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Abstract

In this paper, we examine how the introduction of remote work affects the spatial distribution of workers and firms in cities, social welfare, and their utilities. Developing a New Economic Geography model that incorporates remote work, we explore how transportation costs affect these distributions and the utility levels in equilibrium. We conduct a bifurcation analysis of an equilibrium where all mobile workers agglomerate in the central region of a long narrow economy where an odd number of regions are evenly distributed along a line segment. The bifurcation mechanism, which represents the emergence of remote work after its introduction, is elucidated. Results show that remote work can shift the equilibrium toward two types of equilibria. In one equilibrium, remote workers reside away from the central region, while firms operate in the center. In the other, remote workers reside in the central region, while firms that employ them operate outside the center. In the latter case, even the utility of remote workers declines due to the introduction of remote work.

Keywords: Agglomeration, Bifurcation, Economic geography, Remote work, Welfare.
JEL classification: R10, R12, R23.

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1. Introduction

Both transportation costs and remote work play important roles in determining the spatial market allocation in cities. Theoretical and empirical studies in New Economic Geography (NEG) have shown that transportation costs affect spatial distribution of population in cities (e.g., Krugman, 1991; Redding and Sturm, 2008). Meanwhile, the COVID-19 pandemic introduced remote work as a common work style. Workers across various industries can potentially work remotely (Dingel and Neiman, 2020; Alipour et al., 2023). After the pandemic, some workers in the United States migrated from cities with high land prices to those with lower prices (Brueckner et al., 2023).[1] Thus, remote work can contribute to the intercity migration of remote workers.[2] Such an intercity migration of workers impacts demand for goods transported across cities. In the long term, does the shift in demand induced by remote work generate benefits for both remote and non-remote workers in cities?

In this paper, we aim to elucidate how the introduction of remote work affects the spatial distribution of workers and firms in cities, social welfare, and the utility levels of remote and non-remote workers. Developing an NEG model that incorporates remote work, we explore how transportation costs affect the spatial distribution and the utility levels in equilibrium. In the model, there are two industries and three types of workers: unskilled workers who can neither migrate between cities nor work remotely; skilled workers who can migrate between cities but cannot work remotely; and skilled workers who can migrate and work remotely between cities. Each non-remote skilled worker supplies labor to a firm in one industry, whereas each remote skilled worker supplies labor to a firm in the other industry. Each worker consumes housing measured in terms of floor space. Our paper focuses on the case where skilled workers cannot switch the industry to which they belong.

The introduction of remote work in our model can change the stability of an equilibrium where no skilled workers work remotely. This equilibrium can become unstable due to the presence of remote workers who can migrate to cities with lower housing prices than those in the cities where they reside. We examine how the introduction of remote work affects the stability of equilibrium with no remote workers. Transportation costs can also affect the stability of equilibrium, as these costs affect the demand for goods transported across cities.

[1] Intercity migration of remote workers is reported on https://www.livemint.com/companies/news/tech-workers-take-to-the-mountains-bringing-silicon-valley-with-them-11604301993812.html (accessed on December 8, 2024).

[2] How remote work affects spatial market allocation has been enthusiastically explored since the COVID-19 pandemic (Lee, 2023).

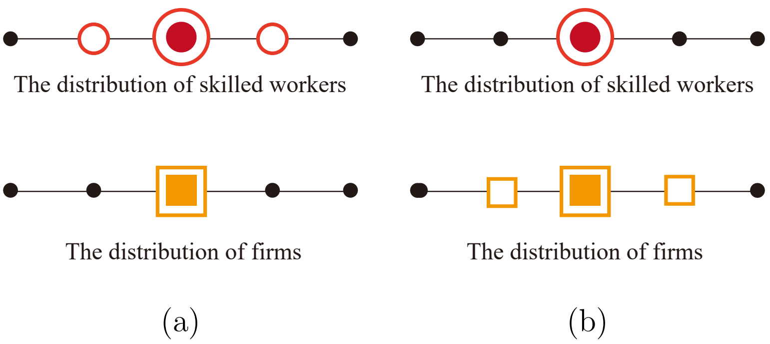

Figure 1: Spatial distribution of workers and firms in the long narrow economy with five regions. ◯ (red): regions where workers who can work remotely reside; • (red): regions where non-remote workers reside; □ (yellow): regions where firms employing workers who can work remotely are located; ■ (yellow): regions where firms employing non-remote workers are located

In the theoretical NEG literature, such costs are known to affect the spatial distribution of workers. In our model, due to transportation costs and the presence of remote work, the spatial distribution of skilled workers can differ from that of firms. Following previous studies in the NEG literature, we explore how stable equilibria changes in response to changes in transportation costs.

We elucidate the mathematical mechanism behind the emergence of equilibria in which some skilled workers work remotely. To explore how the introduction of remote work affects stable equilibria in which skilled workers agglomerate in a large city, we focus on full agglomeration, a population distribution that represents all skilled workers concentrate in a single region and supply labor to firms located there. We elucidate the bifurcation mechanism of the full agglomeration in a long narrow economy where an odd number of regions are evenly distributed along a line segment. The bifurcation we focus on can be interpreted as the emergence of remote workers from the large city. We demonstrate how the introduction of remote work can transform the full agglomeration into the following types of population distributions:

(i) the spatial distribution of workers, some of whom are remote workers residing in cities surrounding the central city and supplying labor to firms in the central city (Figure 1(a)),

(ii) the spatial distribution of workers, some of whom are remote workers residing in the central city and supplying labor to firms in surrounding cities (Figure 1(b)).

A theoretical contribution of our paper is that it elucidates the bifurcation mechanism in the context of remote work. Economic mechanisms (e.g., scale economy) that affect the stability of full agglomeration in a two-region economy have been studied in NEG (Krugman, 1991; Fujita et al., 2001; Baldwin et al., 2011). The bifurcation mechanism within NEG models with a single industry has been theoretically explored on various spatial platforms (e.g., Gaspar et al., 2021; Aizawa et al., 2023; Ikeda et al., 2024). In contrast to these previous studies, our paper elucidates the bifurcation mechanism of the full agglomeration in the long narrow economy with two industries and remote work to explore the intercity impacts of remote work on the spatial distribution of workers and firms in equilibrium.

We show that the introduction of remote work can decrease skilled workers' utilities. This introduction alters the stability of the full agglomeration when skilled workers can earn higher wages or avoid high housing costs by working remotely. Near the unstable full agglomeration, they myopically relocate from the central region to other regions or choose to work in regions other than the central region. Such myopic behavior transforms the unstable full agglomeration into a stable population distribution such as those shown in Figure 1. We demonstrate that remote workers' utilities in the distribution shown in Figure 1(a) are higher than those under the full agglomeration, whereas the utilities in Figure 1(b) are lower. Interestingly, remote workers who behave myopically reduce their own utilities, implying that remote work is not necessarily desirable even for remote workers in the long run.

Our paper is organized as follows. Section 2 presents the NEG model with remote work. Section 3 elucidates the bifurcation mechanism that represents the emergence of remote workers from the full agglomeration. Section 4 examines the bifurcation from the full agglomeration using the NEG model. Section 5 evaluates social welfare and the utility levels of both unskilled and skilled workers. Section 6 concludes.

Related literature. The intracity effects of remote work have been explored in the urban economics literature. How remote work affects agglomeration economies (e.g., externalities due to face-to-face communication) and congestion is examined through numerical analysis (Safirova, 2002; Rhee, 2008). The impacts of remote work on land rents, commuting, and welfare are evaluated using quantitative models that incorporate agglomeration economies and congestion (Delventhal et al., 2022; Monte et al., 2023). The effect of remote work on residents' energy consumption is quantitatively assessed (Larson and Zhao, 2017). Technological changes such as changes in land supply, information and communication technologies, and commuting costs are analytically studied for their effects on choices between hybrid remote work and full-time office work, as well as on welfare (Behrens et al., 2024). The effects of hybrid remote work on wages and on business and residential rents within a city are analytically explored (Brueckner, 2025).

Several studies explore the intercity impacts of remote work. Gokan et al. (2021) examine the spatial distribution of remote workers commuting across cities and non-remote workers in two cities. They show that, as the share of remote work increases, remote workers' utilities can rise, while those of non-remote workers can decline. Brueckner et al. (2023) and Brueckner and Sayantani (2023) explore how the introduction of remote work affects the wages and residential choices of remote workers who supply labor across cities with differing amenity and productivity levels. In particular, Brueckner and Sayantani (2023) show that the introduction of remote work does not change remote workers' utilities. Agrawal and Brueckner (2025) theoretically examine how remote work affects decentralized taxation and public spending.

In contrast to these previous studies, our paper focuses on the intercity impacts of remote work on the spatial distribution of workers and firms in an economy where goods are transported across cities. We explore how remote work and transportation costs affect this distribution. Our paper demonstrates that remote workers' utilities can decrease as a result of the introduction of remote work.

The spatial distribution of workers in an economy with multiple industries or regions has been explored in the NEG literature. The impact of transportation costs on the spatial distribution of firms within industries is explored with two-region economy (Takatsuka and Zeng, 2013) and in racetrack economy where regions are arranged in a circle (Tabuchi and Thisse, 2011). Several theoretical studies focus on how the spatial distribution of workers in multiple regions changes with changes in transportation costs (e.g., Barbero and Zofío, 2016; Gaspar et al., 2018; Ikeda et al., 2018; Takayama et al., 2020). These previous studies do not aim to elucidate the intercity impacts of remote work on the spatial distribution of workers and firms.

2. Model

Following Brueckner and Sayantani (2023), we explore the equilibria of two theoretical models. The common framework for these models is that of the footloose entrepreneur model, which has been used in theoretical analyses of New Economic Geography (Baldwin et al., 2011). One model is the footloose entrepreneur model without remote work, whereas the other model incorporates remote work. To examine how remote work affects workers' location choices, we explore population distribution in equilibria of these models. Moreover, to assess whether remote work is desirable for workers, we compare the social welfare and utility levels in the equilibrium of one model with those of the other model.

2.1. Footloose entrepreneur model with two industries

2.1.1. Basic framework

The footloose entrepreneur model in our paper is the multi-region and two-industry version of the model developed by Pflüger and Südekum (2008). We refer to this model as the FE model. The economy described by the FE model consists of K (≥ 2) regions, skilled and unskilled workers, and three types of sectors: a manufacturing sector, a housing sector, and an agriculture sector. Skilled and unskilled workers are the factor of the production for differentiated goods produced in the manufacturing sector. All workers inelastically supply one unit of labor. Skilled workers can choose their regions of residence among the K regions, whereas unskilled workers cannot. The spatial distribution of the unskilled workers is exogenously given.

The manufacturing sector consists of two types of manufacturing, denoted as M^1 and the M^2 sectors. Each skilled worker is a factor of production in one of the two sectors, and she is immobile between them. The number of the skilled workers employed in the M^m sector $(m = 1, 2)$ in region i is denoted by λ_i^m and the total number of skilled workers in each sector is normalized to be one: $\sum_{i \in \mathcal{K}} \lambda_i^m = 1$, where $\mathcal{K}$ denotes the set of the regions in the economy. Unskilled workers are equally distributed across all regions. The number of unskilled workers in each region is denoted by L. Each skilled worker's labor is the fixed input in manufacturing production, whereas each unskilled worker's labor is the variable input in both manufacturing and agricultural production. The agricultural good is produced under constant returns to scale. Housing is a non-produced consumption good that is supplied in each region. The total housing stock in each region is denoted by H, which is assumed to be constant.

2.1.2. Preferences and demands

All workers have the same preference. The preference of a worker residing in region $i \in \mathcal{K}$ is the following quasi-linear utility:

$$U(C_i^{\mathrm{M}_1}, C_i^{\mathrm{M}_2}, C_i^{\mathrm{H}}, C_i^{\mathrm{A}}) = \alpha_1 \ln C_i^{\mathrm{M}_1} + \alpha_2 \ln C_i^{\mathrm{M}_2} + \beta \ln C_i^{\mathrm{H}} + C_i^{\mathrm{A}}, \tag{1}$$

with $\alpha_1, \alpha_2, \beta > 0$. C_i^{H} denotes the consumption of housing in region i, C_i^{A} denotes the consumption of the agricultural good, and $C_i^{\mathrm{M}_m}$ denotes the CES-consumption index of differentiated goods that firms in M^m-sector produce:

$$C_i^{\mathrm{M}_m} = \left(\sum_{j \in \mathcal{K}} \int_0^{n_j^m} q_{ji}^m(\ell)^{(\sigma^m - 1)/\sigma^m} d\ell \right)^{\sigma^m/(\sigma^m - 1)}.$$

$q_{ji}^m(\ell)$ denotes the consumption in region i of differentiated good $\ell \in [0, n_j^m]$ produced in region $j \in \mathcal{K}$. n_k^m and $\sigma^m > 1$ are the mass of variety in region k and the constant elasticity of substitution between any two differentiated goods, respectively. The budget constraint of the worker residing in region i is

$$\sum_{k\in\mathcal{K}}\left(\int_0^{n_k^1} p_{ki}^1(\ell)q_{ki}^1(\ell)\mathrm{d}\ell+\int_0^{n_k^2} p_{ki}^2(\ell)q_{ki}^2(\ell)\mathrm{d}\ell\right)+p_i^{\mathrm{H}}C_i^{\mathrm{H}}+p_i^{\mathrm{A}}C_i^{\mathrm{A}}=Y_i, \tag{2}$$

where $p_{ki}^m(\ell)$ denotes the price of differentiated good ℓ in region i produced in region k, p_i^{H} denotes the price of the housing in region i, p_i^{A} denotes the price of the agricultural good in region i, and Y_i denotes the income of the worker residing in region i. We assume public ownership of land for simplicity. The income of each worker consists of wage and land revenue:

$$Y_i=\begin{cases} Y_i^{\mathrm{u}}\equiv w_i^{\mathrm{u}}+R & \text{(unskilled worker)},\\ Y_i^1\equiv w_i^1+R & \text{(skilled worker in M}^1\text{ sector)},\\ Y_i^2\equiv w_i^2+R & \text{(skilled worker in M}^2\text{ sector)},\end{cases} \tag{3}$$

where R denotes the equally divided land revenue, w_i^{u} denotes the wage of an unskilled worker in region i, and w_i^m $(m=1,2)$ denotes the wage of a skilled worker supplying labor to a firm in M^m sector.

Each worker residing in region i maximizes the utility (1) subject to the budget constraint (2). Since the utility function is quasi-linear, the demand functions, except for that of agricultural good, are identical across all workers. Using the first-order condition for the utility maximization problem yields the demand functions:

$$C_i^{\mathrm{M}_m}=\frac{\alpha_m}{\rho_i^m},\quad C_i^{\mathrm{H}}=\frac{\beta}{p_i^{\mathrm{H}}},\quad C_i^{\mathrm{A}}=Y_i-\alpha_1-\alpha_2-\beta,\quad q_{ji}^m(\ell)=\frac{\alpha_m(\rho_i^m)^{\sigma_m-1}}{p_{ji}^m(\ell)^{\sigma_m}},$$

where ρ_i^m denotes the price index of the differentiated goods consumed in region i:

$$\rho_i^m=\left(\sum_{j\in\mathcal{K}}\int_0^{n_j^m} p_{ji}^m(\ell)^{1-\sigma_m}d\ell\right)^{1/(1-\sigma_m)}. \tag{4}$$

Substituting the demand functions into the utility (1) yields the indirect utility of a worker residing in region i:

$$U_i=\alpha_1\ln(1/\rho_i^1)+\alpha_2\ln(1/\rho_i^2)+\beta\ln(1/p_i^H)+Y_i+\zeta, \tag{5}$$

where $\zeta=\alpha_1\ln\alpha_1+\alpha_2\ln\alpha_2+\beta\ln\beta-(\alpha_1+\alpha_2+\beta)$ is a constant determined by the exogenous parameters.

2.1.3. Production

Firms in the agricultural sector operate under perfect competition. Each firm requires one unit of the labor of unskilled workers to produce one unit of agricultural good. The agricultural good is assumed to be freely traded across regions and to be the numéraire. In market equilibrium, $p_i^{\mathrm{A}} = w_i^{\mathrm{u}} = 1$ ($\forall i \in \mathcal{K}$) holds.

Differentiated goods produced in M^m sector ($m = 1, 2$) are produced under monopolistic competition with increasing returns to scale. Each firm in M^m sector requires one unit of the labor of skilled workers as the fixed input and c^m units of the labor of unskilled workers as the marginal labor requirement to produce one unit of the differentiated good. The operating profit of the firm in region i is given by

$$\Pi_i^m(\ell) = \sum_{k \in \mathcal{K}} p_{ik}^m(\ell) Q_{ik}^m(\ell) - (c^m x_i^m(\ell) + w_i^m), \tag{6}$$

where $Q_{ik}^m(\ell)$ denotes the total demand in region k for the differentiated good ℓ produced in region i, and $x_i^m(\ell)$ denotes the total output of this differentiated good in region i. $c^m x_i^m(\ell) + w_i^m$ is the cost function of the firm producing the ℓth differentiated good. We assume that the marginal labor requirement c^m is sufficiently small so that firms in each region can employ unskilled workers to produce differentiated goods: $\int_0^{n_i^1} c^1 x_i^1(\ell)\,\mathrm{d}\ell + \int_0^{n_i^2} c^2 x_i^2(\ell)\,\mathrm{d}\ell < L$. Since the total number of workers in region k is $\lambda_k^1 + \lambda_k^2 + L$, $Q_{ik}^m(\ell)$ is given by

$$Q_{ik}^m(\ell) = q_{ik}^m(\ell)(\lambda_k^1 + \lambda_k^2 + L). \tag{7}$$

The differentiated goods are transported between regions with transport costs assumed to take the iceberg form. For one unit of each differentiated good transported from region i to region j, only a fraction $1/\tau_{ij} < 1$ arrives ($\tau_{ii} = 1$). Following Ikeda et al. (2018), we assume $\tau_{ij} = \exp(\tau\, l(i,j))$, which is a function of transport cost parameter $\tau > 0$ and integer $l(i,j)$, which expresses the distance between regions i and j. The iceberg form of the transport cost determines the total production of each differentiated good: $x_i^m(\ell) = \sum_{k \in \mathcal{K}} \tau_{ik} Q_{ik}^m(\ell)$.

The firm in region i determines price $p_{ik}^m(\ell)$ ($k \in \mathcal{K}$) to maximize the profit (6). The first-order condition for the profit maximization problem is

$$\frac{Q_{ik}^m(\ell)}{p_{ik}^m(\ell)} \left(p_{ik}^m(\ell) + (p_{ik}^m(\ell) - c^m \tau_{ik}) \eta_{ik}^m(\ell)\right) = 0,$$

where $\eta_{ik}^m(\ell)$ denotes the price elasticity of the total demand: $\eta_{ik}^m(\ell) = \partial \log Q_{ik}^m(\ell) / \partial \log p_{ik}^m(\ell) = -\sigma_m$ ($\forall \ell \in [0, n_i^m], \forall i, k \in \mathcal{K}$). Using the above first-order condition yields the optimal price of the differentiated good:

$$p_{ik}^m = \frac{\sigma_m}{\sigma_m - 1} \times c\tau_{ik}, \tag{8}$$

where we omit ℓ because the prices of the differentiated goods do not depend on ℓ. This equation implies that $Q_{ik}^m(\ell)$ and $x_i^m(\ell)$ are independent of ℓ. Thus, argument ℓ is omitted subsequently.

2.1.4. Short-run equilibrium

Following standard theoretical analyses using NEG models (e.g., Baldwin et al., 2011), we explore market equilibrium under both short-run and long-run equilibrium conditions. In the short-run equilibrium, spatial distribution of skilled workers $\boldsymbol{\lambda}^m = (\lambda_i^m)_{i\in\mathcal{K}}$ $(m = 1, 2)$ is assumed to be given. The other endogenous variables are determined within the short-run equilibrium. The short-run market equilibrium is characterized by four conditions: the housing market clearing condition, the differentiated goods market clearing condition, the zero-profit condition under the free entry of manufacturing firms, and the labor market clearing condition.

The housing market clearing condition is given by

$$H = C_i^H(\lambda_i^1 + \lambda_i^2 + L). \tag{9}$$

Using the above condition and the demand for housing C_i^{H} yield the equilibrium price of the housing:

$$p_i^{\mathrm{H}} = \frac{\beta(\lambda_i^1 + \lambda_i^2 + L)}{H}. \tag{10}$$

The differentiated good market clearing condition is given by Eq. (7). The zero-profit condition requires that each firm's operating profit, given by Eq. (6), is fully absorbed by wage payment to a skilled worker:

$$w_i^m = \sum_{k\in\mathcal{K}} p_{ik}^m Q_{ik}^m - c^m x_i^m. \tag{11}$$

The labor market clearing condition is expressed as $n_i^m = \lambda_i^m$ which implies that the mass of firms in each region is equals to that of skilled workers residing in the region.[3] Using the price (8), we obtain price index ρ_i^m, shown in Eq. (4), as a function of population distribution $\boldsymbol{\lambda}^m$ and transportation costs:

$$\rho_i^m = \frac{\sigma_m c}{\sigma_m - 1}\left(\sum_{k\in\mathcal{K}} d_{ki}^m \lambda_k^m\right)^{1/(1-\sigma_m)}, \tag{12}$$

where d_{ji}^m is the spatial discounting factor that represents the friction due to transportation between regions j and i:

$$d_{ji}^m = \tau_{ji}^{1-\sigma_m} = \exp\left[-\tau\, l(j,i)(\sigma_m - 1)\right] \in (0,1). \tag{13}$$

[3] The labor market clearing condition for unskilled workers is given by

$$\sum_{i\in\mathcal{K}}\left(\int_0^{n_i^1} c^1 x_i^1(\ell)\,\mathrm{d}\ell + \int_0^{n_i^2} c^2 x_i^2(\ell)\,\mathrm{d}\ell + C_i^A(\lambda_i^1 + \lambda_i^2 + L)\right) = KL.$$

Warlas' law ensures that this condition holds.

As shown by the above equation, this factor decreases with an increase in transportation distance $l(j, i)$. Substituting Eqs. (7), (8), (12), and (13) into Eq. (11) yields the wage of a skilled worker in the short-run equilibrium:

$$w_i^m(\boldsymbol{\lambda}^1, \boldsymbol{\lambda}^2, \tau) = \frac{\alpha_m}{\sigma_m} \sum_{j \in \mathcal{K}} \frac{d_{ij}^m}{\Delta_j^m} (\lambda_j^1 + \lambda_j^2 + L), \tag{14}$$

where Δ_j^m is a function of the population distribution and the spatial discounting factor:

$$\Delta_j^m = \sum_{k \in \mathcal{K}} d_{kj}^m \lambda_k^m.$$

We can rewrite the price index with this factor: $\rho_i^m = (\Delta_i^m)^{1/(1-\sigma_m)} \sigma_m c / (\sigma_m - 1)$.

Indirect utility in the short-run equilibrium can be expressed as a function of population distribution $\boldsymbol{\lambda} = (\boldsymbol{\lambda}^1, \boldsymbol{\lambda}^2)$ with $\boldsymbol{\lambda}^m = (\lambda_i^m)_{i \in \mathcal{K}}$. Let $U_i^{\mathrm{u}}(\boldsymbol{\lambda}^1, \boldsymbol{\lambda}^2, \tau)$ and $U_i^m(\boldsymbol{\lambda}^1, \boldsymbol{\lambda}^2, \tau)$ $(m = 1, 2)$ denote the indirect utility of the unskilled worker and the skilled worker in M^m sector residing in region i, respectively. We explicitly include transportation cost τ in the arguments as a representative exogenous parameter that affects the market equilibrium. Substituting the price of the housing (10), the price index (12) into the indirect utility (5) and using Eq. (3), we obtain the indirect utility with the FE model:

$$U_i^{\mathrm{u}}(\boldsymbol{\lambda}^1, \boldsymbol{\lambda}^2, \tau) = \frac{\alpha_1}{\sigma_1 - 1} \ln \Delta_i^1 + \frac{\alpha_2}{\sigma_2 - 1} \ln \Delta_i^2 - \beta \ln \Lambda_i + 1 + \xi, \tag{15a}$$

$$U_i^1(\boldsymbol{\lambda}^1, \boldsymbol{\lambda}^2, \tau) = \frac{\alpha_1}{\sigma_1 - 1} \ln \Delta_i^1 + \frac{\alpha_2}{\sigma_2 - 1} \ln \Delta_i^2 - \beta \ln \Lambda_i + w_i^1 + \xi, \tag{15b}$$

$$U_i^2(\boldsymbol{\lambda}^1, \boldsymbol{\lambda}^2, \tau) = \frac{\alpha_1}{\sigma_1 - 1} \ln \Delta_i^1 + \frac{\alpha_2}{\sigma_2 - 1} \ln \Delta_i^2 - \beta \ln \Lambda_i + w_i^2 + \xi, \tag{15c}$$

where $\Lambda_i = \lambda_i^1 + \lambda_i^2 + L$ and ξ is a constant determined by exogenous parameters. The first and second terms in each equation in (15) are the effects of the price indices on the utility, the third term is the effect of the housing price, and the fourth term is the effect of the wage. Note that the wage of the unskilled workers is one.

2.1.5. Long-run equilibrium

In the long-run, each skilled worker can decide where to reside. Long-run equilibrium is defined as a spatial distribution of skilled workers $(\boldsymbol{\lambda}^1, \boldsymbol{\lambda}^2) = (\boldsymbol{\lambda}^{1*}, \boldsymbol{\lambda}^{2*})$ which satisfies the following conditions:

$$\begin{cases} U^{m*} - U_i^m(\boldsymbol{\lambda}^{1*}, \boldsymbol{\lambda}^{2*}, \tau) = 0 & \text{if } \lambda_i^{m*} > 0, \\ U^{m*} - U_i^m(\boldsymbol{\lambda}^{1*}, \boldsymbol{\lambda}^{2*}, \tau) \geq 0 & \text{if } \lambda_i^{m*} = 0, \end{cases} \tag{16}$$

and $\sum_{i \in \mathcal{K}} \lambda_i^m = 1$ $(m = 1, 2)$. U^{m*} denotes the equilibrium utility level for skilled workers in M^m sector.

We can explore the long-run equilibrium by solving dynamics that expresses the change in the spatial distribution (Sandholm, 2010). We explore long-run equilibria using the replicator dynamics, which is employed in theoretical studies with NEG models that involve multiple regions (e.g., Gaspar et al., 2018):

$$\frac{\mathrm{d}\boldsymbol{\lambda}}{\mathrm{d}t} = \boldsymbol{F}(\boldsymbol{\lambda}^1, \boldsymbol{\lambda}^2, \tau), \tag{17}$$

where $\boldsymbol{F}(\boldsymbol{\lambda}^1, \boldsymbol{\lambda}^2, \tau) = (F_i^m(\boldsymbol{\lambda}^1, \boldsymbol{\lambda}^2, \tau))_{i\in\mathcal{K}, m\in\{1,2\}}$, and

$$F_i^m(\boldsymbol{\lambda}^1, \boldsymbol{\lambda}^2, \tau) = \lambda_i^m \left[U_i^m(\boldsymbol{\lambda}^1, \boldsymbol{\lambda}^2, \tau) - \overline{U^m}(\boldsymbol{\lambda}^1, \boldsymbol{\lambda}^2, \tau)\right] \quad i \in \mathcal{K}, m \in \{1, 2\}. \tag{18}$$

$\overline{U^m} = \sum_{i\in\mathcal{K}} \lambda_i^m U_i^m$ represents the weighted average utility of skilled workers in M^m sector. Stationary points are defined as the solutions to the following equation:

$$\boldsymbol{F}(\boldsymbol{\lambda}^1, \boldsymbol{\lambda}^2, \tau) = \mathbf{0}. \tag{19}$$

The stable stationary points correspond to stable long-run equilibria (Sandholm, 2010). The stability of a stationary point is determined by the eigenvalues of the Jacobian matrix of $\boldsymbol{F}$: $\boldsymbol{J} = \partial\boldsymbol{F}/\partial\boldsymbol{\lambda}$. If all the real parts of the eigenvalues of $\boldsymbol{J}$ are negative, then the associated stationary point is stable, implying that it is a stable long-run equilibrium.

2.2. Footloose entrepreneur model with remote work

To explore how equilibrium changes when skilled workers can remotely supply labor (i.e., remote work) across regions, we develop a footloose entrepreneur model that incorporates the remote work of skilled workers. We refer to this model as the FER model. We compare the equilibrium of the FER model with that of the FE model to examine the impacts of remote work on the spatial distribution of skilled workers in Sections 4 and 5.

2.2.1. Framework of the FER model

The difference between the FER model and the FE model is only the location choice of each skilled workers with respect to remote work. The other assumptions of the FER model are the same as those of the FE model. Following Brueckner and Sayantani (2023), we express remote work in the FER model as follows. In the FER model, only skilled workers in the M^1 sector can work remotely. Skilled workers in the M^1 sector can freely supply their labor to any region, whereas skilled workers in the M^2 sector can supply their labor to only the region where they reside. This assumption can be interpreted as heterogeneity among industries regarding the possibility of introducing remote work in the real world.

Preference and budget constraint of each worker in the FER model are the same as those in the FE model (i.e., Eqs. (1) and (2)). Let w_{ij}^1 denote the wage of skilled workers in the the M^1 sector residing in region i and supply their labor to firms operating in region j. The difference in the wage between the FE and FER models can alter the population distribution in the long-run equilibrium in the FE model. The number of skilled workers residing in region i and supplying labor to firms operating in region j is denoted by λ_{ij}^1 with the normalizing constant $\sum_{i,j\in\mathcal{K}} \lambda_{ij}^1 = 1$. The income of each worker is given by

$$Y_i = \begin{cases} Y_i^{\mathrm{u}} \equiv w_i^{\mathrm{u}} + R & \text{(unskilled worker)}, \\ Y_{ij}^1 \equiv w_{ij}^1 + R & \text{(skilled worker in } \mathrm{M}^1 \text{ sector)}, \\ Y_i^2 \equiv w_i^2 + R & \text{(skilled worker in } \mathrm{M}^2 \text{ sector)}. \end{cases} \tag{20}$$

The above definition implies that the total number of the location choices of skilled workers in the M^1 sector with the FER model is $K \times K$, whereas the total with the FE model is K (see Eq. (3)).

The production of firms in the M^1 sector in the FER model differs from that in the FE model. Let $\Pi_{ij}^1(\ell)$ denote the profit of a firm that operates in region i, employs a skilled worker residing in region j, and supplies the ℓth differentiated good. Note that this definition implies that $\Pi_{ii}^1(\ell)$ represents the operating profit of a firm employing a skilled worker who does not work remotely, and $\Pi_{ij}^1(\ell)$ $(i \neq j)$ is that of a firm employing a remote worker. The profit of a firm in M^m sector is given by

$$\Pi_{ij}^1(\ell) = \sum_{k\in\mathcal{K}} p_{ik}^1(\ell) Q_{ik}^1(\ell) - \left(c^1 x_i^1(\ell) + w_{ji}^1\right), \tag{21a}$$

$$\Pi_i^2(\ell) = \sum_{k\in\mathcal{K}} p_{ik}^2(\ell) Q_{ik}^2(\ell) - \left(c^2 x_i^2(\ell) + w_i^2\right), \tag{21b}$$

where $c^1 x_i^1(\ell) + w_{ji}^1$ and $c^2 x_i^2(\ell) + w_i^2$ are cost functions of the firms in M^1 and the M^2 sectors, respectively.

2.2.2. Short-run equilibrium

As in the FE model, the population distribution of skilled workers is given in the short-run equilibrium. The distributions of skilled workers in M^1 and the M^2 sectors are given by $\boldsymbol{\lambda}^1 = (\lambda_{ij}^1)_{i,j\in\mathcal{K}}$ and $\boldsymbol{\lambda}^2 = (\lambda_i^2)_{i\in\mathcal{K}}$, respectively. The housing market clearing condition and the differentiated good market clearing condition are the same as those introduced in Section 2.1.4 (i.e., Eqs. (7) and (9)). The zero-profit condition under free entry requires that the operating profits of the firms, given by Eqs. (21a) and (21b), are fully absorbed by the wage payments to the skilled workers:

$$w_{ij}^1 = \sum_{k\in\mathcal{K}} p_{jk}^1 Q_{jk}^1 - c^1 x_j^1, \tag{22a}$$

$$w_i^2 = \sum_{k\in\mathcal{K}} p_{ik}^2 Q_{ik}^2 - c^2 x_i^2. \tag{22b}$$

Equation (22a) indicates that the wages paid to skilled workers supplying labor to firms in region j do not depend on the regions in which they reside. The labor market clearing conditions are given by $n_i^1 = \sum_{k\in\mathcal{K}} \lambda_{ki}^1$ and $n_i^2 = \lambda_i^2$.

Indirect utility can be expressed as a function of the spatial distribution of workers in the FE model. Let $V_i^{\mathrm{u}}(\boldsymbol{\lambda}^1, \boldsymbol{\lambda}^2, \tau)$, $V_{ij}^1(\boldsymbol{\lambda}^1, \boldsymbol{\lambda}^2, \tau)$, and $V_i^2(\boldsymbol{\lambda}^1, \boldsymbol{\lambda}^2, \tau)$ denote the indirect utility of an unskilled worker, a skilled worker in the the M^1 sector residing in region i and supplying labor to a firm in region j, and a skilled worker in the the M^2 sector residing in region i, respectively. These functions are derived using the market equilibrium conditions

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$$V_i^{\mathrm{u}}(\boldsymbol{\lambda}^1, \boldsymbol{\lambda}^2, \tau) = \frac{\alpha_1}{\sigma_1 - 1} \ln N_i^1 + \frac{\alpha_2}{\sigma_2 - 1} \ln \Delta_i^2 - \beta \ln \Lambda_i + 1 + \xi, \tag{23a}$$

$$V_{ij}^1(\boldsymbol{\lambda}^1, \boldsymbol{\lambda}^2, \tau) = \frac{\alpha_1}{\sigma_1 - 1} \ln N_i^1 + \frac{\alpha_2}{\sigma_2 - 1} \ln \Delta_i^2 - \beta \ln \Lambda_i + w_{ij}^1 + \xi, \tag{23b}$$

$$V_i^2(\boldsymbol{\lambda}^1, \boldsymbol{\lambda}^2, \tau) = \frac{\alpha_1}{\sigma_1 - 1} \ln N_i^1 + \frac{\alpha_2}{\sigma_2 - 1} \ln \Delta_i^2 - \beta \ln \Lambda_i + w_i^2 + \xi, \tag{23c}$$

where $N_i^1 = \sum_{o,k\in\mathcal{K}} d_{oi}^1 \lambda_{ko}^1$,

$$w_{ij}^1(\boldsymbol{\lambda}^1, \boldsymbol{\lambda}^2, \tau) = \frac{\alpha_1}{\sigma_1} \sum_{k\in\mathcal{K}} \left[\frac{d_{jk}^1}{N_k^1} \left(\lambda_k^1 + \lambda_k^2 + L \right) \right], \tag{24a}$$

$$w_i^2(\boldsymbol{\lambda}^1, \boldsymbol{\lambda}^2, \tau) = \frac{\alpha_2}{\sigma_2} \sum_{k\in\mathcal{K}} \frac{d_{ik}^2}{\Delta_k^2} (\lambda_k^1 + \lambda_k^2 + L). \tag{24b}$$

Here, $\lambda_j^1 = \sum_{m\in\mathcal{K}} \lambda_{jm}^1$ is the total number of skilled workers in the M^1 sector residing in region j. As in the FE model, the first and second terms in each equation in (23) are the effects of the price indices on the utility, the third term is the effect of the housing price, and the fourth term is the effect of the wage.

2.2.3. Long-run equilibrium

The long-run equilibrium is defined as a spatial distribution of skilled workers $(\boldsymbol{\lambda}^1, \boldsymbol{\lambda}^2) = (\boldsymbol{\lambda}^{1*}, \boldsymbol{\lambda}^{2*})$ which satisfies the following conditions:

$$\begin{cases} V^{1*} - V^1_{ij}(\boldsymbol{\lambda}^{1*}, \boldsymbol{\lambda}^{2*}, \tau) = 0 & \text{if } \lambda^{1*}_{ij} > 0, \\ V^{1*} - V^1_{ij}(\boldsymbol{\lambda}^{1*}, \boldsymbol{\lambda}^{2*}, \tau) \geq 0 & \text{if } \lambda^{1*}_{ij} = 0, \end{cases} \tag{25a}$$

$$\begin{cases} V^{2*} - V^2_{i}(\boldsymbol{\lambda}^{1*}, \boldsymbol{\lambda}^{2*}, \tau) = 0 & \text{if } \lambda^{2*}_{i} > 0, \\ V^{2*} - V^2_{i}(\boldsymbol{\lambda}^{1*}, \boldsymbol{\lambda}^{2*}, \tau) \geq 0 & \text{if } \lambda^{2*}_{i} = 0, \end{cases} \tag{25b}$$

$\sum_{i,j\in\mathcal{K}} \lambda^1_{ij} = 1$, and $\sum_{i\in\mathcal{K}} \lambda^2_i = 1$. V^{m*} denotes the equilibrium utility level of skilled workers in the M^m sector. We explore stable spatial equilibria using the replicator dynamics:

$$\frac{\mathrm{d}\boldsymbol{\lambda}}{\mathrm{d}t} = \boldsymbol{F}(\boldsymbol{\lambda}^1, \boldsymbol{\lambda}^2, \tau), \tag{26}$$

where $\boldsymbol{F}(\lambda, \tau) = (F^1_{ij}, F^2_i)_{i,j\in\mathcal{K}, m\in\{1,2\}}$ and

$$F^1_{ij}(\boldsymbol{\lambda}, \tau) = \lambda^1_{ij}\,[V^1_{ij}(\boldsymbol{\lambda}^1, \boldsymbol{\lambda}^2, \tau) - \overline{V^1}(\boldsymbol{\lambda}^1, \boldsymbol{\lambda}^2, \tau)] \quad i, j \in \mathcal{K}, \tag{27a}$$

$$F^2_{i}(\boldsymbol{\lambda}, \tau) = \lambda^2_{i}\,[V^2_{i}(\boldsymbol{\lambda}^1, \boldsymbol{\lambda}^2, \tau) - \overline{V^2}(\boldsymbol{\lambda}^1, \boldsymbol{\lambda}^2, \tau)] \quad i \in \mathcal{K}. \tag{27b}$$

Here, $\overline{V^1} = \sum_{i,j\in\mathcal{K}} \lambda^1_{ij} V^1_{ij}$ and $\overline{V^2} = \sum_{i\in\mathcal{K}} \lambda^2_i V^2_i$ are the weighted average utilities with the FER model. Stationary points are defined as the solutions to $\boldsymbol{F}(\boldsymbol{\lambda}^1, \boldsymbol{\lambda}^2, \tau) = \boldsymbol{0}$.

A stable market equilibrium in the FE model is not necessarily the same as that in the FER model, since the Jacobian matrix differs between the two models. At an unstable equilibrium, some skilled workers have an incentive to relocate to other regions or change their region for work. Such an incentive can be explained as follows. Let $\left(\widetilde{\boldsymbol{\lambda}}^1_{\mathrm{FE}}, \widetilde{\boldsymbol{\lambda}}^2_{\mathrm{FE}}\right)$ denote a market equilibrium of the FE model. We define the components of $\widetilde{\boldsymbol{\lambda}}^1_{\mathrm{FE}}$ and $\widetilde{\boldsymbol{\lambda}}^2_{\mathrm{FE}}$ as follows.

$$\widetilde{\boldsymbol{\lambda}}^1_{\mathrm{FE}} = \left(\widetilde{\lambda}^1_1, \widetilde{\lambda}^1_2, \ldots, \widetilde{\lambda}^1_K\right), \quad \widetilde{\boldsymbol{\lambda}}^2_{\mathrm{FE}} = \left(\widetilde{\lambda}^2_1, \widetilde{\lambda}^2_2, \ldots, \widetilde{\lambda}^2_K\right).$$

Using these components, we define the associated population distribution in the FER model:

$$\widetilde{\boldsymbol{\lambda}}^1_{\mathrm{FER}} = \left(\widetilde{\boldsymbol{\lambda}}^1_1, \widetilde{\boldsymbol{\lambda}}^1_2, \ldots, \widetilde{\boldsymbol{\lambda}}^1_K\right), \quad \widetilde{\boldsymbol{\lambda}}^1_i = \left(\underbrace{0, \ldots, 0}_{i-1}, \widetilde{\lambda}^1_i, 0, \ldots, 0\right), \quad \widetilde{\boldsymbol{\lambda}}^2_{\mathrm{FER}} = \widetilde{\boldsymbol{\lambda}}^2_{\mathrm{FE}}.$$

Using these vectors and the definitions of the indirect utilities in the FE and FER models, we obtain the indirect utility of skilled workers in the M^1 sector residing in region i and supplying their labor to firms in the region (i.e., non-remote workers): $V^1_{ii}\left(\widetilde{\boldsymbol{\lambda}}^1_{\mathrm{FER}}, \widetilde{\boldsymbol{\lambda}}^2_{\mathrm{FER}}, \tau\right) = U^1_i\left(\widetilde{\boldsymbol{\lambda}}^1_{\mathrm{FE}}, \widetilde{\boldsymbol{\lambda}}^2_{\mathrm{FE}}, \tau\right)$.

If a remote worker can obtain higher indirect utility than a non-remote worker with the spatial distribution $\left(\widetilde{\boldsymbol{\lambda}}^1_{\mathrm{FER}}, \widetilde{\boldsymbol{\lambda}}^2_{\mathrm{FER}}\right)$ (i.e., $V^1_{ii}\left(\widetilde{\boldsymbol{\lambda}}^1_{\mathrm{FER}}, \widetilde{\boldsymbol{\lambda}}^2_{\mathrm{FER}}, \tau\right) < V^1_{ij}\left(\widetilde{\boldsymbol{\lambda}}^1_{\mathrm{FER}}, \widetilde{\boldsymbol{\lambda}}^2_{\mathrm{FER}}, \tau\right)$ $(\exists j \neq i)$), then the equilibrium condition (25a) does not hold with this spatial distribution. For example, if a remote worker can obtain a high wage or avoid incurring a high housing cost, then the equilibrium condition (25a) does not necessarily hold with the spatial distribution $\left(\widetilde{\boldsymbol{\lambda}}^1_{\mathrm{FER}}, \widetilde{\boldsymbol{\lambda}}^2_{\mathrm{FER}}\right)$. We numerically demonstrate that a stable equilibrium in the FE model is unstable in the FER model in Section 4.2.

2.2.4. Social welfare

We evaluate the welfare impact of the introduction of remote work using the Bentham welfare function:

$$W(\boldsymbol{\lambda}^1, \boldsymbol{\lambda}^2, \tau) = \sum_{i \in \mathcal{K}} \left(L V^{\mathrm{u}}_i(\boldsymbol{\lambda}^1, \boldsymbol{\lambda}^2, \tau) + \sum_{k \in \mathcal{K}} \left(\lambda^1_{ik} V^1_{ik}(\boldsymbol{\lambda}^1, \boldsymbol{\lambda}^2, \tau)\right) + \lambda^2_i V^2_i(\boldsymbol{\lambda}^1, \boldsymbol{\lambda}^2, \tau) \right). \tag{28}$$

This function has been used in welfare analyses with NEG models (e.g., Ottaviano et al., 2002; Pflüger and Südekum, 2008).

3. Emergence of remote workers from an unstable equilibrium in the FER model

In our paper, we define the introduction of remote work as a change in equilibrium condition. Before remote work is introduced, equilibrium is determined under the FE model (i.e., condition (16)), as skilled workers in the M^1 sector cannot work remotely. Once remote work is introduced, however, these workers can supply labor to firms operating in locations outside their region of residences. Equilibrium is determined under the FER model (i.e., condition (25)). Thus, the emergence of remote workers represents a transition from an (unstable) equilibrium in the FE model to a stable equilibrium that represents the presence of remote workers in the FER model.

We explore how the introduction of remote work changes the spatial distribution of workers in a setting characterized by a large city surrounded by smaller peripheral cities. To model such a spatial structure, we adopt a spatial platform where regions are evenly distributed along a line segment. This platform is used in theoretical analyses of NEG models with multiple regions (Ikeda et al., 2017, 2024) as it captures how small cities are spatially separated from a central large city. We refer to this platform as a long narrow economy. We assume that the central region lies at the midpoint of the segment, and that the long narrow economy consists of an odd number of regions as illustrated in Figure 2. The set of regions in long narrow economy is represented as

$$\mathcal{K} = \left\{-\hat{k}, -\hat{k}+1, \ldots, -1, 0, 1, \ldots, \hat{k}-1, \hat{k}\right\} \quad \left(\hat{k} = 1, 2, \ldots\right).$$

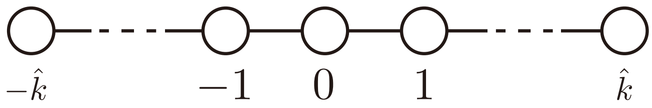


Figure 2: Long narrow economy.

The central region is labeled with $0 \in \mathcal{K}$. Distance between regions and the spatial discount factor are given by, respectively,

$$l(i,j) = |i-j| \quad (i,j \in \mathcal{K}), \tag{29}$$

$$d_{ji}^m = \exp\left(-\tau\,|i-j|\,(\sigma_m - 1)\right). \tag{30}$$

A spatial distribution of skilled workers representing a large city and peripheral small cities can be expressed as $\boldsymbol{\lambda}_{\mathrm{FA}} = (\boldsymbol{\lambda}_{\mathrm{FA}}^1, \boldsymbol{\lambda}_{\mathrm{FA}}^2)$, where

$$\boldsymbol{\lambda}_{\mathrm{FA}}^1 = \left(\underbrace{\mathbf{0}, \ldots, \mathbf{0}}_{\hat{k}}, \boldsymbol{\lambda}_{\mathrm{FA}}, \underbrace{\mathbf{0}, \ldots, \mathbf{0}}_{\hat{k}} \right), \quad \boldsymbol{\lambda}_{\mathrm{FA}} = \left(\underbrace{0, \ldots, 0}_{\hat{k}}, 1, \underbrace{0, \ldots, 0}_{\hat{k}} \right), \quad \boldsymbol{\lambda}_{\mathrm{FA}}^2 = \boldsymbol{\lambda}_{\mathrm{FA}}.$$

$\mathbf{0}$ is the $2\hat{k}+1$-dimensional vector whose components are all zero. This spatial distribution represents that all skilled workers are agglomerated in the central region. This central region is the large city in the FER model. We refer to this distribution as the full agglomeration in the FER model. Full agglomeration, in which all the mobile workers concentrate in a single region, has been studied in theoretical analyses using NEG models (e.g., Fujita et al., 2001; Takayama et al., 2020).

The full agglomeration $\boldsymbol{\lambda}_{\mathrm{FA}}$ is a stationary point for any values of the exogenous parameters:

$$\begin{aligned}
&F_{00}^1(\boldsymbol{\lambda}_{\mathrm{FA}}, \tau) = V_{00}^1(\boldsymbol{\lambda}_{\mathrm{FA}}, \tau) - V_{00}^1(\boldsymbol{\lambda}_{\mathrm{FA}}, \tau) = 0, \\
&F_{ij}^1(\boldsymbol{\lambda}_{\mathrm{FA}}, \tau) = 0 \times [V_{ij}^1(\boldsymbol{\lambda}_{\mathrm{FA}}, \tau) - V_{00}^1(\boldsymbol{\lambda}_{\mathrm{FA}}, \tau)] = 0 \quad (i,j) \neq (0,0), \\
&F_0^2(\boldsymbol{\lambda}_{\mathrm{FA}}, \tau) = V_0^2(\boldsymbol{\lambda}_{\mathrm{FA}}, \tau) - V_0^2(\boldsymbol{\lambda}_{\mathrm{FA}}, \tau) = 0, \\
&F_i^2(\boldsymbol{\lambda}_{\mathrm{FA}}, \tau) = 0 \times [V_i^2(\boldsymbol{\lambda}_{\mathrm{FA}}, \tau) - V_0^2(\boldsymbol{\lambda}_{\mathrm{FA}}, \tau)] = 0 \quad (i \neq 0).
\end{aligned}$$

The stability of the full agglomeration is related to the indirect utility of the skilled workers. The stability of the full agglomeration $\boldsymbol{\lambda}_{\mathrm{FA}}$ is determined by the signs of the real parts of the eigenvalues of Jacobian matrix $\boldsymbol{J}$. We can properly permute the components of $\boldsymbol{\lambda}_{\mathrm{FA}}$ and $\boldsymbol{F}$ to arrive at $\hat{\boldsymbol{\lambda}} = (1,1,0,\ldots,0)$ and $\hat{\boldsymbol{F}} = (F_{00}^1, F_0^2, \ldots)$. The associated Jacobian matrix for these vectors can be rearranged (Ikeda et al., 2018)

$$\hat{\boldsymbol{J}} = \frac{\partial \hat{\boldsymbol{F}}}{\partial \hat{\boldsymbol{\lambda}}} = \begin{pmatrix} \boldsymbol{J}_+ & \boldsymbol{J}_{+0} \\ \boldsymbol{O} & \boldsymbol{J}_0 \end{pmatrix},$$

where $\boldsymbol{J}_+$ and $\boldsymbol{J}_{+0}$ are matrices whose components are generally non-zero, and

$$\boldsymbol{J}_0 = \text{diag}\left(V^1_{-\hat{k},0} - V_{00}, V^1_{\hat{k},0} - V_{00}, \dots, V^1_{-1,0} - V_{00}, V^1_{1,0} - V_{00},\right.$$
$$\left. V^2_{-\hat{k}} - V^2_0, V^2_{\hat{k}} - V^2_0, \dots, V^2_{-1} - V^2_0, V^2_1 - V^2_0\right),$$

where diag$(\cdot)$ denotes a diagonal matrix with the elements in the parentheses. The full agglomeration $\boldsymbol{\lambda}_{\text{FA}}$ is stable if all the eigenvalues of both $\boldsymbol{J}_+$ and $\boldsymbol{J}_0$ have negative real part.

As shown by J_0, the stability of the full agglomeration $\boldsymbol{\lambda}_{\text{FA}}$ is determined by the differences in the utility levels. The differences in utility levels between non-remote and remote work affect the stability of the full agglomeration $\boldsymbol{\lambda}_{\text{FA}}$ (i.e., $V^1_{i,0} - V_{00}$ and $V^1_{0,i} - V_{00}$). If $V^1_{i,0} - V_{00} > 0$ or $V^1_{0,i} - V_{00} > 0$, then the full agglomeration $\boldsymbol{\lambda}_{\text{FA}}$ becomes unstable. Thus, the stability condition of the full agglomeration in the FER model differs from that in the FE model. The unstable full agglomeration dynamically changes to a population distribution over time due to a slight shock to the population distribution and the myopic behavior of each skilled worker.

The indirect utility of skilled workers in the FER model has symmetry properties with full agglomeration $\boldsymbol{\lambda}_{\text{FA}}$. In the long narrow economy, the symmetry conditions for the indirect utilities are expressed as

$$V^1_{-i,-i}(\boldsymbol{\lambda}_{\text{FA}}, \tau) = V^1_{i,i}(\boldsymbol{\lambda}_{\text{FA}}, \tau) \quad \forall i \in \mathcal{K}, \tag{31a}$$

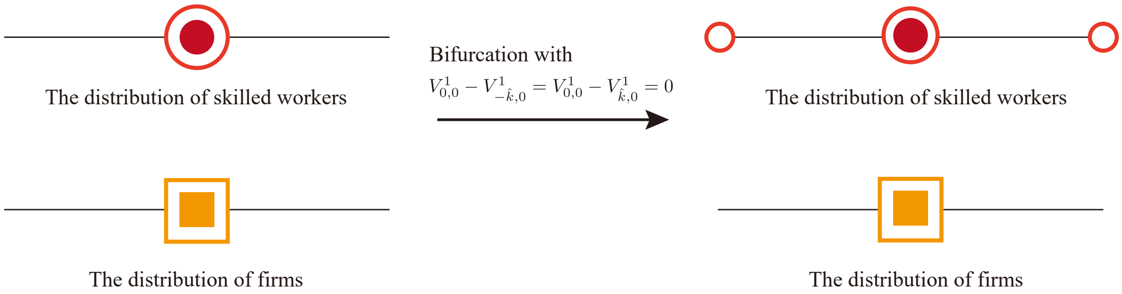
$$V^2_{-i}(\boldsymbol{\lambda}_{\text{FA}}, \tau) = V^2_{i}(\boldsymbol{\lambda}_{\text{FA}}, \tau) \quad \forall i \in \mathcal{K}, \tag{31b}$$

$$V^1_{-i,0}(\boldsymbol{\lambda}_{\text{FA}}, \tau) = V^1_{i,0}(\boldsymbol{\lambda}_{\text{FA}}, \tau) \quad \forall i \in \mathcal{K}, \tag{31c}$$

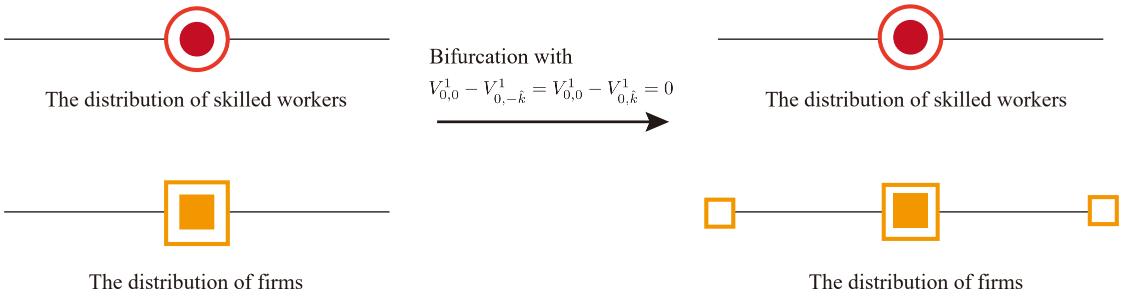
$$V^1_{0,-i}(\boldsymbol{\lambda}_{\text{FA}}, \tau) = V^1_{0,i}(\boldsymbol{\lambda}_{\text{FA}}, \tau) \quad \forall i \in \mathcal{K}, \tag{31d}$$

where we insert commas in the indirect utilities to distinguish whether the notation i refers to the region where a worker resides or the location of the firm to which she belongs. Conditions (31a) and (31b) imply that the utility levels of skilled workers choosing peripheral regions are determined by the distance from the central region with the full agglomeration. Condition (31c) implies that the utility levels of remote workers planning to reside in peripheral regions are determined by the distance between the central region and the regions where they reside, whereas condition (31d) implies that the utility levels of remote workers planning to reside in the central region are determined by the distance between the central region and the regions where the firms to which remote workers belong are located.

The emergence of remote workers from full agglomeration $\boldsymbol{\lambda}_{\text{FA}}$ can be expressed as the emergence of bifurcation solutions. If the difference in indirect utilities $V^1_{i,j} - V^1_{00}$ $(\exists i, j)$ is equal to zero, then the Jacobian matrix is singular. That is, the condition of stability with the full



(a) Emergence of remote workers supplying labor to firms in the central region



(b) Emergence of remote workers supplying labor to firms in peripheral regions

Figure 3: Examples of theoretically predicted bifurcation behavior

agglomeration is related to the condition of the emergence of bifurcation solution from it. The following proposition implies that the condition for the singularity of the Jacobian matrix is a sufficient condition for the emergence of a bifurcating solution under the replicator dynamics. The symmetry condition (31) plays an important role in the proof of the proposition.

Proposition 1. *We have the following theoretical results:*

(i) *If* $V_{j,0} - V_{0,0} = 0$ *holds at* $(\boldsymbol{\lambda}_{\mathrm{FA}}, \tau^*)$*, then a bifurcation solution with* $\lambda_{0,0}, \lambda_{j,0} = \lambda_{-j,0} > 0$ *emerges from* $(\boldsymbol{\lambda}_{\mathrm{FA}}, \tau^*)$*.*

(ii) *If* $V_{0,j} - V_{0,0} = 0$ *holds at* $(\boldsymbol{\lambda}_{\mathrm{FA}}, \tau^*)$*, then a bifurcation solution with* $\lambda_{0,0}, \lambda_{0,j} = \lambda_{0,-j} > 0$ *emerges from* $(\boldsymbol{\lambda}_{\mathrm{FA}}, \tau^*)$*.*

Proof. See Appendix A. □

An intuitive explanation of Proposition 1 is as follows. Each skilled worker myopically switches their location choices to achieve a higher utility level. If skilled workers can achieve higher utility levels by conducting remote work with the full agglomeration, the number of remote workers increases.

Examples of predicted bifurcation solutions are shown in Figure 3. If $V_{0,0} - V_{-\hat{k},0} = 0$, then a bifurcation solution emerges that represents the population distribution with remote workers residing in regions $-\hat{k}$ and $\hat{k}$ (Figure 3(a)). With this solution, remote workers supply labor to firms operating in the central large city (i.e., region 0). On the other hand, if $V_{0,0}-V_{0,\hat{k}}=0$, then a bifurcation solution emerges that represents the population distribution with remote workers supplying labor to firms operating in region $-\hat{k}$ and $\hat{k}$ (Figure 3(b)). With this solution, remote workers are employed by firms operating in regions away from the central city.

There are two points to note regarding Proposition 1. First, this proposition, which demonstrates the existence of the bifurcation solution, is an extension of the theoretical bifurcation analysis of NEG models with a single industry (Ikeda et al., 2024). Second, the proof of this proposition relies solely on the differentiability and symmetry of the indirect utility function in terms of regions, as shown in Eq. (31). Thus, the theoretical result is applicable to other economic models that describe the location choices of economic agents.

4. Emergence of bifurcation solutions from the full agglomeration

4.1. Qualitative analysis of the full agglomeration

Using Proposition 1 and the indirect utility function (23b), we explore how the transportation cost parameter τ affects the emergence of remote workers from the full agglomeration. The emergence of remote workers residing in peripheral regions and supplying labor to the central region is determined by the difference in the utility levels between remote and non-remote workers under the full agglomeration:

$$V^1_{j0}(\boldsymbol{\lambda}_{\text{FA}},\tau)-V^1_{00}(\boldsymbol{\lambda}_{\text{FA}},\tau)=\frac{\alpha_1}{\sigma_1-1}\ln d^1_{0j}+\frac{\alpha_2}{\sigma_2-1}\ln d^2_{0j}+\beta\ln\left(\frac{L+2}{L}\right). \tag{32}$$

The first and second terms represent the effects of relocating to region j on the price indices, and the third term is the effect on housing price. As shown by the third term, the relocation has a positive impact on the utility level through the housing price (i.e., $\ln\left((L+2)/L\right)>0$). The transportation cost parameter τ does not affect the third term, but it does affect the first and second terms (i.e., prices of differentiated goods). Substituting Eq. (30) into Eq. (32) yields

$$V^1_{j0}(\boldsymbol{\lambda}_{\text{FA}},\tau)-V^1_{00}(\boldsymbol{\lambda}_{\text{FA}},\tau)=-\tau j(\alpha_1+\alpha_2)+\beta\ln\left(\frac{L+2}{L}\right).$$

As shown by the above equation, the full agglomeration is unstable for low transportation parameter τ. This intuitively implies that, for low transportation costs, the negative effect of remote work on the price index is relatively small compared to the positive effect on the housing price. $V^1_{j0}-V^1_{00}=0$ holds at $\tau=\beta(j(\alpha_1+\alpha_2))^{-1}\ln\left((L+2)/L\right)$. At this parameter, the bifurcation solution representing the presence of remote workers residing in region j (or $-j$) emerges (Proposition 1(i)).

Whether the bifurcation solution, which represents remote workers residing in the central region and supplying labor to firms in peripheral regions, emerges from the full agglomeration can be examined using the following equation:

$$V^1_{0j}(\boldsymbol{\lambda}_{\mathrm{FA}}, \tau) - V^1_{00}(\boldsymbol{\lambda}_{\mathrm{FA}}, \tau) = w^1_{0j}(\boldsymbol{\lambda}_{\mathrm{FA}}, \tau) - w^1_{00}(\boldsymbol{\lambda}_{\mathrm{FA}}, \tau). \tag{33}$$

The above equation implies that the difference in the utility levels does not depend on the price indices and the housing price, but it depends on the wage. Substituting the wage (24a) into the RHS of Eq. (33) yields

$$w^1_{0j}(\boldsymbol{\lambda}_{\mathrm{FA}}, \tau) - w^1_{00}(\boldsymbol{\lambda}_{\mathrm{FA}}, \tau) = \frac{\alpha_1}{\sigma_1}\left[2(d^1_{j0} - 1) + L\sum_{k\in\mathcal{K}}\left(\frac{d^1_{jk}}{d^1_{0k}} - 1\right)\right]. \tag{34}$$

As shown by the above equation, both the transportation cost parameter τ and the number of unskilled workers L affect whether the wage of remote workers planning to reside in the central region and supply labor to region j is higher than that of non-remote workers residing in the central region. The full agglomeration $\boldsymbol{\lambda}_{\mathrm{FA}}$ is unstable for high transportation costs, as the wage of remote work exceeds that with no remote work:

$$\begin{aligned}\lim_{\tau\to\infty}(w^1_{0j} - w^1_{00}) &= \frac{\alpha_1}{\sigma_1}\left[\lim_{\tau\to\infty} 2(d^1_{j0} - 1) + \lim_{\tau\to\infty} L\sum_{k\in\mathcal{K}}\left(\frac{d^1_{jk}}{d^1_{0k}} - 1\right)\right] \\ &= \frac{\alpha_1}{\sigma_1}(-2 + \infty) > 0 \quad (j \neq 0),\end{aligned} \tag{35}$$

where we omit the argument $(\boldsymbol{\lambda}_{\mathrm{FA}}, \tau)$. This equation intuitively implies that skilled workers conducting remote work can earn higher wages, as firms operating in peripheral regions can generate more revenue from unskilled workers due to higher transportation costs than those in the central region. With sufficiently small τ, by applying Taylor expansion to Eq. (34), we obtain

$$w^1_{0j}(\boldsymbol{\lambda}_{\mathrm{FA}}, \tau) - w^1_{00}(\boldsymbol{\lambda}_{\mathrm{FA}}, \tau) \approx -\frac{\alpha_1\tau(\sigma_1 - 1)}{\sigma_1}\left(2j + L\sum_{k\in\mathcal{K}}|k - j|\right) < 0, \tag{36}$$

implying that wages in the central region are higher than those in peripheral regions. Equation (36) implies that firms operating in the central region generate higher revenue than those in peripheral regions at low transportation costs. The intermediate value theorem ensures the existence of the transportation cost parameter τ such that $w^1_{0j}(\boldsymbol{\lambda}_{\mathrm{FA}}, \tau) - w^1_{00}(\boldsymbol{\lambda}_{\mathrm{FA}}, \tau) = 0$ (i.e., $V^1_{0j}(\boldsymbol{\lambda}_{\mathrm{FA}}, \tau) - V^1_{00}(\boldsymbol{\lambda}_{\mathrm{FA}}, \tau) = 0$). At this value of the parameter, the bifurcation solution emerges from the full agglomeration (Proposition 1(ii)).

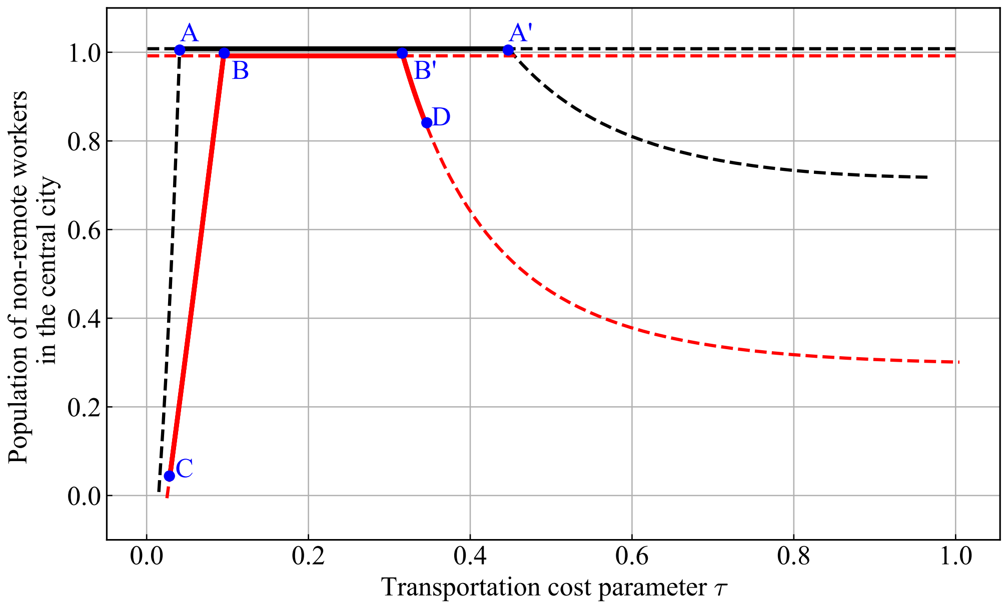
Predicted bifurcation solutions are not necessarily stable since Proposition 1 only states the emergence of bifurcation solutions that represent the presence of remote workers. The stability of each bifurcation solution is determined by the eigenvalues of the Jacobian matrix $\partial \boldsymbol{F}/\partial \boldsymbol{\lambda}$. We numerically examine the stability of the bifurcation solutions to explore the effect of introducing remote work on the population distribution of the stable equilibrium (Section 4.2).

4.2. Numerical bifurcation analysis

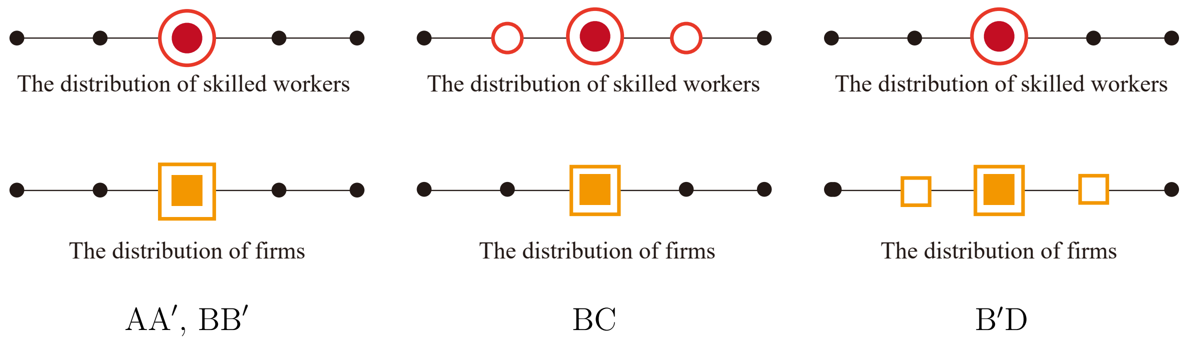
By conducting numerical analysis of the stable equilibria in the FE and FER models, we examine the impact of remote work on the population distribution $\boldsymbol{\lambda}$ in equilibrium. As explained in Sections 2.2.3 and 3, the stability conditions of equilibria differ between the FE and FER models . The stable full agglomeration in the FE model is not necessarily stable in the FER model. To explore how the full agglomeration is affected by remote work, we conduct a bifurcation analysis of the full agglomeration $\boldsymbol{\lambda}_{\text{FA}}$.

Figure 4 shows the stationary points of the dynamics under the FE and FER models with parameters set as $\alpha_i = 0.3$, $\beta = 0.1$, $\sigma_i = 3$, $L = 2.6$. With this parameter setting, the wage of skilled workers in the central region, w^1_{00}, under the full agglomeration is solved as 1.5 (see Appendix B.1 for details). The full agglomeration is stable for $\tau \in (0.041, 0.45)$ under the FE model, as shown by path AA′. In contrast, it is stable for $\tau \in (0.095, 0.32)$ under the FER model, as shown by path BB′. For $\tau \in (0.028, 0.096) \cup (0.32, 0.35)$ under the FER model, the full agglomeration is unstable. The results shown in Figure 4 indicate that in an economy where skilled workers can engage in remote work, the full agglomeration under the FE model is not necessarily stable.

Spatial distribution of skilled workers, some of whom are engaged in remote work, can emerge from an unstable full agglomeration. As shown in Figure 4(a), bifurcation solutions emerge from points B and B′. The bifurcation solution from point B under the FER model represents a spatial distribution of skilled workers, some of whom reside in region 1 (or −1) and supply labor to firms operating in region 0 (see Figure 4(b)). Some skilled workers relocate from the central region to peripheral regions for $\tau \in (0.028, 0.095)$. Moreover, the bifurcation solution from point B′ under the FER model represents a population distribution emerges in which skilled workers reside in region 0 and supply labor to firms operating in region 1 (or −1). In this case, skilled workers do not relocate, while some firms relocate from the central region to region 1 (or −1) for $\tau \in (0.32, 0.35)$. In the numerical results, the wages of the skilled workers under the bifurcation solutions are the same as those under the full agglomeration.



(a) Solutions curves of the replicator dynamics

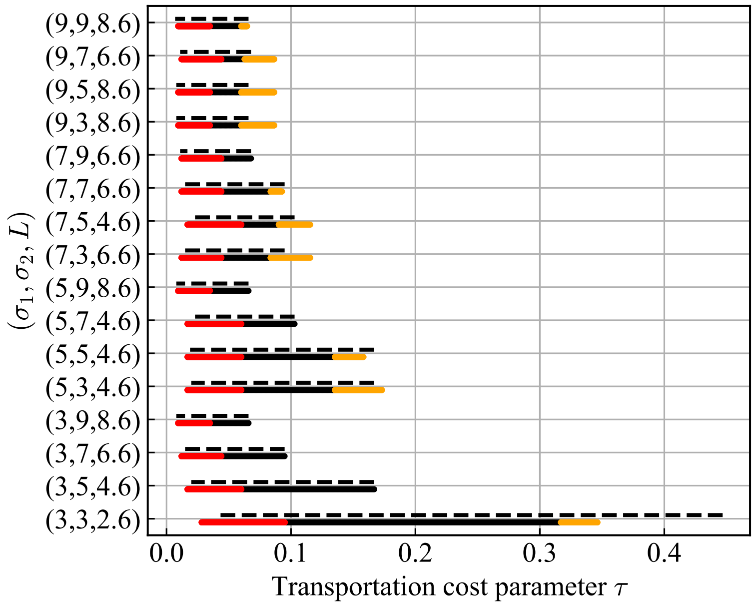


(b) Spatial distribution of skilled workers and firms in the M^1 sector

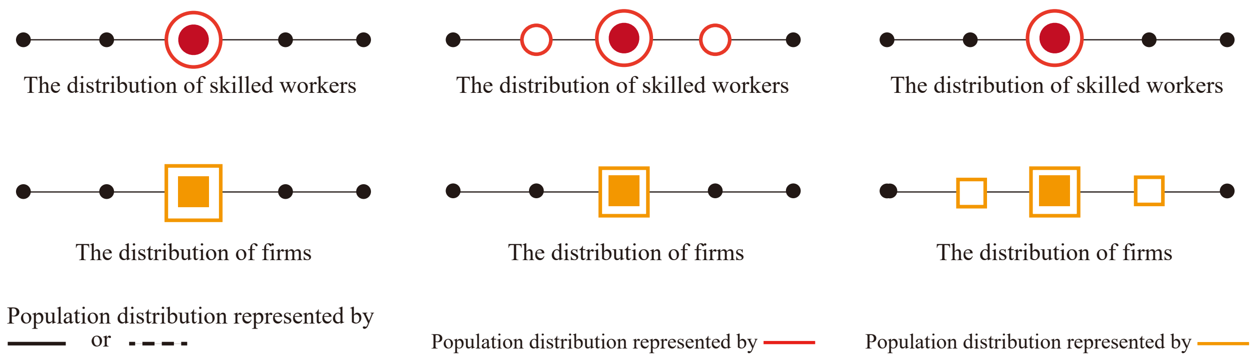
Figure 4: Bifurcating solutions from the full agglomeration $\boldsymbol{\lambda}_{\mathrm{FA}}$ with 5 regions. —: stable equilibrium in the FE model; - - -: unstable equilibrium in the FE model; —: stable equilibrium in the FER model; - - -: unstable equilibrium in the FER model. ○ (red): regions where workers who can work remotely reside; • (red): regions where non-remote workers reside; □ (yellow): regions where firms employing workers who can work remotely are located; ■ (yellow): regions where firms employing non-remote workers are located

By conducting sensitivity analysis of the stable equilibria in the FE and FER models, we examine how exogenous parameters affect their existence. We consider several values for the elasticities of substitution σ_1 and σ_2, as well as the mass of unskilled worker L. Specifically, σ_1 and σ_2 are selected from the set $\{3, 5, 7, 9\}$. The parameters L and others affect the skilled workers' wages w^1_{ij}. We set α_k ($k = 1, 2$) and β to 0.3 and 0.1, respectively. Given specific values for σ_1, σ_2, α_1, α_2, and β, we choose L such that w^1_{00} (i.e., the wage of skilled workers who can work remotely) or w^2_0 falls within the set $\{1.05, 1.5, 2\}$ and $w^1_{00}, w^2_0 > 1$ holds under the full agglomeration.[4] Figure 5 shows the stable equilibria of the FE and FER models for the parameter sets in which $w^1_{00} = 1.5$ or $w^2_0 = 1.5$ holds under the full agglomeration. A spatial distribution of skilled workers, some of whom are remote workers residing in peripheral regions, exists stably for low values of the transportation cost parameter τ. For these values, the full agglomeration becomes unstable across all parameter sets shown in 5(a). A spatial distribution of skilled workers, some of whom are remote workers residing in the central region, exists stably for high values of τ. For these values, the full agglomeration becomes unstable for some parameter sets. These results indicate that stable equilibria involving remote workers can arise under a variety of parameter sets. The results for $w^1_{00} = 1.05$ or $w^1_{00} = 2$ are quantitatively similar to those shown in Figure 5(a) (see Appendix B.1).

[4]We choose parameter sets so that the wages of the skilled workers exceed those of unskilled workers.



(a) Stable equilibrium



(b) Spatial distribution of skilled workers and firms in the M^1 sector

Figure 5: Sensitivity analysis of stable equilibria (solid line in (a): stable equilibrium of the FER model; dashed line in (a): stable equilibrium of the FE model). ◯ (red): regions where workers who can work remotely reside; • (red): regions where non-remote workers reside; □ (yellow): regions where firms employing workers who can work remotely are located; ■ (yellow): regions where firms employing non-remote workers are located

5. The impact of introducing remote work on workers' utilities

5.1. Qualitative analysis of the full agglomeration

The introduction of remote work can marginally affect the utility levels of both skilled and unskilled workers. Changes in population distribution induced by remote work affect price indices, housing prices, and the wages of skilled workers. Welfare change resulting from remote work is the total change in the unskilled and skilled workers' utilities (see Eq. (28)).

We focus on marginal changes in unskilled workers' utilities caused by the emergence of remote workers from the full agglomeration $\boldsymbol{\lambda}_{\mathrm{FA}}$. The emergence of remote workers from the full agglomeration can be expressed as $-\mathrm{d}\lambda^1_{00}\boldsymbol{e}_{00} + \mathrm{d}\lambda_j\boldsymbol{e}_{j0} + \mathrm{d}\lambda_j\boldsymbol{e}_{-j,0}$ $(\mathrm{d}\lambda^1_{00}, \mathrm{d}\lambda_j > 0)$, where $\boldsymbol{e}_{ij}$ denotes the standard basis whose component associated with a remote worker residing in region i and supplying labor to a firm in region j is one. $-\mathrm{d}\lambda^1_{00}\boldsymbol{e}_{00}+\mathrm{d}\lambda_j\boldsymbol{e}_{j0}+\mathrm{d}\lambda_j\boldsymbol{e}_{-j,0}$ represents the appearance of remote workers residing in region j or $-j$ and supplying labor to firms in the central region. Using Eqs. (26) and (27a) yields the constraint on the total population in terms of the marginal change in population distribution: $2\mathrm{d}\lambda_j - \mathrm{d}\lambda^1_{00} = 0$. As shown in Eq. (23a), the indirect utility of each unskilled worker V^{u}_i is composed of price index, housing price and exogenous variables. Using this equation, we obtain the marginal change in each unskilled worker' utility in region i resulting from the marginal change $-\mathrm{d}\lambda^1_{00}\boldsymbol{e}_{00} + \mathrm{d}\lambda_j\boldsymbol{e}_{j0} + \mathrm{d}\lambda_j\boldsymbol{e}_{-j,0}$:

$$\mathrm{d}V^{\mathrm{u}}_0 = \frac{\beta}{\Lambda_0}\frac{\partial\Lambda_0}{\partial\lambda^1_{00}}\mathrm{d}\lambda^1_{00} > 0, \tag{37a}$$

$$\mathrm{d}V^{\mathrm{u}}_i = -\frac{\beta}{\Lambda_i}\frac{\partial\Lambda_i}{\partial\lambda^1_{j,0}}\mathrm{d}\lambda^1_j \quad (i \neq 0). \tag{37b}$$

These equations show that the marginal change $-\mathrm{d}\lambda^1_{00}\boldsymbol{e}_{00} + \mathrm{d}\lambda_j\boldsymbol{e}_{j0} + \mathrm{d}\lambda_j\boldsymbol{e}_{-j,0}$ does not affect the price indices, but does affect the housing prices. The marginal change in the utility level of unskilled workers in the central region is positive due to a decrease in the housing price there, whereas for unskilled workers residing in the region where remote workers reside, it is negative. For unskilled workers in all other regions, this marginal change is zero.

Next, we focus on the marginal change in population distribution which represents the emergence of remote workers residing in the central region: $-\mathrm{d}\lambda^1_{00}\boldsymbol{e}_{00}+\mathrm{d}\lambda_j\boldsymbol{e}_{0j}+\mathrm{d}\lambda_j\boldsymbol{e}_{0,-j}$ $(\mathrm{d}\lambda^1_{00}, \mathrm{d}\lambda_j > 0, 2\mathrm{d}\lambda_j - \mathrm{d}\lambda^1_{00} = 0)$. The marginal change in the utility level of unskilled workers in the central region associated with $-\mathrm{d}\lambda^1_{00}\boldsymbol{e}_{00} + \mathrm{d}\lambda_j\boldsymbol{e}_{0j} + \mathrm{d}\lambda_j\boldsymbol{e}_{0,-j}$ is given by

$$\mathrm{d}V^{\mathrm{u}}_0 = \frac{\alpha_1}{\sigma_1 - 1}\frac{1}{N^1_0}(-d^1_{00}\mathrm{d}\lambda_0 + d^1_{j0}\mathrm{d}\lambda_j + d^1_{-j,0}\mathrm{d}\lambda_j) - \frac{\beta}{\Lambda_0}(-\mathrm{d}\lambda_0 + 2\mathrm{d}\lambda_j). \tag{38}$$

The first and second terms represent the changes in the price indices and the change in the housing price, respectively. The second term is zero because of the population constraint, whereas the first term is negative as shown by the following inequality:

$$-d^1_{00}\mathrm{d}\lambda_0 + d^1_{j0}\mathrm{d}\lambda_j + d^1_{-j,0}\mathrm{d}\lambda_j < -d^1_{00}\mathrm{d}\lambda_0 + d^1_{00}\mathrm{d}\lambda_j + d^1_{00}\mathrm{d}\lambda_j = 0.$$

Using this inequality yields $\mathrm{d}V^{\mathrm{u}}_0 < 0$. On the other hand, the marginal change in the utility level of unskilled workers in a peripheral region is given by

$$\mathrm{d}V^{\mathrm{u}}_i = \frac{\alpha_1}{\sigma_1 - 1}\frac{1}{N^1_i}(-d^1_{0i}\mathrm{d}\lambda_0 + d^1_{ji}\mathrm{d}\lambda_j + d^1_{-j,i}\mathrm{d}\lambda_j) - \frac{\beta}{\Lambda_i}\underbrace{(-\mathrm{d}\lambda_0 + 2\mathrm{d}\lambda_j)}_{=0} \quad (i \neq 0). \tag{39}$$

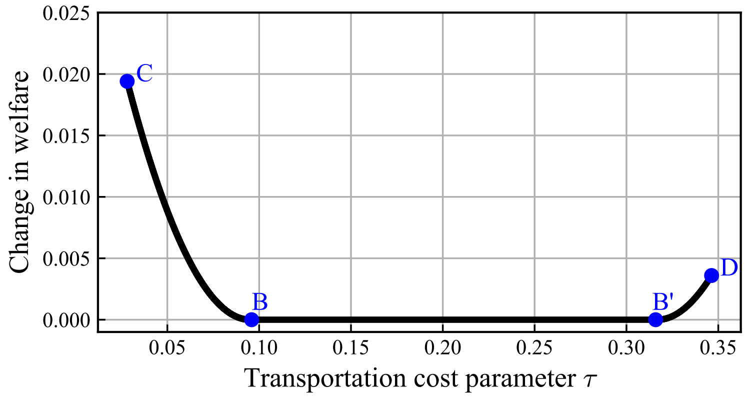
The sign of the above marginal change depends on transportation cost parameter τ.

The marginal change in each skilled worker's utility consists of changes in price indices, housing price, and wage. As the functional forms of the indirect utilities (23b) and (23c) indicate, the effects of the price index and housing price on this utility are the same as those on the utility of unskilled workers. The effect of the wage is added to the change in the utility level. As shown by the wage (24), this effect generally depends on the transportation cost parameter τ and the population distribution $\boldsymbol{\lambda}$.

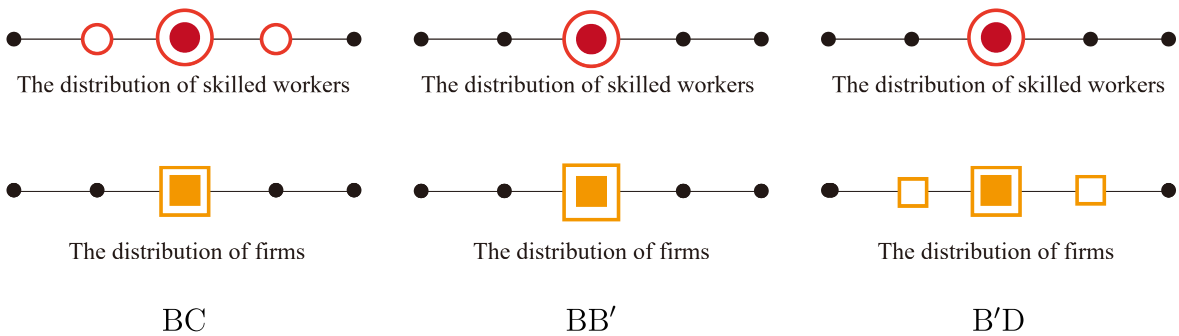
5.2. Numerical welfare analysis

We numerically explore the effects of introducing remote work on social welfare and the utility levels of skilled workers. To assess whether remote work is desirable for workers, we compare the welfare under the full agglomeration and that under stable equilibrium of the FER model. The stable equilibria of the FER model are shown in Figure 4 in Section 4.2.

Figure 6 shows the change in the welfare within the range of stable equilibria of the FER model. Exogenous parameters are the same as those used in the numerical analysis shown in Figure 4. Paths BC and B′D represent changes in the welfare resulting from the emergence of remote workers. Along path BC (B′D), the welfare under the population distribution with remote work exceeds that under the full agglomeration from zero to 0.019 (0.0036). Thus, the introduction of remote work can be desirable from a welfare perspective.



(a) Change in the social welfare $W(\boldsymbol{\lambda}, \tau) - W(\boldsymbol{\lambda}^{\mathrm{FA}}, \tau)$

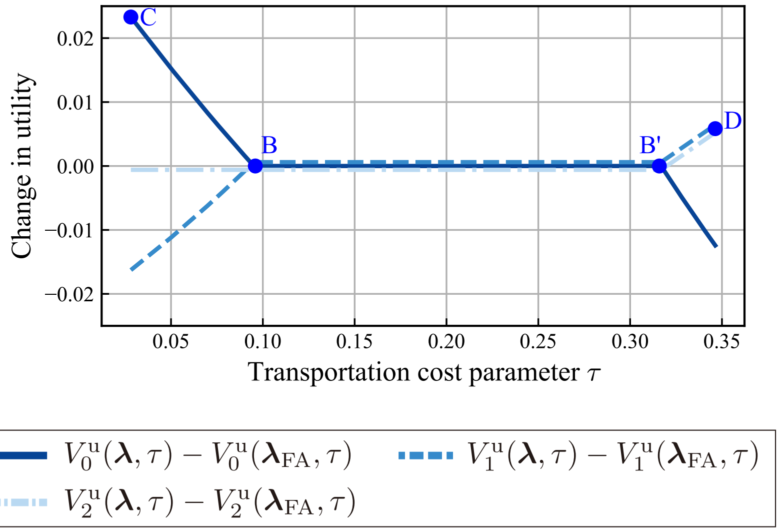


(b) Spatial distribution of skilled workers and firms in the M^1 sector

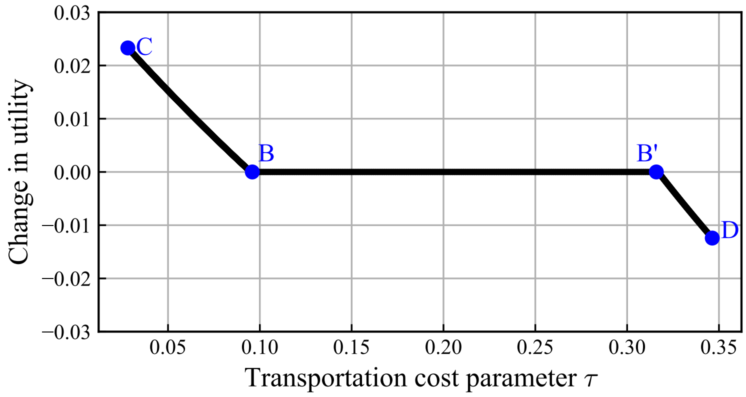
Figure 6: Effect of a change in population distribution on social welfare

Some unskilled workers' utilities decrease as shown in Section 5.1. Figure 7(a) illustrates the changes in the utility levels of unskilled workers in each region resulting from the introduction of remote work. As shown along path BC, the relocation of remote workers to region ± 1 increases the utility level of unskilled workers in the central region, but decreases those of unskilled workers in the two neighboring regions. There is no change in the utility level of the unskilled workers in the two outermost regions (i.e., region ± 2). These changes result from changes in housing prices (see Eq. (37)). On the other hand, as shown along path B′D, the emergence of remote workers residing in the central region increases the utility levels of unskilled workers in the regions other than the central region, but decreases those in the central region.

Figure 7(b) shows the change in the utility level of skilled workers in the M^1 sector resulting from the introduction of remote work. Path AB in the figure shows that the utility level increases as the number of remote workers residing in the surrounding regions increases, for $\tau \in (0.0281, 0.0959)$. In contrast, path B′D shows that the utility level decreases as the number of remote workers supplying labor to firms operating in the surrounding regions increases, for $\tau \in (0.316, 0.346)$ This implies that the introduction of remote work is not desirable even for remote workers themselves in terms of utility. These changes mirror those in the utility levels of unskilled workers in the central region. This is because, in the numerical results, the wages of remote workers remain unchanged compared to the full agglomeration. In this case, the emergence of remote workers residing in peripheral regions increases the utility levels, whereas that of workers residing in the central region decreases them (see Appendix B.2 for details).



(a) Change in each unskilled worker's utility



(b) Change in the skilled worker's utility in the M^1 sector: $V_{0,0}^1(\boldsymbol{\lambda}, \tau) - V_{0,0}^1(\boldsymbol{\lambda}_{\mathrm{FA}}, \tau)$

Figure 7: Effect of a change in population distribution on workers' utilities. $\boldsymbol{\lambda}$: the stable population distribution of the FER model

5.3. Sensitivity analysis

We conduct a sensitivity analysis of the impact of the introduction of remote work on social welfare and the utility levels of skilled workers. As the analysis shown in Figure 5, we select σ_1 and σ_2 from the set $\{3, 5, 7, 9\}$. The parameters α_1, α_2, and β are held constant at $\alpha_1 = 0.3$, $\alpha_2 = 0.3$, and $\beta = 0.1$. Given the selected values of σ_1, σ_2, α_1, α_2, and β, we choose L such that w^1_{00} or w^2_0 takes a value in $\{1.05, 1.5, 2.0\}$ under the full agglomeration $\boldsymbol{\lambda}_{\text{FA}}$ (see Appendix B.1 for details). The total number of parameter sets is 48.

Figure 8 shows the changes in social welfare resulting from the introduction of remote work across 48 parameter sets. Each solid line in the figure represents the change in welfare $W(\boldsymbol{\lambda}, \tau) - W(\boldsymbol{\lambda}^{\text{FA}}, \tau)$ for a given parameter set. The straight lines marked with circles ($\circ$) and triangles ($\triangle$) lie on the zero line because the stable equilibrium is the full agglomeration (i.e., $\boldsymbol{\lambda} = \boldsymbol{\lambda}^{\text{FA}}$). The curves ending in a circle marker ($\circ$) represent population distributions including skilled workers residing in peripheral regions and supplying labor to firms operating in the central region. For all parameter sets, such distributions emerge as bifurcation solutions at a lower transport cost than that indicated by the circle marker. The curves ending in a triangle marker ($\triangle$) represent population distributions including skilled workers residing in the central region and supplying labor to firms in the peripheral regions. For 30 parameter sets, this distribution emerges as a bifurcation solution at a higher transport cost than that indicated by the triangle marker. As shown in Figure 8, social welfare under population distributions involving remote work by some skilled workers exceeds that under full agglomeration.

As shown in Figure 7(b), the introduction of remote work is not necessarily desirable for skilled workers. Figure 9 illustrates how the utility level of skilled workers changes as a result of the introduction of remote work. Each solid line in this figure represents the change in utility: $V^1_{0,0}(\boldsymbol{\lambda}, \tau) - V^1_{0,0}(\boldsymbol{\lambda}_{\text{FA}}, \tau)$. The utility level increases for population distributions including skilled workers residing in peripheral regions and supplying labor to firms in the central region. In contrast, it decreases for distributions including skilled workers residing in the central region and supplying labor to firms in peripheral regions.

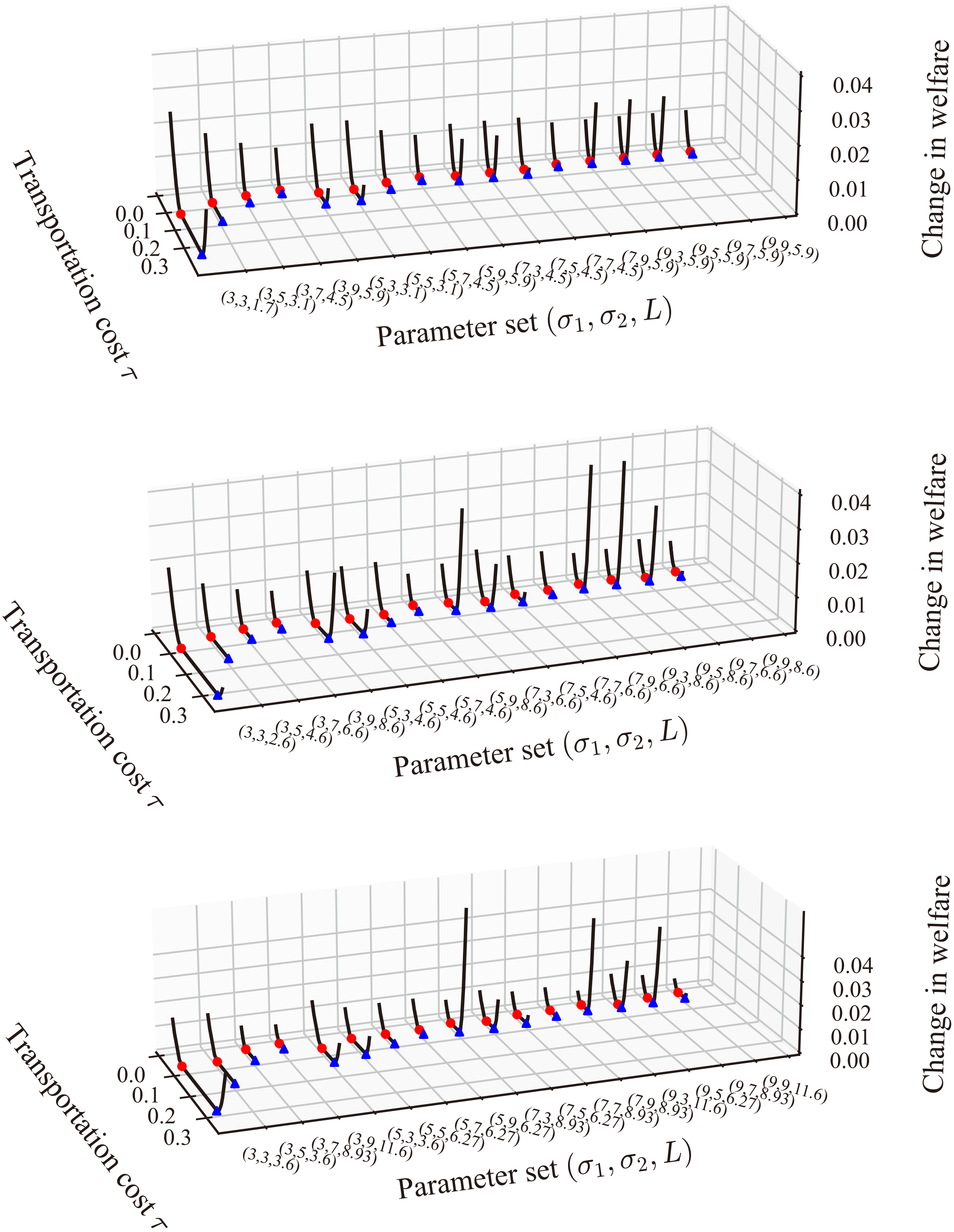


Figure 8: The change in social welfare $W(\boldsymbol{\lambda}, \tau) - W(\boldsymbol{\lambda}^{\mathrm{FA}}, \tau)$ for the parameter set (σ_1, σ_2, L)

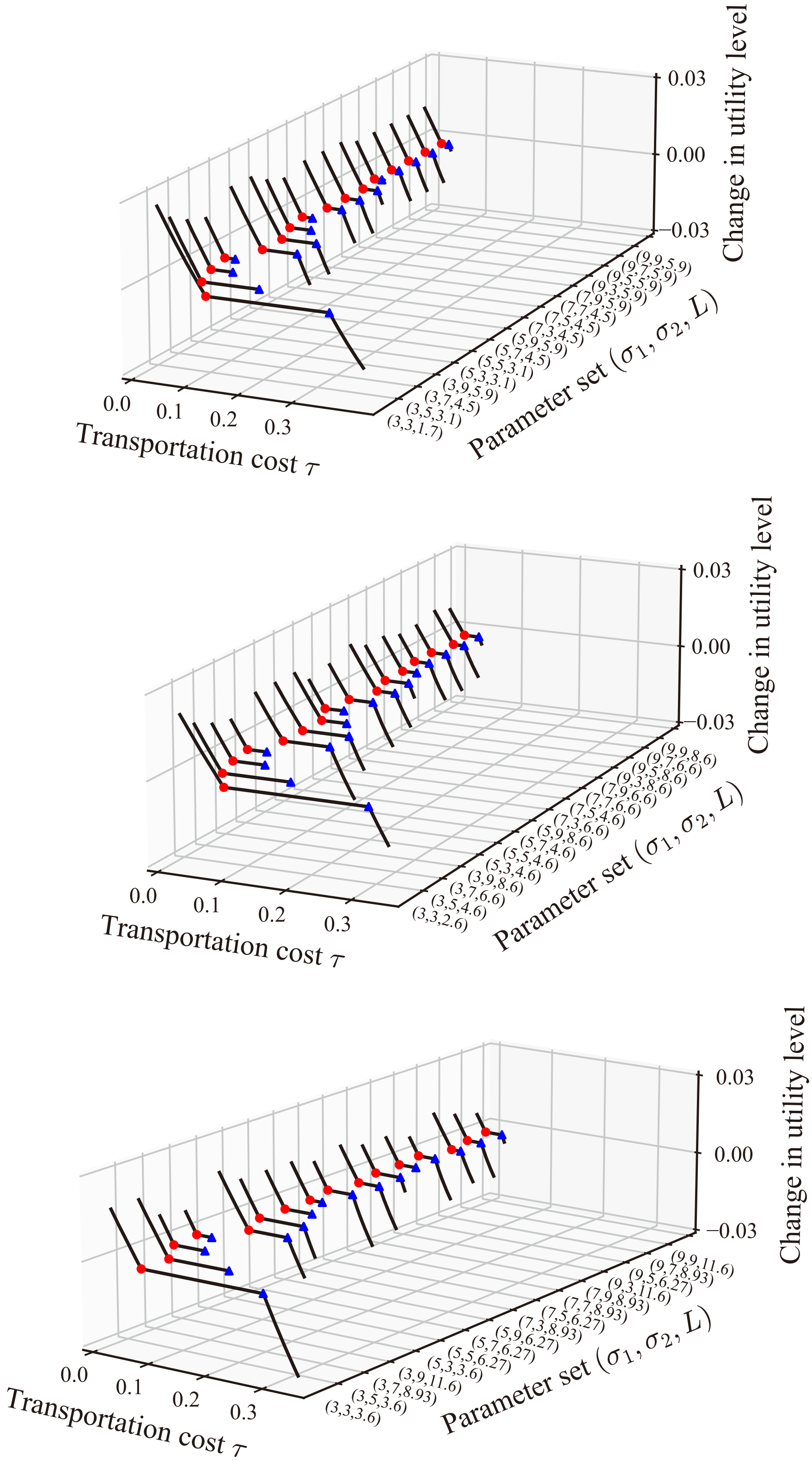


Figure 9: The change in the utility level of the skilled workers in the M^1 sector, $V^1_{0,0}(\boldsymbol{\lambda}, \tau) - V^1_{0,0}(\boldsymbol{\lambda}_{\mathrm{FA}}, \tau)$, for the parameter set (σ_1, σ_2, L)

6. Conclusion

We have explored how the introduction of remote work affects the spatial distribution of workers and firms in cities, social welfare, and the utility levels of remote and non-remote workers. By conducting a bifurcation analysis of equilibrium for the Footloose Entrepreneur models with and without remote work, we examine the stability of full agglomeration and the bifurcation solutions that represent spatial distributions of workers, some of whom are remote workers. Our findings are summarized as follows: (1) remote work can generate two types of spatial distributions of skilled workers: one in which some remote workers reside in small cities, and the other in which some remote workers supply labor to firms in small cities, and (2) if skilled workers behave myopically, remote work can lead to a decrease in the utility of skilled workers, while social welfare increases.

From the perspective of equity, policymakers need to implement policies to manage population distribution. Location-based policies are viable options for mitigating the decline in utility caused by the introduction of remote work. These policies have been theoretically explored in the literature on land use regulation (e.g., Kono and Joshi, 2019) and location-based policies (e.g., Aizawa and Kono, 2023). Welfare and utility analyses of such policies remain future work.

Appendix

A. Proof of Proposition 1

We provide only the proof of Proposition 1 (i) since the proof of Proposition 1 (ii) is almost identical. By conducting the Liapunov–Schmidt reduction for the governing equation (27) (see e.g., Ikeda and Murota, 2019, Chapter 8), and using the symmetry condition (31c) together with the population constraint $\sum_{i,j\in\mathcal{K}} \lambda^1_{ij} = 1$, we obtain the following bifurcation equation:

$$\widehat{F}_{-1}(\lambda^1_{-i,0}, \lambda^1_{i,0}, \tau) = \lambda^1_{-i,0}\left[V^1_{-i,0}(\lambda^1_{-i,0}, \lambda^1_{i,0}, \tau) - \overline{V^1}(\lambda^1_{-i,0}, \lambda^1_{i,0}, \tau)\right], \tag{40a}$$

$$\widehat{F}_{1}(\lambda^1_{-i,0}, \lambda^1_{i,0}, \tau) = \lambda^1_{i,0}\left[V^1_{i,0}(\lambda^1_{-i,0}, \lambda^1_{i,0}, \tau) - \overline{V^1}(\lambda^1_{-i,0}, \lambda^1_{i,0}, \tau)\right], \tag{40b}$$

where

$$\begin{aligned}\overline{V^1}(\lambda^1_{-i,0}, \lambda^1_{i,0}, \tau) &= (1 - \lambda^1_{-i,0} - \lambda^1_{i,0})V^1_{00}(\lambda^1_{-i,0}, \lambda^1_{i,0}, \tau)\\ &\quad + \lambda^1_{-i,0}V^1_{-i,0}(\lambda^1_{-i,0}, \lambda^1_{i,0}, \tau) + \lambda^1_{i,0}V^1_{i,0}(\lambda^1_{-i,0}, \lambda^1_{i,0}, \tau).\end{aligned}$$

We search for a bifurcation solution satisfying $\lambda^1_{00} = 1 - \lambda^1_{-i,0} - \lambda^1_{i,0}$ and $\lambda^1_{ij} = 0$ $((i,j) \neq (0,0), (-i,0), (i,0))$ in a neighborhood of the bifurcation point $(\boldsymbol{\lambda}_{\mathrm{FA}}, \tau^*)$. In the bifurcation equation (40), only the components relevant to the bifurcation analysis are included. The solutions of the bifurcation equation (40), $(\lambda^1_{-i,0}, \lambda^1_{i,0}, \tau)$ satisfying $\widehat{F}_{-1} = \widehat{F}_1 = 0$, are those that bifurcate from the point.

We explore a solution of the bifurcation equation using its symmetry conditions and the implicit function theorem. Due to the bilateral symmetry between regions $-i$ and i, the terms in the bifurcation equation satisfy the following symmetry conditions:

$$V^1_{00}(\lambda^1_{i,0}, \lambda^1_{-i,0}, \tau) = V^1_{00}(\lambda^1_{i,0}, \lambda^1_{-i,0}, \tau), \tag{41a}$$

$$V^1_{-i,0}(\lambda^1_{i,0}, \lambda^1_{-i,0}, \tau) = V^1_{i,0}(\lambda^1_{-i,0}, \lambda^1_{i,0}, \tau). \tag{41b}$$

Given the condition (41b), $\widehat{F}_{-1}$ and $\widehat{F}_1$ are identical for $(\lambda^1_{-i,0}, \lambda^1_{i,0}) = (\lambda, \lambda)$ with $\lambda \geq 0$:

$$\widehat{F}_{-1}(\lambda, \lambda, \tau) = \widehat{F}_1(\lambda, \lambda, \tau) = \lambda\left[V^1_{-i,0}(\lambda, \lambda, \tau) - \overline{V^1}(\lambda, \lambda, \tau)\right]. \tag{42}$$

We apply the implicit function theorem to $V^1_{-i,0}(\lambda, \lambda, \tau) - \overline{V^1}(\lambda, \lambda, \tau)$, since the assumption in the Proposition implies that $V^1_{-i,0}(\lambda, \lambda, \tau) - \overline{V^1}(\lambda, \lambda, \tau) = 0$ holds at $(0, 0, \tau^*)$. This theorem ensures the existence of (λ, λ, τ) such that $\widehat{F}_{-1}(\lambda, \lambda, \tau) = \widehat{F}_1(\lambda, \lambda, \tau) = 0$ holds in a neighborhood of the bifurcation point $(\boldsymbol{\lambda}_{\mathrm{FA}}, \tau^*)$. By applying Taylor expansion to Eq. (42), we obtain

$$\lambda\left[V^1_{-i,0}(\lambda, \lambda, \tau) - \overline{V^1}(\lambda, \lambda, \tau)\right] = \lambda(\gamma_0\widetilde{\tau} + \gamma_1\lambda + \text{higher order terms}), \tag{43}$$

where $\widetilde{\tau} = \tau - \tau^*$ is the incremental parameter and γ_0 and γ_1 are expansion coefficients. The bifurcation solution is asymptotically expressed as $\lambda \approx -\gamma_0\widetilde{\tau}/\gamma_1$.

B. Supplementary discussion on the numerical analyses in Sections 4 and 5

B.1. Skilled workers' wages under the full agglomeration

Substituting the full agglomeration $\boldsymbol{\lambda}_{\text{FA}}$ into wage (24a) yields the wage of skilled workers in the M^1 sector:

$$w_{ij}^1(\boldsymbol{\lambda}_{\text{FA}}, \tau) = \frac{\alpha_1}{\sigma_1} \sum_{k \in \mathcal{K}} \frac{d_{jk}^1}{d_{0k}^1} (\lambda_k^1 + \lambda_k^2 + L). \tag{44}$$

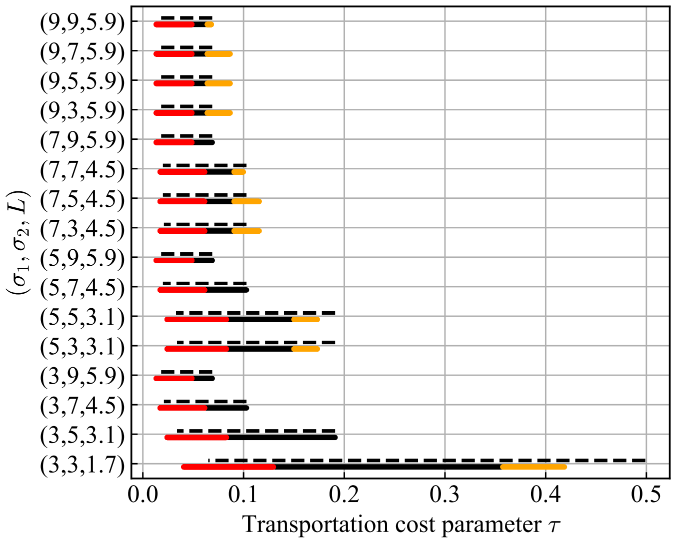
As shown in the above equation, the wages of skilled workers supplying labor to firms operating in the central region are identical:

$$w_{i0}^1(\boldsymbol{\lambda}_{\text{FA}}, \tau) = \frac{\alpha_1}{\sigma_1} ((2\hat{k} + 1)L + 2). \tag{45}$$

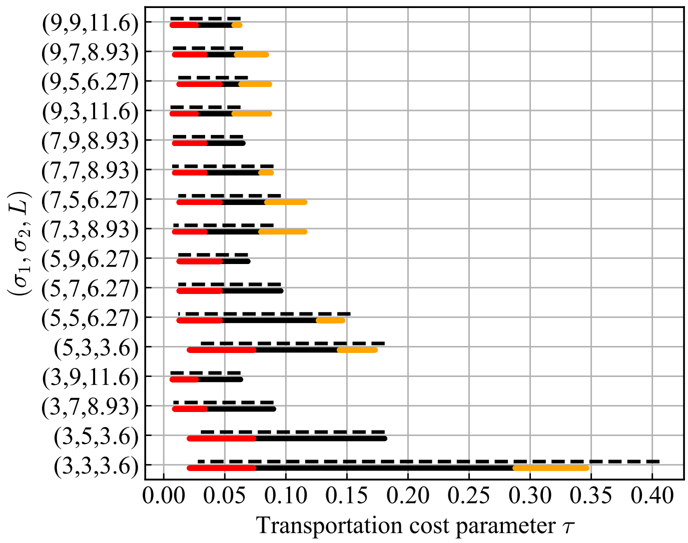
Under the full agglomeration, the wage is determined by α_1, σ_1, $\hat{k}$, and L. Conversely, given α_1, σ_1, $\hat{k}$, and w_{i0}^1, the number of unskilled workers L is uniquely determined:

$$L = \frac{1}{2\hat{k} + 1} \left(\frac{\sigma_1 w_{i0}^1}{\alpha_1} - 2 \right). \tag{46}$$

Figure 10 shows the stable equilibria of the FE and FER models for parameter sets under the full agglomeration. The results in this figure are similar to those in Figure 5(a) in terms of how the stable spatial distribution of skilled workers changes with the transportation cost parameter τ.



(a) Stable equilibria for $w_{00}^1 = 1.05$ or $w_0^2 = 1.05$ under the full agglomeration



(b) Stable equilibria for $w_{00}^1 = 2$ or $w_0^2 = 2$ under the full agglomeration

Figure 10: Sensitivity analysis of stable equilibria (solid line in (a) and (b): stable equilibrium of the FER model; dashed line in (a) and (b): stable equilibrium of the FE model). —: the full agglomeration; —: the spatial distribution of skilled workers, some of whom residing in peripheral regions; —: the spatial distribution of skilled workers, some of whom residing in the central region

B.2.1. Spatial distribution of skilled workers, some of whom reside in peripheral regions

We examine the shifts in skilled workers' utilities due to the change from the full agglomeration to the spatial distribution of skilled workers, some residing in peripheral regions. These shifts are numerically examined, as shown by path BC in Figure 7(b). Let $\boldsymbol{\lambda}^1_{\text{remote},j}$ denote the spatial distribution of skilled workers, some residing in peripheral regions. It can be expressed as $\boldsymbol{\lambda}^1_{\text{remote},j} = ((\lambda^1_{ij})_{i,j}, \boldsymbol{\lambda}^2_{\text{FA}})$ with

$$\lambda^1_{ij} = \begin{cases} 1-2\lambda & ((i,j)=(0,0)), \\ \lambda & ((i,j)\in\{(-j,0),(j,0)\}), \\ 0 & ((i,j)\notin\{(0,0),(-j,0),(j,0)\}), \end{cases}$$

and $0<\lambda<1/2$.

We focus on only the indirect utility of non-remote workers residing in the central region because all remote workers have the same utility levels under the equilibrium condition (25a). The shift in the utility level of skilled workers in the central region can be expressed as the following line integral:

$$V^1_{00}(\boldsymbol{\lambda}^1_{\text{remote}},\tau) - V^1_{00}(\boldsymbol{\lambda}_{\text{FA}},\tau) = \int_{\mathcal{C}} \frac{\partial V^1_{00}}{\partial \lambda^1_{00}} \mathrm{d}\lambda^1_{00} + \frac{\partial V^1_{00}}{\partial \lambda^1_{j0}} \mathrm{d}\lambda^1_{j0} + \frac{\partial V^1_{00}}{\partial \lambda^1_{-j0}} \mathrm{d}\lambda^1_{-j0}, \tag{47}$$

where $\mathcal{C} = \{\boldsymbol{\lambda}_{\text{FA}} + \varphi(\boldsymbol{\lambda}^1_{\text{remote},j} - \boldsymbol{\lambda}_{\text{FA}}) \mid 0 \le \varphi \le 1\}$ is the line segment whose endpoints are the full agglomeration $\boldsymbol{\lambda}_{\text{FA}}$ and the spatial distribution $\boldsymbol{\lambda}^1_{\text{remote},j}$. Substituting the distribution $\boldsymbol{\lambda}^1_{\text{remote},j}$ into the wage yields $w_{00}(\boldsymbol{\lambda}^1_{\text{remote},j},\tau) = (\alpha_1/\sigma_1)((2\hat{k}+1)L+2)$. This equation implies $w_{00}(\boldsymbol{\lambda}^1_{\text{remote},j},\tau) = w^1_{00}(\boldsymbol{\lambda}_{\text{FA}},\tau)$ by virtue of Eq. (45). Thus, the line integral corresponding to the change in the wage is zero:

$$\int_{\mathcal{C}} \frac{\partial w_{00}}{\partial \lambda^1_{00}} \mathrm{d}\lambda^1_{00} + \frac{\partial w_{00}}{\partial \lambda^1_{j0}} \mathrm{d}\lambda^1_{j0} + \frac{\partial w_{00}}{\partial \lambda^1_{-j0}} \mathrm{d}\lambda^1_{-j0} = w_{00}(\boldsymbol{\lambda}^1_{\text{remote},j},\tau) - w_{00}(\boldsymbol{\lambda}_{\text{FA}},\tau) = 0. \tag{48}$$

Using the above equation and Eq. (23b) yield:

$$\begin{aligned} &\int_{\mathcal{C}} \frac{\partial V^1_{00}}{\partial \lambda^1_{00}} \mathrm{d}\lambda^1_{00} + \frac{\partial V^1_{00}}{\partial \lambda^1_{j0}} \mathrm{d}\lambda^1_{j0} + \frac{\partial V^1_{00}}{\partial \lambda^1_{-j0}} \mathrm{d}\lambda^1_{-j0} = \\ &\quad \int_0^1 \left(\frac{\alpha_1}{N^1_0(\sigma_1-1)}\right) \underbrace{\left(\frac{\mathrm{d}\lambda_{00}}{\mathrm{d}\varphi} + \frac{\mathrm{d}\lambda_{j0}}{\mathrm{d}\varphi} + \frac{\mathrm{d}\lambda_{-j0}}{\mathrm{d}\varphi}\right)}_{=0} \mathrm{d}\varphi + \int_0^1 \underbrace{\left(-\frac{\beta}{\Lambda_0}\frac{\mathrm{d}\lambda_{00}}{\mathrm{d}\varphi}\right)}_{>0} \mathrm{d}\varphi > 0. \end{aligned} \tag{49}$$

The above equation implies that the utility level of the remote workers increases due to the introduction of remote work: $V^1_{00}(\boldsymbol{\lambda}^1_{\text{remote}},\tau) > V^1_{00}(\boldsymbol{\lambda}_{\text{FA}},\tau)$. Thus, the utility level of all skilled workers in the M^1 sector increases by the introduction of remote work.

B.2.2. Spatial distribution of skilled workers, some of whom reside in the central region

We examine the shifts in skilled workers' utilities due to the change from the full agglomeration to the spatial distribution of skilled workers, some residing in the central region. These shifts are numerically examined, as shown by path B′D in Figure 7(b). Let $\boldsymbol{\lambda}^2_{\text{remote},j}$ denote the population distribution of skilled workers. It can be expressed as $\boldsymbol{\lambda}^2_{\text{remote},j} = ((\lambda^1_{ij})_{i,j}, \boldsymbol{\lambda}^2_{\text{FA}})$ with

$$\lambda^1_{ij} = \begin{cases} 1-2\lambda & ((i,j)=(0,0)), \\ \lambda & ((i,j)\in\{(0,-j),(0,j)\}), \\ 0 & ((i,j)\notin\{(0,0),(0,-j),(0,j)\}), \end{cases}$$

and $0 < \lambda < 1/2$.

We focus on the following line integral:

$$V^1_{00}(\boldsymbol{\lambda}^2_{\text{remote},j}, \tau) - V^1_{00}(\boldsymbol{\lambda}_{\text{FA}}, \tau) = \int_{\mathcal{C}} \frac{\partial V^1_{00}}{\partial \lambda^1_{00}} \mathrm{d}\lambda^1_{00} + \frac{\partial V^1_{00}}{\partial \lambda^1_{j0}} \mathrm{d}\lambda^1_{j0} + \frac{\partial V^1_{00}}{\partial \lambda^1_{-j0}} \mathrm{d}\lambda^1_{-j0}, \tag{50}$$

where $\mathcal{C} = \{\boldsymbol{\lambda}_{\text{FA}} + \varphi(\boldsymbol{\lambda}^2_{\text{remote},j} - \boldsymbol{\lambda}_{\text{FA}}) \mid 0 \le \varphi \le 1\}$ is the line segment whose endpoints are full agglomeration $\boldsymbol{\lambda}_{\text{FA}}$ and the population distribution with remote workers $\boldsymbol{\lambda}^2_{\text{remote},j}$. As explained in Section 5.2, in the case where the wages of remote workers remain unchanged from those under the full agglomeration (i.e., $w_{00}(\boldsymbol{\lambda}^2_{\text{remote},j}, \tau) = w^1_{00}(\boldsymbol{\lambda}_{\text{FA}}, \tau))$, we can analytically evaluate the changes in their utilities $V^1_{00}(\boldsymbol{\lambda}^2_{\text{remote},j}, \tau) - V^1_{00}(\boldsymbol{\lambda}_{\text{FA}}, \tau)$:[5]

$$\begin{aligned}
\int_{\mathcal{C}} \frac{\partial V^1_{00}}{\partial \lambda^1_{00}} \mathrm{d}\lambda^1_{00} + \frac{\partial V^1_{00}}{\partial \lambda^1_{j0}} \mathrm{d}\lambda^1_{j0} + \frac{\partial V^1_{00}}{\partial \lambda^1_{-j0}} \mathrm{d}\lambda^1_{-j0} = \\
\int_0^1 \frac{\alpha_1}{N^1_0(\sigma_1 - 1)} \left(d^1_{00} \frac{\mathrm{d}\lambda_{00}}{\mathrm{d}\varphi} + d^1_{j0} \frac{\mathrm{d}\lambda_{j0}}{\mathrm{d}\varphi} + d^1_{-j,0} \frac{\mathrm{d}\lambda_{-j0}}{\mathrm{d}\varphi} \right) \mathrm{d}\varphi \\
+ \int_0^1 \left(-\frac{\beta}{\Lambda_0} \right) \underbrace{\left(\frac{\mathrm{d}\lambda_{00}}{\mathrm{d}\varphi} + \frac{\mathrm{d}\lambda_{j0}}{\mathrm{d}\varphi} + \frac{\mathrm{d}\lambda_{-j0}}{\mathrm{d}\varphi} \right)}_{=0} \mathrm{d}\varphi. \\
+ \underbrace{\left(w_{00}(\boldsymbol{\lambda}^2_{\text{remote},j}, \tau) - w^1_{00}(\boldsymbol{\lambda}_{\text{FA}}, \tau) \right)}_{=0}
\end{aligned} \tag{52}$$

The integrand in the first term on the RHS is negative by virtue of Eq. (30):

$$d^1_{00} \frac{\mathrm{d}\lambda_{00}}{\mathrm{d}\varphi} + d^1_{j0} \frac{\mathrm{d}\lambda_{j0}}{\mathrm{d}\varphi} + d^1_{-j,0} \frac{\mathrm{d}\lambda_{-j0}}{\mathrm{d}\varphi} = -2d^1_{00} + d^1_{j0} + d^1_{-j,0} < -2d^1_{00} + d^1_{00} + d^1_{00} = 0. \tag{53}$$

Substituting Eqs. (52) and (53) into Eq. (50) yields $V^1_{00}(\boldsymbol{\lambda}^2_{\text{remote}}, \tau) < V^1_{00}(\boldsymbol{\lambda}_{\text{FA}}, \tau)$, implying that the utility level of all skilled workers in the M^1 sector decreases by the introduction of remote work.

[5] For the bifurcation solutions expressed as $\boldsymbol{\lambda}^2_{\text{remote}}$, the wages of skilled workers can remain unchanged from those under the full agglomeration. To examine this, we focus on the following governing equation.

$$G^1_{ij}(\boldsymbol{\lambda}^1, \boldsymbol{\lambda}^2, \tau) = \lambda^1_{ij}(w^1_{ij}(\boldsymbol{\lambda}^1, \boldsymbol{\lambda}^2, \tau) - w^1_{00}(\boldsymbol{\lambda}_{\text{FA}}, \tau)) = 0, \tag{51a}$$

$$G^2_{i}(\boldsymbol{\lambda}^1, \boldsymbol{\lambda}^2, \tau) = \lambda^2_{i}(w^2_{i}(\boldsymbol{\lambda}^1, \boldsymbol{\lambda}^2, \tau) - w^2_{0}(\boldsymbol{\lambda}_{\text{FA}}, \tau)) = 0. \tag{51b}$$

According to Eq. (45), $w^1_{00}(\boldsymbol{\lambda}_{\text{FA}}, \tau)$ is constant with respect to the transportation cost parameter τ. As shown in Eq. (23b), the solution $\boldsymbol{\lambda}^2_{\text{remote}}$ of the above equations is also a solution of the governing equation (27). This implies that a sufficient condition for $\boldsymbol{\lambda}^2_{\text{remote},j}$ to be a solution of the governing equation (27) is that it is a solution of the equation (51). For this solution, $w_{00}(\boldsymbol{\lambda}^2_{\text{remote},j}, \tau) = w^1_{00}(\boldsymbol{\lambda}_{\text{FA}}, \tau)$ holds.

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