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Abstract

The paper aims to model the underground economy using two different models: one based on the labor supply method and a generalized model for the allocation of time. The model based on the labor supply method is conceived as a simulating one in order to determine some reasonable thresholds of the underground sector extension based only on the available macroeconomic statistical data. The generalized model for the allocation of time is a model based on direct approach which estimates the underground economy through extrapolation of the data collected from a limited number of households. Developing the Lemieux model, the map of the entire process of allocation of time was obtained.

Keywords: underground economy, labor supply method, Laffer curve, allocation of time, Lemieux model
JEL Classification: C61, D10, E62, H31, J22, O17, P36

1. A Model based on the Labor Supply Method

Firstly, we consider a national economy having only two sectors: visible (or official) sector and invisible (or underground) sector. In the case of visible sector the registered GDP is supposed as having the following components:

$$Yv = A + Sv + Bv \quad (1)$$

where Yv is the GDP produced in the visible sector; A - consumption of fixed capital (only in the visible sector); Sv - wages of employees in the visible sector; Bv - profit of entrepreneurs (capitalists) in the visible sector. In the invisible sector the produced GDP will be:

$$Ya = Sa + Ba \quad (2)$$

where Ya is the GDP produced in the underground sector; Sa - wages of employees in the invisible sector; Ba - profit of entrepreneurs (capitalists) in the invisible sector. In the case of invisible sector it is supposed that there is not fixed capital.

Also, we consider that available time fund and number of labour force are distributed between the two sectors as follows:

$$F = Fv + Fa \quad (3)$$

$$L = Lv + La \quad (4)$$

where F is the total available time for work fund within a calendar year; L - number of total potential working persons; Fv - time used for work in visible sector, by year; Fa - potential available time used by the persons having status of employees in the visible sector to also work in a second job in the underground sector; Lv - number of employees working in the visible sector; La - potential number of employees working in the underground sector.

We mention that the available working time is estimated as average number of hours during a calendar year. For instance, in the case of a person, it was considered the average number of hours worked by him/her in a year.

Now, we express the GDP created by every sector as functions of productivity, which here they are considered as linear ones:

$$Yv = Lv \cdot Fv \cdot wv \quad (5)$$

$$Ya = (La \cdot F + Lv \cdot Fa) \cdot wa \quad (6)$$

where wv and wa are the average level of hourly productivity (calculated per person) in the visible and the invisible sector, respectively .

In order to obtain the total number of hours yearly worked in the underground sector, we considered two categories: persons that are working full-time in the underground sector (La · F) and persons employed in the visible sector, but also simultaneously working in a second job in the underground sector (Lv · Fa), respectively.

What is interesting for the agents or people is the level of the disposable income or available GDP obtained as in follows:

$$Ydv = Yv - T = Yv \cdot (1 - t) \quad (7)$$
$$Yda = Ya \quad (8)$$

The relations can be also written:

$$Ydv = Lv \cdot Fv \cdot wv \cdot (1 - t) \quad (9)$$
$$Yda = (Lv \cdot Fa + La \cdot F) \cdot wa \quad (10)$$

where Ydv, Yda are the available income in the visible sector and the invisible sector, respectively ; T is the total amount of taxes (or obligatory levies) paid to authorities; t - average tax rate relatively to Yv.

Taking into account the structural relations (3) and (4), now we can write the expression of total available income as follows:

$$Yd = Lv \cdot Fv \cdot wv \cdot (1 - t) + [Lv \cdot (F - Fv) + (L - Lv) \cdot F] \cdot wa \quad (11)$$
$$Yd = (L - La) \cdot (F - Fa) \cdot wv \cdot (1 - t) + [(L - La) \cdot Fa + La \cdot F] \cdot wa \quad (12)$$

The first relation facilitates to analyse the impact of the number of persons working in the visible sector (Lv), and of their corresponding number of hours worked in this sector (Fv) on the total available income at national level. The second one makes the same, but concerning the number of persons working in the invisible sector

(La), and the number of hours worked in the invisible sector by persons having the status of employees in the visible sector (Fa), respectively. We remember that in the case of the persons actually having a status of non-employees in the visible sector (but having a potential to work, taking into account their age and disposable free time criteria), it is supposed that they allocated their entire available working time to work in the invisible sector (F). At the same time, the persons actually having the status of employees in the visible sector are forced to divide the same entire disposable working time (F) between work in the visible sector (Fv) and work in the invisible sector (Fa), respectively.

Maybe this total available income, higher than the available income in the visible sector, is responsible for some unexplained macroeconomic non-correlation registered between the "official" indicators.

Now, we define the yearly national potential by the following formula:

$$P = F \cdot L \cdot wv \tag{13}$$

where P is the maximum level (or potential) of yearly GDP.

Because on rule the productivity level in the invisible sector is supposed to be lower than in the visible sector, the following non-equality resulted:

$$Yv + Ya < P \tag{14}$$

Now, we can express the actual total available income obtained in a year as:

$$Yd = P \cdot [\, m + lv \cdot fv \cdot (1 - t - m) \,] \tag{15}$$

where m is the ratio between productivity in invisible sector and productivity in visible sector (wa/wv); lv - share of employees in visible sector in the total number of potential working persons (Lv/L); fv - share of time to work in the visible sector in the yearly total available working time (Fv/F). On the other hand, if the whole activity would be allocated in the visible sector, the maximum level of available GDP would be:

$$Yd^* = P \cdot (1 - t) \tag{16}$$

At this stage of analysis, we suppose that people chose actual situation or equivalent of the actual distribution of total capacity to work between sectors. This hypothesis produces an available income greater than or at least equal to that that would be produced by the above hypothetical case. Therefore, there will be the following restriction:

$$Yd > Yd^* \tag{17}$$

After some algebraic operations the following equivalent restrictions can be obtained:

$$m > 1 - t \tag{18}$$
$$wa > wv \cdot (1 - t) \tag{19}$$

We have to already mention that the model was conceived as a simulating one, in order to determine some reasonable thresholds of the underground sector extension, only based on the available macroeconomic statistical data. In this case, we considered the absolute values both for the total potential number of workers (L) and for the total potential number of working hours during a year (F). This may produce some larger estimates than in the case of considering only the visible sector (Lv and Fv).

In the case of invisible sector, we consider that the resulting levels of some indicators - productivity, profit rate, etc. – continue to be smaller than the real situation. So, this is because beside the comprehension to obtain available income, in our model there is in an implicit manner included the comprehension to leisure of people already having a non-active available labour force. For instance, the actual available income computed by our simulation model is greater than the level that would be obtained in the case of a full-time work in the visible sector ($Yd > Yd^*$). The difference must be considered as implicitly including the satisfaction problem of leisure comprehension.

Now, we focus on estimation of a bounded channel for the variance of underground economy dimension. In this way, we write the share of the invisible sector in the national economy as:

$$ya\,(wa) = Ya\,(wa) / Y\,(wa) \tag{20}$$

where Ya (wa) is given by relation (6) and Y is the total yearly GDP:

$$Y\,(wa) = Yv + Ya\,(wa) \tag{21}$$

For the productivity in invisible sector we chose the following two extreme values:

$$wa_{min} = (\,1 - t\,) \cdot wv \tag{22}$$
$$wa_{max} = wv \tag{23}$$

and their corresponding extreme shares of invisible sector in national economy:

$$ya_{min} = 1 - \{\,(\,lv \cdot fv\,) / [\,1 - t \cdot (\,1 - lv \cdot fv\,)\,]\,\} \tag{24}$$
$$ya_{max} = 1 - lv \cdot fv \tag{25}$$

Within this interval some alternative scenarios could be analysed, based on various ratios between the two sectors as regards some main indicators, such as average salary and average profit, respectively.

In the case of the visible sector, in order to evaluate average wage and average rate of profit we considered the following relations:

$$sv = [\,Sv / (\,Lv \cdot Fv\,)\,] = [\,Yv - (\,T + Bv + A\,)\,] / (\,Lv \cdot Fv\,) \tag{26}$$
$$bv = [\,Bv / (\,T + A + Sv\,)\,] = [\,Yv - (\,T + Sv + A\,)\,] / (\,T + A + Sv\,) \tag{27}$$

where sv is the average wage per person and per hour of work; bv - the average profit rate in the visible sector; Sv - total amount of yearly salaries paid in the visible sector; Bv - total amount of yearly profit obtained in the visible sector.

On the other hand, in the invisible sector the corresponding relations are:

$$sa = [\ Sa / (\ La \cdot F + Lv \cdot Fa\ )\ ] = (\ Ya - Ba\ ) / (\ La \cdot F + Lv \cdot Fa\ ) \quad (28)$$

$$ba = [\ Ba / Sa\ ] = (\ Ya - Sa\ ) / Sa = (\ wa - sa\ ) / sa \quad (29)$$

where sa is the average wage per person and per hour of work; ba - the average profit rate in the invisible sector; Sa - total amount of yearly salaries paid in the invisible sector; Ba - total amount of yearly profit in the invisible sector.

2. A Generalized Model for the Allocation of Time

Models Based on Direct Approaches

Many times, the estimations of the underground economy are directly obtained through the extrapolation of data collected from a limited number of households by surveys and samples. But, in the recent years, the modern theoretical models on tax evasion, which started with the work of Allingham and Sandmo (1972) and were continued by Cowell (1990) and other studies already mentioned, were developed in a quantifying way. The support is provided by some rigorously organized empirical studies.

The main impediment to the latter is the difficulty to collect information on the number of hours worked by persons who illegally evade taxes, which makes impossible to measure the effect of taxes on the allocation of time. To remedy these impediments, some studies empirically analysed the labor-supply decisions in the underground economy using micro data from rigorously organized surveys, such as that conducted in Québec by Fortin and Fréchette.

Coming from surveys they identified some key empirical regularities about the work in the untaxed sector, and then they built adequate quantifying models. However, their survey seems to be less accurate for tax evaders operating on the margins of being detected. In this case, the data based on extensive audits is more revealing for this tax evades (Lemieux et al., 1994). The main conclusions deduced by Lemieux, Fortin and Fréchette from these empirical findings were the following three: 1 - labour earnings in the underground sector are concentrated among workers with low earnings in the regular sector, while expenditures on goods and services produced in the underground sector are typically undertaken by people with high earnings in the regular sector; 2 - the wage rate in the regular sector and the wage rate in the underground sector are positively correlated with hours worked in the regular sector but negatively correlated with hours worked in the underground sector; 3 - earnings in the regular sector are a linear or slightly convex function of regular-sector hours, while earnings in the underground sector are a concave function of underground-sector hours.

Then, they developed a model based on the idea that labour earnings in the underground sector are a convex function of hours of work, while in the regular sector labour earnings are a linear function of hours of work. The convexity of earning function in the underground sector implies that the marginal revenues of underground producers decrease as producers reach the limits of the informal markets in which they operate. By contrast, the wage rate of a worker in the regular sector does not vary with the number of hours worked.

The results of Lemieux, Fortin and Fréchette's study suggest that the hours worked in the underground sector are quite responsive to the changes in the net wage in the regular sector. Most important, relating to our interest in this paper, their model

also provides a natural link between the slope of the relationship between tax revenues and tax rates (the "Laffer curve"), and a more conventional measure of the marginal excess burden of taxes due to the misallocation of productive resources from the regular to the underground sector.

The Lemieux's model is based on a concave Cobb-Douglas earnings function in the underground sector

$$Y1 = A1\ h1^{\alpha} \tag{30}$$

where $\alpha < 1$,

and on a linear earnings function in the regular sector

$$Y0 = W0\ h0 \tag{31}$$

The following Figure presents a graphical representation of the model. The main implication of this model, also largely used in literature, is that in point M on the graphic (which means the intersection of the two curves, Y1 and Y0) the average salary in the underground sector equals to that in the visible sector. The following relation can express this, translated in the terms of the first exposed model:

$$SaM = A1\ hv^{\alpha} = SvM = sv\ hv \tag{32}$$

On the graphic, also one may observe that the difference between Sa and Sv is maximal only in a single point of abscise noted ha*, which represents the optimal number of hours worked in the underground sector.

To evaluate this optimal level of underground work, we write the function that must be maximized as:

$$G(ha) = A1\ ha^{\alpha} - sv\ ha \tag{33}$$

and the first-order derivative as:

$$G'(ha) = A1\ ha^{\alpha-1} - sv \tag{34}$$

However, at the national level, the function that concerns is:

$$H(ha) = S - S^* = (A1\ ha^{\alpha} + sv\ ha) - sv\ (hv + ha) \tag{35}$$

Here, we can observe the equality between H and G. So, the two functions have the same solution for maximum.

After some algebraic operations, we obtained the following remarkable value for ha:

$$h^* = hv\ \alpha^{1/(1-\alpha)} \tag{36}$$

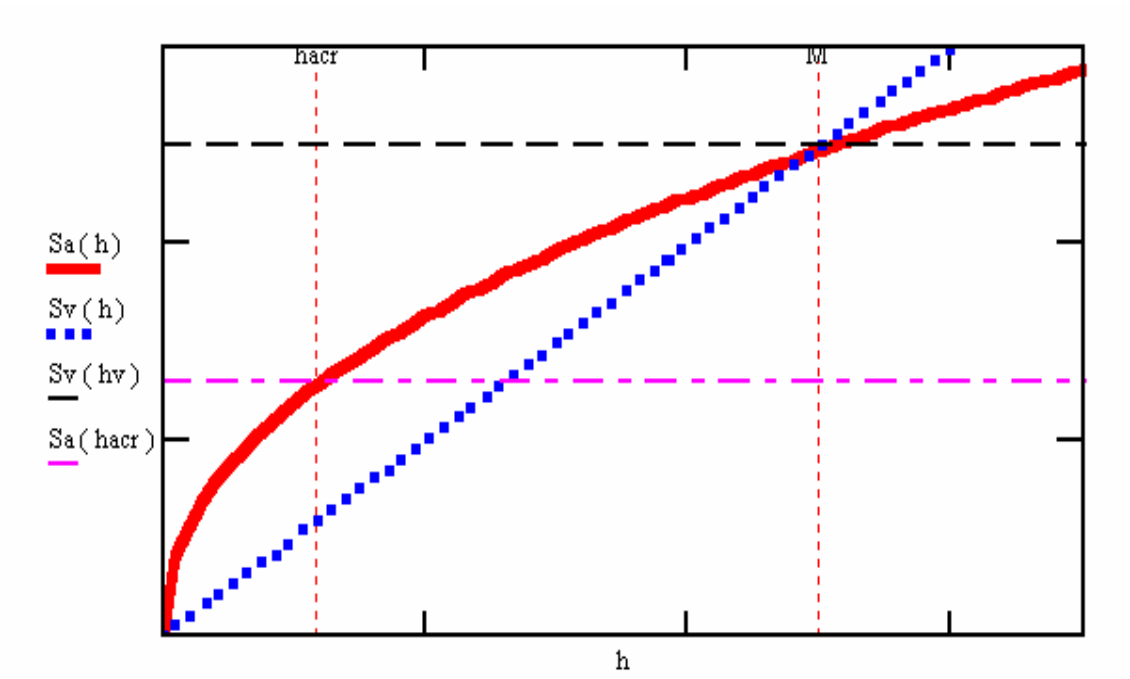
that then allowed us to express the optimal proportion of the underground economy within the national economy by the following relation:

$$h^* / h = \alpha^{1/(1-\alpha)} / [\, 1 + \alpha^{1/(1-\alpha)}] \qquad (37)$$

where, h is total number of hours worked in a country in a year.

On the basis of this relation we calculated the share of the underground economy for various values of parameter α. An interesting result is that, at limit (when $\alpha = 1$), the maximum share of the underground economy (in the considered optimal conditions) is only about 26.9% (the precise value of the limit is 1/(1+e), where e is the base of natural logarithms).

Developing this model, we can obtain the map of the entire process of allocation of time, as shown in Figure (where h* is denoted by hacr). Also, in the context of this model, can be interpreted here the migration of some cohorts from the so-called "army of reserve" (leisure time) to the underground sector. For instance, this may be provoked by an increase in the number of hours worked in the visible sector, which will move to a higher level the curve of wage in the underground sector. This will make the work in the underground activity more attractive and it may lead to a re-allocation of the available time of households.



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