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Abstract

An existing study examines an international mixed duopoly involving a state-owned public firm and a foreign private firm, focusing on their timing choices for quantities and showing that the state-owned public firm should act as the leader. This result differs from that for an endogenous-timing mixed duopoly model where a state-owned public firm coexists with a domestic private firm. We investigate the endogenous order of moves in a mixed duopoly model where a state-owned public firm competes with a private firm that is partially foreign-owned. Specifically, we explore the desirable role of the state-owned public firm, either as a leader or a follower, and present the equilibrium outcome of the model. Our findings reveal that the equilibrium differs depending on whether the foreign ownership ratio of the private firm is low or high.

Keywords: Endogenous timing, mixed oligopoly, partial foreign ownership, Stackelberg

JEL classification numbers: C72, D21, F23, L13, L32

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1. Introduction

In a seminal paper, Hamilton and Slutsky (1990) examine the novel issue of endogenous timing in two-player games and provide important modelling implications for several models in industrial economics. In a pre-play stage, players decide whether to choose actions in the basic game at the first opportunity or to wait until observing their rivals' first-period actions. In one extended game, players first decide when to choose actions without committing to actions in the basic game. The equilibrium has a simultaneous play subgame unless the payoffs of a sequential play subgame achieve Pareto dominance over those of a simultaneous play subgame. In another extended game, opting to act at the first turn requires committing to an action. It is then shown that both sequential play outcomes are equilibria only in undominated strategies. Pal (1998) addresses the issue of endogenous order of moves in a mixed market by adopting the observable delay game of Hamilton and Slutsky (1990) in the context of a Cournot mixed oligopoly market where state-owned and capitalist firms first decide when to choose their quantities, and demonstrates that the results differ strikingly from those in a corresponding Cournot oligopoly market with all profit-maximising capitalist firms. Matsumura (2003) and Lu (2007) investigate endogenous timing in mixed oligopoly models where state-owned public and foreign private firms compete with each other, and show that in the equilibrium outcome of mixed duopoly competition consisting of a single foreign private firm, the state-owned public firm becomes the leader. In addition, Amir and De Feo (2014) analyse the endogenous timing in a Cournot mixed duopoly model, where a state-owned public firm competes with a private firm, either domestic or foreign. It is shown that simultaneous play never emerges as a subgame-perfect Nash equilibrium of the endogenous timing duopoly game. It is also shown that the most plausible outcome is public leadership when the private firm is foreign, and public or private leadership when the private firm is domestic.

We examine a Cournot mixed duopoly market where a private firm is partially foreign-owned. Fjell and Pal (1996) consider a mixed oligopoly model in which a state-owned public firm competes with both domestic and foreign private firms and examine the effects of (partial) ownership of domestic private firms by foreign nationals. It is then shown that foreign acquisition of domestic private firms increases consumer surplus but decreases profit. Fernández-Ruiz (2009) examines firms' decisions to hire managers in a mixed duopoly where a state-owned public firm competes with a foreign private firm. He considers a setting in which a part of the foreign private firm's profits can be included in the domestic welfare function, and demonstrates that the result is in contrast with that of the case where the state-owned public firm competes with the full domestic private firm. Ohnishi (2012) examines the welfare effects of domestic and foreign ownership of the private firm in a quantity-setting mixed duopoly model, and shows that economic welfare is maximised by full domestic ownership of the private firm while consumer surplus is maximised by full foreign ownership of the private firm. It is also shown that neither consumer surplus nor economic welfare is maximised by partial foreign ownership of the private firm. In addition, Ohnishi (2021) investigates the welfare effects of domestic production subsidies in a mixed Cournot duopoly model where a state-owned public firm competes with a partially foreign-owned private firm. The following four games are considered: unsubsidised mixed duopoly, subsidised mixed duopoly, unsubsidised private duopoly, and subsidised private duopoly. As a result, it is shown that when the subsidised mixed duopoly game and the subsidised private duopoly game are compared, optimally chosen subsidies decrease the partially foreign-owned private firm's output and increase the state-owned public firm's output and domestic economic welfare.

In this paper, we investigate endogenous timing in a mixed Cournot duopoly model where a state-owned public firm competes with a private firm that is partially foreign-owned. The sequence of events is as follows. In stage 1, each firm independently and simultaneously selects either 'stage 2' or 'stage 3'. In this context, stage 2 denotes that a firm produces in stage 2, whereas stage 3 signifies that it produces in stage 3. If a firm opts for stage 2, its output is determined in this stage. If a firm chooses stage 3, it decides on its output at this stage. At the conclusion of the game, the market opens, and each firm sells its output. To the best of the author's knowledge, this economic scenario has not been previously addressed in the literature. We examine a desirable role (either leader or follower) of the state-owned public firm and present the equilibrium outcome of the model.

The remainder of this paper proceeds as follows. In Section 2, we explain the model. Section 3 examines three games of fixed timing. Section 4 presents the equilibrium of the model introduced in Section 2. Finally, a conclusion is stated in Section 5.

2. Model

Let us consider an industry comprising one state-owned public firm (firm S) and one partially foreign-owned private firm (firm P). Both firms produce imperfectly substitutable goods. Throughout this paper, subscripts S and P represent firm S and firm P, respectively. In addition, when i and j are used to represent firms in an expression, they should be understood to refer to S and P with $i \neq j$. We do not consider the possibility of entry or exit.

The inverse demand function is given by $p(Q)$, where p is the market price, and Q is the total quantity of output in the market. We assume that $p'' \leq 0$. Therefore, firm i's profit is given by

$$\pi_i = p(Q)q_i - c_i q_i \qquad (i = \mathrm{S}, \mathrm{P}), \tag{1}$$

where q_i represents firm i's output, and c_{P} is firm i's constant marginal cost. We assume that firm S is less efficient than firm P, i.e. $0 < c_{\mathrm{P}} < c_{\mathrm{S}}$.[1] For simplicity, we normalise c_{P} to zero. Firm P aims to maximise its own profit given by (1).

Domestic economic welfare W is given by

$$W = \int_0^Q p(x)dx - p(Q)Q + \pi_{\mathrm{S}} + \lambda \pi_{\mathrm{P}}, \tag{2}$$

where $\lambda \in [0,1]$ denotes the level of domestic ownership. If $\lambda = 1$, firm P is fully domestically owned. On the other hand, if $\lambda = 0$, then firm P is foreign-owned and its profit is excluded from domestic economic welfare. Firm S aims to maximise domestic economic welfare.

The timing of the game is as follows. In the first stage, each firm i independently and simultaneously selects $t_i \in (2,3)$, where t_i denotes the timing of production for the non-negative output q_i. Choosing $t_i = 2$ indicates that firm i produces in the second stage, while choosing $t_i = 3$ signifies production in the third stage. At the end of the first

[1] This assumption is justified by Gunderson (1974) and Nett (1993, 1994) and is often used in the literature studying mixed markets (see, for instance, Fjell and Pal, 1996; Pal, 1998; Lu, 2006, 2007; Fernández-Ruiz, 2009; Ohnishi, 2012). Let us assume that firm S is at least as efficient as firm P. In this case, since firm S, which prioritises domestic economic welfare, has a greater incentive to underbid an opponent's price than firm P, firm S produces where price equals marginal cost. This assumption is introduced to rule out a trivial solution.

stage, each firm observes t_S and t_P. In the second stage, any firm opting for $t_i = 2$ determines its output q_i. At the end of this stage, firm i is able to observe firm j's output, provided that firm j has chosen to produce in the second stage. In the third stage, any firm selecting $t_i = 3$ determines its output q_i. Finally, at the end of the game, the market opens, and each firm i sells its output q_i. In this paper, we derive the subgame perfect Nash equilibria of the model.

3. Fixed timing games

In this section, following Matsumura (2003), we consider the following three games: firm S is the leader, firm P is the leader, and both firms act as Cournot duopolists.

First, we consider the game where firm S is the leader. Firm S chooses q_S and firm P chooses q_P after observing q_S. The first order condition for firm P is

$$p + p'q_\mathrm{P} = 0\,, \tag{3}$$

and the second order condition is

$$2p' + p''q_\mathrm{P} < 0\,. \tag{4}$$

Therefore, the reaction function for firm P under Cournot competition is

$$R'_\mathrm{P} = -\frac{p' + p''q_\mathrm{P}}{2p' + p''q_\mathrm{P}} < 0\,. \tag{5}$$

Firm S chooses q_S so as to maximise $W(q_\mathrm{S}, R_\mathrm{P}(q_\mathrm{S}))$. The first order condition is

$$p - c_\mathrm{S} - p'q_\mathrm{P} + \lambda p'q_\mathrm{P} - p'q_\mathrm{P}R'_\mathrm{P} = 0\,. \tag{6}$$

We assume that $W(q_\mathrm{S}, R_\mathrm{P}(q_\mathrm{S}))$ is concave with respect to q_S. We define $W^L \equiv W(q_\mathrm{S}^L, q_\mathrm{P}^L)$, where q_i^L denotes the equilibrium output of firm i in the game when firm S is the leader.

Second, we consider the game where firm S is the follower. Firm P chooses q_P and firm S choose q_S after observing q_P. The first order condition for firm S is

$$p - c_\mathrm{S} - p'q_\mathrm{P} + \lambda p'q_\mathrm{P} = 0\,, \tag{7}$$

and the second order condition is

$$p'' - p''q_\mathrm{P} + \lambda p''q_\mathrm{P} < 0\,. \tag{8}$$

Firm P chooses q_P so as to maximise $\pi_\mathrm{P}(R_\mathrm{S}(q_\mathrm{P}), q_\mathrm{P})$, and the first order condition is

$$p + p'q_\mathrm{P} + p'q_\mathrm{P}R'_\mathrm{S} = 0\,. \tag{9}$$

We assume that $\pi_\mathrm{P}(R_\mathrm{S}(q_\mathrm{P}), q_\mathrm{P})$ is concave with respect to q_P. We define

$W^F \equiv W(q_S^F, q_P^F)$, where q_i^F denotes the equilibrium output of firm i in the game when firm S is the follower.

Third, we consider the Cournot game where each firm i independently and simultaneously chooses q_i. We define $W^C \equiv W(q_S^C, q_P^C)$, where q_i^C denotes the equilibrium output of firm i in this game.

Now we can state the following proposition.

Proposition 1:
(i) $W^L, W^F > W^C$ if $\lambda(p' + p''q_P) < p''q_P$;
(ii) $W^L > W^C \geq W^F$ if $\lambda(p' + p''q_P) \geq p''q_P$.

Proof: (i) First, we prove that $W^L > W^C$. Since the leader (firm S) maximises domestic economic welfare and it can chooses $q_S = q_S^C$, we obtain $W^L \geq W^C$. We now prove that $W^L \neq W^C$ by showing that $q_S^L \neq q_S^C$. When firm S is the leader, it maximizes $W(q_S, R_P(q_S))$ with respect to q_S. The first condition is (6), where $p', R_P' < 0$, so that $-p'q_P R_P' < 0$. If we consider (7), then $p - c_S - p'q_P + \lambda p' q_P$ must be positive to satisfy (6). Hence, $q_S^L < q_S^C$.

Next, we show that $W^F > W^C$. We consider the game where firm S is the follower. From (7), we obtain

$$R_S' = -\frac{\lambda p' + \lambda p'' q_P - p'' q_P}{p'' - p'' q_P + \lambda p'' q_P}, \tag{10}$$

where $R_S' < 0$ if and only if $\lambda(p' + p''q_P) < p''q_P$. Differentiating $\pi_P(R_S(q_P), q_P)$ at q_P, we obtain (9), where $p'q_P R_S' > 0$. From the concavity of firm P's profit, we obtain $q_P^F > q_P^C$. From (1), We obtain $dW(R_S(q_P), q_P)/dq_P = -p'q_P(1-\lambda) + \lambda p > 0$. Since $W(q_S, R_P(q_S))$ is increasing in q_P, $W^F > W^C$ is proved.
(ii) This proof is essentially the same as that of Proposition 1 (i) and thus is omitted. QED

Proposition 1 (i) indicates that if the ratio of foreign capital in firm P is low, then domestic economic welfare will increase more than in the Cournot game, regardless of whether firm S acts as the leader or the follower. Proposition 1 (ii) states that if firm P has a higher share of foreign capital and firm S acts as the follower, domestic economic welfare is lower than in the Cournot game.

We consider the following lemma.

Lemma 1:

(i) $q_P^F \geq q_P^C$ if $\lambda\left(p' + p''q_P\right) \leq p''q_P$;

(ii) $q_P^F \leq q_P^C$ if $\lambda\left(p' + p''q_P\right) \geq p''q_P$;

(iii) $q_P^L > q_P^C$.

Proof: (i) If firm P is a Stackelberg leader, it maximises $\pi_P(R_S(q_P), q_P)$ with respect to q_P. The first order condition for Stackelberg profit maximisation is (9), where if $\lambda\left(p' + p''q_P\right) < p''q_P$, then $R_S' < 0$, so that $p'q_P R_S' > 0$. If we consider (3), then $p + p'q_P$ must be negative to satisfy (9).

In addition, from (10), if $\lambda\left(p' + p''q_P\right) = p''q_P$, then $R_S' = 0$. Hence, $q_P^F = q_P^C$.

(ii) This proof is similar to that of Lemma 1 (i) and thus is omitted.

(iii) We consider the game where firm S is the leader. When firm S is a Stackelberg leader, it maximizes $W(q_S, R_P(q_S))$ with respect to q_S. The first order condition is (6), where $p', R_P' < 0$, so that $-p'q_P R_P' < 0$. If we consider (7), then $p - c_S - p'q_P + \lambda p'q_P$ must be positive to satisfy (6). Hence, $q_S^L < q_S^C$. Since $R_P' < 0$, if $q_S^L < q_S^C$, then $q_P^L > q_P^C$. QED

Lemma 1 (ii) indicates that if firm P has a high proportion of foreign capital, even when it can act as the Stackelberg leader, its production is reduced below its Cournot output.

At the end of this section, we consider the profit of firm P. In the following of this paper, (q_S^F, q_P^F) denotes the outcome when firm S is the Stackelberg follower, (q_S^L, q_P^L) is the outcome when firm S is the Stackelberg leader, and (q_S^C, q_P^C) is the outcome of the Cournot-Nash game. We present the following proposition.

Proposition 2:

(i) $\pi_P(q_S^F, q_P^F) \geq \pi_P(q_S^C, q_P^C)$;

(ii) $\pi_P(q_S^L, q_P^L) > \pi_P(q_S^C, q_P^C)$.

Proof: (i) This follows from Lemma 1 (i) and (ii).

(ii) $\partial\pi_P(q_S, q_P)/\partial q_S = p'q_P < 0$. In the proof of Proposition 1, we show that $q_S^L < q_S^C$. Thus, Lemma 1 (ii) is derived. QED

This proposition is the same as Lemma 1 (i) and (ii) in Matsumura (2003). In other words, it can be stated that both a partially foreign private firm and a foreign private firm have the same result with respect to profit.

4. Equilibrium

In this section, we present the equilibrium of the model introduced in Section 2. The equilibrium of this study is presented in the following proposition.

Proposition 3: In the equilibrium of the observable delay game:
(i) $(t_{\mathrm{S}}, t_{\mathrm{P}}) = (2,3)$ and $(t_{\mathrm{S}}, t_{\mathrm{P}}) = (3,2)$ if $\lambda\left(p' + p'' q_{\mathrm{P}}\right) < p'' q_{\mathrm{P}}$;
(ii) $(t_{\mathrm{S}}, t_{\mathrm{P}}) = (2,3)$ if $\lambda\left(p' + p'' q_{\mathrm{P}}\right) \geq p'' q_{\mathrm{P}}$.

Proof: (i) In the first stage, each firm i independently and simultaneously chooses $t_i \in (2,3)$. At the end of the first stage, each firm observes t_{S} and t_{P}. In the second stage, firm i choosing $t_i = 2$ selects its output q_i. At the end of the second stage, each firm observes the output of the rival if the rival chooses to produce in the second stage, In the third stage, firm i choosing $t_i = 3$ selects its output q_i. At the end of the game, the market opens and domestic economic welfare and firm P's profit are decided. Hence, we can consider the following table.

		Firm P	
		Stage 2	Stage 3
Firm S	Stage 2	W^C, π_{P}^C	W^L, π_{P}^L
	Stage 3	W^F, π_{P}^F	W^C, π_{P}^C

Table 1: Two-player Game with Two Action Sets (Stage 2 and Stage 3).

In $\lambda\left(p' + p'' q_{\mathrm{P}}\right) < p'' q_{\mathrm{P}}$, from Proposition 1 (i) and Proposition 2, $W^F > W^C$, $W^L > W^C$, $\pi_{\mathrm{P}}^F \geq \pi_{\mathrm{P}}^C$, and $\pi_{\mathrm{P}}^L > \pi_{\mathrm{P}}^C$. Thus, the strategy profiles (Stage 2, Stage 3) and (Stage 3, Stage 2) are both equilibria.
(ii) We also consider Table 1. Here, from Proposition 1 (ii) and Proposition 2, $W^C \geq W^F$, $W^L > W^C$, $\pi_{\mathrm{P}}^F \geq \pi_{\mathrm{P}}^C$, and $\pi_{\mathrm{P}}^L > \pi_{\mathrm{P}}^C$. Thus, the equilibrium occurs at 'Stage 2' for firm S and 'Stage 3' for firm P. QED

If $\lambda\left(p' + p''q_{\mathrm{P}}\right) < p''q_{\mathrm{P}}$, there exist two equilibrium solutions: firm S becomes the leader and firm P becomes the leader. Otherwise, there is only an equilibrium in which firm S acts as the leader.

5. Conclusion

We have investigated the endogenous order of moves in a mixed duopoly model, where a state-owned public firm competes with a partially foreign-owned private firm. From this analysis, we have shown the following. The state-owned public firm always assumes the role of leader in output when coexisting with a private firm with high foreign ownership. However, when the private firm's foreign ownership ratio is low, the state-owned public firm may either take the role of leader or follow as the subordinate.

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