Cultural Difference, Social Identity, and Redistribution

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Abstract

Economic disparity between different ethnic groups is substantial in many countries. In developing countries, ethnic inequality appears to significantly impede economic development. In developed countries, the economic and cultural integration of increasing numbers of immigrants into society is a major concern. Income redistribution in a broader sense is a natural candidate for addressing ethnic inequality. Empilical study suggests that it also seems to contribute to development by reducing inequality and increasing educational investment among the poor.

However, the scale of redistribution limited in many countries. Empirical findings suggest two potential explanations: the presence of culturally diverse ethnic groups and weak national identity.

These findings raise various questions. Under what conditions do inter-ethnic cultural differences diminish over time? Is cultural convergence necessary to expand redistribution and reduce inter-ethnic income disparity? Does it always lead to such outcomes? Under what circumstances do large segments of the population share national identity? Is national identity necessary to increase redistribution? Does it always result in increased redistribution? How do cultural differences and national identity interact?

To address these questions, this paper develops and examines a dynamic model of income redistribution and educational investment augmented with cultural change and social identification.

Keywords: culture, development, ethnic inequality, human capital, redistribution, social identity
JEL classification numbers: I25, J15, J24, O15, Z13

1 Introduction

Economic disparity between different ethnic groups is substantial in many countries. In developing countries, ethnic inequality appears to significantly impede economic development: Alesina, Michalopoulos, and Papaioannou (2016) discover that inter-ethnic inequality in economic well-being is strongly and negatively related to development, after controlling for inequality across administrative regions, based on cross-sectional data from 173 countries. In developed countries, the economic and cultural integration of increasing numbers of immigrants into society is a major concern.

Income redistribution in a broader sense, including public spending on mandatory education, is a natural candidate for addressing ethnic inequality. It also seems to contribute to development by reducing inequality and increasing educational investment among the poor: the empirical study by Berg et al. (2018) indicates that income redistribution, unless very large-scale, increases economic growth by lowering income inequality, and that lower inequality is associated with higher levels of human capital.

However, the scale of redistribution is limited and has not increased in many countries. Why is this the case? This paper explores two potential explanations.

One is the presence of culturally diverse ethnic groups. Empirical studies show that ethnic heterogeneity negatively influences redistribution, particularly when groups are culturally different. Desmet, Ortuño-Ortín, and Weber (2008), based on cross-country data for 105 countries, reveal statistically and economically significant negative relationships between measures of linguistic diversity and redistribution when the measures take into account between-group linguistic distances (proxies for cultural heterogeneity).[1] Tabellini (2020), using U.S. city-level data from 1910 to 1930, shows that European immigration led to reduced public spending, particularly on items that primarily benefit poor immigrants, and this effect is more pronounced when cultural differences between immigrants and natives are greater.[2]

The other is weak national identity. Empirical evidence suggests national identity has a positive effect on redistribution (Transue, 2007; Chen and Li, 2009; Singh, 2015). Transue (2007), through a survey experiment on American whites, finds that those who feel close to the nation are more supportive of a tax increase to improve educational opportunities for minorities, compared to those who feel close to their racial group. Further, he finds that making American identity salient increases their support for the policy. Chen and Li (2009) conduct lab experiments to examine the impact of induced group identity on social preferences and find that participants are more averse to payoff differences to groups they identify with.

These findings raise various questions. Under what conditions do inter-ethnic cultural differences diminish over time? Is cultural convergence necessary to expand redistribution and reduce inter-ethnic income disparity? Does it always lead to such outcomes? Under what circumstances do large segments of the population share national identity? Is national identity necessary to increase redistribution? Does it always result in increased redistribution? How do cultural differences and national identity interact?

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[1] This contrasts with insignificant results of theirs and preceding studies for indexes that do not take into account linguistic distances.

[2] Further, Alesina, Murard, and Rapoport (2021), analyzing regional data for 16 Western European countries, discover that native support for redistribution is lower when the share of immigrants in their region is higher, with a stronger negative relationship observed when immigrants are less skilled than natives and originate from Middle Eastern or Eastern European countries. In contrast, Alesina, Miano, and Stantcheva (2023) conduct large-scale surveys and experiments in six countries (France, Germany, Italy, Sweden, the UK, and the US) and find that making immigration salient to respondents led to decreased support for redistribution, but perceived cultural distance between respondents and immigrants seems to play a minor role in explaining the result.

To address these questions, this paper develops and examines a dynamic model of income redistribution and educational investment augmented with cultural change and social identification, primarily drawing on the models by Shayo (2009) and Yuki (2023).

The seminal work by Shayo (2009) extends the standard political economy model of redistributive taxation (Meltzer and Richard, 1981) by incorporating socio-psychological factors. In the model, there are two classes (the poor and the rich), and the government imposes a proportional tax on incomes to provide a lump-sum transfer, with the tax rate determined through voting. What sets the Shayo model apart from the standard one is that individual utility depends not only on disposable income (consumption) but also negatively on *the perceived distance between oneself and the group one identifies with* (their class or the nation) and positively on *the group's status*. That is, individuals incur a cognitive cost when they differ from other group members in income level and non-economic attributes, while deriving high utility from membership when the group's status, determined by an exogenous factor and average income level, is high. These socio-psychological components are major determinants of social identification and intergroup behaviors, as indicated by influential theories in social psychology (Tajfel and Turner, 1986; Turner et al., 1987) and empirical evidence (Manning and Roy, 2010; Hett, Mechtel, and Kröll, 2020; Fouka, Mazumder, and Tabellini, 2022).[3] Importantly, because these components vary depending on which group one identifies with, their social identities influence the tax rate and disposable incomes. Meanwhile, social identity is *endogenously determined*: one chooses the identity that yields higher utility. Hence, identity, individual outcomes, and aggregate outcomes interact with one another.

The present model diverges from Shayo (2009) mainly in the following respects.[4] First, the focus is on ethnic groups rather than economic classes. In the model, there are two ethnic groups, the majority and the minority. Everyone in the majority is skilled, while some in the minority are unskilled and poor.[5] Thus, redistribution reduces inter-ethnic income inequality. Individuals choose to identify either with their ethnic group or with the nation.

Second, a non-economic attribute of individuals, called *culture*, evolves endogenously over time. Parents shape their child's culture taking into account the cultural distance between the child and the identity group, as well as the cultural distance between the child and themselves.[6] In other words, parents want their child to develop a cultural affinity with the identity group while also preserving a cultural connection with the parent.

Third, the model is dynamic, and variables such as economic class (skilled or unskilled), social identity, tax rate, and culture change endogenously over time. Similar to Yuki (2023), which is also a dynamic extension of Shayo (2009) but does not incorporate ethnicity and endogenous culture,[7] the dynamic part of the model largely draws on Galor and Zeira (1993) and Yuki (2007, 2008). In the

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[3]Evidence suggests that perceived distance and status influence identity. For example, Hett, Mechtel, and Kröll (2020) find that participants in a lab experiment exhibit a preference for groups with smaller social distances and higher social status, with their social identity preferences aligning with their choices in dictator games.

[4]Another important difference from Shayo (2009) is that perceived distance depends on the difference in disposable income between oneself and the group one identifies with, rather than the difference in pre-tax pre-transfer income. In contrast, status does depend on disposable income in Shayo's model too.

[5]To be precise, as explained just below, whether an individual is skilled or unskilled is determined endogenously through educational investment.

[6]The assumption that parents determine their child's culture is made for analytical tractability. However, as explained later, it can capture a significant portion of cultural influences from environments such as neighborhoods and schools on children in the real society.

[7]Yuki (2023) differs from Shayo (2009) and the present model in several respects. First, pre-tax pre-transfer incomes are endogenously determined. Second, the tax rate is determined based on a probabilistic voting model rather than majority voting. Third, Yuki (2023) also examines the effect of exogenous skill biased technological change on identity, redistribution, and economic outcomes.

model, individuals, who vary in wealth inherited from their parent, decide on educational spending that must be self-financed and is needed to become skilled workers. As wealth accumulates, the proportion of those who can afford education and become skilled workers may increase over time, thereby influencing the evolution of other variables. Conversely, wealth accumulation and the proportion of skilled workers are affected by the rate of redistributive tax and other variables.

Based on such a model, the paper examines the dynamics and long-run outcomes of culture, identity, redistribution, and development. Main results are summarized as follows.

The rate of redistributive taxation is positive only when the majority identify with the nation. That is, the majority's national identity is essential for redistribution. This result is consistent with the above-mentioned empirical findings (Transue, 2007; Chen and Li, 2009; Singh, 2015).

Social identity affects cultural investment. Children become culturally closer to the other ethnic group when their parent identify with the nation rather than with their ethnic group.

As mentioned above, social identity is endogenously determined. One's identity is affected by variables such as one's culture and disposable income, as well as cultural and economic environments of their ethnic group and the nation. Interactions among identity and other variables shape the dynamics and long-run outcomes.

In particular, exogenous parameters such as the exogenous component of *national status* and the prominence of culture in the perceived distance largely determine the dynamics and long-run outcomes.[8] In the actual society, the exogenous component of national status would be high when people believe that they share a glorious history, rich cultural heritage, or a "right" sense of values, because they feel proud of belonging to such a nation.

When the exogenous factor of national status is very high, everyone always identifies with the nation, and income redistribution is consistently implemented. As a result, the disposable income of unskilled workers is relatively high, leading to a rapid increase in the proportion of skilled workers in the minority and a swift decline in inter-ethnic economic inequality. In the long run, everyone is skilled and income equality is achieved. Inter-ethnic cultural differences also decrease over time, and *cultural integration* eventually occurs, where the integrated culture contains elements from *both minority and majority origins* in proportion to their population shares. This scenario is more likely to occur when people are less concerned about cultural differences, the exogenous factor of the majority's status is lower, or the initial share of the minority accessible to education is higher.

When the exogenous component of national status is lower, initially, the majority identify with their group and redistribution is not implemented. Long-run outcomes differ greatly depending on the levels of exogenous parameters.

If national status is relatively high due to exogenous reasons, all or skilled minorities identify with the nation. Consequently, they are culturally influenced by the majority, leading to a decrease in inter-ethnic cultural distance over time. When the cultural distance decreases sufficiently, the majority adopt a national identity, and income redistribution begins. Subsequently, the share of the skilled minority increases rapidly, and inter-ethnic economic inequality declines fast. In the long run, income equality is achieved, and cultural integration occurs, although the integrated culture contains a greater share of the majority-origin element compared to the previous case. Income equality is achieved faster when the exogenous factor of national status is higher and the other parameters satisfy similar conditions as the previous case.

Otherwise, the majority consistently identify with their group, and redistribution is never carried out. Thus, the skilled minority's share grows only slowly or does not increase, and inter-ethnic economic inequality diminishes slowly or persists. If national status is very low due to

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[8]National (ethnic) status also depends on the average disposable income of the nation (ethnic group) and thus is endogenous.

exogenous reasons, both ethnic groups identity with their group and *cultural segregation* persists, where the ethnic groups remain culturally dissimilar. This outcome is more likely when cultural differences are more concerning to people or the minority's pride in their group is stronger for exogenous reasons. Otherwise, all or skilled minorities identify with the nation and gradually adopt the majority's culture. The long-run outcome is *cultural assimilation*, in which everyone shares the culture originating *solely from the majority*.

These results indicate that national identities of the majority and the minority have distinct effects on inter-ethnic cultural and economic convergence. The majority's national identity is essential for economic convergence, as income redistribution is implemented only when they identify with the nation. However, this is realized from the beginning only under very favorable conditions (e.g., when the exogenous factor of national identity is very high). When conditions are not very, but relatively favorable, the minority's national identity contributes to economic convergence through their gradual adoption of the majority's culture, which eventually leads to the majority's switch to a national identity and adoption of redistribution. In contrast, under less favorable conditions, the minority's national identity does not foster economic convergence and results in their cultural assimilation into the majority.

Empirical studies (Blouin and Mukand, 2019; Cáeres-Delpiano et al., 2021; Chen, Lin, and Yang, 2023) indicate that *nation-building policies*, such as school education and government propaganda that emphasize shared history, culture, and values, as well as policies promoting inter-group contact, can effectively strengthen national identity. According to the model, these policies elevate national status or deemphasize inter-group cultural differences. The results imply that the effectiveness of nation-building policies in promoting inter-ethnic economic convergence depends on the pre-policy conditions. When conditions are not very, but relatively favorable (e.g., when the exogenous factor of national identity is not very high, but relatively so), these policies would foster economic convergence and cultural integration. In contrast, under unfavorable conditions, the policies would not contribute to economic convergence and result in cultural assimilation.

This paper contributes to the theoretical literature on the relationship between social identity and redistribution (Shayo 2009; Lindqvist and Östling, 2013; Holm, 2016; Dhami, Manifold, and al-Nowaihi, 2021; and Ghiglino, Júarez-Lunam, and Müller, 2021; Yuki, 2023). The relations with closely-related papers, Shayo (2009) and Yuki (2023), are elucidated above (see footnotes 4 and 7 for additional details). Diverging from existing works, this paper investigates the interactions among cultural and human capital investment, social identity, and redistribution.

More broadly, this paper contributes to the theoretical literature on the relationship between identity and economic behaviors (Akerlof and Kranton, 2000; Shayo, 2009; Benabou and Tirole, 2011; Bernard, Hett, and Mechtel, 2016; Carvalho and Dippel, 2020; Grossman and Helpman, 2020; Bonomi, Gennaioli, and Tabellini, 2021; Yuki, 2021).[9] Generalizing the pioneering work of Akerlof and Kranton (2000), the above-mentioned Shayo (2009) constructs the basic analytical framework. Shayo's (2009) framework has been applied to various issues. For instance, Grossman and Helpman (2020) develop a political economy model of trade policy with social identification, motivated by a recent reversal of in trade policies in some Western countries seemingly influenced by the rise of populism and ethnic tensions. They explore how policies are affected by changes in identification patterns triggered by events such as increased ethnic tensions.

This work is also related to the large literature on cultural transmission models, which is initiated by Bisin and Verdier (2000) and surveyed by Bisin and Verdier (2011, 2023a). Recent contributions include Spiro (2020), Bisin and Verdier (2023b), Hiller, Wu, and Zhang (2023), and

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[9]In addition to those already mentioned, recent empirical and experimental studies on identity include Dehdari and Gehring (2022), Assouad (2023), and Carlitz et al. (2024).

Carvalho, Koyama, and Williams (2024). Apart from investigating different issues, the current study diverges from existing works by incorporating cultural evolution into the Shayo-type model of social identification.

The rest of the paper is organized as follows. To facilitate understanding, Section 2 presents and Section 3 examines a simpler model without social mobility. Section 4 introduces the full-fledged model with social mobility, and Section 5 analyzes the model and interpret the results. The section also discusses policy implications. Section 6 concludes. Appendix A presents a part of the full-fledged model, and Appendix B contains lemmas used for proving propositions in Sections 3 and 5. Online Appendix C contains proofs.

2   Model without social mobility

For the sake of clarity, this section presents a simplified model. The full-fledged model is presented in Section 4. Consider a society composed of the majority (group 1), the minority (group 2), and a government. The government levies a proportional tax on earnings to finance lump-sum transfers. The minority are poorer than the majority on average, so the income redistribution lowers inter-ethnic economic inequality. The tax-transfer policy is determined through majority voting, effectively controlled by group 1. Utility depends not only on one's disposable income (consumption) but also on socio-psychological components influenced by their social identity. This means that social identity affects the tax-transfer policy. Social identity is determined endogenously: one chooses to identify with the group (their ethnic group or the nation) that yields higher utility.

The model is dynamic: it is a deterministic, discrete-time, and OLG world in which individuals live for two periods, first as a child, then as an adult. Each adult has a single child, so the population is constant over time. Children are born with an ethnicity inherited from their parent and a culture that is shaped by, but may differ from, the parent. In this simplified model, children inherit their parent's class (skilled or unskilled) and make no decisions. Adults choose social identities, work and receive earnings, vote on the policy, pay taxes and receive transfers, and mold their child's culture. Time-evolving cultures play important roles because they affect the socio-psychological components in utility, thereby influencing the evolution of social identity and the redistributive policy. Although the model is dynamic, a time subscript is *not* used in variables in this section to avoid unnecessary complications.

2.1   Environment

In each generation, the total population is 1, of which N_1 $(1 - N_1)$ individuals belong to group 1 (group 2), where $N_1 > \frac{1}{2}$. All individuals in group 1 are skilled, while group 2 consists of both skilled and unskilled workers. The assumption on group 1 is made for analytical tractability, but similar results would be obtained as long as the proportion of skilled workers in the majority is higher than that in the minority. Because children inherit their parent's class, the proportion of skilled workers in group 2, H_2, is constant in this section; H_2 is endogenized in the full-fledge model presented in Section 4. Workers supply 1 unit of labor to receive earnings, pay the proportional tax on earnings, receive the lump-sum transfer, and spend disposable income on consumption. Individual i's disposable income is given by

$$y_i = (1 - \tau)w_i + T, \tag{1}$$

where w_i is her earnings (exogenous in this section), $\tau \in [0, 1)$ is the tax rate, and T is the transfer.

The government uses tax revenue entirely for the lump-sum tranfer, but taxation involves deadweight loss. The deadweight loss is assumed to be quadratic, so the governmental budget constraint can be expressed as

$$T = \left(\tau - \frac{1}{2} \tau^2 \right) \overline{w}, \tag{2}$$

where $\overline{w}$ is the average earnings of the population.

2.2 Preferences

As in Shayo (2009), individual utility depends not only on disposable income (consumption) but also negatively on *the perceived distance to a group with which they identify* (either their ethnic group or the nation) in cultural and economic dimensions, and positively on *the group's status*. In other words, individuals incur a mental cost when they are culturally or economically different from others in the group, but take pride in belonging to the group when its status is high. Moreover, the utility has a component related to *their child's culture*, reflecting the parental desire for their child to become culturally similar to them while also being culturally close to the group with which they identify. Perceived distance and status are major determinants of social identification and intergroup behaviors, according to influential theories in social psychology (Tajfel and Turner, 1986; Turner et al., 1987) and empirical evidence in economics (Manning and Roy, 2010; Hett, Mechtel, and Kröll, 2020; Fouka, Mazumder, and Tabellini, 2022).[10]

An individual's perception of the distance or proximity to a social group (her ethnic group or the nation) is based on the difference between her disposable income and the group's average disposable income, and the difference between her culture and the group's average culture. The *perceived distance* of individual i, who belongs to ethnic group J ($J = 1, 2$) and class C ($C = S, U$; S [U] is for skilled [unskilled]), to group G ($G = J, N$; N is for the nation) is represented by

$$
\begin{aligned}
d^i_{JCG} &\equiv \underbrace{|y_C - \overline{y}_G|}_{\text{Economic distance}} + \omega_q \underbrace{\left(q^i_{JC} - \overline{q}_G \right)^2}_{\text{Cultural distance}} \\
&= (1 - \tau)|w_C - \overline{w}_G| + \omega_q \left(q^i_{JC} - \overline{q}_G \right)^2 \quad \text{(from(1))}, \tag{3}
\end{aligned}
$$

where y_C and w_C are respectively the disposable income and earnings of the class, and $\overline{y}_G$ and $\overline{w}_G$ are the average income and earnings of the group; $q^i_{JC} \in [0, 1]$ and $\overline{q}_G \in [0, 1]$ represent her culture and the group's average culture, respectively; and ω_q is the weight on the cultural component. For analytical tractability, the economic distance is measured in absolute value following Ghiglino, Júarez-Lunam, and Müller (2021), while the cultural distance is measured in squared distance.

The *status* of social group G ($G = J, N$) one identifies with, S_G, depends on the exogenous component $\widetilde{S_G}$ and the average disposable income of the group:

$$S_G = \delta \widetilde{S_G} + \overline{y}_G, \tag{4}$$

where δ is the weight on the exogenous component.[11]

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[10]For the United Kingdom, Manning and Roy (2010) find that the non-white individuals, whose perceived distance to the "average national" would be greater than that of the whites, are less likely to think of themselves as British. They also find that immigrants from poorer and less democratic (i.e., lower status) countries assimilate faster into a British identity. Hett, Mechtel, and Kröll (2020), based on a lab experiment, find that participants in the experiment prefer groups to which they have a smaller social distance and which have a higher social status, and their social identity preferences are related to their choices in dictator games. Fouka, Mazumder, and Tabellini (2022) find that migrations of African Americans from the South to non-southern metropolitan areas stimulated assimilation of European immigrants for the years 1910−30. Further, they provide evidence suggesting that higher integration resulted from decreased perceived distance of native whites to European immigrants.

[11]Similar to works such as Grossman and Helpman (2021), status is an absolute measure. By contrast, in Shayo (2009), status is a relative measure and is defined as the difference from the reference group. The main results remain unchanged under the alternative specification.

The exogenous component of *national status*, $\widetilde{S_N}$, would be high when people of the nation believe that they share a glorious history or rich cultural heritage, or when the nation achieves commendable performance in international sports competitions, because this fosters a sense of pride among them. In contrast, the exogenous factor of *ethnic group J's status*, $\widetilde{S_J}$, would be high when people of the group share such beliefs. It would be reasonable to suppose that it is higher for the majority, i.e., $\widetilde{S_1} > \widetilde{S_2}$, given the historical dominance of the majority in most societies.

Finally, individual utility depends negatively on *the perceived distance of the child's culture*, $(q^i_{JC})'$, to one's own culture q^i_{JC}, and to the average culture of the group with which one identifies, $\overline{q}_G$. The composite distance is given by

$$D^i_{JCG} \equiv \chi \underbrace{\left[(q^i_{JC})' - \overline{q}_G\right]^2}_{\text{Distance to the identity group}} + (1-\chi) \underbrace{\left[(q^i_{JC})' - q^i_{JC}\right]^2}_{\text{Distance to oneself (the parent)}} , \tag{5}$$

where $\chi \in [0,1]$ is the weight on the cultural distance to the identity group. A high value of χ indicates that the parent wants her child to become culturally close to the identity group rather than to her, who might be quite different from the identity group culturally.

Hence, the utility of an individual from ethnic group J and class C when she identifies with group G is given by

$$u^i_{JCG} = \underbrace{y_C}_{\text{Disposable income}} - \beta \underbrace{d^i_{JCG}}_{\text{Perceived distance}} + \gamma \underbrace{S_G}_{\text{Status}} - \rho \underbrace{D^i_{JCG}}_{\text{Perceived distance of child's culture}} , \quad \beta > 1, \gamma, \rho > 0. \tag{6}$$

$\beta > 1$ is assumed to ensure that the rate of redistributive tax is positive under certain situations.[12]

By substituting $(1)-(5)$ into the above equation, the utility for a group J individual of each class-identity type can be expressed as (note that $\overline{w}_N$ and $\overline{q}_N$ are denoted as $\overline{w}$ and $\overline{q}$, respectively)

$$u^i_{JSN} = (1-\tau)w_s + T - \beta\left[(1-\tau)(w_s - \overline{w}) + \omega_q(q^i_{JS} - \overline{q})^2\right] + \gamma\left[\delta\widetilde{S_N} + (1-\tau)\overline{w} + T\right] - \rho\left\{\chi\left[(q^i_{JS})' - \overline{q}\right]^2 + (1-\chi)\left[(q^i_{JS})' - q^i_{JS}\right]^2\right\}$$

$$= (1-\tau)w_s + (1+\gamma)T - \beta\left[(1-\tau)(w_s - \overline{w}) + \omega_q(q^i_{JS} - \overline{q})^2\right] + \gamma\left[\delta\widetilde{S_N} + (1-\tau)\overline{w}\right] - \rho\left\{\chi\left[(q^i_{JS})' - \overline{q}\right]^2 + (1-\chi)\left[(q^i_{JS})' - q^i_{JS}\right]^2\right\}, \tag{7}$$

$$u^i_{JSJ} = (1-\tau)w_s + (1+\gamma)T - \beta\left[(1-\tau)(w_s - \overline{w}_J) + \omega_q(q^i_{JS} - \overline{q}_J)^2\right] + \gamma\left[\delta\widetilde{S_J} + (1-\tau)\overline{w}_J\right] - \rho\left\{\chi\left[(q^i_{JS})' - \overline{q}_J\right]^2 + (1-\chi)\left[(q^i_{JS})' - q^i_{JS}\right]^2\right\}, \tag{8}$$

$$u^i_{JUN} = (1-\tau)w_u + (1+\gamma)T - \beta\left[(1-\tau)(\overline{w} - w_u) + \omega_q(q^i_{JU} - \overline{q})^2\right] + \gamma\left[\delta\widetilde{S_N} + (1-\tau)\overline{w}\right] - \rho\left\{\chi\left[(q^i_{JU})' - \overline{q}\right]^2 + (1-\chi)\left[(q^i_{JU})' - q^i_{JU}\right]^2\right\}, \tag{9}$$

$$u^i_{JUJ} = (1-\tau)w_u + (1+\gamma)T - \beta\left[(1-\tau)(\overline{w}_J - w_u) + \omega_q(q^i_{JU} - \overline{q}_J)^2\right] + \gamma\left[\delta\widetilde{S_J} + (1-\tau)\overline{w}_J\right] - \rho\left\{\chi\left[(q^i_{JU})' - \overline{q}_J\right]^2 + (1-\chi)\left[(q^i_{JU})' - q^i_{JU}\right]^2\right\}. \tag{10}$$

2.3 Timing of decisions

The timing of decisions by adult individuals in each period is as follows. First, they choose their social identity, either their ethnic group or the nation, and work to receive earnings.[13] Then, they

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[12]Under probabilistic voting, as adopted by Ghiglino, Júarez-Lunam, and Müller (2021) and Yuki (2023), the tax rate is generally positive even when $\beta \leq 1$, but this makes the present model analytically intractable.

[13]By assumption, individuals do not identify with the nation and their ethnic group simultaneously. Conversely, in the model of Grossman and Helpman (2021), individuals always identify with their class and also identify with the nation if the additional identity increases their utility, which depends on the sum of the perceived distance to and the status of each group with which they identify. The present paper does not adopt this specification owing to the complexities associated with the additional terms and the difficulties in analyzing the model.

vote on the tax-transfer policy. Finally, they choose their child's culture $(q_{JC}^i)'$. Similar to Shayo (2009), the policy is determined through majority voting, hence by group 1.

The assumption that parents determine their child's culture is made for analytical tractability, but it can capture a large part of cultural influences from environments such as neighborhoods and schools on children in the real society. For instance, a parent with a national identity may expose her child to an ethnically diverse environment in which the child is culturally influenced by the other ethnic group as well; this influence can be captured by $-\rho\chi\left[(q_{JC}^i)'-\overline{q}\right]^2$ in the utility function. In contrast, when she has an ethnic identity, the child would be raised in an ethnically segregated environment, influenced by their own ethnic group alone, which would be captured by $-\rho\chi\left[(q_{JC}^i)'-\overline{q}_J\right]^2$.

3 Analysis of the model without social mobility

3.1 Child's culture

Because the model can be solved by backward induction, the determination of a child's culture is examined first. For the initial generation, assume that $q_{1S}^i = 1$ and $q_{2S}^i = q_{2U}^i = 0$ for any i. That is, each group is culturally homogenous, while the inter-ethnic cultural distance is highest in the initial period. In subsequent periods, the child's culture is determined by parental investment $(q_{JC}^i)'$, which becomes q_{JC}^i in the next period. From $(7)-(10)$, the parent chooses $(q_{JC}^i)'$ to minimize the weighted average of its distance to the average culture of the identity group and the distance to her culture, q_{JC}^i,[14] which equals

When the parent identifies with her ethnic group: $\chi\left[(q_{JC}^i)'-\overline{q}_J\right]^2+(1-\chi)\left[(q_{JC}^i)'-q_{JC}^i\right]^2,$ (11)

When the parent identifies with the nation: $\chi\left[(q_{JC}^i)'-\overline{q}\right]^2+(1-\chi)\left[(q_{JC}^i)'-q_{JC}^i\right]^2.$ (12)

Under the ethnic identity, from the first-order condition, $(q_{JC}^i)'$ is determined by

$$2\chi\left[(q_{JC}^i)'-\overline{q}_J\right]+2(1-\chi)\left[(q_{JC}^i)'-q_{JC}^i\right]=0.$$
$$\Leftrightarrow (q_{JC}^i)'=\chi\overline{q}_J+(1-\chi)q_{JC}^i. \tag{13}$$

Under the national identity,
$$(q_{JC}^i)'=\chi\overline{q}+(1-\chi)q_{JC}^i. \tag{14}$$

Thus, the child's culture is a weighed average of the average culture of the group with which the parent identifies and the parent's own culture, where the weight on the former is χ.

3.2 Tax rate

The tax-transfer policy is determined through majority voting. By substituting (2) and (14) into (7), the utility of group 1 individual i when she identifies with the nation is given by

$$u_{1SN}^i = (1-\tau)w_s+(1+\gamma)\left(\tau-\frac{\tau^2}{2}\right)\overline{w}-\beta(1-\tau)(w_s-\overline{w})+\gamma\left[\delta\widetilde{S_N}+(1-\tau)\overline{w}\right]-\left[\rho\chi(1-\chi)+\beta\omega_q\right]\left(q_{1S}^i-\overline{q}\right)^2. \tag{15}$$

Her preferred tax rate is obtained by maximizing this equation:

$$(1-\tau)(1+\gamma)\overline{w}-w_s+\beta(w_s-\overline{w})-\gamma\overline{w}=0,$$
$$\Leftrightarrow \tau=1-\frac{w_s-\beta(w_s-\overline{w})+\gamma\overline{w}}{(1+\gamma)\overline{w}}=\frac{\beta-1}{1+\gamma}\frac{w_s-\overline{w}}{\overline{w}}. \tag{16}$$

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[14]For analytical simplicity, the good cost of cultural investment is assumed to be 0.

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By substituting (2) and (13) into (8), her utility under the ethnic identity is given by

$$u^i_{1S1} = (1-\tau)w_s + (1+\gamma)\left(\tau - \frac{\tau^2}{2}\right)\overline{w} + \gamma\left[\delta\widetilde{S_1} + (1-\tau)w_s\right] - [\rho\chi(1-\chi) + \beta\omega_q](q^i_{1S} - \overline{q}_1)^2. \qquad (17)$$

In this case, the preferred tax rate is 0 because $(1-\tau)(1+\gamma)\overline{w} - w_s - \gamma w_s < 0$.

The preferred tax rate is common among all individuals in group 1 because, as shown below, they share the same identity. Since the tax-transfer policy is determined through majority voting, this implies that the implemented tax rate is equal to their preferred rate. The result is summarized as the following lemma.

Lemma 1 *When the majority identify with the nation, the implemented tax rate τ equals $\frac{\beta-1}{1+\gamma}\frac{w_s - \overline{w}}{\overline{w}} > 0$, while when they identify with their ethnic group, $\tau = 0$.*

When the majority identify with the nation, the rate of redistributive tax is positive, whereas when they identify with their ethnic group, the tax rate is 0 and there is no redistribution. This result is consistent with empirical findings that show a positive relationship between national identity and redistribution (Transue, 2007; Chen and Li, 2009; Singh, 2015).[15]

Under national identity, the tax rate increases with β because a greater concern for the perceived distance to the "average national" leads the majority to narrow the income gap with the minority more. In contrast, the tax rate decreases with γ because a greater concern for national status induces them to mitigate the negative effect of taxation on the national average disposable income and thus national status. The negative effect of the strength of concern for national status, or national pride, on the tax rate aligns with Gustavsson (2019), who finds that national pride is negatively related to support for reducing income inequality after controlling for national identity and political ideology, based on Dutch survey data.[16] The tax rate is 0 under ethnic identity since the tax reduces the majority's disposable income, does not affect the perceived distance to other majority members, and lowers the group's status.

The next assumption is imposed to ensure that the rate of redistributive tax is less than $\frac{1}{3}$.[17]

Assumption 1 $\frac{\beta-1}{1+\gamma} \leq \frac{1}{3}$.

This assumption is plausible, considering that the tax revenue is solely used for income redistribution in the model. The assumption guarantees that redistribution always increases the disposable income of the unskilled minority.

3.3 Social identity

Finally, the determination of social identities is examined. The choice of social identity affects one's utility through the perceived distance term, the status term, and the term associated with the child's culture. Thus, one chooses the identity with the higher sum of these terms.[18] Let p^i_{1S} be the indicator of the national identity for individual i of group 1: $p^i_{1S} = 1$ ($p^i_{1S} = 0$) if she identifies

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[15] Transue (2007), based on a survey experiment on American whites, finds that those who feel close to the nation are more supportive of a tax increase to improve educational opportunities for minorities, compared to those who feel close to their racial group. Further, he finds that making American identity salient increases their support for the policy. Chen and Li (2009) conduct lab experiments to examine the effects of induced group identity on social preferences and find that participants are more averse to payoff differences to groups they identify with. Singh (2015), based on statistical and comparative historical analysis of Indian states, shows that states with a stronger sense of shared identity tend to allocate higher public spending to education and health that would greatly benefit the poor.

[16] It is also consistent with Shayo (2009), who, using survey data, finds that national pride is negatively associated with support for redistribution after controlling for income and education in most of the countries examined.

[17] This is because $\frac{w_s}{\overline{w}} < 2$ when $N_1 > \frac{1}{2}$.

[18] When the level of utility is equal under both identities, it is assumed that one chooses the national identity.

with the nation (her ethnic group). Similarly, let p_{2C}^i $(C=S,U)$ be the indicator of the national identity for individual i from group 2 and class C.

From (15)−(17), $p_{1S}^i = 1$ $(p_{1S}^i = 0)$, i.e., a majority individual identifies with the nation (her ethnic group) iff

$$u_{1SN}^i \geq (<) u_{1S1}^i \Leftrightarrow \gamma\delta\widetilde{S_N} \geq (<)\gamma\delta\widetilde{S_1} + (\beta+\gamma)(1-\tau)(w_s - \overline{w}) + [\rho\chi(1-\chi)+\beta\omega_q](\overline{q}_1 - \overline{q})(2q_{1S}^i - \overline{q}_1 - \overline{q})$$

$$\Leftrightarrow \widetilde{S}_N \geq (<)\widetilde{S}_1 + \frac{1}{\gamma\delta}\left\{(\beta+\gamma)(1-N_1)(1-H_2)(1-\tau)(w_s - w_u) + [\rho\chi(1-\chi)+\beta\omega_q](\overline{q}_1 - \overline{q})^2\right\}, \qquad \text{(M)}$$

$$\text{where } \tau = \frac{\beta-1}{1+\gamma}\frac{w_s - \overline{w}}{\overline{w}} \quad (\tau = 0).$$

All majority individuals face the same condition and thus make the same identity choice, because $q_{1S}^i = \overline{q}_1$ for any period under the initial condition $q_{1S}^i = 1$.[19] Thereafter, p_{1S}^i and q_{1S}^i are used without superscript i unless necessary.

By substituting (2), (13), and (14) into (7)−(10), the utility for a group 2 individual of each class-identity type is expressed as

$$u_{2SN}^i = (1-\tau)w_s + (1+\gamma)\left(\tau - \frac{\tau^2}{2}\right)\overline{w} - \beta(1-\tau)(w_s - \overline{w}) + \gamma\left[\delta\widetilde{S}_N + (1-\tau)\overline{w}\right] - [\rho\chi(1-\chi)+\beta\omega_q](\overline{q} - q_{2S}^i)^2, \qquad (18)$$

$$u_{2S2}^i = (1-\tau)w_s + (1+\gamma)\left(\tau - \frac{\tau^2}{2}\right)\overline{w} - \beta(1-\tau)(w_s - \overline{w}_2) + \gamma\left[\delta\widetilde{S}_2 + (1-\tau)\overline{w}_2\right] - [\rho\chi(1-\chi)+\beta\omega_q](\overline{q}_2 - q_{2S}^i)^2, \qquad (19)$$

$$u_{2UN}^i = (1-\tau)w_u + (1+\gamma)\left(\tau - \frac{\tau^2}{2}\right)\overline{w} - \beta(1-\tau)(\overline{w} - w_u) + \gamma\left[\delta\widetilde{S}_N + (1-\tau)\overline{w}\right] - [\rho\chi(1-\chi)+\beta\omega_q](\overline{q} - q_{2U}^i)^2, \qquad (20)$$

$$u_{2U2}^i = (1-\tau)w_u + (1+\gamma)\left(\tau - \frac{\tau^2}{2}\right)\overline{w} - \beta(1-\tau)(\overline{w}_2 - w_u) + \gamma\left[\delta\widetilde{S}_2 + (1-\tau)\overline{w}_2\right] - [\rho\chi(1-\chi)+\beta\omega_q](\overline{q}_2 - q_{2U}^i)^2. \qquad (21)$$

Thus, from (20), (21), and (16), $p_{2U}^i = 1$ $(p_{2U}^i = 0)$, i.e., an unskilled minority identifies with the nation (her ethnic group) iff

$$u_{2UN}^i \geq (<) u_{2U2}^i \Leftrightarrow \gamma\delta\widetilde{S}_N \geq (<)\gamma\delta\widetilde{S}_2 + (\beta-\gamma)(1-\tau)(\overline{w} - \overline{w}_2) + [\rho\chi(1-\chi)+\beta\omega_q](\overline{q} - \overline{q}_2)[(\overline{q}_2 - q_{2U}^i) + (\overline{q} - q_{2U}^i)]$$

$$\Leftrightarrow \widetilde{S}_N \geq (<)\widetilde{S}_2 + \frac{1}{\gamma\delta}\left\{(\beta-\gamma)N_1(1-H_2)(1-\tau)(w_s - w_u) + [\rho\chi(1-\chi)+\beta\omega_q](\overline{q} - \overline{q}_2)[(\overline{q}_2 - q_{2U}^i) + (\overline{q} - q_{2U}^i)]\right\}, \qquad \text{(mU)}$$

$$\text{where } \tau = \frac{\beta-1}{1+\gamma}\frac{w_s - \overline{w}}{\overline{w}} \text{ when } p_{1S} = 1, \text{ while } \tau = 0 \text{ when } p_{1S} = 0.$$

Similary, from (18), (19), and (16), $p_{2S}^i = 1$ $(p_{2S}^i = 0)$, i.e., a skilled minority identifies with the nation (her ethnic group) iff

$$u_{2SN}^i > (<) u_{2S2}^i$$

$$\Leftrightarrow \widetilde{S}_N \geq (<)\widetilde{S}_2 + \frac{1}{\gamma\delta}\left\{-(\beta+\gamma)N_1(1-H_2)(1-\tau)(w_s - w_u) + [\rho\chi(1-\chi)+\beta\omega_q](\overline{q} - \overline{q}_2)[-(q_{2S}^i - \overline{q}_2) + (\overline{q} - q_{2S}^i)]\right\}, \qquad \text{(mS)}$$

$$\text{where } \tau = \frac{\beta-1}{1+\gamma}\frac{w_s - \overline{w}}{\overline{w}} \text{ when } p_{1S} = 1, \text{ while } \tau = 0 \text{ when } p_{1S} = 0.$$

In the model with constant H_2, all minority individuals with a given skill level make the same identity choice, because $q_{2C}^i = \overline{q}_{2C}$ $(C = S,U)$ for any period under the initial condition $q_{2C}^i = 0$.[20] Thus, thereafter in this section, p_{2C}^i and q_{2C}^i are also used without superscript i.

To simplify analysis, the following assumption is imposed on the relationship among several terms in the above equations.

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[19]To be precise, $q_{1S}^i = \overline{q}_1$ holds in the current period because they make the same identity choices in the previous periods based on (M), starting from the initial period.

[20]When H_2 is endogenous, identity choices and q_{2C}^i can be different among group 2 individuals.

Assumption 2 $\gamma\delta\left(\widetilde{S_1}-\widetilde{S_2}\right)>[\rho\chi(1-\chi)+\beta\omega_q](N_1)^2$

The assumption holds when the exogenous component of the majority's status $(\widetilde{S_1})$ is sufficiently greater than that of the minority $(\widetilde{S_2})$, $N_1(>\frac{1}{2})$ is relatively small, or the importance of the exogenous factor of group status $(\gamma\delta)$ is large compared to that of the cultural terms $(\rho\chi(1-\chi)+\beta\omega_q)$ in one's utility. The main results remain mostly unchanged qualitatively even when this condition does not hold, although the analysis becomes much more complicated.

3.3.1 Majority-minority difference in the tendency of national identity

Among the majority, the skilled minority, and the unskilled minority, which group is more likely to identify with the nation? The next proposition examines this question.

Proposition 1 (*i*) *Skilled workers in group 2 are more likely to identify with the nation than unskilled workers.*

(*ii*) *Under Assumption 2:*

(*a*) *Group 1 individuals are less likely to identify with the nation than group 2's skilled workers.*

(*b*) *They are also less likely to identify with the nation than group 2's unskilled workers except when $\beta>\gamma$, $N_1>\frac{\beta+\gamma}{2\beta}$, and H_2 is small, in which case they can be more likely to have a national identity than the unskilled.*

Proof. See Appendix C. ∎

Among the minority, skilled workers are more likely to identify with the nation than unskilled workers. This is because the disposable income of skilled workers is at the same level as that of the majority, and thus their perceived distance to the "average national" is smaller compared to the unskilled.

On the other hand, the majority are *less likely* to identify with the nation than the skilled minority and except when $\beta>\gamma$, $N_1>\frac{\beta+\gamma}{2\beta}$, and H_2 is small, than the unskilled minority (in such a case, the opposite can be true).[21] This is mainly because the higher status of the majority compared to the minority: the majority have greater pride in their group than the minority for exogenous factors $(\widetilde{S_1}>\widetilde{S_2})$, and their income exceeds the national average. Further, the fact that the income of the skilled minority is closer to the national average than to the minority's average, and the opposite for the majority, also contributes to the majority's weaker inclination toward a national identity compared to the skilled minority. In contrast, the majority's income and cultural distances to the national averages are smaller than those of the unskilled minority (due to the majority's larger population size). Hence, the majority's propensity toward a national identity can be stronger than that of the unskilled minority when the status effect is weak and the distance effect is strong, i.e., $\beta>\gamma$, $N_1>\frac{\beta+\gamma}{2\beta}$, and H_2 is small.

Note that the majority's weaker inclination toward a national identity implies their lesser concerns with the perceived distance to and the welfare of the other ethnic group compared to the minority. In this model, the majority's pride in the nation, without considering the minority as part of it, *is not regarded as a form of national identity but rather ethnic identity.*

Empirical studies examining the majority-minority difference in the tendency of national identity are scarce, and the indicators used to measure national identity vary across studies. Masella (2013) employs a measure that is closest to the concept of national identity in this model: the national identity dummy that equals 1 if, in U.S. data, a respondent selects "I am an American

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[21]When Assumption 2 is not satisfied, the result may not hold (that is, the majority may have a stronger tendency to have a national identity) especially when H_2 is large in the early periods, but it does hold in the longer term.
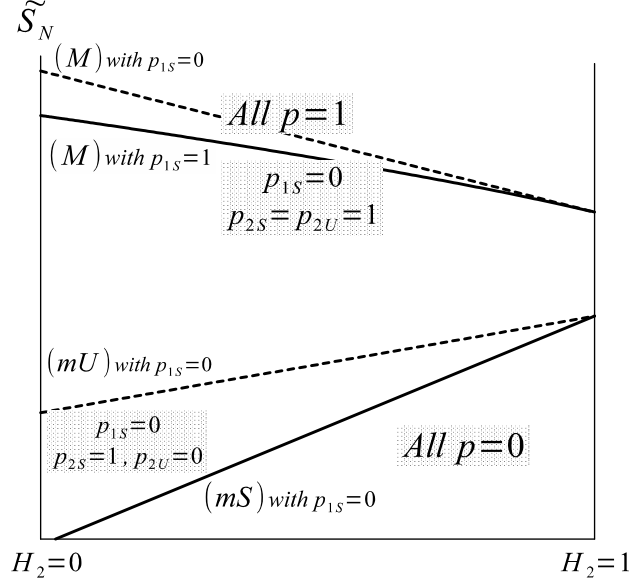
Figure 1: Identity in the initial period when $\beta \leq \gamma$

first and a member of some ethnic group second," over alternatives such as "Above all I am a White American." in a multiple choice question. Using survey data from 21 countries, he finds that when ethnic diversity is relatively low, minority groups are more likely to identify with the nation than the "majority" (the largest ethnic group), and the opposite is true when ethnic diversity is relatively high. Further, when the sample is divided into the subsample consisting solely of "majority" individuals and the one comprising only minorities, national identity is *negatively* related to the size of the respondent's ethnic group for both subsamples. Hence, the theoretical result aligns with his findings, particularly when ethnic diversity is relatively low.[22]

3.3.2 Dependence of social identity on H_2 and $\widetilde{S_N}$

The equations (M), (mU), and (mS) indicate that an individual's choice of identity depends on exogenous parameters such as H_2 and $\widetilde{S_N}$. Based on the equations, Proposition 1, and Lemma A1 in Appendix B, we can visually explore how identity choices of different types of individuals depend on H_2 and $\widetilde{S_N}$. Figure 1 illustrates identity choices when $\beta \leq \gamma$ on the $(H_2, \widetilde{S_N})$ plane, assuming that society is in the initial period where $q_1^i = 1$ and $q_{2S}^i = q_{2U}^i = 0$.[23] Note that the following results hold in subsequent periods as well.

According to the figure, the proportion of individuals identifying with the nation increases with the level of the exogenous component of national status, $\widetilde{S_N}$. Given H_2, when $\widetilde{S_N}$ is high (the region on and above the upper solid line), everyone identifies with the nation, i.e., $p_{1S} = p_{2S} = p_{2U} = 1$.

[22]In contrast, using perceived closeness to one's country as a measure of national identity, which diverges from the concept of national identity in the present model, Staerklé et al. (2010) find that majority individuals are more likely to identify with the nation than minorities, although the majority-minority difference is small.

[23](mU) is located above (mS) and (M) is located above (mU) on the $(H_2, \widetilde{S_N})$ plane from Proposition 1 (i) and (ii)(b). The sign of the slope of each of (M) and (mU) when $p_{1S} = 1$ is the same as the sign when $p_{1S} = 0$ from Lemma A1. In the initial period, $\bar{q}_1 = 1$, $\bar{q}_2 = 0$, and $\bar{q} = N_1$, and the last term on the RHS of (M) equals $\frac{1}{\gamma\delta} [\rho\chi(1-\chi) + \beta\omega_q](1-N_1)^2$, while for (mU) and (mS), it equals $\frac{1}{\gamma\delta} [\rho\chi(1-\chi) + \beta\omega_q](N_1)^2$.

When $\widetilde{S_N}$ is low (the region below the lower solid line), everyone identifies with their ethnic group, i.e., $p_{1S} = p_{2S} = p_{2U} = 0$. When $\widetilde{S_N}$ falls within the intermediate range, the majority identify with their ethnic group, while for relatively high $\widetilde{S_N}$ (the region enclosed by the dotted lines), the minority identify with the nation, i.e. $p_{1S} = 0$, $p_{2S} = p_{2U} = 1$,[24] and for relatively low $\widetilde{S_N}$ (the region between the lower dotted line and the lower solid line), the skilled (unskilled) minority identify with the nation (their ethnic group), i.e., $p_{1S} = 0$, $p_{2S} = 1$, $p_{2U} = 0$. As demonstrated in Proposition 1, the figure indicates that the minority are more likely to identify with the nation than the majority, and within the minority, the skilled are more likely to identify with the nation than the unskilled: $p_{1S} = 1$ only if $p_{2S} = p_{2U} = 1$ and $p_{2U} = 1$ only if $p_{2S} = 1$.

The figure also shows that as the proportion of skilled workers in the minority, H_2, is higher, the regions of mixed national and ethnic identities, i.e., $p_{1S} = 0$, $p_{2S} = 1$, $p_{2U} = 1$ or 0, shrink, whereas when $\widetilde{S_N}$ is high, the region of universal national identity expands, and when $\widetilde{S_N}$ is low, the region of universal *ethnic* identity expands. That is, as the average skill level of the minority is higher and thus the inter-ethnic skill gap is smaller, universal national (*ethnic*) identity becomes more likely when the exogenous factor of national status is high (low). As H_2 is higher, between-group income disparity is smaller. Thus, the majority's perceived distance to the "average national" is smaller, and national status relative to the majority's status is higher. In contrast, the skilled minority's perceived distance to the "average minority" is smaller, and the relative status of the minority to the nation is higher. When $\widetilde{S_N}$ is high (low), the effect on the majority (minority) dominates, and universal national (ethnic) identity emerges.

An increase in the exogenous element of the majority's status $\widetilde{S_1}$ shifts (M) upward on the $(H_2, \widetilde{S_N})$ plane, making universal national identity less likely to be realized. Conversely, an increase in $\widetilde{S_2}$ shifts (mS) downward, making universal ethnic identity more likely to be realized.[25]

Taken together with the results on the tax rate in Section 3.2, these findings indicate that income redistribution is implemented, and thus the inter-ethnic disparity in disposable income is reduced, only when $\widetilde{S_N}$ is sufficiently high. Such a favorable outcome is more likely when H_2 is high and $\widetilde{S_1}$ is low. In other words, when the belief that people share a glorious history or rich cultural heritage is weak and thus national status is low, income redistribution narrowing inter-ethnic income inequality is not implemented. This is especially so when there is a significant inter-ethnic skill gap or when the majority holds strong pride in their cultural legacy and history.

Figure 2 presents a similar graph for the case where $\beta > \gamma$ and $N_1 \leq \frac{\beta+\gamma}{2\beta}$. Unlike Figure 1, (mU) increases with H_2, but the previous results remain unchanged. Figure 3 presents a graph when $\beta > \gamma$, $N_1 > \frac{\beta+\gamma}{2\beta}$, and (mU) is located above (M) for small H_2.[26] Apart from a small difference, the above results remain true.[27]

These results are summarized in the following proposition.

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[24]The figure shows that in the region between the upper dotted line and the upper solid line, both $p_{1S} = p_{2S} = p_{2U} = 1$ and $p_{1S} = 0$, $p_{2S} = p_{2U} = 1$ are equilibria.

[25]An increase in N_1 has multiple effects. It shifts (mS) downward on the $(H_2, \widetilde{S_N})$ plane, making the skilled minority more likely to identify with the nation, thereby reducing the likelihood of universal ethnic identity. An increase in N_1 also shifts (M) downward, increasing the likelihood of universal national identity. However, in the real society, higher N_1 is typically associated with higher $\widetilde{S_1}$, decreasing the likelihood of universal national identity. Hence, the overall effect of N_1 on the realization of universal national identity is ambiguous.

[26]It is possible that (mU) is located above (M) for small H_2 from Proposition 1 (ii)(b). When (M) is above (mU) for any H_2, a figure similar to Figure 2 can be drawn.

[27]Since the relative position of (mU) to (M) is different for small and large H_2, a new case arises in which the majority and skilled minority identify with the nation, while the unskilled minority identify with their ethnic group, i.e., $p_{1S} = p_{2S} = 1$, $p_{2U} = 0$, when $\widetilde{S_N}$ is relatively high and H_2 is small.
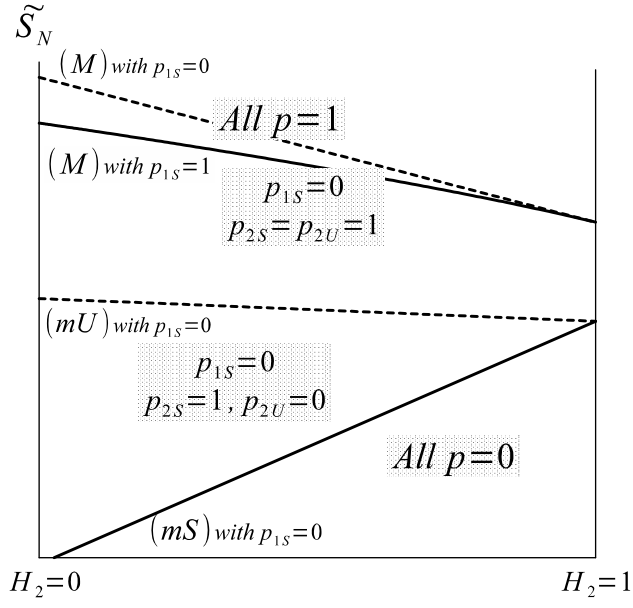
Figure 2: Identity in the initial period when $\beta > \gamma$ and $N_1 \leq \frac{\beta+\gamma}{2\beta}$
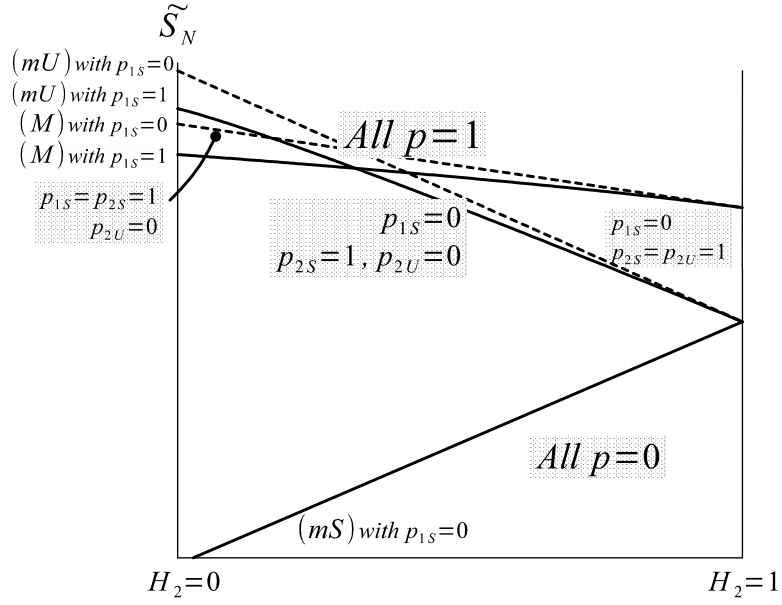

Figure 3: Initial identity when $\beta > \gamma$, $N_1 > \frac{\beta+\gamma}{2\beta}$, and (mU) is located above (M) for small H_2

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Proposition 2 (i) When $\widetilde{S_N}$ is high, everyone identifies with the nation, i.e., $p_{1S} = p_{2S} = p_{2U} = 1$ (except when $\beta > \gamma$ and $N_1 > \frac{\beta+\gamma}{2\beta}$, in which $p_{1S} = p_{2S} = 1$, $p_{2U} = 0$ can occur for small H_2); when $\widetilde{S_N}$ is low, everyone identifies with their ethnic group, i.e., $p_{1S} = p_{2S} = p_{2U} = 0$; and when $\widetilde{S_N}$ is in the intermediate range, the majority identify with their ethnic group, while all or skilled minority individuals identify with the nation, i.e, $p_{1S} = 0$, $p_{2S} = 1$, $p_{2U} = 1$ or 0.

(ii) As H_2 is higher, universal national identity is more likely to be realized when $\widetilde{S_N}$ is high, while when $\widetilde{S_N}$ is low, universal ethnic identity is more likely to be realized.

(iii) Higher $\widetilde{S_1}$ makes universal national identity less likely to occur, while higher $\widetilde{S_2}$ makes universal ethnic identity more likely.

(iv) Income redistribution is implemented only when $\widetilde{S_N}$ is sufficiently high. The policy is more likely to be carried out when H_2 is high and $\widetilde{S_1}$ is low.

3.4 Dynamics and long-run outcomes

The previous section examined how identity choices depend on exogenous parameters such as H_2 and $\widetilde{S_N}$ at a given point in time. However, identity choices evolve over time due to changes in cultural variables. Specifically, as (13) and (14) show, cultural variables, q_{1S}, q_{2S}, and q_{2U}, vary over time, and these changes shift the graphs of (M), (mU), and (mS) on the $(H_2, \widetilde{S_N})$ plane, influencing the identity choices of each type of individuals, p_{1S}, p_{2S}, and p_{2U}. Further, this affects the redistributive tax rate, thereby influencing inter-ethnic income disparity, and cultural variables in the next period. Thus, the dynamics of cultural variables and identity choices interact with each other. How does this interaction shape social identity and culture in the long run?

Based on Lemmas A2 and A3 in Appendix B and taking into account these interactions, the next proposition presents steady-state levels of the identity and cultural variables for each type of individuals. In the following, superscript $*$ is used to denote steady-state variables.

Proposition 3 (i) When $\widetilde{S_N}$ is high,[28] everyone identifies with the nation in the long run, i.e., $p_{1S}^* = p_{2S}^* = p_{2U}^* = 1$, unless $\beta > \gamma$, $N_1 > \frac{\beta+\gamma}{2\beta}$, and H_2 is low, in which case $p_{1S}^* = p_{2S}^* = 1, p_{2U}^* = 0$ can happen. Cultural integration occurs, i.e., when $p_{1S}^* = p_{2S}^* = p_{2U}^* = 1$, $q_{1S}^* = q_{2S}^* = q_{2U}^* = \overline{q}^\# \in$ (0,1), where $\overline{q}^\#$ is the value of $\overline{q}$ in the first period after which $p_{1S} = p_{2S} = p_{2U} = 1$ continues to hold.[29] As $\widetilde{S_N}$ is higher, the proportion of the minority element in the integrated culture is higher, i.e., when $p_{1S}^* = p_{2S}^* = p_{2U}^* = 1$, $\overline{q}^\#$ is smaller, with the maximum proportion being $1 - N_1$.

(ii) When $\widetilde{S_N}$ is low,[30] everyone identifies with their ethnic group in the long run, i.e., $p_{1S}^* = p_{2S}^* = p_{2U}^* = 0$, and cultures remain segregated, i.e., $q_{1S}^* = 1$ and $q_{2S}^* = q_{2U}^* = 0$.

(iii) When $\widetilde{S_N}$ is in the intermediate range,[31] the majority identify with their ethnic group, while all or skilled minority individuals identify with the nation in the long run, i.e., $p_{1S}^* = 0$, $p_{2S}^* = 1, p_{2U}^* = 0$ or 1, and cultural assimilation occurs, i.e., $q_{1S}^* = q_{2S}^* = q_{2U}^* = 1$.

(iv) (i) is more likely to occur when $\widetilde{S_1}$ is lower, while (ii) is more likely when ω_q and $\widetilde{S_2}$ are higher. As H_2 is higher, (i) [(ii)] is more likely to be realized when $\widetilde{S_N}$ is high (low). Further, $\overline{q}^\#$ in (i) is smaller as $\widetilde{S_1}$ and ω_q are lower and H_2 is higher.

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[28] To be precise, when $(H_2, \widetilde{S_N})$ is on or above steady-state (M) with $p_{1S} = 1$ on the $(H_2, \widetilde{S_N})$ plane, where steady-state (M) refers to (M) with steady-state levels of cultural variables.

[29] When $p_{1S}^* = p_{2S}^* = 1$, $p_{2U}^* = 0$, $q_{1S}^* = q_{2S}^* = q_{2U}^* = \overline{q}^\dagger \equiv \frac{1}{1-(1-H_2)N_1}\left[H_2 N_1 \overline{q}_1^\dagger + (1-N_1)\overline{q}_2^\dagger\right]$, where $\overline{q}_1^\dagger$ $(\overline{q}_2^\dagger)$ is the value of $\overline{q}_1$ $(\overline{q}_2)$ in the first period after which $p_{1S} = p_{2S} = 1$, $p_{2U} = 0$ continues to hold.

[30] To be accurate, when $(H_2, \widetilde{S_N})$ is below initial (mS) with $p_{1S} = 0$.

[31] To be precise, when $(H_2, \widetilde{S_N})$ is on or above initial (mS) with $p_{1S} = 0$ and below steady-state (M) with $p_{1S} = 0$.

Proof. See Appendix C. ∎

The relationships between parameters such as $\widetilde{S_N}$ and H_2 and the long-run social identities are qualitatively similar to the static result of Proposition 2. When $\widetilde{S_N}$ is high, everyone identifies with the nation in the long run (unless $\beta > \gamma$, $N_1 > \frac{\beta+\gamma}{2\beta}$, and H_2 is low, in which case it is possible that the majority and the skilled minority have a national identity and the unskilled minority have an ethnic identity). When $\widetilde{S_N}$ is low, everyone identifies with their ethnic group. When $\widetilde{S_N}$ is in the intermediate range, the majority identify with their ethnic group, while all or skilled minority individuals identify with the nation. Universal national identity is more likely to occur when $\widetilde{S_1}$ is lower, and universal ethnic identity is more likely to arise when $\widetilde{S_2}$ and ω_q (the weight on the cultural component in perceived distance) are higher. As H_2 is higher, universal national (ethnic) identity is more likely when $\widetilde{S_N}$ is high (low).

However, combinations of $\widetilde{S_N}$ and H_2 that lead to specific levels of p_{1S}, p_{2S}, and p_{2U} in the long run generally do not coincide with combinations of the parameters that yield the same levels of identity variables in the initial or transition periods. This is because changes in the levels of cultural variables shift the positions of graphs (M), (mU), and (mS) over time.

Specifically, when $\widetilde{S_N}$ is high, the region with universal national identity on the $(H_2, \widetilde{S_N})$ plane *expands over time* as (M) shifts downward (see, for example, Figure 1). Initially, universal national identity is achieved only under highly favorable conditions, i.e., when $\widetilde{S_N}$ is very high or both $\widetilde{S_N}$ and H_2 are high. Under less but still relatively favorable conditions (i.e., when $\widetilde{S_N}$ is relatively but not very high, or when $\widetilde{S_N}$ is high but H_2 is low), national identity is initially held by all or skilled minorities, while the majority identify with their ethnic group, i.e., $p_{1S} = 0$, $p_{2S} = 1$, $p_{2U} = 0$ or 1. Since some or all minorities identify with the nation, they are culturally influenced by the majority, and over time, their culture becomes more aligned with the majority' culture, i.e., q_{2S} and q_{2U} increase and become closer to $q_{1S} = 1$.[32] This decrease in inter-ethnic cultural distance causes (M) to shift downward, leading the majority to switch from ethnic to national identity at some point, resulting in universal national identity.[33]

Parameters such as $\widetilde{S_N}$ and H_2 also influences the long-run cultural composition. When $\widetilde{S_N}$ is sufficiently high to achieve universal national identity in the long run, *cultural integration* occurs, where everyone shares the same culture that contains elements from both minority and majority origins, i.e., $q_{1S}^* = q_{2S}^* = q_{2U}^* \in (0,1)$. The proportion of the minority element in the integrated culture increases with $\widetilde{S_N}$ (also increases with H_2 and decreases with $\widetilde{S_1}$ and ω_q), with the maximum proportion being its population share under universal national identity.[34] When $\widetilde{S_N}$ is low enough that universal ethnic identity is realized in the long run, *cultural segregation* persists, implying that the ethnic groups remain culturally dissimilar, i.e., $q_{1S}^* = 1$ and $q_{2S}^* = q_{2U}^* = 0$. When $\widetilde{S_N}$ is in the intermediate range so that the majority identify with their ethnic group and all or skilled minority individuals identify with the nation, *cultural assimilation* occurs, where everyone shares the culture originating *solely from the majority*. Cultural integration is more likely to arise when $\widetilde{S_1}$ (the exogenous component of the majority's status) is lower, while cultural segregation is more likely when ω_q and $\widetilde{S_2}$ are higher. Further, as H_2 is higher, cultural integration (segregation) is more likely to occur when $\widetilde{S_N}$ is high (low).

Universal national identity leads to cultural integration because both ethnic groups are cultur-

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[32]When the unskilled minority identify with their ethnic group, they are culturally influenced by the majority through the skilled minority, who identify with the nation.

[33]When $p_{1S} = 0$, $p_{2S} = 1$, $p_{2U} = 0$, this occurs after switching to $p_{1S} = 0$, $p_{2S} = p_{2U} = 1$, which is a result of a decrease in the cultural distance among the minority.

[34]When $\beta > \gamma$, $N_1 > \frac{\beta+\gamma}{2\beta}$, and H_2 is low, $p_{1S}^* = p_{2S}^* = 1$, $p_{2U}^* = 0$ is possible, in which case the maximum proportion equals $\frac{1-N_1}{1-(1-H_2)N_1}$, which is higher than the minority's population share.

ally influenced by the other group. As $\widetilde{S_N}$ is higher, the integrated culture has a higher share of the minority element because the periods during which the majority identify with their ethnic group and thus cultural assimilation proceeds (i.e., q_{2S} and q_{2U} increase with q_{1S} unchanged) are shorter. In contrast, universal ethnic identity leads to cultural segregation because there are no cultural interactions between the ethnic groups. Cultural assimilation occurs when national identity is held by some or all minority individuals but not by the majority, because cultural influence only flows from the majority to the minority.

Finally, the above result has the following implications for the tax rate, the disposable incomes of skilled and unskilled workers, and inter-ethnic economic inequality.

Corollary 1 *When $\widetilde{S_N}$ is high, $p_{1S} = 1$ and $\tau = \frac{\beta-1}{1+\gamma}\frac{w_s-\overline{w}}{\overline{w}}$ always or after some point in time, whereas when $\widetilde{S_N}$ is not high, $p_{1S} = 0$ and $\tau = 0$ always. The disposable income of unskilled (skilled) workers is higher (lower) and ethnic inequality is lower when $p_{1S} = 1$. $\tau = \frac{\beta-1}{1+\gamma}\frac{w_s-\overline{w}}{\overline{w}}$ is more likely to occur when $\widetilde{S_1}$ is low and H_2 is high.*

When $\widetilde{S_N}$ is high, the majority identify with the nation and the rate of redistributive tax is positive from the beginning, or they switch to a national identity and the tax rate becomes positive after a certain period. Conversely, when $\widetilde{S_N}$ is not high, they identify with their ethnic group and the tax rate remains zero. Consequently, when $\widetilde{S_N}$ is high, the disposable income of unskilled (skilled) workers is higher (lower) and thus the inter-ethnic inequality in disposable income is smaller. This positive outcome is more likely to occur when $\widetilde{S_1}$ is low and H_2 is high.

The national identity of two ethnic groups has different effects on disposable incomes and inter-ethnic economic inequality. The national identity of the majority induces them to be concerned about the disparity with the minority and the minority's welfare, leading to the adoption of redistributive taxation, which affects the economic outcomes. By contrast, the national identity of the minority leads them to gradually adopt the majority's culture, narrowing inter-ethnic cultural differences. When $\widetilde{S_N}$ is relatively but not very high (and $\widetilde{S_1}$ is not very high), this results in the adoption of national identity by the majority and the implementation of income redistribution.

While social identity has only a static economic effect, i.e., the just-mentioned effect on current disposable incomes and inequality, in the model with constant H_2, it has dynamic economic effects in the model with endogenous H_2, as will be shown later.

4 Model with social mobility

So far, for ease of exposition, the model assumed that children inherit their parents' classes and thus H_2 is constant. This section removes this unrealistic assumption and introduces educational investment that can alter H_2 over time. The newly introduced elements of the model are based on Galor and Zeira (1993) and Yuki (2007, 2008), in which individuals with varying levels of wealth inherited from their parents decide on educational spending that must be self-financed and is required to become skilled workers. In this model, H_2 may evolve over time and social identity has important dynamic effects on economic outcomes.

4.1 Model

The basic structure of the model is similar to the previous model, but several additional settings are introduced to incorporate assets that are transferred intergenerationally and are used for educational investment. Appendix A presents the production part of the model and shows that the skilled wage w_s, the unskilled wage w_u, and the interest rate r are constant.

Lifetime of an individual is described as follows. In childhood, she receives a transfer b from her parent and allocates it to assets a and educational expenditure e, which is required for becoming

a skilled worker, to maximize the utility given by (22) below.[35] She considers not just the impact of her investment decision on future income but also on the socio-psychological components of her utility. The educational investment is binary (i.e. taking education or not), costs $\bar{e}$, and yields a gross economic return of $w_s - w_u$. The investment must be self-financed due to the absence of credit markets. Thus, when $b < \bar{e}$, she does not expend on education, i.e., $e = 0$, and becomes an unskilled worker. In adulthood, the individual earns income from assets and work and spends it on consumption c and a transfer to her child b'. As in the previous model, she also chooses a group with which she identifies, votes on the tax-transfer policy, and molds her child's culture.

When she belongs to ethnic group J ($J = 1, 2$) and class C ($J = S, U$), and identifies with group G ($G = J, N$), she maximizes the following utility subject to the budget constraint:

$$\max \ v^i_{JCG} = \frac{1}{(\lambda)^\lambda (1-\lambda)^{1-\lambda}}(b')^\lambda (c)^{1-\lambda} - \beta d^i_{JCG} + \gamma S_G - \rho D^i_{JCG}, \ \ \lambda \in (0,1), \tag{22}$$

$$s.t. \quad c + b' = (1-\tau)w_C + T + (1+r)a, \tag{23}$$

where, as before, d^i_{JCG} is her perceived distance, S_G is the status of the identity group, and D^i_{JCG} is the composite distance of the child's culture.

By solving the maximization problem, the following consumption and transfer rules are obtained.

$$c = (1-\lambda)[(1-\tau)w_C + T + (1+r)a], \tag{24}$$

$$b' = \lambda[(1-\tau)w_C + T + (1+r)a]. \tag{25}$$

The results on identity choice, the tax rate, and cultural investment remain the same as before, since the indirect utility function equals the utility function of the original model plus $(1 + r)a$.[36]

4.2 Determination of H_1 and H_2

From the above setting, H_2 is equal to the proportion of group 2 individuals who receive $b \geq \bar{e}$ *and* spend $e = \bar{e}$ in childhood. Let F_2 be the proportion of individuals who receive $b \geq \bar{e}$, i.e., those who can afford education. Then, if the utility gain from educational investment is non-negative even when everyone with $b \geq \bar{e}$ takes education, $H_2 = F_2$ holds, whereas if the utility gain is negative with $H_2 = F_2$, H_2 is smaller than F_2. The proportion of skilled workers in the majority, H_1, is also endogenously determined, under the assumption that $F_1 = 1$ in the initial period.

The following assumption is imposed regarding the economic return to education.

Assumption 3 $\frac{2}{3}(w_s - w_u) - (1+r)\bar{e} > 0.$

The assumption posits that taking education raises disposable income, in other words, the economic return to education is positive, even when the rate of redistributive tax is $\frac{1}{3}$, the highest possible value under Assumption 1.

The next lemma shows how H_1 and H_2 are determined.

Lemma 2 *Suppose $F_1 = 1$. Under Assumption 3:*

(i) $H_1 = 1$.[37]

(ii) (a) When $p_{2S} = p_{2U} = 1$ or when $p_{1S} = 1, p_{2S} = 1, p_{2U} = 0$, $H_2 = F_2$.

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[35]For individuals in the initial generation, b is given.

[36]Asset income is assumed to be untaxed to keep these results unchanged and to be consistent with the fact that it is less heavily taxed than labor income.

[37]$H_1 = 0$ is also an equilibrium because the utility gain from education becomes negative when everyone with $b \geq \bar{e}$ does not take education. However, this equilibrium appears unrealistic and uninteresting, thus is not considered.
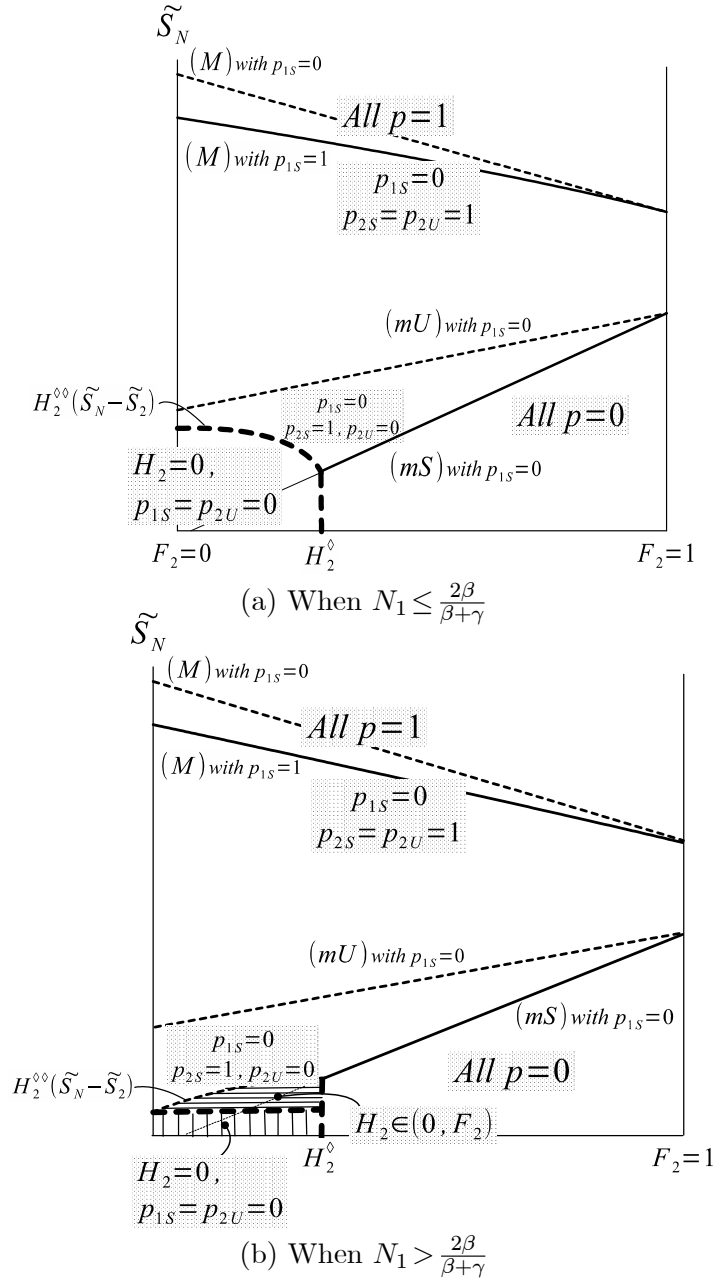
$\widetilde{S}_N$

$(M)\,with\,p_{1S}=0$

All p=1

$(M)\,with\,p_{1S}=1$

$p_{1S}=0$
$p_{2S}=p_{2U}=1$

$(mU)\,with\,p_{1S}=0$

$H_2^{\Diamond\Diamond}(\widetilde{S}_N-\widetilde{S}_2)$

$p_{1S}=0$
$p_{2S}=1,p_{2U}=0$

All p=0

$(mS)\,with\,p_{1S}=0$

$H_2=0,$
$p_{1S}=p_{2U}=0$

$F_2=0$ $H_2^{\Diamond}$ $F_2=1$

(a) When $N_1 \leq \frac{2\beta}{\beta+\gamma}$

$\widetilde{S}_N$

$(M)\,with\,p_{1S}=0$

All p=1

$(M)\,with\,p_{1S}=1$

$p_{1S}=0$
$p_{2S}=p_{2U}=1$

$(mU)\,with\,p_{1S}=0$

$(mS)\,with\,p_{1S}=0$

$H_2^{\Diamond\Diamond}(\widetilde{S}_N-\widetilde{S}_2)$

$p_{1S}=0$
$p_{2S}=1,p_{2U}=0$

All p=0

$H_2\in(0,F_2)$

$H_2=0,$
$p_{1S}=p_{2U}=0$

$H_2^{\Diamond}$ $F_2=1$

(b) When $N_1 > \frac{2\beta}{\beta+\gamma}$

Figure 4: Identity and H_2 for the full-fledged model when $\beta<\gamma$

(b) When $p_{1S}=0, p_{2S}\in[0,1], p_{2U}=0$,[38] if $\widetilde{S}_N-\widetilde{S}_2$ is relatively large, $H_2=F_2$, while if $\widetilde{S}_N-\widetilde{S}_2$ is relatively small, $H_2=0$ for small F_2 and $H_2=F_2$ for greater F_2. Further, when $\beta<\gamma$ and $N_1 > \frac{2\beta}{\beta+\gamma}$, if $\widetilde{S}_N-\widetilde{S}_2$ is in the intermediate range, $H_2=F_2$ for small and large F_2, and $H_2\in(0,F_2)$ for intermediate F_2.[39]

Proof. See Appendix C. ∎

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[38]A more precise statement is that when $p_{1S}=0, p_{2S}\in[0,1], p_{2U}=0$ is realized in adulthood *if* $H_2=F_2$ *holds in childhood*. Actually, when $H_2=0$ is true, $p_{1S}=0, p_{2U}=0$ is realized. $p_{2S}\in(0,1)$ when group 2 skilled individuals are culturally heterogenous, i.e., q_{2S}^i is heterogenous, which can occur despite the group being initially homogenous.

[39]To be more precise, as illustrated in Figure 4 (a), when $\beta\geq\gamma$ or $N_1 \leq \frac{2\beta}{\beta+\gamma}$, and $\widetilde{S}_N-\widetilde{S}_2$ is relatively low, $H_2=0$ for $F_2 < H_2^{\Diamond\Diamond}(\widetilde{S}_N-\widetilde{S}_2)$ when $p_{1S}=0, p_{2S}=1, p_{2U}=0$ and for $F_2 < H_2^{\Diamond}$ when $p_{1S}=p_{2S}=p_{2U}=0$, where $H_2^{\Diamond\Diamond\prime}(\widetilde{S}_N-\widetilde{S}_2)<0$. When $\beta<\gamma$ and $N_1 > \frac{2\beta}{\beta+\gamma}$ (the case illustrated in Figure 4 (b)), and $\widetilde{S}_N-\widetilde{S}_2$ is in the intermediate range, $H_2=H_2^{\Diamond\Diamond}(\widetilde{S}_N-\widetilde{S}_2)$ for $F_2\in(H_2^{\Diamond\Diamond}(\widetilde{S}_N-\widetilde{S}_2), H_2^{\Diamond})$ (where $H_2^{\Diamond\Diamond\prime}(\widetilde{S}_N-\widetilde{S}_2)>0$), whereas when $\widetilde{S}_N-\widetilde{S}_2$ is relatively small, $H_2=0$ for $F_2<H_2^{\Diamond}$. See the proof for the definitions of $H_2^{\Diamond\Diamond}(\widetilde{S}_N-\widetilde{S}_2)$ and $H_2^{\Diamond}$ and the critical levels of $\widetilde{S}_N-\widetilde{S}_2$.

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As for the majority, $H_1 = 1$ when $F_1 = 1$. The result for the minority differs depending on social identity. Figure 4 illustrates identity and H_2 when $\beta < \gamma$, where $H_2 = F_2$ unless otherwise noted.[40] When $p_{2S} = p_{2U} = 1$ or $p_{1S} = 1, p_{2S} = 1, p_{2U} = 0$, $H_2 = F_2$ for any F_2. In contrast, when $p_{1S} = 0, p_{2S} \in [0, 1], p_{2U} = 0$ (the region below (mU) with $p_{1S} = 0$ in the figure), if $\widetilde{S_N} - \widetilde{S_2}$ (the difference between the exogenous component of national status and that of the minority's status) is relatively large, $H_2 = F_2$; while if $\widetilde{S_N} - \widetilde{S_2}$ is relatively small, $H_2 = 0$ for small F_2 and $H_2 = F_2$ for larger F_2. (Further, when $\beta < \gamma$ and $N_1 > \frac{2\beta}{\beta + \gamma}$ [panel (b)], and $\widetilde{S_N} - \widetilde{S_2}$ is in the intermediate range, $H_2 = F_2$ for small and large F_2 and $H_2 \in (0, F_2)$ for intermediate F_2.) That is, the minority may not take education despite a *positive* economic return to education. This occurs because their socio-psychological return to education can be negative when the proportion of those able to afford education is low, due to the large perceived distance of an educated minority to her coethnics.[41]

4.3   Dynamics of F_2 and H_2

Given the distribution of b in the initial period and thus the initial value of F_2, the dynamics of F_2 are determined by the dynamics of b of each lineage. Consider an individual who is born in period $t-1$ and spends her adulthood in period t. Her investment decision depends on the received transfer and the sign of the utility return to education (henceforth, subscript t represents variables for those who become adults in period t):

$$\text{If } b_t < \bar{e} \text{ or the utility return to education is negative,} \quad a_t = b_t, \ e_t = 0, \tag{26}$$

$$\text{If } b_t \geq \bar{e} \text{ } and \text{ the utility return to education is positive,} \quad a_t = b_t - \bar{e}, \ e_t = \bar{e}. \tag{27}$$

By substituting (26) into (25), the equation linking the received transfer b_t to the transfer to her child b_{t+1} when she does not take education and thus remains unskilled equals

$$b_{t+1} = \lambda[(1 - \tau_t)w_u + T_t + (1 + r)b_t]. \tag{28}$$

Similarly, the corresponding equation when she is a skilled worker is

$$b_{t+1} = \lambda[(1 - \tau_t)w_s + T_t + (1 + r)(b_t - \bar{e})]. \tag{29}$$

Consider the case $H_{2,t} = F_{2,t}$. $F_{2,t+1} \geq F_{2,t}$ holds iff all the children of skilled workers can afford education, i.e., for any lineage satisfying $b_t \geq \bar{e}$, $b_{t+1} \geq \bar{e}$. From (29), this is the case if

$$\lambda[(1 - \tau_t)w_s + T_t] \geq \bar{e}. \tag{30}$$

This is true under Assumption 4 (i) below.

Then, $F_{2,t+1} > F_{2,t} \Leftrightarrow H_{2,t+1} > H_{2,t}$ iff there exist lineages satisfying $b_t < \bar{e}$ and $b_{t+1} \geq \bar{e}$. From (28), this is true only if $\lambda\{(1 - \tau_t)w_u + T_t + (1 + r)b_t\} \geq \bar{e}$ holds for some $b_t < \bar{e}$, that is, ($\lambda(1 + r) < 1$ is assumed)

$$\frac{\lambda}{1 - \lambda(1 + r)}[(1 - \tau_t)w_u + T_t] > \bar{e}. \tag{31}$$

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[40]The result for H_2 when $\beta \geq \gamma$ is qualitatively similar to panel (a) of the figure. $p_{1S} = 1, p_{2S} = 1, p_{2U} = 0$ occurs only when $\beta > \gamma$ and $N_1 > \frac{\beta + \gamma}{2\beta}$.

[41]To be more precise, when $p_{1S} = 0, p_{2S} \in (0, 1], p_{2U} = 0$ (the region above (mS) with $p_{1S} = 0$ and below (mU) with $p_{1S} = 0$ in the figure) and $\widetilde{S_N} - \widetilde{S_2}$ is relatively small, this occurs because the small perceived distance of an uneducated minority to her coethnics (due to low F_2) *and* the minority's high status make it psychologically attractive to forgo education and identify with her ethnic group rather than with the nation.

In contrast, $F_{2,t+1} = F_{2,t} \Leftrightarrow H_{2,t+1} = H_{2,t}$ if $b_{t+1} = \lambda\{(1-\tau_t)w_u + T_t + (1+r)b_t\} < \overline{e}$ for any $b_t < \overline{e}$:

$$\frac{\lambda}{1-\lambda(1+r)}\left[(1-\tau_t)w_u + T_t\right] \le \overline{e}. \tag{32}$$

Next, consider the case $H_{2,t} < F$, where $p_{1S,t} = 0$ from Lemma 2 (ii). $F_{2,t+1} > F_{2,t}$ only if (31) holds with $\tau_t = T_t = 0$, i.e., $\frac{\lambda}{1-\lambda(1+r)}w_u > \overline{e}$, while $F_{2,t+1} \le F_{2,t}$ if $\frac{\lambda}{1-\lambda(1+r)}w_u \le \overline{e}$. In this case, the level of H_2 is determined as described in Lemma 2.

Hereafter, the following assumption is imposed to simplify analysis.

Assumption 4 (i) *β or w_s is large enough or γ is small enough that $\frac{\lambda}{1-\lambda(1+r)}[(1-\tau)w_u + T]$ at $H_2 = 0$ when $p_{1S} = 1$ is sufficiently larger than $\overline{e}$.*[42]

(ii) *A not-small proportion of group 2 individuals do not have wealth in the initial period.*

The first part of the assumption implies that the scale of redistribution when $p_{1S} = 1$ is large enough (due to sufficiently large $\tau = \frac{\beta-1}{1+\gamma}\frac{w_s-\overline{w}}{\overline{w}}$) that descendants of unskilled workers can gradually accumulate wealth sufficient for education, unless H_2 is quite large, where τ is close to 0. This ensures that H_2 increases over time for not large H_2. The second part states that a not-small proportion of minority individuals in the initial generation do not have wealth.

The next lemma demonstrates that the second part of the assumption, combined with the first part, ensures that the tax-redistribution policy leads to $H_2 = 1$ in the long run. It also proves that H_2 non-decreases over time even without redistribution, except in one specific situation described in the lemma, where H_2 could decrease. Further, the lemma shows that $H_1 = 1$ always, which is because $F_1 = 1$ for all periods when $F_1 = 1$ initially under Assumption 4 (i).

Lemma 3 (i) *With $F_1 = 1$ in the initial period and Assumption 4 (i), $H_1 = 1$ always.*

(ii) *Under Assumption 4, if $p_{1S} = 1$ and thus $\tau = \frac{\beta-1}{1+\gamma}\frac{w_s-\overline{w}}{\overline{w}}$, H_2 increases over time and becomes 1 in the long run, while if $p_{1S} = 0$ and thus $\tau = 0$, H_2 non-decreases over time, except when $\beta < \gamma$, $N_1 > \frac{2\beta}{\beta+\gamma}$, and $\widetilde{S_N} - \widetilde{S_2}$ is not very, but relatively, low so that $p_{2S} \in (0,1], p_{2U} = 0$, in which case $H_2 \in (0, F_2)$ may decrease.*

Proof. See Appendix C. ∎

5 Main results of the model with social mobility

Based on Lemmas 2 and 3, this section presents and discusses the main results of the model with social mobility. Because H_2 is an endogenous variable and may evolve, inter-ethnic inequality in pre-tax income may change over time. Social identity influences the dynamics of H_2 and thus the dynamics of inter-ethnic earnings inequality.

5.1 When $\frac{\lambda}{1-\lambda(1+r)}w_u \le \overline{e}$

This subsection examines the case in which the unskilled wage w_u is low enough or the cost of education $\overline{e}$ is high enough that $\frac{\lambda}{1-\lambda(1+r)}w_u \le \overline{e}$ holds. In this case, F_2 and H_2 increase over time only when $p_{1S} = 1$ and thus $\tau > 0$.

The next proposition, which corresponds to Proposition 3 for the model with constant H_2, shows how social identities and cultural variables in the steady state depend on exogenous parameters such as $\widetilde{S_N}$ and ω_q (the weight on the cultural component in the perceived distance), as well as F_2 in the initial period.

Proposition 4 *Suppose that $\frac{\lambda}{1-\lambda(1+r)}w_u \le \overline{e}$ holds.*

[42]To be precise, $\frac{\lambda}{1-\lambda(1+r)}\left\{w_u + \frac{\beta-1}{1+\gamma}\frac{(1-N_1)(w_s-w_u)^2}{N_1 w_s + (1-N_1)w_u}\left[N_1 - \frac{1}{2}\frac{\beta-1}{1+\gamma}(1-N_1)\right]\right\}$ is sufficiently greater than $\overline{e}$.

(a) When $N_1 \leq \frac{2\beta}{\beta+\gamma}$

Within figure (a) labels:
$\widetilde{S}_N$ (vertical axis)
$SS\,(M)\,with\,p_{1S}=0$
$SS\,(M)\,with\,p_{1S}=1$
$All\ p^*=1,\,H_2^*=1$
$p_{1S}^*=0$
$p_{2S}^*=1,\,p_{2U}^*=1$
$\widetilde{S}_1$
$\widetilde{S}_2$
$H_2^{\diamond\diamond}(\widetilde{S}_N-\widetilde{S}_2)$
$All\ p^*=0$
$SS\,(mU)\,with\,p_{1S}=0$
$H_2^*=0$
$p_{1S}^*=p_{2U}^*=0$
$Initial\,(mS)\,with\,p_{1S}=0$
$H_2^{\diamond}$
$p_{1S}^*=0,\,p_{2S}^*=1,\,p_{2U}^*=0$
$F_2=1$

(b) When $N_1 > \frac{2\beta}{\beta+\gamma}$

Within figure (b) labels:
$\widetilde{S}_N$ (vertical axis)
$SS\,(M)\,with\,p_{1S}=0$
$SS\,(M)\,with\,p_{1S}=1$
$All\ p^*=1,\,H_2^*=1$
$p_{1S}^*=0$
$p_{2S}^*=1,\,p_{2U}^*=1$
$\widetilde{S}_1$
$\widetilde{S}_2$
$All\ p^*=0$
$SS\,(mU)\,with\,p_{1S}=0$
$H_2^{\diamond\diamond}(\widetilde{S}_N-\widetilde{S}_2)$
$Initial\,(mS)\,with\,p_{1S}=0$
$H_2^*=0$
$p_{1S}^*=p_{2U}^*=0$
$H_2^{\diamond}$
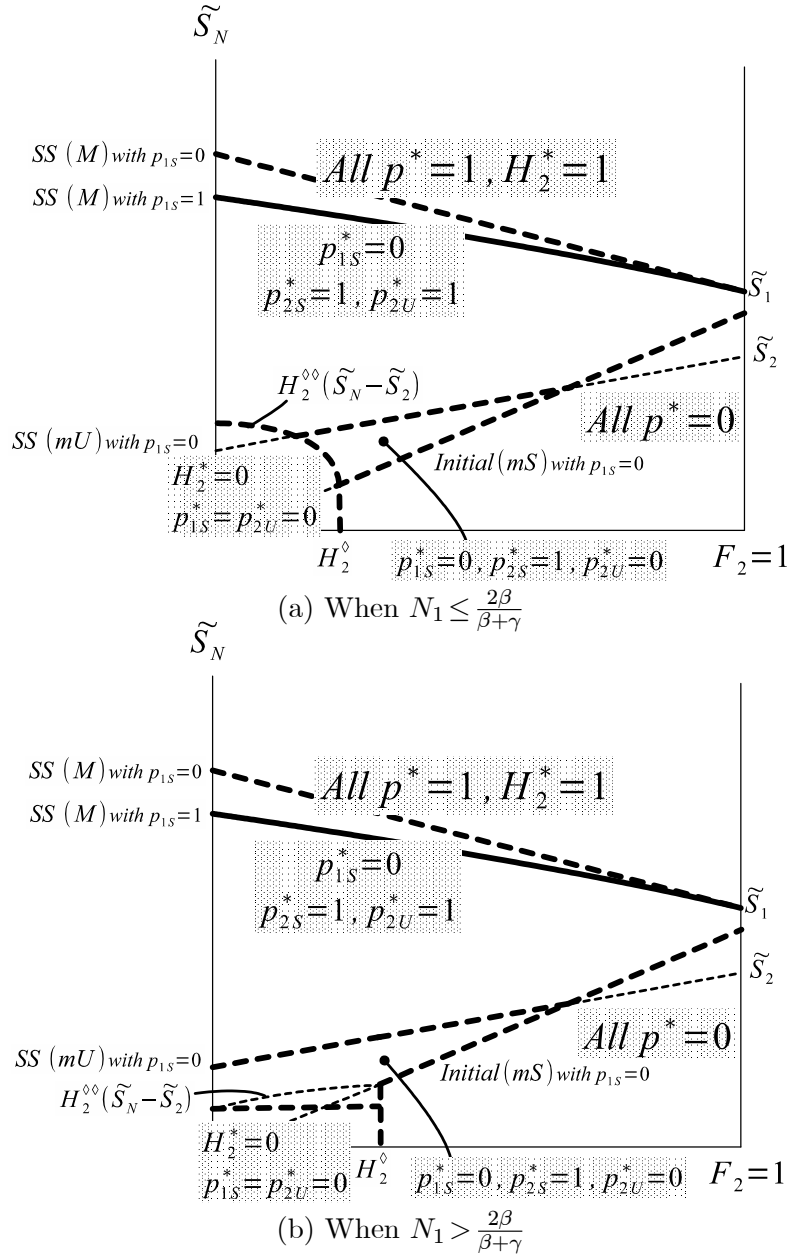$p_{1S}^*=0,\,p_{2S}^*=1,\,p_{2U}^*=0$
$F_2=1$

Figure 5: Relationship between initial $(F_2, \widetilde{S}_N)$ and steady-state outcomes for the full-fledged model with $\frac{\lambda}{1-\lambda(1+r)}w_u \leq \overline{e}$ when $\beta < \gamma$

(i) When $\widetilde{S}_N$ is high,[43] everyone identifies with the nation in the long run, i.e., $p_{1S}^* = p_{2S}^* = p_{2U}^* = 1$. Cultural integration occurs, i.e., $q_{1S}^* = q_{2S}^* = q_{2U}^* = \overline{q}^* = \overline{q}^{\#} \in (0,1)$, where $\overline{q}^{\#}$ is the value of $\overline{q}$ in the first period after which $p_{1S} = p_{2S} = p_{2U} = 1$ continues to hold. As $\widetilde{S}_N$ is higher, $\overline{q}^{\#}$ is lower, with the minimum value being N_1.

(ii) When $\widetilde{S}_N$ is low,[44] everyone identifies with their ethnic group in the long run, i.e., $p_{1S}^* = p_{2S}^* = p_{2U}^* = 0$, or $H_2^* = 0$ and $p_{1S}^* = p_{2U}^* = 0$, and cultures remain segregated, i.e., $q_{1S}^* = 1, q_{2S}^* = q_{2U}^* = 0$.

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[43]To be precise, $(F_2, \widetilde{S}_N)$ in the initial period is on or above steady-state (M) with $p_{1S} = 1$ on the $(F_2, \widetilde{S}_N)$ plane.

[44]To be precise, as illustrated in Figure 5, when $\beta \geq \gamma$ or $N_1 \leq \frac{\beta+\gamma}{2\beta}$, this is the case when initial $(F_2, \widetilde{S}_N)$ is below initial (mS) with $p_{1S} = 0$ or when F_2 is smaller than initial $H_2^{\diamond\diamond}(\widetilde{S}_N - \widetilde{S}_2)$, where $H_2^{\diamond\diamond\prime}(\widetilde{S}_N - \widetilde{S}_2) \leq 0$. When $\beta < \gamma$ and $N_1 > \frac{\beta+\gamma}{2\beta}$, the condition is that initial $(F_2, \widetilde{S}_N)$ is below initial (mS) with $p_{1S} = 0$ for $F_2 \geq H_2^{\diamond}$ and $\widetilde{S}_N - \widetilde{S}_2$ is smaller than the level such that initial $H_2^{\diamond\diamond}(\widetilde{S}_N - \widetilde{S}_2)$ equals 0 for $F_2 < H_2^{\diamond}$, where $H_2^{\diamond\diamond\prime}(\widetilde{S}_N - \widetilde{S}_2) > 0$.

(iii) When $\widetilde{S_N}$ is in the intermediate range,[45] the majority identify with their ethnic group, while all or skilled minority individuals identify with the nation in the long run, i.e., $p_{1S}^*=0$, $p_{2S}^*=1$, $p_{2U}^*=0$ or 1, and cultural assimilation occurs, i.e., $q_{1S}^*=q_{2S}^*=q_{2U}^*=1$.

(iv) (i) is more likely to occur when $\widetilde{S_1}$ is lower and (ii) is more likely to be realized when ω_q and $\widetilde{S_2}$ are higher. (i) and (ii) are more likely when initial F_2 is higher. $\overline{q}^\#$ in (i) is lower as initial F_2 is higher and $\widetilde{S_1}$ and ω_q are lower.

Proof. See Appendix C. ∎

Similar to the model with constant H_2, when the exogenous component of the national status $\widetilde{S_N}$ is high, everyone identifies with the nation in the long run, i.e., $p_{1S}^*=p_{2S}^*=p_{2U}^*=1$, and *cultural integration* occurs, i.e. $q_{1S}^*=q_{2S}^*=q_{2U}^*\in(0,1)$, where the proportion of the minority element in the integrated culture increases with $\widetilde{S_N}$ (also increases with initial F_2 and decreases with $\widetilde{S_1}$ and ω_q), with the maximum proportion being its population share; when $\widetilde{S_N}$ is low, everyone identifies with their ethnic group in the long run, i.e., $p_{1S}^*=p_{2S}^*=p_{2U}^*=0$ or $H_2^*=0$ and $p_{1S}^*=p_{2U}^*=0$, and *cultures remain segregated*, i.e., $q_{1S}^*=1, q_{2S}^*=q_{2U}^*=0$; and when $\widetilde{S_N}$ falls within the intermediate range, the majority identify with their ethnic group, while all or skilled minority individuals identify with the nation in the long run, i.e., $p_{1S}^*=0$, $p_{2S}^*=1$, $p_{2U}^*=0$ or 1, and *cultural assimilation* occurs, i.e., $q_{1S}^*=q_{2S}^*=q_{2U}^*=1$.

Universal national identity and cultural integration are more likely when the exogenous factor of the majority's status $\widetilde{S_1}$ is lower. In contrast, universal ethnic identity and cultural segregation are more likely when ω_q and $\widetilde{S_2}$ are higher, i.e., people are more concerned about cultural differences and the minority are more proud of their group for exogenous reasons. As initial F_2 is higher, universal *ethnic* (national) identity is more prone to arise when $\widetilde{S_N}$ is relatively low (high) [see Figure 5]. That is, smaller inter-ethnic economic disparity renders universal *ethnic* (national) identity and thus cultural *segregation* (integration) more likely when the exogenous factor of national status is low (high). This occurs because smaller economic differences make the identity choices of the two groups more similar and more strongly affected by non-economic factors such as national status.

Figure 5 illustrates the relationship between $(F_2, \widetilde{S_N})$ in the initial period and social identities in the steady state when $\frac{\lambda}{1-\lambda(1+r)}w_u \leq \overline{e}$ and $\beta < \gamma$. Panels (a) and (b) depict the relationship for the case $N_1 \leq \frac{2\beta}{\beta+\gamma}$ and $N_1 > \frac{2\beta}{\beta+\gamma}$, respectively. (The relationship when $\beta \geq \gamma$ is shown in Figures **??** and **??** in the proof of the proposition.) There are two differences from the the model with constant H_2. First, when $\widetilde{S_N}$ is low, $H_2^*=0$ and $p_{1S}^*=p_{2U}^*=0$ hold for low F_2. This is because the socio-psychological return to education is significantly negative when $\widetilde{S_N}$ and F_2 are low, as explained after Lemma 2. Second, when $\widetilde{S_N}$ is high, $p_{1S}^*=p_{2S}^*=1, p_{2U}^*=0$ is not the steady state. This occurs as H_2 increases over time and becomes 1 in the long run due to $p_{1S}=1$ and thus $\tau > 0$.

The next proposition examines how the dynamics and long-run level of H_2 and ethnic inequality depend on exogenous parameters such as $\widetilde{S_N}$ and ω_q.

Proposition 5 *Suppose that* $\frac{\lambda}{1-\lambda(1+r)}w_u \leq \overline{e}$ *holds.*

(i) *When $\widetilde{S_N}$ is very high so that $p_{1S}=p_{2S}=1, p_{2U}=0$ or 1 in the initial period:*[46]

    (a) *The majority identify with the nation, i.e., $p_{1S}=1$, and $\tau > 0$ always. $H_2=F_2$ increases and inter-ethnic economic inequality decreases over time, and everyone becomes skilled workers, i.e., $H_2^*=1$, and income equality is achieved in the long run.*

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[45]To be precise, when initial $(F_2, \widetilde{S_N})$ is below steady-state (M) with $p_{1S}=0$ and above the region for $p_{1S}^*=p_{2S}^*=p_{2U}^*=0$ or $H_2^*=0$ and $p_{1S}^*=p_{2U}^*=0$.

[46]To be precise, when $(F_2, \widetilde{S_N})$ in the initial period is on or above initial (M) with $p_{1S}=1$ on the $(F_2, \widetilde{S_N})$ plane.

(b) This case is more likely to occur when ω_q and $\widetilde{S_1}$ are lower and initial F_2 is higher.

(ii) When $\widetilde{S_N}$ is not very high, the majority identify with their ethnic group initially.

(a) If $\widetilde{S_N}$ is relatively high,[47] the majority switch to a national identity, τ becomes positive, $H_2 = F_2$ starts increasing, and inter-ethnic economic inequality begins to decline at some point. In the long run, $H_2^* = 1$ and ethnic inequality disappears. The increase of $H_2 = F_2$ starts earlier when ω_q and $\widetilde{S_1}$ are lower and $\widetilde{S_N}$ and initial F_2 are higher.

(b) Otherwise,[48] $\tau = 0$, F_2 is constant or decreases, H_2 is usually constant (it may decrease when $\beta < \gamma$ and $N_1 > \frac{\beta+\gamma}{2\beta}$), $H_2^* = F_2^* < 1$, and inter-ethnic economic inequality persists.

(iii) (i) or (ii)(a) [(ii)(b)] is more likely to occur when $\widetilde{S_1}$ is lower (higher) and initial F_2 is higher (lower).

Proof. See Appendix C. ∎

When $\widetilde{S_N}$ is very high, everyone always identifies with the nation and income redistribution is consistently implemented. Thus, the disposable income of unskilled workers is high enough for their descendants to gradually accumulate wealth sufficient for education. Consequently, the proportion of the skilled minority increases and inter-ethnic income inequality declines over time. In the long run, everyone is skilled, i.e., $H_2^* = 1$, and income equality is attained. This case is more likely to occur when ω_q and $\widetilde{S_1}$ are lower and initial F_2 is higher, in other words, when people are less concerned about cultural differences, the majority's pride in their group is weaker for exogenous reasons, and the initial share of the minority accessible to education is higher.

If $\widetilde{S_N}$ is not very high, but relatively so, initially, the majority identify with their group and redistribution is not carried out. Hence, none of the children of unskilled workers can afford education and H_2 stays constant, implying that inter-ethnic economic inequality remains unchanged. In contrast, since all or skilled minorities identify with the nation, they are culturally influenced by the majority, leading to a decrease in inter-group cultural distance over time. When the cultural distance diminishes sufficiently, due to the relatively high $\widetilde{S_N}$, the majority switches to a national identity, and income redistribution begins. Thereafter, the proportion of the skilled minority increases and inter-ethnic inequality decreases over time, and $H_2^* = 1$ and income equality are achieved eventually. The increase of $H_2 = F_2$ starts earlier when $\widetilde{S_N}$ is higher and the other parameters satisfy similar conditions as the previous case, i.e., ω_q and $\widetilde{S_1}$ are lower and initial F_2 is higher.

Otherwise, the majority always identity with their group, redistribution is never implemented, and the share of the minority accessible to education is constant or decreases. Hence, some or all minorities are unskilled and ethnic inequality persists. This case is more likely as $\widetilde{S_1}$ is higher and initial F_2 is lower.

5.2 When $\frac{\lambda}{1-\lambda(1+r)}w_u > \overline{e}$

When $\frac{\lambda}{1-\lambda(1+r)}w_u > \overline{e}$, F_2 always increases over time and $H_2^* = 1$. The next proposition analyzes social identities and cultural compositions in the steady state.

Proposition 6 *Suppose that $\frac{\lambda}{1-\lambda(1+r)}w_u > \overline{e}$ holds.*

[47]To be precise, when initial $(F_2, \widetilde{S_N})$ is on or above steady-state (M) with $p_{1S} = 1$ and below initial (M) with $p_{1S} = 0$.

[48]To be precise, when initial $(F_2, \widetilde{S_N})$ is below steady-state (M) with $p_{1S} = 0$.

(i) When $\widetilde{S}_N$ is high,[49] $p_{1S}^* = p_{2S}^* = p_{2U}^* = 1$. $q_{1S}^* = q_{2S}^* = q_{2U}^* = \overline{q}^* = \overline{q}^\# \in (0,1)$, where $\overline{q}^\#$ is the value of $\overline{q}$ in the first period after which $p_{1S} = p_{2S} = p_{2U} = 1$ continues to hold. As $\widetilde{S}_N$ is higher, $\overline{q}^\#$ is lower, with the minimum value being N_1.

(ii) When $\widetilde{S}_N$ is low,[50] $p_{1S}^* = p_{2S}^* = p_{2U}^* = 0$. $q_{1S}^* = 1$, $q_{2S}^* = q_{2U}^* = \overline{q}_2^* = \overline{q}_2^\flat \in [0,1)$, where $\overline{q}_2^\flat$ is the value of $\overline{q}_2$ in the first period after which $p_{2S} = p_{2U} = 0$ continues to hold.

(iii) When $\widetilde{S}_N$ is in the intermediate range,[51] $p_{1S}^* = 0$, $p_{2S}^* = p_{2U}^* = 1$ and $q_{1S}^* = q_{2S}^* = q_{2U}^* = 1$.

(iv) (i) is more likely when $\widetilde{S}_1$ is lower, and (ii) is more likely when ω_q and $\widetilde{S}_2$ are higher. $\overline{q}^\#$ in (i) is lower as initial F_2 is higher and $\widetilde{S}_1$ and ω_q are lower.

Proof. See Appendix C. ∎

Similar to the case $\frac{\lambda}{1-\lambda(1+r)} w_u \leq \overline{e}$, when $\widetilde{S}_N$ is high, universal national identity and cultural integration occur in the long run, which is more likely when $\widetilde{S}_1$ is lower; when $\widetilde{S}_N$ is low, universal ethnic identity and cultural segregation occur in the long run, which is more likely when ω_q and $\widetilde{S}_2$ are higher; and when $\widetilde{S}_N$ falls within the intermediate range, the majority (minority) identify with their ethnic group (the nation) and the minority are culturally assimilated into the majority in the long run. However, since F_2 increases over time and $H_2^* = 1$, unlike the previous case, only three identity equilibria are possible in the steady state: $p_{1S}^* = p_{2S}^* = p_{2U}^* = 1$ when $\widetilde{S}_N$ is high; $p_{1S}^* = p_{2S}^* = p_{2U}^* = 0$ when $\widetilde{S}_N$ is low; and $p_{1S}^* = 0, p_{2S}^* = p_{2U}^* = 1$ when $\widetilde{S}_N$ is in the intermediate range. Further, compared to the previous case, the ranges of initial $(F_2, \widetilde{S}_N)$ leading to $p_{1S}^* = p_{2S}^* = p_{2U}^* = 1$ and $p_{1S}^* = p_{2S}^* = p_{2U}^* = 0$ are greater because F_2 increases over time (see Figure 5).

While F_2 consistently increases over time, leading to $H_2^* = 1$, the speed at which $H_2^* = 1$ and inter-ethnic economic equality are achieved varies depending on exogenous parameters, as demonstrated in the next proposition.

Proposition 7 Suppose that $\frac{\lambda}{1-\lambda(1+r)} w_u > \overline{e}$ holds.

(i) When $\widetilde{S}_N$ is very high so that $p_{1S} = p_{2S} = 1, p_{2U} = 0$ or 1 in the initial period,[52] $p_{1S} = 1$ and $\tau > 0$ always, and $H_2^* = 1$ and inter-ethnic economic equality are achieved fastest. This case is more likely to occur when ω_q and $\widetilde{S}_1$ are lower and initial F_2 is higher.

(ii) When $\widetilde{S}_N$ is not very high:

    (a) If $\widetilde{S}_N$ is relatively high,[53] the shift from $p_{1S} = 0$ and $\tau = 0$ to $p_{1S} = 1$ and $\tau > 0$ occurs at some point, and convergence to $H_2^* = 1$ accelerates. $H_2^* = 1$ and inter-ethnic economic equality are achieved faster when ω_q and $\widetilde{S}_1$ are lower and $\widetilde{S}_N$ and initial F_2 are higher.

    (b) Otherwise, $p_{1S} = 0$ and $\tau = 0$ always, and convergence to $H_2^* = 1$ occurs slowly.

(iii) (i) or (ii)(a) [(ii)(b)] is more likely be realized when $\widetilde{S}_1$ is lower (higher).

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[49]The exact condition is $\widetilde{S}_N \geq \widetilde{S}_1$.

[50]$p_{1S}^* = p_{2S}^* = p_{2U}^* = 0$ if initial $(F_2, \widetilde{S}_N)$ is below initial (mS) with $p_{1S} = 0$ or $\widetilde{S}_N < \widetilde{S}_2$. Otherwise, it may be realized when $\widetilde{S}_N < \widetilde{S}_2 + \frac{1}{\gamma\delta}[\rho\chi(1-\chi) + \beta\omega_q](N_1)^2$ (the level of $\widetilde{S}_N$ at the intersection of initial (mS) with $p_{1S} = 0$ and $F_2 = 1$).

[51]$p_{1S}^* = 0, p_{2S}^* = p_{2U}^* = 1$ for certain when $\widetilde{S}_N \in \left[\widetilde{S}_2 + \frac{1}{\gamma\delta}[\rho\chi(1-\chi) + \beta\omega_q](N_1)^2, \widetilde{S}_1\right)$ and is possible when $\widetilde{S}_N \in \left[\widetilde{S}_2, \widetilde{S}_2 + \frac{1}{\gamma\delta}[\rho\chi(1-\chi) + \beta\omega_q](N_1)^2\right)$.

[52]To be precise, when initial $(F_2, \widetilde{S}_N)$ is on or above initial (M) with $p_{1S} = 1$ on the $(F_2, \widetilde{S}_N)$ plane.

[53]To be precise, when initial $(F_2, \widetilde{S}_N)$ is below initial (M) with $p_{1S} = 0$ and $\widetilde{S}_N \geq \widetilde{S}_1$.

Proof. See Appendix C. ∎

When $\widetilde{S_N}$ is very high, the majority always identify with the nation, and redistribution is consistently implemented. Thus, the proportion of the skilled minority increases fast, and $H_2^* = 1$ and inter-ethnic economic equality are achieved most rapidly. As in the previous case, this scinario is more likely to occur when ω_q and $\widetilde{S_1}$ are lower and initial F_2 is higher. When $\widetilde{S_N}$ is not very high, but relatively so, the majority identify with their group and no redistribution is carried out initially, but all or skilled minorities identify with the nation. Hence, as the proportion of the skilled minority increases and the minority's culture becomes closer to the majority's, the perceived distance of the majority to the minority decreases, eventually leading the majority to adopt a national identity. Consequently, redistribution begins, accelerating the increase in H_2 and the decrease in inter-ethnic inequality. As when $\frac{\lambda}{1-\lambda(1+r)}w_u \leq \overline{e}$, $H_2^* = 1$ and income equality are achieved faster when ω_q and $\widetilde{S_1}$ are lower and $\widetilde{S_N}$ and initial F_2 are higher. When $\widetilde{S_N}$ is low, the majority always identify with their group and redistribution is never carried out, resulting in a slower attainment of $H_2^* = 1$ and income equality. This case is more likely when $\widetilde{S_1}$ is higher.

5.3 Summary

The main results can be summarized as follows. When the exogenous component of national status $\widetilde{S_N}$ is very high, everyone always identifies with the nation and income redistribution is consistently implemented. As a result, the proportion of skilled workers in the minority increases rapidly and inter-ethnic income inequality declines swiftly. Eventually, everyone is skilled and income equality is achieved. Inter-ethnic cultural differences also diminish over time, leading to cultural integration, where the integrated culture contains elements from both minority and majority origins in proportion to their population shares. This scenario is more likely to occur when people are less concerned about cultural differences, i.e., ω_q is smaller, the exogenous factor of the majority's status $\widetilde{S_1}$ is lower, or the initial share of the minority accessible to education F_2 is higher.

When $\widetilde{S_N}$ is lower, initially, the majority identify with their group and redistribution is not carried out. Long-run outcomes differ greatly depending on the levels of exogenous parameters.

When $\widetilde{S_N}$ is relatively high, all or skilled minorities identify with the nation. Consequently, they are culturally influenced by the majority, leading to a decrease in inter-ethnic cultural distance over time. When the cultural distance decreases sufficiently, the majority switch to a national identity, and income redistribution begins. Subsequently, H_2 increases rapidly, and inter-ethnic economic inequality declines quickly. In the long run, income equality is achieved, and cultural integration occurs, although the integrated culture contains a greater share of the majority-origin element compared to the previous case. $H_2^* = 1$ and economic equality are achieved faster when $\widetilde{S_N}$ is higher and the other parameters satisfy similar conditions as the previous case, i.e., ω_q and $\widetilde{S_1}$ are lower and initial F_2 is higher. These two scenarios leading to income redistribution are more likely to occur when $\widetilde{S_1}$ is lower and initial F_2 is higher.

In contrast, when $\widetilde{S_N}$ is low, the majority always identity with their group, and redistribution is never carried out. Thus, H_2 grows only slowly and inter-ethnic inequality diminishes slowly (when $\frac{\lambda}{1-\lambda(1+r)}w_u > \overline{e}$), or H_2 non-increases and ethnic inequality persists (when $\frac{\lambda}{1-\lambda(1+r)}w_u \leq \overline{e}$). If $\widetilde{S_N}$ is very low, both ethnic groups identity with their group and cultural segregation persists. This negative outcome is more likely when $\widetilde{S_2}$ is higher or when ω_q is greater, i.e., when the minority's pride in their group is stronger for exogenous reasons or when people are concerned more about cultural differences. Otherwise, all or skilled minorities identify with the nation and gradually adopt the majority's culture. The long-run outcome is cultural assimilation, in which everyone shares the culture originating solely from the majority.

These results indicate that national identities of the majority and of the minority have distinct effects on inter-ethnic cultural and economic convergence. The majority's national identity is essential for inter-ethnic economic convergence, as income redistribution is implemented only when they identify with the nation. However, this is realized from the beginning only under very favorable conditions (e.g., when $\widetilde{S_N}$ is very high or ω_q is very low). When conditions are not very, but relatively favorable, the minority's national identity contributes to economic convergence through their gradual adoption of the majority's culture, which eventually leads to the majority's switch to a national identity and adoption of redistribution. In contrast, under less favorable conditions, the minority's national identity results in their cultural assimilation into the majority but does not foster economic convergence.

5.4   Policy Implications

Empirical studies find that *nation-building policies*, such as school education and government propaganda emphasizing common history, culture, and values, as well as policies promoting inter-group contact, are important for fostering national identity and unity (Blouin and Mukand, 2019; Cáeres-Delpiano et al., 2021; Chen, Lin, and Yang, 2023).[54] According to the model, these policies may be can be seen as enhancing national status (increasing $\widetilde{S_N}$) or reducing the salience of inter-ethnic cultural differences in people's minds (decreasing ω_q).

The above results imply that the effectiveness of nation-building policies for promoting inter-ethnic economic convergence depends on the pre-policy levels of parameters such as $\widetilde{S_N}$ and ω_q. When $\widetilde{S_N}$ is relatively, but not very high (or ω_q is relatively, but not very small), these policies would foster economic convergence and cultural integration. In contrast, when $\widetilde{S_N}$ is low (or ω_q is large), the policies would not contribute to economic convergence and lead to cultural assimilation of the minority.

6   Conclusion

[To be added]

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Appendix A   Production part of the model with social mobility

This Appendix presents the production part of the model with social mobility and shows that wages and the interest rate are constant. The model economy is small open and, following Galor and Zeira (1993), has two production technologies to produce final goods that can be used for consumption or investment. One technology uses skilled labor and capital to produce the good, which is expressed as

$$Y_S = F(K, L_S), \tag{33}$$

where Y_S is output, K is physical capital, $L_S (= N_1 + (1-N_1)H_2$ in equilibrium) is labor input, and the function F is concave and constant returns to scale (CRS).

The other technology uses unskilled labor as the sole input, which is given by

$$Y_U = w_u L_U, \tag{34}$$

where Y_U is output, L_U is labor input $(= (1 - N_1)(1 - H_2)$ in equilibrium), and w_u is constant productivity. Thus, the unskilled wage is constant and equals w_u.

Capital is freely mobile internationally. Let r be the time-invariant world interest rate. From the profit maximization condition of a representative firm with the CRS technology,

$$r = F_K(K, L_S). \tag{35}$$

Thus, $\frac{K}{L_S}$ is constant. Consequently, the skilled wage w_s is also time-invariant.

Appendix B   Lemmas A1−A5

B.1   Lemma A1

This subsection presents Lemma A1 that is used to prove the main results. The equations (M), (mU), and (mS) in Section 3.3 show that an individual's choice of identity depends on H_2. For proving the results, it is important to examine the relationship between H_2 and the RHS of each equation. The lemma proves that the sign of the relationship when $p_{1S} = 1$, in which case H_2 affects the RHS through $\tau = \frac{\beta-1}{1+\gamma}\frac{w_s - \overline{w}}{\overline{w}}$ as well, is the same as when $p_{1S} = 0$ under Assumption 1.

Lemma A1 *Under Assumption 1, the sign of the relationship between H_2 and the RHS of each of (M), (mU), and (mS) when $p_{1S} = 1$ is the same as the sign when $p_{1S} = 0$.*

Proof. See Appendix C. ■

B.2 Lemmas A2−A5

This subsection presents lemmas essential for proving Proposition 3, which examines steady-state levels of the identity and cultural variables for each type of individuals for the model with constant H_2, and a similar proposition for the model with endogenous H_2 (Proposition 4). The next lemma examines the directions of shifts of graphs of (M), (mU), and (mS) on the $(H_2, \widetilde{S_N})$ plane for every possible combination of p_{1S}, p_{2S}, and p_{2U} for the model with constant H_2.[55]

Lemma A2 *In the model with constant H_2, under the initial condition $q_1^i = 1$ and $q_2^i = 0$ for any i, the following holds as levels of the cultural variables change over time.*

(i) *(M) and (mS) in the initial period are located at higher positions than or the same positions as those in subsequent periods on the $(H_2, \widetilde{S_N})$ plane.*

(ii) *When $p_{1S} = p_{2S} = p_{2U} = 1$, (M) and (mU) shift downward over time on the $(H_2, \widetilde{S_N})$ plane. (mS) shifts downward (upward) when $\bar{q}_2 + \bar{q} - 2q_{2S} > (<)0$ and shifts downward in the long run.*

(iii) *When $p_{1S} = 1$, $p_{2S} = 1$, $p_{2U} = 0$ or when $p_{1S} = 0$, $p_{2S} = p_{2U} = 1$, (M) and (mU) shift downward. The direction of the shift of (mS) is ambiguous in the short run, but in the long run, it shifts downward.*

(iv) *When $p_{1S} = 0$, $p_{2S} = 1$, $p_{2U} = 0$, (M) shifts downward. (mU) and (mS) shift downward in the long run.*

(v) *When $p_{1S} = p_{2S} = p_{2U} = 0$, (M) does not shift, while (mU) shifts downward and (mS) shifts upward unless $q_{2S} = q_{2U}$, in which case they do not shift.*

Proof. See Appendix C. ∎

The next lemma examines steady-state levels of the cultural variables for every possible steady-state combination of p_{1S}, p_{2S}, and p_{2U} when H_2 is constant.

Lemma A3 *Suppose that H_2 is constant.*

(i) *When $p_{1S}^* = p_{2S}^* = p_{2U}^* = 1$, $q_{1S}^* = q_{2S}^* = q_{2U}^* = \bar{q}^\#$, where $\bar{q}^\#$ is the value of $\bar{q}$ in the first period after which $p_{1S} = p_{2S} = p_{2U} = 1$ continues to hold.*

(ii) *When $p_{1S}^* = p_{2S}^* = 1$, $p_{2U}^* = 0$, $q_{1S}^* = q_{2S}^* = q_{2U}^* = \bar{q}^\dagger \equiv \frac{1}{1-(1-H_2)N_1} \left[H_2 N_1 \bar{q}_1^\dagger + (1-N_1)\bar{q}_2^\dagger \right]$, where $\bar{q}_1^\dagger$ $(\bar{q}_2^\dagger)$ is the value of $\bar{q}_1(\bar{q}_2)$ in the first period after which $p_{1S} = p_{2S} = 1$, $p_{2U} = 0$ continues to hold.*

(iii) *When $p_{1S}^* = 0$ and at least one of p_{2S}^* and p_{2U}^* equals 1, $q_{1S}^* = q_{2S}^* = q_{2U}^* = \bar{q}_1^\flat$.*

(iv) *When $p_{1S}^* = p_{2S}^* = p_{2U}^* = 0$, $q_{1S}^* = \bar{q}_1^\flat$ and $q_{2S}^* = q_{2U}^* = \bar{q}_2^\flat$, where $\bar{q}_1^\flat$ $(\bar{q}_2^\flat)$ is the value of $\bar{q}_1$ $(\bar{q}_2)$ in the first period after which $p_{1S} = 0$ $(p_{2S} = p_{2U} = 0)$ continues to hold.*

Proof. See Appendix C. ∎

The next lemma presents similar results as Lemma A2 for the model with endogenous H_2. Note that F_2 rather than H_2 appears in several parts of the lemma because H_2 is endogenously determined mainly depending on the state variable F_2.

Lemma A4 *In the model with endogenous H_2, under the initial condition $q_1^i = 1$ and $q_2^i = 0$ for any i, the following holds as levels of the cultural variables change over time.*

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[55]When there exist multiple equilibria for a given combination of H_2 and $\widetilde{S_N}$, it is assumed that the equilibrium chosen in the previous period is selected in the present period as well.

(i) *Initial* (M) *and* (mS) *with* $H_2 = F_2$ *are located at higher positions than or the same positions as those in subsequent periods on the* $(F_2, \widetilde{S_N})$ *plane.*

(ii) *When* $p_{1S} = p_{2S} = p_{2U} = 1$, (M) *and* (mU) [*with* $H_2 = F_2$] *shift downward over time on the* $(F_2, \widetilde{S_N})$ *plane.* (mS) [*with* $H_2 = F_2$] *shifts downward (upward) when* $\bar{q}_2 + \bar{q} - 2q_{2S}^i > (<)0$ *and shifts downward in the long run.*

(iii) *When* $p_{1S} = 1$, $p_{2S} = 1$, $p_{2U} = 0$ *or when* $p_{1S} = 0$, $p_{2S} = p_{2U} = 1$, (M) *and* (mU) *shift downward.* (mS) *shifts downward in the long run.*

(iv) *When* $p_{1S} = 0$, $p_{2S} \in (0, 1]$, $p_{2U} = 0$, (M) *shifts downward.* (mU) *and* (mS) *shift downward in the long run.*

(v) *When* $p_{1S} = p_{2S} = p_{2U} = 0$, (M) *does not shift, while* (mU) *shifts downward and* (mS) *shifts upward unless* $q_2^i = \bar{q}_2$ *for any i, in which case they do not shift.*

(vi) *When* $H_2 = 0, p_{1S} = p_{2U} = 0$, (M), (mU), *and* (mS) *do not shift.*

Proof. See Appendix C. ∎

The lemma is similar to the one for the model with constant H_2, with two differences. First, the new case, $H_2 = 0, p_{1S} = p_{2U} = 0$, arises, in which (M), (mU), and (mS) do not shift. Second, when H_2 changes over time, q_{2S}^i becomes heterogenous among lineages with different periods of becoming skilled. As a result, $p_{1S} = 0$, $p_{2S} \in (0, 1)$, $p_{2U} = 0$ can occur, in which the result is same as when $p_{1S} = 0$, $p_{2S} = 1$, $p_{2U} = 0$.

Finally, the next lemma presents similar results as Lemma A3 for the model with endogenous H_2 under the initial condition $q_1^i = 1$ and $q_2^i = 0$ for any i.

Lemma A5 *Consider the model with endogenous* H_2 *under the initial condition* $q_1^i = 1$ *and* $q_2^i = 0$ *for any i.*

(i) *When* $p_{1S}^* = p_{2S}^* = p_{2U}^* = 1$, $q_{1S}^* = q_{2S}^* = q_{2U}^* = \bar{q}^* = \bar{q}^\# \in (0, 1)$, *where* $\bar{q}^\#$ *is the value of* $\bar{q}$ *in the first period after which* $p_{1S} = p_{2S} = p_{2U} = 1$ *continues to hold.*

(ii) *When* $p_{1S}^* = 0$ *and at least one of* p_{2S}^* *and* p_{2U}^* *equals* 1, $q_{1S}^* = q_{2S}^* = q_{2U}^* = \bar{q}_1^\flat$.

(iii) *When* $p_{1S}^* = p_{2S}^* = p_{2U}^* = 0$, $q_{1S}^* = \bar{q}_1^\flat$ *and* $q_{2S}^* = q_{2U}^* = \bar{q}_2^\flat$, *where* $\bar{q}_1^\flat$ $(\bar{q}_2^\flat)$ *is the value of* $\bar{q}_1$ $(\bar{q}_2)$ *in the first period after which* $p_{1S} = 0$ $(p_{2S} = p_{2U} = 0)$ *continues to hold. When* $H_2^* = 0$ *and* $p_{1S}^* = p_{2U}^* = 0$, $q_{1S}^* = \bar{q}_1^\flat$ *and* $q_{2U}^* = \bar{q}_2^\flat$

Proof. See Appendix C. ∎

The differences from the constant H_2 case are: $p_{1S}^* = p_{2S}^* = 1$, $p_{2U}^* = 0$ does not occur; when $p_{1S}^* = p_{2S}^* = p_{2U}^* = 1$, $q_{1S}^* = q_{2S}^* = q_{2U}^* = \bar{q}^* \geq \bar{q}^\#$, where $\bar{q}^\#$ is the value of $\bar{q}$ in the first period after which $p_{1S} = p_{2S} = p_{2U} = 1$ continues to hold, unless $p_{2S} = p_{2U} = 1$ initially, in which case $\bar{q}^* = \bar{q}^\#$ as before; and when $p_{1S}^* = p_{2S}^* = p_{2U}^* = 0$, $q_{2S}^* = q_{2U}^* \geq \bar{q}_2^\flat$, where $\bar{q}_2^\flat$ is the value of $\bar{q}_2$ in the first period after which $p_{2S} = p_{2U} = 0$ continues to hold.