How Does the Middle-Class Share Affect Growth and Distribution in a Three-Class Economy?

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Abstract

This study presents a three-class economy model (workers, middle class, and capitalists) and investigates how the middle-class share evolves over time. It also examines the relationship between the middle-class share and economic growth. Depending on the parameters and initial conditions, three different long-run situations arise: (i) an Anti-Dual equilibrium, in which workers and capitalists coexist while the middle class vanishes; (ii) a Pasinetti equilibrium, in which all three classes coexist; and (iii) a Dual equilibrium, in which workers and the middle class coexist while capitalists vanish. An expanding middle-class share either increases or decreases economic growth depending on the conditions.

Keywords: classical growth model; middle class; income distribution;
JEL Classification: E24; E25; E32

1 Introduction

The decline of the middle class has been extensively discussed.[1] In 2019, the Organisation for Economic Co-operation and Development (OECD) published *Under Pressure: The Squeezed Middle Class*, revealing that the population of the middle class in the OECD economies declined relative the 30 years prior; an increase in the middle

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[1] Analyzing US data, Schettino and Khan (2020) reveal the hollowing out of the middle class and income polarization during 1998–2018. For the middle class in developing countries, see Banerjee and Duflo (2008).

class's income was slower than the increase in the upper class's income, and therefore the income share of the middle class declined, whereas those of the upper and lower classes increased.[2] On the OECD average, the population share of the middle class declined from 64% to 61% between the middle of the 1980s to the middle of the 2010s; that is, the share declined by one percent point every 10 years. This report posits that the expansion of the middle class contributes to higher economic growth.

Temin (2017) describes the fall of the middle class in the U.S. in *The Vanishing Middle Class: Prejudice and Power in a Dual Economy*. Milanovic's (2016) *Global Inequality: A New Approach for the Age of Globalization* extends the Kuznets inverted U-curve hypothesis such that as an economy develops, income inequality increases and then decreases, suggesting an elephant curve such that, as the economy develops further, income inequality increases again. This finding suggests sluggish growth in the income of the middle class. Moreover, in Japan, the Kishida administration proposed a slogan–a virtuous cycle between growth and distribution–to reconstruct the rigid middle class.

The above-mentioned shrinkage of the middle class in developed countries can be related to the declining trend in wage share. As Piketty's (2014) *Capital in the Twenty-First Century* reveals, in many developed countries, the wage share has been declining. Piketty's data are slightly older, but more recent data also show a declining trend in wage share. For instance, the *Annual Report on the Japanese Economy and Public Finance* published by the Cabinet Office of Japan presents the time series of wage share in France, Germany, the U.K., the U.S., and Japan during 1980–2022 (Figure 2-1-19). This figure shows that in all countries, the wage share in 2022 is lower than that in 1980.

A growing middle class is expected to lead to high growth, which is desirable.[3] However, the mechanism by which this occurs is not clear. An increase in the middle-class income share implies a decrease in the capitalists' income share. In general, capitalists have a higher propensity to save than the middle class. Thus, the expansion of the middle class might decrease economy-wide savings, which lowers capital accumulation and leads to a decrease in economic growth. Accordingly, the expansion of the middle class does not necessarily lead to higher economic growth. A theoretical model is required to examine how middle class expansion affects economic growth.

Based on these observations, this study presents a three-class classical growth model

[2] This report defines "middle-income class" as the population living in households with incomes ranging between 75% and 200% of the national median. The population in households with income below 75% of the median are the "lower-income class" and those with income above 200% of the median are the "upper-income class."

[3] Easterly (2001) reveals that countries that have a larger middle class tend to grow faster as long as they are not too ethnically diversified.

with workers, the middle class, and capitalist to investigate the relationship between the share of the middle class and economic growth. We also investigate the dynamics of each class's income share. "Classical" means that Say's law holds, and hence, a demand shortage problem never occurs, and that the real wage rate is determined institutionally; hence, it is not determined at the labor market clearing level, which creates unemployment (Foley et al., 2019).

In our model, workers obtain wages, the middle class obtains wages and profits, and capitalists obtain profits. Hence, the middle class acts as both workers and capitalists. Workers spend all their wages on consumption and do not save, while the middle class and capitalists save from their income. For the saving behavior of the middle class, we assume Pasinetti-type saving, that is, the middle class owns capital through saving.

The novelty of this study is that it relates the expansion of the middle class to wage share. Specifically, we assume that the share of the middle class in total employment increases with an increase in wage share. Here, class consciousness and wage share are assumed to be interrelated; an increase in wage share causes an increase in class consciousness, which leads to an actual expansion of the middle class. The middle class share expands when the wage share exceeds a threshold value, whereas it decreases when the wage share is lower than the threshold.

Our analysis is divided into short- and long-term analyses. In the short run, wage share is assumed to be constant, whereas in the long run, it endogenously changes. In the long run, by employing Goodwin's dynamics, we analyze the dynamics of four variables: the wage share, employment rate, capitalist wealth share, and middle-class share. In the long-run analysis, we endogenize technological progress using the theory of induced innovation and, hence, assume that the growth rate of labor productivity is an increasing function of wage share. Then, we reveal that depending on the initial conditions and parameters, three kinds of long-run situations arise as stable steady states: Pasinetti, Dual, and Anti-Dual equilibria.

As the later numerical simulation shows, during the transition to the steady state, in some cases, the middle class share increases, while the economic growth rate decreases. Thus, a growing middle class does not necessarily increase the economic growth rate. Moreover, when the middle class income share increased, the income shares of workers and capitalists decreased. In other words, the expansion of the middle class entails a tradeoff between efficiency (economic growth) and equality (income distribution).

The remainder of this paper is organized as follows. Section 2 surveys the related literature. Section 3 presents our model and the short-run analysis. Section 4 presents the long-term analysis. Section 5 investigates the dynamics of the main variables and the

income distribution using a numerical simulation. Finally, Section 6 concludes.

2 Literature survey

This study presents a three-class classical growth model. As stated above, the middle class assumes Pasinetti-type saving. In the long run, we introduce Goodwin's dynamics and endogenize technological progress. Accordingly, this section conducts a literature survey on three-class growth models, Pasinetti-type saving, Goodwin models, and endogenous technological progress and explains their relationship to our study.

Three-class model

Tavani and Vasudevan (2014) also considered the middle class. They elaborate on a demand-led Kaleckian model comprising three classes: workers, managers, and capitalists. In their model, the wage gap between workers and managers changed endogenously. Capitalists and managers save. Although managers save money, they do not own capital. This issue relates to worker savings.

Palley (2015) presents a Kaleckian model with workers, middle managers, and top managers. He considers managers to act as both workers and capitalists. In our model, the middle class plays the roles of both workers and capitalists. His analysis focuses on the steady state and does not investigate whether the middle class expands over time as we do.

Dutt and Veneziani (2019) present a three-class classical growth model with high-skilled workers, low-skilled workers, and capitalists. Education investment transforms low-into high-skilled workers. They consider the dynamics of human and capital stocks. If we interpret high-skilled workers as middle class, the share of the middle class increases over time through education and approaches a constant value. In our model, the middle class share increases when the wage share exceeds an institutionally determined threshold level. In their model, the wage gap between high- and low-skilled workers is endogenously determined. By contrast, in our model, both workers and the middle class obtain the same real wage.

Workers' saving

The debates between Pasinetti (1962) and Samuelson and Modigliani (1966) on worker savings are important. Kaldor (1956) suggests a saving function in which the propensity to save wages and profits differs. However, Pasinetti (1962) criticizes Kaldor because his approach is a specification in which a different saving rate corresponds to a different

source of income rather than a specification in which a different saving rate corresponds to a different class.

Pasinetti (1962) proposes the Cambridge equation in which even in an economy in which workers save, the long-run profit rate is determined by the ratio of the natural growth rate to the saving rate of capitalists, which is known as the Pasinetti theorem. Conversely, Samuelson and Modigliani (1966) criticize Pasinetti and state that Pasinetti's theorem is true when the savings rate of capitalists is much higher than that of workers, but false when the savings rate of capitalists is not as large as that of workers. Moreover, they claim that unless Pasinetti's assumption holds, another equilibrium, that is, the Dual equilibrium, emerges, and the Cambridge equation does not hold. In summary, theoretically, both Pasinetti and Dual equilibria occur, depending on the conditions.

Darity (1981) presents a third equilibrium, in which capitalists' capital stock shares approach unity; hence, workers do not own capital stock. This equilibrium is called the Anti-Dual equilibrium.

In our model, these three types of equilibria are obtained as long-run situations depending on the parameters and initial conditions, and they can be stable.

Goodwin dynamics

Goodwin (1967) portrays the interaction between the accumulation of capital and functional income distribution in a two-class economy: when savings out of profit increase, capital accumulation increases, leading to an increase in the demand for labor. This means that the working class has more power to bargain wages, which increases the wage share. Consequently, profits and capital accumulation decrease. This process creates an endless cycle of employment and labor share around their steady-state values.

The original Goodwin model does not consider worker savings or the resultant holding of capital stocks. Van der Ploeg (1984) and Sordi (2001) consider the savings and capital accumulation of Pasinetti-type workers in the Goodwin model. Both studies reveal that introducing worker savings can destabilize the dynamics.

Petach and Tavani (2020) construct a classical growth model incorporating a Pasinetti-type saving function and investigate the effect of capital taxation on the economy. They endogenize the growth rates of labor and capital productivity using the theory of induced innovation and wage-share dynamics based on Goodwin (1967). An increase in the tax rate in the steady state increases workers' wealth share but decreases the economic growth rate; thus, a tradeoff between growth and inequality occurs.

Stamegna (2024) considers low- and high-skilled workers in the Goodwin model. Unlike our model, it introduces markup pricing over unit labor costs and endogenizes

wage inequality between the two types of workers. Hence, it assumes an imperfectly competitive goods market whereas the original Goodwin model assumes a perfectly competitive market. In addition to capitalists, high-skilled workers save wages but do not own capital because their savings are not of the Pasinetti type.

Our model considers middle-class savings and capital accumulation. The middle class obtains both wages and profits, and thus plays a dual role: worker and capitalist. Therefore, it is reasonable to assume that the middle class obtains capital through savings.

Induced innovation

Induced innovation is a hypothesis first introduced by Hicks (1932), which stipulates that firms tend to choose innovation that uses less of the factors for which the relative cost is more expensive. Kennedy (1964) proposed an innovation possibility frontier (IPF), in which a decreasing and concave curve describes the inverse correlation between the attainable growth rate of labor productivity and capital productivity. Under the constraints imposed by the IPF, firms innovate in a direction that reduces their unit cost of production the most.

Goodwin-type models with induced innovation include those of Shah and Desai (1981), van der Ploeg (1987), Foley (2003), Julius (2005), Tavani et al. (2011), and Stamegna (2024). Generally, the introduction of induced innovation stabilizes Goodwin's dynamics. The growth rate of labor productivity increases with the wage share. Accordingly, an increase in the wage share increases labor productivity growth, which decreases the wage share. Hence, a negative feedback effect stabilizes the dynamics.

We also use induced innovation because we regard a change in the wage share as an important factor in inequality and the evolution of the middle class. Indeed, in our model, the dynamics of wage share are crucial to our results.

3 Model and short-run analysis

We consider a one-sector economy with three classes: workers, the middle class, and capitalists. The inputs are labor L and capital K. The final outputs are used for both consumption and savings. The production function is expressed as the Leontief production function. At full capacity, the output is

$$Y = \min\{aL, bK\}. \tag{1}$$

Here, a and b are the output-labor and output-capital ratios, respectively. With cost minimization, the relation $aL = bK$ holds; hence, a denotes labor productivity, and b denotes capital productivity. We assume that labor productivity increases through time at

a constant rate $g_a \geq 0$ while capital productivity stays constant $g_b = 0$, where $g_x = \dot{x}/x$ denotes the growth rate of a variable x and $\dot{x} = dx/dt$ denotes the time derivative of x. This study does not include the influence of aggregate demand in the analysis and assumes that production is always limited to the capital level.

Next, we discuss these three classes. Workers have no capital and derive their income only from wages w. Capitalists have abundant capital and can survive without working; therefore, in this study, they earn income only from capital. The middle class can benefit from economic opportunities inaccessible to ordinary workers and, at some point in time, earns more than subsistence consumption and begins to save. Therefore, middle class income comprises both wages and profits. Production function (1) takes a new form.

$$Y = \min\{a(L_w + L_m), b(K_m + K_c)\} \tag{2}$$

Here, K_m and K_c are the amount of capital owned by the middle class and capitalists, respectively. In what follows, we define $k_c \equiv K_c/K$ and $k_m \equiv K_m/K = 1 - k_c$, and call k_c and k_m capitalists' wealth share and middle class wealth share, respectively. L_m and L_w are the amount of employment belonging to the middle and working classes, respectively. The total population comprises employed workers L_w, employed middle-class L_m, unemployed workers U, and capitalists.

Let the total population of this economy, labor force population, and number of capitalists be P, N, and P_c, respectively. Then, we have $P = N + P_c$. Suppose that the share of the labor force population and that of capitalists in the total population are constant. If the growth rate of P is assumed to be $n > 0$ and constant, then the growth rates of N and P_c are also n. Then, we have $L + U = N$. As $L = L_w + L_m$, we obtain $L_w + L_m + U = N$. As we show later, in the long run, the employment rate $e = L/N$ is constant; hence, $(L_w + L_m)/N$ will also be constant. If we define $l_m = L_m/L$, that is, the ratio of the employed middle class to the employed labor, which we call the employed middle-class share, then the ratio of the middle class to total population, that is, the middle-class share, is given by $\theta_m = L_m/N = l_m e$.

According to Erikson and Goldthorpe (1992), upward social mobility is widely observed. Capitalists are an exclusive class that cannot be joined by other classes. Thus, the middle class may consist partly of former workers. First, the middle class emerges when a set of ideologies differing from the viewpoints of other classes is formed in the social and political fields, that is, when class consciousness is formed. Economically, when the middle class is first recognized as both a social and an economic class, $L_m(0) > 0$ and $K_m(0) > 0$ hold. The dynamics of the employed middle-class share $l_m \equiv L_m/L$ is specified as

$$\dot{l}_m = m(\omega) l_m;\ m'(\omega) \geq 0,\ l_m \leq \varphi \in (0,1). \tag{3}$$

When the wage share ω decreases, the level of inequality also grows. We describe this relationship by assuming that the employed middle class share l_m is an increasing function of the wage share, as in equation (3). Wilkinson and Pickett (2009) conduct a comprehensive study of developed countries and discover a link between high inequality and low social mobility. Corak (2013) refers to a similar relationship as "The Great Gatsby Curve," demonstrating that the greater the wealth inequality in a society, the slower its mobility. In this study, the relationship between wealth inequality and social mobility is depicted by a positive relationship between the wage share and the growth rate of the employed middle class. A sufficiently low level of social inequality entails the necessary economic opportunities for more of the working class to move up to the middle class and vice versa.

Moreover, the employed middle class will expand until its share reaches a social constraint represented by $\varphi \in (0,1]$. If this ratio is lower than unity, a portion of society will always remain in the working class, do manual jobs, and receive only subsistence wages. If this ratio is unity, the entire employment can become middle class, at which point the economy reverts to a two-class economy.

Suppose that function $m(\omega)$ has the following property:

$$\exists \bar{\omega} \in (0,1) \text{ such that } m(\bar{\omega}) = 0 \text{ and } \begin{cases} \text{for } m \in (0, \bar{\omega}], m(\omega) \leq 0 \Longrightarrow \dot{l}_m \leq 0 \\ \text{for } m \in (\bar{\omega}, 1), m(\omega) > 0 \Longrightarrow \dot{l}_m > 0 \end{cases}. \quad (4)$$

Then, the employed middle class chare expands when the wage share is larger than the threshold value $\bar{\omega}$ while it shrinks when the wage share is smaller than the threshold value. This threshold value reflects the cost of living. If the income of the working class exceeded this level, the middle class expanded. In addition, as stated in the Introduction, an increase in the wage share reflects class consciousness, and when class consciousness increases, the share of the employed middle class expands.

Because the total income is distributed between wages and profits, we have $Y = wL + vK$. Here, w is the real wage and v is the gross profit rate before accounting for the depreciation of capital. The relationship between variables in the income scene is given by $w = a\omega$ and $v = b(1-\omega)$. When ω, a, and b is constant, w and v is also constant. When ω is constant and a grows, w grows at the same rate as a. In the short run, wage share remains constant. How output is divided into wages and profits is subject to class-level bargaining between workers, the middle class, and capitalists, and is thus exogenously given. It is important to keep in mind that our arguments on a constant wage share reflect the power balance among the three classes.

Next, g and g_K are the output and capital growth rates, respectively. As expressed in equation (1), g and g_K are equal because production is limited by the amount of

capital used, and capital productivity is constant. Furthermore, to maintain equilibrium in the final goods market, total saving S must equal to total investment $I = \dot{K} + \delta K$. From this, we obtain

$$g = g_K \equiv \frac{S}{K} - \delta = \frac{I}{K} - \delta \tag{5}$$

where $\delta \in [0,1]$ is the depreciation rate of capital.

Next, we consider the dynamics of middle class capital K_m and capitalist capital K_c. Capitalists save a portion of their profits, the middle class saves a portion of their wages and capital, and workers do not save because they consume all their income. Let $s_m \in (0,1)$ be the rate of saving of the middle class and $s_c \in (0,1)$ be the rate of saving of capitalists. For realism, we assume that $s_m < s_c$. The savings of the middle class S_m and capitalist S_c are

$$S_m = \dot{K}_m + \delta K_m = s_m(wL_m + vK_m) \tag{6}$$

$$S_c = \dot{K}_c + \delta K_c = s_c v K_c, \tag{7}$$

where $S = S_m + S_c$.

Using equations (5), (6), and (7), we obtain the growth rate of the entire economy:

$$\begin{aligned} g &= s_m \left(w\frac{L_m}{K} + v\frac{K_m}{K} \right) + s_c v \frac{K_c}{K} - \delta \\ &= b\{s_m \omega l_m + (1-\omega)[s_m(1-k_c) + s_c k_c]\} - \delta. \end{aligned} \tag{8}$$

The economic growth rate is decreasing in ω and increasing in k_c. When the wage share is large, the profit rate is low, which leads to low economic growth. When the capitalists' wealth share is large, their savings are large, and their savings rate is high, which leads to high economic growth. Moreover, the economic growth rate increases with the employed middle-class share, l_m. When the middle-class share is larger, the total savings of the economy are also large, which leads to high economic growth.

From equations (6) and (7), we can derive the dynamics of K_m and K_c:

$$g_{K_m} \equiv \frac{\dot{K}_m}{K_m} = s_m \left(w\frac{L_m}{K_m} + v \right) - \delta = s_m b \left[\omega \frac{l_m}{1-k_c} + (1-\omega) \right] - \delta \tag{9}$$

$$g_{K_c} \equiv \frac{\dot{K}_c}{K_c} = s_c v - \delta = s_c b(1-\omega) - \delta. \tag{10}$$

If l_m is zero, $g_{K_c} > g_{K_m}$ because $s_c > s_m$. This property is used in the long-run analysis described in Section 4.1.

We consider the income distribution for each class. Let σ_w, σ_m, and σ_c be the income shares of workers, middle class, and capitalists, respectively.

$$\sigma_w = \frac{wL_w}{Y} = (1 - l_m)\omega \tag{11}$$

$$\sigma_m = \frac{wL_m + vK_m}{Y} = l_m\omega + (1 - k_c)(1 - \omega) \tag{12}$$

$$\sigma_c = \frac{vK_c}{Y} = k_c(1 - \omega) \tag{13}$$

Workers' income share σ_w depends on the employed middle-class share l_m. The middle-class income share σ_m depends on the employed middle-class share and middle-class' wealth share $k_m = 1 - k_c$. Similarly, the capitalists' income share depends on their wealth share, k_c. In the long run, ω and k_c evolve through time, and thus, each class' income share also evolves through time.

4 Long-run analysis: Incorporating Goodwin dynamics

4.1 Four-dimensional dynamics and steady states

The most important deviation of the long-run from the short-run analysis is that the wage share is no longer constant. This occurs through changes in the bargaining positions of the three classes. In the long run, we introduce the endogenous technological progress arising from induced innovation.

According to the theory of induced innovation, the growth rate of labor productivity becomes an increasing function of wage share.

$$g_a = g_a(\omega);\ g_a'(\omega) > 0 \tag{14}$$

We obtain the dynamics of employment rate e. From the production function given by equation (1), $L = (b/a)K$ holds; hence, we obtain $e = L/N = (bK)/(aN)$. Differentiating this with respect to time yields

$$\frac{\dot{e}}{e} = g - g_a - n. \tag{15}$$

Next, we obtain the dynamics of the wage share. Following Goodwin (1967), we assume that the rate of change in real wages is an increasing function of the employment rate.

$$\frac{\dot{w}}{w} = f(e);\ f'(e) > 0 \tag{16}$$

Since the wage share is given by $\omega = w/a$, the rate of change in the wage share is given by

$$\frac{\dot{\omega}}{\omega} = \frac{\dot{w}}{w} - g_a. \tag{17}$$

Finally, we obtain the dynamics of the capitalists' wealth shares. Differentiating $k_c = K_c/K$ with respect to time, we obtain:

$$\frac{\dot{k}_c}{k_c} = g_{K_c} - g. \tag{18}$$

From equations (8), (10), and (14)–(18), we obtain the following four-dimensional dynamic system.

$$\dot{e} = \{b\big[s_m\omega l_m + (1-\omega)[s_m(1-k_c) + s_c k_c]\big] - g_a(\omega) - n - \delta\}e \tag{19}$$

$$\dot{\omega} = [f(e) - g_a(\omega)]\omega \tag{20}$$

$$\dot{k}_c = b[(s_c - s_m)(1-\omega)(1-k_c) - s_m\omega l_m]k_c \tag{21}$$

$$\dot{l}_m = m(\omega)l_m \tag{22}$$

We next obtain the steady state of the dynamical system.

From equations (4) and (22), the employed middle-class share will be constant when either l_m is zero, when it reaches the upper limit φ, or when the wage share becomes $\omega = \bar{\omega}$. However, the outcome $\omega = \bar{\omega}$ is a coincidence, and hence, generally, we have $\omega \neq \bar{\omega}$. Therefore, we consider the two cases: $l_m = 0$ and $l_m = \varphi$ in order.[4]

Whether the employed middle-class share expands or shrinks depends on the relative size between the wage share and the threshold $\bar{\omega}$. The long-run situation is either $l_m = 0$ or $l_m = \varphi$, and depending on initial conditions and parameters, the employed middle-class share either expands or shrinks. After the employed middle-class share becomes zero or reaches the upper limit φ, our model reduces to the three-variable Goodwin model with endogenous technical change and workers' saving.

Case when $l_m = 0$

First, we consider the case when $l_m = 0$. In this case, from $\dot{k}_c = 0$, we obtain $\omega = 1$, $k_c = 0$, and $k_c = 1$.

When $\omega = 1$, the income share of workers is unity, the profit rate is zero, and the income shares of the middle class and capitalists are zero. Thus, there are no savings in such an economy; hence, reproduction never occurs. Therefore, this case is excluded from the study.

The cases where $k_c = 0$ and $k_m = 1$ is also ruled out. When $l_m = 0$, the middle class only saves profits. However, because the saving rate of capitalists is higher than that of the middle class, their capital accumulation rate exceeds that of the middle class; hence, $k_c = 0$ and $k_m = 1$ are never obtained.

The case of $k_c = 1$ is valid. In this case, from $\dot{e} = 0$, we obtain

$$g_a(\omega) = -bs_c\omega + bs_c - n - \delta. \tag{23}$$

The left-hand side (LHS) of equation (23) is an increasing function of the wage share,

[4] Indeed, as the later numerical simulation reveals, we do not obtain $\omega = \bar{\omega}$ as a long-run situation.

while the right-hand side (RHS) is a decreasing function of the wage share. Under the appropriate conditions, the two curves have a unique intersection that provides the steady-state wage share. This wage share depends on the savings rate of capitalists but not on that of the middle class. Moreover, when capitalists' savings rate increases, the steady-state wage share increases. Substituting this steady-state wage share into $\dot{\omega} = 0$, we obtain

$$f(e) = -bs_c\omega + bs_c - n - \delta. \tag{24}$$

From equation (24), we obtain the steady-state employment rate. Because the steady-state wage share does not depend on the savings rate of the middle class, the steady-state employment rate also does not depend on it and depends on the savings rate of the capitalists. According to Darity (1981), this case of $k_c = 1$ can be called an Anti-Dual equilibrium. At the Anti-Dual equilibrium, workers and capitalists coexist, whereas the middle class vanishes. Each income share is given by

$$\sigma_w^{AD} = \omega^{AD} \tag{25}$$

$$\sigma_m^{AD} = 0 \tag{26}$$

$$\sigma_c^{AD} = 1 - \omega^{AD}. \tag{27}$$

As stated above, an increase in the savings rate of capitalists increases ω^{AD}. Accordingly, at the Anti-Dual equilibrium, an increase in the savings rate of capitalists increases the income share of workers but decreases that of capitalists.

The Anti-Dual equilibrium is obtained when $l_m = 0$, in this case, the middle class saves not from wages but only from profits. Capitalists also save profits, and their savings rate is higher than that of the middle class. Therefore, in the long run, capitalists hold all wealth and an Anti-Dual equilibrium is attained.

Case when $l_m = \varphi$

This case is divided into $k_c = 0$ and $k_c \neq 0$.

First, when $k_c = 0$ in equation $\dot{k}_c = 0$, substituting $k_c = 0$ into $\dot{e} = 0$, we obtain

$$g_a(\omega) = -bs_m(1-\varphi)\omega + bs_m - n - \delta \tag{28}$$

The LHS of equation (28) is an increasing function of the wage share, whereas the RHS is a decreasing function. Under the appropriate conditions, the two curves have a unique intersection that provides the steady-state wage share. This wage share depends on the saving rate of the middle class but does not depend on that of the capitalists. Moreover, an increase in the savings rate of the middle class increases the steady-state wage share. Substituting this wage share into $\dot{\omega} = 0$, we obtain the steady-state employment rate. This employment rate depends on the saving rate of the middle class but does not depend on that of capitalists. The case when $k_c = 0$ can be called a Dual equilibrium, according

to Samuelson and Modigliani (1966). At the Dual equilibrium, workers and the middle class coexist, but capitalists vanish. Each income share is

$$\sigma_w^D = (1-\varphi)\omega^D \tag{29}$$

$$\sigma_m^D = \varphi\omega^D + (1-\omega^D) \tag{30}$$

$$\sigma_c^D = 0. \tag{31}$$

As stated above, an increase in the savings rate of the middle class increases ω^D. Hence, an increase in the saving rate of the middle class increases the income share of workers, while decreasing that of the middle class. Moreover, an increase in the upper limit of the middle-class share φ decreases the income share of workers but increases that of the middle class.

Second, when $k_c \neq 0$ in equation $\dot{k}_c = 0$, we obtain

$$(s_c - s_m)(1-\omega)(1-k_c) - s_m\omega\varphi = 0. \tag{32}$$

Moreover, from equation $\dot{e} = 0$, we obtain

$$b\big[s_m\omega\varphi + (1-\omega)[s_m(1-k_c) + s_c k_c]\big] - g_a(\omega) - n - \delta = 0. \tag{33}$$

Combining equations (32) and (33), we obtain the steady-state values of ω and k_c under appropriate conditions. Further calculations yield

$$g_a(\omega) = -bs_c\omega + bs_c - n - \delta. \tag{34}$$

This is exactly the same as in equation (23). Therefore, an increase in capitalists' savings rate increases the steady-state wage share. Moreover, it was independent of the savings rate of the middle class. Then, steady-state capitalists' wealth share is

$$k_c = \frac{(s_c - s_m)(1-\omega) - s_m\omega\varphi}{(s_c - s_m)(1-\omega)}. \tag{35}$$

Although this value is not necessarily positive, we assume it is positive. For this issue, we will explain soon below. By substituting the steady-state wage share into equation (35), we obtain the steady-state capitalist wealth share. Further, substituting the steady-state wage share into $\dot{\omega} = 0$, we obtain the steady-state employment rate. This employment rate depends on the savings rate of the capitalists, but the node depends on that of the middle class. This case of $k_c \in (0,1)$ can be called the Pasinetti equilibirum according to Pasinetti (1962). At Pasinetti equilibrium, all three classes coexist. Each income share is given by

$$\sigma_w^P = (1-\varphi)\omega^P \tag{36}$$

$$\sigma_m^P = \varphi\omega^P + (1-k_c^P)(1-\omega^P) = \frac{s_c\varphi\omega^P}{s_c - s_m} \tag{37}$$

$$\sigma_c^P = k_c^P(1-\omega^P) = 1 - \frac{[s_c - (1-\varphi)s_m]\omega^P}{s_c - s_m}. \tag{38}$$

As previously stated, an increase in the savings rate of capitalists increases the steady-

state wage share. Hence, an increase in the savings rate of capitalists increases workers' income share. However, the effects of an increase in the capitalists' savings rate on the income share of the middle class and capitalists are ambiguous. On the contrary, an increase in the saving rate of the middle class does not affect the income share of workers, but increases that of the middle class and decreases that of capitalists. Moreover, an increase in the upper limit of the middle class share decreases the income share of workers, increases that of the middle class, and decreases that of capitalists.

Let us explain the condition under which capitalists' wealth shares approach zero. This amounts to whether there exists a positive k_c that satisfies $(s_c - s_m)(1-\omega)(1-k_c) - s_m\omega\varphi = 0$ in equation of $\dot{k}_c = 0$ with ω given. By solving this equation for k_c, we obtain equation (35). If the numerator is positive, then there exists a positive k_c. In contrast, if the numerator is negative, then there never exists a positive k_c, hence, we have $k_c = 0$. This condition leads to

$$\omega^* < \frac{s_c - s_m}{s_c - s_m + \varphi s_m} \equiv \tilde{\omega} \Longrightarrow k_c^* \in (0,1] \tag{39}$$

$$\omega^* > \tilde{\omega} \Longrightarrow k_c^* = 0. \tag{40}$$

Here, ω^* and k_c^* denote the steady-state wage share and capitalist wealth share, respectively. This condition states that the steady-state capitalists' wealth share will be positive when the steady-state wage share is below the threshold value and zero when the steady-state wage share is above the threshold value. This condition corresponds to $(1-\omega)s_c > s_w$ provided by Samuelson and Modigliani (1966); that is, the saving rate of capitalists' is much higher than that of workers when the Pasinetti equilibrium is obtained. Indeed, letting $\varphi = 1$ in equation (44), we obtain exactly the same condition.

Summarizing the above discussions, we obtain the following proposition.

Proposition 1. *Depending on the combination of capitalists' wealth share* k_c *and the middle class share* l_m*, long-run situations become either the* (*i*) *Pasinetti equilibrium:* $k_c^P \in (0,1)$ *and* $l_m = \varphi$*;* (*ii*) *Dual equilibrium:* $k_c^D = 0$ *and* $l_m = \varphi$*; or* (*iii*) *Anti-Dual equilibrium:* $k_c^{AD} = 1$ *and* $l_m = 0$*.*

4.2 Local stability of steady states

This section briefly describes the local stability of each steady-state. Provided that the middle-class share reaches either $l_m = 0$ or $l_m = \varphi$, it is enough to examine the local stability of the system of the differential equations of e, ω, and k_c.

A detailed analysis is provided in the appendix,[5] from which we obtain the following

[5] The mathematical appendix that investigates the local stability of the long-run equilibrium is

three propositions.

Proposition 2. *The Dual equilibrium is locally and asymptotically stable as long as it exists.*

Proposition 3. *If the saving rate of the middle class is sufficiently close to zero or sufficiently close to the threshold value, then the Pasinetti equilibrium is locally and asymptotically stable. Limit cycles can occur when the saving rate of the middle class lies within some interval, and there exists a continuous family of non-constant periodic solutions.*

Proposition 4. *The Anti-Dual equilibrium is locally and asymptotically stable as long as it exists.*

5 Numerical examples

This section numerically investigates the time series of each variable and income share of each class.[6] The analysis in Section 4.2 was conducted on the assumption that $l_m = 0$ or $l_m = \varphi$, and hence, global stability is not necessarily warranted. In this numerical simulation, we show that an economy starting from an arbitrary initial state converges to one of three types of steady states depending on the initial conditions and parameters.

For the numerical analysis, we specify the reserve army effect, induced innovation, and the dynamics of the employed middle-class share as follows:

$$\frac{\dot{w}}{w} = f(e) = \varepsilon_0 + \varepsilon_1 e \tag{41}$$

$$\frac{\dot{a}}{a} = g_a(\omega) = \phi_0 + \phi_1 \omega \tag{42}$$

$$\dot{l}_m = m(\omega) l_m = \gamma(m_1 \omega - m_0) l_m. \tag{43}$$

All parameters are positive. For clarity, we assume that all functional forms are linear. The parameter γ in equation (43) represents an adjustment parameter of l_m, which is not essential but is useful to adjust convergence speed. Finally, from equation (43), the threshold value of the wage share is given by $\bar{\omega} = m_0/m_1$.

5.1 Anti-dual case: Coexistence of workers and capitalists

available upon request from the corresponding author.

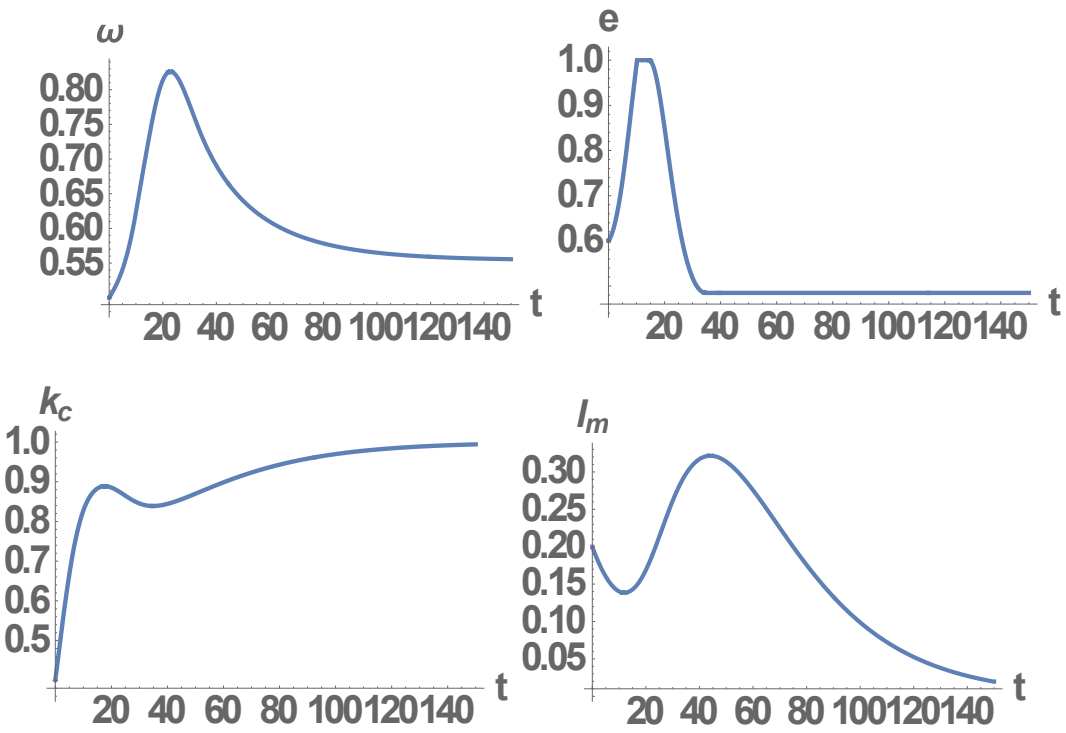
[6] For the numerical simulations, we use Wolfram *Mathematica* 10.

We set the parameters and initial values as follows:

$$\varepsilon_0 = 0.1, \varepsilon_1 = 0.08, n = 0.01, \delta = 0, b = 1, s_w = 0.1, s_c = 0.6,$$
$$m_0 = 0.2, m_1 = 0.3, \phi_0 = 0.1, \phi_1 = 0.07, \gamma = 1, \varphi = 0.8,$$
$$e(0) = 0.6, \omega(0) = 0.5, k_c(0) = 0.4, l_m(0) = 0.2.$$

In this example, we have $\bar{\omega} = m_0/m_1 = 0.67$ and $\tilde{\omega} = 0.86$. This satisfies the conditions under which the employed middle-class share shrinks and capitalists' wealth share increases ($\omega^{AD} < \bar{\omega}$ and $\omega^{AD} < \tilde{\omega}$). The difference between the savings rates of the middle class and capitalists is relatively large. Here, the upper limit of the middle class is set to $\varphi = 0.8$.

Figure 3 shows the time series for each variable. The employed middle-class share increases for some time but eventually decreases and reaches zero. Capitalists' wealth share approaches unity. Hence, in the long run, the middle class vanishes and workers and capitalists coexist.



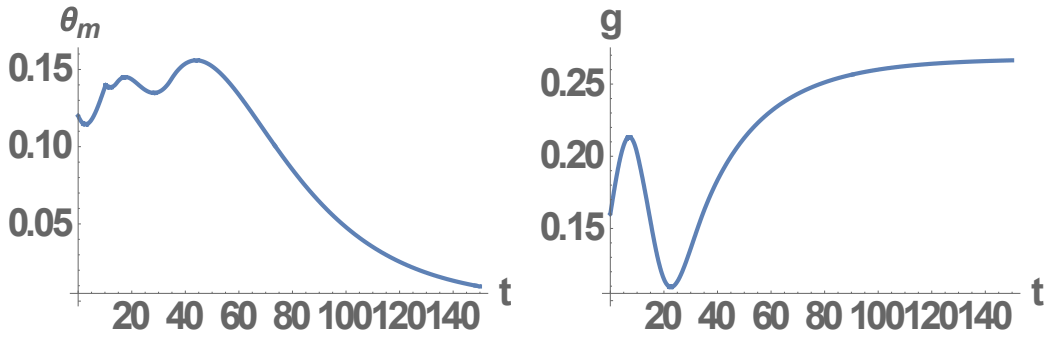


Figure 3: Dynamics in the Anti-Dual case

The income share of each class is as follows. In Figure 4, the dotted blue, solid red, and broken green lines correspond to workers, the middle class, and capitalists, respectively. The income shares of workers and capitalists approach positive values, whereas those of the middle class approach zero.

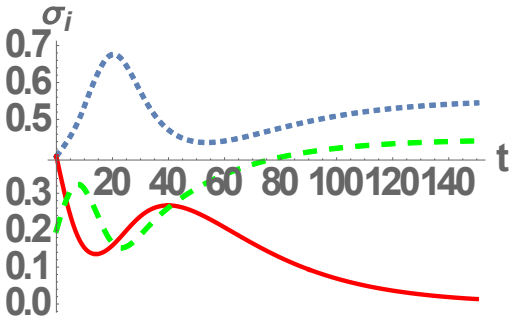


Figure 4: Income share for each class in the Anti-Dual case (blue, red, and green lines correspond to workers, middle class, and capitalists, respectively)

5.2 Pasinetti case: Coexistence of three classes

We present an example of the Pasinetti equilibrium. This example has exactly the same parameters as the Anti-Dual case, but has a different initial value. Let us change the initial value of the wage share from $\omega(0) = 0.5$ to $\omega(0) = 0.65$.

Figure 5 shows the time series for each variable. The middle-class share reaches its upper limit $\varphi = 0.8$. Subsequently, all three classes coexist. This is an interesting case study. Since all the parameters are the same as those of the above Anti-Dual case, they satisfy the conditions under which the middle-class share decreases and capitalists' wealth share increases ($\omega^{AD} < \bar{\omega}$ and $\omega^{AD} < \tilde{\omega}$). However, when an initial value is different, that is, the initial value of the wage share is relatively high, the middle-class share can expand even if $\omega(0) < \bar{\omega}$. This is possible because along the transition, the wage share

fluctuates and satisfies $\omega > \bar{\omega}$ at some time.

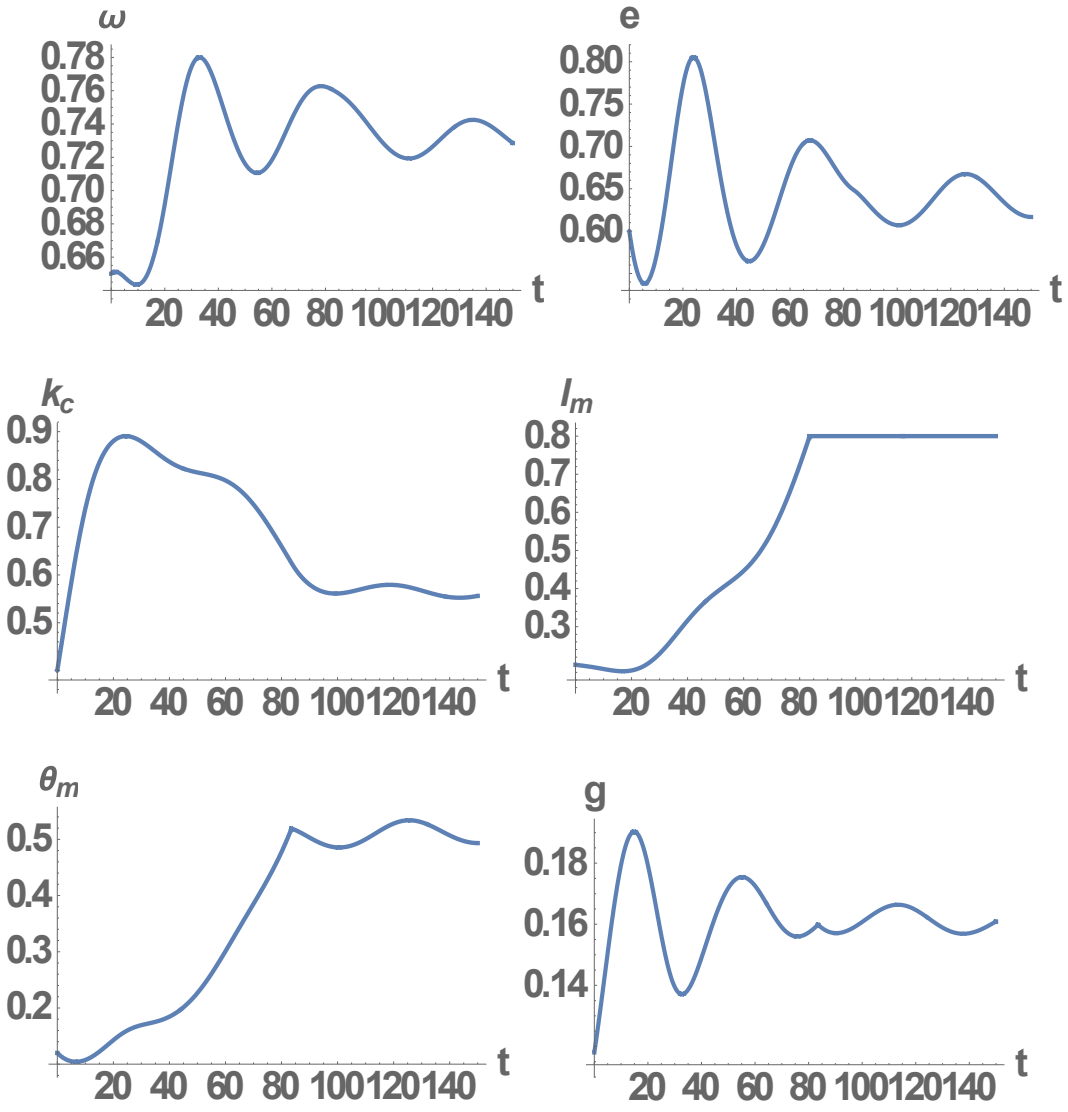


Figure 5: Dynamics in the Pasinetti case

Figure 6 illustrates the income share of each class. The income share of the middle class expands, while that of workers and capitalists contracts. All income shares converge to positive values.

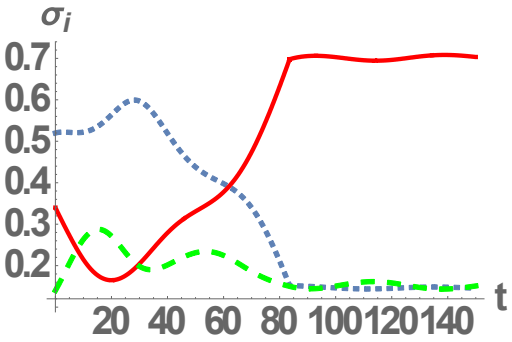


Figure 6: Income share for each class in the Pasinetti case (blue, red, and green lines correspond to workers, middle class, and capitalists, respectively)

As stated above, if the initial value of the wage share is relatively large, the employed middle-call share will continue to increase. Since the difference between the Anti-Dual and Pasinetti cases lies only in the initial value of the wage share, this situation suggests path dependency, such that a different initial situation leads to a different long-run situation.

5.3 Dual case: Coexistence of workers and middle-class

We set the parameters and initial values as follows:

$$\varepsilon_0 = 0.1, \varepsilon_1 = 0.2, n = 0.01, \delta = 0, b = 1, s_w = 0.3, s_c = 0.6,$$
$$m_0 = 0.2, m_1 = 0.4, \phi_0 = 0.1, \phi_1 = 0.3, \gamma = 1, \varphi = 0.9,$$
$$e(0) = 0.6, \omega(0) = 0.65, k_c(0) = 0.6, l_m(0) = 0.1.$$

This example leads to $\bar{\omega} = m_0/m_1 = 0.5$ and $\tilde{\omega} = 0.53$, which satisfy the conditions under which the middle-class share expands and the Dual equilibirum is obtained ($\omega^D > \bar{\omega}$ and $\omega^D > \tilde{\omega}$). The difference between the savings rates of the middle class and capitalists is relatively small. The results are shown in Figure 7. The rate of change in l_m is negative in a period of time but becomes positive in the long run; hence, the employed middle-class share reaches 0.9. Capitalists' wealth share approaches zero. Hence, workers and the middle class coexist, whereas capitalists disappear.

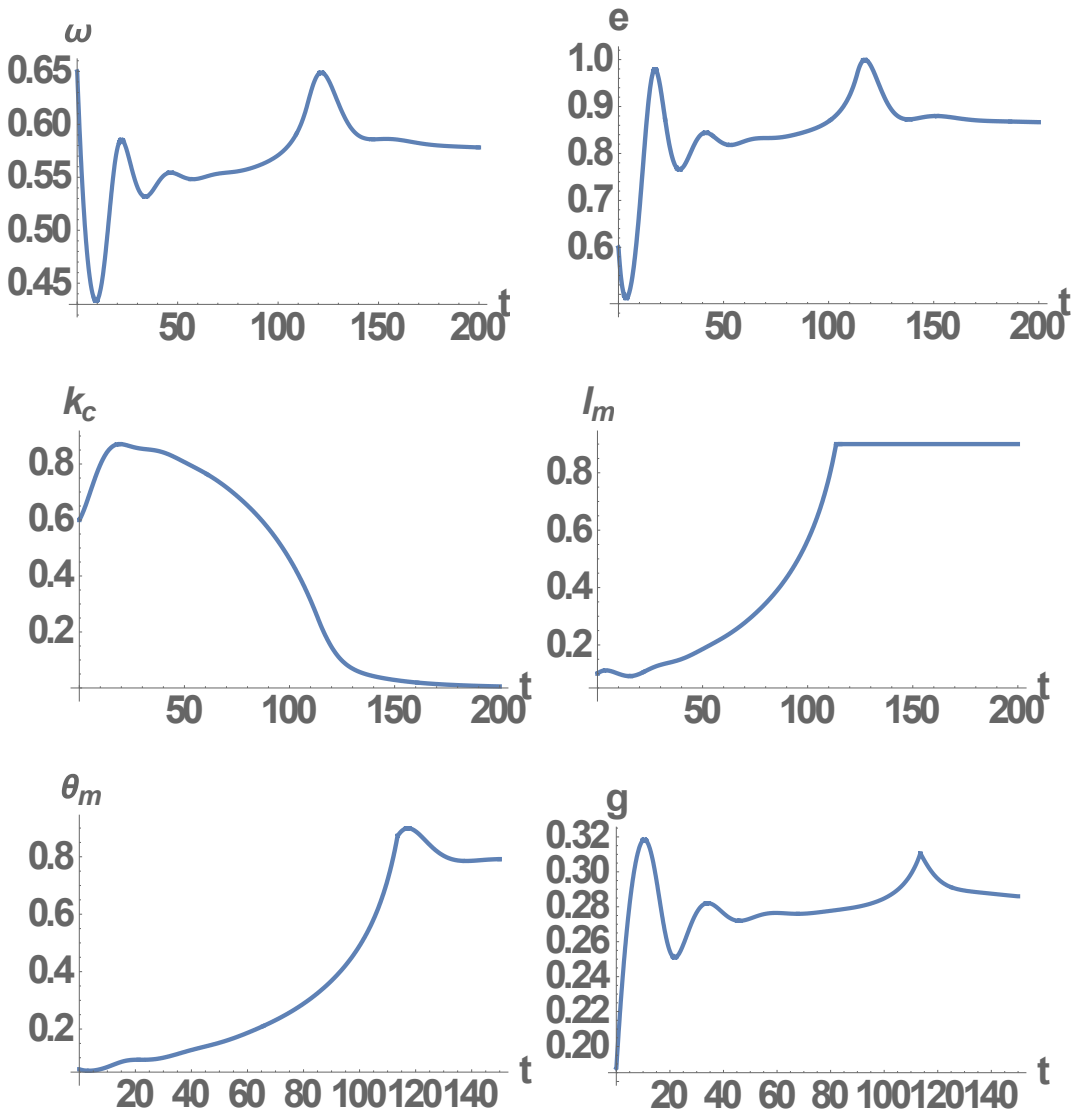


Figure 7: Dynamics in the Dual case

Figure 8 shows the income share of each class. In this case, the income shares of workers and the middle class approach positive values, whereas those of capitalists converge to zero.

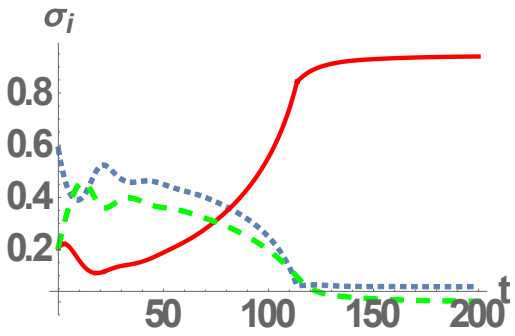


Figure 8: Income share for each class in the Dual case (blue, red, and green lines correspond to workers, middle class, and capitalists, respectively)

5.4 Summary

We summarize the results of the numerical simulation by focusing on the relationship between the middle class share and the economic growth rate.

In the Anti-Dual case, the middle class share decreases, while the economic growth rate increased over time. In this case, the middle-class income share decreased. In the Pasinetti case, the middle-class share increases and the economic growth rate either increases or decreases. In this case, the middle class income share increased. In the Dual case, both the middle-class share and the economic growth rate increase. In this case, the middle class income share increased.

From these observations, the middle class and middle-class income shares are positively correlated. In contrast, the middle-class share and economic growth rate are not necessarily positively correlated. In addition, the middle class income share and economic growth rate are not necessarily positively correlated.

To summarize, the expansion of the middle class did not necessarily lead to higher economic growth. As equation (8) shows, the economic growth rate along the transitional dynamics is a decreasing function of the wage share, an increasing function of the capitalists' wealth share, and an increasing function of the middle-class share. Hence, if all other factors are equal, an expansion of the middle class will increase the economic growth rate. However, because other things are not equal along the transitional dynamics, in some cases, an expansion of the middle class decreases the economic growth rate.

In both the Anti-Dual and Pasinetti cases, an expansion of the middle class lowers the economic growth rate as follows. An increase in the middle-class income share decreases the capitalists' income share. The propensity to save capitalists is higher than that of the middle class. Accordingly, a decrease in the capitalists' income share decreases economy-wide savings and investments, which lowers the economic growth rate.

In the Dual case, the expansion of the middle class increases the economic growth rate as follows: In this case, the presence of capitalists, with the highest propensity to save, diminishes, whereas the presence of the middle class increases savings. Hence, the expansion of the middle class contributes to an increase in economy-wide savings, which increases the economic growth rate.

As the numerical simulation shows, we obtain path dependency, such that the path of development of an economy differs if its history differs. This finding suggests that the share of the middle class either expands or shrinks, even in similar economies. The initial values cannot be changed since they are historically given, but policies can change the threshold value of the wage share $\bar{\omega} = m_0/m_1$. The OECD (2019) points out that increases in the cost of living, such as housing, education, health, and long-term care, depress the middle class; hence, people are dropping out of the middle class. As stated above, the threshold $\bar{\omega}$ reflects class consciousness, and if the cost of living is related to class consciousness, aid for such expenditures contributes to lowering the threshold $\bar{\omega}$, and eventually leads to the expansion of the middle class. However, as stated above, the expansion of the middle class does not necessarily contribute to an increase in the economic growth rate.

In our model, the long-run economic growth rate is given by $g^* = g_a(\omega^*) + n$, that is, the natural growth rate. Since the growth rate of labor productivity $g_a(\omega^*)$ is an increasing function of the ware share, the economic growth becomes larger if the steady-state value of the wage share is larger. From the analysis in Section 4.1, we find that an increase in the savings rate of capitalists at the Pasinetti and Anti-Dual equilibria and an increase in the saving rate of the middle class at the Dual equilibrium increase the steady-state wage share. Accordingly, these changes in savings rates increase the long-run economic growth rate at their respective equilibria. Thus, in our model, the wage-led growth regime holds in the long run, even in the supply driven classical growth model.[7]

6 Conclusions

This study presents a three-class classical growth model in which workers, the middle class, and capitalists coexist and investigates whether the middle-class share expands or shrinks, and the economic growth and income distribution.

In the long-run analysis, depending on the initial value of the wage share, the sizes of the two kinds of threshold values of the wage share, and the long-run value of the wage

[7] Petach and Tavani (2020), Michl and Tavani (2022), Cruz and Tavani (2023), and Rada et al. (2023) also apply similar mechanisms.

share, we obtain Pasinetti's dual and Anti-Dual equilibria. At the Pasinetti equilibrium, all three classes coexist; at the Dual equilibrium, workers and the middle class coexist, while capitalists vanish; and at the Anti-Dual equilibrium, workers and capitalists coexist, while the middle class vanishes.

The main determinant of the expansion of the middle class share is whether the long-term wage share exceeds the threshold value. The middle class share expands if the long-run wage share exceeds the threshold value, whereas it decreases if the long-run wage share is below the threshold value. However, an expansion in the middle-class share does not necessarily lead to economic growth. Moreover, the expansion of the middle class inevitably entails a decline in the income shares of workers and capitalists. Thus, a tradeoff between efficiency and inequality arises. Therefore, the path and long-term situation that should be selected depend on the social norms and national consensus of the corresponding economy.

Our numerical simulation demonstrates the realization of the three types of long-run equilibria and examines how the main variables and income shares evolve in the three cases. However, we do not analyze the pattern corresponding to the real economy. By estimating the parameters using actual data, we can conduct numerical simulations that correspond to the real economy more accurately.

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