Revisiting China's gradualistic economic approach and financial market

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Abstract

We develop a model economy with active financial markets, in which the policymaker's adoption of a gradualistic approach is a Bayesian Nash equilibrium. In addition to its financing role, the financial market also creates a channel for information revelation, encouraging the policymaker to take small policy steps. Smaller policy steps lead to more precise information about the productivity shock. Acquiring more information - both on the extensive margin and the intensive margin - provides sufficient incentives for the policymaker to consistently follow the gradualistic approach. This result holds robust for both exogenous and endogenous information models.

Keywords: the gradualistic approach; active financial markets; information acquisition; endogenous information

JEL Classifications: O2, G1

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1 Introduction

A key approach successfully employed by China to reform its economy over the past 40 years is encapsulated by the aphorism “crossing the river by touching the stones”, a gradualistic approach that optimizes policy through experimentation. This approach involves implementing reforms in small, incremental steps to avoid the risks of large-scale upheaval and to learn from the economy’s reactions to policy changes before making further adjustments. It also allowed the Chinese government to incrementally address market imperfections over time while actively intervening in resource allocation through credit and other policies. In the context of under-developed financial market, this approach has been a key factor in China’s sustained economic growth over the past four decades. However, the question arises: Can China continue to use its gradualistic approach in the presence of fully developed financial markets? Brunnermeier et al. (2017) raise this question and offer a pessimistic perspective, suggesting that as financial markets provide participants greater flexibility in securing financing, they may also foster speculation about future policy shifts, rendering the government’s gradualistic approach ineffective.

In our view, their pessimisitic perspective may stem from overlooking the information-revelation role of financial markets. In their model, Brunnermeier et al. (2017) focus solely on the financing function of financial markets, dismissing their role in information discovery. By doing so, they eliminate a key channel through which the government can gather information, thereby undermining its motivation to pursue small, consistent policy step. This, in turn, contributes to the ineffectiveness of the gradualist approach. In this paper, we aim to explore the following question: If we account for the information-revelation role of financial markets, can policymakers continue to follow the gradualist approach in a time-consistent manner? To address this, we incorporate the information-revelation function into Brunnermeier et al. (2017) and investigate the feasibility of a time-consistent gradualist approach in an economy with active financial markets.

In the baseline model, where financial markets are not fully developed, firms must rely on state-controlled banks for financing, which will only provide loans once the policymaker has chosen her policy. This sets up a game between the policymaker and private agents, where the policymaker moves first. In this equilibrium, the policymaker adops a gradualistic approach by taking small policy steps. Intuitively, choosing smaller policy steps reduces the noise in the firm value signal about economic fundamentals, thereby increasing the signal’s informativeness. By taking small policy steps in the current period, the policymaker acquires high-quality signals, which help improve her policy decisions in future periods. This ability to gather high-quality information provides strong incentives for the policymaker to continue following the gradualistic approach.

In the complete model with well-developed financial markets, private agents have the financial flexibility to make investment decisions even before the policymaker chooses her policy action. Instead of relying on state-controlled banks for financing, they can obtain funding directly from financial markets. However, competition among private agents for limited real investment opportunities forces them to act before the policymaker. The presence of financial markets significantly alters the game between the policymaker and private agents. In this case, agents make investment decisions ahead of the government's policy. To manage the economy, the policymaker first puts forward a policy proposal (ex ante policy) and then chooses policy actions afterward (ex post policy). In the scenario with exogenous information of financial investors, we show that choosing the gradualistic approach consistently is a Bayesian Nash equilibrium. Intuitively, the financial market serves two functions in our model: providing financing for firms and generating information. The policymaker learns about the productivity shocks not only from firms' investment and production decisions but also from financial prices. Gathering more information along the extensive margin provides additional incentives for the policymaker to follow the gradualistic approach in a time-consistent manner. By extending Brunnermeier et al. (2017) to incorporate the information-production role of financial markets, we generalize the conditions under which the gradualistic approach remains effective.

Furthermore, we show that the policymaker obtains more precise signals about the productivity shock by taking smaller policy steps. The intuition is as follows: Smaller policy steps reduce the total uncertainty in the economy caused by both the cost and demand shocks. On the one hand, with less uncertainty, firms make better investment and production decisions, reducing noise in the firm value signal and thereby increasing its informativeness. On the other hand, all financial traders - both informed and uninformed - knowing the productivity shock perfectly, trade more aggressively based on their private information, injecting more information into the financial price signal and increasing its informativeness.

In the full model with information acquisition, the policymaker still adopts the gradualistic approach, and small policy steps increase the informativeness of both the financial price signal and the firm value signal. The intuition behind these results are similar to the earlier model.

However, the gradualistic policy has no net effect on the informativenss of the financial price signal. Intuitively, smaller policy steps reduces the total uncertainty induced by the cost shock and demand shocks. As a result, informed traders trade more aggressively, increasing the informativeness of the financial price signal (i.e, the intensive margin). On the other hand, with more precise price signals, some informed traders have incentives to switch to being uninformed, which reduces the price informativeness of the financial price signal (i.e., the extensive margin). These two opposing effects - the positive intensive margin and the negative extensive margin - cancel each other out, meaning that changes in policy steps have no overall effect on the informativeness of the financial price signal.

Literature review. Our paper is closely related to two theoretical works. Brunnermeier et al. (2017) is a seminal paper that examines the desirability of China's gradualistic approach under different environments, providing a definition of this approach and an analytical framework to explore this question. Benhabib et al. (2019) develop a model of informational interdependence between financial markets and the real economy, investigating how mutural learning between these two sectors leads to self-fulfilling uncertainties in the macroeconomy. Our paper incorporates the mutural learning mechanism developed by Benhabib et al. (2019) into the framework of Brunnermeier et al. (2017) and revisits the desirability of the gradualistic approach in an economy with well-developed financial markets.

Our paper argues that financial prices provide additional information that compels policymakers to follow a gradualistic approach in a time-consistent manner. Hence, our work also relates to the large literature emphasizing the information-revelation function of financial prices, including Hayek (1945), Grossman (1976), Hellwig (1980), Grossman and Stiglitz (1980), Kyle (1985), and Vives (1988, 2008, 2014), among others. In particular, we cite Hayek (1945)'s famous statements,

"*We must look at the price system as such a mechnism for communicating information if we want to understand its real function....It is more than a metaphor to describe the price system as a kind of machinery for registering change, or a system of telecommunications which enables individual producers to watch merely the movement of a few pointers, as an engineer might to watch the hands of a few dials, in order to adjust their activities to changes of which they may never know more than is reflected in the price movement.*"

Moreover, many researchers have empirically examined how the gradualistic approach has contributed to China's macroeconomic development. Chen and Zha (2024) provide an analytical review for these studies.

The paper is organized as follows: Section 2 presents the model; Section 3 examines the baseline model without fully developed financial markets; Section 4 investigates the complete model with active financial markets and exogenous information for financial traders; Section 5 explores the full model with active financial markets and endogenous information; Section 6 concludes; and the mathematical proofs of the propositions are provided in Section 7.

2 The model

2.1 The model setup

Consider an economy with three dates $t = 0, 1, 2$, in which there are three types of agents: a large strategic policymaker, firm j, and a group of financial traders (speculators). There are two types of goods: an intermediate good and a final good. The production of the final good is exogenously, Y, and the price of the final good is normalized as the numeraire, $P \equiv 1$.

Policymaker. Given her information set $\mathcal{I}^G$, the policymaker makes two policy choices, a_1 and a_2, at dates 1 and 2, respectively, to maximize a quadratic objective function:

$$U_0 = \max_{\{a_1, a_2\}} E\left[-(\theta - a_1)^2 - (\theta - a_2)^2 \left|\mathcal{I}^G\right.\right]. \tag{1}$$

The common productivity shock, $\theta \sim N\left(\overline{\theta}, \tau_\theta^{-1}\right)$ is unobservable to the policymaker. In order to make optimal policy decisions, the policymaker extracts information about the productivity shock θ through two information sources: the firm value and the financial price.

Intermediate goods firm. Firm j is an intermediate goods firm. It produces the intermediate good Y_j using the final good as an input according to the production function:

$$Y_j = e^{\theta} k_j, \tag{2}$$

subject to the quadratic cost function:

$$c(k_j) = \frac{1}{2} e^{-a_1 \varepsilon_j} k_j^2, \tag{3}$$

where θ is the common productivity shock to the whole economy, ε_j is the idiosyncratic cost shock specific to firm j, and k_j is the investment input. The input k_j fully depreciates after production. Notice that the policymaker's policy action a_1, which private agents either observe or anticipate, also influences firms' investment costs.

The market demand function for the intermediate good Y_j is assumed to be:

$$Y_j = \left(\frac{1}{P_j}\right)^{\sigma} e^{a_1 \xi_j} Y, \tag{4}$$

where P_j is the price of intermediate good j, and ξ_j represents the idiosyncratic demand shock to intermediate good j. Y is an exogenous constant corresponding to aggregate output (real GDP, denote $y \equiv \log Y$), whereas parameter $\sigma > 1$ measures the price elasticity of demand for good j. Notice that the policymaker's policy a_1 also influences the demand for good j.

In summary, given information $\mathcal{I}_j$, firm $j's$ optimization problem is to maximize its expected profit:

$$E\left[P_j Y_j - c\left(k_j\right) | \mathcal{I}_j\right], \tag{5}$$

subject to its production technology (2), cost function (3), and market demand (4).

Financial markets and traders. Following Benhabib et al. (2019), we assume that a financial asset exists where speculators trade a financial asset (a derivative) contingent on the firm $j's$ firm value, defined as its total income[1]:

$$V_j = P_j Y_j. \tag{6}$$

Specifically, we assume that the payoff of the risk asset (the financial derivative contract) takes the form

$$v_j = \log V_j = \log P_j Y_j. \tag{7}$$

Let q_j denote the market trading price of risky asset associated with firm j. That is, taking a long position in one unit of this financial asset (derivative) requires an initial outlay of q_j and entitles the holder to receive the risky payoff v_j at a later time.

The utility function of speculators is assumed to be

$$U\left(W^i\right) = -\exp\left(-\gamma W^i\right), \tag{8}$$

[1] Assuming that the underlying asset of the derivative is is to ensure the payoff the underlying asset following a log-normal distribution and thus to achieve tractability. This is along the line of the assumption in the literature that a firm's asset value or sales revenue follows a geometric Brownian motion (see, e.g., Merton (1973) and He and Xiong (2012)). This also parallels the modeling device that assumes a specific function form of noisy trading as in Goldstein, Ozdenoren, and Yuan (2013), Sockin and Xiong (2015), and Goldstein and Yang (2019).

where W^i represents the final wealth of speculator $i \in [0,1]$, and γ is the constant coefficient of absolute risk aversion (CARA). The initial wealth of a speculator is assumed to be W_0 and the risk-free (gross) rate is $R_f \, (\equiv 1)$. Each speculator invests in a portfolio composed of the risk-free asset and the risky asset associated with firm j. If speculator i takes a position of x^i units in the financial asset, his final wealth is given by

$$W^i = \left(W_0 - q_j x^i\right) R_f + v_j x^i - \psi = W_0 - \psi + \left(v_j - q_j\right) x^i, \tag{9}$$

where $\psi \in \{0, c\}$ represents the information cost: if the speculator pays $c \, (> 0)$, he becomes an informed trader; otherwise, he remains uninformed.

The net aggregate supply of the financial asset is assumed to be zero. The demand of noise/liquidity traders in the financial market is denoted by u_j, which follows the distribution $u_j \sim N\left(0, \tau_u^{-1}\right)$, where u_j is independent of all other random variables.

Uncertainties and information. The economy is subject to three types of uncertainties: a demand shock ξ_j and two supply shocks. The supply shocks include a common (aggregate) productivity shock θ and an idiosyncratic capital cost shock ε_j. Their prior distributions are given by: $\theta \sim N\left(\overline{\theta}, \tau_\theta^{-1}\right)$, $\varepsilon_j \sim N\left(0, \tau_\varepsilon^{-1}\right)$, and $\xi_j \sim N\left(0, \tau_\xi^{-1}\right)$. The shocks θ, ε_j, and ξ_j are mutually independent, and their prior distributions are publicly known. The common productivity shock θ affects the payoffs of both the policymaker and firm j, whereas the idiosyncratic shocks ξ_j and ε_j directly impact only firm j.

The policymaker has no prior private information about all the shocks. However, she extracts information about the common productivity shock θ from the signals about the firm value and financial prices. Firm j possesses perfect information about the two supply shocks and learns information about its demand shocks from the financial prices in the financial market. The financial traders has imperfect information regarding the demand shocks ξ_j and perfect information about the common productivity shock θ.

In the financial market, as in Grossman and Stiglitz (1980), there is a continuum of traders with unit mass. Traders are categorized into two types: informed and uninformed. By paying a fixed cost $c > 0$, an informed trader i acquires a noisy private signal about the demand shock ξ_j, given by $\chi_j^i = \xi_j + \varrho_j^i$, where $\varrho_j^i \sim N\left(0, \tau_\chi^{-1}\right)$. The noise term ϱ_j^i is independent across different informed traders i. Uninformed traders, on the other hand, do not receive any private signal regarding ξ_j. The proportion of informed traders, denoted by λ, is exogenously given in Section 4 but endogenously determined in Section 5. For our purpose, we assume that all financial traders have perfect information about the common productivity shock, which creates a new channel for the government learning information about the productivity shock.

2.2 Defining gradualism

Given the policymaker's quadratic objective function, one might expect it to choose an action in both periods that corresponds to the best prediction of θ. Similar to Brunnermeier et al. (2017), gradualism in our context refers to the policymaker's decision to deliberately underreact to its best prediction of θ based on the available information.

Definition 1 *Under the gradualistic approach, the policymaker's action at $t = 1$ is below the best prediction of θ based on the available information, i.e., $a_1 < E\left(\theta|\mathcal{I}^{G_1}\right)$.*

Definition 1 generalizes the definition of the gradualistic approach by Brunnermeier et al. (2017). In their model, the financial market plays no role of information production, and the policymaker cannot acquire information from the financial market, implying that $\mathcal{I}^{G_1} = \emptyset$. The gradualistic approach, therefore, implies that the policymaker's action at $t = 1$ is below the prior mean of θ, i.e., $a_1 < \overline{\theta} = E(\theta)$. In our model, however, the policymaker acquires new information about the productivity shock from the financial markets, implying that $\mathcal{I}^{G_1}$ is not an empty set. In the model with active financial markets, we will establish that if the policymaker acquires information from the financial market, she will choose a time-consistent gradualistic economic approach, namely, $a_1 < E\left(\theta|\mathcal{I}^{G_1}\right)$.

3 Government gradualism absent financial markets: benchmark

In this section, we examine a benchmark in which the financial market is absent. In the absence of fully developed financial markets, private firms must receive financing from state-controlled banks, which wait until the policymaker has chosen its policy a_1 at $t = 1$. That is, firms do not make investment decisions until the policymaker has chosen its policy action first. This corresponds to a Stackelberg game between the policymaker and firms, where the policymaker moves first. The timeline is as follows:

$\underline{t = 0}$:

Event 01: The shocks θ, ξ_j, ε_j are realized and unobservable to the policymaker, firm j sees θ and ε_j perfectly.

$\underline{t=1}$:

Event 11: The policymaker chooses its action a_1.

Event 12: Firm j makes its investment decision, k_j, based on available information $\{\theta, \varepsilon_j, a_1\}$.

$\underline{t=2}$:

Event 21: The firm value, v_j, is realized, and the payoff of the financial contract is delivered.

Event 22: The policymaker observes v_j ($\equiv \log P_j Y_j$), updates its belief about θ, and chooses its action a_2.

The equilibrium is composed of firm $j's$ investment decisions k_j, and the policymaker's policy actions (a_1, a_2). In period 1, after the policymaker chose her policy action a_1, firm $j's$ information set has been enlarged as $\mathcal{I}_j = \{\theta, \varepsilon_j, a_1\}$. Then, firm j chooses its investment decisions k_j to maximize its expected profit (5) subject to the production function (2), cost function (3), and market demand (4). Substituting (2), (3), and (4) into (5) leads to the unconstrained optimizing problem:

$$\max_{\{k_j\}} E\left[e^{\frac{a_1}{\sigma}\xi_j} Y^{\frac{1}{\sigma}} e^{\theta\left(1-\frac{1}{\sigma}\right)} k_j^{1-\frac{1}{\sigma}} - \frac{1}{2} e^{-a_1\varepsilon_j} k_j^2 | \mathcal{I}_j\right].$$

Solving the first-order condition (FOC) wrt k_j gives us

$$k_j = \left(1-\frac{1}{\sigma}\right)^{\Theta} Y^{\frac{\Theta}{\sigma}} \exp\left\{\theta\left(1-\frac{1}{\sigma}\right)\Theta + a_1\Theta\varepsilon_j + \frac{\Theta}{2}\left(\frac{a_1}{\sigma}\right)^2 \tau_\xi^{-1}\right\}, \tag{10}$$

where $\Theta \equiv \frac{1}{1+1/\sigma}$. The firm value is thus

$$v_j \equiv \log P_j Y_j = \frac{a_1}{\sigma}\xi_j + \frac{y}{\sigma} + \theta\left(1-\frac{1}{\sigma}\right) + \left(1-\frac{1}{\sigma}\right)\log k_j. \tag{11}$$

Next we solve the policymaker's problem by backward induction. At the beginning of $t=2$, the firm value v_j is realized. The policymaker observes v_j, which serves as a noisy signal about θ, namely,

$$\widetilde{v}_j \equiv \frac{v_j - \frac{2\Theta}{\sigma}y - \left(1-\frac{1}{\sigma}\right)\Theta\log\left(1-\frac{1}{\sigma}\right) - \left(1-\frac{1}{\sigma}\right)\frac{\Theta}{2}\left(\frac{a_1}{\sigma}\right)^2\tau_\xi^{-1}}{2\left(1-\frac{1}{\sigma}\right)\Theta} = \theta + \epsilon_0, \tag{12}$$

where

$$\epsilon_0 \equiv \frac{a_1}{2}\varepsilon_j + \frac{a_1}{2(\sigma-1)\Theta}\xi_j,$$

with $\epsilon_0 \sim N\left(0, \tau_{\epsilon_0}^{-1}\right)$ and $\tau_{\epsilon_0}^{-1} = a_1^2\left(1/4\tau_\varepsilon + 1/4(\sigma-1)^2\Theta^2\tau_\xi\right)$. Given the information set

$\mathcal{I}^{G_2} = \{\widetilde{v}_j, a_1\}$, the policymaker solves

$$\max_{\{a_2\}} E\left[-(\theta - a_2)^2 | \mathcal{I}^{G_2}\right].$$

The first-order condition with respect to a_2 leads to

$$a_2^* = E\left(\theta|\mathcal{I}^{G_2}\right) = \frac{\overline{\theta}\tau_\theta + \widetilde{v}_j\tau_{\epsilon 0}}{\tau_\theta + \tau_{\epsilon 0}} = \overline{\theta} + \frac{\tau_{\epsilon 0}}{\tau_\theta + \tau_{\epsilon 0}}\left(\widetilde{v}_j - \overline{\theta}\right). \tag{13}$$

Notice that $\tau_{\epsilon 0}/(\tau_\theta + \tau_{\epsilon 0})$ is the signal-to-noise ratio of the firm value signal, which decreases in a_1. That is, a smaller a_1 leads to a more precise signal about θ, which helps the policymaker improve her policy choice a_2 in $t = 2$.

By backward induction, the policymaker chooses a_1 at $t = 1$ to solve

$$\max_{\{a_1\}} E\left[-(\theta - a_1)^2 - (\theta - a_2^*)^2\right], \tag{14}$$

by internalizing the impact of a_1 on the precision of the firm value signal. Notice that a_2^* is the policymaker's optimal action in period 2, satisfying (13). Substituting (13) into (14) and rearranging, we obtain the policymaker's problem to be solved in period 2:

$$\max_{\{a_1\}} \left[-\tau_\theta^{-1} - \left(\overline{\theta} - a_1\right)^2 - \frac{1}{\tau_\theta + \tau_{\epsilon 0}}\right].$$

The first-order condition with respect to a_1 gives rise to the equation that pins down a_1^*:

$$\left(\overline{\theta} - a_1\right)\left(\frac{\tau_\theta}{\tau_{\epsilon 0}} + 1\right)^2 = \left(\frac{1}{4}\tau_\varepsilon^{-1} + \frac{1}{4(\sigma - 1)^2\Theta^2}\tau_\xi^{-1}\right)a_1. \tag{15}$$

Appendix A shows that equation (15) has a unique solution $a_1^* \in \left(0, \overline{\theta}\right)$. Thus, we have the following:

Proposition 1 *Equation (15) has a unique solution $a_1^* \in \left(0, \overline{\theta}\right)$. That is, in the equilibrium of the economy without fulled developed financial markets, the policymaker follows the gradualistic approach, i.e., $a_1^* < \overline{\theta} = E(\theta)$.*

Proof The proof is provided in Appendix A.□

Proposition 1 shows that in the equilibrium without financial markets, the policymaker follows the gradualistic economic approach. The intuition is as follows. A lower choice of a_1

reduces the noise in the firm value signal about θ and hence increases the informativeness of the firm value signal, i.e., $\partial\tau_{\epsilon_0}/\partial a_1 = -2\tau_{\epsilon_0}a_1^{-1} < 0$. By taking small policy steps (i.e., $a_1 < \overline{\theta}$), the policymaker acquires high-quality firm value signals $\widetilde{v}_j$ and improves her policy choice in $t = 2$. In equilibrium, she chooses $a_1^* < \overline{\theta}$ at $t = 1$, and $a_2^* = E\left(\theta|\mathcal{I}^{G_2}\right)$ at $t = 2$ to match the updated expectation of θ after observing $\widetilde{v}_j$. In a word, acquiring appropriate information provides sufficient incentives for the policymaker to follow the gradualistic approach.

4 Equilibrium with active financial markets

Now, we examine the complete model with active financial markets. In this model, firms have two financing options: state-owned banks and the stock market. Since the credit rationing behaviors of state-owned banks depend on the policymaker's decisions, firms cannot obtain credit funds until the policymaker selects her policy actions. However, firms can finance their investments directly through the stock market. Financial markets provide private agents with the flexibility to make investment decisions even before the policymaker selects policy action a_1. They can access financial markets for financing, rather than relying solely on state-controlled banks. Furthermore, competition among firms for limited real investment opportunities compels them to act before the policymaker chooses a_1. Therefore, the existence of financial markets alters the strategic interaction between the policymaker and private agents.

In the context of gradualistic reform, the policymaker seeks to influence the economy through government policies. To do so, we assume that the policymaker proposes a policy $\widehat{a}_1$ (i.e., ex ante policy) before firms make their investment decisions. After observing this proposal, firms make investment decisions based on this proposal and other available information. Subsequently - either after or simultaneously - the policymaker selects the actual policy action a_1 (i.e., ex post policy), which may or may not align with the initial proposal. We will examine whether the ex anti policy coinciding with the ex post policy (i.e., $a_1 = \widehat{a}_1$) is the policymaker's optimal policy choices. Based on the above analysis, the timeline of the model economy is thus as follows:

<u>$t = 0$</u>:

Event 01: The shocks θ, ξ_j, ε_j are realized but remain unobservable to the policymaker. Firm j perfectly observes two supply shocks θ and ε_j. Investors observe the productivity shock θ.

Event 02: The policymaker announces a policy proposal $\widehat{a}_1$ (ex ante policy).

Event 03: Investor i decides whether to acquire information and observes a signal about the demand shock for good j, i.e., $\chi_j^i = \xi_j + \varrho_j^i$. (Event 03 will be introduced in Section 5 with endogenous information.)

$\underline{t=1}$:

Event 11: Financial market trading occurs, and the financial price q_j is determined.

Event 12: Firm j makes its investment decision k_j, based on available information $\mathcal{I}_j = \{\theta, \varepsilon_j, q_j, \widehat{a}_1\}$.

Event 13: The policymaker selects the policy action a_1 (ex post policy), based on information $\mathcal{I}^{G_1} = \{q_j, \widehat{a}_1\}$.

$\underline{t=3}$:

Event 31: The asset value v_j, is realized and the financial contract payoff is executed.

Event 32: The policymaker observes v_j ($\equiv \log P_j Y_j$), updates its belief about θ, and selects the policy action a_2, based on information set $\mathcal{I}^{G_1} = \{q_j, v_j, \widehat{a}_1, a_1\}$.

4.1 Optimal decisions

In this subsection, we analyze the optimal decisions of firms, financial investors, and the policymaker.

Firm j's investment decisions at $t=1$. Given its information set $\mathcal{I}_j = \{\theta, \varepsilon_j, q_j, \widehat{a}_1\}$, firm j maximizes its expected profit, (5), subject to the production function (2), cost function (3), and market demand (4). Substituting (2), (3), and (4) into (5) results in the unconstrained optimizition problem:

$$k_j\left(\theta, \varepsilon_j, q_j, \widehat{a}_1\right) = \arg\max_{\{k_j\}} E\left[e^{\frac{\widehat{a}_1}{\sigma}\xi_j} Y^{\frac{1}{\sigma}} e^{\left(1-\frac{1}{\sigma}\right)\theta} k_j^{1-\frac{1}{\sigma}} - \frac{1}{2} e^{-\widehat{a}_1 \varepsilon_j} k_j^2 | \mathcal{I}_j\right].$$

The first-order condition with respect to k_j gives rise to the optimal investment decision

$$k_j\left(\theta, \varepsilon_j, q_j, \widehat{a}_1\right) = \left(1 - \frac{1}{\sigma}\right)^{\Theta} Y^{\frac{\Theta}{\sigma}} e^{\left(1-\frac{1}{\sigma}\right)\theta\Theta + \Theta \widehat{a}_1 \varepsilon_j} \left[E\left(e^{\frac{\widehat{a}_1}{\sigma}\xi_j} | \mathcal{I}_j\right)\right]^{\Theta}, \tag{16}$$

where $\Theta \equiv 1/\left(1 + 1/\sigma\right)$. Once the optimal investment decision is made, firm j's optimal output is determined,

$$Y_j = Y_j\left(\theta, \varepsilon_j, q_j, \widehat{a}_1\right) = e^{\theta} k_j\left(\theta, \varepsilon_j, q_j, \widehat{a}_1\right). \tag{17}$$

Financial market decisions. The share of informed traders in the financial market, λ, is now exogenously given. The information set of informed traders is $\mathcal{I}^{I_i} = \left\{\theta, q_j, \chi^i_j, \widehat{a}_1\right\}$, while that of uninformed traders is $\mathcal{I}^{U_i} = \{\theta, q_j, \widehat{a}_1\}$. An informed trader chooses his risky asset holdings, x^{I_i}, to maximize expected utility,

$$x^{I_i}\left(\theta, q_j, \chi^i_j, \widehat{a}_1\right) = \arg\max_{\{x^{I_i}\}} E\left[-e^{\gamma W^{I_i}} | \theta, q_j, \chi^i_j, \widehat{a}_1\right], \tag{18}$$

where $W^{I_i} = (W_0 - c) + x^{I_i}(v_j - q_j)$, and c denotes a constant cost to acquire information. The first-order condition with respect to x^{I_i} gives rise to his optimal trading position:

$$x^{I_i} = \frac{E\left(v_j|\mathcal{I}^{I_i}\right) - q_j}{\gamma var\left(v_j|\mathcal{I}^{I_i}\right)}, \tag{19}$$

where the firm value v_j depends on the policymaker's policy proposal $\widehat{a}_1$, namely,

$$v_j = v_j(\widehat{a}_1) = \log P_j(\widehat{a}_1) Y_j(\widehat{a}_1) = \frac{y}{\sigma} + \left(1 - \frac{1}{\sigma}\right)\theta + \frac{\widehat{a}_1}{\sigma}\xi_j + \left(1 - \frac{1}{\sigma}\right)\log k_j(\theta, \varepsilon_j, q_j, \widehat{a}_1). \tag{20}$$

The second-order condition with respect to x^{I_i} is satisfied, namely, $-\gamma var\left(v_j|\mathcal{I}^{I_i}\right) < 0$.

Similarly, an uninformed trader chooses his risky asset holdings, x^{U_i}, to maximize his expected utility,

$$x^{U_i}(\theta, q_j, \widehat{a}_1) = \arg\max_{\{x^{U_i}\}} E\left[-e^{\gamma W^{U_i}} | \theta, q_j, \widehat{a}_1\right], \tag{21}$$

where $W^{U_i} = W_0 + x^{I_i}(v_j - q_j)$. The first-order condition with respect to x^{U_i} gives rise to his optimal trading position:

$$x^{U_i} = \frac{E\left(v_j|\mathcal{I}^{U_i}\right) - q_j}{\gamma var\left(v_j|\mathcal{I}^{U_i}\right)}. \tag{22}$$

The associated second-order condition with respect to x^{U_i} is also satisfied, namely, $-\gamma var\left(v_j|\mathcal{I}^{U_i}\right) < 0$.

Policymaker's policy decisions. At $t = 2$, given her information $\mathcal{I}^{G_2} = \{q_j, v_j, \widehat{a}_1, a_1\}$, the policymaker chooses a_2 to maximize her expected utility

$$\max_{\{a_2\}} E\left[-(\theta - a_2)^2 |\mathcal{I}^{G_2}\right]. \tag{23}$$

The first-order condition with respect to a_2 leads to her optimal policy in period 2:

$$a_2 = E\left(\theta|\mathcal{I}^{G_2}\right) \equiv \widehat{\widehat{\theta}}, \text{ with } \tau^{-1}_{\widehat{\widehat{\theta}}} \equiv var\left(\theta|\mathcal{I}^{G_2}\right). \tag{24}$$

At $t=1$, the policymaker with information $\mathcal{I}^{G_1}=\{q_j,\widehat{a}_1\}$ chooses a_1 to solve

$$\max_{\{a_1\}} E\left[-(\theta-a_1)^2-(\theta-a_2)^2\,|\mathcal{I}^{G_1}\right], \tag{25}$$

where a_2 is her optimal policy action at $t=2$, satisfying (24). Substituting (24) into (25) and rearranging, we rewrite the policymaker's problem as follows:

$$\max_{\{a_1\}}\left[-\tau_{\widehat{\theta}}^{-1}-\left(\widehat{\theta}-a_1\right)^2-\tau_{\widehat{\widehat{\theta}}}^{-1}\right], \tag{26}$$

where

$$\widehat{\theta}\equiv E\left(\theta|\mathcal{I}^{G_1}\right), \tau_{\widehat{\theta}}^{-1}\equiv var\left(\theta|\mathcal{I}^{G_1}\right).$$

Solving the first-order condition with respect to a_1 gives rise to the equation which pinning down a_1:

$$2\left(\widehat{\theta}-a_1\right)-\frac{\partial\tau_{\widehat{\widehat{\theta}}}^{-1}}{\partial a_1}=0. \tag{27}$$

Definition 2 *An equilibrium consists of the financial price function $q_j=q\left(\theta,\xi_j,u_j\right)$, the firm's investment decision function $k_j=k\left(\theta,\varepsilon_j,q_j,\widehat{a}_1\right)$, and the policymaker's policies $a_1=a_1\left(q_j,\widehat{a}_1\right)$ and $a_2=a_2\left(q_j,v_j,\widehat{a}_1,a_1\right)$, such that: (i) The price function $q_j=q\left(\theta,\xi_j,u_j\right)$ clears the financial market at $t=1$:*

$$\lambda\int_{i=0}^{1}x^{I_i}di+(1-\lambda)\int_{i=0}^{1}x^{U_i}di+u_j=0, \tag{28}$$

where, for a given $k_j=k\left(\theta,\varepsilon_j,q_j,\widehat{a}_1\right)$ and the policymaker's policy proposal $\widehat{a}_1$, the optimal choices x^{I_i} and x^{U_i} satisfy (18) and (21), respectively. (ii) Given the price function $q_j=q\left(\theta,\xi_j,u_j\right)$, the investment decision decision function $k_j=k\left(\theta,\varepsilon_j,q_j,\widehat{a}_1\right)$ solves the firm's problem (5), while the policy actions $a_1=a_1\left(q_j,\widehat{a}_1\right)$ and $a_2=a_2\left(q_j,v_j,\widehat{a}_1,a_1\right)$ satisfy (26) and (23), respectively.

4.2 Characterization of equilibrium

First, we characterize the financial market equilibrium. We conjecture that the financial price $q_j=q\left(\theta,\xi_j,u_j\right)$ follows a linear price function:

$$q_j=\beta_0+\beta_1\left(\xi_j+\beta_2u_j+\beta_3\theta\right), \tag{29}$$

where β_0, β_1, β_2, and β_3 are undetermined coefficients. Since both firms and financial traders have perfect information about the productivity shock θ, they interpret the price of the risky asset, q_j, as a noisy signal about the demand shock ξ_j, namely,

$$\widetilde{q}_j\left(q_j,\theta\right) \equiv \frac{q_j-\beta_0}{\beta_1}-\beta_3\theta=\xi_j+\beta_2 u_j \equiv \xi_j+\varrho_j^q, \tag{30}$$

where $\varrho_j^q \sim N\left(0,\tau_q^{-1}\right)$ with $\tau_q=\beta_2^{-2}\tau_u$. The information set $\{q_j,\theta\}$ is a one-to-one mapping to $\{\widetilde{q}_j,\theta\}$. The information sets of informed and uninformed traders are updated to $\mathcal{I}^{I_i}=\left\{\theta,\widetilde{q}_j,\chi_j^i,\widehat{a}_1\right\}$ and $\mathcal{I}^{U_i}=\{\theta,\widetilde{q}_j,\widehat{a}_1\}$, respectively. Their optimal trading positions, (19) and (22), are thus updated accordingly:

$$\begin{aligned} x^{I_i} &= \frac{\frac{y}{\sigma}+\left(1-\frac{1}{\sigma}\right)\theta+\frac{\widehat{a}_1}{\sigma}\frac{\chi_j^i\tau_\chi+\widetilde{q}_j\tau_q}{\tau_\xi+\tau_\chi+\tau_q}+\left(1-\frac{1}{\sigma}\right)E\left(\log k_j|\mathcal{I}^{I_i}\right)-q_j}{\gamma\left[\left(\frac{\widehat{a}_1}{\sigma}\right)^2\frac{1}{\tau_\xi+\tau_\chi+\tau_q}+\left(1-\frac{1}{\sigma}\right)^2 var\left(\log k_j|\mathcal{I}^{I_i}\right)\right]}, & (31)\\ x^{U_i} &= \frac{\frac{y}{\sigma}+\left(1-\frac{1}{\sigma}\right)\theta+\frac{\widehat{a}_1}{\sigma}\frac{\widetilde{q}_j\tau_q}{\tau_\xi+\tau_q}+\left(1-\frac{1}{\sigma}\right)E\left(\log k_j|\mathcal{I}^{U_i}\right)-q_j}{\gamma\left[\left(\frac{\widehat{a}_1}{\sigma}\right)^2\frac{1}{\tau_\xi+\tau_q}+\left(1-\frac{1}{\sigma}\right)^2 var\left(\log k_j|\mathcal{I}^{U_i}\right)\right]}. & (32)\end{aligned}$$

Substituting (31) and (32) into the market clearing condition, (28) leads to

$$\begin{aligned} 0 &= u_j+\lambda\frac{\frac{y}{\sigma}+\left(1-\frac{1}{\sigma}\right)\theta+\frac{\widehat{a}_1}{\sigma}\frac{\xi_j\tau_\chi+\widetilde{q}_j\tau_q}{\tau_\xi+\tau_\chi+\tau_q}+\left(1-\frac{1}{\sigma}\right)E\left(\log k_j|\mathcal{I}^{I_i}\right)-q_j}{\gamma\left[\left(\frac{\widehat{a}_1}{\sigma}\right)^2\frac{1}{\tau_\xi+\tau_\chi+\tau_q}+\left(1-\frac{1}{\sigma}\right)^2 var\left(\log k_j|\mathcal{I}^{I_i}\right)\right]} \\ &\quad +\left(1-\lambda\right)\frac{\frac{y}{\sigma}+\left(1-\frac{1}{\sigma}\right)\theta+\frac{\widehat{a}_1}{\sigma}\frac{\widetilde{q}_j\tau_q}{\tau_\xi+\tau_q}+\left(1-\frac{1}{\sigma}\right)E\left(\log k_j|\mathcal{I}^{U_i}\right)-q_j}{\gamma\left[\left(\frac{\widehat{a}_1}{\sigma}\right)^2\frac{1}{\tau_\xi+\tau_q}+\left(1-\frac{1}{\sigma}\right)^2 var\left(\log k_j|\mathcal{I}^{U_i}\right)\right]}. \end{aligned} \tag{33}$$

Second, we characterize firm $j's$ investment decision at $t=1$. Given the information set $\mathcal{I}_j=\{\theta,\varepsilon_j,\widetilde{q}_j,\widehat{a}_1\}$, firm $j's$ optimal investment rule (16) is thus:

$$\log k_j=\log k_j\left(\theta,\varepsilon_j,\widetilde{q}_j,\widehat{a}_1\right)=\phi_0+\phi_1\widehat{a}_1\widetilde{q}_j+\phi_2\widehat{a}_1\varepsilon_j+\phi_3\theta, \tag{34}$$

where

$$
\begin{aligned}
\phi_0 &= \Theta \log\left(1-\frac{1}{\sigma}\right)+\frac{\Theta}{\sigma}y+\frac{\Theta}{2}\left(\frac{\widehat{a}_1}{\sigma}\right)^2\frac{1}{\tau_\xi+\tau_q}, \\
\phi_1 &= \frac{\Theta}{\sigma}\frac{\tau_q}{\tau_\xi+\tau_q}, \\
\phi_2 &= \Theta, \\
\phi_3 &= \Theta\left(1-\frac{1}{\sigma}\right).
\end{aligned}
$$

Plugging (2), (4), and (34) in $v_j = \log V_j = \log P_j Y_j$ yields us

$$
v_j = \frac{y}{\sigma}+\left(1-\frac{1}{\sigma}\right)\theta+\frac{\widehat{a}_1}{\sigma}\xi_j+\left(1-\frac{1}{\sigma}\right)\left(\phi_0+\phi_1\widehat{a}_1\widetilde{q}_j+\phi_2\widehat{a}_1\varepsilon_j+\phi_3\theta\right). \tag{35}
$$

Substituting (34) into (31), (32), and (33), we obtain

$$
x^{I_i} = \frac{\frac{y}{\sigma}+\left(1-\frac{1}{\sigma}\right)\theta+\frac{\widehat{a}_1}{\sigma}\frac{\chi_j^i\tau_\chi+\widetilde{q}_j\tau_q}{\tau_\xi+\tau_\chi+\tau_q}+\left(1-\frac{1}{\sigma}\right)\left(\phi_0+\phi_1\widehat{a}_1\widetilde{q}_j+\phi_3\theta\right)-q_j}{\gamma\left[\left(\frac{\widehat{a}_1}{\sigma}\right)^2\frac{1}{\tau_\xi+\tau_\chi+\tau_q}+\left(1-\frac{1}{\sigma}\right)^2\phi_2^2\widehat{a}_1^2\tau_\varepsilon^{-1}\right]}, \tag{36}
$$

$$
x^{U_i} = \frac{\frac{y}{\sigma}+\left(1-\frac{1}{\sigma}\right)\theta+\frac{\widehat{a}_1}{\sigma}\frac{\widetilde{q}_j\tau_q}{\tau_\xi+\tau_q}+\left(1-\frac{1}{\sigma}\right)\left(\phi_0+\phi_1\widehat{a}_1\widetilde{q}_j+\phi_3\theta\right)-q_j}{\gamma\left[\left(\frac{\widehat{a}_1}{\sigma}\right)^2\frac{1}{\tau_\xi+\tau_q}+\left(1-\frac{1}{\sigma}\right)^2\phi_2^2\widehat{a}_1^2\tau_\varepsilon^{-1}\right]}, \tag{37}
$$

$$
\begin{aligned}
0 &= u_j+\lambda\frac{\frac{y}{\sigma}+\left(1-\frac{1}{\sigma}\right)\theta+\frac{\widehat{a}_1}{\sigma}\frac{\xi_j\tau_\chi+\widetilde{q}_j\tau_q}{\tau_\xi+\tau_\chi+\tau_q}+\left(1-\frac{1}{\sigma}\right)\left(\phi_0+\phi_1\widehat{a}_1\widetilde{q}_j+\phi_3\theta\right)-q_j}{\gamma\left[\left(\frac{\widehat{a}_1}{\sigma}\right)^2\frac{1}{\tau_\xi+\tau_\chi+\tau_q}+\left(1-\frac{1}{\sigma}\right)^2\phi_2^2\widehat{a}_1^2\tau_\varepsilon^{-1}\right]} \\
&\quad+(1-\lambda)\frac{\frac{y}{\sigma}+\left(1-\frac{1}{\sigma}\right)\theta+\frac{\widehat{a}_1}{\sigma}\frac{\widetilde{q}_j\tau_q}{\tau_\xi+\tau_q}+\left(1-\frac{1}{\sigma}\right)\left(\phi_0+\phi_1\widehat{a}_1\widetilde{q}_j+\phi_3\theta\right)-q_j}{\gamma\left[\left(\frac{\widehat{a}_1}{\sigma}\right)^2\frac{1}{\tau_\xi+\tau_q}+\left(1-\frac{1}{\sigma}\right)^2\phi_2^2\widehat{a}_1^2\tau_\varepsilon^{-1}\right]}.
\end{aligned} \tag{38}
$$

Third, we examine the policymaker's decisions. Although the policymaker cannot observe the realizations of all exogenous shocks, she learns information about the productivity shock θ from the equilibrium financial price q_j and the realized firm value v_j. Unlike firms and financial traders, the policymaker interprets q_j as a noisy signal about the productivity shock θ, namely:

$$
\widehat{q}_j = \widehat{q}_j\left(q_j\right) = \frac{q_j-\beta_0}{\beta_1\beta_3} = \theta+\frac{1}{\beta_3}\xi_j+\frac{\beta_2}{\beta_3}u_j \equiv \theta+\varphi_1\xi_j+\varphi_2 u_j, \tag{39}
$$

with the precision

$$\tau_{\widehat{q}} \equiv \left[var\left(\varphi_1 \xi_j + \varphi_2 u_j\right)\right]^{-1} = \frac{1}{\left(\varphi_1^2 \tau_\xi^{-1} + \varphi_2^2 \tau_u^{-1}\right)}, \tag{40}$$

where

$$\varphi_1 \equiv \frac{1}{\beta_3}\ , \varphi_2 \equiv \frac{\beta_2}{\beta_3}.$$

At $t = 2$, the policymaker observes the realizations of the firm value v_j. Unlike financial traders, who only know the policy proposal $\widehat{a}_1$, the policymaker knows both the policy proposal (ex ante policy) $\widehat{a}_1$ and the policy action (ex post policy) a_1. From the policymaker's perspective, firms had made investment and production decisions at $t = 1$ that depend on the policy proposal $\widehat{a}_1$ (that is, Y_j depends on the policy proposal, i.e., $Y_j(\widehat{a}_1)$), and she had chosen the policy action a_1 at $t = 1$ to adjust the real demand (through the demand curve (4)) and to clear the goods market (that is, P_j depends on the policy action a_1, i.e., $P_j(a_1)$). Altogether, the policymaker takes v_j as a function of both $\widehat{a}_1$ and a_1, namely:

$$\begin{aligned} v_j &= v_j(a_1, \widehat{a}_1) = \log P_j(a_1) Y_j(\widehat{a}_1) = \log e^{\frac{a_1}{\sigma}\xi_j} Y^{\frac{1}{\sigma}} Y_j(\widehat{a}_1)^{1-\frac{1}{\sigma}} \\ &= \frac{y}{\sigma} + \frac{a_1}{\sigma}\xi_j + \left(1 - \frac{1}{\sigma}\right)\left[\phi_0 + \phi_1 \widehat{a}_1 \widehat{q}_j + \phi_2 \widehat{a}_1 \varepsilon_j + (1 + \phi_3)\theta\right]. \end{aligned} \tag{41}$$

By observing the realized firm value v_j at $t = 2$, the policymaker interprets it as a noisy signal of the productivity shock θ, namely,

$$\widehat{v}_j = \widehat{v}_j(\widehat{a}_1\ , a_1) = \frac{v_j - \frac{y}{\sigma} - \left(1 - \frac{1}{\sigma}\right)\left(\phi_0 + \phi_1 \widehat{a}_1 \frac{q_j - \beta_0}{\beta_1}\right)}{\left(1 - \frac{1}{\sigma}\right)(1 + \phi_3 - \phi_1 \widehat{a}_1 \beta_3)} = \theta + \varphi_3 \xi_j + \varphi_4 \varepsilon_j, \tag{42}$$

with the precision

$$\tau_{\widehat{v}} \equiv \left[var\left(\varphi_3 \xi_j + \varphi_4 \varepsilon_j\right)\right]^{-1} = \frac{1}{\varphi_3^2 \tau_\xi^{-1} + \varphi_4^2 \tau_\varepsilon^{-1}}, \tag{43}$$

where

$$\varphi_3 = \frac{a_1/\sigma}{\left(1 - \frac{1}{\sigma}\right)(1 + \phi_3 - \phi_1 \widehat{a}_1 \beta_3)}, \varphi_4 = \frac{\phi_2 \widehat{a}_1}{(1 + \phi_3 - \phi_1 \widehat{a}_1 \beta_3)}.$$

Thus, the policymaker's information sets in both periods are equivalently transformed into $\mathcal{I}^{G_1} = \{\widehat{q}_j, \widehat{a}_1\}$ and $\mathcal{I}^{G_2} = \{\widehat{q}_j, \widehat{v}_j(\widehat{a}_1\ , a_1), \widehat{a}_1, a_1\}$, respectively. Using these tranformed information sets, we solve the policymaker's problem and derive the unique equillibrium in Appendix B. Thus, we obtain the following

Proposition 2 *In the economy with active financial markets, a unique equilibrium exists in* *which the policymaker chooses a time-consistent gradualistic policy, i.e.,* $a_1 = \widehat{a}_1 < E\left(\theta|\mathcal{I}^{G_1}\right)$*. Specifically, the equilibrium price function* $q_j = q\left(\theta, \xi_j, u_j\right)$ *follows:*

$$q_j = \beta_0 + \beta_1\left(\xi_j + \beta_2 u_j + \beta_3\theta\right),$$

where

$$\beta_2 = \sqrt[3]{-\frac{n}{2}+\sqrt{\Delta}} + \sqrt[3]{-\frac{n}{2}-\sqrt{\Delta}} - \frac{p}{3}, \tag{44}$$

$$\beta_1 = \frac{\begin{pmatrix} \lambda\gamma\left[\left(\frac{\widehat{a}_1}{\sigma}\right)^2 \frac{1}{\tau_\xi+\tau_\chi+\beta_2^{-2}\tau_u} + \left(1-\frac{1}{\sigma}\right)^2\phi_2^2\widehat{a}_1^2\tau_\varepsilon^{-1}\right]\frac{\widehat{a}_1}{\sigma}\frac{\tau_\chi+\beta_2^{-2}\tau_u}{\tau_\xi+\tau_\chi+\beta_2^{-2}\tau_u} \\ +\left(1-\lambda\right)\gamma\left[\left(\frac{\widehat{a}_1}{\sigma}\right)^2\frac{1}{\tau_\xi+\beta_2^{-2}\tau_u} + \left(1-\frac{1}{\sigma}\right)^2\phi_2^2\widehat{a}_1^2\tau_\varepsilon^{-1}\right]\frac{\widehat{a}_1}{\sigma}\frac{\beta_2^{-2}\tau_u}{\tau_\xi+\beta_2^{-2}\tau_u}\end{pmatrix}}{\begin{pmatrix} \lambda\gamma\left[\left(\frac{\widehat{a}_1}{\sigma}\right)^2\frac{1}{\tau_\xi+\beta_2^{-2}\tau_u} + \left(1-\frac{1}{\sigma}\right)^2\phi_2^2\widehat{a}_1^2\tau_\varepsilon^{-1}\right] + \\ \left(1-\lambda\right)\gamma\left[\left(\frac{\widehat{a}_1}{\sigma}\right)^2\frac{1}{\tau_\xi+\tau_\chi+\beta_2^{-2}\tau_u} + \left(1-\frac{1}{\sigma}\right)^2\phi_2^2\widehat{a}_1^2\tau_\varepsilon^{-1}\right]\end{pmatrix}} + \left(1-\frac{1}{\sigma}\right)\phi_1\widehat{a}_1, \tag{45}$$

$$\beta_3 = \left(1-\frac{1}{\sigma}\right)\left(1+\phi_3\right)\beta_1^{-1}, \tag{46}$$

$$\beta_0 = \frac{y}{\sigma} + \left(1-\frac{1}{\sigma}\right)\phi_0, \tag{47}$$

with

$$\Delta = \frac{\gamma(\sigma-1)^2\phi_2^2\frac{\widehat{a}_1}{\sigma}\tau_\varepsilon^{-1}\tau_u}{27\lambda\tau_\chi}\left(\frac{\gamma\frac{\widehat{a}_1}{\sigma}\left[1+(\sigma-1)^2\phi_2^2\tau_\varepsilon^{-1}\left(\tau_\xi+\tau_\chi\right)\right]}{\lambda\tau_\chi}\right)^3 + \frac{1}{4}\left(\frac{\gamma(\sigma-1)^2\phi_2^2\frac{\widehat{a}_1}{\sigma}\tau_\varepsilon^{-1}\tau_u}{\lambda\tau_\chi}\right)^3,$$

$$n = -\frac{2}{27}\left(\frac{\gamma\frac{\widehat{a}_1}{\sigma}\left[1+(\sigma-1)^2\phi_2^2\tau_\varepsilon^{-1}\left(\tau_\xi+\tau_\chi\right)\right]}{\lambda\tau_\chi}\right)^3 - \frac{\gamma(\sigma-1)^2\phi_2^2\frac{\widehat{a}_1}{\sigma}\tau_\varepsilon^{-1}\tau_u}{\lambda\tau_\chi},$$

$$p = -\frac{\gamma\frac{\widehat{a}_1}{\sigma}\left[1+(\sigma-1)^2\phi_2^2\tau_\varepsilon^{-1}\left(\tau_\xi+\tau_\chi\right)\right]}{\lambda\tau_\chi}.$$

The optimal investment decisions follow

$$\log k_j = \log k_j\left(\theta, \varepsilon_j, \widetilde{q}_j, \widehat{a}_1\right) = \phi_0 + \phi_1\widehat{a}_1\widetilde{q}_j + \phi_2\widehat{a}_1\varepsilon_j + \phi_3\theta,$$

where

$$\phi_0 = \Theta\log\left(1-\frac{1}{\sigma}\right) + \frac{\Theta}{\sigma}y + \frac{\Theta}{2}\left(\frac{\widehat{a}_1}{\sigma}\right)^2\frac{1}{\tau_\xi+\beta_2^{-2}\tau_u}, \phi_1 = \frac{\Theta}{\sigma}\frac{\beta_2^{-2}\tau_u}{\tau_\xi+\beta_2^{-2}\tau_u}, \phi_2 = \Theta, \phi_3 = \Theta\left(1-\frac{1}{\sigma}\right).$$

The policymaker's optimal policy actions in two periods, a_1 and a_2, are determined by

$$2\left(\widehat{\theta}-a_1\right)=\frac{\tau_\theta^{-2}}{var\left(\widehat{q}_j\right)}\frac{\Psi_1}{\Psi_2^2}\left[2\frac{\partial cov\left(\widehat{v}_j,\widehat{q}_j\right)}{\partial a_1}\Psi_2+\Psi_1\Psi_3\right], \tag{48}$$

and

$$a_2=E\left(\theta|\mathcal{I}^{G_2}\right)\equiv\widehat{\widehat{\theta}}, \tag{49}$$

respectively, where

$$\begin{aligned}
\Psi_1 &\equiv var\left(\widehat{q}_j\right)-cov\left(\widehat{v}_j,\widehat{q}_j\right), \\
\Psi_2 &\equiv var\left(\widehat{v}_j\right)var\left(\widehat{q}_j\right)-cov\left(\widehat{v}_j,\widehat{q}_j\right)^2, \\
\Psi_3 &\equiv var\left(\widehat{q}_j\right)\frac{\partial var\left(\widehat{v}_j\right)}{\partial a_1}-2cov\left(\widehat{v}_j,\widehat{q}_j\right)\frac{\partial cov\left(\widehat{v}_j,\widehat{q}_j\right)}{\partial a_1}.
\end{aligned}$$

Proof The proof is provided in Appendix B. □

Proposition 2 establishes that, in an economy with fully developed financial markets, there exists a Bayesian Nash equilibrium in which the policymaker chooses a time-consistent gradualistic policy. By extending the model developed by Brunnermeier et al. (2017), we generalize the conditions under which the gradualistic approach remains valid. In their seminal paper, Brunnermeier et al. (2017) present a pessimistic viewpoint: the gradualistic approach will fail once China fully develops its financial markets. They focus on the financing role of financial markets while neglecting their role in information production. In their framework, the policymaker learns information from firms' investment decisions but not from the financial market. If the financial market is inactive, firms make investment decisions after the policymaker implements credit rationing (carried out by state-owned banks). In this case, there is no time-consistency problem, and the policymaker adopts a gradualistic approach to gather more information from the real sector of the economy. However, if the financial market is fully developed, competitive firms make investment decisions before the policymaker takes action on credit rationing. Although the policymaker may propose a policy, she has no incentive to follow a time-consistent gradualistic approach because she has already observed signals from the real economy and cannot gain additional information. Antipating this, private agents choose to front-run the policymaker. This front-running by private agents renders the policymaker's gradualistic approach ineffective.

In our model economy, the financial market serves two functions: providing financing for firms and generating information. The policymaker learns information about the productivity shock not only from firms' investment and production decisions but also from financial prices. Acquiring more information on both the extensive and intensive margins provides additional incentives for the policymaker to consistently take small policy steps. Through the gradualistic approach, the government learns more information from the economy's reactions to her current policy actions and makes better policy adjustments in the future. Proposition 2 offers an optimistic perspective: even if China fully develops its financial markets, the policymaker still follow a time-consistent gradualistic approach. Our findings extend the conditions under which the gradualistic approach remains valid, thereby deepening the insights developed by Brunnermeier et al. (2017).

Proposition 3 *In the equilibrium of the model with active financial markets, for a given* λ, τ_q, $\tau_{\widehat{q}}$, and $\tau_{\widehat{v}}$, *these variables increase as* $\widehat{a}_1$ *decreases. That is,*

$$\frac{\partial \tau_q}{\partial \widehat{a}_1} < 0; \frac{\partial \tau_{\widehat{q}}}{\partial \widehat{a}_1} < 0, \frac{\partial \tau_{\widehat{v}}}{\partial \widehat{a}_1} < 0.$$

Proof The proof is provided in Appendix C. □

Proposition 3 shows that a smaller policy step, $\widehat{a}_1$, leads to a more precise information about the demand shock ξ_j, i.e., $\partial \tau_q / \partial \widehat{a}_1 < 0$. Intuitively, for financial traders and firms, the total uncertainty over the firm value, v_j, consists of uncertainties from both the cost shock, ε_j, and the demand shock, ξ_j, and τ_q measures the informativeness of the financial price signal, $\widetilde{q}_j$. A smaller policy step reduces the total uncertainty from both shocks. With less uncertainty, informed traders more aggressively on their private information about the demand shock, which overweighes the influence of noise traders, thereby enhancing the informativeness of the financial price.

Proposition 3 also shows that smaller policy steps lead to more precise information about the productivity shock θ, i.e., $\partial \tau_{\widehat{q}} / \partial \widehat{a}_1 < 0$, $\partial \tau_{\widehat{v}} / \partial \widehat{a}_1 < 0$. In our model, the policymaker receives two types of signals about the productivity shock θ: the financial price signal ($\widehat{q}_j$) and the firm value signal $\widehat{v}_j$. Smaller policy steps reduce the overall uncertainty in the economy caused by both the cost shock ε_j and the demand shock ξ_j. On one hand, with less uncertainty, all financial traders (both informed and uninformed), who perfectly observe the productivity shock θ will trade more aggressively based on their private information. As a result, more information about the productivity shock is reflected in the financial price q_j, thereby increasing the informativeness of the financial price signal $\widehat{q}_j$, i.e., $\partial\tau_{\widehat{q}}/\partial\widehat{a}_1 < 0$. On the other hand, for the policymaker, the total uncertainty regarding the firm value v_j stems from the uncertainties of the productivity shock θ, the cost shock ε_j, and the demand shock ξ_j. With less uncertainty induced by ε_j and ξ_j, firms make better investment and production decisions, which reduces the noise in the firm value signal, thereby increasing the informativeness of the firm value signal $\widehat{v}_j$, i.e., $\partial\tau_{\widehat{v}}/\partial\widehat{a}_1 < 0$.

5 Endogenous information

In this section, we extend the complete model with active financial markets to allow for endogenous information acquisition of the financial market. The purpose is to shed light on how the information acquisition influences the full equilibrium and optimal policy of the model.

5.1 Setup

We add Event 03 to the timeline at $t = 0$. After the shocks θ, ξ_j, ε_j are realized and the policymaker announces a policy proposal $\widehat{a}_1$, speculators in the financial market make their information acquisition decisions.

In the financial market, a trader can choose to be informed or uninformed. By paying an information acquisition cost $c > 0$, a trader receives a private signal $\chi_j^i = \xi_j + \varrho_j^i$ with $\varrho_j^i \sim N\left(0, \tau_\chi^{-1}\right)$, as specified in the baseline model; otherwise, he receives no signal. The proportion of informed traders, λ, is thus endogenous.

5.2 Full equilibrium

Now the financial trader's problem has two steps: information acquisition and financial trading. We solve it by backward induction. First, the financial trading problem has been examined in Section 4. Second, we investigate the information acquisition problem of financial traders. The proportion λ is determined such that an uninformed trader and an informed trader have the same expected utility:

$$\frac{EV\left(W^{Ii}\right)}{EV\left(W^{Ui}\right)} = 1, \tag{50}$$

where $V\left(W^i\right) \equiv EU\left(W^i|\mathcal{I}_i\right)$. Similar to Grassman and Stiglitz (1980), we change the equilibrium condition (50) into

$$e^{\gamma c} = \sqrt{\frac{var\left(v_j|\mathcal{I}^{U_i}\right)}{var\left(v_j|\mathcal{I}^{I_i}\right)}} = \sqrt{\frac{\left(\frac{1}{\sigma}\right)^2\left(\tau_\xi+\tau_q\right)^{-1}+\left(1-\frac{1}{\sigma}\right)^2\phi_2^2\tau_\varepsilon^{-1}}{\left(\frac{1}{\sigma}\right)^2\left(\tau_\xi+\tau_\chi+\tau_q\right)^{-1}+\left(1-\frac{1}{\sigma}\right)^2\phi_2^2\tau_\varepsilon^{-1}}}. \tag{51}$$

Proposition 4 *In the full equilibrium of the economy with active financial markets and endogenous information, there exists a unique equilibrium in which the policymaker chooses a time-consistent gradualistic policy, i.e.,* $a_1 = \widehat{a}_1 < E\left(\theta|\mathcal{I}^{G_1}\right)$.

Proof The proof is provided in Appendix D. □

Proposition 5 *In the full equilibirum of the economy with active financial markets and endogenous informatoin, we have the comparative statics:*

$$\frac{\partial\tau_q}{\partial\widehat{a}_1} = 0, \frac{\partial\lambda}{\partial\widehat{a}_1} > 0; \frac{\partial\tau_{\widehat{q}}}{\partial\widehat{a}_1} < 0, \frac{\partial\tau_{\widehat{v}}}{\partial\widehat{a}_1} < 0.$$

Proof The proof is provided in Appendix E. □

Proposition 4 shows that the policymaker still opts for the gradualistic approach in the full equilibrium with endogenous λ. The intuition is similar to Proposition 2. However, in this case, with endogenous information, we cannot derive the explicit expressions for those $\beta's$, as listed in (44)-(47).

Proposition 5 states that, when taking endogenous λ into account, the gradualistic policy induces more traders to become uninformed (i.e., $\partial\lambda/\partial\widehat{a}_1 > 0$), while having no (net) effect on the informativeness of the financial price signal (i.e., $\partial\tau_q/\partial\widehat{a}_1 = 0$). The intuition behind the comparatice statics is as follows. There are two driving forces under the comparative statics $\partial\tau_q/\partial\widehat{a}_1 = 0$. On the one hand, smaller policy steps reduce the total uncertainty induced by the cost shock ε_j and the demand shock ξ_j, causing existing informed traders to trade more aggressively, thus increasing the informativeness of the financial price. This is the intensive margin. On the other hand, observing smaller policy steps and higher precision price signals provides incentives for some informed traders to switch bo being uninformed (i.e., $\partial\lambda/\partial\widehat{a}_1 > 0$). As fewer traders acquire information, price informativeness decreases. This negative extensive margin counteracts the positive intensive margin, leaving the informativeness of the financial price unchanged.

Proposition 5 also shows that smaller policy steps improve the precision of both the financial price signal ($\widehat{q}_j$) and the firm value signal ($\widehat{v}_j$). The intuition is similar to that of Proposition 3. Since smaller policy steps reduce the uncertainty induced by the cost shock and demand shock, all financial traders will trade more aggressively based on their information, injecting more information about the productivity shock into the financial price q_j, thereby increasing the precision of the financial price signal $\widehat{q}_j$. Meanwhile, smaller policy steps decrease the uncertainty induced by ε_j and ξ_j, causing the firm value signal to incorporates less noise, and thus, it has higher informativeness.

6 Conclusion

In this paper, we construct a model economy with active financial markets to examine the optimality of the gradualistic approach. In our model, the financial market serves both as a financing tool and as a channel for information revelation. As a result, the policymaker has two channels to acquire information about economic fundamentals: the real economy and financial markets. The presence of multiple information extraction channels motivates the policymaker to adopt small, incremental policy steps. In the model with exogenous information for financial investors, the policymaker's choice of the gradualistic approach is the unique Bayesian Nash equilibrium. By taking smaller policy steps, the policymaker is able to extract more precise information about the economic fundamentals. In the model with endogenous information, the gradualistic approach remains optimal and beneficial for the policymaker's information acquisition. Altogether, we develop a model with well-developed financial markets in which following the gradualistic approach is a Bayesian Nash equilibrium for the policymaker.

7 Appendix

7.1 Appendix A

Proof of Proposition 1. First, we solve the firm's problem. Given its information set $\mathcal{I}_j = \{\theta, \varepsilon_j, a_1\}$, firm j maximizes (5) subject to the production function (2), cost function (3), and market demand (4). By substitution, the constrained optimizing problem is transformed into the following unconstrained one:

$$\max_{\{k_j\}} E\left[e^{\frac{a_1}{\sigma}\xi_j} Y^{\frac{1}{\sigma}} e^{\theta\left(1-\frac{1}{\sigma}\right)} k_j^{1-\frac{1}{\sigma}} - \frac{1}{2} e^{a_1\varepsilon_j} k_j^2 | \mathcal{I}_j\right] = Y^{\frac{1}{\sigma}} e^{\theta\left(1-\frac{1}{\sigma}\right)} k_j^{1-\frac{1}{\sigma}} E\left[e^{\frac{a_1}{\sigma}\xi_j} | \mathcal{I}_j\right] - \frac{1}{2} e^{a_1\varepsilon_j} k_j^2. \quad (52)$$

Solving the first-order condition with respect to k_j gives us

$$k_j = \left(1 - \frac{1}{\sigma}\right)^{\Theta} Y^{\frac{\Theta}{\sigma}} e^{\theta\left(1-\frac{1}{\sigma}\right)\Theta + a_1\Theta\varepsilon_j} \left[E\left(e^{\frac{a_1}{\sigma}\xi_j} | \mathcal{I}_j\right)\right]^{\Theta}, \quad (53)$$

where

$$\begin{aligned} E\left(e^{\frac{a_1}{\sigma}\xi_j} | \mathcal{I}_j\right) &= \exp\left\{\frac{1}{2}\left(\frac{a_1}{\sigma}\right)^2 \tau_\xi^{-1}\right\}, && (54) \\ \Theta &\equiv \frac{1}{1+\frac{1}{\sigma}}. && (55) \end{aligned}$$

We thus have that

$$\log k_j = \Theta \log\left(1 - \frac{1}{\sigma}\right) + \frac{\Theta}{\sigma} y + \left(1 - \frac{1}{\sigma}\right)\Theta\theta + a_1\Theta\varepsilon_j + \frac{\Theta}{2}\left(\frac{a_1}{\sigma}\right)^2 \tau_\xi^{-1}, \quad (56)$$

$$\begin{aligned} v_j &\equiv \log P_j Y_j = \frac{a_1}{\sigma}\xi_j + \frac{y}{\sigma} + \left(1 - \frac{1}{\sigma}\right)\theta + \left(1 - \frac{1}{\sigma}\right)\log k_j && (57) \\ &= \left(\begin{array}{c} \frac{2}{\sigma}\Theta y + 2\left(1-\frac{1}{\sigma}\right)\Theta\theta + \left(1-\frac{1}{\sigma}\right)\Theta\log\left(1-\frac{1}{\sigma}\right) + \\ \left(1-\frac{1}{\sigma}\right)\frac{\Theta}{2}\left(\frac{a_1}{\sigma}\right)^2\tau_\xi^{-1} + \frac{a_1}{\sigma}\xi_j + \left(1-\frac{1}{\sigma}\right)a_1\Theta\varepsilon_j \end{array}\right). \end{aligned}$$

At the beginning of $t = 2$, the policymaker observes v_j and takes it as a noisy signal about θ, namely,

$$\widetilde{v}_j \equiv \frac{v_j - \frac{2\Theta}{\sigma}y - \left(1-\frac{1}{\sigma}\right)\Theta\log\left(1-\frac{1}{\sigma}\right) - \left(1-\frac{1}{\sigma}\right)\frac{\Theta}{2}\left(\frac{a_1}{\sigma}\right)^2\tau_\xi^{-1}}{2\left(1-\frac{1}{\sigma}\right)\Theta} = \theta + \epsilon_0, \quad (58)$$

where

$$\epsilon_0 \equiv \frac{a_1}{2}\varepsilon_j + \frac{a_1}{2(\sigma-1)\Theta}\xi_j,$$

with

$$\epsilon_0 \sim N\left(0, \tau_{\epsilon_0}^{-1}\right), \tau_{\epsilon_0}^{-1} = \frac{a_1^2}{4}\tau_\varepsilon^{-1} + \frac{a_1^2}{4(\sigma-1)^2\Theta^2}\tau_\xi^{-1}. \quad (59)$$

Second, we solve the policymaker's problem by backward induction. In period 2, given the information set $\mathcal{I}^{G_2} = \{\widetilde{v}_j, a_1\}$, the policymaker solves

$$\max_{\{a_2\}} E\left[-(\theta - a_2)^2 |\mathcal{I}^{G_2}\right]. \tag{60}$$

The first-order condition with respect to a_2 leads to

$$a_2^* = E\left(\theta | I^{G_2}\right) = \frac{\overline{\theta}\tau_\theta + \widetilde{v}_j \tau_{\epsilon 0}}{\tau_\theta + \tau_{\epsilon 0}}. \tag{61}$$

In period 1, the policymaker with no information chooses a_1 to solve

$$\max_{\{a_1\}} E\left[-(\theta - a_1)^2 - (\theta - a_2^*)^2\right], \tag{62}$$

where a_2^* is the optimal action in period 2, satisfying (61). Substituting (61) into (62) and rearranging, we obtain the policymaker's problem to be solved in period 2:

$$\max_{\{a_1\}} \left[-\tau_\theta^{-1} - \left(\overline{\theta} - a_1\right)^2 - \frac{1}{\tau_\theta + \tau_{\epsilon 0}}\right].$$

The first-order condition with respect to a_1 is

$$2\left(\overline{\theta} - a_1\right) + \left(\tau_\theta + \tau_{\epsilon 0}\right)^{-2} \frac{\partial \tau_{\epsilon 0}}{\partial a_1} = 0, \tag{63}$$

where

$$\frac{\partial \tau_{\epsilon 0}}{\partial a_1} = -2\tau_{\epsilon 0} a_1 < 0. \tag{64}$$

Putting (64) in (63) gives rise to the equation that pins down a_1^*:

$$\left(\overline{\theta} - a_1\right)\left(\frac{\tau_\theta}{\tau_{\epsilon 0}} + 1\right)^2 = \left(\frac{1}{4\tau_\varepsilon} + \frac{1}{4(\sigma - 1)^2 \Theta^2 \tau_\xi}\right) a_1. \tag{65}$$

Define a continuous function $f(a_1)$ in the closed interval $\left[0, \overline{\theta}\right]$:

$$f(a_1) \equiv \left(\overline{\theta} - a_1\right)\left(\frac{\tau_\theta}{\tau_{\epsilon 0}} + 1\right)^2 - \left(\frac{1}{4\tau_\varepsilon} + \frac{1}{4(\sigma - 1)^2 \Theta^2 \tau_\xi}\right) a_1.$$

It is easy to know that

$$f(0) = \overline{\theta}\left(\frac{\tau_\theta}{\tau_{\epsilon 0}} + 1\right)^2 > 0, f\left(\overline{\theta}\right) = -\left(\frac{1}{4\tau_\varepsilon} + \frac{1}{4(\sigma - 1)^2 \Theta^2 \tau_\xi}\right)\overline{\theta} < 0,$$

which imply that equation (65) has a solution $a_1 \in \left(0, \overline{\theta}\right)$. Moreover, the function $f(a_1)$ decreases in a_1 in the open interval $\left(0, \overline{\theta}\right)$, namely,

$$f'(a_1) = -\left(\frac{\tau_\theta}{\tau_{\epsilon_0}} + 1\right)^2 + 4\left(\overline{\theta} - a_1\right)\left(\frac{\tau_\theta}{\tau_{\epsilon_0}} + 1\right)\tau_\theta a_1^{-1} - \left(\frac{1}{4\tau_\varepsilon} + \frac{1}{4(\sigma-1)^2\Theta^2\tau_\xi}\right) < 0,$$

which establishes that the solution is unique. The proof is thus completed. □

7.2 Appendix B

Proof of Proposition 2. We prove Proposition 2 through three steps: Step 1, Optimum; Step 2, Financial market equilibrium; Step 3, good market equilibrium and optimal government policy.

Step 1. Optimum. First, we solve the firm's investment decision. Given its information set $I_j = \{\theta, \varepsilon_j, q_j, \widehat{a}_1\}$, firm j maximizes its optimization problem:

$$\max_{\{P_j, Y_j, k_j\}} E\left[P_j Y_j - c(k_j)\,|\mathcal{I}_j\right], \tag{66}$$

subject to the production function (2), cost function (3), and market demand (4). Substituting (2), (3), and (4) into (66) leads to the corresponding unconstrained problem:

$$\max_{\{k_j\}} E\left[e^{\frac{\widehat{a}_1}{\sigma}\xi_j} Y^{\frac{1}{\sigma}} e^{\left(1-\frac{1}{\sigma}\right)\theta} k_j^{1-\frac{1}{\sigma}} - \frac{1}{2}e^{-\widehat{a}_1\varepsilon_j} k_j^2 |\mathcal{I}_j\right].$$

The first-order condition with respect to k_j gives us

$$k_j = k_j(\theta, \varepsilon_j, q_j, \widehat{a}_1) = \left(1 - \frac{1}{\sigma}\right)^\Theta Y^{\frac{\Theta}{\sigma}} e^{\left(1-\frac{1}{\sigma}\right)\theta\Theta + \Theta\widehat{a}_1\varepsilon_j}\left[E\left(e^{\frac{\widehat{a}_1}{\sigma}\xi_j}|\mathcal{I}_j\right)\right]^\Theta, \tag{67}$$

where $\Theta \equiv \frac{1}{1+\frac{1}{\sigma}}$. Once the investment decision is made, firm $j's$ optimal production is determined

$$Y_j = Y_j(\theta, \varepsilon_j, q_j, \widehat{a}_1) = e^\theta k_j(\theta, \varepsilon_j, q_j, \widehat{a}_1).$$

Second, we examine financial traders' decisions. Given his information set $\mathcal{I}^{I_i} = \left\{\theta, q_j, \chi_j^i, \widehat{a}_1\right\}$, an informed trader chooses his risky asset holdings, x^{I_i}, to solve,

$$x^{I_i}\left(\theta, q_j, \chi_j^i, \widehat{a}_1\right) = \arg\max_{\{x^{I_i}\}} E\left[-e^{\gamma W^{I_i}}|\theta, q_j, \chi_j^i, \widehat{a}_1\right].$$

The first-order condition with respect to x^{I_i} gives us

$$x^{I_i} = \frac{E\left(v_j|\mathcal{I}^{I_i}\right) - q_j}{\gamma var\left(v_j|\mathcal{I}^{I_i}\right)}, \tag{68a}$$

where

$$v_j = \log P_j\left(\widehat{a}_1\right) Y_j\left(\widehat{a}_1\right) = \frac{y}{\sigma} + \left(1 - \frac{1}{\sigma}\right)\theta + \frac{\widehat{a}_1}{\sigma}\xi_j + \left(1 - \frac{1}{\sigma}\right)\log k_j\left(\theta, \varepsilon_j, q_j, \widehat{a}_1\right). \tag{69}$$

The second-order condition with respect to x^{I_i} is satisfied, namely, $-\gamma var\left(v_j|\mathcal{I}^{I_i}\right) < 0$. Similarly, given his information set $\mathcal{I}^{U_i} = \{\theta, q_j, \widehat{a}_1\}$, an uninformed trader chooses his risky asset holdings, x^{U_i}, to maximize his utility,

$$x^{U_i}\left(\theta, q_j, \widehat{a}_1\right) = \arg\max_{\{x^{U_i}\}} E\left[-e^{\gamma W^{U_i}}|\mathcal{I}^{U_i}\right].$$

The first-order condition with respect to x^{U_i} leads to

$$x^{U_i} = \frac{E\left(v_j|\mathcal{I}^{U_i}\right) - q_j}{\gamma var\left(v_j|\mathcal{I}^{U_i}\right)}. \tag{70}$$

The second-order condition with respect to x^{U_i} is also satisfied, namely, $-\gamma var\left(v_j|\mathcal{I}^{U_i}\right) < 0$.

Third, we solve the policymaker's problem by backward induction. At $t = 2$, given her information $\mathcal{I}^{G_2} = \{q_j, v_j, \widehat{a}_1, a_1\}$, she chooses a_2 to solve

$$\max_{\{a_2\}} E\left[-\left(\theta - a_2\right)^2|\mathcal{I}^{G_2}\right].$$

The first-order condition gives us:

$$a_2 = E\left(\theta|\mathcal{I}^{G_2}\right) \equiv \widehat{\widehat{\theta}}, \text{ with } \tau_{\widehat{\widehat{\theta}}}^{-1} \equiv var\left(\theta|\mathcal{I}^{G_2}\right). \tag{71}$$

At $t = 1$, the policymaker with information $\mathcal{I}^{G_1} = \{q_j, \widehat{a}_1\}$ chooses a_1 to solve

$$\max_{\{a_1\}} E\left[-\left(\theta - a_1\right)^2 - \left(\theta - a_2\right)^2|\mathcal{I}^{G_1}\right], \tag{72}$$

where a_2 is the policymaker's optimal action in period 2, satisfying (71). Using (71) and (72), we change the policymaker's problem at $t=1$ into:

$$\max_{\{a_1\}}\left[-\tau_{\widehat{\theta}}^{-1}-\left(\widehat{\theta}-a_1\right)^2-\tau_{\widehat{\widehat{\theta}}}^{-1}\right],$$

where

$$E\left(\theta|\mathcal{I}^{G_1}\right)\equiv\widehat{\theta},\tau_{\widehat{\theta}}^{-1}\equiv var\left(\theta|\mathcal{I}^{G_1}\right). \tag{73}$$

The first-order condition with respect to a_1 gives rise to

$$2\left(\widehat{\theta}-a_1\right)-\frac{\partial\tau_{\widehat{\widehat{\theta}}}^{-1}}{\partial a_1}=0. \tag{74}$$

Step 2. Financial market equilibirum. To solve the financial market equilibrium, we conjecture a linear price function

$$q_j=\beta_0+\beta_1\left(\xi_j+\beta_2u_j+\beta_3\theta\right), \tag{75}$$

where β_0, β_1, β_2, and β_3 are undetermined coefficients. Knowing θ perfectly, firms and financial traders take q_j as a noisy signal about the demand shock ξ_j,

$$\widetilde{q}_j\left(q_j,\theta\right)=\frac{q_j-\beta_0}{\beta_1}-\beta_3\theta=\xi_j+\beta_2u_j\equiv\xi_j+\varrho_j^q, \tag{76}$$

where $\varrho_j^q\sim N\left(0,\tau_q^{-1}\right)$ with $\tau_q=\beta_2^{-2}\tau_u$. Using (68a), (70), (72), (74), and (75), we know that the optimal trading positions of the informed and uninformed traders are

$$\begin{aligned}
x^{I_i} &= \frac{\frac{y}{\sigma}+\left(1-\frac{1}{\sigma}\right)\theta+\frac{\widehat{a}_1}{\sigma}E\left(\xi_j|\mathcal{I}^{I_i}\right)+\left(1-\frac{1}{\sigma}\right)E\left(\log k_j|\mathcal{I}^{I_i}\right)-q_j}{\gamma\left[\left(\frac{\widehat{a}_1}{\sigma}\right)^2 var\left(\xi_j|\mathcal{I}^{I_i}\right)+\left(1-\frac{1}{\sigma}\right)^2 var\left(\log k_j|\mathcal{I}^{I_i}\right)\right]}, && (77)\\
x^{U_i} &= \frac{\frac{y}{\sigma}+\left(1-\frac{1}{\sigma}\right)\theta+\frac{\widehat{a}_1}{\sigma}E\left(\xi_j|\mathcal{I}^{U_i}\right)+\left(1-\frac{1}{\sigma}\right)E\left(\log k_j|\mathcal{I}^{U_i}\right)-q_j}{\gamma\left[\left(\frac{\widehat{a}_1}{\sigma}\right)^2 var\left(\xi_j|\mathcal{I}^{U_i}\right)+\left(1-\frac{1}{\sigma}\right)^2 var\left(\log k_j|\mathcal{I}^{U_i}\right)\right]}, && (78)
\end{aligned}$$

where

$$\begin{aligned}
E\left(\xi_j|\mathcal{I}^{I_i}\right) &= \frac{\chi_j^i\tau_\chi+\widetilde{q}_j\tau_q}{\tau_\xi+\tau_\chi+\tau_q}, var\left(\xi_j|\mathcal{I}^{I_i}\right)=\frac{1}{\tau_\xi+\tau_\chi+\tau_q}, && (79a)\\
E\left(\xi_j|\mathcal{I}^{U_i}\right) &= \frac{\widetilde{q}_j\tau_q}{\tau_\xi+\tau_q}, var\left(\xi_j|\mathcal{I}^{U_i}\right)=\frac{1}{\tau_\xi+\tau_q}. && (79b)
\end{aligned}$$

Combining (28) and equations (76)-(79b), we have that

$$\begin{aligned} 0 &= u_j + \lambda \frac{\frac{y}{\sigma} + \left(1-\frac{1}{\sigma}\right)\theta + \frac{\widehat{a}_1}{\sigma}\frac{\xi_j\tau_\chi + \widetilde{q}_j\tau_q}{\tau_\xi+\tau_\chi+\tau_q} + \left(1-\frac{1}{\sigma}\right)E\left(\log k_j|\mathcal{I}^{I_i}\right) - q_j}{\gamma\left[\left(\frac{\widehat{a}_1}{\sigma}\right)^2\frac{1}{\tau_\xi+\tau_\chi+\tau_q} + \left(1-\frac{1}{\sigma}\right)^2 var\left(\log k_j|\mathcal{I}^{I_i}\right)\right]} \\ &+ (1-\lambda)\frac{\frac{y}{\sigma} + \left(1-\frac{1}{\sigma}\right)\theta + \frac{\widehat{a}_1}{\sigma}\frac{\widetilde{q}_j\tau_q}{\tau_\xi+\tau_q} + \left(1-\frac{1}{\sigma}\right)E\left(\log k_j|\mathcal{I}^{U_i}\right) - q_j}{\gamma\left[\left(\frac{\widehat{a}_1}{\sigma}\right)^2\frac{1}{\tau_\xi+\tau_q} + \left(1-\frac{1}{\sigma}\right)^2 var\left(\log k_j|\mathcal{I}^{U_i}\right)\right]}. \end{aligned} \tag{80}$$

Next, using (76), firm $j's$ information set $\mathcal{I}_j = \{\theta, \varepsilon_j, q_j, \widehat{a}_1\}$ is equivalently transformed into $\mathcal{I}_j = \{\theta, \varepsilon_j, \widetilde{q}_j, \widehat{a}_1\}$. Thus, we know that

$$\begin{aligned} E\left(e^{\frac{\widehat{a}_1}{\sigma}\xi_j}|\mathcal{I}_j\right) &= \exp\left\{\frac{\widehat{a}_1}{\sigma}E\left(\xi_j|\theta,\varepsilon_j,\widetilde{q}_j,\widehat{a}_1\right) + \frac{1}{2}\left(\frac{\widehat{a}_1}{\sigma}\right)^2 var\left(\xi_j|\theta,\varepsilon_j,\widetilde{q}_j,\widehat{a}_1\right)\right\} \\ &= \exp\left\{\frac{\widehat{a}_1}{\sigma}\frac{\widetilde{q}_j\tau_q}{\tau_\xi+\tau_q} + \frac{1}{2}\left(\frac{\widehat{a}_1}{\sigma}\right)^2\frac{1}{\tau_\xi+\tau_q}\right\}. \end{aligned} \tag{81}$$

Plugging (81) in (67) gives us

$$k_j = \left(1-\frac{1}{\sigma}\right)^\Theta Y^{\frac{\Theta}{\sigma}} \exp\left\{\left(1-\frac{1}{\sigma}\right)\Theta\theta + \Theta\widehat{a}_1\varepsilon_j + \Theta\left[\frac{\widehat{a}_1}{\sigma}\frac{\widetilde{q}_j\tau_q}{\tau_\xi+\tau_q} + \frac{1}{2}\left(\frac{\widehat{a}_1}{\sigma}\right)^2\frac{1}{\tau_\xi+\tau_q}\right]\right\}, \tag{82}$$

which leads to

$$\log k_j = \log k_j\left(\theta,\varepsilon_j,\widetilde{q}_j,\widehat{a}_1\right) = \phi_0 + \phi_1\widehat{a}_1\widetilde{q}_j + \phi_2\widehat{a}_1\varepsilon_j + \phi_3\theta, \tag{83}$$

where

$$\begin{aligned} \phi_0 &= \Theta\log\left(1-\frac{1}{\sigma}\right) + \frac{\Theta}{\sigma}y + \frac{\Theta}{2}\left(\frac{\widehat{a}_1}{\sigma}\right)^2\frac{1}{\tau_\xi+\tau_q}, \\ \phi_1 &= \frac{\Theta}{\sigma}\frac{\tau_q}{\tau_\xi+\tau_q}, \phi_2 = \Theta, \phi_3 = \Theta\left(1-\frac{1}{\sigma}\right). \end{aligned}$$

Putting (83) in (69), (77), (78), and (80), respectively, we have that

$$v_j = \log P_jY_j = \frac{y}{\sigma} + \left(1-\frac{1}{\sigma}\right)\theta + \frac{\widehat{a}_1}{\sigma}\xi_j + \left(1-\frac{1}{\sigma}\right)\left(\phi_0 + \phi_1\widehat{a}_1\widetilde{q}_j + \phi_2\widehat{a}_1\varepsilon_j + \phi_3\theta\right), \tag{84}$$

$$x^{I_i} = \frac{\frac{y}{\sigma} + \left(1-\frac{1}{\sigma}\right)\theta + \frac{\widehat{a}_1}{\sigma}\frac{\chi_j^i\tau_\chi + \widetilde{q}_j\tau_q}{\tau_\xi+\tau_\chi+\tau_q} + \left(1-\frac{1}{\sigma}\right)\left(\phi_0 + \phi_1\widehat{a}_1\widetilde{q}_j + \phi_3\theta\right) - q_j}{\gamma\left[\left(\frac{\widehat{a}_1}{\sigma}\right)^2\frac{1}{\tau_\xi+\tau_\chi+\tau_q} + \left(1-\frac{1}{\sigma}\right)^2\phi_2^2\widehat{a}_1^2\tau_\varepsilon^{-1}\right]}, \tag{85}$$

$$x^{U_i} = \frac{\frac{y}{\sigma} + \left(1 - \frac{1}{\sigma}\right)\theta + \frac{\widehat{a}_1}{\sigma}\frac{\widetilde{q}_j \tau_q}{\tau_\xi + \tau_q} + \left(1 - \frac{1}{\sigma}\right)\left(\phi_0 + \phi_1 \widehat{a}_1 \widetilde{q}_j + \phi_3 \theta\right) - q_j}{\gamma\left[\left(\frac{\widehat{a}_1}{\sigma}\right)^2 \frac{1}{\tau_\xi + \tau_q} + \left(1 - \frac{1}{\sigma}\right)^2 \phi_2^2 \widehat{a}_1^2 \tau_\varepsilon^{-1}\right]}, \tag{86}$$

$$\begin{aligned} 0 \; = \; & u_j + \lambda \frac{\frac{y}{\sigma} + \left(1 - \frac{1}{\sigma}\right)\theta + \frac{\widehat{a}_1}{\sigma}\frac{\xi_j \tau_\chi + \widetilde{q}_j \tau_q}{\tau_\xi + \tau_\chi + \tau_q} + \left(1 - \frac{1}{\sigma}\right)\left(\phi_0 + \phi_1 \widehat{a}_1 \widetilde{q}_j + \phi_3 \theta\right) - q_j}{\gamma\left[\left(\frac{\widehat{a}_1}{\sigma}\right)^2 \frac{1}{\tau_\xi + \tau_\chi + \tau_q} + \left(1 - \frac{1}{\sigma}\right)^2 \phi_2^2 \widehat{a}_1^2 \tau_\varepsilon^{-1}\right]} \\ & + (1 - \lambda)\frac{\frac{y}{\sigma} + \left(1 - \frac{1}{\sigma}\right)\theta + \frac{\widehat{a}_1}{\sigma}\frac{\widetilde{q}_j \tau_q}{\tau_\xi + \tau_q} + \left(1 - \frac{1}{\sigma}\right)\left(\phi_0 + \phi_1 \widehat{a}_1 \widetilde{q}_j + \phi_3 \theta\right) - q_j}{\gamma\left[\left(\frac{\widehat{a}_1}{\sigma}\right)^2 \frac{1}{\tau_\xi + \tau_q} + \left(1 - \frac{1}{\sigma}\right)^2 \phi_2^2 \widehat{a}_1^2 \tau_\varepsilon^{-1}\right]}. \end{aligned} \tag{87}$$

Comparing the coefficients of (75) and (87), we obtain the following four algebraic equations pinning down $(\beta_0, \beta_1, \beta_2, \beta_3)$:

$$\beta_0 = \frac{\begin{pmatrix} \lambda A_2 \left[\frac{y}{\sigma} - \frac{\widehat{a}_1}{\sigma}\frac{\beta_2^{-2}\tau_u}{\tau_\xi + \tau_\chi + \beta_2^{-2}\tau_u}\frac{\beta_0}{\beta_1} + \left(1 - \frac{1}{\sigma}\right)\left(\phi_0 - \phi_1 \widehat{a}_1 \frac{\beta_0}{\beta_1}\right)\right] + \\ (1 - \lambda) A_1 \left[\frac{y}{\sigma} - \frac{\widehat{a}_1}{\sigma}\frac{\beta_2^{-2}\tau_u}{\tau_\xi + \beta_2^{-2}\tau_u}\frac{\beta_0}{\beta_1} + \left(1 - \frac{1}{\sigma}\right)\left(\phi_0 - \phi_1 \widehat{a}_1 \frac{\beta_0}{\beta_1}\right)\right] \end{pmatrix}}{\begin{pmatrix} \lambda A_2 \left[1 - \frac{\widehat{a}_1}{\sigma}\frac{\beta_2^{-2}\tau_u}{\tau_\xi + \tau_\chi + \beta_2^{-2}\tau_u}\frac{1}{\beta_1} - \left(1 - \frac{1}{\sigma}\right)\phi_1 \widehat{a}_1 \frac{1}{\beta_1}\right] + \\ (1 - \lambda) A_1 \left[1 - \frac{\widehat{a}_1}{\sigma}\frac{\beta_2^{-2}\tau_u}{\tau_\xi + \beta_2^{-2}\tau_u}\frac{1}{\beta_1} - \left(1 - \frac{1}{\sigma}\right)\phi_1 \widehat{a}_1 \frac{1}{\beta_1}\right] \end{pmatrix}}, \tag{88}$$

$$\beta_1 = \frac{\lambda A_2 \frac{\widehat{a}_1}{\sigma}\frac{\tau_\chi}{\tau_\xi + \tau_\chi + \beta_2^{-2}\tau_u}}{\begin{pmatrix} \lambda A_2 \left[1 - \frac{\widehat{a}_1}{\sigma}\frac{\beta_2^{-2}\tau_u}{\tau_\xi + \tau_\chi + \beta_2^{-2}\tau_u}\frac{1}{\beta_1} - \left(1 - \frac{1}{\sigma}\right)\phi_1 \widehat{a}_1 \frac{1}{\beta_1}\right] + \\ (1 - \lambda) A_1 \left[1 - \frac{\widehat{a}_1}{\sigma}\frac{\beta_2^{-2}\tau_u}{\tau_\xi + \beta_2^{-2}\tau_u}\frac{1}{\beta_1} - \left(1 - \frac{1}{\sigma}\right)\phi_1 \widehat{a}_1 \frac{1}{\beta_1}\right] \end{pmatrix}}, \tag{89}$$

$$\beta_1 \beta_2 = \frac{A_1 A_2}{\begin{pmatrix} \lambda A_2 \left[1 - \frac{\widehat{a}_1}{\sigma}\frac{\beta_2^{-2}\tau_u}{\tau_\xi + \tau_\chi + \beta_2^{-2}\tau_u}\frac{1}{\beta_1} - \left(1 - \frac{1}{\sigma}\right)\phi_1 \widehat{a}_1 \frac{1}{\beta_1}\right] + \\ (1 - \lambda) A_1 \left[1 - \frac{\widehat{a}_1}{\sigma}\frac{\beta_2^{-2}\tau_u}{\tau_\xi + \beta_2^{-2}\tau_u}\frac{1}{\beta_1} - \left(1 - \frac{1}{\sigma}\right)\phi_1 \widehat{a}_1 \frac{1}{\beta_1}\right] \end{pmatrix}}, \tag{90}$$

$$\beta_1 \beta_3 = \frac{\begin{pmatrix} \lambda A_2 \left[-\frac{\widehat{a}_1}{\sigma}\frac{\beta_2^{-2}\tau_u}{\tau_\xi + \tau_\chi + \beta_2^{-2}\tau_u}\beta_3 + \left(1 - \frac{1}{\sigma}\right)\left(1 - \phi_1 \widehat{a}_1 \beta_3 + \phi_3\right)\right] + \\ (1 - \lambda) A_1 \left[-\frac{\widehat{a}_1}{\sigma}\frac{\beta_2^{-2}\tau_u}{\tau_\xi + \beta_2^{-2}\tau_u}\beta_3 + \left(1 - \frac{1}{\sigma}\right)\left(1 - \phi_1 \widehat{a}_1 \beta_3 + \phi_3\right)\right] \end{pmatrix}}{\begin{pmatrix} \lambda A_2 \left[1 - \frac{\widehat{a}_1}{\sigma}\frac{\beta_2^{-2}\tau_u}{\tau_\xi + \tau_\chi + \beta_2^{-2}\tau_u}\frac{1}{\beta_1} - \left(1 - \frac{1}{\sigma}\right)\phi_1 \widehat{a}_1 \frac{1}{\beta_1}\right] + \\ (1 - \lambda) A_1 \left[1 - \frac{\widehat{a}_1}{\sigma}\frac{\beta_2^{-2}\tau_u}{\tau_\xi + \beta_2^{-2}\tau_u}\frac{1}{\beta_1} - \left(1 - \frac{1}{\sigma}\right)\phi_1 \widehat{a}_1 \frac{1}{\beta_1}\right] \end{pmatrix}}, \tag{91}$$

where

$$
\begin{aligned}
A_1 &\equiv \gamma\left[\left(\frac{\widehat{a}_1}{\sigma}\right)^2\frac{1}{\tau_\xi+\tau_\chi+\tau_q}+\left(1-\frac{1}{\sigma}\right)^2\phi_2^2\widehat{a}_1^2\tau_\varepsilon^{-1}\right],\\
A_2 &\equiv \gamma\left[\left(\frac{\widehat{a}_1}{\sigma}\right)^2\frac{1}{\tau_\xi+\tau_q}+\left(1-\frac{1}{\sigma}\right)^2\phi_2^2\widehat{a}_1^2\tau_\varepsilon^{-1}\right].
\end{aligned}
$$

Dividing both sides of equation (90) by both sides of equation (89), we obtain a three-order polynomial equation pinning down β_2:

$$
\beta_2^3+p\beta_2^2+q\beta_2+r=0, \tag{92}
$$

where

$$
p=-\frac{\gamma\frac{\widehat{a}_1}{\sigma}\left[1+(\sigma-1)^2\phi_2^2\tau_\varepsilon^{-1}(\tau_\xi+\tau_\chi)\right]}{\lambda\tau_\chi}, q=0, r=-\frac{\gamma\frac{\widehat{a}_1}{\sigma}(\sigma-1)^2\phi_2^2\tau_\varepsilon^{-1}\tau_u}{\lambda\tau_\chi}. \tag{93}
$$

Setting $\beta_2=\widetilde{\beta}_2-p/3$, we change equation (92) into a polynomial about $\widetilde{\beta}_2$:

$$
\widetilde{\beta}_2^3+m\widetilde{\beta}_2+n=0, \tag{94}
$$

where

$$
m = q-\frac{p^2}{3}=-\frac{1}{3}\left(\frac{\gamma\frac{\widehat{a}_1}{\sigma}\left[1+(\sigma-1)^2\phi_2^2\tau_\varepsilon^{-1}(\tau_\xi+\tau_\chi)\right]}{\lambda\tau_\chi}\right)^2, \tag{95}
$$

$$
n = \frac{2}{27}p^3-\frac{pq}{3}+r=-\frac{2}{27}\left(\frac{\frac{\gamma\widehat{a}_1}{\sigma}\left[1+(\sigma-1)^2\phi_2^2\tau_\varepsilon^{-1}(\tau_\xi+\tau_\chi)\right]}{\lambda\tau_\chi}\right)^3-\frac{\frac{\gamma\widehat{a}_1}{\sigma}(\sigma-1)^2\phi_2^2\tau_\varepsilon^{-1}\tau_u}{\lambda\tau_\chi} \tag{96}
$$

The determinant of equation (94) is positive, i.e.,

$$
\begin{aligned}
\Delta &= \left(\frac{n}{2}\right)^2+\left(\frac{m}{3}\right)^3\\
&= \frac{1}{27}\left(\frac{\frac{\gamma\widehat{a}_1}{\sigma}\left[1+(\sigma-1)^2\phi_2^2\tau_\varepsilon^{-1}(\tau_\xi+\tau_\chi)\right]}{\lambda\tau_\chi}\right)^3\frac{\frac{\widehat{a}_1}{\sigma}\gamma(\sigma-1)^2\phi_2^2\tau_\varepsilon^{-1}\tau_u}{\lambda\tau_\chi}+\frac{1}{4}\left(\frac{\frac{\gamma\widehat{a}_1}{\sigma}(\sigma-1)^2\phi_2^2\tau_\varepsilon^{-1}\tau_u}{\lambda\tau_\chi}\right)^2\\
&> 0,
\end{aligned}
$$

which implies that equation (94) has one real root and two conjugate complex roots. The real root is given by the formula

$$\widetilde{\beta}_2 = \sqrt[3]{-\frac{n}{2}+\sqrt{\Delta}} + \sqrt[3]{-\frac{n}{2}-\sqrt{\Delta}}.$$

By substitution, we have that the unique real root of equation (92) is

$$\beta_2 = \widetilde{\beta}_2 - \frac{p}{3} = \sqrt[3]{-\frac{n}{2}+\sqrt{\Delta}} + \sqrt[3]{-\frac{n}{2}-\sqrt{\Delta}} - \frac{p}{3}. \tag{97}$$

Using equations (89), (91), and (88), we solve for the expressions of β_1, β_3, and β_0, respectively, which are listed in Proposition 2.

Step 3. Good market equilibrium and optimal policy. First of all, we solve for the policymaker's decisions. Unlike firms and financial traders, the policymaker translates q_j and v_j as two noisy signals about the productivity shock θ, respectively,

$$\begin{aligned} \widehat{q}_j &= \widehat{q}_j(q_j) = \frac{q_j-\beta_0}{\beta_1\beta_3} = \theta + \frac{1}{\beta_3}\xi_j + \frac{\beta_2}{\beta_3}u_j \equiv \theta + \varphi_1\xi_j + \varphi_2 u_j, && (98)\\ \widehat{v}_j &= v_j(\widehat{a}_1, a_1) = \frac{v_j - \frac{y}{\sigma} - \left(1-\frac{1}{\sigma}\right)\left(\phi_0 + \phi_1\widehat{a}_1\frac{q_j-\beta_0}{\beta_1}\right)}{\left(1-\frac{1}{\sigma}\right)\left(1+\phi_3-\phi_1\widehat{a}_1\beta_3\right)} = \theta + \varphi_3\xi_j + \varphi_4\varepsilon_j, && (99) \end{aligned}$$

where

$$\varphi_1 \equiv \frac{1}{\beta_3}, \varphi_2 \equiv \frac{\beta_2}{\beta_3}, \varphi_3 = \frac{a_1/\sigma}{\left(1-\frac{1}{\sigma}\right)\left(1+\phi_3-\phi_1\widehat{a}_1\beta_3\right)}, \varphi_4 = \frac{\left(1-\frac{1}{\sigma}\right)\phi_2\widehat{a}_1}{\left(1-\frac{1}{\sigma}\right)\left(1+\phi_3-\phi_1\widehat{a}_1\beta_3\right)}.[2]$$

Hence, using the projection theorem, we have that

$$\begin{aligned} \tau_{\widehat{\widehat{\theta}}}^{-1} &\equiv var\left(\theta|\mathcal{I}^{G_2}\right) = var\left(\theta|\widehat{q}_j,\widehat{v}_j\right) = var\left(\theta|\widehat{q}_j\right) - \frac{cov\left(\theta,\widehat{v}_j|\widehat{q}_j\right)^2}{var\left(\widehat{v}_j|\widehat{q}_j\right)} \\ &= var(\theta) - \frac{cov\left(\theta,\widehat{q}_j\right)^2}{var\left(\widehat{q}_j\right)} - \frac{\left[cov\left(\theta,\widehat{v}_j\right) - \frac{cov(\theta,\widehat{q}_j)cov(\widehat{v}_j,\widehat{q}_j)}{var(\widehat{q}_j)}\right]^2}{var\left(\widehat{v}_j\right) - \frac{cov(\widehat{v}_j,\widehat{q}_j)^2}{var(\widehat{q}_j)}} \\ &= \tau_\theta^{-1} - \frac{\tau_\theta^{-2}}{var\left(\widehat{q}_j\right)} - \frac{\tau_\theta^{-2}}{var\left(\widehat{q}_j\right)}\frac{\left(var\left(\widehat{q}_j\right) - cov\left(\widehat{v}_j,\widehat{q}_j\right)\right)^2}{var\left(\widehat{v}_j\right)var\left(\widehat{q}_j\right) - cov\left(\widehat{v}_j,\widehat{q}_j\right)^2}. \end{aligned} \tag{100}$$

Taking partial derivatives on both sides of (100) with respect to a_1 gives rise to

$$\frac{\partial\tau_{\widehat{\widehat{\theta}}}^{-1}}{\partial a_1} = -\frac{\tau_\theta^{-2}}{var\left(\widehat{q}_j\right)}\frac{\Psi_1}{\Psi_2^2}\left[-2\frac{\partial cov\left(\widehat{v}_j,\widehat{q}_j\right)}{\partial a_1}\Psi_2 - \Psi_1\Psi_3\right], \tag{101}$$

[2] Notice that only φ_3 relates to the policy action a_1, while φ_1, φ_2, and φ_4 have nothing to do with the policy action a_1.

where

$$\Psi_1 \equiv var(\widehat{q}_j) - cov(\widehat{v}_j, \widehat{q}_j),$$
$$\Psi_2 \equiv var(\widehat{v}_j)\, var(\widehat{q}_j) - cov(\widehat{v}_j, \widehat{q}_j)^2,$$
$$\Psi_3 \equiv var(\widehat{q}_j)\frac{\partial var(\widehat{v}_j)}{\partial a_1} - 2cov(\widehat{v}_j, \widehat{q}_j)\frac{\partial cov(\widehat{v}_j, \widehat{q}_j)}{\partial a_1}.$$

Plugging (101) in (74) leads to

$$2\left(\widehat{\theta} - a_1\right) = \frac{\tau_\theta^{-2}}{var(\widehat{q}_j)}\frac{\Psi_1}{\Psi_2^2}\left[2\frac{\partial cov(\widehat{v}_j, \widehat{q}_j)}{\partial a_1}\Psi_2 + \Psi_1\Psi_3\right], \tag{102}$$

which determines how the government's policy action a_1 depends on her policy proposal $\widehat{a}_1$, namely, $a_1 = f(\widehat{a}_1)$. Next we will prove that $a_1 = f(\widehat{a}_1) = \widehat{a}_1$ is a solution of (102).

Secondly, we provide the following claim, which shows that when $a_1 = \widehat{a}_1$, the three terms Ψ_1, Ψ_2, and Ψ_3 are all nonnegative.

Claim 1 *If $a_1 = \widehat{a}_1$, then $\Psi_1 \geq 0$, $\Psi_2 \geq 0$, and $\Psi_3 \geq 0$. Namely, $\Psi_1|_{a_1=\widehat{a}_1} \geq 0$, $\Psi_2|_{a_1=\widehat{a}_1} \geq 0$, and $\Psi_3|_{a_1=\widehat{a}_1} \geq 0$.*

Proof Comparing the coefficients between the conjectured price function ((75)) and the market clearing condition ((87)), we obtain the following two equations:

$$\frac{1}{\beta_2} = \frac{\lambda\tau_x}{\gamma\tau_q}\frac{\nu_1}{\nu_4}, \tag{103}$$
$$\frac{\beta_3}{\beta_2} = \lambda\frac{-\beta_3\nu_1 - \beta_3\nu_2 + \nu_3}{\gamma\nu_4} + (1-\lambda)\frac{-\beta_3\nu_5 - \beta_3\nu_2 + \nu_3}{\gamma\nu_6}, \tag{104}$$

where

$$\nu_1 \equiv \frac{\widehat{a}_1}{\sigma}\frac{\tau_q}{\tau_\xi + \tau_\chi + \tau_q}, \nu_2 \equiv \left(1 - \frac{1}{\sigma}\right)\phi_1\widehat{a}_1,$$
$$\nu_3 \equiv \left(1 - \frac{1}{\sigma}\right)(1 + \phi_3), \nu_4 \equiv \left(\frac{\widehat{a}_1}{\sigma}\right)^2\frac{1}{\tau_\xi + \tau_\chi + \tau_q} + \left(1 - \frac{1}{\sigma}\right)^2\phi_2^2\widehat{a}_1^2\tau_\varepsilon^{-1},$$
$$\nu_5 \equiv \frac{\widehat{a}_1}{\sigma}\frac{\tau_q}{\tau_\xi + \tau_q}, \nu_6 \equiv \left(\frac{\widehat{a}_1}{\sigma}\right)^2\frac{1}{\tau_\xi + \tau_q} + \left(1 - \frac{1}{\sigma}\right)^2\phi_2^2\widehat{a}_1^2\tau_\varepsilon^{-1}.$$

From (104), we know that

$$\beta_3 = \varphi_5\widehat{a}_1^{-1}, \tag{105}$$

where

$$\varphi_5 \equiv \frac{\left(\begin{array}{c} \frac{\lambda}{\sigma}\frac{\tau_q}{\tau_\xi+\tau_\chi+\tau_q}\frac{\tau_\chi}{\tau_q}\left[\left(\frac{1}{\sigma}\right)^2\frac{1}{\tau_\xi+\tau_q}+\left(1-\frac{1}{\sigma}\right)^2\phi_2^2\tau_\varepsilon^{-1}\right]+\left(\frac{1}{\sigma}\right)^3\frac{\tau_q}{\tau_\xi+\tau_\chi+\tau_q}\frac{1}{\tau_\xi+\tau_q}+ \\ \left(1-\frac{1}{\sigma}\right)^3\phi_1\phi_2^2\tau_\varepsilon^{-1}+\frac{\lambda}{\sigma}\left(1-\frac{1}{\sigma}\right)^2\phi_2^2\frac{\tau_q}{\tau_\xi+\tau_\chi+\tau_q}\tau_\varepsilon^{-1}+\lambda\left(\frac{1}{\sigma}\right)^2\frac{1}{\tau_\xi+\tau_q}\left(1-\frac{1}{\sigma}\right)\phi_1 \\ +\left(1-\lambda\right)\left(1-\frac{1}{\sigma}\right)^2\phi_2^2\frac{1}{\sigma}\frac{\tau_q}{\tau_\xi+\tau_\chi+\tau_q}\tau_\varepsilon^{-1}+\left(1-\lambda\right)\left(\frac{1}{\sigma}\right)^2\frac{1}{\tau_\xi+\tau_\chi+\tau_q}\left(1-\frac{1}{\sigma}\right)\phi_1 \end{array}\right)}{\left(1-\frac{1}{\sigma}\right)\left(1+\phi_3\right)\left[\lambda\left(\frac{1}{\sigma}\right)^2\frac{1}{\tau_\xi+\tau_q}+\left(1-\lambda\right)\left(\frac{1}{\sigma}\right)^2\frac{1}{\tau_\xi+\tau_\chi+\tau_q}+\left(1-\frac{1}{\sigma}\right)^2\phi_2^2\tau_\varepsilon^{-1}\right]} \geq 0. \tag{106}$$

From (103), we know that

$$\beta_2 = \varphi_6\widehat{a}_1, \tag{107}$$

where

$$\varphi_6 \equiv \frac{\gamma\left[\left(\frac{1}{\sigma}\right)^2\frac{1}{\tau_\xi+\tau_\chi+\tau_q}+\left(1-\frac{1}{\sigma}\right)^2\phi_2^2\tau_\varepsilon^{-1}\right]}{\lambda\frac{1}{\sigma}\frac{\tau_\chi}{\tau_\xi+\tau_\chi+\tau_q}} \geq 0. \tag{108}$$

Putting (105) in the term $\varphi_7 \equiv \frac{1}{1+\phi_3-\phi_1\widehat{a}_1\beta_3}$ gives us

$$\varphi_7 = \frac{\left(\begin{array}{c} \frac{\lambda}{\sigma}\frac{\tau_q}{\tau_\xi+\tau_\chi+\tau_q}\frac{\tau_\chi}{\tau_q}\left[\left(\frac{1}{\sigma}\right)^2\frac{1}{\tau_\xi+\tau_q}+\left(1-\frac{1}{\sigma}\right)^2\phi_2^2\tau_\varepsilon^{-1}\right]+\left(\frac{1}{\sigma}\right)^3\frac{\tau_q}{\tau_\xi+\tau_\chi+\tau_q}\frac{1}{\tau_\xi+\tau_q}+ \\ \left(1-\frac{1}{\sigma}\right)^3\phi_1\phi_2^2\tau_\varepsilon^{-1}+\frac{\lambda}{\sigma}\left(1-\frac{1}{\sigma}\right)^2\phi_2^2\frac{\tau_q}{\tau_\xi+\tau_\chi+\tau_q}\tau_\varepsilon^{-1}+\lambda\left(\frac{1}{\sigma}\right)^2\frac{1}{\tau_\xi+\tau_q}\left(1-\frac{1}{\sigma}\right)\phi_1 \\ +\left(1-\lambda\right)\left(1-\frac{1}{\sigma}\right)^2\phi_2^2\frac{1}{\sigma}\frac{\tau_q}{\tau_\xi+\tau_\chi+\tau_q}\tau_\varepsilon^{-1}+\left(1-\lambda\right)\left(\frac{1}{\sigma}\right)^2\frac{1}{\tau_\xi+\tau_\chi+\tau_q}\left(1-\frac{1}{\sigma}\right)\phi_1 \end{array}\right)}{\left(1+\phi_3\right)\left(\begin{array}{c} \frac{\lambda}{\sigma}\frac{\tau_\chi}{\tau_\xi+\tau_\chi+\tau_q}\left[\left(\frac{1}{\sigma}\right)^2\frac{1}{\tau_\xi+\tau_q}+\left(1-\frac{1}{\sigma}\right)^2\phi_2^2\tau_\varepsilon^{-1}\right]+\left(\frac{1}{\sigma}\right)^3\frac{\tau_q}{\tau_\xi+\tau_\chi+\tau_q}\frac{1}{\tau_\xi+\tau_q} \\ +\frac{\lambda}{\sigma}\left(1-\frac{1}{\sigma}\right)^2\phi_2^2\frac{\tau_q}{\tau_\xi+\tau_\chi+\tau_q}\tau_\varepsilon^{-1}+\left(1-\lambda\right)\left(1-\frac{1}{\sigma}\right)^2\frac{\tau_q}{\tau_\xi+\tau_q}\frac{1}{\sigma}\phi_2^2\frac{\tau_q}{\tau_\xi+\tau_q} \end{array}\right)} \geq 0. \tag{109}$$

Now we prove those three inequalities one by one. First, using equation (105), (106), and (109), we know that

$$1-\frac{\varphi_3}{\varphi_1} = 1-\frac{\frac{1}{\sigma}\frac{a_1}{\widehat{a}_1}\left(\lambda\left(\frac{1}{\sigma}\right)^2\frac{1}{\tau_\xi+\tau_q}+\left(1-\lambda\right)\left(\frac{1}{\sigma}\right)^2\frac{1}{\tau_\xi+\tau_\chi+\tau_q}+\left(1-\frac{1}{\sigma}\right)^2\phi_2^2\tau_\varepsilon^{-1}\right)}{\left(\begin{array}{c} \frac{\lambda}{\sigma}\frac{\tau_\chi}{\tau_\xi+\tau_\chi+\tau_q}\left[\left(\frac{1}{\sigma}\right)^2\frac{1}{\tau_\xi+\tau_q}+\left(1-\frac{1}{\sigma}\right)^2\phi_2^2\tau_\varepsilon^{-1}\right]+\left(\frac{1}{\sigma}\right)^3\frac{\tau_q}{\tau_\xi+\tau_\chi+\tau_q}\frac{1}{\tau_\xi+\tau_q} \\ +\frac{\lambda}{\sigma}\left(1-\frac{1}{\sigma}\right)^2\phi_2^2\frac{\tau_q}{\tau_\xi+\tau_\chi+\tau_q}\tau_\varepsilon^{-1}+\left(1-\lambda\right)\left(1-\frac{1}{\sigma}\right)^2\frac{\tau_q}{\tau_\xi+\tau_q}\frac{1}{\sigma}\phi_2^2\frac{\tau_q}{\tau_\xi+\tau_q} \end{array}\right)}, \tag{110}$$

whose sign is indeterminate. Taking $a_1 = \widehat{a}_1$ in equation (101), we obtain

$$\left(1-\frac{\varphi_3}{\varphi_1}\right)|_{a_1=\widehat{a}_1} = \frac{-\left(\frac{1}{\sigma}\right)^2\tau_\xi\tau_\varepsilon-\left(1-\frac{1}{\sigma}\right)^2\phi_2^2\tau_\xi\left(\tau_q+\tau_\xi+\left(1-\lambda\right)\tau_\chi\right)}{\left(\begin{array}{c} \left(\frac{1}{\sigma}\right)^2\tau_\varepsilon\left(\tau_q+\lambda\tau_\chi\right)+\lambda\left(1-\frac{1}{\sigma}\right)^2\phi_2^2\tau_\xi\tau_\chi \\ +\left(1-\frac{1}{\sigma}\right)^2\phi_2^2\tau_q\left(\tau_q+\tau_\chi+\tau_\xi\right) \end{array}\right)} \leq 0, \tag{111}$$

which also implies that

$$\left(1-\frac{\varphi_1}{\varphi_3}\right)|_{a_1=\widehat{a}_1}\geq 0. \tag{112}$$

Taking together, we have

$$\begin{aligned}
&\Psi_1|_{a_1=\widehat{a}_1} && (113)\\
\equiv\ & [var(\widehat{q}_j)-cov(\widehat{v}_j,\widehat{q}_j)]|_{a_1=\widehat{a}_1}=\tau_\xi^{-1}\tau_q^{-1}\phi_1^2\left[\left(1-\frac{\varphi_3}{\varphi_1}\right)|_{a_1=\widehat{a}_1}\tau_q+\tau_\xi\right]\\
=\ & \frac{\tau_\xi^{-1}\tau_q^{-1}\varphi_1^2\left[\left(\frac{1}{\sigma}\right)^2\lambda\tau_\xi\tau_\chi\tau_\varepsilon+\left(1-\frac{1}{\sigma}\right)^2\phi_2^2\lambda\tau_\xi\tau_\chi\left(\tau_q+\tau_\xi\right)\right]}{\left(\left(\frac{1}{\sigma}\right)^2\tau_\varepsilon\left(\tau_q+\lambda\tau_\chi\right)+\lambda\left(1-\frac{1}{\sigma}\right)^2\phi_2^2\tau_\xi\tau_\chi+\left(1-\frac{1}{\sigma}\right)^2\phi_2^2\tau_q\left(\tau_q+\tau_\chi+\tau_\xi\right)\right)}\\
\geq\ & 0.
\end{aligned}$$

Second, using the expressions in (98) and (99), we can show straightforward that

$$\begin{aligned}
\Psi_2 &\equiv var(\widehat{v}_j)\,var(\widehat{q}_j)-cov(\widehat{v}_j,\widehat{q}_j)^2 && (114)\\
&= \left(\tau_\theta^{-1}+\varphi_1^2\tau_\xi^{-1}+\varphi_2^2\tau_u^{-1}\right)\left(\tau_\theta^{-1}+\varphi_3^2\tau_\xi^{-1}+\varphi_4^2\tau_\varepsilon^{-1}\right)-\left(\tau_\theta^{-1}+\varphi_1\varphi_3\tau_\xi^{-1}\right)^2\\
&= (\varphi_1-\varphi_3)^2\tau_\theta^{-1}\tau_\xi^{-1}+\left(\varphi_2^2\tau_u^{-1}+\varphi_4^2\tau_\varepsilon^{-1}\right)\tau_\theta^{-1}+\varphi_1^2\varphi_4^2\tau_\xi^{-1}\tau_\varepsilon^{-1}+\varphi_2^2\varphi_3^2\tau_u^{-1}\tau_\xi^{-1}+\varphi_2^2\varphi_4^2\tau_u^{-1}\tau_\varepsilon^{-1}\\
&\geq 0.
\end{aligned}$$

Third, using (98) and (99), we derive that

$$\begin{aligned}
\Psi_3 &\equiv var(\widehat{q}_j)\frac{\partial var(\widehat{v}_j)}{\partial a_1}-2cov(\widehat{v}_j,\widehat{q}_j)\frac{\partial cov(\widehat{v}_j,\widehat{q}_j)}{\partial a_1}\\
&= 2\left(\tau_\theta^{-1}+\varphi_1^2\tau_\xi^{-1}+\varphi_2^2\tau_u^{-1}\right)\left(\frac{\varphi_7}{\sigma-1}\right)^2a_1\tau_\xi^{-1}-2\left(\tau_\theta^{-1}+\varphi_1\varphi_3\tau_\xi^{-1}\right)\varphi_1\varphi_3\tau_\xi^{-1}a_1^{-1}\\
&= 2\left(\frac{\varphi_7}{\sigma-1}\right)^2\tau_\xi^{-1}\left[\tau_\theta^{-1}\left(1-\frac{\varphi_1}{\varphi_3}\right)+\varphi_2^2\tau_u^{-1}\right]a_1.
\end{aligned}$$

Using (112) and taking $a_1=\widehat{a}_1$ in the above equation, we obtain

$$\Psi_3|_{a_1=\widehat{a}_1}=2\left(\frac{\varphi_7}{\sigma-1}\right)^2\tau_\xi^{-1}\left[\tau_\theta^{-1}\left(1-\frac{\varphi_1}{\varphi_3}\right)|_{a_1=\widehat{a}_1}+\varphi_2^2\tau_u^{-1}\right]a_1\geq 0. \tag{115}$$

Thus, we complete the proof of Claim 1. ■

Thirdly and finally, we prove the existence of a time-consistent gradualistic policy satisfying

$a_1 = \widehat{a}_1 < \widehat{\theta}$. When $a_1 = \widehat{a}_1$, equation (102) turns out to

$$\left(\widehat{\theta} - \widehat{a}_1\right) = \widehat{a}_1 \left(\frac{\tau_\theta^{-2}}{var\left(\widehat{q}_j\right)} \frac{\Psi_1}{\Psi_2^2} \left[\frac{\varphi_3 \varphi_7}{\sigma - 1} \tau_\xi^{-1} \Psi_2 + \left(\frac{\varphi_7}{\sigma - 1} \right)^2 \Psi_1 \tau_\xi^{-1} \left(\left(1 - \frac{\varphi_1}{\varphi_3} \right) \tau_\theta^{-1} + \varphi_3^2 \tau_u^{-1} \right) \right] \right). \tag{116}$$

Using Claim 1 and equations (113), (114), and (115), we know that the righthand side of equation (116) is nonnegative. Define a continuous function $f\left(\widehat{a}_1\right)$ about $\widehat{a}_1$ on the interval $\left[0, \widehat{\theta}\right]$ as follows:

$$f\left(\widehat{a}_1\right) \equiv \left(\widehat{\theta} - \widehat{a}_1\right) - \widehat{a}_1 \left(\frac{\tau_\theta^{-2}}{var\left(\widehat{q}_j\right)} \frac{\Psi_1}{\Psi_2^2} \left[\frac{\varphi_3 \varphi_7}{\sigma - 1} \tau_\xi^{-1} \Psi_2 + \left(\frac{\varphi_7}{\sigma - 1} \right)^2 \Psi_1 \tau_\xi^{-1} \left(\left(1 - \frac{\varphi_1}{\varphi_3} \right) \tau_\theta^{-1} + \varphi_3^2 \tau_u^{-1} \right) \right] \right), \tag{117}$$

which satisfies

$$f(0) = \widehat{\theta} \equiv E\left(\theta | \mathcal{I}^{G_1}\right) = \frac{\overline{\theta} \tau_\theta + \widehat{q}_j \tau_{\widehat{q}_j}}{\tau_\theta + \tau_{\widehat{q}_j}} > 0,$$

and

$$f\left(\widehat{\theta}\right) = -\widehat{\theta} \left(\frac{\tau_\theta^{-2}}{var\left(\widehat{q}_j\right)} \frac{\Psi_1}{\Psi_2^2} \left[\frac{\varphi_3 \varphi_7}{\sigma - 1} \tau_\xi^{-1} \Psi_2 + \left(\frac{\varphi_7}{\sigma - 1} \right)^2 \Psi_1 \tau_\xi^{-1} \left(\left(1 - \frac{\varphi_1}{\varphi_3} \right) \tau_\theta^{-1} + \varphi_3^2 \tau_u^{-1} \right) \right] \right) < 0.$$

By the intermediate value theorem, we know that the continuous function $f\left(\widehat{a}_1\right)$ has a solution $\widehat{a}_1$ in the open interval, namely, i.e., $\widehat{a}_1 \in \left(0, \widehat{\theta}\right)$. Therefore, we complete the proof of Proposition 2 □

7.3 Appendix C

Proof of Proposition 3. We prove Proposition 3 through three steps. Step 1, we prove that $\partial \tau_q / \partial \widehat{a}_1 < 0$. Dividing both sides of equation (90) by the ones of equation (89), we have that

$$\beta_2 \lambda \frac{1}{\theta} \frac{\tau_\chi}{\tau_\xi + \tau_\chi + \tau_q} = \gamma \left[\left(\frac{1}{\sigma} \right)^2 \widehat{a}_1 \frac{1}{\tau_\xi + \tau_\chi + \tau_q} + \left(1 - \frac{1}{\sigma} \right)^2 \phi_2^2 \widehat{a}_1 \tau_\varepsilon^{-1} \right]. \tag{118}$$

Define a function $F\left(\beta_2, \widehat{a}_1\right)$ as follows:

$$F\left(\beta_2, \widehat{a}_1\right) \equiv \left[\beta_2 \lambda \frac{1}{\theta} \tau_\chi - \gamma \left(\frac{1}{\sigma} \right)^2 \widehat{a}_1 \right] \frac{1}{\tau_\xi + \tau_\chi + \tau_q} - \gamma \left(1 - \frac{1}{\sigma} \right)^2 \phi_2^2 \widehat{a}_1 \tau_\varepsilon^{-1} = 0. \tag{119}$$

Taking the partial derivatives with respect to β_2 and $\widehat{a}_1$ leads to

$$\begin{aligned}\frac{\partial F}{\partial \beta_2} &= \lambda\tau_\chi \frac{1}{\theta}\frac{1}{\tau_\xi+\tau_\chi+\tau_q} + \gamma\left(1-\frac{1}{\sigma}\right)^2 \phi_2^2 \widehat{a}_1 \tau_\varepsilon^{-1} \frac{2\beta_2^{-3}\tau_u}{\tau_\xi+\tau_\chi+\tau_q} > 0,\\ \frac{\partial F}{\partial \widehat{a}_1} &= -\beta_2\lambda\tau_\chi \widehat{a}_1^{-1}\frac{1}{\theta}\frac{1}{\tau_\xi+\tau_\chi+\tau_q} < 0,\end{aligned}$$

both of which establishes that

$$\frac{\beta_2}{\widehat{a}_1} > \frac{\partial \beta_2}{\partial \widehat{a}_1} = -\frac{\partial F/\partial \widehat{a}_1}{\partial F/\partial \beta_2} = \frac{\beta_2 \lambda \tau_\chi \widehat{a}_1^{-1}\frac{1}{\theta}}{\lambda\tau_\chi \frac{1}{\theta} + 2\gamma\left(1-\frac{1}{\sigma}\right)^2 \phi_2^2 \widehat{a}_1 \tau_\varepsilon^{-1}\beta_2^{-3}\tau_u} > 0. \tag{120}$$

Hence, we know that

$$\frac{\partial \tau_q}{\partial \widehat{a}_1} = -2\beta_2^{-3}\tau_u \frac{\partial \beta_2}{\partial \widehat{a}_1} < 0. \tag{121}$$

Step 2, we prove that $\frac{\partial \tau_{\widehat{q}}}{\partial \widehat{a}_1} < 0$. From the expression of (105), i.e, , we know that

$$\frac{1}{\beta_3} = \frac{\widehat{a}_1}{2\theta}\frac{\tau_q}{\tau_\xi+\tau_q} + \frac{\widehat{a}_1}{2(\theta-1)\Theta} A_3, \tag{122}$$

where

$$A_3 \equiv \frac{\left(\begin{array}{c}\left(\frac{1}{\theta}\right)^2(\lambda\tau_\chi\tau_\varepsilon + \tau_q\tau_\varepsilon) + \lambda\left(1-\frac{1}{\theta}\right)^2\phi_2^2(\tau_\chi+\tau_q)(\tau_\xi+\tau_q)\\ +(1-\lambda)\left(1-\frac{1}{\theta}\right)^2\phi_2^2\tau_q(\tau_\xi+\tau_\chi+\tau_q)\end{array}\right)}{\left(\begin{array}{c}\lambda\left(\frac{1}{\theta}\right)^2(\tau_\xi+\tau_\chi+\tau_q)\tau_\varepsilon + (1-\lambda)\left(\frac{1}{\theta}\right)^2\phi_2^2(\tau_\xi+\tau_q)\tau_\varepsilon\\ +\left(1-\frac{1}{\theta}\right)^2\phi_2^2(\tau_\xi+\tau_\chi+\tau_q)(\tau_\xi+\tau_q)\end{array}\right)}. \tag{123}$$

From (122), we derive that

$$\begin{aligned}\left(\frac{1}{\beta_3}\right)^2\frac{\tau_\xi+\tau_q}{\tau_\xi\tau_q} &= \frac{1}{4\theta^2\tau_\xi}\left(\widehat{a}_1^2\frac{\tau_q}{\tau_\xi+\tau_q}\right) + \frac{\tau_\xi}{4\Theta^2(\theta-1)^2}\left(\widehat{a}_1^2\frac{\tau_q}{\tau_\xi+\tau_q}\right)\left(\frac{\tau_\xi+\tau_q}{\tau_\xi\tau_q}A_3\right)^2 \\ &\quad + \frac{1}{2\theta(\theta-1)^2\Theta}\left(\widehat{a}_1^2\frac{\tau_q}{\tau_\xi+\tau_q}\right)\left(\frac{\tau_\xi+\tau_q}{\tau_\xi\tau_q}A_3\right)^2,\end{aligned} \tag{124}$$

which includes two terms associated with $\widehat{a}_1$: $\widehat{a}_1^2\frac{\tau_q}{\tau_\xi+\tau_q}$ and $\frac{\tau_\xi+\tau_q}{\tau_\xi\tau_q}A_3$.

It is easy to derive that

$$\begin{aligned}\frac{\partial\left(\widehat{a}_1^2\frac{\tau_q}{\tau_\xi+\tau_q}\right)}{\partial \widehat{a}_1} &= 2\widehat{a}_1\frac{\tau_q}{\tau_\xi+\tau_q}\left(1-\frac{\tau_\xi\tau_q}{\tau_\xi+\tau_q}\frac{\widehat{a}_1}{\beta_2}\frac{\partial\beta_2}{\partial\widehat{a}_1}\right) \\ &> 2\widehat{a}_1\frac{\tau_q}{\tau_\xi+\tau_q}\left(1-\frac{\tau_\xi\tau_q}{\tau_\xi+\tau_q}\right) \text{ (due to (120))} \\ &= 2\widehat{a}_1\left(\frac{\tau_q}{\tau_\xi+\tau_q}\right)^2 > 0.\end{aligned} \tag{125}$$

We rewrite the term $\frac{\tau_\xi+\tau_q}{\tau_\xi\tau_q}A_3$ as follows:

$$\frac{\tau_\xi+\tau_q}{\tau_\xi\tau_q}A_3=\frac{1}{\tau_\xi}+A_4, \tag{126}$$

where

$$A_4=\frac{\left(\frac{1}{\theta}\right)^2\lambda\tau_\chi\tau_\varepsilon+\lambda\left(1-\frac{1}{\theta}\right)^2\phi_2^2\left(\tau_\xi+\tau_q\right)\tau_\chi}{\left(\begin{array}{c}\lambda\left(\frac{1}{\theta}\right)^2\left(\tau_\xi+\tau_\chi+\tau_q\right)\tau_q\tau_\varepsilon+(1-\lambda)\left(\frac{1}{\theta}\right)^2\phi_2^2\left(\tau_\xi+\tau_q\right)\tau_q\tau_\varepsilon\\+\left(1-\frac{1}{\theta}\right)^2\phi_2^2\left(\tau_\xi+\tau_\chi+\tau_q\right)\left(\tau_\xi+\tau_q\right)\tau_q\end{array}\right)}.$$

It is easy to derive that

$$\begin{aligned}&\frac{\partial\left(\frac{\tau_\xi+\tau_q}{\tau_\xi\tau_q}A_3\right)}{\partial\widehat{a}_1} \qquad (127)\\
=&-\frac{\left(\begin{array}{c}\left(\frac{1}{\theta}\right)^2\left(\tau_\xi+\tau_q\right)^2\left(\tau_\xi+\tau_q\right)+\left(1-\frac{1}{\theta}\right)^2\phi_2^2\left(\tau_\xi+\tau_\chi+\tau_q\right)\left(\tau_\xi+\tau_q\right)\tau_\xi\\+\left(\frac{1}{\theta}\right)^2\tau_\varepsilon\lambda\tau_\xi\tau_\chi+\left(1-\frac{1}{\theta}\right)^2\phi_2^2\left(2\tau_\xi+\tau_\chi+2\tau_q\right)\left(\tau_\xi+\tau_q\right)\tau_q+\\\left(\frac{1}{\theta}\right)^2\lambda\tau_\xi\tau_\chi\left(\begin{array}{c}\left(\frac{1}{\theta}\right)^2\left(\tau_\xi+2\tau_q\right)\tau_\varepsilon+\left(1-\frac{1}{\theta}\right)^2\phi_2^2\left(\tau_\xi+\tau_\chi+\tau_q\right)\left(\tau_\xi+\tau_q\right)\\+\left(\frac{1}{\theta}\right)^2\lambda\tau_\varepsilon\tau_\chi+\left(1-\frac{1}{\theta}\right)^2\phi_2^2\tau_q\left(2\tau_\xi+\tau_\chi+2\tau_q\right)\end{array}\right)\end{array}\right)\frac{\partial\tau_q}{\partial\widehat{a}_1}}{\left(\begin{array}{c}\lambda\left(\frac{1}{\theta}\right)^2\left(\tau_\xi+\tau_\chi+\tau_q\right)\tau_q\tau_\varepsilon+(1-\lambda)\left(\frac{1}{\theta}\right)^2\phi_2^2\left(\tau_\xi+\tau_q\right)\tau_q\tau_\varepsilon\\+\left(1-\frac{1}{\theta}\right)^2\phi_2^2\left(\tau_\xi+\tau_\chi+\tau_q\right)\left(\tau_\xi+\tau_q\right)\tau_q\end{array}\right)^2}\\
>&\ 0.\end{aligned}$$

Combining (124), (125), and (127), we obtain that

$$\frac{\partial}{\partial\widehat{a}_1}\left(\left(\frac{1}{\beta_3}\right)^2\frac{\tau_\xi+\tau_q}{\tau_\xi\tau_q}\right)>0. \tag{128}$$

Combining (40) and (128) gives rise to

$$\frac{\partial\tau_{\widehat{q}}}{\partial\widehat{a}_1}=-\left(\left(\frac{1}{\beta_3}\right)^2\frac{\tau_\xi+\tau_q}{\tau_\xi\tau_q}\right)^{-2}\frac{\partial}{\partial\widehat{a}_1}\left(\left(\frac{1}{\beta_3}\right)^2\frac{\tau_\xi+\tau_q}{\tau_\xi\tau_q}\right)<0. \tag{129}$$

Step 3, we prove that $\frac{\partial\tau_{\widehat{v}}}{\partial\widehat{a}_1}<0$. Using the definition of $\varphi_7\equiv\frac{1}{1+\phi_3-\phi_1\widehat{a}_1\beta_3}$ and (109), we have that

$$\frac{\widehat{a}_1}{1+\phi_3-\phi_1\widehat{a}_1\beta_3}=\frac{\widehat{a}_1}{1+\phi_3}\left[1+\left(1-\frac{1}{\sigma}\right)\Theta A_5\right], \tag{130}$$

where

$$
\begin{aligned}
A_5 &\equiv \frac{\begin{pmatrix} \frac{1}{\sigma}\left(1-\frac{1}{\sigma}\right)^2 \phi_2^2 \left(\tau_\xi+\tau_\chi+\tau_q\right)\left(\tau_\xi+\tau_q\right)\tau_q \\ +\lambda\left(\frac{1}{\sigma}\right)^3 \tau_\chi\tau_\varepsilon\tau_q+\left(\frac{1}{\sigma}\right)^3\left(\tau_\xi+\tau_q\right)\tau_q\tau_\varepsilon \end{pmatrix}}{\begin{pmatrix} \left(\frac{1}{\sigma}\right)^3\left(\tau_\xi+\tau_q\right)\left(\lambda\tau_\chi\tau_\varepsilon+\tau_q\tau_\varepsilon\right)+ \\ \frac{1}{\sigma}\left(1-\frac{1}{\sigma}\right)^2\phi_2^2\left[\lambda\tau_\chi\tau_\xi\left(\tau_\xi+\tau_q\right)+\tau_q\left(\tau_\xi+\tau_q\right)\left(\tau_\xi+\tau_\chi+\tau_q\right)\right] \end{pmatrix}} \qquad (131) \\
&= 1-A_6>0,
\end{aligned}
$$

with

$$
A_6 \equiv \frac{\frac{1}{\sigma}\left(1-\frac{1}{\sigma}\right)^2\phi_2^2\lambda\tau_\xi\tau_\chi\left(\tau_\xi+\tau_q\right)\tau_q+\left(\frac{1}{\sigma}\right)^3\lambda\tau_\chi\tau_\varepsilon\tau_\xi}{\begin{pmatrix} \left(\frac{1}{\sigma}\right)^3\left(\tau_\xi+\tau_q\right)\left(\lambda\tau_\chi\tau_\varepsilon+\tau_q\tau_\varepsilon\right)+ \\ \frac{1}{\sigma}\left(1-\frac{1}{\sigma}\right)^2\phi_2^2\left[\lambda\tau_\chi\tau_\xi\left(\tau_\xi+\tau_q\right)+\tau_q\left(\tau_\xi+\tau_q\right)\left(\tau_\xi+\tau_\chi+\tau_q\right)\right] \end{pmatrix}}. \qquad (132)
$$

Taking the derivative with respect to $\widehat{a}_1$ on both sides of (131) leads to

$$
\begin{aligned}
&\frac{\partial A_5}{\partial \widehat{a}_1} \qquad (133) \\
=\ & -\frac{\partial A_6}{\partial \widehat{a}_1} \\
=\ & -\frac{\begin{pmatrix} \left(\frac{1}{\sigma}\right)^4\lambda\tau_\xi\tau_\chi\tau_\varepsilon\left(1-\frac{1}{\sigma}\right)^2\phi_2^2\left[\lambda\tau_q\tau_\chi+\left(\tau_\xi+\tau_q\right)\left(\tau_\xi+\tau_\chi+\tau_q\right)+\tau_q\left(2\tau_\xi+\tau_\chi+2\tau_q\right)\right] \\ +\left(\frac{1}{\sigma}\right)^6\lambda\tau_\xi\tau_\chi\tau_\varepsilon\left(\lambda\tau_\chi\tau_\varepsilon+2\tau_q\tau_\varepsilon+\tau_\xi\tau_\varepsilon\right)+\left(\frac{1}{\sigma}\right)^4\left(1-\frac{1}{\sigma}\right)^2\phi_2^2\lambda\tau_\xi\tau_\chi\tau_\varepsilon\left(\tau_\xi+\tau_q\right)^2+ \\ \left(\frac{1}{\sigma}\right)^2\left(1-\frac{1}{\sigma}\right)^4\phi_2^2\lambda\tau_\xi^2\tau_\chi\left[\left(\tau_\xi+\tau_q\right)\left(\tau_\xi+\tau_\chi+\tau_q\right)+\tau_q\left(2\tau_\xi+\tau_\chi+2\tau_q\right)\right] \end{pmatrix}\beta_2^3\tau_u\frac{\partial\beta_2}{\partial\widehat{a}_1}}{\left[\left(\frac{1}{\sigma}\right)^3\left(\tau_\xi+\tau_q\right)\left(\lambda\tau_\chi\tau_\varepsilon+\tau_q\tau_\varepsilon\right)+\frac{1}{\sigma}\left(1-\frac{1}{\sigma}\right)^2\phi_2^2\left[\lambda\tau_\chi\tau_\xi\left(\tau_\xi+\tau_q\right)+\tau_q\left(\tau_\xi+\tau_q\right)\left(\tau_\xi+\tau_\chi+\tau_q\right)\right]\right]^2} \\
<\ & 0.
\end{aligned}
$$

Taking the derivative with respect to $\widehat{a}_1$ on both sides of (130) gives us

$$
\begin{aligned}
&\frac{\partial}{\partial\widehat{a}_1}\left(\frac{\widehat{a}_1}{1+\phi_3-\phi_1\widehat{a}_1\beta_3}\right) \qquad (134) \\
=\ & \frac{1}{1+\phi_3}\left[1+\left(1-\frac{1}{\sigma}\right)\Theta\left(1-A_6\right)\widehat{a}_1+2\frac{\partial A_6}{\partial\widehat{a}_1}\tau_q\left(1-\frac{1}{\sigma}\right)\Theta\frac{\widehat{a}_1}{\beta_2}\frac{\partial\beta_2}{\partial\widehat{a}_1}\right] \\
>\ & \frac{\left(1-\frac{1}{\sigma}\right)\Theta}{1+\phi_3}\left(2-A_6+2\frac{\partial A_6}{\partial\tau_q}\tau_q\frac{\widehat{a}_1}{\beta_2}\frac{\partial\beta_2}{\partial\widehat{a}_1}\right) \\
>\ & \frac{\left(1-\frac{1}{\sigma}\right)\Theta}{1+\phi_3}\left(2-A_6+2\frac{\partial A_6}{\partial\tau_q}\tau_q\right),\text{ due to }\frac{\partial A_6}{\partial\tau_q}>0\text{ and (120).}
\end{aligned}
$$

We rewrite (132) as follows:

$$A_6 = \frac{B_1 + B_2}{B_1 + B_2 + B_3 + B_4 + B_5}, \tag{135}$$

where

$$\begin{aligned}
B_1 &\equiv \left(\frac{1}{\sigma}\right)^3 \lambda\tau_\chi\tau_\varepsilon\tau_\xi + \frac{1}{\sigma}\left(1-\frac{1}{\sigma}\right)^2 \phi_2^2\lambda\tau_\chi\tau_\xi^2, \\
B_2 &\equiv \frac{1}{\sigma}\left(1-\frac{1}{\sigma}\right)^2 \phi_2^2\lambda\tau_\chi\tau_\xi\tau_q, \\
B_3 &\equiv \left(\frac{1}{\sigma}\right)^3 \left(\lambda\tau_\chi\tau_\varepsilon + \tau_\xi\tau_\varepsilon\right)\tau_q, \\
B_4 &\equiv \left(\frac{1}{\sigma}\right)^3 \tau_\varepsilon\tau_q^2, \\
B_5 &\equiv \frac{1}{\sigma}\left(1-\frac{1}{\sigma}\right)^2 \phi_2^2\tau_q\left(\tau_\xi+\tau_q\right)\left(\tau_\xi+\tau_\chi+\tau_q\right).
\end{aligned}$$

Thus, we know that

$$2 - A_6 + 2\frac{\partial A_6}{\partial \tau_q}\tau_q > 0 \Longleftrightarrow \tag{136}$$

$$\begin{pmatrix} 2\left(B_1+B_2+B_3+B_4+B_5\right)^2 - \left(B_1+B_2\right)\left(B_1+B_2+B_3+B_4+B_5\right) + \\ 2\left[B_2\left(B_1+B_2+B_3+B_4+B_5\right) - \left(B_1+B_2\right)\left(B_2+B_3+2B_4+\frac{\partial B_5}{\partial \tau_q}\tau_q\right)\right] \end{pmatrix} > 0 \Longleftrightarrow$$

$$\begin{pmatrix} \left(B_1+B_2\right)\left(B_1+B_2+B_3+B_4+B_5\right) + \\ \left(B_3+B_4+B_5\right)\left(B_1+B_2+B_3+B_4+B_5\right) + \\ 2\left[B_2\left(B_1+B_2+B_3+B_4+B_5\right) - \left(B_1+B_2\right)\left(B_2+B_3+2B_4+\frac{\partial B_5}{\partial \tau_q}\tau_q\right)\right] \end{pmatrix} > 0 \Longleftrightarrow$$

$$\begin{pmatrix} \left(B_1+B_2\right)\left(B_1+B_2+B_3+B_4+B_5\right) + \\ 2\left(B_3B_4 + B_4^2 + 2B_4B_5 + 2B_2B_5\right) + \\ 2\left[\left(B_2B_3 + B_3^2 - B_1B_4\right) + \left(2B_3B_5 + B_5^2 + B_1B_5 - B_1\frac{\partial B_5}{\partial \tau_q}\tau_q - B_2\frac{\partial B_5}{\partial \tau_q}\tau_q\right)\right] \end{pmatrix} > 0.$$

It is easy to show that

$$B_2B_3 + B_3^2 - B_1B_4 > 0, 2B_3B_5 + B_5^2 + B_1B_5 - B_1\frac{\partial B_5}{\partial \tau_q}\tau_q - B_2\frac{\partial B_5}{\partial \tau_q}\tau_q > 0. \tag{137}$$

Combining (134), (136), and (137), we obtain that

$$\frac{\partial}{\partial \widehat{a}_1}\left(\frac{\widehat{a}_1}{1+\phi_3-\phi_1\widehat{a}_1\beta_3}\right) > 0. \tag{138}$$

Using (43), (138) and $\widehat{a}_1 = a_1$, we know that

$$\frac{\partial \tau_{\widehat{v}}}{\partial \widehat{a}_1} \equiv -2\left[(\sigma-1)^{-2}\tau_\xi^{-1} + \phi_2^2\tau_\varepsilon^{-1}\right]^{-1}\left(\frac{\widehat{a}_1}{1+\phi_3-\phi_1\widehat{a}_1\beta_3}\right)^{-3}\frac{\partial}{\partial \widehat{a}_1}\left(\frac{\widehat{a}_1}{1+\phi_3-\phi_1\widehat{a}_1\beta_3}\right) < 0. \tag{139}$$

Thus, we complete the proof of Proposition 3. □

7.4 Appendix D

Proof of Proposition 4. Following Grossman and Stiglitz (1980), we change the equilibrium condition (50) into

$$e^{\gamma c} = \sqrt{\frac{var\left(v_j|\mathcal{I}^{U_i}\right)}{var\left(v_j|\mathcal{I}^{I_i}\right)}} = \sqrt{\frac{\left(\frac{1}{\sigma}\right)^2(\tau_\xi+\tau_q)^{-1} + \left(1-\frac{1}{\sigma}\right)^2\phi_2^2\tau_\varepsilon^{-1}}{\left(\frac{1}{\sigma}\right)^2(\tau_\xi+\tau_\chi+\tau_q)^{-1} + \left(1-\frac{1}{\sigma}\right)^2\phi_2^2\tau_\varepsilon^{-1}}}, \tag{140}$$

which includes the unique endogenous variable, β_2. As shown in the proof of Proposition 2, equation (92) determines the unique real root of β_2 for any given λ. In the case of an endogenous λ, we conclude that both equations (92) and (140) simultaneously determine the unique real root of (β_2, λ). Aside from the endogenous λ, the rest of the proof follows the same reasoning as that of Proposition 2. □

7.5 Appendix E

Proof of Proposition 5. First, we prove that $\partial\tau_q/\partial\widehat{a}_1 = 0$ and $\partial\lambda/\partial\widehat{a}_1 > 0$ hold. In the full equilibrium, the endogenous value of β_2 is determined by equaiton (140), in which $\widehat{a}_1$ does not appear. This establishes that β_2 does not depend on $\widehat{a}_1$, i.e., $\partial\beta_2/\partial\widehat{a}_1 = 0$. Since $\tau_q = \beta_2^{-2}\tau_u$, we know that

$$\frac{\partial\tau_q}{\partial\widehat{a}_1} = \frac{\partial\beta_2}{\partial\widehat{a}_1} = 0. \tag{141}$$

Equation (103) tells that $\beta_2 > 0$, and

$$\lambda = \Psi\widehat{a}_1, \tag{142}$$

where

$$\Psi \equiv \frac{\gamma\left[\left(\frac{1}{\sigma}\right)^2\left(\tau_\xi+\tau_\chi+\beta_2^{-2}\tau_u\right)^{-1} + \left(1-\frac{1}{\sigma}\right)^2\phi_2^2\tau_\varepsilon^{-1}\right]}{\frac{\beta_2}{\sigma}\left(\tau_\xi+\tau_\chi+\beta_2^{-2}\tau_u\right)^{-1}\tau_\chi} > 0. \tag{143}$$

Taking the derivatives with respect to $\widehat{a}_1$ on both sides of (142) and using equations (141) and (143), we obtain:

$$\frac{\partial \lambda}{\partial \widehat{a}_1} = \Psi > 0. \tag{144}$$

Second, we prove that $\partial \tau_{\widehat{q}} / \partial \widehat{a}_1 > 0$. Equation (105) tells that $\beta_3 > 0$ and

$$\frac{1}{\beta_3} = \frac{\widehat{a}_1}{2(\sigma-1)\Theta}\varphi_8 + \frac{\widehat{a}_1}{2\sigma}\frac{\tau_q}{\tau_\xi + \tau_q}, \tag{145}$$

where

$$\begin{aligned}
\varphi_8 &\equiv 1 - \frac{\left(\frac{1}{\sigma}\right)^2 \tau_\xi \tau_q + \tau_\chi \tau_\varepsilon + \left(1-\frac{1}{\sigma}\right)^2 \phi_2^2 \left[\tau_\xi\left(\tau_\xi+\tau_q\right) + (1-\lambda)\tau_\xi\tau_\chi\right]}{\left[\left(\frac{1}{\sigma}\right)^2\left(\tau_\xi+\tau_q\right)\tau_\varepsilon + \lambda\left(\frac{1}{\sigma}\right)^2 \tau_\chi\tau_\varepsilon + \left(1-\frac{1}{\sigma}\right)^2\phi_2^2\left(\tau_\xi+\tau_\chi+\tau_q\right)\left(\tau_\xi+\tau_q\right)\right]} \\
&= \frac{\left(\begin{array}{c} \lambda\left(\frac{1}{\sigma}\right)^2\tau_\chi\tau_\varepsilon + \lambda\left(1-\frac{1}{\sigma}\right)^2\phi_2^2\tau_\chi\left(\tau_\xi+\tau_q\right) + \left(\frac{1}{\sigma}\right)^2\tau_q\tau_\varepsilon + \\ (1-\lambda)\left(1-\frac{1}{\sigma}\right)^2\phi_2^2\tau_q\left(\tau_\xi+\tau_\chi+\tau_q\right) + \lambda\left(1-\frac{1}{\sigma}\right)^2\phi_2^2\tau_q\left(\tau_\xi+\tau_q\right) \end{array}\right)}{\left[\begin{array}{c} \lambda\left(\frac{1}{\sigma}\right)^2\left(\tau_\xi+\tau_\chi+\tau_q\right)\tau_\varepsilon + (1-\lambda)\left(\frac{1}{\sigma}\right)^2\left(\tau_\xi+\tau_q\right)\tau_\varepsilon \\ +\left(1-\frac{1}{\sigma}\right)^2\phi_2^2\left(\tau_\xi+\tau_\chi+\tau_q\right)\left(\tau_\xi+\tau_q\right) \end{array}\right]} > 0.
\end{aligned} \tag{146}$$

Equations (144) and (146) tell that

$$\frac{\partial \varphi_8}{\partial \widehat{a}_1} > 0. \tag{147}$$

Taking the derivatives with respect to $\widehat{a}_1$ on both sides of (145) and using (146) and (147) give us

$$\frac{\partial}{\partial \widehat{a}_1}\left(\frac{1}{\beta_3}\right) = \frac{1}{2(\sigma-1)\Theta}\varphi_8 + \frac{\widehat{a}_1}{2(\sigma-1)\Theta}\frac{\partial\varphi_8}{\partial\widehat{a}_1} + \frac{1}{2\sigma}\frac{\tau_q}{\tau_\xi+\tau_q} > 0. \tag{148}$$

Taking the derivatives with respect to $\widehat{a}_1$ on both sides of (40) and using (148) give us

$$\frac{\partial \tau_{\widehat{q}}}{\partial \widehat{a}_1} = -2\beta_3^3\left(\tau_\xi^{-1}+\tau_u^{-1}\right)^{-1}\frac{\partial}{\partial\widehat{a}_1}\left(\frac{1}{\beta_3}\right) < 0. \tag{149}$$

Third, we prove that $\partial \tau_{\widehat{v}} / \partial \widehat{a}_1 > 0$. We rewrite (132) as follows:

$$A_6 = \frac{C_1}{C_1 + C_2 + C_3}, \tag{150}$$

where

$$
\begin{aligned}
C_1 &\equiv \left(\frac{1}{\sigma}\right)^3 \lambda\tau_\chi\tau_\varepsilon\tau_\xi\frac{1}{\sigma}\left(1-\frac{1}{\sigma}\right)^2\phi_2^2\lambda\tau_\xi\tau_\chi\left(\tau_\xi+\tau_q\right)\tau_q, \\
C_2 &\equiv \left(\frac{1}{\sigma}\right)^3 \tau_q\lambda\tau_\chi\tau_\varepsilon, \\
C_3 &\equiv \left(\frac{1}{\sigma}\right)^3 \left(\tau_\xi+\tau_q\right)\tau_q\tau_\varepsilon+\frac{1}{\sigma}\left(1-\frac{1}{\sigma}\right)^2\phi_2^2\tau_q\left(\tau_\xi+\tau_q\right)\left(\tau_\xi+\tau_\chi+\tau_q\right).
\end{aligned}
$$

Taking the derivatives with respect to $\widehat{a}_1$ on both sides of (130), we have that

$$
\begin{aligned}
& \frac{\partial}{\partial\widehat{a}_1}\left(\frac{\widehat{a}_1}{1+\phi_3-\phi_1\widehat{a}_1\beta_3}\right) \\
= & \frac{1}{1+\phi_3}\left[1+\left(1-\frac{1}{\sigma}\right)\Theta\left(1-A_6\right)\right]-\frac{\widehat{a}_1}{1+\phi_3}\left(1-\frac{1}{\sigma}\right)\Theta\frac{\partial A_6}{\partial\widehat{a}_1} \\
> & \frac{\left(1-\frac{1}{\sigma}\right)\Theta}{1+\phi_3}\left[\left(1-A_6\right)+\left(1-\frac{\partial A_6}{\partial\lambda}\frac{\partial\lambda}{\partial\widehat{a}_1}\widehat{a}_1\right)\right] \\
> & 0,
\end{aligned}
\tag{151}
$$

since

$$
\begin{aligned}
1-A_6 &= A_5>0, \\
1-\frac{\partial A_6}{\partial\lambda}\frac{\partial\lambda}{\partial\widehat{a}_1}\widehat{a}_1 &= \frac{C_1^2+C_2^2+C_3^2+2C_1C_2+2C_2C_3+C_1C_3}{\left(C_1+C_2+C_3\right)^2}>0.
\end{aligned}
$$

When $\widehat{a}_1=a_1$, we take the derivatives on both sides of (43), use (151), and derive the result:

$$
\begin{aligned}
\frac{\partial\tau_{\widehat{v}}}{\partial\widehat{a}_1} &\equiv -2\left[(\sigma-1)^{-2}\tau_\xi^{-1}+\phi_2^2\tau_\varepsilon^{-1}\right]^{-1}\left(\frac{\widehat{a}_1}{1+\phi_3-\phi_1\widehat{a}_1\beta_3}\right)^{-3}\frac{\partial}{\partial\widehat{a}_1}\left(\frac{\widehat{a}_1}{1+\phi_3-\phi_1\widehat{a}_1\beta_3}\right) \\
&< 0.
\end{aligned}
\tag{152}
$$

Thus, we complete the proof of Proposition 5. □

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