlight how institutional factors mediate the relationship between innovation and growth, arguing that the distribution of political power affects the adoption of growth-enhancing policies. Their analysis shows that incumbent elites may block innovation to protect existing rents, emphasizing the role of institutional preferences in shaping innovation trajectories.

Acemoglu et al. (2005) explore the nuanced relationship between democracy and economic development, finding that the benefits of democracy on growth vary across sectors, particularly those closer to the technological frontier.

Although the Schumpeterian growth literature focuses on innovation-driven growth and the political economy literature emphasizes institutional determinants, few studies integrate these perspectives. Yet, the interaction between sectoral innovation dynamics and institutional preferences is crucial for understanding growth trajectories, especially in dual-sector economies.

This paper seeks to bridge this gap by developing a model that integrates Schumpeterian growth dynamics, political economy considerations, and income inequality concerns. While existing literature has shown that democracy reduces entry barriers and promotes innovation [e.g. Aghion et al. (2005), Rodrik and Wacziarg (2005)], it has not explored how income inequality itself can serve as an endogenous institutional constraint. In other words, while democratic institutions limit rent-seeking, this paper shows that inequality dynamics, particularly the inverse of the skill premium, can endogenously generate similar constraints.

To develop this argument, the paper builds closely on the framework presented in Chapter 17 of Aghion and Howitt (2008), which represents a seminal contribution to the literature by integrating Schumpeterian growth models with political economy considerations. Their model demonstrates how democracy influences growth by constraining policymakers' ability to collude with incumbents against new entrants. However, the democracy parameter in their model is treated exogenously, leaving unexplored how income inequality forces within the economy might endogenously shape institutional behavior and growth outcomes.

Chapter 17 of Aghion and Howitt (2008) introduces a simple yet powerful model in which democratic institutions limit the ability of incumbent firms to block market entry by offering bribes and political lobbying. The model builds on the Schumpeterian idea of creative destruction, where new firms displace incumbents by introducing superior technologies. However, the entry process is complicated by rent-seeking behaviors, as incumbent firms facing entry threats attempt to secure protection by bribing policymakers. The extent of these anti-competitive behaviors, in turn, depends on the quality of democratic institutions, which the model captures through an exogenous democracy parameter.

When democratic institutions are strong, policymakers' ability to collude with incumbents is constrained, leading to more open markets, higher innovation rates, and faster growth. Conversely, in weaker democracies, rent-seeking behaviors prevail, resulting in stagnation and slower growth. The model also assumes a homogeneous labor force and focuses solely on aggregate growth implications, ignoring how sectoral heterogeneity or income inequality might influence institutional behavior and innovation incentives. This exogenous treatment of democracy and the absence of income inequality considerations present important avenues for extension and refinement.

This paper extends and enriches the Aghion-Howitt framework in several key ways. The first major contribution is the introduction of an unskilled sector into the Schumpeterian framework. By modeling a dual-sector economy, where skilled and unskilled sectors coexist, this paper brings income inequality to the forefront of the analysis. The unskilled sector allows for a detailed examination of how sectoral divergence in innovation adoption can lead to rising wage inequality, with significant implications for social welfare and political stability.

The second major contribution is the endogenization of the institutional constraint that limits rent-seeking behavior. Whereas Aghion and Howitt (2008) rely on an exogenous democracy parameter, this paper introduces income inequality, specifically the inverse of the skill premium, as a key determinant of the policymaker's preferences. In this model, income inequality itself becomes a constraint on policymakers' ability to engage in rent-seeking. A higher skill premium, indicating greater inequality, raises the political cost of favoring incumbents, thereby reducing the effectiveness of bribes. This shift suggests that institutional quality is not an exogenous feature but rather emerges endogenously from underlying income inequality dynamics.

The third key contribution lies in the characterization of three distinct policymaker regimes. In the "Bribe-maximizing Regime," the policymaker prioritizes bribes from incumbent firms, with no concern for inequality. The unskilled sector amplifies inequality when innovation is concentrated in the skilled sector, reflecting unrestricted growth at the cost of rising inequality. In the "Inequality-minimizing Regime," the policymaker focuses solely on minimizing inequality, restricting entry policies that benefit the skilled sector. While income inequality is reduced, innovation incentives are dampened, resulting in slower long-term growth. The "Office-motivated Regime" is a more realistic scenario, where the policymaker balances bribes with electoral incentives tied to income inequality. In this regime, policymakers trade-off between fostering innovation and maintaining inequality within politically acceptable bounds, effectively endogenizing the democracy constraint seen in Aghion and Howitt's model.

These extensions fundamentally re-frame the relationship between innovation, growth, and institutional quality. By demonstrating that income inequality dynamics can act as endogenous institutional constraints, this paper bridges the gap between the Schumpeterian growth literature, which emphasizes technological innovation, and the political economy literature, which focuses on how institutions and special-interest groups shape economic outcomes. In doing so, it provides a more comprehensive framework for understanding the complex interplay between growth, inequality, and institutional behavior.

The results presented in this paper yield several critical insights. First, the presence of an unskilled sector fundamentally alters the growth-inequality trade-off, as innovation in the skilled sector raises the skill premium, exacerbating income inequality. In the "Bribe-maximizing" Regime, growth proceeds unchecked at the cost of rising inequality. The "Inequality-minimizing" Regime sacrifices long-term growth to achieve short-term bridging of income inequality. The "Office-Motivated" Regime, however, strikes a balance, promoting sustainable growth while maintaining inequality within politically acceptable bounds.

Second, the analysis shows that rent-seeking behavior by backward firms leads to sub-optimal innovation outcomes. While advanced firms innovate in response to entry threats, backward firms resort to bribing policymakers to deter competition. The unskilled sector amplifies this distortion, as higher skill premiums make it politically costly for policymakers to ignore income inequality concerns.

Finally, the model reveals that the interaction between income inequality and political incentives plays a crucial role in shaping long-term growth trajectories. Policymakers operating under the "Office-Motivated" Regime design entry policies that balance growth and equity, illustrating how endogenized institutional constraints emerge from income inequality dynamics.

The remainder of this paper is organized as follows. Section 2 provides a detailed description of the consumption and production structures within the economy. Section 3 examines the role of intermediate goods and sectoral output dynamics. Section 4 defines the gross domestic product and inequality in the model, examining how they are affected by innovation and market entry. Section 5 explores the innovation and entry behavior of incumbent firms, distinguishing between advanced and backward firms. Section 6 characterizes the steady-state distribution of advanced firms and its implications for aggregate productivity. Section 7 introduces the policymaker and examines their role in optimizing entry policies to balance welfare objectives and economic efficiency. Section 8 discusses the process of determining the firm entry policy, considering the trade-offs between fostering innovation, minimizing rent-seeking, and reducing income inequality. Section 9 presents the main results of our paper. Section 10 concludes the paper.

2 Consumer Behavior

The economy consists of two types of workers: unskilled workers, denoted by $L_{u,t}$, and skilled workers, denoted by, $L_{s,t}$. Both types of workers supply labor inelastically and derive utility from consuming a unique final consumption good, Y. The preferences of workers are assumed to be identical, regardless of skill level. The lifetime utility of a worker of type $k \in \{u, s\}$, at time t, is given by

$$U_{k,t} = \int_t^{\infty} e^{-rt}\, c_{k,t}\; dt, \quad k \in \{u, s\} \tag{1}$$

where $c_{k,t}$ is the consumption of worker k at time t and r is the discount rate, which due to linear utility is also the interest rate[1].

[1] As in Aghion and Howitt (1992, 1998, and 2008) and Grossman and Helpman (1991), linear utility implies that households are risk-neutral and do not engage in savings or intertemporal consumption smoothing. Consequently, the equilibrium interest rate equals the discount rate, and the marginal utility of consumption remains constant. This decouples household consumption from investment decisions, making firm investment decisions depend solely on expected profits rather than on household savings behavior.

This assumption should not be viewed as a mere simplification for convenience but as a theoretical necessity for sharpening the model's focus on the policymaker's role in shaping innovation incentives. By removing the savings channel, linear utility isolates the effect of entry threats and rent-seeking on innovation behavior, ensuring that the dynamics of growth emerge directly from firm incentives rather than from household savings responses. Aghion and Howitt (1992) demonstrate that under linear utility, the structure of capital markets—whether frictionless, imperfect, or absent—becomes irrelevant to research incentives. Building on this insight, we extend the logic to entry policies, where linear utility ensures that (a) wages translate directly into consumption, and (b) policymaker actions—such as imposing entry costs or encouraging rent-seeking—affect growth solely through their impact on firm incentives, without introducing distortions from shifts in household savings behavior.

Alternatively, we could model households as maximizing Constant Relative Risk Aversion (CRRA) utility

$$U_{k,t} = \int_t^{\infty} e^{-rt} \frac{c_{k,t}^{1-\sigma} - 1}{1-\sigma}\, dt, \;\; 0 < \sigma < 1,$$

subject to the budget constraint

$$\dot{a}_{k,t} = w_{k,t} + i \cdot a_{k,t} - c_{k,t}.$$

where i is the interest rate, $a_{k,t}$ is household wealth, and $w_{k,t}$ is wage income. Under CRRA utility, households engage in intertemporal consumption smoothing, governed by

$$\frac{\dot{c}_{k,t}}{c_{k,t}} = \frac{1}{\sigma}(r - i).$$

The consumption good, Y, is a costless assembly of two distinct final goods: the unskilled sector good, $Y_{u,t}$, and the skilled sector good, $Y_{s,t}$. These goods are competitively produced and combined into the final consumption bundle according to a constant elasticity of substitution (CES) aggregation function

$$Y = [Y_{u,t}^{\rho} + \eta Y_{s,t}^{\rho}]^{1/\rho}, \quad 0 < \rho < 1; \quad \eta > 1, \tag{2}$$

where the elasticity of substitution between $Y_{u,t}$ and $Y_{s,t}$ is $\frac{1}{1-\rho}$. A lower ρ implies a lower substitutability between $Y_{u,t}$ and $Y_{s,t}$, and as $\rho \to 1$, the two goods become perfect substitutes.

The parameter $\eta > 1$ captures the relative weight of the skilled sector good in the consumption bundle, implying that households consume proportionally more of $Y_{s,t}$ than $Y_{u,t}$. The unskilled sector good could be considered akin to a basic homogenous good consumed for subsistence. The skilled sector good is a sophisticated variety of goods that households consume in larger quantities.

The prices of the two final goods are considered to be P_u for the unskilled-sector good and P_s for the skilled-sector good. Therefore, the price of the consumption bundle is given by

$$P_Y = \left[P_u^{\rho} + \eta P_s^{\rho}\right]^{\frac{1}{\rho}}.$$

This price index is the dual of the consumption bundle, capturing how the cost of acquiring a unit of the composite good depends on the prices of its components. It mirrors the structure of the CES utility aggregator, reflecting the trade-off between the two goods and the elasticity of substitution, $\frac{1}{1-\rho}$.

In this formulation, the equilibrium interest rate depends on household consumption growth, introducing a feedback loop between household savings and firm investment. As a result, the intermediate firm's value becomes dependent on the household's consumption path, and the policymaker's optimization problem couples with household savings decisions, making the policymaker's influence on growth indirect and more complex. The model would then require solving a system of differential equations, significantly increasing analytical complexity without advancing the core research question of how entry policies shape innovation incentives and welfare outcomes.

The linear utility assumption, therefore, is not a simplification but a deliberate theoretical choice, consistent with Schumpeterian models, to maintain analytical clarity and isolate the policymaker's direct influence on innovation and growth.

Laborers earn wages for the labor provided, which is the only source of income for households. Therefore, each worker's intertemporal budget constraint is given by

$$\int_0^\infty P_Y \cdot c_{k,t} dt \leq \int_0^\infty w_{k,t} dt, \tag{3}$$

where w_u and w_s denote the wages of the unskilled and skilled labor, respectively. The term $P_Y \cdot c_{k,t}$ represents the total expenditure on consumption by household k at time t, and $w_{k,t}$ represents the corresponding wage income. The budget constraint ensures that the present value of total consumption does not exceed the present value of income.

Since $c_{k,t} = Y$, the household's problem is now to choose $Y_{u,t}$ and $Y_{s,t}$ to maximize their utility. By the principle of duality in consumer theory, utility maximization subject to a budget constraint is equivalent to cost minimization subject to a target utility level. Thus, the relative demand system derived from the cost-minimization problem is identical to what would emerge from the household's direct utility maximization problem. Since the focus of our analysis is on firm behavior and policy outcomes rather than on household intertemporal decisions, adopting the cost-minimization approach simplifies the model without sacrificing rigor.

Given that the prices of $Y_{u,t}$ and $Y_{s,t}$ are P_u and P_s respectively, the cost minimization problem can be formulated as

$$min_{Y_{u,t},Y_{s,t}} \quad P_u Y_{u,t} + P_s \eta Y_{s,t},$$

subject to

$$c_{k,t} = \left[Y_{u,t}^{\rho} + \eta Y_{s,t}^{\rho} \right]^{1/\rho}.$$

We set-up the Lagrangian as

$$\mathcal{L}_k = P_u Y_{u,t} + P_s \eta Y_{s,t} + \mu \Big(c_{k,t} - \left[Y_{u,t}^{\rho} + \eta Y_{s,t}^{\rho} \right]^{1/\rho} \Big),$$

where μ is the Lagrangian multiplier. The first-order conditions are given by

$$\frac{\partial \mathcal{L}_k}{\partial Y_{u,t}} = P_u - \mu \cdot \frac{1}{\rho} \left[Y_{u,t}^{\rho} + \eta Y_{s,t}^{\rho} \right]^{\frac{1}{\rho}-1} \cdot \rho Y_{u,t}^{\rho-1} = 0,$$

and

$$\frac{\partial \mathcal{L}_k}{\partial Y_{s,t}} = P_s - \mu \cdot \frac{1}{\rho}\Big[Y_{u,t}^{\rho} + \eta Y_{s,t}^{\rho}\Big]^{\frac{1}{\rho}-1} \cdot \rho\eta Y_{s,t}^{\rho-1} = 0.$$

The first-order conditions equate the marginal cost of increasing the quantity of consumption of each good to the marginal benefit measured in terms of reduced total cost for achieving the target consumption level. The above equations can be simplified as

$$P_u = \mu \cdot \Big[Y_{u,t}^{\rho} + \eta Y_{s,t}^{\rho}\Big]^{\frac{1}{\rho}-1} \cdot Y_{u,t}^{\rho-1}, \tag{4}$$

and

$$P_s = \mu \cdot \Big[Y_{u,t}^{\rho} + \eta Y_{s,t}^{\rho}\Big]^{\frac{1}{\rho}-1} \cdot \eta Y_{s,t}^{\rho-1}. \tag{5}$$

respectively. On dividing Equation (4) by Equation (5), we obtain the relative demand as

$$\frac{P_u}{P_s} = \frac{\mu \cdot \Big[Y_{u,t}^{\rho} + \eta Y_{s,t}^{\rho}\Big]^{\frac{1}{\rho}-1} \cdot Y_{u,t}^{\rho-1}}{\mu \cdot \Big[Y_{u,t}^{\rho} + \eta Y_{s,t}^{\rho}\Big]^{\frac{1}{\rho}-1} \cdot \eta Y_{s,t}^{\rho-1}},$$

which can be simplified to

$$\frac{P_u}{P_s} = \frac{Y_{u,t}^{\rho-1}}{\eta Y_{s,t}^{\rho-1}},$$

which can be re-written as

$$\frac{P_s}{P_u} = \frac{1}{\eta}\left[\frac{Y_{u,t}}{Y_{s,t}}\right]^{1-\rho}.$$

The above equation gives us the standard relative demand equation for the two final goods. This relationship helps in understanding how the quantities of unskilled and skilled sector goods, Y_u and Y_s, are chosen given their relative prices, P_u and P_s. If P_s rises relative to P_u, households substitute away from the skilled sector good, Y_s, and consume more of the unskilled sector good Y_u. Conversely, if P_u rises relative to P_s, households shift toward consuming more of the skilled sector good. When ρ is close to one (zero), the goods are nearly perfect (poor) substitutes, and their relative consumption will respond strongly (weakly) to changes in their relative prices. A larger η implies that, for the same relative price, households consume more of the skilled sector good relative to the unskilled sector good. Note that the optimal composition of consumption depends only on relative prices and preferences and not on the overall income level.

In our subsequent analysis, we have normalized the price of the final good in the skilled sector, P_s, to be 1. In doing so, we simplify the algebra and make it easier to interpret the model's results without a loss of generality. This normalization implies that the price of the unskilled sector good, P_u, and the price of the consumption bundle, P_Y, are expressed relative to the price of the skilled sector good, P_s. Consequently, the relative demand equation derived earlier now simplifies to

$$\frac{1}{P_u} = \frac{1}{\eta}\left[\frac{Y_{u,t}}{Y_{s,t}}\right]^{1-\rho}. \tag{6}$$

And the price of the consumption bundle, P_Y, will now be

$$P_Y = \left[P_u^{\rho} + \eta\right]^{\frac{1}{\rho}}.$$

This completes the formulation of the consumption side of our model, establishing the price index and relative demand functions that will influence equilibrium outcomes in the production and innovation sectors. We now proceed to the production sector of our model.

3 Sectoral Production and Output Dynamics

As stated earlier, the two final goods, $Y_{u,t}$, and $Y_{s,t}$, are competitively produced. The productivity in the unskilled sector is less than the productivity of the skilled sector. This is captured by the condition

$$A_u < A_{is,t} \leq \bar{A}_{s,t},\ i \in [0, 1]$$

where A_u is the productivity of the unskilled sector, $A_{is,t}$ is the productivity of i^{th} intermediate good in the skilled sector, at time t. $\bar{A}_{s,t}$ is the frontier productivity of the skilled sector at time t. An individual firm, i, in the intermediate sector may be at the frontier of technology, or below it, as will be seen in Section 5.

We now state the production technologies for each of these goods.

3.1 The Unskilled Sector

Output in the unskilled sector is produced using only unskilled labor. The production technology is given by

$$Y_{u,t} = A_u L_{u,t} \tag{7}$$

where $L_{u,t}$ is the labor used in the production of the unskilled good, and A_u is the productivity parameter associated with this sector. Note that

$$\frac{\partial Y_{u,t}}{\partial L_{u,t}} = A_u. \tag{8}$$

Equation (8) shows that the production technology in the unskilled sector exhibits constant returns to scale. The profits of the producer in this sector are given by

$$\Pi_u = P_u Y_{u,t} - w_u L_{u,t}, \tag{9}$$

where P_u is the price of the unskilled good and w_u is the wage rate of the unskilled labor. On substituting for $Y_{u,t}$ from Equation (7) we obtain

$$\Pi_u = P_u A_u L_{u,t} - w_u L_{u,t}.$$

The profit maximization exercise of the producer yields the first-order condition

$$\frac{\partial \Pi_u}{\partial L_{u,t}} = P_u A_u - w_u = 0,$$

which can also be written as

$$w_u = P_u A_u. \tag{10}$$

Equation (10) gives us the optimal wage rate of the unskilled labor engaged in the production of the unskilled good, $Y_{u,t}$, as a function of the price of the final good of this sector, P_u and the productivity parameter of this sector, A_u.

3.2 The Skilled Sector

Output in the skilled sector is produced using both skilled labor and a continuum of intermediary goods. These intermediary goods can be thought of either as inputs used in the manufacture of the final good of this sector or as machines employed in the production of the final good of this sector.

The final good of this sector can be used either for consumption, or as an input in the process of production of intermediate goods, or as investments in the R&D activity. The production technology in this sector is given by

$$Y_{s,t} = L_{s,t}^{1-\alpha} \int_0^1 A_{is,t}^{1-\alpha}\, x_{i,t}^{\alpha}\, di; \quad 0 < \alpha < 1, \tag{11}$$

where $L_{s,t}$ is the labor employed in this sector. $x_{i,t}$ is the quantity of intermediate good i used in the production of $Y_{s,t}$ and $A_{is,t}$ is the productivity associated with intermediate good i, at time t. This term determines the overall efficiency with which skilled labor and intermediate goods are combined to produce the final output $Y_{s,t}$ of the skilled sector. In any period, the productivity will vary across the intermediate goods, depending on whether it is at or behind the frontier and whether the firm decides to innovate in the time period in consideration. The parameter α signifies the elasticity of substitution between skilled labor and intermediate goods in the production process, which reflects the degree to which skilled labor can be substituted for intermediate goods and vice versa, influencing the sector's production dynamics and cost structure. Specifically, a lower value of α indicates a lower elasticity of substitution, implying that skilled labor and intermediate goods are less substitutable. Conversely, a higher value of α signifies a higher elasticity of substitution, suggesting that these inputs can be more easily interchanged in the production process.

Note that

$$\frac{\partial Y_{s,t}}{\partial L_{s,t}} = (1-\alpha)L_{s,t}^{-\alpha} \int_0^1 A_{is,t}^{1-\alpha}\ x_{i,t}^{\alpha}\ di, \tag{12}$$

and

$$\frac{\partial Y_{s,t}}{\partial x_{i,t}} = \alpha L_{s,t}^{1-\alpha} A_{is,t}^{1-\alpha} x_{i,t}^{\alpha-1}. \tag{13}$$

The profits of the producer in this sector are given by

$$\Pi_s = P_s Y_{s,t} - w_s L_{s,t} - \int_0^1 p_{i,t} x_{i,t} di, \tag{14}$$

where P_s is the price of the skilled good (which we have normalized to 1) and w_s is the wage rate of the skilled labor. $p_{i,t}$ is the price of the intermediate good $x_{i,t}$. On substituting for $Y_{s,t}$ from Equation (11), we obtain

$$\Pi_s = L_{s,t}^{1-\alpha} \int_0^1 A_{is,t}^{1-\alpha}\ x_{i,t}^{\alpha}\ di - w_s L_{s,t} - \int_0^1 p_{i,t} x_{i,t} di \tag{15}$$

The profit maximization exercise of the producer yields the first-order conditions

$$\frac{\partial \Pi_s}{\partial L_{s,t}} = (1-\alpha)L_{s,t}^{-\alpha} \int_0^1 A_{is,t}^{1-\alpha}\ x_{i,t}^{\alpha}\ di - w_s = 0,$$

and

$$\frac{\partial \Pi_s}{\partial x_{i,t}} = \alpha \cdot L_{s,t}^{1-\alpha} \cdot A_{is,t}^{1-\alpha} x_{i,t}^{\alpha-1} - p_{i,t} = 0,$$

which can be re-written as

$$w_s = (1-\alpha) L_{s,t}^{-\alpha} \int_0^1 A_{is,t}^{1-\alpha} \; x_{i,t}^{\alpha} \; di, \tag{16}$$

and

$$p_{i,t} = \alpha \cdot L_{s,t}^{1-\alpha} \cdot A_{is,t}^{1-\alpha} x_{i,t}^{\alpha-1}, \tag{17}$$

respectively. Equation (16) and Equation (17) give us the optimal wage rate of the skilled labor, w_s, and optimal price for the intermediate good, $p_{i,t}$. We will re-visit these equations after characterizing and solving the intermediate goods sector.

3.3 The Intermediate Goods Sector

Each intermediate good is uniquely produced by a monopolist in each period, using the final good as the only input. Each unit of the intermediate good uses one unit of the final good of the skilled sector as input. The final output of the skilled sector that is not used for intermediate goods production is available for consumption, and it constitutes the economy's gross domestic product. We will revisit this aspect when we discuss the aggregate economic growth in this economy.

There are two types of firms in the intermediate goods sector: advanced and backward. The type of each firm is determined by its proximity to the technological frontier. We will present this discussion in Section 5. Irrespective of their proximity to the technology frontier, all intermediate firms have the same production and profit structure. The profits of the intermediate good monopolist are given by

$$\pi_{i,t} = p_{i,t} x_{i,t} - x_{i,t}, \tag{18}$$

where, as defined earlier, $p_{i,t}$ is the price of the intermediate good $x_{i,t}$. On substituting for $p_{i,t}$ from the first-order condition of the skilled final good manufacturer given in Equation (17), we obtain

$$\pi_{i,t} = \alpha \cdot L_{s,t}^{1-\alpha} \cdot A_{is,t}^{1-\alpha} x_{i,t}^{\alpha-1} \cdot x_{i,t} - x_{i,t},$$

which simplifies to

$$\pi_{i,t} = \alpha \cdot L_{s,t}^{1-\alpha} \cdot A_{is,t}^{1-\alpha} x_{i,t}^{\alpha} - x_{i,t}. \tag{19}$$

The profit maximization exercise by the intermediate monopolist yields

$$\frac{\partial \pi_{i,t}}{\partial x_{i,t}} = \alpha^2 \cdot L_{s,t}^{1-\alpha} \cdot A_{is,t}^{1-\alpha} x_{i,t}^{\alpha-1} - 1 = 0, \tag{20}$$

which can also be expressed as

$$\alpha^2 \cdot L_{s,t}^{1-\alpha} \cdot A_{is,t}^{1-\alpha} x_{i,t}^{\alpha-1} = 1,$$

which can be further expressed as

$$x_{i,t}^{\alpha-1} = \alpha^{-2} \cdot L_{s,t}^{\alpha-1} \cdot A_{is,t}^{\alpha-1},$$

which can be simplified to

$$x_{i,t} = \alpha^{\frac{-2}{\alpha-1}} \cdot L_{s,t} \cdot A_{is,t},$$

which can also be written as

$$x_{i,t} = \alpha^{\frac{2}{1-\alpha}} \cdot L_{s,t} \cdot A_{is,t}. \tag{21}$$

We substitute $x_{i,t}$ obtained from the above Equation (21) back in the profit function of the intermediate monopolist, given by Equation (19), and obtain

$$\pi_{i,t} = \alpha \cdot L_{s,t}^{1-\alpha} \cdot A_{is,t}^{1-\alpha} \left[\alpha^{\frac{2}{1-\alpha}} \cdot L_{s,t} \cdot A_{is,t}\right]^{\alpha} - \alpha^{\frac{2}{1-\alpha}} \cdot L_{s,t} \cdot A_{is,t}.$$

The above equation can be re-expressed as

$$\pi_{i,t} = \alpha^{\frac{2}{1-\alpha}} \cdot L_{s,t} \cdot A_{is,t} \left[\alpha \cdot L_{s,t}^{1-\alpha} \cdot A_{is,t}^{1-\alpha} \left[\alpha^{\frac{2}{1-\alpha}} \cdot L_{s,t} \cdot A_{is,t}\right]^{\alpha-1} - 1\right],$$

which can further be written as

$$\pi_{i,t} = \alpha^{\frac{2}{1-\alpha}} \cdot L_{s,t} \cdot A_{is,t} \left[\alpha \cdot L_{s,t}^{1-\alpha} \cdot A_{is,t}^{1-\alpha} \left[\alpha^{\frac{2(\alpha-1)}{1-\alpha}} \cdot L_{s,t}^{\alpha-1} \cdot A_{is,t}^{\alpha-1}\right] - 1\right],$$

which can be simplified to

$$\pi_{i,t} = \alpha^{\frac{2}{1-\alpha}} \cdot L_{s,t} \cdot A_{is,t} \left[\alpha \cdot L_{s,t}^{1-\alpha} \cdot A_{is,t}^{1-\alpha} \left[\alpha^{-2} \cdot L_{s,t}^{\alpha-1} \cdot A_{is,t}^{\alpha-1}\right] - 1\right],$$

which can be further simplified to

$$\pi_{i,t} = \alpha^{\frac{2}{1-\alpha}} \cdot L_{s,t} \cdot A_{is,t}\left(\frac{1}{\alpha} - 1\right). \tag{22}$$

Equation (22) gives the profits of the intermediate monopolist as a function of the labor employed in the skilled sector, $L_{s,t}$, the elasticity of substitution between skilled labor and the intermediate goods, α, and the productivity associated with the i^{th} intermediary good, $A_{is,t}$.

We now substitute $x_{i,t}$ obtained in Equation (21) in the optimal wage of the skilled labor, given in Equation (16).

$$w_s = (1-\alpha)L_{s,t}^{-\alpha}\int_0^1 A_{is,t}^{1-\alpha}\left[\alpha^{\frac{2}{1-\alpha}} \cdot L_{s,t} \cdot A_{is,t}\right]^{\alpha} di,$$

which can be simplified as

$$w_s = (1-\alpha)L_{s,t}^{-\alpha}\int_0^1 A_{is,t}^{1-\alpha}\alpha^{\frac{2\alpha}{1-\alpha}} \cdot L_{s,t}^{\alpha} \cdot A_{is,t}^{\alpha} di,$$

which can further be simplified as

$$w_s = (1-\alpha) \cdot \alpha^{\frac{2\alpha}{1-\alpha}} \cdot L_{s,t}^{-\alpha} \cdot L_{s,t}^{\alpha}\int_0^1 A_{is,t}^{1-\alpha}A_{is,t}^{\alpha} di.$$

This can also be written as

$$w_s = (1-\alpha) \cdot \alpha^{\frac{2\alpha}{1-\alpha}}\int_0^1 A_{is,t} di. \tag{23}$$

We express the above Equation (23) as

$$w_s = (1-\alpha) \cdot \alpha^{\frac{2\alpha}{1-\alpha}} A_{s,t}, \tag{24}$$

where

$$A_{s,t} = \int_0^1 A_{is,t} di \tag{25}$$

is the weighted numerical aggregate of all individual productivity parameters in the skilled-good sector.

Substituting $x_{i,t}$ obtained in Equation (21) in the optimal price of the intermediate good obtained in Equation (17), yields

$$p_{i,t} = \alpha \cdot L_{s,t}^{1-\alpha} \cdot A_{is,t}^{1-\alpha}\left[\alpha^{\frac{2}{1-\alpha}} \cdot L_{s,t} \cdot A_{is,t}\right]^{\alpha-1},$$

which can be simplified to

$$p_{i,t} = \alpha \cdot \alpha^{-2} = \frac{1}{\alpha}. \tag{26}$$

Equation (26) indicates that the price charged by the monopolist producer of the intermediate good is a markup over his/her marginal cost.

Finally, we substitute $x_{i,t}$ obtained in Equation (21) in the final output of the skilled good sector, as given in Equation (11), and obtain

$$Y_{s,t} = L_{s,t}^{1-\alpha} \int_0^1 A_{is,t}^{1-\alpha} \left[\alpha^{\frac{2}{1-\alpha}} \cdot L_{s,t} \cdot A_{is,t}\right]^{\alpha} di, \tag{27}$$

which can also be written as

$$Y_{s,t} = L_{s,t}^{1-\alpha} \int_0^1 A_{is,t}^{1-\alpha} \alpha^{\frac{2\alpha}{1-\alpha}} \cdot L_{s,t}^{\alpha} \cdot A_{is,t}^{\alpha} \, di,$$

which can be simplified to

$$Y_{s,t} = \alpha^{\frac{2\alpha}{1-\alpha}} L_{s,t}^{1-\alpha} \cdot L_{s,t}^{\alpha} \int_0^1 A_{is,t}^{1-\alpha} \cdot A_{is,t}^{\alpha} \, di,$$

which can be further simplified as

$$Y_{s,t} = \alpha^{\frac{2\alpha}{1-\alpha}} L_{s,t} \int_0^1 A_{is,t} \, di.$$

We then substitute Equation (25), which defines aggregate productivity in the economy, into the above equation and obtain

$$Y_{s,t} = \alpha^{\frac{2\alpha}{1-\alpha}} L_{s,t} A_{s,t} \tag{28}$$

Equation (28) expresses the final output of the skilled-good sector as a function of the labor employed in the skilled sector, $L_{s,t}$, the elasticity of substitution between skilled labor and the intermediate goods, α, and the aggregate productivity of the skilled sector, $A_{s,t}$. This last term determines the overall efficiency with which skilled labor and intermediate goods are combined to produce the final output, $Y_{s,t}$, of the skilled sector. Higher values of $A_{s,t}$ indicate greater technological advancement or efficiency gains in production, allowing the sector to produce more output, $Y_{s,t}$, for a given input combination of $L_{s,t}$ and $x_{i,t}$. Thus $A_{s,t}$ plays a crucial role in shaping the growth and competitiveness of the skilled sector within the broader economy.

4 Gross Domestic Product and Inequality

We now discuss the gross domestic product (GDP) and inequality in our model. The ensuing analysis of GDP and income inequality leads us to recognize the one route through which the planner can address concerns regarding both growth and income inequality, which is the aggregate productivity of the skilled sector, $A_{s,t}$. In this section, we will show how both the GDP and the skill premium, ω_t, are functions of $A_{s,t}$. In Subsection 7.2, we will then present an analysis of how this aggregate productivity can be affected by the firm entry policy that is set by the policymaker.

4.1 Gross Domestic Product

The Gross Domestic Product (GDP) in this economy represents the market value of all final goods and services produced during a given period. Recall that in each period, the intermediate product is produced by a monopolist using the final good of the skilled good sector as input, one-for-one. The final output of the skilled-good sector, to the extent not used for intermediate production, is available for consumption. Therefore, the GDP or the aggregate value added in this economy is defined as

$$GDP_t = P_u \cdot Y_{u,t} + P_s \cdot Y_{s,t} - \int_0^1 p_{i,t} \cdot x_{i,t} di, \tag{29}$$

where the term $P_u \cdot Y_{u,t}$ is the value of the output in the unskilled sector, $P_s \cdot Y_{s,t}$ is the value of the output in the skilled sector, and $\int_0^1 p_{i,t} \cdot x_{i,t} di$ is the total value of intermediate goods used by the skilled sector. This term is subtracted to avoid double counting, as intermediate goods are produced using the final good of the skilled sector as the input.

In the above equation, we substitute the final output of the unskilled sector, $Y_{u,t}$, from Equation (7), the final output of the skilled sector, Y_s, from Equation (28), the output of the intermediate good sector, $x_{i,t}$, from Equation (21) and the price of the intermediate good, $p_{i,t}$, from Equation (26). We also make use of the fact that the price of the final good in the skilled sector is equal to one ($P_s = 1$) to obtain

$$GDP_t = P_u A_u L_{u,t} + 1 \cdot \alpha^{\frac{2\alpha}{1-\alpha}} L_{s,t} A_{s,t} - \int_0^1 \frac{1}{\alpha} \cdot \alpha^{\frac{2}{1-\alpha}} L_{s,t} A_{is,t} \, di. \tag{30}$$

Substituting for aggregate productivity from Equation (25) and further simplification yields,

$$GDP_t = P_u A_u L_{u,t} + \left[\alpha^{\frac{2}{1-\alpha}}\right]^{\alpha} L_{s,t} A_{s,t} - \frac{1}{\alpha} \cdot \alpha^{\frac{2}{1-\alpha}} L_{s,t} A_{s,t},$$

which can also be written as

$$GDP_t = P_u A_u L_{u,t} + \alpha^{\frac{2}{1-\alpha}} L_{s,t} A_{s,t} \left[\left[\alpha^{\frac{2}{1-\alpha}}\right]^{\alpha-1} - \frac{1}{\alpha}\right].$$

The above equation can also be expressed as

$$GDP_t = G_t = P_u A_u L_{u,t} + \alpha^{\frac{2}{1-\alpha}} L_{s,t} A_{s,t} \Big(\frac{1}{\alpha^2} - \frac{1}{\alpha}\Big),$$

which on factoring out like terms, yields

$$GDP_t = G_t = P_u A_u L_{u,t} + \alpha^{\frac{2}{1-\alpha}} \cdot \frac{1}{\alpha} L_{s,t} A_{s,t} \Big[\frac{1}{\alpha} - 1\Big],$$

which can be reexpressed as

$$GDP_t = G_t = P_u A_u L_{u,t} + \alpha^{\frac{2}{1-\alpha}-1} L_{s,t} A_{s,t} \Big[\frac{1}{\alpha} - 1\Big].$$

The above equation further simplifies to

$$GDP_t = G_t = \underbrace{P_u A_u L_{u,t}}_{(a)} + \underbrace{\alpha^{\frac{1+\alpha}{1-\alpha}} L_{s,t} A_{s,t} \Big[\frac{1}{\alpha} - 1\Big]}_{(b)}. \tag{31}$$

Equation (31) decomposes the economy's output into contributions from the unskilled and skilled sectors. $Term\ (a)$ represents the value of the output of the unskilled sector. Recall that P_u is the price of the unskilled sector good. A decrease in P_u (relative to P_s) means unskilled goods become cheaper, potentially reducing the GDP share from this sector. A_u indicates how efficiently unskilled labor is converted into output. A higher A_u means that each unit of unskilled labor contributes more to total output. The level of unskilled labor, L_u, also directly affects the amount of output produced in this sector.

$Term\ (b)$ represents the value of the output from the skilled sector, which is influenced by three factors. The parameter α is the elasticity of substitution between skilled labor and intermediate goods. A higher α indicates greater substitutability, enabling firms to replace skilled labor with intermediate goods. This reduces reliance on skilled labor and may dampen wage inequality. A lower α implies limited substitutability, making skilled labor indispensable. This enhances the value of skilled labor but may exacerbate wage disparities. The term $\alpha^{\frac{1+\alpha}{1-\alpha}}$ amplifies the contribution of the skilled sector to GDP. The term $\left[\frac{1}{\alpha}-1\right]$ adjusts the contribution of the skilled sector by accounting for the diminishing returns to labor and intermediate inputs.

Skilled labor contributes directly to the production of the skilled sector output, with its impact augmented by the aggregate productivity, $A_{s,t}$, of the skilled sector. It is important to note that the GDP is a linear function of the aggregate productivity in the skilled sector, $A_{s,t}$. The relevance of $A_{s,t}$ becomes clearer when we introduce the planner's optimization problem, where the planner determines the entry probability, θ. The chosen value of θ influences aggregate productivity, $A_{s,t}$, thereby affecting the overall GDP of the economy. We will revisit this discussion in detail when formulating the planner's maximization framework. Additionally, it is worth mentioning that the productivity of the unskilled sector, A_u, is treated as a given parameter and does not evolve over time within the scope of our analysis. Therefore, when we differentiate Equation (31) with respect to time, we obtain

$$\frac{dG_t}{dt} = P_u A_u \frac{dL_{u,t}}{dt} + \alpha^{\frac{1+\alpha}{1-\alpha}} \left(\frac{1}{\alpha} - 1\right) \left[\frac{dL_{s,t}}{dt} A_{s,t} + \frac{dA_{s,t}}{dt} L_{s,t}\right]. \tag{32}$$

An analysis incorporating skill acquisition and resulting changes in labor composition would provide an interesting extension to our work. However, for the purposes of the current model, we assume that the composition of labor remains constant over time. Formally, this implies that

$$\frac{dL_{u,t}}{dt} = 0 \text{ and } \frac{dL_{s,t}}{dt} = 0.$$

Given this assumption, we also drop the time subscript on labor for simplicity. Consequently, Equation (32) can be rewritten as

$$\frac{dG_t}{dt} = \alpha^{\frac{1+\alpha}{1-\alpha}} \left[\frac{1}{\alpha} - 1\right] \frac{dA_{s,t}}{dt} L_s. \tag{33}$$

Equation (33), highlights the determinants of GDP growth in the model. The equation shows that the rate of change of GDP is driven exclusively by changes in aggregate productivity, $A_{s,t}$, in the skilled sector, scaled by three factors. As mentioned earlier, the term $\alpha^{\frac{1+\alpha}{1-\alpha}}$ is the elasticity-driven amplification factor. The exponential term $\frac{1+\alpha}{1-\alpha}$ determines how sensitive GDP growth is to changes in productivity. A higher α implies that technological improvements in the skilled sector can be more easily leveraged, leading to stronger growth responses. A lower α means that productivity gains rely more heavily on skilled labor, potentially dampening the immediate impact of technological advancements.

The term $\left[\frac{1}{\alpha} - 1\right]$ adjusts for diminishing returns in the skilled sector. As α approaches 1, the two inputs become perfect substitutes, reducing this adjustment term. Conversely, when α is lower, the impact of changes in $A_{s,t}$ on GDP growth becomes more pronounced.

A larger skilled labor force amplifies the growth impact of any given increase in productivity. The only driver of GDP growth in the model is the growth rate of aggregate productivity in the skilled sector. This reflects the critical role of innovation and technological advancement in sustaining economic growth. As $A_{s,t}$ increases, the skilled sector becomes more productive, directly translating into higher GDP growth.

The GDP growth equation shows that, under the model's assumptions, all growth is productivity-driven. Since labor composition remains constant, the economy's ability to grow hinges entirely on its capacity for technological progress in the skilled sector. We now turn to the definition of inequality in our model, which constitutes another key consideration for the policymaker when formulating the objective function.

4.2 Inequality

Inequality plays a central role in our model as it directly influences the policymaker's decisions regarding firm entry policies and innovation incentives. While higher innovation can drive economic growth, it may also exacerbate income disparities, especially between skilled and unskilled labor. Therefore, understanding how inequality emerges and evolves is critical for analyzing the trade-offs faced by the policymaker. In this section, we quantify inequality using the skill premium, which captures the wage differential between skilled and unskilled labor.

We define skill premium, ω_t, as

$$\omega_t = \frac{w_s}{w_u}, \tag{34}$$

where w_s represents the wage of skilled labor and w_u represents the wage of unskilled labor.

The skill premium, ω_t, thus measures how much more skilled labor earns compared to unskilled labor. A higher ω_t signifies a greater wage differential, implying that skilled labor is relatively more valuable or scarcer relative to unskilled labor. Substituting from Equation (10) for w_u and from Equation (24) for w_s, we obtain

$$\omega_t = \frac{(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}} A_{s,t}}{P_u A_u},$$

which can be re-written as

$$\omega_t = \frac{(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}} A_{s,t}}{A_u} \cdot \frac{1}{P_u}. \tag{35}$$

From Equation (35) we observe how changes in sectoral productivity influence relative prices. When the productivity of the unskilled sector, A_u, increases, the relative price ratio, $\frac{1}{P_u}$ rises. This implies that P_u decreases relative to $P_s = 1$. In other words, as unskilled labor becomes more productive, the unskilled sector good becomes cheaper compared to the skilled sector good. Conversely, when the productivity of the skilled sector, $A_{s,t}$, increases, the relative price ratio, $\frac{1}{P_u}$, declines. This indicates that P_u increases relative to $P_s = 1$, meaning the skilled sector good becomes cheaper relative to the unskilled sector good.

These results are intuitive. Higher A_u lowers the cost of producing unskilled sector goods, making them cheaper and thereby increasing the relative price ratio $\frac{P_s}{P_u} = \frac{1}{P_u}$. Higher $A_{s,t}$ reduces the cost of skilled sector goods, making them relatively cheaper and thus decreasing the relative price ratio.

Recall from Equation (6) that $\frac{1}{P_u} = \frac{1}{\eta}\left[\frac{Y_{u,t}}{Y_{s,t}}\right]^{1-\rho}$. In this Equation (6), we substitute for $Y_{u,t}$ and $Y_{s,t}$ from Equation (7) and Equation (28) respectively, and obtain

$$\frac{1}{P_u} = \frac{1}{\eta}\left[\frac{A_u L_u}{\alpha^{\frac{2\alpha}{1-\alpha}} L_s A_{s,t}}\right]^{1-\rho}. \tag{36}$$

Substituting Equation (36) in Equation (35) above, we obtain

$$\omega_t = (1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}} \cdot \frac{A_{s,t}}{A_u} \cdot \frac{1}{\eta}\left[\frac{A_u L_u}{\alpha^{\frac{2\alpha}{1-\alpha}} L_s A_{s,t}}\right]^{1-\rho}, \tag{37}$$

which can be re-written as

$$\omega_t = (1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}} \cdot \left(\alpha^{\frac{-2\alpha}{1-\alpha}}\right)^{1-\rho} \cdot \frac{1}{\eta} \cdot \frac{A_{s,t}}{A_u} \cdot \left[\frac{A_{s,t}}{A_u}\right]^{\rho-1} \cdot \left[\frac{L_u}{L_s}\right]^{1-\rho},$$

which can be simplified to

$$\omega_t = (1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha} - \frac{2\alpha(1-\rho)}{1-\alpha}} \cdot \frac{1}{\eta} \cdot \left[\frac{A_{s,t}}{A_u}\right]^{1+\rho-1} \cdot \left[\frac{L_u}{L_s}\right]^{1-\rho},$$

which can further be simplified to

$$\omega_t = (1-\alpha)\alpha^{\frac{2\alpha\rho}{1-\alpha}} \cdot \frac{1}{\eta} \cdot \left[\frac{A_{s,t}}{A_u}\right]^{\rho} \cdot \left[\frac{L_u}{L_s}\right]^{1-\rho}. \tag{38}$$

Equation (38) expresses the skill premium, ω_t, as a function of the elasticity of substitution between skilled labor and the intermediate goods, α, the elasticity of substitution between the unskilled and skilled goods in the consumption bundle, ρ, the consumer's relative preference for the skilled good, η, the unskilled labor force, L_u, the skilled labor force, L_s, the productivity in the unskilled sector, A_u, and the productivity in the skilled sector, $A_{s,t}$.

When α is high, skilled labor and intermediate goods are highly substitutable. This means that firms can adjust their production processes more easily by either using more skilled labor or more intermediate goods, depending on relative prices and productivity. Therefore, firms may substitute skilled labor with intermediate goods more readily when the cost of skilled labor rises relative to intermediate goods. This could potentially lower the demand for skilled labor and thus reduce ω_t. Conversely, if the cost of intermediate goods rises relative to skilled labor, firms may substitute intermediate goods with skilled labor, potentially increasing ω_t. On the other hand, when α is low, skilled labor and intermediate goods are less substitutable. This implies that skilled labor is more specialized or unique in its contribution to production, making it more valuable relative to intermediate goods. This leads to higher wages for the skilled labor and thus a higher value for ω_t, the skill premium.

As $\alpha \to 1$, the term $\alpha^{\frac{2\alpha\rho}{1-\alpha}}$ moderates the influence of productivity differences. The wage gap is less sensitive to productivity advantages because firms can offset skilled labor needs with intermediate goods. Conversely, as $\alpha \to 0$, the term $\alpha^{\frac{2\alpha\rho}{1-\alpha}}$ reduces, making the skill premium more sensitive to productivity gains in the skilled sector.

A higher elasticity of substitution between the unskilled and skilled goods in the consumption bundle, ρ, implies that consumers are more likely to switch their consumption towards the cheaper (in terms of relative productivity) option, thereby affecting the wage premium. A higher value of ρ also increases the sensitivity of ω_t to changes in the productivity ratio, $\frac{A_{s,t}}{A_u}$. The converse is the case with a lower value of ρ. A higher value of η, which is the consumers' preference for the skilled good relative to the unskilled good, indicates a stronger demand for skilled labor, thereby influencing wages.

Recall that $0 < \rho < 1$. Therefore, the term $(1 - \rho)$ is positive. When the supply of skilled labor increases relative to unskilled labor, the ratio $\frac{L_u}{L_s}$ decreases. From Equation (38) this results in a lower skill premium, reducing ω_t. Conversely, when unskilled labor supply increases relative to skilled labor, $\frac{L_u}{L_s}$ rises, leading to a higher skill premium and an increase in ω_t.

Finally, if the aggregate productivity of the skilled sector, $A_{s,t}$ increases relative to the productivity of the unskilled sector, A_u, that is to say, for a higher $\frac{A_{s,t}}{A_u}$, skilled workers are more productive compared to unskilled workers. This productivity advantage can lead to higher wages for skilled workers relative to unskilled workers, thereby increasing the skill premium, ω_t. The converse will be true if A_u increases relative to $A_{s,t}$.

In summary, the skill premium, ω_t, serves as a critical measure of inequality in the model, capturing how differences in productivity, labor composition, consumption preferences, and technological substitutability shape wage disparities between skilled and unskilled labor. These findings underscore a fundamental growth-inequality trade-off faced by the policymaker. Policies that foster innovation and boost skilled sector productivity can accelerate growth but may exacerbate inequality if labor substitutability is limited. Thus, the planner's objective function must carefully balance these competing concerns, ensuring that the benefits of growth are not overshadowed by rising inequality. In subsequent sections, we will explore how the policymaker optimally navigates this trade-off when determining entry probabilities and shaping the economy's long-term growth path.

We now turn to the formulation of the innovation decisions undertaken by intermediate firms, outlining the factors that influence their incentives and strategic choices within the model.

5 Firm Entry and Incumbent Innovation

In this section, we discuss how firms in the intermediate goods sector make their innovation decisions in the light of a threat posed by the entry of a new firm. Let θ be the probability that a potential entrant shows up in the intermediate goods sector. In a subsequent section, this parameter, θ, will be chosen by the policymaker. In doing so, the policymaker exhibits a concern for income inequality in the economy and is also subject to influence by incumbent firms that want to prevent entry.

As stated earlier, within the intermediate sector, there are two types of firms, advanced firms and backward firms, depending on their proximity to the technological frontier, $\bar{A}_{s,t}$, which represents the highest productivity that firms in the intermediate sector can potentially achieve.

Advanced firms are defined as those operating at the frontier level of productivity. In other words, the productivity level of an advanced firm is equal to the highest possible productivity level in the skilled sector at time t. This relationship can be expressed as

$$A_{is,t} = \bar{A}_{s,t}, \tag{39}$$

where $\bar{A}_{s,t}$ represents the frontier productivity level at time t.

On the other hand, backward firms are defined as those operating below the frontier level of productivity. In other words, the productivity level of a backward firm lags behind the highest possible productivity level in the skilled sector at time t, by a factor $\frac{1}{\gamma}$. This relationship is expressed by

$$A_{is,t} = \frac{1}{\gamma}\bar{A}_{s,t}; \quad \gamma > 1 \tag{40}$$

where $\gamma > 1$ is the constant rate at which the frontier productivity level grows over time. Therefore,

$$\bar{A}_{s,t} = \gamma \bar{A}_{s,t-1}. \tag{41}$$

We assume that a potential entrant is always at the technological frontier. When, at time t, an incumbent firm that was at the frontier in time $t-1$ faces the threat of a potential entrant arriving with leading-edge technology, it uses its first mover advantage to block entry and consequently retain its monopoly power. However, if entry occurs and the incumbent firm fails to reach the new technological frontier at time t, the technologically superior new entrant will replace the incumbent in the ensuing Bertrand competition.

For an incumbent firm, we define the cost of investing in research and development (R&D) activity at time t, as

$$c_{i,t} \cdot A_{is,t-1}, \tag{42}$$

where $A_{is,t-1}$ is the incumbent's pre-innovation productivity. We assume $c_{i,t}$ to be random and independently and identically distributed across intermediate sectors. $c_{i,t}$ can take two values

$$c_{i,t} \in \{0, \bar{c}\}, \tag{43}$$

and

$$Pr(c_{i,t} = 0) = Pr(c_{i,t} = \bar{c}). = \frac{1}{2} \tag{44}$$

How incumbent firms react to an entry threat, θ, depends on the marginal benefit that they expect to receive from an innovation, given $c_{i,t}$ and θ. This also varies depending on whether the incumbent is an advanced or a backward firm. In our subsequent sub-sections, we analyze how firms make these decisions.

The payoffs of incumbent firms, both advanced and backward, contingent upon whether they innovate and whether entry occurs, are summarized in Figure 1. We elaborately discuss each of these scenarios is Subsection 5.1 and Subsection 5.2.

We draw attention to the manner in which firms evaluate their innovation decisions in our model. While making the decision to invest in innovation, firms typically compare the present value of lifetime profits with the marginal cost of innovation. However, in our framework, this approach is not directly applicable due to the uncertainty surrounding firm survival. Firms face a constant risk of being displaced, either due to exogenous factors or through policy-induced entry threats. This uncertainty shortens the effective time horizon over which firms expect to realize the returns from innovation. As a result, firms effectively make their innovation decisions on a period-by-period basis, reassessing whether innovation is worthwhile given the current level of entry threat and the risk of displacement[2].

[2]While our framework focuses on a period-by-period decision-making process, one could consider the present value of lifetime profits under continuous-time assumptions. In such a setting, firms would discount

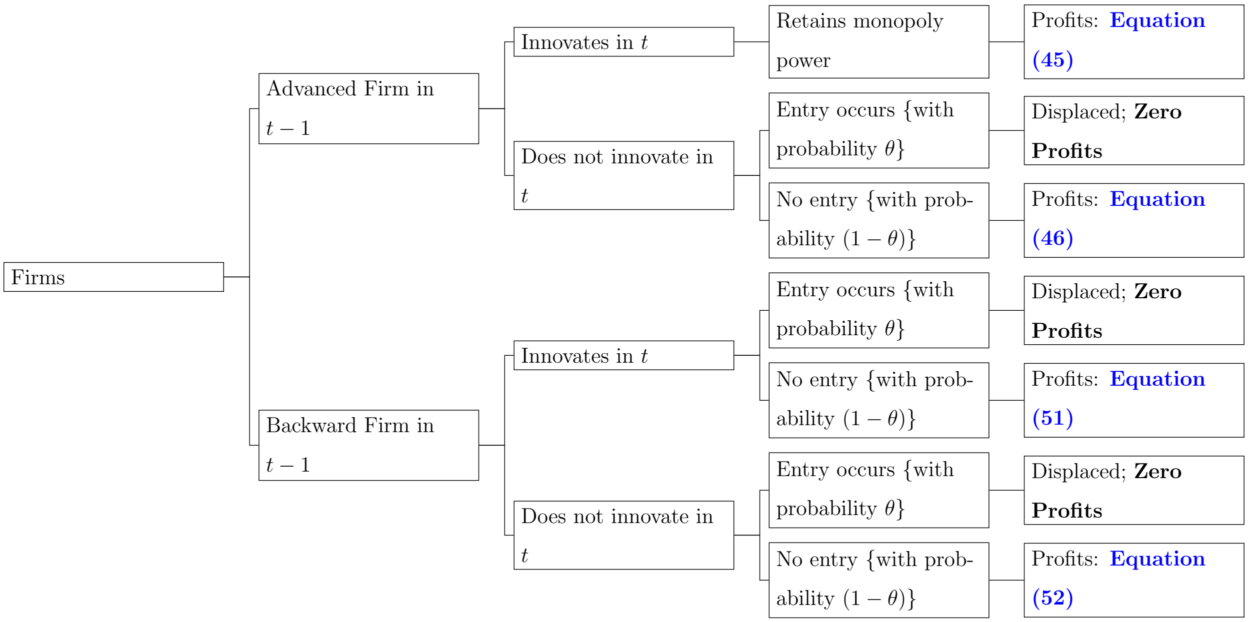

Figure 1: Payoffs of Incumbent Firms.

This approach also reflects the memoryless nature of the displacement process: each period presents a similar decision-making environment regardless of past outcomes. Consequently, the innovation decision simplifies to weighing current expected returns against innovation costs, without requiring explicit calculations of lifetime profit streams.

---

future profits by both the time preference rate, r, and the hazard rate of displacement, λ, which reflects the constant risk of being displaced. The expected present value of profits in this case would be given by

$$\int_0^\infty e^{-rt}\pi e^{-\lambda t}\,dt = \frac{\pi}{r+\lambda}.$$

This expression shows that the effective discount rate is the sum of the time discount rate and the displacement hazard rate, meaning that firms apply a risk-adjusted discount factor to future profits. As λ increases—reflecting higher entry threats or replacement risks—the present value of lifetime profits declines.

Although one could consider the present value of lifetime profits, such an approach would not materially affect our results under the current model's assumptions. The period-by-period evaluation adopted here captures the essential trade-off between current innovation costs and expected future returns, yielding qualitatively similar outcomes.

5.1 Innovation by Advanced Incumbent Firms

We first consider firms that were at the frontier level of technology in the previous time period $t-1$. Accordingly, their productivity level will be

$$A_{is,t-1} = \bar{A}_{s,t-1}.$$

If, in the current time period, t, this firm innovates, it will remain at the frontier in this period, too. This will make it immune to potential entry by an advanced firm. Upon successful innovation, an advanced firm can earn gross profits (i.e., before deducting R&D costs) equal to

$$\pi_{i,t} = \alpha^{\frac{2}{1-\alpha}} \cdot L_{s,t} \cdot \bar{A}_{s,t}\left(\frac{1}{\alpha}-1\right). \tag{45}$$

Note that this is the same as Equation (22) except that the productivity of this firm will be at the frontier, $A_{is,t} = \bar{A}_{s,t}$.

On the other hand, if this firm does not innovate in time period t, then with probability θ, it will be eliminated by a potential entrant and consequently make zero profits. However, with probability $(1-\theta)$, it will survive the entry threat and thereby make a profit of

$$\pi_{i,t} = (1-\theta)\alpha^{\frac{2}{1-\alpha}} \cdot L_{s,t} \cdot \bar{A}_{s,t-1}\left(\frac{1}{\alpha}-1\right). \tag{46}$$

Given these two possibilities, an advanced firm facing a cost of innovation $c_{i,t}\bar{A}_{s,t-1}$ will innovate only when the incremental benefit from the innovation is greater than the cost of innovation itself. This condition is given by

$$\underbrace{\alpha^{\frac{2}{1-\alpha}} \cdot L_{s,t} \cdot \bar{A}_{s,t}\left(\frac{1}{\alpha}-1\right)}_{(a)} - \underbrace{(1-\theta)\alpha^{\frac{2}{1-\alpha}} \cdot L_{s,t} \cdot \bar{A}_{s,t-1}\left(\frac{1}{\alpha}-1\right)}_{(b)} > \underbrace{c_{i,t}\bar{A}_{s,t-1}}_{(c)}. \tag{47}$$

In Equation (47) above, $Term\,(a)$ is the benefit obtained by the firm by innovating. $Term\,(b)$ is the benefit obtained by the firm by not innovating. $Term\,(c)$ is the cost of innovation. By substituting from Equation (39), the above equation can be re-written as

$$\alpha^{\frac{2}{1-\alpha}} \cdot L_{s,t} \cdot \gamma\bar{A}_{s,t-1}\left(\frac{1}{\alpha}-1\right) - (1-\theta)\alpha^{\frac{2}{1-\alpha}} \cdot L_{s,t} \cdot \bar{A}_{s,t-1}\left(\frac{1}{\alpha}-1\right) > c_{i,t}\bar{A}_{s,t-1},$$

and by collecting like terms, it can be simplified to

$$\alpha^{\frac{2}{1-\alpha}} \cdot L_{s,t} \cdot \bar{A}_{s,t-1}\left(\frac{1}{\alpha}-1\right)\Big[\gamma-(1-\theta)\Big] > c_{i,t}\bar{A}_{s,t-1},$$

which on cancelling out the term $\bar{A}_{s,t-1}$ can further be simplified to

$$\alpha^{\frac{2}{1-\alpha}}\left(\frac{1}{\alpha}-1\right) \cdot L_{s,t}\Big[\gamma-1+\theta\Big] > c_{i,t}. \tag{48}$$

Equation (48) captures the condition for an advanced firm to innovate, given the entry probability and cost of innovation. The left-hand side of the inequality represents the marginal benefit of innovation for the firm. The term $\alpha^{\frac{2}{1-\alpha}}\left(\frac{1}{\alpha}-1\right) \cdot L_{s,t}$ captures the combined effect of the elasticity of substitution, the amount of skilled labor, and the frontier productivity level. The factor $\left[\gamma-1+\theta\right]$ adjusts this benefit based on the growth rate of the technological frontier and the entry probability. The right-hand side of the inequality represents the cost of innovation for the firm. For the firm to decide to innovate, the marginal benefit of innovation must exceed the cost of innovation, that is, Equation (48) should hold true.

From Equation (48) it can be seen that with an increase in the entry threat, θ, the term $\left[\gamma-1+\theta\right]$ increases, which means that the marginal benefit of innovation increases as the threat of new entrants becomes more significant. This encourages the advanced incumbent firm to innovate. On the other hand, when θ reduces, the term $\left[\gamma-1+\theta\right]$ reduces, which means that the marginal benefit of innovation decreases as the threat of new entrants becomes less significant. This reduces the incentive for firms to innovate since the risk of displacement by new entrants is lower. Consequently, fewer firms will find it beneficial to innovate, especially if the cost, $c_{i,t}$, is high.

Intuitively, a firm that is at the technological frontier in the previous time period $t-1$, responds to an entry threat in time period t, by innovating and thereby escaping the threat of displacement due to entry. When the probability of new entrants, θ, is high, the competitive pressure motivates incumbent firms to invest in innovation to maintain their leading position. By advancing their technology, these firms can push the frontier further, securing their market dominance and mitigating the risk of being outcompeted by new entrants. Conversely, when θ is low, the likelihood of new entrants is minimal, reducing the immediate threat to the incumbent firms' market position. In such a scenario, the urgency to innovate diminishes, as the lower expected gains may not justify the costs of innovation. This leads to lower investment in R&D and, therefore, potentially leads to slower technological advancement in the long run. Understanding this dynamic highlights the critical role of entry threats in driving innovation and shaping the competitive landscape.

If this firm does innovate, its profits, net of R&D cost will be

$$\pi_{i,t} = \alpha^{\frac{2}{1-\alpha}} \left(\frac{1}{\alpha} - 1 \right) \cdot L_{s,t} \cdot \bar{A}_{s,t} - c_{i,t} \bar{A}_{s,t-1}. \tag{49}$$

And substituting Equation (39) in the Equation (49) above, we obtain

$$\pi_{i,t} = \alpha^{\frac{2}{1-\alpha}} \left(\frac{1}{\alpha} - 1 \right) \cdot L_{s,t} \cdot \gamma \bar{A}_{s,t-1} - c_{i,t} \bar{A}_{s,t-1},$$

which on collecting like terms, can be simplified to

$$\pi_{i,t} = \left[\alpha^{\frac{2}{1-\alpha}} \left(\frac{1}{\alpha} - 1 \right) \cdot \gamma \cdot L_{s,t} - c_{i,t} \right] \bar{A}_{s,t-1}. \tag{50}$$

Equation (50) captures the profits made by an advanced incumbent firm, given θ and $c_{i,t}$.

5.2 Innovation by Backward Incumbent Firms

In this subsection, we consider an incumbent firm that was backward in the previous time period, $t-1$. Accordingly, its productivity will be

$$A_{is,t-1} = \frac{1}{\gamma} \bar{A}_{s,t-1}.$$

Such a firm will remain backward even if it innovates in the current time period, t, since the technological frontier would also have advanced by γ in the current time period. Therefore, this firm will make zero profits if entry occurs with probability θ in time period t, irrespective of whether it innovates. Entry, which is always of a technologically advanced firm, will displace this firm.

On the other hand, if an entry does not occur, the probability for which is given by $(1-\theta)$, the firm does survive and make profits. Note that this is the only case where the firm can make any profits.

If the firm innovates, and entry does not occur, its gross profits (i.e., before deducting R&D costs) will be

$$\pi_{i,t} = \underbrace{\alpha^{\frac{2}{1-\alpha}} \cdot L_{s,t} \cdot A_{is,t} \left(\frac{1}{\alpha} - 1 \right)}_{(a)} \underbrace{\Big[1 - \theta \Big]}_{(b)}. \tag{51}$$

Note that $Term(a)$ in Equation (51) is similar to Equation (22). The productivity parameter $A_{is,t}$ indicates that this firm has innovated in the current time period but is still not at the technological frontier. $Term$ (b) is the probability that an entry does not occur in the intermediate good sector.

On the other hand, if this firm does not innovate and entry does not occur, its profits will be

$$\pi_{i,t} = \underbrace{\alpha^{\frac{2}{1-\alpha}} \cdot L_{s,t} \cdot A_{is,t-1} \left(\frac{1}{\alpha} - 1 \right)}_{(a)} \underbrace{\Big[1 - \theta \Big]}_{(b)}. \tag{52}$$

As earlier, term (a) in Equation (51) is the same as Equation (22). The difference is that the productivity parameter $A_{is,t-1}$ indicates that this firm has not innovated in the current time period but is still not at the technological frontier. Term (b) is, as earlier, the probability that an entry does not occur in the intermediate good sector.

A backward firm, with innovation cost $c_{i,t}A_{is,t-1}$, will innovate only when the incremental benefit from innovation exceeds the innovation cost. This is captured by the condition

$$\alpha^{\frac{2}{1-\alpha}} \cdot L_{s,t} \cdot A_{is,t} \left(\frac{1}{\alpha} - 1 \right) \Big[1 - \theta \Big] - \alpha^{\frac{2}{1-\alpha}} \cdot L_{s,t} \cdot A_{is,t-1} \left(\frac{1}{\alpha} - 1 \right) \Big[1 - \theta \Big] > c_{i,t}A_{is,t-1}. \tag{53}$$

Recall that $A_{is,t} = \gamma A_{is,t-1}$, which we substitute for in the above equation and obtain

$$\alpha^{\frac{2}{1-\alpha}} \cdot L_{s,t} \cdot \gamma A_{is,t-1} \left(\frac{1}{\alpha} - 1 \right) \Big[1 - \theta \Big] - \alpha^{\frac{2}{1-\alpha}} \cdot L_{s,t} \cdot A_{is,t-1} \left(\frac{1}{\alpha} - 1 \right) \Big[1 - \theta \Big] > c_{i,t}A_{is,t-1},$$

which on the collection of like terms can be written as

$$\alpha^{\frac{2}{1-\alpha}} \left(\frac{1}{\alpha} - 1 \right) \cdot L_{s,t} \cdot A_{is,t-1} \Big[1 - \theta \Big] \Big[\gamma - 1 \Big] > c_{i,t}A_{is,t-1},$$

which on canceling out like terms, can be simplified as

$$\alpha^{\frac{2}{1-\alpha}} \left(\frac{1}{\alpha} - 1 \right) L_{s,t} \Big[1 - \theta \Big] \Big[\gamma - 1 \Big] > c_{i,t}. \tag{54}$$

Equation (54) captures the condition under which a backward firm innovates, given θ and $c_{i,t}$. It can be noticed that the backward firm's incentive to innovate has a negative relationship with the entry probability θ. Intuitively, this means that since the firm is far below the technological frontier, it is not going to survive an entry threat, irrespective of whether it innovates. Therefore, it is discouraged from innovating if the entry threat increases because it cannot escape the displacement caused by the entry of a technologically advanced entrant.

If the firm does innovate and survive the entry threat, its profit net of R&D investment will be

$$\pi_{i,t} = \alpha^{\frac{2}{1-\alpha}} \cdot L_{s,t} \cdot A_{is,t}\left(\frac{1}{\alpha} - 1\right)\Big[1 - \theta\Big] - c_{i,t}A_{is,t-1}.$$

In the above equation, we substitute for $A_{is,t} = \gamma A_{is,t-1}$, and obtain

$$\pi_{i,t} = \alpha^{\frac{2}{1-\alpha}} \cdot L_{s,t} \cdot \gamma A_{is,t-1}\left(\frac{1}{\alpha} - 1\right)\Big[1 - \theta\Big] - c_{i,t}A_{is,t-1},$$

which on collecting like terms, can also be written as

$$\pi_{i,t} = \left[\alpha^{\frac{2}{1-\alpha}}\left(\frac{1}{\alpha} - 1\right)L_{s,t} \cdot \gamma\Big[1 - \theta\Big] - c_{i,t}\right]A_{is,t-1}. \tag{55}$$

Equation (55) captures the profits made by a backward firm, net of R&D investment, if it innovates, given θ and $c_{i,t}$.

6 The Steady-State Share of Advanced Firms

In this section, we characterize the steady-state fraction of advanced firms in our model. We make the following two assumptions before we proceed to characterize the steady state.

Assumption 1. ***We assume that initially, the entry threat, θ, is zero.***

Assumption 1 allows for the analysis of firms' natural inclination towards innovation in an environment free from the pressure of potential new entrants. It helps to identify the innovation behavior of firms solely driven by their internal cost structures and productivity levels without external competitive pressure. Without the entry threat, firms' decisions to innovate are influenced only by their innovation costs, denoted by $c_{i,t}$, and the existing technological frontier.

We also make the following additional assumption.

Assumption 2. ***Absent any entry threat, that is to say, when $\theta = 0$, no firm with innovation cost equal to $\bar{c}$, ever innovates. This assumption takes the form***

$$(\gamma - 1)\alpha^{\frac{2}{1-\alpha}}\left(\frac{1}{\alpha} - 1\right) \cdot L_{s,t} < \bar{c}.$$

Assumption 2 mathematically expresses that the expected benefit of innovation, when there is no threat of new entrants, is less than the cost of innovation for firms with the highest cost, $\bar{c}$. This assumption effectively underscores that, without the pressure of potential new entrants, the expected returns from innovation are insufficient to justify the expenditure for firms facing the highest cost of innovation.

Assumption 2 serves as a critical baseline for analyzing the effects of policy-induced entry threats on firm innovation behavior. By stating that no firm with the highest innovation cost, $\bar{c}$, will innovate in the absence of an entry threat ($\theta = 0$), we delineate the conditions under which firms are inert in terms of innovation. This baseline helps in contrasting the innovation behavior when entry threat, $\theta > 0$, is introduced.

Assumption 2 also sets the stage for exploring the impact of policy interventions on innovation. If policymakers increase the probability of entry, θ, they alter the cost-benefit analysis for incumbent firms. This increased threat, $\theta > 0$, creates an additional incentive for firms to innovate, as failure to do so could result in loss of market position and profits. We thereby demonstrate how policy-induced competition can stimulate innovation, especially among firms that would otherwise remain stagnant due to high innovation costs.

We make use of these two assumptions to determine the steady-state fraction of advanced firms in the economy, conditional upon $\theta = 0$. Let β_t denote the fraction of advanced firms at time period t. Suppose that an advanced firm that successfully innovates at date t starts out in time period $t+1$ as an advanced firm, and all other firms start out as backward firms. Additionally, with exogenous probability ε, a backward firm at the end of time period t is replaced by a new, advanced firm at time period $t+1$. The dynamic equation governing the fraction of advanced firms can be written as

$$\underbrace{\beta_{t+1}}_{(a)} = \underbrace{z_A\beta_t}_{(b)} + \underbrace{\varepsilon(1 - z_A\beta_t)}_{(c)}, \tag{56}$$

where $z_A = Pr(c = 0) = \frac{1}{2}$, is the probability that an advanced firm innovates if $\theta = 0$.

Recall that as per Assumption 2, firms do not innovate if $\theta = 0$ and $c \neq 0$. In Equation (56), $Term\,(a)$ is the fraction of advanced firms in time period $t+1$. $Term\,(b)$ is the fraction of advanced firms that started out as advanced firms and have successfully innovated and continued as advanced firms in time period $t+1$. $Term\,(c)$ is the fraction of backward firms that have been replaced by new advanced firms. By substituting for $z_A = \frac{1}{2}$, and the fact that in the steady-state, $\beta_{t+1} = \beta_t = \beta^*$, the steady-state fraction of advanced firms is given by

$$\beta = \frac{1}{2}\beta + \varepsilon\Big(1 - \frac{1}{2}\beta\Big),$$

which can be simplified to

$$\beta - \frac{1}{2}\beta + \frac{\varepsilon}{2}\beta = \varepsilon,$$

which can further be simplified to

$$\beta\Big(1 - \frac{1}{2} + \frac{\varepsilon}{2}\Big) = \beta\Big(1 - \frac{1}{2}(1-\varepsilon)\Big) = \beta\Big(\frac{2-1+\varepsilon}{2}\Big) = \beta\Big(\frac{1+\varepsilon}{2}\Big) = \varepsilon.$$

Therefore, the steady-state fraction of advanced firms in the economy is given by

$$\beta^* = \Big(\frac{2\varepsilon}{1+\varepsilon}\Big) \tag{57}$$

Note from Equation (57) that the steady-state fraction of advanced firms, β^*, is solely determined by the exogenous parameter ε, representing the probability of a backward firm being replaced by an advanced firm[3]. By abstracting away from exogenously determining β^*, we establish a framework to explore the direct implications of policy interventions on

[3]This result follows directly from Assumption 1 and Assumption 2, which collectively ensure that, in the absence of entry threats ($\theta = 0$), the only mechanism for backward firms to become advanced is *via* the exogenous replacement process. Specifically, Assumption 1 ($\theta = 0$) eliminates entry threats, thereby removing any external pressure on incumbent firms to innovate. Assumption 2 further restricts endogenous innovation by stating that firms with the highest innovation cost ($\bar{c}$) do not innovate when $\theta = 0$. Together, these assumptions ensure that the steady-state fraction of advanced firms is determined solely by the exogenous parameter ε, which governs the replacement of backward firms by advanced entrants.

While, in the subsequent analysis, the policymaker sets a positive entry probability ($\theta > 0$), potentially incentivizing incumbent firms to innovate, our current framework abstracts from any direct impact of θ on β^*. This abstraction allows us to focus on the immediate policy effects on aggregate productivity and inequality without altering the long-run composition of firms. In other words, while $\theta > 0$ influences incumbents' short-term innovation incentives, it does not affect the steady-state share of advanced firms in the present firm behavior. This approach enables us to isolate and analyze the causal relationships between policy decisions, entry threats, and innovation incentives. The core objective of our analysis is to explore how the policymaker's entry policy, captured by the entry probability, θ, influences firm behavior. The firm entry policy, θ, is set by the policymaker who not only succumbs to bribes offered by incumbent firms to restrict entry but also reducing income inequality (ω_t). This trade-off underscores the political economy dimension of the model, where the policymaker's entry policy balances income inequality among the two types of labor and rent-seeking pressures from incumbents.

With the steady-state share of advanced firms being given by Equation (57), the aggregate productivity of the skill sector, when there is no entry threat faced by incumbent firms (i.e., $\theta = 0$), is given by

$$A_{s,t} = \beta^* \bar{A}_{s,t} + (1 - \beta^*) \cdot \frac{1}{\gamma} \bar{A}_{s,t}$$

This equation illustrates how the aggregate productivity, $A_{s,t}$, in the skilled sector is determined by the productivity levels of both advanced and backward firms in the steady

framework.

However, one would expect β^* to depend on θ, reflecting endogenous innovation responses to entry threats. A plausible feedback mechanism could involve an increase in the fraction of advanced firms that choose to innovate when the entry threat is positive. In the baseline scenario ($\theta = 0$), only zero-cost firms innovate, yielding $z_A = \frac{1}{2}$. However, when $\theta > 0$, the threat of entry could induce some positive-cost firms to innovate, thereby increasing the overall fraction of innovating advanced firms.

The feedback mechanism could take the form

$$z_A(\theta) = \frac{1}{2} + \delta\theta,$$

where $\delta > 0$ captures the sensitivity of firms' innovation response to the entry threat. The steady-state share of advanced firms would adjust accordingly to

$$\beta^*(\theta) = \frac{2\varepsilon}{1 + \varepsilon - 2\delta\theta(1 - \varepsilon)}.$$

This formulation shows that a higher entry threat, θ, raises the steady-state fraction of advanced firms by expanding the subset of firms willing to innovate, beyond just the zero-cost firms.

While incorporating such a feedback mechanism would enrich the model by linking policy-induced competition directly to the long-run technological composition, it would also introduce non-linearities that complicate analytical tractability.

state. Specifically, $\bar{A}_{s,t}$ represents the frontier productivity level, and γ is the factor by which backward firms lag behind the frontier. The term $\beta^* \bar{A}_{s,t}$ represents the contribution to aggregate productivity from the advanced firms, which operate at the frontier productivity level. The term $(1 - \beta^*) \cdot \frac{1}{\gamma} \bar{A}_{s,t}$ represents the contribution from the backward firms, which operate at a productivity level of $\frac{1}{\gamma} \bar{A}_{s,t}$.

In the absence of an entry threat ($\theta = 0$), the steady-state share of advanced firms, β^*, determines the proportion of firms at the frontier. The aggregate productivity is thus a weighted average of the productivity levels of the advanced and backward firms. The weight for the advanced firms is β^*, while the weight for the backward firms is $1 - \beta^*$, adjusted by their relative productivity level, $\frac{1}{\gamma}$. This equation highlights the impact of the distribution of firms' productivity levels on the overall productivity of the skilled sector.

7 Introducing the Policymaker

We now introduce a policymaker who sets the firm entry policy, θ, in each period. On the one hand, the policymaker responds to bribes offered by incumbent firms to restrict entry. On the other hand, he/she is also concerned about reducing the wage gap between the skilled and unskilled labor force, ω_t[4]. In this section, we now proceed to show how each of these variables—the bribes offered by incumbent firms, the GDP, and the skill premium, ω_t, can be expressed as functions of the firm entry policy, θ. We will then proceed to set the policymaker's objective function where he/she sets the entry policy, θ.

[4] At the end of this section, it will be shown that both bribes and GDP are linear functions of θ. Therefore, we abstract away from considering an increase in the output of the economy as an additional concern for the policymaker since a policymaker that sets an entry policy that maximizes his/her bribes is automatically also maximizing the output of the economy. However, while comparing the impact of the entry policy, we consider both the GDP and the skill premium, which is a measure of inequality in the economy.

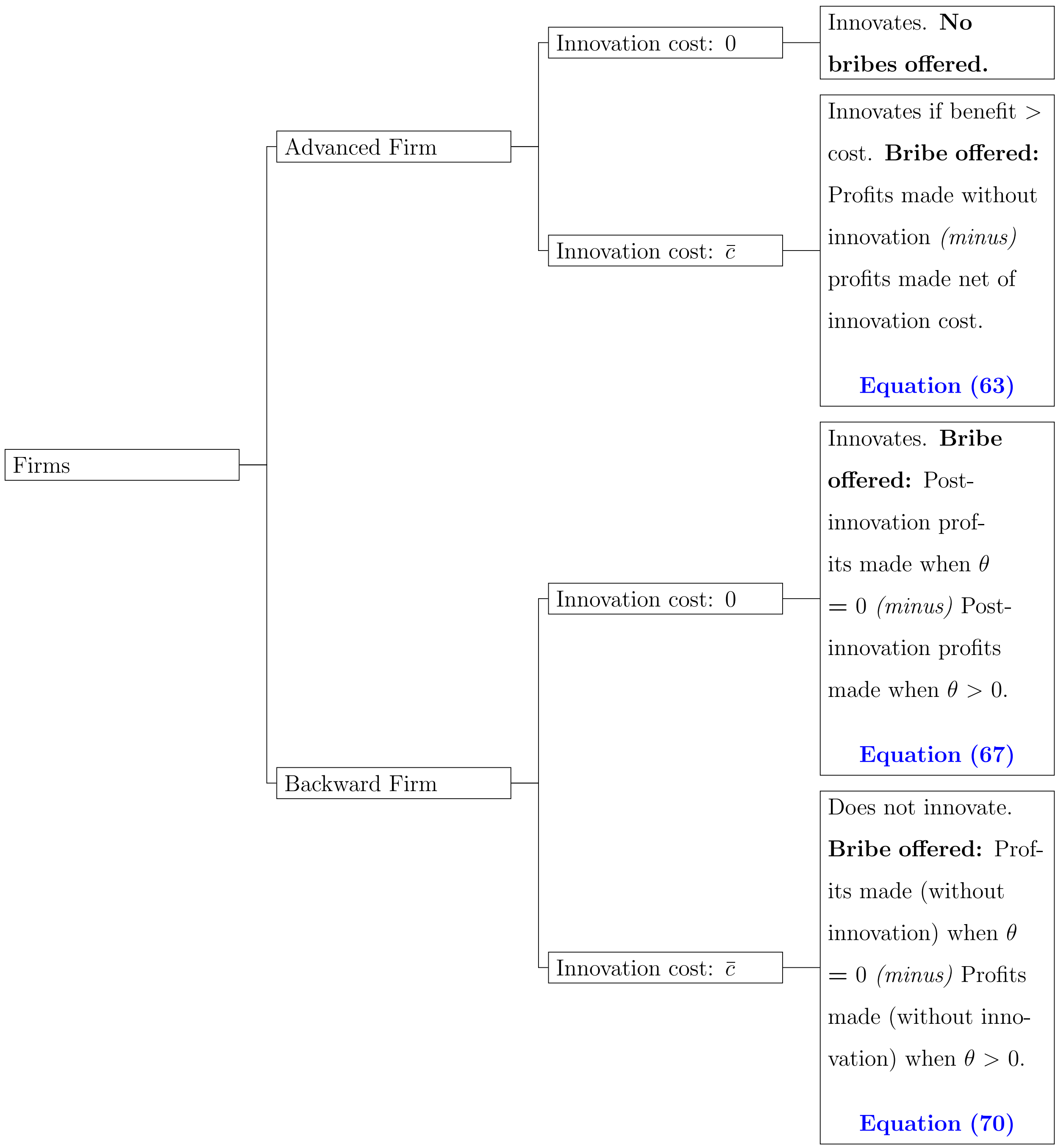

Figure 2: Bribes Offered by Incumbent Firms.

7.1 Bribes Offered by Incumbent Firms

In this section, we compute the maximum[5] bribes that incumbent firms—both advanced and backward—would be willing to pay to the policymaker to prevent a shift in entry probability from $\theta = 0$ to $\theta > 0$. To compute these bribes, we first need to compute the total payoffs of each type of firm, and for each cost realization: $c_{i,t} = 0$ or $c_{i,t} = \bar{c}$. We show that these payoffs are functions of the firm entry policy, θ.

We will now examine the bribes offered in detail, depending on whether the firm is advanced or backward. The discussion on bribes is summarized in Figure 2.

7.1.1 Bribes by Advanced Incumbent Firms

Consider first an incumbent intermediary firm that was advanced in the previous time period. Recall that we have specified two possibilities for the cost of innovation: $c_{i,t} = 0$ or $c_{i,t} = \bar{c}$.

Case 1: Cost of innovation, $c_{i,t}$, is zero.

If the innovation cost in time period t is $c_{i,t} = 0$, an incumbent intermediary firm will always innovate, irrespective of the entry threat, since, by doing so, it will make itself immune to the entry threat. Therefore, for an advanced incumbent firm facing $c_{i,t} = 0$, the post-innovation

[5] This section works out the maximum potential bribe that either the advanced or backward incumbent firm will offer to the policymaker. Since a bargaining game is not explicitly modeled, the sharing of surplus is not worked out. It is important to note that the actual bribes paid by incumbent firms may, in practice, be lower than the theoretical maximum derived in this subsection. The actual amount would likely result from a bargaining process between the policymaker and the firms. While detailed modeling of this bargaining process is beyond the scope of our analysis, it could, for instance, be represented by a Nash bargaining framework, where the division of surplus depends on the relative bargaining powers of the two parties. However, given that the policymaker holds absolute authority over the entry policy, θ, and that an entry probability of $\theta = 1$ could result in the complete displacement of incumbents from the market, the policymaker is likely to possess significant bargaining power. As a result, the policymaker would plausibly be able to extract a larger share of the firms' profits, thereby driving the equilibrium bribes closer to the maximum threshold. For the purposes of this model, we abstract away from the complexities of the bargaining game and focus solely on the maximum bribes as an upper bound on the rents that incumbent firms are willing to offer.

profit will be independent of the entry probability. From Equation (50), we know that the profits of an advanced incumbent firm are given by

$$\pi_{i,t} = \left[\alpha^{\frac{2}{1-\alpha}}\left(\frac{1}{\alpha} - 1\right) \cdot \gamma \cdot L_{s,t} - c_{i,t}\right]\bar{A}_{s,t-1}.$$

Since $c_{i,t} = 0$, the profit for this type of firm would reduce to

$$\pi_{i,t} = \left[\alpha^{\frac{2}{1-\alpha}}\left(\frac{1}{\alpha} - 1\right) \cdot \gamma \cdot L_{s,t}\right]\bar{A}_{s,t-1}. \tag{58}$$

Essentially, if a firm is advanced in the previous period, and if innovation is costless in the current period, it has no incentive to bribe the policymaker. This is because the firm can secure its position and maintain its profits simply by innovating. There is no additional benefit to be gained from bribing the policymaker because the firm's profits are already maximized through innovation.

Case 2: Cost of innovation, $c_{i,t}$, is $\bar{c}$.

An incumbent intermediary firm facing cost, $c_{i,t} = \bar{c}$, will innovate only if the entry threat, θ, becomes sufficiently high that Equation (48) holds, which means that the post-innovation profits, given the entry threat, θ, exceeds the cost of innovation. If it does innovate, it loses a chunk of its profits compared to its pre-innovation profits. Therefore, in time period t, the maximum bribe that an incumbent intermediary firm that was advanced in time period $t-1$, facing cost $c_{i,t} = \bar{c}$, would be willing to pay to the policymaker will be the difference between the profits it would have made without innovation and the profits (net of innovation cost) that it would be making if it innovates.

The pre-innovation profits of an incumbent intermediary firm are given by Equation (28). Note that for an incumbent firm that was advanced in the previous period, $t-1$, and has chosen not to innovate in the current time period, t, the productivity parameter in time period t would be $A_{is,t} = \bar{A}_{s,t-1}$. Therefore, we substitute for this fact in Equation (28) to obtain the current period pre-innovation profit of a firm that was advanced in the previous period and has not chosen to innovate in the current period. This is given by

$$\pi_{i,t} = \alpha^{\frac{2}{1-\alpha}}\left(\frac{1}{\alpha} - 1\right) \cdot L_{s,t} \cdot \bar{A}_{s,t-1}. \tag{59}$$

On the other hand, if the incumbent intermediary firm chooses to innovate, its profits will be given by Equation (50). Substituting for $c_{i,t} = \bar{c}$ in Equation (50), we obtain

$$\pi_{i,t} = \left[\alpha^{\frac{2}{1-\alpha}}\left(\frac{1}{\alpha}-1\right)\cdot\gamma\cdot L_{s,t} - \bar{c}\right]\bar{A}_{s,t-1}. \tag{60}$$

The maximum bribe that an incumbent firm that was advanced in time period $t-1$, which faces an entry threat $\bar{c}$, would be willing to pay to the policymaker would therefore be the difference between Equation (59) and Equation (60). We express it as

$$\alpha^{\frac{2}{1-\alpha}}\left(\frac{1}{\alpha}-1\right)\cdot L_{s,t}\cdot\bar{A}_{s,t-1} - \left[\alpha^{\frac{2}{1-\alpha}}\left(\frac{1}{\alpha}-1\right)\cdot\gamma\cdot L_{s,t} - \bar{c}\right]\bar{A}_{s,t-1}, \tag{61}$$

which can be simplified as

$$\bar{A}_{s,t-1}\left[\alpha^{\frac{2}{1-\alpha}}\left(\frac{1}{\alpha}-1\right)\cdot L_{s,t}(1-\gamma) + \bar{c}\right],$$

which can also be written as

$$\bar{A}_{s,t-1}\underbrace{\left[\bar{c} - (\gamma-1)\alpha^{\frac{2}{1-\alpha}}\left(\frac{1}{\alpha}-1\right)\cdot L_{s,t}\right]}_{(a)}. \tag{62}$$

Note that from Assumption 2, $Term\,(a)$ in Equation (62) is positive. Equation (62) gives us the maximum bribe that each incumbent firm that was advanced in the previous period will be willing to pay to the policymaker to prevent entry in time period t. Since the fraction of advanced firms in the steady state is given by β^*, and the probability that the cost of innovation is $\bar{c}$, is $Pr(c_{i,t} = \bar{c}) = \frac{1}{2}$, the total bribes offered by all incumbent firms that were advanced in the previous period and face an innovation cost of $\bar{c}$, that want the policymaker to restrict entry of new, technologically advanced, firms in time period t, will be

$$B_{a,t} = \bar{A}_{s,t-1}\cdot\beta^*\cdot\frac{1}{2}\left[\bar{c} - (\gamma-1)\alpha^{\frac{2}{1-\alpha}}\left(\frac{1}{\alpha}-1\right)\cdot L_{s,t}\right]\cdot 1_{\Phi},$$

which on substituting for the fact that $\bar{A}_{s,t-1} = \frac{1}{\gamma}\bar{A}_{s,t}$, can also be written as

$$B_{a,t}(\theta) = \frac{1}{\gamma}\bar{A}_{s,t}\cdot\beta^*\cdot\frac{1}{2}\left[\bar{c} - (\gamma-1)\alpha^{\frac{2}{1-\alpha}}\left(\frac{1}{\alpha}-1\right)\cdot L_{s,t}\right]\cdot 1_{\Phi}. \tag{63}$$

where the term 1_Φ is equal to 1 if Equation (48) holds, that is the condition for an advanced firm to innovate, given the entry probability and cost of innovation, holds. If this condition does not hold, the term 1_Φ would take the value zero, and the total bribes given by these firms would also be zero since they would not be engaging in innovation.

Note from Equation (63) that the total bribes that advanced incumbent firms as a group would be willing to pay to the policymaker when faced with an innovation cost of $\bar{c}$ in the current time period is not entirely independent of the entry threat, θ. Essentially, the entry threat, θ, impacts the firm's threshold of innovation decision, as given in Equation (48), but not the resultant profit differential, which is used to determine the bribes. Once the advanced firms decide to innovate so as to stay at the frontier and avoid displacement, their post-innovation profit becomes a constant factor, not influenced by the entry threat θ. It has already been seen in Equation (48) that a higher entry threat, θ, encourages advanced incumbent firms to innovate.

7.1.2 Bribes by Backward Incumbent Firms

We now consider incumbent intermediary firms that were backward in the previous time period, $t-1$. Such firms will innovate in time period t if and only if their cost of innovation is zero, that is, $c_{i,t} = 0$, irrespective of their entry threat. This is because even if a firm that was backward in time period $t-1$ innovates in time period t, it will still remain backward in time period t and will be displaced when entry occurs in time period t.

Case 1: Cost of innovation, $c_{i,t}$, is zero.

Consider, first, the scenario that it faces an innovation cost of $c_{i,t} = 0$. In such a scenario, the incumbent firm that was backward in the previous period would choose to innovate since innovation is costless. However, it would survive in time period t only if entry threat $\theta = 0$. Therefore, such a firm has an incentive to bribe the policymaker to restrict entry. The backward incumbent firm would be willing to pay a maximum bribe of the post-innovation profits that it would be foregoing when moving from $\theta = 0$ to $\theta > 0$. This means that the maximum bribe that this firm would be willing to pay to the policymaker would be the difference between the profits that it makes when it innovates and $\theta = 0$, and the profits that it would make when it innovates and $\theta > 0$. The post-innovation profit of an incumbent backward firm is given by Equation (55). When not faced by an entry threat, that is $\theta = 0$, and when the cost of innovation, $c_{i,t} = 0$, this would be suitably modified and written as

$$\pi_{i,t} = \alpha^{\frac{2}{1-\alpha}} \left(\frac{1}{\alpha} - 1 \right) L_{s,t} \cdot \gamma A_{is,t-1}. \tag{64}$$

On the other hand, when the entry threat is $\theta > 0$, if a backward firm innovates, its post-innovation profits are given by Equation (55). When considering the fact that in the present case $c_{i,t} = 0$, Equation (55) is suitably modified and written as

$$\pi_{i,t} = \alpha^{\frac{2}{1-\alpha}} \left(\frac{1}{\alpha} - 1 \right) L_{s,t} \cdot \gamma A_{is,t-1} \Big[1 - \theta \Big]. \tag{65}$$

As mentioned earlier, the maximum bribe that a firm facing $c_{i,t} = 0$ would be willing to pay would be the difference in post-innovation profits when $\theta = 0$ and when $\theta > 0$. This is given by the difference between Equation (64) and Equation (65).

$$\alpha^{\frac{2}{1-\alpha}} \left(\frac{1}{\alpha} - 1 \right) L_{s,t} \cdot \gamma A_{is,t-1} - \alpha^{\frac{2}{1-\alpha}} \left(\frac{1}{\alpha} - 1 \right) L_{s,t} \cdot \gamma A_{is,t-1} \Big[1 - \theta \Big], \tag{66}$$

which can be simplified as

$$\alpha^{\frac{2}{1-\alpha}} \left(\frac{1}{\alpha} - 1 \right) L_{s,t} \cdot \theta \gamma A_{is,t-1}. \tag{67}$$

Case 2: Cost of innovation, $c_{i,t}$, is $\bar{c}$. If a firm that was backward in the previous time period faces a cost of innovation $c_{i,t} = \bar{c}$ in the current time period, it will not innovate if entry threat $\theta > 0$. This is because, even if it innovates and takes a cut on its profits, it would not be able to survive an entry threat. In such a scenario, the maximum bribe that such a firm would be willing to pay the policymaker would be the difference between profits made when it does not innovate and $\theta = 0$, and the profits made when it does not innovate and $\theta > 0$. The profits of a backward firm, when it does not innovate, is given by Equation (52). When we consider $\theta = 0$, Equation (52) is suitably modified as

$$\pi_{i,t} = \alpha^{\frac{2}{1-\alpha}} \left(\frac{1}{\alpha} - 1 \right) \cdot L_{s,t} \cdot A_{is,t-1}. \tag{68}$$

The profits of a backward firm that does not innovate and faces an entry threat of $\theta > 0$, are given by Equation (52)

$$\pi_{i,t} = \alpha^{\frac{2}{1-\alpha}} \left(\frac{1}{\alpha} - 1 \right) \cdot L_{s,t} \cdot A_{is,t-1} \Big[1 - \theta \Big]$$

Therefore, the maximum bribe that a backward firm facing $c_{i,t} = \bar{c}$, would be willing to pay to the policymaker is given by the difference between the above two equations, which is given by

$$\alpha^{\frac{2}{1-\alpha}} \left(\frac{1}{\alpha} - 1 \right) \cdot L_{s,t} \cdot A_{is,t-1} - \alpha^{\frac{2}{1-\alpha}} \left(\frac{1}{\alpha} - 1 \right) \cdot L_{s,t} \cdot A_{is,t-1} \Big[1 - \theta \Big], \tag{69}$$

which can be simplified as

$$\alpha^{\frac{2}{1-\alpha}} \left(\frac{1}{\alpha} - 1 \right) \cdot L_{s,t} \cdot \theta \cdot A_{is,t-1}. \tag{70}$$

Equation (67) gives the maximum bribe that a backward firm will be ready to pay when $c_{i,t} = 0$ and Equation (70) gives the maximum bribe that a backward firm will be ready to pay when $c_{i,t} = \bar{c}$. Since *ex ante*, the firm has equal probabilities of facing either of the two scenarios, the maximum bribe that the backward firms as a group would be willing to pay the policymaker is given by

$$B_{b,t} = (1-\beta^*) \left[\frac{1}{2} \cdot \alpha^{\frac{2}{1-\alpha}} \left(\frac{1}{\alpha} - 1 \right) L_{s,t} \cdot \theta\gamma \cdot A_{is,t-1} + \frac{1}{2} \cdot \alpha^{\frac{2}{1-\alpha}} \left(\frac{1}{\alpha} - 1 \right) \cdot L_{s,t} \cdot \theta \cdot A_{is,t-1} \right], \tag{71}$$

where $(1-\beta^*)$ is the steady state share of backward firms in the economy. The above equation can also be written as

$$B_{b,t} = (1-\beta^*) A_{is,t-1} \left[\frac{1}{2} \cdot \alpha^{\frac{2}{1-\alpha}} \left(\frac{1}{\alpha} - 1 \right) L_{s,t} \cdot \theta\gamma + \frac{1}{2} \cdot \alpha^{\frac{2}{1-\alpha}} \left(\frac{1}{\alpha} - 1 \right) \cdot L_{s,t} \cdot \theta \right],$$

which, on substituting for $A_{is,t-1} = \frac{1}{\gamma} \bar{A}_{s,t-1} = \frac{1}{\gamma^2} \bar{A}_{s,t}$, can be re-written as

$$B_{b,t} = (1-\beta^*) \frac{1}{\gamma^2} \bar{A}_{s,t} \left[\frac{1}{2} \cdot \alpha^{\frac{2}{1-\alpha}} \left(\frac{1}{\alpha} - 1 \right) L_{s,t} \cdot \theta\gamma + \frac{1}{2} \cdot \alpha^{\frac{2}{1-\alpha}} \left(\frac{1}{\alpha} - 1 \right) \cdot L_{s,t} \cdot \theta \right],$$

which can be simplified as

$$B_{b,t}(\theta) = (1-\beta^*) \frac{1}{\gamma^2} \bar{A}_{s,t} \left[\frac{1}{2} \cdot \alpha^{\frac{2}{1-\alpha}} \left(\frac{1}{\alpha} - 1 \right) L_{s,t} \cdot \theta \right] \Big(\gamma + 1 \Big). \tag{72}$$

Note from Equation (72) that the maximum bribes offered by backward firms change linearly with θ. This reflects the economic reality that backward firms are more willing to pay higher bribes to avoid being displaced when the threat of entry is significant. A higher entry threat, θ, means that these firms are increasingly vulnerable to being displaced from the economy.

7.1.3 Total Bribes Offered by Incumbent Firms

On adding Equation (63) and Equation (72), we get the total bribes that incumbent firms will be willing to pay the policymaker so as to prevent him/her from increasing the entry threat from $\theta = 0$ to $\theta > 0$. Thus, the total bribes will be

$$B_t(\theta) = B_{a,t}(\theta) + B_{b,t}(\theta) = \frac{1}{\gamma}\bar{A}_{s,t} \cdot \beta^* \cdot \frac{1}{2}\left[\bar{c} - \big(\gamma - 1\big)\alpha^{\frac{2}{1-\alpha}}\left(\frac{1}{\alpha} - 1\right) \cdot L_{s,t}\right] \cdot 1_\Phi + (1-\beta^*)\frac{1}{\gamma^2}\bar{A}_{s,t}\left[\frac{1}{2} \cdot \alpha^{\frac{2}{1-\alpha}}\left(\frac{1}{\alpha} - 1\right)L_{s,t} \cdot \theta\right]\Big(\gamma + 1\Big). \quad (73)$$

Equation (73) captures the aggregate behavior of incumbent firms in terms of their willingness to bribe the policymaker to restrict entry. The equation combines the incentives of both advanced and backward firms (where the share of advanced firms in the economy, β^* is exogenous), reflecting their strategic responses to the entry threat parameter, θ. Note that for advanced firms, θ indirectly affects the bribes by affecting their decision to innovate, which is conditional upon Equation (48) holding true. This is captured by the term 1_Φ in Equation (73). For a backward firm, θ directly scales the bribe amount, showing a linear relationship. This indicates that as the entry threat, θ, increases, backward firms are more willing to pay higher bribes to avoid being displaced.

We now analyze how the total bribe paid by incumbent firms changes with a change in the entry threat, θ. The first-order derivative with respect to θ is given by

$$\frac{\partial B_t(\theta)}{\partial \theta} = (1-\beta^*)\frac{1}{\gamma^2}\bar{A}_{s,t}\left[\frac{1}{2} \cdot \alpha^{\frac{2}{1-\alpha}}\left(\frac{1}{\alpha} - 1\right)L_{s,t}\right]\Big(\gamma + 1\Big), \quad (74)$$

which is positive. Note that the term 1_Φ is an indicator function, which can only be 0 or 1, and is therefore constant with respect to θ. The second-order derivative is given by

$$\frac{\partial^2 B_t(\theta)}{\partial \theta^2} = 0. \quad (75)$$

Since $\frac{\partial B_t(\theta)}{\partial \theta} > 0$ and $\frac{\partial^2 B_t(\theta)}{\partial \theta^2} = 0$, it is clear that the total bribes are a linear function of the entry probability, θ. Since θ lies in the range of $[0, 1]$, the policymaker can earn the highest bribes by setting the entry policy as $\theta = 1$.

7.2 The Dynamics of Aggregate Productivity

It has been shown that the GDP and the wage premium are both functions of the aggregate productivity in the skilled goods sector, $A_{s,t}$. In this sub-section, we show how this aggregate productivity is influenced by firms' decision to innovate, which in turn is influenced by the entry threat θ and the cost realization faced by each firm, $c_{i,t}$. Note that, irrespective of whether the firm is advanced or backward, if entry occurs in sector i with probability θ, the productivity in that sector will always be at the frontier since the new entrant comes with frontier technology. Therefore, in such a scenario, the productivity of the i^{th} firm will be $\bar{A}_{s,t}$. When entry does not occur, for which the probability is $(1 - \theta)$, the change in a firm's productivity depends on the type of the firm and the cost of innovation faced by each of it.

The changes in the productivity of firms, conditional upon the entry threat, θ, are summarized in Figure 3.

Case 1: Contributions to Aggregate Productivity by Advanced Firms

In the case of an advanced firm, if entry occurs in the i^{th} sector with probability θ, then the productivity of that sector would be at the frontier since the new entrant comes with frontier technology. Therefore, the contribution to aggregate productivity by sectors in which entry has occurred is given by

$$\beta^* \theta \bar{A}_{s,t}, \tag{76}$$

where β^* is the equilibrium share of advanced firms in the economy.

When there is no entry threat, the decision to innovate depends on the cost of innovation faced by the firm. If the innovation cost faced by the firm is $c_{i,t} = 0$, for which the probability is $\frac{1}{2}$, an advanced firm innovates. Therefore, the contribution to aggregate productivity by advanced firms that have survived an entry threat, faced an innovation cost of $c_{i,t} = 0$, and

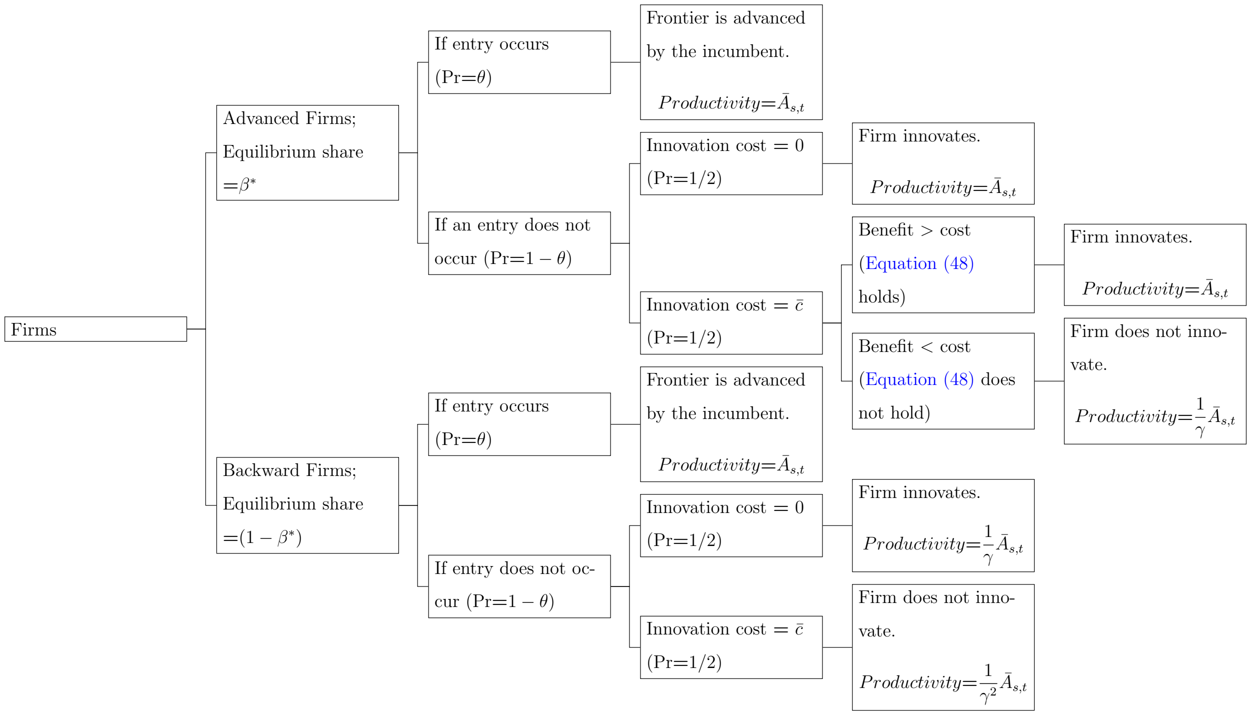

Figure 3: Changes in the Productivity of Firms, Conditional Upon the Entry Threat, θ.

innovated will be

$$\beta^*(1-\theta)\frac{1}{2}\bar{A}_{s,t}. \tag{77}$$

On the other hand, if it faces a cost of innovation $c_{i,t} = \bar{c}$, whether the firm innovates depends on whether the benefit from innovation exceeds the cost of innovation, that is, when Equation (48) holds. In such a scenario, the contribution to aggregate productivity by advanced firms that face an innovation cost of $\bar{c}$ and for whom Equation (48) holds, is given by

$$\beta^*(1-\theta)\frac{1}{2}\bar{A}_{s,t}\cdot 1_\Phi, \tag{78}$$

where the term 1_Φ takes the value one if Equation (48) holds, and zero otherwise. Advanced firms that survive entry threat and face an innovation cost of $\bar{c}$ do not innovate if the benefit from innovation does not exceed the cost of innovation, that is, if Equation (48) does not hold. Consequently, the productivity of the i^{th} firm in this scenario will be the same as the productivity in the previous period; that is, it will be $\bar{A}_{s,t-1} = \frac{1}{\gamma}\bar{A}_{s,t}$. The contribution to aggregate productivity by such advanced firms is given by

$$\beta^*(1-\theta)\frac{1}{2}\bar{A}_{s,t-1}\cdot 1_\Psi = \beta^*(1-\theta)\frac{1}{2}\cdot\frac{1}{\gamma}\bar{A}_{s,t}\cdot 1_\Psi, \tag{79}$$

where the term 1_Ψ takes the value one if Equation (48) does not hold, and zero otherwise. Adding Equation (76), Equation (77), Equation (78), and Equation (79) gives us the total contribution to aggregate productivity by advanced firms. This addition yields

$$A_{s,t,a} = \beta^*\theta\bar{A}_{s,t} + \beta^*(1-\theta)\frac{1}{2}\bar{A}_{s,t} + \beta^*(1-\theta)\frac{1}{2}\bar{A}_{s,t}\cdot 1_\Phi + \beta^*(1-\theta)\frac{1}{2}\cdot\frac{1}{\gamma}\bar{A}_{s,t}\cdot 1_\Psi, \tag{80}$$

which, on collection of terms, can also be written as

$$A_{s,t,a} = \beta^*\left[\theta\bar{A}_{s,t} + (1-\theta)\frac{1}{2}\bar{A}_{s,t} + (1-\theta)\frac{1}{2}\bar{A}_{s,t}\Big[1\cdot 1_\Phi + \frac{1}{\gamma}\cdot 1_\Psi\Big]\right],$$

and on factoring out the term $\bar{A}_{s,t}$ can be simplified as

$$A_{s,t,a} = \beta^*\left[\frac{1}{2}\bar{A}_{s,t} + \theta\frac{1}{2}\bar{A}_{s,t} + (1-\theta)\frac{1}{2}\bar{A}_{s,t}\Big[1\cdot 1_\Phi + \frac{1}{\gamma}\cdot 1_\Psi\Big]\right],$$

and on combining like terms, this reduces to

$$A_{s,t,a}(\theta) = \frac{\beta^*\bar{A}_{s,t}}{2}\left[1+\theta+(1-\theta)\Big[1\cdot 1_\Phi + \frac{1}{\gamma}\cdot 1_\Psi\Big]\right], \tag{81}$$

Note that Equation (81) denotes the contribution made by advanced firms to the aggregate productivity in the economy. This contribution not only depends on the entry threat, θ, but also the growth rate of the frontier, γ, and also on the marginal benefits and costs associated with innovation. Further, it is also a function of the steady-state share of advanced firms, β^*, the frontier productivity level, $\bar{A}_{s,t}$, the indicator function, 1_Φ, which takes value 1 if Equation (48) holds and 0 otherwise, and the indicator function, 1_Ψ which takes the value 1 if Equation (48) does not hold and 0 otherwise.

Case 2: Contributions to Aggregate Productivity by Backward Firms

In the case of a backward firm, once again, if entry occurs in the i^{th} sector, the productivity in that sector will be at the frontier since the entrant comes with frontier technology. The contribution to aggregate productivity by such firms will be given by

$$(1-\beta^*)\theta\bar{A}_{s,t}. \tag{82}$$

When there is no entry threat, for which the probability is given by $(1-\theta)$, firms decide whether to innovate, depending on the cost of innovation, $c_{i,t}$. If the cost of innovation is zero, for which the probability is $\frac{1}{2}$, these firms innovate. The contribution to aggregate productivity by such firms is given by

$$(1-\beta^*)(1-\theta)\frac{1}{2}A_{is,t} = (1-\beta^*)(1-\theta)\frac{1}{2}\cdot\frac{1}{\gamma}\bar{A}_{s,t}. \tag{83}$$

On the other hand, if the cost of innovation is $\bar{c}$, for which the probability is $\frac{1}{2}$, these firms do not innovate. The contribution to aggregate productivity by such firms is given by

$$(1-\beta^*)(1-\theta)\frac{1}{2}A_{s,t-1} = (1-\beta^*)(1-\theta)\frac{1}{2}\cdot\frac{1}{\gamma}\bar{A}_{s,t-1} = (1-\beta^*)(1-\theta)\frac{1}{2}\cdot\frac{1}{\gamma^2}\bar{A}_{s,t}. \tag{84}$$

The total contribution to aggregate productivity by backward firms is given by adding Equation (82), Equation (83), and Equation (84). This addition yields

$$A_{s,t,b} = (1-\beta^*)\theta\bar{A}_{s,t} + (1-\beta^*)(1-\theta)\frac{1}{2}\cdot\frac{1}{\gamma}\bar{A}_{s,t} + (1-\beta^*)(1-\theta)\frac{1}{2}\cdot\frac{1}{\gamma^2}\bar{A}_{s,t}, \tag{85}$$

which, on factoring out $(1-\beta^*)\bar{A}_{s,t}$ can be simplified as

$$A_{s,t,b} = (1-\beta^*)\bar{A}_{s,t}\left[\theta + (1-\theta)\frac{1}{2}\cdot\frac{1}{\gamma}\left[1+\frac{1}{\gamma}\right]\right], \tag{86}$$

which can also be written as

$$A_{s,t,b}(\theta) = (1-\beta^*)\bar{A}_{s,t}\left[\theta + (1-\theta)\frac{1}{2}\cdot\frac{1+\gamma}{\gamma^2}\right] \tag{87}$$

Note that Equation (87) denotes the contribution made by the backward firms to the aggregate productivity in the economy. This contribution is a function of the share of backward firms, $1-\beta^*$, the frontier productivity, $\bar{A}_{s,t}$, the entry threat, θ, and the growth rate of the frontier, γ.

The total contribution to productivity made by advanced firms and backward firms put together is obtained by adding Equation (81) and Equation (87), which yields

$$\begin{aligned} A_{s,t}(\theta) = A_{s,t,a}(\theta) + A_{s,t,b}(\theta) = \frac{\beta^*\bar{A}_{s,t}}{2}&\left[1+\theta+(1-\theta)\left[1\cdot 1_{\Phi} + \frac{1}{\gamma}\cdot 1_{\Psi}\right]\right] \\ &+ (1-\beta^*)\bar{A}_{s,t}\left[\theta + (1-\theta)\frac{1}{2}\cdot\frac{1+\gamma}{\gamma^2}\right]. \end{aligned} \tag{88}$$

Equation (88) encapsulates the complex interplay between aggregate productivity in the skilled sector and the entry probability, θ, innovation costs, $c_{i,t}$, and the rate of growth of the frontier γ. To see how the aggregate productivity of the skilled sector, $A_{s,t}$, behaves with respect to θ, we find the partial derivative of Equation (85) with respect to θ, which yields

$$\frac{\partial A_{s,t}}{\partial \theta} = \underbrace{\frac{\beta^* \bar{A}_{s,t}}{2}\left[1 - \left[1 \cdot 1_\Phi + \frac{1}{\gamma} \cdot 1_\Psi\right]\right]}_{(a)} + \underbrace{(1-\beta^*)\bar{A}_{s,t}\left[1 - \frac{1}{2} \cdot \frac{1+\gamma}{\gamma^2}\right]}_{(b)} \tag{89}$$

Note from Equation (89) that $Term\,(b)$ is always positive, given that $0 < \beta < 1$ and $\gamma > 1$. The sign of $Term\,(a)$ depends on whether Equation (48) holds, that is, whether the benefit from innovation exceeds the cost of innovation for advanced firms. Two scenarios are possible here, depending on whether Equation (48) holds.

Scenario 1: Equation (48) holds

In such a case, the term 1_Φ will be equal to 1 and the term 1_Ψ will be equal to zero. Consequently, $Term\,(a)$ of Equation (86) will be

$$\frac{\beta^* \bar{A}_{s,t}}{2}\left[1 - \left[1 \cdot 1 + \frac{1}{\gamma} \cdot 0\right]\right] = \frac{\beta^* \bar{A}_{s,t}}{2}\left[1 - 1\right] = 0,$$

in which case the right-hand side of Equation (89) will be positive.

Scenario 2: Equation (48) does not hold

In such a case, the term 1_Φ will be equal to zero and the term 1_Ψ will be equal to 1. Consequently, $Term\,(a)$ of Equation (86) will be

$$\frac{\beta^* \bar{A}_{s,t}}{2}\left[1 - \left[1 \cdot 0 + \frac{1}{\gamma} \cdot 1\right]\right] = \frac{\beta^* \bar{A}_{s,t}}{2}\left[1 - \frac{1}{\gamma}\right] = \frac{\beta^* \bar{A}_{s,t}(\gamma - 1)}{2\gamma},$$

in which case, again, the right-hand side of Equation (89) will be positive.

Thus, in both scenarios, the overall sign of Equation (89) is positive. This means that the aggregate productivity of the skilled sector, $A_{s,t}$, increases with respect to θ.

$$\frac{\partial A_{s,t}}{\partial \theta} > 0. \tag{90}$$

Intuitively, a higher θ leads to a more competitive environment where firms are driven to continuously improve their productivity through innovation.

From Equation (89), we further arrive at the second-order derivative, which is given by

$$\frac{\partial^2 A_{s,t}}{\partial \theta^2} = 0. \tag{91}$$

This indicates that the relationship between the aggregate productivity in the skilled sector, $A_{s,t}$, and the entry probability, θ, is linear[6]. This means that the $A_{s,t}$ attains maximum when $\theta = 1$.

7.2.1 Relationship of Aggregate Productivity in the Skilled Sector, $A_{s,t}$ with the GDP

Equation (31) expresses the GDP as a function of, *inter alia*, the aggregate productivity in the skilled sector.

$$GDP_t = G_t = P_u A_u L_{u,t} + \alpha^{\frac{1+\alpha}{1-\alpha}} L_{s,t} A_{s,t} \left[\frac{1}{\alpha} - 1\right].$$

Recall from Equation (31) that the GDP is a linear function of the aggregate productivity of the skilled sector, $A_{s,t}$. We now analyze how GDP behaves with respect to changes in the entry probability, θ. The first-order derivative with respect to θ is given by

$$\frac{\partial G_t(A_{s,t}(\theta))}{\partial \theta} = \alpha^{\frac{1+\alpha}{1-\alpha}} \left[\frac{1}{\alpha} - 1\right] L_s \frac{\partial A_{s,t}(\theta)}{\partial \theta}. \tag{92}$$

Since $\frac{\partial A_{s,t}(\theta)}{\partial \theta} > 0$, the above equation is positive. The second-order derivative with respect to θ is given by

$$\frac{\partial^2 G_t(A_{s,t}(\theta))}{\partial \theta^2} = \alpha^{\frac{1+\alpha}{1-\alpha}} \left[\frac{1}{\alpha} - 1\right] L_s \frac{\partial^2 A_{s,t}(\theta)}{\partial \theta^2}. \tag{93}$$

And since $\frac{\partial^2 A_{s,t}(\theta)}{\partial \theta^2} = 0$, the above equation becomes zero. Therefore, we now have that

$$\frac{\partial G_t(A_{s,t}(\theta))}{\partial \theta} > 0; \quad \frac{\partial^2 G_t(A_{s,t}(\theta))}{\partial \theta^2} = 0,$$

which means that the GDP is a linear function of θ. Since θ lies in the range of $[0, 1]$, the GDP attains maximum when the firm entry policy, θ, is set to 1.

[6] Note that the terms 1_Φ and 1_Ψ are indicator functions, which can only be 0 or 1, and therefore are constants with respect to θ. Recall that these indicators depend on whether Equation (48) holds.

7.2.2 Relationship of Aggregate Productivity in the Skilled Sector, $A_{s,t}$ with the Inverse of Skill Premium

We now proceed to analyze how the skill premium behaves with respect to the entry probability, θ. Recall from Equation (38) that the skill premium is given by

$$\omega_t(A_{s,t}(\theta)) = (1-\alpha)\alpha^{\frac{2\alpha\rho}{1-\alpha}} \cdot \frac{1}{\eta} \cdot \left[\frac{A_{s,t}(\theta)}{A_u}\right]^{\rho} \cdot \left[\frac{L_u}{L_s}\right]^{1-\rho}.$$

We differentiate the above equation with respect to θ and obtain

$$\frac{\partial \omega_t(A_{s,t}(\theta))}{\partial \theta} = (1-\alpha)\alpha^{\frac{2\alpha\rho}{1-\alpha}} \cdot \frac{1}{\eta} \cdot \left[\frac{1}{A_u}\right]^{\rho} \cdot \left[\frac{L_u}{L_s}\right]^{1-\rho} \cdot \rho \cdot \Big[A_{s,t}(\theta)\Big]^{\rho-1} \cdot \frac{\partial A_{s,t}(\theta)}{\partial \theta}. \tag{94}$$

Since we know from Equation (89) that $\frac{\partial A_{s,t}}{\partial \theta} > 0$, and since $0 < \rho < 1$, Equation (94) is positive, that is, $\frac{\partial \omega_t}{\partial \theta} > 0$. The second-order derivative is given by

$$\frac{\partial^2 \omega_t}{\partial \theta^2} = (1-\alpha)\alpha^{\frac{2\alpha\rho}{1-\alpha}} \cdot \frac{1}{\eta} \cdot \left[\frac{1}{A_u}\right]^{\rho} \cdot \left[\frac{L_u}{L_s}\right]^{1-\rho} \cdot \rho \cdot \Bigg[(\rho-1)\Big[A_{s,t}(\theta)\Big]^{\rho-2} \cdot \left[\frac{\partial A_{s,t}(\theta)}{\partial \theta}\right]^2 + \Big[A_{s,t}(\theta)\Big]^{\rho-1} \cdot \frac{\partial^2 A_{s,t}(\theta)}{\partial \theta^2}\Bigg]. \tag{95}$$

From Equation (90), we know that $\frac{\partial^2 A_{s,t}(\theta)}{\partial \theta^2} = 0$. Therefore, Equation (95) can be reduced to

$$\frac{\partial^2 \omega_t}{\partial \theta^2} = (1-\alpha)\alpha^{\frac{2\alpha\rho}{1-\alpha}} \cdot \frac{1}{\eta} \cdot \left[\frac{1}{A_u}\right]^{\rho} \cdot \left[\frac{L_u}{L_s}\right]^{1-\rho} \cdot \rho \cdot \Bigg[(\rho-1)\Big[A_{s,t}(\theta)\Big]^{\rho-2} \cdot \left[\frac{\partial A_{s,t}(\theta)}{\partial \theta}\right]^2\Bigg]. \tag{96}$$

Since $0 < \rho < 1$, $(\rho - 1) < 0$. Therefore, Equation (96) is negative. From Equation (94) and Equation (96) we have that

$$\frac{\partial \omega_t}{\partial \theta} > 0; \quad \frac{\partial^2 \omega_t}{\partial \theta^2} < 0.$$

This shows that the skill premium, ω_t, is a concave function of the entry probability, θ. This means that the skill premium reaches a maximum at the point where $\frac{\partial \omega_t}{\partial \theta} = 0$.

The policymaker is assumed to dislike wage inequality and is interested in reducing the wage differential between skilled and unskilled laborers. To this end, we consider that the inverse of the skill premium enters his/her objective function, which he/she sets to maximize. Therefore, we now proceed to analyze how the inverse of the skill premium behaves with respect to the entry probability, θ. Equation (38) gives the skill premium, from which the inverse of the skill premium is obtained as

$$\omega_t^{-1}(A_{s,t}(\theta)) = \left[(1-\alpha)\alpha^{\frac{2\alpha\rho}{1-\alpha}} \cdot \frac{1}{\eta}\right]^{-1} \cdot \left[\frac{A_u}{A_{s,t}(\theta)}\right]^{\rho} \cdot \left[\frac{L_s}{L_u}\right]^{1-\rho}. \tag{97}$$

Note that ω_t^{-1} is inversely proportional to the aggregate productivity of the skill sector, $A_{s,t}$. This inverse relationship is an essential characteristic of how the inverse of skill premium responds to changes in aggregate productivity in the skilled sector, which we will invoke while discussing our main results.

From Equation (97), we obtain the first-order derivative of ω_t^{-1} with respect to θ as

$$\frac{\partial \omega_t^{-1}(A_{s,t}(\theta))}{\partial \theta} = \left[(1-\alpha)\alpha^{\frac{2\alpha\rho}{1-\alpha}} \cdot \frac{1}{\eta}\right]^{-1} \left[\frac{L_s}{L_u}\right]^{1-\rho} \big[A_u\big]^{\rho} \left[-\rho\big[A_{s,t}(\theta)\big]^{-(\rho+1)} \cdot \frac{\partial A_{s,t}(\theta)}{\partial \theta}\right]. \tag{98}$$

Since $0 < \rho < 1$, and $\frac{\partial A_{s,t}(\theta)}{\partial \theta} > 0$, Equation (98) is negative. The second-order derivative is given by

$$\begin{aligned}\frac{\partial^2 \omega_t^{-1}(A_{s,t}(\theta))}{\partial \theta^2} = &\left[(1-\alpha)\alpha^{\frac{2\alpha\rho}{1-\alpha}} \cdot \frac{1}{\eta}\right]^{-1} \left[\frac{L_s}{L_u}\right]^{1-\rho} \big[A_u\big]^{\rho} \\ &\cdot \left[\rho(\rho+1)\big[A_{s,t}(\theta)\big]^{-(\rho+2)} \cdot \left[\frac{\partial A_{s,t}(\theta)}{\partial \theta}\right]^2 - \rho\big[A_{s,t}(\theta)\big]^{-(\rho+1)} \cdot \frac{\partial^2 A_{s,t}(\theta)}{\partial \theta^2}\right].\end{aligned}$$

Since $\frac{\partial^2 A_{s,t}(\theta)}{\partial \theta^2} = 0$, the above equation reduces to

$$\begin{aligned}\frac{\partial^2 \omega_t(A_{s,t}(\theta))^{-1}}{\partial \theta^2} = &\left[(1-\alpha)\alpha^{\frac{2\alpha\rho}{1-\alpha}} \cdot \frac{1}{\eta}\right]^{-1} \left[\frac{L_s}{L_u}\right]^{1-\rho} \big[A_u\big]^{\rho} \\ &\cdot \left[\rho(\rho+1)\big[A_{s,t}(\theta)\big]^{-(\rho+2)} \cdot \left[\frac{\partial A_{s,t}(\theta)}{\partial \theta}\right]^2\right],\end{aligned} \tag{99}$$

which is positive since $0 < \rho < 1$. Therefore, from Equation (98) and Equation (99), we have

$$\frac{\partial \omega_t^{-1}}{\partial \theta} < 0; \quad \frac{\partial^2 \omega_t^{-1}}{\partial \theta^2} > 0. \tag{100}$$

This means that the inverse of the skill premium is a decreasing and convex function with respect to θ. This suggests that as the entry probability θ increases, the inverse of the skill premium decreases at an increasing rate. Therefore, to maximize the inverse of the skill premium, a lower value of θ would be preferable. Since θ lies in the range of $[0, 1]$, this means that the inverse of skill premium is maximized when $\theta = 0$.

The relationships between the aggregate productivity in the skill sector, $A_{s,t}$, the entry probability, θ, the total bribes, B, the GDP and the inverse of the skill premium, ω_t^{-1}, are summarized in Table 1.

	Aggregate Productivity ($A_{s,t}$)	Entry Probability (θ)
Aggregate Productivity, ($A_{s,t}$)	-	Linear
Bribe (B)	-	Linear
GDP (G_t)	Linear	Linear
Inverse of skill premium (ω_t^{-1})	Inversely Proportional to $A_{s,t}$	Convex and Decreasing with θ

Table 1: Relationship between $A_{s,t}$, θ, Bribes, GDP and ω_t^{-1}.

8 Determining the Planner's Choice of the Firm Entry Policy, θ

We now present the politician's objective function, based on which he/she determines the firm entry policy. The policymaker is not only concerned about addressing inequality, ω_t^{-1} in the economy but also influenced by rent-seeking activities by incumbent firms, who offer bribes to the politicians to restrict the entry of technologically advanced firms. Thus, the objective function[7] of the policymaker is given by

$$\Theta(\theta) = a \cdot B_t(\theta) + (1-a) \cdot \omega_t^{-1}(\theta); \quad a \in [0,1] \tag{101}$$

[7]A more realistic specification of the policymaker's objective function would incorporate GDP (or the growth rate of GDP) as an explicit argument. In the current formulation, the policymaker balances bribes and inequality, implicitly assuming that growth is sufficiently incentivized through the bribe channel due to its positive correlation with entry probability, θ. However, this assumption may not guarantee growth in equilibrium, especially when income inequality concerns dominate. In the absence of an explicit growth term, the policymaker could optimally choose $\theta = 0$, resulting in a no-growth equilibrium due to the absence of new entry threat and, consequently, reasons to invest in innovation.

To address this potential limitation, consider the following alternative formulation of the objective function

$$\Theta(\theta) = \zeta \cdot G_t(\theta) + a \cdot B_t(\theta) + (1-a-\zeta) \cdot \omega_t^{-1}(\theta); \quad \zeta, a \in [0,1], \quad \zeta + a \leq 1,$$

where ζ represents the weight on GDP, reflecting the policymaker's emphasis on economic growth, a captures the weight on bribes, representing rent-extraction incentives, and $1 - a - \zeta$ denotes the weight on inequality reduction.

This extension allows for a richer set of policy regimes, each representing different policymaker priorities.

1. When $a = 1, \text{and } \zeta = 0$, the policymaker maximizes bribes exclusively. Since bribes increase linearly with θ, the equilibrium entry probability would be high, as a higher entry threat incentivizes incumbents to pay substantial bribes to block new entrants. This could be dubbed as the "Rent-seeking" regime.

2. When $a = 0, \text{and } \zeta = 0$, the policymaker exclusively minimizes inequality. Given the convex and decreasing relationship between the inverse of skill premium and θ, the optimal policy would push θ toward zero to minimize inequality, resulting in a no-growth equilibrium. This may be called as the "Inequality-only" regime.

3. When $a = 0, \text{and } \zeta > 0$, the policymaker balances growth and inequality reduction. The linear relationship between GDP and θ ensures a positive entry probability, although inequality concerns limit θ from reaching levels as high as those in the rent-seeking regime. This could be the purely benevolent regime, where the policymaker is not pursuing bribes but purely seeks to have positive growth and reduced income inequality.

4. Having $a > 0, \zeta > 0, \text{and } 1 - a - \zeta > 0$ could be the politically motivated regime where policymaker simultaneously considers growth, rent extraction, and inequality reduction. In this case, the equilibrium θ depends on the relative weights assigned to each component. Given the linear relationships between GDP, bribes, and θ, monotonic responses are ensured. Specifically,

where a is the weight that the policymaker assigns on the total bribes, B_t, offered by the incumbent firms, and $(1-a)$ is the weight assigned by him/her on the inverse of the skill premium, ω^{-1}.

Proposition 1. ***A policymaker who does not have a distributional concern sets a highly competitive entry policy. Accordingly, the aggregate productivity of the skilled sector is at the frontier of technology in the skilled sector.***

Proof. A policymaker who does not have a distributional concern does not aim to reduce the wage inequality between the two types of laborers in the economy. Therefore, he/she sets a weight $a = 1$, and as a result, the weight on the inverse of skill premium is zero. We call this regime as **"Regime 1: The Bribe-maximizing Regime"**. In this regime, the objective function of the policymaker is given by

$$\Theta_{nw}(\theta) = a \cdot B_t(\theta). \tag{102}$$

The first-order and second-order derivatives of this objective function, with respect to θ, are given by

$$\frac{\partial \Theta_{nw}(\theta)}{\partial \theta} = a \cdot \frac{\partial B_t(\theta)}{\partial \theta} > 0,$$

---

i. If $(\zeta + a > 1 - a - \zeta) \implies a + \zeta > \frac{1}{2}$, the combined incentives for growth and rent extraction outweigh inequality concerns, resulting in a higher θ.

ii. Conversely, if $a + \zeta > \frac{1}{2}$, inequality concerns dominate, pulling θ downward.

We conjecture the following ranking of the equilibrium entry probability

$$\theta_{\text{Rent-seeking}} > \theta_{\text{Politically-motivated}} > \theta_{\text{Purely-benevolent}} > \theta_{\text{Inequality-only}}.$$

The rent-seeking regime yields the highest θ due to strong incentives to sustain entry threats for maximum bribe extraction. The politically motivated regime is likely to produce a higher θ than the benevolent regime if $a+\zeta > \frac{1}{2}$, reflecting the joint dominance of growth and rent-extraction incentives over inequality concerns. Conversely, if inequality concerns dominate ($a + \zeta < \frac{1}{2}$), the politically motivated policymaker would set a lower θ than the benevolent counterpart. Finally, the inequality-only regime results in the lowest θ (zero), given the increasing inequality cost associated with higher entry probabilities.

Although the linear relationships simplify some comparative statics, deriving closed-form solutions for the equilibrium θ and analytically ranking GDP, bribes, and inequality across all regimes are leading to intractable derivations, necessitating simulation-based approaches to explore these trade-offs rigorously. We propose this for future work.

and

$$\frac{\partial^2 \Theta_{nw}(\theta)}{\partial \theta^2} = a \cdot \frac{\partial^2 B_t(\theta)}{\partial \theta^2} = 0,$$

respectively. Therefore, the objective function of a policymaker that does not have a distributional concern is a strictly linear increasing function of the entry policy, θ. Given that θ lies in the range $[0, 1]$, the policymaker will choose $\theta = 1$ to maximize $\Theta_{nw}(\theta)$. Setting an entry policy, $\theta = 1$, means that the threat of an entrant entering the economy and displacing the incumbent is high. Consequently, the incumbent firms, recognizing the high risk of displacement, may offer substantial bribes to the policymaker to deter entry. Thus, a policymaker without a distributional concern, focusing solely on maximizing bribes, will prefer a high entry threat, $\theta = 1$. This concludes the proof.

When the entry policy is set to $\theta = 1$, the aggregate productivity in the skill sector is obtained by substituting for $\theta = 1$ in Equation (87). This yields

$$A_{s,t}(\theta = 1) = \frac{\beta^* \bar{A}_{s,t}}{2}\left[1+1+(1-1)\Big[1\cdot 1_\Phi + \frac{1}{\gamma}\cdot 1_\Psi\Big]\right] + (1-\beta^*)\bar{A}_{s,t}\left[1+(1-1)\frac{1}{2}\cdot\frac{1+\gamma}{\gamma^2}\right], \quad (103)$$

which can be simplified to

$$A_{s,t}(\theta = 1) = \frac{\beta^* \bar{A}_{s,t}}{2} \cdot \big(2\big) + (1-\beta^*)\bar{A}_{s,t} \cdot \big(1\big) = \beta^* \bar{A}_{s,t} + (1-\beta^*)\bar{A}_{s,t} = \bar{A}_{s,t}. \quad (104)$$

From Equation (104), it can be seen that the aggregate productivity of the skilled sector, when the policymaker sets $\theta = 1$, is exactly at the frontier. This means that

$$\frac{A_{s,t}(\theta = 1)}{\bar{A}_{s,t}} = 1. \quad (105)$$

Equation (105) captures the fact that a high entry threat induces a highly competitive environment where incumbent firms are continuously pressured to innovate and improve their productivity to avoid being displaced by new entrants. The constant threat of entry ensures that the market remains dynamic, fostering innovation and efficiency among firms. This completes the proof. □

Proposition 2. ***A policymaker that is only concerned about reducing the income inequality in the economy sets the most restrictive entry policy. Accordingly,***

the aggregate productivity of the skill sector is lower than the maximum potential efficiency level that it can reach.

Proof. A policymaker that is solely concerned about reducing the income inequality in the economy sets weights $a = 0$. Consequently, he/she assigns zero-weight to total bribes offered by firms and a weight of 1 on the inverse of skill premium. We call this as **"Regime 2: The Inequality-minimizing Regime"**. Such a policymaker has the following objective function

$$\Theta_{ow}(\theta) = (1 - a) \cdot \omega_t^{-1}(\theta). \tag{106}$$

The first-order and second-order derivatives of this objective function, with respect to θ, are given by

$$\frac{\partial \Theta_{ow}(\theta)}{\partial \theta} = (1 - a) \cdot \frac{\partial \omega_t^{-1}(\theta)}{\partial \theta} < 0,$$

and

$$\frac{\partial^2 \Theta_{ow}(\theta)}{\partial \theta^2} = (1 - a) \cdot \frac{\partial^2 \omega_t^{-1}(\theta)}{\partial \theta^2} > 0$$

respectively. The first-order derivative indicates that the objective function $\Theta_{ow}(\theta)$ is decreasing in θ. The second-order derivative being positive confirms that $\Theta_{ow}(\theta)$ is a convex function. Therefore, to minimize $\Theta_{ow}(\theta)$, the policymaker sets θ to its lower bound, $\theta = 0$, which corresponds to the least competitive, or the most restrictive, entry policy. In this scenario, the incumbent firms do not face the pressure to innovate to avoid displacement by new entrants.

The absence of entry threat, that is, $\theta = 0$, allows the incumbent firms to maintain their market positions without the pressure to continuously innovate. This leads to a more stable but less competitive market environment, leading to lower aggregate productivity growth in the skilled sector. However, the reduction in competitiveness also results in lower wage inequality, as measured by the skill premium, ω. Since the inverse of the skill premium is a decreasing and convex function of θ, a lower θ results in a higher inverse of skill premium, implying that wage inequality declines. Hence, by setting $\theta = 0$, the policymaker successfully reduces income inequality in the economy, achieving the desired objective.

When the entry policy is set to $\theta = 0$, the aggregate productivity in the skill sector is obtained by substituting for $\theta = 0$ in Equation (87). This yields[8]

$$A_{s,t}(\theta = 0) = \frac{\beta^* \bar{A}_{s,t}}{2}\left[1 + 0 + (1-0)\left[\frac{1+\gamma}{2\gamma}\right]\right] + (1-\beta^*)\bar{A}_{s,t}\left[0 + (1-0)\frac{1}{2}\cdot\frac{1+\gamma}{\gamma^2}\right],$$

which can be simplified to

$$\begin{aligned} A_{s,t}(\theta = 0) &= \frac{\beta^* \bar{A}_{s,t}}{2}\left[1 + \frac{1+\gamma}{2\gamma}\right] + (1-\beta^*)\bar{A}_{s,t}\frac{1+\gamma}{2\gamma^2} \\ &= \frac{\bar{A}_{s,t}}{2\gamma}\left[\beta * \cdot \frac{3\gamma+1}{2} + (1-\beta^*)\frac{1+\gamma}{\gamma}\right] = \frac{\bar{A}_{s,t}}{2\gamma}\left[\frac{\beta^*(3\gamma^2+1) + 2(1-\beta^*)(1+\gamma)}{2\gamma}\right], \end{aligned}$$

which is further simplified to

$$\frac{A_{s,t}(\theta = 0)}{\bar{A}_{s,t}} = \frac{1}{4\gamma^2}\left[\beta^*(3\gamma^2+1) + 2(1-\beta^*)(1+\gamma)\right]. \tag{107}$$

Equation (107) gives the aggregate productivity in the skill sector relative to the frontier technology in the skill sector, when $\theta = 0$. This provides a measure of how far the overall productivity is to the maximum achievable productivity. Note that the right-hand side of Equation (107) is linear in terms of β^*. Further, note that,

$$\lim_{\beta^* \to 0} \frac{1}{4\gamma^2}\left[\beta^*(3\gamma^2+1) + 2(1-\beta^*)(1+\gamma)\right] = \frac{1+\gamma}{2\gamma^2} < 1$$

and

$$\lim_{\beta^* \to 1} \frac{1}{4\gamma^2}\left[\beta^*(3\gamma^2+1) + 2(1-\beta^*)(1+\gamma)\right] = \frac{3}{4} + \frac{1}{4\gamma^2} < 1.$$

Therefore, for $\gamma > 1$ and $0 < \beta^* < 1$, the right-hand side of Equation (107) will always be less than one. This shows that when the policymaker sets the entry policy as $\theta = 0$, the aggregate productivity of the skill sector is lower than the maximum potential efficiency level that it can reach, which is captured as

$$\frac{A_{s,t}(\theta = 0)}{\bar{A}_{s,t}} < 1. \tag{108}$$

This completes the proof. □

[8]Note that when $\theta = 0$, if $c_{i,t} = \bar{c}$ then Equation (48) does not hold by virtue of Assumption 2. In such a case, $1_\Phi = 0$ and $1_\Psi = 1$. On the other hand, if $c_{i,t} = 0$, then Equation (48) will hold. In such a case, $1_\Phi = 1$ and $1_\Psi = 0$. Since *ex ante* the probability for either scenarios is $\frac{1}{2}$, the term $\left[1 \cdot 1_\Phi + \frac{1}{\gamma} \cdot 1_\Psi\right]$ will be equal to $\left[\frac{1}{2}\left(0 + \frac{1}{\gamma}\right) + \frac{1}{2}\left(1 + 0\right)\right] = \frac{1+\gamma}{2\gamma}$.

Proposition 3. ***If the policymaker is office-motivated, the aggregate productivity is greater than the aggregate productivity in the "Inequality-minimizing Regime", provided the effective labor employed in the skilled sector is greater than or equal to the effective labor employed in the unskilled sector.***

Proof. We recognize that an "Office-Motivated" policymaker would not only require votes to remain in office (and therefore dislikes wage inequality) but also needs campaign contributions to contest elections. Therefore, he/she would assign equal weights to both total bribes offered by incumbent firms and also to the inverse of the skill premium. We call this as **"Regime 3: The Office-motivated Regime"**. The objective function of such a policymaker is given by Equation (101) with $a = 0.5$, which we term as Θ_{om}. The first-order derivative of the policymaker's objective function, with respect to the firm entry policy, θ, is given by

$$\frac{\partial \Theta_{om}(\theta)}{\partial \theta} = 0.5 \cdot \underbrace{\frac{\partial B_t(\theta)}{\partial \theta}}_{>0} + 0.5 \cdot \underbrace{\frac{\partial \omega_t^{-1}(\theta)}{\partial \theta}}_{<0}. \tag{109}$$

From Equation (74), we know that $\frac{\partial B_t(\theta)}{\partial \theta} > 0$, and from Equation (98), we know that $\frac{\partial \omega_t^{-1}(\theta)}{\partial \theta} < 0$. Therefore, the overall sign of Equation (104) is ambiguous.

The second-order derivative of the policymaker's objective function is given by

$$\frac{\partial^2 \Theta_{om}(\theta)}{\partial \theta^2} = 0.5 \cdot \underbrace{\frac{\partial^2 B_t(\theta)}{\partial \theta^2}}_{=0} + 0.5 \cdot \underbrace{\frac{\partial^2 \omega_t^{-1}(\theta)}{\partial^2 \theta}}_{>0}. \tag{110}$$

From Equation (75), we know that $\frac{\partial^2 B_t(\theta)}{\partial \theta^2} = 0$, and from Equation (99), we know that $\frac{\partial^2 \omega_t^{-1}(\theta)}{\partial \theta^2} > 0$. Therefore, the overall sign of Equation (110) is positive. This shows that the objective function of such a policymaker is convex. The policymaker will set θ to maximize $\Theta(\theta)$, which involves solving for θ where $\frac{\partial \Theta(\theta)}{\partial \theta} = 0$. This is given by

$$0.5 \cdot \frac{\partial B_t(\theta)}{\partial \theta} + \ 0.5 \cdot \frac{\partial \omega_t^{-1}(\theta)}{\partial \theta} = 0,$$

which can also be written as

$$\frac{\partial \omega_t^{-1}(\theta)}{\partial \theta} = -\frac{\partial B_t(\theta)}{\partial \theta}.$$

Substituting for $\frac{\partial \omega_t^{-1}(\theta)}{\partial \theta}$ from Equation (98), we can write the above euqation as

$$\left[(1-\alpha)\alpha^{\frac{2\alpha\rho}{1-\alpha}} \cdot \frac{1}{\eta}\right]^{-1} \left[\frac{L_s}{L_u}\right]^{1-\rho} \left[A_u\right]^{\rho} \left[-\rho\left[A_{s,t}(\theta)\right]^{-(\rho+1)} \cdot \frac{\partial A_{s,t}(\theta)}{\partial \theta}\right] = -\frac{\partial B_t(\theta)}{\partial \theta}, \tag{111}$$

which simplifies to

$$\frac{A_{s,t}(\theta=\theta_{om})}{\bar{A}_{s,t}} = \bar{A}_{s,t}^{-1}\big[\rho A_u\big]^{\frac{\rho}{1+\rho}} \left[\frac{2(1-\beta^*)\cdot(\gamma+1)}{\gamma\beta^*\Big[\gamma+1\Big]+2(1-\beta^*)\Big[2\gamma^2-(1+\gamma)\Big]}\right]^{\frac{-1}{1+\rho}} \cdot \left[(1-\alpha)^2\alpha^{\frac{1+\alpha(2\rho+1)}{1-\alpha}}\cdot\frac{1}{\eta}\right]^{\frac{-1}{1+\rho}} \cdot \left[\frac{\big(L_s\big)^{\rho}}{\big(L_u\big)^{1-\rho}}\right]^{\frac{1}{1+\rho}}. \quad (112)$$

[Refer to Appendix 1 for a detailed derivation.]

The policymaker who assigns non-zero and equal weights to bribes and the inverse of skill premium, sets an entry policy, θ, that satisfies Equation (112). Thus, the optimal entry policy, θ, balances the effects of bribes and the inverse of the skill premium based with equal weights on both. The optimal entry policy, θ is also a function of the frontier productivity, $\bar{A}_{s,t}$, elasticity of substitution between skilled and unskilled labor, α, steady-state fraction of advanced firms, β^*, growth rate of the technological frontier, γ, the skilled labor, L_s, the unskilled labor, L_u, the productivity of unskilled labor, A_u, the preference for the skilled good, η, and the substitution factor between the skilled and unskilled good, ρ.

In Proposition 1, it was shown that in Regime 1, the aggregate productivity of the skilled sector is at the frontier. In Proposition 2, it was shown that in Regime 2, the aggregate productivity of the skilled sector is below the frontier. We now compare the relative productivities in Regimes 2 and 3. We calculate the ratio

$$\frac{A_{s,t}(\theta=0)}{\bar{A}_{s,t}} : \frac{A_{s,t}(\theta=\theta_{om})}{\bar{A}_{s,t}} \implies A_{s,t}(\theta=0) : A_{s,t}(\theta=\theta_{om}), \quad (113)$$

where $\frac{A_{s,t}(\theta=0)}{\bar{A}_{s,t}}$ is given by Equation (107) and $\frac{A_{s,t}(\theta=\theta_{om})}{\bar{A}_{s,t}}$ is given by Equation (112). If this ratio is less than 1, then the aggregate productivity under Regime 3 is greater than that in Regime 2. This result has consequences on growth and income inequality, which we will discuss in Section 9.

We now analyze the condition under which the aggregate productivity in the "Office-Motivated" Regime is higher than the aggregate productivity in the "Inequality-minimizing" Regime by analyzing the ratio in Equation (113). For the given parametric restrictions, if

$$\frac{\big[L_s\big]^{\rho}}{\big[\bar{A}_{s,t}\big]^{1+\rho}} \geq \frac{\big[L_u\big]^{1-\rho}}{\big[\rho A_u\big]^{\rho}}, \quad (114)$$

then,

$$\frac{A_{s,t}(\theta = 0)}{\bar{A}_{s,t}} < \frac{A_{s,t}(\theta = \theta_{om})}{\bar{A}_{s,t}} \implies A_{s,t}(\theta = 0) < A_{s,t}(\theta = \theta_{om}). \tag{115}$$

[*See Appendix 2 for a detailed derivation of this condition.*]

Equation (114) gives the sufficient condition when aggregate productivity is higher in the "Office-Motivated" Regime, *vis-à-vis* the "Inequality-minimizing" Regime. Equation (114) implies that the effective labor employed in the skilled sector must be greater than or equal to the effective labor employed in the unskilled sector. This sufficient condition essentially requires a balanced relationship between the productivity and labor force in the skilled and unskilled sectors.

If the left-hand side of Equation (114) is greater than the right-hand side, it implies that the skilled sector, when accounted for its labor force and the frontier productivity, dominates the unskilled sector. In such a scenario, an office-motivated policymaker—who assigns equal weight to growth (*via* bribes) and inequality reduction—has the incentive to allow more entry. This entry drives aggregate productivity relative to the "Inequality-minimizing" Regime, which restricts entry to minimize inequality.

The second part of Equation (115) reveals

$$A_{s,t}(\theta = 0) < A_{s,t}(\theta = \theta_{om}).$$

This suggests that a policymaker who optimizes entry policies with office-motivated incentives can significantly enhance aggregate productivity in the skilled sector, provided that the skilled sector is relatively more effective, that is to say, Equation (114) holds. This completes the proof of Proposition 3. □

It has been shown in Appendix 2 that if the sufficient condition given by Equation (114) is not met, then it would not be analytically possible to rank the aggregate productivities of Regime 2 and Regime 3[9].

This completes the section on the determination of firm entry policy by the policymaker. We summarize the results of this section in Table 2.

[9] In such a scenario, one may resort to a simulation exercise to determine the relative ranking of productivities across regimes.

Regime	Firm Entry Policy (θ)	Aggregate Productivity relative to the Frontier
Bribe-maximizing	$\theta = 1$	$\frac{A_{s,t}(\theta=1)}{\bar{A}_{s,t}} = 1$
Inequality-minimizing	$\theta = 0$	$\frac{A_{s,t}(\theta=0)}{\bar{A}_{s,t}} < 1$
Office-motivated	$\theta = \theta_{om}$ satisfying Eq. (112)	$\frac{A_{s,t}(\theta=0)}{\bar{A}_{s,t}} < \frac{A_{s,t}(\theta=\theta_{om})}{\bar{A}_{s,t}}$

Table 2: Results of the Policymaker's Maximization Exercise in the Three Regimes.

9 Main Results

We now proceed to present the main results of our paper. Recall from Table 1 that GDP and the aggregate productivity of the skilled sector, $A_{s,t}$, have a linear relationship. On the other hand, the inverse of skill premium, ω^{-1}, and the aggregate productivity of the skilled sector, $A_{s,t}$, have an inverse relationship. We present our main results as the following two propositions.

Proposition 4. ***The Gross Domestic Product and the income inequality in the economy are lower in the "Inequality-minimizing" Regime vis-à-vis the "Office-motivated" Regime if the effective labor employed in the skilled sector is greater than or equal to the effective labor employed in the unskilled sector.***

Proof. From Equation (31), the GDP of the economy can be expressed as

$$G_t = P_u A_u L_{u,t} + \alpha^{\frac{1+\alpha}{1-\alpha}} L_{s,t} A_{s,t} \left[\frac{1}{\alpha} - 1\right].$$

This expression shows that GDP is a linear function of the aggregate productivity in the skilled sector, $A_{s,t}$. Thus, any factor influencing $A_{s,t}$ directly affects GDP.

From Equation (38), the skill premium, ω_t, which captures income inequality, is given by

$$\omega_t = (1-\alpha)\alpha^{\frac{2\alpha\rho}{1-\alpha}} \cdot \frac{1}{\eta} \cdot \left[\frac{A_{s,t}}{A_u}\right]^{\rho} \cdot \left[\frac{L_u}{L_s}\right]^{1-\rho}.$$

Since ω_t is directly proportional to $A_{s,t}$, higher aggregate productivity in the skilled sector leads to greater income inequality, as it increases the returns to skilled labor relative to unskilled labor.

From Proposition 3, we know that if the sufficient condition, given by Equation (114), holds true, then the aggregate productivity of the skill sector in the "Inequality-minimizing" Regime is lower than the aggregate productivity in the "Office-motivated" Regime.

Thus, given that GDP is linearly related to $A_{s,t}$ and skill premium (and thus inequality) is directly proportional to $A_{s,t}$, it follows that both GDP and income inequality are lower in the "Inequality-minimizing" Regime than in the "Office-motivated" Regime, provided that the effective labor in the skilled sector is greater than or equal to that in the unskilled sector. This condition ensures that the skilled sector's productivity gains play a dominant role in determining GDP and inequality outcomes.

Intuitively, in the "Inequality-minimizing" Regime, the policymaker's primary objective is to minimize inequality, which leads to restricted entry into the skilled sector. Since entry fuels innovation and pushes firms toward the technological frontier, limiting entry results in lower aggregate productivity, $A_{s,t}$, in the skilled sector. Lower $A_{s,t}$ reduces GDP as fewer skilled sector innovations materialize. Simultaneously, income inequality falls because the returns to skilled labor (reflected by the skill premium) decline when productivity improvements in the skilled sector are curtailed.

The balance between skilled labor supply and sectoral productivity frontiers determines the severity of the growth-inequality trade-off. The condition

$$\frac{\left[L_s\right]^{\rho}}{\left[\bar{A}_{s,t}\right]^{1+\rho}} \geq \frac{\left[L_u\right]^{1-\rho}}{\left[\rho A_u\right]^{\rho}}$$

provides crucial insights in this context. If the skilled sector's adjusted effective labor supply (left-hand side) dominates the unskilled sector's contribution (right-hand side), skilled sector productivity becomes the key driver of GDP and income inequality. In this scenario, restricting entry (as in the "Inequality-minimizing" Regime) significantly reduces GDP (due to lower skilled sector productivity) but also reduces inequality (by lowering the skill premium). Conversely, when the unskilled sector's adjusted contribution dominates, entry restrictions have a lesser impact on GDP and inequality because the skilled sector's influence on the overall economy is weaker.

In the "Office-motivated" Regime, the policymaker balances rent extraction (through bribes) with inequality concerns. This results in a moderate level of entry, boosting productivity and GDP, but also widening inequality as the skill premium rises.

Proposition 5. ***In the "Bribe-maximizing" Regime, the Gross Domestic Product in the economy is the highest, and the income inequality in the economy is also the highest.***

Proof. Once again, from Equation (31), we know

$$G_t = P_u A_u L_{u,t} + \alpha^{\frac{1+\alpha}{1-\alpha}} L_{s,t} A_{s,t} \Big[\frac{1}{\alpha} - 1 \Big].$$

which reveals the linear relationship between GDP and the aggregate productivity in the skilled sector, $A_{s,t}$. Thus, the maximum GDP occurs when $A_{s,t}$ attains its highest possible level.

From Equation (38), the skill premium ω_t, which captures income inequality, is given by

$$\omega_t = (1-\alpha)\alpha^{\frac{2\alpha\rho}{1-\alpha}} \cdot \frac{1}{\eta} \cdot \left[\frac{A_{s,t}}{A_u} \right]^{\rho} \cdot \left[\frac{L_u}{L_s} \right]^{1-\rho}.$$

Since ω_t is directly proportional to $A_{s,t}$, income inequality also increases with higher skilled sector productivity. (Alternatively, Equation (97) shows that the inverse of skill premium, which is a measure of income equality in the economy, is inversely proportional to the aggregate productivity in the skilled sector.)

From Proposition 1, we know that in the "Bribe-maximizing" Regime, the aggregate productivity, $A_{s,t}$, reaches the technological frontier, $\bar{A}_{s,t}$, which represents the highest achievable productivity in the skilled sector. Hence, given that GDP is linearly increasing in $A_{s,t}$ and income inequality (*via* the skill premium) is also increasing in $A_{s,t}$, it follows that in the "Bribe-maximizing" Regime, where $A_{s,t} = \bar{A}_{s,t}$, GDP is at its maximum, and income inequality is also at its highest.

Intuitively, the "Bribe-maximizing" Regime represents a scenario where the policymaker's sole objective is to maximize bribe extraction, which is positively correlated with the entry probability, θ. This incentivizes the policymaker to allow unrestricted entry, fostering a high level of competition among firms. Unrestricted entry pushes firms to continuously innovate to maintain their market positions, propelling the aggregate productivity, $A_{s,t}$, to the technological frontier, $\bar{A}_{s,t}$. As GDP depends linearly on $A_{s,t}$, achieving the frontier translates to the maximum possible GDP. The skill premium rises as skilled sector productivity improves, increasing wage disparities between skilled and unskilled labor. In a frontier productivity environment, returns to skill are maximized, leading to greater income inequality.

10 Conclusion

This paper developed a comprehensive analytical framework to examine how firm entry policies, innovation incentives, and rent-seeking behavior interact within a dual-sector economy characterized by skilled and unskilled labor. The model explored how incumbent firms respond to entry threats and how these responses, shaped by innovation costs and rent-seeking, influence productivity, growth, and income inequality. Central to the analysis was the role of a policymaker balancing conflicting objectives—maximizing bribes from rent-seeking incumbents while minimizing income inequality.

The core contribution of this paper lies in its extension of traditional Schumpeterian growth models by endogenizing institutional constraints through income inequality. Unlike existing literature that treats institutional factors such as democracy as exogenous, this paper shows that income inequality itself can serve as a powerful institutional constraint. When wage inequality—captured by the skill premium—rises, it becomes politically costly for policymakers to accept bribes from backward firms seeking to restrict entry. Thus, inequality dynamics shape entry policies and long-term growth trajectories.

Several key findings emerge with significant implications for theory and policy. First, entry threats serve as a crucial driver of innovation, especially among advanced firms operating at the technological frontier. High entry probabilities incentivize these firms to innovate and maintain their competitive edge, boosting productivity and growth. However, this growth comes at the cost of increased income inequality, as innovation in the skilled sector raises the skill premium. Conversely, low entry threats weaken innovation incentives, leading to stagnation. This underscores a fundamental trade-off between growth and inequality, where innovation-driven policies may inadvertently widen income disparities.

Second, rent-seeking behavior by backward firms distorts market competition. Unlike advanced firms that innovate under entry threats, backward firms—facing higher innovation costs—prefer to bribe policymakers to deter competition. This behavior results in suboptimal innovation and lowers aggregate productivity. The presence of an unskilled sector amplifies these distortions, as higher skill premiums make it politically costly for policymakers to ignore inequality concerns. This interplay reveals how institutional weaknesses can trap economies in low-growth equilibria with persistent income disparities.

Third, the introduction of three policymaker regimes—"Bribe-maximizing", "Inequality-minimizing", and "Office-Motivated"—provides a nuanced understanding of how institutional preferences shape growth-inequality outcomes. In the "Bribe-maximizing" Regime, where bribe maximization dominates, growth proceeds unchecked at the cost of rising inequality. The "Inequality-minimizing" Regime prioritizes reducing inequality but sacrifices innovation and long-term growth. The "Office-Motivated" Regime, the most realistic scenario, balances bribes with electoral incentives tied to inequality, promoting sustainable growth while keeping inequality within politically acceptable bounds. This regime highlights the role of political incentives in shaping economic trajectories.

These results have important policy implications. Policymakers must carefully navigate the growth-inequality trade-off when designing firm entry policies. Policies that encourage entry threats can spur innovation and growth but may exacerbate inequality. Conversely, policies aimed at reducing inequality by restricting entry may stifle innovation and slow progress. The optimal policy lies in striking a balance—promoting innovation while ensuring that the benefits of growth are broadly shared.

The paper also underscores the broader economic consequences of rent-seeking and entry regulation. Limiting the influence of firms on policymaking through institutional reforms could reduce distortions, promote inclusive growth, and lower income inequality. Strengthening democratic institutions and enhancing the absorptive capacity of backward sectors are essential strategies in this regard.

While the model provides a robust framework for understanding the interplay between innovation, growth, and inequality, it also suggests avenues for future research. One potential extension involves relaxing the assumption of a constant labor composition. Incorporating skill acquisition dynamics would allow a more comprehensive analysis of how labor market transitions influence growth-inequality trade-offs. For instance, enabling unskilled workers to invest in education and transition into the skilled sector could reduce the skill premium and mitigate inequality while sustaining growth.

Additionally, the assumption that potential entrants always operate at the technological frontier could be relaxed. Considering entrants with varying levels of technological sophistication would capture the diverse nature of market entry in real-world economies, where entrants differ in innovation capabilities and competitive impact.

In conclusion, this paper provides a unified framework that bridges the Schumpeterian growth literature with political economy perspectives by endogenizing institutional constraints through income inequality dynamics. The model's insights—highlighting the trade-offs between innovation-driven growth, income inequality, and institutional behavior—offer valuable guidance for policymakers seeking to promote inclusive and sustainable development. By emphasizing balanced entry policies and institutional reforms, this paper lays the groundwork for future research on these complex interdependencies.

Appendices

Appendix 1: Aggregate productivity of the skilled sector relative to the frontier productivity in the *"office-motivated" regime*

Consider Equation (111)

$$\left[(1-\alpha)\alpha^{\frac{2\alpha\rho}{1-\alpha}}\cdot\frac{1}{\eta}\right]^{-1}\left[\frac{L_s}{L_u}\right]^{1-\rho}\Big[A_u\Big]^{\rho}\left[-\rho\Big[A_{s,t}(\theta)\Big]^{-(\rho+1)}\cdot\frac{\partial A_{s,t}(\theta)}{\partial\theta}\right]=-\frac{\partial B_t(\theta)}{\partial\theta},$$

which upon multiplying both sides by (-1), can be re-written as

$$\left[(1-\alpha)\alpha^{\frac{2\alpha\rho}{1-\alpha}}\cdot\frac{1}{\eta}\right]^{-1}\left[\frac{L_s}{L_u}\right]^{1-\rho}\Big[A_u\Big]^{\rho}\left[\rho\Big[A_{s,t}(\theta)\Big]^{-(\rho+1)}\cdot\frac{\partial A_{s,t}(\theta)}{\partial\theta}\right]=\frac{\partial B_t(\theta)}{\partial\theta},$$

and by isolating the term $A_{s,t}(\theta)$, this can be written as

$$\Big[A_{s,t}(\theta)\Big]^{-(\rho+1)}=\left[\frac{\frac{\partial B_t(\theta)}{\partial\theta}}{\rho\big(A_u\big)^{\rho}\cdot\frac{\partial A_{s,t}(\theta)}{\partial\theta}}\right]\cdot\left[(1-\alpha)\alpha^{\frac{2\alpha\rho}{1-\alpha}}\cdot\frac{1}{\eta}\right]\left[\frac{L_u}{L_s}\right]^{1-\rho},$$

which can further be simplified to

$$A_{s,t}(\theta)=\left[\frac{\frac{\partial B_t(\theta)}{\partial\theta}}{\rho\big(A_u\big)^{\rho}\cdot\frac{\partial A_{s,t}(\theta)}{\partial\theta}}\right]^{\frac{-1}{1+\rho}}\cdot\left[(1-\alpha)\alpha^{\frac{2\alpha\rho}{1-\alpha}}\cdot\frac{1}{\eta}\right]^{\frac{-1}{1+\rho}}\left[\frac{L_u}{L_s}\right]^{\frac{-(1-\rho)}{1+\rho}},$$

which can also be written as

$$A_{s,t}(\theta)=\left[\frac{\frac{\partial B_t(\theta)}{\partial\theta}}{\rho\big(A_u\big)^{\rho}\cdot\frac{\partial A_{s,t}(\theta)}{\partial\theta}}\right]^{\frac{-1}{1+\rho}}\cdot\left[(1-\alpha)\alpha^{\frac{2\alpha\rho}{1-\alpha}}\cdot\frac{1}{\eta}\right]^{\frac{-1}{1+\rho}}\left[\frac{L_s}{L_u}\right]^{\frac{1-\rho}{1+\rho}}.$$

By substituting for $A_{s,t}(\theta)$ from Equation (87), we get

$$\begin{aligned}A_{s,t}(\theta)=\frac{\beta^*\bar{A}_{s,t}}{2}\left[1+\theta+(1-\theta)\Big[1\cdot 1_{\Phi}+\frac{1}{\gamma}\cdot 1_{\Psi}\Big]\right]+(1-\beta^*)\bar{A}_{s,t}\left[\theta+(1-\theta)\frac{1}{2}\cdot\frac{1+\gamma}{\gamma^2}\right]\\=\left[\frac{\frac{\partial B_t(\theta)}{\partial\theta}}{\rho\big(A_u\big)^{\rho}\cdot\frac{\partial A_{s,t}(\theta)}{\partial\theta}}\right]^{\frac{-1}{1+\rho}}\cdot\left[(1-\alpha)\alpha^{\frac{2\alpha\rho}{1-\alpha}}\cdot\frac{1}{\eta}\right]^{\frac{-1}{1+\rho}}\left[\frac{L_s}{L_u}\right]^{\frac{1-\rho}{1+\rho}},\end{aligned}$$

which can also be written as

$$\begin{aligned}A_{s,t}(\theta)=\frac{\beta^*\bar{A}_{s,t}}{2}\left[1+\theta+(1-\theta)\Big[1\cdot 1_{\Phi}+\frac{1}{\gamma}\cdot 1_{\Psi}\Big]\right]+(1-\beta^*)\bar{A}_{s,t}\left[\theta+(1-\theta)\frac{1}{2}\cdot\frac{1+\gamma}{\gamma^2}\right]\\=\left[\frac{\big(A_u\big)^{-\rho}}{\rho}\cdot\frac{\frac{\partial B_t(\theta)}{\partial\theta}}{\frac{\partial A_{s,t}(\theta)}{\partial\theta}}\right]^{\frac{-1}{1+\rho}}\cdot\left[(1-\alpha)\alpha^{\frac{2\alpha\rho}{1-\alpha}}\cdot\frac{1}{\eta}\right]^{\frac{-1}{1+\rho}}\left[\frac{L_s}{L_u}\right]^{\frac{1-\rho}{1+\rho}}.\end{aligned}$$

which can further be written as

$$A_{s,t}(\theta) = \frac{\beta^* \bar{A}_{s,t}}{2}\Bigg[1+\theta+(1-\theta)\Big[1\cdot 1_{\Phi}+\frac{1}{\gamma}\cdot 1_{\Psi}\Big]\Bigg] + (1-\beta^*)\bar{A}_{s,t}\Bigg[\theta+(1-\theta)\frac{1}{2}\cdot\frac{1+\gamma}{\gamma^2}\Bigg]$$

$$= \big[A_u\big]^{\frac{\rho}{1+\rho}}\big[\rho\big]^{\frac{1}{1+\rho}}\left[\frac{\frac{\partial B_t(\theta)}{\partial\theta}}{\frac{\partial A_{s,t}(\theta)}{\partial\theta}}\right]^{\frac{-1}{1+\rho}}\cdot\left[(1-\alpha)\alpha^{\frac{2\alpha\rho}{1-\alpha}}\cdot\frac{1}{\eta}\right]^{\frac{-1}{1+\rho}}\left[\frac{L_s}{L_u}\right]^{\frac{1-\rho}{1+\rho}}. \quad (116)$$

From equations (74) and (88), we calculate the ratio

$$\frac{\frac{\partial B_t(\theta)}{\partial\theta}}{\frac{\partial A_{s,t}(\theta)}{\partial\theta}} = \frac{(1-\beta^*)\frac{1}{\gamma^2}\bar{A}_{s,t}\left[\frac{1}{2}\cdot\alpha^{\frac{2}{1-\alpha}}\left(\frac{1}{\alpha}-1\right)L_s\right]\Big(\gamma+1\Big)}{\frac{\beta^*\bar{A}_{s,t}}{2}\left[1-\Big[1\cdot 1_{\Phi}+\frac{1}{\gamma}\cdot 1_{\Psi}\Big]\right]+(1-\beta^*)\bar{A}_{s,t}\left[1-\frac{1}{2}\cdot\frac{1+\gamma}{\gamma^2}\right]},$$

which can be simplified to

$$\frac{\frac{\partial B_t(\theta)}{\partial\theta}}{\frac{\partial A_{s,t}(\theta)}{\partial\theta}} = \frac{\frac{(1-\beta^*)}{2}\frac{\gamma+1}{\gamma^2}\bar{A}_{s,t}\left[\alpha^{\frac{2}{1-\alpha}}\left(\frac{1}{\alpha}-1\right)L_s\right]}{\frac{\bar{A}_{s,t}}{2}\left[\beta^*\left[1-\Big[1\cdot 1_{\Phi}+\frac{1}{\gamma}\cdot 1_{\Psi}\Big]\right]+2(1-\beta^*)\left[1-\frac{1}{2}\cdot\frac{1+\gamma}{\gamma^2}\right]\right]},$$

which upon cancelling out like terms in the numerator and the denominator, can be written as

$$\frac{\frac{\partial B_t(\theta)}{\partial\theta}}{\frac{\partial A_{s,t}(\theta)}{\partial\theta}} = \frac{(1-\beta^*)\cdot\frac{\gamma+1}{\gamma^2}\left[\alpha^{\frac{2}{1-\alpha}}\left(\frac{1}{\alpha}-1\right)L_s\right]}{\beta^*\left[1-\Big[1\cdot 1_{\Phi}+\frac{1}{\gamma}\cdot 1_{\Psi}\Big]\right]+2(1-\beta^*)\left[1-\frac{1}{2}\cdot\frac{1+\gamma}{\gamma^2}\right]}.$$

In footnote 3, it has been shown that the term $\Big[1\cdot 1_{\Phi}+\frac{1}{\gamma}\cdot 1_{\Psi}\Big]$ will be equal to $\Big[\frac{1}{2}\big(0+\frac{1}{\gamma}\big)+\frac{1}{2}\big(1+0\big)\Big]=\frac{1+\gamma}{2\gamma}$. Substituting this in the above equation, we obtain

$$\frac{\frac{\partial B_t(\theta)}{\partial\theta}}{\frac{\partial A_{s,t}(\theta)}{\partial\theta}} = \frac{(1-\beta^*)\cdot\frac{\gamma+1}{\gamma^2}\left[\alpha^{\frac{2}{1-\alpha}}\left(\frac{1}{\alpha}-1\right)L_s\right]}{\beta^*\left[1-\Big[\frac{1+\gamma}{2\gamma}\Big]\right]+2(1-\beta^*)\left[1-\frac{1}{2}\cdot\frac{1+\gamma}{\gamma^2}\right]},$$

which can be simplified to

$$\frac{\frac{\partial B_t(\theta)}{\partial\theta}}{\frac{\partial A_{s,t}(\theta)}{\partial\theta}} = \frac{(1-\beta^*)\cdot\frac{\gamma+1}{\gamma^2}\Big[\alpha^{\frac{2}{1-\alpha}}\Big(\frac{1}{\alpha}-1\Big)L_s\Big]}{\frac{\beta^*}{2\gamma}\Big[2\gamma-(1+\gamma)\Big]+\frac{(1-\beta^*)}{\gamma^2}\Big[2\gamma^2-(1+\gamma)\Big]},$$

which upon factoring out $\frac{1}{2\gamma^2}$ in the denominator, can be written as

$$\frac{\frac{\partial B_t(\theta)}{\partial \theta}}{\frac{\partial A_{s,t}(\theta)}{\partial \theta}} = \frac{(1-\beta^*)\cdot\frac{\gamma+1}{\gamma^2}\Big[\alpha^{\frac{2}{1-\alpha}}\Big(\frac{1}{\alpha}-1\Big)L_s\Big]}{\frac{1}{2\gamma^2}\Bigg[\gamma\beta^*\Big[\gamma+1\Big]+2(1-\beta^*)\Big[2\gamma^2-(1+\gamma)\Big]\Bigg]},$$

and on canceling out γ^2 in the numerator and denominator, the above equation can be written as

$$\frac{\frac{\partial B_t(\theta)}{\partial \theta}}{\frac{\partial A_{s,t}(\theta)}{\partial \theta}} = \frac{2(1-\beta^*)\cdot(\gamma+1)\Big[\alpha^{\frac{2}{1-\alpha}}\Big(\frac{1}{\alpha}-1\Big)L_s\Big]}{\gamma\beta^*\Big[\gamma+1\Big]+2(1-\beta^*)\Big[2\gamma^2-(1+\gamma)\Big]},$$

Substituting the above in Equation (116), we obtain

$$\begin{aligned} A_{s,t}(\theta) &= \frac{\beta^*\bar{A}_{s,t}}{2}\Bigg[1+\theta+(1-\theta)\Big[1\cdot 1_\Phi+\frac{1}{\gamma}\cdot 1_\Psi\Big]\Bigg]+(1-\beta^*)\bar{A}_{s,t}\Bigg[\theta+(1-\theta)\frac{1}{2}\cdot\frac{1+\gamma}{\gamma^2}\Bigg] \\ &= \Big[A_u\Big]^{\frac{\rho}{1+\rho}}\Big[\rho\Big]^{\frac{1}{1+\rho}}\Bigg[\frac{2(1-\beta^*)\cdot(\gamma+1)\Big[\alpha^{\frac{2}{1-\alpha}}\Big(\frac{1}{\alpha}-1\Big)L_s\Big]}{\gamma\beta^*\Big[\gamma+1\Big]+2(1-\beta^*)\Big[2\gamma^2-(1+\gamma)\Big]}\Bigg]^{\frac{-1}{1+\rho}} \\ &\qquad\cdot\Bigg[(1-\alpha)\alpha^{\frac{2\alpha\rho}{1-\alpha}}\cdot\frac{1}{\eta}\Bigg]^{\frac{-1}{1+\rho}}\Bigg[\frac{L_s}{L_u}\Bigg]^{\frac{1-\rho}{1+\rho}}, \end{aligned}$$

which can be simplified to

$$\begin{aligned} A_{s,t}(\theta) &= \frac{\beta^*\bar{A}_{s,t}}{2}\Bigg[1+\theta+(1-\theta)\Big[1\cdot 1_\Phi+\frac{1}{\gamma}\cdot 1_\Psi\Big]\Bigg]+(1-\beta^*)\bar{A}_{s,t}\Bigg[\theta+(1-\theta)\frac{1}{2}\cdot\frac{1+\gamma}{\gamma^2}\Bigg] \\ &= \Big[\rho A_u\Big]^{\frac{\rho}{1+\rho}}\Bigg[\frac{2(1-\beta^*)\cdot(\gamma+1)}{\gamma\beta^*\Big[\gamma+1\Big]+2(1-\beta^*)\Big[2\gamma^2-(1+\gamma)\Big]}\Bigg]^{\frac{-1}{1+\rho}} \\ &\qquad\cdot\Big[\alpha^{\frac{2}{1-\alpha}}\Big(\frac{1}{\alpha}-1\Big)L_s\Big]^{\frac{-1}{1+\rho}}\cdot\Bigg[(1-\alpha)\alpha^{\frac{2\alpha\rho}{1-\alpha}}\cdot\frac{1}{\eta}\Bigg]^{\frac{-1}{1+\rho}}\Bigg[\frac{L_s}{L_u}\Bigg]^{\frac{1-\rho}{1+\rho}}, \end{aligned}$$

which can be re-written as

$$\begin{aligned} A_{s,t}(\theta) &= \frac{\beta^*\bar{A}_{s,t}}{2}\Bigg[1+\theta+(1-\theta)\Big[1\cdot 1_\Phi+\frac{1}{\gamma}\cdot 1_\Psi\Big]\Bigg]+(1-\beta^*)\bar{A}_{s,t}\Bigg[\theta+(1-\theta)\frac{1}{2}\cdot\frac{1+\gamma}{\gamma^2}\Bigg] \\ &= \Big[\rho A_u\Big]^{\frac{\rho}{1+\rho}}\Bigg[\frac{2(1-\beta^*)\cdot(\gamma+1)}{\gamma\beta^*\Big[\gamma+1\Big]+2(1-\beta^*)\Big[2\gamma^2-(1+\gamma)\Big]}\Bigg]^{\frac{-1}{1+\rho}} \\ &\qquad\cdot\Big[\alpha^{\frac{2}{1-\alpha}}\Big(\frac{1}{\alpha}-1\Big)\Big]^{\frac{-1}{1+\rho}}\cdot\Bigg[(1-\alpha)\alpha^{\frac{2\alpha\rho}{1-\alpha}}\cdot\frac{1}{\eta}\Bigg]^{\frac{-1}{1+\rho}}\cdot\Big[L_s\Big]^{\frac{-1}{1+\rho}}\cdot\Bigg[\frac{L_s}{L_u}\Bigg]^{\frac{1-\rho}{1+\rho}}, \end{aligned}$$

which can be simplified to

$$A_{s,t}(\theta) = \frac{\beta^* \bar{A}_{s,t}}{2}\left[1+\theta+(1-\theta)\left[1\cdot 1_\Phi + \frac{1}{\gamma}\cdot 1_\Psi\right]\right] + (1-\beta^*)\bar{A}_{s,t}\left[\theta+(1-\theta)\frac{1}{2}\cdot\frac{1+\gamma}{\gamma^2}\right]$$
$$= \left[\rho A_u\right]^{\frac{\rho}{1+\rho}}\left[\frac{2(1-\beta^*)\cdot(\gamma+1)}{\gamma\beta^*\left[\gamma+1\right]+2(1-\beta^*)\left[2\gamma^2-(1+\gamma)\right]}\right]^{\frac{-1}{1+\rho}}$$
$$\cdot\left[\Big(\frac{1}{\alpha}-1\Big)(1-\alpha)\alpha^{\frac{2(1+\alpha\rho)}{1-\alpha}}\cdot\frac{1}{\eta}\right]^{\frac{-1}{1+\rho}}\cdot\left[\frac{\big(L_s\big)^\rho}{\big(L_u\big)^{1-\rho}}\right]^{\frac{1}{1+\rho}},$$

which can be further simplified to

$$A_{s,t}(\theta) = \frac{\beta^* \bar{A}_{s,t}}{2}\left[1+\theta+(1-\theta)\left[1\cdot 1_\Phi + \frac{1}{\gamma}\cdot 1_\Psi\right]\right] + (1-\beta^*)\bar{A}_{s,t}\left[\theta+(1-\theta)\frac{1}{2}\cdot\frac{1+\gamma}{\gamma^2}\right]$$
$$= \left[\rho A_u\right]^{\frac{\rho}{1+\rho}}\left[\frac{2(1-\beta^*)\cdot(\gamma+1)}{\gamma\beta^*\left[\gamma+1\right]+2(1-\beta^*)\left[2\gamma^2-(1+\gamma)\right]}\right]^{\frac{-1}{1+\rho}}$$
$$\cdot\left[\frac{(1-\alpha)^2}{\alpha}\alpha^{\frac{2(1+\alpha\rho)}{1-\alpha}}\cdot\frac{1}{\eta}\right]^{\frac{-1}{1+\rho}}\cdot\left[\frac{\big(L_s\big)^\rho}{\big(L_u\big)^{1-\rho}}\right]^{\frac{1}{1+\rho}},$$

which can also be written as

$$A_{s,t}(\theta) = \frac{\beta^* \bar{A}_{s,t}}{2}\left[1+\theta+(1-\theta)\left[1\cdot 1_\Phi + \frac{1}{\gamma}\cdot 1_\Psi\right]\right] + (1-\beta^*)\bar{A}_{s,t}\left[\theta+(1-\theta)\frac{1}{2}\cdot\frac{1+\gamma}{\gamma^2}\right]$$
$$= \left[\rho A_u\right]^{\frac{\rho}{1+\rho}}\left[\frac{2(1-\beta^*)\cdot(\gamma+1)}{\gamma\beta^*\left[\gamma+1\right]+2(1-\beta^*)\left[2\gamma^2-(1+\gamma)\right]}\right]^{\frac{-1}{1+\rho}}$$
$$\cdot\left[(1-\alpha)^2\alpha^{\frac{2(1+\alpha\rho)-(1-\alpha)}{1-\alpha}}\cdot\frac{1}{\eta}\right]^{\frac{-1}{1+\rho}}\cdot\left[\frac{\big(L_s\big)^\rho}{\big(L_u\big)^{1-\rho}}\right]^{\frac{1}{1+\rho}},$$

which, on dividing throughout by $\bar{A}_{s,t}$, yields

$$\frac{A_{s,t}(\theta)}{\bar{A}_{s,t}} = \frac{\beta^*}{2}\left[1+\theta+(1-\theta)\left[1\cdot 1_\Phi + \frac{1}{\gamma}\cdot 1_\Psi\right]\right] + (1-\beta^*)\left[\theta+(1-\theta)\frac{1}{2}\cdot\frac{1+\gamma}{\gamma^2}\right]$$
$$= \bar{A}_{s,t}^{-1}\left[\rho A_u\right]^{\frac{\rho}{1+\rho}}\left[\frac{2(1-\beta^*)\cdot(\gamma+1)}{\gamma\beta^*\left[\gamma+1\right]+2(1-\beta^*)\left[2\gamma^2-(1+\gamma)\right]}\right]^{\frac{-1}{1+\rho}}$$
$$\cdot\left[(1-\alpha)^2\alpha^{\frac{1+\alpha(2\rho+1)}{1-\alpha}}\cdot\frac{1}{\eta}\right]^{\frac{-1}{1+\rho}}\cdot\left[\frac{\big(L_s\big)^\rho}{\big(L_u\big)^{1-\rho}}\right]^{\frac{1}{1+\rho}},$$

Appendix 2: Sufficient condition when aggregate productivity is higher in the *"office-motivated"* regime, *vis-à-vis* the *"only-welfare"* regime

The comparison of the ratio mentioned in Equation (113) is a comparison of the terms contained in

$$\frac{A_{s,t}(\theta=0)}{\bar{A}_{s,t}} = \underbrace{\frac{1}{4\gamma^2}\Bigg[\beta^*(3\gamma^2+1)+2(1-\beta^*)(1+\gamma)\Bigg]}_{Term\ 1}, \text{ and}$$

$$\frac{A_{s,t}(\theta=\theta_{om})}{\bar{A}_{s,t}} = \bar{A}_{s,t}^{-1}\left[\rho A_u\right]^{\frac{\rho}{1+\rho}} \underbrace{\left[\frac{2(1-\beta^*)\cdot(\gamma+1)}{\gamma\beta^*\Big[\gamma+1\Big]+2(1-\beta^*)\Big[2\gamma^2-(1+\gamma)\Big]}\right]^{\frac{-1}{1+\rho}}}_{Term\ 2}$$

$$\cdot \underbrace{\left[(1-\alpha)^2\alpha^{\frac{1+\alpha(2\rho+1)}{1-\alpha}}\cdot\frac{1}{\eta}\right]^{\frac{-1}{1+\rho}}}_{Term\ 3}\cdot\left[\frac{\left(L_s\right)^{\rho}}{\left(L_u\right)^{1-\rho}}\right]^{\frac{1}{1+\rho}}.$$

We now analyze each of the terms in the two equations. Consider, first, $term1$.

$$\frac{1}{4\gamma^2}\Bigg[\beta^*(3\gamma^2+1)+2(1-\beta^*)(1+\gamma)\Bigg].$$

Recall that $0<\beta^*<1$ and $\gamma>1$. We evaluate $Term\,1$ at the limits of β^*.

When $\beta^*=1$,

$$\frac{1}{4\gamma^2}(3\gamma^2+1)=\frac{3\gamma^2+1}{4\gamma^2}=\frac{3}{4}+\frac{1}{4\gamma^2}<1.$$

And, when $\beta^*=0$

$$\frac{1}{4\gamma^2}\cdot 2(1+\gamma)=\frac{1+\gamma}{2\gamma^2}<1$$

$Term\,1$ is a convex combination of these two cases, which implies that $Term\,1$ will always be less than 1. We next consider $Term\,2$:

$$\left[\frac{2(1-\beta^*)\cdot(\gamma+1)}{\gamma\beta^*\left[\gamma+1\right]+2(1-\beta^*)\left[2\gamma^2-(1+\gamma)\right]}\right]^{\frac{-1}{1+\rho}}.$$

We first evaluate the expression within the parentheses at the limits of β^*.

When $\beta^*=1$

$$\frac{2(1-1)(\gamma+1)}{\gamma(\gamma+1)}=\frac{0}{\gamma(\gamma+1)}=0.$$

And, when $\beta^* = 0$,

$$\frac{2(\gamma+1)}{2\left[2\gamma^2-(1+\gamma)\right]} = \frac{2(\gamma+1)}{2\left[2\gamma^2-1-\gamma\right]} = \frac{\gamma+1}{2\gamma^2-\gamma-1} < 1$$

For intermediate values of β^* where $0 < \beta^* < 1$,

1. Consider the numerator: $2(1-\beta^*)(\gamma+1)$. Since $0 < \beta^* < 1$, $1-\beta^*$ is positive and lies between 0 and 1. Thus, $2(1-\beta^*)(\gamma+1)$ is positive and lies between 0 and $2(\gamma+1)$.

2. Consider the denominator: $\gamma\beta^*(\gamma+1) + 2(1-\beta^*)\left[2\gamma^2-(1+\gamma)\right]$. Since $0 < \beta^* < 1$, the term $\gamma\beta^*(\gamma+1)$ is positive and lies between 0 and $\gamma(\gamma+1)$, and the term $2(1-\beta^*)\left[2\gamma^2-(1+\gamma)\right]$ is positive and lies between $2[2\gamma^2-(1+\gamma)]$ and 0.

Combining these, the denominator lies between $\gamma(\gamma+1)$ and $2[2\gamma^2-(1+\gamma)]$. Therefore, since the expression is equal to 0 when $\beta^* = 1$, equal to $\frac{\gamma+1}{2\gamma^2-\gamma-1} < 1$ when $\beta^* = 0$, and for intermediate values of β^*, the expression within the parentheses lies between 0 and a positive number less than 1.

$$0 < \frac{2(1-\beta^*)(\gamma+1)}{\gamma\beta^*(\gamma+1)+2(1-\beta^*)[2\gamma^2-(1+\gamma)]} < \frac{\gamma+1}{2\gamma^2-\gamma-1} < 1$$

Since the term inside the parentheses is positive and less than 1 for $0 < \beta^* < 1$ and $\gamma > 1$, raising this fraction to the power $\frac{-1}{1+\rho}$ (where $0 < \rho < 1$) results in a value greater than 1. Therefore, for $0 < \beta^* < 1$, $\gamma > 1$, and $0 < \rho < 1$, the $Term\,2$ is greater than 1.

We next consider $Term\,3$:

$$\left[(1-\alpha)^2\alpha^{\frac{1+\alpha(2\rho+1)}{1-\alpha}} \cdot \frac{1}{\eta}\right]^{\frac{-1}{1+\rho}}.$$

Recall that $0 < \alpha < 1$ and $\eta > 1$. With these conditions, the expression with the parentheses is positive and less than one. Therefore, being raised to a negative power makes the entire expression greater than 1. Therefore, $term3$ is greater than 1 for the given parametric restrictions.

Therefore, a comparison between $\frac{A_{s,t}(\theta=0)}{\bar{A}_{s,t}}$ and $\frac{A_{s,t}(\theta=\theta_{om})}{\bar{A}_{s,t}}$ reduces to

$$Term\ 1\ :\ Term\ 2 \times Term\ 3 \times\ \bar{A}_{s,t}^{-1}\left[\rho A_u\right]^{\frac{\rho}{1+\rho}} \cdot \left[\frac{\left(L_s\right)^{\rho}}{\left(L_u\right)^{1-\rho}}\right]^{\frac{1}{1+\rho}}.$$

On dividing both sides by $Term\ 2 \times Term\ 3$, we obtain

$$\frac{\overbrace{Term\ 1}^{<1}}{\underbrace{Term\ 2}_{>0}\times\underbrace{Term\ 3}_{>1}} : \bar{A}_{s,t}^{-1}\left[\rho A_u\right]^{\frac{\rho}{1+\rho}} \cdot \left[\frac{\left(L_s\right)^{\rho}}{\left(L_u\right)^{1-\rho}}\right]^{\frac{1}{1+\rho}}.$$

The left-hand side of the above ratio is less than 1. If the right-hand side of the above ratio is greater than or equal to 1, then our original comparison results in

$$\frac{A_{s,t}(\theta = 0)}{\bar{A}_{s,t}} < \frac{A_{s,t}(\theta = \theta_{om})}{\bar{A}_{s,t}}.$$

On the other hand, if the right-hand side is also less than 1, then we cannot rank the relative productivities, $\frac{A_{s,t}(\theta=0)}{\bar{A}_{s,t}}$ and $\frac{A_{s,t}(\theta=\theta_{om})}{\bar{A}_{s,t}}$.

Accordingly, for

$$\frac{A_{s,t}(\theta = 0)}{\bar{A}_{s,t}} < \frac{A_{s,t}(\theta = \theta_{om})}{\bar{A}_{s,t}},$$

we need to have

$$\bar{A}_{s,t}^{-1}\left[\rho A_u\right]^{\frac{\rho}{1+\rho}} \cdot \left[\frac{\left(L_s\right)^{\rho}}{\left(L_u\right)^{1-\rho}}\right]^{\frac{1}{1+\rho}} \geq 1,$$

which can also be written as

$$\left[\rho A_u\right]^{\frac{\rho}{1+\rho}} \cdot \left[\frac{\left(L_s\right)^{\rho}}{\left(L_u\right)^{1-\rho}}\right]^{\frac{1}{1+\rho}} \geq \bar{A}_{s,t},$$

and on raising the power by $1+\rho$ on both sides, we obtain

$$\left[\rho A_u\right]^{\rho} \cdot \left[\frac{\left(L_s\right)^{\rho}}{\left(L_u\right)^{1-\rho}}\right] \geq \left[\bar{A}_{s,t}\right]^{1+\rho},$$

which can be re-written as

$$\frac{\left[L_s\right]^{\rho}}{\left[\bar{A}_{s,t}\right]^{1+\rho}} \geq \frac{\left[L_u\right]^{1-\rho}}{\left[\rho A_u\right]^{\rho}}$$

If the above condition holds, then,

$$\frac{A_{s,t}(\theta = 0)}{\bar{A}_{s,t}} < \frac{A_{s,t}(\theta = \theta_{om})}{\bar{A}_{s,t}} \implies A_{s,t}(\theta = 0) < A_{s,t}(\theta = \theta_{om})$$

which is Equation (112) in our model.

References

Daron Acemoglu. Oligarchic versus democratic societies. *Journal of the European Economic Association*, 6(1):1–44, 2008.

Daron Acemoglu and James A. Robinson. *Economic Origins of Dictatorship and Democracy.* Cambridge University Press, Cambridge, 2006.

Daron Acemoglu, Simon Johnson, James A. Robinson, and Pierre Yared. Income and democracy. *CEPR Discussion Paper No. 5273*, 2005.

Philippe Aghion and Peter Howitt. A model of growth through creative destruction. *Econometrica*, 60(2):323–351, 1992.

Philippe Aghion and Peter W Howitt. *The economics of growth.* MIT press, 2008.

Philippe Aghion, Peter Howitt, Maxine Brant-Collett, and Cecilia García-Peñalosa. *Endogenous growth theory.* MIT press, 1998.

Philippe Aghion, Alberto Alesina, and Francesco Trebbi. Endogenous political institutions. *The Quarterly Journal of Economics*, 119(2):565–611, 2004.

Philippe Aghion, Nicholas Bloom, Richard Blundell, Rachel Griffith, and Peter Howitt. Competition and innovation: An inverted-u relationship. *The Quarterly Journal of Economics*, 120(2):701–728, 2005.

Robert J. Barro. Determinants of democracy. *Journal of Political Economy*, 107(S6):S158–S183, 1999.

Gene M. Grossman and Elhanan Helpman. *Innovation and Growth in the Global Economy.* MIT Press, 1991.

Bruce E. Kaufman. How capitalism endogenously creates rising income inequality and economic crisis: The macro political economy model of early industrial relations. *SSRN Electronic Journal*, 2018.

Casey B. Mulligan, Ricard Gil, and Xavier Sala-i Martin. Do democracies have different public policies than nondemocracies? *Journal of Economic Perspectives*, 18(1):51–74, 2004.

Dani Rodrik and Romain Wacziarg. Do democratic transitions lead to economic growth? *The American Economic Review*, 95(2):50–55, 2005.

Joseph A Schumpeter. *Capitalism, socialism and democracy.* New York: Harpeer & Bros., 1942.

Chenggang Xu. *The Problem of Growth-Inequality Nexus: An Analysis based on the Case of China since Reform and Opening Up.* PhD thesis, Technische Universität Darmstadt, 2013.