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Abstruct

This study analyzes a reformed pay-as-you-go pension system in which individual benefits increase proportionally with the number of children. While such a design appears to encourage fertility and address population aging, we use a simple overlapping generations model to show that the reform can reduce overall social welfare. Our results highlight an important trade-off between demographic goals and economic efficiency in pension design.

Keywords— OLG, fertility, pay-as-you-go, pension, population.

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1 Introduction

Many countries currently operate pay-as-you-go (PAYG) pension systems, under which benefits for the elderly are financed by contributions from the working-age population. However, as population size continues to decline, the financial burden on younger generations increases, making it increasingly difficult to sustain the system. Thus, the challenges of maintaining a PAYG pension scheme and addressing population decline are closely interrelated.

This study employs an overlapping generations (OLG) model with endogenous fertility, based on the framework developed by Diamond (1965). Since young and old cohorts coexist in this setting, the OLG model facilitates the analysis of intergenerational issues in PAYG pension systems. It also serves as a useful tool for examining household fertility behavior.

Many studies have investigated the relationship between PAYG pension systems and fertility. Notable contributions on pension systems include Fanti and Gori (2010, 2012), Michel and Pestieau (2013), and Miyazaki (2014), while fertility-related analyses include Boldrin and Jones (2002) and Mizuno and Yakita (2003).

This study particularly builds on previous research that integrates fertility decisions into the PAYG framework. Zhang (1995) analyzed fertility and economic growth in a model incorporating unfunded pension schemes and private intergenerational transfers. Zhang and Zhang (1998) examined the influence of childbearing motives and intergenerational transfer patterns on social security. Wigger (1999) extended an endogenous growth model to include endogenous fertility, evaluating the effects of PAYG pensions on fertility behavior. Miyazaki (2013) incorporated the time cost of childrearing and found that PAYG pensions can promote fertility in the long run. Stauvermann (2013) studied a system in which pension benefits increase proportionally with the number of children under a lump-sum tax regime.

This study proposes a "child-proportional pension system," in which individual benefits increase with the number of children. We compare this system to a conventional PAYG pension scheme that does not account for fertility. While the proposed system can raise fertility rates, our analysis shows that it may reduce overall social welfare. We also derive the conditions under which population decline can be avoided under the child-proportional pension system.

The remainder of the paper is structured as follows. Section 2 presents the model setup. Section 3 analyzes the child-proportional pension system and derives the equilibrium. Section 4 discusses the benchmark case with the existing system. Section 5 compares both systems using numerical simulations. Section 6 concludes.

2 Setup

2.1 Demographics

Let N_t denote the population of the generation born in period t. Individuals decide at a young age how many children they will have, and the number of children is denoted by n_t. The following equation represents the demographic dynamics of the model.

$$N_{t+1} = n_t N_t \tag{1}$$

2.2 Households

We assume that individuals live in two periods: young and old, and that the length of each period is 1. Individuals inelastically supply one unit of labor when they are young and receive disposable income $(1-\tau)w_t$, where w_t is the labor income and $\tau \in [0, 1)$ is the income tax rate. Individuals allocate their disposable income to consumption when young, c_t^y, savings s_t, and childcare expenses, $\delta w_t n_t$. We assume that childcare expenses are a fixed percentage δ of income and increase proportionally with the number of children. Thus, the budget constraint for a young individual is as follows.

$$c_t^y + s_t + \delta w_t n_t = (1-\tau)w_t \tag{2}$$

The individual receives $(1 + r_{t+1})s_t$ in old age, which includes the interest income from savings made at a young age, where $1+r_{t+1}$ is the gross interest rate. Additionally, individuals receive a pension benefit P_{t+1} in old age. Thus, old individuals consume their savings with interest income $(1+r_{t+1})s_t$ and pension benefits P_{t+1}. The budget constraints in old age are as follows:

$$c_{t+1}^o = (1 + r_{t+1})s_t + P_{t+1} \tag{3}$$

From Equations (2) and (3), we obtain the lifetime budget constraint.

$$c_t^y + \delta w_t n_t + \frac{c_{t+1}^o}{1 + r_{t+1}} = (1 - \tau)w_t + \frac{P_{t+1}}{1 + r_{t+1}} \tag{4}$$

Individuals maximize the following utility function under the lifetime budget constraints:

$$u_t = \ln c_t^y + \beta \ln n_t + \gamma \ln c_{t+1}^o \tag{5}$$

where $\beta > 0$ is the altruism factor, and $\gamma \in [0, 1]$ is the discount parameter.

2.3 Production

Firms use capital K_t and labor L_t for production. The labor input corresponds to the population of the young generation, such that $L_t = N_t$. Assume that the production function follows a Cobb-Douglas form: $Y_t = F(K_t, L_t) = K_t^\alpha L_t^{1-\alpha}$, where $\alpha \in (0, 1)$ represents the capital share. In production, we assume that capital fully depreciates at the end of each period and is not carried over to the next. Additionally, we assume that output is sold at a unit price of 1. Let $k_t = K_t/L_t$ and $f(k_t) = {k_t}^\alpha$. From the profit maximization problem, we obtain the following first-order conditions.

$$1 + r_{t+1} = \alpha k_t^{\alpha-1} \tag{6}$$

$$w_t = (1 - \alpha)k_t^\alpha \tag{7}$$

3 The main model

In this section, we present a model that incorporates the child-proportional pension system.

3.1 Pension system

The government offers a PAYG pension system for old individuals. Under this system, the pension benefits an individual receives are directly proportional to the number of children they raised at their young age. Consequently, individuals who raise more children at a young age receive larger pension benefits in old age. Pension benefits are expressed as follows:

$$P_{t+1} = d_{t+1} n_t \tag{8}$$

where d_{t+1} represents the pension benefit for an individual with one child.

The government's balanced budget equation is given as follows:

$$P_{t+1} N_t = \tau w_{t+1} N_{t+1} \tag{9}$$

Using Equation (8), we rewrite (9) as follows:

$$d_{t+1} = \tau w_{t+1} \tag{10}$$

3.2 Utility maximization

Consider the household utility maximization problem. In this model, the pension benefit P_{t+1} is denoted by $d_{t+1} n_t$, meaning that pension benefits depend on the individual's number of children.

By solving this problem, we obtain c_t^y, n_t, and c_{t+1}^o.

$$c_t^y = \frac{(1 - \tau)w_t}{1 + \beta + \gamma} \tag{11}$$

$$n_t = \frac{\beta(1 - \tau)w_t(1 + r_{t+1})}{(1 + \beta + \gamma)[\delta w_t(1 + r_{t+1}) - \tau w_{t+1}]} \tag{12}$$

$$c_{t+1}^o = \frac{\gamma(1 - \tau)w_t(1 + r_{t+1})}{1 + \beta + \gamma} \tag{13}$$

3.3 Equilibrium and steady state

In equilibrium, $n_t k_{t+1} = s_t$ is satisfied. Thus, we have the capital accumulation equation.

$$k_{t+1} = \frac{\alpha\gamma\delta(1-\alpha)}{\alpha\beta + (1-\alpha)(\beta+\gamma)\tau} k_t^{\alpha} \tag{14}$$

In the steady state, the per capita capital stock remains constant in each period, such that $k_t = k_{t+1} = k^*$. Let k^* denote the steady-state capital stock per capita. From Equation (14), we obtain the steady-state capital stock per capita:

$$k^* = \left[\frac{\alpha\gamma\delta(1-\alpha)}{\alpha\beta + (1-\alpha)(\beta+\gamma)\tau}\right]^{\frac{1}{1-\alpha}} \tag{15}$$

From equations (6), (7), (11), (12), and (13), we also obtain the consumption and fertility in the steady state. Here, M is added as a subscript to indicate the results of the main model.

$$c_M^{y*} = \frac{(1-\alpha)(1-\tau)}{1+\beta+\gamma}\left[\frac{\alpha\gamma\delta(1-\alpha)}{\alpha\beta + (1-\alpha)(\beta+\gamma)\tau}\right]^{\frac{\alpha}{1-\alpha}} \tag{16}$$

$$n_M^* = \frac{[\alpha\beta + (1-\alpha)(\beta+\gamma)\tau](1-\tau)}{\delta(1+\beta+\gamma)[\alpha + (1-\alpha)\tau]} \tag{17}$$

$$c_M^{o*} = \frac{\alpha\gamma(1-\alpha)(1-\tau)}{1+\beta+\gamma}\left[\frac{\alpha\gamma\delta(1-\alpha)}{\alpha\beta + (1-\alpha)(\beta+\gamma)\tau}\right]^{\frac{2\alpha-1}{1-\alpha}} \tag{18}$$

4 The comparison model

In this section, we describe the model using an existing pension system.

4.1 Government

Again, the government levies income tax on the younger generation, which is subsequently used to provide pension benefits to the older generation. Thus, the balanced budget equation holds:

$$P_{t+1} N_t = \tau w_{t+1} N_{t+1} \tag{19}$$

From Equation (19), we obtain the pension benefits per person.

$$P_{t+1} = \tau w_{t+1} n_t \tag{20}$$

4.2 Utility maximization

In this model, the pension benefit P_{t+1} is constant for households. This differs from the main model.

By solving this problem, we obtain c_t^y, n_t, and c_{t+1}^o.

$$c_t^y = \frac{\delta(1-\tau)(w_t)^2(1+r_{t+1})}{\delta(1+\beta+\gamma)w_t(1+r_{t+1}) - \beta\tau w_{t+1}} \tag{21}$$

$$n_t = \frac{\beta(1-\tau)w_t(1+r_{t+1})}{\delta(1+\beta+\gamma)w_t(1+r_{t+1}) - \beta\tau w_{t+1}} \tag{22}$$

$$c_{t+1}^o = \frac{\gamma\delta(1-\tau)(w_t)^2(1+r_{t+1})^2}{\delta(1+\beta+\gamma)w_t(1+r_{t+1}) - \beta\tau w_{t+1}} \tag{23}$$

4.3 Equilibrium and steady state

In the equilibrium, again, $n_t k_{t+1} = s_t$ is satisfied. Thus, we have the following capital accumulation equation:

$$k_{t+1} = \frac{\alpha\gamma\delta(1-\alpha)}{\beta[\alpha + (1-\alpha)\tau]} k_t^{\alpha} \tag{24}$$

From Equation (24), we obtain the capital stock per capita in a steady state.

$$k^* = \left[\frac{\alpha\gamma\delta(1-\alpha)}{\beta[\alpha + (1-\alpha)\tau]} \right]^{\frac{1}{1-\alpha}} \tag{25}$$

From equations (6), (7), (21), (22), and (23), we also obtain the consumption and fertility in the steady state. Similar to the main model, C is added as a subscript to indicate that these are the results of the comparison model.

$$c_C^{y*} = \frac{(1-\alpha)[\alpha + (1-\alpha)\tau](1-\tau)}{\alpha(1+\beta+\gamma) + (1-\alpha)(1+\beta)\tau} \left[\frac{\alpha\gamma\delta(1-\alpha)}{\beta[\alpha + (1-\alpha)\tau]} \right]^{\frac{\alpha}{1-\alpha}} \tag{26}$$

$$n_C^* = \frac{\beta[\alpha + (1-\alpha)\tau](1-\tau)}{\delta[\alpha(1+\beta+\gamma) + (1-\alpha)(1+\beta)\tau]} \tag{27}$$

$$c_C^{o*} = \frac{\alpha\gamma(1-\alpha)[\alpha + (1-\alpha)\tau](1-\tau)}{\alpha(1+\beta+\gamma) + (1-\alpha)(1+\beta)\tau} \left[\frac{\alpha\gamma\delta(1-\alpha)}{\beta[\alpha + (1-\alpha)\tau]} \right]^{\frac{2\alpha-1}{1-\alpha}} \tag{28}$$

5 Numerical calculation

Numerical calculations were performed to assess whether the results obtained from the model were realistic. First, the capital share α was set to $\alpha = 0.4$, which is used by Cooley and Prescott (1995). Next, the altruism factor β was set to $\beta = 0.271$, following De La Croix and Doepke (2003). Additionally, we set the discount factor to $\gamma = 0.37$. Finally, we assume $\delta = 0.25$, following the value by Miyazaki (2013).

5.1 Population sustainability

First, numerical calculations are conducted to determine the conditions under which the fertility rate in the main model exceeds one—that is, the population does not decline. Solving the inequality $n_M^* > 1$ for δ yields the following condition:

$$\delta < \frac{[\alpha\beta + (1-\alpha)(\beta+\gamma)\tau](1-\tau)}{(1+\beta+\gamma)[\alpha + (1-\alpha)\tau]} \equiv \hat{\delta} \tag{29}$$

This implies that the population does not decline if $\delta < \hat{\delta}$.

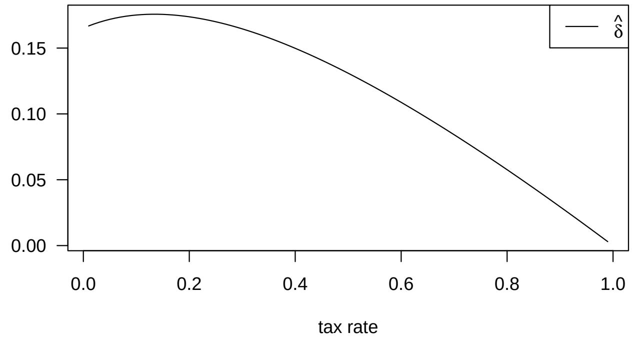
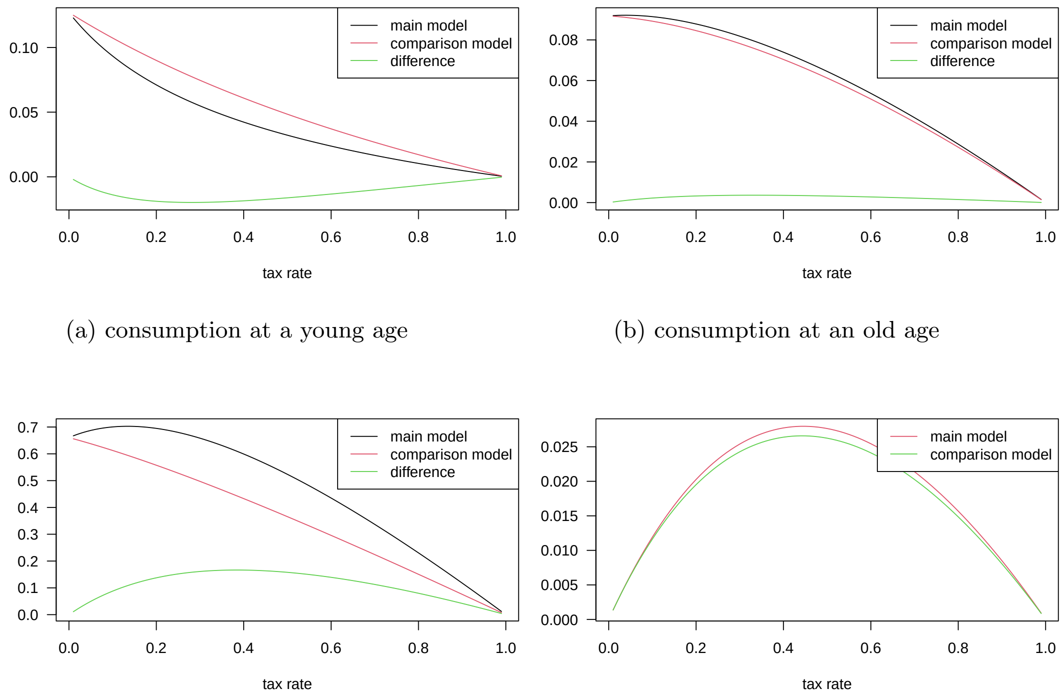


Figure 1: The value of $\hat{\delta}$

The curves in Figure 1 show the values of $\hat{\delta}$. For values of δ that lie below the curve, the fertility rate in the main model exceeds one. The value of $\hat{\delta}$ reaches its maximum around $\tau = 0.2$. For income tax rates above this level, $\hat{\delta}$ declines as the income tax rate increases. These results suggest that, under the main model's pension system, population decline can be avoided if child-rearing costs are kept sufficiently low.

5.2 Result changes

Next, we consider how the model results change with the pension system transition.



(a) consumption at a young age

(b) consumption at an old age

(c) fertility

(d) pension benefits

Figure 2: Comparison of variables in steady state

Here, the “difference” refers to the result of subtracting the comparison model ' s outcome from that of the main model, and it represents the change in each variable caused by the transition in the pension system.

Figure 2-(a) compares steady-state consumption during youth. Across all income tax rates, the consumption level in the main model is consistently lower than that in the comparison model. This indicates that the transition to the new pension system reduces young-age consumption in the steady state. Figure 2-(b) compares steady-state consumption during old age. Unlike for the young, old-age consumption increases with the transition. This result reflects the design of the new pension system: as households are incentivized to have more children, they receive higher pension benefits in old age, which in turn raises their consumption during retirement. Figure 2-(c) shows the steady-state fertility rate. The main model achieves a higher fertility rate than the comparison model. This is because, in the main model, pension benefits are proportional to the number of children. This creates an incentive for households to have more children, in order to boost consumption during old age. Figure 2-(d) compares steady-state pension benefits. Overall, the main model provides higher pension benefits. However, under extremely low or high income tax rates, the benefits in both models are nearly the same. At very low tax rates, government revenues are limited, and thus even in the main model, benefits remain small despite being child-dependent. Conversely, at very high tax rates, households cannot afford to have many children,

which neutralizes the advantage of the child-based pension design—resulting in little difference between the two models.

As these results show, the transition to the new pension system increases fertility. Therefore, it may serve as an effective measure to address the issues of declining birth rates and population aging.

5.3 Lifetime utility

Finally, we examine how the transition to the new pension system affects an individual's lifetime utility. As in the previous sections, the "difference" is defined as the individual's utility in the main model minus that in the steady state of the comparison model.

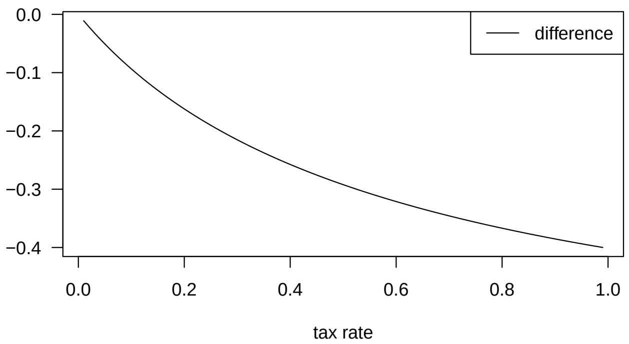


Figure 3: The difference of Lifetime utility

As shown in Figure 3, the transition to the new pension system tends to reduce an individual's lifetime utility. Moreover, as the income tax rate increases, the loss in lifetime utility caused by the transition becomes larger.

6 Conclusion

This study investigates a new pension system within an overlapping generations (OLG) model that endogenizes fertility. The analysis yields three main findings.

First, under the new pension system, if child-rearing costs are sufficiently low, the economy can attain a steady state in which the population does not decline. Second, the new system increases household fertility by strengthening incentives to have children. Third, although the system is effective in addressing population decline, it tends to reduce individual lifetime utility.

References

[1] Boldrin M,Jones L(2002). Mortality, Fertility and Saving in a Malthusian Economy. Rev Econ Dynam,775-814

[2] Bräuninger (2005). Social security, unemployment, and growth. International Tax and Public Finance 12, 423–434.

[3] Cooley TF, Prescott EC (1995). Economic growth and business cycles. Frontiers of business cycle research. Princeton University Press, Princeton, 1–38

[4] Corneo, G. and M. Marquard (2000). Public pensions, unemployment insurance, and growth. Journal of Public Economics 75, 293-311.

[5] Diamond P (1965). National debt in a neoclassical growth model. Am Econ Rev 55,1126–1150

[6] De La Croix D, Doepke M (2003). Inequality and growth: why differential fertility matters. Am Econ Rev 93,1091–1113

[7] Demmel and Keuschnigg (2000). Funded pensions and unemployment. Finanzarchiv 57 (1), 22-38.

[8] Fanti L, Gori L (2012). Fertility and PAYG pensions in the overlapping generations model. J Pop Econ 25,955–961

[9] Miyazaki K (2013). Pay-as-you-go social security and endogenous fertility in a neoclassical growth model. J Pop Econ 26,1233–1250

[10] Mizuno M,Yakita A(2013). Elderly labor supply and fertility decisions in aging-population economies. Economics Letters 121,3,395-399

[11] Ono (2010). Growth and unemployment in an OLG economy with public pensions. Journal of Population Economics 23, 737-767. Econ Res Guard 3,1,61-69

[12] Wigger BU (1999). Pay-as-you-go financed public pensions in a model of endogenous growth and fertility. J Pop Econ.12,625-640

[13] Zhang J(1995). Does unfunded social security also depress output growth?. Econ Lett 49,307-312

[14] Zhang J, Zhang J (1998). Social security, intergenerational transfers, and endogenous growth. Can J Econ 31,1225-1241