Immigration, Demand, Supply and Sectoral Heterogeneity in the UK Labor Market: A Time Series Approach

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Immigration, Demand, Supply and Sectoral Heterogeneity in the UK Labor Market*

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Abstract

Should immigration be regarded as an exogenous shock? If so, what is its effect on native wages? Might any effect differ across different sectors of the economy? In this paper we answer these questions by applying macroeconomic time series methods to a time series of UK labor market variables from 2001-2019 for 35 different sectors. The paper uses a VAR approach to model, for the first time, immigration, native wages and hours worked, as responding to demand, supply and immigration shocks at both aggregate and sectoral levels. The labor market is thereby modeled as being subject to multiple shocks at any one time, with individual shocks reinforcing and offsetting each other. We find that the share of migrant labor is 'Granger caused' by other labor market variables which suggests that immigration is, in part, endogenously determined by aggregate demand and supply. However it also retains a component which has a negative association between immigration and native wages. This component, which may be thought of as a 'migration shock', accounts for most of the change in migration share over the sample period and plays a significant negative role in the determination of native wage growth, particularly in unskilled sectors such as retail and hospitality. However other contemporaneous shocks have offsetting positive associations between immigration and native wages, whose effects differ substantially across sectors. (222 Words)

Keywords: Immigration, Demand, Supply, VAR, Sectoral Heterogeneity

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1 Introduction

The effect of immigration on the labor market is of intrinsic economic interest. It is also the focus of longstanding political attention, with immigration linked empirically to the rise of counter-globalization voting patterns across the world, see Rodrik (2021) and Docquier et al (2023). In this context, establishing whether there may exist adverse labor market effects associated with immigration becomes an even more important concern. The empirical literature to date has consistently found only small effects of immigration on wages.[1] Manacorda, Manning, and Wadsworth (2012), and Ottoviano and Peri (2012) offer imperfect substitutability of immigrant and native labor as one possible explanation for these small wage effects. In this paper, we argue instead that wages, hours and indeed immigration, are determined simultaneously in a labor market subject to multiple shocks at the aggregate and sectoral level. Thus, for example, the observed weak association between wages and immigration may be due to a negative association between wage growth and immigration being offset by a positive association between immigration, wages and aggregate demand. By the same token, effects which have been attributed to migration may in fact be the result of other shocks to the system.

We investigate empirically the relative importance of different types of shocks in explaining variation in key UK labor market variables. The UK is an important exemplar in this regard, exhibiting as it does, large variations in immigration rates, wages and employment across sectors and over time. Our analysis employs a vector autoregression (VAR) approach which models immigration as part of a multivariate stochastic process evolving throughout the sample period. This contrasts with much of the literature which models immigration as an assumed exogenous shock to labor supply.

Moreover, even if immigration at the aggregate level is driven by an exogenous shock, immigration at the sectoral level may not be exogenous. Migrants will tend to flow to those sectors with high demand for their labor all else being equal. Sectors will also likely differ in the nature of the production process, and thereby in their use of different types of labor, and for other supply side reasons. Therefore to analyze the labor market effects of immigration one also needs to take account of likely sectoral heterogeneity as well as the multiplicity of shocks.

It has long been acknowledged that modeling immigration solely as a labor supply shock has limitations. Borjas (1994) noted that "The size and composition of the immigrant flow are jointly determined by supply side considerations ....as well as by factors that determine the host country's demand for immigrants". Borjas' comments relate to the demand constraints at country-level imposed by visa quotas, which are common across industrialized economies.[2] The existence of shortage occupation lists as in Australia, Canada and the UK, can also be viewed as evidence of the importance of demand in influencing the level and type of immigration. There may also be dynamic demand responses to immigration. Ottoviano and

[1] See for example Borjas (2004), or Ottoviano and Peri (2012) for the United States or Dustmann, Frattini and Preston (2013), and Manacorda, Manning, and Wadsworth, (2012) for the UK).

[2] The EU of course allows unrestricted mobility of individuals between member states.

Peri (2012) acknowledge this possibility, stating "We treat immigration as a labor supply shock, omitting any productivity impact that it may produce due, for example, to improved efficiency, choice of better technologies, or scale externalities". Peri, Rury and Wiltshire (2023) state that their results on the effect of immigration following Hurricane Marie are also consistent with a negative labor supply shock, offset by positive consumer demand shocks.

The assumption of exogenous labor supply shocks nevertheless still underlies the identification of immigration's effects on native workers' labor market outcomes in much of the empirical literature. Dustmann, Schönberg, and Stuhler (2016) state, "Any of the approaches we discuss slices the labor market into different sub-labor-markets and uses variation in the inflow of immigrants into these sub-labor-markets as an identification device. We assume here that the allocation of immigrants to these sub-labor-markets is (conditionally) independent of shocks to wages or employment of native workers (which could be achieved either through random allocation of immigrants, or by use of an appropriate instrument)... Studies that slice the labor market into skill groups instead typically assume that immigrant inflows are exogenous, an assumption that may be violated (Llull 2014)." Campo et al (2018) similarly argue "…there is significant consensus that immigrants select into labor markets with more favorable conditions (lower unemployment, higher wages) thus... immigration flows might be higher... to high productivity sectors which are more attractive and likely to be growing."

Alongside this issue of identification there also exists the possibility that immigration effects differ across skill levels. The existing literature has acknowledged the possibility of heterogeneous effects of immigration. Largely this has focussed on different effects by migrant skill level and geographical origin, see e.g. Dustmann et al (2016), Ottoviano and Peri (2012) and Manacorda, Manning, and Wadsworth, (2012). In this paper we follow Mountford and Wadsworth (2023) in distinguishing between skill levels across industries. [3]

We show that these concerns about abstracting away from the effects of demand and sectoral heterogeneity may be well placed. We use a VAR approach where demand, supply and immigration shocks can occur simultaneously in every time period so that there is potential for multiple shocks to either offset or complement each other. The VAR framework has been previously used to study the effects of migration most notably by Blanchard and Katz (1992) for internal migration, and more recently Furlanetto and Robstad (2019) on Norwegian data. However to our knowledge our paper is the first to employ a VAR framework that explicitly incorporates shocks at the sectoral as well as aggregate level.

We employ a six variable VAR using UK data from 2001-2019 for each of 35 different labor market sectors to identify demand, supply and immigration effects on key labor market outcomes of interest at both the aggregate and sectoral level. The six variables are the economy-wide migration share, hours worked and real wage of natives, along with the same variables for each sector. These six variables permit the identification of the six aggregate

[3] Mountford and Wadsworth (2023) find that the effects of skilled immigration on training of the native workforce differs significantly across sectors, with in particular, negative effects of immigration on native training in the skilled non-traded sectors and positive effects in the traded sector. They attribute this to the limited ability of the non-traded sector to increase output in response to supply shocks compared to the traded sector.

and sectoral shocks. The combination of aggregate and sectoral variables in the same VAR echoes the approach of Canova (2005) in modeling the effects of US shocks on smaller Latin American economies and Mumtaz and Surico (2009) on the effects of international shocks on the domestic, (UK), economy.

VARs have long been regarded as a good way of describing the dynamic correlations in the data, see e.g. Sims (2003), Baumeister and Hamilton (2024). This has typically been done using an arbitrary decomposition, namely the Cholesky factorization, of the variance-covariance matrix of the residuals of the VAR. Decomposition allows the creation of basis functions which together are able to reconstruct the observed time series. These reconstructions, denoted 'historical decompositions', implicitly provide narrative descriptions of the evolution of the observed time series, as different basis functions play greater and lesser roles at different times. We describe these historical decompositions of the evolution of native wages hours and immigration in detail in section 5 below. However, for an n variable VAR there are $n!$ different possible Cholesky factorizations, each of which will provide a different implicit narrative. In addition advocating causality based on any Cholesky factorization is problematic due to the strong restrictions it imposes on the responses of the identified shocks, see for example Uhlig (2005) and Baumeister and Hamilton (2015, 2019, 2024).

We therefore also employ the sign restriction identification methodology which has been frequently used in the macroeconomic literature, notably by Canova and De Nicoló (2002), Uhlig (2005), Mumtaz and Surico (2009) and Baumeister and Hamilton (2015, 2019, 2024) and applied to immigration by Furlanetto and Robstad (2019) and Kiguchi and Mountford (2017). In this paper we follow the approach of Baumeister and Hamilton (2015, 2019, 2024) who show how one can incorporate beliefs and incomplete information about the effects of different shocks into the priors for the VAR's parameters in a Bayesian estimation procedure. The choice of which pattern of sign restrictions to impose is nevertheless subjective to some degree. In this paper we follow the macroeconomic literature and use minimal restrictions, so that, for example, a labor demand/business cycle shock is identified as a shock with a positive co-movement of native wages and hours.

The minimal identifying assumptions we use still leave a lot of scope for interpretation. Should a shock which generates a positive co-movement of aggregate migration share, aggregate hours and native wages, be characterized as an exogenous aggregate labor demand shock? Macroeconomists are very confident that such a force should be present in the data, either as a macroeconomic demand or business cycle shock. The historical decompositions also support this interpretation, as this shock explains most of the variation in hours worked over the sample, with strong negative shocks in the period after the 2008 financial crisis. We are therefore happy to label this shock as an aggregate labor demand/business cycle shock, but we cannot rule out other possible interpretations. Similarly, we label shocks with a negative association between aggregate immigration and aggregate native wages as a labor supply/migration shock in line with standard theory. The historical decompositions are also consistent with this interpretation, as this shock explains most of the variation in migrant share over the sample period with large negative shocks after the Brexit referendum. This is very intuitive. Again, however, of course, there are other possible interpretations.

In our final section we put some numbers on the extent of the wage effects of each identified aggregate shock using a counterfactual approach, where the contribution of one of the identified shocks set to zero in an otherwise standard historical decomposition analysis. This exercise shows the estimated contribution of the left out shock to the observed time series. This is done without reference to a deep structural model and so one should not use this analysis to make statements like "If immigration was $x\%$ lower then native wages would be $y\%$ higher". However one can make statements like "At the model's median estimate, the contribution of the aggregate migration labor supply shock to native wage growth in sector A over the sample period was $x\%$ out of a total sectoral native wage growth of $y\%$." Indeed we find for certain sectors, such as the unskilled retail sector, that the absence of the shock that explains most of the variation in migration share, results in a native wage level more than a 15% higher by the end of the sample period. However in many professional sectors the absence of this migration shock has very little effect on the native wage path. This shows that an aggregate shock may have very different effects across sectors.

The paper is organized as follows. We first describe the data that underpin the analysis in section 2. We then describe the sectoral variation in native wage and immigration growth over the sample period in section 3 which is the focus of the paper. Section 4 outlines the estimated models and the respective identification methods used to generate the impulse response functions, historical decompositions and counterfactual time series for immigration and native wages presented in section 5.

Figure 1: Growth rates of immigration share, hours worked and average native real wages in the UK 2001-2020. We have used red lines for the data series in figures throughout this paper.

2 Data and motivation

In order to estimate our VAR models we need aggregate and sectoral level data on the total hours worked, wages of UK-born workers and the concentration of immigrants working. The requisite information is contained in the UK Labour Force Survey (LFS). The LFS is a quarterly random sample of around 40,000 households and the individuals therein. We use data starting in the first quarter of 2001 and end our sample in the last quarter of 2019 so as not to include data subject to the effects of the COVID pandemic.[4]

The LFS contains details of the country of birth of every individual in the sample. An immigrant is defined as anyone who is born outside the UK. The LFS also gives the 3-digit industry and occupation codes of employed workers. Since specific industries contain many occupations and a given occupation can be found across different industries, the definition of a sector in our analysis combines individual occupation and industry affiliation. Sample size constraints determine that a sector is built as a combination of four possible occupations, (Professional/Managerial, Other Non-Manual, Skilled Manual and Manual), and 13 indus-

[4] The LFS sample response rates also decline significantly during the pandemic which adversely affects data analysis using disaggregated units.

tries.[5] For example, in our data, sector 112 is a professional (1-digit SOC code = 1) working in the health industry, (2-digit SIC code = 12). One complication with pooling LFS data over time is that the occupational codes change approximately every 10 years.[6] The industry classifications also change in 2009 but we are able to correct for this using the mapping of Smith.[7] We collate the data by sector for each quarter in each year. This ensures that there is a minimum of 100 observations in each of 35 sectors in each quarter with a median sample cell size of 1122 for hours and 267 for native wages. The hours variable we use in our analysis is 'Total Hours Worked', in the survey reference week includes paid and unpaid overtime. We observe hourly native wages for 40% of the survey respondents and use the median of this at the sectoral level deflated by the CPI price index.[8] The aggregate versions of these variables are the aggregates of the sectoral variables weighted by their LFS population weights.

Granger causality

The idea that the amount of immigrant labor employed in an economy will depend on the demand for and supply of labor is extremely intuitive. Figure 1 plots the year on year growth rates of the share of immigrants in the working age population, the total number of hours worked and the average real wage of natives between 2001-2020 using UK LFS data. The time series appear related most noticeably after the financial crisis of 2007-2008 and the subsequent recovery period. This is borne out by Granger causality tests, reported in Table 1, which show that immigration is Granger caused by the total number of hours worked and that native wages are Granger caused by immigration. These results are generated using a VAR of our three aggregate variables with 4 lags both with and without a time trend, estimated on the entire sample and for the shorter sub-sample 2004q1-2016q2, to demonstrate that the results are not due to Brexit or the sample's initial conditions. Interestingly total hours worked are not Granger caused by either native real wages or immigration in any specification or sample.

These results suggest that the level of immigration and native real wages are related to the total hours worked in the economy, which itself is surely affected by the macroeconomic environment and hence demand and supply effects. Empirical macroeconomics, as explained below in section 4, has developed methods to untangle the individual effects from the multiple influences on a variable's time series. We apply these methods to identify the contribution of labor demand, supply and immigration shocks at both the aggregate and sectoral level, on immigration share, hours worked and native wages in the UK economy.

[5] Production, Construction, Retail, Transport, Food & Hospitality, Media&IT, Finance, Scientific, Transport&Support Services, Public Admin, Education, Health , Other Services.

[6] The latest industry re-coding was 2008 and there were 2010 and 2020 re-coding for occupations. The occupational classifications also change much more significantly in 2001, which makes matching before this period difficult. Using 4 broad occupation codes facilitates comparability over time.

[7] The change in the industry codes is less substantial, see https://warwick.ac.uk/fac/soc/economics/staff/jcsmith/sicmapping/

[8] The LFS only elicits wage information from 40% of each sample in every quarter.

Table 1

Granger causality tests for aggregate labor market variables

	Time Period 2003q1-2019q4		**Time Period 2004q1-2016q2**	
Model:	**VAR(4)**	**VAR(4) with trend**	**VAR(4)**	**VAR(4) with trend**
	Chi-Sq	Chi-Sq	Chi-Sq	Chi-Sq
Immigration				
Exclude hours	21.89***	18.97***	23.91***	23.87***
Exclude real native wage	6.234	4.654	11.48**	8.031*
Exclude both	25.86***	20.90***	33.23***	28.11***
Total hours				
Exclude immig.	3.230	1.342	3.161	1.773
Exclude real native wage	3.961	3.703	5.145	5.484
Exclude both	5.974	4.268	6.594	6.053
Real native wages				
Exclude immig.	12.95**	14.44***	11.15**	11.15**
Exclude hours	5.592	7.017	4.141	6.918
Exclude both	14.53*	16.26**	13.34	15.27*

Notes: The table reports, the Chi-squared values for the Granger causality tests from VARs of the year on year growth rates of the share of immigrants in the working population, the total number of hours worked and the average hourly native wage in the UK. The VARs use 4 lags and are run for the time periods, 2003q1-2019q4 and 2004q1-2016q2. ***,**, and * indicate significance at the 1%,5% and 10% level respectively.

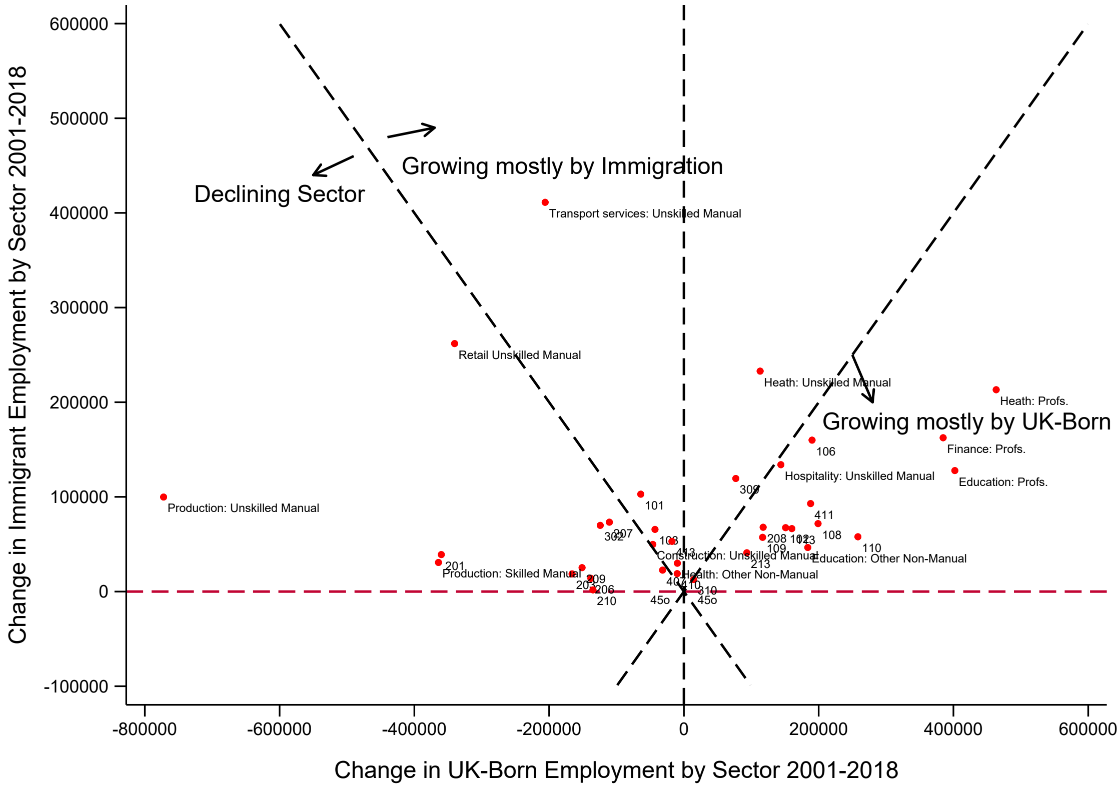


Figure 2: Change in UK-Born & Immigrant Employment by Sector 2001-2018

3 Sectoral variation in immigrant labor

To illustrate the degree of heterogeneity in the use of immigrant labor across sectors, Figure 2 graphs the 18-year change in employment of both UK-born workers and immigrants in each of the 35 sectors in the data set. The backward sloping 45 degree line separates occupations that experience net growth in employment in this period from those that are declining. Any occupation that lies above and to the right of this line is growing. The forward sloping 45 degree line separates occupations that are growing primarily because of immigration - those sectors above the line - from those that are growing mainly due to growth in UK-born employment - those sectors below the line. The figure shows that most sectors grow over this period, but a minority decline (e.g. Unskilled Manual in Production or Unskilled Manual in Retail). In all these declining sectors, the number of immigrants rises while the number of UK-born workers falls. This means that the share of immigrants has risen in all sectors with a net decline in employment.[9] Of the sectors with net employment growth over the period, some grow exclusively because of rising immigrant numbers, (e.g. Transport services: Unskilled Manual) while numbers of UK-born employed fall. Others grow through approximately equal numbers of immigrants and UK-born, (e.g. Unskilled workers in Hospitality) and some grow primarily, though not exclusively, through rising numbers of UK-born workers, (e.g. Education Professionals). There is no sector in which the level of immigration falls over this

[9] This finding also indicates that the immigrant share, a common measure of immigrant concentration in the literature, can also change because of changes in the size of the native workforce.

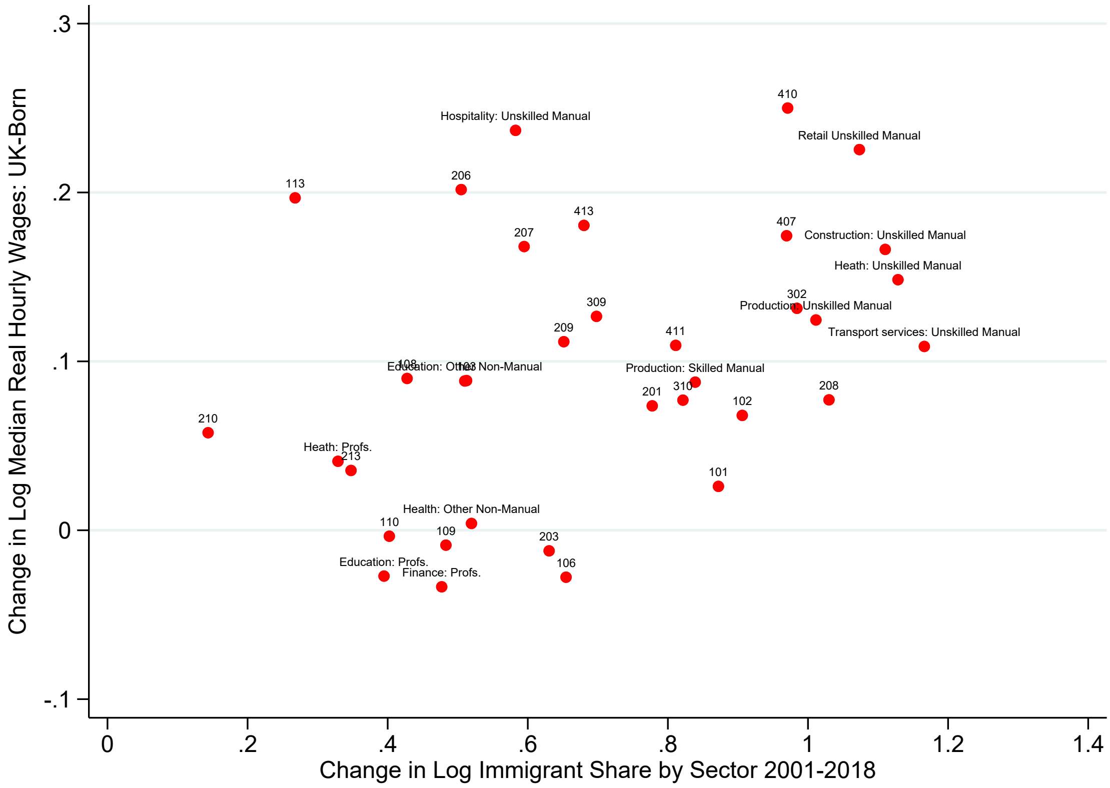


Figure 3: Log change in UK-born log real hourly wage & immigrant employment share by sector 2001-2018

period. Overall the Figure shows that there is substantial heterogeneity in changes in both employment and the immigrant share across sectors over the sample period. This suggests that different sectors are subject to different shocks and/or they react differently to a given shock.

Figure 3 indicates another facet of heterogeneity of experience across sectors by plotting the change in (log) native wages of UK-born workers in each sector over the sample period against the change in the sectoral log immigrant share. For a given change in immigrant share, the graph shows a large variation in wage growth across sectors. In some sectors native wages fall, while in other sectors, for the same immigrant change, native wages rise. Again this suggests that the association between immigration and the labor market experience of native-born workers is unlikely to be the same in all sectors. However, despite this heterogeneity, there does appear to be a positive relationship overall between changes in immigration and native wages. In our analysis below we decompose the extent to which this is caused by supply and demand factors in each sector.

4 Sectoral labor market dynamics with multiple causal factors

We argue in this paper that it is useful to exploit the information contained in the time series dimension of the data to model sectoral labor markets as being subject to multiple forces at any one time. For example, sectoral wages of native workers in period t, w_t^{sec}, may be subject to shocks from a combination of aggregate migration, $\epsilon_{aggM,t}$, aggregate supply, $\epsilon_{aggS,t}$, aggregate demand, $\epsilon_{aggD,t}$, sectoral migration, $\epsilon_{secM,t}$, sectoral supply, $\epsilon_{secS,t}$, and sectoral demand, $\epsilon_{secD,t}$, as in the following equation,

$$w_t^{sec} = \beta x_{t-1} + \alpha_1 \epsilon_{aggM,t} + \alpha_2 \epsilon_{aggS,t} + \alpha_3 \epsilon_{aggD,t} + \alpha_4 \epsilon_{secM,t} + \alpha_5 \epsilon_{secS,t} + \alpha_6 \epsilon_{secD,t} \qquad (1)$$

where α_i are parameters indicating the strength of each shock in determining native wages in this sector and where x_{t-1} is a vector of predetermined variables.

This type of wage equation corresponds to one of the equations in a structural VAR, where the predetermined variables are the constant term and the lags of the variables included in the VAR, denoted y, so that in equation (1), $x_{t-1} = [1, y'_{t-1}, \ldots, y'_{t-p}]'$, and x_{t-1} is an $((np+1) \times 1)$ vector where n is the number of variables and p is the lag length in the VAR.

A structural VAR is described by equation (2),

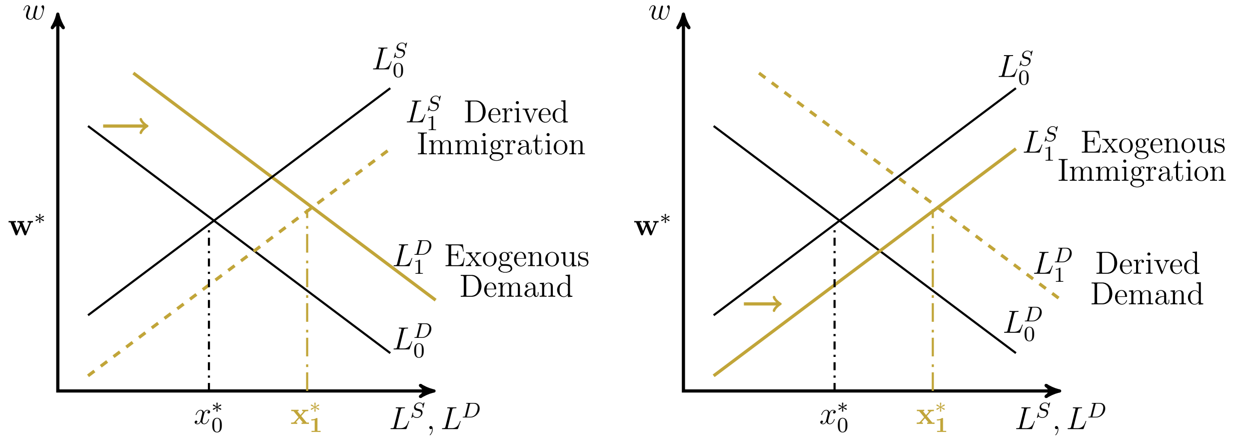
$$Sy_t = C + B_1 y_{t-1} + B_2 y_{t-2} + \cdots + B_p y_{t-p} + \epsilon_t \qquad (2)$$

where S, is the $(n \times n)$ matrix of contemporaneous effects, C, is the $(n \times 1)$ vector of constants, B_i, are the $(n \times n)$ matrix of parameters for lag i, and ϵ_t, is the vector of the fundamental shocks on the VAR. The $(n \times n)$ variance covariance matrix for these shocks, $E[\epsilon_t \epsilon_t'] = D$, is assumed to be diagonal. The diagonal structure implies the shocks are 'fundamental' in the sense of not being associated with each other.

In our VAR model, for each sector we choose $p = 4$, for quarterly data, and $n = 6$, as we are interested in the interaction between 3 aggregate and 3 sectoral shocks on aggregate and sectoral variables. The six variables are the year on year difference of the logs of the economy-wide migration share, M_t^{agg}, economy-wide total hours worked, H_t^{agg}, economy-wide real wages of native workers, W_t^{agg}, the sectoral migration share, m_t^{sec}, total hours worked in the sector, h_t^{sec}, and sectoral real wages of natives, w_t^{sec}. Thus $y_t = (M_t^{\text{agg}}, H_t^{\text{agg}}, W_t^{\text{agg}}, m_t^{\text{sec}}, h_,^{\text{sec}} w_t^{\text{sec}})'$. The aggregate variables allow for the identification of economy-wide aggregate demand, supply and migration shocks while the presence of the sectoral variables allows for the possibility of sectoral demand, supply and migration shocks to also affect the outcome variables.

Baumeister and Hamilton (2015, 2019) show how one can incorporate prior beliefs about the signs and size of the coefficients in the contemporaneous effects matrix, S, into a Bayesian estimation procedure for the VAR. They derive a Metropolis Hasting algorithm for drawing from the posterior distribution for the parameters resulting from these priors. Baumeister and Hamilton (2015) describes this method in detail and so our exposition here can be brief. We use the programs supplied by Baumeister and Hamilton for the replication of Baumeister and Hamilton (2019) to estimate the model.[10] We also impose additional assumptions in the

[10] We make 12,000,000 draws from the posterior for each sector discarding 8,000,000 draws as 'burn in' and



(a) Exogenous demand shock potentially causing increased immigration.

(b) Exogenous immigration shock potentially causing increased demand for labor.

Figure 4: Identification: which shocks matter most for changes in sectoral wages of natives?

prior for the B parameters, namely we impose stationarity and set the prior to be very tight around zero for the parameters associated with the lagged sectoral variables in the aggregate variable equations, following e.g. Blake and Mumtaz (2017). This implies, as we will see below, that the dynamics of the aggregate variables are almost entirely determined by the 3×3 sub-VAR of the aggregate variables.

As a comparator, we also estimate the responses of the VAR using the Cholesky factorization as described below in section 4.1.1. For this case we impose the same restrictions on the B parameters, and assume that the prior and posterior for B and Σ belong to the Normal–Wishart family, which allows one to sample directly from the posterior, following Uhlig (2005).[11]

4.1 Identifying different structural shocks

Given the estimated model parameters and variance covariance matrix, the aim of identification is to define the different fundamental shocks which underlie the movement of each variable. This amounts to choosing the matrix, S, in the structural VAR described in equation (2). Given this matrix S, one can then calculate which shocks are most important for the variation of each variable, and also how they reinforce or offset each other. For example, as illustrated in Figure 4, one may be unsure whether demand shocks or migration shocks are most important in determining sectoral wage rates. An observed significant rise in hours worked occurring alongside little or no change in sectoral wages may be the result of a positive shock to the demand for labor which is responded to by an increase in the labor supply,

retain every 500th draw as the 'thinning' process. We set $\lambda_0 = 100$, $\lambda_1 = 1$, $\lambda_2 = 1$ and $\lambda_3 = 100$ in the Minnesota prior, use the identity matrix for the covariance of the proposal distribution and adjust the jump size during the burn-in phase when it deviates too far from an acceptance rate of 0.35.

[11] We make 100,000 draws from the posterior for each sector discarding 50,000 draws as 'burn in' and retain every 25th draw as the 'thinning' process.

including from immigration. This case is depicted in Panel a) of Figure 4. Equally, as depicted in Panel b), an exogenous shock to the labor supply which increases domestic labor demand in response could generate a similar effect.

Clearly this is not an exhaustive list of potential explanations for this relationship. In Section 5 we compare the contribution of six identified shocks to the variation in native wage growth in each sector. Structural analysis uses economic theory to separate out multiple economic time series into their fundamental economic components. In our case it is often thought that some of migration is an exogenous shock to the system. If one also believes that migration will take one quarter to react to other shocks then the first row of the S matrix in the structural equation (2) will be zeros and the shock to the first equation can be thought of as an exogenous migration shock. This is the case in the Cholesky migration, described below section 4.1.1, where we label the first shock as a migration shock

However the assumption that migration will not react to business cycle shocks and supply shocks within a quarter is a strong one and so we also employ a weaker set of identifying assumptions using the sign restriction following the approach of Baumeister and Hamilton (2017,2024) as described in section 4.1.2.

4.1.1 Using the Cholesky factor

The Cholesky factorization of the reduced form's variance covariance matrix, Σ, is widely seen as a useful and transparent, if arbitrary, way of summarizing the data's dynamics. The reduced form VAR is given by multiplying equation (2) by the inverse of the S matrix,

$$y_t = S^{-1}C + S^{-1}B_1 y_{t-1} + S^{-1}B_2 y_{t-2} + \cdots + S^{-1}B_p y_{t-p} + u_t \tag{3}$$

where, therefore, $u_t = S^{-1}\epsilon_t$. Any symmetric positive-definite matrix, such as a variance covariance matrix, Σ, can be written as the product of a lower triangular matrix, L, known as the Cholesky factor, and its transpose, such that $LL' = \Sigma$. thus setting $S^{-1} = L$ is one possible form for the S matrix. Given an S matrix, the dynamics of the data can be summarized by six independent shocks, $\epsilon_1 \ldots \epsilon_6$. Each shock is assumed to have a zero mean and a unit variance. Defining $\epsilon_t = [\epsilon_{1,t} \ldots \epsilon_{6,t}]$, the independence of the shocks and their normalization implies that $E[\epsilon_t \epsilon_t'] = I_6$. The prediction errors of the VAR, u_t, is therefore mapped into these independent shocks via the equation $u_t = L\epsilon_t$, since $E[u_t u'] = LE[\epsilon_t \epsilon_t']L' = \Sigma$. The Cholesky factorization thus decomposes each variable's time series into the sum of responses to multiple independent shocks, see e.g. Baumeister and Hamilton (2024) or Sims (2003).

However because L is a lower triangular matrix, setting $S^{-1} = L$ implies that only one shock, $\epsilon_{1,t}$, affects all variables contemporaneously.[12] This is often seen as a very strong restriction. The sign restriction approach, described below, imposes looser restrictions so that, in our case, all sectoral variables can be affected contemporaneously by all shocks.

[12] See Appendix A.2

4.1.2 Using Sign Restrictions

Instead of assuming that S^{-1} is lower triangular, we can impose a looser set of restrictions that the VAR is lower block diagonal. Given that y_t is ordered, $y_t = (M_t^{\text{agg}}, H_t^{\text{agg}}, W_t^{\text{agg}}, m_t^{\text{sec}}, h_,^{\text{sec}} w_t^{\text{sec}})'$, this implies that aggregate variables are not contemporaneously affected by sectoral variables, following e.g. the intuition of Liu, Mumtaz and Theophilopoulou (2014). The S matrix therefore has the form

$$S = \begin{bmatrix} 1 & s_{MH} & s_{MW} & 0 & 0 & 0 \\ s_{HM} & 1 & s_{HW} & 0 & 0 & 0 \\ s_{WM} & s_{WH} & 1 & 0 & 0 & 0 \\ s_{mM} & s_{mH} & s_{mW} & 1 & s_{mh} & s_{mw} \\ s_{hM} & s_{hH} & s_{hW} & s_{hm} & 1 & s_{hw} \\ s_{wM} & s_{wH} & s_{wW} & s_{wm} & s_{wh} & 1 \end{bmatrix} = \begin{bmatrix} S_1 & 0 \\ S_2 & S_3 \end{bmatrix} \tag{4}$$

where S_1, S_2 and S_3 are the upper left, bottom left, and bottom right 3×3 submatrices of S. The first column of S gives the contemporaneous effect of aggregate migration on the other variables. Thus s_{HM} is the contemporaneous effect of aggregate migration on aggregate hours. Similarly the first row is the contemporaneous effects of other variables on aggregate migration, so that s_{MH} is the contemporaneous effect of aggregate hours on aggregate migration. Note that in the top left 3×3 block all three aggregate variables can contemporaneously affect each other, and the sectoral variables can be contemporaneously affected by all, aggregate and sectoral, variables.

The impact matrix of the fundamental shocks in the reduced form VAR, equation (3), is the inverse of the S matrix, S^{-1}. The lower block triangular nature of S implies that the determinant of S, $\det(S)$, is given by $\det(S) = \det(S_1)\det(S_3)$ and that S^{-1} can be decomposed into the product of 3×3 matrices.

$$S^{-1} = \begin{bmatrix} S_1^{-1} & 0 \\ -S_3^{-1} S_2 S_1^{-1} & S_3^{-1} \end{bmatrix}$$

Thus S^{-1} can be written

$$S^{-1} = \left[\begin{array}{ccc|ccc} \frac{1 - s_{HW}s_{WH}}{\det(S_1)} & \frac{s_{MW}s_{WH} - s_{MH}}{\det(S_1)} & \frac{s_{MH}s_{HW} - s_{MW}}{\det(S_1)} & 0 & 0 & 0 \\ \frac{s_{WM}s_{HW} - s_{HM}}{\det(S_1)} & \frac{1 - s_{MW}s_{WM}}{\det(S_1)} & \frac{s_{MW}s_{HM} - s_{HW}}{\det(S_1)} & 0 & 0 & 0 \\ \frac{s_{HM}s_{WH} - s_{WM}}{\det(S_1)} & \frac{s_{MH}s_{WM} - s_{WH}}{\det(S_1)} & \frac{1 - s_{MH}s_{HM}}{\det(S_1)} & 0 & 0 & 0 \\ & & & \frac{1 - s_{hw}s_{wh}}{\det(S_3)} & \frac{s_{mw}s_{wh} - s_{mh}}{\det(S_3)} & \frac{s_{mh}s_{hw} - s_{mw}}{\det(S_3)} \\ & -S_3^{-1} S_2 S_1^{-1} & & \frac{s_{wm}s_{hw} - s_{hm}}{\det(S_3)} & \frac{1 - s_{mw}s_{wm}}{\det(S_3)} & \frac{s_{mw}s_{hm} - s_{hw}}{\det(S_3)} \\ & & & \frac{s_{hm}s_{wh} - s_{wm}}{\det(S_3)} & \frac{s_{mh}s_{wm} - s_{wh}}{\det(S_3)} & \frac{1 - s_{mh}s_{hm}}{\det(S_3)} \end{array} \right] \tag{5}$$

Equation (5) demonstrates that a sign restriction on any one element of the S matrix does not necessarily imply a sign for the impact of any shock on any variable. Nevertheless sign restrictions can be imposed using the formulas for impact given in equation (5) during the sampling procedure for the elements of S, so that e.g. the restriction that the 4th ordered shock has a positive impact on sectoral hours and a positive impact on sectoral native wages, are the restrictions that $\frac{s_{wm}s_{hw} - s_{hm}}{\det(S_3)} > 0$ and $\frac{s_{hm}s_{wh} - s_{wm}}{\det(S_3)} > 0$. We order the shocks so that the aggregate migration shock is placed first in the ϵ_t vector, so that $\epsilon_t = (\epsilon_{aggM,t}, \epsilon_{aggS,t}, \epsilon_{aggD,t}, \epsilon_{secM,t}, \epsilon_{secS,t}, \epsilon_{secD,t})'$. Thus the sign restrictions for the aggregate migration shock are in the first column of S^{-1}.

The sign restrictions we impose in the structural model are described in equation (6), which omits the constant and lagged terms in equation (3) for ease of exposition. Thus the sign restrictions for a positive aggregate demand shock, in column3, are that it has a positive impact effect on aggregate immigration, aggregate hours worked and aggregate real wages of natives. The aggregate supply shock in column 2 is identified as a shock which increases aggregate hours worked and reduces the aggregate real wage of natives on impact. The aggregate migration shock in column 1 is identified as a shock which increases aggregate migration share and hours worked on impact. There are no restrictions on the responses of other variables which is often referred to as being 'agnostic' about the responses of unrestricted variables to a shock. Note that the sign restrictions described by equation (6) do not completely pin down the interpretation of these shocks. In particular, as we discuss below in section 5, the sign restrictions do not restrict the response of migration to a supply shock. Thus if the response of migration to the second shock is positive then this too could be interpreted as a migration shock.

$$\underbrace{\begin{bmatrix} \text{Aggregate migration share} \\ \text{Aggregate hours worked} \\ \text{Aggregate native wage} \\ \text{Sectoral migration share} \\ \text{Sectoral hours worked} \\ \text{Sectoral native wage} \end{bmatrix}}_{y_t} = \underbrace{\begin{bmatrix} + & \text{none} & + & \text{none} & \text{none} & \text{none} \\ + & + & + & \text{none} & \text{none} & \text{none} \\ \text{none} & - & + & \text{none} & \text{none} & \text{none} \\ \text{none} & \text{none} & \text{none} & + & \text{none} & \text{none} \\ \text{none} & \text{none} & \text{none} & \text{none} & + & + \\ \text{none} & \text{none} & \text{none} & \text{none} & - & + \end{bmatrix}}_{S^{-1}} \underbrace{\begin{bmatrix} \epsilon_{aggM,t} \text{ (Agg. Migration Shock)} \\ \epsilon_{aggS,t} \text{ (Agg. Supply Shock)} \\ \epsilon_{aggD,t} \text{ (Agg. Demand Shock)} \\ \epsilon_{secM,t} \text{ (Sec. Migration Shock)} \\ \epsilon_{secS,t} \text{ (Sec. Supply Shock)} \\ \epsilon_{secD,t} \text{ (Sec. Demand Shock)} \end{bmatrix}}_{\epsilon_t} \quad (6)$$

We do not impose the same sign restrictions at the sectoral level as at the aggregate level. Thus for immigration, at the aggregate level it is intuitive that an increase in aggregate immigration should increase aggregate hours. However this need not be the case for every sector. As discussed above in section 3, in some sectors increased immigration may be associated with reduced total hours as migrant labor offsets an outflow of domestic labor. Thus the sectoral migration shock is simply restricted to increase the sectoral migrant share in equation (6). Similarly sectoral demand shocks are not required to increase sectoral migration shares.

Different identifying assumptions, will imply different roles for the identified shocks in contributing to the observed time series. Baumeister and Hamilton (2015, 2019) show how one can go further and tighten the identification by incorporating knowledge about the likely sizes of these S matrix parameters (elasticities) into the priors for these parameters. We do not do this in this paper as the incorporation of sector specific information for 35 sectors is beyond the scope of this paper. We therefore employ loose priors for all these coefficients, and leave the analysis employing more informative tighter priors for future work.[13]

[13]The priors are a students t distribution, $t_\nu(\mu, \tau^2, \nu)$ with $\mu = 0, \tau = 100$ and $\nu = 3$.

4.2 The structural model: historical decompositions and counterfactuals

Each estimated draw of the S matrix allows the construction of a time series of fundamental shocks, ϵ_t from the estimated reduced form errors, u_t, via the formula, $\epsilon_t = Su_t$. The observed time series can then be reconstructed using these time series of fundamental shocks by recursive iteration of equation (2) as described in Hamilton (1994) and briefly in Appendix A.1. This is the historical decomposition.

We illustrate our results with 'counterfactual analysis'. The counterfactual error, ϵ_t^{CF} from setting one of the shocks to zero, is simply ϵ_t with the corresponding element set to zero- e.g. the second element of ϵ_t is set to zero for all t if the influence of second fundamental shock on the observed time series is being removed. The counterfactual reduced form residual will then be given by $u_t^{CF} = S^{-1}\epsilon_t^{CF}$. The structural constant terms C in equation (2) can be constructed in the same way by multiplying the estimated reduced form constant by S. The counterfactual constant term, C^{CF} will then be C with the element corresponding to the shock replaced by zero. The counterfactual reduced form constant term is then given by the vector $S^{-1}C^{CF}$. As we have emphasized above this 'counterfactual' should not be interpreted as performing a 'pseudo experiment'. Rather, this counterfactual is simply a different presentation of the standard historical decomposition to quantify the contribution of a specific shock over the sample period. One aspect that is not typical is accounting for the contribution of the specified shock to the steady state growth rate of each variable via its contribution to the constant term in equation of the VAR. This is important and is often ignored by the literature.

5 Results

In this section we discuss the results from the VARs identified under both the Cholesky and sign restriction methodologies described above. We first discuss the results for aggregate variables in section 5.1 and then the results at the sectoral level in section 5.2. In each case we first describe the impulse response functions. These are the building blocks for the discussion of the historical decompositions of the data which will be the focus of our attention. A key motivation for this paper is the idea that different contemporaneous shocks may offset each other. Historical decompositions show in each time period the contribution of each of the six identified shocks, as well as the initial conditions and constant term, to each data series. In section 5.1.3 we also apply the historical decomposition method to put numbers on the contribution of the identified aggregate shocks with a negative association between aggregate migration and native wage growth by removing the influence of these shocks from the constructed time series.

Figure 5: Impulse responses for the unskilled manual sectors using the Cholesky factor approach.

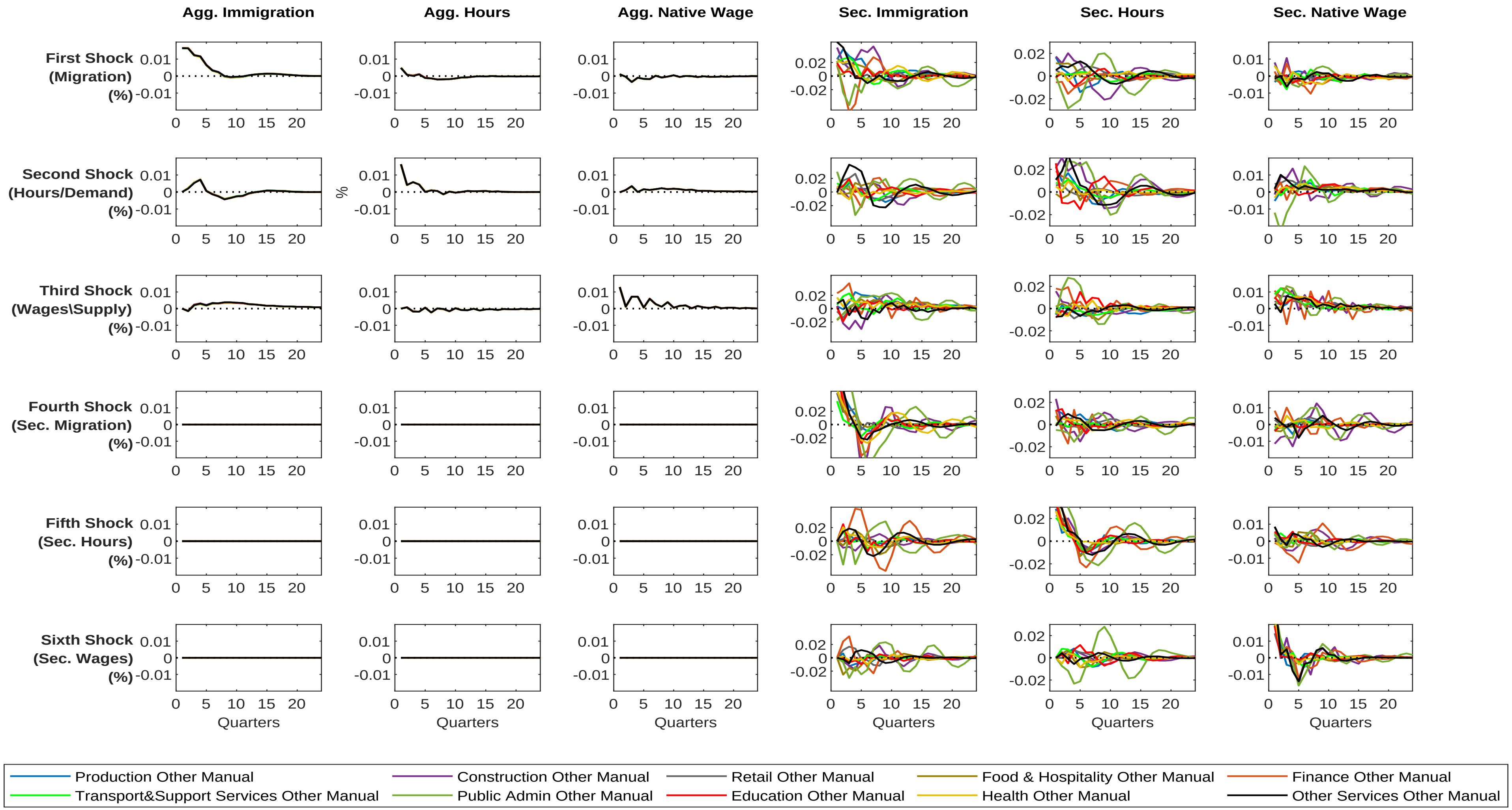


Figure 5 plots the median impulse responses for the unskilled manual sectors using the Cholesky factorization approach. Each row plots the responses to a particular shock (row 1 is the first shock, row 2 is the second shock, etc.) and each column is the responses of a particular variable (column 1 is aggregate migration, column 2 is aggregate hours, etc.). The responses of all 10 sectors are plotted together. By construction, the sectoral variables play very little role in aggregate dynamics and so the aggregate responses are all very similar to each other and appear for the most part to be a thick line.

Figure 6: Impulse responses for the unskilled manual sectors using the sign restriction approach

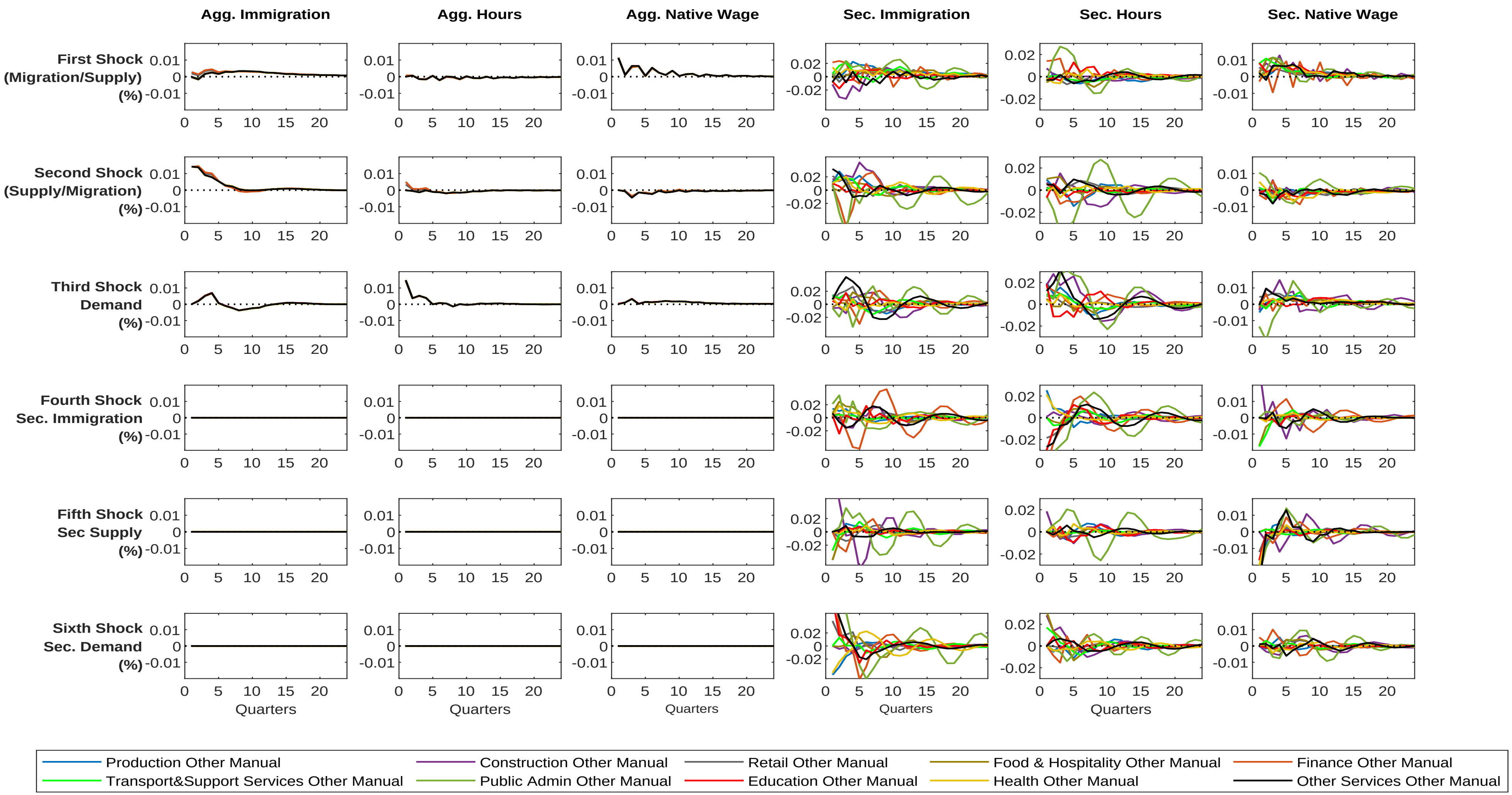


Figure 6 plots the median impulse responses for the unskilled manual sectors using the sign restriction approach of section 4.1.2. Each row plots the responses to a particular shock (row 1 is the first identified shocks, aggregate migration/ supply row 2 is the second identified shock, aggregate supply/migration, etc.) and each column is the responses of a particular variable (column 1 is aggregate migration, column 2 is aggregate hours, etc.). The responses of all 10 sectors are plotted together. By construction, the sectoral variables play very little role in aggregate dynamics and so the aggregate responses are all very similar to each and appear for the most part to be a thick line.

5.1 Results for aggregate shocks

In order to uncover the dynamic effects of our identified shocks on native wages, immigration and hours and the relative importance of each shock, we present impulse response functions and historical decompositions of the data. The impulse responses for the two identification methods - Cholesky and sign restrictions - are shown in Figures 5 and 6 and the historical decomposition of aggregate native wages, migration and hours are shown in Figures 8 and 9.

The impulse response functions map out the dynamic paths of all six variables in the VAR in response to a standard deviation innovation for each of the identified shocks.[14] This is done for each draw from the posterior distribution of the parameters and so results in a distribution of impulse responses. In Figures 5 and 6 we focus on the median response from this distribution and in Figure 7 we discuss the confidence bands to gauge the uncertainty around the central estimates.

5.1.1 Impulse response functions of aggregate shocks

Figures 5 and 6 plot the median impulse responses from the 10 VARs estimated for the 10 unskilled manual sectors in the data-set for the Cholesky and structural sign restrictions approach respectively.[15] Note that these figures plot the median impulse responses for all 10 sectors and yet the impulse responses of the aggregate variable to aggregate shocks appear for the most part to be one thick line. As the VARs for each sector are run independently of each other, this similarity demonstrates the accuracy of the estimation process. The restrictions, described in section 4, that sectoral variables play very little role in determining aggregate variables are evident in the negligible responses of aggregate variables to the sectoral shocks displayed in the lower 3×3 submatrices of each Figure. Thus the top left 3×3 submatrices of Figures 5 and 6 are close to those of an independent 3 dimensional VAR in the aggregate variables.

The responses of the aggregate variables to aggregate shocks are displayed in Figure 7a for the Cholesky case and Figure 7b for the sign restriction approach. These Figures plot the 16th, 50th and 84th quantiles from the posterior from one sector, the unskilled transport services sector, and shades the 16-84th quantile confidence set.[16] What is noticeable about these Figures is how similar the responses are to each other. The first shock in the Cholesky ordering in Figure 7a, looks extremely similar to a supply shock in Figure 7b and we have colored both of these shocks yellow to emphasize this and refer to them as 'migration/supply shocks'. Similarly the second shock in the Cholesky ordering in Figure 7a, looks extremely similar to the demand/business cycle shock in Figure 7b respectively. We have therefore colored both of these shocks blue and refer to them as 'demand shocks' . Finally the third shock in the Cholesky ordering in Figure 7a, looks extremely similar to the supply shock in Figure 7b and these shocks are colored green. The similarity of the shocks is not so surprising

[14] See the Appendix A.1, or Hamilton (1994), for description of impulse responses.

[15] The impulse responses for the other sectors are available on request.

[16] As Figures 5 and 6 show the aggregate responses from other sectors are almost identical.

when one notes that the signs of the impulses of the Cholesky factor approach have a very similar pattern to those imposed by the sign restrictions. One only needs to replace the zero restrictions of the Cholesky factor with small, positive or negative, epsilon perturbations for them to have the same signs as those described in equation (6). Although similar, the sign restriction impulses do give a slightly larger role for the yellow colored shocks, which can be seen in the historical decomposition for native wages in Figure 8.

The impulse responses in Figures 7a and 7b show that both identification approaches still leave a lot of room for interpretation. The role of judgment in the labeling of the underlying structural shocks demonstrates, as in Baumeister and Hamilton (2019) and Uhlig (2005), that macroeconomic theory by itself is not sufficient to tightly pin down the effects of any particular shock. Thus if one is wanting a definitive result on the percentage of native wages 'caused' by migration then you will need to find more information and/or impose tighter restrictions. As described in section 4.1.2, the structural approach of Baumeister and Hamilton (2015, 2019) is designed to be able to incorporate such additional information, although doing so for 35 different sectors would be a large task.

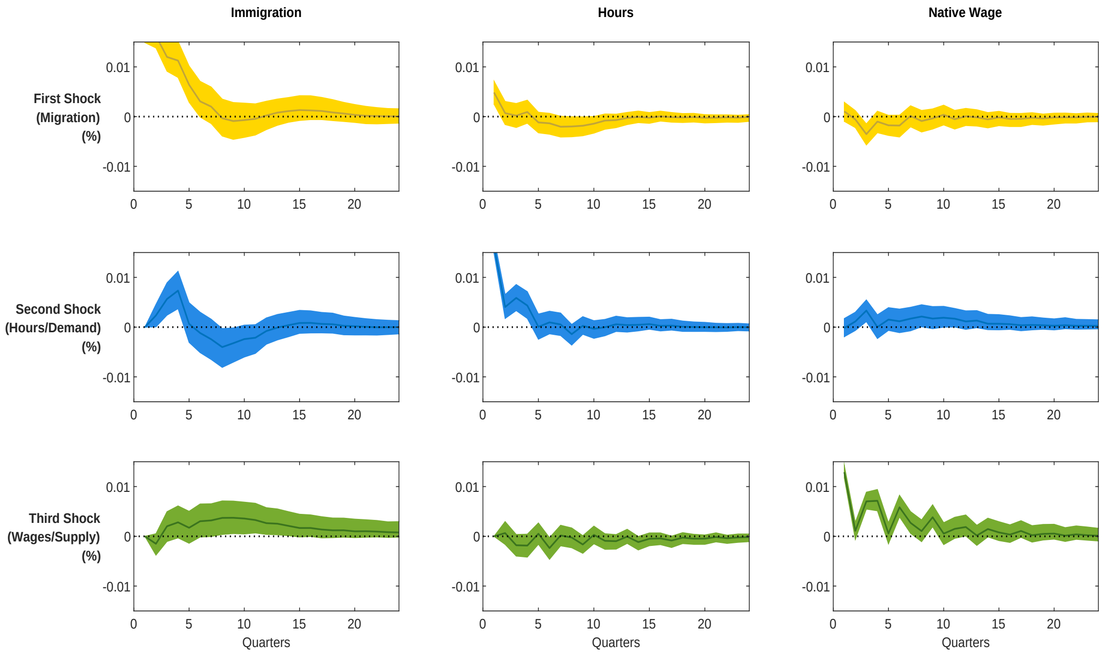
We asked at the outset whether immigration could be thought of as an exogenous shock. The impulses responses in Figures 7a and 7b, suggest that aggregate immigration is influenced by shocks that could be considered aggregate supply, colored green, and aggregate demand/business cycle , colored blue. However in each case there is also a shock which could be thought of as an exogenous migration shock, colored yellow, which is the first shock in the Cholesky case in Figure 7a and the second shock in the structural sign restrictions case in Figure 7b. This is of key importance. It shows that there exists a decomposition of the data where one of the key components, the yellow colored shock, has negative association between immigration and native wages. The historical decompositions described in the section 5.1.2 show how important this shock is in describing the underlying data and in section 5.1.3 we quantify the importance of this shock to native wage growth over the sample period. As we will show in some sectors, particularly unskilled manual sectors, its influence has been considerable.

The top right 3×3 submatrices of Figures 5 and 6 show the responses of the sectoral variables to the three aggregate shocks. These submatrices show the very different reactions across sectors to aggregate shocks with some impulses responding positively and some negatively to the same aggregate shock. Nevertheless there are also similarities across sectors. For example in the response to the second row in Figure 5 and the third row in Figure 6 almost all the sectors have positive responses of native wages, hours and migration with the respective aggregate variables also being initially positive. These shocks therefore have a natural interpretation as macroeconomic demand shocks as we discuss below. It is noticeable that the one sector with a negative native wage response to this shock is in the public sector, the unskilled public administration sector.

While the top right 3×3 submatrices Figures 5 and 6 show the importance of macroeconomic shocks to the sectoral labor markets, the bottom right 3×3 submatrices show that in both identification approaches there remains a lot of variation in sectoral variables after the macroeconomic shocks have been accounted for. Taking the Cholesky case, the shocks on the diagonal of Figure 5 all have strong positive impulses, a consequence of the identification assumptions where each previously ordered variable's response is restricted to be zero on impact. These clear diagonal impulses give a natural labeling of the shocks as, respectively, sectoral migration, hours and native wage shocks. Again the responses of the hours shock could also be interpreted as a sectoral demand shock. The off diagonal responses are more varied and show the significance of sectoral heterogeneity. This shows that an aggregate shock may have very different effects across sectors, so that for example, the effects of an aggregate migration shock may have larger native wage effects in some sectors than in others. We quantify these differences using our counterfactual analysis summarized in Table 2 below.

Figure 7: Impulse responses of aggregate variables to aggregate shocks

(a) Cholesky approach



(b) Sign restriction approach

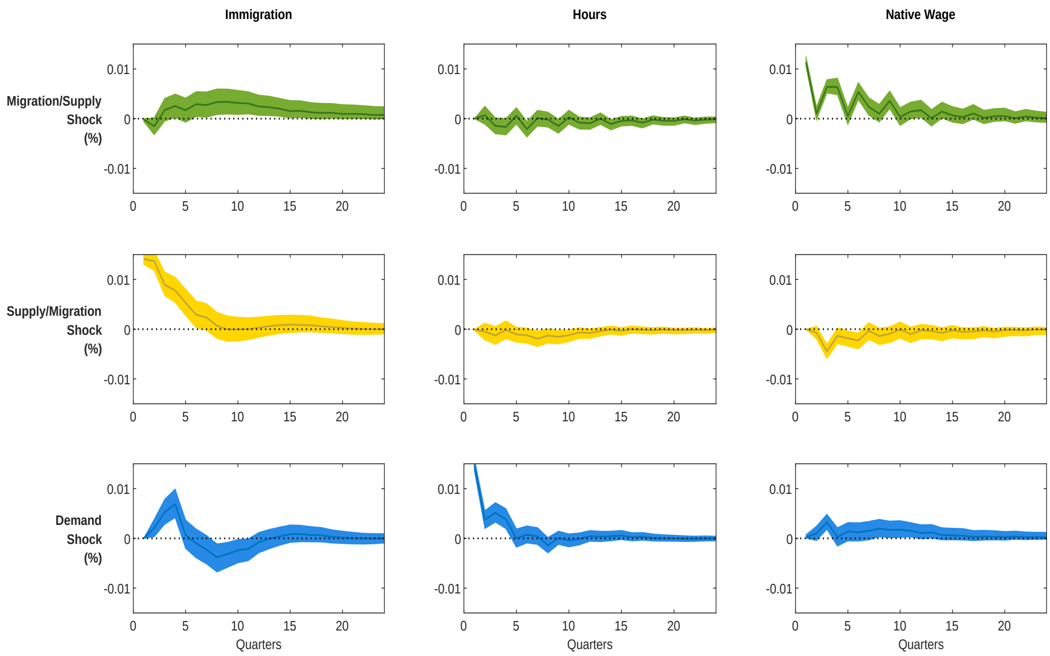


Figure 7 plots the impulse responses of the aggregate variables to aggregate shocks for the unskilled manual sectors using the Cholesky factorization approach in panel a) and the sign restriction approach in panel b). The impulses of similar shocks are colored the same in each panel so that first shock in panel a) and the second shock in panel b) are colored yellow, the of the second shock in panel a) and the third shock in panel b) are colored blue and the remaining shocks are colored green. The shaded region is the area between the 16% and 84% quantiles of the posterior distribution with the median colored in a darker shade. The response are taken from the unskilled transport services sector but as Figure 5 shows the median responses of the aggregate variables for all sectors are very similar.

5.1.2 Historical decomposition of aggregate variables

Historical decompositions show in each time period the contribution of each of the six identified shocks, as well as the initial conditions and constant term, to each data series. Given the estimated parameters, the time series of the data used in the VAR can be decomposed into the contributions from each fundamental shock by iterating on the estimated VAR.[17] These contributions depend on raised powers of the matrix of β parameters and will therefore differ across sectors. The contribution of each shock is highly related to its impulse response function, as described in e.g. Hamilton (1994). Therefore the similarity in the shape of the impulse response functions across the different identification methods will also be present in the historical decompositions. We discuss the historical decomposition of aggregate native wage growth in Figure 8 and of aggregate migration and hours worked growth in Figure 9

The historical decomposition for the aggregate native wage series for both identification methods is shown as a stacked bar chart, using the median responses.[18] In each panel of Figure 8 we have colored in blue the contribution of the shock which most resembles an aggregate demand/business cycle shock. We have used yellow to color the contribution of the shock where there is a negative association between migration and native wages. The growth rate of native wages is mostly determined by the green colored 'supply' shock and the constant term, although the blue colored ' demand' shock plays an important role in the period following the financial crisis of 2008. The Cholesky identification scheme in Figure 8a tells a similar story as would be expected given the similarity of the aggregate responses described above. The yellow colored 'migration' shocks, play a significant part in native wage growth dynamics in both panels, being notably negative around the time of the Brexit referendum (June 2016) and then having a positive, or less negative, contribution afterwards which is intuitive.

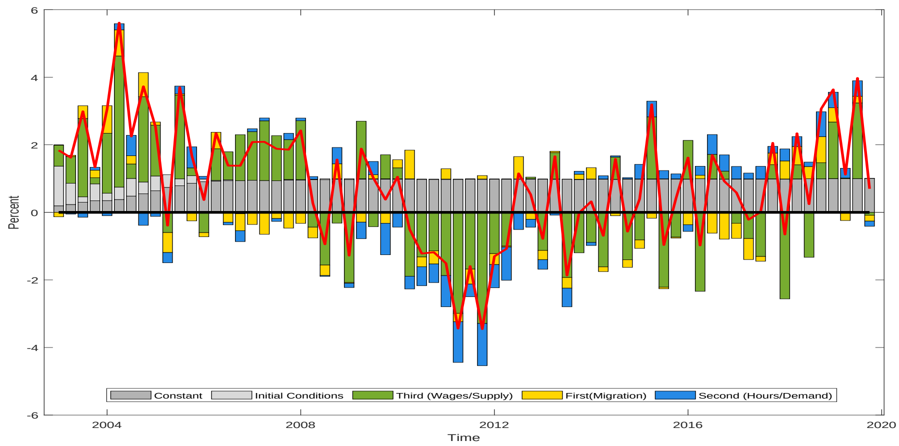
It is very important to note that a positive contribution of a yellow shock means that effect on native wages is less negative than expected. It does not necessarily mean that the total contribution of this shock to native wages is positive. The total contribution of each shock also depends on its contribution to the constant term whose total contribution is shaded grey in Figure 8. Each fundamental shock contribution is related to its constant term in the structural VAR, equation (2), as described e.g in Hamilton (1994). As we show in the next section in our counterfactual analysis, the total contribution of the 'migration shocks', which we have colored yellow, has a substantial negative contribution to long run growth rate of native wages in many sectors.

[17] This is briefly described in the Appendix A.1, where the historical decomposition formula is given by equation (A3).

[18] The sum of the median contributions do not necessarily sum to the original but as Figure 8 shows the match is quite close.

Figure 8: Historical decomposition of aggregate native wage growth using two identification methods

(a) Aggregate native wages - structural cholesky approach



(b) Aggregate native wages - sign restrictions approach

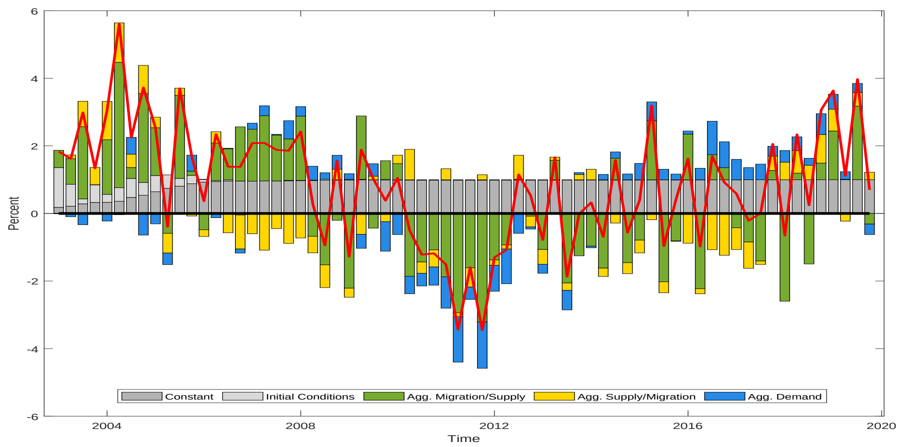
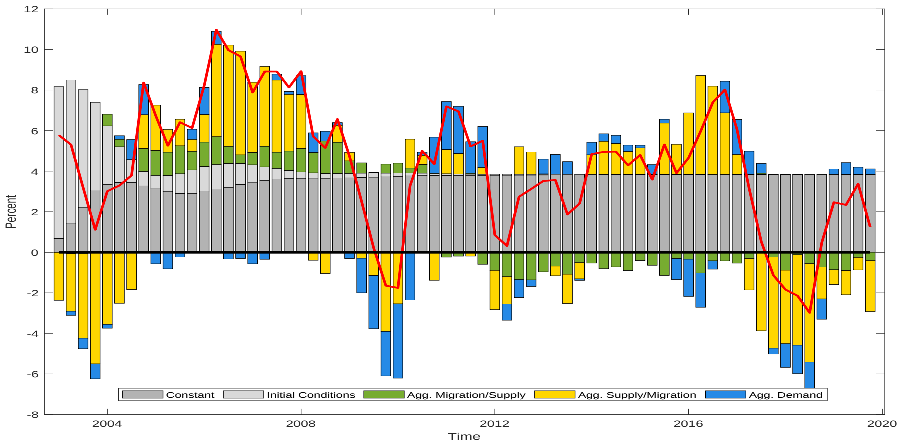


Figure 8 plots the median estimate of the historical contribution to aggregate native wage growth of the aggregate identified shocks, the initial conditions and the constant term, using the two identification techniques described in section 4. Panel a) plots the Cholesky case, Panel b) plots the structural sign restrictions case. This decomposition is taken from the unskilled transport services sector. In each panel the red line plots data for aggregate native wage growth. The colors of the bars relate to the colors of the impulses in Figure 7.

Figure 9: Historical decomposition of growth of aggregate migration and aggregate hours using the sign restriction methods

(a) Aggregate migration



(b) Aggregate hours

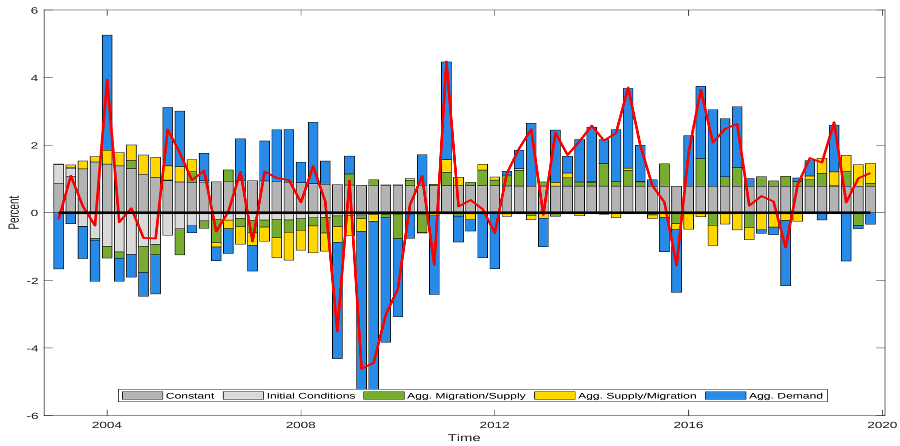


Figure 9 plots the median estimates of the historical decomposition of aggregate migration, panel a) and aggregate hours, panel b) using the sign restrictions methodology. The yellow bars are the supply/migration shocks and the blue bars are demand/business cycle shocks. This decomposition is taken from the unskilled transport services sector. In each panel the red line plots observed series. The colors of the bars relate to the colors of the impulses in Figure 7.

Figure 9 shows the historical decompositions for aggregate migration and aggregate hours. They show that the dominant shock driving variations in the migration share is the yellow colored migration shock and that for aggregate hours is the blue demand shock. These results are consistent with the labels we have applied to these shocks above. One may look at these Figures and think that the effects of the migration and demand/business cycle shocks even out over the sample period as the positive bars seem of a similar size to the negative bars. However this ignores the effect of each shock on the long run growth rate of each variable, which is related to the constant term. In our counterfactual analysis in section 5.1.3 we show that both of these shocks have significant long run effects in the direction indicated by their impulse response functions. The yellow colored 'migration' shock having a negative effect on native wage growth and a positive effect on migration and the demand/business cycle shock having a positive effect on both native wage growth and migration share.

5.1.3 How large are the effects? - a counterfactual analysis

In this section we quantify the effects of the fundamental shocks on the changes to migration share and native wages over the sample. We do this by setting the contribution of each shock, one at a time, to zero and then recalculating each time series as per a historical decomposition as described in section 4.2. The difference between this counterfactual time series and the observed time series highlights the contribution of the left-out shock to the observed time series. We do this for the Cholesky identification approach as drawing directly from the posterior has smaller simulation error.[19] As stressed in section 4.2, this 'counterfactual' is done without reference to a deep structural model and so one should not use the analysis to make statements such as 'If immigration was $x\%$ lower then native wages would be $y\%$ higher'. However one can make statements, such as 'At the model's median estimate, the contribution of the supply/immigration shock to wage growth over the sample period in sector A was $x\%$ out of a total wage growth of $y\%$'. Indeed we find that for several sectors, particularly unskilled manual sector, the supply/immigration (yellow) shock accounts for a negative 15% growth in native wages. i.e. in the counterfactual series, native wages in some sectors are 15% higher at the end of the sample period than in the observed data. We present the results of this exercise in two ways, graphically, in Figure 10, and numerically as in Table 2.

[19] The median percentage counterfactuals do not vary much across runs. This is shown in Figure B1 and Table B1 in the appendix.

Figure 10: Counterfactual for aggregate native wage growth with no migration shock

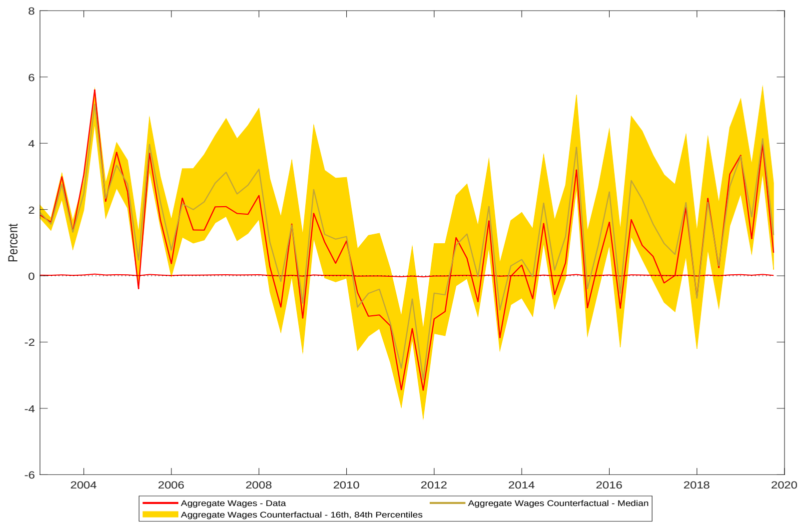


Figure 10 plots the counterfactual time series for native wage growth where the immigration shock using cholesky approach, is set to zero. The red line plots the data series for aggregate wage growth and the shaded region being the area between the 16% and 84% quantiles of the counterfactual posterior distribution with the median plotted in a darker shade. The distribution is mostly above the data over the sample period, illustrating that the absence of the migration/supply shock implies a higher rate of native wage growth in the counterfactual.

Figure 10 plots the counterfactual time series for aggregate native wages with the migration shock set to zero. The median and the 16% and 84% quantile bands of the counterfactual are colored yellow and the actual time series is colored red. As the yellow band is generally above the actual series, this shows that aggregate native wage growth is positively impacted by the absence of the migration shock particularly in time periods before the financial crisis and in the period before Brexit. This is consistent with the historical decomposition displayed in Figure 8 where these periods were those with the strongest negative effects of the migration shock.

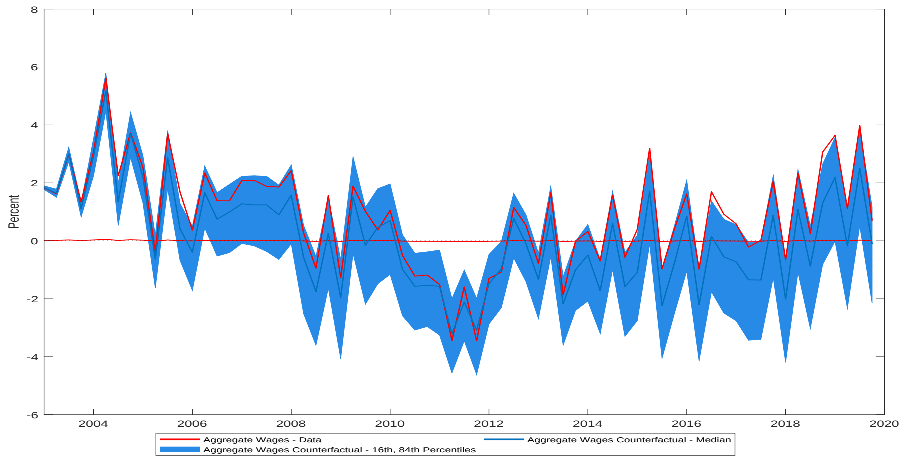
The total contribution of the migration shock also depends on its contribution to the constant term which, as we have shown, is an important determinant of the long run growth of native wages. Figure 10 shows that median counterfactual native wage growth is below the data series in almost every period. These effects compound so that their effect on native wage levels at the end of the sample period is substantial as we detail in Table 2.[20]

Figure 11 plots the equivalent counterfactual time series with a) the demand/business cycle shocks and b) the supply shocks set to zero. The demand/ business cycle shock has a persistently positive effect on native wage growth while the supply shock has initially a positive effect on native wages growth but then a negative effect during the recession following the 2008 financial crisis and a roughly neutral effect thereafter. The supply shock has significant effects which cancel out over time, as is also shown in Figure 8.

[20] The Figure shows a positive effects of the absence of the supply/migration shock.

Figure 11: Counterfactuals for aggregate native wage growth

(a) Counterfactual with no demand/business cycle shock



(b) Counterfactual with no supply shock

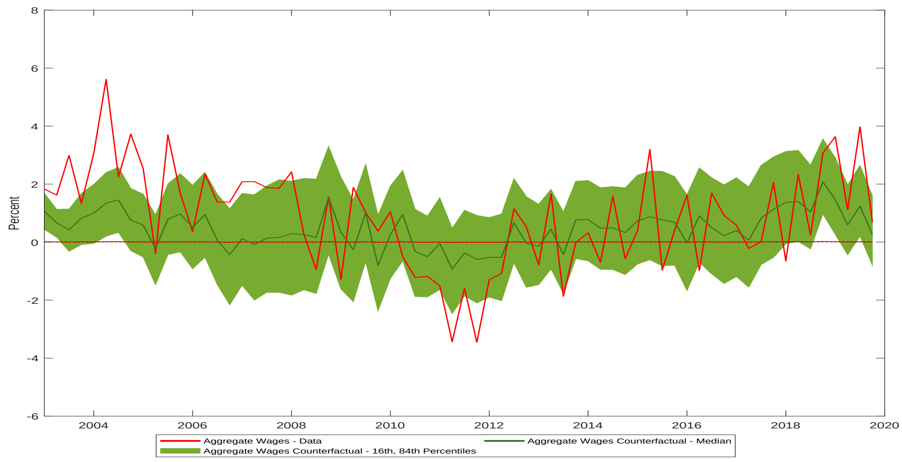


Figure 11 plots the counterfactual time series for native wage growth where in panel a) demand/business cycle shocks and panel b) supply (green) shocks, are set to zero. The red line plots the data series for aggregate native wage growth and the shaded region being the area between the 16% and 84% quantiles of the counterfactual posterior distribution with the median plotted in a darker shade. The distribution for the absence of the demand/business cycle shock is mostly below the data over the sample period, illustrating that the absence of the demand/business cycle shock implies a lower rate of native wage growth in the counterfactual. The colors of the plots relate to the colors of the impulses in Figure 7.

Tables 2 and 3 display the growth of native wages and migration respectively over the sample period for all 35 sectors along with the median of the counterfactual series omitting one of the three aggregate shocks. These tables allow us to gauge the size of the effects described in Figures 10 and 11 and also show the heterogeneity of effects across sectors.

The first column of Table 2 displays the growth of median real native wages in the data. This shows over the sample period 2003-2019 real native wage growth was very small in many sectors. In many professional sectors native real wage growth was negative with education and media professional sectors showing the largest declines. In the unskilled manual sectors by contrast, most sectors experienced positive native wage growth of between 10% and 20% over the sample period. [21]

The second column of Table 2 displays the counterfactual growth of native real wages where the immigration shock is set to zero. This column highlights in bold those sectors where the difference in the native wage growth between the counterfactual and the observed time series was over 15%. The unskilled manual group has the most sectors with a 15% or larger difference. The largest differences are in food and hospitality, retail and transport, but also in sectors with large public employment, education and health. In almost all unskilled manual sectors, the counterfactual native wage was higher than the observed native wage, with the sole exception being the public administration sector. For the professional grouping, native wage growth in some sectors is worse in the counterfactual. These are the education, other services, production and scientific sectors. This is consistent with immigrant labor in these sectors being complementary to domestic labor. Interestingly, education and scientific other non-manual sectors also show this pattern. Five professional sectors show large positive differences. These are the construction, media, public administration, retail and transport professional sectors. This is consistent with immigrant labor in these sectors being a substitute for domestic labor.

The third column of Table 2 displays the counterfactual growth of native real wages with the demand/business cycle shock set to zero. The patterns in this column generally go in the opposite direction to those in column two. This illustrates the countervailing forces acting on native wages. Again there is significant heterogeneity across sectors. Noticeably native wages in the transport sectors at all skill levels appear to be particularly sensitive to demand/business cycle shocks. The counterfactuals for the absence of supply shocks are listed in the fourth column. These are weaker effects than the other two which is consistent with Figure 11b since, as we have seen, the effects of the supply shock cancel out over time.

Table 3 repeats the exercise in Table 2 but for the migration share series. Column one confirms the findings of Figure 2 that the migration share grew across all sectors over the sample period, although with substantial heterogeneity across sectors. When the migration shock is set to zero, aggregate immigration, the final row of Table 3, is much muted but still positive due to the influence of the demand and supply shocks. The scale of the reduction in migration share varies a lot across sectors. Indeed in the public administration sector, the migrant share grows when the aggregate migration shock is set to zero. While the results may be considered an outlier, it should also be noted that this is a sector which experienced a large decline in native employment over the sample period. The absence of demand and supply shocks reduces growth in the migrant share, columns three and four. As before these aggregate shocks manifest themselves differently across sectors.

[21] This is broadly consistent with other data such as the ONS monthly wages and salaries survey, available at https://www.ons.gov.uk/employmentandlabourmarket/peopleinwork/employmentandemployeetypes /bulletins/averageweeklyearningsingreatbritain/january2025

Table 2

Cumulative native wage growth 2003-2019 by sector in the data and under counterfactuals setting aggregate shocks to zero

Sector	**Native wage growth in data**	**Counterfactual 1st Cholesky**	**Counterfactual 2nd Cholesky**	**Counterfactual 3rd Cholesky**
Professional sectors				
Production	0.99	0.94	0.94	0.95
Construction	1.05	**1.24**	0.92	0.96
Retail	0.98	**1.23**	0.93	0.97
Media&IT	0.93	1.06	0.81	0.93
Finance	0.98	0.93	0.80	0.92
Scientific	0.99	0.86	0.92	0.92
Transport&Support Services	1.05	1.23	0.84	1.01
Public Admin	1.00	1.11	0.83	0.95
Education	0.91	0.86	0.91	0.85
Health	1.05	1.02	1.04	0.98
OtherServices	1.08	0.84	1.18	0.99
Other non-manual sectors				
Production	1.11	1.21	1.05	1.11
Retail	1.04	1.10	0.93	1.00
Media&IT	1.03	0.93	1.06	0.98
Finance	1.21	**1.49**	1.18	1.19
Scientific	1.00	0.74	1.19	0.94
Transport&Support Services	1.13	**1.39**	0.85	1.14
Public Admin	1.08	1.15	1.12	1.03
Education	1.07	0.85	1.07	0.99
Health	1.06	1.03	0.92	0.99
OtherServices	1.12	**1.37**	0.87	1.07
Skilled manual sectors				
Production	1.10	1.20	0.90	1.05
Construction	1.11	1.22	0.94	1.03
Transport&Support Services	1.04	1.01	0.80	1.01
Public Admin	1.15	1.27	1.05	1.10
Unskilled manual sectors				
Production	1.13	1.20	0.94	1.08
Construction	1.13	1.08	0.86	1.05
Retail	1.22	**1.39**	0.91	1.18
Food&Hospitality	1.23	**1.39**	1.09	1.16
Finance	1.08	**1.38**	0.87	1.07
Transport&Support Services	1.14	1.24	0.98	1.06
Public Admin	1.19	1.15	1.38	1.11
Education	1.12	**1.27**	0.96	1.05
Health	1.13	1.26	0.93	1.08
OtherServices	1.18	1.32	0.89	1.14
Aggregate wage growth	1.155	1.24	1.02	1.08

Notes: The table reports the cumulative native wage growth 2003-2019 by sector in the data, and for the median in the counterfactual series from the cholesky approach, derived by setting one of the aggregate fundamental shocks to zero. The cumulative native wage growth is calculated as $\prod_{t=1}^{T}(1+\text{native wage growth}_t)^{0.25}$. Sectors with a noticeable difference in the native wage growth between the counterfactual and the observed time series are highlighted in bold.

Table 3

Cumulative migration growth 2003-2019 by sector in the data and under counterfactuals setting aggregate shocks to zero

Sector	Migration Share Growth in Data	Counterfactual 1st Cholesky	Counterfactual 2nd Cholesky	Counterfactual 3rd Cholesky
Professional Sectors				
Production	1.79	1.43	**0.81**	1.77
Construction	1.61	1.56	1.70	1.75
Retail	1.50	**1.04**	1.36	1.58
Media&IT	1.48	**0.69**	1.62	1.48
Finance	1.48	**0.87**	1.87	1.54
Scientific	1.60	**0.91**	1.60	1.54
Transport&Support Services	1.39	**0.98**	1.16	1.35
Public Admin	1.34	**2.81**	**0.73**	1.30
Education	1.45	1.53	1.79	1.51
Health	1.32	1.16	1.10	1.28
OtherServices	1.24	0.79	1.50	1.27
Other Non-Manual Sectors				
Production	1.81	**0.43**	**4.93**	1.69
Retail	1.70	1.90	1.94	1.80
Media&IT	1.27	0.96	1.29	1.23
Finance	1.79	**1.25**	1.93	1.65
Scientific	2.10	**0.96**	1.92	1.87
Transport&Support Services	1.39	**1.17**	1.30	1.17
Public Admin	1.03	**0.35**	1.13	0.92
Education	1.13	0.88	1.19	1.08
Health	1.47	**1.91**	1.45	1.44
OtherServices	1.41	1.35	1.30	1.46
Skilled Manual Sectors				
Production	2.27	2.19	**1.72**	1.97
Construction	2.14	**0.60**	2.02	1.77
Transport&Support Services	1.73	**1.16**	2.06	1.63
Public Admin	1.13	**0.38**	1.18	0.94
Unskilled Manual Sectors				
Production	2.83	0.86	3.08	2.29
Construction	2.28	0.51	2.69	2.47
Retail	2.45	1.40	1.85	2.29
Food&Hospitality	1.72	1.42	1.13	1.60
Finance	2.13	2.51	1.70	1.88
Transport&Support Services	2.72	1.57	2.40	2.31
Public Admin	1.63	**4.07**	0.81	1.68
Education	1.94	1.44	1.56	1.92
Health	2.45	1.31	2.43	2.08
OtherServices	1.53	0.97	0.99	1.51
Aggregate Migration Share Growth	2.03	1.24	1.98	1.92

Notes: The table reports the Cumulative migration growth 2003-2019 by sector in the data, and for the median in the counterfactual series from the cholesky approach, derived by setting one of the aggregate fundamental shocks to sero. The cumulative migration share growth is calculated as $\prod_{t=1}^{T}(1 + \text{migration sharegrowth}_t)^{0.25}$. Sectors with a noticeable difference in the migration share growth between the counterfactual and the observed time series are highlighted in bold.

5.2 Results for sectoral variables

The median impulse responses of the sectroal variables are displayed above in the last three columns of Figures 5 and 6, where the lower right 3×3 submatrix plots the responses of sectoral variables to sectoral shocks. In the Cholesky case Figure 5 shows that the strongest responses are on the diagonals, which naturally leads one to think of the fourth ordered shock as a sectoral migration shock and the fifth and sixth ordered shocks as sectoral hours and sectoral wage shocks respectively. In the sign restrictions case Figure 6 shows that the responses to the sectoral shocks are much more varied than the responses to the aggregate shocks, in particular, the responses of hours and native wages are negative in some sectors and positive in others.

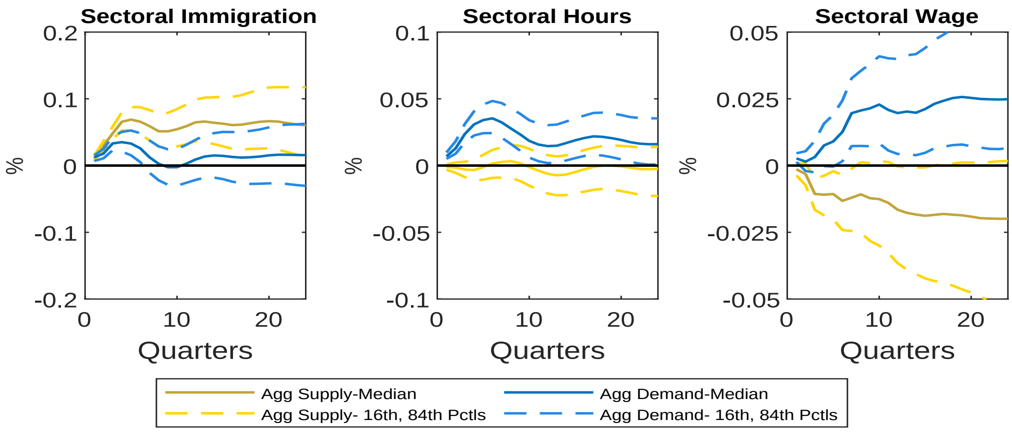
There are two broad features in these figures which immediately stand out. Firstly that, as one would expect, the responses of sectoral variables tend to have a higher variance than the aggregate variables. The second is that the aggregate shocks explain a lot of the variation in the sectoral labor market variables. This is not surprising, it is what one would expect macroeconomic shocks to do.

Sectoral shocks nevertheless still account for a lot of variation in sectoral variables. The heterogeneity across sectors is also apparent and is also in line with expectations. Some sectors appear pro-cyclical, others counter-cyclical. Similarly some sectors experience an increase in sectoral migration share in response to an aggregate migration shock and others a decrease. However, while intuitive, this makes the interpretation of the sectoral decompositions less straightforward because one cannot assume, for example, that an aggregate supply shock should be colored yellow in the historical decompositions figures as in section 5.1, because in some sectors aggregate supply shocks may not have a negative association between sectoral migration and sectoral native wages.

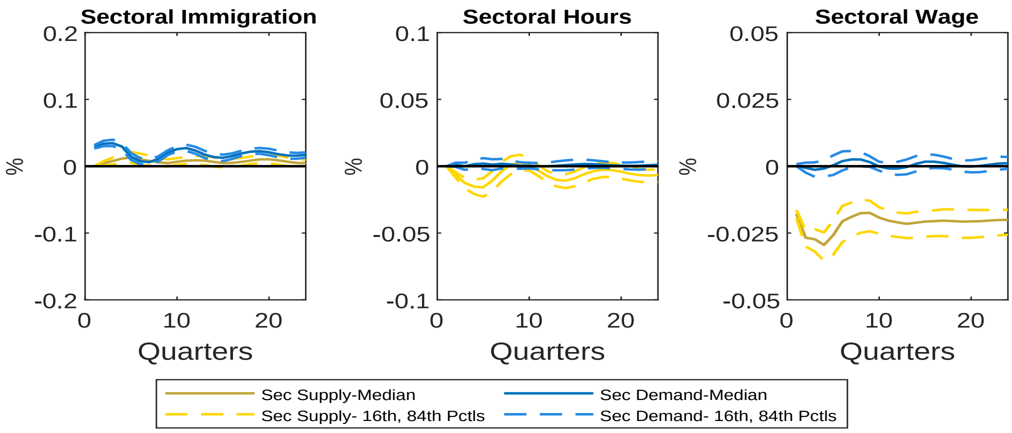
Our approach therefore is to look at the impulse responses of two example sectors, which have experienced large inflows of migrants over the sample period; unskilled food and hospitality and unskilled transport services. These are sectors where one might expect to find a large response of native wage growth to sectoral immigration. We examine the impulses in these sectors which will then allow us to interpret the contribution of these shocks in the historical decomposition of native wage growth in section 5.2.1.

Figure 12: Sectoral impulse responses in the unskilled transport sector

(a) Aggregate shocks: sign restrictions



(b) Sectoral shocks: sign restrictions



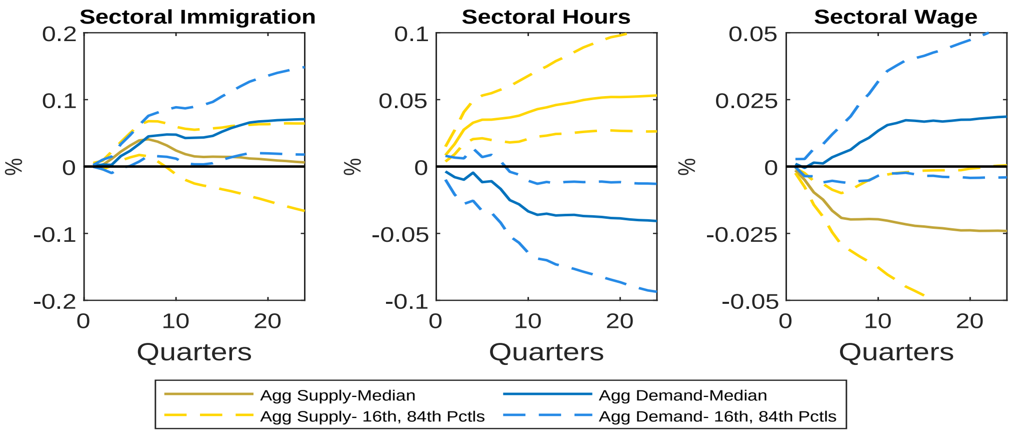
The unskilled transport support services and food and hospitality sectors

Figure 12 plots median cumulative impulse responses, i.e. the levels, - solid lines - as well as the 16th and 84th percentiles - dashed lines- of the sectoral variables to two aggregate and sectoral shocks for the unskilled transport support services sector. We choose to plot the cumulative responses as the non-cumulative responses sometimes oscillate which makes the longer run effects difficult to make out. In contrast the cumulative responses are clear. The responses to aggregate shocks are displayed in panel a) and those to sectoral shocks in panel b). In both panels the shocks are identified using the sign restrictions approach, with the identified shocks corresponding to supply/migration shocks colored yellow and those corresponding to aggregate demand/business cycle shocks colored blue. In both cases the supply/migration shocks have a negative association between sectoral migration share and native wages and the demand/business cycle shocks have a non-negative association. Therefore when looking at the historical decomposition of native wage growth in the unskilled transport support services sector in section 5.2.1 below, it is appropriate to color the sectoral supply shocks yellow and interpret these shocks as having a negative relationship between migration and native wage growth.

Figure 13 has the same structure as Figure 12 but plots the responses for the unskilled Food and Hospitality sector. The responses in the two figures are similar only now the sectoral supply/migration shocks have a very small response. Thus in this sector only the aggregate supply/migration shock should interpreted in the historical decomposition in section 5.2.1 as having a negative relationship between migration and native wage growth. As we will see below, sectoral supply shocks only play a very small role in the historical decompositions plotted in Figure 14b below which is consistent with these impulse response functions.

Figure 13: Sectoral impulse responses in the unskilled food and hospitality sector

(a) Aggregate shocks: sign restrictions



(b) Sectoral shocks: sign restrictions

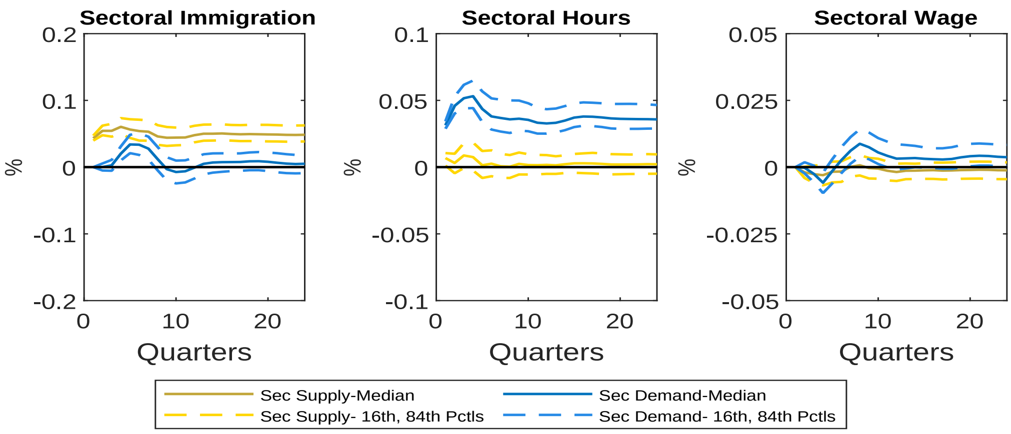
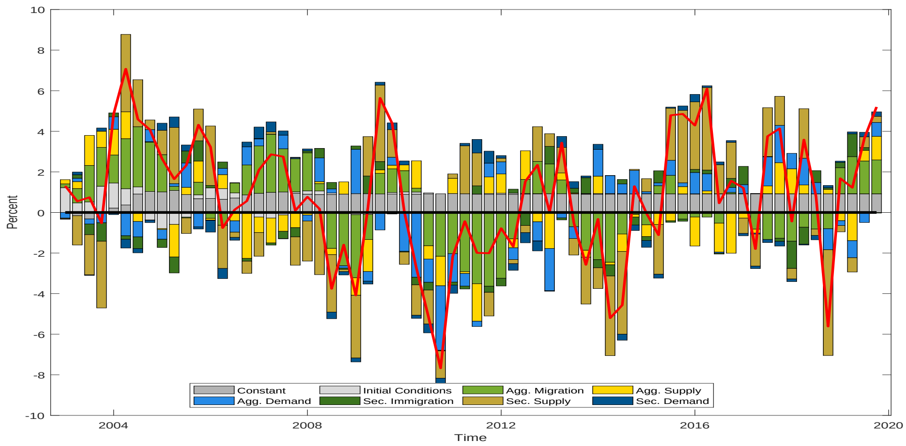


Figure 14: Historical decomposition of native wages in unskilled transport and food and hospitality sectors

(a) Unskilled transport services sector



(b) Unskilled food and hospitality sector

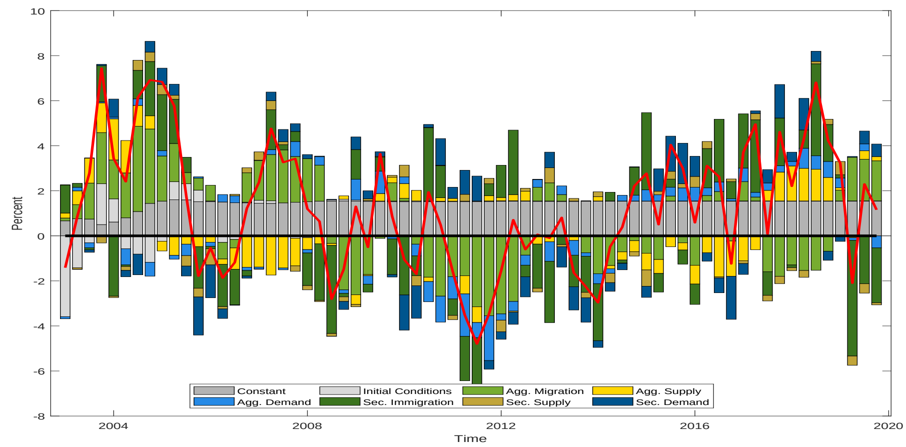


Figure 14 plots the median estimate of the historical contribution of the six identified shocks and the constant term and initial conditions and to native wage growth in the unskilled manual transport and food and hospitality sectors. the shocks are identified using the structural time series case In each panel the red line plots data for sectoral native wage growth. The colors of the bars relate to the colors of the impulses in Figure 7 with darker shades for the sectoral shocks.

5.2.1 Sectoral historical decompositions

The historical decompositions at the sectoral level have eight different contributing factors, as now the three sectoral shocks play a significant role in the dynamics. We have used the same color for the sectoral shocks as their corresponding aggregate shocks but have chosen a darker shade of these colors for the sectoral shocks in order to differentiate them from the aggregate shocks.

Figure 14a for the decomposition of native wage growth in the unskilled transport sector shows a large role for sectoral supply shocks particularly in the years preceding the 2008 and

Brexit as we have seen in section 5.2 these shocks have a negative association between sectoral migration and native wages. Intuitively also aggregate demand/business cycle shocks played a large role in the decline of native wages in following the 2008 financial crisis and native wage growth remaining low on average until about 2016.

Figure 14b for the decomposition of native wage growth in the unskilled food and hospitality sector, shows that aggreghate shocks play the largest role in explaining the variation although the sectoral demand and supply - dark blue and dark green colored -also play a significant role. Consistent with the impulse responses for this sector, sectoral supply/migration shocks - colored dark yellow - play very little role. The aggregate supply/migration shocks - colored yellow - is still important in the years before the 2008 recession and Brexit, as it was in the aggregate native wage discussed in section 5.1.2.

Taken together these figures show how the contribution of aggregate and sectoral demand, supply and migration shocks to native wage growth differs across sectors. It is worth re-itertating that a positive contribution of the yellow shock means that this shock is lower than expected and so its effects on native wages are less negative than expected. It does not mean necessarily that its contribution to native wages is positive. The total contribution will also depend on the shock's contribution to the constant term in the structural VAR, equation (2), as discussed in section 4.2.

We have focused here on the historical decompositions of wage growth in the unskilled transport , and food and hospitality sectors as the impulses for these sectors are quite clear. In other sectors the effects are less strong but as we have shown in Table 2 there are many sectors where the cumulative contribution of the aggregate supply/migration shock is substantially negative and so these results help illustrate these cases.[22]

[22] Decompositions based on the other methods are available on request.

6 Conclusion

We asked at the outset whether immigration could be thought of as an exogenous shock, whether immigration can be plausibly associated with adverse labor market effects, and if so, whether these effects are similar across different sectors of the economy. We have applied established methods of multiple time series analysis to decompose a time series of UK labor market variables into 'fundamental' constituent parts. As argued by Uhlig (2005) and Baumeister and Hamilton (2015), economic theory does not provide sufficient information to definitively identify these fundamental parts. We have therefore applied two different, though plausible, approaches for characterizing them, across 35 different sectors of the UK labor market. We have found in both identification schemes that aggregate immigration is, in part, determined by shocks that could be considered aggregate supply and aggregate demand shocks. Thus what have previously been considered the effects of exogenous shocks to immigration may in fact be the result of multiple underlying causes that sometimes work in opposing directions.

In answer to the question of whether there are adverse labor market effects of immigration, in each identification approach we have found that there are shocks where immigration and native wages are positively associated, and shocks where immigration and native wages are negatively associated. A natural interpretation for the positive association at the aggregate level is a macroeconomic demand or business cycle shock. One interpretation of the negative association is that migration is causing, directly or indirectly, a reduction in native wages. We have shown that this shock accounts for most of the variation in migration and plays a significant role in the determination of native wage growth but that the size of its effect can vary considerably across sectors.

The literature on the labor market effects of immigration has frequently noted that its results are subject to the proviso that they are abstracting away from the effects of demand shocks and sectoral heterogeneities. Our approach has shown that this proviso is indeed justified and that the literature may not be identifying significant adverse effects of immigration on native wages, particularly in certain sectors, because over the same period demand shocks have been working in the opposite direction. Our conclusion therefore echoes that of Uhlig (2005) regarding monetary policy. It is that there are good reasons for being uncertain about the labor market effects of immigration.

References

[1] Baumeister, Christiane and James Hamilton, 2015, Sign Restrictions, Structural Vector Autoregressions, and Useful Prior Information, Econometrica, 83(5), September 2015, 1963-1999

[2] Baumeister, Christiane and James Hamilton, 2019, Structural Interpretation of Vector Autoregressions with Incomplete Identification: Revisiting the Role of Oil Supply and Demand Shocks, American Economic Review, 109(5), May 2019, 1873-1910.

[3] Baumeister, Christiane, and James D. Hamilton, 2024, Advances in Using Vector Autoregressions to Estimate Structural Magnitudes, Econometric Theory, 40(3), 472-510.

[4] Bergholt, Drago, Fabio Canova, Francesco Furlanetto, Nicolò Maffei-Faccioli and Pål Ulvedal ,(2024) "What Drives the Recent Surge in Inflation? : The Historical Decomposition Rollercoaster", *Norges Bank Working Paper*, 07/2024.

[5] Blanchard, Olivier and Lawrence Katz, (1992), "Regional Evolutions", Brookings Papers on Economic Activity (1), 1-75

[6] Borjas, George, (1994). The Economics of Immigration. *Journal of Economic Literature*, 32(4), 1667–1717.

[7] Blake, Andrew, and Haroon Mumtaz, 2017, Applied Bayesian econometrics for central bankers, Centre for Central Banking Studies, Bank of England

[8] Caliendo, Lorenzo, Luca David Opromolla, Fernando Parro, and Alessandro Sforza, (2021), "Goods and Factor Market Integration: A Quantitative Assessment of the EU Enlargement", *Journal of Political Economy*, 129(12): 3491-3545

[9] Campo, Francesco, Giuseppe Forte, Jonathan Portes (2018) The Impact of Migration on Productivity and Native-born Workers' Training, Discussion Paper IZA No. 11833

[10] Canova,Fabio and Gianni De Nicolò, (2002), Monetary disturbances matter for business fluctuations in the G-7, *Journal of Monetary Economics*, 49(6), 1131-1159.

[11] Canova, Fabio, (2005), The Transmission of US shocks to Latin America, Journal of Applied Econometrics, Vol(20), 229– 251

[12] Canova, Fabio (2007), 'Methods for Applied Macroeconomic Research', Princeton University Press.

[13] Docquier, F., L. Guichard, S. Iandolo, H. Rapoport, R. Turati and G Vannoorenberghe, (2023), Populism and the Skill-Content of Globalization: Evidence from the Last 60 Years. mimeo LISER, Luxembourg Institute of Socio-Economic Research.

[14] Dustmann, Christian, Tommaso Frattini and Ian Preston , (2013). "The Effect of Immigration along the Distribution of Wages", Review of Economics Studies, 80, pp 145-173.

[15] Dustmann, Christian, Uta Schönberg, and Jan Stuhler (2016) The Impact of Immigration: Why Do Studies Reach Such Different Results?, Journal of Economic Perspectives—Vol 30 (4): 31–56

[16] Foerster, Andrew T. Pierre-Daniel G. Sarte, and Mark W. Watson, (2011), "Sectoral versus Aggregate Shocks: A Structural Factor Analysis of Industrial Production", *Journal of Political Economy*, 2011 119(1), 1-38

[17] Furlanetto, Francesco and Orjan Robstad, (2019) "Immigration and the macroeconomy: some new empirical evidence", *Review of Economic Dynamics* , Elsevier for the Society for Economic Dynamics, vol. 34, 1-19.

[18] Hamilton, James, (1994), Time Series Analysis, Princeton University Press.

[19] Kiguchi, T., Mountford, A., (2017), "Immigration and unemployment: a macroeconomic approach", *Macroeconomic Dynamics*, 1–27

[20] Liu,Philip, Haroon Mumtaz, Angeliki Theophilopoulou, (2014) The transmission of international shocks to the UK. Estimates based on a time-varying factor augmented VAR, Journal of International Money and Finance, Volume 46.

[21] Mumtaz, Haroon and Paolo Surico, (2009),The Transmission of International Shocks: A Factor-Augmented VAR Approach, Journal of Money, Credit and Banking, vol. 41(s1), pages 71-100, February.

[22] Manacorda, Marco, Manning, Alan and Wadsworth, Jonathan, (2012). The Impact of Immigration on the Structure of Wages: Theory and Evidence from Britain. *Journal of the European Economic Association*, 10 (1), 120-151.

[23] Mountford, Andrew and Jonathan Wadsworth, (2023) "Good Jobs, Training and Skilled Immigration", *Economica* 90 (359), 851-881

[24] Mountford, Andrew and Harald Uhlig, (2009) "What are the effects of fiscal policy shocks?", *Journal of Applied Econometrics* vol 24, pages 960-992.

[25] Ottoviano, Gianmarco and Giovanni Peri, (2012) "Rethinking the Effect of Immigration on Wages" *Journal of the European Economic Association*, 10 (1), 152-197.

[26] Peri, Giovannii, Rury Derek and Justin C. Wiltshire, (2022) , "The Economic Impact of Migrants from Hurricane Maria", *The Journal of Human Resources*, forthcoming

[27] Rodrik, Dani, (2021), Why Does Globalization Fuel Populism? Economics, Culture, and the Rise of Right-Wing Populism Annual Review of Economics, Vol 13(1), 133-170

[28] Sims, C, 2003, Comments on Smets and Wouters, Available at http://sims.princeton.edu/yftp/Ottawa/SWcommentSlides.pdf.

[29] Uhlig, Harald (2005), What are the effects of monetary policy on output? Results from an agnostic identification procedure, Journal of Monetary Economics, 52 381-419.

[30] Uhlig, Harald (2017), Shocks, sign restrictions, and identification, in Advances in economics and econometrics, edited by Bo Honoré, Ariel Pakes, Monika Piazzesi, and Larry Samuelson, 2 95-127, Cambridge University Press

A Appendix

A.1 Formulas used in Calculation

The formulas we use to calculate the impulse response functions and historical decompositions follow those of the literature see e.g. Hamilton (1994), Uhlig (2005) and Baumeister and Hamilton (2018).

The reduced form VAR of n dimensions and p lags can be written

$$\underset{(n\times1)}{y_t} = \underset{(n\times1)}{C} + \underset{(n\times n)}{B_1}\underset{(n\times1)}{y_{t-1}} + \underset{(n\times n)}{B_2}\underset{(n\times1)}{y_{t-2}} + \dots \underset{(n\times n)}{B_p}\underset{(n\times1)}{y_{t-p}} + \underset{(n\times1)}{u_t} \qquad u_t \sim \mathcal{N}(0, \underset{(n\times n)}{\Sigma}) \tag{A1}$$

This can be stacked and written as 1 lag VAR,

$$\underset{(np\times1)}{\widehat{Y_t}} = \underset{(np\times1)}{\widehat{C}} + \underset{(np\times np)}{F}\underset{(np\times1)}{\widehat{Y_{t-1}}} + \underset{(np\times1)}{u_t} \tag{A2}$$

where

$$\widehat{Y_t} = \begin{bmatrix} y_t \\ \vdots \\ y_{t-p+1} \end{bmatrix} \widehat{c} = \begin{bmatrix} C_{n\times1} \\ 0_{(n(p-1)\times1)} \end{bmatrix} \quad F = \begin{bmatrix} B_1 & \dots & B_p \\ I_{n(p-1)} & & 0_{n(p-1),n} \end{bmatrix} \widehat{u_t} = \begin{bmatrix} u_t \\ 0_{n(p-1),1} \end{bmatrix}$$

Iteration of equation (A2) forward implies that the observation $\widehat{Y_{t+s}}$ in period $t+s$ can be decomposed into three contributions, the initial conditions,the constant terms and the innovations in the previous s periods , i.e.

$$\widehat{Y_{t+s}} = \underbrace{F^s\widehat{Y_t}}_{\text{Initial Conditions}} + \underbrace{F^{s-1}\widehat{C} + F^{s-2}\widehat{C} + \dots + \widehat{C}}_{\text{constant terms}} + \underbrace{F^{s-1}\widehat{u_{t+1}} + F^{s-2}\widehat{u_{t+2}} + \dots + \widehat{u_{t+s}}}_{\text{Innovations}} \tag{A3}$$

The historical decomposition and counterfactual exercises are produced using equation (A3) where the contribution of the fundamental innovations use the formula $u_t = S^{-1}\epsilon_t$ following equation (3) in the text.

Iterating backwards into infinite history, $\widehat{Y_t}$ can be expressed as an $MA(\infty)$ process

$$\begin{aligned} \widehat{Y_t} &= \widehat{C} + F\widehat{C} + F^2\widehat{C} + \dots + \widehat{u_t} + F\widehat{u_{t-1}} + F^2\widehat{u_{t-2}} + \dots + \\ \widehat{Y_t} &= \widehat{\mu} + \Psi(L)\widehat{u_t} \end{aligned} \tag{A4}$$

Where $\mu = (I_n - A_1 - \dots A_p)^{-1}C$ and $\widehat{\mu} = [\mu' \ 0_{1\times n(p-1)}]'$ and where $\Psi(L)$ is an $MA(\infty)$ process. These can be written

$$\begin{aligned} y_t &= \mu + u_t + \Psi_1 u_{t-1} + \Psi_2 u_{t-2} + \Psi_3 u_{t-2} + \\ \widehat{Y_t} &= \mu + \Psi(L)u_t \end{aligned} \tag{A5}$$

where Ψ_j is the upper left $n \times n$ clock of matrix F^j, following Hamilton (1994).

A.2 Cholesky Factorization - Structural Interpretation

Cholesky factorization of the reduced form variance-covariance matrix, Σ, also has a structural interpretation. Since $u_t = L\epsilon_t$ premultiplying the reduced form VAR, equation (**??**), by the Cholesky factor L^{-1} gives

$$L^{-1}y_t = L^{-1}\tilde{C} + L^{-1}\tilde{B}_1 y_{t-1} + L^{-1}\tilde{B}_2 y_{t-2} + \ldots L^{-1}\tilde{B}_p y_{t-p} + \epsilon_t \tag{A6}$$

Since L is lower triangular then L^{-1} is also lower triangular and equation (A6) has the form

$$\begin{bmatrix} c_{11} & 0 & 0 & 0 & 0 & 0 \\ c_{21} & c_{22} & 0 & 0 & 0 & 0 \\ c_{31} & c_{32} & c_{33} & 0 & 0 & 0 \\ c_{41} & c_{42} & c_{43} & c_{44} & 0 & 0 \\ c_{51} & c_{52} & c_{53} & c_{54} & c_{55} & 0 \\ c_{61} & c_{62} & c_{63} & c_{64} & c_{65} & c_{66} \end{bmatrix} \begin{bmatrix} \text{Aggregate Migration Share} \\ \text{Aggregate Hours Worked} \\ \text{Aggregate Native Wage} \\ \text{Sectoral Migration Share} \\ \text{Sectoral Hours Worked} \\ \text{Sectoral Native Wage} \end{bmatrix} = L^{-1}C + \text{Lagged terms} + \begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_3 \\ \epsilon_4 \\ \epsilon_5 \\ \epsilon_6 \end{bmatrix} \tag{A7}$$

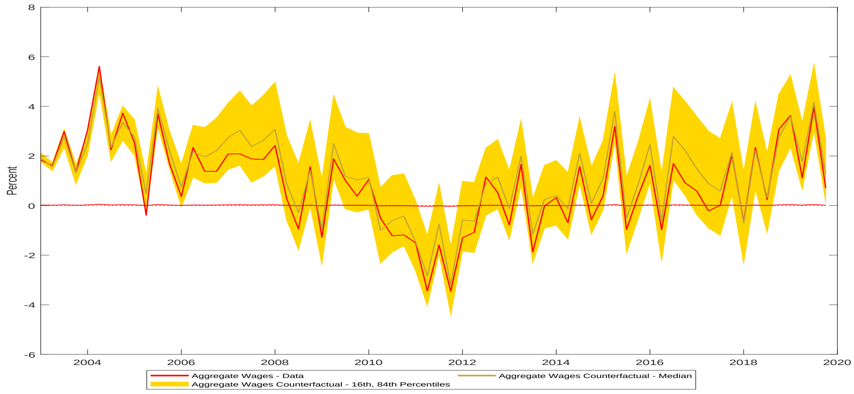
Equation (A7) implies that the variable ordered first in the VAR - aggregate migration share in this case - is only a function of lagged values of the other variables and $\epsilon_{1,t}$. Thus the order that the variables are placed in the VAR has a very significant impact on the interpretation and properties of each shock. In equation (A7), a natural interpretation of the first shock is as an exogenous aggregate migration shock. Similarly, the variable ordered second - Aggregate Hours Worked - is only a function of lagged values of the other variables and the contemporaneous value of aggregate migration (which is a function of $\epsilon_{1,t}$) and also of $\epsilon_{2,t}$. One interpretation of the second shock is therefore as a shock to aggregate hours net of the effects of the shock to aggregate migration. The same logic can be applied to the other shocks so that for the variable ordered last - the sectoral real wage in this case-, depends on all the current values of all variables and is therefore dependent on all contemporanous shocks $\epsilon_{1,t}$, $\epsilon_{2,t}$, $\epsilon_{3,t}$, $\epsilon_{4,t}$, $\epsilon_{5,t}$ and $\epsilon_{6,t}$. This has the same form as equation (1) above. However the property of the Cholesky factorization that only one shock's impulse responses, $\epsilon_{1,t}$, affects all the variables contemporaneously is often seen as a very strong restriction.

B Counterfactual Precision

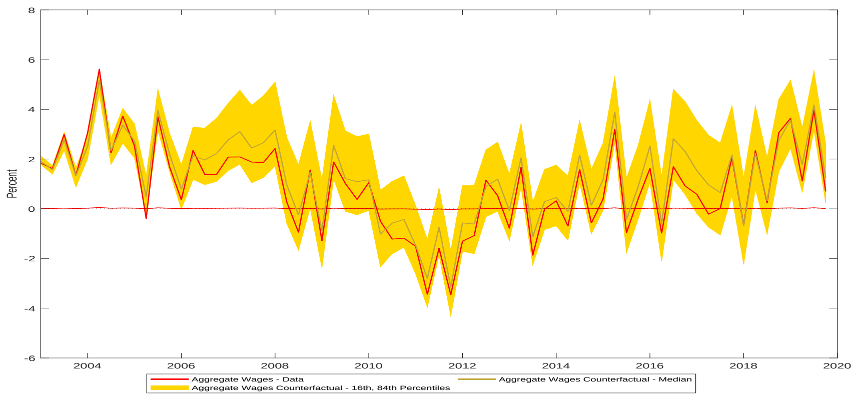
The counterfactual exercizes are cumulative and so simulation variation is compounded. Therefore to check whether the results described in section 5.1.3 are robust to simulation variation we made 3 independent runs of our counterfactual exercize for the unskilled manual sectors, each with 1,000,000 draws from the posterior distribution, a burn in of 250,000 and a thinning factor of 250 leaving a retained sample of 3000 draws. We present the aggregate wage counterfactual for the first cholesky shock in Figure B1. This shows that the counterfactual for aggregate wages is almost identical across the three runs. The counterfactual across the unskilled manual sectors is displayed in Table B1. This shows that the differences in the median counterfactual across runs is very small being at most 2% and with most sectors less than this.

Figure B1: Counterfactuals for aggregate native wage growth where migration shocks set to zero

(a) Counterfactual run 1



(b) Counterfactual run 2



(c) Counterfactual run 3

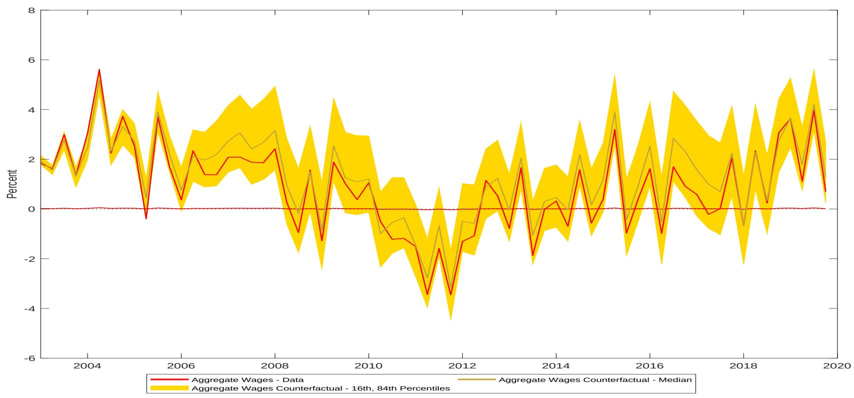


Figure B1 plots the counterfactual time series from three separate and independent runs for native wage growth where the first cholesky shock, the migration shocks, is set to zero. The Figures for the three graphs are almost identical. In each case, the data is taken using the unskilled transport services VAR and the red line plots the data series for aggregate native wage growth and the shaded region being the area between the 16% and 84% quantiles of the counterfactual posterior distribution with the median plotted in a darker shade.

Table B1

Variation across 3 runs of the counterfactual for unskilled manual sectors

Sector	**Wage Growth in Data**	**Counterfactual 1st Cholesky**	**Counterfactual 2nd Cholesky**	**Counterfactual 3rd Cholesky**
<u>**Unskilled Manual Sectors**</u>				
Production	1.13	1.21, 1.20, 1.21	0.94, 0.95, 0.94	1.07, 1.08, 1.08
Construction	1.13	1.08, 1.10, 1.09	0.86, 0.85, 0.85	1.07, 1.05, 1.05
Retail	1.22	1.37, 1.38, 1.37	0.92, 0.92, 0.92	1.18, 1.19, 1.18
Food&Hospitality	1.23	1.38, 1.39, 1.39	1.11, 1.09, 1.10	1.15, 1.15, 1.15
Finance	1.08	1.38, 1.37, 1.38	0.87, 0.87, 0.87	1.07, 1.07, 1.07
Transport&Support Services	1.14	1.22, 1.24, 1.24	0.98, 0.98, 0.96	1.07, 1.07, 1.08
Public Admin	1.19	1.15, 1.15, 1.14	1.37, 1.37, 1.39	1.11, 1.11, 1.11
Education	1.12	1.27, 1.26, 1.27	0.96, 0.96, 0.96	1.06, 1.06, 1.06
Health	1.13	1.27, 1.26, 1.25	0.92, 0.93, 0.93	1.08, 1.08, 1.09
OtherServices	1.18	1.33, 1.32, 1.32	0.89, 0.89 , 0.89	1.12, 1.13, 1.13
Aggregate Wage Growth	1.155	1.24, 1.24, 1.24	1.02, 1.02, 1.02	1.08, 1.08, 1.08

Notes: The table reports the cumulative wage growth 2003-2019 by sector in the data, and in the median of the counterfactual series using the cholesky approach, derived by setting one of the aggregate fundamental shocks to zero, from three separate and independent runs of the sampler. The cumulative native wage growth is calculated as $\prod_{t=1}^{T}(1 + \text{native wage growth}_t)^{0.25}$.