Incremental Innovation by Heterogeneous Incumbents and Economic Growth: relationship between two sources of growth

Ohki, Kazuyoshi

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Incremental Innovation by Heterogeneous Incumbents and Economic Growth:

relationship between two sources of growth *

Kazuyoshi Ohki†

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Abstract

In this paper, we construct a tractable endogenous growth model that incorporates both incremental innovation by heterogeneous incumbents and innovation by entrants. Our model features two endogenous sources of growth: quality improvement (vertical growth) and expansion in the variety of goods (horizontal growth). We then examine the policy effects of a subsidy for incremental innovation by incumbents and a subsidy for innovation by entrants on the overall economic growth rate, as well as on the relationship between the two sources of growth. Our model confirms that incumbents with higher profit flows tend to engage in incremental innovation for a longer duration and incur greater innovation costs, which is consistent with both Schumpeter's hypothesis and the findings of Christensen (1997) Additionally, the model generates counterintuitive results that are not commonly found in the conventional literature. First, a subsidy for incremental innovation by incumbents may reduce the entry of new firms. Second, a subsidy for innovation by entrants may have a negative effect on the overall economic growth rate.

keyword: Economic Growth, R&D, In-house model, Firm-Heterogeneity, Innovation by Incumbents, IPR policy, Incremental Innovation, Sustaining Innovation

JEL classification: O31, O32, O33, O34, O41

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†Faculty of Economics and Management, Institute of Human and Social Sciences, Kanazawa University, Kakuma, Kanazawa 920-1192 JAPAN. emai:kazuyoshi.ohki@gmail.com

1 Introduction

Since the 18th century, the global economy has expanded through capital accumulation, population growth, and technological advancements. In particular, technological progress—primarily driven by Research and Development (R&D) activities—has played a crucial role in economic development, especially in recent years. Recognizing its importance, numerous researchers have sought to analyze the impact of R&D activities by developing endogenous growth models that incorporate R&D-driven innovation. The development of R&D-based growth models has evolved through several key stages in the literature. Early foundational models, such as those by Romer (1990), Segerstrom, Anant, and Dinopoulos (1990), Grossman and Helpman (1991), and Aghion and Howitt (1992), focused exclusively on innovation by entrants (or followers). [1] These models provided valuable insights into the role of new firms in driving technological progress but had a notable limitation: they could not account for the innovative activities of incumbents (or leaders). [2] Subsequent research extended these frameworks to incorporate innovation by incumbents, acknowledging the importance of technological superiority and first-mover advantages. Notable contributions in this area include Thompson and Waldo (1994), Aghion, Harris, and Vickers (1997), Peretto (1998), Segerstrom and Zornierek (1999), Aghion, Harris, Howitt, and Vickers (2001), Etro (2004), Segerstrom (2007), Ledezma (2013), and Kiedaisch (2015). These models allowed for a more comprehensive analysis of innovation by incumbents (or leaders). However, these models still have a limitation in that they either do not account for the heterogeneity of innovative activity among incumbents or, even when considering firm differences, restrict their analysis to R&D activities within a single industry. [3] More recent advancements in R&D-based growth model have sought to address these shortcomings by explicitly incorporating firm heterogeneity across multiple industries. Models developed by Klette and Kortum (2004), Acemoglu and Akcigit (2012), Denicolo and Zanchettin (2012), Acemoglu and Cao (2015), Akcigit and Kerr (2018), Parello (2019), Iwaisako and Ohki (2019), and Ohki(2023) have attempted to examine innovation by both heterogeneous incumbents and entrants (or heterogeneous leaders and followers) across diverse industrial sectors. The present study is part of a broader line of research that contribute to the understanding of endogenous innovation by heterogeneous incumbents

[1] Followers are defined as firms that once held state-of-the-art technology but were later leapfrogged by another firm.

[2] Bartelsman and Doms (2000) and Foster et al (2001) report that 75 % of total factor productivity (TFP) growth stems from R&D activities conducted by incumbent firms.

[3] In reality, multiple industries exist rather than just one, and firms exhibit heterogeneity in their value and efficiency, leading to different responses to the same policy.

and its impact on economic growth. [4]

The present paper considers innovation conducted by both heterogeneous incumbents and entrants. Entrants engage in R&D to develop new goods and become incumbents. These activities contribute to an increase in the variety of goods, as in a variety expansion model (horizontal growth). Incumbents engage in R&D to improve the productivity and quality of their current technology and secure higher profit flows. These activities contribute not only to an increase in their private adjusted quality, but also to an increase in the common adjusted quality (vertical growth). [5] Incumbents are heterogeneous in their ability to conduct innovation to improve their existing technologies. These abilities are drawn from an exogenous distribution when firms invent new goods. Under this setting, incumbent firms differ in terms of quality, and consequently also in terms of output, employment, profits, and R&D expenditures. Using this model, we examine how a subsidy (or a tax) on innovation by entrants and incumbents affect the overall economic growth rate, as well as the relationship between horizontal and vertical growth.

In our model, innovation by incumbents improves their existing technologies, and we refer to this activity as "incremental innovation". Incremental innovation is a cross-cutting concept used across multiple disciplines, including economics, business studies, engineering and technology, and policy research. It is defined as the continuous improvement of existing products, services, or processes over time. It typically involves routine technological advancements and modifications aimed at helping firms maintain their competitiveness in the market. Incremental innovation in our model aligns with the concept of "sustaining innovation" as defined by Christensen (1997), as it is generally regarded as such.

The main results of our analysis is following points. First, incumbents with higher efficiency in incremental innovation tend to engage in incremental innovation for a longer duration, incur greater innovation costs, produce a larger quantity of goods, employ more labor, and generate higher profit flows. This result is consistent with both the Schumpeterian hypothesis and Christensen's notion of sustaining innovation. [6] Second, a subsidy for innovation by entrants has opposing effects on vertical and horizontal growth, and consequently, an ambiguous effect on the economy's overall growth rate. It increases the innovation by entrants (horizontal growth), which intensifies competition, leading to decreases the benefit

[4] Some studies, such as Melitz (2003), Minniti, Parello, and Segerstrom (2013), Chu, Cozzi, Furukawa, and Liao (2017), and Chu, Cozzi, Fan, Furukawa, and Liao (2019), analyze economic growth and welfare in the context of firm heterogeneity. However, they do not explicitly consider endogenous innovative activities by heterogeneous incumbents (or leaders).

[5] Models in which firms producing differentiated goods engage in R&D to improve their own productivity originate with Smulders and Klundert (1995) and Peretto (1996).

[6] Whereas the Schumpeterian hypothesis suggests that larger incumbents tend to be more innovative, Christensen (1997) draws a distinction between sustaining innovation and 'disruptive innovation.' He argues that larger incumbents are generally receptive to sustaining innovation but resistant to disruptive innovation.

of incremental innovation by incumbents (vertical growth). The former (latter) effect dominates, the economy's overall growth rate increases (decreases). This result implies that excessive preferential policies for entrants may not always benefit the overall economy. Third, a subsidy for incremental innovation by incumbents has positive effect on vertical growth and the economy's overall growth, whereas, an ambiguous effect on the horizontal growth. It increases incremental innovation by incumbents (vertical growth), which increases the average value of incumbents when the vertical growth rate is held constant. However, an increase in the vertical growth rate decreases the average value of incumbents by intensifying competition.. When the former (latter) effect dominates, the average value of incumbents increases (decreases), leading to an increase (decrease) in new firm entry, which in turn increases (decreases) the horizontal growth rate. Even when a subsidy for incremental innovation by incumbents reduces the horizontal growth rate, the positive effect on vertical growth always dominates in the present model, and consequently, the overall growth rate of the economy increases. [7]

From the perspective of examining R&D-based growth models in which both heterogeneous incumbents and entrants conduct innovation, the papers most closely related to the present study are Acemoglu and Cao (2015) and Ohki (2023). Acemoglu and Cao (2015) provide an extension of the textbook multisector Schumpeterian growth model. In their model, incumbents conduct incremental innovation, as in the present paper, whereas entrants conduct "radical innovation". They interpret radical innovation as the introduction of innovative and qualitatively superior products that replace existing firms and their products. In their model, radical innovation leads to a greater increase in quality compared to incremental innovation, and successful entrants become the leaders in the targeted industry. These entrants behave similarly to those in a quality-ladder model. In contrast, entrants in our model invent new differentiated goods and become incumbents without fully replacing the existing industry. Their behavior is similar to that in a variety-expansion model. As a result, the model structure in Acemoglu and Cao (2015) differs significantly from that of the present study.

While Ohki (2023) constructs a framework similar to that of the present paper, a notable difference is that their model allows incumbents to engage in disruptive innovation instead of incremental innovation. They interpret disruptive innovation as the act of abandoning profit flows based on existing technologies in order to shift to new technologies—an act that is often referred to as 'cannibalization' in corporate

[7] However, caution is warranted before concluding that a subsidy for incumbents necessarily benefit the overall economy. We provide a preliminary discussion of this point in Section 3.3.

strategy. Ohki (2023) shows that incumbents with higher-quality technologies produce a larger quantity of goods, employ more labor, and generate higher profit flows; however, they are less willing to engage in disruptive innovation. Taken together with the first main result of the present paper, these findings are consistent with Christensen (1997), which argues that large firms tend to be proactive in incremental innovation but reluctant to pursue disruptive innovation. In this sense, the present paper and Ohki (2023) can be seen as complementary to each other.

The present paper can be positioned as an extension of the in-house model that incorporates heterogeneity among incumbents. Although firm heterogeneity is not considered, there are studies that analyze multiple sources of growth. Peretto (1998), Peretto (2003), Connolly and Peretto (2003), and Peretto and Connolly (2007) develop R&D-based growth models featuring both horizontal innovation (an increase in the number of differentiated goods) and vertical innovation (quality improvement), known as in-house model. [8] A number of recent studies continue to use the in-house model to analyze a variety of policy issues, for example, Peretto (2007), Chu, Cozzi, and Galli (2012), and Chu and Peretto (2023). A well-known limitation of conventional in-house models is that horizontal growth is solely determined by the population growth rate. In contrast, the present paper develops a model in which both horizontal and vertical growth are endogenously determined and influenced by policy variables.

The rest of the paper is structured as follows. In Section 2, we construct a general equilibrium model considering incremental innovation by heterogeneous incumbents and innovation by entrants. In Section 3, we derive the equilibrium under an analytically tractable case by setting the parameter that captures the extent to which incumbents' quality growth is affected by externality from vertical growth to a specific value. In Section 3.1, we examine the case in which incumbents' quality growth is not affected at all by the externality from vertical innovation, i.e., $\alpha = 0$. In Section 3.2, we examine the case in which incumbents' quality growth is fully affected by the externality from vertical innovation, i.e., $\alpha = 1$. In Section 3.3, we provide a preliminary discussion of a more general model.

2 Model

We construct an infinite-horizon representative agent model in which households consume differentiated goods that are invented through innovation by entrants. The productivity of production and the quality

[8] In the early stages of the in-house model literature, Young (1998), Dinopoulos and Thompson (1998), and Howitt (1999) also examined both vertical and horizontal innovation in their analyses, with the aim of eliminating scale effects.

of differentiated goods differ across goods. These increase endogenously through incremental innovation by incumbents, but decrease exogenously due to technological obsolescence. Both innovation by entrants and incremental innovation by incumbents are sources of overall economic growth, which are endogenously determined and influenced by policy variables. Using this model, we examine how a subsidy (or a tax) on innovation by entrants and incumbents affect the overall economic growth rate, as well as the relationship between the two sources of growth. We normalize the wage rate as the numeraire, $w(t) = 1$. We focus on the Balanced Growth Path (BGP) equilibrium where all variables grow at a constant rate.

2.1 Households

There are $L(t)$ households at time t, which grows at exogenous rate, g_L. Household members live forever and are endowed with one unit of labor, which is supplied inelastically. Each household maximizes its discounted utility:

$$U = \int_0^\infty \exp\left[-\rho t\right] \ln u(t)\, dt, \tag{1}$$

where ρ is a subjective discount rate and $u(t)$ represents instantaneous utility from consumption at time t. Households derive utility from consuming $n(t)$ differentiated goods, and preferences are expressed as a CES utility function of the Dixit-Stiglitz (1977) form:

$$u(t) = \left[\int_0^{n(t)} q(j,t)\, x(j,t)^{\frac{\sigma-1}{\sigma}} dj\right]^{\frac{\sigma}{\sigma-1}}, \tag{2}$$

where $q(j,t)$ and $x(j,t)$ denote the quality and the consumption volume of incumbent j at time t respectively, and $\sigma > 1$ is the elasticity of substitution of differentiated goods.

Solving static utility maximization problem of the household, we now derive the per capita demand for each differentiated goods:

$$x(j,t) = \left[\frac{q(j,t)}{p(j,t)}\right]^{\sigma} P(t)^{\sigma-1} E(t), \tag{3}$$

where $P(t) = \left[\int_0^{n(t)} q(j,t)^{\sigma}\, p(j,t)^{-(\sigma-1)} dj\right]^{\frac{-1}{\sigma-1}}$ denotes price index of differentiated goods at time t and $E(t)$ is per capita expenditure at time t.

Given (3), inter-temporal utility maximization yields $\frac{\dot{E}(t)}{E(t)} = r(t) - \rho$ where $r(t)$ is the interest rate at time t. In the BGP equilibrium, the growth rate of expenditure g_E is constant. Thus, interest rate is also constant, and expressed as:

$$r = \rho + g_E. \tag{4}$$

2.2 Production

Each differentiated goods are produced by incumbents, which has heterogeneous producing technology and quality of their producing goods. Incumbents having productivity ϕ must hire $\frac{1}{\phi}x$ unit of labor to produce x unit of goods, and there is no fixed cost to produce differentiated goods. The instantaneous profits of incumbent j is expressed as $\pi(j,t) = x(j,t)\,L(t)\,[p(j,t) - 1/\phi(j,t)]$, where $p(j,t)$ is the price of incumbent j. As incumbents supply goods monopolistically, they choose the optimal price $p(j,t) = \frac{\sigma}{\sigma-1}\frac{1}{\phi(j,t)}$.

Using (3), we obtain the incumbents' quantity of production having productivity ϕ and quality q as:

$$x(j,t)\,L(t) = [q(j,t)\,\phi(j,t)]^{\sigma}\left(\frac{\sigma-1}{\sigma}\right)^{\sigma} P(t)^{\sigma-1}\,E(t)\,L(t)\,, \tag{5}$$

and labor demand having productivity ϕ and quality q as:

$$l^D(j,t) = \frac{x(j,t)\,L(t)}{\phi(j,t)} = \left[q(j,t)^{\sigma}\,\phi(j,t)^{\sigma-1}\right]\left(\frac{\sigma-1}{\sigma}\right)^{\sigma} P(t)^{\sigma-1}\,E(t)\,L(t)\,. \tag{6}$$

,and incumbents' instantaneous profit having productivity ϕ and quality q as:

$$\pi(j,t) = \left[q(j,t)^{\sigma}\,\phi(j,t)^{\sigma-1}\right]\frac{1}{\sigma}\left(\frac{\sigma-1}{\sigma}\right)^{\sigma-1} P(t)^{\sigma-1}\,E(t)\,L(t)\,. \tag{7}$$

2.3 Adjusted quality

To explain the model in a simple way, we define the composite factor of incumbents' property (hereafter referred to as the adjusted quality) as follows:

$$\theta(j,t) \equiv q(j,t)^{\sigma}\,\phi(j,t)^{\sigma-1}\,. \tag{8}$$

We suppose that adjusted quality is composed of "private adjusted quality", θ_P, and "common adjusted quality", θ_C, as:

$$\theta(j,t) \equiv \theta_P(j,t)\,\theta_C(j,t)\,. \tag{9}$$

On the one hand, the private adjusted quality represents an incumbent-specific characteristic. It undergoes a decline at an exogenous rate, κ_O, due to technological obsolescence, whereas incumbents have the capacity to improve their private adjusted quality at an exogenous rate, κ_I, through the implementation of incremental Innovation. Thus, the private adjusted quality is determined based on the duration since the commencement of operations and the length of time spent on incremental innovation. We express the private adjusted quality of incumbents that has operated for τ periods as:

$$\theta_P(\tau) = \begin{cases} \theta_P(0)\exp\left[\left[-\kappa_O + \kappa_I\right]\tau\right] & when \quad \tau \leq \hat{T} \\ \theta_P(0)\exp\left[\kappa_I \hat{T}\right]\exp\left[-\kappa_O \tau\right] & when \quad \tau > \hat{T} \end{cases}, \tag{10}$$

where $\hat{T}$ is the moment when incumbents decide to cease incremental innovation, which is determined endogenously.

On the other hand, the common adjusted quality represents a quality characteristic that applies to all incumbents that started operations at the same time. This quality captures the spillover effects from the accumulated research conducted by all previous incumbents. We assume that the common adjusted quality is higher for later generations of incumbents and grows at an endogenous rate, $g_{\theta C}$, as a result of this spillover effect. Additionally, even after operations begin, the common adjusted quality of incumbents continues to improve by a fraction of this rate, $\alpha g_{\theta C}$ $(0 \leq \alpha \leq 1)$. Thus, the common adjusted quality is determined according to the duration of continued operation, τ, and the timing of operation commencement, s. This can be expressed as:

$$\theta_C(\tau, s) = \theta_C(s)\exp\left[\alpha g_{\theta C}\tau\right], \tag{11}$$

where $\theta_C(s)$ is an initial common adjusted quality of incumbents that initiated operations at period s, which endogenously grows as:

$$g_{\theta C} = \frac{\dot{\theta}_C(s)}{\theta_C(s)} = \lambda \left[\int_s^{\infty} \exp\left[-\omega\left[\tau - s\right]\right] \frac{\mathbb{E}[\theta_P(\tau)]}{\theta_P(0)} d\tau \right]^{\gamma}, \tag{12}$$

where λ, γ and ω are parameter, and $\mathbb{E}[\theta_P(\tau)]$ is an expected private adjusted quality of incumbents that has been in operation for τ periods. Equation (12) can be interpreted that the common adjusted quality grows driven by the historical accumulation of innovation efforts across incumbents. [9] A high value of λ enhances this spillover effect. A high value of γ increases the sensitivity to changes in the degree of incremental innovation by incumbents. A high value of ω means that the quality of the new generation has a greater impact on the growth of the common adjusted quality compared to old generation. [10]

Based on the above framework, the adjusted quality of an incumbent that commenced operations in period s and has continued operating for a duration of τ can be expressed as:

$$\theta(\tau, s) = \begin{cases} \theta_P(0)\,\theta_C(s)\exp\left[\left[-\kappa_O + \kappa_I + \alpha g_{\theta E}\right]\tau\right] & when \quad \tau \leq \hat{T} \\ \theta_P(0)\,\theta_C(s)\exp\left[\kappa_I \hat{T}\right]\exp\left[-\left[\kappa_O - \alpha g_{\theta E}\right]\tau\right] & when \quad \tau > \hat{T} \end{cases}. \tag{13}$$

Since all incumbents with the same adjusted quality produce the same quantity of goods, hire same amount of labor and earn the same profit, we will index incumbents by θ instead of j from this point forward. From (13), the adjusted quality θ is a function of the period in which incumbents commenced operations, s, and the duration of their operation, τ, then labor demand and profit of incumbents having adjusted quality θ are also function of s and τ, and can be expressed as: [11]

$$l^D(\theta(\tau, s)) = \begin{cases} \exp\left[-\left[-g_E - g_L + g_Q + \kappa_O - \kappa_I - \alpha g_{\theta C}\right]\tau\right] \frac{\sigma-1}{\sigma} \frac{\theta_P(0)\theta_C(s)E(s)L(s)}{Q(s)} & when \quad \tau \leq \hat{T} \\ \exp\left[\kappa_I \hat{T}\right]\exp\left[-\left[-g_E - g_L + g_Q + \kappa_O - \alpha g_{\theta C}\right]\tau\right] \frac{\sigma-1}{\sigma} \frac{\theta_P(0)\theta_C(s)E(s)L(s)}{Q(s)} & when \quad \tau > \hat{T} \end{cases}, \tag{14}$$

[9] Models in which externalities lead to an increase in aggregate productivity are one of the central ideas of endogenous growth theory. Arrow (1962) posits an externality through which capital accumulation contributes to technological progress via learning by doing. Romer (1986) introduces a spillover effect of capital stock by incorporating knowledge-based elements into physical capital accumulation. Lucas (1988) emphasizes human capital accumulation through education and training as a key driver of growth, highlighting the associated spillover effects. Romer (1990) models the accumulation of knowledge through R&D activities, which in turn leads to increased productivity.

[10] In order to prevent the divergence of Equation (12), the condition $[\kappa_O - \kappa_I + k\psi + \omega] > 0$ must hold.

[11] From (8), price index is expressed as $P(t) = \left[\int_0^{n(t)} \theta(j,t)\,dj\right]^{\frac{-1}{\sigma-1}} \left(\frac{\sigma}{\sigma-1}\right)$.

$$\pi\left(\theta\left(\tau,s\right)\right)=\begin{cases}\exp\left[-\left[-g_E-g_L+g_Q+\kappa_O-\kappa_I-\alpha g_{\theta C}\right]\tau\right]\frac{\theta_P(0)}{\sigma}\frac{\theta_C(s)E(s)L(s)}{Q(s)} & when\quad \tau\leq\hat{T}\\ \exp\left[\kappa_I\hat{T}\right]\exp\left[-\left[-g_E-g_L+g_Q+\kappa_O-\alpha g_{\theta C}\right]\tau\right]\frac{\theta_P(0)}{\sigma}\frac{\theta_C(s)E(s)L(s)}{Q(s)} & when\quad \tau>\hat{T}\end{cases}\tag{15}$$

where $Q\left(t\right)\equiv\int_0^{n(t)}\theta\left(j,t\right)dj=\int_0^{\infty}\dot{n}\left(t-\tau\right)\mathbb{E}\left[\theta\left(\tau,t-\tau\right)\right]d\tau$ is aggregate adjusted quality of all existing incumbents at time t, which is determined endogenously. $\dot{n}\left(t-\tau\right)$ is the number of incumbents that commenced operations in period $t-\tau$, and $\mathbb{E}[\theta\left(\tau,t-\tau\right)]$ is the expected value of the adjusted quality of incumbents that commenced operations in period $t-\tau$ and has continued operating for a duration of τ.

Substituting (5) and (8) into (2) yields:

$$u\left(t\right)=Q\left(t\right)^{\frac{1}{\sigma-1}}\frac{\sigma-1}{\sigma}E\left(t\right).\tag{16}$$

We interpret overall growth rate of this economy as growth rate of instantaneous utility, which is expressed as

$$g_u=\frac{1}{\sigma-1}g_Q+g_E,\tag{17}$$

where g_E and g_Q are growth rate of expenditure per capita and aggregate adjusted quality respectively, which is determined endogenously.

2.4 Entry

Entrants must hire $c_E\theta_C\left(s\right)E\left(s\right)L\left(s\right)/Q\left(s\right)$ unit of labor to invent new differentiated goods at time s. Then the cost of invent new differentiated goods is expressed as:

$$C_E\left(s\right)=s_Ec_E\frac{\theta_C\left(s\right)E\left(s\right)L\left(s\right)}{Q\left(s\right)}.\tag{18}$$

Here, s_E is the subsidy (tax) for innovation by entrants when $s_E<1$ ($s_E>1$). After entry, entrants can perceive their ability to undertake incremental innovation, ξ, which is drawn form a given distribution

$G(\xi)$. We specify distribution assuming Pareto distribution as: [12]

$$G(\xi) = 1 - \xi^{-k} \quad 1 \leq \xi < \infty \ , \tag{19}$$

where expected value of ξ is given by $k/(k-1)$, which decreases in $k > 1$.

After the revelation of their incremental innovation ability, entrants decide whether to commence operations by incurring the initial operating cost or to exit the market. Entrants that choose to begin operations at time s must hire $C_O(s)$ unit of labor. For entrants with lower incremental innovation ability, the discounted value of future profit flows is small. There exists a critical value, ξ^{c1}, at which expected profits fall below operating costs. Entrants that fall below this threshold exit the market, while those that exceed it commence operations.

Once entrants commence operations, they become incumbents. The differentiated goods produced by incumbents are protected by perfect and perpetual intellectual property rights (IPR), which allow them to supply these goods monopolistically without the risk of imitation. As a result, they are able to earn monopoly profits, as expressed in equation (15).

2.5 Incremental Innovation

Incumbents have the opportunity every period to enhance their existing adjusted technology at a rate of κ_I by hiring a specific unit of labor. We call this activity as incremental innovation. Incumbents having ability ξ that commenced operations in period s and have continued operating for a duration of τ must hire $c_I(\tau)\,\theta(\tau, s)\,E(s+\tau)\,L(s+\tau)/\xi Q(s+\tau)$ unit of labor to conduct incremental innovation. Accordingly, the cost of conducting incremental innovation is expressed as:

$$C_I(\xi, \tau, s) = \frac{1}{\xi} s_I c_I(\tau)\,\theta(\tau, s)\,\frac{E(s+\tau)\,L(s+\tau)}{Q(s+\tau)} \tag{20}$$

where s_I is the subsidy (tax) for incremental innovation by incumbents when $s_I < 1$ ($s_I > 1$), and $c_I(\tau)$ express the difficulty of incremental innovation, which increases at exogenous rates ψ. The initial value

[12] In generally, cumulative distribution function of Pareto distribution is expressed as $G(\xi) = 1 - a^k \xi^{-k} \quad a \leq \xi < \infty$. In this paper, we simplify $a = 1$. Then, density function of this is expressed as $g(\xi) = k\xi^{-(k+1)}$.

of the difficulty of incremental innovation ($c_I(0)$) is exogenous given, then we can derive:

$$c_I(\tau) = \exp[\psi\tau]\, c_I(0). \tag{21}$$

From (20) and (21), we can express the cost of incremental innovation as:

$$C_I(\xi,\tau,s) = \exp[[g_E + g_L - g_Q - \kappa_O + \kappa_I + \alpha g_{\theta C} + \psi]\,\tau]\,\frac{1}{\xi} s_I c_I(0)\,\theta_P(0)\,\theta_C(s)\,\frac{E(s)\,L(s)}{Q(s)} \tag{22}$$

Since the cost of incremental innovation increases each period, incumbents eventually cease this activity. We denote this moment as $\hat{T}$, which depends on the ability for incremental innovation, ξ, and is determined endogenously.

Given the settings described so far, the value of incumbents commencement operation at time s, and drawing ξ can be expressed as follows:

$$V(\xi,s) = \begin{array}{l} \int\limits_0^{\hat{T}(\xi)} \left[\begin{array}{l} \exp[-[r - g_E - g_L + g_Q + \kappa_O - \kappa_I - \alpha g_{\theta C}]\,\tau]\,\frac{\theta_P(0)\theta_C(s)}{\sigma}\frac{E(s)L(s)}{Q(s)} \\ -\exp[-[r - g_E - g_L + g_Q + \kappa_O - \kappa_I - \alpha g_{\theta C} - \psi]\,\tau]\,\frac{s_I c_I(0)}{\xi}\theta_P(0)\,\theta_C(s)\,\frac{E(s)L(s)}{Q(s)} \end{array}\right] d\tau \\ + \int\limits_{\hat{T}(\xi)}^{\infty} \exp[-[r - g_E - g_L + g_Q + \kappa_O - \alpha g_{\theta C}]\,\tau]\exp\left[\kappa_I \hat{T}\right]\frac{\theta_P(0)\theta_C(s)}{\sigma}\frac{E(s)L(s)}{Q(s)}d\tau \\ -C_O(s) \end{array}. \tag{23}$$

From the commencement of operations until time $\hat{T}$, incumbents earn instantaneous profits, which grow at the rate of $-[r - g_E - g_L + g_Q + \kappa_O - \kappa I - \alpha g\theta C] > 0$, while incurring the instantaneous cost of incremental innovation. These dynamics are described in lines 1 and 2 of equation (23). From time $\hat{T}$ onward, incumbents earn instantaneous profits, which decrease at the rate of $-[r - g_E - g_L + g_Q + \kappa_O - \alpha g_{\theta C}] < 0$. This dynamics is described in lines 3 of equation (23). Additionally, incumbents incur operating costs prior to the commencement of operations, which is described in line 4 of equation (23).

Incumbents choose the optimal $\hat{T}$ to maximize their value as:

$$\begin{array}{l} \frac{\kappa_I}{[r - g_E - g_L + g_Q + \kappa_O - \alpha g_{\theta C}]}\exp\left[-[r - g_E - g_L + g_Q + \kappa_O - \kappa_I - \alpha g_{\theta C}]\,\hat{T}\right]\frac{\theta_P(0)\theta_C(s)}{\sigma}\frac{E(s)L(s)}{Q(s)} \\ = \exp\left[-[r - g_E - g_L + g_Q + \kappa_O - \kappa_I - \alpha g_{\theta C} - \psi]\,\hat{T}\right]\frac{1}{\xi}s_I c_I(0)\,\theta_P(0)\,\theta_C(s)\,\frac{E(s)L(s)}{Q(s)} \end{array}. \tag{24}$$

This condition is a necessary condition with respect to $\hat{T}$ to maximize (23), and expresses the trade-off between marginal benefit (Left Hand Side: LHS) and marginal cost (Right Hand Side; RHS) from continuation the incremental innovation. When incumbents continue incremental innovation, they can, on the one hand, achieve higher profit flows due to improvements in adjusted quality. On the other hand, they incur additional costs associated with conducting incremental innovation. If the marginal benefit from continuing incremental innovation is greater (smaller) than the marginal cost, incumbents will delay (advance) the timing of ceasing innovation. Therefore, the timing for stopping incremental innovation is determined by the condition where the marginal benefit from continuing the incremental innovation equals the marginal cost.

Equation (24) can be simplified and rewritten to show that the net marginal benefit from continuing incremental innovation equals zero, as follows:

$$\frac{\kappa_I}{[r - g_E - g_L + g_Q + \kappa_O - \alpha g_{\theta C}]} \frac{1}{\sigma} - \exp\left[\psi \hat{T}\right] \frac{1}{\zeta} s_I c_I (0) = 0. \tag{25}$$

A sufficient condition with respect to $\hat{T}$ for maximizing equation (23) is that the net marginal benefit from continuing incremental innovation decreases with $\hat{T}$ at the moment when incumbents cease incremental innovation, which can be derived from (25) as:

$$\psi > 0. \tag{26}$$

In addition to the case of interior solution, we have to consider the case of corner solution. Incumbents drawing low ξ at time s, marginal benefit from continuation the incremental innovation can be strictly less than that of marginal cost, even at the moment when they commence operations, which is expressed as follows:

$$\frac{\kappa_I}{[r - g_E - g_L + g_Q + \kappa_O - \alpha g_{\theta C}]} \frac{1}{\sigma} - \frac{1}{\zeta} s_I c_I (0) < 0.$$

Incumbents drawing such low ξ decide not to conduct incremental innovation at all. We define the critical level of efficiency for incremental innovation ξ^{c2}, determining whether the incumbent engages in incremental innovation or not. This critical level satisfies the following condition:

$$\frac{\kappa_I}{[r - g_E - g_L + g_Q + \kappa_O - \alpha g_{\theta C}]}\frac{1}{\sigma} - \frac{1}{\zeta^{c2}} s_I c_I(0) = 0. \tag{27}$$

From (25) and (27), we can write the timing of ceasing incremental innovation as a function of efficiency of incremental innovation:

$$\begin{array}{rclcl} \hat{T}(\xi) & = & 0 & for & \xi \leq \xi^{c2} \\ \hat{T}(\xi) & = & \frac{1}{\psi}\ln\frac{\xi}{\xi^{c2}} & for & \xi^{c2} < \xi \end{array} \tag{28}$$

,

where

$$\xi^{c2} = \frac{s_I c_I(0)}{\kappa_I}\sigma[r - g_E - g_L + g_Q + \kappa_O - \alpha g_{\theta C}]. \tag{29}$$

Equation (28) and (29) determine the equilibrium value of ξ^{c2} and $\hat{T}$ with given g_Q and $g_{\theta C}$. We can confirm $\frac{\partial \hat{T}(\xi, g_Q)}{\partial \xi} > 0$ and $\frac{\partial^2 \hat{T}(\xi, g_Q)}{\partial \xi \partial \xi} < 0$. We can recognize the positive relationship between ξ and $\tilde{T}$, and then incumbents drawing high ξ tend to engage in incremental innovation for a longer period and incur higher innovation costs, leading to higher adjusted quality. From equations (5), (6), (7), and (8), incumbents with high adjusted quality produce a larger quantity of goods, employ more labor, and earn higher monopolistic profits. This leads to the following proposition.

Proposition 2.1. *Incumbents with higher efficiency in incremental innovation tend to engage in incremental innovation for a longer duration, incur greater innovation costs, produce a larger quantity of goods, employ more labor, and generate higher profit flows.*

On the one hand, this proposition shows positive relationship between firm size and R&D activity, which is consistent with Schumpeter's hypothesis. [13] On the other hand, this proposition contrasts with the findings of Iwaisako and Ohki (2019) and Ohki (2023), which suggest that incumbents with high adjusted quality and substantial profits tend to hesitate in pursuing subsequent innovations. They examine the innovative activity that incumbents seek new profit opportunities at the expense of profits

[13] This hypothesis is also supported by Acemoglu and Cao (2015) and Akcigit and Kerr (2018) in the theoretical context by analyzing incremental innovation by incumbents. With regard to Schumpeter's hypothesis, Cohen and Levin (1989) and Cohen (1995) provide marvelous survey.

gained in existing markets, which is aligns with Christensen's definition of disruptive innovation. These seemingly contradictory results are not inconsistent, as the definition of innovation considered in their model is clearly different. Rather, when viewed alongside our present model, they are consistent with Christensen's idea that large firms tend to engage more actively in sustaining innovation, but are less inclined to pursue disruptive innovation.

In figure 1, we illustrates the dynamics of profit growth based on the value of $\hat{T}(\xi)$ over time, showing how many times the profit has multiplied since the start of operations. [14] The profit of incumbents deciding $\hat{T}(\xi) = 0$ will decline to less than one-tenth of its initial level 17 periods after the start of operations. For incumbents deciding $\hat{T}(\xi) = 10$, profit will rise to 1.8 times its initial level 10 periods after the start of operations, before gradually declining to less than one-tenth of the initial level 31 periods after the start of operations. Similarly, the profit of incumbents deciding $\hat{T}(\xi) = 30$ will increase to six times its initial level 30 periods after the start of operations, followed by a gradual decline, reaching less than one-tenth of the initial level 43 periods after the start of operations. In the case of incumbents deciding $\hat{T}(\xi) = 50$, profit will peak at 20 times its initial level 50 periods after the start of operations, before gradually declining and falling below one-tenth of the initial level 75 periods after the start of operations. [15]

[Insert Figure 1 here]

2.6 The value of incumbents

To simplify our model, we assume that the operating cost is equal to the value of incumbents with $\xi \leq \xi^{c2}$, which can be expressed as follows:

$$C_O(s) = \int_0^\infty \exp\left[-\left[r - g_E - g_L + g_Q + \kappa_O - \alpha g_{\theta C}\right]\tau\right] \pi(0, s)\, d\tau. \tag{30}$$

This assumption allows us to avoid complex case distinctions when deriving the average value of incumbents, as entrants that choose not to engage in incremental innovation will exit the market. Thanks to

[14] Since , production output, and labor demand are proportional to profit, the dynamics of these variables follow the same pattern as profit.

[15] From equation (15), the profit of incumbents, on the one hand, grows at a rate of $-\left[-g_E - g_L + g_Q + \kappa_O - \kappa_I - \alpha g_{\theta C}\right] > 0$ before they cease incremental innovation, i.e., $\tau \leq \hat{T}(\xi)$. Given that the long-term average growth rate of the Earnings Per Share (EPS) of the S&P 500 is approximately 6%, we set $-\left[-g_E - g_L + g_Q + \kappa_O - \kappa_I - \alpha g_{\theta C}\right] = 0.06$. On the other hand, after incumbents cease incremental innovation, i.e., $\tau > \hat{T}(\xi)$, their profits decrease at the rate of $-\left[-g_E - g_L + g_Q + \kappa_O - \alpha g_{\theta C}\right] < 0$. Bessen (2008) estimates that the annual depreciation rate of profits generated by patents is about 14%, so we set $-\left[-g_E - g_L + g_Q + \kappa_O - \alpha g_{\theta C}\right] = -0.14$.

this assumption, the critical value determining whether incumbents decide not to start operations is the same as the critical value determining whether they decide not to conduct incremental innovation as:

$$\xi^C \equiv \xi^{c1} = \xi^{c2}. \tag{31}$$

From (23) and (30), we can express the values of incumbent entering at time s, and drawing ξ as:

$$\begin{aligned}
&V(\xi, s) \\
&= \quad 0 \qquad\qquad for \quad \xi \leq \xi^C \\
&= \int_0^{\hat{T}(\xi)} \left[\begin{array}{l} \exp\left[-\left[r - g_E - g_L + g_Q + \kappa_O - \kappa_I - \alpha g_{\theta C}\right]\tau\right] \frac{\theta_P(0)\theta_C(s)}{\sigma} \frac{E(s)L(s)}{Q(s)} \\ -\exp\left[-\left[r - g_E - g_L + g_Q + \kappa_O - \kappa_I - \alpha g_{\theta C} - \psi\right]\tau\right] \frac{s_I c_I(0)\theta_P(0)\theta_C(s)}{\xi} \frac{E(s)L(s)}{Q(s)} \end{array} \right] d\tau \\
&\quad + \int_{\hat{T}(\xi)}^{\infty} \exp\left[-\left[r - g_E - g_L + g_Q + \kappa_O - \alpha g_{\theta C}\right]\tau\right] \exp\left[\kappa_I \hat{T}\right] \frac{\theta_P(0)\theta_C(s)}{\sigma} \frac{E(s)L(s)}{Q(s)} d\tau \qquad for \quad \xi > \xi^C \\
&\quad - \int_0^{\infty} \exp\left[-\left[r - g_E - g_L + g_Q + \kappa_O - \alpha g_{\theta C}\right]\tau\right] \frac{\theta_P(0)\theta_C(s)}{\sigma} \frac{E(s)L(s)}{Q(s)} d\tau
\end{aligned} \quad , \tag{32}$$

where $\hat{T}(\xi)$ and ξ^c satisfy (28) and (29) respectively.

2.7 Free Entry Condition

When we calculate an expected value of incumbents, we have to consider the magnitude correlation between the minimum efficiency of incremental innovation, $\xi^{\min} = 1$, and the critical level of efficiency of incremental innovation, ξ^c. First, if $\xi^{\min} \leq \xi^c$ is satisfied, we obtain an equilibrium where entrants with $\xi < \xi^c$ do not engage in incremental innovation and exit the market immediately. Conversely, incumbents with $\xi^c \leq \xi$ commence operations and continue incremental innovation until $\hat{T}(\xi)$. Second, if $\xi^{\min} > \xi^c$ is satisfied, all entrants commence operations and continue incremental innovation until $\hat{T}(\xi)$. Then we can express the expected value of incumbents:

$$\begin{aligned}
\mathbb{E}\left[V(\xi, s)\right] &= \int_{\xi^C}^{\infty} V(\xi, s)\, dG(\xi) \quad if \quad \xi^{\min} \leq \xi^C \\
\mathbb{E}\left[V(\xi, s)\right] &= \int_{\xi^{\min}}^{\infty} V(\xi, s)\, dG(\xi) \quad if \quad \xi^{\min} > \xi^C
\end{aligned} . \tag{33}$$

To avoid complex case distinctions, we restrict our analysis to the case where $\xi^{\min} \leq \xi^{c}$ is satisfied. Free entry condition is satisfied in each period, and then expected gain from entry at time s equals to zero. Since the cost of entry is $s_E c_E \frac{\theta_C(s)E(s)L(s)}{Q(s)}$, we obtain the following condition:

$$\int_{\xi^C}^{\infty} V(\xi, s)\, dG(\xi) \leq s_E c_E \frac{\theta_C(s)\, E(s)\, L(s)}{Q(s)}. \tag{34}$$

When an equilibrium value of g_n is positive, (34) is satisfied with equality. When an equilibrium value of g_n is negative, expected gain from entry is too small to enter the market for entrants. Then there is no new entry, and the number of differentiated goods does not increase. Thus, if LHS of (34) is strictly smaller than RHS even when $g_n = 0$, we obtain an interior equilibrium that the number of differentiated goods remains constant, $g_n = 0$.

2.8 Quality Level of Differentiated Goods

Aggregate adjusted quality of differentiated goods, Q, is calculated as: [16]

$$Q(t) = n(t)\, \theta_C(t)\, g_n \Theta, \tag{35}$$

where

$$\Theta = \theta_P(0) \left[\xi^C\right]^{-k} \frac{[g_Q + \kappa_O - \alpha g_{\theta C} + k\psi]}{[g_Q + \kappa_O - \kappa_I - \alpha g_{\theta C} + k\psi]\,[g_Q + \kappa_O - \alpha g_{\theta C}]}.$$

Taking the natural logarithm of equation (35) and differentiating it with respect to time t, we obtain the growth rate of the aggregate adjusted quality of differentiated goods as follows:

$$g_Q = \frac{\dot{n}(t)}{n(t)} + \frac{\dot{\theta}_C(t)}{\theta_C(t)} = g_n + g_{\theta C}. \tag{36}$$

2.9 Labor Market

Aggregate labor demand is the sum of labor demand for production (L^D), labor demand for incremental innovation (L^I), labor demand for commencing operations (L^O), and labor demand for innovation by entrants (L^E). Aggregate labor supply is exogenously given (L), and then the labor market clearing

[16] In appendix B, we derive the equation (35).

condition at time t is:

$$L^{D}(t)+L^{I}(t)+L^{O}(t)+L^{E}(t)=L(t) \tag{37}$$

3 Equilibrium

From labor market clearing condition, we obtain the following equation as: [17]

$$E=\left[\begin{array}{l}\frac{\sigma-1}{\sigma}+\frac{1}{\Theta}\theta_{P}(0)\,c_{I}(0)\,\frac{k}{k+1}\left[\xi^{C}\right]^{-[k+1]}\frac{1}{[g_{Q}+\kappa_{O}-\kappa_{I}+k\psi-\alpha g_{\theta C}]}\\+\frac{1}{\Theta}\frac{1}{\kappa_{I}}\frac{1}{\xi^{C}}\theta_{P}(0)\,s_{I}c_{I}(0)+\frac{1}{\Theta}\left[\xi^{C}\right]^{k}c_{E}\end{array}\right]^{-1}. \tag{38}$$

where

$$\Theta=\theta_{P}(0)\left[\xi^{C}\right]^{-k}\frac{[g_{Q}+\kappa_{O}-\alpha g_{\theta C}+k\psi]}{[g_{Q}+\kappa_{O}-\kappa_{I}-\alpha g_{\theta C}+k\psi]\,[g_{Q}+\kappa_{O}-\alpha g_{\theta C}]}. \tag{39}$$

Since the RHS of equation (38) remains constant at the BGP equilibrium, the LHS of equation (38) must also be constant. As a result, we can confirm that consumption per capita is constant, leading to the following equation:

$$g_{E}=0 \tag{40}$$

Substituting this equation into equation (4), we can verify that the interest rate is equal to the subjective discount rate, resulting in the following equation:

$$r=\rho. \tag{41}$$

From the free entry condition, along with equations (40) and (41), we obtain the following equation: [18]

$$\frac{1}{[\rho-g_{L}+g_{Q}+\kappa_{O}-\kappa_{I}-a g_{\theta C}+k\psi]}\frac{\theta_{P}(0)}{[k+1]}\left[\xi^{C}\right]^{-[k+1]}=\frac{s_{E}c_{E}}{s_{I}c_{I}(0)}. \tag{42}$$

[17] In Appendix C, we present a detailed derivation of this equation.

[18] In Appendix A, we provide a detailed derivation of this equation.

From (29), (31), (40) and (41), the critical value of ξ is expressed as:

$$\xi^C = \frac{s_I c_I(0)}{\kappa_I} \sigma \left[\rho - g_L + g_Q + \kappa_O - \alpha g_{\theta C}\right]. \tag{43}$$

The growth rate of common adjusted quality, as derived from (12), is expressed as: [19]

$$g_{\theta C} = \lambda \left[\xi^C\right]^{-\gamma k} \left[\frac{[\kappa_O + k\psi + \omega]}{[\kappa_O - \kappa_I + k\psi + \omega][\kappa_O + \omega]}\right]^{\gamma}. \tag{44}$$

From (17), (36) and (40), the overall growth rate consists of horizontal growth, g_n and vertical growth,$g_{\theta C}$, and is expressed as follows:

$$g_u = \frac{1}{\sigma - 1} g_Q = \frac{1}{\sigma - 1}\left[g_n + g_{\theta C}\right]. \tag{45}$$

This system consists of nine equations—(36), (38), (39), (40), (41), (42), (43), (44), and (45)—along with nine endogenous variables: g_u, g_Q, g_n, $g_{\theta C}$, g_E, ξ^C, Θ, r and E, and the system is solvable. Particularly, the cases where $\alpha = 0$ and $\alpha = 1$ are analytically very tractable. Here, α represents the relative positive external impact derived from improvements in the common adjusted quality after commencing operations compared to before operations. In the following subsection, we separately analyze the special cases where $\alpha = 0$ and $\alpha = 1$. From equations (40) and (41), $g_E = 0$ and $r = \rho$ hold regardless of the value of α, whereas the other endogenous variables are influenced by α.

3.1 The case where $\alpha = 0$

In this subsection, we analyze the special cases where $\alpha = 0$. When $\alpha = 0$, no positive external effect arises from improvements in the common adjusted quality for incumbents that have already commenced operations.

3.1.1 Equilibrium

From (42) and (43), free entry condition is expressed as:

$$\frac{1}{[\rho - g_L + g_Q + \kappa_O - \kappa_I + k\psi]} \frac{\theta_P(0)}{[k+1]} \left[\xi^C\right]^{-[k+1]} = \frac{s_E c_E}{s_I c_I(0)}, \tag{46}$$

[19] In Appendix B, we provide a detailed derivation of this equation.

where

$$\xi^C = \frac{s_I c_I(0)}{\kappa_I}\sigma\left[\rho - g_L + g_Q + \kappa_O\right]. \tag{47}$$

Equation (46) and (47) determine the equilibrium value of g_Q and ξ^C.

We can rewrite equation (39) as follows:

$$\Theta = \theta_P(0)\left[\xi^C\right]^{-k}\frac{\left[g_Q + \kappa_O + k\psi\right]}{\left[g_Q + \kappa_O - \kappa_I + k\psi\right]\left[g_Q + \kappa_O\right]}. \tag{48}$$

By substituting the equilibrium values of g_Q and ξ^C into equations (44), (45) and (48), we derive the equilibrium values of $g_{\theta C}$, g_u and Θ, respectively. Next, plugging the equilibrium value of g_Q and $g_{\theta C}$ into equation (36) allows us to obtain the equilibrium value of g_n. Finally, substituting the equilibrium value of g_Q, ξ^C, Θ, and $\alpha = 0$ into equation (38) results in the equilibrium value of E.

3.1.2 Policy analysis

We begin by analyzing the impact of a subsidy for innovation by entrants, which corresponds to a decrease in s_E. LHS of equation (46) is a decreasing function of g_Q, while RHS of equation (46) is a increasing function of s_E. Therefore, a decrease in s_E results in an increase in g_Q, which subsequently raises the economic growth rate g_u. According to equation (47), an increase in g_Q raises ξ^C, and a higher ξ^C reduces $g_{\theta C}$, as shown in equation (44). From (36), g_n must increase to maintain equality since g_Q increases while $g_{\theta C}$ decreases. If innovation by entrants is taxed, the results are exactly the opposite.

We summarize above discussion in the following proposition:

Proposition 3.1. *When we set $\alpha = 0$, indicating that no positive external effect arises from improvements in the common adjusted quality for incumbents that have already commenced operations, we obtain the following results.*

- *A subsidy (a tax) for innovation by entrants stimulates (mitigates) innovation by entrants, and then increases (decreases) the horizontal growth rate, g_n.*
- *A subsidy (a tax) for innovation by entrants mitigates (stimulates) incremental innovation by incumbents, and then decreases (increases) the vertical growth rate, $g_{\theta C}$.*

- *A subsidy (a tax) for innovation by entrants increases (decreases) growth rate of the aggregate adjusted quality, g_Q, and thereby increases (decreases) overall economic growth rate, g_u.*

The above proposition can be interpreted as follows. A subsidy for innovation by entrants encourages invention of new differentiated goods, leading to an increase in the growth rate of the number of differentiated goods. This intensifies competition, thereby reducing both the current and future profits of incumbents, which in turn diminishes their incentive to engage in incremental innovation. A reduction in incremental innovation diminishes the average private adjusted quality, thereby weakening the positive externality that enhances the common adjusted quality. Consequently, while the growth rate of the number of differentiated goods increases, the growth rate of the common adjusted quality decreases. However, since the former effect outweighs the latter, the growth rate of the aggregate adjusted quality rises, ultimately leading to an increase in the overall economic growth rate.

Next, we analyze the effect of a subsidy for incremental innovation by incumbents, which corresponds to a decrease in s_I. By substituting (47) into (46), we obtain:

$$\frac{1}{[\rho - g_L + g_Q + \kappa_O - \kappa_I + k\psi]\,[\rho - g_L + g_Q + \kappa_O]^{k+1}} \frac{1}{[k+1]} \left[\frac{\kappa_I}{\sigma}\right]^{k+1} = [s_I c_I(0)]^k \, s_E c_E. \tag{49}$$

LHS of equation (49) is a decreasing function of g_Q, while RHS of equation (49) is a increasing function of s_I. Therefore, a decrease in s_I results in an increase in g_Q, which subsequently raises the economic growth rate g_u. RHS of (46) increases as s_I decreases, while LHS of (46) decreases as g_Q increases; therefore, ξ^C must decrease to maintain equality. A lower ξ^C raises $g_{\theta C}$, as shown in equation (44). From (36), the impact on g_n is ambiguous since both g_Q and $g_{\theta C}$ increase. If innovation by incumbents is taxed, the results are exactly the opposite.

We summarize above discussion in the following proposition:

Proposition 3.2. *When we set $\alpha = 0$, we obtain the following results.*

- *The impact of subsidy (tax) for incremental innovation by incumbents on innovation by entrants is ambiguous; consequently, the impact on the horizontal growth rate, g_n, is ambiguous.*

- *Subsidy (tax) for incremental innovation by incumbents stimulate (mitigates) incremental innovation by incumbents, and then increases (decreases) the vertical growth rate, $g_{\theta C}$.*

- *Subsidy (tax) for incremental innovation by incumbents increases (decreases) growth rate of the aggregate adjusted quality, g_Q, and thereby increases (decreases) overall economic growth rate, g_u.*

The above proposition can be interpreted as follows. A subsidy for incremental innovation by incumbents incentivizes such innovation, thereby increasing the average private adjusted quality. This, in turn, strengthens the positive externality that enhances the common adjusted quality. On one hand, a higher growth rate of the common adjusted quality intensifies competition, which reduces the average value of incumbents. On the other hand, a subsidy for incremental innovation lowers the cost of such innovation, thereby increasing the average value of incumbents. If the former (latter) effect dominates, the average value of incumbents decreases (increases), leading to a decline (rise) in new entrants. As a result, the growth rate of the number of differentiated goods may either increase or decrease. Even if the horizontal growth rate declines, the effect of a higher vertical growth rate always dominates. Consequently, the growth rate of aggregate adjusted quality rises, ultimately leading to an increase in overall economic growth.

We summarize the main result in below table.

$\alpha = 0$	g_Q	g_n	$g_{\theta C}$
$s_E \downarrow$	$\uparrow$	$\uparrow$	$\downarrow$
$s_I \downarrow$	$\uparrow$	?	$\uparrow$

Table 1: Summary of policy analysis when $\alpha = 0$

A notable finding from the analysis in this subsection is that it points to the possibility that subsidies for incremental innovation by incumbents may reduce the entry of new firms. This implies that the two sources of growth in our model, g_n and $g_{\theta C}$, do not necessarily respond in the same direction to a subsidy for incremental innovation by incumbents —in contrast to the result that the two sources of growth always respond in opposite directions to a subsidy for innovation by entrants. It is natural to expect that these respond in the same direction to subsidies for incremental innovation by incumbents, since such subsidies increase the value of incumbents (i.e., increase net profit flows after entry), leading to a higher incentive for entry, which in turn raises the growth rate of the number of differentiated goods. This mechanism is also emphasized by Iwaisako and Ohki (2019) and Ohki (2023), which conclude that subsidies for innovation by incumbents increase expected firm value and, in turn, promote the entry of new firms. Moreover, Acemoglu and Akcigit (2012) highlight the "trickle-down" effect, whereby treating more profitable companies more generously can increase the incentive for companies that have not yet generated significant profits to engage in innovation, ultimately contributing to overall economic growth. [20] In contrast to these views, the mechanism of our model operates as follows. A subsidy for incremental innovation by incumbents stimulates incremental innovation, which, in turn, increases the growth rate of common adjusted quality. An increase in the growth rate of common adjusted quality intensifies competition, thereby reducing the average value of incumbents. If the positive externality of incremental innovation accumulation —whereby it raises the growth rate of common adjusted quality— is strong, the average value of incumbents may decline despite a subsidy for incremental innovation by incumbents. In this case, the incentive for entry also decreases, leading to a reduction in the growth rate of the number of differentiated goods.

3.1.3 Simuration

In this subsection, we conduct a simulation analysis to visually confirm the obtained results in Subsection 3.1.2. Following preceding studies, we set a parameter values as $\rho = 0.04$, $g_L = 0.01$ and $\sigma = 5$. [21] From the discussion related to Figure 1, we verify that $-\left[-g_E - g_L + g_Q + \kappa_O - \kappa_I - \alpha g_{\theta C}\right] = 0.06$ and $-\left[-g_E - g_L + g_Q + \kappa_O - \alpha g_{\theta C}\right] = -0.14$. Accordingly, we set $\kappa_I = 0.2$ as a parameter value. The values specified so far will be consistently utilized in the subsequent analysis of this paper.

To set the remaining model parameters, we draw on actual data concerning long-term economic growth rates, ,the average lifespan of companies, and the finding of Bartelsman and Doms (2000) and Foster et al (2001). World long-term average economic growth rate is approximately 3% to 4%, we set the parameters to ensure that g_u is about 0.03 to 0.04 when $s_E = 1$ and $s_I = 1$. From (45), this implies that we set the parameters to ensure that g_Q is about 0.12 to 0.16 when $s_E = 1$ and $s_I = 1$. When $g_Q = 0.12$, $-\left[-0 - 0.01 + 0.12 + \kappa_O\right] = -0.14$ is satisfied, then we set $\kappa_O = 0.03$. Bartelsman and Doms (2000) and Foster et al. (2001) report that 75% of TFP growth stems from R&D activities conducted by incumbent firms. This result is interpreted as indicating that the growth rate of θ_C contributes 75% of the growth rate of Q, whereas that of n contributes 25%. Thus, we set the parameters to ensure that

[20] In our model, incumbents with greater accumulated incremental innovation and higher private adjusted quality incur higher innovation costs due to the parameter ψ. Since a subsidy is designed to cover a fixed percentage of these costs, high-profit incumbents receive larger subsidy amounts—effectively resulting in more generous treatment for more profitable incumbents. Strictly speaking, this differs from the setting in Acemoglu and Akcigit (2012), who argue that high-profit firms should receive stronger IPR protection. Nevertheless, it is fundamentally consistent with the innovation promotion policy in our model, in the sense that both approaches advocate preferential treatment for high-profit firms.

[21] Following Jones and Williams (2000), we interpret the interest rate as the expected internal rate of return to R&D projects, and we set $\rho = 0.04$. According to the World Development Indicators (World Bank, 2023), the average annual population growth rate in the United States between 1961 and 2023 was approximately 1%, and we set $g_L = 0.01$. Based on Basu (1996) and Norrbin (1993), the average markup of price over marginal cost, $\frac{\sigma}{\sigma-1}$, has been estimated as ranging between 1.05 and 1.4 and we set $\sigma = 5$, which means the markup is 1.25.

g_{θ_C} is approximately 0.09 to 0.12 and g_n is approximately 0.03 to 0.04 when $s_E = 1$ and $s_I = 1$. [22] The average lifespan of incumbents varies across years, ranging from approximately 20 to 35 years, we set parameters to ensure that $\hat{T}$ is about 10 when $s_E = 1$ and $s_I = 1$. [23]

[Insert Figure 2 here]

Figure 2 visually illustrates the effect of a subsidy (or tax) for innovation by entrants. We set the parameters $c_E = 1$, $c_I = 0.3$, $\psi = 0.02$, $\lambda = 0.055$, $\omega = 0.17$, $\gamma = 0.3$, $k = 2$, and $\theta_P(0) = 2$. The growth rates of u, Q, and n increase with a subsidy for entrants, whereas the growth rate of θ_C decreases, which is consistent with Proposition 3.1. We also confirm that the average value of $\hat{T}$ decreases with a subsidy for entrants, which is driven by a decline in incremental innovation by incumbents. Additionally, we observe that per capita expenditure decreases with a subsidy for entrants due to the reallocation of resources from production to innovative activities undertaken by entrants.

[Insert Figure 3 here]

Figure 3 visually illustrates the effect of a subsidy (or tax) for incremental innovation by incumbents. All parameters are identical to those in Figure 2. The growth rates of u, Q, and θ_C increase with a subsidy for incremental innovation by incumbents, which is consistent with Proposition 3.2. The growth rate of n also increases with a subsidy for incremental innovation by incumbents, which can be attributed to the modest rise in the growth rate of θ_C. We also confirm that the average value of $\hat{T}$ increases with a subsidy for incremental innovation by incumbents, which is driven by a increment in incremental innovation by incumbents. Furthermore, we observe that per capita expenditure decreases with a subsidy for incremental innovation by incumbents due to the reallocation of resources from production to innovative activities undertaken by both entrants and incumbents.

[Insert Figure 4 here]

[22] Small and Medium Enterprise Agency (2023) reports the following data: between 2016 and 2020, the average firm entry rate (net entry rate: entry rate minus exit rate) was recorded as follows: 5.0% (1.54%) in Japan, 9.6% (0.74%) in the United States, 12.7% (2.14%) in the United Kingdom, 7.6% (-1.56%) in Germany, and 10.8% (6.26%) in France. Averaging these values across the five countries results in an overall average firm entry rate of 9.1% (1.8%). We confirm that our benchmark parameter settings, g_n is approximately 0.03 to 0.04, are consistent with actual data , in the sense that they fall within the range of entry rates and net entry rates observed across countries.

[23] Hillenbrand et al (2017) show the data that the organizations in the S&P 500 index had been on that list for an average of approximately 35 years in the late 1970s. However recently, the average tenure has been on a declining trend, and the average tenure is closer to 20 years. Tokyo Shoko Research, ltd. reports that in 2023, the average lifespan of bankrupt firms in the manufacturing industry in Japan was 36.3 years. In our model, incumbents with $\hat{T} = 10$ decreases the profit less than half (one-tenth) of the initial level 20 (31) years after the start of operations. We do not strictly define firm exit as occurring when profits decline by a certain percentage. However, based on this general idea, we set the average duration of incremental innovation to 10 years, taking into account both empirical evidence and the behavior of our model. This calibration ensures that the timing at which the average incumbent loses their influence in our model is broadly consistent with observed patterns of firm exit in the real world.

Figure 4 visually illustrates the effect of a subsidy (or tax) for incremental innovation by incumbents when γ is high. We modify the parameters $\lambda = 0.044$, $\gamma = 0.4$, while keeping all other parameters identical to those in Figure 3. These adjustments are intended to enhance the responsiveness of the growth rate of θ_C to an increase in incremental innovation. As pointed out in Proposition 3.2, the growth rate of n decreases with a subsidy for incremental innovation by incumbents, which can be attributed to the significant rise in the growth rate of θ_C.

3.2 The case where $\alpha = 1$

In this subsection, we analyze the special cases where $\alpha = 1$. When $\alpha = 1$, incumbents that have already commenced operations fully benefit from improvements in the common adjusted quality, to the same extent as entrants that have not yet entered the market.

3.2.1 Equilibrium

Noting that $g_Q = g_n + g_{\theta C}$ holds, we can rewrite equation (39), (42), and (43) as follows:

$$\frac{1}{[\rho - g_L + g_n + \kappa_O - \kappa_I + k\psi]} \frac{\theta_P(0)}{[k+1]} \left[\xi^C\right]^{-[k+1]} = \frac{s_E c_E}{s_I c_I(0)}, \tag{50}$$

$$\xi^C = \frac{s_I c_I(0)}{\kappa_I} \sigma \left[\rho - g_L + g_n + \kappa_O\right], \tag{51}$$

$$\Theta = \theta_P(0) \left[\xi^C\right]^{-k} \frac{[g_n + \kappa_O + k\psi]}{[g_n + \kappa_O - \kappa_I + k\psi]\,[g_n + \kappa_O]}. \tag{52}$$

Equation (50) and (51) determine the equilibrium value of g_n and ξ^C. By substituting the equilibrium values of g_n and ξ^C into equations (44) and (52), we derive the equilibrium values of $g_{\theta C}$ and Θ, respectively. Next, plugging the equilibrium value of g_n and $g_{\theta C}$ into equation (36) allows us to obtain the equilibrium value of g_Q. Subsequently, inserting the equilibrium value of g_Q into equation (45) yields the equilibrium value of g_u. Finally, substituting the equilibrium value of g_n, ξ^C, Θ, and $\alpha = 1$ into equation (38) results in the equilibrium value of E.

3.2.2 Policy analysis

We begin by analyzing the impact of a subsidy for innovation by entrants, which corresponds to a decrease in s_E. LHS of equation (50) is a decreasing function of g_n, while RHS of equation (50) is a increasing function of s_E. Therefore, a decrease in s_E results in an increase in g_n. According to equation (51), an increase in g_n raises ξ^C, and a higher ξ^C reduces $g_{\theta C}$, as shown in equation (44). While g_n rises and $g_{\theta C}$ declines, if the effect of the former (latter) dominates, g_Q will increase (decrease). This, in turn, leads to an increase (decrease) in the overall economic growth rate, g_u. If innovation by entrants is taxed, the results are exactly the opposite.

We summarize above discussion in the following proposition:

Proposition 3.3. *When we set $\alpha = 1$, indicating that incumbents can benefit equally from the positive external derived from improvements in the common adjusted quality compared to entrants before operation, we obtain the following results.*

- *A subsidy (a tax) for innovation by entrants stimulates (mitigates) innovation by entrants, and then increases (decreases) the horizontal growth rate, g_n.*
- *A subsidy (a tax) for innovation by entrants mitigates (stimulates) incremental innovation by incumbents, and then decreases (increases) the vertical growth rate, $g_{\theta C}$.*
- *The impact of a subsidy (a tax) for innovation by entrants on growth rate of the aggregate adjusted quality, g_Q, is ambiguous; consequently, the impact on the overall growth rate of economics, g_u, is also ambiguous.*

The intuition behind this proposition follows the same general flow as the explanation of the effect of a subsidy for entrants in the case where $\alpha = 0$. A key difference is that incumbents who have already commenced operations benefit from improvements in the common adjusted quality when $\alpha > 0$. Moreover, when $\alpha = 1$, they benefit fully, to the same extent as entrants that have not yet entered the market. In this special case, the average value of incumbents is not affected by $g_{\theta C}$, because the effect of a higher $g_{\theta C}$ in intensifying competition is exactly offset by the effect of a higher $g_{\theta C}$ in increasing incumbents' adjusted quality when $\alpha = 1$. For the same reason, the incentive to conduct incremental innovation is also not affected by $g_{\theta C}$. From equation (50) and (51), we can confirm that both the free entry condition and the optimality condition for $\hat{T}$ are independent of the value of $g_{\theta C}$.

As shown earlier, a subsidy for innovation by entrants reduces the incentive to conduct incremental innovation. This leads to a decrease in $g_{\theta C}$, which, when $\alpha = 0$, increases the average value of incumbents by reducing competitive pressure. However, this effect does not arise when $\alpha = 1$. Therefore, compared to the case where $\alpha = 0$, the positive effect of a subsidy for entrants on g_n becomes smaller. Moreover, if the negative effect of such subsidy on $g_{\theta C}$ is sufficiently large, the growth rate of aggregate adjusted quality, g_Q, and thus overall economic growth rate,g_u, may decline.

Next, we analyze the effect of a subsidy for incremental innovation by incumbents, which corresponds to a decrease in s_I. By substituting (51) into (50), we obtain:

$$\frac{1}{[\rho - g_L + g_n + \kappa_O - \kappa_I + k\psi]\,[\rho - g_L + g_n + \kappa_O]^{k+1}} \frac{1}{[k+1]} \left[\frac{\kappa_I}{\sigma}\right]^{k+1} = [s_I c_I(0)]^k\, s_E c_E. \tag{53}$$

LHS of equation (53) is a decreasing function of g_n, while RHS of equation (53) is a increasing function of s_I. Therefore, a decrease in s_I results in an increase in g_n. RHS of (50) increases as s_I decreases, while LHS of (50) decreases as g_Q increases; therefore, ξ^C must decrease to maintain equality. A lower ξ^C raises $g_{\theta C}$, as shown in equation (44). From (36), since both g_n and $g_{\theta C}$ increase, the growth rate of the aggregate adjusted quality, g_Q, increases, which subsequently raises the economic growth rate g_u. If innovation by incumbents is taxed, the results are exactly the opposite.

We summarize above discussion in the following proposition:

Proposition 3.4. *When we set $\alpha = 1$, we obtain the following results.*

- *A subsidy (a tax) for incremental innovation by incumbents stimulates (mitigates) innovation by entrants, and then increases (decreases) the horizontal growth rate, g_n.*
- *A subsidy (a tax) for incremental innovation by incumbents stimulate (mitigates) incremental innovation by incumbents, and then increases (decreases) the vertical growth rate, $g_{\theta C}$.*
- *A subsidy (a tax) for incremental innovation by incumbents increases (decreases) growth rate of the aggregate adjusted quality, g_Q, and then increases (decreases) overall growth rate of economy, g_u.*

The intuition behind this proposition follows the same general flow as the explanation of the effect of a subsidy for incumbents in the case where $\alpha = 0$. A key difference is that the average value of incumbents is not affected by $g_{\theta C}$. A subsidy for incremental innovation by incumbents increases the vertical growth rate, $g_{\theta C}$, which, when $\alpha = 0$, reduces the average value of incumbents by strengthening competitive pressure. However, this effect does not arise when $\alpha = 1$. Thus, given a fixed value of g_n, a subsidy for incremental innovation necessarily leads to an increase in the average value of incumbents. As a result, more entrants engage in innovation, leading to a necessary increase in the growth rate of the number of differentiated goods.

We summarize the main result in below table.

$\alpha = 1$	g_Q	g_n	$g_{\theta C}$
$s_E \downarrow$	?	$\uparrow$	$\downarrow$
$s_I \downarrow$	$\uparrow$	$\uparrow$	$\uparrow$

Table 2: Summary of policy analysis when $\alpha = 1$

A notable finding from the analysis in this subsection is that it points to the possibility that a subsidy for innovation by entrants may lower the overall economic growth rate. It is natural that a subsidy for innovation by entrants promotes the entry of new firms, thereby increasing the economic growth rate, and many prior studies have reached the same conclusion. In contrast to this view, Ohki (2023) shows the potential deleterious effects of subsidies provided to new entrants on the economy. [24] Acemoglu and Cao (2015) also reports the possibility that entry barriers or taxes on entrants can have a positive effect on growth. [25] In this paper, we construct a model in which the economy grows through two sources of growth, which respond in opposite directions to subsidies for entrants. Subsidies for entrants increase the growth rate of the number of differentiated goods while reducing the growth rate of common adjusted quality. When the latter effect dominates the former, subsidies for entrants are not beneficial to the economy.

Our result can also be interpreted as suggesting that support for new entrants and the deregulation of entry barriers may have adverse effects on the economy. Regarding patent length, where a longer duration implies higher barriers to entry, previous studies have examined its impact on economic performance. Iwaisako and Futagami (2003), Futagami and Iwaisako (2007), and Acemoglu and Akcigit (2012) argue that a finite patent duration is optimal for the economy, suggesting that excessively low entry barriers

[24] They endogenize the average quality of differentiated goods by incorporating technology switching activity and assume that the entry cost for entrants benefits from a positive externality of the average quality. Subsidies for entrants reduce the incentive to switch technologies, leading to a decline in the average quality of differentiated goods. If the positive externality of innovation from average quality is strong, support for entrants reduces the incentive for entrants to innovate. This, in turn, lowers the growth rate of the number of differentiated goods, which is the only source of economic growth in their model.

[25] Their mechanism is similar to that of the present paper in that the rate of entry of new firms depresses the profitability of incumbent innovation and, through this channel, may reduce aggregate productivity growth. However, the role of entrants differs clearly between their paper and ours. In their model, entrants engage in radical innovation, which displaces existing incumbents. In contrast, in our model, entrants invent new differentiated goods, as in the simple variety expansion model.

can have detrimental effects on economic growth. Similarly, research on patent breadth, which also serves as a measure of entry barriers, has explored its role in fostering or hindering economic expansion. Li (2001), Goh and Olivier (2002), Chu (2011), Saito (2017), and Iwaisako (2020) assert that broader patent protection, which effectively raises entry barriers, contributes positively to economic growth. These findings indicate that while excessive restrictions on market entry may stifle competition and innovation, a certain degree of entry barriers, whether through patent length or breadth, can be beneficial for sustained economic development. [26]

3.2.3 Simuration

In this subsection, we conduct a simulation analysis to visually confirm the obtained results in Subsection 3.2.2. We set the parameters to be as close as possible to those in Subsection 3.1.3, but some parameter values have been modified as follows. When $\alpha = 1$, $g_E = 0$ and $g_L = 0.01$ hold, $-\left[-g_E - g_L + g_Q + \kappa_O - \alpha g_{\theta C}\right] = -\left[-0 - 0.01 + g_n + \kappa_O\right] = -0.14$ is satisfied. We set $\kappa_O = 0.12$, anticipating that the simulation will result in g_n being around 0.03. When $\alpha = 1$ holds, the positive spillover effect is stronger than the case $\alpha = 0$. As a result, both the growth rate and the expected duration of incremental innovation take higher values under the given parameters. Accordingly, we adjust the parameters by setting $\psi = 0.025$ and $\theta_P(0) = 1$. All remaining parameters, except for λ and γ, are identical to those in Subsection 3.1.3.

[Insert Figure 5 here]

Figure 5 visually illustrates the effect of a subsidy (or tax) on innovation by entrants. We set the parameters $\lambda = 0.078$ and $\gamma = 0.3$. The growth rates of n increase with a subsidy for entrants, whereas the growth rate of θ_C decreases, which is consistent with Proposition 3.3. The growth rate of u and Q also increases with a subsidy for entrants, which can be attributed to the modest decline in the growth rate of θ_C. The effect of innovation by entrants on $\hat{T}$ and expenditure are similar to those shown in Figure 2.

[Insert Figure 6 here]

Figure 6 visually illustrates the effect of a subsidy (or tax) on innovation by entrants when γ is high. We modify the parameters $\lambda = 0.016$, $\gamma = 1.4$, while keeping all other parameters identical to those in

[26] Please consider reading comprehensive survey study, Chu (2022), which presents a detailed and well-organized discussion on the topic.

Figure 5. These adjustments are intended to enhance the responsiveness of the growth rate of θ_C to a decrease in incremental innovation. As pointed out in Proposition 3.3, the growth rate of u and Q decreases with a subsidy for entrants, which can be attributed to the significant decrease in the vertical growth rate of θ_C.

[Insert Figure 7 here]

Figure 7 visually illustrates the effect of a subsidy (or tax) on incremental innovation by incumbents. All parameters are identical to those in Figure 5. The growth rates of u, Q, n and θ_C increase with a subsidy for incremental innovation by incumbents, which is consistent with Proposition 3.4. The effect of a subsidy (or tax) on incremental innovation by incumbents on $\hat{T}$ and expenditure are similar to those shown in Figure 3.

3.3 Discussion

In this subsection, we present a preliminary discussion of a more general version of the model. In Subsection 3.1 and 3.2, we examine the analytically tractable case by setting $\alpha = 0$ and $\alpha = 1$ respectively. However, in general, it is feasible to consider the full range $0 \leq \alpha \leq 1$, and our result represents a combination of the results in Subsections 3.1 and 3.2. The anticipated results are summarized in the following table.

$0 \leq \alpha \leq 1$	g_Q	g_n	$g_{\theta C}$
$s_E \downarrow$	?	$\uparrow$	$\downarrow$
$s_I \downarrow$	$\uparrow$	?	$\uparrow$

Table 3: Summary of anticipated policy outcome when $0 \leq \alpha \leq 1$

As shown in the above table, our model suggests that a subsidy for incremental innovation by incumbents consistently enhances overall economic growth. However, can we confidently conclude that such subsidy always have a positive effect on the economy? Further rigorous analysis is required before making such a definitive claim.

We set equation (12) as integral of the average gross growth rate of private quality, $\mathbb{E}\left[\theta_P\left(\tau\right)\right]/\theta_P\left(0\right)$, weighted by an exogenous discount value, $\exp\left[-\omega\tau\right]$, from the past up to the present. This reflects the idea that common adjusted quality evolves through the accumulation of individual innovations. However, to more accurately capture the accumulation of individual innovations, it may be more appropriate to formulate it as the integral of the gross growth rates of private quality across all differentiated goods that commenced in the same period, $\dot{n}(t-\tau)\,\mathbb{E}\left[\theta_P(\tau)\right]/\theta_P(0)$, rather than using the average gross growth rate, weighted by an exogenous discount value and the number of differentiated goods in the reference period , $\exp\left[-\omega\tau\right]/n(t-\bar{\tau})$, from the past up to the present. In this setting, the equation that determines vertical growth is a function of g_n, and can be rewritten as:

$$g_{\theta C} = \lambda \left[\int_0^\infty \exp\left[-\omega\tau\right] \exp\left[g_n\left[\bar{\tau}-\tau\right]\right] g_n \frac{\mathbb{E}\left[\theta_P(\tau)\right]}{\theta_P(0)} d\tau \right]^\gamma .$$

This modification makes $g_{\theta C}$ a function of g_n. This introduces a new mechanism that a subsidy for incremental innovation decreases the growth rate of n, g_n, thereby weakening the growth effect of θ_C, and potentially even reducing it. As a result, the impact of a subsidy to incumbents on overall growth becomes ambiguous, which does not align with the findings of the present paper. This new finding also suggests that excessive preferential policies for incumbents may not always benefit the overall economy, just like a subsidy for entrants.

Based on the above discussion, such an extension would make the model more refined and allow for the derivation of more general results. However, since it would be too complex to analyze analytically, we limit ourselves to briefly discussing the expected outcomes in the present paper.

4 Conclusion

The main contribution of present paper is following points. First, we constructed a tractable model where heterogeneous incumbents conduct innovation in order to improve their current technology, and entrants conduct innovation in order to enter the market. This represents a legitimate extension of the R&D-based endogenous growth model. Second, our present model, when considered together with Ohki (2023), can explain the argument made by Christensen (1997), which argues that large firms tend to be proactive in incremental innovation but reluctant to pursue disruptive innovation. In this sense, we have succeeded in capturing a managerial perspective within the framework of an R&D-based endogenous growth model in economics. Third, we construct an in-house model in which both horizontal and vertical growth are endogenously determined and influenced by policy variables. Empirically, Bartelsman and Doms (2000) present data showing that both entrants and incumbents contribute to TFP growth. Theoretically, Peretto and Connolly (2007) argue that horizontal and vertical innovation have distinct implications. Taken together, these points suggest that our third contribution is also meaningful for the development of R&D-based growth models. Fourth, we derive results that cannot be obtained in conventional models—namely, that a subsidy to entrants do not necessarily lead to positive overall economic growth, nor do a subsidy to incumbents necessarily enhance the entry incentives of potential entrants. Finally, our model is sufficiently simple and tractable to allow for further extensions, which we leave for future work.

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A Appendix

By evaluating the integrals in equation (33), the value of incumbents with $\xi \geq \xi^C$ can be expressed as follows:

$$V(\xi,s) = \begin{array}{l} \frac{1-\exp\left[-[r-g_E-g_L+g_Q+\kappa_O-\kappa_I-ag_{\theta C}]\hat{T}(\xi)\right]}{[r-g_E-g_L+g_Q+\kappa_O-\kappa_I-ag_{\theta C}]}\frac{\theta_P(0)}{\sigma}\frac{\theta_C(s)E(s)L(s)}{Q(s)} \\ -\frac{1-\exp\left[-[r-g_E-g_L+g_Q+\kappa_O-\kappa_I-\psi-ag_{\theta C}]\hat{T}(\xi)\right]}{[r-g_E-g_L+g_Q+\kappa_O-\kappa_I-\psi-ag_{\theta C}]}\frac{s_Ic_I(0)}{\xi}\frac{\theta_P(0)\theta_C(s)E(s)L(s)}{Q(s)} \\ +\frac{\exp\left[-[r-g_E-g_L+g_Q+\kappa_O-\kappa_I-ag_{\theta C}]\hat{T}(\xi)\right]}{[r-g_E-g_L+g_Q+\kappa_O-ag_{\theta C}]}\frac{\theta_P(0)}{\sigma}\frac{\theta_C(s)E(s)L(s)}{Q(s)} \\ -\frac{1}{[r-g_E-g_L+g_Q+\kappa_O-ag_{\theta C}]}\frac{\theta_P(0)}{\sigma}\frac{\theta_C(s)E(s)L(s)}{Q(s)} \end{array}.$$

Substituting equation (28) into the above equation, we obtain:

$$V(\xi,s) = \begin{array}{l} \frac{1-\left[\frac{\xi}{\xi^C}\right]^{-\frac{[r-g_E-g_L+g_Q+\kappa_O-\kappa_I-ag_{\theta C}]}{\psi}}}{[r-g_E-g_L+g_Q+\kappa_O-\kappa_I-ag_{\theta C}]}\frac{\theta_P(0)}{\sigma}\frac{\theta_C(s)E(s)L(s)}{Q(s)} + \frac{\left[\frac{\xi}{\xi^C}\right]^{-\frac{[r-g_E-g_L+g_Q+\kappa_O-\kappa_I-ag_{\theta C}]}{\psi}}}{[r-g_E-g_L+g_Q+\kappa_O-ag_{\theta C}]}\frac{\theta_P(0)}{\sigma}\frac{\theta_C(s)E(s)L(s)}{Q(s)} \\ -\frac{1-\left[\frac{\xi}{\xi^C}\right]^{-\frac{[r-g_E-g_L+g_Q+\kappa_O-\kappa_I-\psi-ag_{\theta C}]}{\psi}}}{[r-g_E-g_L+g_Q+\kappa_O-\kappa_I-\psi-ag_{\theta C}]}\frac{s_Ic_I(0)}{\xi}\frac{\theta_P(0)\theta_C(s)E(s)L(s)}{Q(s)} - \frac{1}{[r-g_E-g_L+g_Q+\kappa_O-ag_{\theta C}]}\frac{\theta_P(0)}{\sigma}\frac{\theta_C(s)E(s)L(s)}{Q(s)} \end{array}$$

By using $\xi^C = \frac{s_Ic_I(0)}{\kappa_I}\sigma\left[r-g_E-g_L+g_Q+\kappa_O-ag_{\theta C}\right]$ and evaluating above equation, we get:

$$V(\xi,s) = \frac{\theta_P(0)\,\theta_C(s)\,E(s)\,L(s)}{Q(s)}s_Ic_I(0)\left[\begin{array}{l} \frac{1-\left[\frac{\xi}{\xi^C}\right]^{-\frac{[r-g_E-g_L+g_Q+\kappa_O-\kappa_I-ag_{\theta C}]}{\psi}}}{[r-g_E-g_L+g_Q+\kappa_O-\kappa_I-ag_{\theta C}]}\frac{1}{\xi^C} \\ -\frac{1-\left[\frac{\xi}{\xi^C}\right]^{-\frac{[r-g_E-g_L+g_Q+\kappa_O-\kappa_I-\psi-ag_{\theta C}]}{\psi}}}{[r-g_E-g_L+g_Q+\kappa_O-\kappa_I-\psi-ag_{\theta C}]}\frac{1}{\xi} \end{array}\right].$$

Substituting this equation into (33) and evaluating it, we obtain:

$$\mathbb{E}\left[V(\xi,s)\right] = \frac{\theta_P(0)\,\theta_C(s)\,E(s)\,L(s)}{Q(s)}\frac{1}{[r-g_E-g_L+g_Q+\kappa_O-\kappa_I-ag_{\theta C}+k\psi]}\frac{s_Ic_I(0)}{[k+1]}\left[\xi^C\right]^{-[k+1]}.$$

Consequently, the free entry condition ($g_n > 0$) can be expressed as follows:

$$\frac{1}{[r-g_E-g_L+g_Q+\kappa_O-\kappa_I-ag_{\theta C}+k\psi]}\frac{\theta_P(0)}{[k+1]}\left[\xi^C\right]^{-[k+1]} = \frac{s_Ec_E}{s_Ic_I(0)}.$$

B Appendix

In this appendix, we derive equation (35) and equation (44). The aggregate adjusted quality of incumbents operating for τ periods is expressed as the product of the number of incumbents operating for τ periods and their expected adjusted quality, as follows:

$$\dot{n}\left(t-\tau\right)\mathbb{E}\left[\theta\left(\tau,t-\tau\right)\right].$$

The number of incumbents operating for τ periods is expressed as:

$$\dot{n}\left(t-\tau\right)=n\left(t-\tau\right)\frac{\dot{n}\left(t-\tau\right)}{n\left(t-\tau\right)}=n\left(t-\tau\right)g_n=\exp\left[-g_n\tau\right]n\left(t\right)g_n.$$

The expected value of the adjusted quality of incumbents operating for τ periods is expressed as the product of the common adjusted quality and the expected private adjusted quality of incumbents:

$$\mathbb{E}\left[\theta\left(\tau,s\right)\right]=\theta_C\left(\tau,t-\tau\right)\mathbb{E}\left[\theta_P\left(\tau\right)\right].$$

The common adjusted quality of incumbents that commenced operations in period $t-\tau$ and continued operating for a duration of τ is expressed as:

$$\theta_C\left(\tau,t-\tau\right)=\theta_C\left(t-\tau\right)\exp\left[\alpha g_{\theta C}\tau\right]=\exp\left[-g_{\theta C}\tau\right]\theta_C\left(t\right)\exp\left[\alpha g_{\theta C}\tau\right]=\theta_C\left(t\right)\exp\left[-\left[1-\alpha\right]g_{\theta C}\tau\right].$$

The expected private adjusted quality of incumbents operating for τ periods is expressed as:

$$\mathbb{E}\left[\theta_P\left(\tau\right)\right]=\int\limits_{\xi^{c1}}^{\xi(\tau)}\theta_P\left(0\right)\exp\left[-\kappa_O\tau\right]\exp\left[\kappa_I\hat{T}\left(\xi\right)\right]dG\left(\xi\right)+\int\limits_{\xi(\tau)}^{\xi^{\max}}\theta_P\left(0\right)\exp\left[-\left[\kappa_O-\kappa_I\right]\tau\right]dG\left(\xi\right).$$

The first term represents the expected private adjusted quality of incumbents that have already ceased incremental innovation, while the second term represents the expected private adjusted quality of incumbents that are still engaged in incremental innovation. Using (19) and (28), we can obtain:

$$\mathbb{E}\left[\theta_P\left(\tau\right)\right]=\theta_P\left(0\right)\frac{\left[\xi^C\right]^{-k}}{\kappa_I-k\psi}\left[\kappa_I\exp\left[-\left[\kappa_O-\kappa_I+k\psi\right]\tau\right]-k\psi\exp\left[-\kappa_O\tau\right]\right].$$

By integrating the aggregate adjusted quality of incumbents from each generation, extending from the present far back into the past, we obtain the aggregate adjusted quality of differentiated goods as follows:

$$Q(t) = \int_0^\infty \dot{n}(t-\tau)\, \mathbb{E}\left[\theta(\tau, t-\tau)\right] d\tau.$$

By calculating above equation, we can derive (35), which is expressed as:

$$Q(t) = n(t)\, \theta_C(t)\, g_n \Theta,$$

where

$$\Theta = \theta_P(0) \left[\xi^C\right]^{-k} \frac{\left[g_Q + \kappa_O - \alpha g_{\theta C} + k\psi\right]}{\left[g_Q + \kappa_O - \kappa_I - \alpha g_{\theta C} + k\psi\right]\left[g_Q + \kappa_O - \alpha g_{\theta C}\right]}.$$

Nex we evaluate (44). Substituting $\mathbb{E}\left[\theta_P(\tau)\right]$ into (12), we obtain the following equation as:

$$g_{\theta C} = \lambda \left[\xi^C\right]^{-\gamma k} \left[\frac{\left[\kappa_O + k\psi + \omega\right]}{\left[\kappa_O - \kappa_I + k\psi + \omega\right]\left[\kappa_O + \omega\right]}\right]^\gamma.$$

C Appendix

From (14), labor demand for production of incumbents with $\theta(\tau, s)$ is $l^D(\theta(\tau, s)) = \theta(\tau, s) \frac{\sigma-1}{\sigma} \frac{E(t)L(t)}{Q(t)}$. Then the aggregate labor demand for production of incumbents, who commenced operations in period $t-\tau$ and have been operating for τ periods is expressed as:

$$l^d(\tau, t-\tau) = \exp\left[-g_n \tau\right] g_n n(t)\, \mathbb{E}\left[\theta(\tau, s)\right] \frac{\sigma-1}{\sigma} \frac{E(t)\, L(t)}{Q(t)}.$$

By integrating the aggregate labor demand for production of incumbents from each generation, extending from the present far back into the past, we obtain the aggregate labor demand for production at time t as follows:

$$L^D(t) = \int_0^\infty \exp\left[-g_n \tau\right] g_n n(t)\, \mathbb{E}\left[\theta(\tau, t-\tau)\right] \frac{\sigma-1}{\sigma} \frac{E(t)\, L(t)}{Q(t)} d\tau = Q(t) \frac{\sigma-1}{\sigma} \frac{E(t)\, L(t)}{Q(t)} = \frac{\sigma-1}{\sigma} E(t)\, L(t).$$

From equation (22), the labor demand for incremental innovation by incumbents with ξ, who commenced operations in period $t-\tau$ and have been operating for τ periods, is given by:

$$l_I(\xi,\tau,t-\tau)=\frac{1}{\xi}c_I(0)\exp\left[-\left[\kappa_O-\kappa_I+[1-\alpha]g_{\theta C}-\psi\right]\tau\right]\frac{\theta_P(0)\theta_C(t)E(t)L(t)}{Q(t)}.$$

From equation (28), after a duration of τ, only incumbents with $\xi\geq\xi(\tau)=\exp[\psi\tau]\xi^C$ continue to engage in incremental innovation. Then the labor demand for incremental innovation by incumbents , who commenced operations in period $t-\tau$ and have been operating for τ periods, is expressed as:

$$\begin{aligned}&l^I(\tau,t-\tau)\\&=\exp[-g_n\tau]g_nn(t)\left[\int\limits_{\xi(\tau)}^{\infty}\tfrac{1}{\xi}c_I(0)\exp\left[-\left[\kappa_O-\kappa_I+[1-\alpha]g_{\theta C}-\psi\right]\tau\right]\tfrac{\theta_P(0)\theta_C(t)E(t)L(t)}{Q(t)}k\xi^{-k-1}d\xi\right].\end{aligned}$$

By calculating above equation, we can derive the following expression:

$$l^I(\tau,t-\tau)=g_nn(t)\frac{k}{k+1}c_I(0)\exp\left[-\left[g_Q+\kappa_O-\kappa_I-\alpha g_{\theta C}+k\psi\right]\tau\right]\left[\xi^C\right]^{-[k+1]}\frac{\theta_P(0)\theta_E(t)E(t)L(t)}{Q(t)}.$$

By integrating the aggregate labor demand for incremental innovation of incumbents from each generation, extending from the present far back into the past, we obtain the aggregate labor demand for incremental innovation at time t as follows:

$$L^I(t)=\int\limits_0^{\infty}l^I(\tau,t-\tau)d\tau=\theta_P(0)c_I(0)\frac{k}{k+1}\left[\xi^C\right]^{-[k+1]}\frac{1}{\left[g_Q+\kappa_O-\kappa_I-\alpha g_{\theta C}+k\psi\right]}\frac{E(t)L(t)}{\Theta}.$$

At time t, $n(t)g_n$ incumbents commence operation. From (30), the labor demand for operation of new incumbents is expressed as:

$$L^O(t)=n(t)g_n\frac{1}{\left[r-g_E-g_L+g_Q+\kappa_O-\alpha g_{\theta C}\right]}\frac{1}{\sigma}\frac{\theta_P(0)\theta_C(t)E(t)L(t)}{Q(t)}.$$

By calculating above equation and using equatioin (43), we can derive the following expression:

$$L^O(t)=\frac{1}{\kappa_I}\frac{1}{\xi^C}\theta_P(0)s_Ic_I(0)\frac{E(s)L(s)}{\Theta}.$$

Since only a proportion of $\int\limits_{\xi^C}^{\infty}k\xi^{-k-1}d\xi=\left[\xi^C\right]^{-k}$ of all entrants commence operations, the number of entrants at time t is given by $\frac{1}{[\xi^C]^{-k}}n(t)g_n$. From (18), the labor demand for invent new differentiated goods of entrants is expressed as:

$$L^{E}(t)=c_{E}\left[\xi^{C}\right]^{k}\frac{E(t)\,L(t)}{\Theta}.$$

Substituting above equation into (37), we obtain the following equation:

$$E(t)=\left[\begin{array}{l}\frac{\sigma-1}{\sigma}+\frac{1}{\Theta}\theta_{P}(0)\,c_{I}(0)\,\frac{k}{k+1}\left[\xi^{C}\right]^{-[k+1]}\frac{1}{[g_{Q}+\kappa_{O}-\kappa_{I}+k\psi-\alpha g_{\theta C}]}\\+\frac{1}{\Theta}\frac{1}{\kappa_{I}}\frac{1}{\xi^{C}}\theta_{P}(0)\,s_{I}c_{I}(0)+\frac{1}{\Theta}\left[\xi^{C}\right]^{k}c_{E}\end{array}\right]^{-1}.$$

D Appendix

In this appendix, we derive the expected value of $\hat{T}$. From (19) and (28), the expected value of $\hat{T}$ is expressed as:

$$\mathbb{E}\left[\hat{T}(\xi)\right]=\int_{\xi^{C}}^{\infty}\frac{1}{\psi}\ln\frac{\xi}{\xi^{C}}k\xi^{-k-1}d\xi.$$

By applying the formula for integration by parts, the expression can be rewritten as follows:

$$\mathbb{E}\left[\hat{T}(\xi)\right]=\left[-\frac{1}{\psi}\xi^{-k}\ln\frac{\xi}{\xi^{C}}\right]_{\xi^{C}}^{\infty}-\int_{\xi^{C}}^{\infty}-\frac{1}{\psi}\xi^{-k-1}d\xi.$$

Since the first term disappears, we obtain:

$$\mathbb{E}\left[\hat{T}(\xi)\right]=-\int_{\xi^{C}}^{\infty}-\frac{1}{\psi}\xi^{-k-1}d\xi=\frac{1}{k\psi}\left[\xi^{C}\right]^{-k}.$$

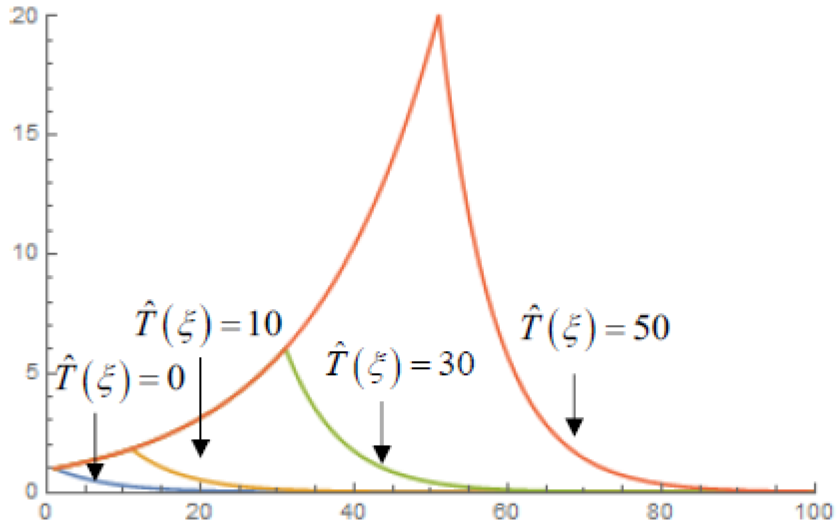

Figure 1: Dynamics of profit based on the value of $\hat{T}\,(\xi)$

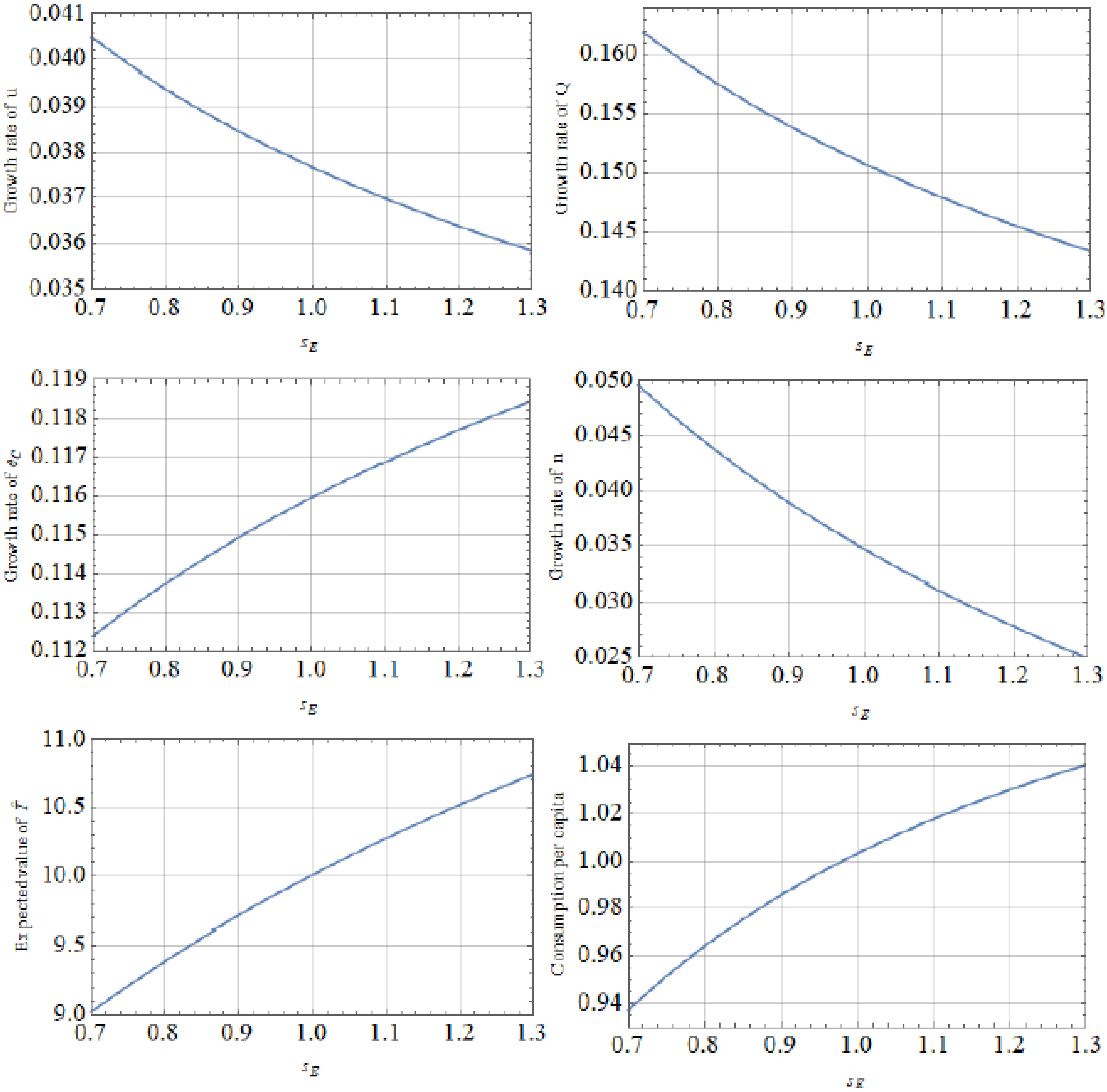

Figure 2: An analysis of the policy effects of entry subsidies and entry taxes when $\alpha = 0$.

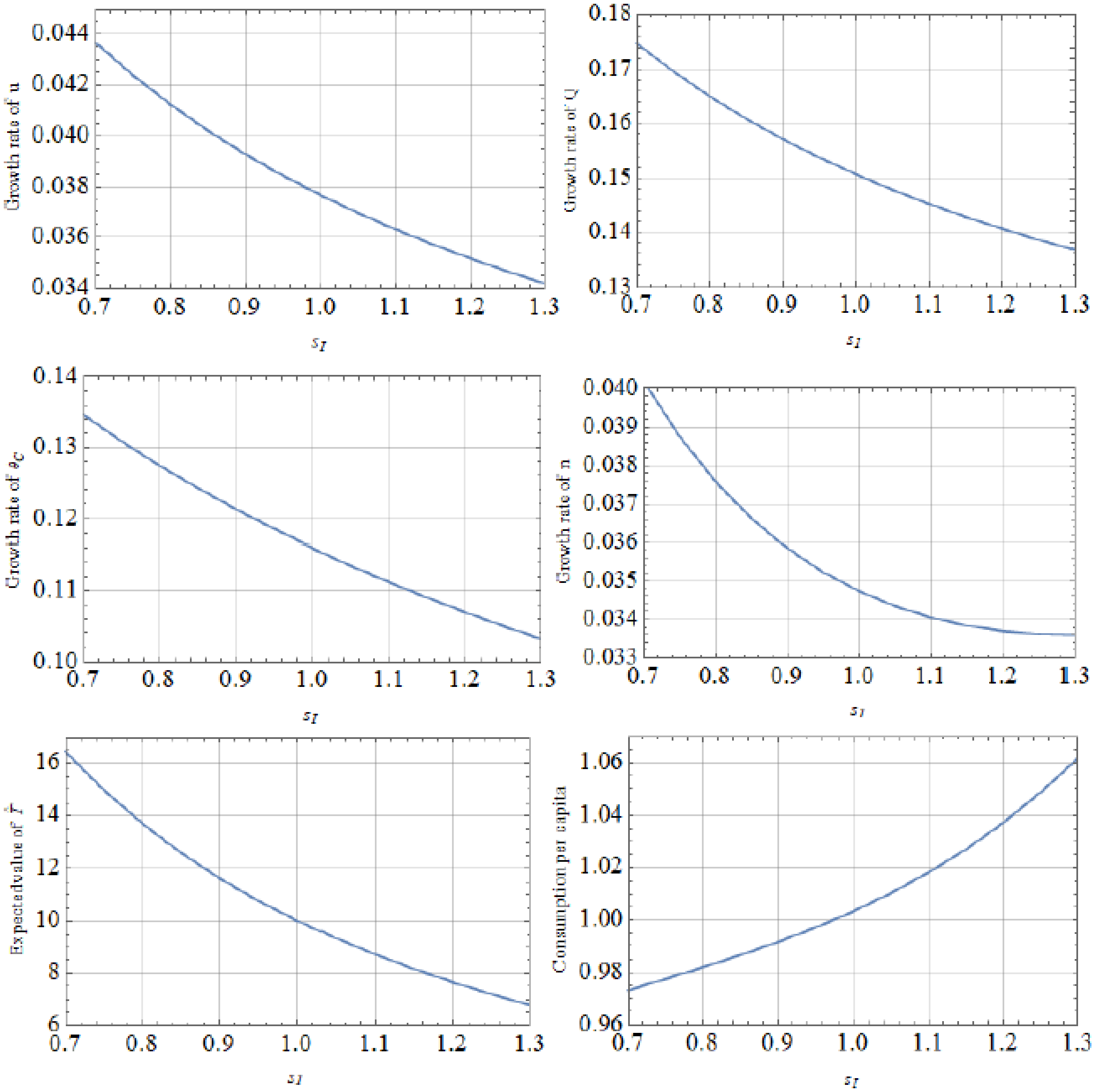

Figure 3: An analysis of the policy effects of subsidies and taxes on incremental innovation when $\alpha = 0$.

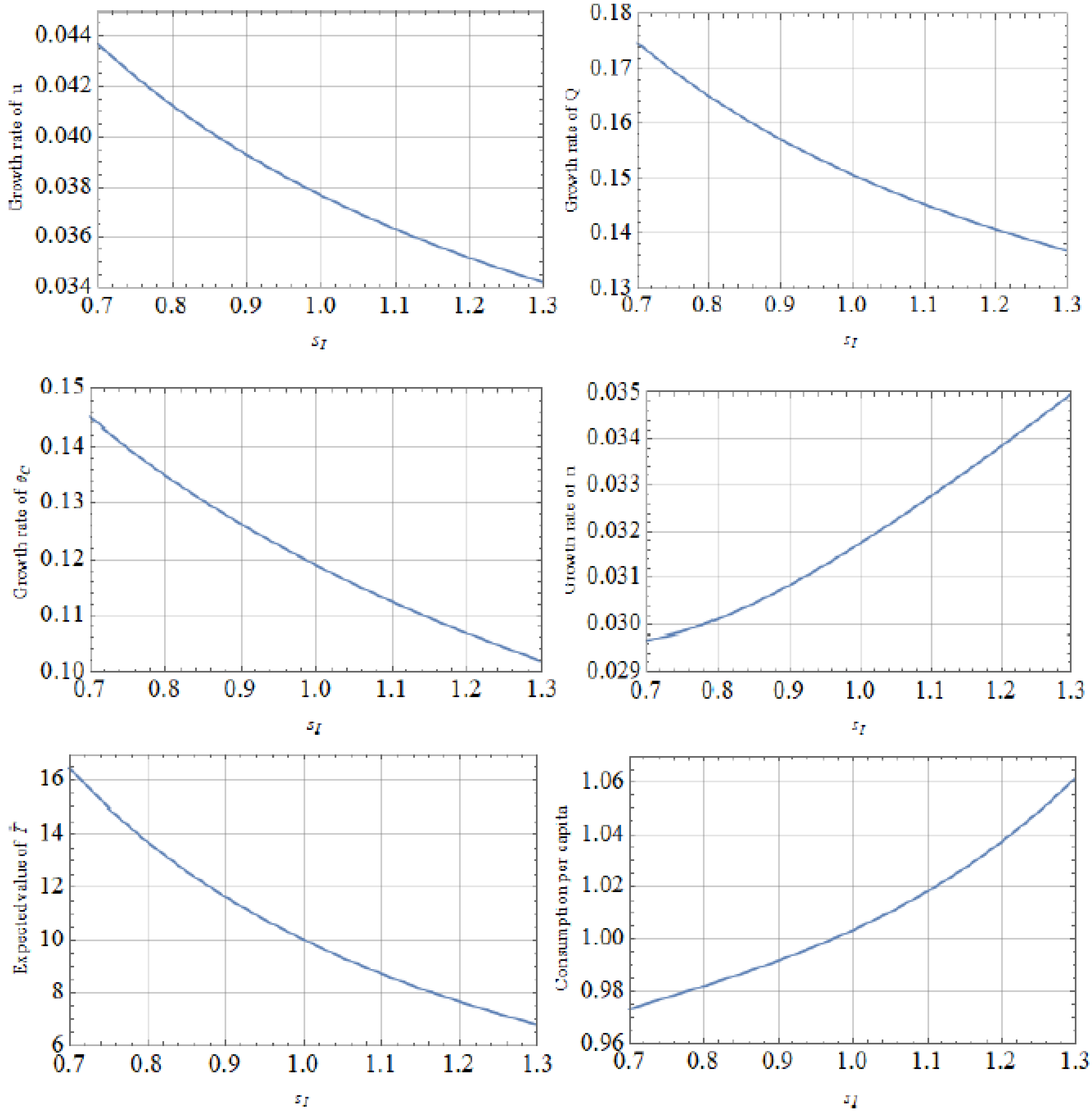

Figure 4: An analysis of the policy effects of subsidies and taxes on incremental innovation when $\alpha = 0$: a large γ.

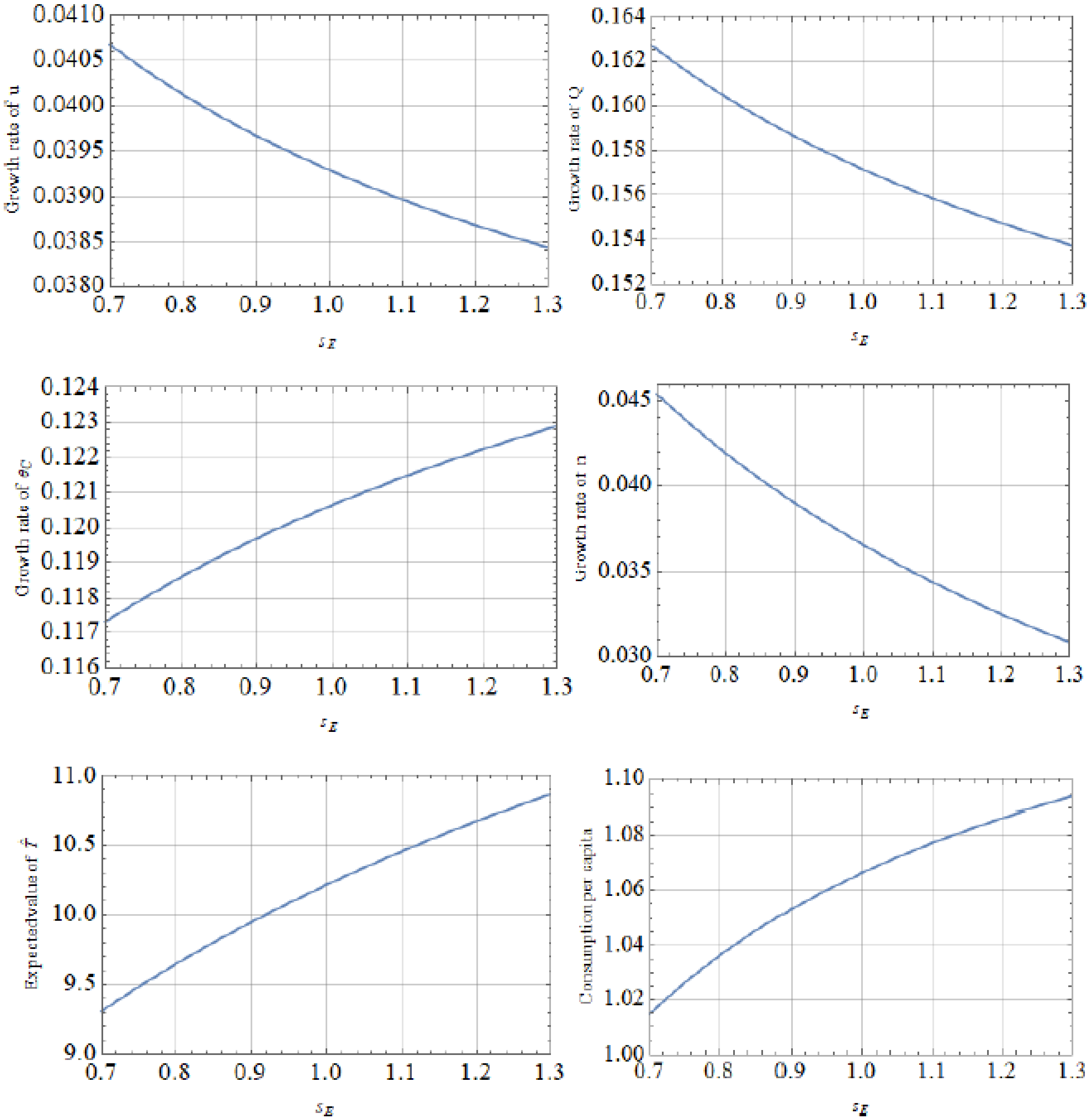

Figure 5: An analysis of the policy effects of entry subsidies and entry taxes when $\alpha = 1$.

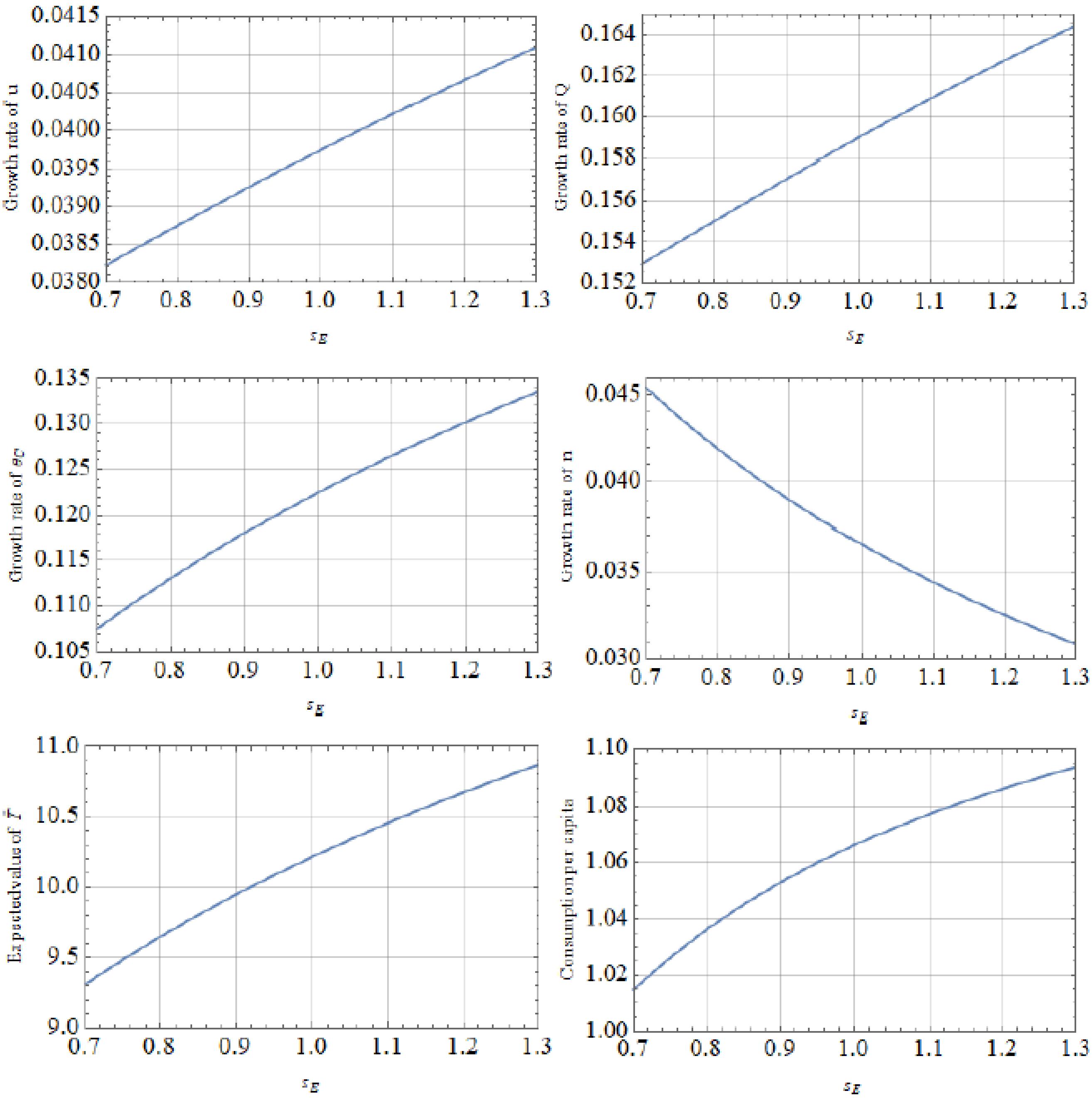

Figure 6: An analysis of the policy effects of entry subsidies and entry taxes when $\alpha = 1$: a large γ.

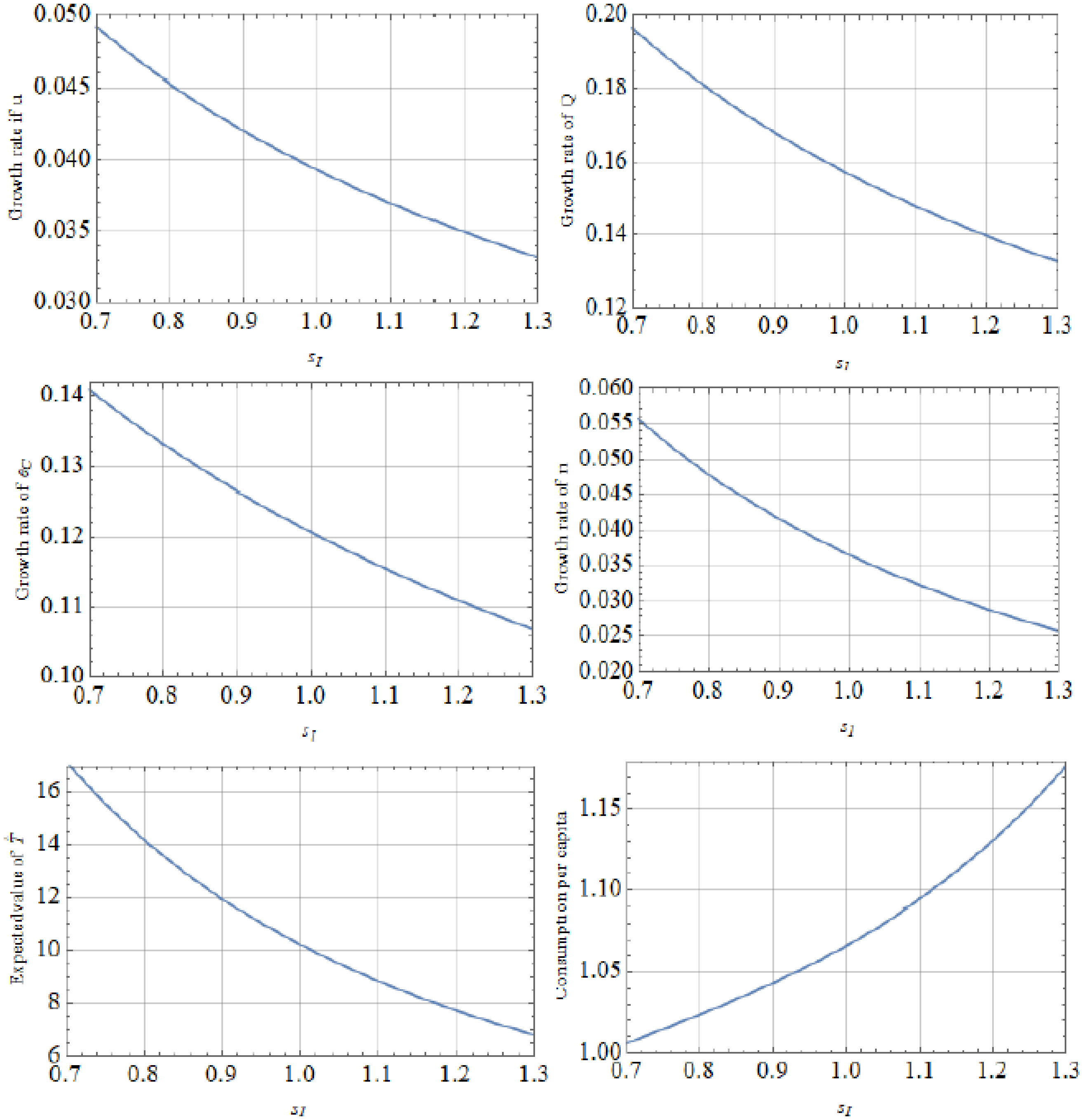

Figure 7: An analysis of the policy effects of subsidies and taxes on incremental innovation when $\alpha = 1$.