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Abstract

This study develops a two-country New Keynesian (NK) model incorporating deep habits in consumption to analyze macroeconomic dynamics under the optimal coordinated monetary policy. The central bank adjusts interest rates more aggressively in response to structural shocks in an open economy than in a closed economy. Deep habits strengthen the central bank's incentive to adjust terms of trade through interest rates due to habit formation and counter-cyclical markup behavior, creating price inelasticity in demand. Deep habits also lead to deviations from the law of one price, reflected in goods-specific real exchange rates, which the degree of home bias influences. Finally, this study compares international policy coordination to noncoordination to analyze welfare gains, showing that they depend on key structural factors like price rigidity, deep habits, and home bias. Policy coordination stabilizes domestic output and inflation by internalizing externalities in terms of trade and consumption.

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Keywords: Optimal monetary policy; Deep habit; Policy coordination; Commitment;

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1 Introduction

Macroeconomic theoretical models have increasingly incorporated the formation of household consumption habits. In particular, standard medium-scale dynamic stochastic general equilibrium (DSGE) models often employ habit formation in household consumption to reproduce the hump-shaped dynamics of endogenous variables in response to structural shocks (Christiano, Eichenbaum and Evans, 2005, Smets and Wouters, 2007).

Ravn, Schmitt-Grohé and Uribe (2006) recently extended the concept of habit formation to "deep habits," a form that inherits the empirically desirable hump-shaped dynamics.[1] Traditional (or "superficial") habits are formed for aggregated goods, while deep habits are formed for goods-by-goods.[2] Deep habits generate additional externalities in models, providing new insights. For example, Ravn et al. (2006) highlighted that deep habits change the demand structure of households; therefore, price setting decisions of final-goods firms and markups of the goods price over their cost could also be altered, bringing crowding-in effects of fiscal policy (Ravn et al., 2006, Zubairy, 2014).[3] Many previous studies focused on the role of deep habits in fiscal policy; however, only a few considered the role of deep habits in optimal monetary policy. In particular, to the best of our knowledge, no open economy NK models with deep habits have considered the optimal monetary policy.

This study develops a two-country NK model with deep habits in household consumption, extending Leith, Moldovan and Rossi (2012, 2015)'s closed economy NK model with deep habits to a two-country version based on Corsetti, Dedola and Leduc (2010), Ravn, Schmitt-Grohe and Uribe (2007), Jacob and Uusküla (2019), and Ida and Okano (2023a). We then examine the central bank's behavior in solving the Ramsey problem, which maximizes both countries' economic welfare in a coordinated fashion. Furthermore, we compare cooperative and noncooperative policies to examine the welfare gains from international coordination.

In a two-country model, the central bank should consider the terms of trade externality

[1]See Kormilitsina and Zubairy (2018) and Zubairy (2014).

[2]Cantore, Levine and Melina (2014) used Bayesian estimation to compare how superficial and deep habits perform regarding data fitting in a DSGE model.

[3]Ravn et al. (2006) highlighted that the markup dynamics generated by deep habits differ significantly from the markup derived from sticky prices in the standard new Keynesian (NK) model. Our model avoids confusion between the two by formulating each markup distinctly, as we demonstrate later.

and consumption externality due to deep habit. When terms of trade externalities exist—for instance, high elasticity of substitution between domestic and foreign goods—improving the terms of trade can increase imported-goods consumption while reducing the domestic labor burden, thereby improving economic welfare (Pappa, 2004, De Paoli, 2009). Therefore, in conducting monetary policy to maximize economic welfare, the central bank must balance consumption externalities arising from deep habits, terms of trade externalities, and the costs associated with nominal rigidities.

Our main findings are as follows. Unlike in the standard NK model, when a deep habit exists, the central bank faces a trade-off between the output gap and inflation stability even in response to a productivity shock. Previous studies have shown that when the central bank conducts the optimal commitment policy, it is more reluctant to move the interest rate aggressively in a closed economy model with deep habits than a model without deep habits. Moreover, unlike in a closed economy, the central bank alters the interest rate more in the two-country open economy model. The key to understanding this finding lies in the asymmetric changes in domestic and foreign markups generated by the deep habits and the resulting changes in relative demand. Combining these changes with the central bank's behavior and the externalities of the terms of trade that are unique to the open economy model leads to markedly different results from those of a closed economy. In an open economy, regardless of deep habits, the central bank is incentivized to internalize the externalities due to the terms of trade. When deep habits exist, the process of habit formation, along with the counter-cyclical markup behavior, introduces a price-inelastic component into the demand function for individual goods. This inelasticity strengthens the incentive to influence the terms of trade, enhancing the central bank's interest rate movements; however, if the home bias in favor of domestic goods is small, the international spillover effects of economic shocks become increasingly pronounced. Thus, the necessity to adjust the terms of trade diminishes, leading to a more moderate response of interest rates.

We also show that the deviations from the law of one price (LOP), or the good-specific real exchange rate, generated by the deep habits are related to the degree of home bias. In particular, the deviations fully disappear without a home bias.

Comparing welfare gains shows that the welfare gains from coordination depend on key structural factors such as price rigidity, deep habits, and home bias. When prices are flexible, coordination benefits are relatively small; however, as price rigidity increases, the gains from coordination become more significant as noncooperative policies fail to adjust prices optimally. Deep habits tend to reduce coordination benefits by naturally smoothing international consumption imbalances.

Impulse response analysis further reveals how policy coordination affects macroeconomic dynamics. Under cooperative policies, interest rate adjustments help stabilize domestic output and inflation at the cost of slightly increasing foreign volatility. This situation suggests that policy coordination can internalize the externalities arising from terms of trade and consumption habit formation.

The remainder of the paper is organized as follows. Section 2 reviews the related literature, Section 3 describes the model structure, and Section 4 describes the central bank's optimal monetary policy. Section 5 demonstrates the impulse responses to the structural shocks, calculates the welfare gains from international policy coordination, and provides economic intuition about the results. Finally, Section 6 concludes our research and summarizes the results.

2 Literature Review

Our study is based on two research strands: deep consumption habits and international monetary-policy coordination.

Many fields have used habit formation to generate real rigidity. For instance, Abel (1990) incorporated the "keeping up with the Joneses effect" in the utility function into the asset pricing model to examine the equity premium puzzle. Previous studies noted the similarities between deep habits and the customer market model (Bils, 1989), as deep habits modify demand behavior at the goods-by-goods level, affecting firm-level pricing like customer markets.[4]

Incorporating deep habits into the NK model has drawn attention to how deep habits produce countercyclical markup. In other words, under the deep habit, firms may respond to an increase in current demand by lowering markups and reducing prices to secure future profits. Thus, "price may magnify, rather than stabilize, demand movements" (Bils, 1989). The influence of countercyclical markups on optimal monetary policy is significant. As Leith et al. (2012) discussed, under a deep habit, the central bank cannot lower interest rates sufficiently

[4]Hong (2019) embedded customer capital due to deep habits into a standard model of firm dynamics with entry and exit.

even when they should (e.g., due to positive productivity shocks). This situation occurs because lowering the interest rate will cause firms to lower their markups, leading to undesirable overconsumption. The dilemma that lowering interest rates to stimulate demand also impacts the supply side generates a trade-off between inflation stability and the output gap, indicating a deviation from the "divine coincidence." In the context of optimal policy, Amato and Laubach (2004) investigated the optimal monetary policy under internal superficial habit in consumption. Givens (2016) explored how deep habits affect welfare gains from commitment relative to discretion, revealing that deep habits weaken the stabilization trade-offs facing a discretionary planner.

Some properties from countercyclical markup may carry over in the two-country model; however, which properties and the extent to which they are inherited in the open economy model are worth investigating. For example, deep habits can affect international price markups. Specifically, Ravn et al. (2007) showed that deep habits could endogenously generate deviations from the LOP. Furthermore, Jacob and Uusküla (2019) showed that deep habits could be the source of incomplete pass-through of exchange rates to international prices.

This paper also considers optimal monetary policy in an open economy model since it incorporates deep habits into a two-country open economy model. An intriguing question for optimal monetary policy in an open economy model is whether each country gains more by coordinating its policies than by pursuing its welfare without cooperation (cooperation vs. noncooperation). There have been many previous studies in this regard; they can be divided by the export pricing behavior of firms: producer currency pricing (PCP) vs. local currency pricing (LCP). PCP models dealing with international cooperation are, for example, Clarida, Galí and Gertler (2002), Benigno and Benigno (2006), Pappa (2004) and Obstfeld and Rogoff (2002). LCP models include Devereux and Engel (2003) and Engel (2011), Fujiwara and Wang (2017).[5] Moreover, De Paoli (2009), Corsetti and Dedola (2005), Corsetti et al. (2010) provide a comprehensive discussion of the economy's behavior and optimal monetary policy in PCP and LCP.

Similar conclusions regarding policy coordination gains are obtained for PCP and LCP; policy coordination gains can be generated, but they are not large.[6] Regarding PCP, Clarida

[5] Fujiwara and Wang (2017) reviews optimal monetary policy in open economy.

[6] Kim (2023) reviewed the occurrence and magnitude of welfare gains in PCPs and LCPs.

et al. (2002) pointed out that the nominal exchange rate is important in a relative price adjustment, meaning that distortions can occur when countries strategically manipulate exchange rates or terms of trade in favor of each other's economy. There is room for policy coordination to eliminate these distortions. Engel (2011) argued that in the case of LCP, nominal exchange rates are no longer helpful for the relative price adjustment; instead, coordination gains are generated by coordinately correcting deviations from the LOP and currency misalignments. Kim (2023), on the other hand, constructs an asymmetric two-country model. One country follows the PCP, and the other follows the LCP. In this case, the asymmetry of the model allows for a not-too-small level of coordinated gains even under conditions where gains would disappear in both the PCP and LCP models.

These models are related to this paper, but no models listed include deep habit. This paper gains new findings by introducing deep habit. The main questions are about how deep habits affect gains and the relationship between home bias and deep habit. Both home bias and deep habits are related to household preferences; however, their interaction has not yet been examined, and it is particularly interesting. We develop the two-country model based on Corsetti et al. (2010)[7], incorporating deep habits in consumption similarly to Ravn et al. (2007), Jacob and Uusküla (2019). Our two-country model allows for different pricing between domestic and foreign markets by reflecting distinct consumption habits at the individual goods level. Therefore, it can reproduce the deviation from the LOP as in Ravn et al. (2007). We investigate the optimal commitment policy following Leith et al. (2012, 2015).

3 The Model

We develop the two-country model based on Bodenstein et al. (2019), incorporating deep habits in consumption following Ravn et al. (2007), Jacob and Uusküla (2019)[8] and discuss optimal monetary policies following Leith et al. (2012, 2015).

The two countries are symmetrical and the same size. Home and foreign households con-

[7]Our model directly refers to the model demonstrated by Bodenstein, Guerrieri and LaBriola (2019), a simplified version of Corsetti et al. (2010) and Benigno and Benigno (2006).

[8]Bodenstein et al. (2019) developed a nonlinear two-country model based on Benigno and Benigno (2006) and Corsetti et al. (2010) to provide an example of macroeconomic policy games (i.e., welfare gains of international policy coordination) between home and foreign central banks.

sume home-produced and foreign-produced goods with home bias and deep habit. Final goods producers maximize profits in monopolistically competitive domestic and foreign markets. Intermediate goods producers produce input for domestic final goods producers with monopolistic competition. The government levies a lump-sum tax on households and subsidizes firms to eliminate distortions from monopolistic competition and deep consumption habits in a steady state. Central banks maximize social welfare cooperatively or noncooperatively, using each domestic inflation as a policy instrument. The model structure of the foreign country is symmetrical to that of the home country. Unless otherwise noted, we denote foreign variables with an asterisk.

We should consider external habit (Smets and Wouters, 2007) rather than internal habit formation (Fuhrer, 2000, Christiano et al., 2005). Households with external habits cannot internalize the externalities of their utility on other households' utility because they care about other households' consumption rather than their past consumption ("keeping up with the Joneses" effect.) [9]

3.1 Households

Households derive utility from the consumption of home and foreign goods, and they form consumption habits at the level of individual goods rather than aggregate goods. When habits are deep, consumer preferences are at the individual goods level over time, generating the habit-persistence at a goods-by-goods level (Jacob and Uusküla, 2019).

A representative household $k \in [0,1]$ maximizes lifetime utility for an infinite period.

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{\left(X_t^k\right)^{1-\sigma}}{1-\sigma} - \chi \frac{\left(N_t^k\right)^{1+\upsilon}}{1+\upsilon} \right] \tag{1}$$

where X_t is the habit-adjusted aggregate consumption of the household. N_t represents hours worked, and β is the discount factor. σ denotes the inverse of the intertemporal elasticities of habit-adjusted consumption, υ is the inverse of the intertemporal elasticities of work, and χ is the relative weight on disutility from time spent working.

X_t^k is a CES composite of habit-adjusted consumption of domestic $X_{D,t}^k$ and imported goods $X_{M,t}^k$:

$$X_t^k = \left(\omega^{\frac{1}{\eta}} (X_{D,t}^k)^{\frac{\eta-1}{\eta}} + (1-\omega)^{\frac{1}{\eta}} (X_{M,t}^k)^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}} \tag{2}$$

[9] See Ravn et al. (2006) and Leith et al. (2012) for more details.

where η is the elasticity of substitution between domestic and imported goods and $\omega \in [0, 1]$ is a degree of home bias in consumption. $X^k_{D,t}$ and $X^k_{M,t}$ are also CES aggregates of the variety of goods $i \in [0, 1]$:

$$X^k_{D,t} = \left[\int_0^1 (C^k_{D,t}(i) - \theta_D \bar{S}_{D,t-1}(i))^{\frac{\epsilon-1}{\epsilon}} di\right]^{\frac{\epsilon}{\epsilon-1}}, \tag{3}$$

$$X^k_{M,t} = \left[\int_0^1 (C^k_{M,t}(i) - \theta_M \bar{S}_{M,t-1}(i))^{\frac{\epsilon-1}{\epsilon}} di\right]^{\frac{\epsilon}{\epsilon-1}} \tag{4}$$

where ϵ is the elasticity of substitution among the variety of goods, and $\bar{S}_{D,t}$ and $\bar{S}_{M,t}$ are the stocks of consumption habits.

Habit stocks are the sum of stocks at the end of the last period and habits that will be newly formed in the current period. The law of motions for habit stocks is given by:

$$\bar{S}_{D,t}(i) = \varrho_D \bar{S}_{D,t-1}(i) + (1 - \varrho_D)\bar{C}_{D,t}(i), \tag{5}$$

$$\bar{S}_{M,t}(i) = \varrho_M \bar{S}_{M,t-1}(i) + (1 - \varrho_M)\bar{C}_{M,t}(i). \tag{6}$$

Note that habit stocks are independent of each household k. $\bar{C}_{D,t}(i) = \int_0^1 C^k_{D,t}(i)dk$ and $\bar{C}_{M,t}(i) = \int_0^1 C^k_{D,t}(i)dk$ are the average consumption of domestic goods and imported goods, respectively. In each period, household k refers to the stock of the average household consumption in the previous period concerning good i and allocates a portion of it to habit formation. Note that habitual consumption stocks do not yield utility by definition.

Demand functions consist of a price-elastic component and an inelastic component due to the stock of habit consumption. The demand functions of domestic and imported goods for each good i are described as follows:

$$C^k_{D,t}(i) = \left(\frac{P_{D,t}(i)}{P_{D,t}}\right)^{-\epsilon} X^k_{D,t} + \theta_D \bar{S}_{D,t-1}(i), \tag{7}$$

$$C^k_{M,t}(i) = \left(\frac{P_{M,t}(i)}{P_{M,t}}\right)^{-\epsilon} X^k_{M,t} + \theta_M \bar{S}_{M,t-1}(i) \tag{8}$$

where $P_{D,t}(i)$ and $P_{M,t}(i)$ are price of domestic goods i and price of imported goods i, respectively. $P_{D,t}$ and $P_{M,t}$ are the aggregate price of domestic goods and imported goods, respectively:

$$P_{D,t} = \left(\int_0^1 P^{1-\epsilon}_{D,t}(i)di\right)^{\frac{1}{1-\epsilon}}, \tag{9}$$

$$P_{M,t} = \left(\int_0^1 P^{1-\epsilon}_{M,t}(i)di\right)^{\frac{1}{1-\epsilon}}. \tag{10}$$

The budget constraint for household k is as follows:

$$\int_0^1 \left[P_{D,t}(i)C^k_{D,t}(i) + P_{M,t}(i)C^k_{M,t}(i)\right] di + E_t\left\{Q_{t,t+1}D^k_{t+1}\right\} \leq D^k_t + W_t N^k_t + \Phi_t - T_t$$

where W_t is the nominal wage, and D_t is the nominal payoff on the portfolio of assets. $Q_{t,t+1}$ is the one-period stochastic discount factor for nominal payoffs relevant to the domestic household, and E_t is the mathematical expectation conditional on information available at time t. Following Clarida et al. (2002), we assume households have free access to a complete set of contingent claims traded internationally. Φ_t is the dividend from firms owned by households and T_t is lump-sum tax (if $T_t \geq 0$) or subsidy (if $T_t < 0$.)

Using (7)–(10), we obtain

$$P_{D,t}X^k_{D,t} + P_{M,t}X^k_{M,t} + \bar{v}_t + E_t\left\{Q_{t,t+1}D^k_{t+1}\right\} \leq D^k_t + W_t N^k_t + \Phi_t - T_t \tag{11}$$

and

$$\bar{v}_t = \theta_D \int_0^1 P_{D,t}(i)\bar{S}_{D,t-1}(i)di + \theta_M \int_0^1 P_{M,t}(i)\bar{S}_{M,t-1}(i)di \tag{12}$$

where $\bar{v}_t$ is the expenditure on the stock of habitual consumption. Note that from Equations (5) and (6), $\bar{v}_t$ include past average consumptions. Due to the term $\bar{v}_t$, the budget constraint includes consumption externality since average consumptions are given for each household k.

Households maximize lifetime utility (1) subject to its budget constraint (11). The first-order condtion, concerning habit-adjusted domestic consumpution $X_{D,t}$, can be defined as follows:

$$\beta E_t\left[\left(\frac{X_{t+1}}{X_t}\right)^{-\sigma+1/\eta}\left(\frac{X_{D,t+1}}{X_{D,t}}\right)^{-1/\eta}\left(\frac{P_{D,t}}{P_{D,t+1}}\right)\right] R_t = 1 \tag{13}$$

where $R_t \equiv 1/E_t\{Q_{t,t+1}\}$ denotes the risk-free gross nominal interest rate between periods t and $t+1$. The superscript k can be dropped in the optimal conditions due to the assumption of homogeneity of the representative households. The form of consumption Euler equation (13) is equivalent to that under traditional (or superficial) habit formation.

The first-order condition for hours worked is:

$$(X_t)^\sigma \left[\omega^{1/\eta}\left(\frac{X_{D,t}}{X_t}\right)^{-1/\eta}\right]^{-1} \chi(N_t)^\upsilon = \frac{W_t}{P_{D,t}}. \tag{14}$$

We can combine the first-order conditions for $X_{D,t}$ and $X_{M,t}$ to derive the following condition:

$$\left(\frac{\omega}{1-\omega}\right)^{1/\eta}\left(\frac{X_{D,t}}{X_{M,t}}\right)^{-1/\eta} = \frac{P_{D,t}}{P_{M,t}}. \tag{15}$$

3.2 Final goods Producers

Final-goods firms produce final goods by aggregating intermediate goods. We assume that final goods producers can flexibly change goods' prices and can make different pricing decisions for home and foreign markets. We also assume that domestic final-goods firms only purchase the intermediate goods produced in their home country. The final-goods firms face monopolistic competition and determine the optimal price and markups conditional on the demand function of each good. Note that the deep habit affects the demand function, as shown in (7). Unlike the standard NK model, the model with deep habits has a demand function that includes the term independent of the current price. This additional term implies that the demand function depends on the past average (or external) demand. In other words, the current pricing behavior of final goods producers can affect current and future expected demands and profits, called the intertemporal effect of a deep habit (Ravn et al., 2006).

Each firm produces final goods $Y(i)$ by CES-aggregating intermediate goods $Y(i,j)$:

$$Y_t(i) = \left(\int_0^1 Y_t(i,j)^{\frac{\xi-1}{\xi}} dj \right)^{\frac{\xi}{\xi-1}}. \tag{16}$$

Then, the final goods are shipped to the home and foreign markets.

$$Y_t(i) = Y_t^D(i) + Y_t^{EX}(i) \tag{17}$$

where $Y_t^D(i)$ is the domestic output provided to domestic consumers, and $Y_t^{EX}(i)$ is an exported output. Thus, the market-clearing condition of domestic output is

$$Y_t^D(i) = C_{D,t}(i), \tag{18}$$

$$Y_t^{EX}(i) = C_{M,t}^*(i). \tag{19}$$

Where $C_{M,t}^*(i)$ is the consumption of foreign households for the final goods i produced in the home country. Hereafter, foreign variables are denoted by a superscript $*$.

3.2.1 Profit maximization for final-goods firms

The profits of final goods firms in our model would be nonzero since the demand function has a constant term independent of its price. We define the profit of a final goods firm as follows:

$$\Phi_t(i) \equiv \Phi_{D,t}(i) + \Phi^*_{M,t}(i), \tag{20}$$

$$\Phi_{D,t}(i) \equiv \left(P_{D,t}(i) - P^m_t(i)\right) C_{D,t}(i), \tag{21}$$

$$\Phi^*_{M,t}(i) \equiv \left(\mathcal{E}_t P^*_{M,t}(i) - P^m_t(i)\right) C^*_{M,t}(i) \tag{22}$$

where $\Phi(i)$ is the total profit of the final good firm i, $\Phi_D(i)$ is the profit from domestic sales, and $\Phi^*_M(i)$ is the profit from sales to foreign households. $P^m_t(i)$ is the price index of intermediate goods for producing final goods i.

When households have deep habits in consumption, demand for goods may differ, and firms would differentiate prices in domestic and foreign markets. Ravn et al. (2007) named this "pricing to habits," showing that deviations from the LOP occur endogenously.[10] Thus, as described in Section 2, our model can be regarded as a sort of LCP.[11]

3.2.2 Optimal price setting of final goods for domestic consumption

Final goods firms maximize the discounted present value of future domestic sales profits as follows:

$$\max_{\{P_{D,t}(i), C_{D,t}(i)\}} E_t \sum_{s=0}^{\infty} Q_{t,t+S} \left(P_{D,t+S}(i) - P^m_{t+S}(i)\right) C_{D,t+S}(i) \tag{23}$$

subject to the demand function (7) and the law of motion of the habit stock (5):

$$C_{D,t+s}(i) = \left(\frac{P_{D,t+s}(i)}{P_{D,t+s}}\right)^{-\epsilon} X_{D,t+s} + \theta_D C_{D,t+s-1}(i) \tag{24}$$

where $\varrho_D = 0$ for simplicity. We derive the first-order conditions of this problem as follows:

$$\Lambda_{D,t}(i) = \left(P_{D,t}(i) - P^m_t(i)\right) + \theta_D E_t \left[Q_{t,t+1} \Lambda_{D,t+1}(i)\right], \tag{25}$$

$$C_{D,t}(i) = \Lambda_{D,t}(i) \left[\epsilon \left(\frac{P_{D,t}(i)}{P_{D,t}}\right)^{-\epsilon-1} (P_{D,t})^{-1} X_{D,t}\right] \tag{26}$$

[10] Monacelli (2005) related incomplete exchange rate pass-through on import prices to the LOP gap in a small-open NK model. Aside from deep habits, several other factors for real rigidities also lead to deviations from the LOP, such as local distribution costs (Corsetti and Dedola, 2005). See Jacob and Uusküla (2019).

[11] For the LCP model, see Engel (2011) and Corsetti et al. (2010). Fujiwara and Wang (2017) extended Engel's model to focus on noncooperative policy games under LCP in a two-country DSGE model.

where $\Lambda_{t,D}(i)$ is the Lagrange multiplier.

3.2.3 Optimal price settings for exports

Similarly, we set up the profit maximization problem for exported goods toward the foreign country, $Y_t^{EX} = C^*_{M,t}$,

$$\max_{\{\mathcal{E}_t P^*_{M,t}(i), C^*_{M,t}(i)\}} E_t \sum_{s=0}^{\infty} Q_{t,t+s} \left(\mathcal{E}_t P^*_{M,t+s}(i) - P^m_{t+s}(i) \right) C^*_{M,t+s}(i) \tag{27}$$

subject to the demand function

$$C^*_{M,t+s}(i) = \left(\frac{P^*_{M,t+s}(i)}{P^*_{M,t+s}} \right)^{-\epsilon} X^*_{M,t+s} + \theta^*_M C^*_{M,t+s-1}(i). \tag{28}$$

Note that $P^m_{t+s}(i)$ is as given when intermediate firms optimize their goods price.

Domestic firms choose the final-goods price in terms of home currency $\mathcal{E}_t P^*_M(i)$; conversely, foreign households determine their consumption behavior in terms of foreign currencies. Moreover, foreign households may have different consumption habits or preferences for identical goods from those in the home country.

The following are this problem's first-order conditions:

$$\Lambda^*_{M,t}(i) = \left(\mathcal{E}_t P^*_{M,t}(i) - P^m_t(i) \right) + \theta^*_M E_t \left[Q_{t,t+1} \Lambda^*_{M,t+1}(i) \right] \tag{29}$$

$$C^*_{M,t}(i) = \Lambda^*_{M,t}(i) \left[\epsilon \left(\frac{P^*_{M,t}(i)}{P^*_{M,t}} \right)^{-\epsilon-1} \left(\mathcal{E}_t P^*_{M,t} \right)^{-1} X^*_{M,t} \right]. \tag{30}$$

3.3 Intermediate goods producers

Following Leith et al. (2012, 2015), the intermediate goods firms facing monopolistic competition derive the marginal cost function by solving the cost minimization problem and determining the optimal price under nominal price rigidity.

NK models incorporating deep habits often adopt Rotemberg (1982)-type adjustment cost instead of Calvo pricing as price stickiness for aggregation convenience. Whether Rotemberg-type or Calvo (1983)-type price stickiness, the same Phillips curve can be obtained up to a first-order approximation (Woodford, 2003a). Conversely, Lombardo and Vestin (2008) showed that the two pricing assumptions could yield different social inflation costs when the steady-state is inefficient. Considering this difference, we choose the Calvo pricing as a source of price stickiness by following Leith, Moldovan and Rossi (2009). Specifically, we deal with these two rigidities separately: intermediate goods firms face Calvo-type nominal rigidities, whereas final goods firms' demand functions are affected by households' deep habits. In such a way, as suggested by Leith et al. (2009), the model can be built with Calvo pricing and deep habits without losing its desirable aggregate properties.

3.3.1 Production function of intermediate goods firms

The production function of the intermediate firm j that produces the intermediate good used for inputs in final goods i, $Y(i,j)$, is defined as follows:

$$Y_t(i,j) = A_t N_t(i,j) \tag{31}$$

where A_t is an exogenous technological progress that is common to individual intermediate goods firms. The (log-linearized) technology follows AR(1): $\ln A_t = \rho_a \ln A_{t-1} + \epsilon_t^a$ where $\rho_a \in (0,1)$ is the coefficient, and ϵ_t^a is independent and identically distributed productivity shock.

From the cost minimization problem for intermediate goods firms, we derive $MC_t = (1-\tau)W_t/A_t$ where MC_t is a nominal marginal cost common across firms. τ is the tax rate imposed on nominal costs chosen to achieve an efficient allocation at the steady-state equilibrium.[12]

Under monopolistic competition, intermediate goods firms face the following demand function:

$$Y_t(i,j) = \left(\frac{P_t^m(i,j)}{P_t^m(i)}\right)^{-\xi} Y_t(i) \tag{32}$$

and

$$P_t^m(i) \equiv \left(\int_0^1 (P_t^m)^{1-\xi}(i,j)dj\right)^{\frac{1}{1-\xi}} \tag{33}$$

where $P_t^m(i,j)$ is the price of the intermediate good j put into the final good i. $P_t^m(i)$ is the price index of the intermediate goods in terms of the final good i. We assume intermediate goods $Y_t(i,j)$ are provided only domestically and not traded internationally; thus, intermediate firms do not differentiate their prices between home and foreign countries.

[12] See 3.5.2 for deriving the optimal tax rate.

3.3.2 Optimal price setting of intermediate goods firms

The intermediate goods firm maximizes the discounted sum of profit in the face of Calvo (1983)-type nominal price rigidity. The nominal profit of the intermediate goods firm is defined as $\Phi_t^m(i,j) \equiv (P_t^m(i,j) - MC_t)Y_t(i,j)$. The profit maximization problem of the intermediate goods firm is as follows:

$$\max_{P_t^{mo}(i,j)} E_t \sum_{s=0}^{\infty} \alpha^s Q_{t,t+s}(P_t^{mo}(i,j) - MC_{t+s})Y_{t+s}(i,j)$$

subject to the demand function for the intermediate goods (32):

$$Y_{t+s}(i,j) = \left(\frac{P_t^{mo}(i,j)}{P_{t+s}^m(i)}\right)^{-\xi} Y_{t+s}(i) \tag{34}$$

where α is the degree of price stickiness, and $P_t^{mo}(i,j)$ is optimal price of intermediate goods j for final goods i.

The first-order condition is as follows:

$$\frac{P_t^{mo}(i,j)}{P_{D,t}} = \left(\frac{\zeta_t}{\zeta_t - 1}\right) \frac{E_t \sum_{s=0}^{\infty}(\alpha\beta)^s X_{t+s}^{-\sigma+1/\eta} X_{D,t+s}^{-1/\eta} mc_{t+s} \left(P_{t+s}^m(i)\right)^{\xi} Y_{t+s}(i)}{E_t \sum_{s=0}^{\infty}(\alpha\beta)^s X_{t+s}^{-\sigma+1/\eta} X_{D,t+s}^{-1/\eta} \left(\frac{P_{D,t+s}}{P_{D,t}}\right)^{-1} \left(P_{t+s}^m(i)\right)^{\xi} Y_{t+s}(i)} \tag{35}$$

where $mc_t \equiv MC_t/P_{D,t}$ is the real marginal cost in terms of domestic prices, and ζ_t is the time-varying elasticity of substitution between intermediate goods. $\zeta_t/(\zeta_t - 1)$ implies the desired markup of the intermediate goods firms. We assume that ζ_t follows AR(1) process: $\ln \zeta_t = (1 - \rho_m) \ln \xi + \rho_m \zeta_{t-1} + \varepsilon_t^m$. An exogenous increase in ζ_t implies a reduction in the desired markup of intermediate goods producers, interpreted as a negative cost-push shock on the supply side.[13]

(35) can be rewritten using the auxiliary variables $K_{1,t}, K_{2,t}$ as follows:

$$\frac{P_t^{mo}(i,j)}{P_{D,t}} = \left(\frac{\zeta_t}{\zeta_t - 1}\right) \frac{K_{1,t}}{K_{2,t}} \tag{36}$$

where

$$K_{1,t} \equiv E_t \sum_{s=0}^{\infty}(\alpha\beta)^s X_{t+s}^{-\sigma+1/\eta} X_{D,t+s}^{-1/\eta} mc_{t+s} \left(\frac{P_{t+s}^m(i)}{P_{D,t}}\right)^{\xi} Y_{t+s}(i),$$
$$K_{2,t} \equiv E_t \sum_{s=0}^{\infty}(\alpha\beta)^s X_{t+s}^{-\sigma+1/\eta} X_{D,t+s}^{-1/\eta} \left(\frac{P_{D,t+s}}{P_{D,t}}\right)^{-1} \left(\frac{P_{t+s}^m(i)}{P_{D,t}}\right)^{\xi} Y_{t+s}(i).$$

[13]See Ireland (2004) for a detailed explanation of the time-varying desired markup of intermediate goods.

By transforming these into recursive forms, we get

$$K_{1,t} = X_t^{-\sigma+1/\eta} X_{D,t}^{-1/\eta} mc_t \left(\frac{P_{D,t}}{P_t^m}\right)^{-\xi} Y_t(i) + \alpha\beta E_t \left[K_{1,t+1}(\Pi_{D,t+1})^{\xi}\right], \tag{37}$$

$$K_{2,t} = X_t^{-\sigma+1\eta} X_{D,t}^{-1/\eta} \left(\frac{P_{D,t}}{P_t^m}\right)^{-\xi} Y_t(i) + \alpha\beta E_t \left[K_{2,t+1}(\Pi_{D,t+1})^{\xi-1}\right]. \tag{38}$$

Here, we can drop the subscript i from its price level; hence, $P_t^m = P_t^m(i)$, assuming symmetric intermediate goods.

3.3.3 Price distribution of intermediate goods

Finally, the following is the distribution of the prices of international goods firms:

$$(P_t^m)^{1-\xi} = \alpha(P_{t-1}^m)^{1-\xi} + (1-\alpha)(P_t^{mo}(j))^{1-\xi}. \tag{39}$$

3.4 Terms of trade and the good-specific real exchange rate

We define the home country's terms of trade as the relative prices of foreign imported goods.

$$TOT_t \equiv \frac{P_{M,t}}{P_{D,t}}. \tag{40}$$

Because deep habits exist, the demand for domestic-produced goods could differ in domestic and foreign markets, $P_{D,t} \neq \mathcal{E}_t P_{M,t}^*$, which implies the LOP does not hold for domestic-produced (and foreign-produced) goods. Following Ravn et al. (2007), we define the gap between the domestic price and the foreign price of domestic-produced goods in terms of home currency, or the *good-specific real exchange rate*, as:[14]

$$\psi_t \equiv \frac{\mathcal{E}_t P_{M,t}^*}{P_{D,t}}. \tag{41}$$

Similarly, the deviation between the price in the foreign and home countries is defined as follows for a foreign-produced good.

$$\psi_t^* \equiv \frac{P_{M,t}/\mathcal{E}_t}{P_{D,t}^*}. \tag{42}$$

[14]The concepts of terms of trade and good-specific real exchange rates are different. $P_{M,t} \neq \mathcal{E}_t P_{M,t}^*$ because P_M is the price of import for the home country, P_M^* is the import price index for the foreign country, and home and foreign countries produce different goods.

By incorporating deep habits into the model, we can no longer define the consumer price index (CPI) and CPI-adjusted exchange rate (i.e., real exchange rate) in a typical manner. Instead, we focus on each price index and good-specific exchange rate.[15]

3.5 Equilibrium

3.5.1 Aggregate output

Aggregating the intermediate goods set for j, with the production function (31) and the demand function (32), we can express the intermediate goods as follows:

$$Y_t(i)\Delta_t(i) = A_t N_t(i) \tag{43}$$

where $\Delta_t(i) \equiv \int_0^1 \left(\frac{P_t^m(i,j)}{P_t^m(i)}\right)^{-\xi} dj$. Following Leith et al. (2015), we can drop the i subscript, with the assumption of final goods-producing sectors being symmetric:

$$Y_t = \frac{A_t}{\Delta_t} N_t, \tag{44}$$

$$\Delta_t = \int_0^1 \left(\frac{P_t^m(j)}{P_t^m}\right)^{-\xi} dj. \tag{45}$$

3.5.2 Perfect financial market

We assume a perfect financial market, which implies the following:

$$Q_{t,t+1} = \beta \left(\frac{X_{t+1}}{X_t}\right)^{-\sigma} \left(\frac{\omega^{1/\eta}(X_{D,t+1}/X_{t+1})^{-1/\eta}}{\omega^{1/\eta}(X_{D,t}/X_t)^{-1/\eta}}\right) \left(\frac{P_{D,t}}{P_{D,t+1}}\right),$$

$$\mathcal{E}_t Q^*_{t,t+1} = \beta \left(\frac{X_{t+1}}{X_t}\right)^{-\sigma} \left(\frac{(1-\omega)^{1/\eta}(X^*_{M,t+1}/X^*_{t+1})^{-1/\eta}}{(1-\omega)^{1/\eta}(X^*_{M,t}/X^*_t)^{-1/\eta}}\right) \left(\frac{\mathcal{E}_t P^*_{M,t}}{\mathcal{E}_{t+1} P^*_{M,t+1}}\right).$$

A perfect capital market equates both countries' stochastic discount factors for nominal payoffs in terms of the same currency denomination, $Q_{t,t+1} = \mathcal{E}_t Q^*_{t,t+1}$. Simplifying these equations allows us to derive an international risk-sharing condition regarding habit-adjusted consumption associated with the good-specific real exchange rate.

$$k_0 \left(\frac{1-\omega}{\omega}\right)^{\frac{1}{\eta}} \left(\frac{X_t^*}{X_t}\right)^{-\sigma+1/\eta} \left(\frac{X^*_{M,t}}{X_{D,t}}\right)^{-\eta} = \psi_t \tag{46}$$

[15] Alternatively, we can define CPI following Ravn et al. (2007) as $P_t \equiv \gamma_p P_{D,t} + (1-\gamma_p)P_{M,t}$, where $\gamma_p = \frac{P_D C_D}{P_D C_D + P_M C_M}$ is the (constant) relative weight of home goods prices at the steady state. We can also define aggregate consumption and real exchange rates as $C_t \equiv \frac{P_{D,t}C_{D,t}+P_{M,t}C_{M,t}}{P_t} = \frac{C_{D,t}+TOT_t C_{M,t}}{\gamma_p+(1-\gamma_p)TOT_t}$ and $q_t \equiv \frac{\mathcal{E}_t P_t^*}{P_t} = \frac{\gamma_p^* TOT_t/\psi_t^* + (1-\gamma_p^*)\psi_t}{\gamma_p+(1-\gamma_p)TOT_t}$ where P_t^* and γ_p^* are foreign CPI and relative price weight, respectively.

where $k_0 \equiv \psi_0 \left(\frac{1-\omega}{\omega}\right)^{\frac{1}{\eta}} \left(\frac{X_0^*}{X_0}\right)^{-\sigma+1/\eta} \left(\frac{X_{M,0}^*}{X_{D,0}}\right)^{-\eta}$ which is normalized to 1 as an assumption.

3.5.3 Aggregate profits

The home country's aggregate nominal profits can be expressed as follows:

$$\begin{aligned}\Phi_t &\equiv \int_0^1 \int_0^1 \Phi_t^m(i,j)djdi + \int_0^1 \Phi_{D,t}(i)di + \int_0^1 \Phi_{M,t}^*(i)di \\ &= P_{D,t}Y_t^D + \mathcal{E}_t P_{M,t}^* Y_t^{EX} - W_t N_t. \end{aligned} \tag{47}$$

We derive (47) in Appendix A.

3.5.4 Aggregate resource constraint

Using (47), we express the aggregate resource constraint as follows:

$$P_{D,t}C_{D,t} + \mathcal{E}_t P_{M,t}^* C_{M,t}^* = P_{D,t}Y_t^D + \mathcal{E}_t P_{M,t}^* Y_t^{EX} = \Phi_t + W_t N_t. \tag{48}$$

3.5.5 Overall markup and optimal tax

As described above, we chose government tax to achieve an efficient steady state: $\tau = 1 - \left(\frac{1}{1-\theta_D\beta}\right)\left(\frac{1}{\mu}\frac{\xi-1}{\xi}\right)$. This approach eliminates multiple distortions: the distortion from monopolistic competition between intermediate goods firms and the distortion of final goods consumption demand generated by deep habit.[16] We consider two types of markups: the markup of intermediate goods firms P_t^{mo}/MC_t and the markup of final goods firms $\mu_t \equiv P_{D,t}/P_t^{mo}$. Thus, the overall markup is the product of the two markups:

$$\frac{P_t^{mo}}{MC_t}\frac{P_{D,t}}{P_t^{mo}} = \frac{P_{D,t}}{MC_t} \equiv mc_t^{-1}.$$

From Appendix C, overall markup at the efficient steady state is derived as:

$$\frac{P^{mo}}{MC}\frac{P_D}{P^{mo}} = mc^{-1} = \left[\left(\frac{\xi-1}{\xi}\right)\frac{1}{\mu}\right]^{-1}$$

which the tax defined above eliminates. See Appendix B for details of equilibrium conditions and Appendix C for the steady states.

[16] See Leith et al. (2009) and Levine, Pearlman and Pierse (2008) for the derivation of optimal tax.

4 Optimal monetary policy

Following Bodenstein et al. (2019), this section considers the case of optimal monetary policy under international cooperation and non-cooperation.[17] Under cooperation, the global authority conducts monetary policy to maximize the economic welfare of both the domestic and foreign countries with instrument variables such as domestic inflation in each country. These variables are constrained by the state of the economic system and optimal conditions for households and firms in both countries. Consider the following Lagrangian:

$$\mathcal{L}_0 = E_0 \sum_{t=0}^{\infty} \beta^t \left[(1-\gamma) U_1(\boldsymbol{y}_t, \boldsymbol{y}_{t-1}, \boldsymbol{u}_t) + \gamma U_2(\boldsymbol{y}_t, \boldsymbol{y}_{t-1}, \boldsymbol{u}_t) + \boldsymbol{\lambda}_t f(\boldsymbol{y}_{t+1}, \boldsymbol{y}_t, \boldsymbol{y}_{t-1}, \boldsymbol{u}_t) \right] \tag{49}$$

where $\boldsymbol{y}_t$ is a vector of endogenous variables, and $\boldsymbol{u}_t$ is a vector of exogenous variables. $\boldsymbol{\lambda}_t$ is a vector of Lagrange multipliers.

$$U_1(\boldsymbol{y}_t, \boldsymbol{y}_{t-1}, \boldsymbol{u}_t) = \left[\frac{X_t^{1-\sigma}}{1-\sigma} - \chi \frac{N_t^{1+\upsilon}}{1+\upsilon} \right], \tag{50}$$

$$U_2(\boldsymbol{y}_t, \boldsymbol{y}_{t-1}, \boldsymbol{u}_t) = \left[\frac{(X_t^*)^{1-\sigma}}{1-\sigma} - \chi \frac{(N_t^*)^{1+\upsilon}}{1+\upsilon} \right] \tag{51}$$

are the household's utility functions in the home and foreign countries, respectively. $f(\cdot) = 0$ is the constraint provided by the equilibrium path of the private economy (See Appendix B). The Ramsey policy is computed by approximating the equilibrium system around the perturbation point where the Lagrange multipliers are at their steady state (see Appendix C). The optimal decision rules are computed around this steady state of the endogenous variables and the Lagrange multipliers.

Under non-cooperative policy, each country conducts monetary policy to maximize its economic welfare with instrument variables such as domestic inflation in each country. In this case, we define the following Lagrangian under open-loop Nash equilibrium:

$$\mathcal{L}_{0,j} = E_0 \sum_{t=0}^{\infty} \beta^t \left[U_j(\boldsymbol{y}_t, \boldsymbol{y}_{t-1}, \boldsymbol{u}_t) + \boldsymbol{\lambda}_{j,t} f(\boldsymbol{y}_{t+1}, \boldsymbol{y}_t, \boldsymbol{y}_{t-1}, \boldsymbol{u}_t) \right] \tag{52}$$

[17] This paper examines the optimal monetary policy under commitment. Givens (2016) investigated the welfare gains of commitment over discretion in a model with deep habits in consumption. Ida and Okano (2021, 2023b) explored government delegation policies in a small open economy to overcome the stabilization bias induced by the discretionary policy. We also consider the case of non-cooperative policy and investigate the welfare gain of cooperation in Section 5.4.

for $j \in \{1, 2\}$. We compute this problem's first-order conditions using Dynare, a software package for solving dynamic general equilibrium models, and a toolbox provided by Bodenstein et al. (2019)[18].

5 Quantitative analysis

In this section, we perform a quantitative analysis using the two-country sticky price model with deep habits developed in this paper. We calibrate the model parameters and assess their fit to real-world data by comparing moments, focusing on the empirical validity of deep habits. We also examine impulse responses to economic shocks and calculate the gains from policy coordination under different parameter settings.

5.1 Calibration

This section describes the parameters used in the simulations. Table 1 summarizes the parameter calibrations. We set the discount rate β to 0.99. The relative risk aversion σ (or the inverse of the habit-adjusted intertemporal substitution of consumption) is set to 2.0, while the elasticity of substitution between home and foreign goods η is set to 2.0. Clarida et al. (2002) and Pappa (2004) showed that a shock occurring in one country has a positive spillover effect on the other country when $\sigma\eta > 1$ in their models. This feature of the spillover effect carries over to our model.

Following Ravn et al. (2007), we assume that the degree of habits is equal among countries and goods.[19] Specifically, we set $\theta_D = \theta_M^* = \theta_M = \theta_D^* = \theta = 0.4$. We use this value as a baseline; the following sections compare impulse responses in cases with and without a deep consumption habit. This value might be lower than that of previous studies. According to Leith et al. (2012, 2015), macro-based estimates of deep habits vary from a relatively low value of 0.53, as in Ravn, Schmitt-Grohé and Uribe (2012), to extremely high values of 0.95–0.97, as in Ravn et al. (2006) and Lubik and Teo (2014). At the same time, micro-based estimates have much lower values of 0.29–0.5 (Ravina, 2005).

[18] More details on how to solve these problems are provided on Dynare's website. https://www.dynare.org/

[19] Following Ravn et al. (2006), we impose symmetry assumptions $\theta_D = \theta_D^*, \theta_M = \theta_M^*$. Moreover, further assumptions $\theta_D = \theta_M$ are imposed to make the steady-state system tractable.

Note that the determinacy problems should be considered as much as empirical plausibility. Models with deep habits are prone to indeterminacy due to the countercyclical markup behavior (Zubairy, 2014, Ravn, Schmitt-Grohe, Uribe and Uuskula, 2010, Jacob and Uusküla, 2019).[20] The interaction caused by deep habits might be more reinforced in our two-country model with sticky prices, raising concerns about model stability. To avoid the problem, we keep the degree of habit somewhat lower. We use $\theta = 0.4$ as a benchmark and compare the results to cases without deep habits ($\theta = 0$).

We set the degree of home bias ω to 0.85, following Bodenstein et al. (2019). For most of the remaining parameters, we followed Leith et al. (2015, 2009). We set the elasticity of substitution for both final ϵ and intermediate goods ξ to 11, the degree of price stickiness α to 0.75, and the Frisch labor supply elasticity (inverse of the intertemporal elasticity of work) υ to 4.0. Furthermore, we set the AR(1) coefficients for technology and cost-push shocks to $\rho_a = 0.9$ and $\rho_m = 0.9625$, respectively. The relative weight of the disutility of labor χ is set to 1, We set the home country weight in the coordinated Ramsey policy to $(1 - \gamma)$ =0.5.

5.2 Moments matching of the model with real economy data

This subsection analyzes the impact of incorporating deep habit formation on business cycle moments. As discussed in Sections 1 and 2, like superficial habits, deep habits generate a hump-shaped response of macroeconomic variables to economic shocks, as shown in Christiano et al. (2005), Leith et al. (2012) and Givens (2016). Moreover, replacing superficial habits with deep habits improves the fit between the impulse responses to monetary shocks and those estimated from structural VAR models, as reported by Ravn et al. (2010) and Givens (2016). Additionally, deep habits induce a crowding-in effect of government spending, aligning with empirical evidence, as highlighted by Ravn et al. (2006, 2010). In open economy models, Chari, Kehoe and McGrattan (2002) emphasized the importance of incorporating consumption persistence to account for the persistence of real exchange rates; habit formation in consumption is a candidate for achieving this persistence.

We evaluate its empirical fit to examine whether these characteristics are carried over in the

[20] In models with deep habits, expectations of higher future demand decrease current markups, increase real wages, and boost current demand, reinforcing future demand expectations and potentially leading to a self-fulfilling equilibrium (Jacob, 2015). Zubairy (2014) and Leith et al. (2012) examined the determinacy of interest rate rules in sticky price models with deep habits.

two-country model presented in this study. Specifically, we follow Backus, Kehoe and Kydtand (1995) and Ravn et al. (2007) and use long-term available data from the United States (US) and Canada. We compute the relative standard deviations and correlation coefficients of output, consumption, and terms of trade, comparing the statistical moments from actual data with those generated by the model. Table 2 compares business cycle moments derived from the benchmark model with those observed in the data.

[Table 2 around here.]

Table 2 compares business cycle moments obtained from the model under different degrees of deep habit formation ($\theta = 0.0, 0.4, 0.6$) with empirical data. All data are from the Federal Reserve Economic Data (FRED) database of the Federal Reserve Bank of St. Louis. For the US, output is measured as per capita real gross domestic product (GDP), which is seasonally adjusted annually and deflated by the consumer price index. Consumption is represented by per capita personal consumption expenditure, also seasonally adjusted at an annual rate. The terms of trade data are seasonally adjusted in a quarterly series. Due to data limitations, we use aggregate terms of trade rather than the bilateral terms of trade between the US and Canada. For Canada, GDP is measured as seasonally adjusted quarterly nominal GDP, deflated by the Canadian consumer price index to obtain real GDP. Consumption is represented by seasonally adjusted nominal household final consumption expenditure quarterly. We apply the Hodrick–Prescott (HP) filter to remove trends with a smoothing parameter of $\lambda = 1600$. The sample period spans 200 quarters (from Q3 1973 to Q2 2023), and the model-generated data also cover 200 periods.

The key statistics include the standard deviation of output σ_y, capturing the magnitude of business cycle fluctuations, as well as the relative volatility of consumption to output σ_c/σ_y, and the relative volatility of terms of trade to output σ_{tot}/σ_y. Additionally, the table reports the autocorrelation of output $\mathrm{Corr}(y_t, y_{t-1})$ and measures the persistence of business cycles and the correlation between domestic and foreign consumption $\mathrm{Corr}(c_t, c_t^*)$, reflecting the degree of international consumption comovement.

The results indicate that the model's standard deviation of output σ_y decreases as θ increases, approaching the empirical value of 1.702, though it remains slightly higher. The model underestimates the relative volatility of consumption σ_c/σ_y compared to the empirical value

(0.922); however, this measure increases with θ, improving the match. Similarly, the relative volatility of terms of trade σ_{tot}/σ_y increases with θ, bringing it closer to the empirical value of 1.844. Regarding the persistence of output fluctuations, the model consistently overestimates $\text{Corr}(y_t, y_{t-1})$, as its values remain higher than the empirical benchmark of 0.802. The international consumption correlation $\text{Corr}(c_t, c_t^*)$ is lower in the model than in the data (0.663); however, increasing θ improves the fit.[21]

While a higher degree of deep habit formation enhances the model's alignment with the data in certain dimensions—particularly in trade volatility and consumption correlation—discrepancies remain in output volatility and persistence. The results suggest that the model with deep habit formation can somewhat replicate the empirical moments of business cycles.

5.3 Impulse responses

This section observes the impulse responses of the foreign and the home country in the case of exogenous shocks that occurred in the foreign country. As in other open models, foreign shocks lead to spillovers to the home country. The spillover effects are affected by parameters specific to open models (e.g., the degree of economic openness). Furthermore, the nature of economic dynamics also depends on the degree of deep habit.; hence, the optimal policy response also varies. In the following, we observe the responses to productivity and cost-push shocks.

5.3.1 Impluse responses of foreign productivity shock

Figure 1 shows the impulse response to a productivity shock occurred in a foreign country. This figure compares impulse responses of the benchmark case ($\theta = 0.4$) with the case of no habit formation ($\theta = 0$).

[Figure 1 around here.]

We first review the effects of foreign productivity shock on the foreign country. The effects of the shock align with the results of previous studies in a closed economy model—a positive productivity shock raises natural output and decreases pressure on inflation. Under optimal

[21] Chari et al. (2002) reported that, for the US from the 1970s to the 1990s, the relative standard deviation of consumption was 0.79, the autocorrelation of GDP was 0.87, and the consumption correlation between the US and Europe was 0.27.

monetary policy, the central bank reduces these fluctuations by lowering interest rates. Thus, without deep habits, the central bank can fully control the fluctuations in inflation and the output gap associated with changes in productivity shocks. The solid blue line in Figure 1 (without deep habit) shows that the domestic inflation in a foreign country $\pi^*_{D,t}$ and output gap x^*_t are fully stabilized to the foreign productivity shocks.

In the case of deep habits, households tend to over-consume due to habit formation, leading to an increase in output deviating from the efficiency level (positive output gap). With deep habits, final goods firms are willing to lower the markups strategically to increase expected future demand, called the countercyclical response of markups or the intertemporal effect of deep habits (Ravn et al., 2006). In other words, final goods firms' pricing behavior amplifies rather than stabilizes fluctuations in demand, which is consistent with the empirical evidence.[22]

The above discussion is consistent with a closed economy case such as Leith et al. (2015); however, the relationship between the degree of deep habits and the magnitude of decreases in nominal interest rate in an open economy model is quite different from that in closed and open economies. In a closed economy with deep habits, the central bank is reluctant to lower interest rates in response to productivity shock because lowering interest rates leads to decreased markups of final goods and increased household overconsumption. However, in an open economy model, the nominal interest rate is lower than without deep habit.

The key difference between closed and open economies is the role of terms of trade. In a closed economy with strong consumption habits, the central conflict arises from the need to stabilize domestic prices while internalizing habit-driven externalities. An additional challenge emerges in an open economy: internalizing the externalities of terms of trade. The central bank aims to limit interest rate cuts to mitigate habit externalities; however, it also faces incentives to lower rates to improve economic welfare through terms of trade effects.[23]

As mentioned, deep habits generate the endogenous trade-off between stabilizing the output gap and inflation in a productivity shock that deviates from the "divine coincidence."[24] Unlike

[22]See, for example, Rotemberg and Woodford (1999).

[23]For further discussions on the effects of terms of trade adjustments, see Section 1, Galí and Monacelli (2005), Pappa (2004), and De Paoli (2009), among others.

[24]Givens (2016) introduced other common ways of overcoming the divine coincidence, aside from deep habits, such as putting a supply shock on the Phillips curve (Clarida, Galí and Gertler, 1999) and adding an interest rate stabilization term to the central bank's objective function (Amato and Laubach, 1999, Woodford, 2003b).

the closed economy case, the greater the degree of habit formation, the stronger this effect will be.

Moreover, in an open economy model, the change in markups caused by productivity shocks and the resulting change in the demand function leads to different pricing at home and foreign destinations. This pricing strategy is based on the different demand curves, or LCP, and is the source of the deviation from the LOP. The deviation of LOP for foreign goods is equivalent to the good-specific real exchange rate for foreign goods ψ_t^*.

With deep habit, the change in the terms of trade is greater, which leads to a more significant spillover effect on the home country. The intuition is as follows. Foreign productivity shocks increase foreign output, which worsens the terms of trade in a foreign country, with or without deep habits. Furthermore, with deep habits, the increase in foreign output causes habit formation and demand, which deviates from the LOP, causing the good-specific real exchange rate (Ravn et al., 2012). Due to the fluctuation of good-specific real exchange rates, the terms of trade deterioration are worse than in the open economy model without deep habit.

5.3.2 The role of degree of home bias in response to foreign productivity shock

We next examine the impulse responses to productivity shocks with and without home bias in consumption while keeping the degree of deep habits at the baseline level of $\theta = 0.4$ in Figure 2. This approach allows us to highlight the impacts specific to an open economy.

[Figure 2 around here.]

When transitioning from a baseline with home bias ($\omega = 0.85$) to a case without home bias ($\omega = 0.5$), increasing foreign production due to a foreign productivity shock results in a greater share of foreign goods being consumed in the home country as imported goods. This situation implies that the required adjustment in terms of trade for a given change in exports and imports is comparatively small. Consequently, the central bank's incentive to manipulate the terms of trade weakens, leading to a more muted interest rate reduction than the case with home bias. As a result, fluctuations in the foreign country's output gap, consumption, and inflation become smaller, improving economic welfare.

Moreover, Ravenna and Walsh (2006) showed that the endogenous cost-push shock could be generated by introducing a cost channel for monetary policy via borrowing constraint, resulting in an endogenous cost-push shock to the NK Phillips curve.

Interestingly, without home bias, i.e., $\omega = 0.5$, the good-specific real exchange rate remains unchanged in response to productivity shocks. The absence of home bias in consumption implies that relative price changes between domestic and foreign goods in response to shocks are identical across countries, ensuring that LOP holds. Conversely, in the presence of home bias, a foreign productivity shock leads to a decline in the price of foreign goods abroad, while demand for and the price of foreign goods in the domestic market remain unchanged. This asymmetry in price responses to shocks generates deviations from the LOP and fluctuations in the good-specific real exchange rate. Hence, the greater the home bias, the larger the fluctuations in the good-specific real exchange rate and the terms of trade.[25]

Without home bias, the spillover effect of foreign shocks on aggregate demand in the home economy intensifies. Thus, the domestic interest rate is more substantially lowered, causing greater volatility in consumption and the output gap in the home economy.

When deep habits exist, productivity shocks create a trade-off between output gap stabilization and inflation stabilization. Moreover, in open-economy models, markups of intermediate goods producers in both countries behave countercyclically, causing deviations from the LOP. These deviations imply that optimal interest rate responses in open economies with deep habits are more significant than in closed economies with deep habits. In contrast, without home bias, such deviations from the LOP do not arise, making the dynamics of an open economy with deep habits resemble a closed economy with deep habits.

5.3.3 Impulse responses of foreign cost-push shock

Figure 3 depicts the impulse responses to a reduction in the desired markup of foreign intermediate goods firms (1% increase in ξ_t^*), or a negative foregin cost-push shock.

[Figure 3 around here.]

A decline in the desired markup incentivizes those adjusting prices to lower them toward the average, reducing inflation and increasing output; shock acts as a negative cost-push shock. Even without deep habits, the central bank faces a trade-off between inflation stability and the output gap.

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[25] Ravn et al. (2007) also indicated that structural shocks endogenously change good-specific real exchange rates; however, they did not discuss the relationship between home bias and deep habits.

As with a productivity shock, deep habits lead final goods producers to lower markups, prioritizing future demand over immediate profits. This results in pricing-to-market behavior, depreciating good-specific real exchange rates and worsening the terms of trade. In response to the negative foreign cost-push shock, the foreign central bank cuts interest rates more aggressively to counteract deflationary pressure; thus, the decline in foreign inflation remains similar to the case without deep habits.

This situation contrasts with closed-economy models, where deep habits prompt the central bank to raise interest rates temporarily to internalize consumption externalities, causing an initial markup increase before it declines. In a two-country model with deep habits, the central bank also considers terms of trade externalities; thus, this temporary markup increase does not occur.

Figure 4 illustrates this by reducing home bias from $\omega = 0.85$ to $\omega = 0.5$. Similar to productivity shocks, the central bank exploits spillover effects through terms of trade adjustments. With lower home bias, the home country imports more foreign goods, allowing the central bank to reduce foreign markups further. Without home bias, firms no longer engage in pricing-to-market, muting good-specific real exchange rate movements; however, foreign shocks significantly impact the domestic economy.

[Figure 4 around here.]

5.4 The role of policy cordination

This subsection calculates the gains from international policy coordination compared to the case of non-cooperative policies by home and foreign countries. Following Fujiwara and Wang (2017), Kim (2023), the gain of coordination is given by

$$E_t\beta^j \sum_{j=0}^{\infty} \Big[(1-\gamma)U_1((1+\Omega)X_{t+j}^{\text{nash}}, N_{t+j}^{\text{nash}}) + \gamma U_2(C_{t+j}^{*\text{nash}}, N_{t+j}^{*\text{nash}})\Big] = (1-\gamma)W_{1,t}^{\text{coop}} + \gamma W_{2,t}^{\text{coop}} \tag{53}$$

where $W_{1,t}^{\text{coop}}$ and $W_{2,t}^{\text{coop}}$ are the cooperative welfare levels of the home and foreign country, respectively. Supersprict "nash" and "coop" indicate non-cooperative open-loop Nash equilibrium and cooperative Ramsey equilibrium, respectively. Ω is the parameter of consumption-equivalent welfare loss, representing the unit of steady-state consumption that must be given up to achieve the same level of welfare as the Nash equilibrium in international cooperation. Appendix D provides the derivation of the solution to this equation concerning Ω.

5.4.1 Welfare gains of policy coordination in the case of symmetric home bias

Table 3 shows welfare gains of policy coordination. The table divides the cases by the degree of stickiness price α and the degree of deep habits θ. Each row shows the exogenous economic shocks: home and foreign productivity, cost-push, and compound shocks. Each column shows a pair of price stickiness and the degree of deep habits. $\alpha^L = 0.01$ (flexible price), $\alpha^H = 0.75$ (sticky price), $\theta^L = 0.0$ (no deep habit), $\theta^M = 0.1$ and $\theta^H = 0.60$. As mentioned, including deep habits in the model when prices are sticky can lead to instability, especially with non-cooperative policies. For example, when price rigidity is 0.75, the degree of deep habits yielding a stable Nash equilibrium solution is $\theta = 0.1$ at most. In contrast, θ can be a relatively large value when prices are flexible.

[Table 3 around here.]

Panel (a) in Table 3 is a benchmark case where home and foreign countries share the same value of home bias, $\omega = \omega^* = 0.85$. Panel (a) shows when the price is flexible (in the case of α^L). When the home bias is symmetric, the gains from international cooperation are equal regardless of whether the shock originates from a home or foreign country since all other conditions are symmetric. The gain is smaller than when prices are sticky α^H, regardless of the type of shock or the presence of deep habits. This situation is unsurprising since price rigidity is one of the primary sources of market distortions. Flexible prices require little relative price adjustment, and even non-cooperation can yield desirable results; however, even when prices are flexible, the gain slightly increases as the degree of deep habits θ goes from 0.0 to 0.6.

When prices are sticky, the change in gains and their differences are marginal; θ can be from 0 to 0.1 at most due to the stability problems discussed above (especially in the non-cooperative case). However, when a deep habit exists, the gain decreases from 0.0796% to 0.0782% when the productivity shock is the source of fluctuation. In contrast, the effect of the degree of deep habits on the gains is negligible in the case of cost-push shocks.

The existence of deep habits with sticky prices reduces gains, indicating that reducing excessive consumption due to habits may benefit foreign countries. Habit consumption includes imported consumption, and reducing imported consumption will suppress foreign overproduction and over-labor. As a result, international desirability is almost achieved in non-cooperative policies, and the gain is smaller, particularly for productivity shocks. This result is also related to the degree of home bias. When the home bias is 0.85, the change is small, ranging from 0.0796% to 0.0782%. Conversely, in the case without home bias, panel (b), the change is relatively large, ranging from 0.0362% to 0.0263%. This result occurs because the effect of import inhibition is larger when there is no home bias.

5.4.2 Welfare gains of policy coordination in the case of no home bias

Panel (b) in Table 3 shows the case without home bias ($\omega = \omega^* = 0.50$). Compared to the case with high home bias panel (a), a substantial difference occurs; however, the gain in the case of cost-push shocks is larger with or without deep habit. The existing discussion of international coordination can explain the difference in gains due to different degrees of home bias. In the case of productivity shocks, no trade-off occurs between inflation and the output gap (in the absence of deep habit); in contrast,cost-push shocks generate a policy trade-off. This trade-off gives countries more incentives to strategically manipulate their terms of trade to improve their economies; thus, there is more room for policy coordination to remove the distortion.

5.4.3 Welfare gains of policy coordination in the case of asymmetric home bias

Panel (c) in Table 3 shows the case of heterogeneous home bias. When the home bias differs between countries, the shock has policy implications regardless of the source. The difference in the origin of the shocks has no significant impact when prices are flexible or during productivity shocks; however, the difference is particularly pronounced in the case of cost-push shocks. For example, when $\omega = 0.85, \omega^* = 0.60, \alpha = 0.75$, and $\theta = 0.10$, the gain with a cost-push shock in the home country is 0.1891; the gain in the case of foreign cost-push shock is 0.0583. Panel (b) shows that when a cost-push shock occurs in the home country, the home central bank attempts to increase its welfare through relative price adjustment by manipulating the terms of trade. In this case, it is relatively easy to cause expenditure switching as long as the home bias of the foreign country is small[26] . This distortion creates room for gains through policy

[26]De Paoli (2009) demonstrated that the greater the degree of substitutability between goods, the larger the expenditure switching effect from changes in the terms of trade, thus, the greater the benefits from improved

coordination. Conversely, when a cost-push occurs in the foreign country, and the home bias is significant in the home country, households in the home country are unwilling to import foreign goods; thus, an expenditure switch in favor of the foreign country cannot occur.

These results remain unchanged regardless of the presence or degree of deep habits. Strong deep habits can have a significant influence; however, our calibrations do not indicate that their presence is sufficient to override the effects of terms of trade manipulation or price stabilization. Nonetheless, the deep habits may reduce the gains from policy coordination when specific conditions are met, such as productivity shocks, price rigidity, and the absence of home bias.

5.4.4 Impulse responses: Ramsey optimal policy and non-cooperative policy

Figure 5 compared impulse responses to a foreign productivity shock under international coordination (Ramsey) and non-coordination (Nash). In a two-country model with price stickiness and deep habits, equilibrium stability is particularly problematic in the Nash case; therefore, Figures 5 and 6 assume $\theta = 0.1$.

[Figure 5 around here.]

Table 3 shows that the difference between coordinated and non-coordinated responses to productivity shocks is small. In contrast, Figure 5 indicates that the Ramsey policy results in a slightly larger interest rate cut than the Nash case. This situation reduces output gap and inflation volatility domestically, at the cost of slightly higher volatility in the foreign economy. In the Nash case, an incentive arises to manipulate terms of trade in favor of the foreign economy, whereas coordination internalizes this externality. Due to international policy coordination, the spillover of productivity shocks from foreign economies to the domestic economy is more limited than the Nash equilibrium, leading to a smaller reduction in domestic interest rates under coordination. The interest rate cut stimulates greater demand and production in the noncooperative case, causing an increase in real wages, a larger output gap, and higher inflation in the domestic economy. This difference in the sign of domestic inflation rates between coordinated and noncoordinated outcomes reflects these dynamics.

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terms of trade. Pappa (2004) showed the parameter conditions for international and intertemporal substitution elasticities under which the objectives of an independent central bank align with those of the social planner. Terms of trade fluctuations had little impact on domestic consumption or inflation, resulting in no gains from international coordination.

Figure 6 examines impulse responses to a negative foreign cost-push shock under coordination and non-coordination, yielding similar insights. Coordination slightly increases the foreign output gap and inflation volatility while stabilizing the domestic output gap; thus, the foundation of international coordination is the monetary policy that internalizes both terms of trade and consumption externalities from deep habits.

[Figure 6 around here.]

The differences in interest rate adjustments between the domestic and foreign economies primarily drive the gains from policy coordination. In the noncooperative case, foreign interest rates are adjusted more aggressively to counter shocks abroad, while in the coordinated case, these changes are more restrained. This situation also results in a more moderate change in domestic interest rates. Consequently, foreign volatility is slightly higher under coordination, while domestic volatility is somewhat lower compared to the non-cooperative outcome. These results reflect the spillover effects of terms of trade adjustments, incorporating the distortions in demand caused by habit formation.

6 Concluding remarks

Our paper shows how the dependence between the deep habits in consumption and optimal monetary policy in the closed NK model may differ when extended to a two-country NK model. We calibrate the model parameters and assess the model's fit to real-world data by comparing moments, focusing on the empirical validity of deep habits. Our findings indicate that incorporating deep habit formation improves the model's fit to empirical data, particularly regarding trade volatility and consumption correlation, suggesting that the model reasonably approximates business cycle dynamics.

Furthermore, in response to the structural shocks, the central bank changes the interest rate aggressively in the two-country open economy model. This result contrasts with a closed economy, where the central bank is reluctant to move interest rates. When there is a deep habit in consumption, the international central bank can exploit the externalities of the terms of trade. Habit formation might boost the expenditure-switching effect, which differentiates the aggressiveness of the central bank between closed and open economies with deep habits. This result occurs because habitual consumption does not generate utility, meaning that intratemporal substitution from aggregate consumption to leisure via exploiting terms of trade may improve welfare efficiently by reducing meaningless habitual consumption.

Furthermore, the deviations from the LOP, or the good-specific real exchange rate, generated by the deep habits are related to the degree of home bias. In particular, the deviations fully disappear with no home bias.

Moreover, our results suggest that international monetary policy coordination offers welfare gains, especially when prices are sticky ($\alpha_H = 0.75$); however, coordination is less beneficial when deep habits are present, particularly in response to productivity shocks. The degree of home bias is also influential, with higher home bias leading to more uniform coordination benefits across shocks. Furthermore, policy coordination can address inefficiencies from price stickiness and consumption habits; however, it is not always indispensable. Deep habits may reduce coordination gains under certain conditions, and non-cooperative outcomes can remain relatively efficient, especially with flexible prices.

Our study considered an optimally coordinated central bank; however, the discretionary policy was not considered. Therefore, investigating the gain in commitment, as in Givens (2016), is one possible avenue for future research.

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Appendix

A Aggregate Profits

Following Leith et al. (2009), we express the aggregate profits of intermediate goods firms as follows:

$$
\begin{aligned}
&\int_0^1 \int_0^1 \Phi_t^m(i,j)djdi \\
&= \int_0^1 \int_0^1 (P_t^m(i,j) - MC_t^n) Y_t(i,j)djdi \\
&= \int_0^1 \int_0^1 \left(P_{D,t}(i,j) - \frac{W_t}{A_t}\right) \left(\frac{P_t^m(i,j)}{P_t^m(i)}\right)^{-\xi} (Y_{D,t}(i) + Y_{EX,t}(i))djdi \\
&= \int_0^1 \int_0^1 \left[\frac{P_t^m(i,j)^{1-\xi}}{P_t^m(i)^{-\xi}}(Y_{D,t}(i) + Y_{EX,t}(i)) - \frac{W_t}{A_t} A_t N_t(i,j)\right] djdi \\
&= \int_0^1 (P_t^m(i))^{\xi}(Y_{D,t}(i) + Y_{EX,t}(i)) \left(\int_0^1 (P_t^m)^{1-\xi}(i,j)dj\right) di - W_t \int_0^1 \int_0^1 N_t(i,j)djdi \\
&= \int_0^1 (P_t^m(i))^{\xi}(Y_{D,t}(i) + Y_{EX,t}(i))(P_t(i)^m)^{1-\xi} di - W_t N_t \\
&= \int_0^1 (P_t^m(i))(Y_{D,t}(i) + Y_{EX,t}(i))di - W_t N_t
\end{aligned}
$$

Aggregate profits of final goods firms are:

$$
\begin{aligned}
&\int_0^1 \Phi_{D,t}(i)di + \int_0^1 \Phi_{M,t}^*(i)di \\
&= \int_0^1 (P_{D,t}(i) - P_t^m(i))Y_{D,t}(i)di + \int_0^1 (\mathcal{E}_t P_{M,t}^*(i) - P_t^m(i))Y_{EX,t}(i)di \\
&= \int_0^1 (P_{D,t}(i)Y_{D,t}(i) + \mathcal{E}_t P_{M,t}^*(i)Y_{EX,t}(i))di - \int_0^1 P_t^m(i)(Y_{D,t}(i) + Y_{EX,t}(i))di.
\end{aligned}
$$

Thus, aggregate profits in a home country are as follows:

$$
\begin{aligned}
\Phi_t \equiv& \int_0^1 \int_0^1 \Phi_t^m(i,j)djdi + \int_0^1 \Phi_{D,t}(i)di + \int_0^1 \Phi_{M,t}^*(i)di \\
=& \int_0^1 (P_t^m(i))(Y_{D,t}(i) + Y_{EX,t}(i))di - W_t N_t \\
+& \int_0^1 (P_{D,t}(i)Y_{D,t}(i) + \mathcal{E}_t P_{M,t}^*(i)Y_{EX,t}(i))di \\
-& \int_0^1 P_t^m(i)(Y_{D,t}(i) + Y_{EX,t}(i))di \\
=& \int_0^1 (P_{D,t}(i)Y_{D,t}(i) + \mathcal{E}_t P_{M,t}^*(i)Y_{EX,t}(i))di - W_t N_t \\
=& P_{D,t}Y_{D,t} + \mathcal{E}_t P_{M,t}^* Y_{EX,t} - W_t N_t
\end{aligned}
$$

The first and second terms on the right-hand side of the last equation can be derived by combining the CES aggregation properties of the production function and its prices with the cost minimization problem from the final goods firm.

B System of Equilibrium Conditions

Complete-market condition

$$\left(\frac{1-\omega}{\omega}\right)^{\frac{1}{\eta}}\left(\frac{X_t^*}{X_t}\right)^{-\sigma+1/\eta}\left(\frac{X_{M,t}^*}{X_{D,t}}\right)^{-\eta}=\psi_t \tag{B.1}$$

Habit-adjusted aggregate consumption

$$X_t=\left(\omega^{\frac{1}{\eta}}X_{D,t}^{\frac{\eta-1}{\eta}}+(1-\omega)^{\frac{1}{\eta}}X_{M,t}^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}} \tag{B.2}$$

$$X_t^*=\left(\omega^{\frac{1}{\eta}}X_{D,t}^{*\frac{\eta-1}{\eta}}+(1-\omega)^{\frac{1}{\eta}}X_{M,t}^{*\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}} \tag{B.3}$$

From habit-adjusted consumption (7), assuming that goods i are symmetric for households, we can write the following:

$$X_{D,t}=C_{D,t}-\theta_D\bar{S}_{D,t-1} \tag{B.4}$$

$$X_{M,t}=C_{M,t}-\theta_M\bar{S}_{M,t-1} \tag{B.5}$$

$$X_{D,t}^*=C_{D,t}^*-\theta_D^*\bar{S}_{D,t-1}^* \tag{B.6}$$

$$X_{M,t}^*=C_{M,t}^*-\theta_M^*\bar{S}_{M,t-1}^* \tag{B.7}$$

The stocks of habit consumption

$$\bar{S}_{H,t}=\varrho_H\bar{S}_{H,t-1}+(1-\varrho_H)\,C_{H,t} \tag{B.8}$$

$$\bar{S}_{F,t}=\varrho_F\bar{S}_{F,t-1}+(1-\varrho_F)\,C_{F,t} \tag{B.9}$$

$$\bar{S}_{H,t}^*=\varrho_H^*\bar{S}_{H,t-1}^*+(1-\varrho_H^*)\,C_{H,t}^* \tag{B.10}$$

$$\bar{S}_{F,t}^*=\varrho_F^*\bar{S}_{F,t-1}^*+(1-\varrho_F^*)\,C_{F,t}^* \tag{B.11}$$

First order conditions

$$\left(\frac{\omega}{1-\omega}\right)^{1/\eta}\left(\frac{X_{D,t}}{X_{M,t}}\right)^{-1/\eta}=\frac{P_{D,t}}{P_{M,t}}=TOT_t^{-1} \tag{B.12}$$

$$\omega^{-1/\eta}\chi(X_t)^{\sigma-1/\eta}X_{D,t}^{1/\eta}N_t^{\upsilon}=\frac{W_t}{P_{D,t}}=w_t \tag{B.13}$$

where $w_t \equiv W_t/P_{D,t}$.

Euler equation of consumption

$$1 = \beta E_t \left[\left(\frac{X_{t+1}}{X_t} \right)^{-\sigma+1/\eta} \left(\frac{X_{D,t+1}}{X_{D,t}} \right)^{-1/\eta} \left(\frac{P_{D,t}}{P_{D,t+1}} \right) \right] R_t \tag{B.14}$$

$$1 = \beta E_t \left[\left(\frac{X^*_{t+1}}{X^*_t} \right)^{-\sigma+1/\eta} \left(\frac{X^*_{D,t+1}}{X^*_{D,t}} \right)^{-1/\eta} \left(\frac{P^*_{D,t}}{P^*_{D,t+1}} \right) \right] R^*_t \tag{B.15}$$

the distribution of intermediate goods prices

$$(P^m_t)^{1-\xi} = \alpha (P^m_{t-1})^{1-\xi} + (1-\alpha)(P^{mo}_t(j))^{1-\xi}$$

Therefore,

$$1 = \alpha \left(\frac{1}{\Pi^m_t} \right)^{1-\xi} + (1-\alpha) \left(\frac{P^{mo}_t}{P^m_t} \right)^{1-\xi} \tag{B.16}$$

The evolution of price dispersion

$$\Delta^m_t = \alpha (\Pi^m_t)^{\xi} \Delta^m_{t-1} + (1-\alpha) \left(\frac{P^{mo}_t}{P^m_t} \right)^{-\xi} \tag{B.17}$$

Optimal intermediate price

$$\frac{P^{mo}_t}{P_{D,t}} = \left(\frac{\zeta_t}{\zeta_t - 1} \right) \frac{K_{1,t}}{K_{2,t}} \tag{B.18}$$

$$\frac{P^{mo*}_t}{P^*_{D,t}} = \left(\frac{\zeta^*_t}{\zeta^*_t - 1} \right) \frac{K^*_{1,t}}{K^*_{2,t}} \tag{B.19}$$

where

$$K_{1,t} = X_t^{-\sigma+1/\eta} X_{D,t}^{-1/\eta} mc_t \left(\frac{P_{D,t}}{P^m_t} \right)^{-\xi} Y_t + \alpha\beta E_t \left[K_{1,t+1} (\Pi_{D,t+1})^{\xi} \right] \tag{B.20}$$

$$K^*_{1,t} = (X^*_t)^{-\sigma+1/\eta} (X^*_{D,t})^{-1/\eta} mc^*_t \left(\frac{P^*_{D,t}}{P^{m*}_t} \right)^{-\xi} Y^*_t + \alpha\beta E_t \left[K^*_{1,t+1} (\Pi^*_{D,t+1})^{\xi} \right] \tag{B.21}$$

and

$$K_{2,t} = X_t^{-\sigma+1/\eta} X_{D,t}^{-1/\eta} \left(\frac{P_{D,t}}{P^m_t} \right)^{-\xi} Y_t + \alpha\beta E_t \left[K_{2,t+1} (\Pi_{D,t+1})^{\xi-1} \right] \tag{B.22}$$

$$K^*_{2,t} = (X^*_t)^{-\sigma+1/\eta} (X^*_{D,t})^{-1/\eta} \left(\frac{P^*_{D,t}}{P^{m*}_t} \right)^{-\xi} Y^*_t + \alpha\beta E_t \left[K^*_{2,t+1} (\Pi^*_{D,t+1})^{\xi-1} \right] \tag{B.23}$$

Where $\Pi_{D,t} = P_{D,t}/P_{D,t-1}$.

When intermediate goods firms can decide their price, subscript j can be removed from its optimal price since all firms behave in the same manner, $P^{mo}_t(j) = P^{mo}_t$ for all j.

Aggregate production function

$$Y_t = Y_{D,t} + Y_{EX,t} = \frac{A_t}{\Delta_t} N_t \tag{B.24}$$

$$Y_t^* = Y_{D,t}^* + Y_{EX,t}^* = \frac{A_t^*}{\Delta_t^*} N_t^* \tag{B.25}$$

Aggregate resource constraints are derived from home and foreign households' budget constraints,

$$P_{D,t}Y_{D,t} + \mathcal{E}_t P_{M,t}^* Y_{EX,t} = P_{D,t}C_{D,t} + \mathcal{E}_t P_{M,t}^* C_{M,t}^*$$

$$\Rightarrow Y_{D,t} + \psi_t Y_{EX,t} = C_{D,t} + \psi_t C_{M,t}^* \tag{B.26}$$

$$P_{D,t}^* Y_{D,t}^* + P_{M,t} Y_{EX,t}/\mathcal{E}_t = P_{D,t}^* C_{D,t}^* + P_{M,t} C_{M,t}/\mathcal{E}_t$$

$$\Rightarrow Y_{D,t}^* + \psi_t^* Y_{EX,t}^* = C_{D,t}^* + \psi_t^* C_{M,t} \tag{B.27}$$

Behavior of deep habit formation

$$\Lambda_{D,t}(i) = (P_{D,t}(i) - P_t^m(i)) + \theta_D E_t \left[Q_{t,t+1} \Lambda_{D,t+1}(i) \right]$$

$$Q_{t,t+1} = \beta \left(\frac{X_{t+1}}{X_t} \right)^{-\sigma+1/\eta} \left(\frac{X_{D,t}}{X_{D,t+1}} \right)^{1/\eta} \left(\frac{P_{D,t}}{P_{t+1}} \right)$$

Dropping i and eliminate $Q_{t,t+1}$ from the first equation,

$$\Lambda_{D,t} = (P_{D,t} - P_t^m) + \beta\theta_D E_t \left[\left(\frac{X_{t+1}}{X_t} \right)^{-\sigma+1/\eta} \left(\frac{X_{D,t}}{X_{D,t+1}} \right)^{1/\eta} \left(\frac{P_{D,t}}{P_{D,t+1}} \right) \Lambda_{D,t+1} \right]$$

$$\Rightarrow \lambda_{D,t} = \left(1 - \frac{P_t^m}{P_{D,t}} \right) + \beta\theta_D E_t \left[\left(\frac{X_{t+1}}{X_t} \right)^{-\sigma+1/\eta} \left(\frac{X_{D,t}}{X_{D,t+1}} \right)^{1/\eta} \lambda_{D,t+1} \right] \tag{B.28}$$

where $\lambda_{D,t} \equiv \Lambda_{D,t}/P_{D,t}$ is Lagrange multiplier in real terms. And,

$$C_{D,t} = \epsilon \lambda_{D,t} X_{D,t} \tag{B.29}$$

Similarly,

$$\Lambda_{M,t}^* = (\mathcal{E}_t P_{M,t}^* - P_t^m) + \beta\theta_M^* E_t \left[\left(\frac{X_{t+1}}{X_t} \right)^{-\sigma+1/\eta} \left(\frac{X_{D,t}}{X_{D,t+1}} \right)^{1/\eta} \left(\frac{P_{D,t}}{P_{D,t+1}} \right) \Lambda_{M,t+1}^* \right]$$

Dividing both sides by $\mathcal{E}_t P_{M,t}^*$ and using $\psi_t = \mathcal{E}_t P_{M,t}^*/P_{D,t}$,

$$\lambda_{M,t}^* = \left(1 - \frac{P_t^m}{\psi_t P_{D,t}} \right) + \beta\theta_M^* E_t \left[\left(\frac{X_{t+1}}{X_t} \right)^{-\sigma+1/\eta} \left(\frac{X_{D,t}}{X_{D,t+1}} \right)^{1/\eta} \left(\frac{\psi_{t+1}}{\psi_t} \right) \lambda_{M,t+1}^* \right] \tag{B.30}$$

where $\lambda^*_{M,t} = \Lambda^*_{M,t}/(\mathcal{E}_t P^*_{M,t})$. Also,

$$C^*_{M,t} = \epsilon \lambda^*_{M,t} X^*_{M,t} \tag{B.31}$$

The demand for goods produced in a foreign country, considering the deep habit, can be derived as follows:

$$\Lambda^*_{D,t}(i) = (P^*_{D,t}(i) - P^{m*}_t(i)) + \theta^*_D E_t \left[Q^*_{t,t+1}\Lambda^*_{D,t+1}(i)\right]$$
$$Q^*_{t,t+1} = \beta \left(\frac{X^*_{t+1}}{X^*_t}\right)^{-\sigma+1/\eta} \left(\frac{X^*_{D,t}}{X^*_{D,t+1}}\right)^{1/\eta} \left(\frac{P^*_{D,t}}{P^*_{t+1}}\right)$$

Dropping i and eliminate $Q^*_{t,t+1}$ from the first equation,

$$\Lambda^*_{D,t} = (P^*_{D,t} - P^{m*}_t) + \beta\theta^*_D E_t \left[\left(\frac{X^*_{t+1}}{X^*_t}\right)^{-\sigma+1/\eta} \left(\frac{X^*_{D,t}}{X^*_{D,t+1}}\right)^{1/\eta} \left(\frac{P^*_{D,t}}{P^*_{D,t+1}}\right)\Lambda^*_{D,t+1}\right]$$
$$\Rightarrow \lambda^*_{D,t} = \left(1 - \frac{P^{m*}_t}{P^*_{D,t}}\right) + \beta\theta^*_D E_t \left[\left(\frac{X^*_{t+1}}{X^*_t}\right)^{-\sigma+1/\eta} \left(\frac{X^*_{D,t}}{X^*_{D,t+1}}\right)^{1/\eta} \lambda^*_{D,t+1}\right] \tag{B.32}$$

where $\lambda^*_{D,t} \equiv \Lambda^*_{D,t}/P^*_{D,t}$ is Lagrange multiplier in real terms. And,

$$C^*_{D,t} = \epsilon \Lambda^*_{D,t}(P^*_{D,t})^{-1} X^*_{D,t} \Rightarrow C^*_{D,t} = \epsilon \lambda^*_{D,t} X^*_{D,t} \tag{B.33}$$

Similarly,

$$\Lambda_{M,t} = (P_{M,t}/\mathcal{E}_t - P^{m*}_t) + \beta\theta_M E_t \left[\left(\frac{X^*_{t+1}}{X^*_t}\right)^{-\sigma+1/\eta} \left(\frac{X^*_{D,t}}{X^*_{D,t+1}}\right)^{1/\eta} \left(\frac{P^*_{D,t}}{P^*_{D,t+1}}\right)\Lambda_{M,t+1}\right]$$

Dividing both sides by $P_{M,t}/\mathcal{E}_t$ and using $\psi^*_t = \frac{P_{M,t}/\mathcal{E}_t}{P^*_{D,t}}$,

$$\frac{\Lambda_{M,t}}{P_{M,t}/\mathcal{E}_t} = \left(1 - \frac{P^{m*}_t}{\psi^*_t P^*_{D,t}}\right) + \beta\theta_M E_t \left[\left(\frac{X^*_{t+1}}{X^*_t}\right)^{-\sigma+1/\eta} \left(\frac{X^*_{D,t}}{X^*_{D,t+1}}\right)^{1/\eta} \left(\frac{\psi^*_{t+1}}{\psi^*_t}\right)\left(\frac{\Lambda^*_{M,t+1}}{\mathcal{E}_{t+1}P^*_{M,t+1}}\right)\right]$$
$$\Rightarrow \lambda_{M,t} = \left(1 - \frac{P^{m*}_t}{\psi^*_t P^*_{D,t}}\right) + \beta\theta_M E_t \left[\left(\frac{X^*_{t+1}}{X^*_t}\right)^{-\sigma+1/\eta} \left(\frac{X^*_{D,t}}{X_{D,t+1}}\right)^{1/\eta} \left(\frac{\psi^*_{t+1}}{\psi^*_t}\right)\lambda_{M,t+1}\right] \tag{B.34}$$

where $\lambda_{M,t} = \Lambda_{M,t}/(P_{M,t}/\mathcal{E}_t)$. Also,

$$C_{M,t} = \epsilon \Lambda_{M,t}\frac{1}{P_{M,t}/\mathcal{E}_t} X_{M,t}$$
$$\Rightarrow C_{M,t} = \epsilon \lambda_{M,t} X_{M,t} \tag{B.35}$$

Inflation, relative prices and real variables

Terms of trade

$$TOT_t = \frac{P_{M,t}}{P_{D,t}} \tag{B.36}$$

$$TOT_t^* = \frac{P_{M,t}^*}{P_{D,t}^*} \tag{B.37}$$

Good-specific real exchange rate

$$\psi_t = \frac{\mathcal{E}_t P_{M,t}^*}{P_{D,t}} \tag{B.38}$$

$$\psi_t^* = \frac{P_{M,t}/\mathcal{E}_t}{P_{D,t}^*} \tag{B.39}$$

Terms of trade and good-specific real exchange rates are combined as follows:

$$\psi_t \psi_t^* = \frac{\mathcal{E}_t P_{M,t}^* P_{M,t}/\mathcal{E}_t}{P_{D,t} P_{D,t}^*} = TOT_t \cdot TOT_t^* \tag{B.40}$$

Optimal relative intermediate goods price

$$p_t^{mo} \equiv \frac{P_t^{mo}}{P_{D,t}} \tag{B.41}$$

$$p_t^{mo*} \equiv \frac{P_t^{mo*}}{P_{D,t}^{m*}} \tag{B.42}$$

Price markup of final goods firms

$$\mu_t \equiv \frac{P_{D,t}}{P_t^m} \tag{B.43}$$

$$\mu_t^* \equiv \frac{P_{D,t}^*}{P_t^{m*}} \tag{B.44}$$

We can express the relative optimal price to the average price of intermediate goods as follows:

$$\frac{P_t^{mo}}{P_t^m} = \frac{P_t^{mo}}{P_{D,t}} \frac{P_{D,t}}{P_t^m} = p_t^{mo} \mu_t \tag{B.45}$$

$$\frac{P_t^{mo*}}{P_t^{m*}} = p_t^{mo*} \mu_t^* \tag{B.46}$$

Intermediate goods price inflation

$$\Pi_t^m = \frac{P_t^m}{P_{t-1}^m} \tag{B.47}$$

$$\Pi_t^{m*} = \frac{P_t^{m*}}{P_{t-1}^{m*}} \tag{B.48}$$

Domestic price inflation

$$\Pi_{D,t} \equiv \frac{P_{D,t}}{P_{D,t-1}} = \frac{P_{D,t}}{P_t^m}\frac{P_{t-1}^m}{P_{D,t-1}}\frac{P_t^m}{P_{t-1}^m} = \frac{\mu_t}{\mu_{t-1}}\Pi_t^m \tag{B.49}$$

$$\Pi_{D,t}^* = \frac{\mu_t^*}{\mu_{t-1}^*}\Pi_t^{m*} \tag{B.50}$$

Real wage

$$w_t = \frac{W_t}{P_{D,t}} \tag{B.51}$$

$$w_t^* = \frac{W_t^*}{P_{D,t}^*} \tag{B.52}$$

Real marginal cost

$$mc_t = \frac{MC_t}{P_{D,t}} = \frac{w_t}{A_t} \tag{B.53}$$

$$mc_t^* = \frac{w_t^*}{A_t^*} \tag{B.54}$$

C Steady state

$$\left(\frac{1-\omega}{\omega}\right)^{\frac{1}{\eta}} \left(\frac{X^*}{X}\right)^{-\sigma+1/\eta} \left(\frac{X_M^*}{X_D}\right)^{-\eta} = \psi \tag{C.1}$$

$$X = \left(\omega^{\frac{1}{\eta}} X_D^{\frac{\eta-1}{\eta}} + (1-\omega)^{\frac{1}{\eta}} X_M^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}} \tag{C.2}$$

$$X^* = \left(\omega^{\frac{1}{\eta}} X_D^{*\frac{\eta-1}{\eta}} + (1-\omega)^{\frac{1}{\eta}} X_M^{*\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}} \tag{C.3}$$

$$X_D = C_D - \theta_D S_D \tag{C.4}$$

$$X_M = C_M - \theta_M S_M \tag{C.5}$$

$$X_D^* = C_D^* - \theta_D^* S_D^* \tag{C.6}$$

$$X_M^* = C_M^* - \theta_M^* S_M^* \tag{C.7}$$

$$S_D = C_D \tag{C.8}$$

$$S_M = C_M \tag{C.9}$$

$$S_D^* = C_D^* \tag{C.10}$$

$$S_M^* = C_M^* \tag{C.11}$$

$$\left(\frac{\omega}{1-\omega}\right)^{1/\eta}\left(\frac{X_D}{X_M}\right)^{-1/\eta} = TOT^{-1} \tag{C.12}$$

$$\left(\frac{\omega}{1-\omega}\right)^{1/\eta}\left(\frac{X_D^*}{X_M^*}\right)^{-1/\eta} = TOT(\psi\psi^*)^{-1} \tag{C.13}$$

$$\omega^{-1/\eta}\chi(X)^{\sigma-1/\eta}(X_D)^{1/\eta}(N)^{\upsilon} = w \tag{C.14}$$

$$\omega^{-1/\eta}\chi(X^*)^{\sigma-1/\eta}(X_D^*)^{1/\eta}(N^*)^{\upsilon} = w^* \tag{C.15}$$

$$1 = \beta\Pi_D^{-1}R \tag{C.16}$$

$$1 = \beta\Pi_D^{*-1}R^* \tag{C.17}$$

$$1 = \alpha(\Pi^m)^{-1+\xi} + (1-\alpha)(p^{mo}\mu)^{1-\xi} \tag{C.18}$$

$$1 = \alpha(\Pi^{m*})^{-1+\xi} + (1-\alpha)(p^{mo*}\mu^*)^{1-\xi} \tag{C.19}$$

$$\Delta^m = \frac{(1-\alpha)(p^{mo}\mu)^{-\xi}}{1-\alpha(\Pi^m)^{\xi}} \tag{C.20}$$

$$\Delta^{m*} = \frac{(1-\alpha)(p^{mo*}\mu^*)^{-\xi}}{1-\alpha(\Pi^{m*})^{\xi}} \tag{C.21}$$

$$p^{mo} = \left(\frac{\zeta}{\zeta-1}\right)\frac{K_1}{K_2}$$

$$p^{mo*} = \left(\frac{\zeta^*}{\zeta^*-1}\right)\frac{K_1^*}{K_2^*}$$

$$K_1 = \frac{(X)^{-\sigma+1/\eta}(X_D)^{-1/\eta}mc(\mu)^{-\xi}Y}{1-\alpha\beta(\Pi_D)^{\xi}} \tag{C.22}$$

$$K_1^* = \frac{(X^*)^{-\sigma+1/\eta}(X_D^*)^{-1/\eta}mc^*(\mu^*)^{-\xi}Y^*}{1-\alpha\beta(\Pi_D^*)^{\xi}} \tag{C.23}$$

$$K_2 = \frac{(X)^{-\sigma+1/\eta}(X_D)^{-1/\eta}\mu^{-\xi}Y}{1-\alpha\beta(\Pi_D)^{\xi-1}} \tag{C.24}$$

$$K_2^* = \frac{(X^*)^{-\sigma+1/\eta}(X_D^*)^{-1/\eta}(\mu^*)^{-\xi}Y^*}{1-\alpha\beta(\Pi_D^*)^{\xi-1}} \tag{C.25}$$

$$\Rightarrow (p^{mo})^{-1} = \left(\frac{\zeta}{\zeta - 1}\right) \frac{1 - \alpha\beta(\Pi_D)^{\xi-1}}{1 - \alpha\beta(\Pi_D)^{\xi}} mc \tag{C.26}$$

$$(p^{mo*})^{-1} = \left(\frac{\zeta^*}{\zeta^* - 1}\right) \frac{1 - \alpha\beta(\Pi_D^*)^{\xi-1}}{1 - \alpha\beta(\Pi_D^*)^{\xi}} mc^* \tag{C.27}$$

$$Y = \frac{A}{\Delta^m} N \tag{C.28}$$

$$Y^* = \frac{A^*}{\Delta^{m*}} N^* \tag{C.29}$$

$$mc = \frac{w}{A} \tag{C.30}$$

$$mc^* = \frac{w^*}{A^*} \tag{C.31}$$

$$Y_D + \psi Y_{EX} = C_D + \psi_t C_M^* \tag{C.32}$$

$$Y_D^* + \psi_t^* Y_{EX}^* = C_D^* + \psi^* C_M \tag{C.33}$$

$$\mu = [1 - (1 - \beta\theta_D)\lambda_D]^{-1} \tag{C.34}$$

$$\psi\mu = [1 - (1 - \beta\theta_M^*)\lambda_M^*]^{-1} \tag{C.35}$$

$$\mu^* = [1 - (1 - \beta\theta_D^*)\lambda_D^*]^{-1} \tag{C.36}$$

$$\psi^*\mu^* = [1 - (1 - \beta\theta_M)\lambda_M]^{-1} \tag{C.37}$$

$$C_D = \epsilon\lambda_D X_D \tag{C.38}$$

$$C_M^* = \epsilon\lambda_M^* X_M^* \tag{C.39}$$

$$C_D^* = \epsilon\lambda_D^* X_D^* \tag{C.40}$$

$$C_M = \epsilon\lambda_M X_M \tag{C.41}$$

$$A = 1 \tag{C.42}$$

$$A^* = 1 \tag{C.43}$$

$$\zeta = \xi \tag{C.44}$$

$$\zeta^* = \xi^* \tag{C.45}$$

$$\Pi^m = \Pi_D \tag{C.46}$$

$$\Pi^{*m} = \Pi_D^* \tag{C.47}$$

For tractability, we make some simplifying and symmetry assumptions.

Eliminating λ, C in both countries

$$\Rightarrow \mu = \left[1 - \frac{1-\beta\theta_D}{(1-\theta_D)\epsilon}\right]^{-1}$$
$$\psi\mu = \left[1 - \frac{1-\beta\theta_M^*}{(1-\theta_M^*)\epsilon}\right]^{-1}$$
$$\mu^* = \left[1 - \frac{(1-\beta\theta_D^*)}{(1-\theta_D^*)\epsilon}\right]^{-1}$$
$$\psi^*\mu^* = \left[1 - \frac{(1-\beta\theta_M)}{(1-\theta_M)\epsilon}\right]^{-1}$$

Symmetry assumptions, $\theta_D = \theta_D^*, \theta_M = \theta_M^*$.

$$\Rightarrow \mu = \mu^*$$
$$\psi = \psi^*$$

Further assumptions, $\theta_D = \theta_M$. $\psi = \psi^* = 1$

$$\left(\frac{1-\omega}{\omega}\right)^{\frac{1}{\eta}} \left(\frac{X_M^*}{X_D}\right)^{-\eta} = 1$$

Finally, assuming $\Pi^m = \Pi_D = 1$, steady state prices are determined, and the deterministic steady state system equations are uniquely determined analytically.

D Welfare gains from international cooperation

We derive the gains from international cooperation in terms of steady-state consumption.[27]

$$E_t \sum_{j=0}^{\infty} \beta^j \left[(1-\gamma)U((1+\Omega)X_{t+j}^b, N_{t+j}^b) + \gamma U^*(C_{t+j}^{*b}, N_{t+j}^{*b})\right] = (1-\gamma)W_t^a + \gamma W_t^{*a}$$

where superscript b and a indicate Nash equilibrium and cooperative equilibrium, respectively. Ω represents the unit of steady-state consumption that must be given up to achieve the same level of welfare as the Nash equilibrium in international cooperation.

Define the global welfare as follows:

$$W_{w,t}^i \equiv (1-\gamma)W_t^i + \gamma W_t^{*i}, \text{ for } i \in [b, a].$$

[27] This appendix is based on Landi (2021).

By using this, the welfare gain of cooperation from non-cooperation can be calculated as follows:

$$
\begin{aligned}
& W^a_{w,t} - W^b_{w,t} \\
=& E_t \sum_{j=0}^{\infty} \beta^j \left[(1-\gamma) U((1+\Omega) X^b_{t+j}, N^b_{t+j}) + \gamma U(C^{*b}_{t+j}, N^{*b}_{t+j}) \right] \\
& - E_t \sum_{j=0}^{\infty} \beta^j \left[(1-\gamma) U(X^b_{t+j}, N^b_{t+j}) + \gamma U(C^{*b}_{t+j}, N^{*b}_{t+j}) \right] \\
=& E_t \sum_{j=0}^{\infty} \beta^j \left[(1-\gamma) \left\{ U((1+\Omega) X^b_{t+j}, N^b_{t+j}) - U(X^b_{t+j}, N^b_{t+j}) \right\} \right]
\end{aligned}
$$

We specify the period utility function as $U(X_t, N_t) = \dfrac{(X_t)^{1-\sigma}}{1-\sigma} - \chi \dfrac{(N_t)^{1+\upsilon}}{1+\upsilon}$,

$$
\begin{aligned}
& W^a_{w,t} - W^b_{w,t} \\
&= E_t \sum_{j=0}^{\infty} \beta^j \left[(1-\gamma) \left\{ \frac{\left((1+\Omega) X^b_{t+j}\right)^{1-\sigma}}{1-\sigma} - \chi \frac{\left(N^b_{t+j}\right)^{1+\upsilon}}{1+\upsilon} - \frac{\left(X^b_{t+j}\right)^{1-\sigma}}{1-\sigma} + \chi \frac{\left(N^b_{t+j}\right)^{1+\upsilon}}{1+\upsilon} \right\} \right] \\
&= E_t \sum_{j=0}^{\infty} \beta^j \left[(1-\gamma) \left\{ \frac{\left((1+\Omega) X^b_{t+j}\right)^{1-\sigma}}{1-\sigma} - \frac{\left(X^b_{t+j}\right)^{1-\sigma}}{1-\sigma} \right\} \right] \\
&= E_t \sum_{j=0}^{\infty} \beta^j \left[(1-\gamma) \left\{ \frac{\left((1+\Omega)^{1-\sigma} - 1\right) (X^b_{t+j})^{1-\sigma}}{1-\sigma} \right\} \right]
\end{aligned}
$$

Evaluating this at the steady state, $X^b_{t+j} = X$,

$$
\begin{aligned}
W^a_{w,t} - W^b_{w,t} &= E_t \sum_{j=0}^{\infty} \beta^j \left[(1-\gamma) \left\{ \frac{\left((1+\Omega)^{1-\sigma} - 1\right) (X)^{1-\sigma}}{1-\sigma} \right\} \right] \\
&= \left[(1-\gamma) \left\{ \frac{\left((1+\Omega)^{1-\sigma} - 1\right) (X)^{1-\sigma}}{1-\sigma} \right\} \right] E_t \sum_{j=0}^{\infty} \beta^j \\
&= \frac{(1-\gamma) \left((1+\Omega)^{1-\sigma} - 1\right) (X)^{1-\sigma}}{(1-\sigma)(1-\beta)}
\end{aligned}
$$

Therefore,

$$
\Omega = \left[(W^a_{w,t} - W^b_{w,t})(1-\sigma)(1-\beta)(1-\gamma)^{-1} X^{\sigma-1} + 1 \right]^{\frac{1}{1-\sigma}} - 1
$$

Parameter	Value	Description
β	0.99	Subjective discount factor
σ	2.0	Inverse of the intertemporal elasticities of habit-adjusted consumption
ω, ω^*	0.85	Degree of home bias in consumption
ϵ	11	Elasticity of substitution across final goods
ξ	11	Elasticity of substitution across intermediate goods
η	2.0	Elasticity of substitution between home and foreign goods
θ_i, θ_i^*	0.4	Degree of habit persistence ($i = D, M$)
ϱ_i, ϱ_i^*	0.0	Persistence of habit stock ($i = D, M$)
υ	4.0	Inverse of the intertemporal elasticities of work
χ	1	Relative weight on disutility from time spent working
α	0.75	Degree of price stickiness
ρ_m	0.9625	Persistence of cost-push shock
ρ_a	0.9	Persistence of technology shock
γ	0.5	Weight on home country in Ramsey policy

Table 1: structural parameter values used in simulations

	$\theta = 0.0$	$\theta = 0.4$	$\theta = 0.6$	Data
σ_y	1.869	1.847	1.839	1.702
σ_c/σ_y	0.657	0.670	0.764	0.922
σ_{tot}/σ_y	1.261	1.433	1.797	1.844
$\mathrm{Corr}(y_t, y_{t-1})$	0.865	0.883	0.892	0.802
$\mathrm{Corr}(c_t, c_t^*)$	0.363	0.412	0.494	0.663

Table 2: Empirical validation of a two-country sticky price model with deep habits: Business cycle moments

Note: σ_y denotes the standard deviation of output. σ_c/σ_y and σ_{tot}/σ_y represent the relative volatilities of consumption and terms of trade to output, respectively. $\mathrm{Corr}(y_t, y_{t-1})$ captures the autocorrelation of output. $\mathrm{Corr}(c_t, c_t^*)$ measures the correlation between domestic and foreign consumption. θ denotes the degree of deep habit. All data are from the Federal Reserve Economic Data (FRED) database of the Federal Reserve Bank of St. Louis. For the US, output is measured as per capita real GDP, seasonally adjusted annually, deflated by the consumer price index. Consumption is represented by per capita personal consumption expenditure, also seasonally adjusted at an annual rate. The terms of trade data are seasonally adjusted in a quarterly series. Due to data limitations, we use aggregate terms of trade rather than the bilateral terms of trade between the US and Canada. For Canada, GDP is measured as seasonally adjusted quarterly nominal GDP, deflated by the Canadian consumer price index to obtain real GDP. Consumption is represented by seasonally adjusted nominal household final consumption expenditure quarterly. We apply the Hodrick–Prescott (HP) filter to remove trends with a smoothing parameter of $\lambda = 1600$. The sample period spans 200 quarters, from Q3 1973 to Q2 2023, and the model-generated data also cover 200 periods.

(a) $\omega = \omega^* = 0.85$: benchmark case

	(α^L, θ^L)	(α_L, θ^M)	(α_L, θ^H)	(α_H, θ_L)	(α_H, θ_M)
home productivity shock a_t	0.0001	0.0001	0.0004	0.0796	0.0782
foreign productivity shock a_t^*	0.0001	0.0001	0.0004	0.0796	0.0782
home cost-push shock ζ_t	0.0004	0.0004	0.0004	0.0568	0.0568
foreign cost-push shock ζ_t^*	0.0004	0.0004	0.0004	0.0568	0.0568
home shocks a_t, ζ_t	0.0005	0.0005	0.0008	0.1365	0.1351
foreign shocks a_t^*, ζ_t^*	0.0005	0.0005	0.0008	0.1365	0.1351
all shocks $a_t, a_t^*, \zeta_t, \zeta_t^*$	0.0010	0.0010	0.0016	0.2735	0.2705

(b) $\omega = \omega^* = 0.50$: no home-bias

	(α^L, θ^L)	(α_L, θ^M)	(α_L, θ^H)	(α_H, θ_L)	(α_H, θ_M)
home productivity shock a_t	0.0000	0.0000	0.0018	0.0362	0.0263
foreign productivity shock a_t^*	0.0000	0.0000	0.0018	0.0362	0.0263
home cost-push shock ζ_t	0.0019	0.0019	0.0018	0.2679	0.2680
foreign cost-push shock ζ_t^*	0.0019	0.0019	0.0018	0.2679	0.2680
home shocks a_t, ζ_t	0.0019	0.0019	0.0036	0.3044	0.2944
foreign shocks a_t^*, ζ_t^*	0.0019	0.0019	0.0036	0.3044	0.2944
all shocks $a_t, a_t^*, \zeta_t, \zeta_t^*$	0.0038	0.0038	0.0073	0.6106	0.5905

(c) $\omega = 0.85, \omega^* = 0.60$: heterogeneity of home-bias

	(α^L, θ^L)	(α_L, θ^M)	(α_L, θ^H)	(α_H, θ_L)	(α_H, θ_M)
home productivity shock a_t	0.0001	0.0001	0.0005	0.0802	0.0712
foreign productivity shock a_t^*	0.0001	0.0001	0.0002	0.0808	0.0795
home cost-push shock ζ_t	0.0013	0.0013	0.0013	0.1890	0.1891
foreign cost-push shock ζ_t^*	0.0004	0.0004	0.0004	0.0584	0.0583
home shocks a_t, ζ_t	0.0014	0.0014	0.0018	0.2695	0.2606
foreign shocks a_t^*, ζ_t^*	0.0005	0.0005	0.0006	0.1394	0.1379
all shocks $a_t, a_t^*, \zeta_t, \zeta_t^*$	0.0019	0.0019	0.0024	0.4096	0.3992

Note: $\alpha^L = 0.01,\ \alpha^H = 0.75,\ \theta^L = 0.00,\ \theta^M = 0.10,\ \theta^H = 0.60$
and $\theta_D = \theta_D^* = \theta_M = \theta_M^* = \theta^i$ for $i \in \{L, M, H\}$.

Table 3: Welfare gains from policy coordination in a two-country model with sticky prices and deep habits

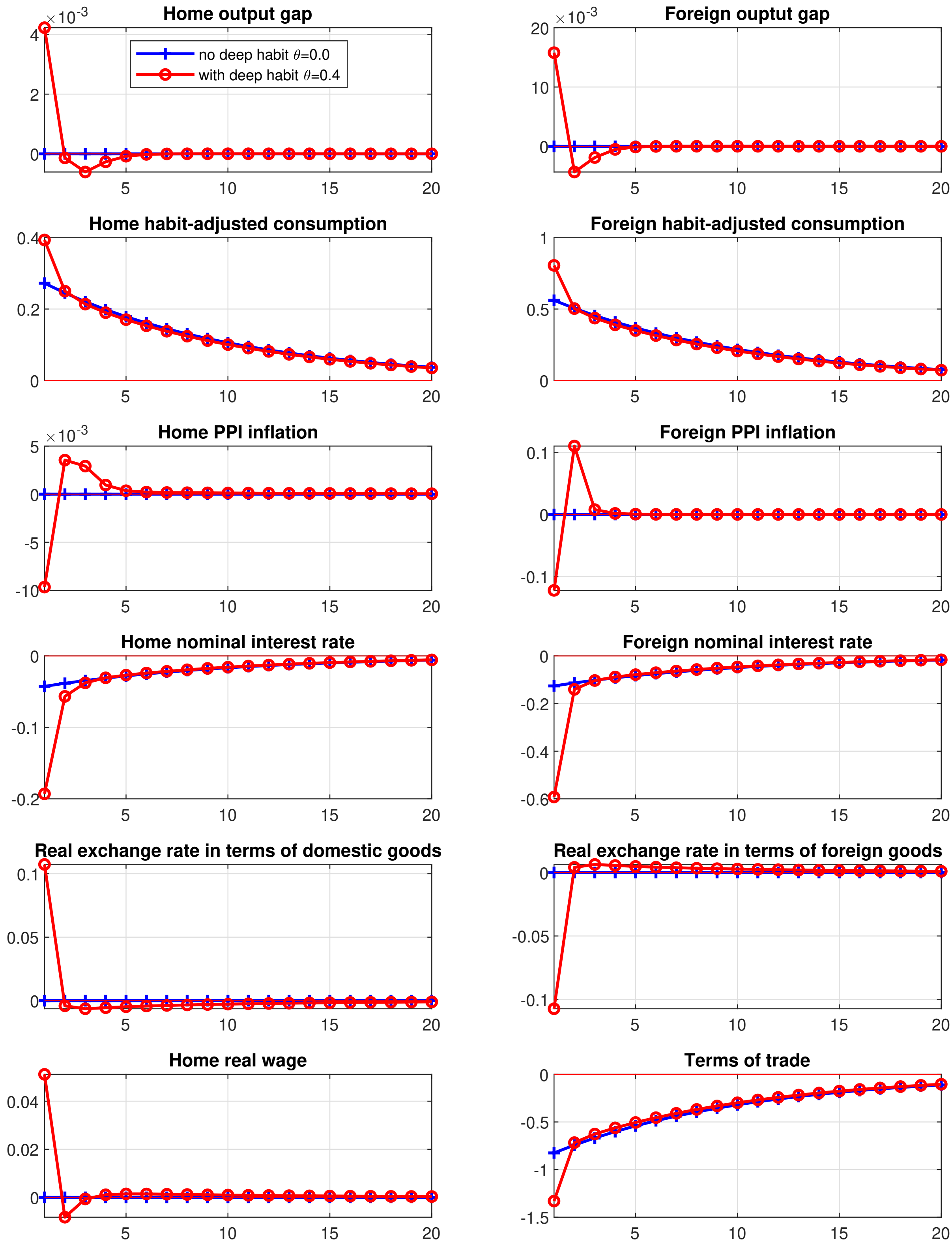

Figure 1: Impulse responses to a foreign productivity shock: comparison with and without deep habit

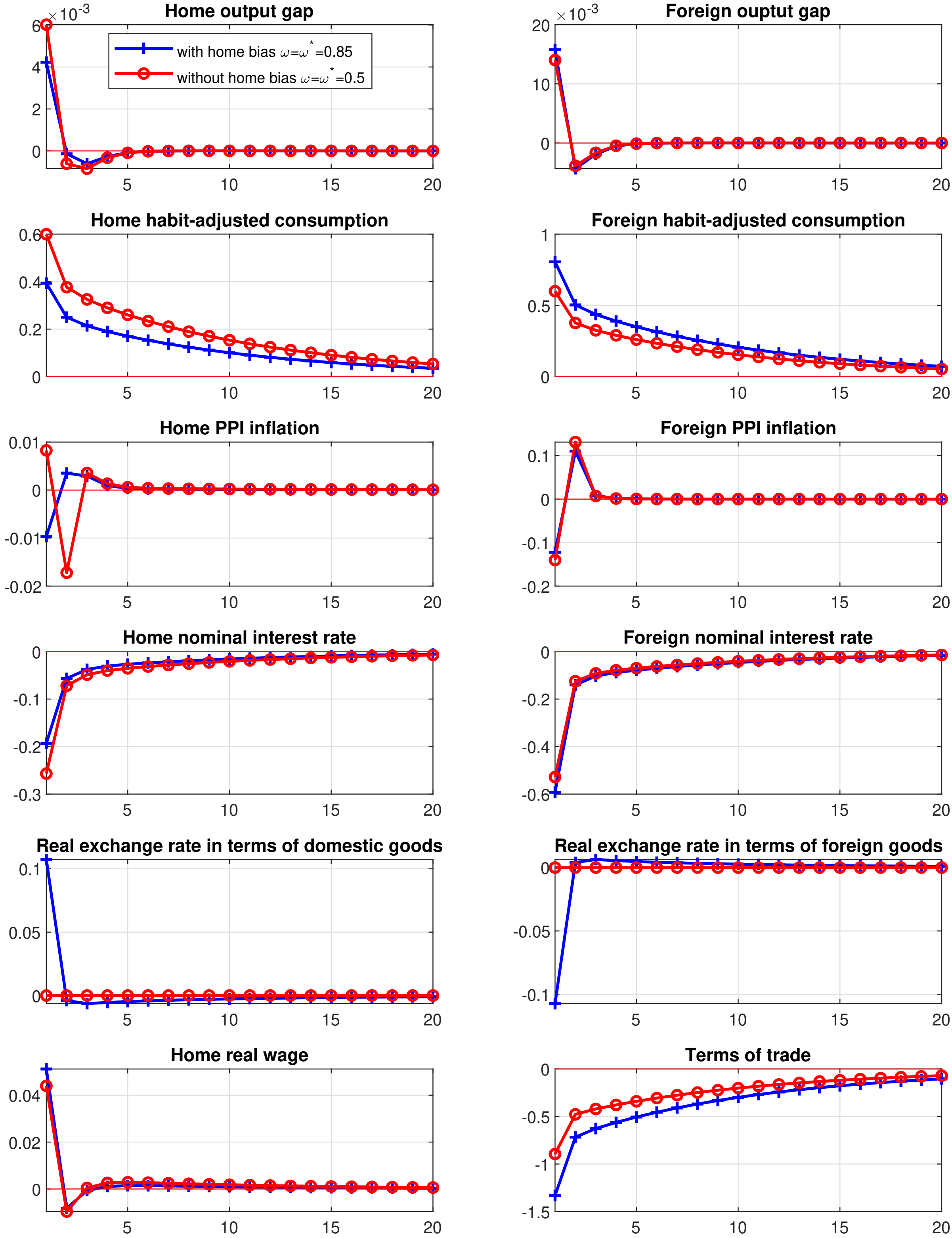

Figure 2: Impulse responses of a foreign productivity shock concerning changing the degree of home bias

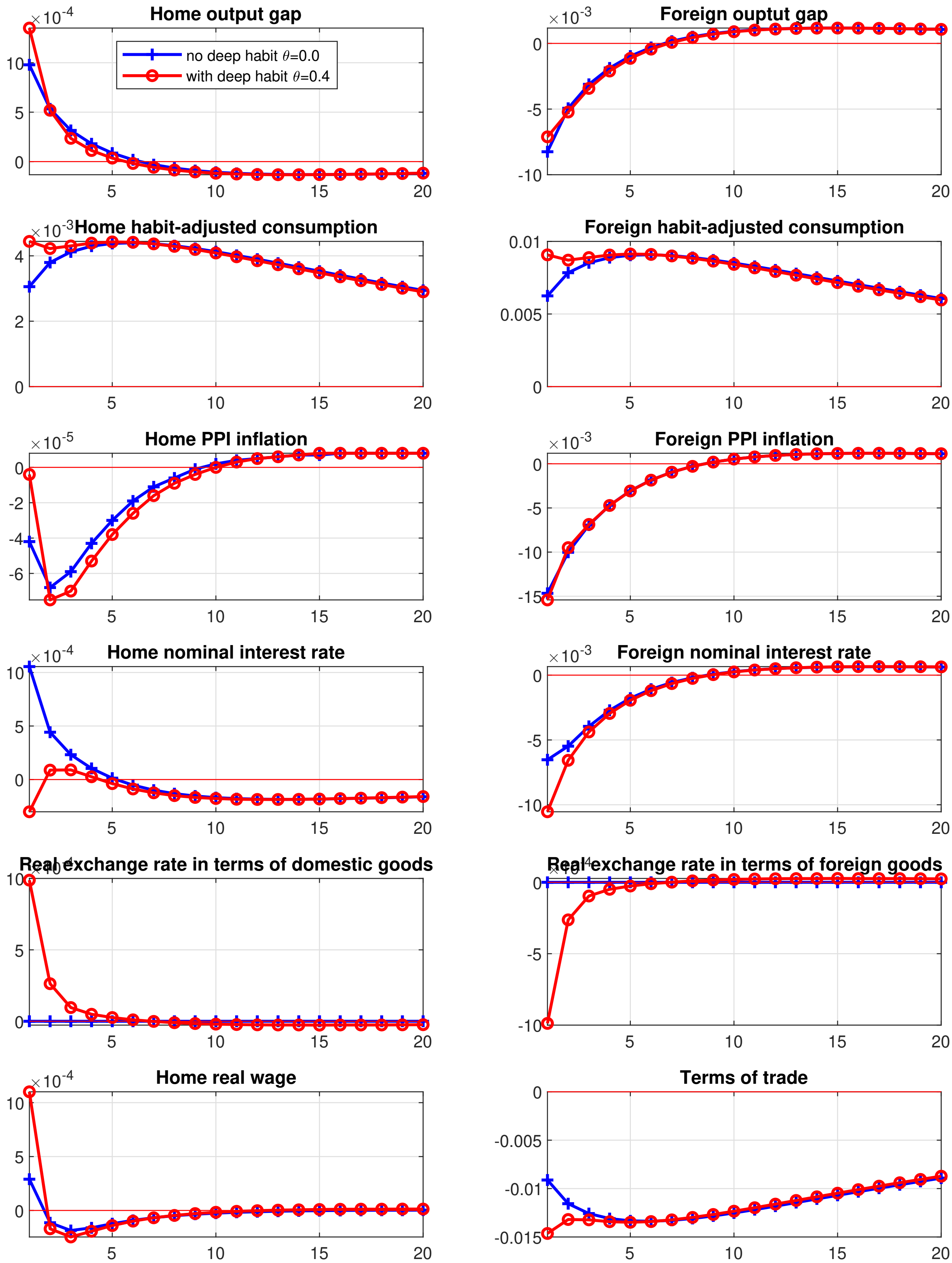

Figure 3: Impulse responses of a negative foreign cost-push shock: comparison with and without deep habit

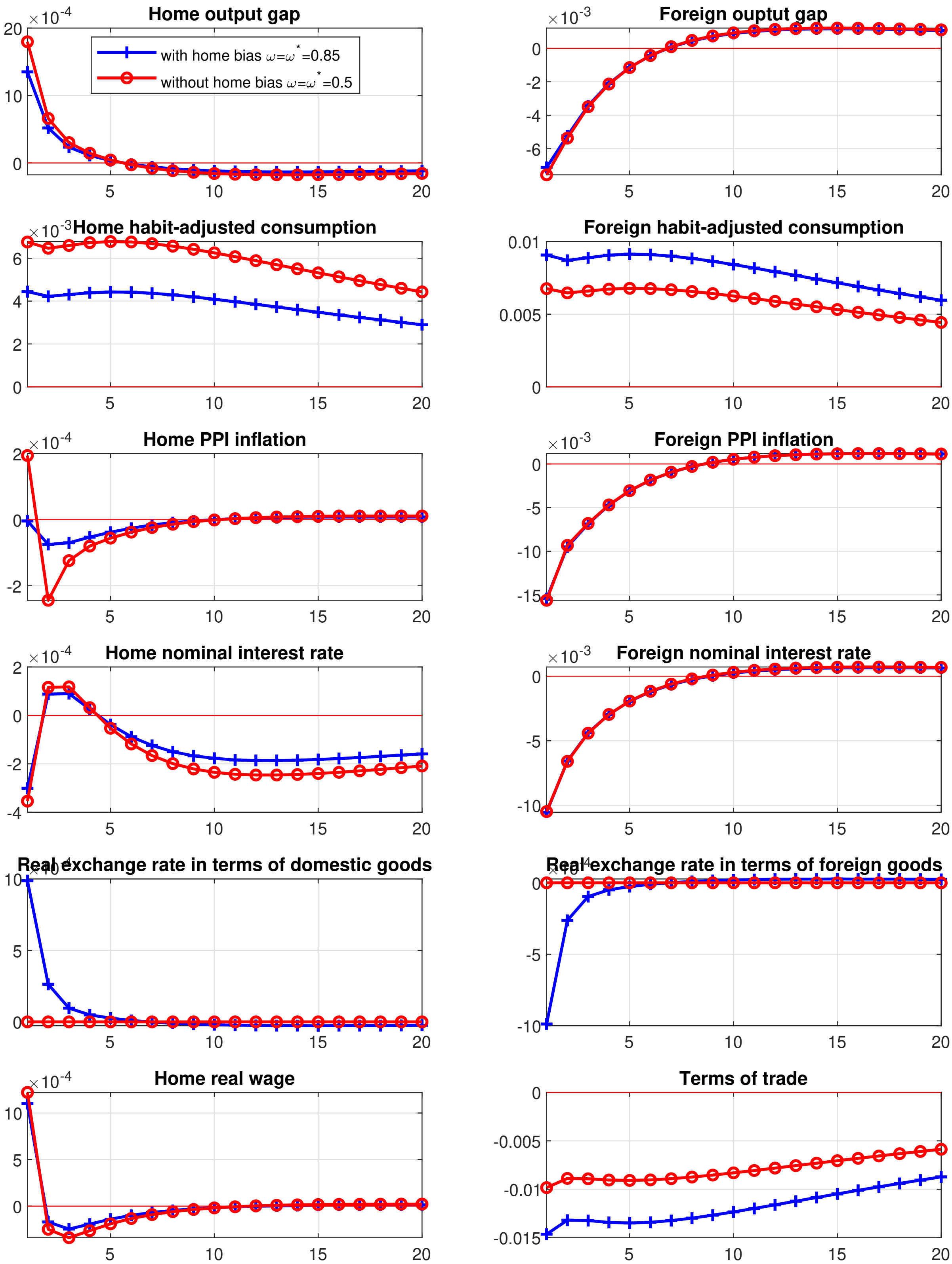

Figure 4: Impulse responses of a negative foreign cost-push shock concerning changing the degree of home bias

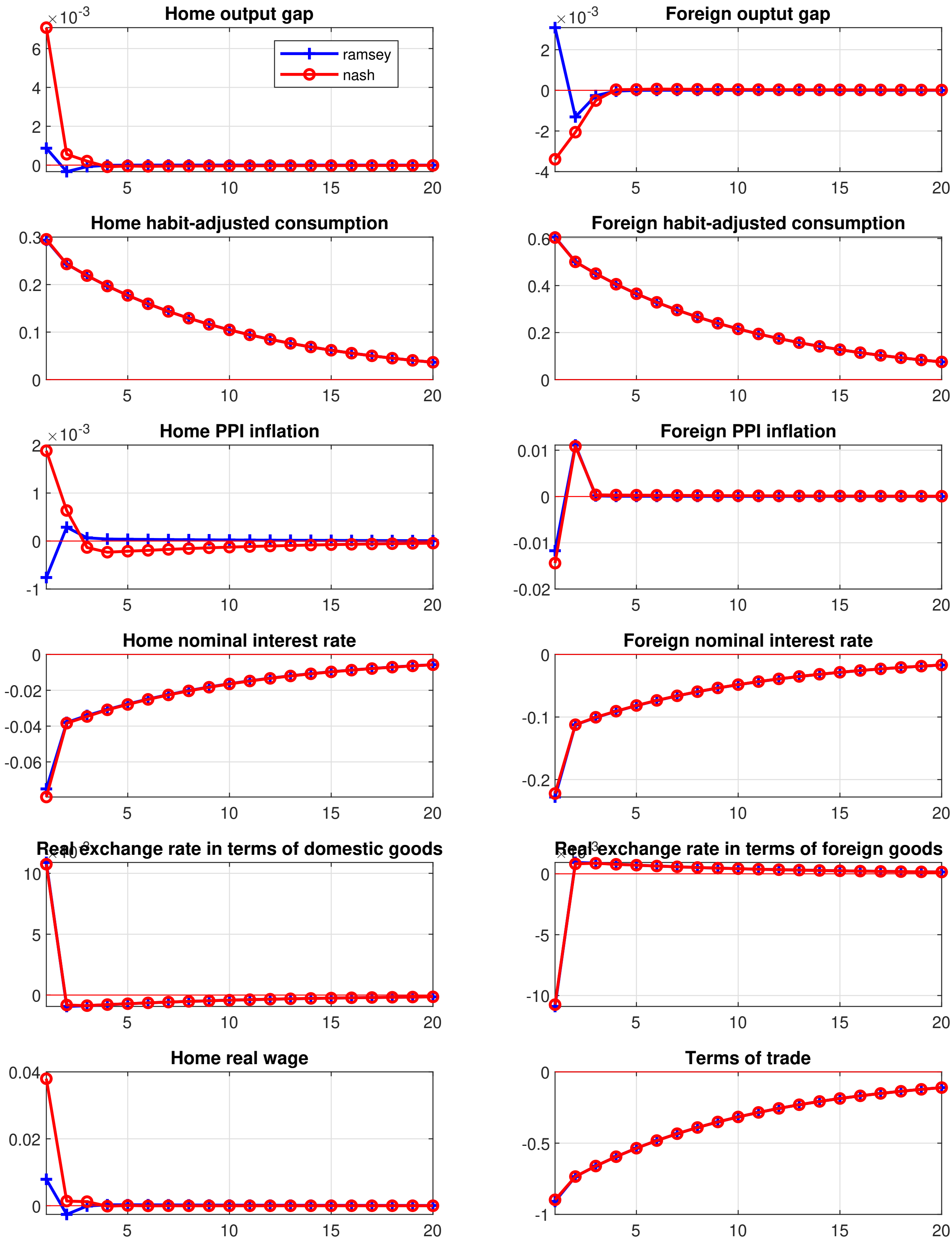

Figure 5: Impulse responses to a foreign productivity shock: cooperative Ramsey vs. non-cooperative open-loop Nash equilibrium

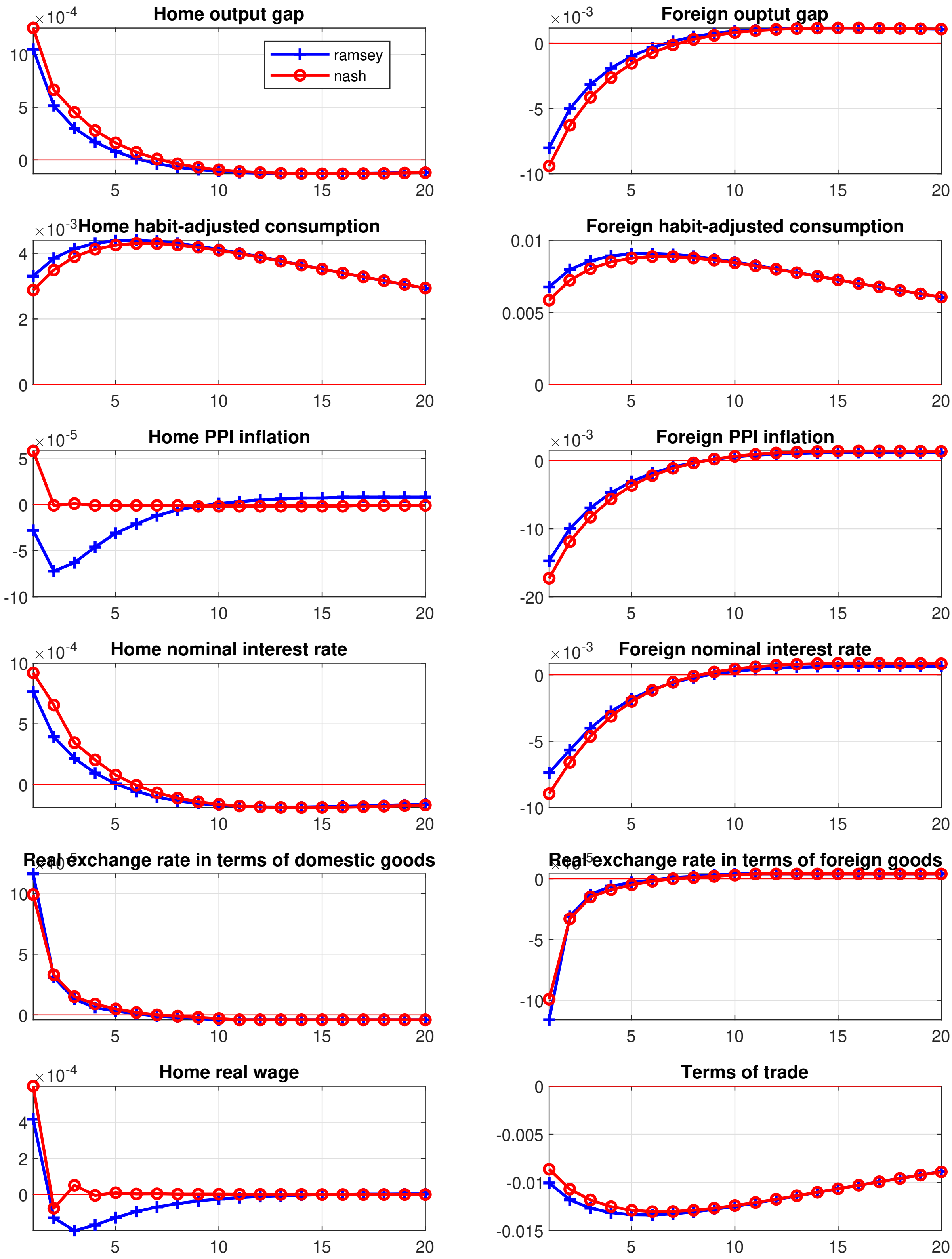

Figure 6: Impulse responses of a negative foreign cost-push shock: cooperative Ramsey vs. non-cooperative open-loop Nash equilibrium