Fertility Decisions under Coexisting Pay-as-You-Go Pensions and Unemployment Insurance

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Abstruct

This study investigates the effects of increased tax rates on fertility decisions, pension benefits, and unemployment benefits in an economy with both a pay-as-you-go (PAYG) pension system and unemployment insurance. By incorporating voluntary unemployment, the model highlights how higher tax rates reduce households' willingness to work, thereby affecting the social security system through the production channel. The analysis also reveals differing impacts of the two types of labor income taxes.

Keywords— OLG, fertility, PAYG, pension, unemployment.

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1 Introduction

In many developed countries, pay-as-you-go (PAYG) pension systems and unemployment insurance are operated simultaneously. While the PAYG system redistributes income across generations, unemployment insurance is an intergenerational redistribution mechanism. But both systems affect each other through the labor market. For instance, if stronger unemployment insurance is provided, more people may leave their jobs and choose to live by accepting coverage. It may lead to a decrease in labor supply and decrease the pension benefits that are provided for old agents.

In this study, we use an overlapping generations (OLG) model originally developed by Diamond (1965), which is extended to include endogenous fertility. This study analyzes the interaction between a pay-as-you-go pension system and unemployment insurance by endogenizing the household's choice of time allocated to employment and unemployment. This study describes the aspect of reduced willingness to work following a tax rate increase by endogenizing the labor decision by households. This allows for a more comprehensive analysis of the impact of a tax rate increase on the social insurance system, which also takes into account the reduction in the labor force.

Our analysis shows that, under certain conditions, an increase in the unemployment insurance tax rate can raise both the steady-state capital stock per capita and unemployment benefits. Conversely, an increase in the pension tax rate may reduce both the steady-state capital stock per capita and unemployment benefits. Furthermore, we find that the increase in tax rates leads to a decline in the steady-state fertility rate and pension benefits.

Several studies have analyzed household fertility decisions in the context of pension systems. [1] Miyazaki (2013) indicates that the expansion of social security may increase the fertility rate in a neoclassical growth model with endogenous fertility, where child-rearing costs are captured through both time and monetary expenses. Wigger (1999) shows that a small-scale public pension system can stimulate per capita income growth. Zhang and Zhang (1998) examine the effects of a pension system on fertility and savings in a model that incorporates transfers from children to parents. Zhang (1995) investigates how a pension system affects economic growth through its influence on fertility and savings. [2]

Most of the previous studies that have identified the relationship between unemployment and pensions have focused on analyzing involuntary unemployment, modeling wage bargaining by labor unions. Corneo and Marquardt (2000) show that the unemployment rate does not depend on the contribution rate of the pension system and is not correlated with capital accumulation. Ono (2010) extends the model in Corneo and Marquardt (2000) to be more consistent with empirical studies and shows that a higher contribution rate makes the unemployment rate lower. Demmel and Keuschnigg (2000) analyze how the pension reform from unfunded to funded affects the efficiency gains through unemployment. Bräuninger (2005) uses an overlapping generations model in which a pension system and unemployment insurance exist, and analyzes the impact of each on economic growth.

The remainder of this paper is organized as follows. Chapter 2 presents the model and derives the steady-state values of variables. Chapter 3 examines how changes in tax rates affect capital per capita, fertility, unemployment benefits, and pension benefits. Chapter 4 investigates whether the conditions derived in Chapter 3 evaluate the plausibility of the conditions derived from the model using numerical simulations. Chapter 5 concludes.

2 The model

2.1 Demographics

The generation born in period t is denoted by N_t. Each individual has n_t children during their youth. Thus, the following equation holds:

$$N_{t+1} = n_t N_t \tag{1}$$

2.2 Households

The individual lives for two periods: young and old.

[1] Fanti and Gori (2012) analyze the impact of changes in fertility on the pension system by introducing child care costs into the model.

[2] Other studies that have examined fertility decisions include Boldrin and Jones (2002) and Mizuno and Yakita (2013).

In the young period, they choose their level of consumption c_t^y, savings s_t, and the number of children n_t. Individuals are required to pay a fraction $\delta \in [0,1)$ of the wage to raise their children. Accordingly, the total cost of raising n_t children is $\delta w_t n_t$. In this study, we add voluntary unemployment. Thus, they choose how much time to allocate to work l_t, and to unemployment $1-l_t$. If they work, they earn a wage w_t. However, the pension tax rate τ_p and the unemployment insurance tax rate τ_u are imposed on their wage income. If they are unemployed, they receive unemployment insurance benefits d_t. Nevertheless, even if they do not work, they are still required to pay the pension tax.

In the old period, individuals receive the interest income $(1+r_{t+1})s_t$ and the pension benefits P_{t+1}, both of which are used for consumption c_{t+1}^o. Here, r_{t+1} denotes the net interest rate.

Therefore, the budget constraints during the young and old periods are as follows.

$$c_t^y + s_t + \delta w_t n_t = (1-\tau_p-\tau_u)l_t w_t + (1-\tau_p)(1-l_t)d_t \tag{2}$$

$$c_{t+1}^o = (1+r_{t+1})s_t + P_{t+1} \tag{3}$$

From equations (2) and (3), we obtain the lifetime budget constraint.

$$c_t^y + \frac{c_{t+1}^o}{1+r_{t+1}} + \delta w_t n_t = (1-\tau_p-\tau_u)l_t w_t + (1-\tau_p)(1-l_t)d_t + \frac{P_{t+1}}{1+r_{t+1}} \tag{4}$$

The individual maximizes the following utility function under the lifetime budget constraint.

$$U_t = \ln c_t + \beta \ln n_t + \gamma \ln c_{t+1}^o \tag{5}$$

where $\beta > 0$ is the altruism factor and $, \gamma \in (0,1]$ is the discount parameter.

2.3 Production

Assume that the production function is a Cobb-Douglas production function given by $F(K_t, L_t) = K_t^\alpha L_t^{1-\alpha}$, where $\alpha \in (0,1)$ represents the capital share. We assume that capital is fully depreciated at the end of each period and is not carried over to the next. In addition, output is sold at price normalized to one. Let $k_t = K_t/L_t$ and $f(k_t) = k^\alpha$. The firm's profit maximization yields the following first-order conditions.

$$1 + r_t = \alpha k_t^{\alpha-1} \tag{6}$$

$$w_t = (1-\alpha)k_t^\alpha \tag{7}$$

2.4 Government

The government operates two systems: a pension system and an unemployment insurance system. It is assumed that the budget for each system is balanced in each period.

The balanced-budget equation for unemployment insurance in period $t+1$ is given by:

$$(1-l_{t+1})d_{t+1}N_{t+1} = \tau_u l_{t+1} w_{t+1} N_{t+1}$$

From the above equation, the per capita amount of unemployment insurance benefits can be derived as:

$$d_{t+1} = \frac{l_{t+1}}{1-l_{t+1}} \tau_u w_{t+1} \tag{8}$$

Similarly, the balanced-budget equation for the pension system in period $t+1$ is:

$$P_{t+1}N_t = \tau_p \Big[l_{t+1} w_{t+1} + (1-l_{t+1})d_{t+1} \Big] N_{t+1}$$

Using equation (8), the per capita pension benefit can be derived as follows:

$$P_{t+1} = \tau_p(1+\tau_u) l_{t+1} w_{t+1} n_t \tag{9}$$

2.5 Utility maximization

Consider the household ' s utility maximization problem, in which the utility function (5) is maximized with respect to c_t^y, c_{t+1}^o, n_t, and l_t, subject to the lifetime budget constraint (4). First, the Lagrangian for this problem is formulated as follows:

$$\begin{aligned}\mathscr{L} &= \ln c_t^y + \beta \ln n_t + \gamma \ln c_{t+1}^o \\ &+ \lambda \left[(1-\tau_p-\tau_u) l_t w_t + (1-\tau_p)(1-l_t) d_t + \frac{P_{t+1}}{1+r_{t+1}} - c_t^y - \frac{c_{t+1}^o}{(1+r_{t+1})} - \delta w_t n_t \right]\end{aligned} \tag{10}$$

By differentiating the Lagrange function (10) for $c_t^y, n_t, c_{t+1}^o, l_t$, and λ respectively. The following first-order conditions for utility maximization can be derived.

$$\frac{\partial \mathscr{L}}{\partial c_t^y} = \frac{1}{c_t^y} - \lambda = 0 \tag{11}$$

$$\frac{\partial \mathscr{L}}{\partial n_t} = \frac{\beta}{n_t} - \lambda \delta w_t = 0 \tag{12}$$

$$\frac{\partial \mathscr{L}}{\partial c_{t+1}^o} = \frac{\gamma}{c_{t+1}^o} - \frac{\lambda}{r_{t+1}} = 0 \tag{13}$$

$$\frac{\partial \mathscr{L}}{\partial l_t} = \lambda \left[(1-\tau_p-\tau_u) w_t - (1-\tau_p) d_t \right] = 0 \tag{14}$$

From (11), (12) and (13), we obtain

$$c_t^y = \frac{1}{\beta} \delta w_t n_t \tag{15}$$

$$c_{t+1}^o = \frac{1}{\beta} \gamma \delta w_t (1+r_{t+1}) n_t. \tag{16}$$

From (14) and (8), we obtain the following equation.

$$l_t = \frac{1-\tau_p-\tau_u}{1-\tau_p-\tau_p\tau_u} < 1 \tag{17}$$

From lifetime budget constraint (4) and (15), (16), and (17), and production's first-order conditions (7) and (6), we obtain the solutions decided by the households.

$$c_t^y = \frac{\alpha\delta(1-\alpha)(1-\tau_p-\tau_u)(1-\tau_p-\tau_p\tau_u)k_t^{2\alpha}}{\alpha\delta(1+\beta+\gamma)(1-\tau_p-\tau_p\tau_u)k_t^\alpha - \beta(1+\tau_u)(1-\tau_p-\tau_u)\tau_p k_{t+1}} \tag{18}$$

$$n_t = \frac{\alpha\beta(1-\tau_p-\tau_u)(1-\tau_p-\tau_p\tau_u)k_t^{\alpha}}{\alpha\delta(1+\beta+\gamma)(1-\tau_p-\tau_p\tau_u)k_t^\alpha - \beta(1+\tau_u)(1-\tau_p-\tau_u)\tau_p k_{t+1}} \tag{19}$$

$$c_{t+1}^o = \frac{\alpha^2\gamma\delta(1-\alpha)(1-\tau_p-\tau_u)(1-\tau_p-\tau_p\tau_u)k_t^{2\alpha}k_{t+1}^{\alpha-1}}{\alpha\delta(1+\beta+\gamma)(1-\tau_p-\tau_p\tau_u)k_t^\alpha - \beta(1+\tau_u)(1-\tau_p-\tau_u)\tau_p k_{t+1}} \tag{20}$$

2.6 Equilibrium and steady state

In this Section, we consider the steady state. In equilibrium, the capital market and the labor market are cleared. Thus, the following conditions are held.

$$K_{t+1} = s_t N_t \tag{21}$$

$$L_t = l_t N_t \tag{22}$$

Transforming (21) with using (18), (19), (20), and (22), we obtain the capital accumulation equation.

$$k_{t+1} = \frac{\alpha\gamma\delta(1-\alpha)(1-\tau_p-\tau_p\tau_u)}{\beta(1-\tau_p-\tau_u)[\alpha+(1-\alpha)(1+\tau_u)\tau_p]} k_t^\alpha \tag{23}$$

In the steady state, $k_{t+1} = k_t = k^*$ holds. Thus, from (23), we obtain the steady state capital stock per capita.

$$k^* = \Big[\frac{\alpha\gamma\delta(1-\alpha)(1-\tau_p-\tau_p\tau_u)}{\beta(1-\tau_p-\tau_u)[\alpha+(1-\alpha)(1+\tau_u)\tau_p]}\Big]^{\frac{1}{1-\alpha}} \tag{24}$$

From (6) (7), (18), (19), and (20), we also obtain the consumption, fertility, and labor in the steady state.

$$c^{y*} = \frac{(1-\alpha)(1-\tau_p-\tau_u)[\alpha+(1-\alpha)(1+\tau_u)\tau_p]}{\alpha(1+\beta+\gamma)+(1-\alpha)(1+\beta)(1+\tau_u)\tau_p}k^{*\alpha} \tag{25}$$

$$n^* = \frac{\beta(1-\tau_p-\tau_u)[\alpha+(1-\alpha)(1+\tau_u)\tau_p]}{\alpha\delta(1+\beta+\gamma)+\delta(1-\alpha)(1+\beta)(1+\tau_u)\tau_p} \tag{26}$$

$$c^{o*} = \frac{\alpha\gamma(1-\alpha)(1-\tau_p-\tau_u)[\alpha+(1-\alpha)(1+\tau_u)\tau_p]}{\alpha(1+\beta+\gamma)+(1-\alpha)(1+\beta)(1+\tau_u)\tau_p}k^{*2\alpha-1} \tag{27}$$

$$l^* = \frac{1-\tau_p-\tau_u}{1-\tau_p-\tau_p\tau_u} \tag{28}$$

Similarly, we can calculate the pension benefit and the unemployment benefit in the steady state.

$$d^* = \frac{(1-\alpha)(1-\tau_p-\tau_u)}{1-\tau_p}k^{*\alpha} \tag{29}$$

$$P^* = \frac{\beta(1-\alpha)(1+\tau_u)(1-\tau_p-\tau_u)\tau_p[\alpha+(1-\alpha)(1+\tau_u)\tau_p]}{\alpha\delta(1-\tau_p-\tau_p\tau_u)[\alpha(1+\beta+\gamma)+(1-\alpha)(1+\beta)(1+\tau_u)\tau_p]}k^{*\alpha} \tag{30}$$

3 The effect of the increase tax rates

3.1 The increase of pension tax rate

In this section, we examine the impact of increasing tax rates.

First, by differentiating equation (24) with respect to τ_p, we obtain the following results:

$$\frac{\partial k^*}{\partial \tau_p} = -\frac{1}{1-\alpha}\frac{\alpha\gamma\delta(1-\alpha)\varepsilon}{\beta(1-\tau_p-\tau_u)^2[\alpha+(1-\alpha)(1+\tau_u)\tau_p]^2}k^{*\alpha} \tag{31}$$

where

$$\varepsilon = -\alpha\tau_u^2 + (1-\alpha)(1+\tau_u)(1-\tau_p)(1-\tau_p-\tau_u-\tau_u^2)$$

Here, $\varepsilon > 0$ holds if the following condition is satisfied.

$$\alpha < \frac{(1+\tau_u)(1-\tau_p)(1-\tau_p-\tau_u-\tau_u^2)}{\tau_u^2+(1+\tau_u)(1-\tau_p)(1-\tau_p-\tau_u-\tau_u^2)} \equiv \hat{\alpha}$$

Thus, the inequality below is satisfied.

$$\frac{\partial k^*}{\partial \tau_p} \lesseqgtr 0 \quad \text{if} \quad \alpha \lesseqgtr \hat{\alpha}$$

Therefore, increasing the pension tax rate lowers the capital stock per capita in the steady state.

Next, by differentiating (26) with respect to τ_p, we obtain the following results.

$$\frac{\partial n^*}{\partial \tau_p} = \frac{\beta\delta\eta}{\delta^2[\alpha(1+\beta+\gamma)+(1-\alpha)(1+\beta)(1+\tau_u)\tau_p]^2} \tag{32}$$

where

$$\begin{aligned}\eta =& -(1-\alpha)^2(1+\beta)(1+\tau_u)^2\tau_p^2 - 2\alpha(1-\alpha)(1+\beta+\gamma)(1+\tau_u)\tau_p \\ &+ \alpha\gamma(1-\alpha)(1+\tau_u)(1-\tau_u) - \alpha^2(1+\beta+\gamma)\end{aligned}$$

Thus, the inequality below is satisfied.

$$\frac{\partial n^*}{\partial \tau_p} < 0 \quad \text{if} \quad \eta < 0$$

Increasing the pension tax rate causes a lower fertility rate in the steady state.

Similarly, by differentiating (29) and (30), we obtain the following results.

$$\frac{\partial d^*}{\partial \tau_p} = (1-\alpha)\left(-\frac{(1-\alpha)\tau_u}{(1-\tau_p)^2} + \frac{\alpha(1-\tau_p-\tau_u)}{1-\tau_p}\frac{\partial k^*}{\partial \tau_p}k^{*-1}\right)k^{*\alpha} \tag{33}$$

$$\frac{\partial P^*}{\partial \tau_p} = (1-\alpha)\left(-\frac{(1+\tau_u)^2(1-\tau_p)^2}{(1-\tau_p-\tau_p\tau_u)^2}n^*k^{*\alpha} + \frac{\tau_p(1+\tau_u)(1-\tau_p-\tau_u)}{1-\tau_p-\tau_p\tau_u}\left(\frac{\partial n^*}{\partial \tau_p}k^{*\alpha} + \alpha\frac{\partial k^*}{\partial \tau_p}n^*k^{*\alpha-1}\right)\right) \tag{34}$$

From (33) and (34), we obtain the following results.

$$\frac{\partial d^*}{\partial \tau_p} < 0, \quad \text{if} \quad \alpha < \hat{\alpha}$$
$$\frac{\partial P^*}{\partial \tau_p} < 0 \quad \text{if} \quad \alpha < \hat{\alpha} \quad \text{and} \quad \eta < 0$$

Given the above results, under certain conditions, increasing the pension tax rate decreases both the unemployment insurance and pension benefits.

3.2 The increase of unemployment tax rate

In this section, we examine the impact of increasing tax rates.

First, by differentiating equation (24) with respect to τ_u, we obtain the following results:

$$\frac{\partial k^*}{\partial \tau_u} = \frac{1}{1-\alpha}\frac{\alpha\gamma\delta(1-\alpha)\zeta}{\beta(1-\tau_p-\tau_u)^2[\alpha+(1-\alpha)(1+\tau_u)\tau_p]^2}k^{*\alpha} > 0 \tag{35}$$

where

$$\zeta = \alpha(1-\tau_p)^2 + (1-\alpha)\tau_p^2\tau_u^2 + 2(1-\alpha)(1-\tau_p-\tau_p\tau_u)\tau_p\tau_u > 0$$

Next, by differentiating (26) with respect to τ_u, we obtain the following results.

$$\frac{\partial n^*}{\partial \tau_u} = -\frac{\beta\delta\theta}{\delta^2[\alpha(1+\beta+\gamma)+(1-\alpha)(1+\beta)(1+\tau_u)\tau_p]^2} < 0 \tag{36}$$

where

$$\theta = (1-\alpha)^2(1+\beta)(1+\tau_u)^2\tau_p^2 + \alpha(1-\alpha)[2(1+\beta)(1+\tau_u)+\gamma(\tau_p+2\tau_u)]\tau_p + \alpha^2(1+\beta+\gamma) > 0$$

From the above discussion, increasing the unemployment tax rate increases the capital stock per capita in the steady state and decreases the steady-state fertility rate.

Similarly, by differentiating (29) and (30), we obtain the following results.

$$\frac{\partial d^*}{\partial \tau_u} = -\frac{1-\alpha}{1-\tau_p}\mu k^{*\alpha} \tag{37}$$

where

$$\mu = 1 - \alpha(1-\tau_p-\tau_u)\frac{\partial k^*}{\partial \tau_u}k^{*-1}$$

$$\frac{\partial P^*}{\partial \tau_u} = \frac{1-\alpha}{1-\tau_p-\tau_p\tau_u}\left(-2\tau_u n^*k^{*\alpha} + \tau_p(1+\tau_u)(1-\tau_p-\tau_u)\nu\right) \tag{38}$$

where

$$\nu = \frac{\partial n^*}{\partial \tau_u}k^{*\alpha} + \alpha\frac{\partial k^*}{\partial \tau_u}n^*k^{*\alpha-1}$$

From (37) and (38), we obtain the following results.

$$\frac{\partial d^*}{\partial \tau_u} > 0 \quad \text{if} \quad \mu > 0$$
$$\frac{\partial P^*}{\partial \tau_u} < 0 \quad \text{if} \quad \nu < 0$$

Given the above results, under certain conditions, increasing the unemployment tax rate increases the unemployment insurance and decreases the pension benefits.

4 Numeric calculation

Next, we use numerical calculations to examine whether the results obtained from the model are realistic. First, the capital share α is set to $1/3$, which is lower than the setting of 0.4 in Cooley and Prescott (1995). ext, the altruism factor β is set to $\beta = 0.271$, which is given in De La Croix and Doepke (2003). And we set the discount factor as $\gamma = 0.37$. Next, we assume that $\delta = 0.25$. This is a setting given by Miyazaki (2013).

First, we check the value of ϵ. The result is indicated in Fig.1.

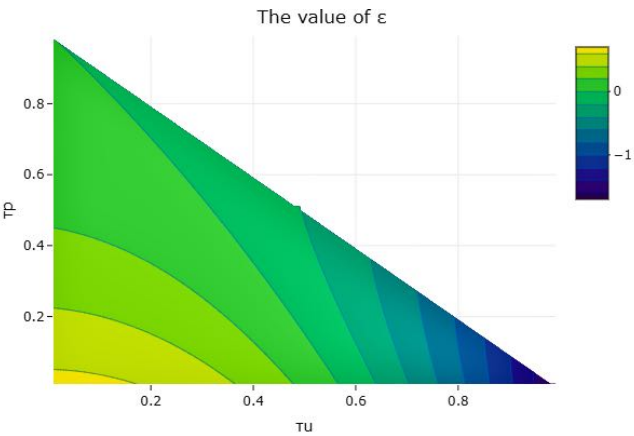

Figure 1: The value of ϵ

It can be seen that the value of ϵ is positive in the left-hand region and negative in the right-hand region in Figure 1. Thus, when the unemployment tax rate is low, the value of epsilon is positive.

Next, we show the value of η.

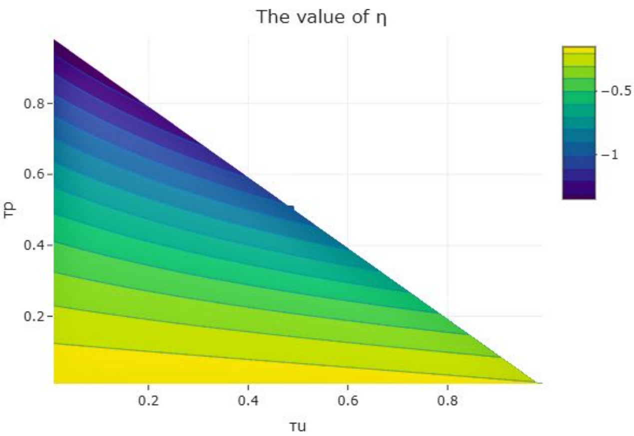

Figure 2: The value of η

In this numerical setting, the value of η is negative in all regions in Figure 2. It can be seen that the value of η decreases as the pension tax rate increases.

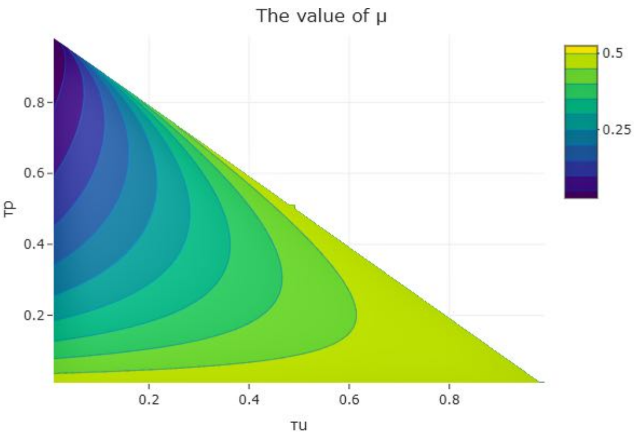

Figure 3: The value of μ

In Figure 3, the value of μ is indicated. Under this setting, μ is positive in all regions. This value increases if the unemployment tax rate increases.

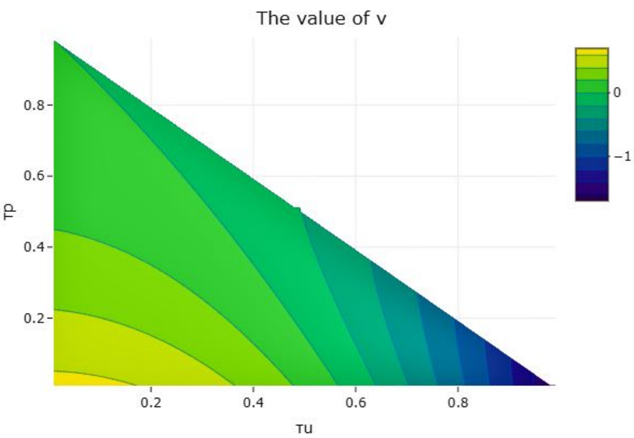

Figure 4: The value of ν

In Figure 4, the values of ν are shown. As in Figure 1, ν takes positive values in the left region and negative values in the right region. As the unemployment tax rate increases, the value of ν decreases.

5 Conclusion

This study analyzes the effects of higher tax rates on household fertility, unemployment benefits, and pension benefits within an overlapping generations (OLG) model that incorporates a pay-as-you-go (PAYG) pension system and voluntary unemployment decisions by households. The main findings of the study are as follows:

First, an increase in the pension tax rate reduces the steady-state capital stock per capita, whereas an increase in the unemployment insurance tax rate may lead to an increase in it under certain conditions. Second, both higher pension and unemployment tax rates lower the steady-state fertility rate. Third, an increase in the pension tax rate decreases steady-state unemployment benefits, while an increase in the unemployment tax rate may raise them. Fourth, both tax increases reduce steady-state pension benefits.

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