Social Identity, Redistribution, and Development

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Abstract

Empirical studies suggest that income redistribution promotes economic growth and development by reducing inequality and increasing educational investment among the poor. However, the scale of redistribution is limited in many developing countries. Why is the scale of redistribution small? This paper examines the role of social identity, whose importance in redistribution and development is supported in existing empirical research. Under what conditions does national identity emerge, and how does it influence the economic outcomes?

To answer the questions, this paper develops a dynamic model of income redistribution and educational investment augmented with social identification and explores the interaction among identity, redistribution, and development theoretically. Specificaly, it examines how two key drivers of development—endogenous human capital accumulation and exogenous, increasingly skill-biased technological change—shape identity, redistribution, and development.

Keywords: social identity, redistribution, economic development, national identity, nation-building policies

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1 Introduction

Cross-country differences in economic growth and development are substantial. Empirical evidence shows that income and asset inequalities are negatively related to these performances (Deininger and Squire, 1998; Easterly, 2007). This suggests that redistributive policies may stimulate growth and development by reducing inequality. Indeed, the empirical study by Berg et al. (2018) indicates that income redistribution, unless implemented on a very large-scale, increases economic growth by lowering income inequality. They also find that lower inequality is associated with higher levels of human capital. Further, Hanushek and Woessmann (2012a) find that an increase in educational achievement, measured by cognitive skills, has a substantial impact on growth. These findings suggest that redistribution may foster economic growth by reducing inequality and increasing educational investment among the poor.

However, Berg et al. (2018, Figure 5) show that the scale of redistribution—more precisely, the inequality-reducing effect of taxes and transfers—is much smaller in many developing countries compared to developed countries. Goni, Lopez, and Serven (2011) find that although market inequality is not very different between Latin American and Western European countries, after-tax, after-transfer inequality is much higher in the former. This remains true even when public expenditures on education and health are taken into account.

Given the potential benefits of redistribution for growth and development, why is its scale so limited in many developing countries? This paper examines the role of social identity. The lack of a shared national identity—that is, the dominance of subnational identities over national identity—is often blamed for the poor economic performance of socially diverse countries (Collier, 2009; Michalopoulos and Papaioannou, 2015; Fukuyama, 2018). Empirical evidence suggests that national identity positively affects redistribution (Chen and Li, 2009; Transue, 2007; Singh, 2015). Under what conditions does national identity emerge, and how does it influence redistribution, educational investment, and development?

Technological change, a major driving force of growth and development alongside the accumulation of human and physical capital, is increasingly skill-biased. The technology available to today's developing countries is far more skill-biased than what developed countries used during their periods of modernization and industrialization. What are the implications of skill-biased technological change (SBTC) for identity, redistribution, and development?

This paper theoretically explores the interaction between social identity, income redistribution, and economic development. To this end, it develops a dynamic model of income redistribution and educational investment, augmented with social identification, primarily building on the framework of Shayo (2009). Specificaly, the paper examines how two key drivers of development—endogenous human capital accumulation and exogenous, increasingly skill-biased technological change—shape identity, redistribution, and development.

Shayo (2009) extends the standard political economy model of redistributive taxation (Meltzer and Richard, 1981) by incorporating socio-psychological factors. In the model, there are two classes, the poor and the rich, and the government imposes a proportional tax on their incomes to provide a lump-sum transfer, with the tax rate determined through voting. What distinguishes this model from the standard one is that individual utility depends not only on disposable income (consumption) but also negatively on *the perceived distance between oneself and the group one identifies with* (their class or the nation) and positively on *the group's status*. Specifically, individuals incur a cognitive cost when they differ from other group members in income level and non-economic attributes, while they take pride in belonging to a high-status group, where status depends on an exogenous factor and the group's average income. These socio-psychological factors are major determinants of social identification and intergroup behaviors, according to influential theories in social psychology (Tajfel and Turner, 1986; Turner et al., 1987) and empirical evidence (Manning and Roy, 2010; Hett, Mechtel, and Kröll, 2020; Fouka, Mazumder, and Tabellini, 2022).[1] Since these components vary depending on which group individuals identify with, their social identities influence the tax rate and disposable incomes. At the same time, social identity is *endogenously determined*: individuals choose the identity that maximizes their utility. Hence, identity and individual as well as aggregate outcomes interact with each other.

The present model differs from Shayo (2009) mainly in two respects.[2] First, pre-tax, pre-transfer incomes are endogenously determined. The rich (poor) are skilled (unskilled) workers, and their earnings depend on the proportion of skilled workers. Second, the model is dynamic, with variables such as the share of skilled workers, earnings, social identity, and the tax rate evolving endogenously over time, while technology advances exogenously with rising skill bias. The dynamic part of the model draws on Galor and Zeira (1993) and Yuki (2007, 2008), in which individuals with varying levels of inherited wealth decide on educational spending that must be self-financed and is necessary to become skilled workers. The initial proportion of individuals with sufficient wealth for education is a critical determinant of the dynamics. If this proportion is sufficiently high, the share of skilled workers increase over time, total output grows rapidly, the earnings gap between skilled and unskilled workers narrows, and welfare levels eventually equalize. Conversely, if the proportion is low, the economy is stuck in a "poverty trap," where the skilled share remains low, output growth is slow, and disparities in earnings and welfare persist.

Based on such a model, the paper examines the dynamics and long-run outcomes of the skilled workers' share, identity, redistribution, and development. Main results are summarized as follows.

First, for a given share of skilled workers, the rate of redistributive taxation is higher as more individuals, especially the skilled, identify with the nation. This result contrasts with Shayo (2009), but is consistent with the thoretical result of Ghiglino, Júarez-Lunam, and Müller (2021) and empirical findings such as Chen and Li (2009), Transue (2007), Singh (2015), and Cappelen, Enke, and Tungodden (2025).[3]

Second, when considering the endogenous evolution of skilled worker share alongside exogenous *skill-neutral* technical change as drivers of development, the dynamics and long-run outcomes differ depending on the exogenous component of *national status* (compared to class status), as well as inter-class differences in non-economic attributes, which may be interpreted as culture, norms, or values, and their prominence in perceived distance.[4] In the actual society, the exogenous component of national status would be high when people believe they share a glorious history, rich culture, or a "right" sense of values, as this fosters a sense of pride in belonging to the nation.

[1]For example, Hett, Mechtel, and Kröll (2020) find that participants in a lab experiment prefer to identify with groups to which they have a smaller social distance and which have a higher social status, and their social identity preferences are related to their choices in dictator games.

[2]Other important differences from Shayo (2009), which lead to different outcomes for the tax rate, are the following. First, the perceived distance depends on the difference in disposable income between oneself and the group one identifies with, rather than the difference in pre-tax, pre-transfer income. Second, the tax rate is determined based on a probabilistic voting model (Lindbeck and Weibull, 1987), as opposed to majority voting. These settings are shared with Ghiglino, Júarez-Lunam, and Müller (2021), but unlike their model, social identities are endogenously determined in the present model.

[3]Unlike this paper, Ghiglino, Júarez-Lunam, and Müller (2021) treat social identities as exogenous. The differing results from Shayo (2009) stem from the model settings mentioned in footnote 2: in contrast to Shayo (2009), the perceived distance depends on the difference in disposable income rather than pre-tax pre-transfer income, and the tax rate is determined based on a probabilistic voting model rather than majority voting. He provides empirical results consistent with the theoretical result, which are discussed in Section 3.1.

[4]National (class) status also depends on the average disposable income of the nation (class) and thus is endogenous.

In particular, when the exogenous component of national status is higher, or when inter-class cultural differences are smaller or less salient, society is less likely to be stuck in a "poverty trap." Moreover, under favorable initial conditions of wealth distribution that avoid such a trap, the skilled worker share and output grow faster, leading to the earlier attainment of welfare equality. This is because these exogenous factors shape redistribution by influencing identity.[5]

What is notable is the case where the exogenous factors are neither very high nor very low, and the initial conditions are favorable. When the share of skilled workers and thus the level of development are low, skilled workers identify with their class while unskilled workers identify with the nation (consistent with the empirical finding of Cappelen, Enke, and Tungodden, 2025), and the dynamics do not depend on the exogenous factors. When the skilled share becomes sufficiently high, society generally undergoes a shift in social identity, significantly influencing subsequent dynamics. If national status is relatively high due to exogenous factors, or if inter-class cultural differences are small or not salient, the educated shift from class to national identity, leading to *universal national identity*. This expands redistribution and accelerates upward mobility for the poor through education. Otherwise, the uneducated switch from national to class identity and *universal class identity* results, reducing redistribution and *slowing or stalling* upward mobility. That is, under the former (latter) situation, an increasing skilled worker share ultimately has a positive (*negative*) effect on national identity, redistribution, and the pace of development. This occurs because declining inter-class disparities in income and population make the economic determinants of identity more alike across groups, causing identity to be increasingly shaped by non-economic factors such as the exogenous component of national status and inter-class cultural differences.

Third, when skill-biased technical change (SBTC) serves as an engine of development, it adversely affects upward mobility and long-term outcomes by widening inter-class wage disparity, altering social identity, and weakening redistribution. Unlike an increasing share of skilled workers, the impact of SBTC is consistently negative regardless of non-economic determinants of identity such as the exogenous component of national identity. As SBTC advances, society is more prone to fall into a "poverty trap". Even under favorable initial conditions that avert this trap, the skilled worker share grows more slowly, delaying welfare equality. Moreover, when SBTC continues, society generally shifts to an equilibrium where fewer individuals identify with the nation. This shift reduces support for income redistribution, amplifying SBTC's adverse effects.

The findings indicate that, when the expansion of skilled labor is a major driver of development, large cross-country disparities in the level and pace of development may stem from differences in the exogenous component of national status, inter-class divides in culture, norms, and values, or the degree to which people are concerned about these divides. In many developing countries, the belief that people share a glorious history, rich culture, or "correct" values is weak, and inter-class cultural, normative, and value differences are large or perceived to be serious. According to the model, such circumstances lower national status or widen the perceived distance to the other class, hindering the formation of a common national identity. As a result, redistribution is limited, upward mobility through education is constrained, and developmental progress is slow. Various empirical studies (Blouin and Mukand, 2019; Londoño-Vélez, 2022; Chen, Lin, and Yang, 2023) reveal that *nation-building policies*, such as school education and government propaganda emphasizing shared history, culture, and values, as well as policies facilitating inter-group contact, may effectively strengthen national identity or increase support for redistribution. According to the model, these policies can elevate national status or decrease (or deemphasize) inter-class differences

[5]That is, the proportion of people identifying with the nation and the rate of redistributive tax are higher, when the exogenous component of national status (compared to that of class status) is greater or inter-class cultural differences are smaller or less prominent in people's minds.

in culture, norms, and values, potentially playing a critical role in divided societies.

The results on SBTC suggest that as technology becomes more skill-biased, establishing national identity becomes more challenging, and redistributive policies lose effectiveness. Consequently, upward mobility and developmental progress slow down. This may be another reason why the pace of development, especially among the poor, is slower in many developing countries compared to the era when today's advanced economies underwent industrialization and modernization. The result may also explain the lack of increased demand for and scale of redistribution in advanced economies over the last several decades (Kenworthy and McCall, 2008; Ashok et al., 2016).

Finally, classic modernization theories in political science (Deutsch, 1953; Gellner, 1983; Weber, 1979), based on Europe's historical experiences, argue that modernization (including industrialization and universal education) fosters widespread national identity at the expense of subnational identities (Robinson, 2014). However, the model's findings on social identity, particularly the shift in identity, suggest that these theories are applicable only when national status is relatively high or when inter-class differences in culture, norms, and values are relatively small or of little concern.

This paper contributes to the theoretical literature on the relationship between social identity and redistribution (Shayo 2009; Lindqvist and Östling, 2013; Holm, 2016; Dhami, Manifold, and al-Nowaihi, 2021; and Ghiglino, Júarez-Lunam, and Müller, 2021). Besides Shayo (2009), several settings of the model follow Ghiglino, Júarez-Lunam, and Müller (2021) (footnote 2), who consider a society with two ethnicities and three income groups, but unlike their model, identities are endogenous. Neither work explores the relationship among identity, redistribution, and development.

More broadly, the paper adds to the theoretical literature on the relationship between identity and economic behaviors (Akerlof and Kranton, 2000; Shayo, 2009; Benabou and Tirole, 2011; Bisin et al., 2011; Sambanis and Shayo, 2013; Bernard, Hett, and Mechtel, 2016; Carvalho and Dippel, 2020; Grossman and Helpman, 2020; Bonomi, Gennaioli, and Tabellini, 2021; Yuki, 2021).[6] Generalizing the pioneering work of Akerlof and Kranton (2000), the aforementioned Shayo (2009) constructs the basic analytical framework. Shayo's (2009) framework has been applied to various issues, with Yuki (2021) being particularly relevant to the present work. Drawing on the seminal work of Sambanis and Shayo (2013) on the interaction between social identity and ethnic conflict, Yuki (2021) examines how the shift of economic activities from ethnically-segregated traditional sectors to the integrated modern sector, driven by the increased productivity of the latter, affects identity, ethnic conflict, and development. Unlike the present model, his model does not consider educational investment and redistribution, as well as intergenerational linkages. Another notable application is Grossman and Helpman (2020), who, inspired by a recent reversal of trade policies in some Western countries seemingly influenced by the rise of populism and ethnic tensions, develops a political economy model of trade policy with social identification and examine how policies are affected by changes in identification patterns triggered by events such as increased ethnic tensions.

The rest of the paper is organized as follows. Section 2 presents and Section 3 examines the static model. Section 4 introduces and analyzes the dynamic model, while Section 5 presents and explains the main results. Section 6 concludes. Appendix A contains propositions used in Section 3, Appendix B provides supplementary analysis for Section 4, and Appendix C contains proofs.

2 Static model

For clarity, this section presents a static model, with the full-fledged model introduced in Section 4. The model society comprises skilled workers, unskilled workers, and a government. The government

[6] In addition to those already mentioned, recent empirical and experimental studies on identity include Dehdari and Gehring (2021) and Assouad (2021).

imposes a proportional tax on earnings to finance a lump-sum transfer, with the tax-transfer policy determined by a probabilistic voting model. Individual utility depends not only on one's disposable income (consumption) but also on socio-psychological components that depend on her social identity. Consequently, social identity affects the tax-transfer policy. Identity is determined endogenously, with individuals choosing to identify with either their economic class or the nation. Markets are competitive.

2.1 Economic environment

The total population is 1, and the number of skilled workers is H, which is constant in this static model. Workers supply 1 unit of labor to receive earnings, pay a proportional tax on earnings, receive a lump-sum transfer, and spend disposable income on consumption. The disposable income of individual i is denoted by

$$y_i = (1-\tau)w_i + T, \tag{1}$$

where w_i is her earnings, $\tau \in [0,1)$ is the tax rate, and T is the transfer.

The government uses tax revenue entirely for the lump-sum transfer, but taxation involves deadweight loss. The deadweight loss is assumed to be quadratic, thus the governmental budget constraint equals

$$T = \left(\tau - \frac{1}{2}\tau^2\right)\overline{w}, \tag{2}$$

where $\overline{w}$ is average earnings of the population.

The final good is produced using skilled labor and unskilled labor as inputs. The production function takes the following CES form:

$$Y(=\overline{w}) = \left\{\alpha(A_s H)^{\frac{\sigma-1}{\sigma}} + (1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}\right\}^{\frac{\sigma}{\sigma-1}},\ \alpha \in (0,1),\ \sigma \in (1,3], \tag{3}$$

where A_s (A_u) represents skilled (unskilled) labor augmenting technology, and σ is the elasticity of substitution between skilled and unskilled workers. $\sigma \in (1,3]$ is assumed following Autor, Goldin, and Katz (2020), who estimate $\sigma = 1.62$ using U.S. data and state that estimates in the literature typically fall within the 1 to 2.5 range. The CES function is used to analyze the effects of skill-biased technological change (SBTC).

From first-order conditions of the profit maximization problem of a representative firm, the wage of skilled workers and that of unskilled workers respectively equal

$$w_s = \left\{\alpha(A_s H)^{\frac{\sigma-1}{\sigma}} + (1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}\right\}^{\frac{\sigma}{\sigma-1}-1} \alpha(A_s H)^{\frac{\sigma-1}{\sigma}} \frac{1}{H}, \tag{4}$$

$$w_u = \left\{\alpha(A_s H)^{\frac{\sigma-1}{\sigma}} + (1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}\right\}^{\frac{\sigma}{\sigma-1}-1} (1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}} \frac{1}{1-H}. \tag{5}$$

2.2 Preferences

As in Shayo (2009), individual utility depends not only on one's disposable income (consumption) but also negatively on *the perceived distance between oneself and the group with which one identifies* (their economic class—skilled or unskilled—or the nation) and positively on *the group's status.* In other words, one incurs a mental cost when they differ from others in the group in relevant attributes, while takes pride in belonging to the group when its status is high. These socio-psychological components, rooted in influential theories of social psychology (Tajfel and Turner, 1986; Turner et al., 1987), are key determinants of social identification and intergroup behaviors (Manning and Roy, 2010; Hett, Mechtel, and Kröll, 2020; Fouka, Mazumder, and Tabellini, 2022).[7]

An individual perceives the distance or proximity to a group (her class or the nation) based on the difference between her disposable income and the group's average disposable income. The *perceived distance* for an individual in class C (where C represents skilled [S] or unskilled [U]) to group G (where G represents class C or the nation [N]) is defined as:[8]

$$\begin{aligned} d_{CG} &= |y_C - y_G| \\ &= (1-\tau)\,|w_C - w_G| \quad (\text{from}(1)), \end{aligned} \tag{6}$$

where y_G and w_G are respectively the average disposable income and earnings of group G, while y_C and w_C are those of the class. For analytical tractability, the distance is measured in absolute value, following Ghiglino, Júarez-Lunam, and Müller (2021). In Shayo's (2009) model, perceived distance also depends on differences in non-economic attributes that may be interpreted as class-specific culture, norms, values etc. For ease of presentation, this dependence is not modeled here but is considered in Section 5.1.1.

The *status* of group G ($G = C, N$), S_G, depends on an exogenous component, $\widetilde{S_G}$, and the group's average disposable income:

$$S_G = \delta \widetilde{S_G} + y_G, \tag{7}$$

where δ is the weight on the exogenous component.[9] The term y_G reflects altruism toward the group. To simplify the analysis, the exogenous component of *class status* is assumed to be identical for both classes, i.e., $\widetilde{S_S} = \widetilde{S_U} \equiv \widetilde{S_C}$, which would not affect the main results qualitatively.[10]

The exogenous component of *national status,* $\widetilde{S_N}$, captures non-economic factors that foster a sense of pride in belonging to the nation, such as shared historical achievements, cultural reservoir, widely held values, or successes in international arenas like sports or diplomacy. For instance, $\widetilde{S_N}$ would be high when people believe they share a glorious history, rich cultural wealth, or a "right" sense of values.

Finally, the utility of an individual in class C who identifies with group G is given by:

$$u_{CG} = y_C - \beta d_{CG} + \gamma S_G, \qquad \beta, \gamma > 0, \tag{8}$$

where β and γ are weights on perceived distance and group status, respectively. That is, she cares not only about her own disposable income (consumption) but also about the difference in

[7]For the United Kingdom, Manning and Roy (2010) find that non-whites, whose perceived distance to the "average national" would be greater than that of whites, are less likely to think of themselves as British. They also find that immigrants from poorer and less democratic (i.e., lower status) countries assimilate faster into a British identity. Hett, Mechtel, and Kröll (2020) find that participants in a lab experiment prefer groups to which they have a smaller social distance and which have a higher social status, and their social identity preferences are related to their choices in dictator games. Fouka, Mazumder, and Tabellini (2022) find that migrations of African Americans from the South to non-southern metropolitan areas stimulated assimilation of European immigrants for the years 1910−30. Further, they provide evidence suggesting that the higher integration resulted from decreased perceived distance of native whites to European immigrants.

[8]The concept of perceived distance is developed in cognitive psychology to study how a person categorizes information that comes in to her (stimuli) (Nosofsky, 1986). Turner et al. (1987) apply the concept to the categorization of people, including oneself, into social groups, in constructing an influential social psychological theory, self-categorization theory. The theory attempts to explain psychological basis of social identification.

[9]Status is an absolute measure, following works such as Grossman and Helpman (2021), while in Shayo's (2009) model, status is a relative measure defined as the difference from the reference group. The main results remain unchanged under the alternative specification.

[10]Since $y_S > y_U$, assuming $\widetilde{S_S} > \widetilde{S_U}$ would strengthen the qualitative results.

disposable income between herself and the average group member, the group's average disposable income, and the exogenous component of the group's status.

By substituting (1), (2), (6), and (7) into the above equation, the utility for each combination of class and identity can be expressed as

$$\begin{aligned} u_{SN} &= (1-\tau)w_s+\left(\tau-\frac{1}{2}\tau^2\right)\overline{w}-\beta(1-\tau)(w_s-\overline{w})+\gamma\left[\delta\widetilde{S_N}+(1-\tau)\overline{w}+\left(\tau-\frac{1}{2}\tau^2\right)\overline{w}\right] \\ &= (1+\gamma)\left(\tau-\frac{1}{2}\tau^2\right)\overline{w}+(1-\tau)w_s-\beta(1-\tau)(w_s-\overline{w})+\gamma\left[\delta\widetilde{S_N}+(1-\tau)\overline{w}\right], \end{aligned} \tag{9}$$

$$u_{SS} = (1+\gamma)\left(\tau-\frac{1}{2}\tau^2\right)\overline{w}+(1-\tau)w_s+\gamma\left[\delta\widetilde{S_C}+(1-\tau)w_s\right], \tag{10}$$

$$u_{UN} = (1+\gamma)\left(\tau-\frac{1}{2}\tau^2\right)\overline{w}+(1-\tau)w_u-\beta(1-\tau)(\overline{w}-w_u)+\gamma\left[\delta\widetilde{S_N}+(1-\tau)\overline{w}\right], \tag{11}$$

$$u_{UU} = (1+\gamma)\left(\tau-\frac{1}{2}\tau^2\right)\overline{w}+(1-\tau)w_u+\gamma\left[\delta\widetilde{S_C}+(1-\tau)w_u\right]. \tag{12}$$

In the model, one's social identity, that is, the group with which one identifies, is not fixed. Between the nation and their class, one chooses the group that brings higher utility because of a shorter perceived distance or higher status.[11] One's identity may change when the levels of variables influencing utility, either directly or indirectly through the choices of others, change.

2.3 Political environment and Timing of decisions

Similar to Grossman and Helpman (2021) and Ghiglino, Júarez-Lunam, and Müller (2021), the political environment is based on a probabilistic voting model (Lindbeck and Weibull, 1987). Two parties, parties 1 and 2, that differ in the non-policy dimension ("ideology") compete for public office by announcing their electoral platforms on the tax rate, τ_1 and τ_2. They propose platforms to maximize the probability of wining the majority election. Individuals care about both the policy platforms and the "ideologies" of the competing parties and vote sincerely.

Individual i in class C who identifies with group G prefers party 1 if

$$u_{CG}(\tau_1) \geq u_{CG}(\tau_2) + \eta_i + \mu, \tag{13}$$

where $u_{CG}(\tau_j)$ $(j = 1,2)$ is the "non-ideology" component of the utility given by (8) when party j implements the tax rate τ_j. η_i is an individual-specific parameter that measures the voter's "ideological" bias toward party 2 and has a uniform distribution on $\left[-\frac{1}{2\phi}, \frac{1}{2\phi}\right]$, where $\phi > 0$. A negative value of η_i implies that the individual has an "ideological" bias in favor of party 1. μ measures the average popularity of party 2 relative to party 1 in the population and has a uniform distribution on $\left[-\frac{1}{2\psi}, \frac{1}{2\psi}\right]$, where $\psi > 0$.

The timing of decisions is as follows: first, workers choose their social identities; then, the parties announce their policy platforms independently; finally, workers vote for the party that offers higher utility, and the winning party implements the proposed policy.

[11] By assumption, one does not simultaneously identify with the nation and their class. Conversely, in Grossman and Helpman's (2021) model, an individual always identifies with her class and also with the nation if doing so increases her utility, where the utility depends on the sum of the perceived distance to, and the status of, each group with which she identifies. The present paper does not adopt this specification due to the complexities introduced by the additional terms and the difficulties in analyzing the model.

3 Analysis of the static model

3.1 Tax rate

Because the model can be solved by backward induction, the determination of the tax rate τ is examined first.

From (13), a class C ($C = S, U$) individual identifying with group G ($G = C, N$) is indifferent between the two parties when η_i equals $\eta_{CG} \equiv u_{CG}(\tau_1) - u_{CG}(\tau_2) - \mu$. Thus, all individuals in this category with $\eta_i < \eta_{CG}$ prefer party 1. Hence, the share of votes party 1 obtains equals

$$\begin{aligned} s_1 &= H\phi\left[p\left(\eta_{SN}+\frac{1}{2\phi}\right)+(1-p)\left(\eta_{SS}+\frac{1}{2\phi}\right)\right]+(1-H)\phi\left[q\left(\eta_{UN}+\frac{1}{2\phi}\right)+(1-q)\left(\eta_{UU}+\frac{1}{2\phi}\right)\right] \\ &= \frac{1}{2}-\phi\mu+\phi\begin{pmatrix} H\{p[u_{SN}(\tau_1)-u_{SN}(\tau_2)]+(1-p)[u_{SS}(\tau_1)-u_{SS}(\tau_2)]\} \\ +(1-H)\{q[u_{UN}(\tau_1)-u_{UN}(\tau_2)]+(1-q)[u_{UU}(\tau_1)-u_{UU}(\tau_2)]\} \end{pmatrix}, \end{aligned} \tag{14}$$

where $p \in [0, 1]$ ($q \in [0, 1]$) is the proportion of skilled (unskilled) workers identifying with the nation. s_1 depends on μ and thus is a random variable.

From the above equation, the probability that party 1 wins the election is

$$\begin{aligned} \Pr\left[s_1 \geq \frac{1}{2}\right] &= \Pr\left[\mu \leq \begin{pmatrix} H\{p[u_{SN}(\tau_1)-u_{SN}(\tau_2)]+(1-p)[u_{SS}(\tau_1)-u_{SS}(\tau_2)]\} \\ +(1-H)\{q[u_{UN}(\tau_1)-u_{UN}(\tau_2)]+(1-q)[u_{UU}(\tau_1)-u_{UU}(\tau_2)]\} \end{pmatrix}\right] \\ &= \frac{1}{2}+\psi\begin{pmatrix} H\{p[u_{SN}(\tau_1)-u_{SN}(\tau_2)]+(1-p)[u_{SS}(\tau_1)-u_{SS}(\tau_2)]\} \\ +(1-H)\{q[u_{UN}(\tau_1)-u_{UN}(\tau_2)]+(1-q)[u_{UU}(\tau_1)-u_{UU}(\tau_2)]\} \end{pmatrix}. \end{aligned} \tag{15}$$

Because the probability that party 2 wins the election equals $1 - \Pr\left[s_1 \geq \frac{1}{2}\right]$ and τ_1 and τ_2 enter symmetrically in the above equation, the unique equilibrium is such that the two parties propose the same tax rate, τ, that maximizes the utilitarian social welfare function:

$$\begin{aligned} & H\{pu_{SN}(\tau)+(1-p)u_{SS}(\tau)\} + (1-H)\{qu_{UN}(\tau)+(1-q)u_{UU}(\tau)\} \\ &= (1+\gamma)\left(\tau-\frac{1}{2}\tau^2\right)\overline{w}+(1-\tau)\begin{pmatrix} \overline{w}+\gamma\{H[p\overline{w}+(1-p)w_s]+(1-H)[q\overline{w}+(1-q)w_u]\} \\ -\beta[Hp(w_s-\overline{w})+(1-H)q(\overline{w}-w_u)] \end{pmatrix}+\text{constants}, \end{aligned} \tag{16}$$

where (9)–(12) are used to derive the last equation.

From the above equation, the proposed tax rate equals

$$\tau = 1-\frac{1}{(1+\gamma)\overline{w}}\begin{pmatrix} \overline{w}+\gamma\{H[p\overline{w}+(1-p)w_s]+(1-H)[q\overline{w}+(1-q)w_u]\} \\ -\beta\{Hp(w_s-\overline{w})+(1-H)q(\overline{w}-w_u)\} \end{pmatrix}, \tag{17}$$

if the right-hand side of the equation is positive, otherwise, $\tau = 0$. Note that when $\beta = \gamma = 0$, i.e., socio-psychological factors do not affect utility, $\tau = 0$ holds due to the linear utility, the utilitarian social welfare, and the cost of taxation.

The next lemma shows that only $(p, q) = (0, 0), (1, 1), (0, 1), (1, 0)$ can be stable equilibria. In other words, (p, q) with p or $q \in (0, 1)$, i.e, workers with the same skill level have different identities, cannot constitute a stable equilibrium.

Lemma 1 *Only* $(p, q) = (0, 0), (1, 1), (0, 1), (1, 0)$ *can be stable equilibria.*

Proof. See Appendix C. ■

Based on this lemma, the following proposition summarizes how the tax rate depends on identity choices of the two groups and H.

Proposition 1 *(i) (a)* $\tau = 0$ *when* $p = q = 0$*, i.e, everyone identifies with their class.*

(b) $\tau = \frac{2\beta}{1+\gamma}(a(H) - H)$*, where* $a(H) \equiv \frac{\alpha(A_s H)^{\frac{\sigma-1}{\sigma}}}{\alpha(A_s H)^{\frac{\sigma-1}{\sigma}} + (1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}} \geq H$ *from* $w_s \geq w_u$*, when* $p = q = 1$*, i.e., everyone identifies with the nation.*

(c) When $p = 0$ *and* $q = 1$*, i.e., the skilled identify with their class and the unskilled identify with the nation,* $\tau = \frac{\beta-\gamma}{1+\gamma}(a(H) - H)$ *if* $\beta > \gamma$ *and* $\tau = 0$ *if* $\beta \leq \gamma$*.*

(d) $\tau = \frac{\beta+\gamma}{1+\gamma}(a(H) - H)$ *when* $p = 1$ *and* $q = 0$*, i.e., the skilled identify with the nation and the unskilled identify with their class. This is not an equilibrium when* $\beta \leq \gamma$*.*

(ii) Given H*,* τ *is highest when* $p = q = 1$*, and if* $\beta > (\leq)\gamma$*, it is lowest when* $p = q = 0$ *(when* $p = q = 0$ *and* $p = 0$*,* $q = 1$*).*

(iii) Given p *and* q*, there exists an* $H^+ \in (0, \overline{H})$*, where* $\overline{H}$ *satisfies* $\overline{H} = a(\overline{H}) \Leftrightarrow w_s = w_u$*, such that* $\frac{d\tau}{dH} > (<) 0$ *for* $H < (>) H^+$*.*

Proof. See Appendix C. ■

The tax rate is 0 when $p=q=0$, i.e, everyone identifies with their class, and if $\beta \leq \gamma$, when $p=0$, $q=1$ as well, i.e., the skilled identify with their class and the unskilled identify with the nation.[12] In other cases, τ equals a constant times $a(H) - H$, where $a(H) \equiv \frac{\alpha(A_s H)^{\frac{\sigma-1}{\sigma}}}{\alpha(A_s H)^{\frac{\sigma-1}{\sigma}} + (1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}}$.

Given H, the tax rate is highest when $p=q=1$, i.e, everyone identifies with the nation, while if $\beta > (\leq)\gamma$, it is lowest when $p=q=0$ (when $p=q=0$ and $p=0$, $q=1$). Further, the tax rate when $p = 1$, $q = 0$ is higher than when $p = 0$, $q = 1$ if $\beta > \gamma$. (If $\beta \leq \gamma$, $p = 1$, $q = 0$ can be realized.)

Hence, given the share of skilled workers, the rate of redistributive taxation is higher as more individuals, especially the skilled, identify with the nation. This result aligns with Ghiglino, Júarez-Lunam, and Müller (2021) and contrasts with Shayo (2009).[13,14] National identity has a positive effect on the tax rate for the following reasons. First, individuals identifying with the nation are concerned with the perceived distance to the "average national", which decreases with increased redistribution. Second, their concern with national status has a negative effect on the tax rate because tax lowers the average disposable income (due to the taxation cost) and thus national status, but this effect is dominated by the effect through the perceived distance.[15] The result is generally consistent with empirical findings regarding the relationship between national identity and

[12] Ghiglino, Júarez-Lunam, and Müller (2021) allow β and γ to differ between the classes. In this case, $\tau > 0$ holds even when $p=q=0$ if γ (the importance of status in the utility) for the unskilled is greater than that for the skilled. In the present paper, β and γ are assumed to be common to both classes to make the subsequent analysis tractable.

[13] The same result holds for the model of Ghiglino, Júarez-Lunam, and Müller (2021), in which β and γ can differ between the classes and social identities are *exogenous*, except that τ when $p=q=0$ is higher than when $p=0$, $q=1$ iff $\beta \leq \gamma$ for the unskilled and their γ is strictly greater than γ for the skilled.

[14] Shayo (2009) shows that the tax rate preferred by the poor is *lower* under national identity than under class identity. Since the poor determine tax policy in his model, this also applies to the implemented tax rate. In contrast, in the present model, τ preferred by the unskilled is higher (lower) under national identity iff $\beta > (<)\gamma$, i.e., perceived distance weighs more (less) than status in one's utility, and the implemented τ is *always* (weakly) *higher* when more individuals identify with the nation. The reason why τ preferred by the unskilled can be higher under national identity is that perceived distance depends on differences in *disposable income*. If it depends on differences in *pre-tax, pre-transfer* income as in Shayo (2009), their preferred τ is lower under national identity. (In contrast, status does depend on *disposable* income in his model too.) The result on the implemented τ holds because not only the unskilled but also the skilled, whose preferred τ is *always higher* under national identity, influence the tax policy.

[15] To be precise, when $\beta < \gamma$ and $p = 0$, i.e., status is more important than perceived distance in one's utility and the skilled identify with their class, this effect dominates the effect through the perceived distance for the unskilled and thus their preferred tax rate is lower when $q = 1$ than when $q = 0$. However, since the implemented τ equals 0 when $p = q = 0$, $\tau = 0$ holds when $p = 0$, $q = 1$ as well.

redistribution (Chen and Li, 2009; Transue, 2007; Singh, 2015; Cappelen, Enke, and Tungodden, 2025),[16] but it contrasts with the empirical finding of Shayo (2009).[17]

Unlike Shayo (2009) and Ghiglino, Júarez-Lunam, and Müller (2021), pre-tax, pre-transfer incomes are endogenous and depend on H, causing the tax rate to vary with H. The relationship between the share of skilled workers and the tax rate is non-monotonic; namely, τ increases with H for relatively small H and decreases with H for relatively large H. However, this result is not important for understanding the main results below.

To ensure that a higher tax rate always increases the disposable income of unskilled workers across the possible range of τ, the following assumption is imposed.

Assumption 1 $\frac{2\beta}{1+\gamma} \leq 1$.

The following corollary shows how the disposable incomes of skilled and unskilled workers and the inter-class inequality in disposable income depend on their social identities.

Corollary 1 (*i*) *Given H, the disposable income of skilled workers and the inter-class inequality in disposable income are lowest when $p = q = 1$, and are highest when $p = q = 0$ (when $p = q = 0$ and $p = 0$, $q = 1$), if $\beta > (\leq)\gamma$.*

(*ii*) *Under Assumption 1, given H, the disposable income of unskilled workers is highest when $p = q = 1$, and is lowest when $p = q = 0$ (when $p = q = 0$ and $p = 0$, $q = 1$), if $\beta > (\leq)\gamma$.*

Proof. See Appendix C. ■

Given H, the disposable income of unskilled workers is highest, while that of skilled workers and the inter-class disparity in disposable income are lowest when $p = q = 1$. On the other hand, if $\beta > (\leq)\gamma$, the unskilled income is lowest, while the skilled income and the inter-class inequality are highest when $p = q = 0$ (when $p = q = 0$ and $p = 0$, $q = 1$). That is, as more individuals, particularly the skilled, identify with the nation, the disposable income of the unskilled (skilled) increases (decreases), leading to a decrease in inter-class income inequality.

3.2 Social identity

The choice of social identity affects the perceived distance term and the status term in the utility function. Thus, individuals choose the identity that maximizes the combined value of these two

[16] Chen and Li (2009) conduct lab experiments to examine the effects of induced group identity on social preferences and find that subjects are more averse to payoff differences to groups they identify with. Transue (2007), based on a survey experiment on American whites, shows that those who feel close to the nation are more supportive of a tax increase to improve educational opportunities for minorities, compared to those who feel close to their racial group. Further, he finds that making American identity salient increases their support for the tax increase. Singh (2015), based on statistical and comparative historical analysis of Indian states, shows that states with a stronger sense of shared identity spend more on education and health. Cappelen, Enke, and Tungodden (2025) conduct surveys across 60 countries to measure universalism, defined as the degree to which altruism is invariant to the group membership of others. They find that an individual's level of universalism is positively correlated with support for governmental programs to reduce inequality.

[17] Using survey data from democratic countries, Shayo (2009) finds that the extent of redistribution to the poor is smaller in countries with a *higher* median level of a national identity measure, a result that diverges from other cited works. Several factors may contribute to this discrepancy. First, the national identity measure is constructed from answers to six questions, none of which ask respondents to compare their country to subnational groups (e.g., class), and all but one of which involve comparisons to *other countries*. Answers may differ when the comparison groups are subnational. Second, the modeling of identity choice highlights the need for variables that measure the strength of national identity *relative to* class identity when examining the theoretical result: it is possible that both the absolute strength of national identity and the relative strength of class identity are large simultaneously.

terms. Hence, from (9)−(12) and $H \leq \overline{H}$, i.e., $w_s \geq w_u$, the following conditions are obtained ($\Delta\widetilde{S_N} \equiv \widetilde{S_N} - \widetilde{S_C}$). Note that $H \geq \frac{\beta+\gamma}{2\beta}$ holds only when $\beta > \gamma$.

[$\mathbf{p} = \mathbf{q} = \mathbf{1}$] This occurs iff

$$\gamma\delta\Delta\widetilde{S_N} \geq (1-\tau)(w_s - w_u)\max[(\beta+\gamma)(1-H), (\beta-\gamma)H].$$

The condition can be split into two cases:

$$\gamma\delta\Delta\widetilde{S_N} \geq (1-\tau)(\beta+\gamma)(1-H)(w_s - w_u) \text{ for } H \leq \min\left\{\frac{\beta+\gamma}{2\beta}, \overline{H}\right\}, \tag{18}$$

$$\gamma\delta\Delta\widetilde{S_N} \geq (1-\tau)(\beta-\gamma)H(w_s - w_u) \text{ for } H \in \left[\frac{\beta+\gamma}{2\beta}, \overline{H}\right] \text{ when } \overline{H} > \frac{\beta+\gamma}{2\beta}, \tag{19}$$

where τ is the tax rate when $p = q = 1$, i.e., $\tau = \frac{2\beta}{1+\gamma}(a(H) - H)$ from Proposition 1 (i)(b).

[$\mathbf{p} = \mathbf{q} = \mathbf{0}$] This occurs iff

$$\gamma\delta\Delta\widetilde{S_N} \leq (w_s - w_u)\min[(\beta+\gamma)(1-H), (\beta-\gamma)H],$$

which can be split into:

$$\gamma\delta\Delta\widetilde{S_N} \leq (\beta-\gamma)H(w_s - w_u) \text{ for } H \leq \min\left\{\frac{\beta+\gamma}{2\beta}, \overline{H}\right\}, \tag{20}$$

$$\gamma\delta\Delta\widetilde{S_N} \leq (\beta+\gamma)(1-H)(w_s - w_u) \text{ for } H \in \left[\frac{\beta+\gamma}{2\beta}, \overline{H}\right] \text{ when } \overline{H} > \frac{\beta+\gamma}{2\beta}, \tag{21}$$

because $\tau = 0$ from from Proposition 1 (i)(a).

[$\mathbf{p} = \mathbf{0}, \mathbf{q} = \mathbf{1}$] This occurs iff $H \leq \min\left\{\frac{\beta+\gamma}{2\beta}, \overline{H}\right\}$,

$$\gamma\delta\Delta\widetilde{S_N} \leq (1-\tau)(\beta+\gamma)(1-H)(w_s - w_u), \text{ and } \gamma\delta\Delta\widetilde{S_N} \geq (1-\tau)(\beta-\gamma)H(w_s - w_u), \tag{22}$$

where $\tau = \frac{\beta-\gamma}{1+\gamma}(a(H) - H)$ if $\beta > \gamma$ and $\tau = 0$ if $\beta \leq \gamma$ from Proposition 1(i)(c). This occurs only for $H \leq \min\left\{\frac{\beta+\gamma}{2\beta}, \overline{H}\right\}$, as the RHS of the first condition must be greater than that of the second one.

[$\mathbf{p} = \mathbf{1}, \mathbf{q} = \mathbf{0}$] This occurs iff $H \in \left[\frac{\beta+\gamma}{2\beta}, \overline{H}\right]$,

$$\gamma\delta\Delta\widetilde{S_N} \geq (1-\tau)(\beta+\gamma)(1-H)(w_s - w_u), \text{ and } \gamma\delta\Delta\widetilde{S_N} \leq (1-\tau)(\beta-\gamma)H(w_s - w_u), \tag{23}$$

where $\tau = \frac{\beta+\gamma}{1+\gamma}(a(H) - H)$ from Proposition 1 (i)(d).

Propositions in Appendix A examine combinations of H and $\Delta\widetilde{S_N}$ under which each of these conditions holds. To prove the propositions, the following assumption is imposed.

Assumption 2 $\tau = \frac{2\beta}{1+\gamma}(a(H) - H) < \frac{1}{2}$ *at* $H = H^+$, *where* H^+ *satisfies* $a'(H^+) - 1 = 0$.

This assumption states that the maximum possible tax rate is less than $\frac{1}{2}$. It is not restrictive, as the portion of tax revenue allocated to redistribution (rather than non-redistributive governmental expenditure) is much lower than half of aggregate labor income in the real economy.

3.2.1 Result

The following analysis focuses on the more interesting case $\beta > \gamma$; the analysis when $\beta \leq \gamma$ is presented in Appendix A.[18] Based on Proposition A2 (i) in the appendix, Figure 1 shows combinations

[18] The other important reason for focusing on the case $\beta > \gamma$ is that when $\beta \leq \gamma$, the figure illustrating combinations of H and $\Delta\widetilde{S_N}$ such that each equilibrium exists (Figure 5 in Appendix A) changes greatly and becomes closer to the corresponding figure for $\beta > \gamma$, when, as assumed in Section 5.1.1 and Shayo (2009), perceived distance also depends on inter-class differences in non-economic attributes that represent culture, norms, and values.

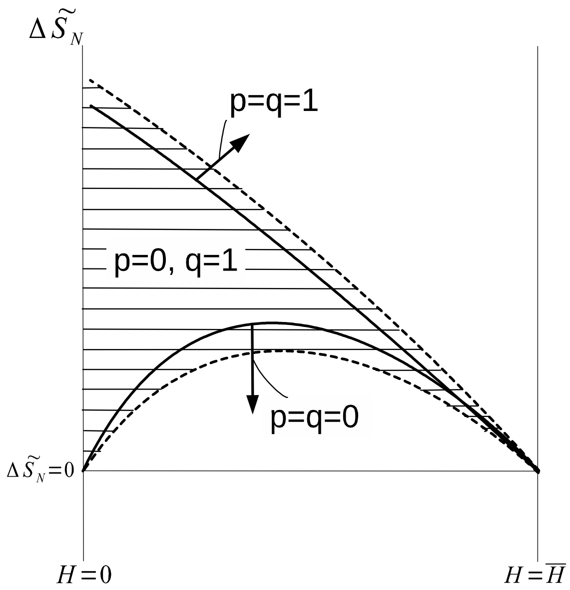

Figure 1: Equilibria when $\beta > \gamma$ and $\frac{A_s}{A_u}$ is relatively low

of H and $\Delta\widetilde{S_N} \equiv \widetilde{S_N} - \widetilde{S_C}$ (the difference in the exogenous component of national status and that of class status) each equilibrium exists when $\frac{A_s}{A_u}$ is sufficiently low to satisfy $\overline{H} \leq \frac{\beta+\gamma}{2\beta}$. ($\overline{H}$ represents the H satisfying $H = a(H) \Leftrightarrow w_s = w_u$.) $p = q = 1$, i.e., universal national identity, is realized when $(H, \Delta\widetilde{S_N})$ is in the region on or to the right of the solid downward-sloping curve. $p = q = 0$, i.e., universal class identity, is realized when $(H, \Delta\widetilde{S_N})$ is in the region on or below the solid convex curve. $p = 0$, $q = 1$, i.e., skilled workers identity with their class and unskilled workers identify with the nation, holds when $(H, \Delta\widetilde{S_N})$ is in the region with horizontal lines.[19]

Given H, everyone identifies with the nation (their class) when $\Delta\widetilde{S_N}$ is high (low), that is, when people are (are not) very proud of the nation relative to their class for non-economic reasons, such as culture and history. When $\Delta\widetilde{S_N}$ is in the intermediate range, educated workers identity with their class and uneducated workers identity with the nation, which aligns with the empirical finding of Cappelen, Enke, and Tungodden (2025).[20]

Realized equilibria depend on H unless $\Delta\widetilde{S_N}$ is very high or low. When H is small, $p = 0$, $q = 1$ (the skilled [unskilled] identify with their class [the nation]) is the only equilibrium for a wide range of $\Delta\widetilde{S_N}$. However, as H increases, $p = q = 1$, i.e., *universal national identity* ($p = q = 0$, i.e., *universal class identity*) becomes more dominant when $\Delta\widetilde{S_N}$ is relatively high (low). This result becomes important in later analysis.

The relationship between realized equilibria and H is mainly driven by the fact that the wage gap, $w_s - w_u$, decreases with H. When H is low, skilled workers identify with their class because they perceive a large distance to the "average national," and their class status is high relative to national status. This is due to a large wage differential and their small population share. In contrast, unskilled workers identify with the nation when H is low because their large population

[19]The lower dividing line for $p = 0$, $q = 1$ increases (decreases) with H for small (large) H, but when H is intermediate, the relation with H is not analytically clear.

[20]Cappelen, Enke, and Tungodden (2025), based on surveys for 60 countries, find that universalism (the degree to which altruism is invariant to the group membership of others) tends to be lower for college-educated respondents.

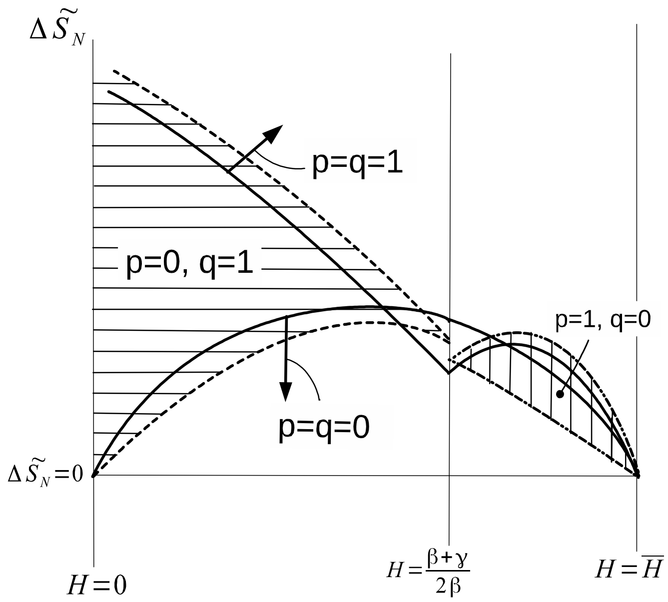

Figure 2: Equilibria when $\beta > \gamma$ and $\frac{A_s}{A_u}$ is relatively high

share makes the perceived distance to the "average national" small, and the large wage gap makes national status high relative to their class status. As H increases, the wage gap narrows, and the population size differential declines, reducing inter-class differences in the perceived distance to the "average national" and in class status. That is, the economic factors influencing the identity choices of the two groups become more similar. As a result, identity choices are increasingly shaped by *non-economic factors* such as the exogenous component of national status. Thus, when H becomes sufficiently large, if $\Delta\widetilde{S_N}$ is relatively high (low), skilled (unskilled) workers switch from class (national) to national (class) identity, leading to universal national (class) identity.[21]

The figure shows the presence of regions with multiple equilibria. This arises from two-way positive causations between national identity and tax rate: as the share of those identifying with the nation is higher, so is the tax rate (Proposition 1), while as the tax rate is higher, the inter-class disparity in disposable income is lower, leading to more people identifying with the nation.

Figure 2 depicts equilibria when the skill-biasedness of technology, $\frac{A_s}{A_u}$, is high enough that $\overline{H} > \frac{\beta+\gamma}{2\beta}$, based on Proposition A2 (ii) and (iii) in Appendix A. The figure is similar to Figure 1 for $H \leq \frac{\beta+\gamma}{2\beta}$. For $H > \frac{\beta+\gamma}{2\beta}$, unlike the previous case, $p=0$, $q=1$ is not an equilibrium and $p=1$, $q=0$, i.e., the educated identify with *the nation* and the uneducated identify with *their class*, is realized when $\Delta\widetilde{S_N}$ is relatively, but not extremely, low (in the region with vertical lines). This is consistent with the finding of Cappelen, Enke, and Tungodden (2025) for recent developed countries.

3.3 Summary

Combining the results on the tax rate and social identity, the findings can be summarized as follows. First, given H, the rate of redistributive tax is high (low) when the exogenous status difference $\Delta\widetilde{S_N} \equiv \widetilde{S_N} - \widetilde{S_C}$ is relatively high (low), because the proportion of those identifying with the nation

[21]To be precise, when $\Delta\widetilde{S_N}$ is relatively low, unskilled workers revert to national identity at a very large H. However, as will be made clear in Section 5, this scenario is not very relevant.

is high (low). Second, unless $\Delta\widetilde{S_N}$ is very high or low, identity and the tax rate vary with the share of skilled workers and thus with the level of economic development. When H is low, skilled workers identify with their class, unskilled workers identify with the nation, and the tax rate is low. When H is sufficiently high, for relatively high $\Delta\widetilde{S_N}$, everyone identifies with the nation and the tax rate is high, while for relatively low $\Delta\widetilde{S_N}$, everyone identifies with their class and the tax rate is at least as low as when H is low. These results highlight the importance of $\Delta\widetilde{S_N}$ (which would be high when people believe that they share a glorious history, rich cultural heritage, or a "right" sense of values in the real world) and H for national identity and redistributive taxation.[22]

In the real economy, the share of skilled workers evolves, and sectoral productivities grow over time. Hence, the next section introduces a dynamic version of the model in which H is determined endogenously and A_s and A_u increase exogenously.

4 Dynamics

The rest of the paper examines how social identity, redistribution, earnings, and inter-class inequalities change over time. This section presents a dynamic version of the model and examines the dynamics of H. The dynamic part of the model draws on Galor and Zeira (1993) and Yuki (2007, 2008), in which individuals with varying levels of inherited wealth decide on educational spending needed to become skilled workers.

4.1 Model

For simplicity, A_s and A_u are fixed until Section 5.1.2. Consider a deterministic, discrete-time, and small-open OLG economy in which individuals live for two periods, first as a child, then as an adult. Each adult has one child, thus the population is constant over time. The population of each generation is normalized to be 1. The lifetime of an individual is as follows.

In childhood, the individual receives a transfer b from her parent and spends it on assets a (interest rate is r) and educational expenditure e, which is required to become a skilled worker, to maximize the utility given by (24) below. Her investment decision takes into account not only the impact on future income but also the influence on the socio-psychological components of utility. The educational investment is binary (i.e. taking education or not), costs $\overline{e}$, and yields a gross economic return of $w_s - w_u$. It must be self-financed due to the absence of credit markets. Thus, when $b < \overline{e}$, she cannot afford education, i.e., $e = 0$, and becomes an unskilled worker.

In adulthood, the individual earns income from assets and work, allocating it between consumption c and a transfer to her child b'. As before, she also chooses a group to identify with and votes for a party that maximizes her utility. When she belongs to class C ($C\!=\!S,U$) and identifies with group G ($G\!=\!C,N$), she maximizes the following (the "non-ideology" component of) utility subject to the budget constraint:

$$\max\ v_{CG} = \frac{1}{(\lambda)^{\lambda}(1-\lambda)^{1-\lambda}}(b')^{\lambda}(c)^{1-\lambda} - \beta d_{CG} + \gamma S_G, \quad \lambda \in (0,1), \tag{24}$$

$$s.t. \quad c + b' = (1-\tau)w_C + T + (1+r)a, \tag{25}$$

where w_C is the wage for class C. By solving the maximization problem, the following consumption

[22] Section 5.1.1 examines a modified model in which perceived distance also depends on differences in non-economic attributes that would represent culture, norms, or values, as in Shayo (2009). The analysis there shows that inter-class distances in these attributes and their prominence in perceived distance too are important for the results.

and transfer rules are obtained.

$$c = (1-\lambda)[(1-\tau)w_C + T + (1+r)a], \tag{26}$$

$$b' = \lambda[(1-\tau)w_C + T + (1+r)a]. \tag{27}$$

Results on identity choice and the tax rate remain unchanged from the static model, since the indirect utlity function equals the utility function of the original model plus $(1+r)a$.[23]

From the above setup, H is equal to the proportion of individuals who receive $b \geq \overline{e}$ *and* choose to spend $e = \overline{e}$ in childhood. Let F be the proportion of those who receive $b \geq \overline{e}$. If the utility gain from educational investment is non-negative *even when* everyone with $b \geq \overline{e}$ takes education, $H = F$ holds; this is the case when F is not large, as shown in Appendix B. If the utility gain is negative with $H = F$, which is true when F is sufficiently large, H is smaller than F and is determined so that individuals are indifferent between making the educational investment and not. Denote such H by $H^* \in (0, \overline{H})$, whose determination is explained in Appendix B.

4.2 Dynamics of F and H

Given the distribution of b in the initial period and the corresponding initial value of F, the dynamics of F are determined by the evolution of b for each lineage. Consider an individual who is born in period $t-1$ and reaches adulthood in period t. Her investment decision depends on the received transfer and the sign of the utility return to education (henceforth, subscript t represents variables for those who become adults in period t):

$$\text{If } b_t < \overline{e} \text{ or the utility return to education is negative,} \quad a_t = b_t,\ e_t = 0, \tag{28}$$

$$\text{If } b_t \geq \overline{e} \text{ and the utility return to education is positive,} \quad a_t = b_t - \overline{e},\ e_t = \overline{e}. \tag{29}$$

By substituting (28) into (27), the dynamic equation linking the received transfer b_t to the transfer to her child b_{t+1} when she does not take education and thus is an unskilled worker equals

$$b_{t+1} = \lambda\left[(1-\tau_t)w_{ut} + T_t + (1+r)b_t\right]. \tag{30}$$

Similarly, the corresponding equation when she is a skilled worker equals

$$b_{t+1} = \lambda\left[(1-\tau_t)w_{st} + T_t + (1+r)(b_t-\overline{e})\right]. \tag{31}$$

Now, the evolutions of F_t (the proportion of those who can afford education) and H_t (the proportion of those who invest in education and become skilled workers) are examined for the case $F_t < H^*$, where $H_t = F_t$ holds. (When $F_t \geq H^*$, $H_t = H^*$, which is time-invariant.) $F_{t+1} \geq F_t \Leftrightarrow H_{t+1} \geq H_t$ holds iff all children of skilled workers can afford education, i.e., for any lineage satisfying $b_t \geq \overline{e}$, $b_{t+1} \geq \overline{e}$. From (31), this is the case if

$$\lambda\left[(1-\tau_t)w_{st} + T_t\right] \geq \overline{e}. \tag{32}$$

When this condition holds, $H_{t+1} = F_{t+1} > H_t = F_t$ is true iff some children of unskilled workers gain access to education, i.e., there exist lineages satisfying $b_t < \overline{e}$ and $b_{t+1} \geq \overline{e}$. From (30), this occurs iff $b_{t+1} = \lambda\{(1-\tau_t)w_{ut} + T_t + (1+r)b_t\} \geq \overline{e}$ for a lineage with the highest $b_t < \overline{e}$. This is the case only if ($\lambda(1+r) < 1$ is assumed)

$$\frac{\lambda}{1-\lambda(1+r)}\left[(1-\tau_t)w_{ut} + T_t\right] > \overline{e}. \tag{33}$$

[23]To maintain consistency with the static results and reflect the real-world fact that asset income is taxed less heavily than labor income, it is assumed that asset income is not taxed.

By contrast, $F_{t+1} = F_t \Leftrightarrow H_{t+1} = H_t$ holds iff $b_{t+1} = \lambda\{(1-\tau_t)w_{ut} + T_t + (1+r)b_t\} < \overline{e}$ for a lineage with the highest $b_t < \overline{e}$, which is the case if

$$\frac{\lambda}{1-\lambda(1+r)}\left[(1-\tau_t)w_{ut} + T_t\right] \leq \overline{e}. \tag{34}$$

In summary, when $F < H^*$ and the disposable income (net of asset income) of skilled workers is sufficiently high, $H(=F)$ increases over time if the net disposable income of unskilled workers is also high enough; otherwise, it remains constant. Since disposable incomes depend on $H = F$, whether H increases or not is determined by the level of F. The next lemma shows that, for *given values* of p and q and under certain conditions, if the initial level of F is sufficiently high, H increases over time and eventually reaches H^*; otherwise, it stays low.

Lemma 2 *Suppose that Assumption 1 holds, and* $\overline{e}$ (λ) *is sufficiently but not extremely small* (*large*), *with fixed values of* p *and* q. *Then,*

(*i*) *There exists* $\underline{F} \in (0, H^*)$ *such that when* $F_0 \in (\underline{F}, H^*)$, H *increases over time and converges to* H^*, *and when* $F_0 \leq \underline{F}$, $H_t \leq \underline{F}$ *for any* t.[24] *When* $F_0 \in (\underline{F}, H^*)$, *the utility of everyone converges to the same level in the long run.*

(*ii*) *When* $\beta > (\leq)\gamma$, $\underline{F}$ *is lowest when* $p = q = 1$ *and is highest when* $p = q = 0$ (*when* $p = q = 0$ *and* $p = 0$, $q = 1$).[25]

Proof. See Appendix C. ■

When the initial proportion of individuals with sufficient wealth for education is high enough, i.e., $F_0 \in (\underline{F}, H^*)$, the share of skilled workers and thus the unskilled wage are relatively high from the start. This allows unskilled workers with relatively large wealth to transfer enough resources for their children to obtain education and become skilled, as long as the transfer motive λ is strong enough or the cost of education $\overline{e}$ is sufficiently low. This leads to an increase in $H = F$, which in turn raises the unskilled wage and further promotes upward mobility of the children of unskilled workers. In this way, H increases over time and eventually reaches H^* at which one is indifferent between taking education and not.[26,27] Further, since within-class disparities in transfers decrease over time from (30) and (31), the welfare level of everyone equalizes in the long run.

In contrast, when $F_0 < \underline{F}$, generally, the unskilled wage is too low for the children of unskilled workers to receive sufficient wealth for education. Consequently, H does not increase continuously and H stays low, i.e., $H < \underline{F} < H^*$.[28] This implies that skilled workers enjoy higher utility than unskilled workers even in the long run.

[24] When $F_0 \leq \underline{F}$, if $p = q = 0$ or if $\beta \leq \gamma$ and $p = 0$, $q = 1$, where $\tau = 0$, $H_t = F_0$ for any t. For other values of p and q, where $\tau > 0$, one cannot rule out the possibility that $H \leq \underline{F}$ *temporarily* increases or decreases. (See the proof of the lemma for details.) Further, the possibility of multiple values of H^* cannot be ruled out in these cases. If multiple values of H^* exist, H converges to the highest H^* when $F_0 \in (\underline{F}, H^*)$. Contrary to the assumption of the lemma, if $\overline{e}$ (λ) is extremely small (large), from any $F_0 < H^*$, H increases over time and converges to H^*.

[25] Magnitude relations among H^* for different values of p and q are clear when $\frac{A_s}{A_u}$ is small enough that $\overline{H} \leq \frac{\beta+\gamma}{2\beta}$ (as in Figure 3 below), or when $\overline{H} > \frac{\beta+\gamma}{2\beta}$ and H^* for $p = q = 1$ is smaller than $\frac{\beta+\gamma}{2\beta}$. In these cases, H^* for $p = q = 1$ is smallest, H^* for $p = q = 0$ is largest, and H^* for $p = 0$, $q = 1$, which decreases with $\Delta\widetilde{S_N}$, falls in between. When $\overline{H} > \frac{\beta+\gamma}{2\beta}$ and H^* for $p = q = 1$ is greater than $\frac{\beta+\gamma}{2\beta}$ (as in Figure 4 below), the relations are generally ambiguous.

[26] After H reaches H^*, F continues to increase and converges to 1.

[27] When $p = q = 0$ holds at $H = H^*$, the net *economic* return to education is negative; in other words, the earnings of skilled workers, after deducting educational expenditure, are *lower* than the earnings of unskilled workers, i.e., $w_s - (1+r)\overline{e} < w_u$ at $H = H^*$. This occurs because education confers an non-economic benefit, i.e., higher status, as well. For other values of p and q, whether the long-run net economic return is positive or not is unclear.

[28] As mentioned in footnote 24, $H < \underline{F}$ might temporarily increase or decrease.

Note that the threshold level of F, $\underline{F}$, varies depending on the values of p and q. It is lowest when $p=q=1$, and if $\beta > (\leq)\gamma$, it is highest when $p=q=0$ (when $p=q=0$ and $p=0$, $q=1$). In other words, under universal national (class) identity, the minimum level of F_0 required for upward mobility and long-run welfare equalization is lowest (highest). This is because the tax rate is highest (zero) and thus income transfers to the unskilled are largest (not conducted).

5 Main results

Based on Lemma 2 and findings from Section 3 (many of which are illustrated in Figures 1 and 2), this section examines how two key drivers of economic development—the endogenous evolution of skilled workers' share and exogenous skill-biased technical change—shape identity, redistribution, and development. Henceforth, the following assumption on identity dynamics is imposed.

Assumption 3 *When society is in equilibrium with specific values of p and q, it stays in that equilibrium in subsequent periods, as long as the equilibrium continues to exist.*

This means that the values of p and q do not change as long as they are equilibrium values.

5.1 Effect of H on identity, redistribution, and development

For ease of analysis, the effect of H is first examined for given levels of productitivies A_s and A_u. As in Section 3.2, the analysis focuses on the case $\beta > \gamma$; however, the following results are mostly unchanged when $\beta \leq \gamma$.

The next proposition examines how the dynamics and long-run levels of H, earnings, and earnings inequality, as well as the long-run interclass disparity in welfare, depend on F_0 and $\Delta\widetilde{S_N} \equiv \widetilde{S_N} - \widetilde{S_C}$. Unlike Lemma 2, p and q are *endogernized* by incorporating the results from Section 3.

Proposition 2 *Suppose that $\beta > \gamma$, Assumptions 1–3, and the existence conditions for $\underline{F}$ in Lemma 2 are satisfied. Consider a society starting with $F_0 < H^*$.*[29]

(*i*) *When $F_0 \leq \underline{F}$, $H_t \leq \underline{F}$ for any t. Because H is low, Y remains low, w_s is high, w_u is low, and inter-class disparities in earnings and welfare are large.*

(*ii*) *When $F_0 > \underline{F}$, H increases over time and almost always converges to H^*.*[30] *Accordingly, Y rises, w_s decreases, w_u increases, the earnings disparity narrows over time, and in the long run, the welfare level of everyone becomes equal.*

(*iii*) *As $\Delta\widetilde{S_N}$ is higher, $\underline{F}$ is lower, making $F_0 > \underline{F}$ more likely. In particular, when $\Delta\widetilde{S_N}$ is very high* (*very low*), $p=q=1$ ($p=q=0$), *leading to the lowest* (*highest*) $\underline{F}$.

Proof. See Appendix C. ■

When the initial proportion of those who can afford education is sufficiently low, i.e., $F_0 \leq \underline{F}$, the share of skilled workers remains low, i.e., $H_t \leq \underline{F}$ for all t. Hence, output (and per capita income) remains low, the skilled wage is high while the unskilled wage is low, and inter-class disparities in earnings and welfare are large. Society is caught in a "poverty trap". In contrast, when the initial proportion of such individuals is sufficiently high, i.e., $F_0 > \underline{F}$, the skilled workers' share increases over time and almost always reaches H^*. Consequently, output rises, the skilled wage

[29] H^* refers to the value of H^* for the values of p and q in the initial period. The same applies to $\underline{F}$ below.

[30] As explained in footnote 34 in the next proposition, when society starts with $p=0$, $q=1$, H could stop increasing after shifting to $p=q=0$.

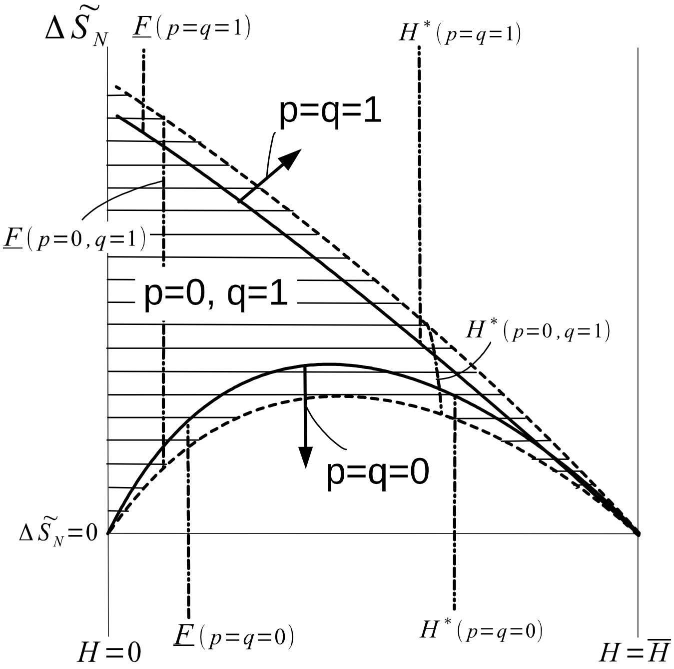

Figure 3: Dynamics when $\beta > \gamma$ and $\frac{A_s}{A_u}$ is relatively low

decreases while the unskilled wage increases, the earnings gap narrows over time, and eventually, the welfare level of everyone becomes equal. This result is standard in Galor-Zeira type models that incorporate indivisibilities in human capital investment and credit constraints, and is consistent with empirical findings such as Litschig and Lombardi (2019) and Aiyar and Ebeke (2020).[31]

The likelihood of achieving the latter favorable scenario increases with $\Delta\widetilde{S_N} \equiv \widetilde{S_N} - \widetilde{S_C}$, the difference between the exogenous components of national and class status: when $\Delta\widetilde{S_N}$ is higher, $\underline{F}$ is lower, implying that $F_0 > \underline{F}$ holds with a smaller F_0, i.e., under a worse initial condition. Specifically, when $\Delta\widetilde{S_N}$ is very high (very low), everyone identifies with the nation (their class), and the tax rate is highest (zero). Consequently, $\underline{F}$ is lowest (highest), and the good outcomes are most (least) likely to be realized.

The previous proposition shows that the dynamics and long-run outcomes depend on F_0 and $\Delta\widetilde{S_N}$, as these variables determine whether $F_0 > \underline{F}$ or not. Focusing on the case $F_0 > \underline{F}$, the next proposition shows that $\Delta\widetilde{S_N}$ also impacts the *evolution of social identity and redistributive taxation*, thereby influencing the dynamics and long-run outcomes.

[31] Litschig and Lombardi (2019), using Brazilian sub-national data from 1970 to 2000, show that sub-national units with a higher share of income going to the middle quintile at the expense of the bottom quintile in 1970 grew faster subsequently, while areas with a higher initial share of income going to the top at the expense of the middle did not. Further, they find that the positive effect of a higher middle quintile share is observed only in places in which people in the middle quintile are poor. The model can yield a similar result if the initial distribution of wealth is such that many individuals in the middle quintile have b slightly below $\bar{e}$ and those in the top quintile have $b > \bar{e}$. Aiyar and Ebeke (2020), using cross-county data, find that the negative effect of income inequality on growth is stronger when the degree of the inequality of opportunity, which is measured by father-son correlations of income and education, is higher. In the model, when $F < H^*$, the intergenerational correlations are high, and the effect of increased inequality through lowered τ on H is negative, whereas when $F \geq H^*$, the correlations are low, particularly for those with $b \geq \bar{e}$, and the effect of increased inequality on H is zero or small.

Proposition 3 *Suppose the assumptions and conditions in Proposition 2 hold, and society starts with* $F_0 \in (\underline{F}, H^*)$.[32]

(*i*) *If* $\Delta\widetilde{S_N}$ *is very high* (*low*), $p{=}q{=}1$ ($p{=}q{=}0$), *and* τ *is high* ($\tau{=}0$) *at all times. Consequently, the disposable income of unskilled workers is high* (*low*) *for a given* H, *and* H *converges to* H^* *rapidly* (*slowly*).

(*ii*) *Otherwise, when* $\Delta\widetilde{S_N}$ *is relatively high* (*low*), *society generally shifts from* $p = 0$, $q = 1$ *to* $p{=}q{=}1$ ($p{=}q{=}0$) *eventually.*[33] *The shift increases* τ (*decreases* τ *to* 0) *and accelerates* (*slows or halts*) *the convergence to* H^*.[34]

(*iii*) *When* $\Delta\widetilde{S_N}$ *is not very high or low, multiple equilibria may exist for given* $\Delta\widetilde{S_N}$ *and* $H{=}F$. *The dynamics and long-run outcomes differ depending on which equilibrium is initially realized.*

Proof. See Appendix C. ■

Figure 3, based on Figure 1 in Section 3.2, is useful for understanding the dynamics when $\frac{A_s}{A_u}$ is small enough that $\overline{H} \leq \frac{\beta+\gamma}{2\beta}$. The proposition examines the situation in which F_0 is greater than the $\underline{F}$ corresponding to p and q in the initial period. In the figure, $H = F$ moves rightward as long as p and q remain unchanged and $H < H^*$. When the initial values of p and q are no longer equilibrium values, $H{=}F$ continues to move rightward if H falls between the new $\underline{F}$ and H^* for the updated values of p and q.

If $\Delta\widetilde{S_N}$ is very high (low), everyone always identifies with the nation (their class), i.e., $p = q = 1$ ($p = q = 0$), and as a result, redistributive taxation is implemented on a large scale (not implemented). Hence, the disposable income of unskilled workers is high (low) for a given H, leading to rapid (slow) convergence of H to H^* and welfare equalization.

Otherwise, when $\Delta\widetilde{S_N}$ is relatively high (low), society generally shifts from the equilibrium in which the skilled identify with their class and the unskilled identify with the nation, i.e., $p{=}0$, $q{=}1$, to the one in which *everyone identifies with the nation* (*their class*) eventually. Before the shift, the tax rate, and thus the dynamics, are not affected by $\Delta\widetilde{S_N}$. However, post-shift dynamics diverge depending on the level of $\Delta\widetilde{S_N}$. When $\Delta\widetilde{S_N}$ is relatively high, the educated shift from class to national identity and begin supporting redistribution, which expands the redistributive policy and *accelerates* convergence toward H^*. In contrast, when $\Delta\widetilde{S_N}$ is relatively low, the uneducated switch from national to class identity, reducing the demand for redistribution. As a result, redistribution ceases, *slowing or halting* convergence.[35,36,37] That is, when $\Delta\widetilde{S_N}$ is relatively high (low), an increasing share of skilled workers has a positive (*negative*) effect on national identity, redistribution, and the pace of development ultimately.

Multiple equilibria may exist for given $\Delta\widetilde{S_N}$ and $H{=}F$, unless $\Delta\widetilde{S_N}$ is very high or low. The figure illustrates that when $\Delta\widetilde{S_N}$ is particularly high (low) within the range where $p{=}0$, $q{=}1$ is

[32] $\underline{F}$ and H^* refer to the values corresponding to p and q in the initial period.

[33] As illustrated in Figure 3 (Figure 4), when $\frac{A_s}{A_u}$ is small (large) enough that $\overline{H} \leq (>) \frac{\beta+\gamma}{2\beta}$ and $\Delta\widetilde{S_N}$ is intermediate for the range of $\Delta\widetilde{S_N}$ considered in (ii), society may settle in $p = 0$, $q = 1$ ($p = 1$, $q = 0$) in the long run.

[34] The increase of H could stop after the shift to $p = q = 0$ when $\Delta\widetilde{S_N}$ falls within the range in which $p_0 = 0$, $q_0 = 1$, and F_0 is greater than $\underline{F}$ for $p = 0, q = 1$ but smaller than $\underline{F}$ for $p = q = 0$. See Figures 3 and 4.

[35] When $\beta \leq \gamma$, the identity shift has no effect on redistribution: $\tau = 0$ both when $p = 0$, $q = 1$ and when $p = q = 0$.

[36] The result of $\tau = 0$ after the shift to $p = q = 0$ arises from assumptions to make the model analytically tractable. Specifically, if ϕ for the unskilled is greater than that for the skilled, i.e., the distribution of σ_i (the parameter capturing a voter's "ideological" bias toward party 2) is narrower for the unskilled, the weight on the unskilled in the social welfare function is greater, resulting in $\tau > 0$. Nevertheless, the qualitative results remain unchanged.

[37] The increase of H may stop after the shift when $p_0{=}0$, $q_0{=}1$ and F_0 is greater than $\underline{F}$ for $p{=}q{=}0$ but smaller than $\underline{F}$ for $p{=}0, q{=}1$.

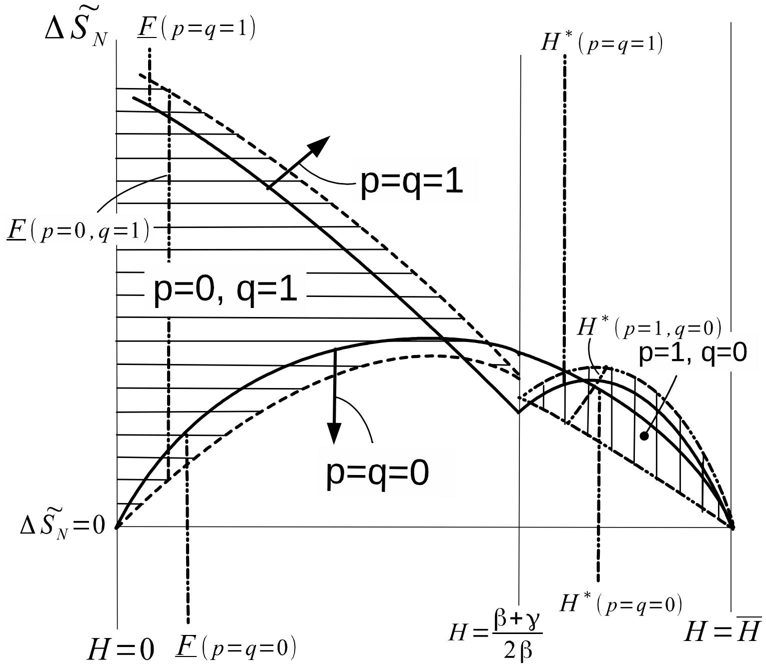

Figure 4: Dynamics when $\beta > \gamma$, $\frac{A_s}{A_u}$ is relatively high, and γ is small

an equilibrium, i.e., in the region near the upper (lower) dividing line for $p=0$, $q=1$, both $p=q=1$ ($p=q=0$) and $p=0$, $q=1$ are equilibria. The dynamics and long-run outcomes differ depending on which equilibrium *happens to* be realized initially. When society is within the upper (lower) region with multiple equilibria and $p_0=0$, $q_0=1$ happens to be the initial equilibrium, convergence to H^* is less (more) likely to occur, i.e., $\underline{F}$ is greater (smaller), and the speed of convergence is slower (faster) than when $p_0=q_0=1$ ($p_0=q_0=0$) is initially realized.

Figure 4, based on Figure 2, is a corresponding figure when the skill-biasedness of technology, $\frac{A_s}{A_u}$, is large enough that $\overline{H} > \frac{\beta+\gamma}{2\beta}$ *and* the degree of status concern γ is small enough that $H^* > \frac{\beta+\gamma}{2\beta}$.[38] (When γ is not small, the dynamics are very similar to the previous case.) Although the figure appears more complicated, the above results remain unchanged. The main difference is that $p=1$, $q=0$, i.e., the educated [uneducated] identify with the nation [their class], rather than $p=0$, $q=1$, could hold in the long run when $\Delta\widetilde{S_N}$ falls within the intermediate range. This aligns with the empirical finding of Cappelen, Enke, and Tungodden (2025) for recent developed countries.

The figures show that society generally shifts from $p=0$, $q=1$ to $p=q=1$ ($p=q=0$) when $\Delta\widetilde{S_N}$ is relatively, but not extremely, large (small). This can be explained as follows (see Section 3.2.1 for details). As H increases, the wage gap between skilled and unskilled workers narrows, and the population disparity between the classes decreases. This reduces inter-class differences in the perceived distance to the "average national" and class status, making the economic determinants of identity choice more similar across the groups. As a result, identity choices are increasingly shaped by *non-economic factors,* such as the exogenous component of national status. Thus, when H becomes sufficiently large, if the exogenous factor of national status is high (low) or that of class

[38]For the case illustrated in Figure 4, the relationships among H^* for different values of p and q are ambiguous. The exception is that, as shown in the figure, H^* for $p=1$, $q=0$ at the intersection with the upper (lower) dividing line for $p=1$, $q=0$ is greater than H^* for $p=q=1$ (smaller than H^* for $p=q=0$).

status is low (high), i.e., $\Delta\widetilde{S_N} \equiv \widetilde{S_N} - \widetilde{S_C}$ is large (small), skilled (unskilled) workers switch from class (national) to national (class) identity, leading to universal national (class) identity.

5.1.1 Dependence of perceived distance on non-economic attributes

So far, for the sake of clarity, the perceived distance term of the utility function depends solely on the difference in disposable income between oneself and the identity group. However, it would be more realistic to assume that, as in Shayo (2009), the perceived distance also depends on differences in non-economic attributes that would represent culture, norms of behavior, values, etc..

Suppose, for simplicity, that individuals belonging to a particular class share the same non-economic attributes. Then, whether an individual in class C ($C = S, U$; S [U] is for skilled [unskilled]) possesses the class-specific non-economic characteristics can be expressed by the following indicators:

$$q_C^S = 1 (= 0) \text{ and } q_C^U = 0 (= 1) \text{ for } C = S (= U). \tag{35}$$

The perceived distance between an individual in class C ($C = S, U$) and group G ($G = C, N$; N is for the nation) is then given by

$$d_{CG} = \omega_q \left(\left| q_C^S - q_G^S \right| + \left| q_C^U - q_G^U \right| \right) + \left| y_C - y_G \right|, \tag{36}$$

where ω_q is the weight on differences in the indicators, and q_G^S (q_G^U) denotes the average value of the indicator for class S (class U) specific attributes in group G. Specifically, $q_S^S = 1$, $q_S^U = 0$ ($q_U^S = 0$, $q_U^U = 1$), and $q_N^S = H$, $q_N^U = 1 - H$.

A decrease in ω_q implies that people care less about inter-class differences in culture, norms of behavior, values, and so forth. A lower ω_q may also be interpreted as a decline in class-specific culture, norms, and values, and the *homogenization of these attributes across the classes*, given that the indicators do not capture their quantitative importance. The next proposition shows that a decrease in ω_q has similar effects to an increase in $\Delta\widetilde{S_N}$ on dynamics and long-run outcomes.

Proposition 4 *Suppose that the assumptions and conditions of Proposition 2 hold.*

(*i*) *When ω_q is smaller, $\underline{F}$ is lower, making it more likely that $F_0 \geq \underline{F}$ holds. In particular, if ω_q is very small* (*large*), $p = q = 1$ ($p = q = 0$), *resulting in the lowest* (*highest*) $\underline{F}$.

(*ii*) *Suppose that society starts with $F_0 \in (\underline{F}, H^*)$.*

(*a*) *If ω_q is very small* (*large*), *then $p = q = 1$ ($p = q = 0$) and τ is high ($\tau = 0$) at all times. Consequently, the disposable income of unskilled workers is high* (*low*) *for a given H, and H converges to H^* rapidly* (*slowly*).

(*b*) *Otherwise, when ω_q is relatively small* (*large*), *society generally shifts from $p = 0$, $q = 1$ to $p = q = 1$ ($p = q = 0$) eventually. The shift increases τ* (*decreases τ to* 0) *and speeds up* (*slows or stops*) *the convergence to H^*.*

(*c*) *When ω_q is not very small or very large, multiple equilibria may exist for given ω_q and $H = F$. The dynamics and long-run outcomes differ depending on which equilibrium is realized initially.*

Proof. See Appendix C. ■

Graphically, this result holds because all dividing lines in Figures 3 and 4 shift downward as ω_q decreases.

5.1.2 Time-varying exogenous variables

Thus far, the productitivies A_s and A_u, as well as the cost of education $\overline{e}$, have been time-invariant. The qualitative results remain unchanged when these variables grow over time at the same constant rate (thus, the skill-biasedness of technology, $\frac{A_s}{A_u}$, is time-invariant), as long as the model is adjusted so that the exogenous components of status, $\widetilde{S_N}$ and $\widetilde{S_C}$, are multiplied by a variable that also grows at the same rate in the utility function.[39] It would be reasonable to suppose that $\overline{e}$ grows at the same rate as the productivities and thus earnings, given that the main cost of education is the cost of hiring teachers and other staff. The assumption on the status variables would also be plausible, because the importance of exogenous factors influencing status, such as culture, history, and values, in one's welfare does not appear to diminish with economic growth in the real world.

5.1.3 Implications

This section has explored how the endogenous evolution of the skilled worker share alongside exogenous *skill-neutral* technological change affect identity, redistribution, and development. The findings have important implications for the real economy when skill upgrading is a major driving force of development. The results indicate that large cross-country disparities in the level and pace of development may be attributed to differences in the exogenous element of national status, inter-class divides in culture, norms, and values, or people's concerns about these divides, as well as differences in the initial distributions of wealth. In many developing countries, the belief that people share a glorious history, rich culture, or "right" sense of values is weak, and inter-class cultural, normative, and value differences are large or perceived to be serious. According to the model, such situations lead to low national status or a large perceived distance to the other class, hindering the formation of a common national identity. Consequently, the scale of redistribution is limited, the upward mobility of the poor through education is constrained, and the pace of development is slow.

This implication of the model aligns with empirical findings. First, empirical research indicates that national identity promotes growth and development by increasing income redistribution and stimulating educational investment. Various studies (Chen and Li, 2009; Transue, 2007; Singh, 2015; Cappelen, Enke, and Tungodden, 2025) suggest that national identity has a positive effect on redistribution (see footnote 16 in Section 3.1 for details). Berg et al. (2018), based on cross-country data covering numerous countries, indicates that income redistribution, unless very large-scale, makes growth faster and more sustainable by reducing income inequality. They also find that lower inequality is associated with higher years of education. Hanushek and Woessmann (2012a) find that educational achievement, measured by cognitive skills, has a large effect on growth, using data from 64 countries. Second, there is suggestive evidence for Latin American countries that income redistribution is limited in scale and difficult to expand due to a weak sense of common identity stemming from severe social divisions. Goni, Lopez, and Serven (2011) observe that while market inequality is not very different between Latin American and Western European countries, after-tax

[39]Suppose that these variables grow at rate g, and let $A_{st} = g^t A_s$, $A_{ut} = g^t A_u$, and $\overline{e_t} = g^{t-1}\overline{e}$ (note that $\overline{e_t}$ is the cost in period $t-1$). Then, given H_t, w_{st}, w_{ut}, and T_t also grow at g. Denote detrended endogenous variables with a tilde, e.g., $\widetilde{w_{st}} \equiv \frac{w_{st}}{g^t}$ and $\widetilde{b_t} \equiv \frac{b_t}{g^{t-1}}$. By dividing both sides of the equations by g^t, (28) and (29) can be respectively expressed as $\widetilde{b_{t+1}} = \lambda\{(1-\tau_t)\widetilde{w_{ut}} + \widetilde{T_t} + (1+r)\frac{\widetilde{b_t}}{g}\}$ and $\widetilde{b_{t+1}} = \lambda\{(1-\tau_t)\widetilde{w_{st}} + \widetilde{T_t} + (1+r)\frac{1}{g}(\widetilde{b_t} - \overline{e})\}$. Then, the conditions for $H_{t+1} = F_{t+1} > H_t = F_t$ become $\lambda[(1-\tau_t)\widetilde{w_{st}} + \widetilde{T_t}] \geq \overline{e}$ and $\frac{\lambda}{1-\frac{\lambda}{g}(1+r)}\left[(1-\tau_t)\widetilde{w_{ut}} + \widetilde{T_t}\right] > \overline{e}$, which are very similar to (32) and (33), respectively. Results on identity choice and the tax rate do not change because given H, all terms in the utility function grow at g. Results on earnings and welfare disparities do not change when relative measures are used. Results on output, earnings, disposable incomes, and the long-run welfare level remain valid when detrended.

after-transfer inequality is much higher in Latin America due to small and poorly targeted transfers. This holds true even when public spending on education and health is considered.[40] Blofield and Juan Pablo (2011), using World Values Survey data, find that a significant portion of the population in Latin American countries favors even *higher* income inequalities than those currently prevalent (though a sizable segment supports lower inequalities), whereas Europeans generally accept the status quo. Experts on Latin America (O'Donnell, 1998; Vilas, 1997) argue that implementing policies to seriously address poverty and inequality is challenging because people do not have a sense of common belonging or broad solidarity due to sharp social divisions or polarization.

The results indicate the critical importance of *nation-building policies*, such as school education and government propaganda emphasizing common history, culture, and values, as well as policies promoting inter-group contact, in countries with low national status or large perceived inter-class cultural, normative, and value divides. According to the model, these policies enhance national status or reduce (or deemphasize) the inter-class divisions, thereby contributing to the formation of a shared identity and fostering economic development. Studies indicate that nation-building policies can increase support for income redistribution or strengthen national identity. Londoño-Vélez (2022) finds that a Colombian financial aid reform that increased the share of low-income students at an elite university prompted cross-class interaction and led to greater support for progressive redistribution among high-income students. Chen, Lin, and Yang (2023) examine a curriculum reform that introduced a large amount of Taiwan-related contents into the history subject for junior high school students and find that students under the new curriculum are much more likely to hold an exclusive Taiwanese identity rather than dual identities of Taiwanese and Chinese. Blouin and Mukand (2019), based on lab-in-the-field experiments in post-genocide Rwanda, show that exposure to government radio propaganda that emphasizes a new Rwandan identity weakened ethnic identity and increased interethnic trust and cooperation.

Finally, according to the model, when the share of skilled workers and thus the level of development become high, typically, everyone identifies with the nation (their class) when $\Delta\widetilde{S_N}$ is high (low) or ω_q is low (high). Specifically, if $\Delta\widetilde{S_N}$ is relatively, but not extremely, high or ω_q is relatively, but not extremely, low, society shifts from the equilibrium in which the skilled identify with their class and the unskilled identify with the nation to universal national identity; otherwise, it shifts to universal class identity. Classic modernization theories in political science (Deutsch, 1953; Gellner, 1983; Weber, 1979), based on Europe's past experiences, argue that modernization (including industrialization and universal education) fosters widespread national identity at the expense of subnational identities (Robinson, 2014). However, the result suggests that these theories hold true *only when* national status is relatively high or perceived inter-class cultural, normative, and value divides are relatively small. The result also shows that the identity shift associated with modernization positively impacts redistribution and development pace if these conditions are met, but *negatively* impacts the outcomes otherwise.

5.2 Effect of SBTC on identity, redistribution, and development

This section explores the impact of skill-biased technical change (SBTC), i.e., increasing $\frac{A_s}{A_u}$, which has been a key engine of economic growth in both developed and developing economies over the past

[40]Hanushek and Woessmann (2012b) find that the poor growth and development performance of Latin American nations is primarily attributed to poor educational achievement. This finding, along with the limited scale of redistribution, suggests that greater redistribution, including increased public expenditures on education, could improve economic outcomes by enabling the poor to access quality education.

several decades (Schulte, 2021).[41] In the model, SBTC is assumed exogenous, as most technological progress originates from foreign economies, except in very large economies like the U.S. To simplify the analysis, exogenous variables such as A_u and $\Delta\widetilde{S_N}$ are fixed: as mentioned in Section 5.1.2, the qualitative results remain unchanged when A_u grows at a constant rate and the exogenous components of status, $\widetilde{S_N}$ and $\widetilde{S_C}$, are multiplied by a variable that grows at the same rate. Unlike the original setting, $\bar{e}$ is assumed to be proportional to w_s, as the main cost of education in the real world is the cost of hiring teachers (i.e., skilled workers) and w_s now grows over time due to SBTC. The following proposition summarizes the findings.

Proposition 5 *Suppose that the assumptions and conditions of Proposition 2 hold. Then, for a given level of H, an increase in* $\frac{A_s}{A_u}$ *has the following effects.*

(*i*) *The inter-class disparity in earnings increases.*

(*ii*) *All dividing lines for identity choice shift upward on the* $(H, \Delta\widetilde{S_N})$ *plane. Consequently, if* $\frac{A_s}{A_u}$ *continues to rise, society eventually shifts to an equilibrium with weaker national identity, unless* $\Delta\widetilde{S_N}$ *is small:*

(*a*) *If* $\frac{A_s}{A_u}$ *is small enough that* $\overline{H} \leq \frac{\beta+\gamma}{2\beta}$ *(as in Figure 3) or* $H < \frac{\beta+\gamma}{2\beta}$, *society shifts from* $p=q=1$ *to* $p=0$, $q=1$ *(from* $p=0, q=1$ *to* $p=q=0$*) when* $\Delta\widetilde{S_N}$ *is relatively large (small).*[42]

(*b*) *Otherwise, it shifts from* $p = q = 1$ *to* $p = q = 0$, *possibly via* $p = 1, q = 0$, *when* $\Delta\widetilde{S_N}$ *is relatively small.*

(*iii*) *Given p and q,* τ *increases, but the inter-class disparity in welfare widens. When the identity shift occurs,* τ *falls, exacerbating the welfare disparity.*

(*iv*) *When the cost of education is sufficiently high,*[43] $\underline{F}_t$ *increases, and if* $F_0 > \underline{F}_0$, *the speed of convergence to* H^*, *where* H^* *increases with* $\frac{A_s}{A_u}$, *slows down. This slowdown is particularly pronounced when the identity shift occurs.*

Proof. See Appendix C. ■

SBTC widens the wage gap between skilled and unskilled workers. To counteract this growing disparity, the tax rate is raised, but the expansion of redistribution is insufficient to fully offset the rise in inequality. Hence, the perceived distances to the other class, and thus to the "average national," increase. SBTC also raises (lowers) the status of the skilled (unskilled) class relative to that of the nation, due to the growing inter-class disparity in disposable income. These changes contribute to weakening the national identity of skilled workers in the sense that the utility gain from identifying with the nation rather than their class decreases. For unskilled workers, the lowered relative status of their class partially offsets the increased distance to the "average national", but the latter effect dominates (due to $\beta > \gamma$), leading to a weakened national identity for them as well. As a result, all dividing lines for identity choice in Figures 3 and 4 shift upward. Therefore, if $\frac{A_s}{A_u}$ continues to increase, society eventually shifts to an equilibrium with weaker national identity and *lower* redistributive taxation (unless $\Delta\widetilde{S_N}$ is small and thus everyone identifies with their class).

The growing inequality in disposable income exacerbates the disparity in welfare. This effect is particularly large when the identity shift occurs, as it reduces the scale of redistribution.

[41]In particular, Schulte (2021), using panel data from 40 advanced and emerging economies for the period 1995-2007, shows that skill-biased technology diffusion, the spread of skill-biased technology from one country to others, is an important part of SBTC in developing countries, accounting for about 43% of SBTC.

[42]To be precise, when $\overline{H} > \frac{\beta+\gamma}{2\beta}$ and $H(< \frac{\beta+\gamma}{2\beta})$ is close to $\frac{\beta+\gamma}{2\beta}$, shifts from $p = q = 1$ to $p = q = 0$, as well as from $p = 0$, $q = 1$ to $p = q = 1$, and then to $p = q = 0$, are also possible (see Figure 4).

[43]To be accurate, this is true when the constant s in $\bar{e}_t = sw_{st-1}$ is sufficiently large.

SBTC also influences the dynamics and long-run outcomes. It increases $\underline{F}_t$ by raising the cost of education, which is proportional to the skilled wage, at a higher rate than the unskilled wage. Consequently, escaping the "poverty trap" becomes more challenging for societies starting with a relatively small share of individuals able to afford education. Further, the pace of convergence to H^* slows down when $F_0 > \underline{F}_0$, particularly when the identity shift occurs and the scale of redistribution shrinks.

Regardless of the levels of non-economic determinants of identity such as $\Delta\widetilde{S_N}$, SBTC ultimately has a negative effect on national identity, redistribution, and the pace of development. This is because SBTC increases inter-class disparities in earnings and disposable income, thereby weakening national identity. This contrasts with the impact of an increase in H, which differs depending on the levels of $\Delta\widetilde{S_N}$ and ω_q (the weight on non-economic attributes in perceived distance). A higher share of skilled workers makes the economic determinants of identity more similar across classes, increasing the importance of non-economic factors. Consequently, when $\Delta\widetilde{S_N}$ is relatively high (low) or ω_q is relatively small (large), an increasing share of skilled workers positively (negatively) influences national identity, redistribution, and the pace of development.

5.2.1 Implications

The result suggests that as technology becomes more skill-biased, establishing national identity becomes more challenging, and redistribution becomes less effective. Consequently, achieving upward mobility for the poor and equalizing welfare becomes more difficult or slower. SBTC has advanced in both developing and developed economies in recent decades (Schulte, 2021). The higher skill bias of current technology, along with a weaker sense of shared history, culture, and values, as well as greater inter-class cultural and normative divides, may explain why many developing countries experience slower upward mobility and development compared to the rates seen in developed countries during their modernization periods.

In advanced economies, income inequality has risen significantly in recent decades, yet the demand for and scale of income redistribution have not increased; in some measures, they have even decreased (Kenworthy and McCall, 2008; Ashok et al., 2016). To explain this phenomenon, Windsteiger (2022) develops a model in which individuals are segregated according to income and mainly interact with those at similar income levels, resulting in biased information on income inequality. Consequently, under certain conditions, increased inequality leads to a fall in perceived inequality and thus a decrease in support for and the level of redistribution. With a slight modification, the present model can also explain this phenomenon. Suppose that, unlike the original setting, individuals are heterogeneous in the weight on the exogenous component of group status in utility, δ, with a continuous distribution. Then, unless $\Delta\widetilde{S_N}$ is small or very large, as $\frac{A_s}{A_u}$ increases, p and q, and thus τ, could decrease *continuously*.

6 Conclusion

This paper developed a dynamic model of income redistribution and educational investment incorporating social identification and theoretically explored the interaction among identity, redistribution, and development. Analysis showed that, given a skilled workers' share, the rate of redistributive taxation is higher as the proportion of individuals (especially the skilled) identifying with the nation is higher. When the exogenous component of national status is higher (which would be the case when people have stronger pride in the nation, for example, due to a shared belief in a glorious national history), or when inter-class differences in culture, norms, and values are smaller or less of a concern, the proportion of people having a national identity and the redistributive tax rate are higher. Consequently, society is less likely to fall into a "poverty trap", and under favorable initial conditions that avoid this trap, the share of skilled workers and output increase faster. As the proportion of skilled workers rises, society generally undergoes a shift in social identity, significantly influencing subsequent dynamics. When these exogenous factors are favorable (unfavorable), an increasing skilled share ultimately has a positive (negative) effect on national identity, redistribution, and the pace of development. Unlike an increasing share of skilled workers, the impact of skill-biased technical change (SBTC) is consistently negative regardless of non-economic determinants of identity such as the exogenous component of national identity. As SBTC proceeds, society becomes more prone to a "poverty trap", and under favorable initial conditions, the upward mobility of the poor slows down. Further, when SBTC continues, society generally shifts to an equilibrium where fewer people identify with the nation, leading to reduced redistribution, thereby intensifying SBTC's negative effects.

The result suggests that large cross-country differences in the level and pace of development may partly stem from differences in the exogenous component of national status, as well as in inter-class divides in culture, norms, and values or people's concerns about these divides. In many developing countries, the belief that people share a glorious history, rich culture, or a "right" sense of values is weak and inter-class differences in culture, norms, and values are large or perceived to be serious. According to the model, these factors make the formation of a common national identity difficult, and as a result, the scale of redistribution limited, the upward mobility of the poor and the pace of development slow. For these countries, nation-building policies, such as school education and government propaganda emphasizing common history, culture, and values and policies promoting between-group contact, would be crucial for good outcomes. Technology available for present developing countries is much more skill-biased than what developed countries used when they underwent the modernization and industrialization of their economies. The result on SBTC suggests that this may be another reason for the slower pace of development, particularly of the poor, in many developing countries.

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Appendix A Propositions A1 and A2

This Appendix presents results on shapes of dividing lines for quilibria, which are the basis for Figures 1 and 2 in Section 3.2. The next proposition presents the result when $\beta \leq \gamma$.

Proposition A1 *Suppose that $\beta \leq \gamma$ and Assumption 2 holds.*

(i) $p = q = 1$, $p = 0$, $q = 1$, and $p = q = 0$ can be stable equilibria.

(ii) The dividing line for $p = q = 1$ and the upper dividing line for $p = 0$ and $q = 1$ decrease with H, go to $+\infty$ as $H \to 0$, and go to 0 as $H \to \overline{H}$, where $\overline{H}$ is H satisfying $H = a(H) \Leftrightarrow w_s = w_u$, on the $(H, \Delta\widetilde{S_N})$ plane.

(iii) There exists an $H^\sharp \in (0, \overline{H})$ such that the dividing line for $p = q = 0$ and the lower dividing line for $p = 0$ and $q = 1$ decrease (increase) with H for $H < (>) H^\sharp$. They go to 0 as $H \to 0$ and $H \to \overline{H}$.

(iv) On the $(H, \Delta\widetilde{S_N})$ plane, the dividing line for $p = q = 0$ and the lower dividing line for $p = 0$ and $q = 1$ are the same; they are located below the dividing line for $p = q = 1$ and the upper dividing line for $p = 0$ and $q = 1$; the dividing line for $p = q = 1$ is located below the upper dividing line for $p = 0$ and $q = 1$.

Proof. See Appendix C. ■

Based on the proposition, Figure 5 illustrates combinations of H and $\Delta\widetilde{S_N}$ each equilibrium exists when $\beta \leq \gamma$. There are two differences from the figures when $\beta > \gamma$ (Figures 1 and 2 in Section 3.2). First, the dividing line for $p = q = 0$ decreases (increases) with H for relatively small (large) H. That is, the relation with H is opposite to the case $\beta > \gamma$. This is because the effect

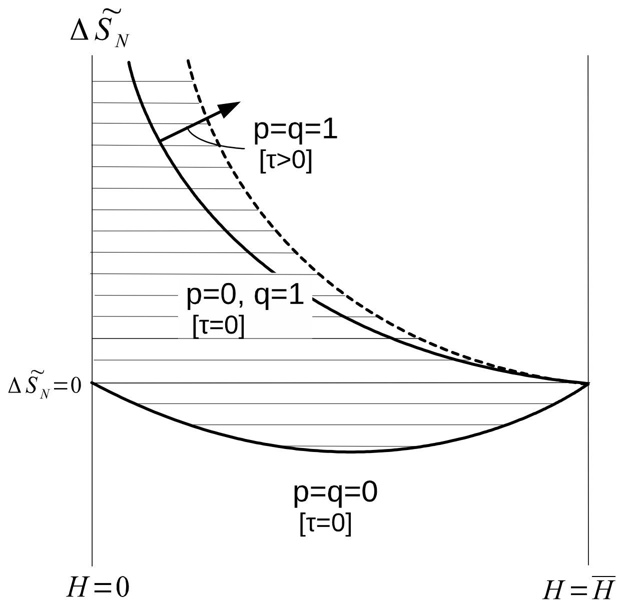

Figure 5: Equilibria when $\beta \leq \gamma$

of H on status dominates the effect on perceived distance when $\beta \leq \gamma$, where the former effect is that, when H is relatively small (large), an increase in H increases (decreases) national status relative to class status and thus the utility from identifying with the nation. Second, the lower dividing line for $p = 0$, $q = 1$ coincides with the dividing line for $p = q = 0$ because τ when $p = 0$, $q = 1$ is 0 when $\beta \leq \gamma$. Except these points, the figure is similar to the one when $\beta > \gamma$ and A_s is relatively low (Figure 1).

The next proposition presents the result when $\beta > \gamma$. Based on (i) [(ii) and (iii)] of the proposition, Figure 1 [Figure 2] in Section 3.2 illustrates combinations of H and $\Delta\widetilde{S_N}$ each equilibrium exists when $\frac{A_s}{A_u}$ is relatively low [high].

Proposition A2 *Suppose that $\beta > \gamma$ and Assumption 2 holds.*

(i) When $\frac{A_s}{A_u}$ is small enough that $\overline{H} \leq \frac{\beta+\gamma}{2\beta}$, Proposition A1 applies except the following.

(a) The dividing line for $p = q = 0$ and the lower dividing line for $p = 0$ and $q = 1$ are different. The former increases (decreases) with H for $H < (>) H^{\sharp}$. The latter increases (decreases) with H for small (large) enough H.[44]

(b) On the $(H, \Delta\widetilde{S_N})$ plane, the dividing line for $p = q = 0$ is located above the lower dividing line for $p = 0$ and $q = 1$; when H is relatively high, the dividing line for $p = q = 0$ could be located above the dividing line for $p = q = 1$ and the upper dividing line for $p = 0$ and $q = 1$.

(ii) When $\frac{A_s}{A_u}$ is large enough that $\overline{H} > \frac{\beta+\gamma}{2\beta}$ and $H \leq \frac{\beta+\gamma}{2\beta}$, the results are same as (i) except the following.

(a) When A_s is large enough that $H^{\sharp} \geq \frac{\beta+\gamma}{2\beta}$, the dividing line for $p = q = 0$ and the lower dividing line for $p = 0$ and $q = 1$ increase with H.

[44]When H is intermediate, the relationship with H is not analytically clear.

(b) *When H is relatively high, the dividing line for $p=q=0$ is located above the dividing line for $p=q=1$ and the upper dividing line for $p=0$ and $q=1$. The lower and upper dividing lines for $p=0$ and $q=1$ intersect at $H=\frac{\beta+\gamma}{2\beta}$.*

(iii) *When $\overline{H}>\frac{\beta+\gamma}{2\beta}$ and $H>\frac{\beta+\gamma}{2\beta}$,*

(a) *$p=q=1$, $p=1$ and $q=0$, and $p=q=0$ can be stable equilibria.*

(b) *The dividing line for $p=q=0$ and the lower dividing line for $p=1$ and $q=0$ decrease with H and go to 0 as $H\to\overline{H}$ on the $(H,\Delta\widetilde{S_N})$ plane.*

(c) *The dividing line for $p=q=1$ and the upper dividing line for $p=1$ and $q=0$ decrease with H for sufficiently large H and go to 0 as $H\to\overline{H}$. They could increase with H for sufficiently small H.*[45]

(d) *The dividing line for $p=q=0$ is located above the lower dividing line for $p=1$ and $q=0$; the dividing line for $p=q=1$ is located below the upper dividing line for $p=1$ and $q=0$; when H is relatively low (high), the dividing line for $p=q=0$ is located above (below) the dividing line for $p=q=1$ and the upper dividing line for $p=1$ and $q=0$; the lower and upper dividing lines for $p=1$ and $q=0$ intersect at $H=\frac{\beta+\gamma}{2\beta}$.*[46]

Proof. See Appendix C. ■

Appendix B Determination of H^* and Proof on the value of H

This Appendix formally explains how H^* is determined and proves that $H=F$ ($H=H^*$) holds when F is small (large). By substituting (26) and (27) into (24), the indirect utility function equals

$$v_{CG}=(1-\tau)w_C+T+(1+r)a-\beta d_{CG}+\gamma S_G. \tag{37}$$

From this equation, (9)–(12), (28), and (29),

$$\begin{aligned}
v_{SN} &= (1+\gamma)\left(\tau-\frac{1}{2}\tau^2\right)\overline{w}+(1-\tau)w_s-\beta(1-\tau)(w_s-\overline{w})+\gamma\left[\delta\widetilde{S_N}+(1-\tau)\overline{w}\right]+(1+r)a,\\
&= (1+\gamma)\left(\tau-\frac{1}{2}\tau^2\right)\overline{w}+(1-\tau)w_s-\beta(1-\tau)(w_s-\overline{w})+\gamma\left[\delta\widetilde{S_N}+(1-\tau)\overline{w}\right]+(1+r)(b-\overline{e}), \quad (38)\\
v_{SS} &= (1+\gamma)\left(\tau-\frac{1}{2}\tau^2\right)\overline{w}+(1-\tau)w_s+\gamma\left[\delta\widetilde{S_C}+(1-\tau)w_s\right]+(1+r)(b-\overline{e}), \quad (39)\\
v_{UN} &= (1+\gamma)\left(\tau-\frac{1}{2}\tau^2\right)\overline{w}+(1-\tau)w_u-\beta(1-\tau)(\overline{w}-w_u)+\gamma\left[\delta\widetilde{S_N}+(1-\tau)\overline{w}\right]+(1+r)b, \quad (40)\\
v_{UU} &= (1+\gamma)\left(\tau-\frac{1}{2}\tau^2\right)\overline{w}+(1-\tau)w_u+\gamma\left[\delta\widetilde{S_C}+(1-\tau)w_u\right]+(1+r)b. \quad (41)
\end{aligned}$$

H^* when $p=q=1$ is H satisfying $v_{SN}=v_{UN}$, thus from (38) and (40), H satisfying

$$\begin{aligned}
&(1-\tau)w_s-\beta(1-\tau)(w_s-\overline{w})-(1+r)\overline{e}=(1-\tau)w_u-\beta(1-\tau)(\overline{w}-w_u)\\
&\Leftrightarrow [1-\beta(1-2H)](1-\tau)(w_s-w_u)-(1+r)\overline{e}=0, \quad (42)\\
&\text{where } \tau=\frac{2\beta}{1+\gamma}(a(H)-H).
\end{aligned}$$

[45]When H is intermediate, the relationship with H is not analytically clear.

[46]When H is intermediate, the relative positions of these dividing lines are not analytically clear.

H^* when $p=q=0$ is H satisfying $v_{SS}=v_{UU}$, thus from (39) and (41), H satisfying

$$(1+\gamma)(w_s-w_u)-(1+r)\overline{e}=0. \tag{43}$$

H^* when $p=0$, $q=1$ is H satisfying $v_{SS}=v_{UN}$, thus from (39) and (40), H satisfying

$$\begin{aligned}(1-\tau)w_s+\gamma\left[\delta\widetilde{S_C}+(1-\tau)w_s\right]-(1+r)\overline{e} &= (1-\tau)w_u-\beta(1-\tau)(\overline{w}-w_u)+\gamma\left[\delta\widetilde{S_N}+(1-\tau)\overline{w}\right]\\ \Leftrightarrow [1+\beta H+\gamma(1-H)](1-\tau)(w_s-w_u)&-(1+r)\overline{e}-\gamma\delta\Delta\widetilde{S_N}=0, \\ \text{where } \tau=\tfrac{\beta-\gamma}{1+\gamma}(a(H)-H) &\text{ when } \beta>\gamma \text{ and } \tau=0 \text{ when } \beta\le\gamma.\end{aligned} \tag{44}$$

H^* when $p=1$, $q=0$ is H satisfying $v_{SN}=v_{UU}$, thus from (38) and (41), H satisfying

$$\begin{aligned}(1-\tau)w_s-\beta(1-\tau)(w_s-\overline{w})+\gamma\left[\delta\widetilde{S_N}+(1-\tau)\overline{w}\right]-(1+r)\overline{e} &= (1-\tau)w_u+\gamma\left[\delta\widetilde{S_C}+(1-\tau)w_u\right]\\ \Leftrightarrow [1-\beta(1-H)+\gamma H](1-\tau)(w_s-w_u)&-(1+r)\overline{e}+\gamma\delta\Delta\widetilde{S_N}=0, \\ \text{where } \tau=\tfrac{\beta+\gamma}{1+\gamma}&(a(H)-H).\end{aligned} \tag{45}$$

H^* exists because the LHSs of (42)−(45) go to $+\infty$ as $H\to 0$ from $\lim_{H\to 0}(w_s-w_u)=+\infty$, and the LHSs become negative as $H\to\overline{H}$ from $\lim_{H\to\overline{H}}(w_s-w_u)=0$. To be more precise, the latter is true when $p=0$, $q=1$, which is an equlibrium only when $\beta>\gamma$ from Propositions A1 and A2, because $\Delta\widetilde{S_N}$ in (44) is positive from (22). It is true when $p=1$, $q=0$ because the LHS of (45) is smaller than $[1-\beta(1-H)+\gamma H](1-\tau)(w_s-w_u)-(1+r)\overline{e}+(\beta-\gamma)(1-\tau)H(w_s-w_u)=[1-\beta(1-2H)](1-\tau)(w_s-w_u)-(1+r)\overline{e}$ from (23).

Proof that $H=F$ ($H=H^*$) when F is small (large)

When $p=q=0$, $H=F$ ($H=H^*$) holds when $v_{SS}\ge(<)v_{UU}$ is satisfied with $H=F$, i.e., $(1+\gamma)(w_s-w_u)-(1+r)\overline{e}\ge(<)0$ with $H=F$ from (43). Because w_s-w_u decreases with H, this is the case when $F\le(>)H^*$. Similarly, when $p=0$, $q=1$ and $\beta\le\gamma$ (thus $\tau=0$), $H=F$ ($H=H^*$) holds when $F\le(>)H^*$ from (44).

For other cases, the relation between H and the LHSs of the above equations determining H^* is generally unclear. Whether $H=F$ or $H=H^*$ is clear only for sufficiently small or large F. As for the range of F satisfying $H=F$, the following is true. When $p=0$, $q=1$ and $\beta>\gamma$, from (22), the LHS of (44) is greater than $[1-\beta(1-2H)](1-\tau)(w_s-w_u)-(1+r)\overline{e}>(1-\beta)(1-\tau)(w_s-w_u)-(1+r)\overline{e}$. Hence, $H=F$ holds at least for F weakly smaller than the smallest H satisfying $(1-\beta)(1-\tau)(w_s-w_u)-(1+r)\overline{e}=0$, which is smaller than H^* for $p=q=0$. It is easy to see that a similar result holds when $p=q=1$ from (42). When $p=1$, $q=0$, from (23), the LHS of (45) is greater than $(1+\gamma)(1-\tau)(w_s-w_u)-(1+r)\overline{e}$. Hence, $H=F$ holds at least for F weakly smaller than the smallest H satisfying $(1+\gamma)(1-\tau)(w_s-w_u)-(1+r)\overline{e}=0$, which is smaller than H^* for $p=q=0$.

As for the range of F satisfying $H=H^*$, the following is true. When $p=0$, $q=1$ and $\beta>\gamma$, $H=H^*$ at least for F weakly greater than H^* for $p=q=0$. This is because, from (22), the LHS of (44) is smaller than $(1+\gamma)(1-\tau)(w_s-w_u)-(1+r)\overline{e}$, which is smaller than the LHS of (43). Similarly, when $p=q=1$ and $\beta\le\gamma$, $H=H^*$ at least for F weakly greater than H^* for $p=q=0$ because the LHS of (42) is smaller than that of (43) from $[1-\beta(1-2H)](1-\tau)(w_s-w_u)<(1+\gamma)(w_s-w_u)$. When $p=q=1$ and $\beta>\gamma$, the LHS of (42) is smaller than $(1+\beta)(w_s-w_u)-(1+r)$, thus $H=H^*$ at least for F weakly greater than H satisfying $(1+\beta)(w_s-w_u)-(1+r)=0$, which is greater than H^* for $p=q=0$. A similar result holds when $p=1$, $q=0$ also because the LHS of (45) is smaller than $[1-\beta(1-2H)](1-\tau)(w_s-w_u)-(1+r)\overline{e}$, which is smaller than $(1+\beta)(w_s-w_u)(w_s-w_u)-(1+r)\overline{e}$.

Appendix C Proofs of lemmas, propositions, and a corollary

Proof of Lemma 1. Suppose that $q \in (0,1)$ is an equilibrium, which implies that unskilled workers are indifferent between the two identities. Then, from (11) and (12),

$$-\beta(1-\tau)(\overline{w}-w_u)+\gamma\left[\delta\widetilde{S_N}+(1-\tau)\overline{w}+T\right]=\gamma\left[\delta\widetilde{S_C}+(1-\tau)w_u+T\right]$$
$$\Leftrightarrow \gamma\delta\Delta\widetilde{S_N}=(\beta-\gamma)(1-\tau)(\overline{w}-w_u), \tag{46}$$

where from (17),

$$\tau = 1-\frac{1}{(1+\gamma)\overline{w}}\begin{pmatrix}[Hw_s+(1-H)w_u]+\gamma\{H[p\overline{w}+(1-p)w_s]+(1-H)[q\overline{w}+(1-q)w_u]\}\\ -\beta\{Hp(w_s-\overline{w})+(1-H)q(\overline{w}-w_u)\}\end{pmatrix}.$$

From the above equation, τ increases (decreases) with q when $\beta > (<)\gamma$. Thus, when q increases, the RHS of (46) decreases and thus identifying with the nation becomes more attractive than identifying with their class. This means that the equilibrium is unstable. (Further, when $\beta = \gamma$, (46) holds only when $\Delta\widetilde{S_N} = 0$.) $p \in (0,1)$ is not a stable equilibrium can be proved similarly. ■

Proof of Proposition 1. (i) Values of τ are obtained from (17). $a(H) \equiv \frac{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}}{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}+(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}} \geq H \Leftrightarrow w_s \geq w_u$ from (4) and (5). (ii) Straightforward from (i) except that $p = 1, q = 0$ is not an equilibrium when $\beta \leq \gamma$, which is shown in the proof of Proposition A1 (i). (iii) From (i)(b), $a(H) - H = 0$ at $H = 0, a(H)$, and

$$\begin{aligned}
a'(H)&=\frac{\sigma-1}{\sigma}\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}\frac{\left\{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}+(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}\right\}\frac{1}{H}-\left\{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}\frac{1}{H}-(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}\frac{1}{1-H}\right\}}{\left\{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}+(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}\right\}^2}\\
&=\frac{\sigma-1}{\sigma}\frac{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}}{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}+(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}}\frac{(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}}{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}+(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}}\frac{1}{H(1-H)}\\
&=\frac{\sigma-1}{\sigma}\frac{a(H)[1-a(H)]}{H(1-H)} > 0,
\end{aligned} \tag{47}$$

which implies that

$$\begin{aligned}
\lim_{H\to 0}[a'(H)-1] &= \frac{\sigma-1}{\sigma}\lim_{H\to 0}\left[\frac{a(H)}{H}\right]-1\\
&= \frac{\sigma-1}{\sigma}\lim_{H\to 0}\left\{\frac{1}{H}\frac{1}{1+\frac{1-\alpha}{\alpha}\left[\frac{A_u(1-H)}{A_sH}\right]^{\frac{\sigma-1}{\sigma}}}\right\}-1 = +\infty,
\end{aligned} \tag{48}$$

$$\lim_{H\to\overline{H}}[a'(H)-1] = -\frac{1}{\sigma} < 0. \tag{49}$$

Further,

$$\begin{aligned}
a''(H) &= \frac{\sigma-1}{\sigma}\frac{H(1-H)a'(H)[1-2a(H)]-a(H)[1-a(H)](1-2H)}{[H(1-H)]^2}\\
&= \frac{\sigma-1}{\sigma}a(H)[1-a(H)]\frac{\frac{\sigma-1}{\sigma}[1-2a(H)]-(1-2H)}{[H(1-H)]^2}\\
&= \frac{\sigma-1}{\sigma}a(H)[1-a(H)]\frac{-\frac{1}{\sigma}[1-2a(H)]-2(a(H)-H)}{[H(1-H)]^2} < 0.
\end{aligned} \tag{50}$$

Hence, there exists unique $H^+ \in (0,\overline{H})$ satisfying $a'(H)-1=0$, and $a(H)-H$ increases (decreases) with H for $H<(>)H^+$. This implies that $\frac{d\tau}{dH}>(<)0$ for $H<(>)H^+$. ■

Proof of Corollary 1 . (i) The result on inequality is from $(1-\tau)w_s+T-[(1-\tau)w_u+T]=(1-\tau)(w_s-w_u)$ and Proposition 1 (ii). The result on the disposable income of skilled workers holds because $(1-\tau)w_s+T=(1-\tau)w_s+\left(\tau-\frac{1}{2}\tau^2\right)\overline{w}$ decreases with τ from $-w_s+(1-\tau)\overline{w}<0$. (ii) The derivative of $(1-\tau)w_u+T=(1-\tau)w_u+\left(\tau-\frac{1}{2}\tau^2\right)\overline{w}$ with respect to τ equals $-w_u+(1-\tau)\overline{w}$, which is positive under Assumption 1 because

$$\begin{aligned} -w_u+(1-\tau)\,\overline{w}>0 &\Leftrightarrow \tau<\frac{\overline{w}-w_u}{\overline{w}} \\ &\Leftrightarrow \tau<\frac{H\left\{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}\frac{1}{H}-(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}\frac{1}{1-H}\right\}}{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}+(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}} \text{ (from (3) and (5))} \\ &\Leftarrow \frac{2\beta}{1+\gamma}(a(H)-H)<\frac{1}{1-H}(a(H)-H) \text{ (from Proposition 1),} \end{aligned} \tag{51}$$

where the last inequality is true under Assumption 1. ■

Proof of Lemma 2. (i) When $p=q=0$ or when $\beta\leq\gamma$ and $p=0$, $q=1$, $\tau=0$ from Proposition 1 and thus $(1-\tau)w_s+T=w_s$, which equals $w_u+\frac{(1+r)\overline{e}}{1+\gamma}$ $(w_u+\frac{(1+r)\overline{e}+\gamma\delta\Delta\widetilde{S_N}}{1+\beta H+\gamma(1-H)})$ at $H=H^*$ when $p=q=0$ from (43) (when $\beta\leq\gamma$ and $p=0$, $q=1$ from (44)). Because w_s decreases with H from (4), (32) is satisfied for any $H\leq H^*$ if it holds at $H=H^*$, where the condtion can be expressed as $\lambda w_u\geq\left[1-\frac{\lambda(1+r)}{1+\gamma}\right]\overline{e}$ when $p=q=0$ and as $\lambda\left[w_u+\frac{\gamma\delta\Delta\widetilde{S_N}}{1+\beta H+\gamma(1-H)}\right]\geq\left[1-\frac{\lambda(1+r)}{1+\beta H+\gamma(1-H)}\right]\overline{e}$ when $p=0$, $q=1$. Because w_s-w_u decreases with H and w_u increases with H from (4) and (5), H^* and w_u at $H=H^*$ decrease with $\overline{e}$. Hence, (32) holds when $\overline{e}$ is sufficiently small or λ is sufficiently large. Because $w_s<w_u+(1+r)\overline{e}$ at $H=H^*$, (33) too holds at $H=H^*$. For $H<H^*$, the LHS of (33) increases with H. Thus, if (34) holds at $H=0$, there exists $\underline{F}\in(0,H^*)$ such that $\frac{\lambda}{1-\lambda(1+r)}w_u=\overline{e}$ at $H=\underline{F}$; $\frac{\lambda}{1-\lambda(1+r)}w_u>(\leq)\overline{e}$ and thus $H=F<H^*$ increases over time (is time-invariant) for $H>(\leq)\underline{F}$. Because $w_u=(1-\alpha)^{\frac{\sigma}{\sigma-1}}A_u$ at $H=0$, the condition holds if $\overline{e}$ is not too small or λ is not too large so that $\frac{\lambda}{1-\lambda(1+r)}(1-\alpha)^{\frac{\sigma}{\sigma-1}}A_u<\overline{e}$ is true.

When values of p and q are such that $\tau>0$, whether $(1-\tau)w_s\geq(1-\tau)w_u+(1+r)\overline{e}$ at $H=H^*$ or not is unclear from (42), (44), and (45). Consider the case in which $(1-\tau)w_s\geq(1-\tau)w_u+(1+r)\overline{e}$ for any $H\leq H^*$ first. In this case, $\underline{F}$ smaller than the one for $\tau=0$ exists if the above conditions hold. This is because (34) holds at $H=0$ from $(1-\tau)w_u+T=w_u$ at $H=0$, $\frac{\lambda}{1-\lambda(1+r)}[(1-\tau)w_u+T]>\frac{\lambda}{1-\lambda(1+r)}w_u\geq\overline{e}$, i.e., (33), holds for H weakly greater than $\underline{F}$ for $\tau=0$, where $(1-\tau)w_u+T>w_u$ under Assumption 1 from the proof of Corollary 1 (ii), and (32) holds from $(1-\tau)w_s\geq(1-\tau)w_u+(1+r)\overline{e}$. (Since the relation between $(1-\tau)w_u+T$ and H is unclear, the possibility that there exist multiple levels of H satisfying $\frac{\lambda}{1-\lambda(1+r)}[(1-\tau)w_u+T]=\overline{e}$ cannot be excluded; by definition, $\underline{F}$ is the highest H satisfying the equation. Because the relation between $(1-\tau)w_s+T$ and H too is ambiguous, the possible existence of $H<\underline{F}$ satisfying $\lambda[(1-\tau)w_s+T]=\overline{e}$ too cannot be ruled out, although this is unlikely given that w_s is large for small H. Thus, the possibility that $H<\underline{F}$ increases or decreases temporarily cannot be ruled out.)

Next, consider the case in which $(1-\tau)w_s<(1-\tau)w_u+(1+r)\overline{e}$ for some $H\leq H^*$. As in the previous case, if the conditions for values of p and q such that $\tau=0$ are satisfied, (34) holds at $H=0$ and (33) holds for H weakly greater than $\underline{F}$ for $\tau=0$. By contrast, these conditions do not assure that (32) is true. Because $(1-\tau)w_s+T=(1-\tau)w_s+\left(\tau-\frac{1}{2}\tau^2\right)\overline{w}>\frac{1}{2}\left(w_s+\frac{3}{4}\overline{w}\right)>\frac{7}{8}\overline{w}$ from Assumption 2 and $\overline{w}$ increases with H from (3), (4), and (5), (32) holds if $\lambda\frac{7}{8}\overline{w}\geq\overline{e}$ holds for the minimum

H satisfying $(1-\tau)w_s = (1-\tau)w_u + (1+r)\overline{e}$, which is denoted by $H^\dagger$. Because $(1-\tau)(w_s - w_u)$ decreases with H at $H = H^\dagger$, $H^\dagger$ decreases with $\overline{e}$. Hence, (32) is true if $\overline{e}$ is sufficiently small or λ is sufficiently large. The result on the utility is true because within-class disparities in transfers diminish over time from (30) and (31).

(ii) The result for $\underline{F}$ is straightforward from the proof of (i) and Corollary 1 (ii), and the result for H^* is straightforward from the definition of H^* and Proposition 1. ■

Proof of Proposition 2. (i) The result on H is straightforward from Lemma 2 (i) when values of p and q are time-invariant, which is the case when $p = q = 0$ in the initial period from footnote 24 attached to the lemma. For other values of p_0 and q_0, as mentioned in the footnote, the possibility that $H \leq \underline{F}$ increases or decreases temporarily cannot be ruled out. Then, from Figures 1 and 2 based on Proposition A2, values of p and q may change. When the shift from $p = 0, q = 1$ to $p = q = 0$ occurs, H remains smaller than $\underline{F}$ for $p = 0, q = 1$, because H is time-invariant after the shift to $p = q = 0$ from footnote 24. The same is true for the shift from $p = q = 1$ to $p = q = 0$. When the shift from $p = q = 1$ to $p = 0, q = 1$ occurs, H remains smaller than $\underline{F}$ for $p = q = 1$, because $\underline{F}$ for $p = 0, q = 1$ is greater than $\underline{F}$ for $p = q = 1$ from Lemma 2 (ii). By contrast, when $p_0 = 0, q_0 = 1$ and F_0 is smaller than $\underline{F}$ for $p = 0, q = 1$ and greater than $\underline{F}$ for $p = q = 1$, the possibility that the shift from $p = 0, q = 1$ to $p = q = 1$ occurs and H converges to H^* cannot be ruled out. (The shift from or to $p = 1, q = 0$ is not considered, because $p = 1, q = 0$ is realized only for large H when $\beta > \gamma$ and $\frac{A_s}{A_u}$ is relatively high,)

The results on Y, w_s, w_u, and the earnings disparity are straightforward from (3)−(5). The welfare disparity is greater than the long-run level for $F_0 > \underline{F}$, which is 0 from (ii). Thus, the disparity is large in the sense that it is greater than the one for $F_0 > \underline{F}$ when H is sufficiently large.

(ii) The result on H is from Figures 1 and 2 based on Proposition A2, Lemma 2 (i), and Assumption 3. The result on the long-run welfare is from Lemma 2 (i), and the results on w_s, w_u, and the earnings disparity are from (4) and (5). The result on Y is from $\frac{dY}{dH} > 0$, which is proved as follows. $\frac{dY}{dH} > 0 \Leftrightarrow w_s > w_u$ because, from (3),

$$\begin{aligned}\frac{dY}{dH} &= \left\{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}} + (1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}\right\}^{\frac{\sigma}{\sigma-1}-1}\left\{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}\frac{1}{H} - (1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}\frac{1}{1-H}\right\} \\ &= w_s - w_u \text{ (from (4) and (5))}, \qquad (52)\end{aligned}$$

where $w_s > w_u$ from $H < \overline{H}$.

(iii) The results are from Lemma 2 (ii) and Proposition A2 (Figures 1 and 2). ■

Proof of Proposition 3. (i) and (ii) The results on identities and τ are from Figures 1 and 2 that are based on Proposition A2, Proposition 1, and Assumption 3. Convergence to H^* is from Proposition 2 (ii), except the stop of the convergence when $\Delta\widetilde{S_N}$ is relatively low, which is explained in footnote 34 attached to the proposition. (iii) Existence of multiple equilibria are from Figures 1 and 2 (Proposition A2). Assumption 3 implies the persistent effect of the initial equilibrium on the subsequent dynamics and long-run outcomes. ■

Proof of Proposition 4. First, it is proved that shapes of the dividing lines for each combination of p and q are similar to those under the original setting. From (35) and (36), the utility of an individual in class C ($C = S, U$) identifying with social group G ($G = C, N$), u_{CG}, which is given by (9)−(12) under the original setting, is expressed as:

$$u_{SN} = (1+\gamma)\left(\tau - \frac{1}{2}\tau^2\right)\overline{w} + (1-\tau)w_s - \beta\left[2\omega_q(1-H) + (1-\tau)(w_s - \overline{w})\right] + \gamma\left[\delta\widetilde{S_N} + (1-\tau)\overline{w}\right], \qquad (53)$$

$$u_{SS} = (1+\gamma)\left(\tau-\frac{1}{2}\tau^2\right)\overline{w}+(1-\tau)w_s+\gamma\left[\delta\widetilde{S_C}+(1-\tau)w_s\right], \quad (54)$$

$$u_{UN} = (1+\gamma)\left(\tau-\frac{1}{2}\tau^2\right)\overline{w}+(1-\tau)w_u-\beta\left[2\omega_q H+(1-\tau)(\overline{w}-w_u)\right]+\gamma\left[\delta\widetilde{S_N}+(1-\tau)\overline{w}\right], \quad (55)$$

$$u_{UU} = (1+\gamma)\left(\tau-\frac{1}{2}\tau^2\right)\overline{w}+(1-\tau)w_u+\gamma\left[\delta\widetilde{S_C}+(1-\tau)w_u\right]. \quad (56)$$

From these equations, the condition for $p = q = 0$ equals ($\tau = 0$ from Proposition 1 (i)(a))

$$p=q=0 \text{ iff } \gamma\delta\Delta\widetilde{S_N} \le \min\{[(w_s-w_u)(\beta+\gamma)+2\beta\omega_q](1-H), [(w_s-w_u)(\beta-\gamma)+2\beta\omega_q]H\}$$

$$\Leftrightarrow \gamma\delta\Delta\widetilde{S_N} \le [(w_s-w_u)(\beta-\gamma)+2\beta\omega_q]H \quad \text{for } H \le \min\left\{H^\flat_{00}, \overline{H}\right\} \quad (57)$$

$$\text{and } \gamma\delta\Delta\widetilde{S_N} \le [(w_s-w_u)(\beta+\gamma)+2\beta\omega_q](1-H) \quad \text{for } H \in \left[H^\flat_{00}, \overline{H}\right] \text{when } \overline{H} > H^\flat_{00}, \quad (58)$$

$$\text{where } H^\flat_{00} \text{ is } H \text{ satisfying } H = \frac{1}{2}\left\{1+\frac{\gamma}{\beta}\frac{w_s-w_u}{(w_s-w_u)+2\omega_q}\right\}.$$

$H^\flat_{00}$ is unique because $\frac{1}{2}\left\{1+\frac{\gamma}{\beta}\frac{w_s-w_u}{(w_s-w_u)+2\omega_q}\right\}$ equals $\frac{\beta+\gamma}{2\beta}$ at $H=0$ and decreases with H. Note that $\overline{H} \le (>) H^\flat_{00} \Leftrightarrow \overline{H} \le (>) \frac{1}{2}$ and $\frac{\beta+\gamma}{2\beta}$ of the original model is replaced by $H^\flat_{00}$.

As with the corresponding equations of the original model, the RHS of (57) when $\beta > \gamma$ increases (decreases) with H for small (large) H and the RHS of (58) decreases with H. The former can be shown as follows. The proof of (iii) of Proposition A1 shows that there exists $H^\sharp \in (0, \overline{H})$ such that $\frac{d[(w_s-w_u)H]}{dH} > (<) 0$ for $H < (>) H^\sharp$. Further, $\frac{d^2[(w_s-w_u)H]}{dH^2} < 0$ for $H \ge H^\sharp$ can be easily proven from the equations in the proof. Hence, unless $\beta\omega_q$ is very large, there exists $H^{\sharp\sharp} \in (H^\sharp, \overline{H})$ such that the RHS of (57) when $\beta > \gamma$ increases (decreases) with H for $H < (>) H^{\sharp\sharp}$.

The condition for $p = q = 1$ equals

$$p=q=1 \text{ iff } \gamma\delta\Delta\widetilde{S_N} \ge \max\{[(1-\tau)(w_s-w_u)(\beta+\gamma)+2\beta\omega_q](1-H), [(1-\tau)(w_s-w_u)(\beta-\gamma)+2\beta\omega_q]H\}$$

$$\Leftrightarrow \gamma\delta\Delta\widetilde{S_N} \ge [(1-\tau)(w_s-w_u)(\beta+\gamma)+2\beta\omega_q](1-H) \quad \text{for } H \le \min\left\{H^\flat_{11}, \overline{H}\right\} \quad (59)$$

$$\text{and } \gamma\delta\Delta\widetilde{S_N} \ge [(1-\tau)(w_s-w_u)(\beta-\gamma)+2\beta\omega_q]H \quad \text{for } H \in \left[H^\flat_{11}, \overline{H}\right] \text{when } \overline{H} > H^\flat_{11}, \quad (60)$$

$$\text{where } \tau = \frac{2\beta}{1+\gamma}(a(H)-H) \text{ and } H^\flat_{11} \text{ is } H \in (0, H^\flat_{00}) \text{ satisfying } H = \frac{1}{2}\left\{1+\frac{\gamma}{\beta}\frac{(1-\tau)(w_s-w_u)}{(1-\tau)(w_s-w_u)+2\omega_q}\right\}.$$

$H^\flat_{11} < H^\flat_{00}$ because $\frac{1}{2}\left\{1+\frac{\gamma}{\beta}\frac{(1-\tau)(w_s-w_u)}{(1-\tau)(w_s-w_u)+2\omega_q}\right\} < \frac{1}{2}\left\{1+\frac{\gamma}{\beta}\frac{w_s-w_u}{(w_s-w_u)+2\omega_q}\right\}$. $\overline{H} \le (>) H^\flat_{11} \Leftrightarrow \overline{H} \le (>) \frac{1}{2}$.

As with the corresponding equations of the original model, the RHS of (59) decreases with H because it equals the original equation plus $2\beta\omega_q(1-H)$ and the RHS of (60) increases with H for small H and decreases with H for H close to $\overline{H}$. To be precise, the RHS of (60) increases with H at least for $H \le H^{\sharp\sharp} \in (H^\sharp, \overline{H})$ from $\frac{d[(1-\tau)(w_s-w_u)H]}{dH} > 0$ at least for $H \le H^\sharp$ (the proof of Proposition A2 (i)(a)) and the above proof on the RHS of (57), and it decreases with H for H close to $\overline{H}$ from $\frac{d[(1-\tau)(w_s-w_u)H]}{dH} = \frac{d[(w_s-w_u)H]}{dH}$ at $H = \overline{H}$ and the proof on the RHS of (57).

The condition for $p = 0, q = 1$ equals

$$p=0, q=1 \text{ iff } \gamma\delta\Delta\widetilde{S_N} \le [(1-\tau)(w_s-w_u)(\beta+\gamma)+2\beta\omega_q](1-H) \quad (61)$$

$$\text{and } \gamma\delta\Delta\widetilde{S_N} \ge [(1-\tau)(w_s-w_u)(\beta-\gamma)+2\beta\omega_q]H, \text{ where } \tau = \frac{\beta-\gamma}{1+\gamma}(a(H)-H). \quad (62)$$

This occurs only for $H \leq \min\{H^\flat_{01}, \overline{H}\}$, where $H^\flat_{01}$ is $H \in (H^\flat_{11}, H^\flat_{00})$ satisfying $H = \frac{1}{2}\left\{1 + \frac{\gamma}{\beta}\frac{(1-\tau)(w_s - w_u)}{(1-\tau)(w_s-w_u)+2\omega_q}\right\}$ with $\tau = \frac{\beta-\gamma}{1+\gamma}(a(H)-H)$. From the above proofs on the equations for $p=q=0$ and $p=q=1$, the relations between the RHSs of (61) and (62) and H are similar to those under the original setting.

Finally, the condition for $p=1, q=0$ equals

$$p=1, q=0 \text{ iff } \gamma\delta\Delta\widetilde{S_N} \geq [(1-\tau)(w_s-w_u)(\beta+\gamma)+2\beta\omega_q](1-H) \tag{63}$$

$$\text{and } \gamma\delta\Delta\widetilde{S_N} \leq [(1-\tau)(w_s-w_u)(\beta-\gamma)+2\beta\omega_q]H, \text{ where } \tau = \frac{\beta+\gamma}{1+\gamma}(a(H)-H). \tag{64}$$

This happens only for $H \in \left[H^\flat_{10}, \overline{H}\right]$, where $H^\flat_{10}$ is $H \in (H^\flat_{11}, H^\flat_{01})$ satisfying $H = \frac{1}{2}\left\{1 + \frac{\gamma}{\beta}\frac{(1-\tau)(w_s-w_u)}{(1-\tau)(w_s-w_u)+2\omega_q}\right\}$ with $\tau = \frac{\beta+\gamma}{1+\gamma}(a(H)-H)$, thus only when $\overline{H} > H^\flat_{10} \Leftrightarrow \overline{H} > \frac{1}{2}$. The relations between the RHSs of these equations and H are similar to those under the original setting from the above proofs on the equations for $p = q = 0$ and $p = q = 1$.

From these results, the figure that illustrates combinations of H and $\Delta\widetilde{S_N}$ such that each equilibrium exists when $\overline{H} \leq \frac{1}{2}$ is very similar to Figures 1 and 3. (The difference is that the highest value of H such that the dividing line for $p = q = 0$ is upward sloping and the corresponding H for the lower dividing line for $p=0, q=1$ are greater.) Shapes of the dividing lines of the figure when $\overline{H} > \frac{1}{2}$ too are similar to those of Figures 2 and 4, although, unlike these figures, the critical value of H at which the equation for the dividing line for $p=q=1$ changes, the one above which $p=1, q=0$ holds, the one below which $p=0, q=1$ holds, and the one at which the equation for the dividing line for $p=q=1$ changes are all different, i.e., $H^\flat_{11} < H^\flat_{10} < H^\flat_{01} < H^\flat_{00}$.

A decrease in ω_q decreases the RHSs of (57)−(64). Hence, given H, $\Delta\widetilde{S_N}$ satisfying the equations decrease, i.e., all the dividing lines shift downward on the $(H, \Delta\widetilde{S_N})$ plane. ■

Proof of Proposition 5. (i) From (4) and (5),

$$\begin{aligned}
\frac{dw_s}{d\left(\frac{A_s}{A_u}\right)^{\frac{\sigma-1}{\sigma}}} - \frac{dw_u}{d\left(\frac{A_s}{A_u}\right)^{\frac{\sigma-1}{\sigma}}} &= A_u\alpha(H)^{\frac{\sigma-1}{\sigma}}(\Gamma)^{\frac{\sigma}{\sigma-1}-2}\left\{\begin{array}{c}\frac{1}{H}\left[\alpha\left(\frac{A_s}{A_u}\right)^{\frac{\sigma-1}{\sigma}}(H)^{\frac{\sigma-1}{\sigma}} + (1-\alpha)(1-H)^{\frac{\sigma-1}{\sigma}} + \frac{1}{\sigma-1}\left(\frac{A_s}{A_u}\right)^{\frac{\sigma-1}{\sigma}}\alpha(H)^{\frac{\sigma-1}{\sigma}}\right] \\ -\frac{1}{\sigma-1}(1-\alpha)(1-H)^{\frac{\sigma-1}{\sigma}}\frac{1}{1-H}\end{array}\right\} \\
&= A_u\alpha(H)^{\frac{\sigma-1}{\sigma}}(\Gamma)^{\frac{\sigma}{\sigma-1}-2}\left\{\begin{array}{c}\frac{1}{H}\left[\alpha\left(\frac{A_s}{A_u}\right)^{\frac{\sigma-1}{\sigma}}(H)^{\frac{\sigma-1}{\sigma}} + (1-\alpha)(1-H)^{\frac{\sigma-1}{\sigma}}\right] \\ +\frac{1}{\sigma-1}\left[\left(\frac{A_s}{A_u}\right)^{\frac{\sigma-1}{\sigma}}\alpha(H)^{\frac{\sigma-1}{\sigma}}\frac{1}{H} - (1-\alpha)(1-H)^{\frac{\sigma-1}{\sigma}}\frac{1}{1-H}\right]\end{array}\right\} > 0,
\end{aligned} \tag{65}$$

where the last inequality sign is from $w_s > w_u$.

(ii) The RHSs of the conditions for identity choice, (18)−(23), are expressed as $(1-\tau)(w_s-w_u)$ times an expression that does not depend on A_s and A_u. Consider the case $p = q = 1$, in which $\tau = \frac{2\beta}{1+\gamma}(a(H)-H)$ from Proposition 1 (i). (Other cases can be proved similarly.) From (65) and the proposition ($\Gamma \equiv \alpha\left(\frac{A_s}{A_u}H\right)^{\frac{\sigma-1}{\sigma}} + (1-\alpha)(1-H)^{\frac{\sigma-1}{\sigma}}$),

$$\begin{aligned}
\frac{d[(1-\tau)(w_s-w_u)]}{d\left(\frac{A_s}{A_u}\right)^{\frac{\sigma-1}{\sigma}}} &= -\frac{2\beta}{1+\gamma}\frac{\alpha(H)^{\frac{\sigma-1}{\sigma}}(1-\alpha)(1-H)^{\frac{\sigma-1}{\sigma}}}{\left[\alpha\left(\frac{A_s}{A_u}H\right)^{\frac{\sigma-1}{\sigma}} + (1-\alpha)(1-H)^{\frac{\sigma-1}{\sigma}}\right]^2}A_u(\Gamma)^{\frac{\sigma}{\sigma-1}-1}\left[\alpha\left(\frac{A_s}{A_u}H\right)^{\frac{\sigma-1}{\sigma}}\frac{1}{H} - (1-\alpha)(1-H)^{\frac{\sigma-1}{\sigma}}\frac{1}{1-H}\right] \\
&+ \left[1 - \frac{2\beta}{1+\gamma}(a(H)-H)\right]A_u\alpha(H)^{\frac{\sigma-1}{\sigma}}(\Gamma)^{\frac{\sigma}{\sigma-1}-2}\left\{\begin{array}{c}\frac{1}{H}\left[\alpha\left(\frac{A_s}{A_u}\right)^{\frac{\sigma-1}{\sigma}}(H)^{\frac{\sigma-1}{\sigma}} + (1-\alpha)(1-H)^{\frac{\sigma-1}{\sigma}}\right] \\ +\frac{1}{\sigma-1}\left[\left(\frac{A_s}{A_u}\right)^{\frac{\sigma-1}{\sigma}}\alpha(H)^{\frac{\sigma-1}{\sigma}}\frac{1}{H} - (1-\alpha)(1-H)^{\frac{\sigma-1}{\sigma}}\frac{1}{1-H}\right]\end{array}\right\}
\end{aligned}$$

$$
= A_u(\Gamma)^{\frac{\sigma}{\sigma-1}-2}\alpha(H)^{\frac{\sigma-1}{\sigma}}\left\{\begin{array}{c} -\frac{2\beta}{1+\gamma}(1-a(H))\left[\alpha\left(\frac{A_s}{A_u}H\right)^{\frac{\sigma-1}{\sigma}}\frac{1}{H}-(1-\alpha)(1-H)^{\frac{\sigma-1}{\sigma}}\frac{1}{1-H}\right] \\ +\left[1-\frac{2\beta}{1+\gamma}(a(H)-H)\right]\frac{1}{H}\left[\alpha\left(\frac{A_s}{A_u}\right)^{\frac{\sigma-1}{\sigma}}(H)^{\frac{\sigma-1}{\sigma}}+(1-\alpha)(1-H)^{\frac{\sigma-1}{\sigma}}\right] \\ +\frac{1}{\sigma-1}\left[1-\frac{2\beta}{1+\gamma}(a(H)-H)\right]\left[\left(\frac{A_s}{A_u}\right)^{\frac{\sigma-1}{\sigma}}\alpha(H)^{\frac{\sigma-1}{\sigma}}\frac{1}{H}-(1-\alpha)(1-H)^{\frac{\sigma-1}{\sigma}}\frac{1}{1-H}\right] \end{array}\right\}
$$

$$
= A_u(\Gamma)^{\frac{\sigma}{\sigma-1}-2}\alpha(H)^{\frac{\sigma-1}{\sigma}}\left\{\begin{array}{c} \left[1-\frac{2\beta}{1+\gamma}(1-H)\right]\alpha\left(\frac{A_s}{A_u}H\right)^{\frac{\sigma-1}{\sigma}}\frac{1}{H}+\frac{2\beta}{1+\gamma}(1-a(H))(1-\alpha)(1-H)^{\frac{\sigma-1}{\sigma}}\frac{1}{1-H} \\ +\left[1-\frac{2\beta}{1+\gamma}(a(H)-H)\right]\frac{1}{H}(1-\alpha)(1-H)^{\frac{\sigma-1}{\sigma}} \\ +\frac{1}{\sigma-1}\left[1-\frac{2\beta}{1+\gamma}(a(H)-H)\right]\left[\left(\frac{A_s}{A_u}\right)^{\frac{\sigma-1}{\sigma}}\alpha(H)^{\frac{\sigma-1}{\sigma}}\frac{1}{H}-(1-\alpha)(1-H)^{\frac{\sigma-1}{\sigma}}\frac{1}{1-H}\right] \end{array}\right\} > 0, \quad (66)
$$

where $1-\frac{2\beta}{1+\gamma}(1-H) > 0$ from Assumption 1. Hence, the dividing lines for identity choice shift upward. The result on the identity shift is from Proposition A2 in Appendix A or Figures 3 and 4.

(iii) [Result on τ] From Proposition 1, when $p = q = 0$ is not true, τ equals a constant times $a(H)-H$. The derivatives of $a(H)-H$ with respect to $\left(\frac{A_s}{A_u}\right)^{\frac{\sigma-1}{\sigma}}$ equal

$$
\frac{d(a(H)-H)}{d\left(\frac{A_s}{A_u}\right)^{\frac{\sigma-1}{\sigma}}} = \frac{1}{\left\{\alpha\left(\frac{A_s}{A_u}H\right)^{\frac{\sigma-1}{\sigma}}+(1-\alpha)(1-H)^{\frac{\sigma-1}{\sigma}}\right\}^2}
$$

$$
\times\left\{\left[\alpha\left(\frac{A_s}{A_u}H\right)^{\frac{\sigma-1}{\sigma}}+(1-\alpha)(1-H)^{\frac{\sigma-1}{\sigma}}\right]\alpha(H)^{\frac{\sigma-1}{\sigma}}(1-H)-\left[\alpha\left(\frac{A_s}{A_u}H\right)^{\frac{\sigma-1}{\sigma}}(1-H)-(1-\alpha)(1-H)^{\frac{\sigma-1}{\sigma}}H\right]\alpha(H)^{\frac{\sigma-1}{\sigma}}\right\}
$$

$$
= \frac{\alpha(H)^{\frac{\sigma-1}{\sigma}}(1-\alpha)(1-H)^{\frac{\sigma-1}{\sigma}}}{\left\{\alpha\left(\frac{A_s}{A_u}H\right)^{\frac{\sigma-1}{\sigma}}+(1-\alpha)(1-H)^{\frac{\sigma-1}{\sigma}}\right\}^2} > 0. \quad (67)
$$

When the identity shift occurs, the result is straightforward from (ii) and Proposition 1 (ii).

[Result on the welfare disparity] The difference in welfare between skilled and unskilled workers equal to the LHSs of (42)−(45) in Appendix B. Given values of p and q, an increase in $\frac{A_s}{A_u}$ raises $(1-\tau)(w_s-w_u)$ of the LHSs from (66) in the proof of (ii) and thus the inter-class welfare disparity. (Increased $\frac{A_s}{A_u}$ does not affect $\bar{e}$ because it is proportional to w_s in the previous period.) The proof of the result when the identity shift occurs is as follows.

Shift from $p = q = 1$ to $p = q = 0$ (case mentioned in footnote 42): When $H \leq \frac{\beta+\gamma}{2\beta}$, the LHS of (43) is greater than that of (42), thus, together with the fact that w_s-w_u increases with $\frac{A_s}{A_u}$ from (i), the identity shift increases the inter-class welfare disparity. When $H > \frac{\beta+\gamma}{2\beta}$, because the condition for $p = q = 1$ is (19) and the one for $p = q = 0$ is (21),

$$
(w_{s,00}-w_{u,00})(\beta+\gamma)(1-H) \geq (1-\tau_{11})(w_{s,11}-w_{u,11})(\beta-\gamma)H, \quad (68)
$$

where subscript 00 is for $p = q = 0$ and subscript 11 is for $p = q = 1$. The difference between the LHS of (43) and that of (42) equals

$$
\begin{aligned}
& (1+\gamma)(w_{s,00}-w_{u,00})-[1-\beta(1-2H)](1-\tau_{11})(w_{s,11}-w_{u,11}) \\
\geq\ & (1+\gamma)(w_{s,00}-w_{u,00})-[1-\beta(1-2H)](w_{s,00}-w_{u,00})\frac{(\beta+\gamma)(1-H)}{(\beta-\gamma)H} \quad \text{(from (68))} \\
=\ & \frac{w_{s,00}-w_{u,00}}{(\beta-\gamma)H}\{(1+\gamma)(\beta-\gamma)H-[1-\beta(1-2H)](\beta+\gamma)(1-H)\} \\
>\ & \frac{w_{s,00}-w_{u,00}}{H}\frac{\beta+\gamma}{2\beta}\{(1+\gamma)-(1-\beta+\beta+\gamma)\} = 0, \ \text{(from } H > \frac{\beta+\gamma}{2\beta}) \quad (69)
\end{aligned}
$$

where $[1-\beta(1-2H)](1-H)$ decreases with H from $H > \frac{\beta+\gamma}{2\beta}$.

Shift from $p = q = 1$ to $p = 0$, $q = 1$: From Propositions A1 and A2, $p = 0$, $q = 1$ is realized only for $H \leq \frac{\beta+\gamma}{2\beta}$. Given H, the LHS of (44) is lowest when $\gamma\delta\Delta\widetilde{S_N} = (\beta+\gamma)(1-\tau)(1-H)(w_s - w_u)$ from (22), in which case the LHS equals

$$\begin{aligned}&[1+\beta H+\gamma(1-H)]\,(1-\tau)(w_s-w_u)-(1+r)\overline{e}-(\beta+\gamma)(1-\tau)(1-H)(w_s-w_u)\\&=[1-\beta(1-2H)](1-\tau)(w_s-w_u)-(1+r)\overline{e},\end{aligned} \tag{70}$$

which is greater than the LHS of (42) because τ is lower when $p = 0$, $q = 1$ from Proposition 1 (ii) and $w_s - w_u$ being increasing in $\frac{A_s}{A_u}$.

Shift from $p = 0$, $q = 1$ to $p = q = 0$: Given H, the LHS of (44) is highest when $\gamma\delta\Delta\widetilde{S_N} = (\beta-\gamma)(1-\tau)H(w_s - w_u)$ from (22), in which case the LHS equals

$$\begin{aligned}&[1+\beta H+\gamma(1-H)](1-\tau)(w_s-w_u)-(1+r)\overline{e}-(\beta-\gamma)(1-\tau)H(w_s-w_u)\\&=(1+\gamma)(1-\tau)(w_s-w_u)-(1+r)\overline{e},\end{aligned} \tag{71}$$

which is smaller than the LHS of (43) from $\tau > 0$ and $w_s - w_u$ being increasing in $\frac{A_s}{A_u}$.

Shift from $p = q = 1$ to $p = 1$, $q = 0$: From Propositions A1 and A2, $p = 1$, $q = 0$ is realized only for $H > \frac{\beta+\gamma}{2\beta}$. The difference between the LHS of (45) and that of (42) is

$$\begin{aligned}&[1-\beta(1-H)+\gamma H](1-\tau_{10})(w_{s,10}-w_{u,10})+\gamma\delta\Delta\widetilde{S_N}-[1-\beta(1-2H)](1-\tau_{11})(w_{s,11}-w_{u,11})\\ \geq\ &[1-\beta(1-H)+\gamma H](1-\tau_{10})(w_{s,10}-w_{u,10})\\&+(1-\tau_{11})(w_{s,11}-w_{u,11})(\beta-\gamma)H-[1-\beta(1-2H)](1-\tau_{11})(w_{s,11}-w_{u,11})\ \text{(from (19))}\\ =\ &[1-\beta(1-H)+\gamma H][(1-\tau_{10})(w_{s,10}-w_{u,10})-(1-\tau_{11})(w_{s,11}-w_{u,11})]>0,\end{aligned} \tag{72}$$

where the last inequality holds because $\tau_{11} > \tau_{10}$ from Proposition 1 (ii) and $w_{s,10} - w_{u,10} > w_{s,11} - w_{u,11}$ from $w_s - w_u$ being increasing in $\frac{A_s}{A_u}$.

Shift from $p = 1$, $q = 0$ to $p = q = 0$: The difference in the LHS of (43) and that of (45) equals

$$\begin{aligned}&(1+\gamma)(w_{s,00}-w_{u,00})-\left\{[1-\beta(1-H)+\gamma H](1-\tau_{10})(w_{s,10}-w_{u,10})+\gamma\delta\Delta\widetilde{S_N}\right\}\\&\geq(1+\gamma)(w_{s,00}-w_{u,00})-(w_{s,00}-w_{u,00})(\beta+\gamma)(1-H)-[1-\beta(1-H)+\gamma H](1-\tau_{10})(w_{s,10}-w_{u,10})\ \text{(from (21))}\\&=[1-\beta(1-H)+\gamma H][(w_{s,00}-w_{u,00})-(1-\tau_{10})(w_{s,10}-w_{u,10})]>0,\end{aligned} \tag{73}$$

where the last inequality holds from $w_s - w_u$ being increasing in $\frac{A_s}{A_u}$.

(iv) [Result on the speed of convergence] From (30) and $\overline{e}_{t+1} = sw_{st}$, where s is a constant, in order for the child of an unskilled worker to be financially accessible to education, the following must hold for b_t the worker receives.

$$\begin{aligned}&\lambda\{(1-\tau_t)w_{ut}+T_t+(1+r)b_t\}\geq sw_{st}\\&\Leftrightarrow\lambda(1+r)b_t\geq sw_{st}-\lambda\left[(1-\tau_t)w_{ut}+T_t\right].\end{aligned} \tag{74}$$

If the RHS of the above equation increases with $\frac{A_s}{A_u}$, increased $\frac{A_s}{A_u}$ slows down the upward mobility of children of unskilled workers. When $p_t = q_t = 0$ and thus $\tau_t = 0$, the condition for the slowed mobility is (henceforth, time subscripts are omitted unless necessary) $s\frac{dw_s}{d\left(\frac{A_s}{A_u}\right)} - \lambda\frac{dw_u}{d\left(\frac{A_s}{A_u}\right)} > 0 \Leftrightarrow \frac{dw_s}{d\left(\frac{A_s}{A_u}\right)^{\frac{\sigma-1}{\sigma}}} \Big/ \frac{dw_u}{d\left(\frac{A_s}{A_u}\right)^{\frac{\sigma-1}{\sigma}}} > \frac{\lambda}{s}$, where the LHS of the last equation equals, from (4), (5), and (65),

$$\frac{\frac{dw_s}{d\left(\frac{A_s}{A_u}\right)^{\frac{\sigma-1}{\sigma}}}}{\frac{dw_u}{d\left(\frac{A_s}{A_u}\right)^{\frac{\sigma-1}{\sigma}}}} = \frac{\frac{1}{H}\left[\alpha\left(\frac{A_s}{A_u}\right)^{\frac{\sigma-1}{\sigma}}(H)^{\frac{\sigma-1}{\sigma}}+(1-\alpha)(1-H)^{\frac{\sigma-1}{\sigma}}+\frac{1}{\sigma-1}\left(\frac{A_s}{A_u}\right)^{\frac{\sigma-1}{\sigma}}\alpha(H)^{\frac{\sigma-1}{\sigma}}\right]}{\frac{1}{\sigma-1}(1-\alpha)(1-H)^{\frac{\sigma-1}{\sigma}}\frac{1}{1-H}}$$

$$= \frac{1-H}{H}\left[\sigma-1+\sigma\frac{\alpha\left(\frac{A_s}{A_u}\right)^{\frac{\sigma-1}{\sigma}}(H)^{\frac{\sigma-1}{\sigma}}}{(1-\alpha)(1-H)^{\frac{\sigma-1}{\sigma}}}\right]. \quad (75)$$

From (43) in Appendix B,

$$\begin{aligned} H_t &< H^* \Leftrightarrow (1+\gamma)(w_{st}-w_{ut}) > (1+r)sw_{st-1} \\ &\Rightarrow (1+\gamma)(w_{st}-w_{ut}) > (1+r)sw_{st} \text{ (since } H_t \geq H_{t-1}) \\ &\Leftrightarrow \frac{1-H}{H}\frac{\alpha\left(\frac{A_s}{A_u}\right)^{\frac{\sigma-1}{\sigma}}(H)^{\frac{\sigma-1}{\sigma}}}{(1-\alpha)(1-H)^{\frac{\sigma-1}{\sigma}}} > \frac{1+\gamma}{(1+\gamma)-(1+r)s} \text{ (from (4) and (5)).} \quad (76) \end{aligned}$$

Thus,

$$\begin{aligned} \frac{\frac{dw_s}{d\left(\frac{A_s}{A_u}\right)^{\frac{\sigma-1}{\sigma}}}}{\frac{dw_u}{d\left(\frac{A_s}{A_u}\right)^{\frac{\sigma-1}{\sigma}}}} &> \frac{1-H}{H}(\sigma-1)+\sigma\frac{1+\gamma}{(1+\gamma)-(1+r)s} \\ &> \frac{1+\gamma}{(1+\gamma)-(1+r)s} \text{ (from } \sigma \in (1,3]). \quad (77) \end{aligned}$$

Hence, the condition for the slowed upward mobility holds if

$$\frac{1+\gamma}{(1+\gamma)-(1+r)s} \geq \frac{\lambda}{s} \Leftrightarrow (1+\gamma)s - [(1+\gamma)-(1+r)s]\lambda \geq 0, \quad (78)$$

where $s \leq \lambda$ must be true because from (31) and $\tau_t = 0$, $b_{t+1} = \lambda\{w_{st}+(1+r)(b_t-\bar{e}_t)\} \geq \bar{e}_{t+1} = sw_{st}$ must hold for children of skilled workers to be accessible to education, which is necessary for H_t to non-decrease over time. The above inequality holds (does not hold) at $s = \lambda$ ($s = 0$) and the LHS of the second inequality increases with s. Hence, if s is sufficiently high, increased $\frac{A_s}{A_u}$ slows down the upward mobility of children of unskilled workers when $p = q = 0$.

When $p = q = 0$ does not hold and thus $\tau > 0$, the condition for the decreased mobility is $\frac{dw_s}{d\left(\frac{A_s}{A_u}\right)^{\frac{\sigma-1}{\sigma}}} / \frac{d[(1-\tau)w_u+T]}{d\left(\frac{A_s}{A_u}\right)^{\frac{\sigma-1}{\sigma}}} > \frac{\lambda}{s}$ from (74). This condition is less likely to hold than the condition when $p = q = 0$ because $\frac{d[(1-\tau)w_u+T]}{d\left(\frac{A_s}{A_u}\right)^{\frac{\sigma-1}{\sigma}}} > \frac{dw_u}{d\left(\frac{A_s}{A_u}\right)^{\frac{\sigma-1}{\sigma}}}$ from $\frac{d[(1-\tau)w_u+T]}{d\left(\frac{A_s}{A_u}\right)^{\frac{\sigma-1}{\sigma}}} = \frac{dw_u}{d\left(\frac{A_s}{A_u}\right)^{\frac{\sigma-1}{\sigma}}} + \frac{\partial(T-\tau w_u)}{\partial\tau}\frac{d\tau}{d\left(\frac{A_s}{A_u}\right)^{\frac{\sigma-1}{\sigma}}} + \frac{\partial(T-\tau w_u)}{\partial\left(\frac{A_s}{A_u}\right)^{\frac{\sigma-1}{\sigma}}}$, where $\frac{d(T-\tau w_u)}{d\tau} > 0$ from the proof of Corollary 1 (ii), $\frac{d\tau}{d\left(\frac{A_s}{A_u}\right)^{\frac{\sigma-1}{\sigma}}} > 0$ from (iii), and $\frac{\partial(T-\tau w_u)}{\partial\left(\frac{A_s}{A_u}\right)^{\frac{\sigma-1}{\sigma}}} = A_u\alpha(H)^{\frac{\sigma-1}{\sigma}}(\Gamma)^{\frac{\sigma}{\sigma-1}-2}\left\{\left(1-\frac{1}{2}\tau\frac{\sigma}{\sigma-1}\right)\Gamma+\frac{1}{\sigma-1}\left[\left(\frac{A_s}{A_u}\right)^{\frac{\sigma-1}{\sigma}}\alpha(H)^{\frac{\sigma-1}{\sigma}}-(1-\alpha)(1-H)^{\frac{\sigma-1}{\sigma}}\frac{H}{1-H}\right]\right\} > 0$ from $\sigma \leq 3$ and $\tau < \frac{1}{2}$ (Assumption 2). But the condition does hold when s is sufficiently high because $\frac{dw_s}{d\left(\frac{A_s}{A_u}\right)^{\frac{\sigma-1}{\sigma}}} > \frac{d[(1-\tau)w_s+T]}{d\left(\frac{A_s}{A_u}\right)^{\frac{\sigma-1}{\sigma}}} > \frac{d[(1-\tau)w_u+T]}{d\left(\frac{A_s}{A_u}\right)^{\frac{\sigma-1}{\sigma}}}$, where $\frac{dw_s}{d\left(\frac{A_s}{A_u}\right)^{\frac{\sigma-1}{\sigma}}} > \frac{d[(1-\tau)w_s+T]}{d\left(\frac{A_s}{A_u}\right)^{\frac{\sigma-1}{\sigma}}}$ from $\frac{d(-\tau w_s+T)}{d\left(\frac{A_s}{A_u}\right)^{\frac{\sigma-1}{\sigma}}} =$

$-\frac{d\left[\tau(w_s-w_u)(1-H)+\frac{1}{2}\tau^2(w_sH+w_u(1-H))\right]}{d\left(\frac{A_s}{A_u}\right)^{\frac{\sigma-1}{\sigma}}}<0$ ($\frac{d(w_s-w_s)}{d\left(\frac{A_s}{A_u}\right)^{\frac{\sigma-1}{\sigma}}}>0$ from (i)) and $\frac{d[(1-\tau)w_s+T]}{d\left(\frac{A_s}{A_u}\right)^{\frac{\sigma-1}{\sigma}}}>\frac{d[(1-\tau)w_u+T]}{d\left(\frac{A_s}{A_u}\right)^{\frac{\sigma-1}{\sigma}}}$ from $\frac{d[(1-\tau)(w_s-w_s)]}{d\left(\frac{A_s}{A_u}\right)^{\frac{\sigma-1}{\sigma}}}>0$, (66) in the proof of (ii).

When the identity shift occurs, the slowed upward mobility is more likely, i.e., it happens with smaller s, because τ falls and thus the change in $(1-\tau)w_u+T$, which could be positive or negative, is smaller than the change when τ is constant from $\frac{d(T-\tau w_u)}{d\tau}>0$.

[Result on H^*] H^* for different values of p and q are solutions to (42)−(45) in Appendix B. An increase in $\frac{A_s}{A_u}$ raises $(1-\tau)(w_s-w_u)$ in the LHSs of the equations from (66) in the proof of (ii). (Increased $\frac{A_s}{A_u}$ does not affect $\bar{e}$, which is proportional to w_s in the previous period.) While the relation between $(1-\tau)(w_s-w_u)$ and H is generally unclear, the fact that the LHSs of these equations equal $+\infty$ at $H=0$ and $-(1+r)\bar{e}<0$ at $H=\overline{H}$ implies that the LHSs decrease with H at $H=H^*$ (or when multiple levels of H^* exist, at the highest H^*, to which H converges). Hence, increased $\frac{A_s}{A_u}$ raises H^*.

[Result on $\underline{F}$] From (30), there exist lineages satisfying $b_t<\bar{e}_t=sw_{st-1}$ and $b_{t+1}\geq\bar{e}_{t+1}=sw_{st}$ only if $\lambda\{(1-\tau_t)w_{ut}+T_t+(1+r)b_t\}\geq sw_{st}$ for some $b_t<sw_{st-1}$, which is the case when

$$\begin{aligned}&\lambda\{(1-\tau_t)w_{ut}+T_t+(1+r)sw_{st-1}\}-sw_{st}>0\\ \Leftrightarrow\ &\lambda(1+r)sw_{st-1}>sw_{st}-\lambda[(1-\tau_t)w_{ut}+T_t].\end{aligned}\tag{79}$$

The equation corresponds to (33) when $\bar{e}$ is time-invariant, thus H_t satisfying it with equality is $\underline{F}_t$, though unlike before, it depends on H_{t-1}. Because the relation between the RHS of (79) and H_t is unclear, multiple values of H_t satisfying the equation with equality could exist; by definition, $\underline{F}_t$ is the highest value of such H_t, whose existence can be proved in a similar way as the proof of Lemma 2 (i) for the constant $\bar{e}$ case. Since the RHS equals $(s-\lambda)w_{st}\leq 0$ at $H_t=\overline{H}$ from $\tau_t=0$ (Proposition 1) and $\lambda\geq s$ (see the proof on the speed of convergence), the RHS decreases with H_t at $H_t=\underline{F}_t$. From the proof on the speed of convergence, the RHS of (79) increases with $\frac{A_{st}}{A_{ut}}$ when s is sufficiently high. Hence, $\underline{F}_t$ increases with $\frac{A_{st}}{A_{ut}}$. ■

Proof of Proposition A1. (i) $p=1$, $q=0$ cannot hold because the two conditions of (23) do not hold simultaneously when $\beta\leq\gamma$. (ii) Since $\beta\leq\gamma$, $\frac{\beta+\gamma}{2\beta}\geq 1$ and thus $H<\frac{\beta+\gamma}{2\beta}$ always holds. Hence, the RHS of the condition for $p=q=1$, (18), and that of the first condition for $p=0$, $q=1$, (22), equal $(\beta+\gamma)(1-\tau)(1-H)(w_s-w_u)$, where τ equals a constant times $a(H)-H$ from Proposition 1. In the following, the proof for the condition for $p=q=1$, where $\tau=\frac{2\beta}{1+\gamma}(a(H)-H)$, is provided.

From (47) in the proof of Proposition 1,

$$a'(H)=\frac{\sigma-1}{\sigma}\frac{a(H)[1-a(H)]}{H(1-H)}>0.\tag{80}$$

From (4) and (5) ($\Omega\equiv\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}+(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}$),

$$\begin{aligned}\frac{dw_s}{dH}&=\frac{1}{\sigma}\Omega^{\frac{\sigma}{\sigma-1}-2}\alpha(A_s)^{\frac{\sigma-1}{\sigma}}(H)^{-\frac{1}{\sigma}}\left(\begin{array}{l}\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}\frac{1}{H}-(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}\frac{1}{1-H}\\ -\left\{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}+(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}\right\}\frac{1}{H}\end{array}\right)\\ &=-\frac{1}{\sigma}\Omega^{\frac{\sigma}{\sigma-1}-2}(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}\alpha(A_s)^{\frac{\sigma-1}{\sigma}}(H)^{-\frac{1}{\sigma}}\frac{1}{H(1-H)}<0.\end{aligned}\tag{81}$$

$$\frac{dw_u}{dH} = \frac{1}{\sigma}\Omega^{\frac{\sigma}{\sigma-1}-2}(1-\alpha)(A_u)^{\frac{\sigma-1}{\sigma}}(1-H)^{-\frac{1}{\sigma}}\begin{pmatrix}\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}\frac{1}{H}-(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}\frac{1}{1-H}\\+\left\{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}+(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}\right\}\frac{1}{1-H}\end{pmatrix}$$
$$= \frac{1}{\sigma}\Omega^{\frac{\sigma}{\sigma-1}-2}(1-\alpha)(A_u)^{\frac{\sigma-1}{\sigma}}(1-H)^{-\frac{1}{\sigma}}\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}\frac{1}{H(1-H)}>0. \quad (82)$$

From the above two equations,

$$\frac{d(w_s-w_u)}{dH} = -\frac{1}{\sigma}\Omega^{\frac{\sigma}{\sigma-1}-2}\alpha(A_s)^{\frac{\sigma-1}{\sigma}}(H)^{-\frac{1}{\sigma}}(1-\alpha)(A_u)^{\frac{\sigma-1}{\sigma}}(1-H)^{-\frac{1}{\sigma}}\frac{1}{H(1-H)}$$
$$= -\frac{1}{\sigma}\Omega^{\frac{\sigma}{\sigma-1}}\frac{a(H)(1-a(H))}{H^2(1-H)^2}. \quad (83)$$

Hence, from the above equation and (80),

$$\frac{d[(1-\tau)(1-H)(w_s-w_u)]}{dH} = -\frac{2\beta}{1+\gamma}\left\{\frac{\sigma-1}{\sigma}\frac{a(H)[1-a(H)]}{H(1-H)}-1\right\}$$
$$\times(1-H)\left\{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}+(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}\right\}^{\frac{\sigma}{\sigma-1}-1}\left\{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}\frac{1}{H}-(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}\frac{1}{1-H}\right\}$$
$$-\left[1-\frac{2\beta}{1+\gamma}(a(H)-H)\right]\left\{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}+(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}\right\}^{\frac{\sigma}{\sigma-1}}\frac{1}{H^2(1-H)}\left[\frac{1}{\sigma}a(H)(1-a(H))+H(a(H)-H)\right]$$
$$= -\frac{\Omega^{\frac{\sigma}{\sigma-1}}}{H^2(1-H)}\begin{pmatrix}\frac{2\beta}{1+\gamma}(a(H)-H)\left\{\frac{\sigma-1}{\sigma}a(H)[1-a(H)]-H(1-H)\right\}\\+\left[1-\frac{2\beta}{1+\gamma}(a(H)-H)\right]\left[\frac{1}{\sigma}a(H)(1-a(H))+H(a(H)-H)\right]\end{pmatrix}$$
$$= -\frac{\Omega^{\frac{\sigma}{\sigma-1}}}{H^2(1-H)}\begin{pmatrix}\frac{2\beta}{1+\gamma}(a(H)-H)\left\{\begin{matrix}a(H)[1-a(H)]-H(1-H)+H(a(H)-H)\\-\left[\frac{1}{\sigma}a(H)(1-a(H))+H(a(H)-H)\right]\end{matrix}\right\}\\+\left[1-\frac{2\beta}{1+\gamma}(a(H)-H)\right]\left[\frac{1}{\sigma}a(H)(1-a(H))+H(a(H)-H)\right]\end{pmatrix}$$
$$= -\frac{\Omega^{\frac{\sigma}{\sigma-1}}}{H^2(1-H)}\left\{\begin{matrix}\frac{2\beta}{1+\gamma}(a(H)-H)^2[1-a(H)]\\+\left[1-\frac{4\beta}{1+\gamma}(a(H)-H)\right]\left[\frac{1}{\sigma}a(H)(1-a(H))+H(a(H)-H)\right]\end{matrix}\right\}, \quad (84)$$

which is negative since $1-\frac{4\beta}{1+\gamma}(a(H)-H)\geq 0$ from Assumption 2.

Further, from the first and second lines of (84) ($\overline{H}$ is H satisfying $H = a(H)$),

$$\lim_{H\to 0}[(1-\tau)(1-H)(w_s-w_u)] = +\infty, \quad \lim_{H\to\overline{H}}[(1-\tau)(1-H)H(w_s-w_u)] = 0. \quad (85)$$

(iii) The RHS of the condition for $p = q = 0$, (20), and that of the second condition for $p = 0$, $q = 1$, (22), equal $(\beta-\gamma)H(w_s-w_u) < 0$ since $\tau = 0$ in both cases when $\beta \leq \gamma$. From (83) in the proof of (ii) ($\Omega \equiv \alpha(A_sH)^{\frac{\sigma-1}{\sigma}}+(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}$),

$$\frac{d[H(w_s-w_u)]}{dH} = \Omega^{\frac{\sigma}{\sigma-1}-1}\left\{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}\frac{1}{H}-(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}\frac{1}{1-H}\right\}$$
$$-\frac{1}{\sigma}\Omega^{\frac{\sigma}{\sigma-1}-2}\alpha(A_s)^{\frac{\sigma-1}{\sigma}}(H)^{-\frac{1}{\sigma}}(1-\alpha)(A_u)^{\frac{\sigma-1}{\sigma}}(1-H)^{-\frac{1}{\sigma}}\frac{1}{1-H}$$
$$= \Omega^{\frac{\sigma}{\sigma-1}-1}\left\{\begin{matrix}\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}\frac{1}{H}-(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}\frac{1}{1-H}\\-\frac{1}{\sigma}\frac{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}}{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}+(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}}\frac{1}{H(1-H)^2}\end{matrix}\right\}, \quad (86)$$

where ($\overline{H}$ is H satisfying $H=a(H)$)

$$\lim_{H\to 0} H(w_s-w_u)=0,\ \lim_{H\to 0}\frac{d[H(w_s-w_u)]}{dH}=+\infty, \tag{87}$$

$$\lim_{H\to \overline{H}} H(w_s-w_u)=0,\ \lim_{H\to \overline{H}}\frac{d[H(w_s-w_u)]}{dH}=-\frac{1}{\sigma}\Omega^{\frac{\sigma}{\sigma-1}}\frac{1}{1-H}<0. \tag{88}$$

In the following, it is proved that there exists $H^\sharp\in(0,\overline{H})$ such that the first term of inside the big parenthesis of (86) is greater (smaller) than the second term for $H<(>)H^\sharp$. This implies that $\frac{d[H(w_s-w_u)]}{dH}>(<)0$ for $H<(>)H^\sharp$ and thus, when $\beta\le\gamma$, the RHS of the condition for $p=q=0$, (20), and that of the second condition for $p=0$, $q=1$ decrease (increase) with H for $H<(>)H^\sharp$.

The derivative of the first term with respect to H equals

$$-\frac{1}{\sigma}\frac{1}{H^2(1-H)^2}\left\{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}(1-H)^2+(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}H^2\right\}<0. \tag{89}$$

The derivative of the second term with respect to H equals

$$\begin{aligned}&\frac{1}{\sigma}\frac{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}}{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}+(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}}\frac{3H-1}{H^2(1-H)^3}\\
&+\frac{1}{\sigma}\frac{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}}{H(1-H)^2}\frac{\sigma-1}{\sigma}\frac{\left\{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}+(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}\right\}\left(\frac{1}{H}-\frac{1}{1-H}\right)-\left\{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}\frac{1}{H}-(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}\frac{1}{1-H}\right\}}{\left\{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}+(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}\right\}^2}\\
&=\frac{1}{\sigma}\frac{1}{H(1-H)^2}\frac{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}}{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}+(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}}\left\{\frac{\sigma-1}{\sigma}\frac{-\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}\frac{1}{1-H}+(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}\frac{1}{H}}{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}+(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}}+\frac{3H-1}{H(1-H)}\right\},\end{aligned} \tag{90}$$

which is negative (positive) for small (large) H.

The difference between the derivative of the first term and that of the second term is proportional to

$$\begin{aligned}&-(1-H)\left\{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}(1-H)^2+(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}H^2\right\}\\
&-\frac{\alpha(1-\alpha)(A_sH)^{\frac{\sigma-1}{\sigma}}[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}}{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}+(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}}\left\{\frac{\sigma-1}{\sigma}\frac{-\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}H+(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}(1-H)}{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}+(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}}-(1-3H)\right\}.\end{aligned} \tag{91}$$

In the following, it is proved that the difference is negative. This fact, together with the fact that the first term inside the big parenthesis of (86) is greater than the second term when $H\to 0$,

$$\begin{aligned}&\lim_{H\to 0}\left\{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}\frac{1}{H}-(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}\frac{1}{1-H}\right\}=\alpha(A_s)^{\frac{\sigma-1}{\sigma}}\lim_{H\to 0}\left(\frac{1}{H}\right)^{\frac{1}{\sigma}}-(1-\alpha)(A_u)^{\frac{\sigma-1}{\sigma}}\\
&>\lim_{H\to 0}\left\{\frac{1}{\sigma}\frac{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}}{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}+(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}}\frac{1}{H(1-H)^2}\right\}=\frac{1}{\sigma}\alpha(A_s)^{\frac{\sigma-1}{\sigma}}\lim_{H\to 0}\left(\frac{1}{H}\right)^{\frac{1}{\sigma}},\end{aligned} \tag{92}$$

implies that the first term is greater than the second term for $H<(>)H^\sharp$.

Let $J\equiv(A_sH)^{\frac{\sigma-1}{\sigma}}$ and $K\equiv[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}$. If $-\alpha JH+(1-\alpha)K(1-H)\ge 0$, (91) is smaller than

$$\frac{1}{\alpha J+(1-\alpha)K}\left\{-(1-H)\left[\alpha J(1-H)^2+(1-\alpha)KH^2\right][\alpha J+(1-\alpha)K]+\alpha(1-\alpha)JK(1-3H)\right\}, \tag{93}$$

which is negative when $1-3H\le 0$. When $1-3H>0$, if $J\ge K$, (93) is weakly smaller than

$$\frac{1}{\alpha J+(1-\alpha)K}\Big\{-(1-H)\Big[\alpha J(1-H)^2+(1-\alpha)KH^2\Big]K+\alpha(1-\alpha)JK(1-3H)\Big\}$$
$$=\frac{1}{\alpha J+(1-\alpha)K}\Big\{\alpha JK\Big[-(1-H)^3+(1-\alpha)(1-3H)\Big]-(1-H)(1-\alpha)K^2H^2\Big\}$$
$$=\frac{1}{\alpha J+(1-\alpha)K}\left\{\alpha JK\left[-\alpha(1-3H)-2H^2-(1-H)H^2\right]-(1-H)(1-\alpha)K^2H^2\right\}<0. \tag{94}$$

If $J<K$, (93) equals

$$\frac{1}{\alpha J+(1-\alpha)K}\Big(-(1-H)\Big\{\Big[\alpha J(1-H)^2+(1-\alpha)KH^2\Big]\alpha J+[(1-\alpha)KH]^2\Big\}-\alpha(1-\alpha)JK\left[(1-H)^3-(1-3H)\right]\Big)$$
$$=\frac{1}{\alpha J+(1-\alpha)K}\Big(-\alpha(1-\alpha)JK\left[2H^2+(1-H)H^2\right]-(1-H)\Big\{\Big[\alpha J(1-H)^2+(1-\alpha)KH^2\Big]\alpha J+[(1-\alpha)KH]^2\Big\}\Big)<0. \tag{95}$$

If $-\alpha JH+(1-\alpha)K(1-H)<0$, (91) is smaller than

$$\frac{1}{\alpha J+(1-\alpha)K}\Big\{-(1-H)\Big[\alpha J(1-H)^2+(1-\alpha)KH^2\Big][\alpha J+(1-\alpha)K]-\alpha(1-\alpha)JK\Big[\frac{-\alpha JH+(1-\alpha)K(1-H)}{\alpha J+(1-\alpha)K}-(1-3H)\Big]\Big\}$$
$$<\frac{1}{\alpha J+(1-\alpha)K}\Big\{-(1-H)\Big[\alpha J(1-H)^2+(1-\alpha)KH^2\Big]\frac{(1-\alpha)K}{H}+\alpha(1-\alpha)JK\left[H+(1-3H)\right]\Big\}$$
$$=\frac{1}{\alpha J+(1-\alpha)K}\frac{1}{H}\Big\{\alpha(1-\alpha)JK\Big[H(1-2H)-(1-H)^3\Big]-(1-H)[(1-\alpha)HK]^2\Big\}$$
$$=\frac{1}{\alpha J+(1-\alpha)K}\frac{1}{H}\Big(\alpha(1-\alpha)JK\{H(1-2H)-\left[1-3H+2H^2+(1-H)H^2\right]\}-(1-H)[(1-\alpha)HK]^2\Big)$$
$$=-\frac{1}{\alpha J+(1-\alpha)K}\frac{1}{H}\left\{\alpha(1-\alpha)JKH\left[(2H-1)^2+(1-H)H^2\right]+(1-\alpha)^2H^2(1-H)K^2\right\}<0. \tag{96}$$

(iv) The RHS of the condition for $p=q=0$ and that of the second condition for $p=0$, $q=1$ are the same from (20) and (22) because $\tau=0$ in both cases when $\beta\le\gamma$. Hence, the dividing line for $p=q=0$ and the lower dividing line for $p=0$, $q=1$ are the same. Because the RHS of the condition for $p=q=0$ (and of the second condition for $p=0$ and $q=1$) is non-positive from $\beta\le\gamma$, it is always smaller than the RHS of the condition for $p=q=1$ and that of the first condition for $p=0$, $q=1$. Thus, the dividing line for $p=q=0$ is located below the dividing line for $p=q=1$ and the upper dividing line for $p=0$, $q=1$ on the $(H,\Delta\widetilde{S_N})$ plane. From (18) and (22), the RHS of the condition for $p=q=1$ and that of the first condition for $p=0$, $q=1$ are the same except the value of τ, which is higher when $p=q=1$ from Proposition 1. Hence, the RHS of the former condition is smaller than that of the latter condition, that is, the dividing line for $p=q=1$ is located below the upper dividing line for $p=0$, $q=1$. ■

Proof of Proposition A2. (i) If H satisfying $H=a(H)$ is smaller than $\frac{\beta+\gamma}{2\beta}$, the equations for the dividing lines are the same as when $\beta\le\gamma$. Hence, Proposition A1 applies except the following. (a) Because the RHS of the condition for $p=q=0$, (20), is positive from $\beta>\gamma$, the dividing line for $p=q=0$ increases (decreases) with H for $H<(>)H^\sharp$. Since $\beta>\gamma$, the RHS of the second condition for $p=0$, $q=1$, (22), equals $(\beta-\gamma)(1-\tau)H(w_s-w_u)$, where $\tau=\frac{\beta-\gamma}{1+\gamma}(a(H)-H)$ from Proposition 1.

From (80) and (83) in the proof of Proposition A1 (ii),

$$\frac{d[(1-\tau)H(w_s-w_u)]}{dH}=\left\{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}+(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}\right\}^{\frac{\sigma}{\sigma-1}-1}$$

$$\times\left(\begin{array}{c}-\frac{\beta-\gamma}{1+\gamma}\left\{\frac{\sigma-1}{\sigma}\frac{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}}{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}+(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}}\frac{(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}}{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}+(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}}\frac{1}{H(1-H)}-1\right\}\\ \times H\left\{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}\frac{1}{H}-(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}\frac{1}{1-H}\right\}\\ +\left[1-\frac{\beta-\gamma}{1+\gamma}(a(H)-H)\right]\left\{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}\frac{1}{H}-(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}\frac{1}{1-H}-\frac{1}{\sigma}\frac{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}}{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}+(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}}\frac{1}{H(1-H)^2}\right\}\end{array}\right). \tag{97}$$

where the expression inside the big parenthesis equals

$$\begin{aligned}&\left[1-\frac{2(\beta-\gamma)}{1+\gamma}(a(H)-H)\right]\left\{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}\frac{1}{H}-(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}\frac{1}{1-H}-\frac{1}{\sigma}\frac{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}}{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}+(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}}\frac{1}{H(1-H)^2}\right\}\\&-\frac{\beta-\gamma}{1+\gamma}a(H)\left\{\frac{\sigma-1}{\sigma}\frac{(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}}{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}+(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}}\frac{1}{1-H}-1\right\}\left\{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}\frac{1}{H}-(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}\frac{1}{1-H}\right\}\\&\qquad-\frac{\beta-\gamma}{1+\gamma}(a(H)-H)\frac{1}{\sigma}\frac{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}}{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}+(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}}\frac{1}{H(1-H)^2}\\&=\left[1-\frac{2(\beta-\gamma)}{1+\gamma}(a(H)-H)\right]\left\{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}\frac{1}{H}-(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}\frac{1}{1-H}-\frac{1}{\sigma}\frac{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}}{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}+(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}}\frac{1}{H(1-H)^2}\right\}\\&\qquad-\frac{\beta-\gamma}{1+\gamma}a(H)\left[\frac{1-a(H)}{1-H}-1\right]\left\{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}\frac{1}{H}-(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}\frac{1}{1-H}\right\}\\&\quad+\frac{1}{\sigma}\frac{\beta-\gamma}{1+\gamma}\frac{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}}{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}+(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}}\frac{1}{1-H}\left\{-\frac{a(H)-H}{H(1-H)}+\frac{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}\frac{1}{H}-(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}\frac{1}{1-H}}{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}+(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}}\right\}\\&=\left[1-\frac{2(\beta-\gamma)}{1+\gamma}(a(H)-H)\right]\left\{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}\frac{1}{H}-(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}\frac{1}{1-H}-\frac{1}{\sigma}\frac{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}}{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}+(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}}\frac{1}{H(1-H)^2}\right\}\\&\qquad+\frac{\beta-\gamma}{1+\gamma}a(H)\frac{a(H)-H}{1-H}\left\{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}\frac{1}{H}-(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}\frac{1}{1-H}\right\},\end{aligned} \tag{98}$$

where $1-\frac{2(\beta-\gamma)}{1+\gamma}(a(H)-H)\geq 0$ from Assumption 2.

Hence, the above expression and thus $\frac{d[(1-\tau)H(w_s-w_u)]}{dH}$ are positive at least when $H\leq H^\sharp\in(0,\overline{H})$ in which the first term of (98) is non-negative from the proof of Proposition A1 (iii), and they are negative when H is close to $\overline{H}$.

(b) Because $\tau>0$ when $p=0$, $q=1$ from $\beta>\gamma$, the RHS of the second condition for $p=0$, $q=1$, (22), is smaller than that of the condition for $p=q=0$, (20). The RHS of the condition for $p=q=0$ equals $(\beta-\gamma)(1-\tau)H(w_s-w_u)$, while the RHS of the condition for $p=q=1$, (18), and that of the first condition for $p=0$, $q=1$, (22), equal $(\beta+\gamma)(1-\tau)(1-H)(w_s-w_u)$. Because $(\beta-\gamma)H=(\beta+\gamma)(1-H)$ at $H=\frac{\beta+\gamma}{2\beta}$ and $\tau=0$ when $p=q=0$, for relatively high H, the RHS of the condition for $p=q=0$ could be greater than the RHSs of the other two conditions. By contrast, as Proposition A1, when H is relatively low, the RHS of the condition for $p=q=0$ is smaller than the RHSs of the other two conditions because from (ii) and (iii) of the proposition, the RHS of the former goes to 0 as $H\to 0$, while the RHSs of the other conditions go to $+\infty$ as $H\to 0$.

(ii) When $\overline{H}>\frac{\beta+\gamma}{2\beta}$ and $H\leq\frac{\beta+\gamma}{2\beta}$, similar to (i), the equations for the dividing lines are the same as when $\beta\leq\gamma$ and thus the results of (i) hold except the two points. (a) From (86) in the proof of Proposition A1 (iii),

$$H=H^\sharp \Leftrightarrow \alpha\left(\frac{A_s}{A_u}H\right)^{\frac{\sigma-1}{\sigma}}\frac{1}{H}-(1-\alpha)(1-H)^{\frac{\sigma-1}{\sigma}}\frac{1}{1-H}-\frac{1}{\sigma}\frac{\alpha\left(\frac{A_s}{A_u}H\right)^{\frac{\sigma-1}{\sigma}}(1-\alpha)(1-H)^{\frac{\sigma-1}{\sigma}}}{\alpha\left(\frac{A_s}{A_u}H\right)^{\frac{\sigma-1}{\sigma}}+(1-\alpha)(1-H)^{\frac{\sigma-1}{\sigma}}}\frac{1}{H(1-H)^2}=0, \quad (99)$$

where the LHS of the equation decreases with H from the proof. Further, the derivative of the LHS with respect to $\alpha\left(\frac{A_s}{A_u}H\right)^{\frac{\sigma-1}{\sigma}}$ equals

$$\begin{aligned}
&\frac{1}{H}-\frac{1}{\sigma}\frac{\left\{\alpha\left(\frac{A_s}{A_u}H\right)^{\frac{\sigma-1}{\sigma}}+(1-\alpha)(1-H)^{\frac{\sigma-1}{\sigma}}\right\}-\alpha\left(\frac{A_s}{A_u}H\right)^{\frac{\sigma-1}{\sigma}}}{\left\{\alpha\left(\frac{A_s}{A_u}H\right)^{\frac{\sigma-1}{\sigma}}+(1-\alpha)(1-H)^{\frac{\sigma-1}{\sigma}}\right\}^2}\frac{(1-\alpha)(1-H)^{\frac{\sigma-1}{\sigma}}}{H(1-H)^2} \\
= \;&\frac{1}{H}\left[1-\frac{1}{\sigma}\frac{(1-a(H))^2}{(1-H)^2}\right]>0. \qquad (100)
\end{aligned}$$

Thus, $H^\sharp$ increases with $\frac{A_s}{A_u}$. From (98) in the proof of (i)(a) and (99), $\frac{d[(1-\tau)H(w_s-w_u)]}{dH}>0$ when $H\le H^\sharp$. Hence, when $\frac{A_s}{A_u}$ is large enough that $H^\sharp\ge\frac{\beta+\gamma}{2\beta}$, the dividing line for $p=q=0$ and the lower dividing line for $p=0$, $q=1$ increase with H for $H\le\frac{\beta+\gamma}{2\beta}$.

(b) When H is relatively high, the dividing line for $p=q=0$ is definitely located above the other two dividing lines because given τ, the RHS of the condition for $p=q=0$ is the same as the RHS of the condition for $p=q=1$ and that of the first condition for $p=0$, $q=1$ at $H=\frac{\beta+\gamma}{2\beta}$ from (20) and (22) and $\tau=0$ when $p=q=0$. The last result is straightfowrard from (22).

(iii) (a) $p{=}0$, $q{=}1$ cannot hold because the two conditions of (22) do not hold simultaneously when $\beta>\gamma$. (b) When $H>\frac{\beta+\gamma}{2\beta}$, the dividing line for $p{=}q{=}0$ and the lower dividing line for $p{=}1$, $q{=}0$ equal $(\beta{+}\gamma)(1{-}\tau)(1{-}H)(w_s{-}w_u)$ from (21) and (23). Hence, the proof of Proposition A2 (ii) applies for them. (c) When $H>\frac{\beta+\gamma}{2\beta}$, the dividing line for $p{=}q{=}1$ and the upper dividing line for $p{=}1$, $q{=}0$ equal $(\beta{-}\gamma)(1{-}\tau)H(w_s{-}w_u)$ from (19) and (23). Hence, the proof of (i)(b) and (ii)(b) applies for them. When $\frac{A_s}{A_u}$ is small enough that $\overline{H}$ is close to $\frac{\beta+\gamma}{2\beta}$, the dividing lines decrease with H from the proof of (i)(a); hence, the term "could" is used in the last sentence of (c).

(d) From (21) and (23), the RHS of the condition for $p=q=0$ and that of the second condition for $p=1$, $q=0$ are the same except the value of τ, which is 0 when $p=q=0$. Hence, the RHS of the former condition is greater than that of the latter condition, that is, the dividing line for $p=q=0$ is located above the lower dividinding line for $p=1$, $q=0$. From (19) and (23), the RHS of the condition for $p=q=1$ and that of the first condition for $p=1$, $q=0$ are the same except the value of τ, which is higher when $p=q=1$ from $\beta>\gamma$. Hence, the RHS of the former condition is smaller than that of the latter condition. From (21), (19), and (23), given τ, the RHS of the condition for $p=q=0$ is smaller than the RHS of the condition for $p=q=1$ and that of the first condition for $p=1$, $q=0$ when $H>\frac{\beta+\gamma}{2\beta}$ and they are equal at $H=\frac{\beta+\gamma}{2\beta}$, while $\tau=0$ when $p=q=0$. Hence, when H is relatively low, the dividing line for $p=q=0$ is located above the dividling line for $p=q=1$ and the upper dividinding line for $p=1$, $q=0$. From (4) and (5), the RHSs of all the conditions go to 0 as $H\to\overline{H}$. Further, because $\overline{H}>\frac{\beta+\gamma}{2\beta}>\frac{1}{2}$, $\left|\lim_{H\to\overline{H}}\frac{d[(1-H)(w_s-w_u)]}{dH}\right|<\left|\lim_{H\to\overline{H}}\frac{d[(1-\tau)H(w_s-w_u)]}{dH}\right|$ from (84) and (88). Hence, when H is relatively high, the dividing line for $p=q=0$ is located below the other two dividling lines. The last result is straightfowrard from (23). ■