Aging and its macroeconomic consequences: An inverted U-shaped endogenous economic growth

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Abstract

This study theoretically analyzes population aging and its impacts on economic growth, wealth inequality, and fiscal sustainability. We introduce lifetime uncertainty to the overlapping generations model with heterogeneous households with varied intertemporal preferences, where unintended bequests caused by death are inherited by offspring. Aging can have both positive and negative impacts on economic growth and fiscal sustainability: *saving-enhancing effects* based on the life cycle theory and *wealth-depletion effects* caused by extended longevity. When aging advances, saving-enhancing effects are offset by wealth-depletion effects, which eventually outweigh the former. The results show an "inverted U-shaped" relationship between life expectancy and economic growth rate, or fiscal sustainability. Numerical simulation reveals that aging can produce a trade-off between economic growth and wealth inequality. We also show that a rise in deficit or government expenditure ratios exacerbate fiscal instability, economic growth, and wealth inequality under certain conditions.

Keywords: Population Aging; Secular Stagnation; Inequality; Fiscal Sustainability

JEL classification: D63; H63; J14; O11

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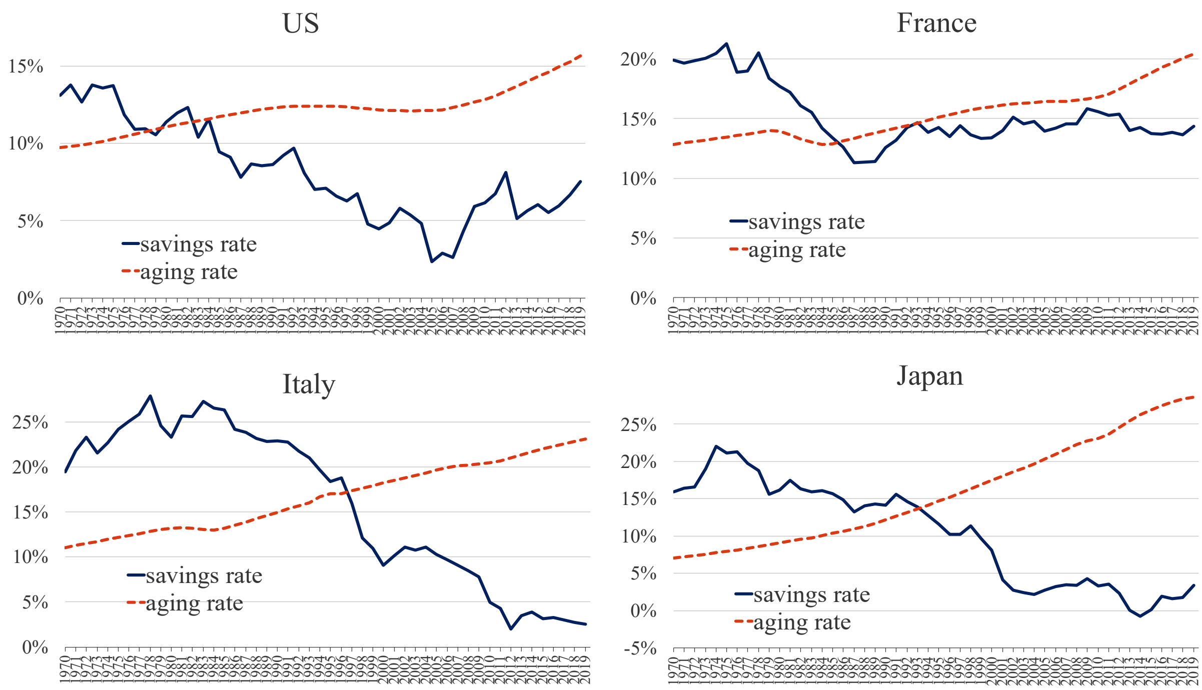
1. Introduction

The global economy is in the midst of a rapid demographic transition: population aging. For an economy, aging is an inevitable, unprecedented global phenomenon that most countries sooner or later face, and its diverse impacts on the macroeconomy are yet to be fully elucidated. As of 2020, the share of population aged 65 and above out of total exceeded 20% in European countries, such as France, Germany, Italy, Spain, Finland, Sweden, Austria, Netherlands, Denmark, and Hungary (World Bank 2025). Even in the United States, with a massive influx of immigrants, the aging rate already reached 17.4% in 2023.

Although population aging is most conspicuous in industrialized economies, numerous developing countries would follow a similar demographic trail, with a lag, as an even more rapid, concentrated form. The global population over 65 years old is projected to more than double by 2050, estimated at more than 15% of the world's population (United Nations 2023).

Aging has imposed pronounced economic distress on various economies. First, the decline of the savings rate with aging has been observed in advanced economies. Figure 1 shows the variation of aging and savings rates of the household section for the last 50 years, where the savings rate has been on a consistent downward trend since the 1980s. Although the savings rate is a complex indicator affected by taxation, social security systems, financial crisis, or natural disasters, its trend has presented a direction opposite to that of aging for years. Similarly, Bloom et al. (2015) suggest that high-income economies with higher aging rates show a negative association between aging and savings rate, while this association is slightly positive in low- and middle-income countries with relatively lower aging rates.



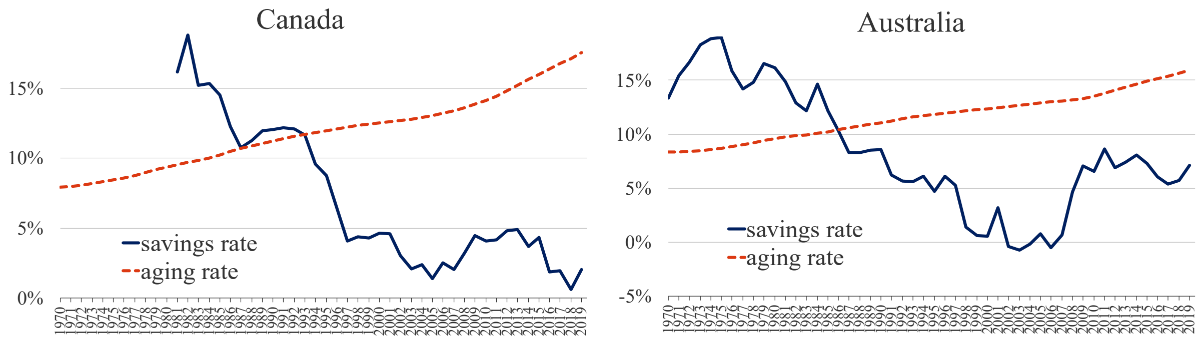


Figure 1. Aging and the decline in savings rate
Source: World Bank (2025)

Regarded as the most aged among major advanced economies, with 29.6% of the ageing rate in 2023, Japan has suffered from tremendous accumulated public debt, lower savings rate, and stagnated economic growth. These features can be highlighted in other major economies as well. Some studies have empirically identified the causal effects of aging on the savings rate in Japan (Braun et al. 2009; Koga 2006). In particular, Unayama and Ohno (2017a; 2017b; 2018) reveal that the decline in savings rate is not due to an increase in the proportion of the elderly with low savings rates but to a considerable fall in the elderly's savings rate itself, which is estimated to account for 65% at least of the total decline in the savings rate observed in Japan. These findings imply that the savings rate lowered by ageing affects economic growth through capital investment, partly leading to secular stagnation.

Moreover, the demographic change initiated by longer longevity has stressed the fiscal balance of government to finance growing social security costs such as pension and medical care, jeopardizing its fiscal sustainability. In advanced economies, government debt has rapidly augmented with a chronic budget deficit, whereas wealth inequality tends to expand. Figure 2 indicates that the public debt–GDP ratio has sharply risen in the US and Japan for a quarter of century even before the COVID-19 pandemic, and the Gini coefficient of wealth appears to show an increasing trend for the same period, especially in the US and EU. This trend is consistent with that shown by Piketty (2014), who indicates the global trend of expansion of income inequality.

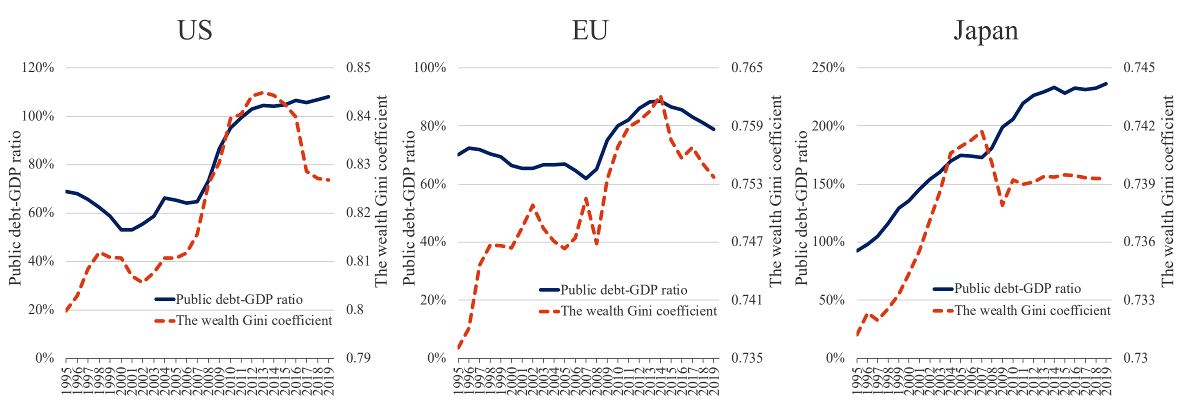


Figure 2. Public debt–GDP ratio and wealth inequality
Source: International Monetary Fund (2025a; 2025b) and The World Inequality Lab (2025)

Literature review

Numerous studies have been conducted on the impact of aging on the economy, although its impacts on the macro economy are manifold and still controversial in both theoretical and empirical discussions. Pecchenino and Pollard (1997), assuming a two-period overlapping generations model and introducing a measure of stochastic individual mortality, show that a rise in the survival rate increases the optimal savings level and leads to economic growth. Futagami and Nakajima (2001) prove that their argument holds in the continuous time model as well and that the aging of society increases both the savings and economic growth rates. Yakita (2008a) analyzes the public investment policy maximizing economic growth in the model with stochastic mortality and no heritable bequests.

Using panel data for 68 countries, Bloom et al. (2003) empirically show that an increase in life expectancy result in higher savings rate at all ages, although such increase is offset in the long run by a higher share of the elderly with a lower savings rate under constant population.

Some theoretical studies have analyzed the impact of aging on human capital and found that an increase in life expectancy facilitates investment in human capital and increases output. Kalemli-Ozcan et al. (2000) and Cervellati and Sunde (2005) depict an increase in optimal years of education as longer life expectancy allows individuals to enjoy the benefits of education for a longer period.[1] Lee and Mason (2010) empirically reveal that lower mortality and fertility that cause population aging can promote human capital per capita and capital deepening, raising labor productivity.

Futhermore, Prettner (2013) and Prettner and Trimborn (2017) introduce mortality and fertility into research and development (R&D) models. They show that longevity increases the economic growth rate in both cases of Romer (1990), which assumes strong externalities of accumulated ideas, and Jones (1995), which assumes relatively weak externalities.

Meanwhile, other empirical studies present a non-positive relationship between aging and economic growth. Acemoglu and Johnson (2007) conduct an empirical study of aging and its effects on key macroeconomic variables and find that an increase in life expectancy increases both the population and GDP, but GDP growth is relatively lower than population growth and does not lead to an increase in output per capita.[2] Additionally, Bloom et al. (2011) and Hansen and Lønstrup (2015) empirically show that longer life expectancy in advanced countries has a negative impact on economic growth. Further, Maestas et al. (2023), with US data from 1980 to 2010, estimate that GDP per capita decreased by 5.5% for every 10% increase in the share of the population aged 60 and over.

Acemoglu and Restrepo (2017) argue that no negative association between aging and GDP per capita in the United States from 1990 to 2015 is partially due to adopting automation

[1] Hazan and Zoabi (2006) indicate that in a model in which the children's educational level is determined by their parents, an increase in the average life expectancy of children increases both fertility and the returns from their educational level and, consequently, may not change the relative level, which does not promote the accumulation of human capital per capita.

[2] Bloom et al. (2014) criticize the trade-off between improved health and output per capita suggested in Acemoglu and Johnson (2007). However, Acemoglu and Johnson (2014) refute their claims and re-argue that the main conclusion of their paper in 2007 still holds even after taking into account the points made by Bloom et al. (2014).

technology such as AI and robots, which compensates for labor scarcity accompanied by population aging and countervails its possible adverse effects.[3] However, Eggertson et al. (2019), revisiting their analysis, indicate that such relationship between aging and GDP per capita was not maintained from 2008 to 2015, and aging had a negative effect on output growth under secular stagnation regime, where advanced economies faced the zero lower bound and nominal interest rate could no longer flexibly adjust downwards to accommodate capital deepening.

Minamimura and Yasui (2019) show—both theoretically and empirically—that declining mortality accelerates the substitution of physical capital by human capital and, consequently, that aging can have a negative impact on output per capita in countries with low levels of education. In this sense, their results support the discussion of Acemoglu and Johnson (2007). Additionally, Futagami and Sunaga (2022) analyze the relationship between risk aversion and economic growth; based on an empirical study that finds the elderly to be more risk averse than younger people, they show, in their theoretical model, that a rise in aging rate hinders capital accumulation and economic growth.

Moreover, empirical studies suggest that population aging can have both positive and negative effects on economic growth. Cervellati and Sunde (2011) define a turning point based on the demographic transition theory, by which population growth eventually reaches a declining phase as mortality and fertility decline, and claim that aging may reduce output per capita before the turning point but increase it after.

Conversely, Lee and Shin (2019) draw a somewhat different conclusion from the panel data for 142 countries from 1960 to 2014: the impact of a rise in the aging rate on economic growth depends on whether or not the working-age population is increasing at the same time, and aging slows down economic growth when aging and decline in working-age population occur simultaneously in an economy with a high aging rate. Therefore, aging appears to promote economic growth in some countries while hindering it in others.

This paper theoretically analyzes aging and its non-monotonic causal effects on economic growth, wealth inequality, and fiscal sustainability based on the heterogeneous agent model proposed by Maebayashi and Konishi (2021). They introduce an indicator of inequality into the public debt model of Carlberg (1995) and Bräuninger (2005), which explicitly captures the various paths for government debt issuance to distort households' savings allocation.[4]

We extend their heterogeneous agent model further in the following two ways. First, we capture the productive aspect of government expenditure by assuming that public investment contributes to output and can be amassed as public capital as in Yakita (2008b). We derive a system of difference equations on three endogenous variables—ratio of public debt to private capital, ratio of public to private capital, and wealth inequality—and analyze their complicated interaction with comparative statics in the multiple balanced growth paths (BGPs).

[3] Acemoglu (2010) and Acemoglu and Resterepo (2018) support that labor scarcity encourages automation. Moreover, Acemoglu and Resterepo (2022) suggest that countries undergoing more rapid aging tend to adopt further automation technology.

[4] The public debt model proposed by Carlberg (1995) and Bräuninger (2005) is still undergoing various developments: Arai (2011), Minea and Villieu (2012), Teles and Mussolini (2014), Agénor and Yilmaz (2017), Futagami and Konishi (2022), and Hagiwara (2024).

Second, we introduce lifetime uncertainty to the overlapping generations model with heterogeneous households with varied intertemporal preferences, the rich and the poor, where unintended bequests caused by death are inherited by offspring. In our model, individuals are assumed to survive to old age with a certain probability at the end of youth, or to die otherwise. When an individual dies, the savings stored in the youth period are passed on to children as unintended bequests, and otherwise, no bequests are left.

Aging, the rise in survival probability, can have both positive and negative impacts on economic growth, as empirical studies suggest (Cervellati and Sunde 2011; Lee and Shin 2019). On one hand, households with longer life expectancy save more to equalize consumption even after retirement, which enhances economic growth through capital accumulation. We call this *saving-enhancing effects* of aging. On the other hand, aging also has *wealth-depletion effects* accompanied by the longer retirement period; individuals with extended longevity consume more wealth for existence, which hinders economic growth and reduces bequests for offspring.[5]

Moreover, saving-enhancing effects reduce the relative ratio of accumulated public debts to private capital through more unintended bequests, leading to fiscal consolidation. Meanwhile, wealth-depletion effects aggravate fiscal conditions with a more relative burden of public debt. Hence, aging has various influences on fiscal sustainability as well as economic growth, and its effects change depending on the level of aging.

When survival probability is relatively low, saving-enhancing effects are more dominant, and aging facilitates economic growth and enhances fiscal sustainability. As an economy ages, saving-enhancing effects are gradually offset by wealth-depletion effects, and the latter eventually surpasses the former. The results show an "inverted U-shaped" relationship between life expectancy and economic growth rate, as well as a threshold of aging level that maximizes economic growth.

Additionally, numerical simulation based on advanced economies such as the U.S. and EU countries suggests that when aging exceeds a certain threshold, a trade-off emerges between economic growth and equality: the economic growth rate decreases with aging whereas wealth inequality improves.

Furthermore, comparative statics reveals that the rise in government bond insurance–GDP and government expenditures–GDP ratios exacerbates fiscal instability, economic growth, and wealth inequality simultaneously when an elasticity of public debt-to-capital ratio toward each ratio is greater than a certain level. Therefore, in advanced countries that have faced serious budget deficits and aging, the effect to aggravate inequality is supposed to be dominant enough to surpass the inequality-improving effects of aging depicted in the simulation.

The paper is structured as follows. Section 2 presents the model framework, the system of difference equations, and numerical simulation based on advanced economies. Section 3 presents comparative statics of aging, deficit ratio, and government expenditure ratio in the multiple BGPs. Finally, Section 4 concludes the paper.

[5] Wealth-depletion effects capture the savings behavior of households, consistent with the results in Unayanama and Ohno (2017a; 2017b; 2018).

2 Model

2.1 Production Sector

We assume an economy producing a final good with labor, private capital, and productive public capital. Government expenditure, accumulated as stock, contributes to output as a production factor. In this economy, numerous identical firms manufacture a single commodity, and the aggregated production function takes the following Cobb-Douglas production function, which exhibits constant scale to returns:

$$Y_t = \Gamma {G_t}^{\theta} {K_t}^{\gamma} (e_t N)^{1-\theta-\gamma}, \tag{1}$$

where Y_t denotes output, Γ total factor productivity,[6] G_t public capital, K_t private capital, and N labor. The t index represents the period.[7] Each worker provides one unit of labor inelastically, and we assume that N is constant and normalized as one. $\theta, \gamma \in (0,1)$ denote the elasticity of public capital and private capital share, respectively. Following Romer (1986), $e_t \equiv \frac{K_t}{N}$ implies a labor efficiency based on learning-by-doing and is proportionate to private capital per worker. As in Futagami et al. (1993), we also define the public–private capital ratio as $\Omega_t \equiv \frac{G_t}{K_t}$, which represents the production efficiency of private capital. Therefore, the production function can be simplified to

$$Y_t = \Gamma {G_t}^{\theta} {K_t}^{\gamma} (e_t N)^{1-\theta-\gamma}. \tag{2}$$

The goods and factors markets are perfectly competitive. As ${G_t}^{\alpha}$ and e_t in production function (1) are externalities for firms, the profit maximization conditions are

$$r_t = \Gamma \gamma \Omega_t^{\theta}, \tag{3}$$

$$w_t = \Gamma (1-\theta-\gamma) \Omega_t^{\theta} K_t, \tag{4}$$

where r_t and w_t denote the rental price of capital and real wage rate, respectively. Equations (3) and (4) show that the relative increase of public capital G_t for private capital K_t induces capital deepening through a rise in r_t and causes the real wage rate w_t to increase.

2.2 Household Sector

We introduce lifetime uncertainty π as an aging indicator to an overlapping generations model with heterogenous agents presented by Maebayashi and Konishi (2021). In our model, individuals live for two periods. Young workers earn wage income, consume part of it, and save the remaining. Upon aging, they retire and receive capital income from private and public assets. The return they earn from one asset is equivalent to that from the other under non-arbitrage conditions.

Each generation consists of two heterogeneous households, the rich and poor, denoted by R and P, respectively. They have varied intertemporal preferences $\alpha_i \in \left(0, \frac{1}{2}\right)$ toward future consumption, and we assume that $\alpha_R > \alpha_P$ ($i = R$ or P), which implies that the relatively "patient" rich with a higher discount factor save more than the poor. In this model, the share of the rich in generation is given as δ, and therefore, that of the poor is $1 - \delta$.

[6] Total factor productivity Γ is constant and exogenously given.

[7] We assume that one period is approximately 35 years, which is long enough for the government to adjust the tax rate.

Moreover, we introduce lifetime uncertainty π to the OLG model, as in Pecchenino and Pollard (1997). In our model, individuals are assumed to survive to old age with probability π at the end of youth, or to die with probability $1-\pi$. If an individual survives with probability π, then they consume savings with no bequests left over. Meanwhile, if an individual dies and exits the economy with probability $1-\pi$, the savings stored in the youth period are passed on to children as unintended bequests κ_{t+1}^{i}. Under the efficient insurance, individuals can inherit, on average, the π portion of savings as bequests upon the death of their parents' generation.[8]

The individual's utility function depends on the consumption per worker in the working and retirement periods, $c_{i,t}^{Y}$ and $c_{i,t+1}^{O}$, respectively, and individuals cognizant of survival probability π maximize their utility without foreseeing their own death beforehand.

$$u_i = (1-\alpha_i)\log c_{i,t}^{Y} + \pi\alpha_i \log c_{i,t+1}^{O}. \tag{5}$$

The intertemporal budget constraint is $c_{i,t}^{Y} + \frac{c_{i,t+1}^{O}}{1+(1-\tau_{t+1})r_{t+1}} = (1-\tau_t)w_t + (1-\pi)\kappa_t^i$, where τ_t denotes the constant income tax rate. Unintended bequests κ_t^i, accompanying the early deaths of parents' generation, is given as $[1+(1-\tau_t)r_t]s_{t-1}^{i}$, and therefore, savings $s_t^i = (1-\tau_t)w_t - c_{i,t}^{Y}$ can be represented as follows:

$$s_t^i = \hat{\alpha}_i(1-\tau_t)w_t + \tilde{\alpha}_i[1+(1-\tau_t)r_t]s_{t-1}^{i}, \tag{6}$$

where we define $\hat{\alpha}_i \equiv \frac{\pi\alpha_i}{1-\alpha_i+\pi\alpha_i}$ and $\tilde{\alpha}_i \equiv \frac{(1-\pi)\pi\alpha_i}{1-\alpha_i+\pi\alpha_i}$. The variations of $\hat{\alpha}_i$ and $\tilde{\alpha}_i$ in response to π are examined as follows. The first term in Equation (6) represents the optimal savings allocation in disposable labor income, and the rise in π, regarded as extended average longevity of the economy, necessarily increases its savings rate as $\frac{\partial\hat{\alpha}_i}{\partial\pi} = \frac{\alpha_i(1-\alpha_i)}{(1-\alpha_i+\pi\alpha_i)^2} > 0$. The results are consistent with life cycle theory because individuals anticipating to live longer save more to equalize consumption between youth and old as the economy ages.

The second term indicates the savings allocation in unintended bequests from parents' generation, and the effect of π is not monotonic depending on the level of aging. From $\frac{\partial\tilde{\alpha}_i}{\partial\pi} = \frac{\alpha_i(1-\alpha_i)(1-2\pi)-{\alpha_i}^2\pi^2}{(1-\alpha_i+\pi\alpha_i)^2}$, $\frac{\partial\tilde{\alpha}_i}{\partial\pi} > 0$ holds when $0 < \pi < \pi_i^*$ where $\pi_i^* = \frac{\sqrt{1-\alpha_i}-(1-\alpha_i)}{\alpha_i} < \frac{1}{2}$, and $\frac{\partial\tilde{\alpha}_i}{\partial\pi} > 0$ when $\pi_i^* < \pi < 1$. Accordingly, when the level of aging is relatively low, a rise in π causes individuals to increase their savings rate of the wealth inherited from their parents. Meanwhile, when the economy ages beyond a certain threshold π_i^*, it turns to decline as more of the old consume and deplete their wealth for existence. In other words, the eventual savings

[8] Consider the efficient insurance: an individual contracts insurance with an insurance company to pay a certain premium h_t^i in exchange for receiving the bequests $(1-\pi)\kappa_t^i + m_t^i$, regardless of whether the parent generation dies early or not. Note that $m_t^R > m_t^P > 0$. The expected bequest before insurance contract $E_B[\kappa_t^i]$ is $(1-\pi)\kappa_t^i$, and the expected bequest after insurance contract $E_A[\kappa_t^i]$ is $(1-\pi)\kappa_t^i + m_t^i - h_t^i$. Then, the minimum premium satisfying $E_B[\kappa_t^i] \leq E_A[\kappa_t^i]$ is $h_t^i = m_t^i$, and thus, the individual will obtain the insurance benefit $(1-\pi)\kappa_t^i$ equivalent to the expected insurance with this contract. Under the risk-aversion utility function, as our model supposes, rational individuals are willing to contract this insurance as assured benefits necessarily increase their utility. The insurance company offers insurance in each period and distributes bequests $(1-\pi)\kappa_t^i$ equally among each class, the rich or the poor. Note that the rich and the poor cannot choose insurance products across classes because the bequests received by them differ.

rate is dictated by the conflicting effects of aging: saving-enhancing effects based on the life cycle theory and wealth-depletion effects caused by extended longevity. [9]

2.3 Government Sector

The government balances total revenues and expenditures by adjusting the tax rate τ_t.

$$gY_t + r_t D_t = \lambda Y_t + \tau_t (Y_t + r_t D_t). \tag{7}$$

The government spends a share of the national income gY_t as public investment for roads, airports, and other social infrastructures, where $g \in (0, 1)$ is given exogenously. D_t denotes public debt accumulated up to the beginning of the current period t, and the government pays interest $r_t D_t$ to households.

With regard to the revenues, the government issues new government bond λY_t to finance the current fiscal year, where $\lambda \in (0, 1)$. Moreover, the government taxes the gross income $Y_t + rD_t$ with the tax rate τ_t. Defining the public debt-to-capital ratio as $x_t \equiv \frac{D_t}{K_t}$, the level of the tax rate $\tau_t{}^*$ that satisfies Equation (7) can be expressed as follows:

$$\tau_t^* = 1 - \frac{1 + \lambda - g}{1 + \gamma x_t}. \tag{8}$$

The public capital at the next period, G_{t+1}, is defined as the addition of the current government investment gY_t to the public capital accumulated up to the period t:

$$G_{t+1} = G_t + gY_t. \tag{9}$$

Maintenance costs are assumed to be autonomously covered in a self-supporting accounting system adopted as in highways.

2.4 The system of difference equations

The total savings of the whole economy is given as $A_t \equiv \delta s_t^R N + (1 - \delta) s_t^P N$, where δ is the ratio of the rich to the population. From Equations (6) and (8),

$$A_t = \hat{\alpha}(1 - \tau_t) w_t N + [1 + (1 - \tau_t) r_t][\, \tilde{\alpha}_R s_{t-1}^R \delta N + \tilde{\alpha}_P s_{t-1}^P (1 - \delta) N], \tag{10}$$

where $\hat{\alpha} \equiv \delta\, \hat{\alpha}_R + (1 - \delta)\hat{\alpha}_P$. The total savings are divided into public debt and private capital in the subsequent period:

$$A_t = D_{t+1} + K_{t+1}. \tag{11}$$

From Equation (11), we obtain $A_{t-1} = (1 + x_t) K_t$. Therefore, the gross growth rate of total savings can be expressed as

$$\frac{A_t}{A_{t-1}} = \frac{\hat{\alpha}\mu\,(1 - \theta - \gamma)\Omega_t^\theta}{(1 + \gamma x_t)(1 + x_t)} + \left[\frac{1 + \gamma x_t + \mu\gamma\Omega_t^\theta}{1 + \gamma x_t}\right][(\tilde{\alpha}_R - \tilde{\alpha}_P)\Phi_{t-1} + \tilde{\alpha}_P], \tag{12}$$

where $\mu \equiv \Gamma(1 + \lambda - g)$. Here, Φ_t indicates the wealth inequality between heterogeneous households and defined as $\Phi_t \equiv \frac{\delta s_t^R N}{A_t}$, the share of savings of the rich out of total savings. Φ_{t-1} in Equation (12) implies that as wealth inequality reflects the previous generation's savings inherited as unintended bequests, inequality over generations is maintained through inheritance in this model.

The public debt in the current period is refinanced with part of total savings A_t and

[9] Note that the levels of disposal income and unintended bequests would eventually change in response to higher π.

accumulated over periods. The difference between D_{t+1} and D_t corresponds to the new government bond: $D_{t+1} - D_t = \lambda Y_t$. Accordingly,

$$\frac{D_{t+1}}{D_t} = 1 + \frac{\lambda \Gamma \Omega_t^{\theta}}{x_t}. \tag{13}$$

From Equation (11) and (9), the gross growth rate of private and public capital is, respectively,

$$\frac{K_{t+1}}{K_t} = (1 + x_t)\frac{A_t}{A_{t-1}} - \left(x_t + \lambda \Gamma \Omega_t^{\theta}\right), \tag{14}$$

$$\frac{G_{t+1}}{G_t} = 1 + g\Gamma \Omega_t^{\theta-1}. \tag{15}$$

Moreover, from Equation (6),

$$\frac{s_t^R}{s_{t-1}^R} = \hat{\alpha}_R \left[\frac{\delta\mu(1-\theta-\gamma)\Omega_t^{\theta}}{\Phi_{t-1}(1+\gamma x_t)(1+x_t)}\right] + \tilde{\alpha}_R \left[\frac{1 + \gamma x_t + \mu\gamma\Omega_t^{\theta}}{1+\gamma x_t}\right]. \tag{16}$$

Now, from Equations (13), (14), (15), and (16), we derive the system of difference equations on three endogenous variables: x, Ω, and Φ.

$$x_{t+1} = \frac{x_t + \lambda\Gamma\Omega_t^{\theta}}{(1+x_t)\frac{A_t}{A_{t-1}} - \left(x_t + \lambda\Gamma\Omega_t^{\theta}\right)}, \tag{17}$$

$$\Omega_{t+1} = \frac{\Omega_t + g\Gamma\Omega_t^{\theta}}{(1+x_t)\frac{A_t}{A_{t-1}} - \left(x_t + \lambda\Gamma\Omega_t^{\theta}\right)}, \tag{18}$$

$$\Phi_t = \frac{\hat{\alpha}_R \left[\frac{\delta\mu(1-\theta-\gamma)\Omega_t^{\theta}}{(1+\gamma x_t)(1+x_t)}\right] + \tilde{\alpha}_R \left[\frac{\Phi_{t-1}\left(1+\gamma x_t + \mu\gamma\Omega_t^{\theta}\right)}{1+\gamma x_t}\right]}{\frac{A_t}{A_{t-1}}}. \tag{19}$$

In the steady state, $x_{t+1} = x_t \Leftrightarrow \frac{K_{t+1}}{K_t} = \frac{D_{t+1}}{D_t}$ holds. From Equations (13) and (14), the $x_{t+1} = x_t$ locus is expressed as

$$\Phi_{t-1} = \left[\frac{1}{\tilde{\alpha}_R - \tilde{\alpha}_P}\right]\left[\frac{(1+x_t)\left(1 + \frac{\lambda\Gamma\Omega_t^{\theta}}{x_t}\right)(1+\gamma x_t) - \hat{\alpha}\mu(1-\theta-\gamma)\Omega_t^{\theta}}{(1+x_t)(1+\gamma x_t + \gamma\mu\Omega_t^{\theta})} - \tilde{\alpha}_P\right]. \tag{20}$$

Similarly, in the steady state, $\Omega_{t+1} = \Omega_t \Leftrightarrow \frac{K_{t+1}}{K_t} = \frac{G_{t+1}}{G_t}$ holds. From Equations (14) and (15), we obtain the $\Omega_{t+1} = \Omega_t$ locus as

$$\Phi_{t-1} = \left[\frac{1}{\tilde{\alpha}_R - \tilde{\alpha}_P}\right]\left[\frac{\left(1 + g\Gamma\Omega_t^{\theta-1} + x_t + \lambda\Gamma\Omega_t^{\theta}\right)(1+\gamma x_t) - \hat{\alpha}\mu\,(1-\theta-\gamma)\Omega_t^{\theta}}{(1+x_t)(1+\gamma x_t + \gamma\mu\Omega_t^{\theta})} - \tilde{\alpha}_P\right]. \tag{21}$$

Finally, in the steady state, $\Phi_t = \Phi_{t-1} \Leftrightarrow \frac{A_t}{A_{t-1}} = \frac{s_t^R}{s_{t-1}^R}$ holds. From Equations (12) and (16), the $\Phi_t = \Phi_{t-1}$ locus can be represented as

$$\frac{1}{\mu\Omega_t^{\theta}}\,(1+x_t)\left(1+\gamma x_t + \gamma\mu\Omega_t^{\theta}\right) = \frac{1-\theta-\gamma}{(\tilde{\alpha}_R - \tilde{\alpha}_P)(1-\Phi_{t-1})}\left[\hat{\alpha} - \frac{\delta\hat{\alpha}_R}{\Phi_{t-1}}\right]. \tag{22}$$

We gain three conditional equations of steady state on x, Ω and Φ. Equations (20), (21), and (22) all depend on x, Ω, and Φ and constitute three-dimensional surfaces. In this system of difference equations, three multiple BGPs exist, which are given as the intersections of three surfaces.

2.5 Multiple balanced growth paths

In this section, we discuss the existence of multiple BGPs. Although three-dimensional surfaces constituted by Equations (20), (21), and (22) are difficult to grasp, we can visually analyze multiple steady states by unifying these equations.

First, from Equations (20) and (21), we obtain

$$\lambda\Omega_t = gx_t. \tag{23}$$

Equation (23) suggests that two variables, x_t and Ω_t, hold the linear relationship in the steady states. Since this equation does not depend on Φ_{t-1}, Equation (23) is described as a plane parallel to the Φ_{t-1} axis.

Next, substituting the above Equation into Equations (20) and (22) yields the following two equations, respectively, the intersections of which identify x^* and Φ^* of equilibria.

$$\Phi_{t-1} = \left(\frac{1}{\tilde{\alpha}_R - \tilde{\alpha}_P}\right)\left[\frac{(1+x_t)\left[1 + \frac{\lambda\Gamma\left(\frac{g}{\lambda}x_t\right)^\theta}{x_t}\right](1+\gamma x_t) - \hat{\alpha}\mu(1-\theta-\gamma)\left(\frac{g}{\lambda}x_t\right)^\theta}{(1+x_t)\left[1+\gamma x_t + \gamma\mu\left(\frac{g}{\lambda}x_t\right)^\theta\right]} - \tilde{\alpha}_P\right], \tag{24}$$

$$\frac{1}{\mu}\left[\frac{(1+x_t)(1+\gamma x_t)}{\left(\frac{g}{\lambda}x_t\right)^\theta} + (1+x_t)\gamma\mu\right] = \left[\frac{1-\theta-\gamma}{(\tilde{\alpha}_R - \tilde{\alpha}_P)(1-\Phi_{t-1})}\right]\left(\hat{\alpha} - \frac{\delta\hat{\alpha}_R}{\Phi_{t-1}}\right). \tag{25}$$

Since both equations do not include Ω_t, they are pictured as two surfaces parallel to the Ω_t axis. We define the right-hand side of Equation (24) as $\Theta(x_t)$. Then, $\Theta(x_t)$ is a convex curve downward with two asymptotes: $\Phi_{t-1} = \frac{1-\tilde{\alpha}_P}{\tilde{\alpha}_R - \tilde{\alpha}_P}$ and $x_t = 0$ (see Appendix A). Similarly, we define the left- and right-hand sides of Equation (25) as $\Psi_L(x_t)$ and $\Psi_R(\Phi_{t-1})$, respectively. Equation (25) is a convex curve downward with the asymptote $\Phi_{t-1} = 1$, extremely skewed to right (see Appendix B).[10]

[10] Except extreme cases not realized in the scope of reasonable parameters, Equations (24) and (25) can have intersections in the range of Equation (25) monotonically increasing for x_t as in Figure 3.

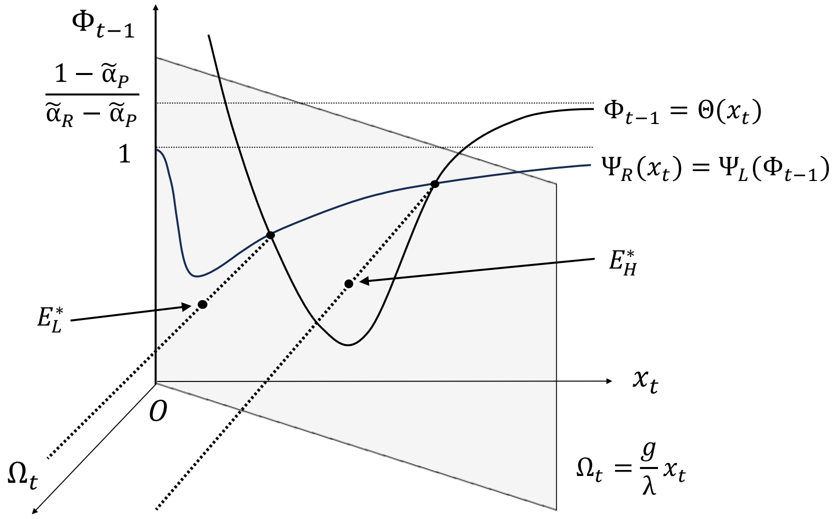


Figure 3. The existence of multiple steady states

Figure 3 is the diagram depicting outlines of Equations (23), (24), and (25) in three dimensions. In this model, the steady states are presented as the equilibria where Equations (23), (24), and (25) hold. In the case that Equations (24) and (25) intersect in (x_t, Φ_{t-1}) plane, the set of these intersections can be pictured as the two lines parallel to the Ω_t axis. Since the $\lambda\Omega_t = gx_t$ plane necessarily crosses these two lines, two points identified as the intersections of two lines and one plane represent the steady state equilibria of this model. As Figure 3 indicates, the values of x^*, Ω^*, and Φ^* at one equilibrium are higher than those of the other; thus, the equilibrium close to the origin O is called lower equilibrium E_L, and the other is called higher equilibrium E_H.

In the equilibria where x and Ω converge, $\frac{K_{t+1}}{K_t} = \frac{D_{t+1}}{D_t} = \frac{G_{t+1}}{G_t}$ holds. Moreover, $\frac{Y_{t+1}}{Y_t}$ depends on $\frac{K_{t+1}}{K_t}$ from Equation (2), and $\frac{K_{t+1}}{K_t}$ coincides with gross saving growth rate $\frac{A_{t+1}}{A_t}$ from Equation (14). Let the growth rate in the steady state be ρ, and the following equation holds:

$$\rho = \frac{Y_{t+1}}{Y_t} = \frac{K_{t+1}}{K_t} = \frac{D_{t+1}}{D_t} = \frac{G_{t+1}}{G_t} = \frac{A_{t+1}}{A_t}. \tag{26}$$

Thus, this model has multiple BGPs. ρ can be presented from Equations (13), (23), and (26) as

$$\rho = 1 + \frac{\Gamma\lambda^{1-\theta} g^{\theta}}{x^{*1-\theta}}. \tag{27}$$

Equation (27) reveals that the rise in the burden of public debt on households distorts the saving allocation for private capital and reduces economic growth rate in BGPs. Since the growth rate decreases with x^* and $\lambda\Omega^* = gx^*$ holds in BGPs, it implies that the positive effects of the rise in Ω^*, public capital accumulation, on growth rate are always offset by the negative effects of x^*, the burden of public debt, in this model.

Proposition 1

Assume an economy with heterogeneous households with varied intertemporal preferences, the rich and the poor, that produces a final good with labor, private capital, and productive

public capital:
(i) the system of difference equations on public debt–capital ratio x_t, public–private capital ratio Ω_t and wealth inequality Φ_t has multiple BGPs: lower equilibrium E_L and higher equilibrium E_H.
(ii) the economic growth rate in the BGPs is determined by the levels of x^* or Ω^*, not depending on Φ^*.
(iii) the linear relationship holds between x^* and Ω^*, and the effects of the rise in Ω^* on economic growth are offset by the negative effects of x^*.

2.6 Numerical simulation

In this section, we present numerical simulations for advanced economies such as the U.S. and EU countries to ascertain whether each of multiple equilibria is stable or unstable. Table 1 quantifies the variables of this model.

Table 1. Calibration of variables

Variables	θ	γ	g	λ	δ	α_R	α_P	π
Values	0.20	0.25	0.20	0.03	0.50	0.45	0.35	0.50

Since the seminal paper of Aschauer (1989), who estimated the elasticity of public capital α to be 0.39, a large number of empirical studies have been conducted.[11] Especially Bom and Ligthartghart (2014) conducted a cross-sectional survey analysis of 67 empirical studies from 1983 to 2008 and found that the average elasticity of public capital is 0.146, while the level rises to 0.268 per country when spillover effects across regions over time are taken into account. In this paper, we assume $\theta = 0.20$, considering the fact that one period is 35 years long.

The labor share in factor values in major industrialized countries, including the U.S. and European countries, has remained stable at around two-thirds over the past 10 years.[12] Therefore, capital and labor shares are set to 0.25 and 0.55, respectively.

The government spending–output ratio g can be estimated by dividing outlays (excluding interest payments) by nominal GDP based on Equation (7). In the U.S., the average ratio of government spending to GDP is 18.9% from FY2013 to FY2019 (see Appendix B). Therefore, we assume $g = 0.2$. Note that total expenditures in FY2020 and beyond are extremely high owing to fiscal policies associated with COVID-19 and excluded in the identification of variables.

The deficit ratio λ is assumed to be 0.03 with reference to the fiscal conditions in U.S. and EU countries. The average deficit ratio in the U.S., which divides the budget deficit by the

[11] Munnnell (1990) regards hospital, school and police as well as the social infrastructure as broadly defined public capital and estimated α to be 0.33. According to a study by Arslanalp et al. (2010) covering 22 OECD countries over a 40-year period from 1960 to 2001, the elasticity of public capital is estimated to be 0.132.

[12] The six countries covered are the United States, the United Kingdom, Germany, France, Canada, and Japan. The average capital share for 10 years from 2010 to 2019 is 60.9% (Feenstra et al. 2015).

nominal GDP in each year, is 3.1% from FY2013 to FY2019 (see Appendix C).[13] Moreover, in EU countries, the deficit ratio of general government is constrained up to 3% of GDP in each country under the Maastricht Treaty of 1993.[14]

The ratio of the rich to population δ is set to 0.5. Although the rich population is thought to be smaller than the poor one in the actual economy, we can identify the degree of wealth inequality without the distortion of class share by assuming that the economy consists of the same number of rich and poor. When wealth inequality $\Phi_t = \frac{\delta s_t^R N}{A_t}$ in the BGPs deviates from 0.5, the disparity can be regarded as the difference in the savings per capita between the rich and poor.

As regards the intertemporal weights, we set $\alpha_R = 0.45$ and $\alpha_P = 0.35$, referring to the estimate of social time preference rate for major developed countries by Evans and Sezer (2004). They estimate the long-term time preference rates for EU countries (the U.K., Germany, and France) and non-EU countries (Japan, the U.S., and Australia) and find the former to be 1.0% and the latter to be 1.5%, considering disaster risk. As one period lasts 35 years, the estimated time preference rate is $\left(\frac{1}{1.015}\right)^{35} \approx 0.594$. The average discount factor in the economy satisfies $1: 0.594 = 1 - \alpha: \alpha$ and can be estimated to be approximately 0.4. Thus, we set the intertemporal weights of the rich and poor to 0.45 and 0.35, respectively.

Lifetime uncertainty π is assumed to be 0.5 in the base case. π means the survival probability with which individuals survive from youth to old age and can be interpreted as an indicator of aging rate. Additionally, the total factor productivity Γ is set to 18 in this simulation.

Table 2. Results of numerical simulation

The lower equilibrium E_L^*				The higher equilibrium E_H^*			
x_L^*	Ω_L^*	Φ_L^*	Growth rate	x_H^*	Ω_H^*	Φ_H^*	Growth rate
0.623	4.152	0.617	2.21%	1.972	13.146	0.638	1.08%
e_L^1	e_L^2	e_L^3	–	e_H^1	e_H^2	e_H^3	–
0.624	0.465	0.302	–	1.650	0.686	0.435	–

Table 2 shows the results of x^*, Ω^*, Φ^*, economic growth rate, and eigenvalues in the lower equilibrium E_L^* and higher equilibrium E_H^*. Since the absolute values of eigenvalues e_L are all less than 1, lower equilibrium is identified as locally and asymptotically stable (see Appendix D and E). Meanwhile, higher equilibrium is unstable, and as $0 < e_H^2 < e_H^3 < 1 < e_H^1$, E_H^* is a saddle point. These properties on stability hold in all other simulations provided later, and we

[13] Since the global financial crisis in 2008, the fiscal deficit in the U.S. has been particularly high, and the deficit ratio has exceeded 5% until FY2012. Therefore, taking into account the impact of the financial crisis and COVID-19, the average is calculated based on the fiscal data from FY2013 to FY2019.

[14] In contrast, Japan has recorded considerably high deficit ratios, and the average deficit ratio is estimated at around 7% through the period (Economic and Social Research Institute, Cabinet Office, Government of Japan 2025). As of 2025, Japan is the country with the highest public debt–GDP ratio, which has exceeded 230%.

focus mainly on the analysis of the stable equilibrium.

Result 1
In the numerical simulation based on advanced economies, lower equilibrium E_L^* is locally and asymptotically stable, whereas higher equilibrium E_H^* is a saddle point.

3. Comparative statics

3.1 Population aging

We examine a rise in a lifetime uncertainty π and its manifold impacts: saving-enhancing effects and wealth-depletion effects. In this model, the optimal savings are determined under these conflicting effects. Since many advanced economies have experienced the decline in savings rate with aging, this section focuses especially on the range of $\pi \in (\pi_p^*, 1)$ where $\frac{\partial \tilde{\alpha}_i}{\partial \pi}|_{\pi > \pi_p^*} < 0$ and $\pi_R^* < \pi_P^* < \frac{1}{2}$: wealth-depletion effects emerge in the savings behavior of the rich and poor.

The effects of π on BGPs can be captured with the shifts of those two curves and one plane given by Equations (23), (24), and (25). The total differentiation of Equation (27) yields

$$\frac{d\rho}{d\pi} = -\frac{(1-\theta)\Gamma\lambda^{1-\theta}g^{\theta}}{x^{*2-\theta}}\frac{\partial x^*}{\partial \pi}. \tag{28}$$

Therefore, the effects of aging on economic growth in the BGPs depend on $\frac{\partial x^*}{\partial \pi}$, and when aging causes the rise in x^*, it hinders economic growth, and vice versa.

Next, the total differentiation of Equation (25) yields

$$\frac{\partial \Phi_{t-1}}{\partial \pi} = -\frac{\frac{1-\theta-\gamma}{1-\Phi_{t-1}}\left[\left(-\frac{\frac{\partial(\tilde{\alpha}_R-\tilde{\alpha}_P)}{\partial \pi}}{(\tilde{\alpha}_R-\tilde{\alpha}_P)^2}\right)\left(\hat{\alpha}-\frac{\delta\hat{\alpha}_R}{\Phi_{t-1}}\right)+\left(\frac{1}{\tilde{\alpha}_R-\tilde{\alpha}_P}\right)\left(\frac{\partial\hat{\alpha}}{\partial\pi}-\frac{\delta}{\Phi_{t-1}}\frac{\partial\hat{\alpha}_R}{\partial\pi}\right)\right]}{\frac{1-\theta-\gamma}{\tilde{\alpha}_R-\tilde{\alpha}_P}\left(\frac{1}{1-\Phi_{t-1}}\right)\left[\left(\frac{1}{1-\Phi_{t-1}}\right)\left(\hat{\alpha}-\frac{\delta\hat{\alpha}_R}{\Phi_{t-1}}\right)+\frac{\delta\hat{\alpha}_R}{\Phi_{t-1}^2}\right]} < 0. \tag{29}$$

Equation (29) shows that the rise in π shifts $\Psi_R(\Phi_{t-1}) = \Psi_L(x_t)$ downward (see Appendix F). Meanwhile, Equation (23) is independent of π, and the $\lambda\Omega_t = gx_t$ plane remains unchanged.

Finally, we consider the effects of aging on Equation (24). We define the right-hand side of Equation (21) as $\Theta(x_t) \equiv \left(\frac{1}{\tilde{\alpha}_R-\tilde{\alpha}_P}\right)(\omega_1 - \hat{\alpha}\omega_2 - \tilde{\alpha}_P)$, and differentiating $\Theta(x_t)$ with π yields the following condition:

$$\frac{\partial\Theta}{\partial\pi} \gtreqless 0 \Leftrightarrow -\frac{\frac{\partial(\tilde{\alpha}_R-\tilde{\alpha}_P)}{\partial\pi}}{\tilde{\alpha}_R-\tilde{\alpha}_P} \gtreqless \frac{\omega_2\frac{\partial\hat{\alpha}}{\partial\pi}+\frac{\partial\tilde{\alpha}_P}{\partial\pi}}{\omega_1-\hat{\alpha}\omega_2-\tilde{\alpha}_P}, \tag{30}$$

where $\omega_1 \equiv \frac{\left[1+\frac{\lambda\Gamma\left(\frac{g}{\lambda}x_t\right)^{\theta}}{x_t}\right](1+\gamma x_t)}{1+\gamma x_t+\gamma\mu\left(\frac{g}{\lambda}x_t\right)^{\theta}}$ and $\omega_2 \equiv \frac{\mu(1-\theta-\gamma)\left(\frac{g}{\lambda}x_t\right)^{\theta}}{(1+x_t)\left[1+\gamma x_t+\gamma\mu\left(\frac{g}{\lambda}x_t\right)^{\theta}\right]}$. Equation (30) suggests that the effects of aging on $\Theta(x_t)$ differ depending on π, and the rise in π induces a twisted shift of $\Theta(x_t)$ (see Appendix G).

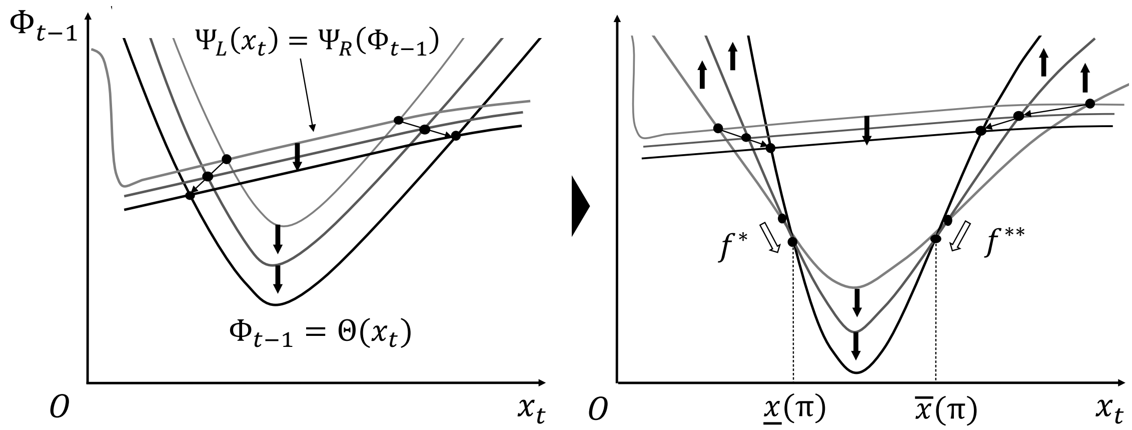


Figure 4. The twisted shifts with higher π

Figure 4 shows the dynamics of transition points f^* and f^{**}, where $\frac{\partial\Theta}{\partial\pi}$ switches from positive to negative and from negative to positive, respectively. When the level of x_t of f^* is defined as $\underline{x}(\pi)$ and that of f^* as $\overline{x}(\pi)$, $\frac{\partial\Theta}{\partial\pi} < 0$ holds in the range of $x_t \in (\underline{x}, \overline{x})$, whereas $\frac{\partial\Theta}{\partial\pi} > 0$ in other areas (see Appendix G). When aging proceeds, $\underline{x}(\pi)$ increases, while $\overline{x}(\pi)$ decreases, which diminishes the distance between $\underline{x}(\pi)$ and $\overline{x}(\pi)$.

When π is relatively low, Equation (24) and (25) have intersections lower than f^* and f^{**}; a rise in π shifts both curves downward in the vicinity of equilibria, with the shift width of $\Psi_L(x_t) = \Psi_R(\Phi_{t-1})$ rather diminutive.[15]. As a result, lower x_L^* and Ω_L^* in the stable equilibrium E_L facilitates economic growth, for Equation (8) suggests that the decline in x_L^* implies the reduction of relative public debts, leading to the lower tax rate and more disposal income and savings for private capital investment. Thus, when π is small, saving-enhancing effects prevail, and aging facilitates economic growth and fiscal sustainability.

Meanwhile, when the economy ages further, the transition points move below $\Psi_L(x_t) = \Psi_R(\Phi_{t-1})$ and the shifting direction of $\Phi_{t-1} = \Theta(x_t)$ switches around equilibria; a rise in π shifts $\Phi_{t-1} = \Theta(x_t)$ upward in the range of $x_t \in (0, \underline{x})$ and $(\overline{x}, \infty)$, increasing the level of x_L^* in the stable equilibrium E_L and hindering economic growth. Economic growth stagnates because more of the old deplete their wealth for existence, which suppresses savings growth and capital deepening.

Hence, an "inverted U-shaped" relationship occurs between life expectancy and economic growth rate, and a threshold of aging level that maximizes economic growth exists. This is because saving-enhancing effects, based on the life cycle theory, are gradually offset by wealth-depletion effects, caused by extended longevity, and eventually, the latter surpasses the former when aging proceeds. Table 3 presents the results of comparative statics.

Table 3. Comparative statics

[15] Note that the shift width is theoretically limited to the small range of $\Phi_{t-1} \in (\frac{1}{2}, 1)$.

Variables	Equation			Lower equilibrium E_L^*			Higher equilibrium E_H^*		
	(23)	(24)	(25)	x_L^*	Ω_L^*	Φ_L^*	x_H^*	Ω_H^*	Φ_H^*
π	Unchanged	Down	Down	−	−	−	±	±	±
		Up		+	+	±	−	−	−
λ	Slope↓	Up	Up	+	±	+	−	−	±
g	Slope↑	Up	Up	+	+	+	−	±	±

Table 4. The results of numerical simulation with π

Variables		Lower equilibrium E_L^*				Higher equilibrium E_H^*			
		x_L^*	Ω_L^*	Φ_L^*	Growth rate	x_H^*	Ω_H^*	Φ_H^*	Growth rate
π	0.70	0.430	2.868	0.592	2.71%	2.151	14.343	0.606	1.02%
	0.80	0.407	2.712	0.581	2.79%	1.941	12.941	0.590	1.10%
	0.85	0.403	2.689	0.576	2.803%	1.806	12.043	0.582	1.15%
	0.90	0.405	2.700	0.571	2.799%	1.661	11.075	0.575	1.21%
	0.95	0.411	2.740	0.567	2.78%	1.510	10.067	0.568	1.29%

Figure 4 shows the results of numerical simulation of the rise in π with parameters set based on advanced economies such as U.S. and EU countries. When an aging indicator π exceeds 0.85, x_L^* turns to rise and lowers the economic growth rate. These outcomes are consistent with those of many developed countries that have experienced aging, slowing economic growth, and deteriorating fiscal conditions simultaneously. Figure 5 is the graph of the relationship between π and the economic growth rate, equivalent to the growth rate of savings in this model, and it represents an inverted U-shaped relationship with $\pi = 0.85$, the supposed transition point.

Additionally, a trade-off between economic growth and equality can be observed beyond the transition point; the economic growth rate decreases with aging, while inequality is improved further. Wealth inequality Φ_L^* decreases because when average life expectancy rises, the rich with higher intertemporal weight consume relatively more wealth and reduce more bequests for longer existence than the poor. Thus, the effects of wealth-depletion become greater for the rich as their discount rate is higher than that of the poor, and the disparity of wealth between these two types of households is reduced.

In this simulation, inequality improves with aging. As stated in Section 1, however, wealth inequality in advanced economies tends to widen. Therefore, the implication is that some mechanism, like the rise in λ or g as discussed later on, would work in the actual economy that aggravates inequality to exceed the inequality-improving effects of aging as depicted above.

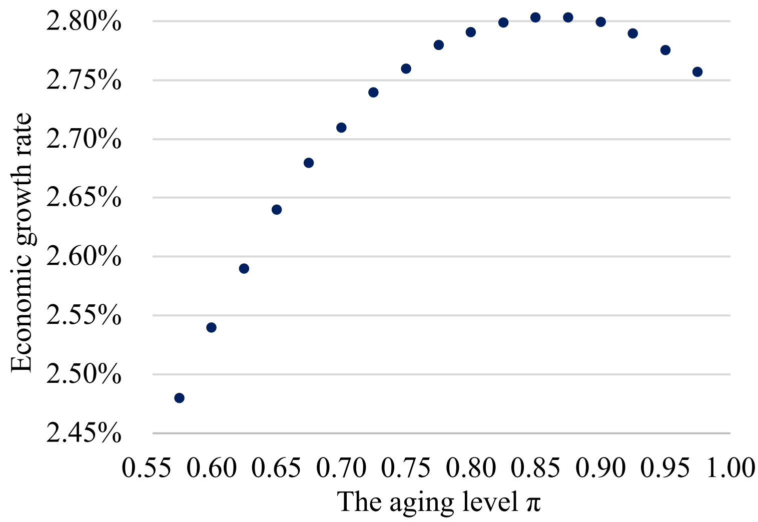


Figure 5. An inverted U-shaped relationship

Proposition 2

(i) When aging proceeds, saving-enhancing effects based on the life cycle theory are offset by the wealth-depletion effects caused by extended longevity, and eventually the latter surpass the former.

(ii) An "inverted U-shaped" relationship occurs between life expectancy and economic growth rate, and a threshold of aging level that maximizes economic growth exists under the assumption that $\frac{\partial \Phi_{t-1}}{\partial \pi}$ is diminutive.

Result 2

Economic growth and equality are achieved simultaneously with aging until π reaches the transition point, whereas the trade-off can be observed beyond it as the economic growth rate decreases, while inequality is improved further.

3.2 Government bond insurance

Next, we consider the effects of government bond insurance on BGPs. Differentiating Θ in Equation (24) with the deficit ratio λ yields

$$\begin{aligned}\frac{\partial \Theta}{\partial \lambda} = &-\left(\frac{1}{\tilde{\alpha}_R - \tilde{\alpha}_P}\right)\frac{(1+\gamma x_t)\left(1+\Gamma\lambda^{1-\theta}g^{\theta}x_t^{\theta-1}\right)\gamma\Gamma g^{\theta}x_t^{\theta}}{\left[1+\gamma x_t+\gamma\mu\left(\frac{g}{\lambda}x_t\right)^{\theta}\right]^2}[\lambda-\theta(1+\theta-g)] \\ &+\mathcal{W}_1\left(\frac{1}{\tilde{\alpha}_R - \tilde{\alpha}_P}\right)(1+x_t)(1+\gamma x_t)(1-\theta)\Gamma\lambda^{-\theta}g^{\theta}x_t^{\theta-1} \\ &-\mathcal{W}_1\left(\frac{1}{\tilde{\alpha}_R - \tilde{\alpha}_P}\right)\hat{\alpha}(1-\theta-\gamma)\Gamma\lambda^{-\theta-1}g^{\theta}x_t^{\theta} \\ &\times\left[1-\frac{\gamma\mu\left(\frac{g}{\lambda}x_t\right)^{\theta}}{1+\gamma x_t+\gamma\mu\left(\frac{g}{\lambda}x_t\right)^{\theta}}\right][\lambda-\theta(1+\theta-g)]>0,\end{aligned} \tag{31}$$

where $\mathcal{W}_1 \equiv \frac{1}{(1+x_t)\left[1+\gamma x_t+\gamma\mu\left(\frac{g}{\lambda}x_t\right)^{\theta}\right]} > 0$ and $\lambda - \theta(1+\theta-g) < 0$.[16]

Similarly, total differentiating Equation (25) with λ yields

$$\frac{\partial \Phi_{t-1}}{\partial \lambda} = \frac{\frac{\partial \Psi_L}{\partial \lambda}}{\frac{\partial \Psi_R}{\partial \Phi_{t-1}}} = -\frac{\frac{(1+x_t)(1+\gamma x_t)\Gamma g^{\theta} x_t^{\theta}[\lambda-\theta(1+\lambda-g)]}{\left[\mu\left(\frac{g}{\lambda}x_t\right)^{\theta}\right]^2 \lambda^{1+\theta}}}{\frac{1-\theta-\gamma}{\tilde{\alpha}_R-\tilde{\alpha}_P}\left(\frac{1}{1-\Phi_{t-1}}\right)\left[\left(\frac{1}{1-\Phi_{t-1}}\right)\left(\hat{\alpha}-\frac{\delta\hat{\alpha}_R}{\Phi_{t-1}}\right)+\frac{\delta\hat{\alpha}_R}{\Phi_{t-1}^2}\right]} > 0. \quad (32)$$

Equation (32) reveals that a rise in λ shifts $\Psi_L(x_t) = \Psi_R(\Phi_{t-1})$ downward, with its shift width diminutive.[17]

Moreover, the effect of λ on Equation (23) is

$$\frac{\partial \Omega_t}{\partial \lambda} = -\frac{g}{\lambda^2}x_t < 0. \quad (33)$$

Thus, as Figure 6 shows, the rise in λ shifts Equations (24) and (25) upward and makes the slope of (x_t, Ω_t) plane less steep. As a result, both x_L^* and Φ_L^* in the lower stable E_L increase, which suggests that fiscal conditions deteriorate with more relative public debts, and wealth inequality between the rich and poor widens (see Table 3).

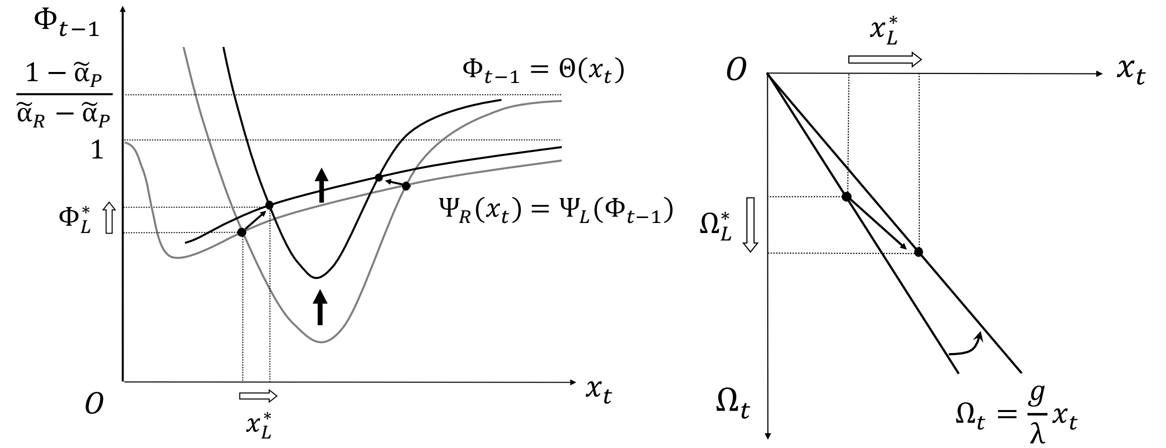


Figure 6. The shifts with higher λ

Furthermore, the total differentiation of Equation (27) yields

$$\frac{d\rho}{d\lambda} = \frac{(1-\theta)\Gamma g^{\theta}}{\lambda^{\theta} x^{*1-\theta}}(1-\varepsilon_{\lambda}), \quad (34)$$

where $\varepsilon_{\lambda} \equiv \frac{\lambda}{x^*}\frac{\partial x^*}{\partial \lambda}$ is the deficit ratio elasticity of x^*. Equation (34) suggests that when $\varepsilon_{\lambda} > 1$ ($\varepsilon_{\lambda} < 1$), the rise in λ decreases (increases) the economic growth rate. In the model, λ has various paths to affect the savings of households. On one hand, the rise in λ distorts savings allocation through further government bond insurance and has negative effects that hinder

[16] We reasonably assume $\lambda - \theta(1+\theta-g) < 0$ because the deficit ratio λ, several percent, is considerably small compared with θ and g in the real advanced economy.

[17] The shift width is theoretically limited to the small range of $\Phi_{t-1} \in (\frac{1}{2}, 1)$, and the denominator of $\frac{\partial \Psi_L}{\partial \lambda}$ is considerably large compared with the numerator in the base case.

private capital accumulation. Moreover, higher x_L^* leads to larger interest expense on government bonds, which is reflected as the increase in γx_L^* in Equation (8). Thus, the higher tax rate is realized in the equilibrium, depressing disposable income and therefore savings. On the other hand, the rise in λ has positive effects in that it augments households' savings by decreasing the equilibrium tax rate and therefore increasing disposable income, as shown in Equation (8).

Table 5. The results of numerical simulation with λ

Variables		The lower equilibrium E_L^*				The higher equilibrium E_H^*			
		x_L^*	Ω_L^*	Φ_L^*	Growth rate	x_H^*	Ω_H^*	Φ_H^*	Growth rate
λ	0.015	0.200	2.673	0.610	2.81%	3.768	50.245	0.664	0.42%
	0.020	0.298	2.976	0.612	2.66%	3.110	31.107	0.655	0.59%
	0.025	0.426	3.410	0.614	2.47%	2.539	20.312	0.646	0.80%
	0.030	0.623	4.152	0.617	2.21%	1.972	13.146	0.638	1.08%

In the numerical simulation based on advanced economies, elasticity ε_λ is calculated to be greater than 1, and the economic growth rate declines with higher λ. Table 5 shows that the rise in λ aggravates fiscal instability, hinders economic growth, and exacerbates wealth inequality in the stable equilibrium. The rise in wealth inequality Φ_L^*, the share of the rich in the total savings of the economy, indicates that the rich moderate the decline of the savings growth rate in response to further government bond insurance, while the poor show a relatively large decrease in savings growth rate when higher λ causes the equilibrium tax rate to rise and disposable income to decrease. Most advanced countries have experienced aging and public debt expansions at the same time, the latter of which would predominantly affect wealth inequality in the economy.

Additionally, the rise in λ facilitates the relative accumulation of public capital with higher Ω_L^*. This is because more government bond insurance distorts savings allocation to hinder private capital accumulation, increasing the relative amount of public capital to private capital. Furthermore, the results show that higher λ decreases the levels of x_{H}^*, Ω_{H}^* and Φ_{H}^* in the higher unstable equilibrium E_H^*. Therefore, a lower E_H^* approaching origin O can be regarded as the reduction of stable area leading to convergence, defined as the distance between O and E_H^*. Thus, a potentially unstable economy emerges.

Proposition 3

(i) the rise in the deficit ratio $\boldsymbol{\lambda}$ exacerbates fiscal instability and wealth inequality with higher public debt–capital ratio $\boldsymbol{x_L^*}$ and wealth inequality $\boldsymbol{\Phi_L^*}$ under the assumption that $\boldsymbol{\frac{\partial \Phi_{t-1}}{\partial \lambda}}$ is diminutive.

(ii) when the elasticity of $\boldsymbol{x^*}$ with respect to $\boldsymbol{\lambda}$, $\boldsymbol{\varepsilon_\lambda}$, is greater (smaller) than 1, the rise in $\boldsymbol{\lambda}$

hinders (facilitates) economic growth in the stable BGP.
(iii) when ε_λ is greater than 1, the rise in λ exacerbates economic growth and wealth inequality simultaneously.

Result 3
In the numerical simulation based on advanced economies,
(i) the rise in λ exacerbates fiscal instability, economic growth, and wealth inequality with higher x_L^*, public-private capital ratio Ω_L^* and wealth inequality Φ_L^*.
(ii) the rise in λ produces potentially unstable economy with lower x_H^*, Ω_H^*, and Φ_H^* in higher equilibrium E_H^*.

3.3 Public investment

We consider the effects of public investment on BGPs. Differentiating Θ in Equation (24) with the government expenditure ratio g yields

$$\begin{aligned}\frac{\partial\Theta}{\partial g} = &-\left(\frac{1}{\tilde{\alpha}_R-\tilde{\alpha}_P}\right)\frac{(1+\gamma x_t)\left(1+\Gamma\lambda^{1-\theta}g^{\theta}x_t^{\theta-1}\right)\gamma\Gamma x_t^{\theta}}{\lambda^{\theta}g^{1-\theta}\left[1+\gamma x_t+\gamma\mu\left(\frac{g}{\lambda}x_t\right)^{\theta}\right]^2}[\theta(1+\lambda-g)-g] \\ &+\mathcal{W}_1\left(\frac{1}{\tilde{\alpha}_R-\tilde{\alpha}_P}\right)(1+x_t)(1+\gamma x_t)\theta\Gamma\lambda^{1-\theta}g^{\theta-1}x_t^{\theta-1} \\ &-\mathcal{W}_1\left(\frac{1}{\tilde{\alpha}_R-\tilde{\alpha}_P}\right)\hat{\alpha}(1-\theta-\gamma)\Gamma\lambda^{-\theta}g^{\theta-1}x_t^{\theta} \\ &\times\left[1-\frac{\gamma\mu\left(\frac{g}{\lambda}x_t\right)^{\theta}}{1+\gamma x_t+\gamma\mu\left(\frac{g}{\lambda}x_t\right)^{\theta}}\right][\theta(1+\lambda-g)-g]>0,\end{aligned} \tag{35}$$

where $\theta(1+\lambda-g)-g<0$.[18]

Total differentiating Equation (25) with g yields

$$\frac{\partial\Phi_{t-1}}{\partial g}=\frac{\frac{\partial\Psi_L}{\partial g}}{\frac{\partial\Psi_R}{\partial\Phi_{t-1}}}=-\frac{\dfrac{(1+x_t)(1+\gamma x_t)\Gamma x_t^{\theta}[\theta(1+\lambda-g)-g]}{\left[\mu\left(\frac{g}{\lambda}x_t\right)^{\theta}\right]^2\lambda^{\theta}g^{1-\theta}}}{\frac{1-\theta-\gamma}{\tilde{\alpha}_R-\tilde{\alpha}_P}\left(\frac{1}{1-\Phi_{t-1}}\right)\left[\left(\frac{1}{1-\Phi_{t-1}}\right)\left(\hat{\alpha}-\frac{\delta\hat{\alpha}_R}{\Phi_{t-1}}\right)+\frac{\delta\hat{\alpha}_R}{\Phi_{t-1}^2}\right]}>0. \tag{36}$$

Equation (36) shows that a rise in g shifts $\Psi_L(x_t)=\Psi_R(\Phi_{t-1})$ downward, with its shift width diminutive.[19]

The effects of λ on Equation (23) is

$$\frac{\partial\Omega_t}{\partial g}=\frac{1}{\lambda}x_t>0. \tag{37}$$

Thus, as Figure 7 shows, the rise in g shifts Equations (24) and (25) upward and makes the slope of (x_t,Ω_t) plane steeper, which increases x_L^*, Ω_L^*, and Φ_L^* in the lower stable E_L. Accordingly, more public investment exacerbates fiscal conditions and wealth inequality while facilitating public capital accumulation (see Table 3).

[18] We can reasonably assume $\lambda<\theta\le g$ in the real advanced economies.
[19] Note that the shift width is theoretically limited to the small range of $\Phi_{t-1}\in(\frac{1}{2},1)$ and the denominator of $\frac{\partial\Psi_L}{\partial g}$ is considerably large compared with the numerator in the base case.

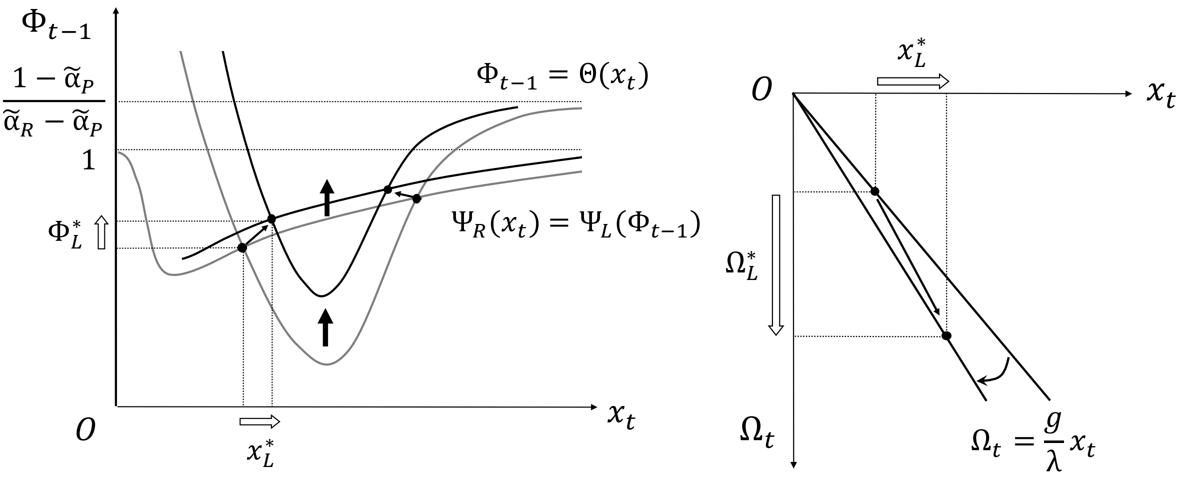


Figure 7. The shifts with higher g

Total differentiation of Equation (27) yields

$$\frac{d\rho}{dg}=\frac{\theta\Gamma\lambda^{1-\theta}}{g^{1-\theta}x^{*1-\theta}}\left(1-\frac{1-\theta}{\theta}\varepsilon_g\right), \tag{38}$$

where $\varepsilon_g \equiv \frac{g}{x_t}\frac{\partial x^*}{\partial g}$ is the government expenditure ratio elasticity of x^*. Equation (38) shows that when $\frac{\theta}{1-\theta}>\varepsilon_g$ ($\frac{\theta}{1-\theta}<\varepsilon_g$), the rise in g decreases (decreases) the economic growth rate.[20] On one hand, the rise in g means more public investment and increases the relative public capital Ω_L^*. Higher Ω_L^* promotes economic growth because it facilitates private capital accumulation by increasing the interest rate r_t from Equation (3). On the other hand, more government expenditure reduces households' savings by raising the equilibrium tax rate. As a result, it prevents private capital accumulation and decreases the economic growth rate. Public investment also leads to larger interest expense r_tD_t on government bonds as it has an externality effect to increase the interest rate r_t, decreasing the disposable income with a higher tax rate.

Table 6. The results of numerical simulation with g

Variables		The lower equilibrium E_L^*				The higher equilibrium E_H^*			
		x_L^*	Ω_L^*	Φ_L^*	Growth rate	x_H^*	Ω_H^*	Φ_H^*	Growth rate
g	0.22	0.668	4.895	0.617	2.16%	1.870	13.712	0.636	1.14%
	0.24	0.727	5.813	0.618	2.08%	1.743	13.944	0.634	1.21%
	0.26	0.812	7.034	0.620	1.98%	1.581	13.703	0.632	1.31%
	0.28	0.972	9.073	0.622	1.79%	1.336	12.466	0.628	1.63%

[20] Previous studies show that the elasticity of public capital toward output, θ, is estimated to be considerably below 0.5 and therefore, $\frac{\theta}{1-\theta}$ is supposed to be smaller than 1.

In the numerical simulation, elasticity ε_g is calculated to be greater than $\frac{\theta}{1-\theta} = 0.25$, which causes the economic growth rate to decline with higher g. Table 6 shows that public investment aggravates fiscal instability, hinders economic growth, and exacerbates wealth inequality in the stable equilibrium while facilitating the relative accumulation of public capital. As in the previous section 3.2, the rise in wealth inequality Φ_L^* indicates that the rich suppress the decline of the savings growth rate compared to the poor, who face a relatively large decrease in it.

Proposition 4
(i) the rise in the deficit ratio $\boldsymbol{\lambda}$ exacerbates fiscal instability and wealth inequality with higher public debt–capital ratio $\boldsymbol{x_L^*}$ and wealth inequality $\boldsymbol{\Phi_L^*}$, while public investment increases public–private capital ratio $\boldsymbol{\Omega_L^*}$ under the assumption that $\boldsymbol{\frac{\partial \Phi_{t-1}}{\partial g}}$ is diminutive.

(ii) when the elasticity of $\boldsymbol{x^*}$ with respect to $\boldsymbol{g}$, $\boldsymbol{\varepsilon_g}$, is greater (smaller) than $\boldsymbol{\frac{\theta}{1-\theta}}$, the rise in $\boldsymbol{g}$ hinders (facilitates) economic growth in the stable BGP.
(iii) when $\boldsymbol{\varepsilon_\lambda}$ is greater than $\boldsymbol{\frac{\theta}{1-\theta}}$, the rise in $\boldsymbol{g}$ exacerbates economic growth and wealth inequality simultaneously.

Result 4
In the numerical simulation based on advanced economies, the rise in $\boldsymbol{g}$ exacerbates fiscal instability, economic growth, and wealth inequality with higher $\boldsymbol{x_L^*}$, public–private capital ratio $\boldsymbol{\Omega_L^*}$, and wealth inequality $\boldsymbol{\Phi_L^*}$.

4. Conclusion

This study theoretically analyzes population aging and its impacts on economic growth, wealth inequality, and fiscal sustainability. We introduce lifetime uncertainty to the overlapping generations model, where heterogeneous households with varied intertemporal preferences survive from youth to old age with a certain probability, and unintended bequests caused by death are inherited by offspring.

Aging can have both positive and negative impacts on economic growth and fiscal sustainability: saving-enhancing effects and wealth-depletion effects. On one hand, households with longer life expectancy save more to equalize consumption after retirement, which induces economic growth through capital accumulation. This saving-enhancing effect also reduces the relative ratio of public debts to private capital, leading to fiscal consolidation. On the other hand, aging also has wealth-depletion effects accompanied by a longer retirement period; individuals with extended longevity consume more wealth for existence, which hinders economic growth through less bequests for offspring and undermines fiscal sustainability with more relative public burden.

When aging proceeds, saving-enhancing effects based on the life cycle theory are offset by wealth-depletion effects caused by extended longevity, and eventually, the latter surpass the former. Accordingly, an "inverted U-shaped" relationship occurs between life expectancy and economic growth rate, which suggests the existence of a threshold of aging level that maximizes economic growth. The findings have insightful implications because these two countervailing effects of aging depicted in this model would reflect the controversial consequences of aging suggested in previous empirical studies.

Additionally, numerical simulation based on advanced economies such as U.S. and EU countries suggests that when aging exceeds a certain threshold, a trade-off emerges between economic growth and equality; the economic growth rate decreases with aging, whereas wealth inequality improves.

Moreover, we investigate the effects of government bond insurance and public investment on the macro economy. When the elasticity of public debt–capital ratio toward each ratio is greater than a certain level, higher deficit or government expenditure ratios exacerbate fiscal instability, economic growth, and wealth inequality simultaneously. Therefore, in the advanced economies that have faced serious budget deficits so far, the effect to aggravate inequality is supposed to outweigh inequality-improving effects of aging as presented in the simulation.

This study has two implications for policy on aging. First, under the assumption that social security and fiscal policies can indirectly affect the population's health status and life expectancy, the government should make policies with an understanding of the optimal level of aging for the macroeconomy. Although the optimal policy in economics does not necessarily coincide with the politically or ethically correct one, recognizing the divergence between the economic implications and actual policymaking is at least beneficial.

Second, aging can produce a trade-off between economic growth and equality. Therefore, the government is expected to compare both positive and negative effects of population aging and fully consider the long-term effects on the macroeconomy. In this sense, superficial measures against aging may not necessarily lead to desirable consequences, and they may bring unexpected side effects to the economy.

The possible future research directions are two. Since this paper examines the dynamics of households' savings influenced by longer life expectancy with inelastic labor supply, this model abstracts from the households' possible responses to adjust its labor supply accordingly. Additionally, population aging can have advantageous effects on human capital accumulation, which may prevent stagnated economic growth with further aging. Future research should scrutinize how the main results of this paper change when such counter behavior and positive effects of aging are considered.

Appendices

Appendix A (The outline of Equation (24))

The right-hand side of Equation (24) is defined as $\Theta(x_t)$;

$$\Theta(x_t)=\left(\frac{1}{\tilde{\alpha}_R-\tilde{\alpha}_P}\right)\left[\frac{(1+x_t)\left[1+\frac{\lambda\Gamma\left(\frac{g}{\lambda}x_t\right)^{\theta}}{x_t}\right](1+\gamma x_t)-\hat{\alpha}\mu(1-\theta-\gamma)\left(\frac{g}{\lambda}x_t\right)^{\theta}}{(1+x_t)\left[1+\gamma x_t+\gamma\mu\left(\frac{g}{\lambda}x_t\right)^{\theta}\right]}-\tilde{\alpha}_P\right]. \quad \text{(A1)}$$

From this equation, it follows that $\lim_{x_t\to 0}\Theta(x_t)=\infty$ and $\lim_{x_t\to\infty}\Theta(x_t)=\frac{1-\tilde{\alpha}_P}{\tilde{\alpha}_R-\tilde{\alpha}_P}>1$.

Next, we define part of $\Theta(x_t)$ as $\eta_n(x_t)\equiv(1+x_t)\left[1+\frac{\lambda\Gamma\left(\frac{g}{\lambda}x_t\right)^{\theta}}{x_t}\right](1+\gamma x_t)-\hat{\alpha}\mu(1-\theta-\gamma)\left(\frac{g}{\lambda}x_t\right)^{\theta}$ where $\lim_{x_t\to 0}\Theta(x_t)=\infty$ and $\lim_{x_t\to\infty}\eta_n(x_t)=\infty$. Accordingly, $\eta_n(x_t)$ takes the minimum value in the range $0<x_t<\infty$. Similarly, we define part of $\Theta(x_t)$ as $\eta_d(x_t)\equiv(1+x_t)\left[1+\gamma x_t+\gamma\mu\left(\frac{g}{\lambda}x_t\right)^{\theta}\right]$. $\eta_d(x_t)$ is a monotonically increasing function of x_t and $\frac{\partial\eta_d}{\partial x_t}>0$ holds in the range of $x_t>0$. Figure 8 shows that $\frac{\eta_n(x_t)}{\eta_d(x_t)}$ takes the minimum value in the range $0<x_t<\infty$ and $\Theta(x_t)$ is a convex curve downward with the asymptotes $\Phi_{t-1}=\frac{1-\tilde{\alpha}_P}{\tilde{\alpha}_R-\tilde{\alpha}_P}$ and $x_t=0$.

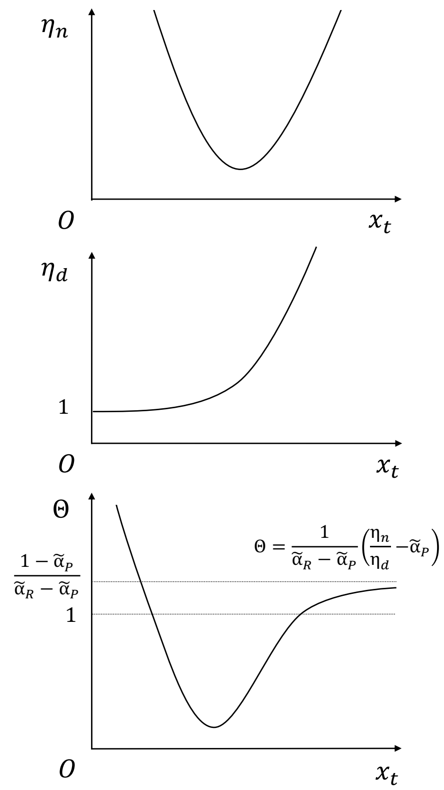


Figure 8. The outline of $\Theta(x_t)$

Appendix B

Table 7. Annual fiscal data of the U.S.

(In millions of dollars)

Fiscal year	2013	2014	2015	2016
(1)Nominal GDP	16,784,849	17,527,164	18,238,301	18,745,076
(2)Outlays	3,454,881	3,506,284	3,691,850	3,852,616
(3)Reciepts	2,775,106	3,021,491	3,249,890	3,267,965
(4)Interest payments (net)	220,885	228,956	223,181	240,033
Government spending/GDP [(2)-(4)]/(1)	19.3%	18.7%	19.0%	19.3%
Deficit ratio [(2)-(3)]/(1)	4.0%	2.8%	2.4%	3.1%

(Table continued)

Fiscal year	2017	2018	2019	Average
(1)Nominal GDP	19,542,979	20,611,861	21,433,225	18,983,351
(2)Outlays	3,981,630	4,109,044	4,446,956	3,863,323
(3)Reciepts	3,316,184	3,329,907	3,463,364	3,203,415
(4)Interest payments	262,551	324,975	375,158	267,963
Government spending/GDP [(2)-(4)]/(1)	19.0%	18.4%	19.0%	18.9%
Deficit ratio [(2)-(3)]/(1)	3.4%	3.8%	4.6%	3.1%

The United States Government Publishing Office (2025)
World Bank (2025)

Appendix C (The outline of Equation (25))

The left- and right-hand sides of Equation (25) are defined as $\Psi_L(x_t)$ and $\Psi_R(\Phi_{t-1})$, respectively. From $\Psi_L(x_t) = \left(\frac{\lambda}{g}\right)^\theta \left[\frac{1}{\mu}\left(\frac{1}{x_t^\theta} + x_t^{1-\theta}\right)\left(\frac{1}{x_t^\theta} + \gamma x_t^{1-\theta}\right)\right] + (1 + x_t)\gamma$, it follows that $\lim_{x_t \to 0} \Psi_L(x_t) = \infty$, $\lim_{x_t \to \infty} \Psi_L(x_t) = \infty$ and $\Psi_L(x_t) > 0$ in the range of $x_t > 0$. Meanwhile, $\Psi_R(\Phi_{t-1}) = \left[\frac{1-\theta-\gamma}{(\tilde{\alpha}_R - \tilde{\alpha}_P)(1-\Phi_{t-1})}\right]\left(\hat{\alpha} - \frac{\delta \hat{\alpha}_R}{\Phi_{t-1}}\right)$ where $\lim_{\Phi_{t-1} \to 1} \Psi_L(x_t) = \infty$.

Therefore, when $x_t \to 0$, $\Psi_L(x_t) \to \infty$, it follows that $\Phi_{t-1} \to 1$ in $\Psi_R(\Phi_{t-1})$ because $\Psi_R(\Phi_{t-1})$ necessarily increases to satisfy $\Psi_L(x_t) = \Psi_R(\Phi_{t-1})$. Similarly, when

$x_t \to 0$, $\Psi_L(x_t) \to \infty$, it follows that $\Phi_{t-1} \to 1$ in $\Psi_R(\Phi_{t-1})$ under the equality. Furthermore, differentiating Equation (25) with x_t yields $\frac{\partial \Psi_L(x_t)}{\partial x_t} = 0$, and the level of x_t satisfying this corresponds to the x_t coordinate of the vertex in Equation (25). From $\frac{\partial \Psi_L(x_t)}{\partial x_t} = 0$, we obtain that $(1+\gamma)(1-\theta)x_t + \gamma(2-\theta)x_t^2 + \mu\gamma\left(\frac{\lambda}{g}\right)^{\theta} x_t^{1+\theta} = \theta$, and the level of x_t satisfying this is supposed to be considerably diminutive as $\theta \in (0, 1)$. Accordingly, Equation (25) is a convex curve downward with the asymptotes $\Phi_{t-1} = 1$, extremely skewed to right.

Appendix D (Stability analysis)

We analyze the stability of equilibria. We denote $\mathcal{F}_A$ and $\mathcal{F}_R$ as follows:

$$\mathcal{F}_A \equiv \frac{A_t}{A_{t-1}} = \frac{\hat{\alpha}\mu\,(1-\theta-\gamma)\Omega_t^{\theta}}{(1+\gamma x_t)(1+x_t)} + \left[\frac{1+\gamma x_t + \mu\gamma\Omega_t^{\theta}}{1+\gamma x_t}\right][(\tilde{\alpha}_R - \tilde{\alpha}_P)\Phi_{t-1} + \tilde{\alpha}_P], \tag{D1}$$

$$\mathcal{F}_R \equiv \frac{s_t^R}{s_{t-1}^R} = \hat{\alpha}_R\left[\frac{\delta\mu(1-\theta-\gamma)\Omega_t^{\theta}}{\Phi_{t-1}(1+\gamma x_t)(1+x_t)}\right] + \tilde{\alpha}_R\left[\frac{1+\gamma x_t + \mu\gamma\Omega_t^{\theta}}{1+\gamma x_t}\right]. \tag{D2}$$

The Jacobian matrix $\mathcal{J}$ is defined as

$$\mathcal{J} \equiv \begin{bmatrix} J_{11} & J_{12} & J_{13} \\ J_{21} & J_{22} & J_{23} \\ J_{31} & J_{32} & J_{33} \end{bmatrix}. \tag{D3}$$

The components of the matrix are given by

$$J_{11} \equiv \frac{\partial x_{t+1}}{\partial x_t} = \frac{(1-\lambda\Gamma\Omega_t^{\theta})\mathcal{F}_A - (1+x_t)(x_t + \lambda\Gamma\Omega_t^{\theta})\frac{\partial \mathcal{F}_A}{\partial x_t}}{\left[(1+x_t)\mathcal{F}_A - (x_t + \lambda\Gamma\Omega_t^{\theta})\right]^2}, \tag{D4}$$

$$J_{12} \equiv \frac{\partial x_{t+1}}{\partial \Omega_t} = \frac{\theta\lambda\Gamma\Omega_t^{\theta-1}(1+x_t)\mathcal{F}_A - (1+x_t)(x_t + \lambda\Gamma\Omega_t^{\theta})\frac{\partial \mathcal{F}_A}{\partial \Omega_t}}{\left[(1+x_t)\mathcal{F}_A - (x_t + \lambda\Gamma\Omega_t^{\theta})\right]^2}, \tag{D5}$$

$$J_{13} \equiv \frac{\partial x_{t+1}}{\partial \Phi_{t-1}} = -\frac{(1+x_t)(x_t + \lambda\Gamma\Omega_t^{\theta})\frac{\partial \mathcal{F}_A}{\partial \Phi_{t-1}}}{\left[(1+x_t)\mathcal{F}_A - (x_t + \lambda\Gamma\Omega_t^{\theta})\right]^2}, \tag{D6}$$

$$J_{21} \equiv \frac{\partial \Omega_{t+1}}{\partial x_t} = -\frac{(\Omega_t + g\Gamma\Omega_t^{\theta})\left[\mathcal{F}_A + (1+x_t)\frac{\partial \mathcal{F}_A}{\partial x_t} - 1\right]}{\left[(1+x_t)\mathcal{F}_A - (x_t + \lambda\Gamma\Omega_t^{\theta})\right]^2}, \tag{D7}$$

$$\begin{aligned} J_{22} \equiv \frac{\partial \Omega_{t+1}}{\partial \Omega_t} &= \frac{(1+\theta g\Gamma\Omega_t^{\theta-1})(1+x_t)\mathcal{F}_A - (1+\theta g\Gamma\Omega_t^{\theta-1})(x_t + \lambda\Gamma\Omega_t^{\theta})}{\left[(1+x_t)\mathcal{F}_A - (x_t + \lambda\Gamma\Omega_t^{\theta})\right]^2} \\ &\quad - \frac{(\Omega_t + g\Gamma\Omega_t^{\theta})(1+x_t)\frac{\partial \mathcal{F}_A}{\partial \Omega_t} + \theta\lambda\Gamma\Omega_t^{\theta-1}(\Omega_t + g\Gamma\Omega_t^{\theta})}{\left[(1+x_t)\chi_A - (x_t + \lambda\Gamma\Omega_t^{\theta})\right]^2}, \end{aligned} \tag{D8}$$

$$J_{23} \equiv \frac{\partial \Omega_{t+1}}{\partial \Phi_{t-1}} = -\frac{(\Omega_t + g\Gamma\Omega_t^{\theta})(1+x_t)\frac{\partial \mathcal{F}_A}{\partial \Phi_{t-1}}}{\left[(1+x_t)\mathcal{F}_A - (x_t + \lambda\Gamma\Omega_t^{\theta})\right]^2}, \tag{D9}$$

$$J_{31} \equiv \frac{\partial \Phi_t}{\partial x_t} = \frac{\Phi_{t-1}\frac{\partial \mathcal{F}_R}{\partial x_t} - \Phi_{t-1}\frac{\partial \mathcal{F}_A}{\partial x_t}}{\mathcal{F}_A}, \tag{D10}$$

$$J_{32} \equiv \frac{\partial \Phi_t}{\partial \Omega_t} = \frac{\Phi_{t-1}\frac{\partial \mathcal{F}_R}{\partial \Omega_t} - \Phi_{t-1}\frac{\partial \mathcal{F}_A}{\partial \Omega_t}}{\mathcal{F}_A}, \tag{D11}$$

$$J_{33} \equiv \frac{\partial \Phi_t}{\partial \Phi_{t-1}} = \frac{\chi_R + \Phi_{t-1}\frac{\partial \mathcal{F}_R}{\partial \Phi_{t-1}} - \Phi_{t-1}\frac{\partial \mathcal{F}_A}{\partial \Phi_{t-1}}}{\mathcal{F}_A}, \tag{D12}$$

where

$$\frac{\partial \mathcal{F}_A}{\partial x_t} = -\frac{\hat{\alpha}(1+\gamma+2\gamma x_t)\mu\,(1-\theta-\gamma)\Omega_t^{\theta}}{(1+\gamma x_t)^2(1+x_t)^2} - \frac{\gamma^2\mu\Omega_t^{\theta}[\Phi_{t-1}\tilde{\alpha}_R + (1-\Phi_{t-1})\tilde{\alpha}_P]}{(1+\gamma x_t)^2}, \tag{D13}$$

$$\frac{\partial \mathcal{F}_A}{\partial \Omega_t} = \frac{\hat{\alpha}\theta\mu\,(1-\theta-\gamma)\Omega_t^{\theta-1}}{(1+\gamma x_t)(1+x_t)} + \frac{\theta\gamma\mu\Omega_t^{\theta-1}[(\tilde{\alpha}_R-\tilde{\alpha}_P)\Phi_{t-1}+\tilde{\alpha}_P]}{1+\gamma x_t}, \tag{D14}$$

$$\frac{\partial \mathcal{F}_A}{\partial \Phi_{t-1}} = \frac{(1+\gamma x_t+\gamma\mu\Omega_t^{\theta})(\tilde{\alpha}_R-\tilde{\alpha}_P)}{1+\gamma x_t}, \tag{D15}$$

$$\frac{\partial \mathcal{F}_R}{\partial x_t} = -\frac{\tilde{\alpha}_R\gamma^2\mu\Omega_t^{\theta}}{(1+\gamma x_t)^2}, \tag{D16}$$

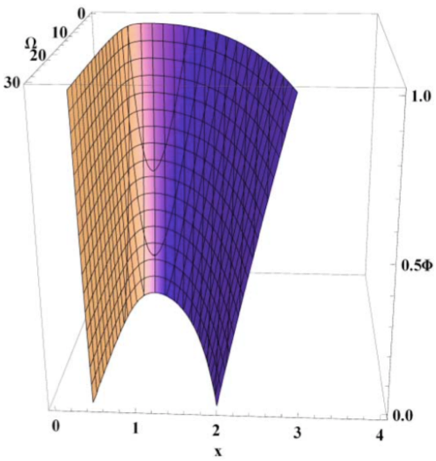
$$\frac{\partial \mathcal{F}_R}{\partial \Omega_t} = \frac{\delta\hat{\alpha}_R\theta\mu(1-\theta-\gamma)\Omega_t^{\theta-1}}{\Phi_{t-1}(1+\gamma x_t)(1+x_t)} + \frac{\tilde{\alpha}_R\gamma\mu\Omega_t^{\theta-1}}{1+\gamma x_t}, \tag{D17}$$

$$\frac{\partial \mathcal{F}_R}{\partial \Phi_{t-1}} = -\frac{\delta\hat{\alpha}_R\theta\mu(1-\theta-\gamma)\Omega_t^{\theta}}{\Phi_{t-1}^2(1+\gamma x_t)(1+x_t)}. \tag{D18}$$

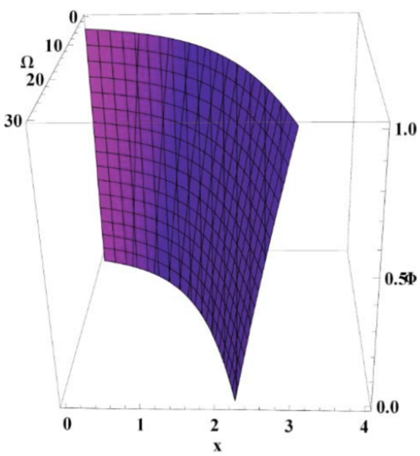
In the numerical simulation, we calculate the eigenvalues at each BGP and discriminate the stability of each equilibrium. As a result, the lower equilibrium E_L^* is locally and asymptotically stable. Meanwhile, the higher equilibrium is a saddle point.

Appendix E (Numerical simulation)

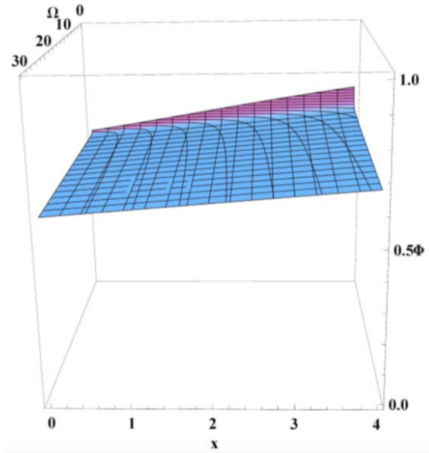
Figure 9 shows the loci of $x_{t+1} = x_t$, $\Omega_{t+1} = \Omega_t$ and $\Phi_t = \Phi_{t-1}$ in the steady states and multiple equilibria emerging in the intersections: lower stable equilibrium E_L and higher unstable equilibrium E_H.

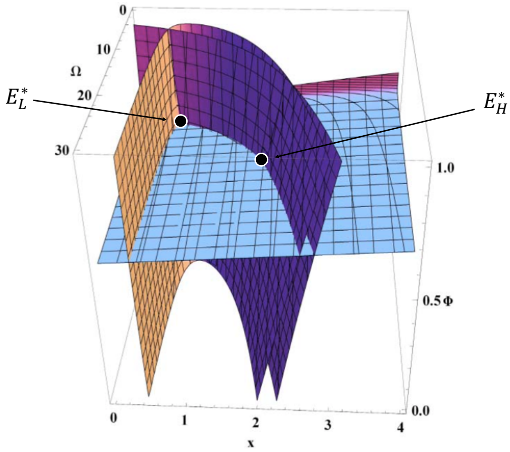


$x_{t+1} = x_t$ locus



$\Omega_{t+1} = \Omega_t$ locus





$\Phi_t = \Phi_{t-1}$ locus multiple equilibria

Figure 9. Difference equation system and multiple BGPs

Appendix F (Aging)

We denote the right-hand side of Equation (25) as $\Psi_R(\Phi_{t-1})$. Differentiating this with π yields

$$\frac{\partial \Psi_R}{\partial \pi} = \frac{1-\theta-\gamma}{1-\Phi_{t-1}}\left[\left(-\frac{\frac{\partial(\tilde{\alpha}_R-\tilde{\alpha}_P)}{\partial \pi}}{(\tilde{\alpha}_R-\tilde{\alpha}_P)^2}\right)\left(\hat{\alpha}-\frac{\delta\hat{\alpha}_R}{\Phi_{t-1}}\right) + \left(\frac{1}{\tilde{\alpha}_R-\tilde{\alpha}_P}\right)\left(\frac{\partial\hat{\alpha}}{\partial \pi}-\frac{\delta}{\Phi_{t-1}}\frac{\partial\hat{\alpha}_R}{\partial \pi}\right)\right]. \tag{F1}$$

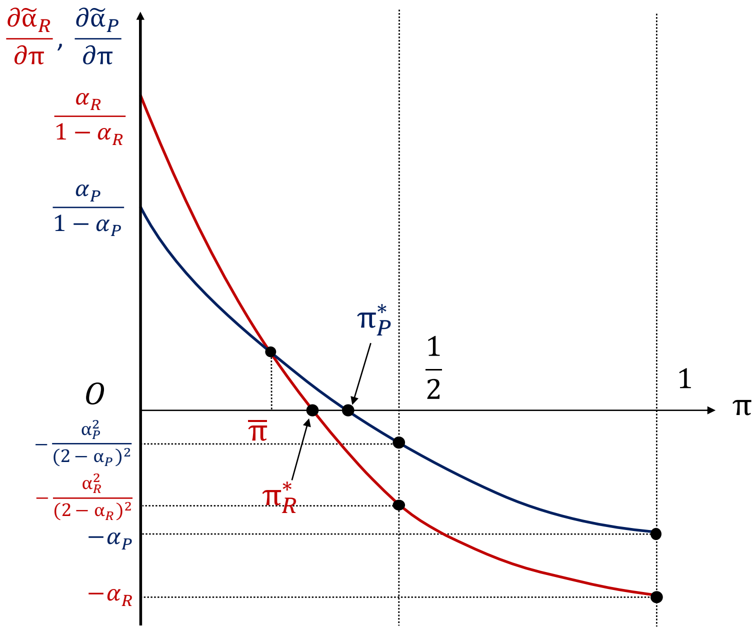


Figure 10. The effects of π on $\tilde{\alpha}_R$ and $\tilde{\alpha}_P$

We focus on the range of $\pi \in (\pi_p^*, 1)$, and Figure 10 shows that $\frac{\partial(\tilde{\alpha}_R - \tilde{\alpha}_P)}{\partial \pi}|_{\pi > \pi_p^*} < 0$ holds. In the BGPs, $\Psi_R(\Phi_{t-1})$ should be positive; thus, $\hat{\alpha} - \frac{\delta \hat{\alpha}_R}{\Phi_{t-1}} > 0 \Leftrightarrow \frac{1}{\Phi_{t-1}} < 1 + \frac{1-\delta}{\delta}\frac{\hat{\alpha}_P}{\hat{\alpha}_R}$ holds. Consequently, we obtain,

$$\begin{aligned}\frac{\partial \hat{\alpha}}{\partial \pi} - \frac{\delta}{\Phi_{t-1}}\frac{\partial \hat{\alpha}_R}{\partial \pi} &> (1-\delta)\left(\frac{\partial \hat{\alpha}_P}{\partial \pi} - \frac{\partial \hat{\alpha}_R}{\partial \pi}\frac{\hat{\alpha}_P}{\hat{\alpha}_R}\right) \\ &= (1-\delta)\frac{\alpha_P}{1-\alpha_P+\pi\alpha_P}\left[\frac{1-\alpha_P}{1-\alpha_P+\pi\alpha_P} - \frac{1-\alpha_R}{1-\alpha_R+\pi\alpha_R}\right] \\ &= (1-\delta)\frac{\alpha_P}{1-\alpha_P+\pi\alpha_P}\frac{\pi(\alpha_R-\alpha_P)}{(1-\alpha_R+\pi\alpha_R)(1-\alpha_P+\pi\alpha_P)} > 0.\end{aligned} \tag{F2}$$

Accordingly, $\frac{\partial \Psi_R}{\partial \pi} > 0$. Total differentiation of Equation (25) yields

$$\begin{aligned}\frac{\partial \Phi_{t-1}}{\partial \pi} &= -\frac{\frac{\partial \Psi_R}{\partial \pi}}{\frac{\partial \Psi_R}{\partial \Phi_{t-1}}} \\ &= -\frac{\frac{1-\theta-\gamma}{1-\Phi_{t-1}}\left[\left(-\frac{\frac{\partial(\tilde{\alpha}_R-\tilde{\alpha}_P)}{\partial \pi}}{(\tilde{\alpha}_R-\tilde{\alpha}_P)^2}\right)\left(\hat{\alpha} - \frac{\delta\hat{\alpha}_R}{\Phi_{t-1}}\right) + \left(\frac{1}{\tilde{\alpha}_R-\tilde{\alpha}_P}\right)\left(\frac{\partial \hat{\alpha}}{\partial \pi} - \frac{\delta}{\Phi_{t-1}}\frac{\partial \hat{\alpha}_R}{\partial \pi}\right)\right]}{\frac{1-\theta-\gamma}{\tilde{\alpha}_R-\tilde{\alpha}_P}\left(\frac{1}{1-\Phi_{t-1}}\right)\left[\left(\frac{1}{1-\Phi_{t-1}}\right)\left(\hat{\alpha} - \frac{\delta\hat{\alpha}_R}{\Phi_{t-1}}\right) + \frac{\delta\hat{\alpha}_R}{\Phi_{t-1}^2}\right]} < 0.\end{aligned} \tag{F3}$$

Thus, the rise in π shifts $\Psi_R(\Phi_{t-1}) = \Psi_L(x_t)$ downward. Note that when $\pi = \bar{\pi}$, $\frac{\partial \Phi_{t-1}}{\partial \pi} < 0$ as $\frac{\partial(\tilde{\alpha}_R - \tilde{\alpha}_P)}{\partial \pi} = 0$, whereas when $\pi \to 0$, $\frac{\partial \Phi_{t-1}}{\partial \pi} \to \infty$. This implies that $\frac{\partial \Phi_{t-1}}{\partial \pi}$ is positive when π is small and has a transition point from positive to negative somewhere in the range of $\pi \in (0, \bar{\pi})$.

Appendix G (The twisted shift)

We express the right-hand side of Equation (24) $\Theta(x_t)$ as $\Theta(x_t) = \left(\frac{1}{\tilde{\alpha}_R - \tilde{\alpha}_P}\right)(\omega_1 - \hat{\alpha}\omega_2 - \tilde{\alpha}_P)$ where $\omega_1 \equiv \frac{\left[1+\frac{\lambda\Gamma\left(\frac{g}{\lambda}x_t\right)^\theta}{x_t}\right](1+\gamma x_t)}{1+\gamma x_t+\gamma\mu\left(\frac{g}{\lambda}x_t\right)^\theta}$ and $\omega_2 \equiv \frac{\mu(1-\theta-\gamma)\left(\frac{g}{\lambda}x_t\right)^\theta}{(1+x_t)\left[1+\gamma x_t+\gamma\mu\left(\frac{g}{\lambda}x_t\right)^\theta\right]}$. Differentiating $\Theta(x_t)$ with π yields

$$\frac{\partial \Theta}{\partial \pi} = \left(-\frac{\frac{\partial(\tilde{\alpha}_R-\tilde{\alpha}_P)}{\partial \pi}}{(\tilde{\alpha}_R-\tilde{\alpha}_P)^2}\right)(\omega_1 - \hat{\alpha}\omega_2 - \tilde{\alpha}_P) + \left(\frac{1}{\tilde{\alpha}_R-\tilde{\alpha}_P}\right)\left(-\omega_2\frac{\partial \hat{\alpha}}{\partial \pi} - \frac{\partial \tilde{\alpha}_P}{\partial \pi}\right). \tag{G4}$$

Therefore, we obtain

$$\frac{\partial \Theta}{\partial \pi} \gtreqless 0 \Leftrightarrow -\frac{\frac{\partial(\tilde{\alpha}_R-\tilde{\alpha}_P)}{\partial \pi}}{\tilde{\alpha}_R-\tilde{\alpha}_P} \gtreqless \frac{\omega_2\frac{\partial \hat{\alpha}}{\partial \pi} + \frac{\partial \tilde{\alpha}_P}{\partial \pi}}{\omega_1 - \hat{\alpha}\omega_2 - \tilde{\alpha}_P}. \tag{G5}$$

Equation (G5) indicates that $\frac{\partial \Theta}{\partial \pi}$ depends on the rate of change in $\tilde{\alpha}_R - \tilde{\alpha}_P$ and $\omega_1 - \hat{\alpha}\omega_2 - \tilde{\alpha}_P$

when π is changed. Now, we define the left- and right-hand sides of Equation (G5) as $\chi_L(\pi) \equiv -\frac{\frac{\partial(\tilde{\alpha}_R-\tilde{\alpha}_P)}{\partial\pi}}{\tilde{\alpha}_R-\tilde{\alpha}_P}$ and $\chi_R(\pi, x_t) \equiv \frac{\omega_2\frac{\partial\hat{\alpha}}{\partial\pi}+\frac{\partial\tilde{\alpha}_P}{\partial\pi}}{\omega_1-\hat{\alpha}\omega_2-\tilde{\alpha}_P}$, respectively.

From $\lim_{x_t\to 0}\omega_1 = \infty$, $\lim_{x_t\to\infty}\omega_1 = 1$, $\lim_{x_t\to 0}\omega_2 = 0$ and $\lim_{x_t\to\infty}\omega_2 = 0$, it follows that $\lim_{x_t\to\infty}\chi_R(\pi, x_t) = \frac{\frac{\partial\tilde{\alpha}_P}{\partial\pi}}{1-\tilde{\alpha}_P}$ and $\lim_{x_t\to 0}\chi_R(\pi, x_t) = 0$. Therefore, $\chi_R(\pi, x_t)$ is an upward convex curve as $\frac{\partial\tilde{\alpha}_P}{\partial\pi} < 0$ when $\pi_P^* < \pi$. In the BGPs, $\Theta(x_t)$ should be positive; thus, we first consider the case of $\omega_1 - \hat{\alpha}\omega_2 - \tilde{\alpha}_P > 0$ for any x_t.

Case (A) $\boldsymbol{\omega_1 - \hat{\alpha}\omega_2 - \tilde{\alpha}_P > 0}$ for any $\boldsymbol{x_t}$.

Differentiating $\chi_L(\pi)$ and $\chi_R(\pi, x_t)$ with π yields, respectively,

$$\frac{\partial\chi_L}{\partial\pi} = -\left[\frac{\frac{\partial^2(\tilde{\alpha}_R-\tilde{\alpha}_P)}{\partial\pi^2}(\tilde{\alpha}_R-\tilde{\alpha}_P) - \left(\frac{\partial(\tilde{\alpha}_R-\tilde{\alpha}_P)}{\partial\pi}\right)^2}{(\tilde{\alpha}_R-\tilde{\alpha}_P)^2}\right] > 0, \tag{G6}$$

$$\frac{\partial\chi_R}{\partial\pi} = \frac{\left(\omega_2\frac{\partial^2\hat{\alpha}}{\partial\pi^2}+\frac{\partial^2\tilde{\alpha}_P}{\partial\pi^2}\right)(\omega_1-\hat{\alpha}\omega_2-\tilde{\alpha}_P) + \left(\omega_2\frac{\partial\hat{\alpha}}{\partial\pi}+\frac{\partial\tilde{\alpha}_P}{\partial\pi}\right)^2}{(\omega_1-\hat{\alpha}\omega_2-\tilde{\alpha}_P)^2}. \tag{G7}$$

$\frac{\partial\chi_L}{\partial\pi}$ is always positive as $\frac{\partial^2(\tilde{\alpha}_R-\tilde{\alpha}_P)}{\partial\pi^2} = -\frac{2\alpha_R(1-\alpha_R)}{(1-\alpha_R+\pi\alpha_R)^3} + \frac{2\alpha_P(1-\alpha_P)}{(1-\alpha_P+\pi\alpha_P)^3} < 0$. $\lim_{\pi\to 0}\frac{\partial\chi_L}{\partial\pi} = \lim_{\pi\to 1}\frac{\partial\chi_L}{\partial\pi} = \infty$ shows that $\chi_L(\pi)$ diverges negatively when π approaches 0 from positive, and conversely, it diverges positively when π approaches 1. Meanwhile, $\frac{\partial\chi_R}{\partial\pi}$ takes a constant value, although it can be both positive and negative as $\frac{\partial^2\hat{\alpha}}{\partial\pi^2} < 0$ and $\frac{\partial^2\tilde{\alpha}_i}{\partial\pi^2} = -\frac{2\alpha_i(1-\alpha_i)}{(1-\alpha_i+\pi\alpha_i)^3} < 0$. Accordingly, from the discussion stated above, it follows that the line $\chi_L(\pi)$ completely exceeds $\chi_R(\pi, x_t)$ from below when $\pi \to 1$. As Figure shows, $\chi_L(\pi)$ and $\chi_R(\pi, x_t)$ have multiple intersections when $\chi_L(\pi)$ gradually rises, and these intersections provide the levels of x_t where $\frac{\partial\Theta}{\partial\pi}$ switches from negative to positive and vice versa; $\frac{\partial\Theta}{\partial\pi} < 0$ in the range of $x_t \in (\underline{x}, \overline{x})$ where $\underline{x} < \overline{x}$ and $\frac{\partial\Theta}{\partial\pi} > 0$ in other areas. When aging proceeds, $\underline{x}(\pi)$ increases while $\overline{x}(\pi)$ decreases, which diminishes the distance between $\underline{x}(\pi)$ and $\overline{x}(\pi)$. Eventually, $\chi_L(\pi)$ exceeds $\chi_R(\pi, x_t)$, and these intersections disappear.

As Figure 11 shows, we define transition points where $\frac{\partial\Theta}{\partial\pi}$ turns from positive to negative and from negative to positive as f^* and f^{**}, respectively, where the level of x_t of f^* is given as $\underline{x}(\pi)$ and likewise that of f^* as $\overline{x}(\pi)$. The rise in π induces a twisted shift of $\Theta(x_t)$ and diminishes the distance between f^* and f^{**}, which implies that the effects of aging differ depending on the level of π.

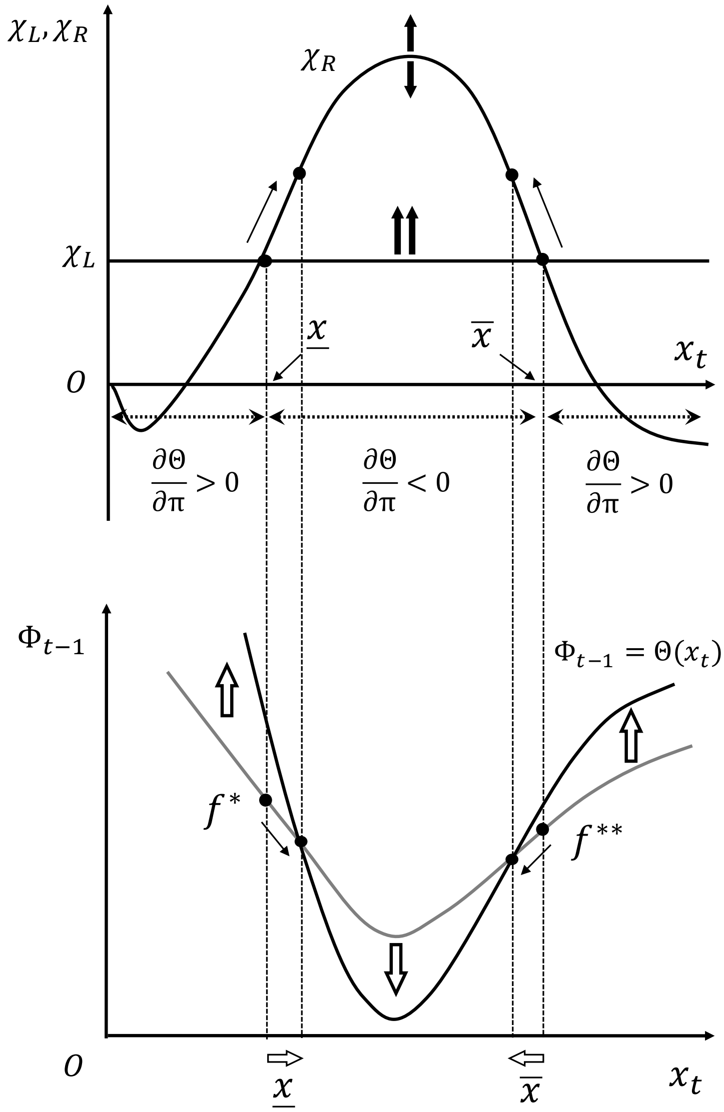


Figure 11. The twisted shift with Case (A)

Case (B) $\boldsymbol{\omega_1 - \hat{\alpha}\omega_2 - \tilde{\alpha}_P < 0}$ for certain x_t.

We consider the case that $\omega_1 - \hat{\alpha}\omega_2 - \tilde{\alpha}_P$ can take negative value depending on the level of x_t. When $\omega_1 - \hat{\alpha}\omega_2 - \tilde{\alpha}_P < 0$, $\Theta(x_t) < 0$ because $\tilde{\alpha}_R - \tilde{\alpha}_P > 0$, and $\chi_R(\pi, x_t) = \frac{\omega_2 \frac{\partial\hat{\alpha}}{\partial\pi} + \frac{\partial\tilde{\alpha}_P}{\partial\pi}}{\omega_1 - \hat{\alpha}\omega_2 - \tilde{\alpha}_P}$ is also negative in the area of x_t where $\Theta(x_t) < 0$. Therefore, as Figure 12 shows, $\chi_R(\pi, x_t)$ diverges to infinity as the denominator turns from positive to negative. Thus, the same discussion holds in Case (A).

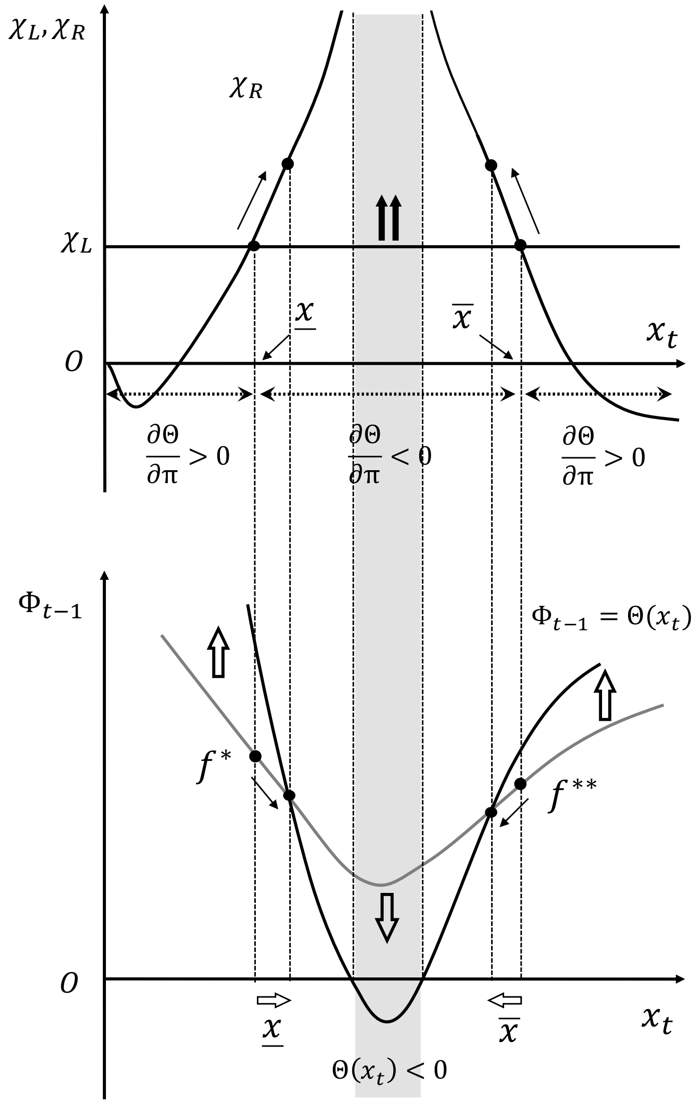


Figure 12. The twisted shift with Case (B)

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