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Patent Policy at a Tipping Point: Why Stronger Patent Protection May Not Foster Economic Growth

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Abstract

According to the available evidence, stronger patent protection exerts, at best, a modest positive effect on economic growth. Different variants of Schumpeterian growth models predict a wide range of outcomes, from strongly positive to strongly negative effects. We propose a Schumpeterian growth model with endogenous innovation scale, a generalized innovation function that combines R&D lab-equipment and labor-embodied technical knowledge, and a complexity-of-innovation effect. Consistent with the evidence, plausible calibrations of the model suggest that a typical OECD economy lies near the peak of an inverted-U-shaped curve, where the effect of stronger patent protection on growth is close to zero. In some cases, this effect may even be negative.

JEL codes: O30, O40

Keywords: Endogenous innovation scale, Endogenous markups, Patent breadth, Economic growth, Innovation.

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1 Introduction

Does stronger patent protection foster economic growth? According to a canonical Schumpeterian growth model with patent breadth, the answer is unambiguously positive (Chu, 2024). Strengthening patent protection increases labor employment in the R&D sector, thereby boosting innovation and growth. The available empirical evidence, however, is more mixed. While some studies report positive effects (e.g., Gould and Gruben, 1996), others find statistically insignificant results (e.g., Ginarte and Park, 1997) or even negative effects (e.g., Adams, 2009). Overall, the literature suggests, at best, a modest impact (Neves et al., 2021). Recent extensions of the basic Schumpeterian growth model produce a wide range of predictions, from strongly positive to strongly negative effects. We develop a Schumpeterian growth model that predicts an inverted-U-shaped relationship between patent protection and growth. Consistent with the evidence, our quantitative analysis shows that a typical OECD economy lies close to the peak of this curve, where the growth effects of stronger patent protection are modest. In some countries, the effect may even be slightly negative.

To study the growth effects of patent protection, we extend a standard Schumpeterian growth model with patent breadth in several directions. First, and most important, we allow R&D firms to optimally choose the innovation scale (or step size) of new designs (Yu and Lai, 2025). Second, we consider a generalized innovation function combining lab-equipment with labor-embodied technical knowledge (Chu and Liao, 2025). Third, we incorporate a complexity-of-innovation effect in the innovation function, whereby the rate of arrival of new innovations decreases with the scale of the latest innovation (Yu and Lai, 2025). In this framework, stronger patent protection gives an incentive to R&D firms to choose smaller innovation scales, which, in turn, lowers the complexity of innovation. We show, therefore, that stronger patent protection affects growth through three channels: (i) by reducing the step size of new innovations, (ii) by lowering the complexity of innovation, and (iii) by stimulating R&D labor employment. While the first effect is negative, the second and third effects are positive. We also show that the smaller step size reduces the R&D labor effect. Hence, although stronger patent protection unambiguously increases R&D labor and innovation, its ultimate effect on economic growth is uncertain and depends on parameter values.

We then calibrate the model to an average OECD economy and investigate its quantitative implications. We find that the relationship between economic growth and patent protection follows an inverted-U shape: stronger patent protection increases growth up to a point, beyond which it becomes growth-reducing. Under our baseline calibration, the average OECD economy is located near the peak of the curve, where additional patent protection still increases growth, but only marginally. In this case, the negative step-size effect and the positive complexity-of-innovation effect approximately offset each other, leaving the R&D labor effect as the primary driver of the growth response. The R&D labor effect is substantially weaker with endogenous step size, however, which explains why the overall growth effect is considerably smaller in our setting than under exogenous step size.

We also find that varying key parameters of the model—such as increasing the steady-state arrival rate of innovations or the markup factor—within plausible ranges, can move the economy to the downward-sloping segment of the curve, where stronger patent protection reduces growth. According to our model, therefore, negative growth effects of stronger patent protection are to be expected in some countries. These results help reconcile Schumpeterian growth theory with the empirical evidence.

The endogenous step-size feature plays a key role in our model, reducing the growth effect of patent protection by about two thirds. Previous studies have highlighted the potential dependence of growth effects on the nature of innovation choices. Ginarte and Park (1997) find that patent rights, rather than directly explaining international differences in growth rates, promote growth indirectly by encouraging the research sector to invest in riskier projects. Hall and Ziedonis (2001) suggest that increased patenting can reduce innovation scale, leading to lower average patent quality following patent policy reforms. There is also evidence that entrepreneurs weigh risks and benefits when determining the optimal scale of innovation. Shenhar (1993) and Robertson and Gatignon (1998) provide evidence that innovation scale is chosen strategically by entrepreneurs who take into account the broader economic environment. Akcigit and Kerr (2018) further suggest that the scale of innovation is inversely related to firm size.

This paper is closely related to a line of research studying innovation policy within Schumpeterian endogenous growth models. Pioneered by Romer (1990), endogenous growth models emphasize the creation of new products through R&D as the engine of innovation and growth. Schumpeterian growth models, by contrast, focus on quality improvements, where firms compete to develop higher-quality products that replace older ones, thus generating innovation and growth through a process of creative destruction. Foundational contributions to this strand of growth theory include Segerstrom et al. (1990), Grossman and Helpman (1991), and Aghion and Howitt (1992). Schumpeterian growth models have since been extended to study innovation policy by incorporating patent breadth as a measure of patent protection. Li (2001) develops what can be considered the canonical Schumpeterian growth model with patent breadth. In this model, economic growth increases unambiguously with stronger patent protection.

Li's (2001) model has been extended in a number of ways, with important implications for the relationship between patent protection and growth. Chu et al. (2013) incorporate human capital accumulation and endogenous fertility, while Chu et al. (2016) and Chu et al. (2021) allow for endogenous market structure. All three studies predict a negative long-run relationship between patent breadth and economic growth—a result that has received little empirical support. A second group of studies considers extensions that yield an inverted-U relationship. This category includes Chu et al. (2012), who obtain an inverted-U curve in a model with endogenous labor supply and a lab-equipment innovation specification; Iwaisako and Futagami (2013) and Yang (2021), who find a similar shape in models with capital accumulation; and Chu and Liao (2025), who find an inverted-U relationship in a model with a general innovation specification. By identifying mechanisms that may dampen the growth effect of patent protection, these important studies have helped to bring theory closer to the evidence. However, plausible calibrations of these models still produce relatively large positive growth effects from patent protection, suggesting that some important missing element may be at play.

Despite its key role in our study, we are not the first to use an endogenous formulation of the innovation step size in a Schumpeterian growth model. Lu et al. (2024) incorporate this feature to examine the effects of patent policy on growth and inequality. Yu and Lai (2025) adopt a similar approach in a monetary Schumpeterian model to investigate the interaction between patent breadth and monetary policy. Like us, both studies find an inverted-U relationship between patent breadth and growth. Unlike us, however, they report strongly negative growth effects of patent protection under baseline parameter values. This difference reflects variations in both calibration strategies and model structure. Our model differs from theirs by adopting a generalized innovation function that combines knowledge-driven and lab-equipment-based specifications. Compared to Lu et al. (2024), our model also includes a complexity-of-innovation effect.

The remainder of this study is structured as follows. Section 2 discusses the empirical evidence on the relationship between patent protection and growth. Section 3 describes the theoretical Schumpeterian growth framework. Section 4 explores the quantitative implications of the model. Sections 5 and 6 discuss the main findings and conclude.

2 Empirical evidence

The empirical literature on the relationship between patent rights and economic growth shows considerable variation, reflecting differences in reported effects, relevant channels, methodological approaches, and country-specific contexts across studies. Findings range from positive to negative, with many studies reporting small or statistically insignificant effects. Moreover, some studies suggest that the relationship between patent protection and growth is not direct or at least not exclusively so, potentially operating through more complex channels. This raises questions about the appropriate empirical specification, which depends on the functional form of the relationship and the confounding factors that must be controlled for. Hence, the role of patent protection in fostering growth seems neither straightforward nor universal.

The typical approach in the empirical literature is to estimate a linear regression model of economic growth on a measure of patent protection and a set of control variables for confounding factors. Economic growth is usually measured by the rate of change in real GDP per capita. The most common measure of patent protection is the Ginarte-Park (1997) index, which combines five categories of patent rights: (a) patent duration, (b) coverage, (c) enforcement mechanisms, (d) restrictions on patent scope, and (e) membership in international treaties. Each category is scored from zero to one, with higher values indicating stronger protection. Some studies instead use the Rapp-Rozek (1990) index. The estimated coefficient of patent protection in a growth regression indicates its empirical impact on economic growth. Most studies rely on cross-country regressions, though some use industry- or firm-level data.

Table 1 provides a selected list of empirical studies on the relationship between patent protection and growth, including brief descriptions of the data and methods employed, and the main results reported.[1] The direction, magnitude, and statistical significance of the effect seems to vary both within and between studies, depending on methods, specifications and data samples used. While studies such as Gould and Gruben's (1996) and Kim et al.'s (2012) report mostly positive and statistically significant estimates, Ginarte and Park (1997) obtain mostly negative and statistically insignificant effects. Somewhere in between, Thompson and Rushing (1996), Hall and Ziedonis (2001), Schneider (2005), and Mrad (2017) arrive at mostly positive but economically modest and statistically insignificant

[1] The list contains most of the studies included in a recent meta-analysis by Neves et al. (2021). It is not intended to be exhaustive or systematic but rather to illustrate the variety of results reported in the literature.

effects. In sum, the empirical literature reports mixed findings, suggesting at best a small positive effect of patent protection on growth.

Our assessment of the empirical literature is supported by three recent meta-analyses that summarize the evidence on the link between patent protection and economic growth. Neves et al. (2021) analyze 256 estimates from 22 studies and find that patent protection has a positive but economically small effect on innovation and growth after accounting for differences across studies (e.g., sample size, data structure, level of analysis, and measure of patent rights). Churchill et al. (2022) and Panda and Sharma (2020) also report modest but statistically insignificant effects of patent protection on growth. Based on their findings, Churchill et al. (2022) argue that claims about the economic benefits of intellectual property protection are not supported by the available evidence. A recent survey by Chu (2024) on the theoretical and empirical literature similarly concludes that evidence on the link between patent rights and growth remains weak. Overall, the meta-analytic consensus suggests that patent protection probably has a positive but modest effect on growth.

Some studies have used non-linear threshold regressions to examine how a country's level of economic development influences the impact of patent protection on growth. For example, Thompson and Rushing (1996, 1999) and Falvey et al. (2006) find that the effects vary significantly with the level of development. Similarly, Chu et al. (2014) show that the impact of intellectual property rights on growth depends on a country's distance to the technology frontier. Hence, although patent protection may have a modest growth effect in developed countries, its effect in developing countries seems even smaller.

3 Theoretical framework

This section examines the effect of patent protection on economic growth from a theoretical perspective using a Schumpeterian growth model. Section 3.1 presents the structure of the model, and Section 3.2 discusses its balanced-growth properties. Section 3.3 analyzes the growth effect of patent protection and the channels through which it operates. Section 4 provides a quantitative analysis of these channels.

3.1 Model

We develop a Schumpeterian growth model in which innovation drives economic growth by improving the quality of intermediate goods (see Grossman and Helpman, 1991; and Aghion and Howitt, 1992). We extend the standard Schumpeterian model with patent breadth in three directions. First, we allow R&D firms to maximize net payoffs by optimizing the innovation scale of new designs, as in Yu and Lai (2025). Second, following Chu and Liao (2025), we introduce a general innovation specification that nests both the knowledge-driven and lab-equipment approaches. Third, we incorporate a complexity-of-innovation effect, in which the arrival rate of new innovations decreases with the step size of the most recent innovation.

3.1.1 Households

The economy is inhabited by a representative household with the following lifetime utility function:

$$U = \int_0^\infty e^{-\rho t} \ln C_t \, dt, \tag{1}$$

where C_t is consumption at time t, and $\rho > 0$ is the subjective discount rate. We assume that population is constant and normalize it to one without loss of generality. The household is endowed with one unit of time, which is supplied inelastically as labor. Labor is fully mobile across sectors. The final good serves as the numeraire, with its price set to one. The household maximizes (1) subject to the asset accumulation constraint:

$$\dot{A}_t = r_t A_t + w_t - C_t, \tag{2}$$

where A_t is the value of financial assets (consisting of shares in monopolistic firms), r_t is the rate of return on these assets, and w_t is the wage rate. Standard dynamic optimization yields the optimality condition for consumption:

$$\frac{\dot{C}_t}{C_t} = r_t - \rho. \tag{3}$$

3.1.2 Final goods sector

The final goods sector consists of perfectly competitive firms that produce a homogeneous output using a Cobb-Douglas aggregator over a unit continuum of differentiated intermediate goods, $x_t(j)$, for $j \in [0, 1]$:

$$y_t = e^{\int_0^1 \ln x_t(j)\, dj}. \tag{4}$$

The conditional demand for intermediate good j at time t is

$$x_t(j) = \frac{y_t}{p_{x,t}(j)}, \tag{5}$$

where $p_{x,t}(j)$ is the price of intermediate good j relative to the final good at time t.

3.1.3 Intermediate goods sector

Producers of intermediate goods operate under monopolistic competition. Each sector j is temporarily led by a firm holding the patent for the latest innovation. This firm produces the most recent vintage of intermediate goods in that sector until an entrant with a superior innovation replaces it. The production function of the current producer in industry j is

$$x_t(j) = z^{q_t(j)} L_{x,t}(j), \tag{6}$$

where $z > 1$ is the innovation scale (or step size) of productivity improvement, which proxies the innovation plan chosen by R&D firms. We assume that firms choose z optimally (see below). The variable $q_t(j)$ denotes the number of productivity improvements in industry j between time 0 and t, and $L_{x,t}(j)$ is the labor employed in industry j.

The marginal cost of producing one unit of intermediate goods is

$$MC_t(j) = \frac{1}{z^{q_t(j)}} w_t. \tag{7}$$

Under standard Bertrand competition, the profit-maximizing price $p_{x,t}(j)$ is a markup over marginal cost

$$\mu z = \frac{p_{x,t}(j)}{MC_t(j)} \geq 1, \tag{8}$$

where $\mu \geq 1/z$ captures the degree of patent breadth, following Li (2001), Iwaisako (2020), Furukawa et al. (2024), and Yu and Lai (2025). This is the key parameter in our analysis, reflecting the strength of patent protection and the ability of the patent holder to charge a price above marginal cost while retaining market leadership.

We do not constrain the markup charged by the new leader, except by the assumption of incomplete market breadth, since the previous leader exits the market. When $\mu = 1/z$, the firm sets its price equal to marginal cost. The monopolistic profit of each intermediate-good producer at time t is

$$\Pi_{x,t} = \Pi_{x,t}(j) = p_{x,t}(j)x_t(j) - MC_t(j)x_t(j) = \left(\frac{\mu z - 1}{\mu z}\right) y_t, \tag{9}$$

where the first equality reflects identical profit flows across industries, and the third equality follows from equations (5) and (8). The wage income earned by workers in industry j at time t is

$$w_t L_{x,t}(j) = \frac{1}{\mu z} p_{x,t}(j)x_t(j) = \frac{y_t}{\mu z}. \tag{10}$$

3.1.4 R&D sector

There is a unit continuum of perfectly competitive R&D firms, indexed by $i \in [0,1]$. Let $v_t(j)$ be the expected value of owning the most recent innovation in the intermediate-good sector. Since, from equation (9), $\Pi_{x,t}(j) = \Pi_{x,t}$, the value of inventions is equal across industries, so that $v_t(j) = v_t$ for all $j \in [0,1]$. Following Cozzi et al. (2007), we focus on a symmetric equilibrium in which the Poisson arrival rate of innovation is identical across sectors, $\lambda_t(i) = \lambda_t$. The no-arbitrage (HJB) condition that determines v_t is

$$r_t v_t = \Pi_{x,t} + \dot{v}_t - \lambda_t v_t. \tag{11}$$

This expression equates the interest rate r_t to the rate of return on v_t, which is the sum of monopolistic profit ($\Pi_{x,t}$), capital gain ($\dot{v}_t$), and expected capital loss due to creative destruction ($\lambda_t v_t$).

The firm-level innovation arrival rate per unit of time is

$$\lambda_t(i) = \left(\frac{\varphi}{\kappa Z_t^{\alpha}}\right) R_t(i)^{\alpha} L_{r,t}(i)^{1-\alpha}. \tag{12}$$

The aggregate arrival rate of innovation is a combination of R_t units of final output and R&D labor $L_{r,t}$:

$$\lambda_t = \int_0^1 \lambda_t(i)\, di = \frac{\varphi}{\kappa}\left(\frac{R_t}{Z_t}\right)^{\alpha} (L_{r,t})^{1-\alpha}, \tag{13}$$

where $\varphi > 0$ is a productivity parameter of the R&D sector, and $\kappa = z^{\phi}$ reflects the level of complexity in innovation. As noted by Shenhar (1993), Robertson and Gatignon (1998), and Yu and Lai (2025), a larger innovation scale increases complexity, reducing the arrival rate of innovation. Thus, in equation (13), innovation arrives more slowly when innovation is more complex. This complexity-of-innovation effect plays a central role in the link between patent protection and growth (see below). The parameter $\phi > 0$ is the inverse measure of the professional knowledge of R&D firms, indicating the sensitivity of the innovation arrival rate to changes in the innovation scale. A smaller ϕ implies greater professional knowledge and less sensitivity to complexity.

In equation (13), Z_t denotes the aggregate technology level, capturing an increasing-difficulty effect of R&D. $R_t = \int_0^1 R_t(i)\, di$ is total final output used in R&D, and $L_{r,t} = \int_0^1 L_{r,t}(i)\, di$ is labor employment in R&D. Under the general innovation specification, the aggregate arrival rate shows decreasing returns to both inputs. The parameter $\alpha \in (0,1)$

determines the intensity of R_t relative to $L_{r,t}$ in R&D. This innovation specification encompasses the 'knowledge-driven' case as $\alpha \to 0$ (Romer, 1990) and the 'lab-equipment' case as $\alpha \to 1$ (Rivera-Batiz and Romer, 1991) as special cases.

The expected profit of an R&D firm is

$$\Pi_{RD,t} = v_t \lambda_t - w_t L_{r,t} - R_t. \tag{14}$$

Hence, the profit-maximizing conditions are

$$\alpha \lambda_t v_t = R_t, \tag{15}$$

$$(1-\alpha)\lambda_t v_t = w_t L_{r,t}. \tag{16}$$

In a symmetric equilibrium, each entrepreneur chooses the innovation scale z to maximize expected profit. The optimality condition is

$$\frac{\partial \Pi_{RD,t}}{\partial z} = \lambda_t \frac{\partial v_t}{\partial z} + v_t \frac{\partial \lambda_t}{\partial z} = 0, \tag{17}$$

where $\partial v_t / \partial z = \frac{y_t}{\mu} \left(r_t + \lambda_t - \frac{\dot{v}_t}{v_t} \right) z^2 > 0$ and $\partial \lambda_t / \partial z = -\frac{\phi \lambda_t}{z} < 0$. Raising the innovation scale creates two opposing effects. It increases the expected value of the intermediate firm (v_t) but also raises complexity, reducing the arrival rate (λ_t). The optimal step size balances these effects. Focusing solely on one of them yields a suboptimal equilibrium.

Using (17) and noting from (9) that $\Pi_{x,t}/y_t = (\mu z - 1)/\mu z$, the optimal innovation scale chosen by the entrepreneur is

$$\bar{z} = \frac{1+\phi}{\mu \phi}. \tag{18}$$

Clearly, $\bar{z}$ depends on professional knowledge of R&D firms (ϕ) and patent breadth (μ). Lower professional knowledge (higher ϕ) reduces the optimal innovation scale (lower $\bar{z}$) due to the slower arrival rate of innovation. Conversely, stronger patent breadth (higher μ) reduces the optimal step size of innovation (lower $\bar{z}$). Stronger patent protection raises the expected value of an innovation and, therefore, the potential loss if the innovation fails. To mitigate this risk, the entrepreneur chooses a smaller innovation scale. Note that, since $\bar{z} > 1$, it follows that $\mu < (1+\phi)/\phi$.

Substituting equation (18) into the expression of κ yields the equilibrium level of complexity:

$$\bar{\kappa} = \bar{z}^{\phi} = \left(\frac{1+\phi}{\mu \phi} \right)^{\phi}. \tag{19}$$

Thus, stronger patent protection reduces innovation complexity, as R&D firms respond by choosing a smaller innovation scale.

3.1.5 Equilibrium

The equilibrium is a time path of allocations $\{A_t,\, C_t,\, y_t,\, L_{x,t}(j),\, L_{r,t}(i),\, R_t(i)$, and $x_t(j)\}$ and prices $\{r_t,\, p_{x,t}(j),\, w_t,\, v_t\}$ such that:

- the representative household maximizes its lifetime utility, taking $\{r_t,\, w_t\}$ as given;
- competitive firms produce final goods y_t to maximize profits, taking $\{p_{x,t}(j)\}$ as given;
- each monopolistic firm j produces differentiated intermediate goods $\{x_t(j)\}$ and maximizes profits by choosing $\{L_{x,t}(j), p_{x,t}(j)\}$ and taking $\{w_t\}$ as given;
- competitive R&D firms choose $\{z,\, R_t(i),\, L_{r,t}(i)\}$ to maximize expected profits, taking $\{w_t, v_t\}$ as given;
- the capital market clears, meaning the total market value of assets equals the total market value of firms:
$$A_t = \int_0^1 v_t(j)\, dj;$$
- the labor market clears, meaning total labor supply equals total labor demand:
$$L_{x,t} + \int_0^1 L_{r,t}(i)\, di = 1;$$
- the final goods market clears, meaning total output equals total consumption and R&D spending:
$$y_t = C_t + \int_0^1 R_t(i)\, di.$$

3.2 Aggregate economy and balanced growth path

This section characterizes the decentralized equilibrium and shows that the economy follows a unique, saddle-point stable balanced growth path with a positive growth rate (Lemma 1) and stationary labor allocations in the intermediate goods and R&D sectors (Lemma 2). It also derives the expression for the growth rate of technology, which equals the growth rate of output along the balanced growth path.

We define the aggregate technology level in the R&D sector as

$$Z_t = e^{\int_0^1 q_t(j)\, dj \ln \bar{z}} = e^{\int_0^t \lambda_\tau\, d\tau \ln \bar{z}}, \tag{20}$$

where the second equality follows from the law of large numbers. Log-differentiating Z_t with respect to time gives the growth rate of aggregate technology:

$$g_t = \frac{\dot{Z}_t}{Z_t} = \lambda_t \ln \bar{z}, \tag{21}$$

where $\bar{z}$ is the equilibrium step size.

Substituting equations (20) and (6) into equation (4) yields the aggregate production function:

$$y_t = Z_t L_{x,t}, \tag{22}$$

where $L_{x,t}(j) = L_{x,t}$ for all $j \in [0, 1]$.

Lemma 1. *The aggregate economy always jumps to a unique and saddle-point balanced growth path.*

Proof. See Appendix A.1. □

Substituting equation (22) and the optimal innovation step size into equation (10) gives $w_t = \frac{1}{\mu\bar{z}} Z_t$. Combining equations (15) and (16) gives

$$\frac{\alpha}{1-\alpha} = \frac{R_t}{w_t L_{r,t}}, \tag{23}$$

which, using $w_t = \frac{1}{\mu\bar{z}} Z_t$, leads to

$$\frac{R_t}{Z_t} = \frac{\alpha}{(1-\alpha)\mu\bar{z}} L_{r,t}. \tag{24}$$

Substituting this expression into equation (13) gives the equilibrium innovation arrival rate. Substituting again into equation (21) yields the growth rate of aggregate technology:

$$g_t = \lambda_t \ln \bar{z} = \frac{\varphi}{\kappa} \Theta L_{r,t} \ln \bar{z}, \tag{25}$$

where we define $\Theta \equiv \left(\frac{\alpha}{(1-\alpha)\mu\bar{z}}\right)^{\alpha} > 0$. Importantly, when R&D firms optimally choose the innovation scale, the term $\mu\bar{z}$ in Θ remains fized, since equation (18) implies $\mu\bar{z} = (1+\phi)/\phi$. In this case, $\Theta \equiv \left(\frac{\alpha\phi}{(1-\alpha)(1+\phi)}\right)^{\alpha} > 0$, and the direct effect of μ on g disappears from equation (25). This direct channel remains, however, if the step size is exogenous.

Lemma 2. *On the balanced growth path, the equilibrium allocations of labor are stationary.*

Proof. See Appendix A.2. □

By Lemma 2 and equation (22), the growth rate of aggregate technology in equation (25) equals the growth rate of output along the balanced growth path.

3.3 The effects of patent protection

We now study the effects of an increase in patent protection, as measured by μ, on the optimal allocation of R&D labor, innovation, and the output growth rate along the balanced growth path. Section 3.3.1 analyzes the effects on R&D labor and innovation, while Section 3.3.2 examines the effects on growth.

3.3.1 Patent protection, R&D labor, and innovation

In the model, strengthening patent protection unequivocally increases the share of labor devoted to R&D activities. This result is summarized in Proposition 1.

Proposition 1. *The allocation of labor to the R&D sector, L_r, is a monotonically increasing function of the strength of patent protection, μ, along the balanced growth path.*

Proof. See Appendix A.3. □

Intuitively, strengthening patent breadth increases the markup of intermediate firms along the balanced growth path, as described in equation (9). This motivates R&D firms to adopt a smaller innovation step size, which reduces the complexity of innovation. As a result, expected profits rise, encouraging R&D firms to allocate more labor to research.

The endogenous step-size feature of the model alters the mechanism through which patent protection affects the share of R&D labor. To see this, consider the steady-state share of R&D labor given in equation (A.9) of Appendix A.1. With an endogenous step size, $\bar{z}$ adjusts in response to μ to maintain the markup factor at $\mu\bar{z} = (1+\phi)/\phi$. As a result, patent protection influences the R&D labor share only through $\bar{\kappa}$, as described in equation (A.13). Conversely, with an exogenous step size, $\bar{z}$ remains fixed and the markup factor $\mu\bar{z}$ varies with μ, which introduces additional transmission channels (see equation (A.14) in Appendix A.4). It is a priori unclear whether the R&D labor effect of patent protection is smaller or larger in the endogenous step-size model. As shown in Appendix A.4, however, this effect is smaller under the sufficient condition $\mu\bar{z} < [\alpha/(1-\alpha)]^2$, which is easily met under the parameter values considered in the model's quantitative analysis (see Section 4).

Because R&D labor increases with patent protection, so does the innovation rate. Proposition 2 summarizes this result.

Proposition 2. *The innovation rate, λ, is a monotonically increasing function of the strength of patent protection, μ, along the balanced growth path.*

Proof. See Appendix A.5. □

3.3.2 Patent protection and growth

While patent protection has a clearly positive effect on R&D labor and innovation along the balanced growth path, its impact on growth is less straightforward. Proposition 3 outlines the mechanisms through which patent protection affects growth.

Proposition 3. *Strengthening patent protection (i.e., increasing μ) affects growth positively by reducing the complexity of innovations (lower $\bar{\kappa}$) and increasing the R&D labor share (higher L_r), but negatively by reducing the innovation step size (lower $\bar{z}$). The net effect on growth depends on the relative magnitudes of these partial effects.*

Proof. Using the chain rule to differentiate g with respect to μ in equation (25) yields:

$$\frac{\partial g}{\partial \mu} = \underbrace{\frac{\partial g}{\partial \Theta}\frac{\partial \Theta}{\partial \mu}}_{\text{direct effect}} + \underbrace{\frac{\partial g}{\partial \bar{\kappa}}\frac{\partial \bar{\kappa}}{\partial \mu}}_{\kappa \text{ effect}} + \underbrace{\frac{\partial g}{\partial L_r}\frac{\partial L_r}{\partial \mu}}_{L_r \text{ effect}} + \underbrace{\frac{\partial g}{\partial \bar{z}}\frac{\partial \bar{z}}{\partial \mu}}_{z \text{ effect}}. \tag{26}$$

Since $\partial\Theta/\partial\mu = 0$ (eliminating the direct effect), $\partial g/\partial\bar{\kappa} = -g/\bar{\kappa}$, $\partial\bar{\kappa}/\partial\mu = -\phi\bar{\kappa}/\mu$, $\partial g/\partial L_r = g/L_r$, $\partial L_r/\partial\mu$ is given by equation (A.13), $\partial g/\partial\bar{z} = g/(\bar{z}\ln\bar{z})$, and $\partial\bar{z}/\partial\mu = -\bar{z}/\mu$. Substituting and rearranging gives

$$\frac{\partial g}{\partial \mu} = \frac{g}{\mu}\Big[\underbrace{\phi}_{\substack{\kappa \text{ effect} \\ (+)}} + \underbrace{\frac{\phi^2\bar{\kappa}\rho}{\varphi(1-\alpha+\phi)\Theta L_r}}_{\substack{L_r \text{ effect} \\ (+)}} - \underbrace{\frac{1}{\ln\bar{z}}}_{\substack{z \text{ effect} \\ (-)}}\Big]. \tag{27}$$

Clearly, the κ and L_r effects are both positive, while the z effect is negative. The net impact on growth ambiguous and depends on parameter values. □

Equation (27) identifies three channels through which patent protection affects growth. The first, the κ effect, works through innovation complexity: stronger patent protection reduces complexity by inducing R&D firms to adopt smaller innovation steps, which positively affects growth. The second, the L_r effect, operates through R&D labor allocation: stronger patent protection raises expected profits, motivating firms to increase R&D employment, which also boosts growth. The third, the z effect, reflects the smaller innovation steps that result from stronger patent protection, which slows technological progress and reduces growth. Whether patent protection ultimately promotes or hinders growth depends on whether the positive κ and L_r effects outweigh the negative z effect.

The endogenous step-size feature of the model has three key implications for how patent protection affects growth. First, it eliminates the direct negative effect that would arise through Θ under an exogenous step-size model. Second, it dampens the positive L_r effect (see Section 3.3.1). Finally, it introduces two additional effects with opposing signs: the positive k effect and the negative z effect. Section 4 quantifies these effects by calibrating the model.

4 Quantitative analysis

This section presents a quantitative analysis of the relationship between patent protection and growth implied by the model. Section 4.1 outlines the calibration strategy. Section 4.2 reports the baseline quantitative results. Section 4.3 examines the sensitivity of the findings to changes in key parameters. Section 4.4 introduces endogenous labor supply. Finally, Section 4.5 explores results under an alternative calibration for developing countries.

4.1 Calibration

We calibrate the model to match parameter values from empirical studies and key macroeconomic features of a typical advanced OECD economy, such as the growth rate and the share of R&D labor. Our calibration strategy is exactly identified. Table 3 reports the parameter values used in the baseline calibration. In what follows, we compare the results from the endogenous and exogenous step-size versions of the model. The same baseline calibration applies to both.

From a quantitative standpoint, a key parameter is λ, the innovation arrival rate. We set the baseline value of λ to 10%, which lies between the 4% estimate of Caballero and Jaffe (1993) and the 17% estimate of Laitner and Stolyarov (2013).[2] Following Acemoglu and Akcigit (2012), who obtain a 33% rate in a calibrated growth model, some recent studies use larger values (e.g., Chu and Liao, 2025). We explore alternative values of 7.5% and 15% in the sensitivity analysis (see Section 4.3).

Our calibration targets the average per capita growth rate of an average OECD economy. According to the World Development Indicators of the World Bank, the average annual growth rate of GDP per capita in OECD countries from 1961 to 2023 was 2.4%.[3]

[2] This value is also consistent with Lanjouw (1998), who estimates the probability of obsolescence for four technology sectors in West Germany during 1953–1988 to be between 7% and 12%.

[3] For former Soviet republics, data are only available from 1991 onward.

Using the first equality in equation (25), a growth rate of $g = 0.024$ and $\lambda = 0.1$ imply a steady-state value of $\bar{z} = 1.271$.

We set the baseline value of μ to imply a 25% markup, or a markup factor of 1.25, consistent with De Loecker et al. (2020) and Impullitti and Rendahl (2025). Other studies report values ranging from 2.8% (Chu and Cozzi, 2018) to 33% (Yu and Lai, 2025), broadly consistent with the 5%–40% range in Jones and Williams (2000). Yang (2018) focuses on markup rates between 10% and 40%. Our baseline lies near the midpoint of these estimates. In the model, market power is measured by $\mu\bar{z}$, which we set to 1.25. Given $\bar{z} = 1.271$, the implied baseline value of μ is 0.983. We consider markup factors of 1.15 and 1.35 in the sensitivity analysis.

The subjective discount rate ρ is set to the standard value of 0.05 (see, e.g., Yu and Lai, 2025; Chu and Liao, 2025). Given $\bar{z}$ and μ, equation (18) determines $\phi = 4$, and equation (19) gives $\bar{\kappa} = 2.612$. The remaining parameters, α and φ, are jointly pinned down by the second equality in equation (25) and the steady-state condition (A.9), to match the empirical share of R&D labor. According to the OECD Main Science and Technology Indicators, this share roughly doubled in OECD countries over the past three decades, rising from 0.79% in 1981 to 1.54% in 2021.[4] We target the most recent value (2021) in the baseline calibration. The implied values of α and φ are 0.906 and 2.661, respectively.

4.2 Results

We begin by examining the relationship between the steady-state growth rate, g, and the level of patent protection, μ. Figure 1 illustrates this relationship under baseline parameter values. The blue solid line represents the endogenous step-size model, while the red dashed line represents the exogenous step-size model. The vertical line, marked with 'c', denotes the calibration value of μ. Panel (a) shows the level of g, while panel (b) presents the partial derivative of g with respect to μ, i.e., the slope of the g curve at each value of μ.

Both models exhibit an inverted-U relationship between g and μ. However, this pattern is not apparent in the exogenous step-size model because the turning point lies at unrealistically high values of μ (not shown). As a result, the calibration point falls on the fast-increasing portion of the curve, indicating a relatively large marginal effect of patent protection on growth. In contrast, in the endogenous step-size model, the inverted-U shape is clearly visible within plausible values of μ. Although the calibration point still lies on

[4]Indicator 'Total R&D personnel per thousand total employment' (OECD.MSTI.TP_TTXEM). Data are available for 17 OECD countries in 1981 and 26 in 2021.

the upward-sloping segment, it is much closer to the peak, implying a significantly smaller marginal effect. This is the paper's main quantitative result: a plausibly calibrated endogenous step-size model to an average OECD economy predicts a positive but modest effect of patent protection on growth.

Table 3 reports the numerical values of this effect (column 'Total effect'). In the exogenous step-size model, the partial derivative of g with respect to μ at the baseline is 0.146, about three times larger than the value of 0.044 in the endogenous model. To interpret this difference, consider an increase in μ that raises the markup factor by 0.1 (e.g., from 1.25 to 1.35), assuming $\bar{z}$ remains fixed at its baseline value of 1.271. This implies a change in μ of approximately 0.079 (= 0.1/1.271). A derivative of 0.146 implies that growth increases by 0.011 (= 0.079×0.146)—over one percentage point, or nearly half the baseline growth rate. In contrast, in the endogenous step-size model, the same change in μ increases growth by only 0.0035 (= 0.079×0.044), or about a third of a percentage point.

To understand why the endogenous model predicts a much smaller growth effect, consider the decomposition of the total effect in Table 3. In the exogenous step-size model, the total effect includes a negative direct effect (from the presence of μ in the term Θ; see equation (25)) and a positive indirect effect through L_r. Under the baseline calibration, the positive L_r effect dominates, yielding a large overall effect. In the endogenous model, the direct effect is muted, but two additional effects arise: a positive complexity-of-innovation effect (the κ effect) and a negative step-size effect (the z effect). These effects are roughly equal in magnitude and largely cancel each other out. As a result, the total growth effect is once again driven by the L_r effect, as in the exogenous model, but its magnitude is considerably smaller. In summary, the smaller impact of patent protection on growth in the endogenous model is primarily due to its weaker influence on R&D labor.

4.3 Sensitivity

This section examines the sensitivity of the results to recalibrations of the model using different values for the innovation arrival rate, λ, and the markup factor, $\mu\bar{z}$. We focus on these two parameters due to both their quantitative importance (as shown below) and the considerable uncertainty surrounding their values in the literature. In each alternative calibration, we vary one parameter while following the calibration strategy outlined in Section 4.1.

We consider two variants for λ: a lower value (0.075) and a higher value (0.15), labeled variant 1 and variant 2, respectively. For the markup factor $\mu\bar{z}$, we consider values of 1.15

and 1.35, labeled variant 3 and variant 4. Table 4 reports the parameter values for these alternative calibrations. Note that these values represent relatively small deviations from the baseline and fall well within the range used in the literature (see Section 4.1).

Figure 2 shows the relationship between patent protection and growth for different values of λ. It is important to note that the calibration value of μ (marked 'c') depends on λ. While the shape of the inverted-U curve remains broadly unchanged, its implications for the marginal effect of patent protection do not. Since the baseline calibration lies near the peak of the curve, a modest increase in λ is enough to shift the economy to the downward-sloping segment, implying a negative marginal effect of patent protection on growth. A similar observation holds for changes in the markup factor, as shown in Figure 3. These findings suggest that the growth effect of patent protection is generally small and may be either positive or negative, depending on parameter values. Given cross-country and intertemporal variation in λ and $\mu\bar{z}$, this result may help explain the mixed empirical evidence on the growth effects of patent protection.

Table 5 decomposes the total growth effect across the four alternative calibrations, comparing the endogenous and exogenous step-size models. In the exogenous model, patent protection has a consistently positive impact on growth, primarily driven by the L_r effect. In contrast, the overall effect in the endogenous model is more nuanced and depends on the parameter values. Changes in λ mainly influence the z effect, while changes in $\mu\bar{z}$ affect the κ effect. In all variants, the L_r effect remains the smallest among the three channels.

4.4 Endogenous labor

This section modifies the baseline model described in Section 3.1 to allow for endogenous labor supply. The modified model is described in Appendix B, and the quantitative results are based on equation (B.12).

Allowing for endogenous labor supply introduces a new parameter, γ, which denotes the weight of leisure in instantaneous utility. We set $\gamma = 1.85$, which implies a steady-state labor supply of approximately one-third—a standard value in the literature. We then recalibrate φ to ensure that $g = 0.024$, yielding $\varphi = 7.987$. All other parameters are kept at their baseline values.

Figure 4 presents the results. The differences between the exogenous and endogenous labor supply cases in the relationship between patent protection and growth are quantitatively minor. This is intuitive, since R&D labor—the relevant labor input for growth in the model—accounts for only a small fraction of total labor.

4.5 Developing countries

As noted in Section 2, the empirical literature on the effects of patent protection on economic growth appears to suggest an even weaker impact in developing countries. Although it is debatable whether Schumpeterian dynamics can adequately explain economic growth in these economies (Aghion et al., 2014), we nevertheless attempt to calibrate the baseline model to a typical developing country in this section.

Data on R&D labor in developing countries are limited and not directly comparable to the OECD data used for developed economies. One way to address this limitation is to use data for non-OECD countries available in the OECD dataset.[5] Among these countries, the average R&D labor share ranges from 0.5% to 1%, substantially lower than in OECD countries. Since R&D intensity in other developing countries is likely even lower, we use the lower bound of this range (0.5%) as our calibration target.

For economic growth, we again use data on real GDP per capita growth from the World Development Indicators of the World Bank. We define a developing country broadly as any economy not part of the OECD. While this definition includes some upper-middle-income and high-income countries (according to World Bank classification), it allows a consistent comparison with the results in Section 4.2. The average annual growth rate of non-OECD countries between 1961 and 2023 was 1.76%.[6]

Although the arrival rate of innovation and markup levels are likely to differ between developed and developing economies, we lack reliable data to guide these values. For this reason, we retain the baseline values of these parameters. Calibrating the model to match $g = 0.0176$ and $L_r = 0.005$ implies the following values: $\bar{z} = 1.192$, $\mu = 1.048$, $\bar{\kappa} = 2.022$, $\alpha = 0.965$, and $\varphi = 1.761$.

Figure 5 compares the relationship between growth and patent protection for developed and developing countries. The calibration point (marked 'c') differs across groups because the calibrated value of μ changes. As a result, the growth effect of patent protection differs not only due to differences in the underlying curves but also because the slopes are evaluated at different points. For developed economies, the effect is positive but small. For developing economies, the calibration point lies nearly at the peak of the curve, implying that the marginal effect of patent protection on growth is virtually zero.

[5] This group includes Argentina, China, Romania, the Russian Federation, Singapore, Taiwan, and South Africa.

[6] Restricting the sample to countries classified by the World Bank as low- or lower-middle-income yields a slightly lower but comparable growth rate of 1.28%.

5 Discussion

Our survey of the empirical literature on the effects of patent protection on economic growth reveals mixed findings. The effect appears weak at best in developed countries and may be entirely absent in developing ones. This so-called ‘patent puzzle’ has been difficult to reconcile with standard Schumpeterian growth models. We develop a Schumpeterian growth model featuring a complexity-of-innovation channel and a general innovation specification, in which R&D firms optimally choose the scale of their innovations. In the model, the optimal size of quality improvements is inversely related to the degree of patent protection—stronger patent rights lead firms to reduce the scale of their innovations. The model implies an inverted-U relationship between economic growth and patent protection.

Plausible model calibrations suggest that an average OECD economy is likely located in the flat region near the peak of the inverted-U curve, implying modest effects—if any—from further strengthening patent rights. Moreover, deviations within plausible ranges of key parameters, such as the innovation arrival rate or the markup factor, may result in zero or even negative growth effects. A comparable Schumpeterian growth model with an exogenous step size also predicts an inverted-U relationship. However, our quantitative analysis shows that the peak of this curve occurs at an implausibly high level of patent protection. According to that model, an average OECD economy would experience substantial growth gains from additional increases in patent protection. By accounting for both the modest average effect of patent protection on growth and the heterogeneity of findings reported in the literature, including negative effects, our model helps reconcile Schumpeterian growth theory and evidence.

In our model, patent protection affects growth through three channels: (i) it stimulates R&D labor (positive), (ii) it reduces the optimal innovation scale (negative), and (iii) it lowers the complexity of innovation (positive). Quantitatively, however, effects (ii) and (iii) tend to offset each other, leaving the R&D labor channel as the main driver of the overall effect. In the exogenous step-size model—which lacks channels (ii) and (iii)—the R&D labor effect is also the primary determinant of growth. In the endogenous step-size model, by contrast, the innovation-scale and complexity-of-innovation effects partially offset the R&D response, reducing its contribution to growth by a factor of three to six, depending on parameter values. Hence, it is the magnitude of the R&D labor response to patent protection that ultimately differentiates the quantitative growth effects in the two models.

What, then, is the empirical effect of patent protection on R&D labor? To the best of our knowledge, no empirical study has directly addressed this question. The available evidence is indirect, focusing instead on the broader impact of patent protection on R&D spending. Several studies provide some evidence that stronger patent protection increases R&D spending, but the effect tends to be economically modest (see, e.g., Kanwar and Evenson (2003), Almeida and Teixeira (2007), Das (2020), Arqué-Castells and Fons-Rosen (2024)). Sakakibara and Branstetter (2001) examine a 1998 patent law reform in Japan that significantly expanded and strengthened patent rights. They find that the reform did not lead to a significant increase in R&D employment or spending. Taken together, these studies suggest that patent protection is unlikely to substantially increase R&D spending or employment—and, consequently, is unlikely to be a major driver of growth. Boldrin and Levine (2013), for example, argue that there is no conclusive empirical evidence that patent protection promotes innovation, productivity, or economic growth beyond its effect on patent counts.

6 Conclusion

In this study, we examine the growth effects of patent protection in a generalized Schumpeterian growth model that features an endogenous innovation step size and a flexible innovation structure incorporating a complexity-of-innovation effect. The model predicts an inverted-U relationship between the growth rate and the strength of patent protection, driven by the interaction of three channels: a complexity-of-innovation effect, an R&D-labor effect, and an innovation step-size effect. The overall impact on growth is theoretically ambiguous and depends on parameter values.

Our quantitative analysis shows that a typical OECD economy is likely located in the flat region near the peak of the inverted-U curve, where the growth effects of stronger patent protection are modest. These findings help reconcile Schumpeterian growth theory with the mixed results found in the empirical literature, which suggest, at best, a weak relationship between patent protection and growth.

From a policy perspective, our results indicate that innovation strategies based solely on strengthening patent protection may have limited effectiveness in promoting growth. Additional policy instruments, such as R&D subsidies and tax incentives, may be needed to achieve stronger growth outcomes. Moreover, patent protection offers only partial coverage of intellectual property rights. Integrating complementary instruments—such as licenses, trademarks, and other forms of IP—may be more effective in fostering innovation and economic growth.

This study also opens several avenues for future research. We model patent protection through patent breadth. Future extensions could incorporate other dimensions such as patent length, blocking patents, or an endogenous patent system. For developing countries, where growth may rely more on imitation than innovation, it may be important to explicitly account for the distance to the technology frontier. On the empirical side, analyzing the relationship between R&D labor and patent protection may shed further light on the mechanisms linking IP policy and growth. We leave these questions for future research.

Table 1: Empirical evidence on the relationship between patent rights and growth: selected studies

Study	Sample	Method	Patent protection measure	Results
Thompson & Rushing (1996)	112 countries, 1970-1985	OLS, MLE	Rapp-Rozek	Positive but insignificant[a]
Gould & Gruben (1996)	90 countries, 1960-1988	OLS, IV	Rapp-Rozek	Mostly positive and significant
Ginarte & Park (1997)	60 countries, 1960-1990	SUR	Ginarte-Park	Mostly negative and insignificant
Thompson & Rushing (1999)	55 countries, 1975-1990	SUR	Rapp-Rozek	Inconclusive[a]
Hall & Ziedonis (2001)	95 semi-conductor US firms, 1979-1995	OLS	Author's own	Mostly positive and insignificant
Schneider (2005)	47 countries, 1970-1990	OLS	Ginarte-Park	Mostly positive and insignificant
Falvey et al. (2006)	79 countries, 1975-1994	OLS	Ginarte-Park	Mostly positive and significant[a]
Adams (2009)	73 developing countries, 1985-2003	SUR	Ginarte-Park	Negative and significant
Kim et al. (2012)	70 countries, 1975-2003	OLS, IV	Ginarte-Park	Mostly positive and significant
Hu & Png (2013)	54 manufacturing industries in 72 countries, 1976-2000	OLS	Author's own	Mostly positive and significant
Chu et al. (2014)	92 countries, 1970-2005	OLS	Ginarte-Park	Mostly negative and significant[b]
Mrad (2017)	48 developing countries, 1970-2009	SUR	Ginarte-Park	Positive and insignificant

Notes: [a]Depending on level of development. [b]Depending on distance to technology frontier.

Table 2: Baseline calibration values

Parameter	Description	Value	Source
λ	Rate of arrival of innovations	0.1	Caballero and Jaffe (1993); Laitner and Stolyarov (2013)
g	Growth rate	0.024	Average in OECD economies (1961-2023)
$\bar{z}$	Innovation step size	1.271	Implied by λ and g
$\mu\bar{z}$	Markup factor	1.25	De Loecker et al. (2020)
μ	Patent protection	0.983	Implied by $\mu\bar{z}$ and $\bar{z}$
ρ	Subjective discount rate	0.05	Yu and Lai (2024); Chu and Liao (2025)
ϕ	R&D professional knowledge	4	Implied by $\mu\bar{z}$
$\bar{\kappa}$	Complexity of innovation	2.612	Implied by $\bar{z}$ and ϕ
L_r	R&D labor share	0.0154	Average in OECD economies (1981-2021)
α	R&D labor intensity	0.906	Implied by g and L_r
φ	R&D productivity	2.661	Implied by g and L_r

Table 3: Decomposition of the effect of patent protection on growth: baseline calibration

	Partial effects				
	Direct effect	L_r effect	κ effect	z effect	Total effect
Exogenous step size	-0.022	0.168			0.146
Endogenous step size		0.048	0.098	-0.102	0.044

Table 4: Alternative calibrations

Parameter	Variant 1 $\lambda = 0.075$	Variant 2 $\lambda = 0.15$	Variant 3 $\mu\bar{z} = 1.15$	Variant 4 $\mu\bar{z} = 1.35$
λ	0.075	0.15	0.1	0.1
$\bar{z}$	1.377	1.174	1.271	1.271
$\mu\bar{z}$	1.25	1.25	1.15	1.35
μ	0.908	1.065	0.905	1.062
ϕ	4	4	6.667	2.857
$\bar{\kappa}$	3.597	1.897	4.953	1.985
α	0.896	0.917	0.844	0.933
φ	3.115	2.518	8.727	1.462

Notes: In all variants, the calibration strategy is as discussed in Section 4.1, targeting $g = 0.024$ and $L_r = 0.0154$. Only those parameter values that change relative to the baseline calibration are shown.

Table 5: Decomposition of the effect of patent protection on growth: alternative calibrations

	Partial effects				
	Direct effect	L_r effect	κ effect	z effect	Total effect
Variant 1: $\lambda = 0.075$					
Exogenous step size	-0.024	0.199			0.176
Endogenous step size		0.069	0.106	-0.083	0.092
Variant 2: $\lambda = 0.15$					
Exogenous step size	-0.021	0.140			0.120
Endogenous step size		0.029	0.090	-0.141	-0.021
Variant 3: $\mu\bar{z} = 1.15$					
Exogenous step size	-0.022	0.287			0.265
Endogenous step size		0.086	0.177	-0.111	0.153
Variant 4: $\mu\bar{z} = 1.35$					
Exogenous step size	-0.021	0.117			0.096
Endogenous step size		0.032	0.065	-0.094	0.002

Figure 1: Patent protection and growth: endogenous versus exogenous step size

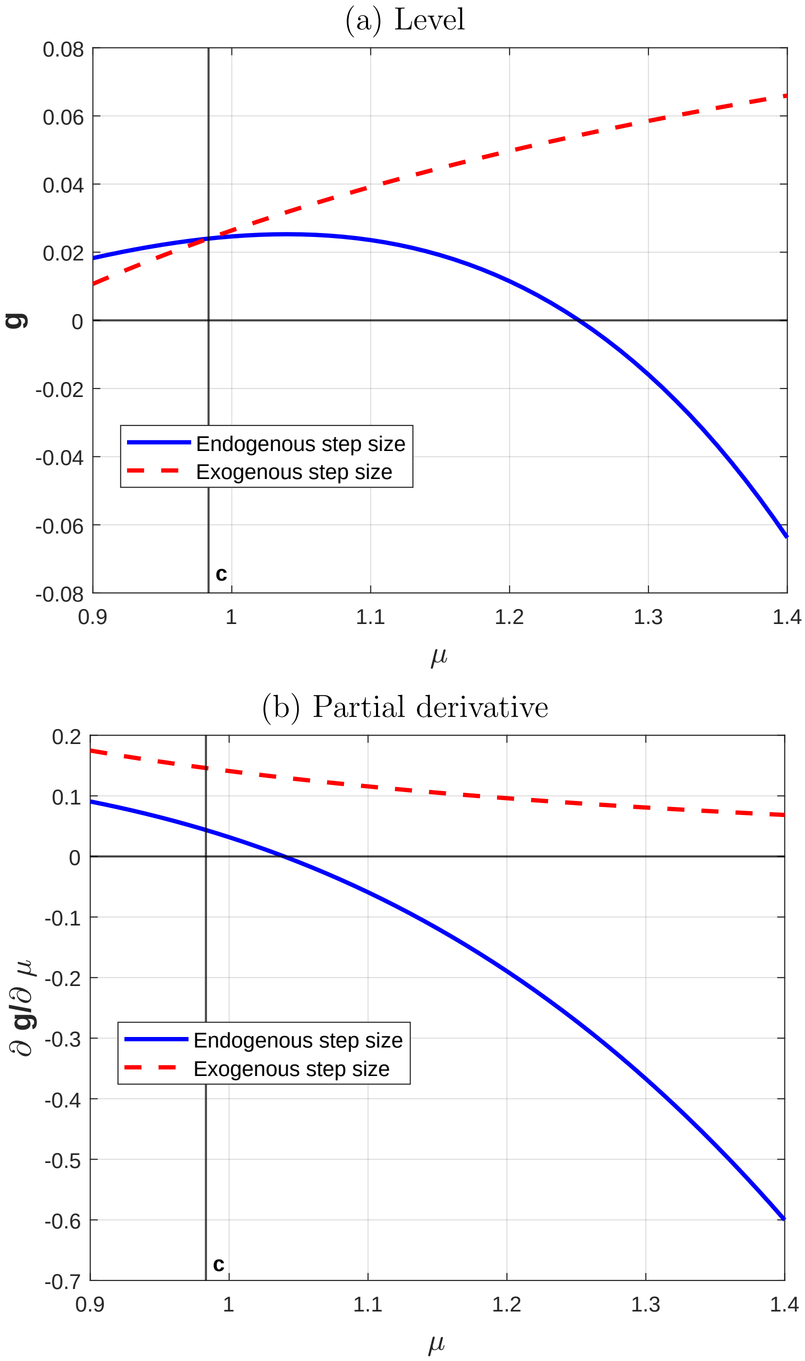
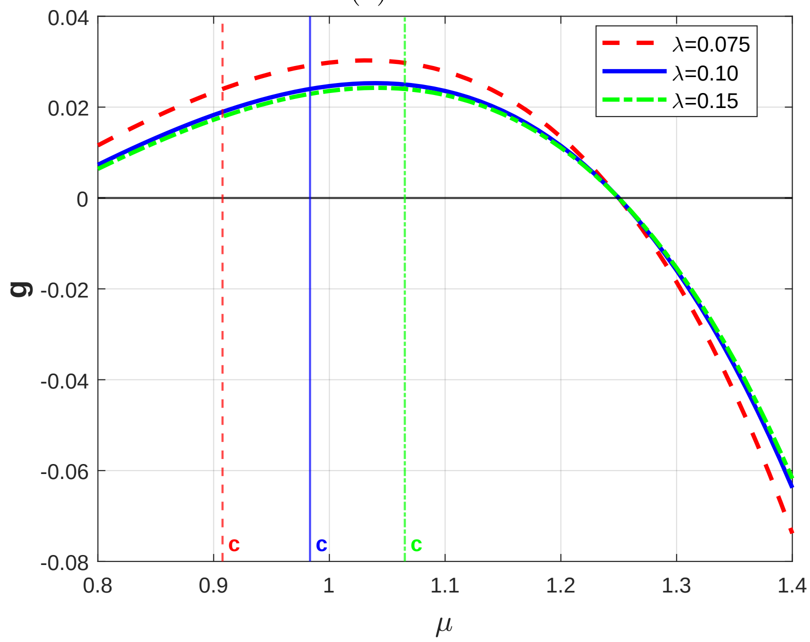


Figure 2: Patent protection and growth: varying the innovation rate (λ)

(a) Level



(b) Partial derivative

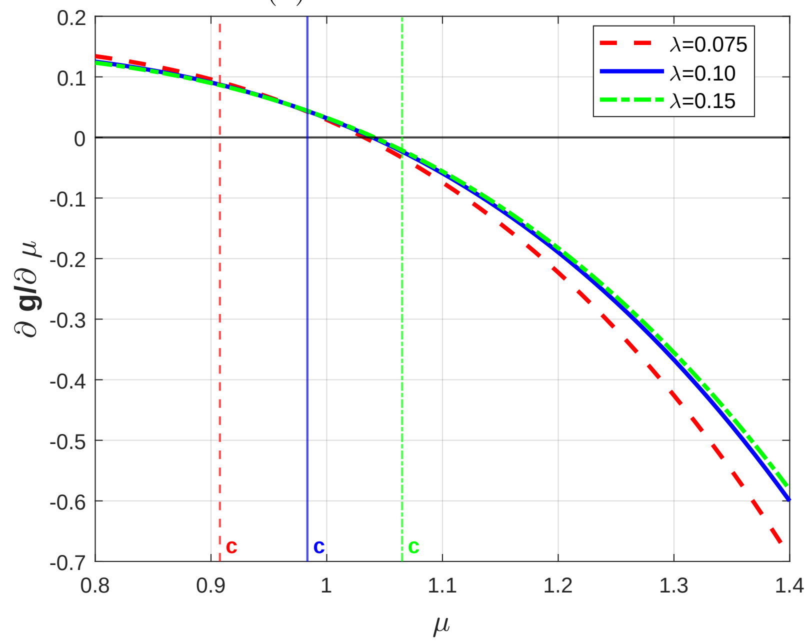


Figure 3: Patent protection and growth: varying the markup rate ($\mu\bar{z}$)

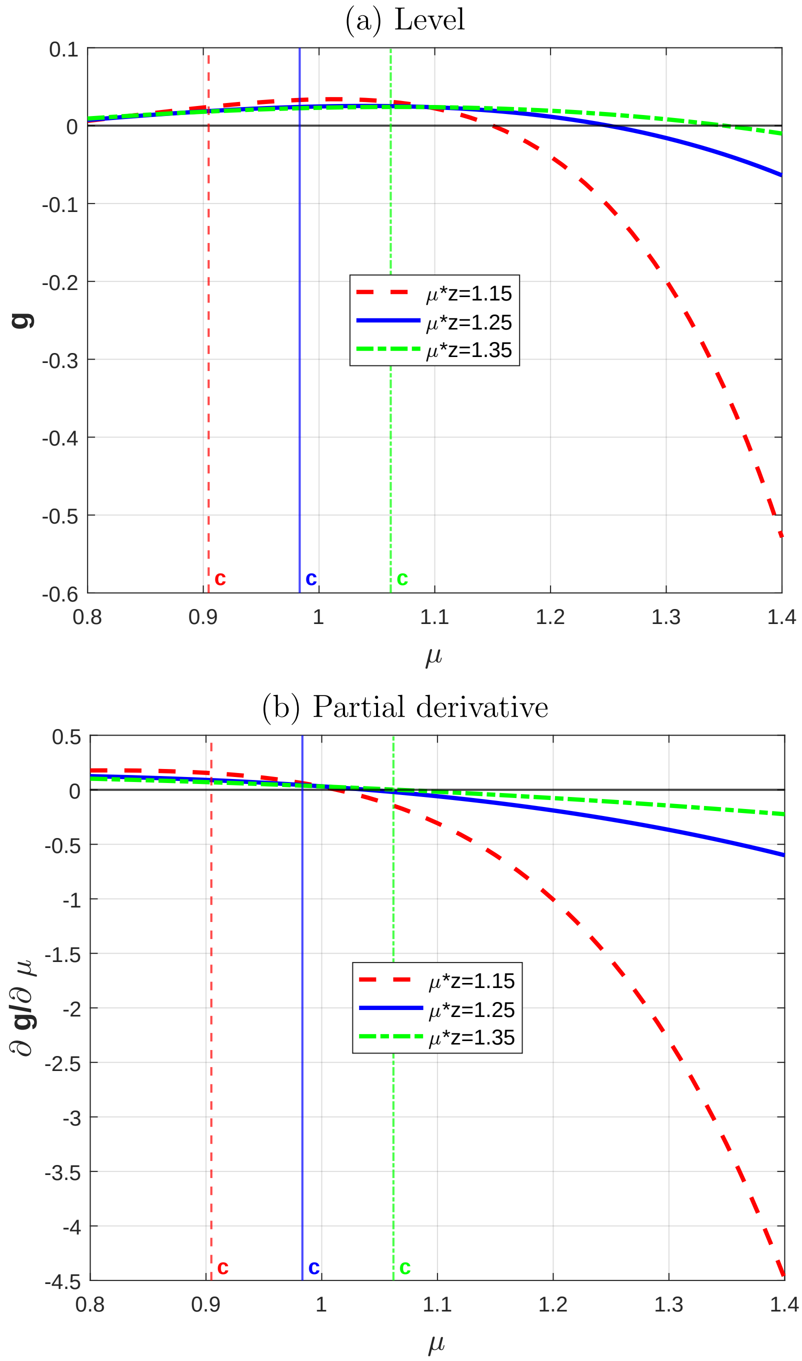


Figure 4: Patent protection and growth: endogenous versus exogenous labor

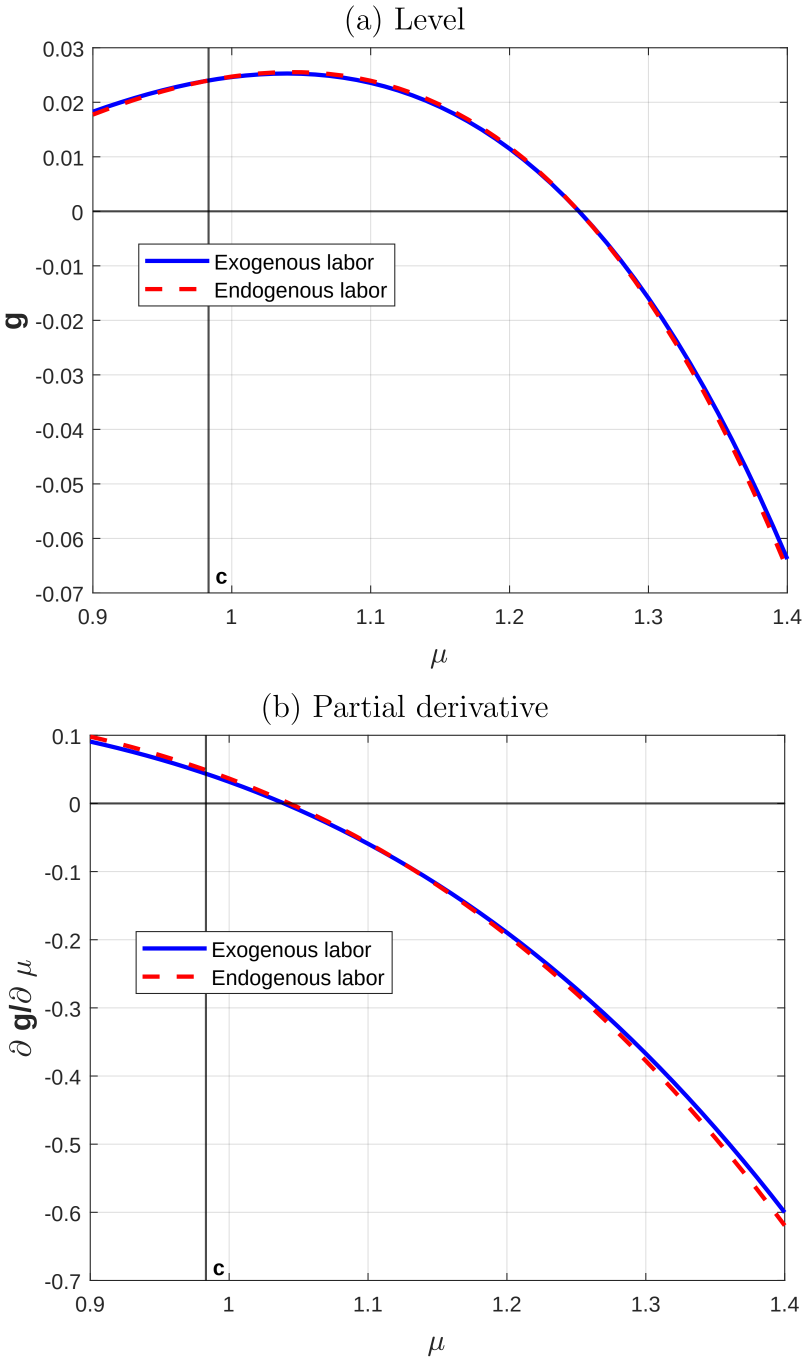
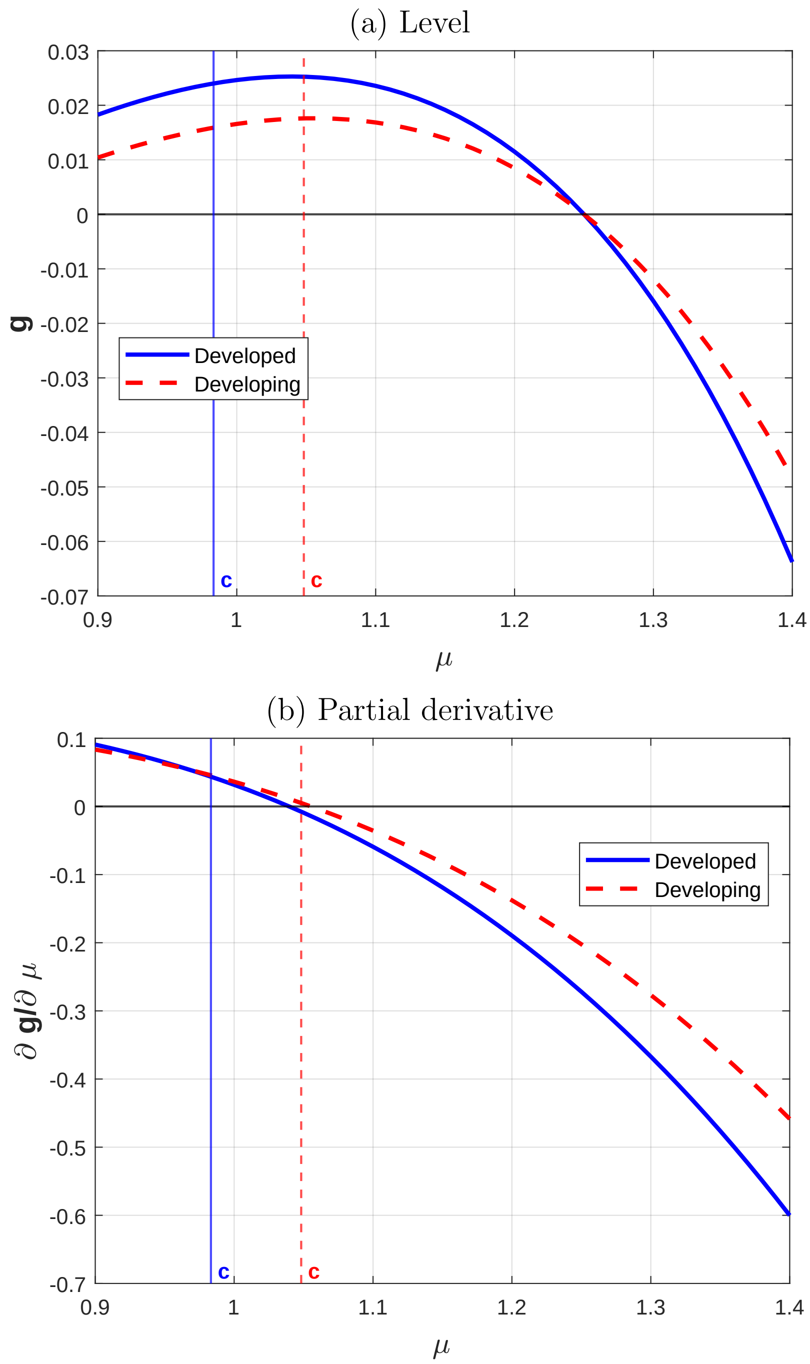


Figure 5: Patent protection and growth: developed versus developing countries



Declaration of generative AI and AI-assisted technologies in the writing process

During the preparation of this work the author(s) used ChatGPT-4o in order to improve the readability and language of the manuscript. After using this tool/service, the author(s) reviewed and edited the content as needed and take(s) full responsibility for the content of the published article.

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Appendix

A Derivations

A.1 Proof of Lemma 1

Proof. Taking the log-derivative of condition (16) with respect to time yields

$$\frac{\dot{\lambda}_t}{\lambda_t} + \frac{\dot{v}_t}{v_t} = \frac{\dot{w}_t}{w_t} + \frac{\dot{L}_{r,t}}{L_{r,t}}. \tag{A.1}$$

Using equations (10) and (22) yields $w_t = \frac{Z_t}{\mu\bar{z}}$. Then, combining equations (15) and (16) we obtain: $\alpha/(1-\alpha) = R_t/w_t L_{r,t}$ and using $w_t = \frac{Z_t}{\mu\bar{z}}$ we obtain $\alpha L_{r,t}/(1-\alpha)\mu\bar{z} = R_t/Z_t$ as shown in equation (24). Substituting equation (24) into (13) yields

$$\lambda_t = \frac{\varphi}{\bar{\kappa}}\Theta L_{r,t}, \tag{A.2}$$

where $\bar{\kappa} = \bar{z}^{\phi}$ and $\Theta \equiv \left(\frac{\alpha}{(1-\alpha)\mu\bar{z}}\right)^{\alpha}$. Log-differentiating (A.2) with respect to time yields $\dot{\lambda}_t/\lambda_t = \dot{L}_{r,t}/L_{r,t}$. Substituting this expression into (A.1) yields $\dot{v}_t/v_t = \dot{w}_t/w_t$ and using $w_t = \frac{Z_t}{\mu\bar{z}}$ yields the following identity

$$\frac{\dot{v}_t}{v_t} = \frac{\dot{w}_t}{w_t} = \frac{\dot{Z}_t}{Z_t}. \tag{A.3}$$

Using (A.2) and the HJB condition of the R&D sector in equation (10), (A.3) can be rewritten as:

$$\frac{\dot{Z}_t}{Z_t} = r_t + \frac{\varphi}{\bar{\kappa}}\Theta L_{r,t} - \frac{\Pi_{x,t}}{v_t}. \tag{A.4}$$

Using equations (9) and (16) yields $\Pi_{x,t}/v_t = (\mu\bar{z}-1)(1-\alpha)\lambda_t y_t/\mu\bar{z}w_t L_{r,t}$. Substituting the expression in equation (10) for y_t/w_t, yields $\Pi_{x,t}/v_t = (\mu\bar{z}-1)(1-\alpha)\lambda_t L_{x,t}/L_{r,t}$. Finally substituting this expression into (A.4), using (A.2) and the labor-market clearing condition yields:

$$\frac{\dot{Z}_t}{Z_t} = r_t + \frac{\varphi}{\bar{\kappa}}\Theta L_{r,t} - (1-\alpha)(\mu\bar{z}-1)\frac{\varphi}{\bar{\kappa}}\Theta(1-L_{r,t}). \tag{A.5}$$

We proceed by defining a new transformed variable $\Psi_t = \frac{C_t}{Z_t}$. Then, log-differentiating this variable with respect to time yields $\dot{\Psi}_t/\Psi_t = \dot{C}_t/C_t - \dot{Z}_t/Z_t$. Combining this expression with equation (3) yields $r_t = \dot{\Psi}_t/\Psi_t + \dot{Z}_t/Z_t + \rho$. Substituting this into (A.5) yields

$$\frac{\dot{\Psi}_t}{\Psi_t} = (1-\alpha)(\mu\bar{z}-1)\frac{\varphi}{\bar{\kappa}}\Theta - \frac{\varphi}{\bar{\kappa}}[1+(1-\alpha)(\mu\bar{z}-1)]\Theta L_{r,t} - \rho. \tag{A.6}$$

From the final goods market clearing condition we obtain that $\Psi_t = (y_t - R_t)/Z_t$. Using equations (22) and (24) for y_t/Z_t and R_t/Z_t respectively, yields

$$\Psi_t = 1 - \frac{(1-\alpha)\mu\bar{z} + \alpha}{(1-\alpha)\mu\bar{z}} L_{r,t}. \tag{A.7}$$

Differentiating (A.7) with respect to time, and substituting it into (A.6) yields

$$\dot{L}_{r,t} = \frac{(1-\alpha)\mu\bar{z}\Psi_t}{(1-\alpha)\mu\bar{z}+\alpha} \left\{ \frac{\varphi}{\bar{\kappa}}[1+(1-\alpha)(\mu\bar{z}-1)]\Theta L_{r,t} - (1-\alpha)(\mu\bar{z}-1)\frac{\varphi}{\bar{\kappa}}\Theta + \rho \right\} \tag{A.8}$$

Expression (A.8) is a one-dimensional differential equation in $L_{r,t}$. There exists a single steady-state equilibrium point:

$$L_r = \frac{(1-\alpha)(\mu\bar{z}-1)\frac{\varphi}{\bar{\kappa}}\Theta - \rho}{\frac{\varphi}{\bar{\kappa}}[1+(1-\alpha)(\mu\bar{z}-1)]\Theta}, \tag{A.9}$$

where the following condition

$$\rho < (1-\alpha)(\mu\bar{z}-1)\frac{\varphi}{\bar{\kappa}}\Theta < [1+(1-\alpha)(\mu\bar{z}-1)] + \rho, \tag{A.10}$$

ensures that on the balanced growth path, $0 < L_r < 1$. Evaluating the derivative of (A.8) with respect to $L_{r,t}$ for the steady-state L_r yields: $\left.\frac{\partial \dot{L}_{r,t}}{\partial L_{r,t}}\right|_{L_{r,t}=L_r} > 0$. Given that $L_{r,t}$ is a jump variable, the dynamics is characterized with saddle-point stability, such that $L_{r,t}$ must jump to the unique and stable steady-state value L_r. □

A.2 Proof of Lemma 2

Proof. From (A.9) it is evident that the labor allocation to R&D is constant on the balanced growth path. Using the labor-market clearing condition $L_x = 1 - L_r$, yields the equilibrium labor allocation to production

$$L_x = 1 - \frac{(1-\alpha)(\mu\bar{z}-1)\frac{\varphi}{\bar{\kappa}}\Theta - \rho}{\frac{\varphi}{\bar{\kappa}}[1+(1-\alpha)(\mu\bar{z}-1)]\Theta}. \tag{A.11}$$

□

A.3 Proof of Proposition 1

Proof. Equation (A.9) can be straightforwardly manipulated to give

$$L_r = \frac{(1-\alpha)(\mu\bar{z}-1)}{1+(1-\alpha)(\mu\bar{z}-1)} - \frac{\rho\bar{\kappa}}{\varphi[1+(1-\alpha)(\mu\bar{z}-1)]\Theta}, \tag{A.12}$$

Note that, under endogenous step size, $\mu\bar{z}$ is fixed at $(1+\phi)/\phi$, keeping $\bar{\kappa}$ as the only term depending on μ in (A.12). Taking the partial derivative of L_r with respect to μ gives

$$\frac{\partial L_r}{\partial \mu} = \frac{\partial L_r}{\partial \bar{\kappa}}\frac{\partial \bar{\kappa}}{\partial \mu} = \frac{\phi^2 \rho \bar{\kappa}}{\varphi(1-\alpha+\phi)\mu\Theta} > 0 \tag{A.13}$$

The imposed parameter restrictions, namely $\alpha \in (0,1)$ and $\rho, \varphi, \phi > 0$, guarantee the strict positiveness of $\partial L_r/\partial\mu$. □

A.4 Patent protection and R&D labor with exogenous step size

In the exogenous step-size model, since $\bar{z}$ is fixed, the markup factor $\mu\bar{z}$ varies with μ. Denote the numerator and denominator of the right-hand side of (A.9) as

$$n \equiv (1-\alpha)(\mu\bar{z}-1)\frac{\varphi}{\bar{\kappa}}\Theta - \rho \quad \text{and} \quad d \equiv \frac{\varphi}{\bar{\kappa}}[1+(1-\alpha)(\mu\bar{z}-1)]\Theta.$$

The partial derivative of L_r with respect to μ in the exogenous step-size model can then be expressed as

$$\frac{\partial L_r}{\partial \mu} = \frac{\varphi(1-\alpha)\Theta}{\bar{\kappa}\mu}\frac{1}{d} - \frac{\varphi\Theta}{\bar{\kappa}\mu}\frac{n}{d^2}\left[(1-\alpha)^2\mu\bar{z} - \alpha^2\right] + \frac{\partial L_r}{\partial \bar{\kappa}}\frac{\partial \bar{\kappa}}{\partial \mu}, \tag{A.14}$$

where the first and second terms on the right-hand side arise from the exogenous step size, while the final term corresponds to the expression in equation (A.13).

Note that the first and third terms on the right-hand side are necessarily positive. The second term is also positive as long as the markup factor is not excessively high, specifically if $\mu\bar{z} < [\alpha/(1-\alpha)]^2$. This condition is therefore sufficient for the overall effect of μ on L_r to be greater in the exogenous step-size model (equation (A.14)) than in the endogenous step-size model (equation (A.13)). This condition is easily met in the parametrized model, given the range of α values considered (see Sections 4.1–4.3).

A.5 Proof of Proposition 2

Proof. Substituting equations (18), (19) and (24) into (13) yields, after rearranging, the steady-state rate of innovation:

$$\lambda = \left(\frac{\alpha}{1-\alpha}\right)^{\alpha}\left(\frac{\phi}{\phi+1}\right)^{\phi+1}\varphi L_r \mu^{\phi}. \tag{A.15}$$

Taking the partial derivative with respect to μ, while using (A.14), and rearranging gives

$$\frac{\partial \lambda}{\partial \mu} = \left(\frac{\alpha}{1-\alpha}\right)^{\alpha}\left(\frac{\phi}{\phi+1}\right)^{\phi+1}\varphi\phi\mu^{\phi-1}\left[L_r + \frac{\phi\rho\bar{\kappa}}{\varphi(1-\alpha+\phi)\Theta}\right], \tag{A.16}$$

which, given the parameter restrictions, is unambiguously positive. □

B Extension: endogenous labor

This section extends the baseline model by incorporating a leisure-labor choice in the households optimization problem, thereby giving rise to an endogenous supply of labor.

The representative household has preferences over consumption and leisure:

$$U = \int_0^\infty e^{-\rho t} \left[\ln C_t + \gamma \ln(1 - L_t)\right] dt, \tag{B.1}$$

where C_t is consumption at time t, L_t is labor supply, $\gamma > 0$ is leisure preference and $\rho > 0$ is the subjective discount rate. The household faces the following budget constraint

$$\dot{A}_t = r_t A_t + w_t L_t - C_t. \tag{B.2}$$

The optimality condition for consumption is still given by (3). The consumption-leisure optimality condition is now

$$L_t = 1 - \gamma \frac{C_t}{w_t}. \tag{B.3}$$

The production sectors of the economy (final good producers, intermediate good producers and the R&D sector) remain identical. The equilibrium path of the economy includes a modified market-clearing condition for the labor market: $L_{x,t} + \int_0^1 L_{r,t}(i)\, di = L_t$.

The existence and uniqueness of the balanced growth path and the dynamic behaviour of the system can be followed just like in the baseline model of exogenous labor. By doing so, we arrive at equations (A.1)-(A.4) in Appendix A.1.

Equation (A.5) now becomes:

$$\frac{\dot{Z}_t}{Z_t} = r_t + \frac{\varphi}{\bar{\kappa}} \Theta L_{r,t} - (1-\alpha)(\mu \bar{z} - 1) \frac{\varphi}{\bar{\kappa}} \Theta (L_t - L_{r,t}), \tag{B.4}$$

where, again, $\Theta \equiv \left(\frac{\alpha}{(1-\alpha)\mu\bar{z}}\right)^\alpha$. Similarly, equation (A.6) changes to:

$$\frac{\dot{\Psi}_t}{\Psi_t} = (1-\alpha)(\mu \bar{z} - 1) \frac{\varphi}{\bar{\kappa}} \Theta (L_t - L_{r,t}) - \frac{\varphi}{\bar{\kappa}} \Theta L_{r,t} - \rho. \tag{B.5}$$

Following the consumption-leisure optimality condition, we can derive the following expression for the aggregate labor supply:

$$L_t = 1 - \gamma \frac{C_t}{w_t} = 1 - \gamma \mu \bar{z} \Psi_t, \tag{B.6}$$

where the second equality comes from the fact that $w_t = \frac{Z_t}{\mu \bar{z}}$. Note that L_t is a linear function of Ψ_t. From the final goods market clearing condition $\Psi_t = (y_t - R_t)/Z_t$. Using equations (22) and (24) for y_t/Z_t and R_t/Z_t respectively, yields

$$\Psi_t = (L_t - L_{r,t}) - \frac{\alpha}{(1-\alpha)\mu\bar{z}} L_{r,t} = L_t - \left(\frac{(1-\alpha)\mu\bar{z} + \alpha}{(1-\alpha)\mu\bar{z}}\right) L_{r,t}. \tag{B.7}$$

Combining this expression with (B.6) gives

$$\Psi_t = \frac{1}{1+\gamma\mu\bar{z}} - \left(\frac{(1-\alpha)\mu\bar{z} + \alpha}{(1-\alpha)\mu\bar{z}(1+\gamma\mu\bar{z})}\right) L_{r,t}. \tag{B.8}$$

Substituting the expression for L_t into $L_t - L_{r,t}$ yields

$$L_t - L_{r,t} = \frac{1}{1+\gamma\mu\bar{z}} + \frac{\alpha\gamma + \alpha - 1}{(1-\alpha)(1+\gamma\mu\bar{z}} L_{r,t}. \tag{B.9}$$

Differentiating (B.8) with respect to time, substituting it into (B.7), and using (B.9) gives

$$\begin{aligned} \dot{L}_{r,t} = & \frac{(1-\alpha)\mu\bar{z}(1+\gamma\mu\bar{z})\Psi_t}{(1-\alpha)\mu\bar{z} + \alpha} \\ & \times \left\{ \frac{\varphi}{\bar{\kappa}} \left[\frac{(1+\gamma\mu\bar{z}) - (\alpha\gamma + \alpha - 1)(\mu\bar{z} - 1)}{1+\gamma\mu\bar{z}} \right] \Theta L_{r,t} - \frac{\varphi}{\bar{\kappa}} \frac{(1-\alpha)(\mu\bar{z} - 1)}{1+\gamma\mu\bar{z}} \Theta + \rho \right\} \end{aligned} \tag{B.10}$$

The steady-state R&D labor allocation is then

$$L_r = \frac{(1-\alpha)(\mu\bar{z} - 1)}{1+\gamma\mu\bar{z} + (1-\alpha-\alpha\gamma)(\mu\bar{z} - 1)} - \frac{\rho(1+\gamma\mu\bar{z})\bar{\kappa}}{\varphi\left[1+\gamma\mu\bar{z} + (1-\alpha-\alpha\gamma)(\mu\bar{z} - 1)\right]\Theta}. \tag{B.11}$$

Given (B.11), it follows from (B.8) that Ψ_t is constant on the balanced growth path. Then from (B.6) it follows that L_t is constant and given the labor-market clearing condition, $L_{x,t}$ is also constant on the balanced growth path. Therefore the labor allocations to each sector are stationary. Substituting (B.11) in equation (25) gives the following closed-form expression of strengthening patent policy on economic growth:

$$\frac{\partial g}{\partial \mu} = \frac{g}{\mu}\left[\phi + \frac{\phi\bar{\kappa}\rho(1+\gamma\mu\bar{z})}{L_r\varphi\left[1+\gamma\mu\bar{z} + (1-\alpha-\alpha\gamma)(\mu\bar{z} - 1)\right]\Theta} - \frac{1}{\ln\bar{z}}\right], \tag{B.12}$$